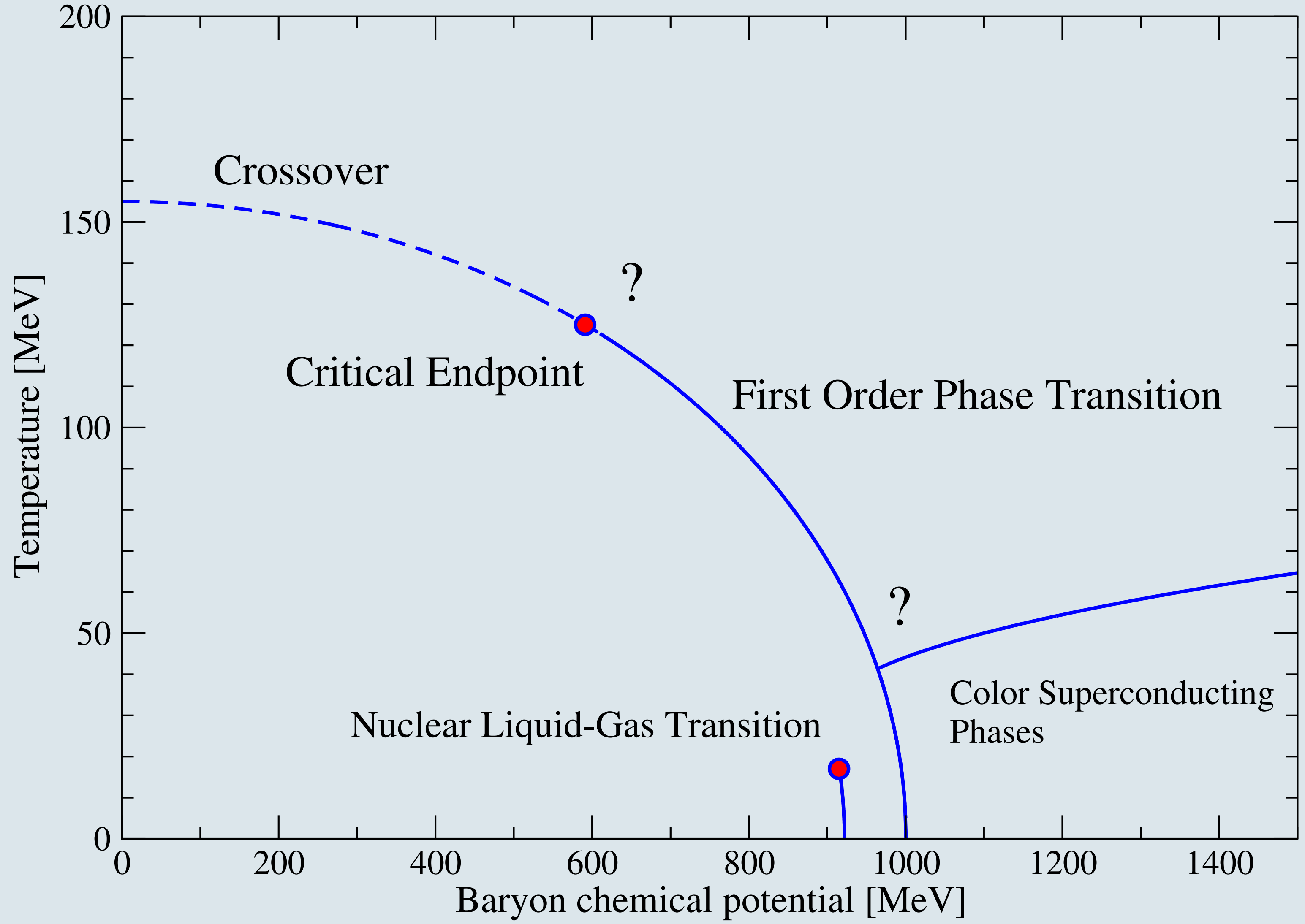


QCD-phase diagram with functional methods

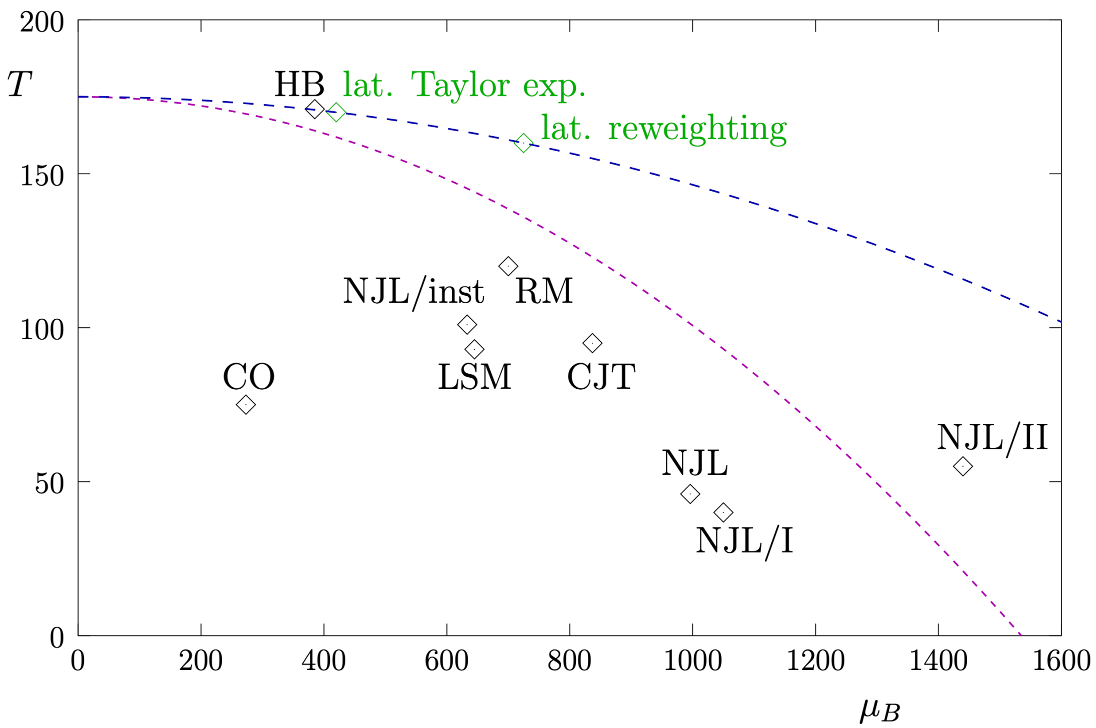
Review: CF, PPNP 105 (2019) [1810.12938]

Bernhardt and CF, PRD 108 (2023) 11, 114018

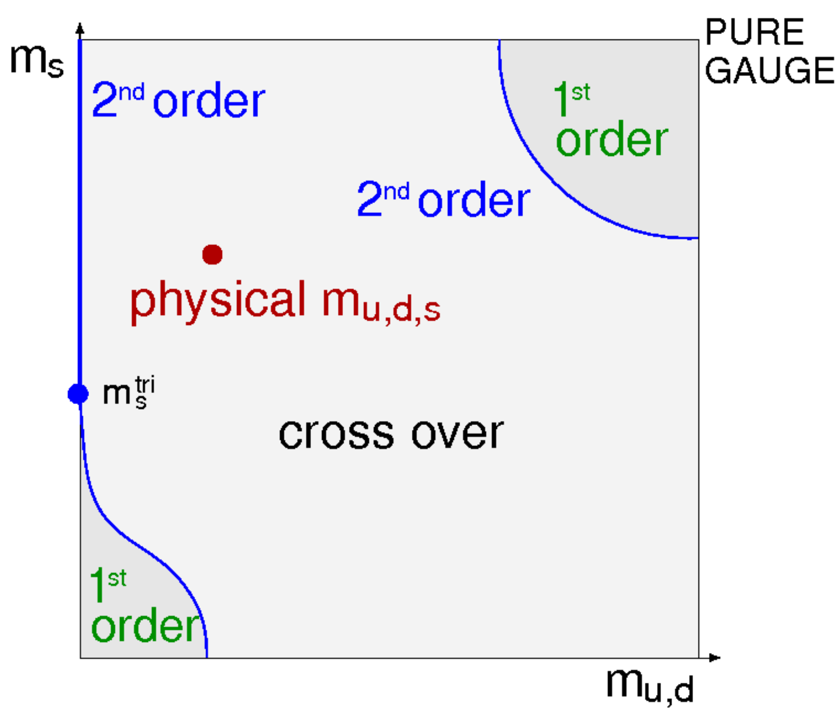
Bernhardt and CF, EPJA 59 (2023) 8, 181

Take-home-message

2004

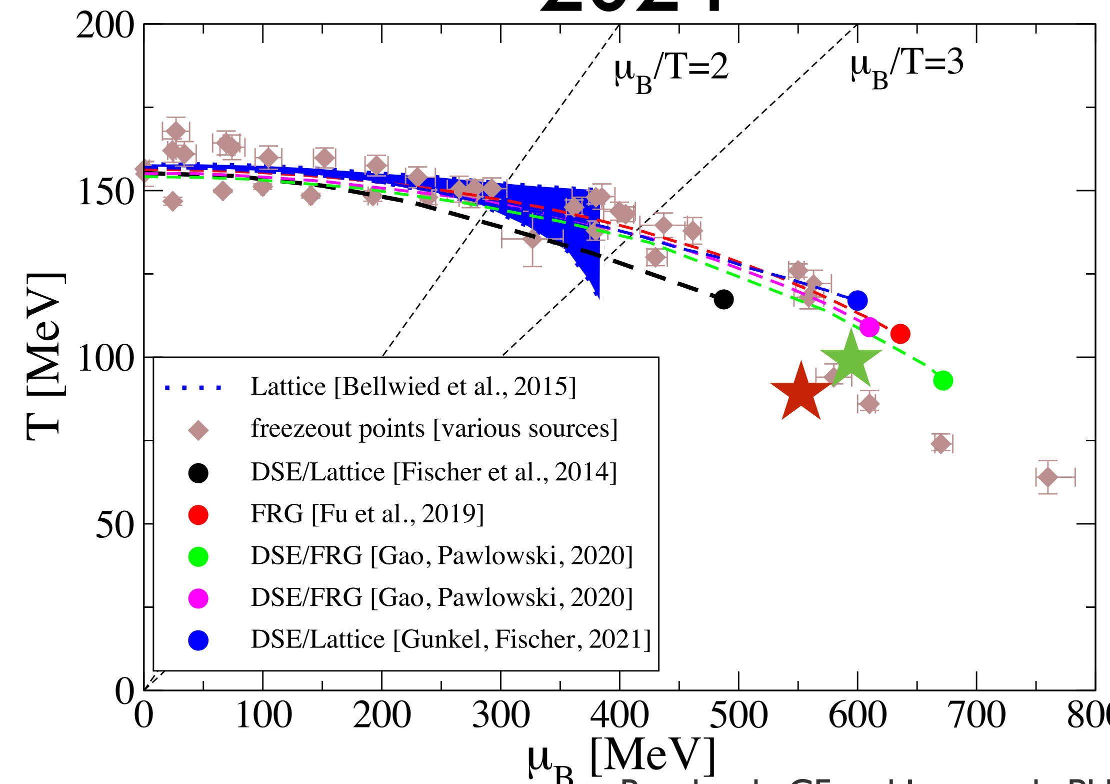


Stephanov, Prog. Theor. Phys. Suppl. 153 (2004)

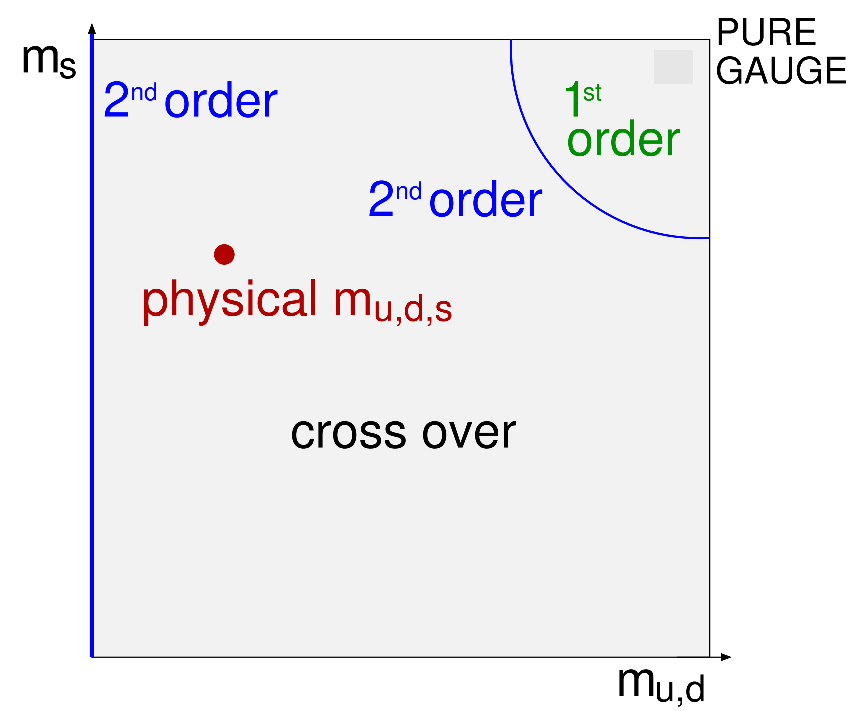


Pisarski and Wilczek, PRD 29 (1984), 338-341

2024

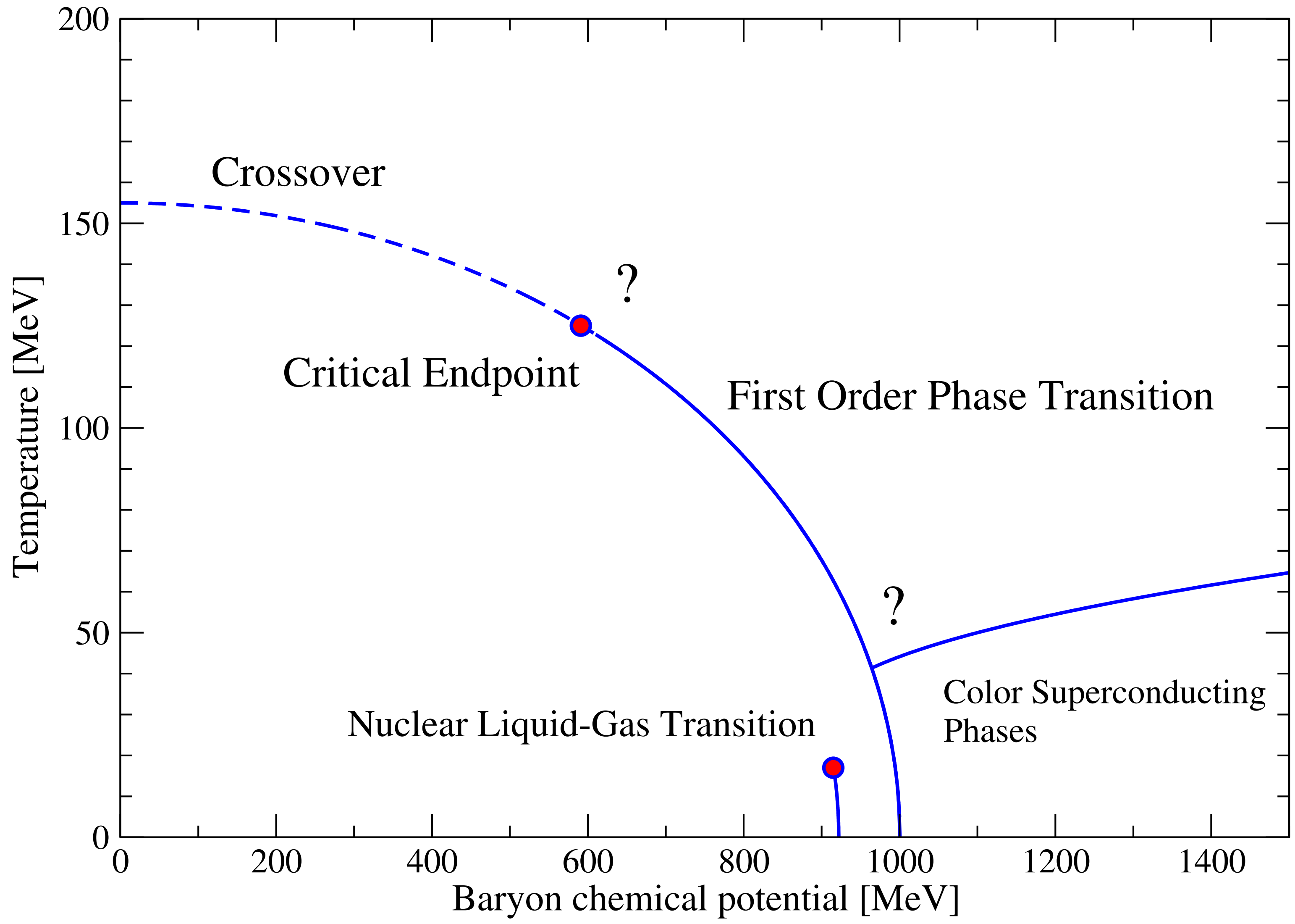


Bernhardt, CF and Isserstedt, PLB 841 (2023)
G. Basar, arXiv:2312.06952
D.A. Clarke, et al. PoS LATTICE2023 (2024), 168

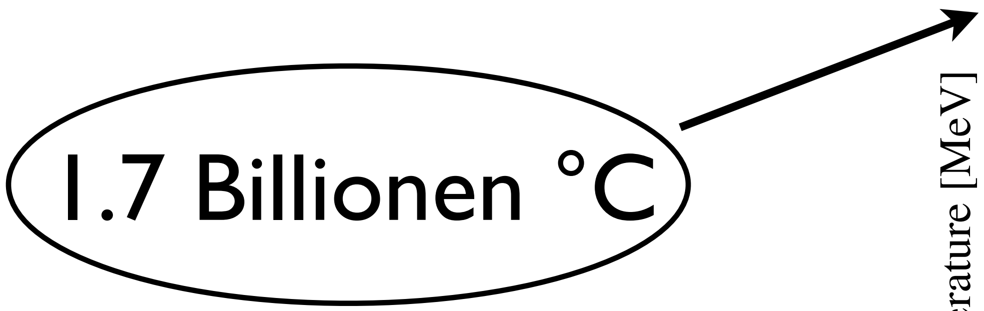


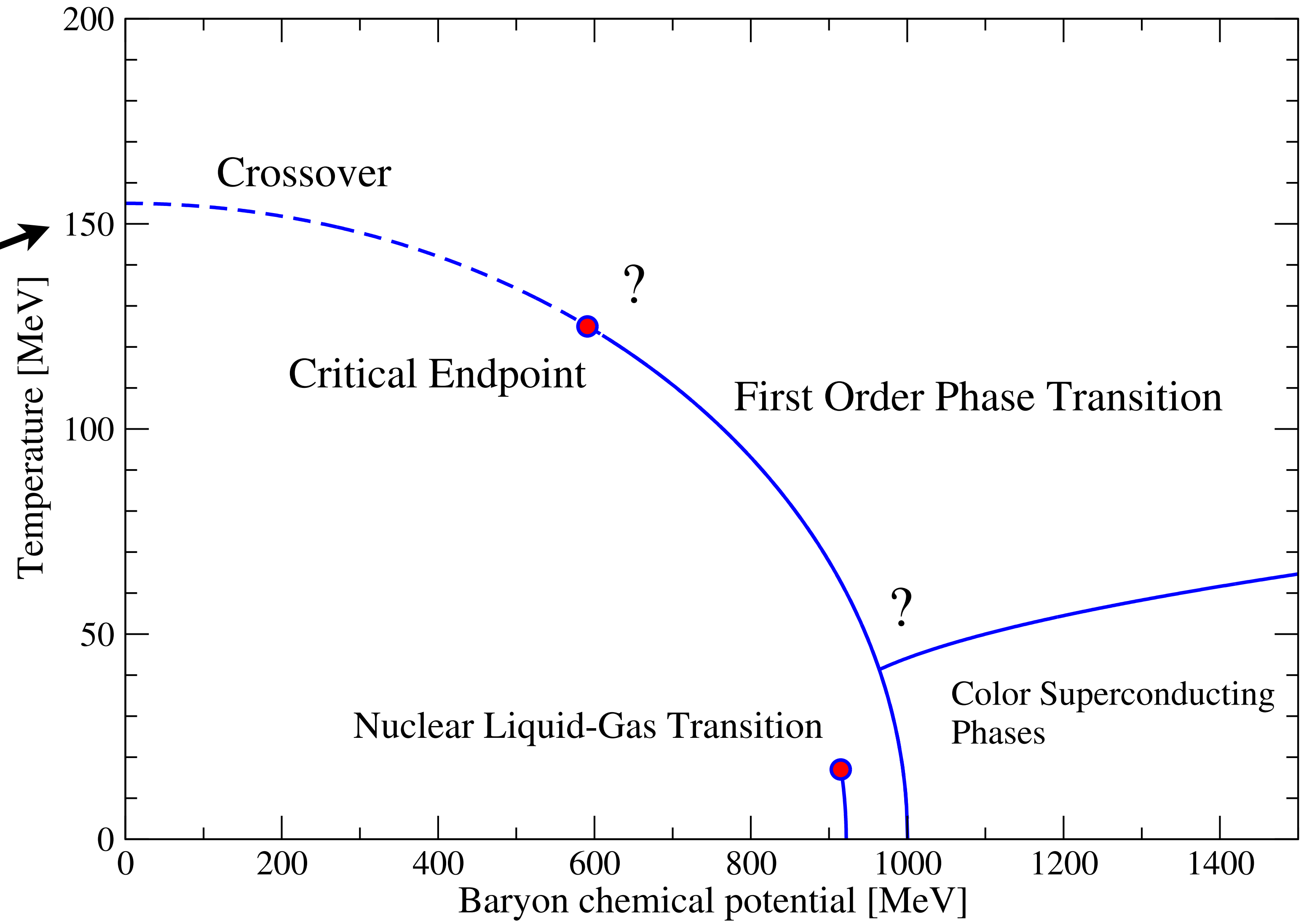
Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141
Bernhardt and CF, PRD 108 (2023) no.11, 114018

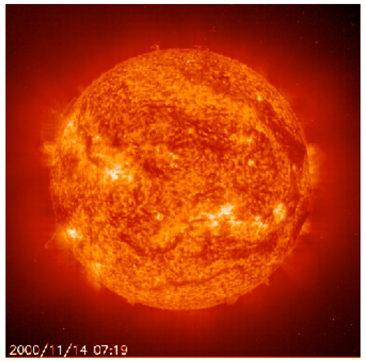
Phase diagram of quark matter: QCD



Phase diagram of quark matter: QCD

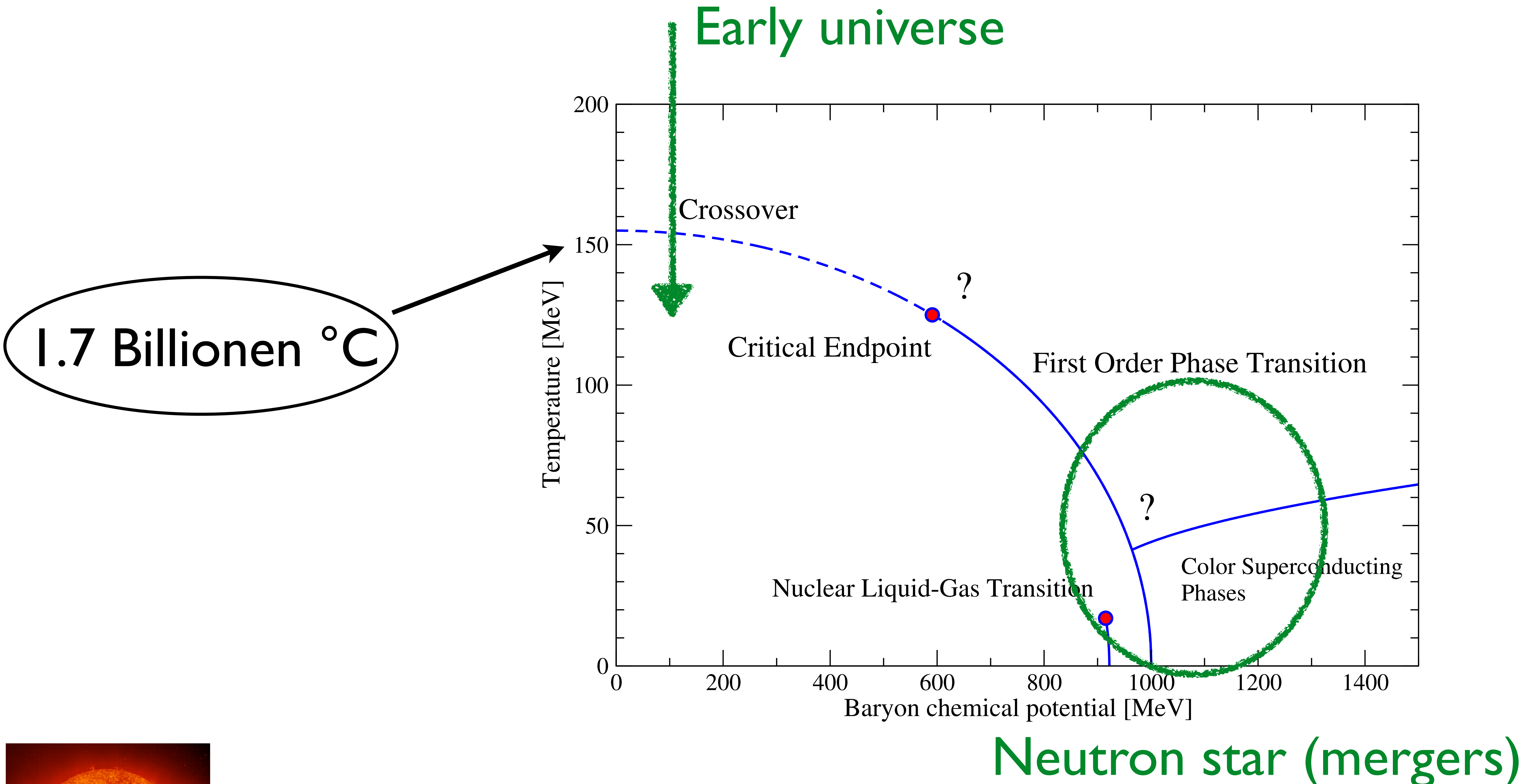


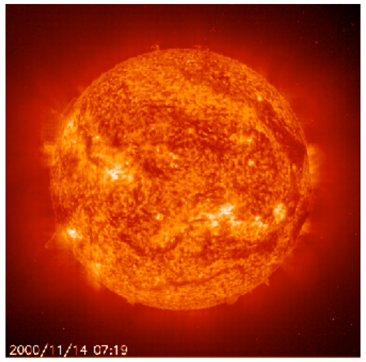




15 Millionen °C

Phase diagram of quark matter: QCD





15 Millionen °C

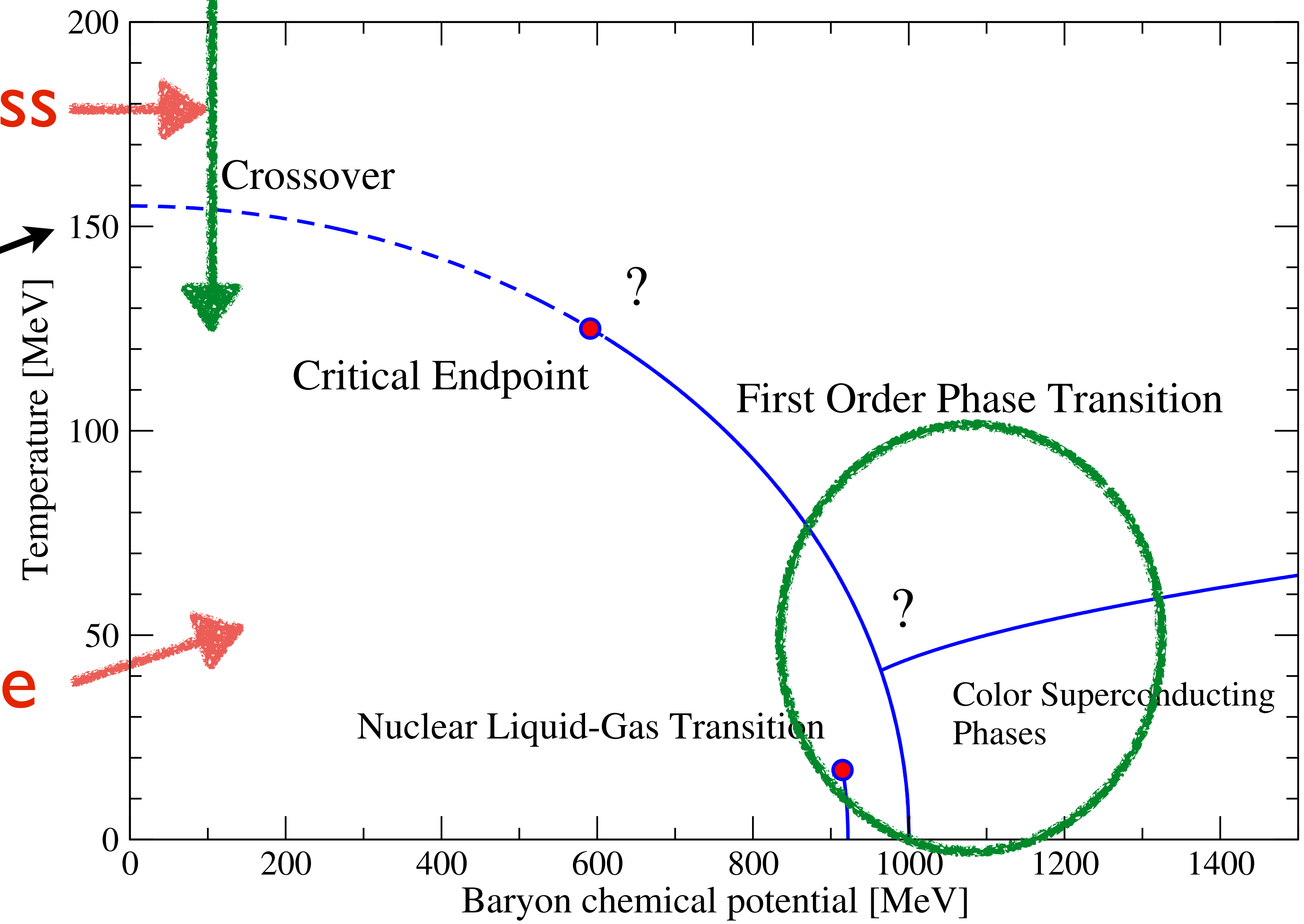
Phase diagram of quark matter: QCD

Early universe

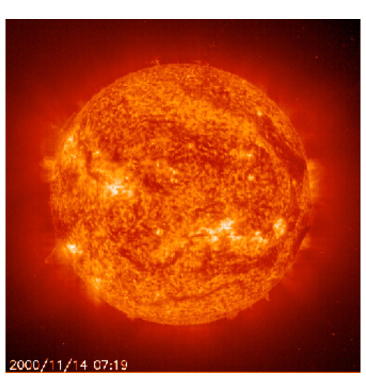
Quarks (almost) massless

1.7 Billionen °C

Quarks massive



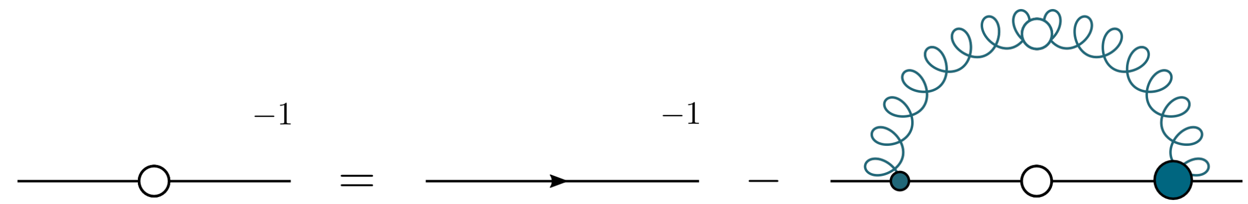
Neutron star (mergers)



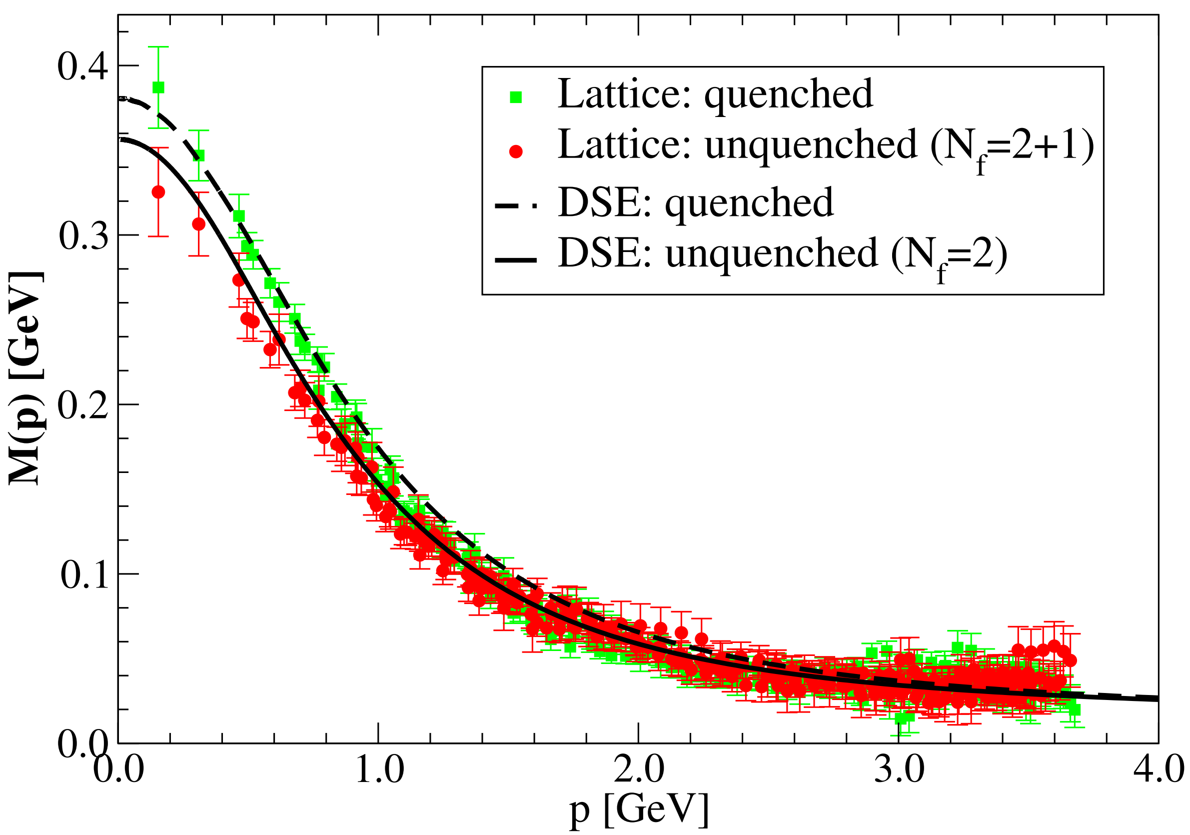
15 Millionen °C

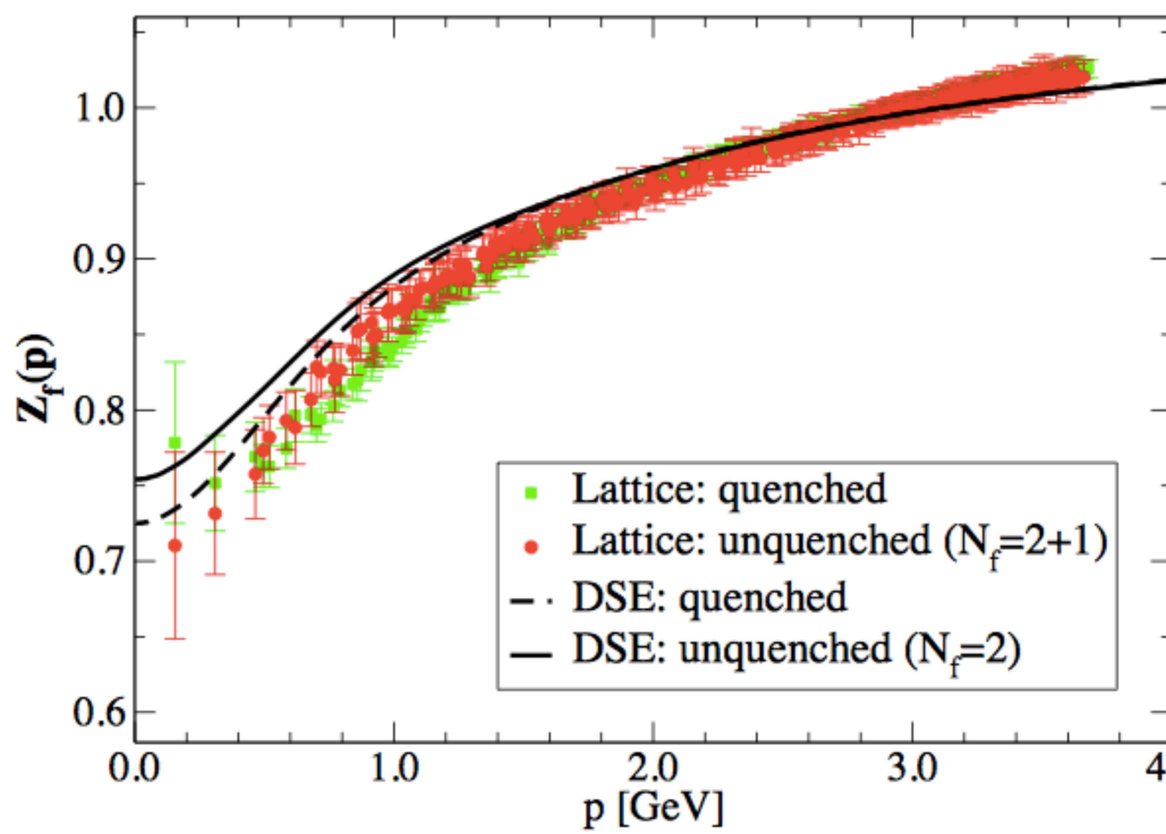
Dynamical mass generation

$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





Dynamical mass generation

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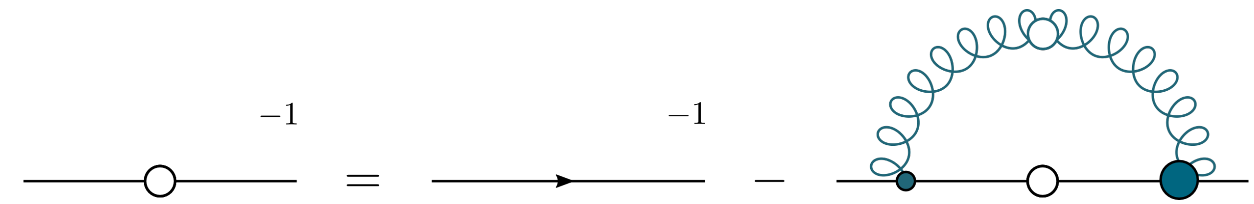

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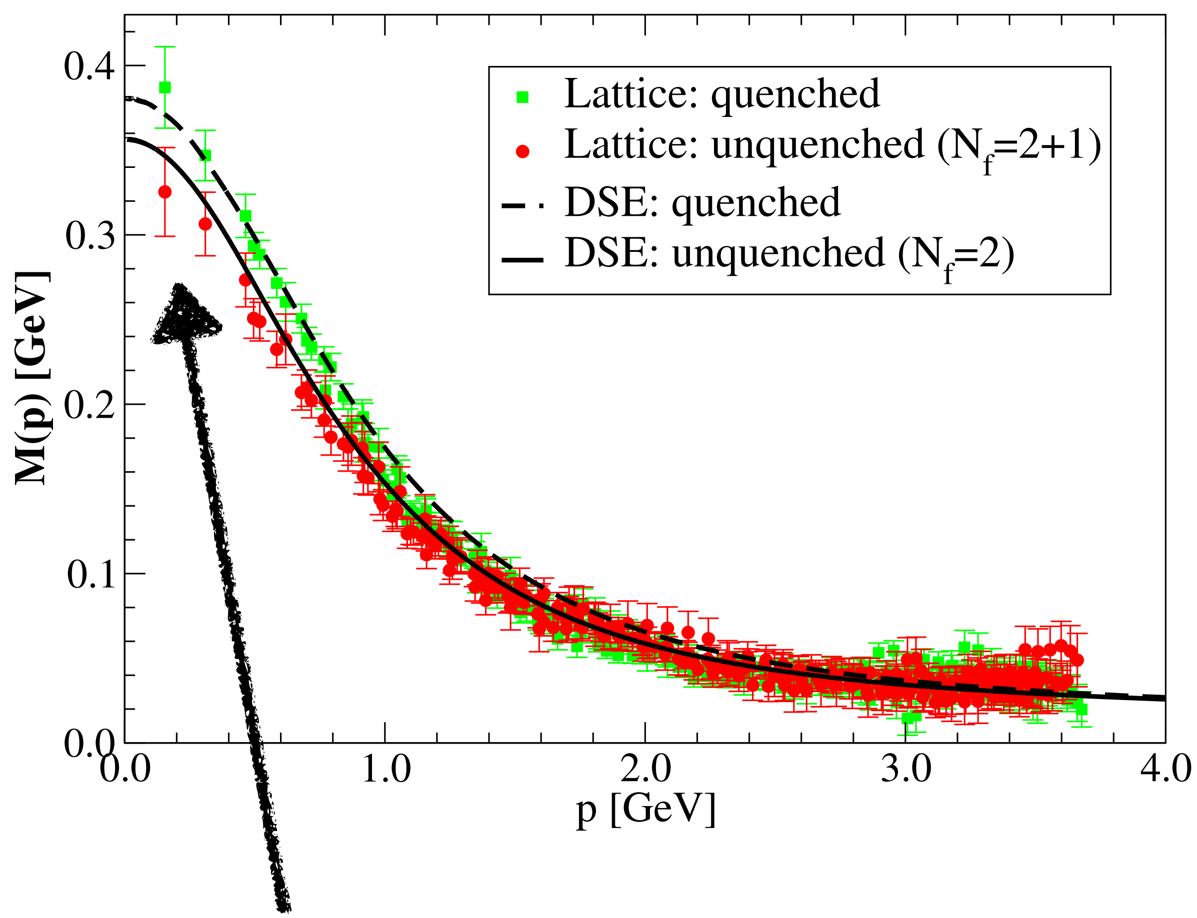
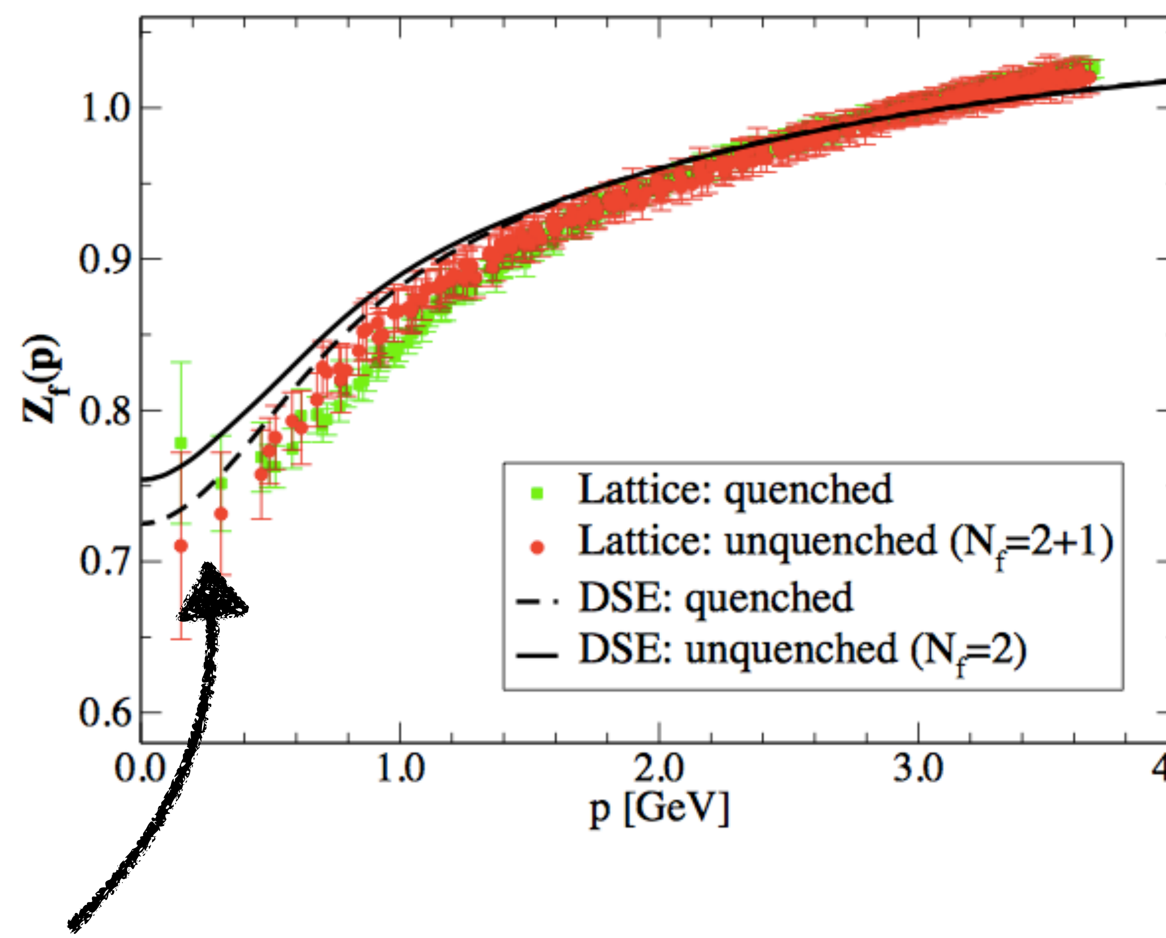
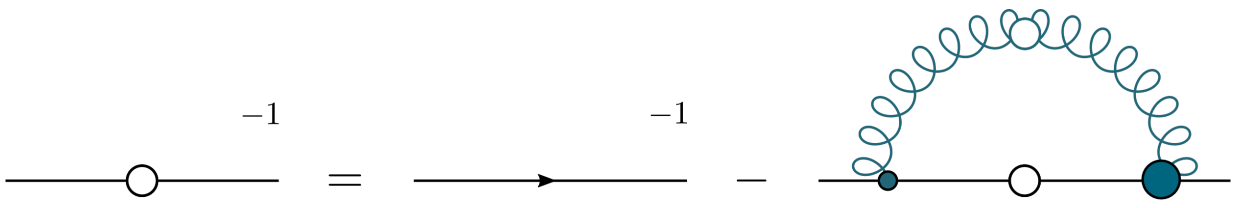




‘constituent quark’: large mass - very composite

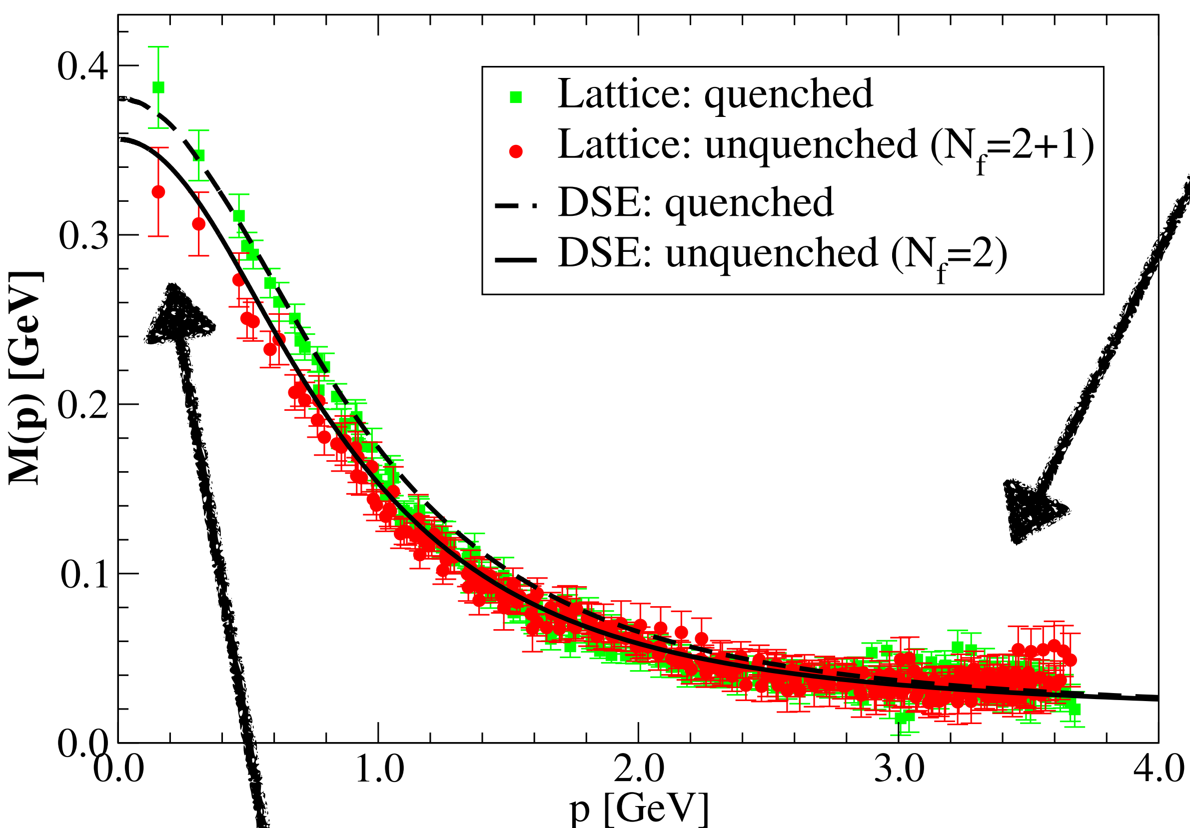
Dynamical mass generation

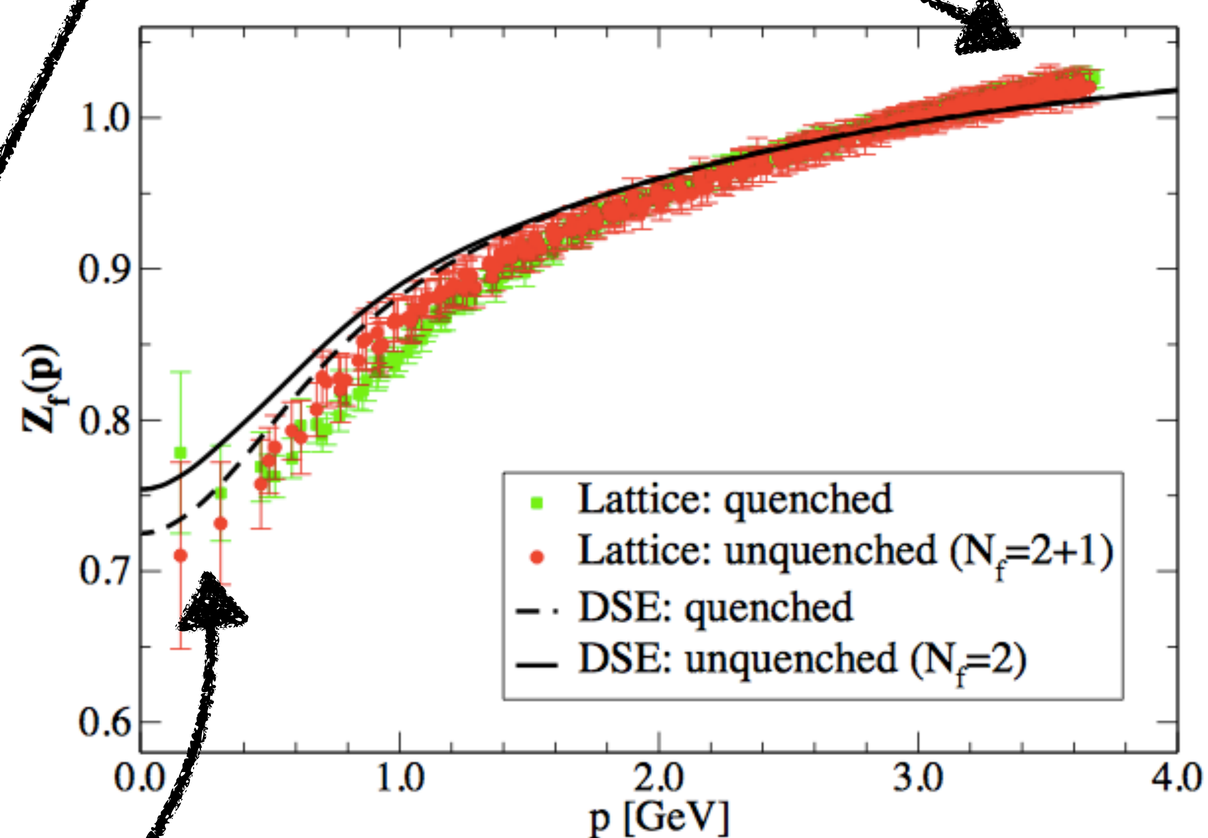
$$S^{-1}(p) = \frac{(i\not{p} + M(p^2))}{Z_f(p^2)}$$



DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507

‘current quark’: small mass; non-composite



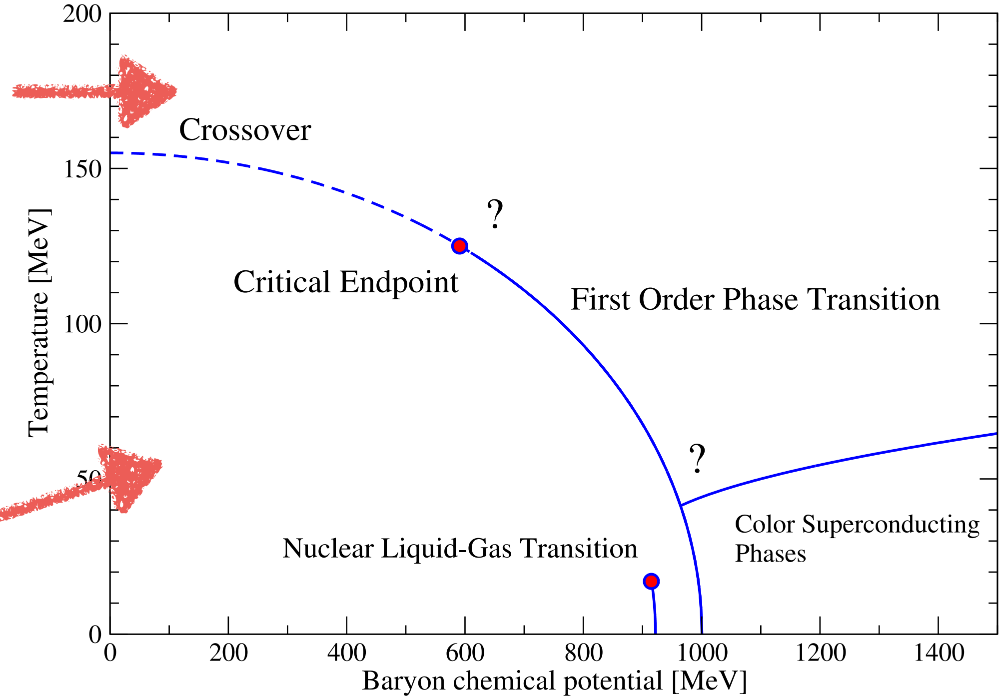


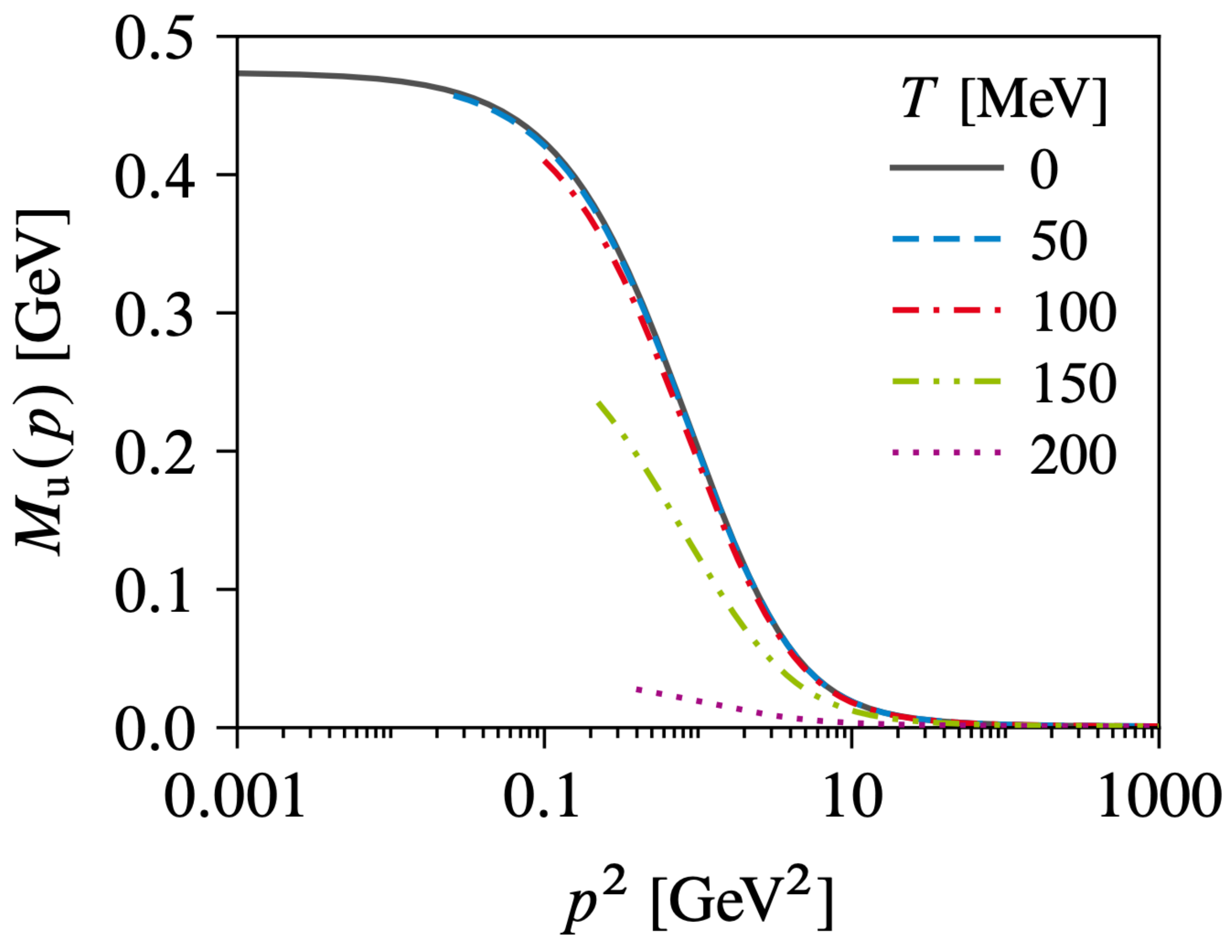
‘constituent quark’: large mass - very composite

QCD phase transitions: 2+1 quark flavors

Quarks (almost) massless

Quarks massive





QCD phase transitions: 2+1 quark flavors

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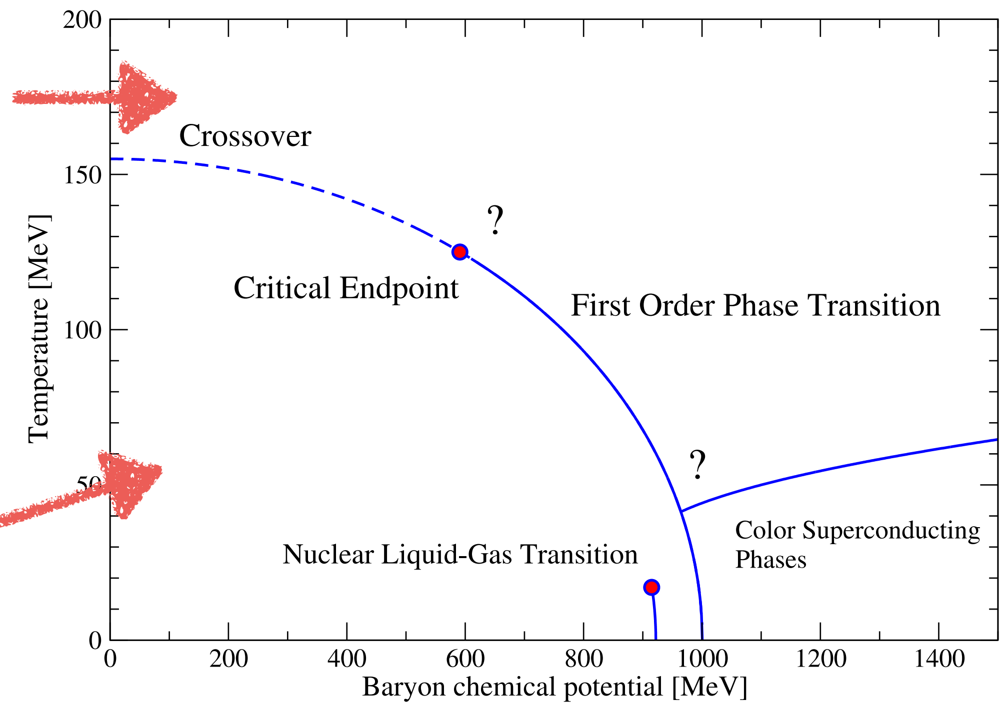

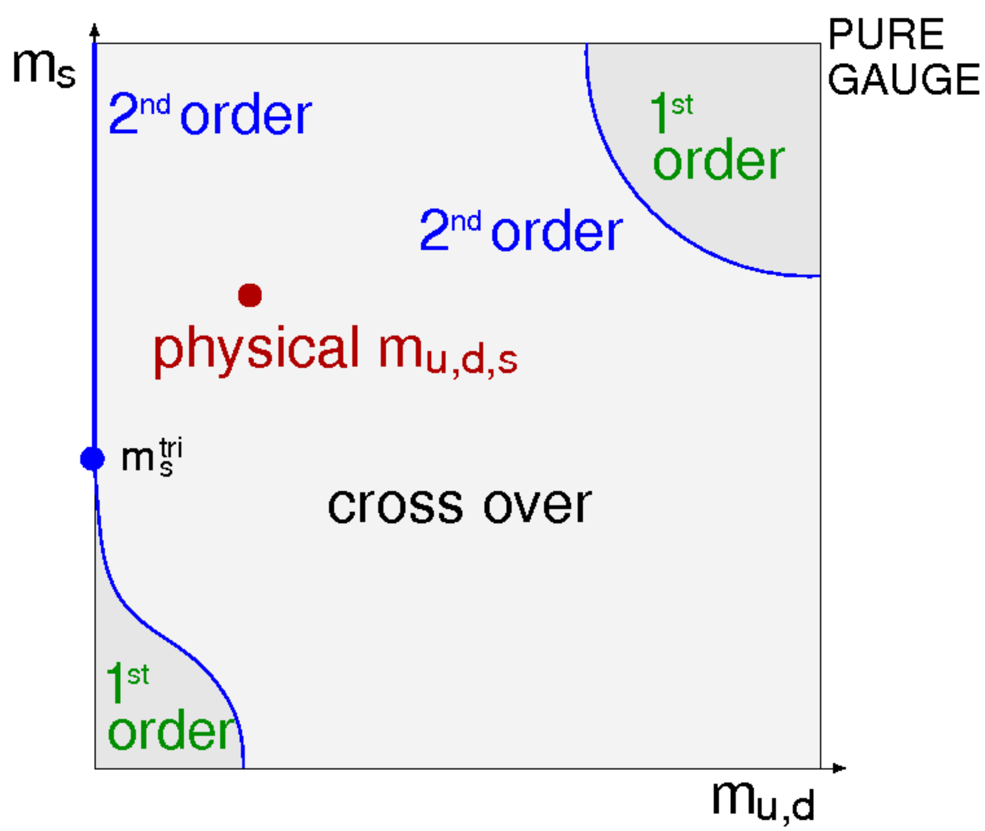

QCD phase transitions: 2+1 quark flavors

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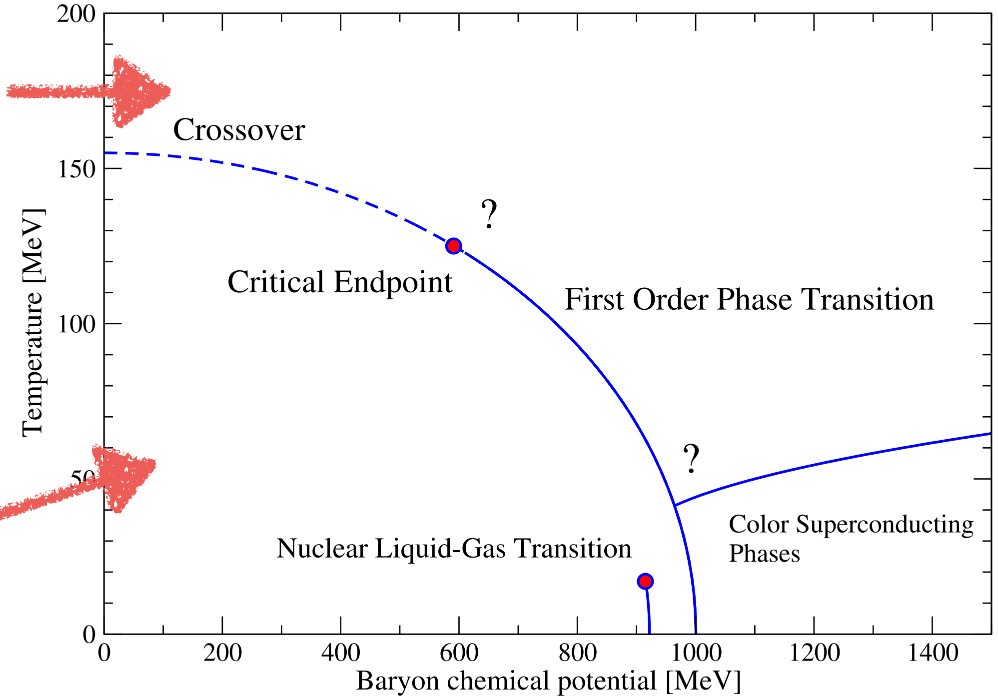
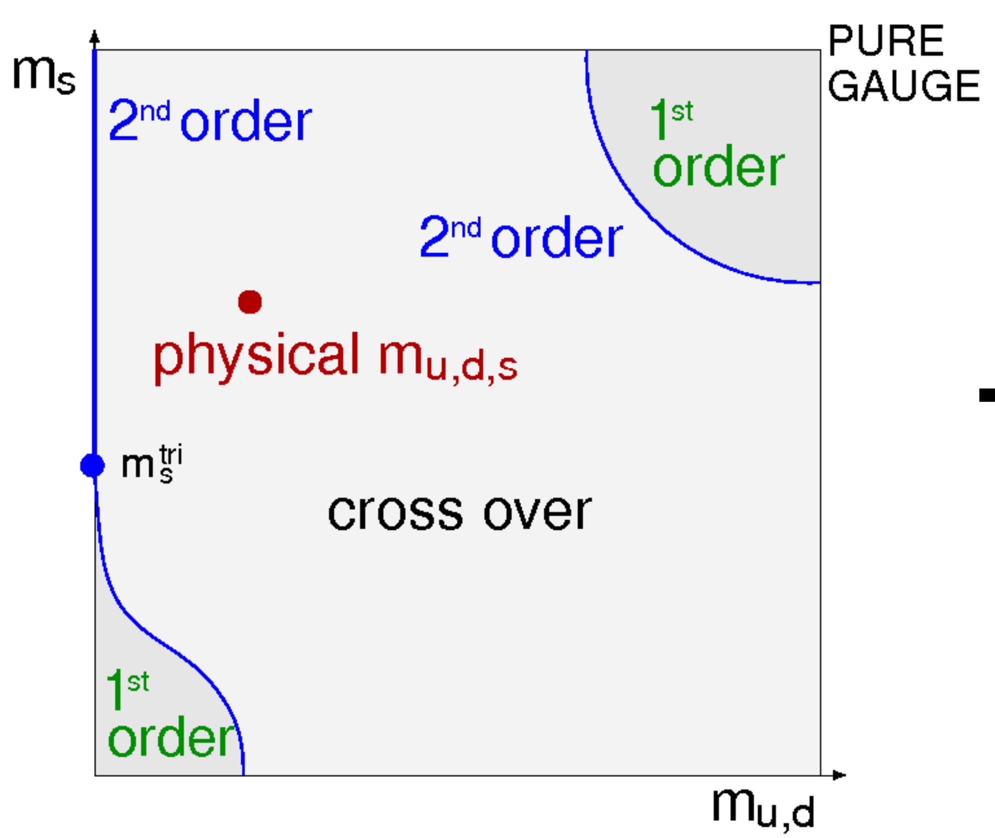
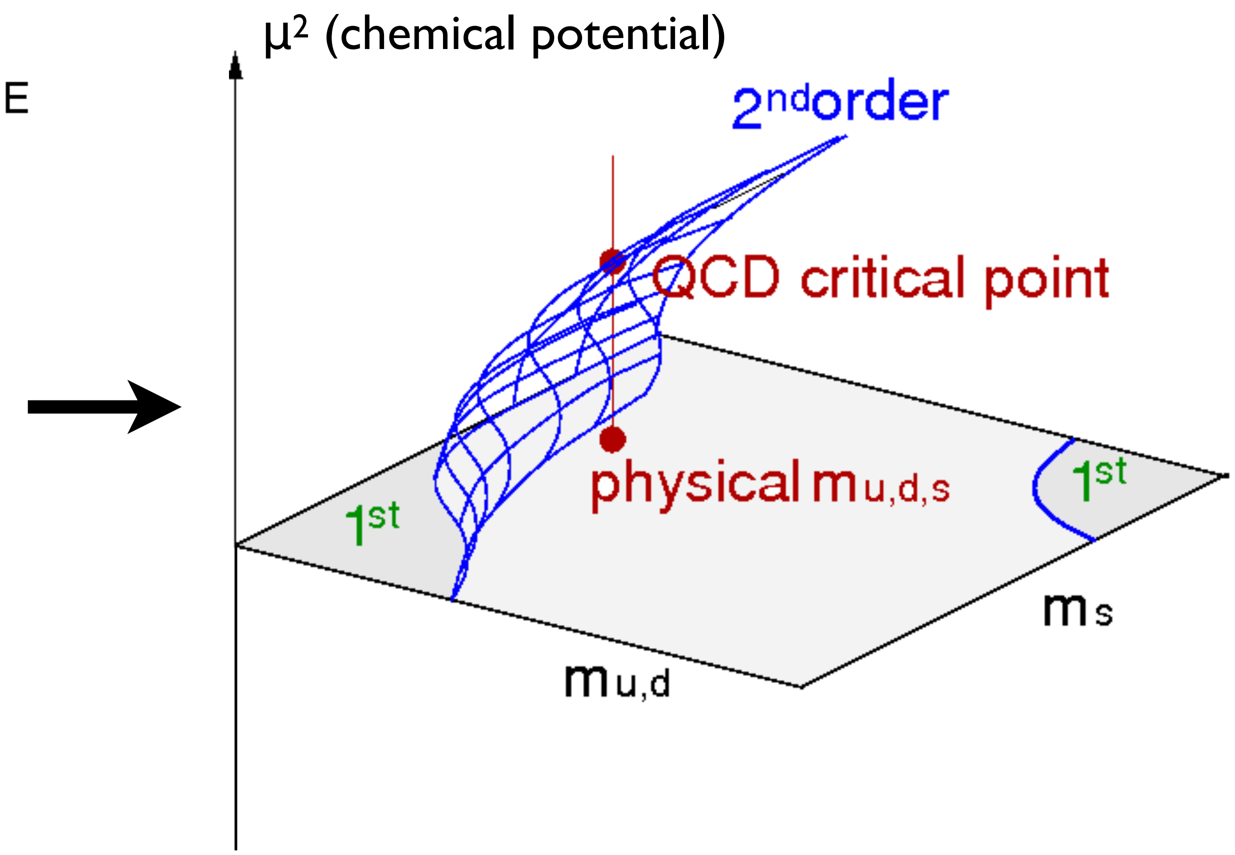
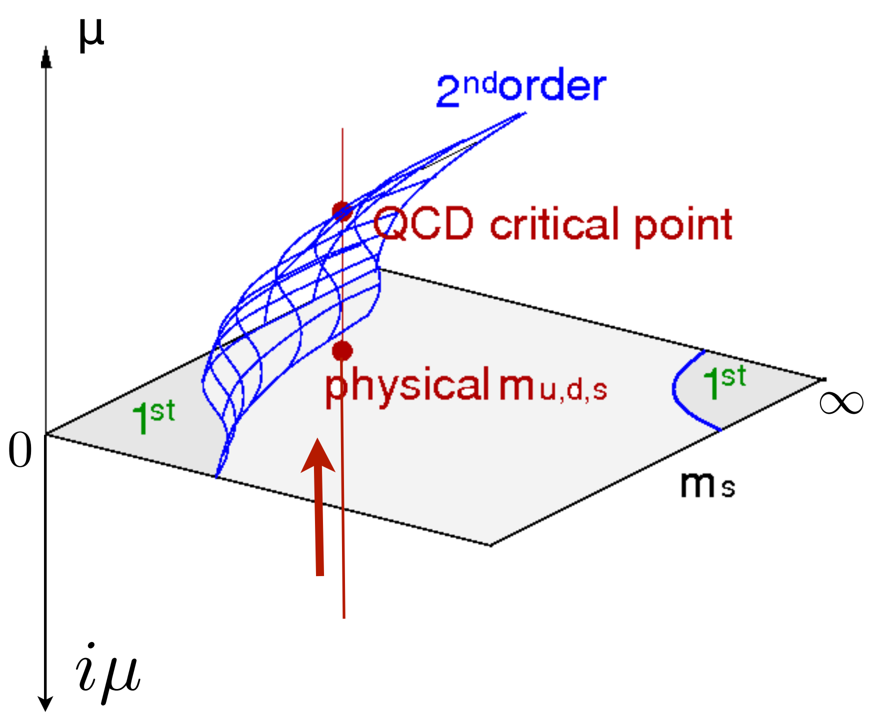
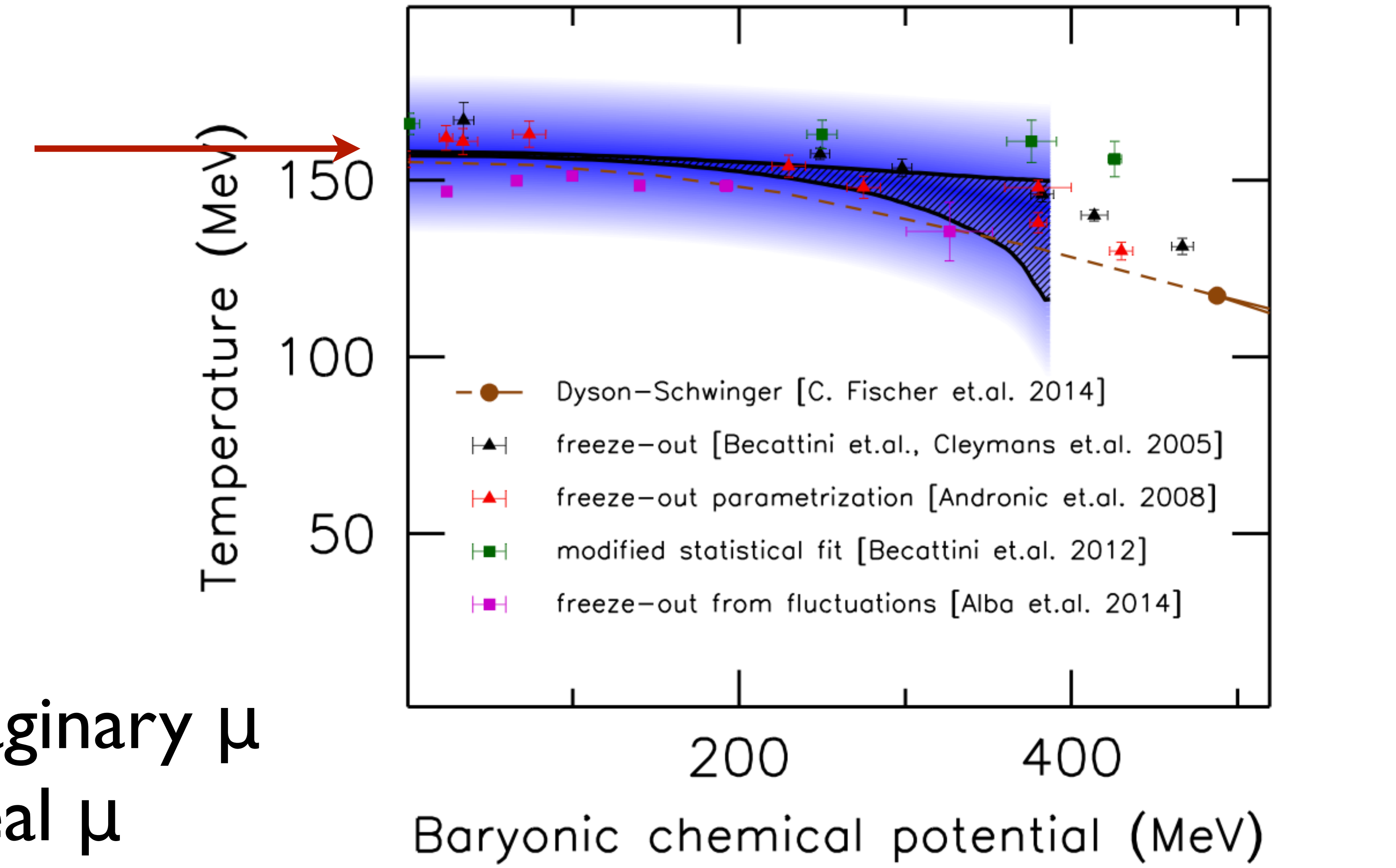






Chiral transition line from analytic continuation





Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB 751 (2015) 559

Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

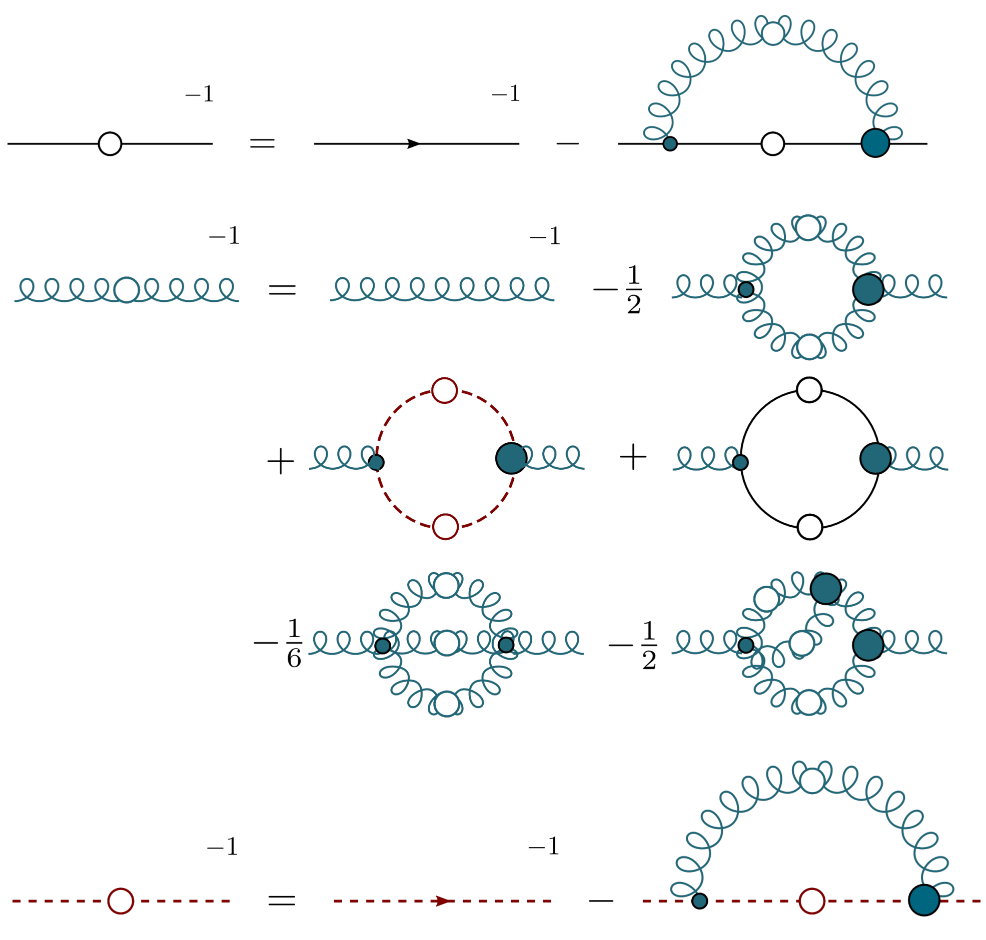
HOT-QCD: similar results

Main result:

- No transition for $\mu_B/T < 2\text{-}3$

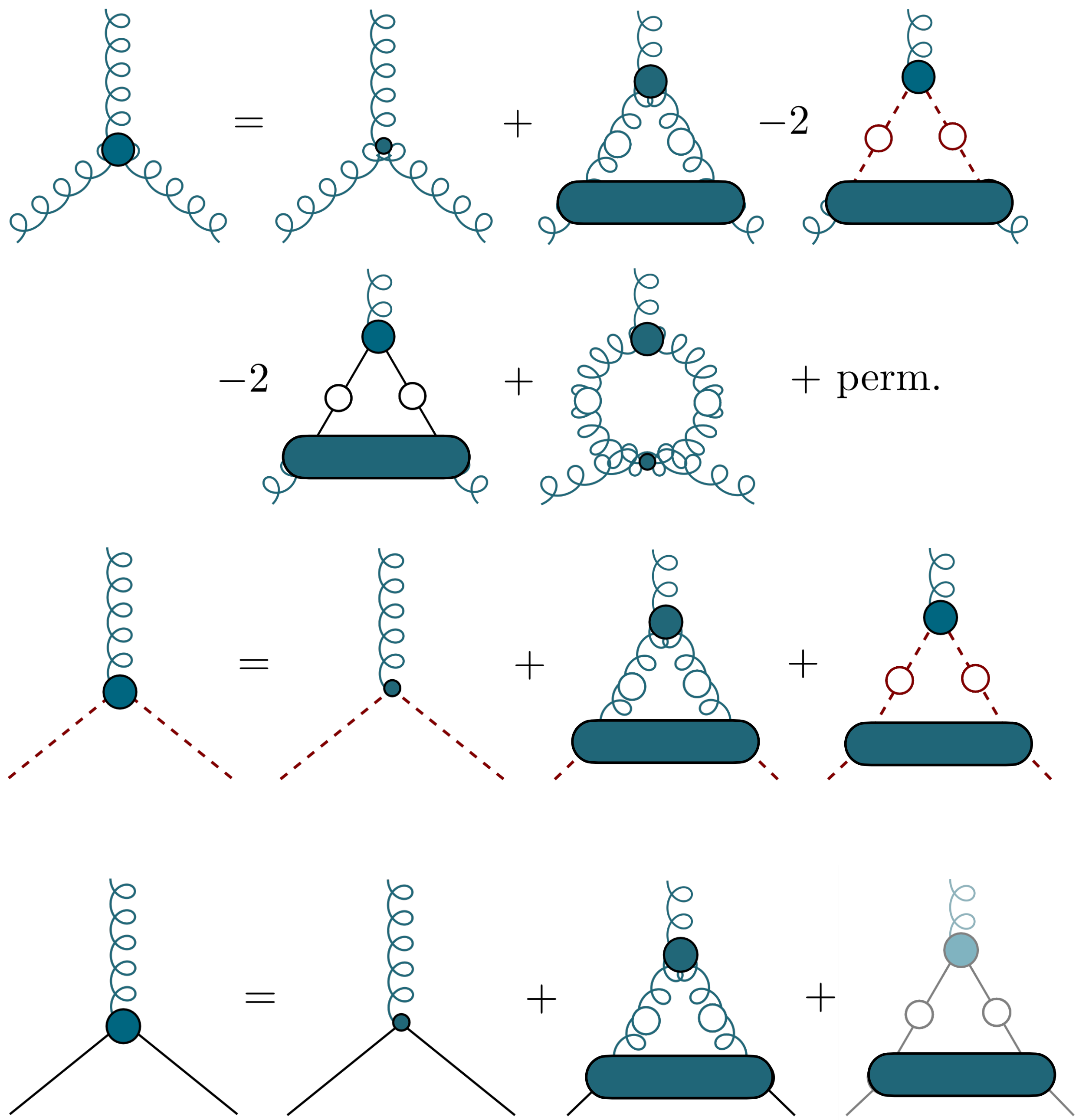
QCD with functional methods (T=0, μ=0)

propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

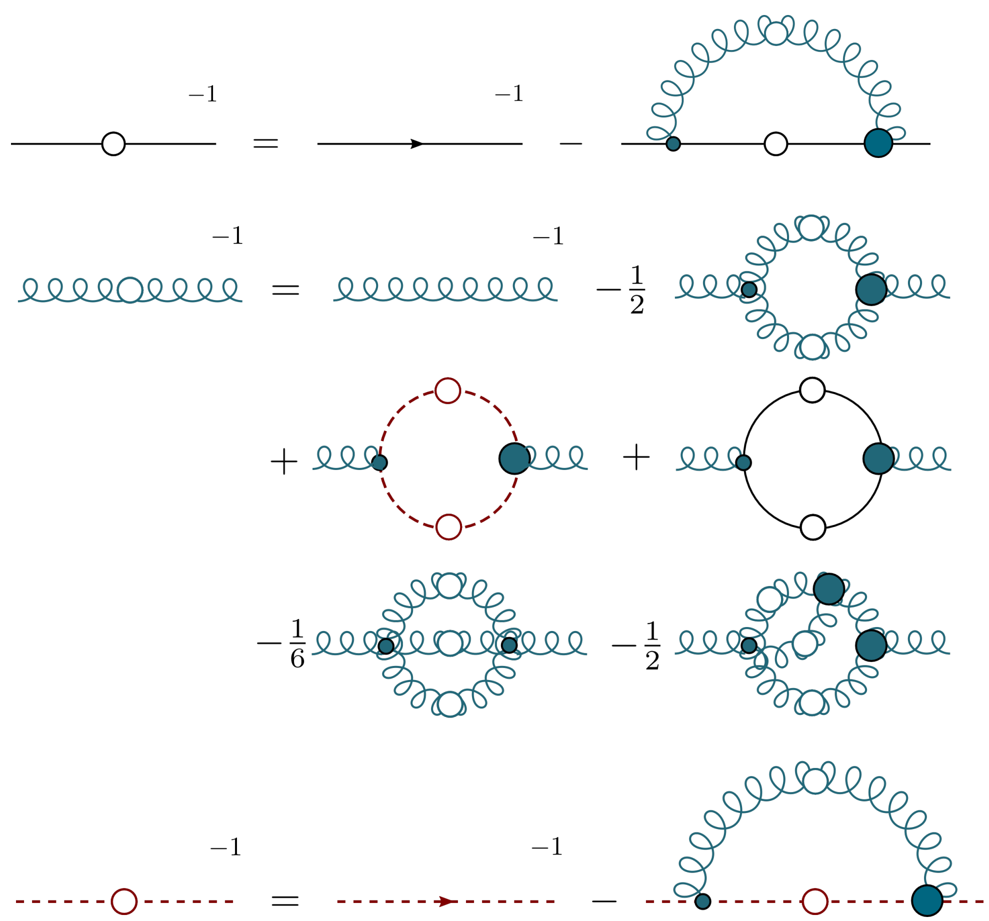
vertices



Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

propagators



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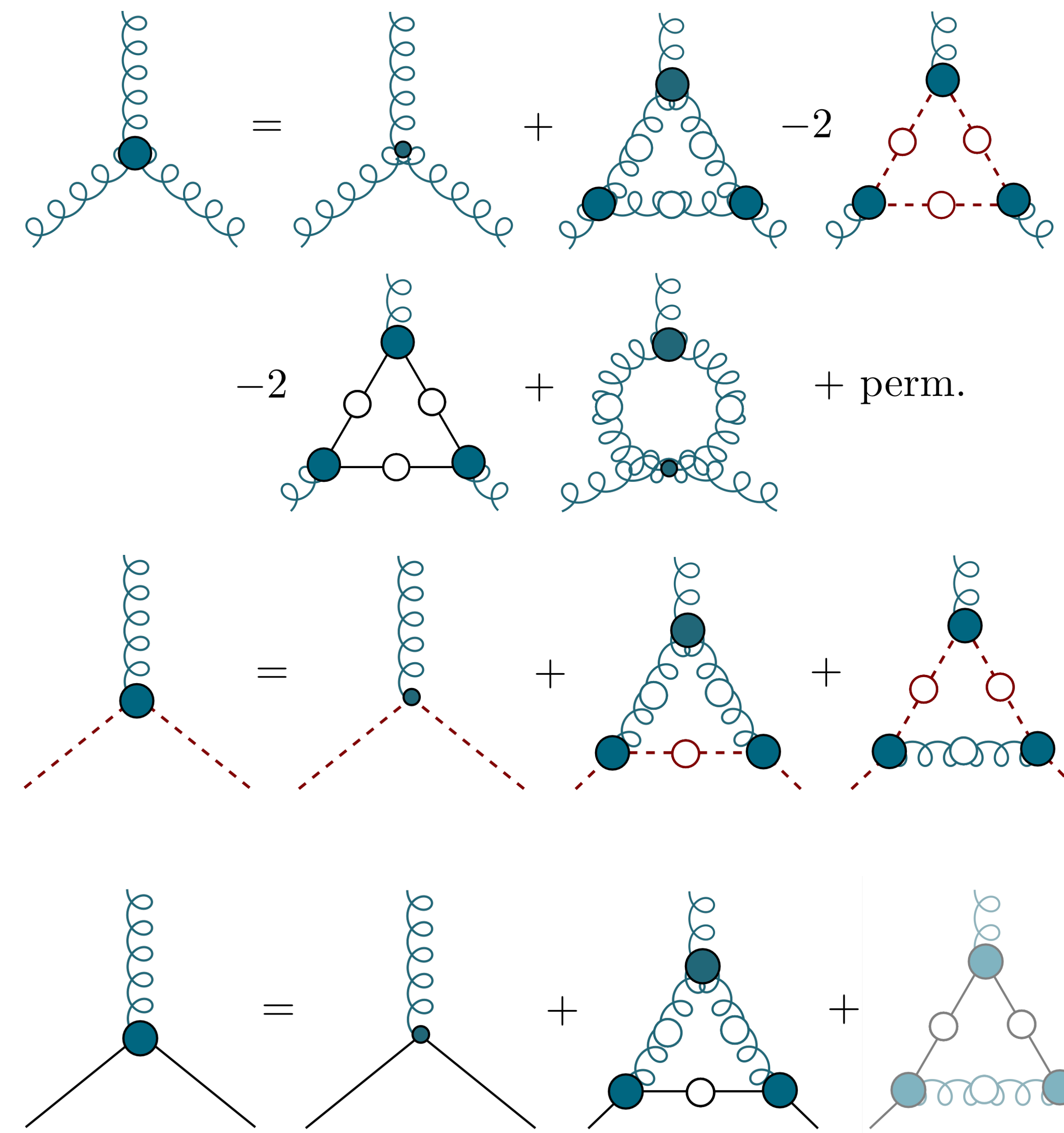
vertices

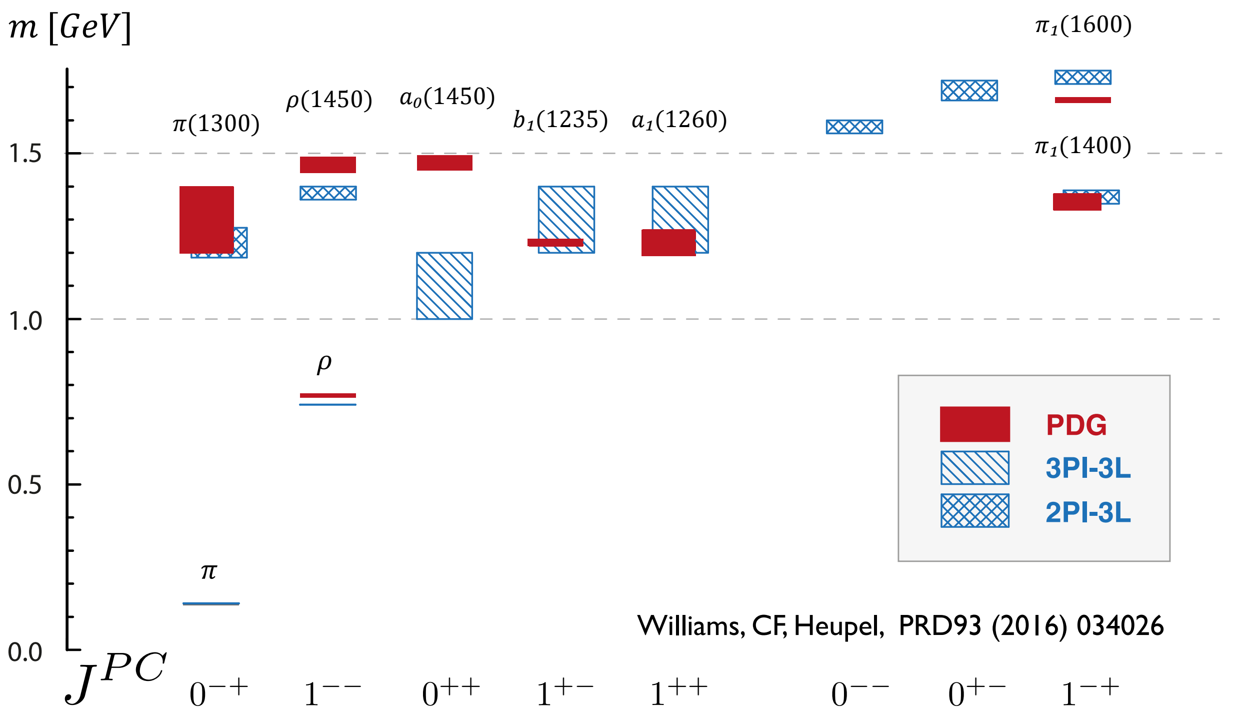


CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

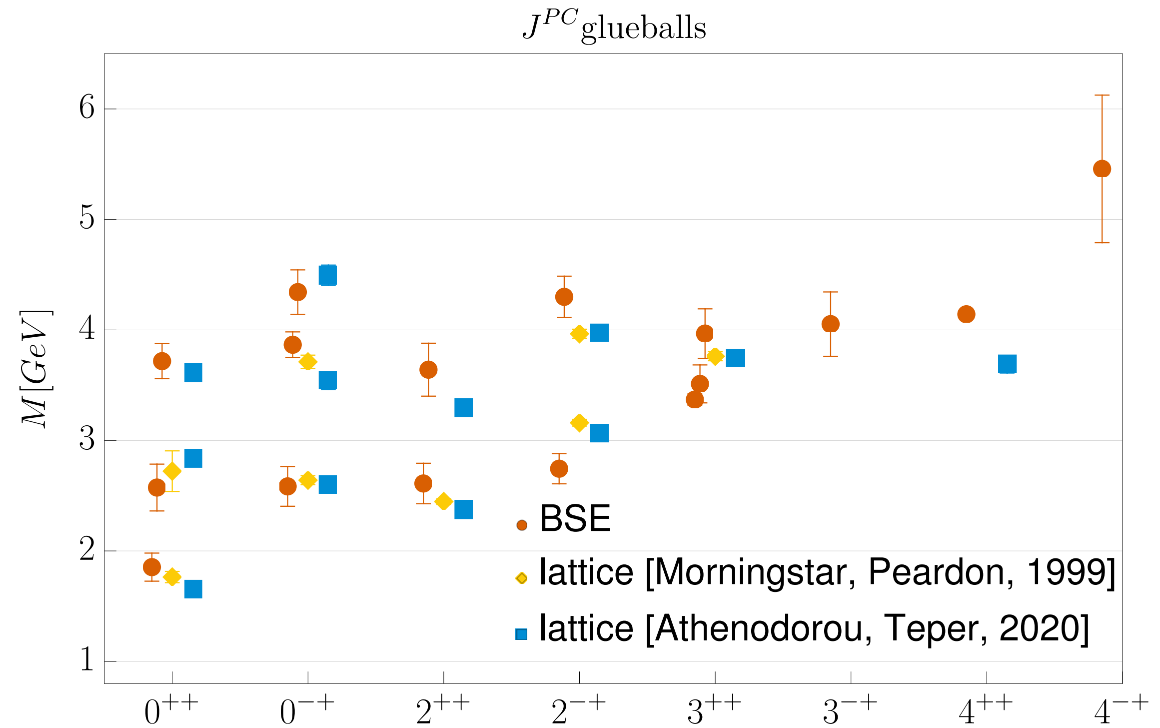
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

Hadron spectra: mesons, baryons, glueballs

Mesons:

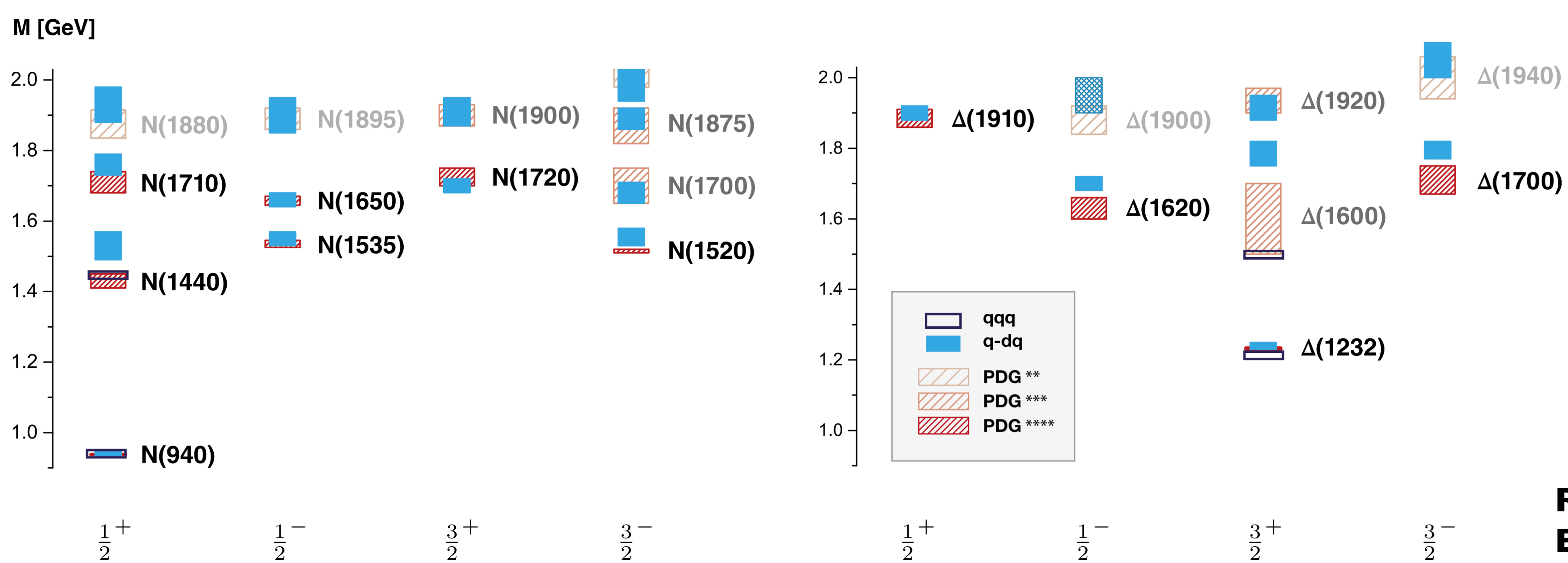


Glueballs:



CF, Huber, Sanchis-Alepuz, EPJC 80 (2020)
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021)

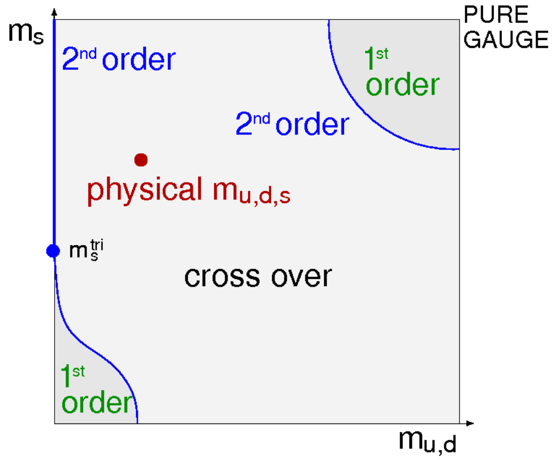
Baryons:

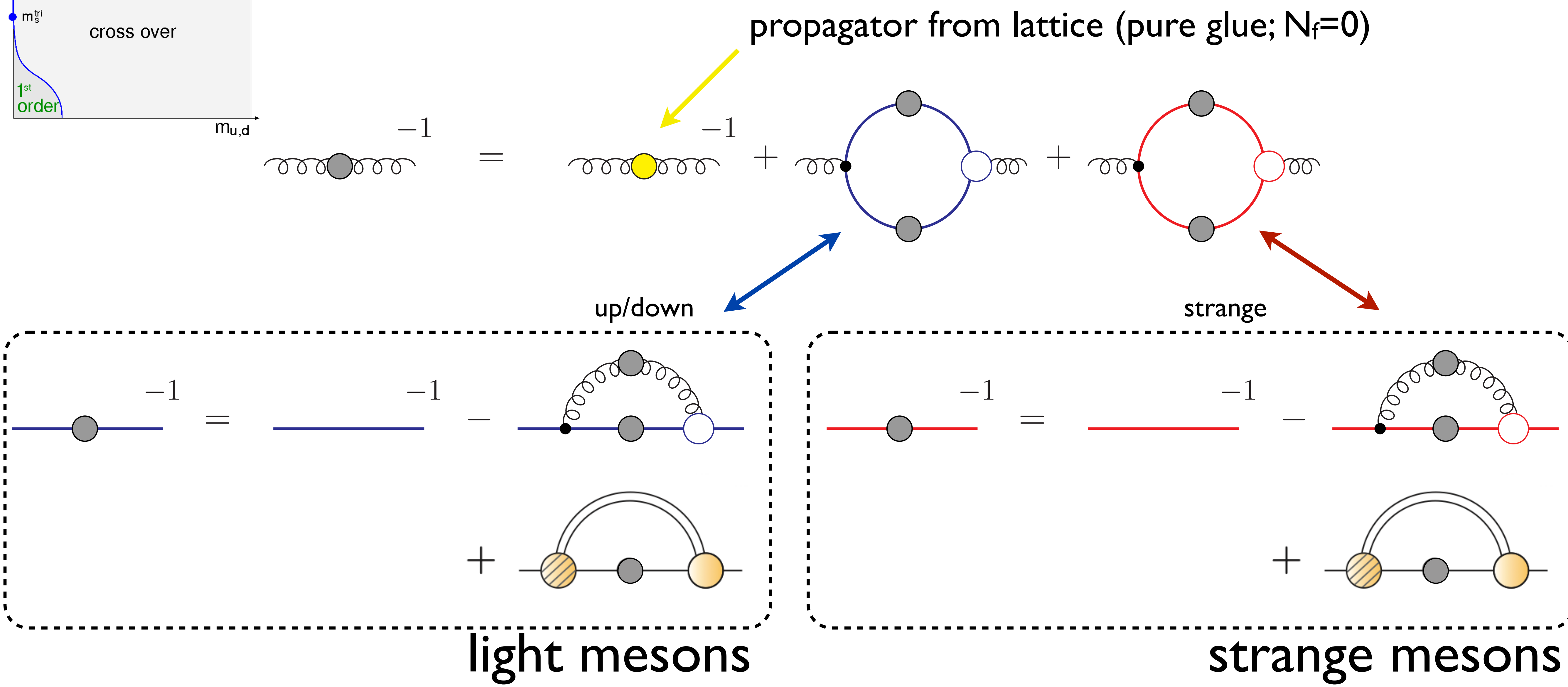


Review:
Eichmann et al. PPNP 91 (2016)

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

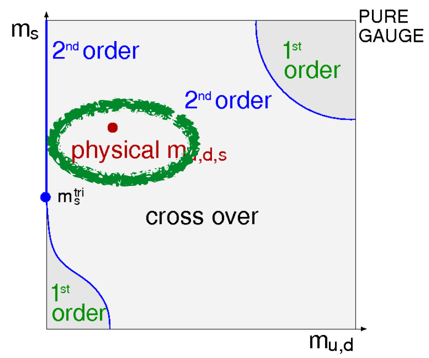
N_f=2+1-QCD with DSEs and meson backcoupling

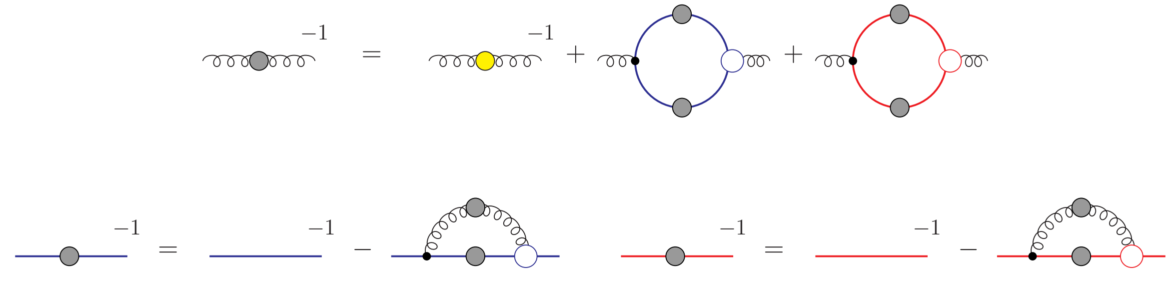


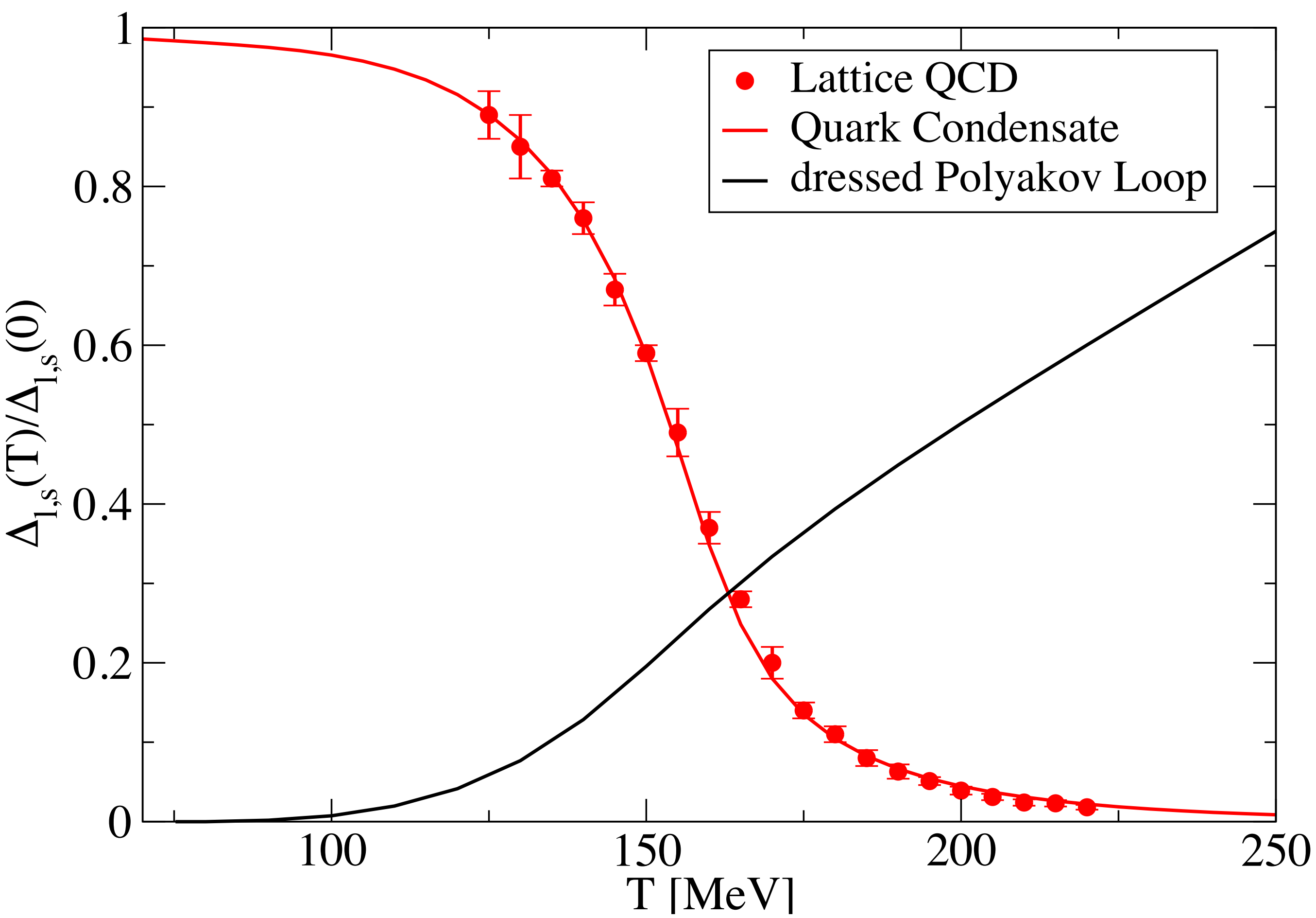


N_f= 2 + 1

N_f=2+1, μ=0, physical point

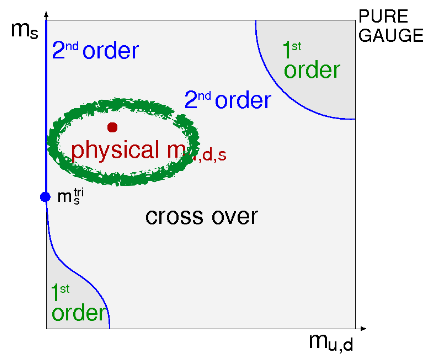


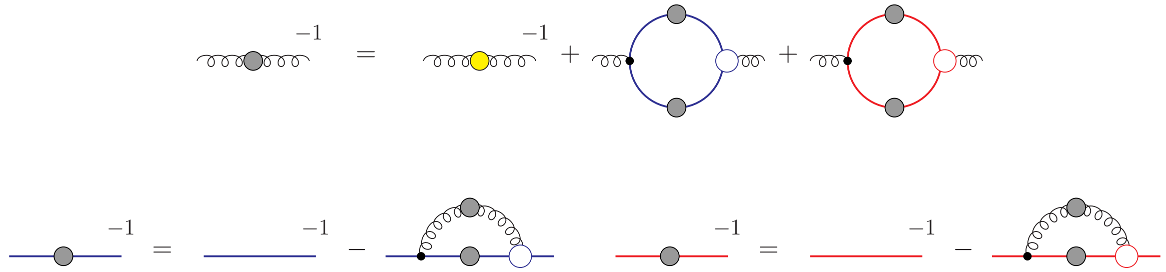


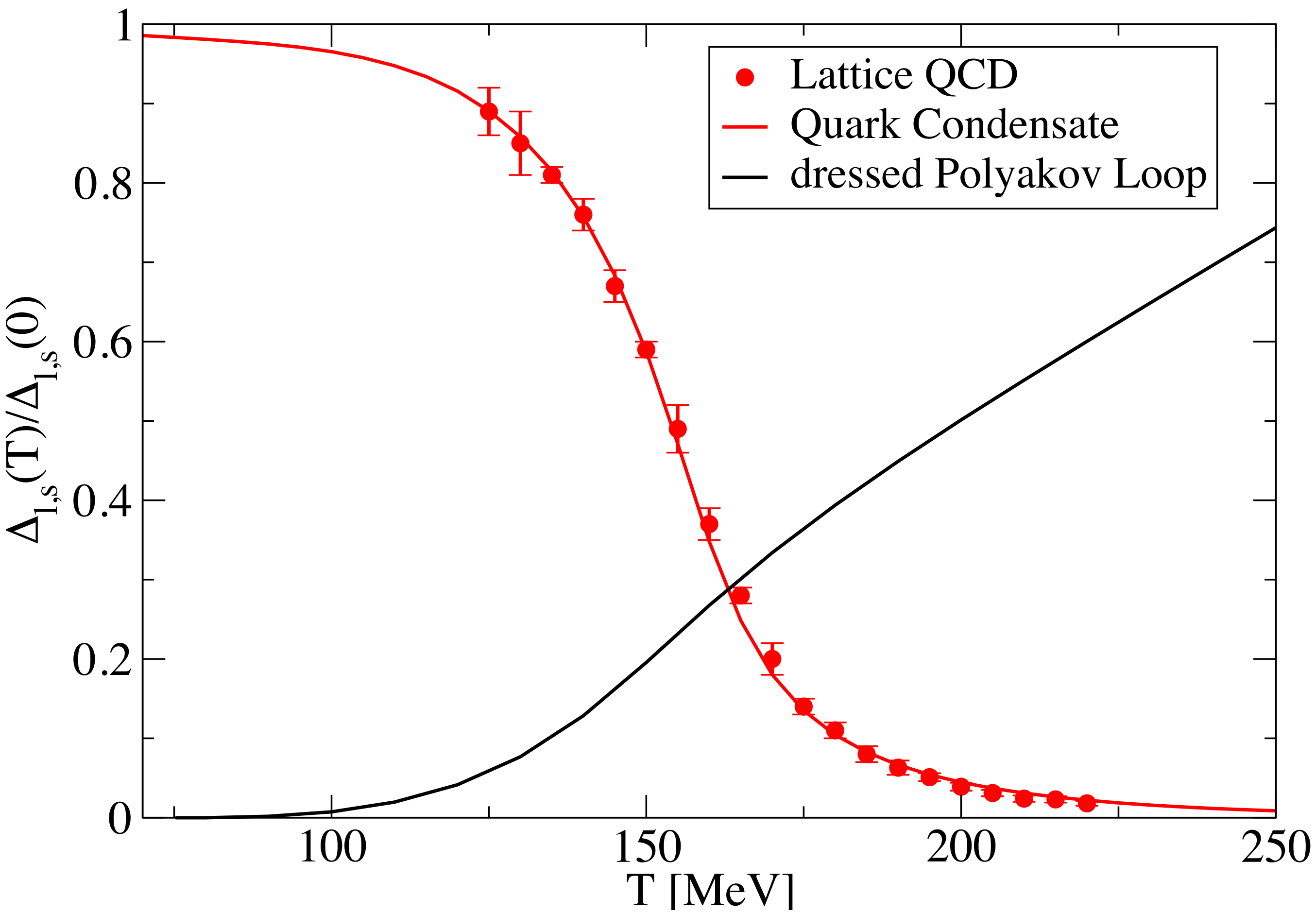


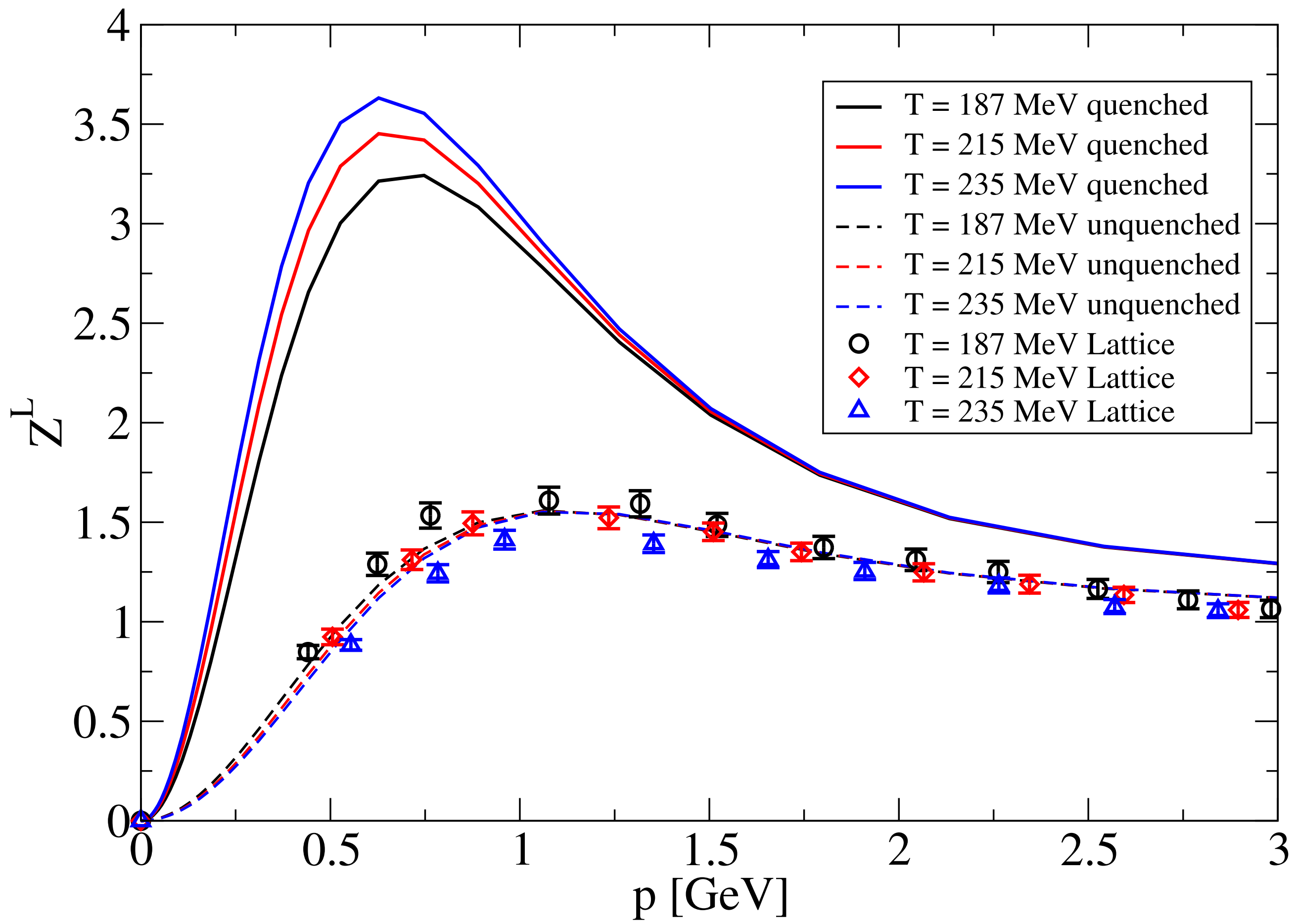
Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

N_f=2+1, μ=0, physical point









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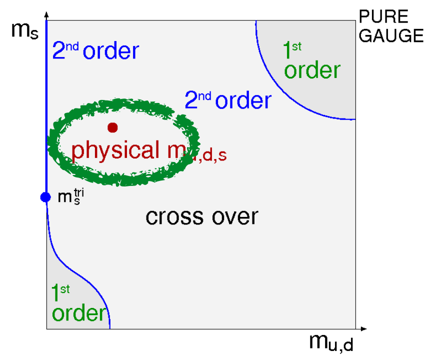
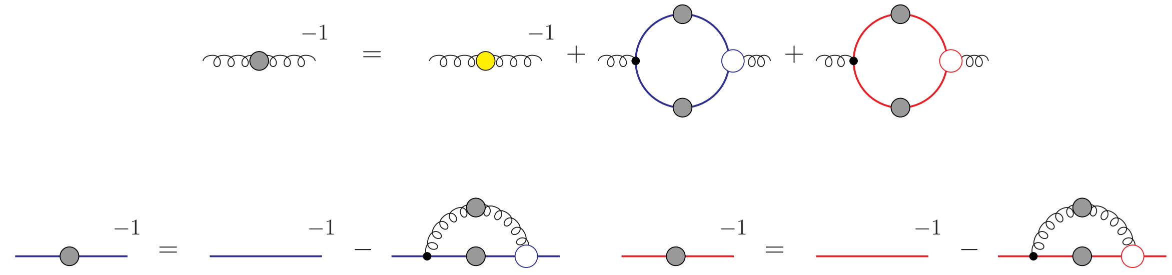
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

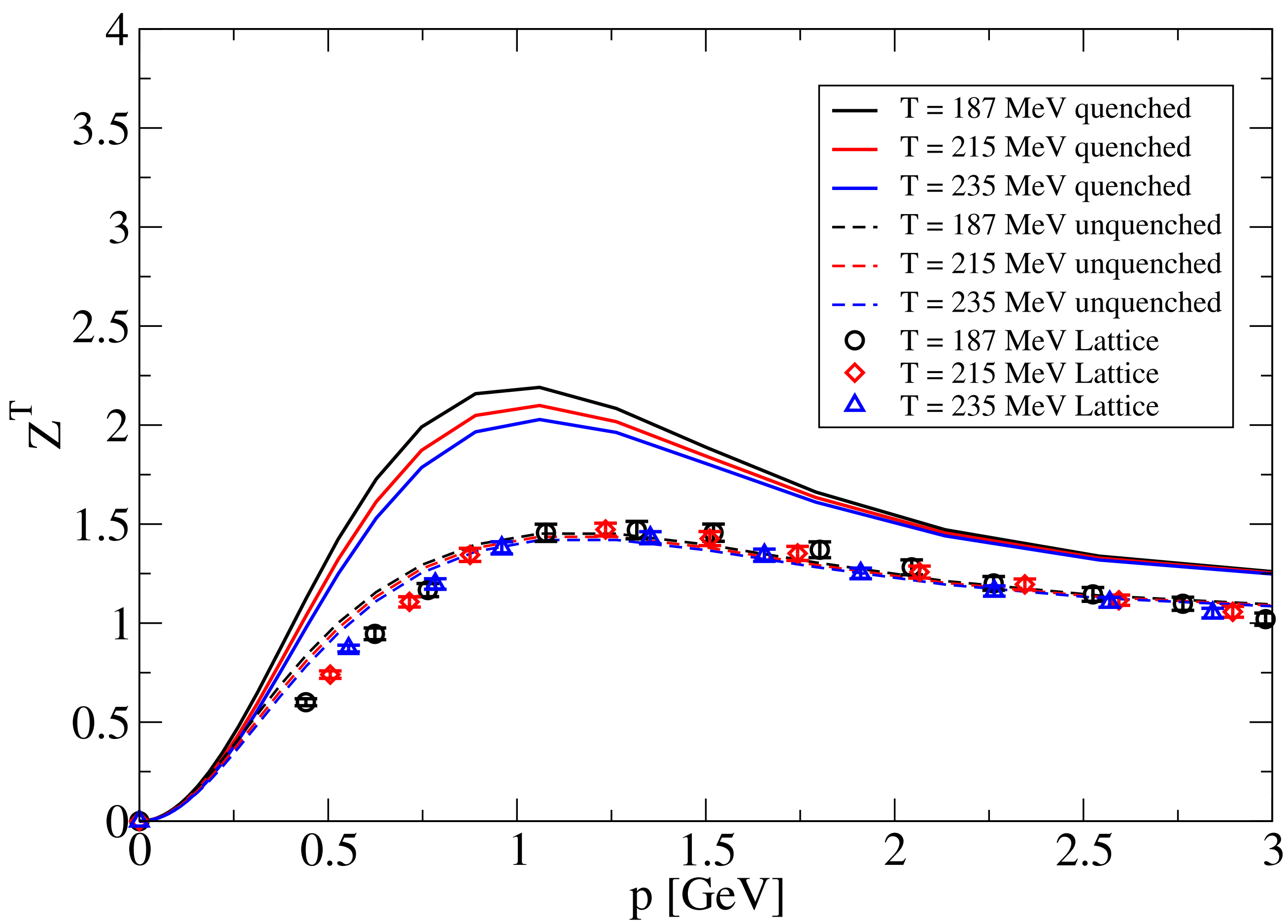
Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

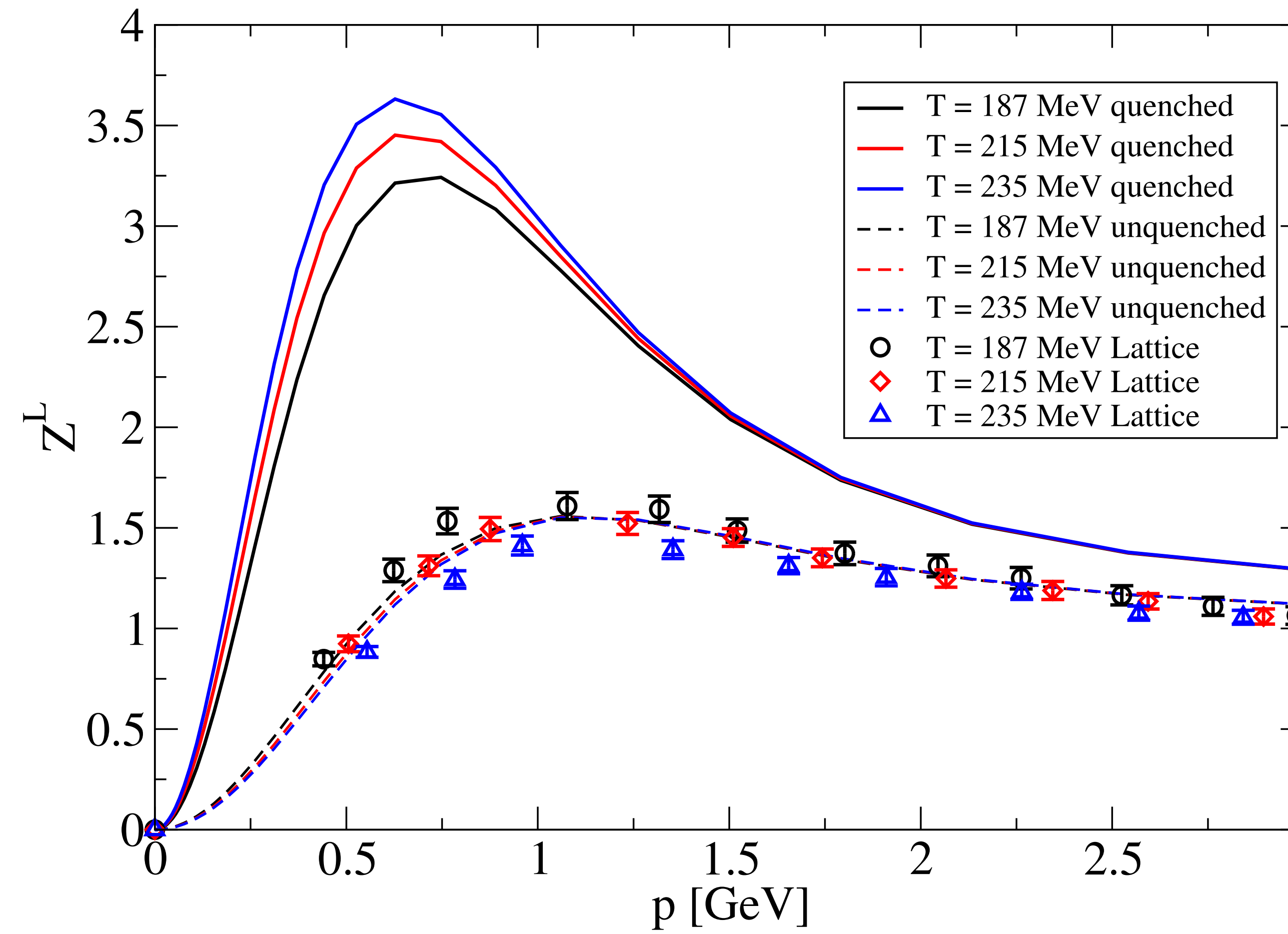
- quantitative agreement: DSE prediction verified by lattice

N_f=2+1, μ=0, physical point





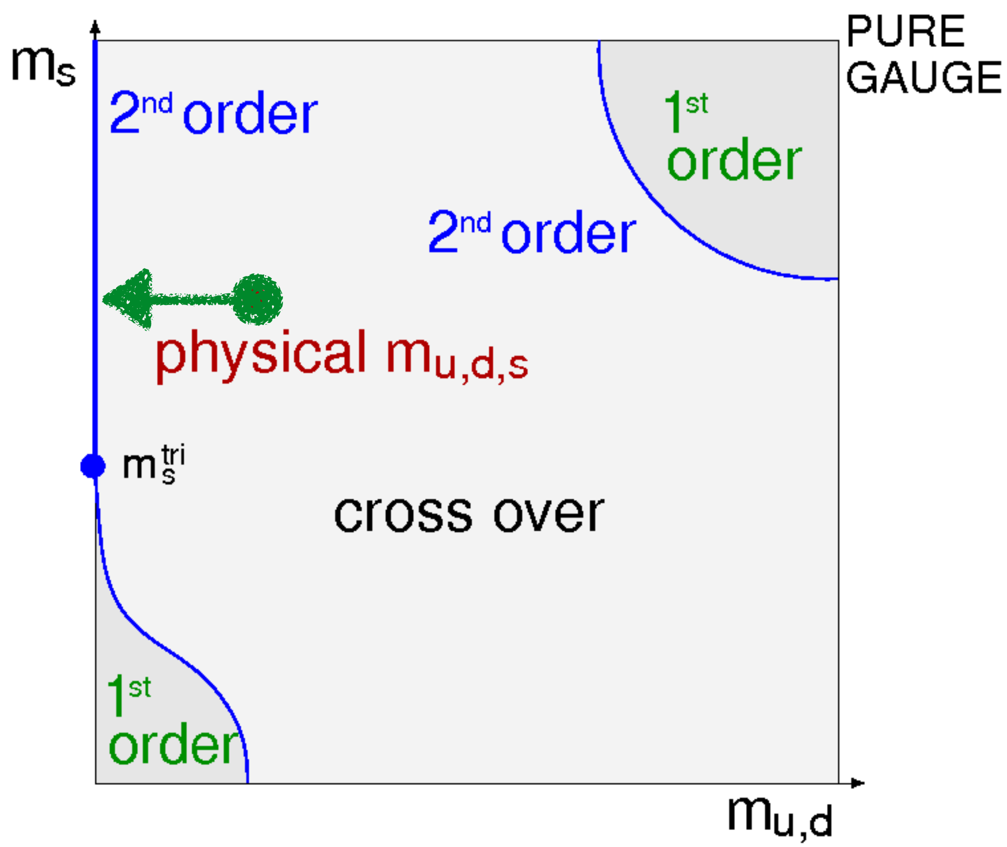


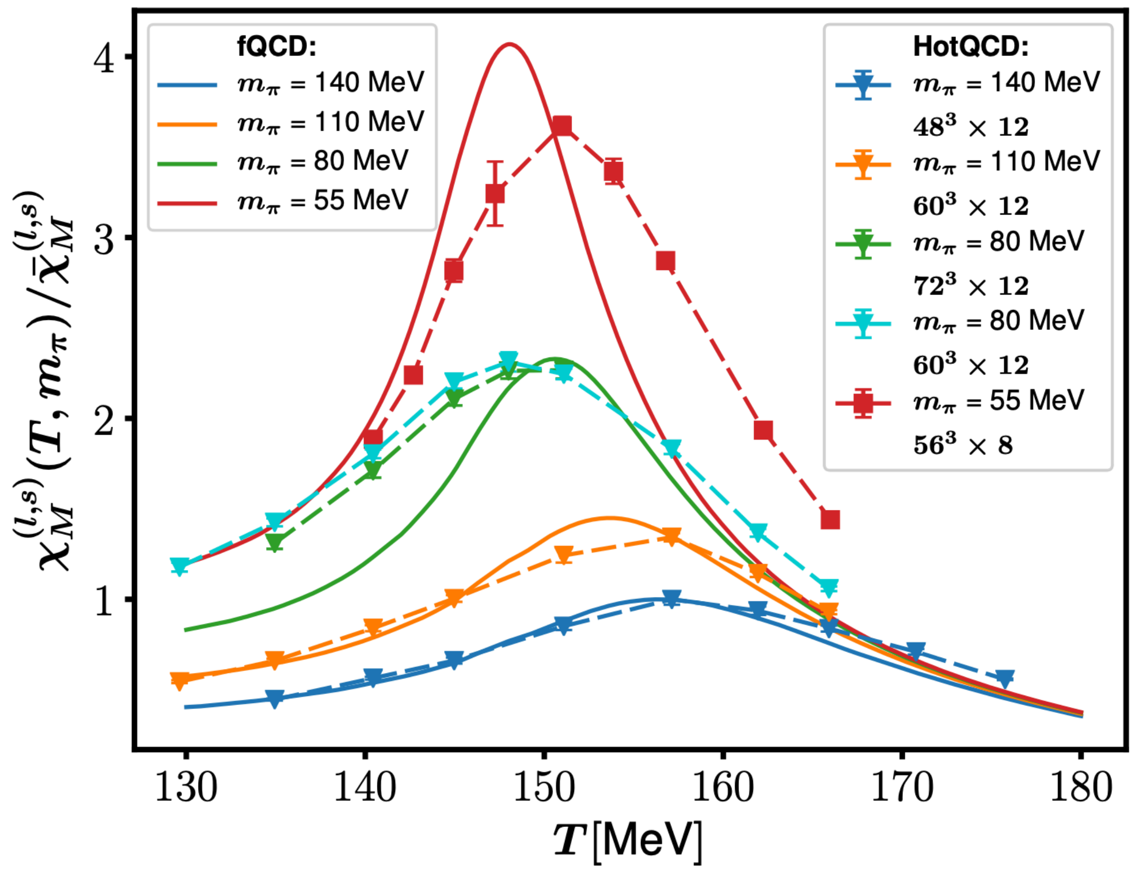


Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- quantitative agreement: DSE prediction verified by lattice

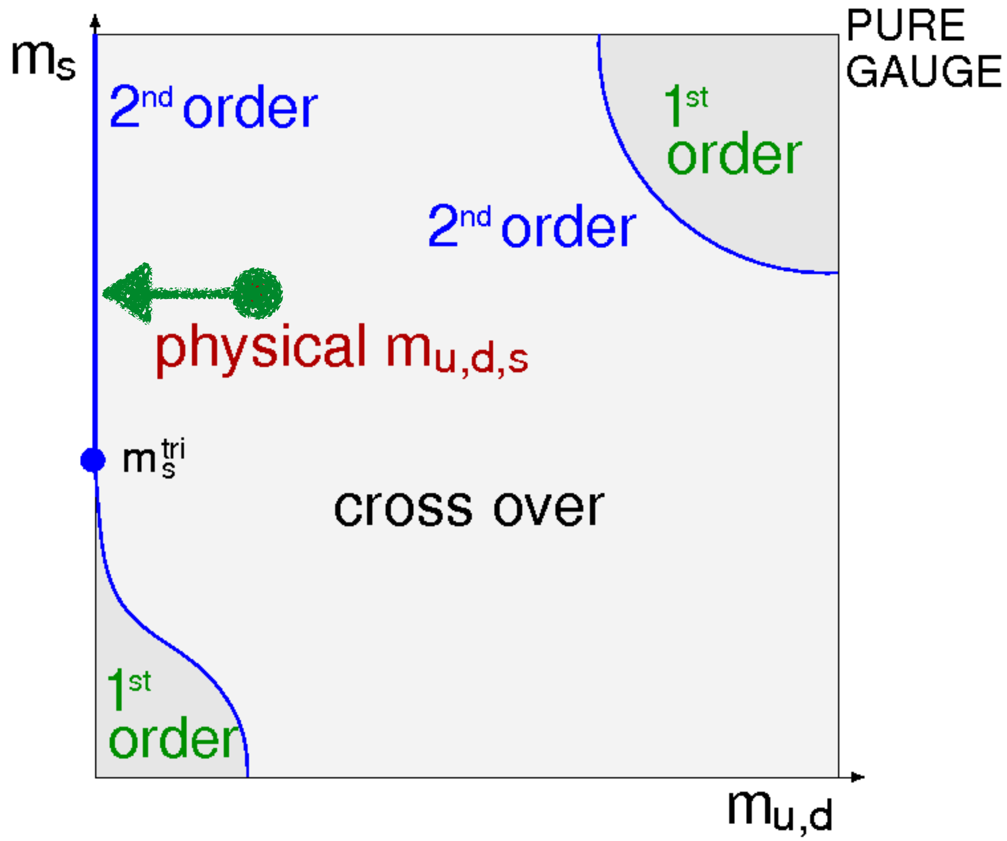
Towards the chiral limit…

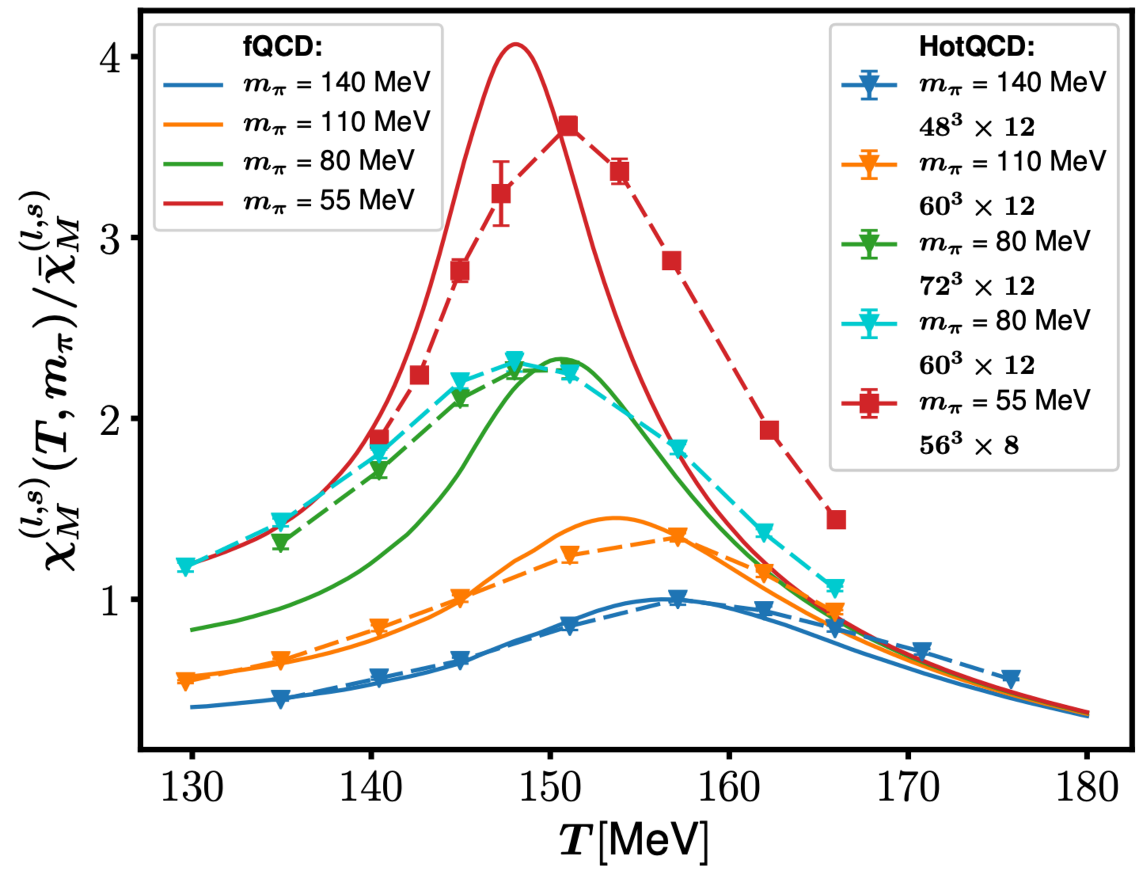


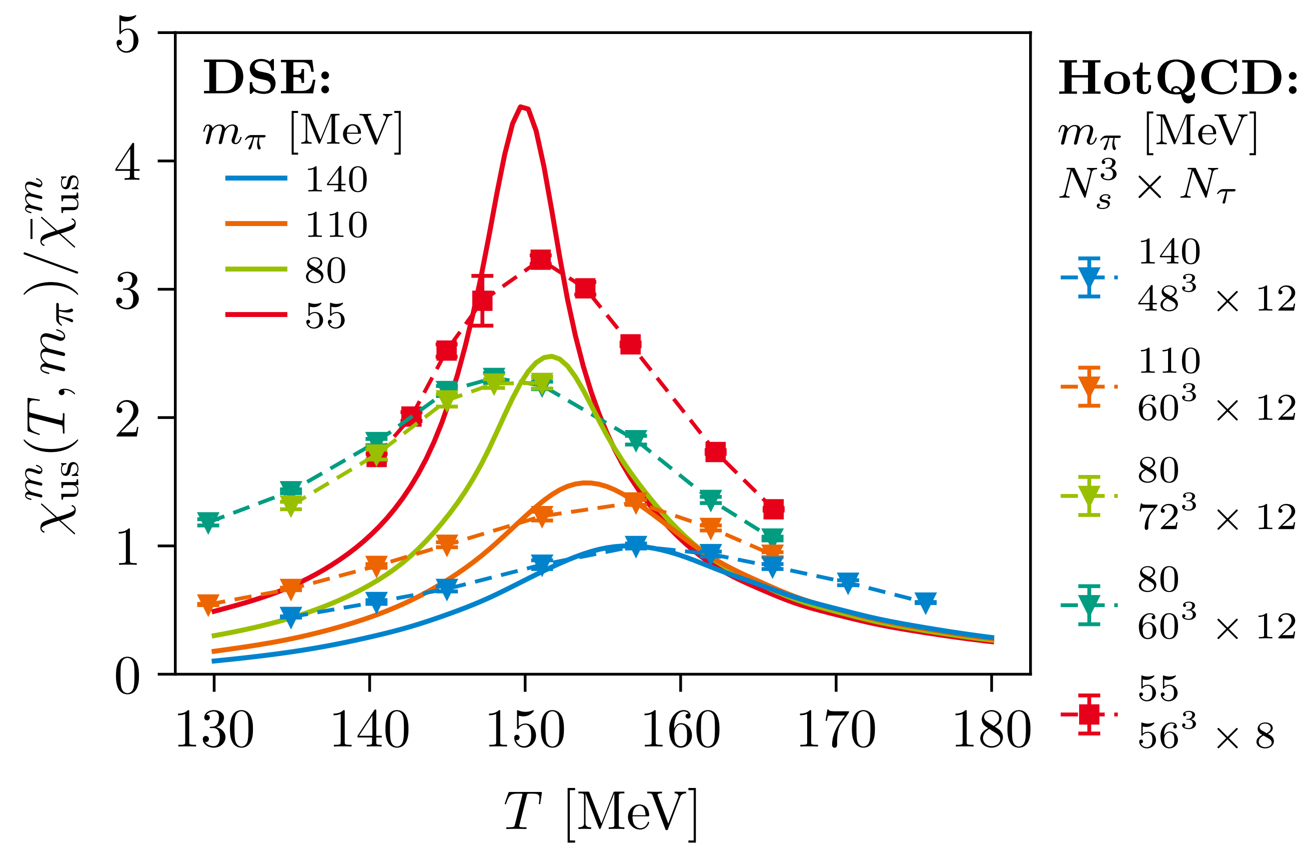


HotQCD: Ding et al. PRL 123, 062002 (2019)
FRG: Braun et al, PRD 102 (2020) 5, 056010
DSE: Bernhardt and CF, PRD 108 (2023) 114018

Towards the chiral limit…







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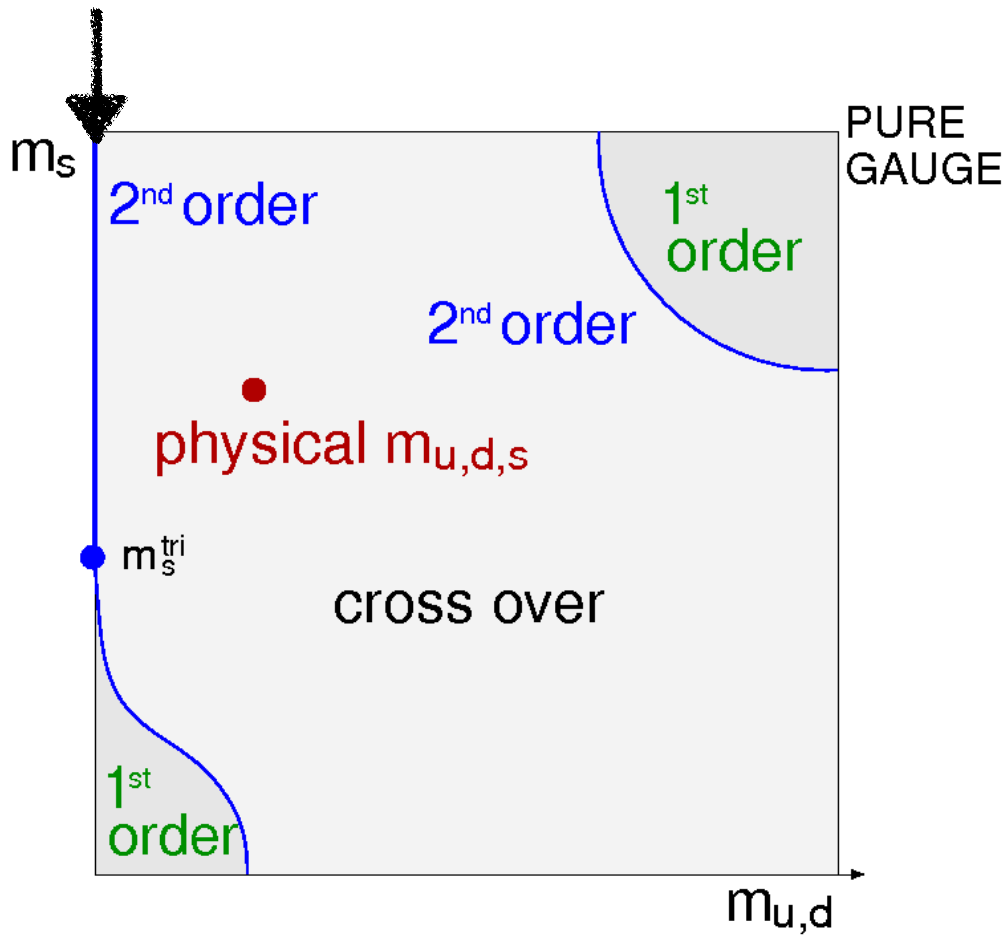
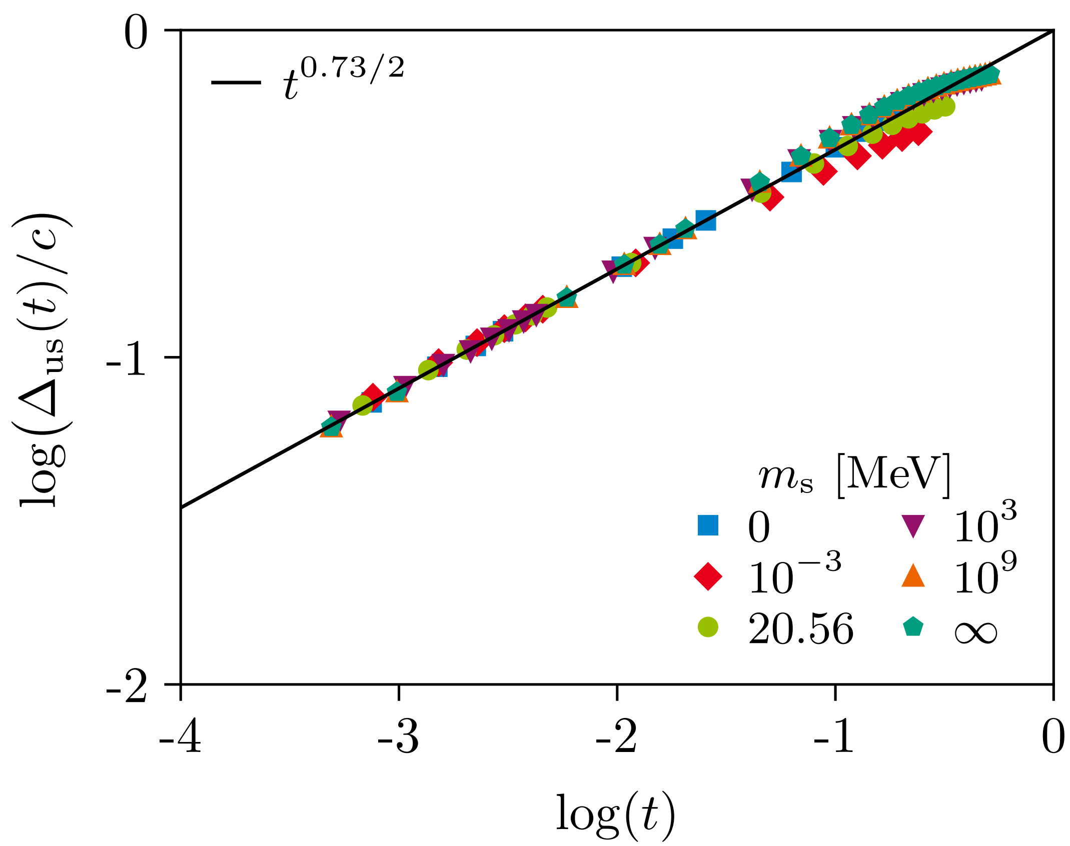
At the chiral limit…





Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141

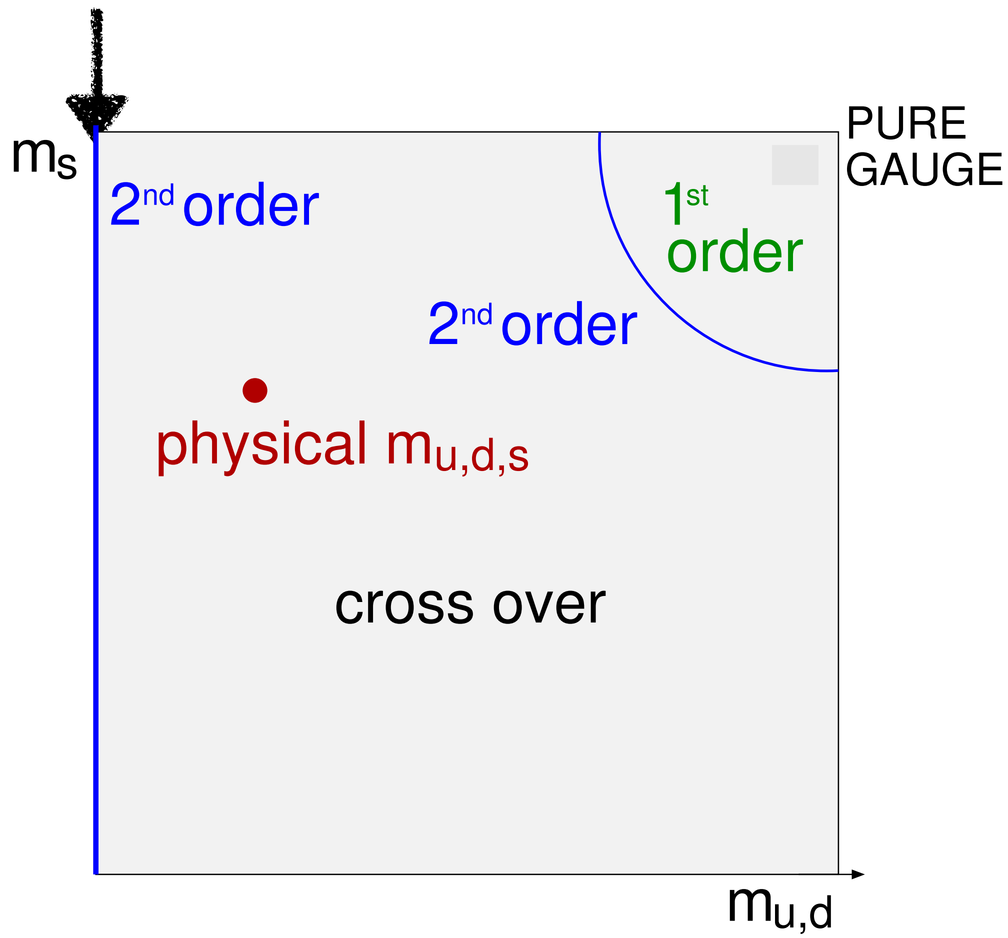
Bernhardt and CF, PRD 108 (2023) no.11, 114018

reproduce CF and Mueller, PRD 84 (2011) 054013

DSE: Bernhardt and CF, PRD 108 (2023) 114018

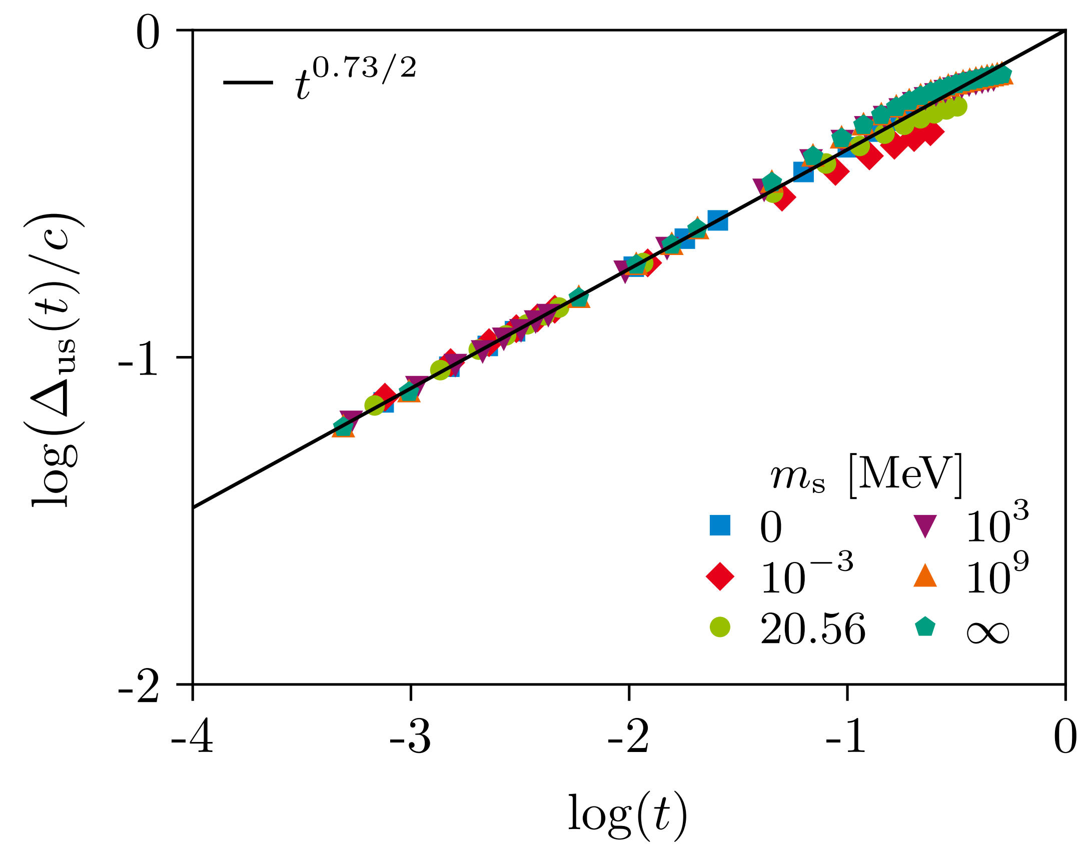
Lattice: Dini, et al, PRD 105 (2022) no.3, 034510
Ding et al. PRL 123, 062002 (2019)
Bornyakov et al. PRD 82, 014504 (2010)

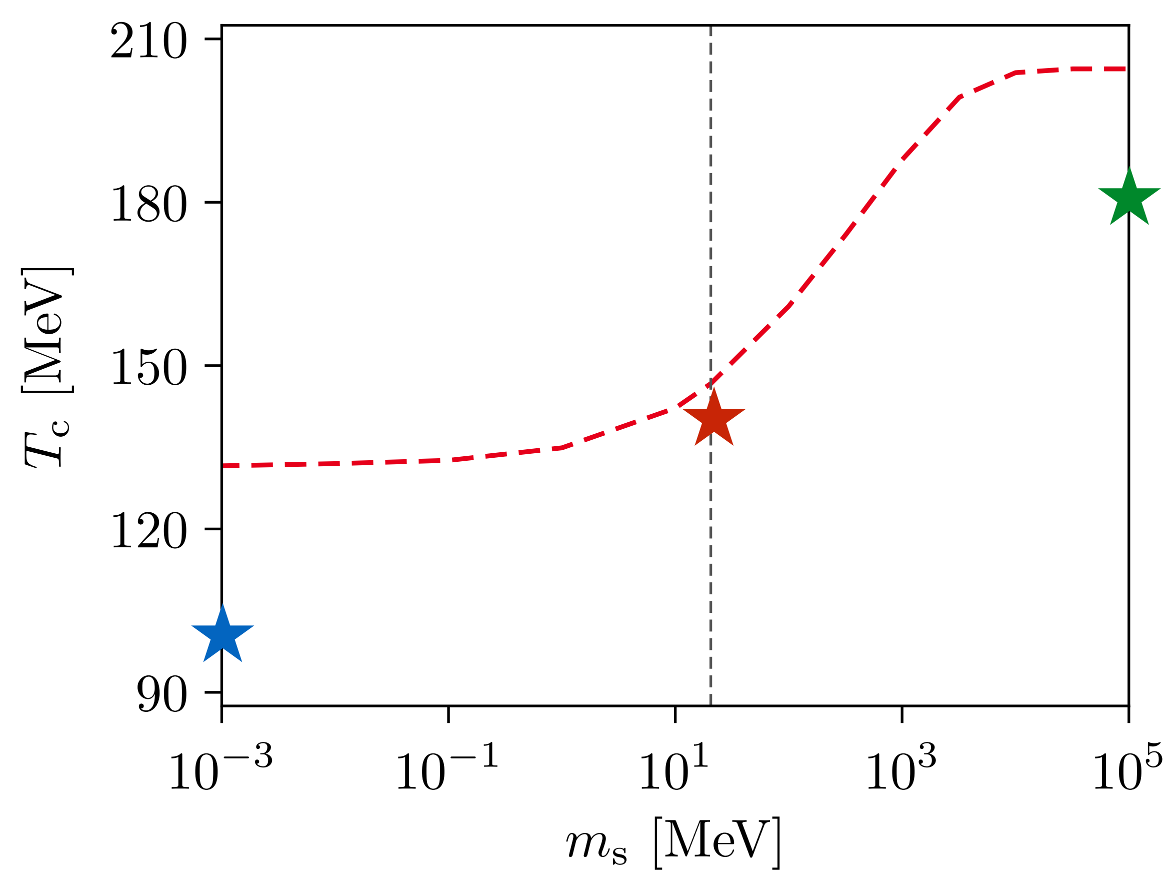
At the chiral limit…



Cuteri, Philipsen and Sciarra, JHEP 11 (2021), 141

Bernhardt and CF, PRD 108 (2023) no.11, 114018





reproduce CF and Mueller, PRD 84 (2011) 054013

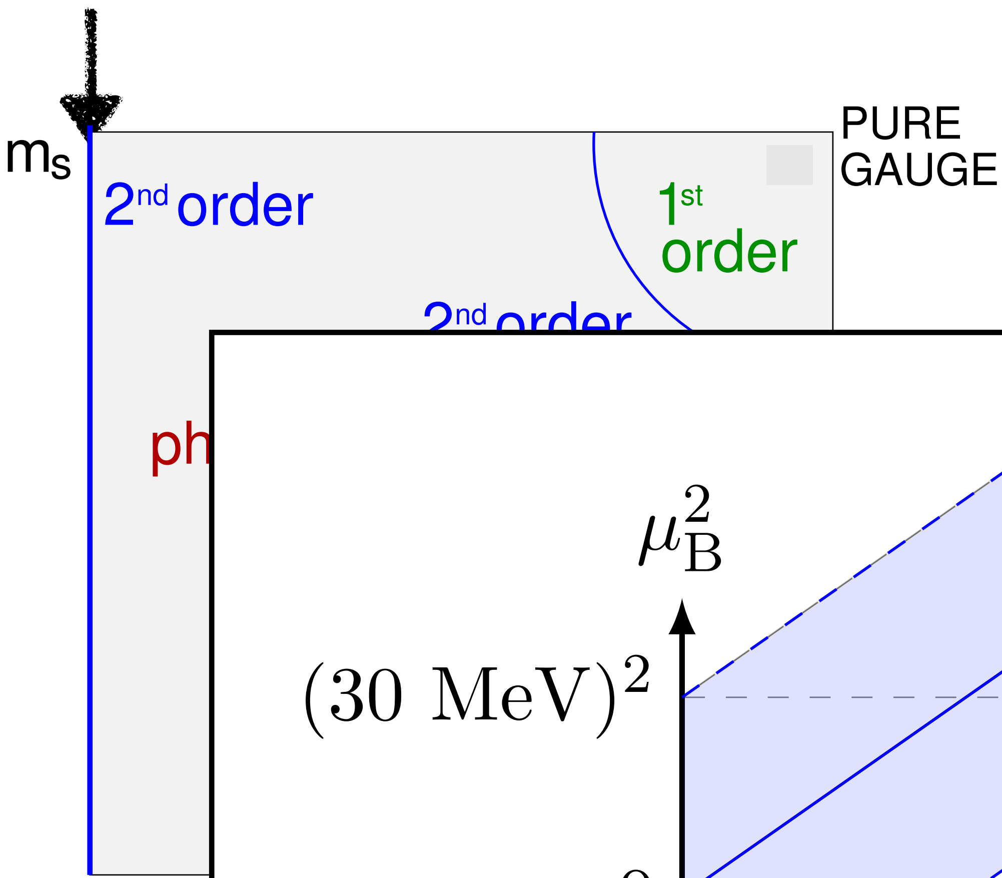
DSE: Bernhardt and CF, PRD 108 (2023) 114018

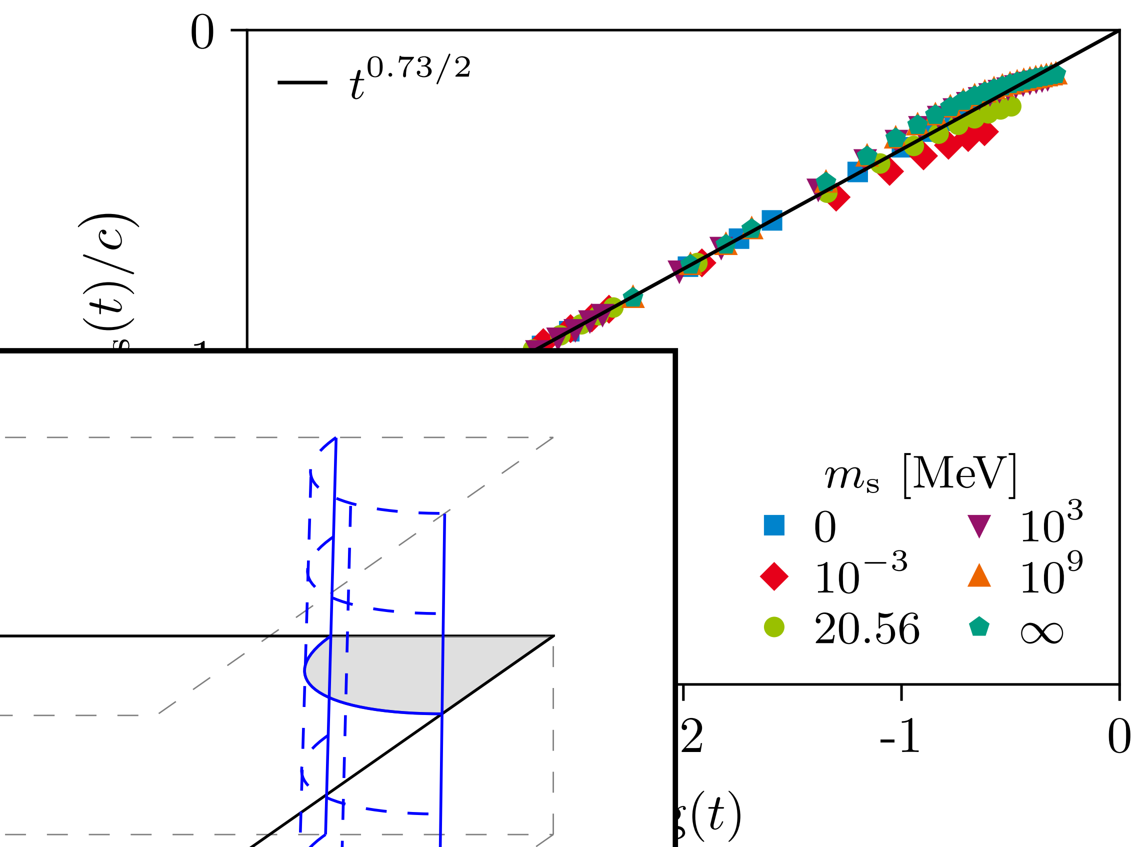
Lattice: Dini, et al, PRD 105 (2022) no.3, 034510

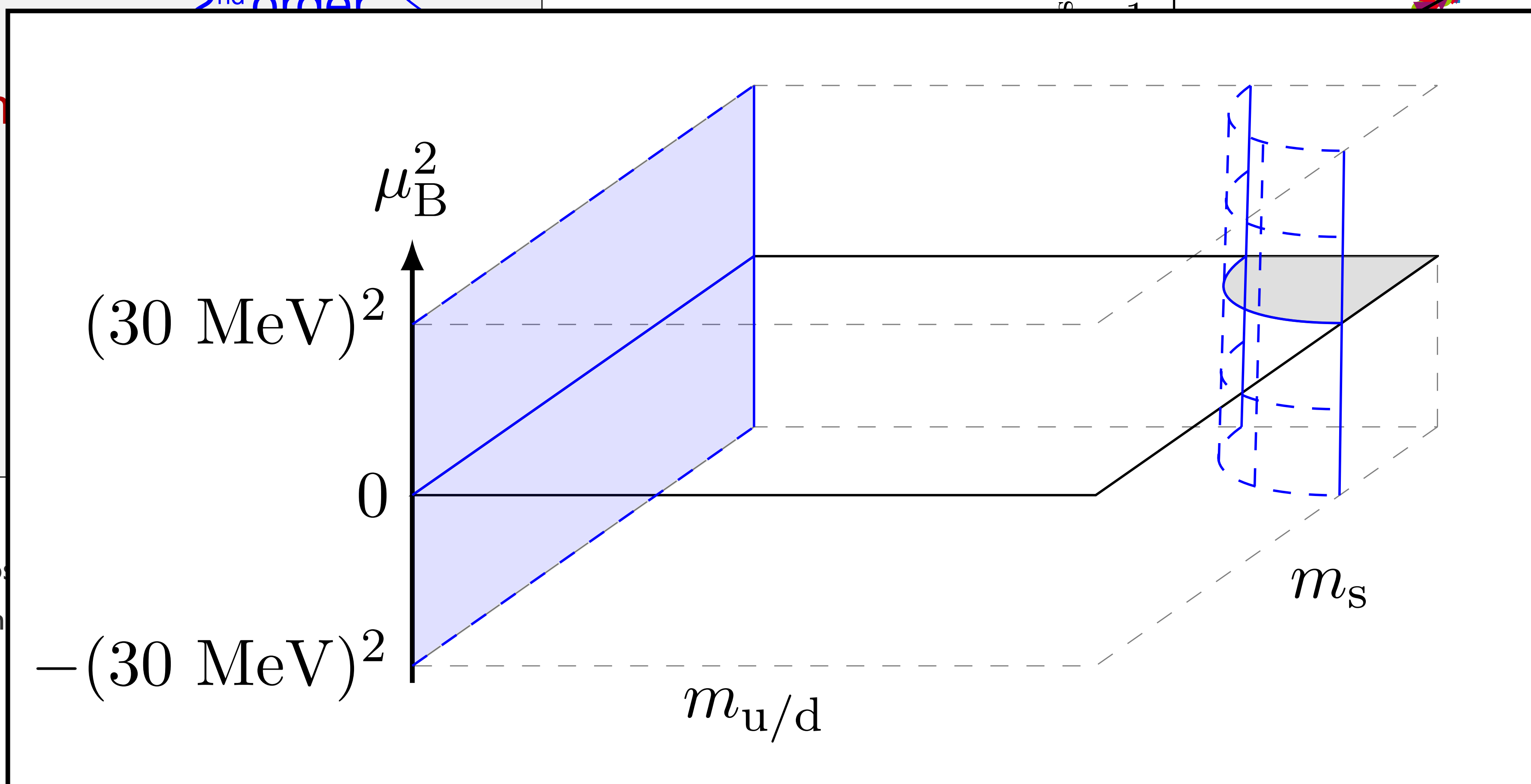
Ding et al. PRL 123, 062002 (2019)

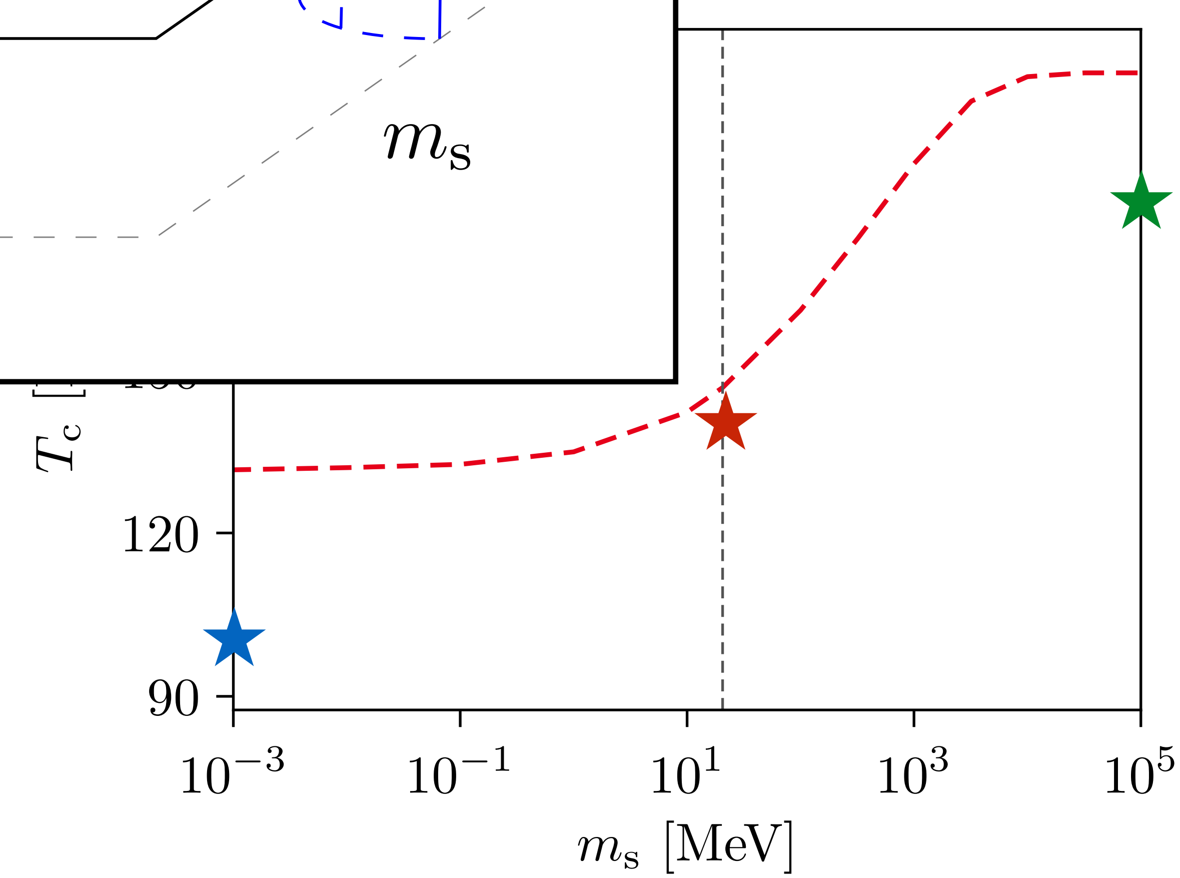
Bornyakov et al. PRD 82, 014504 (2010)

At the chiral limit…









Cuteri, Philip

Bernhardt an

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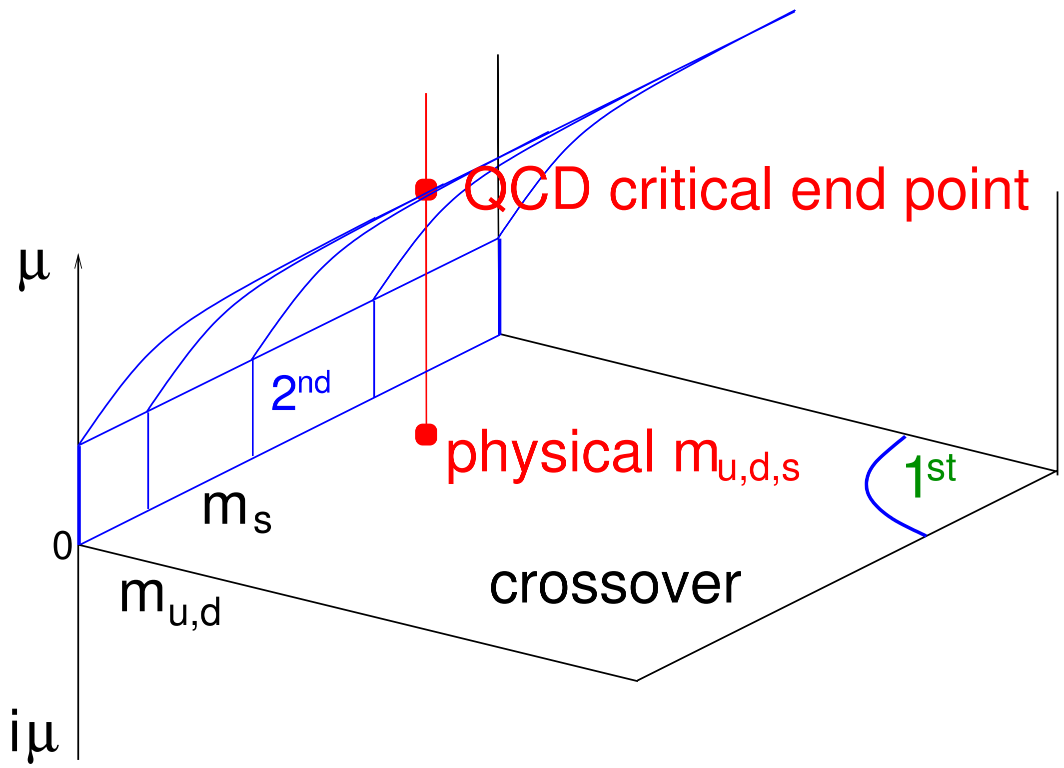
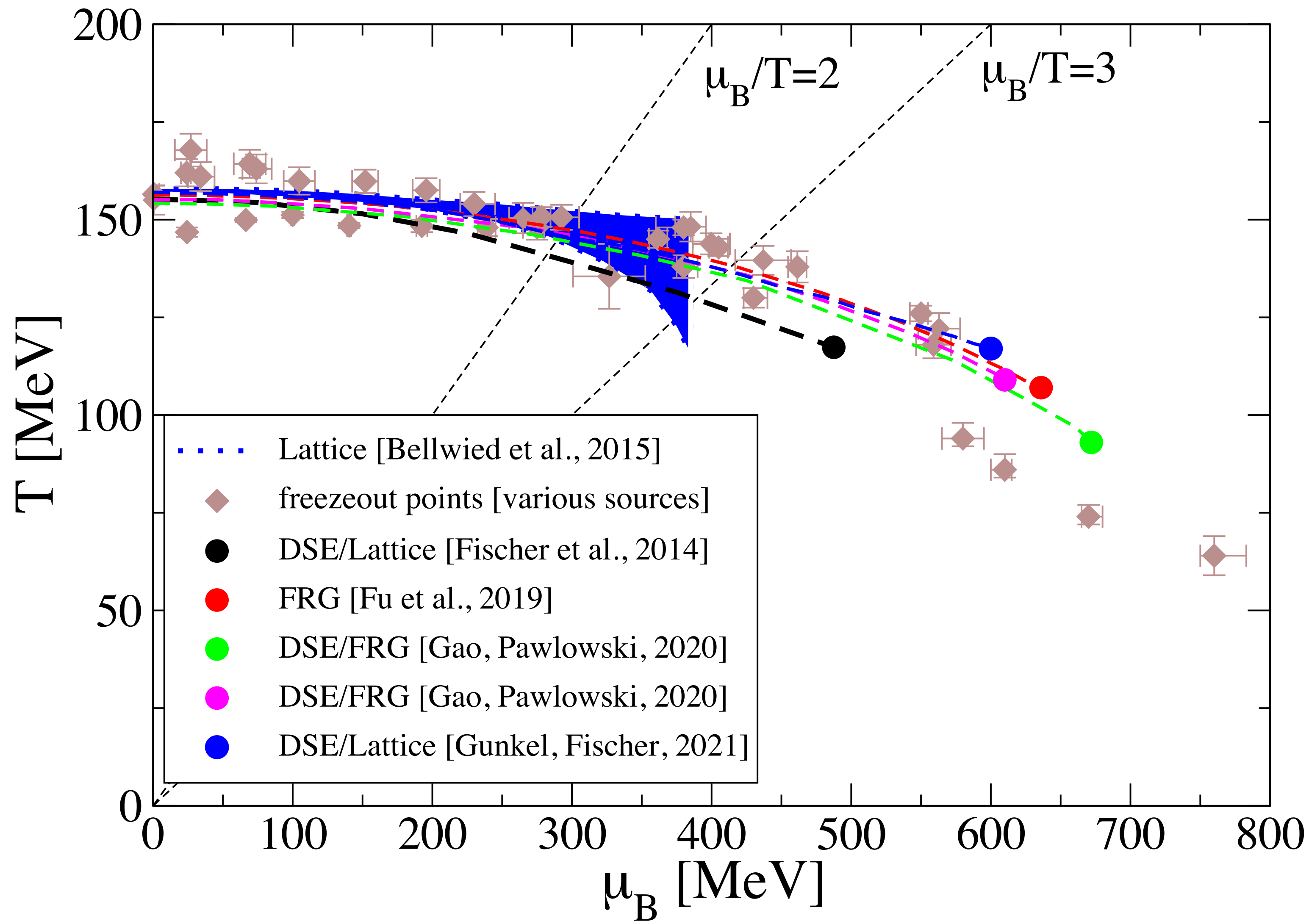
Location of CEP





Bernhardt, CF and Isserstedt, PLB 841 (2023)

- how stable is this result ??
  - crosscheck with FRG ✔

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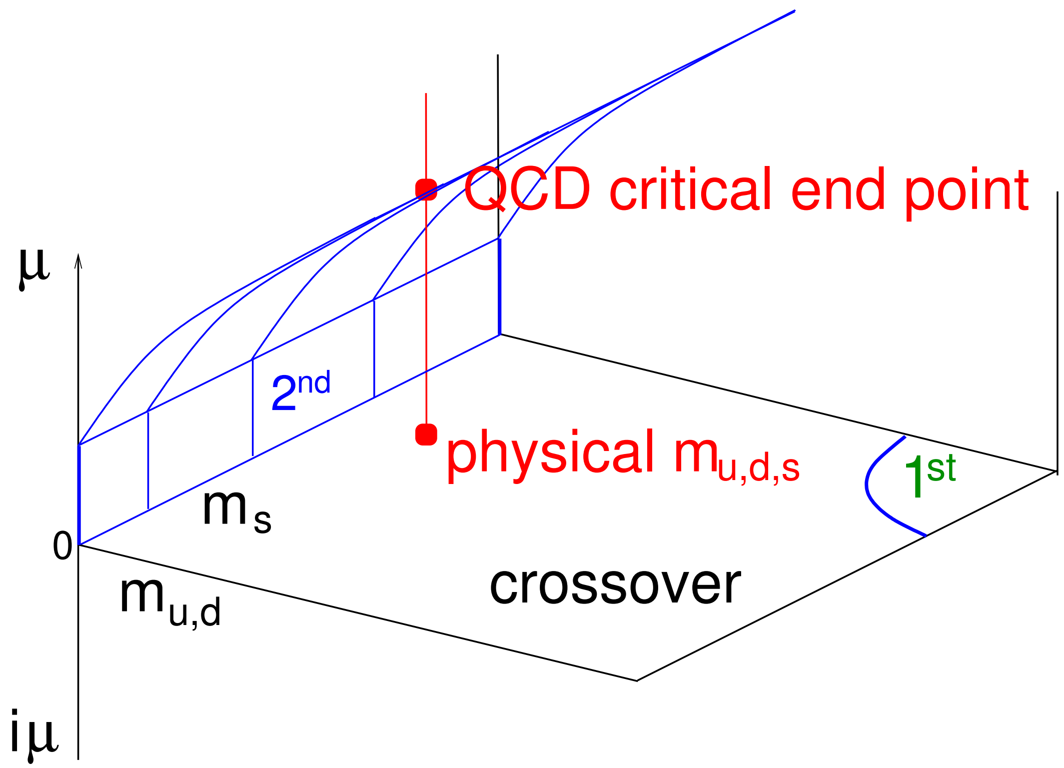

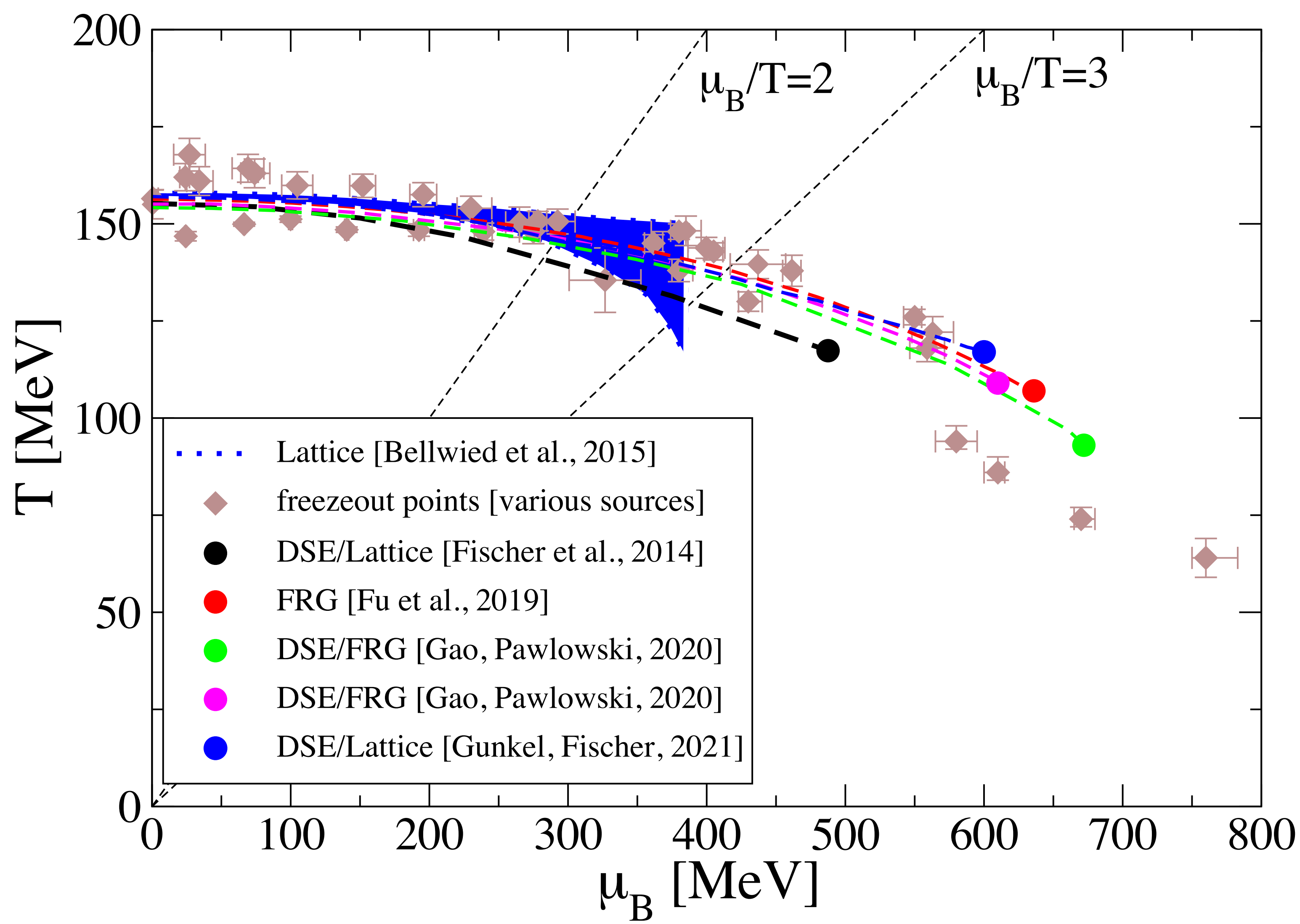

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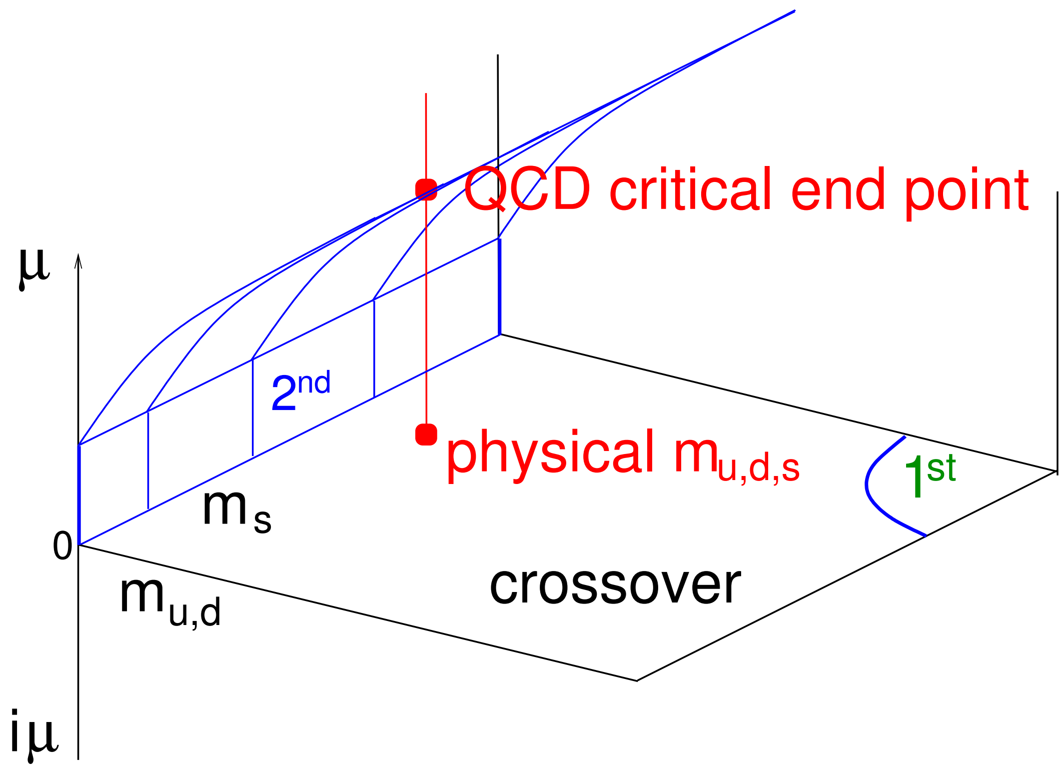
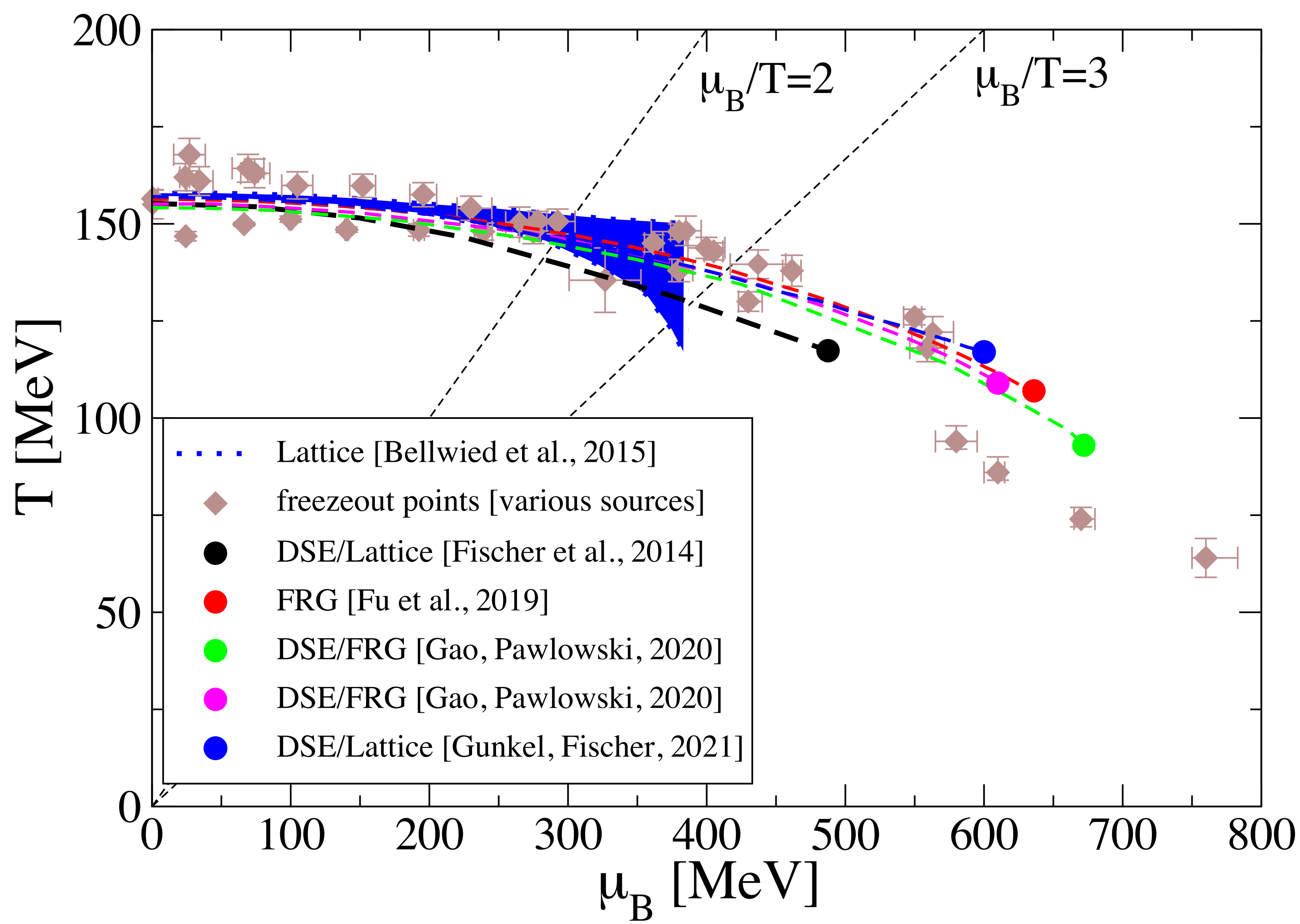
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Location of CEP





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  - baryon and meson effects ✔ Eichmann, CF, Welzbacher, PRD93 (2016) Gunkel and CF, PRD104 (2021)

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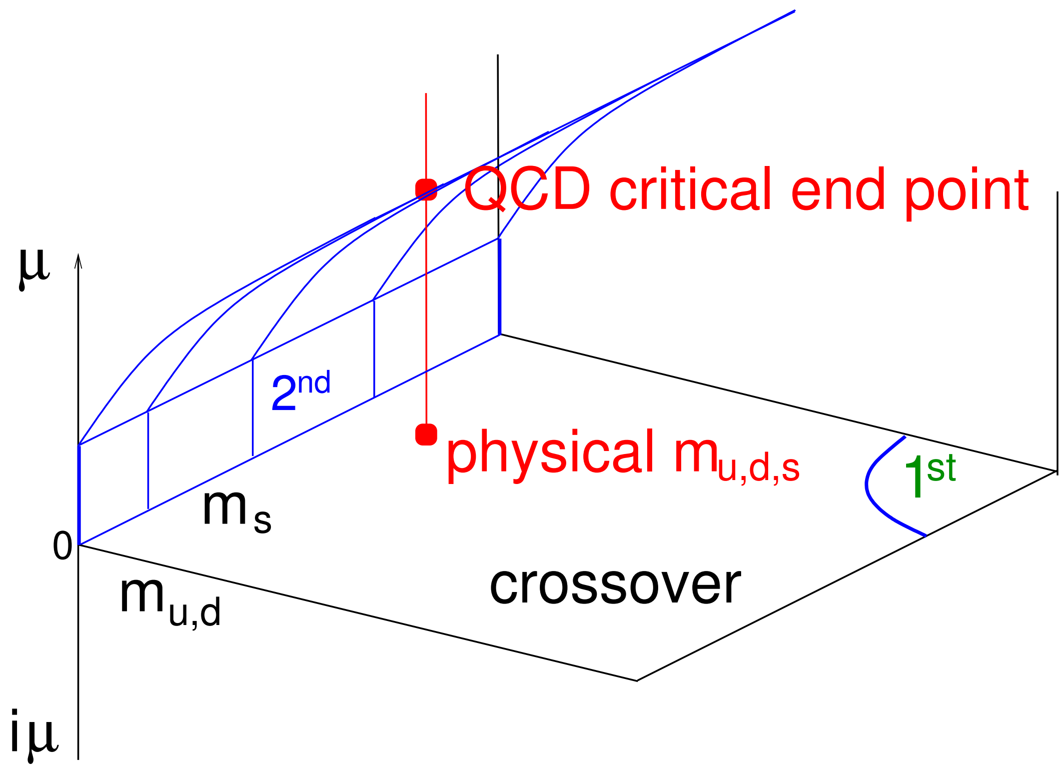

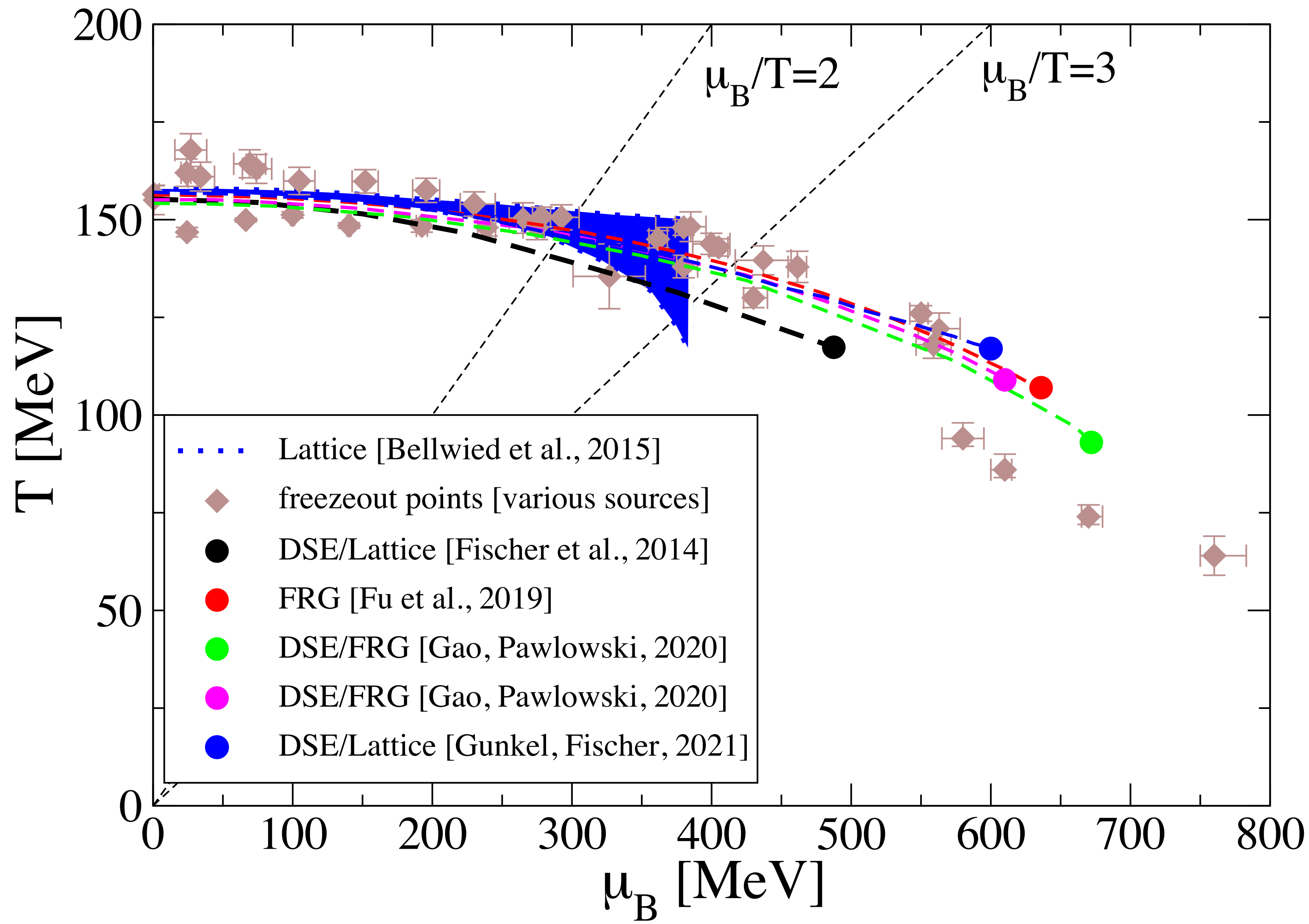

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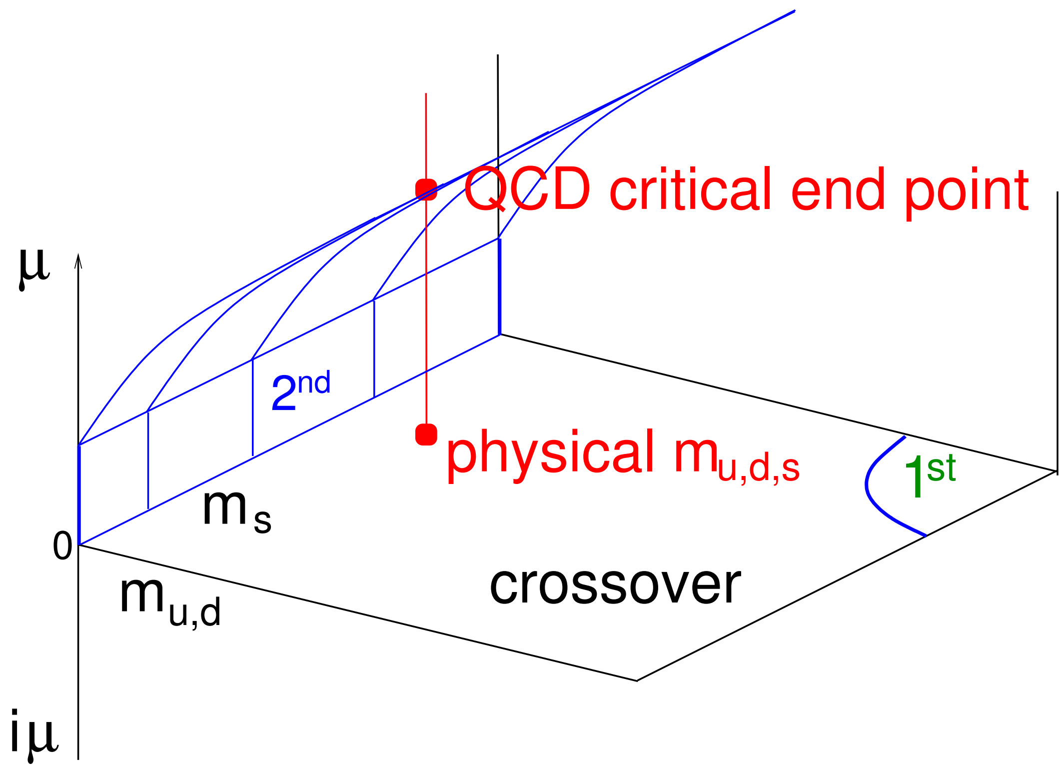

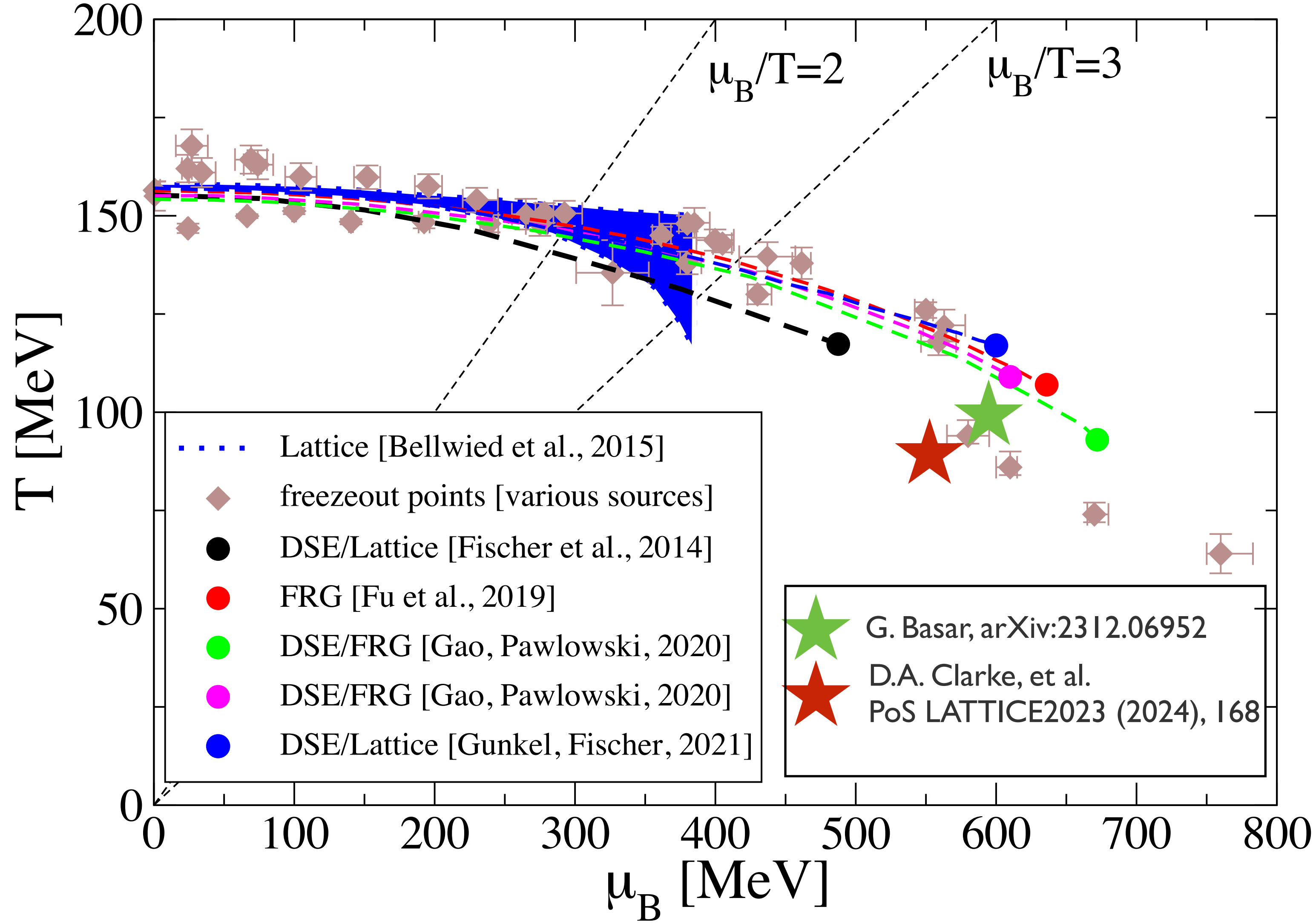

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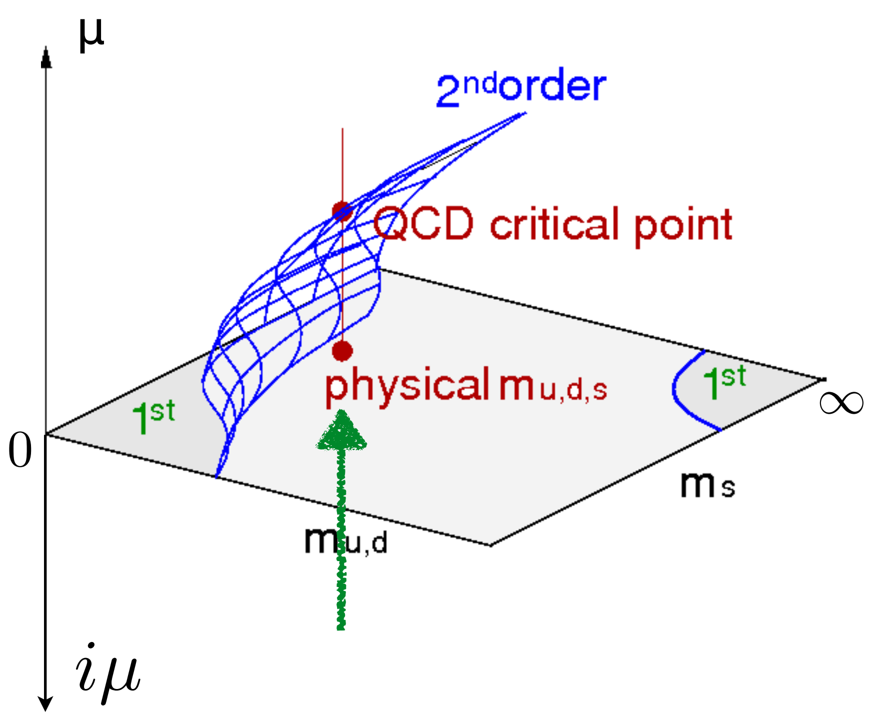
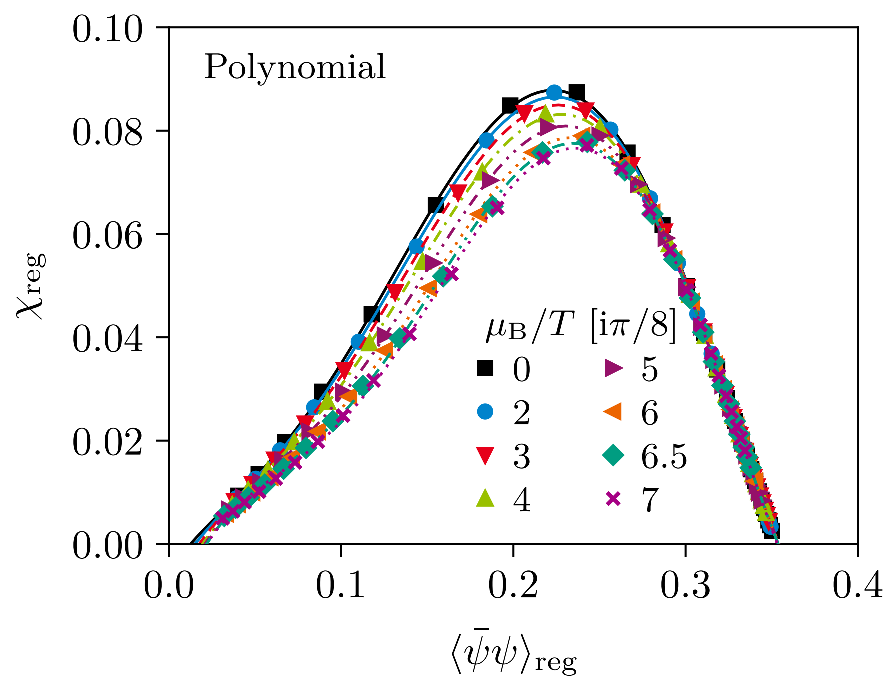
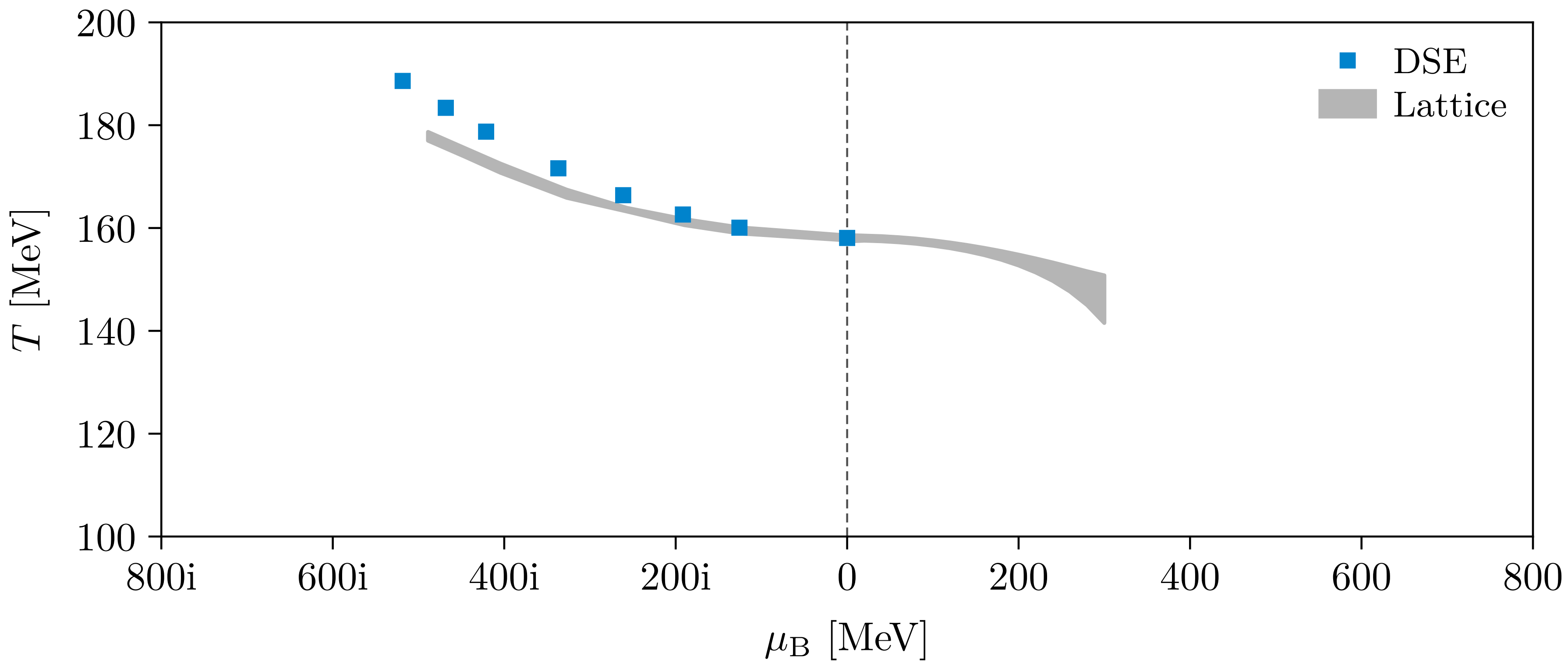
  - cross-check with lattice ✔

Extrapolation from imaginary chemical potential





$$\chi(T) = \frac{\partial\langle\bar{\psi}\psi\rangle(T)}{\partial m_{\mathrm{u}}}$$



Lattice: Borsanyi et al. PRL 125 052001 (2020)
DSE: Bernhardt, CF, EPJA 59 (2023) 8, 181

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2\left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_c}\right)^4$$

$$\kappa_2^{\mathrm{poly}} = 0.0196\,, \quad \kappa_4^{\mathrm{poly}} = 0.00015\,,$$

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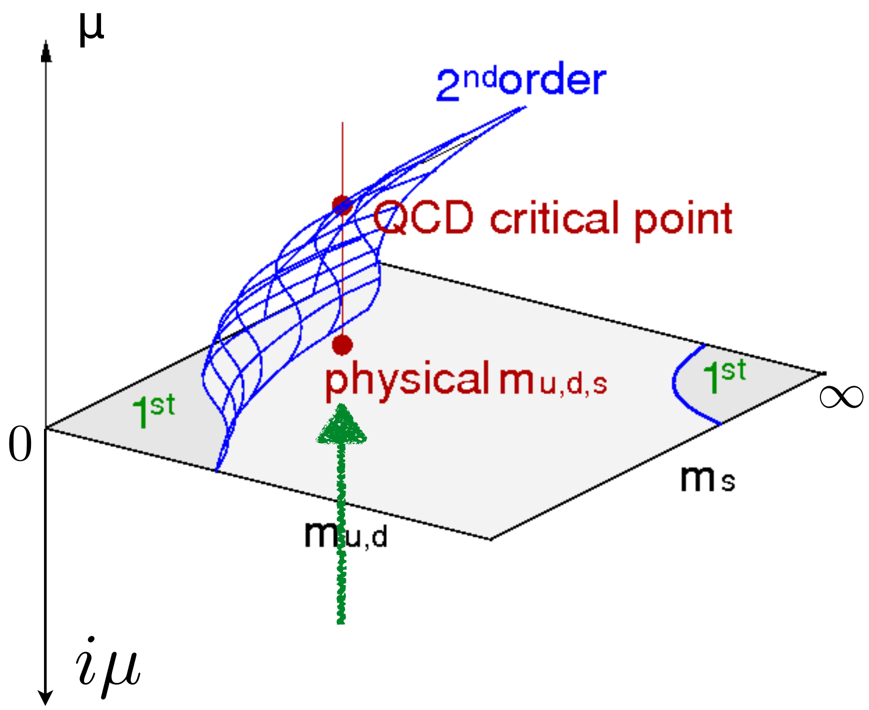

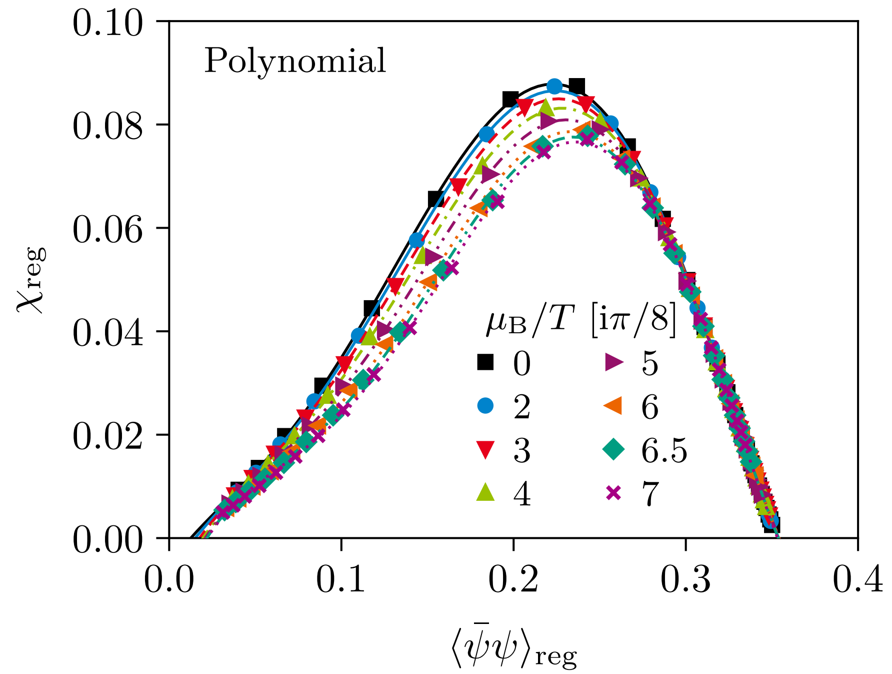

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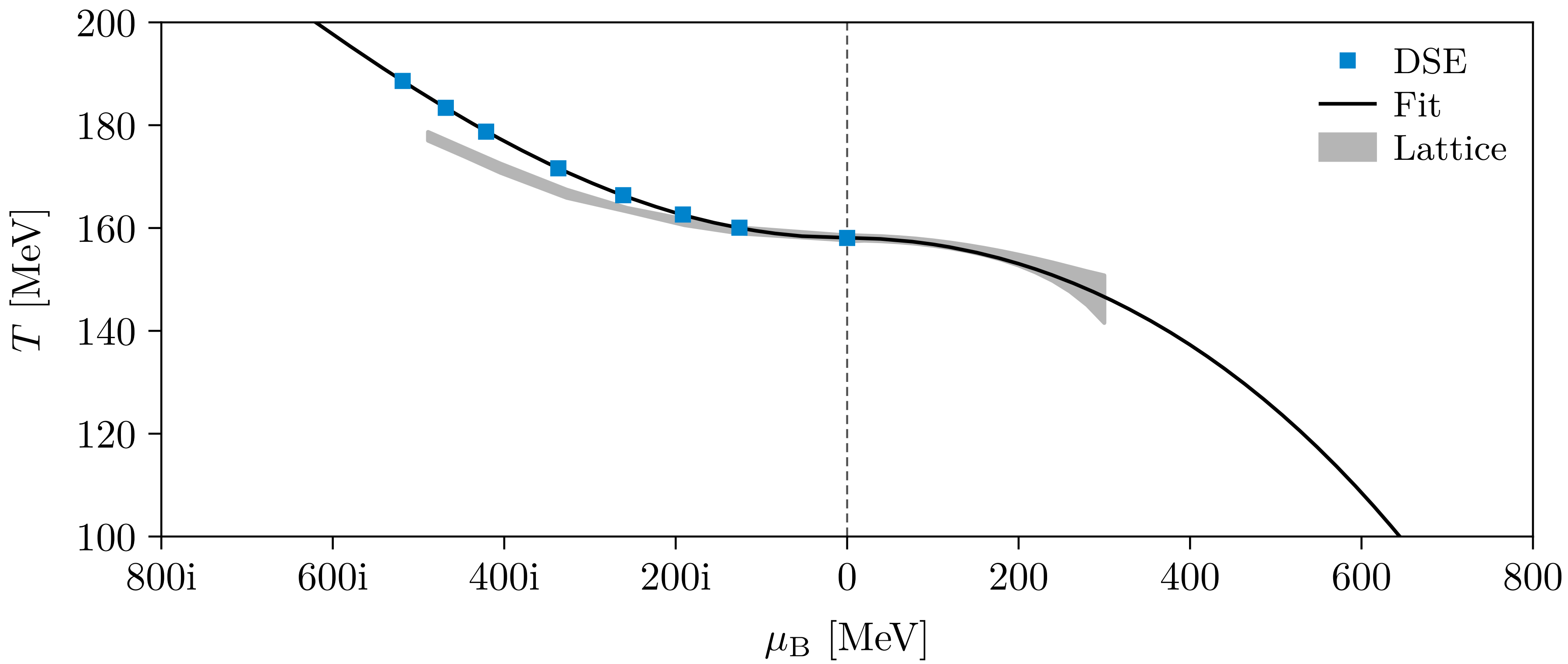

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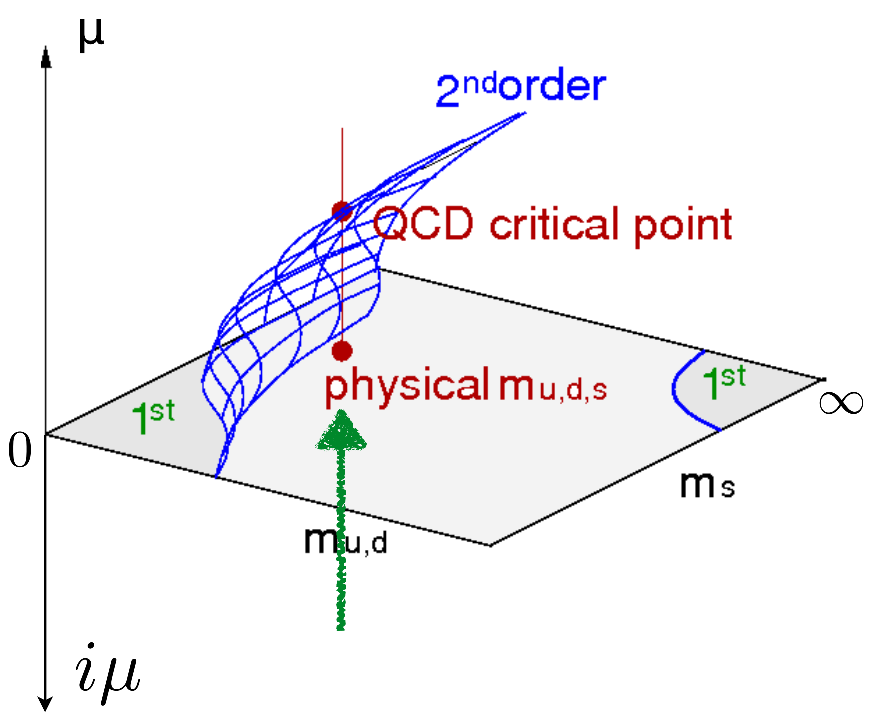

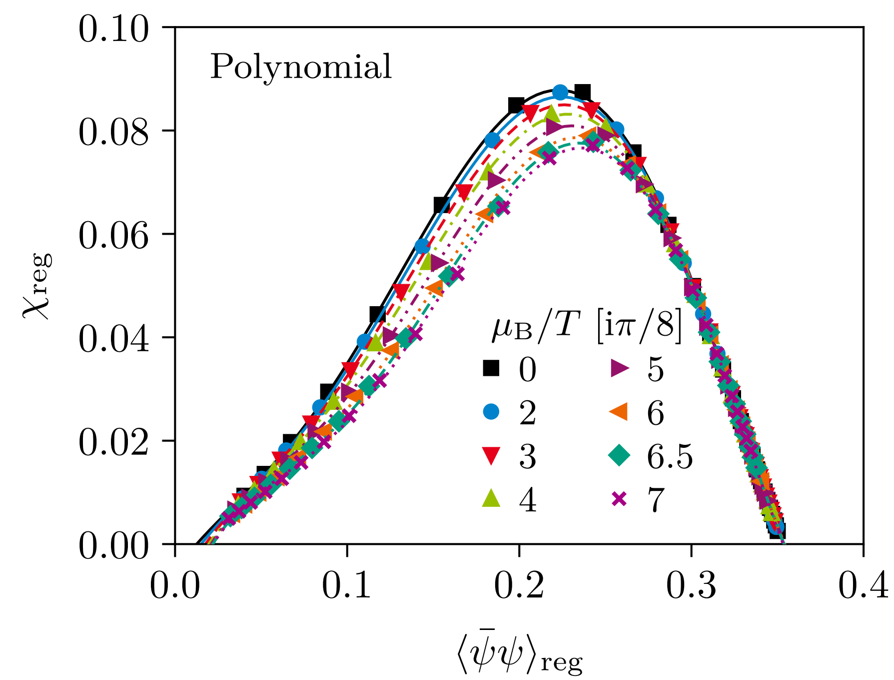

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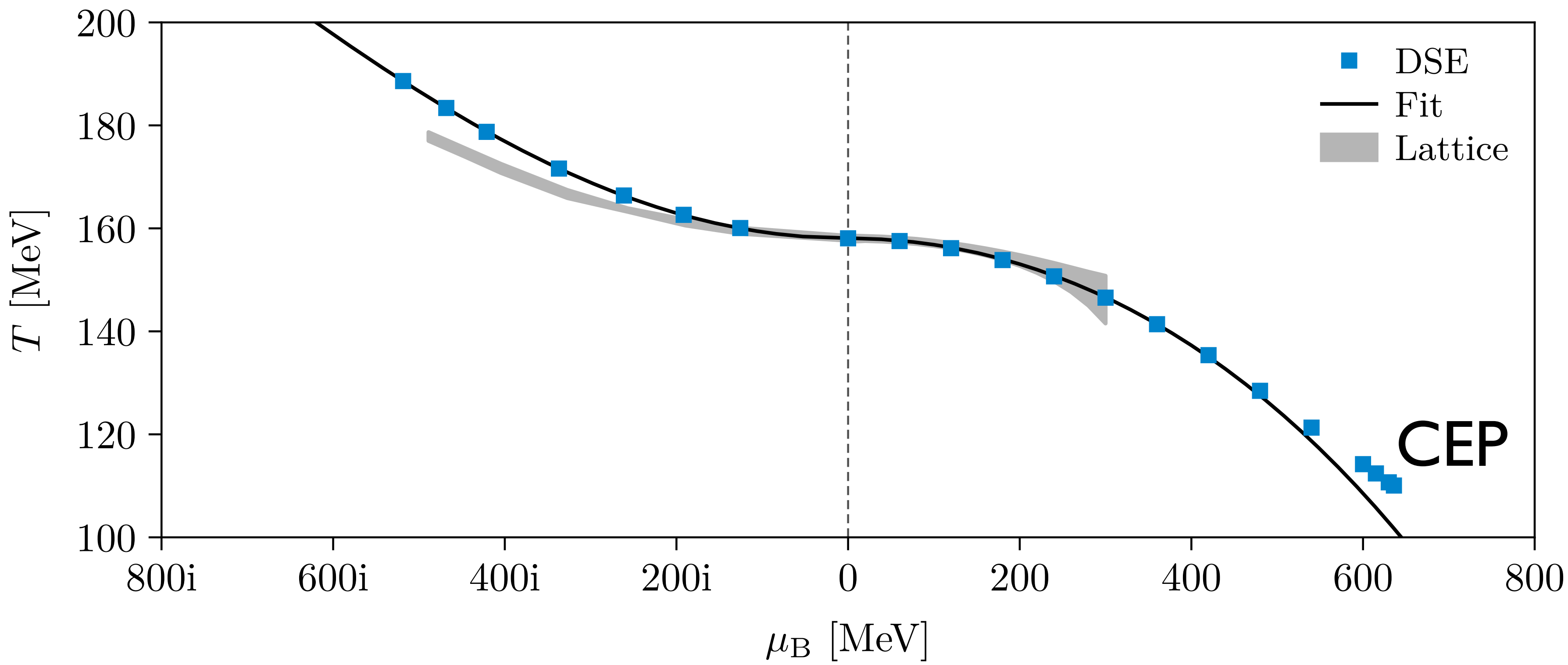

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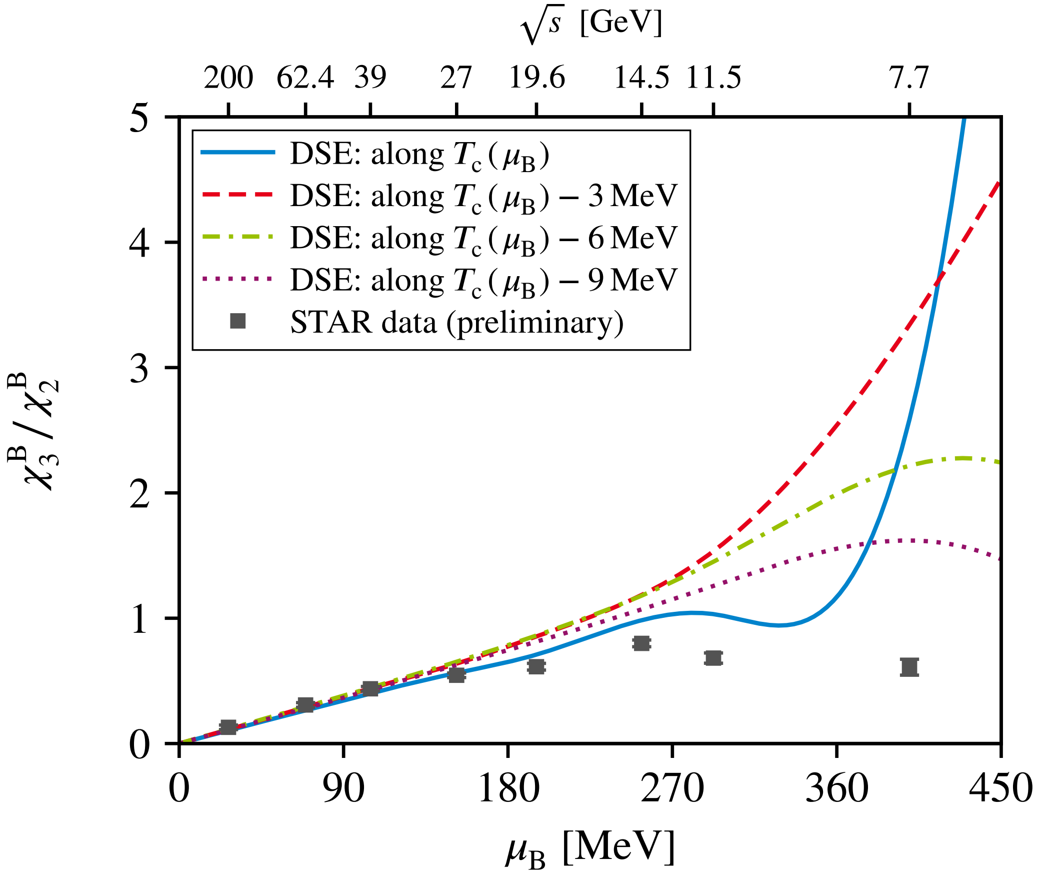
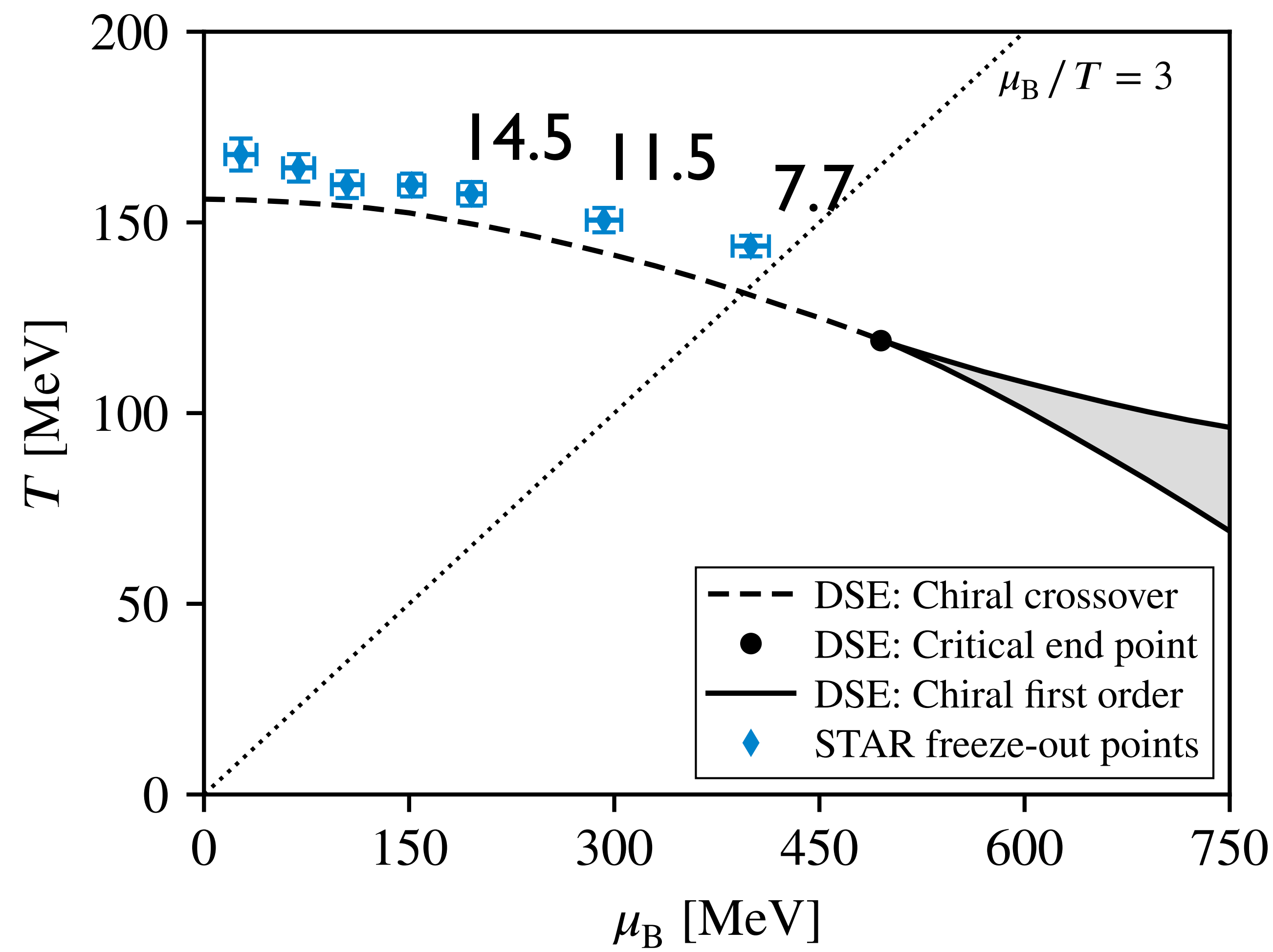
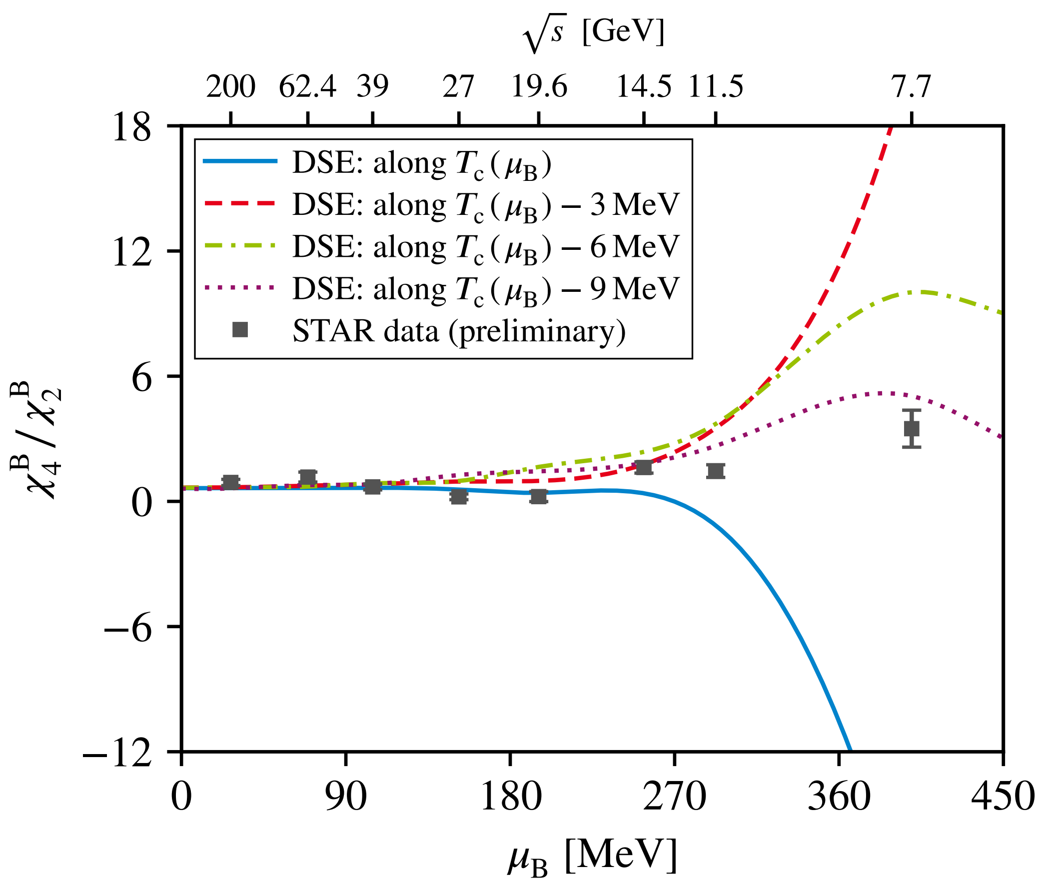
- Extrapolation works very well!

Contact with experiment: skewness and curtosis







$\sqrt{s} \geq 14.5$: good agreement

$\sqrt{s} = 11.5$: trend ok !

$\sqrt{s} \leq 7.7$: freezeout line $\neq$ transition line ?!

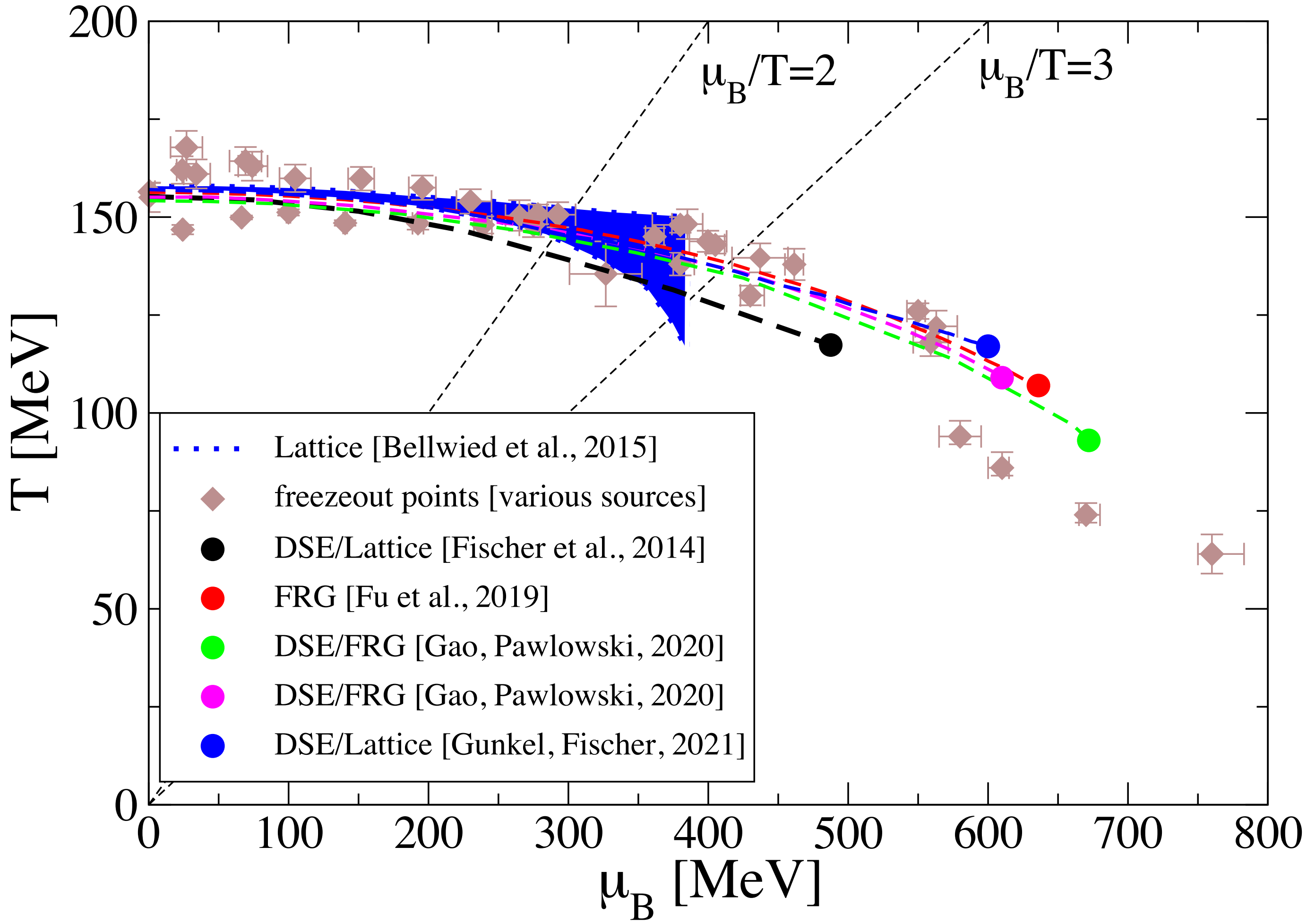
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

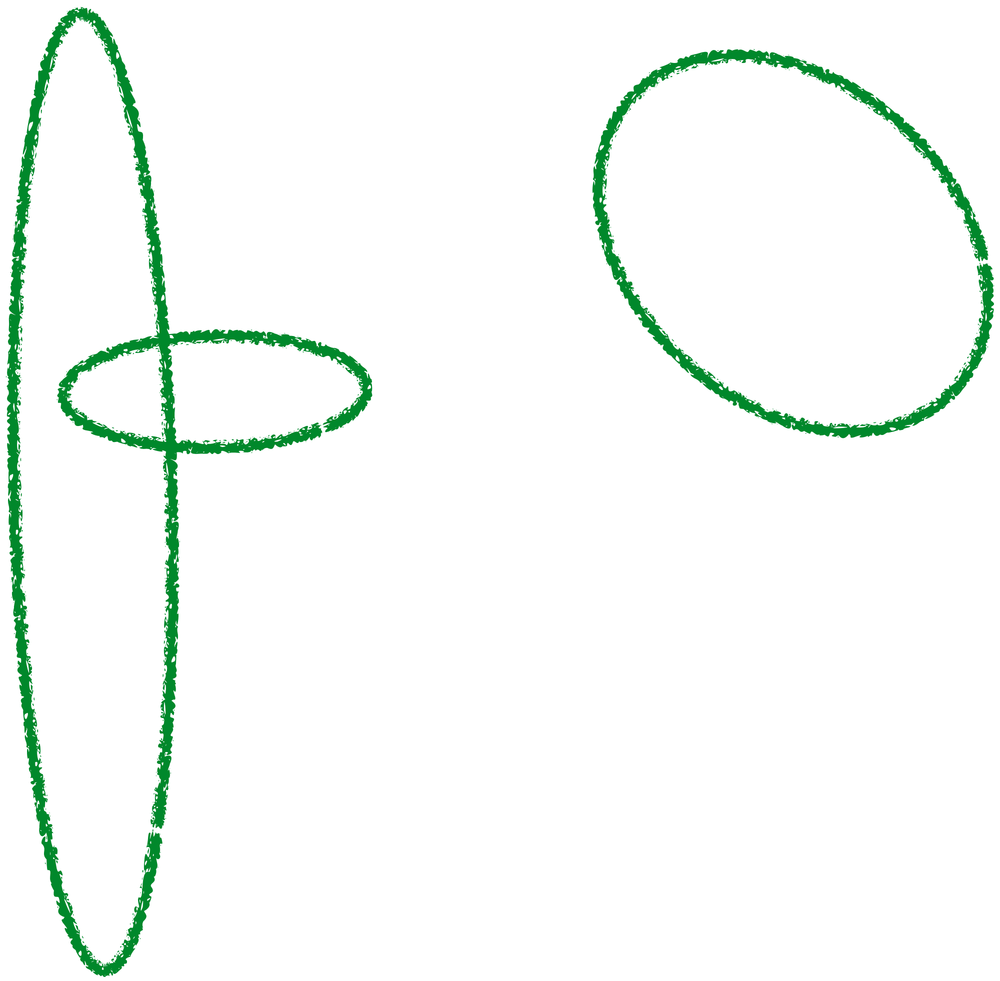
Summary: QCD with functional methods

Main goals:

- **one** framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- **same** framework for QCD phase diagram

Main results:





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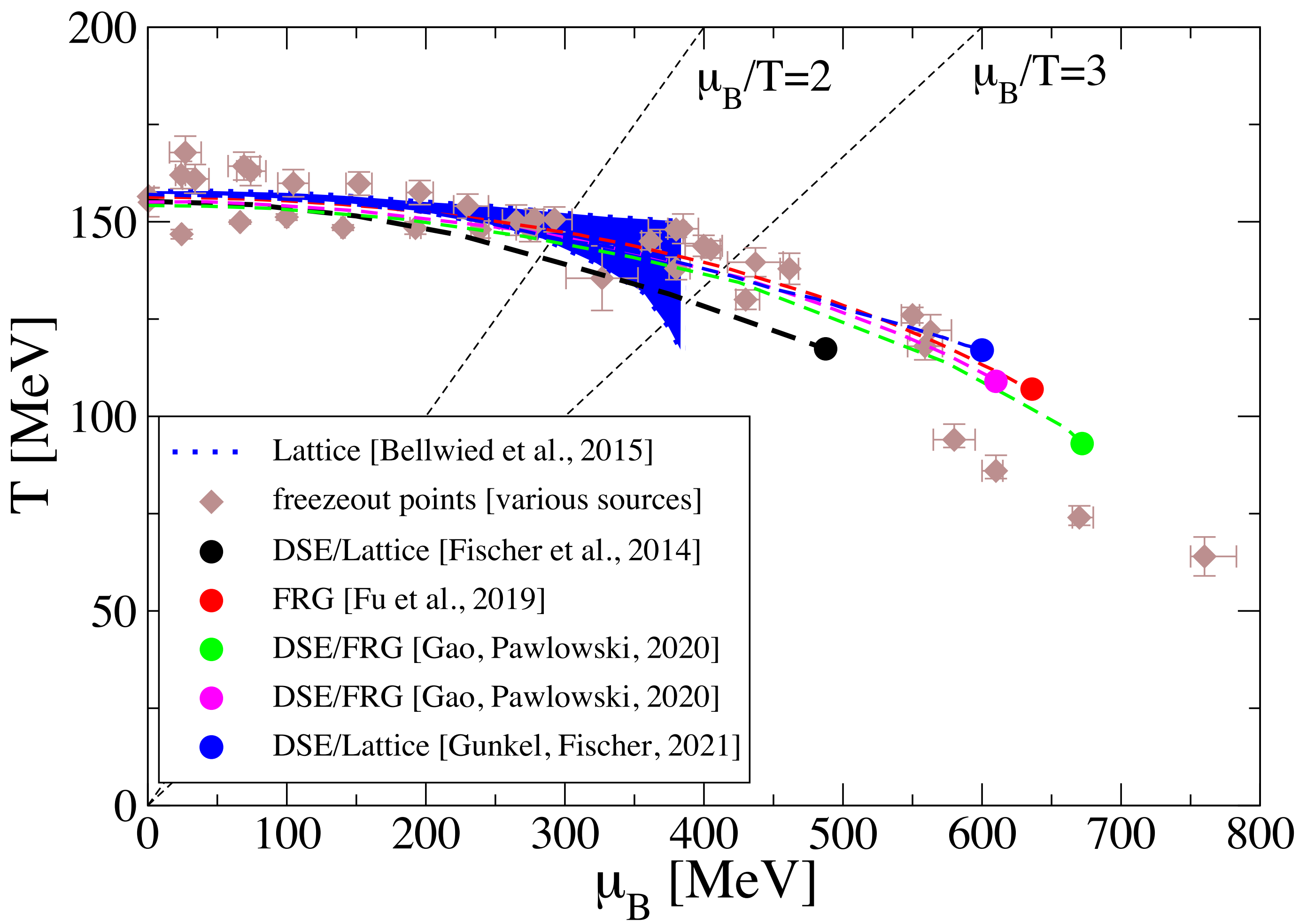
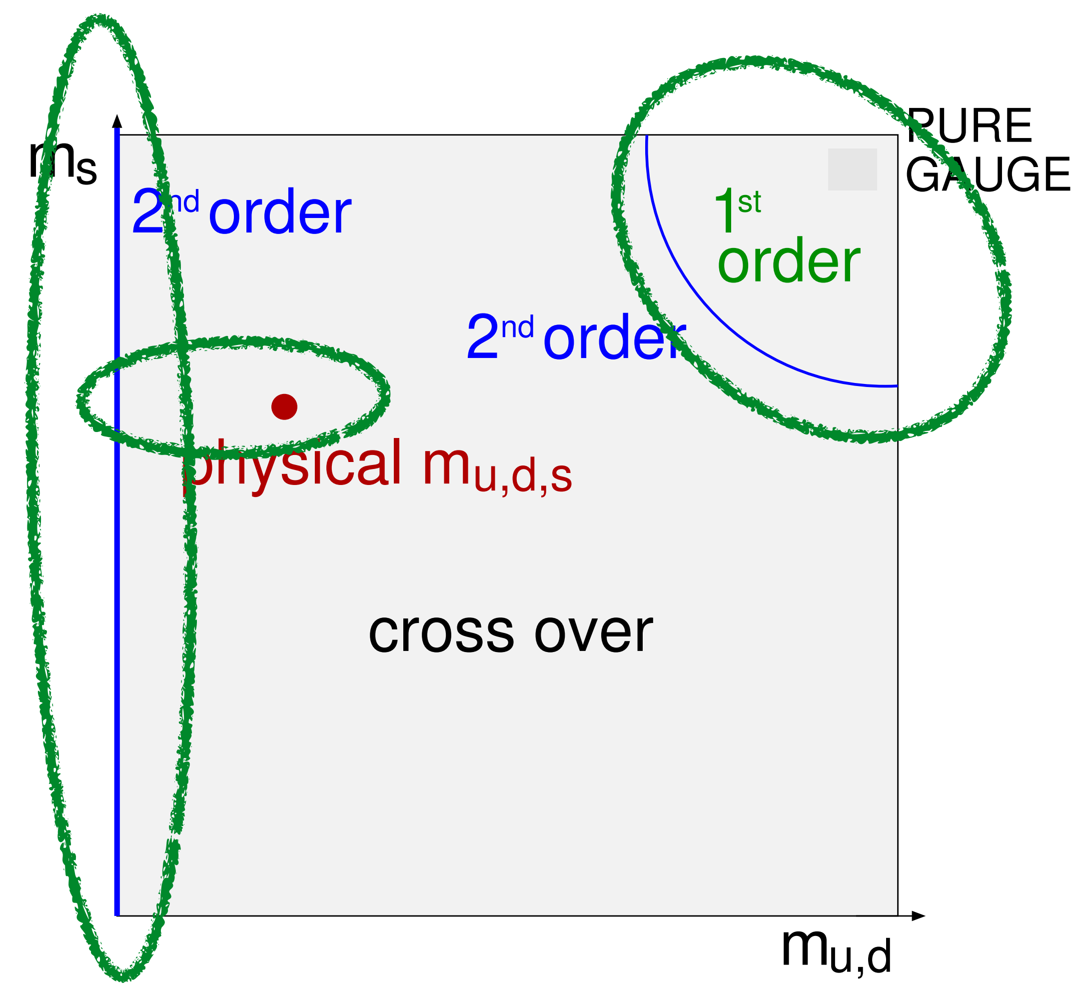




Backup

Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet available…
- Diagrammatics: vertex-DSE (see later…)

explicit solutions at T=0: Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

- Slavnov-Taylor identity: T,μ,m-dependent vertex

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$

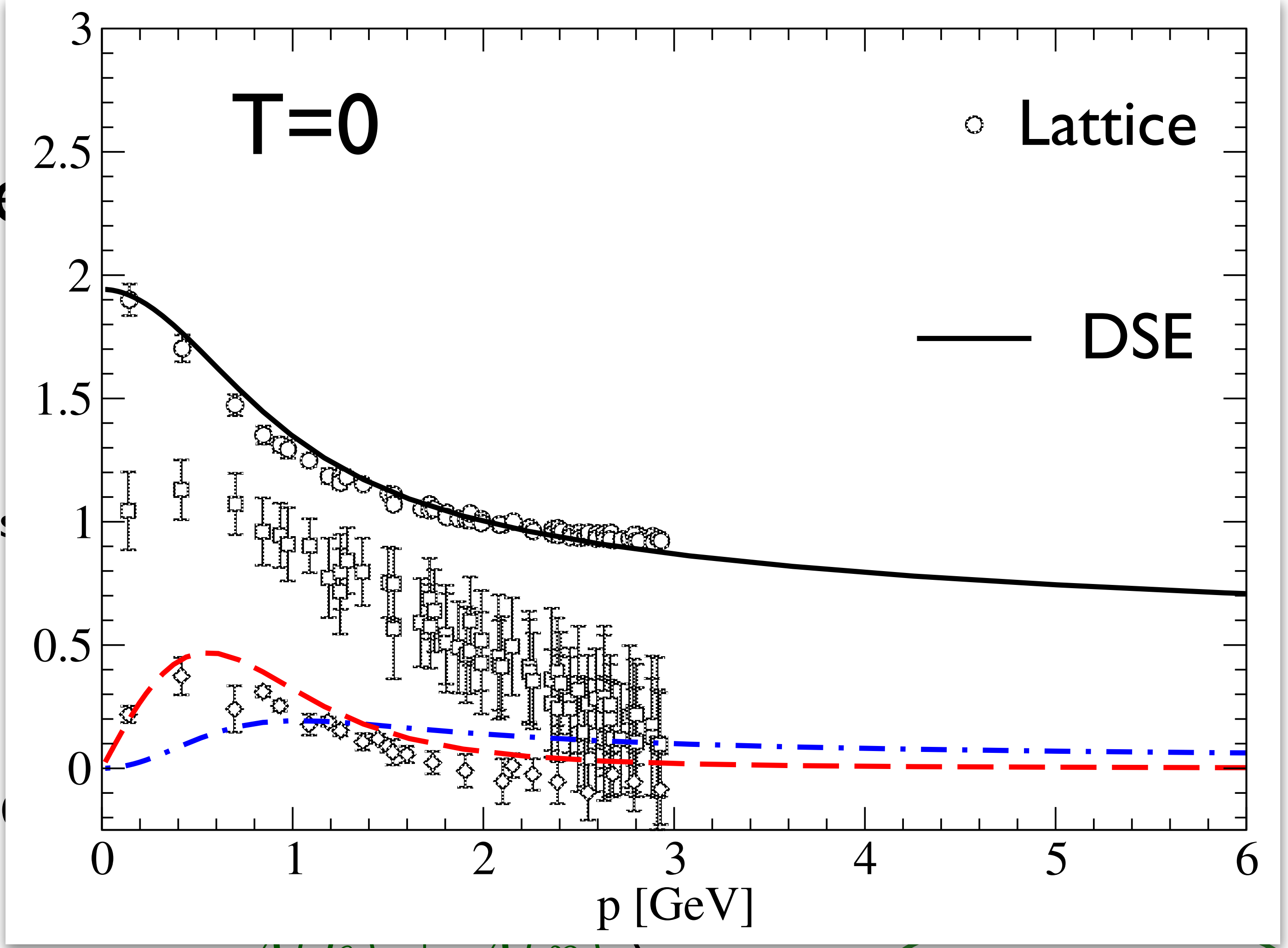
$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

STI

PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

Approximation for Quark-Gluon interaction



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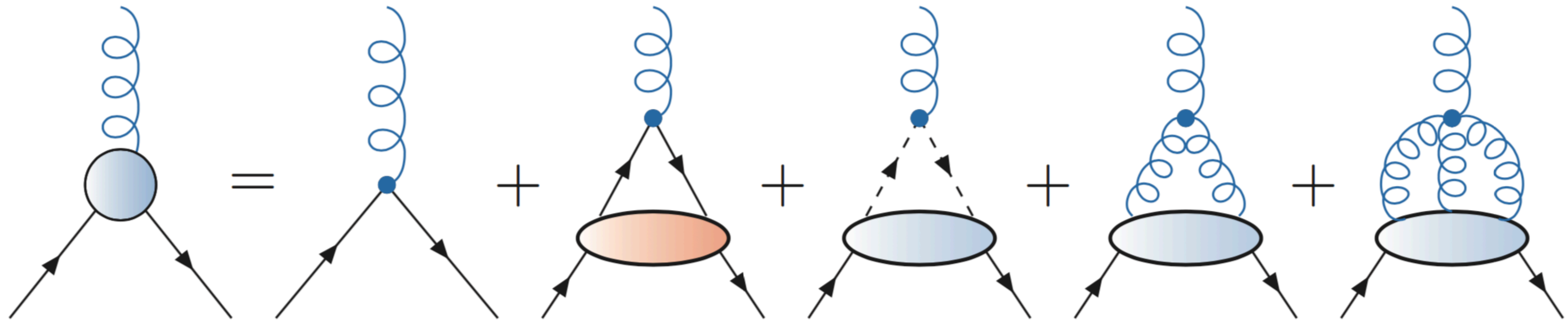
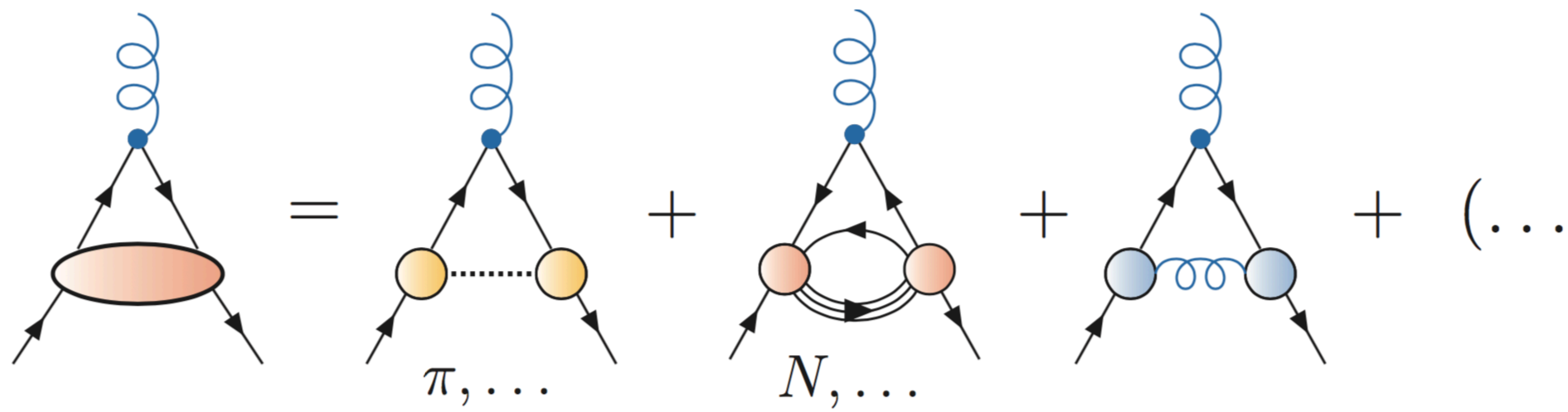
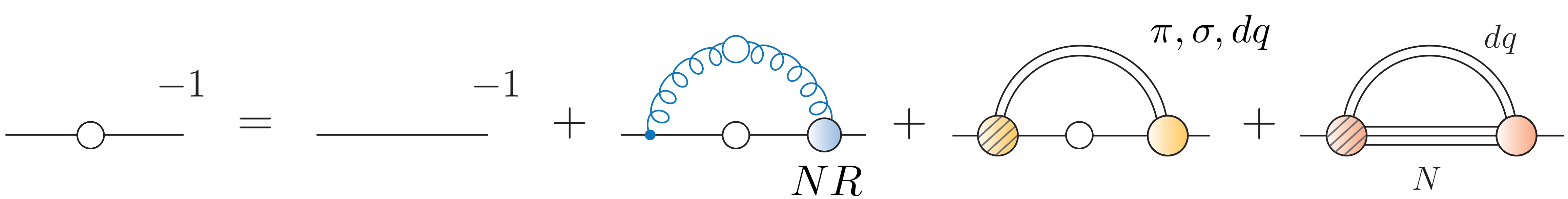
Hadron effects in quark-gluon interaction

quark-gluon vertex:





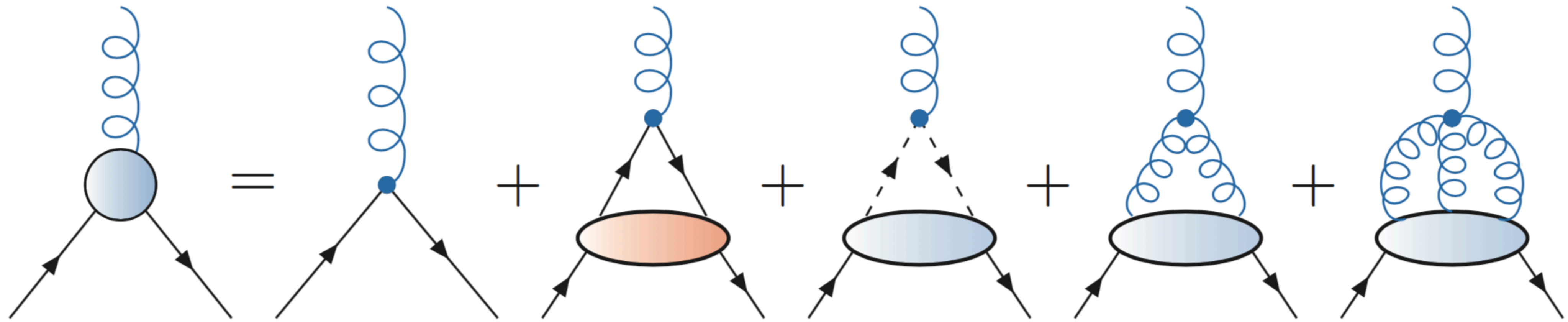
quark:

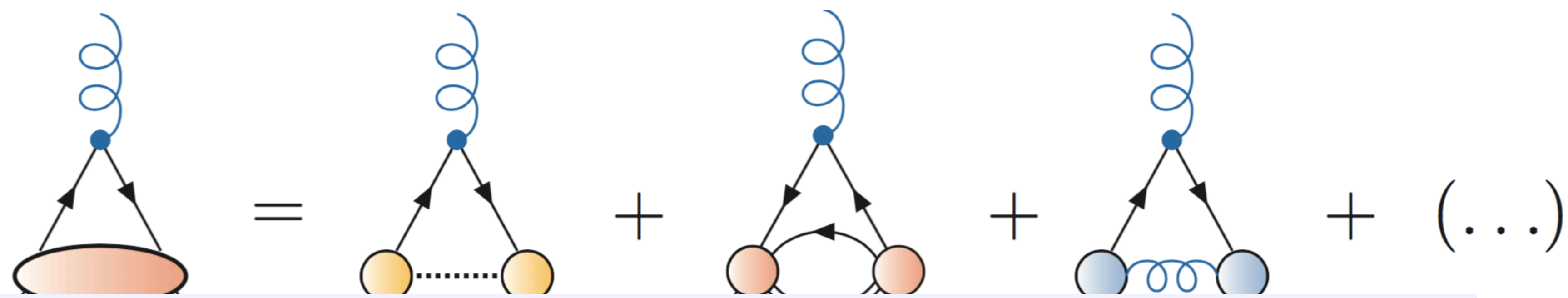


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

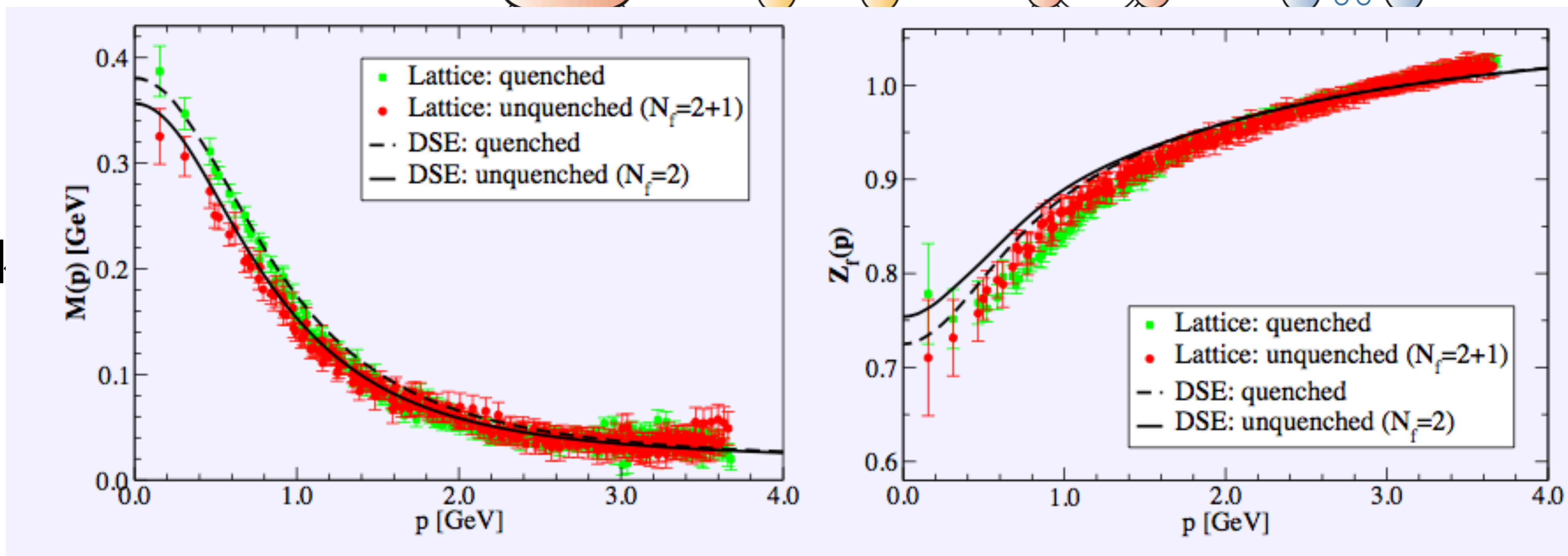
Hadron effects in quark-gluon interaction

quark-gluon vertex:





quar

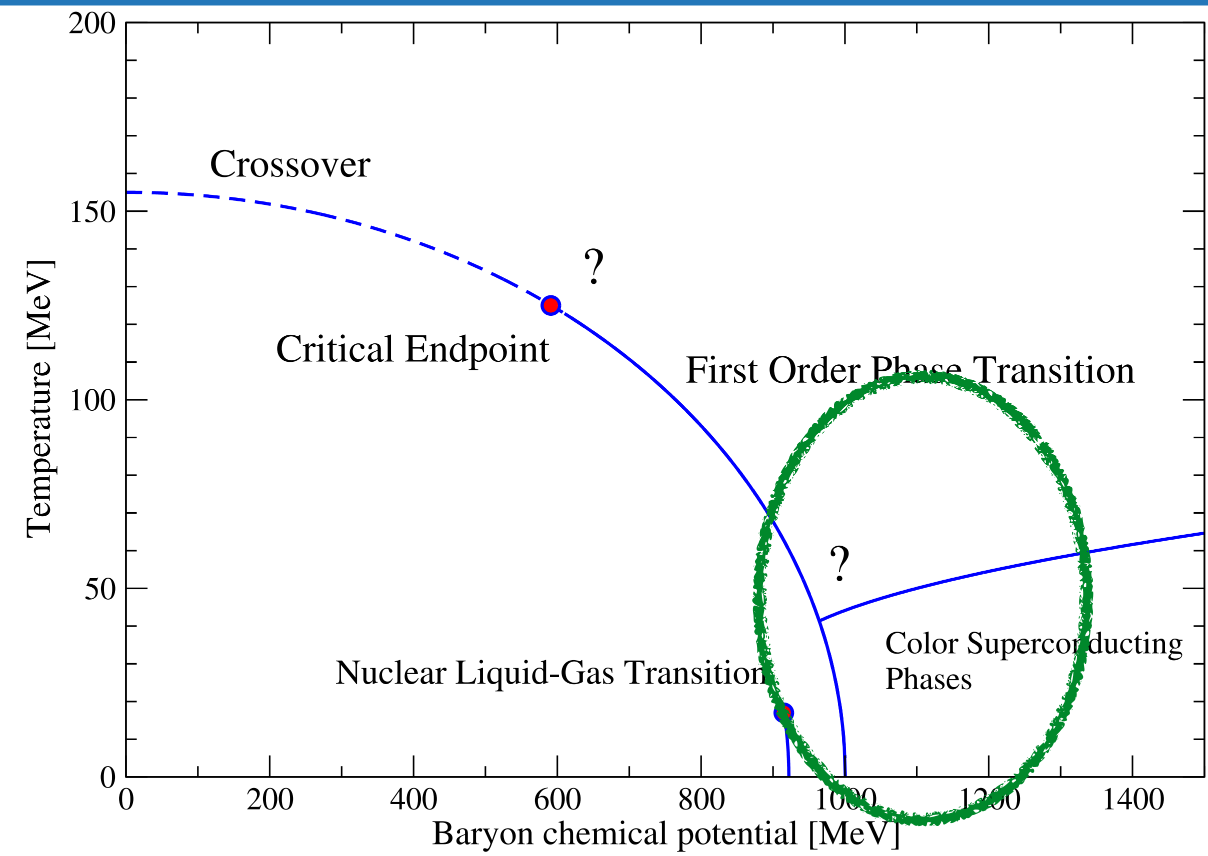


CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

2]

Equation of state from QCD

Gefördert durch DFG Deutsche Forschungsgemeinschaft

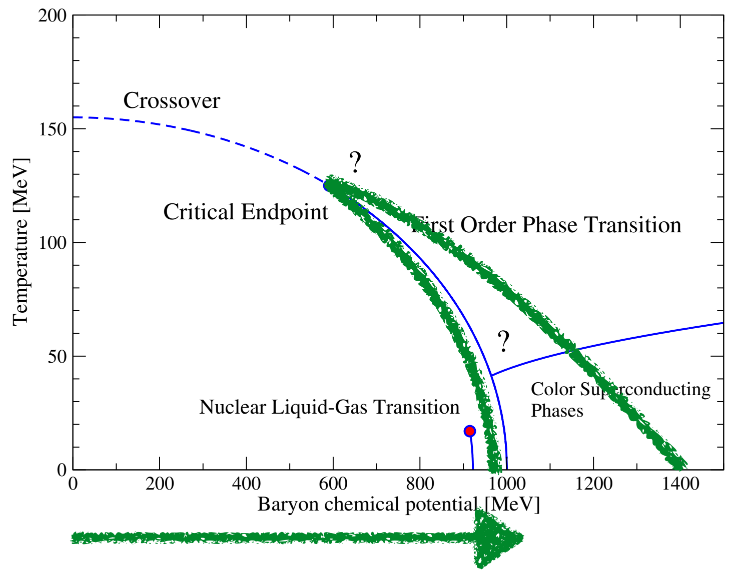


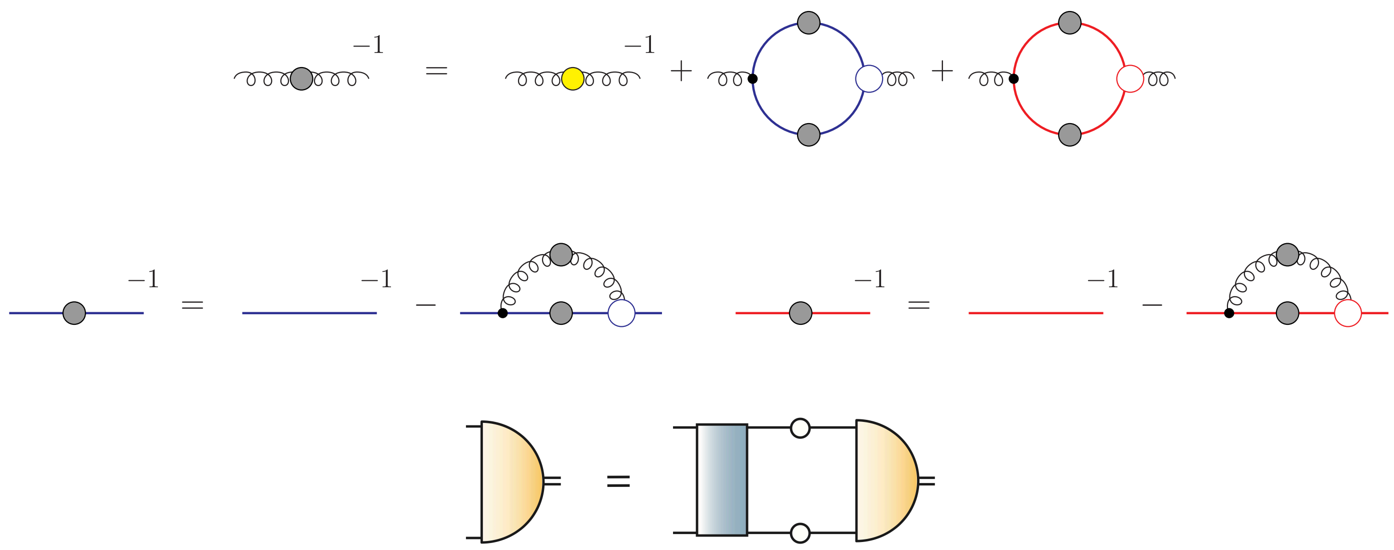
EoS from microscopic QCD (functional approach):

- chirally broken phase
  - quarks, mesons ✔ our work
  - baryons work in progress (DFG-ind.)
- superconducting phase(s) ✔ Buballa et al. Müller, Buballa, Wambach, arXiv:1603.02865
- inhomogenuous broken ('cristaline') phase(s) work in progress (CRC, A03) Motta, Bernhardt, Buballa, CF, arXiv:2306.09749

see talk of Theo Motta

Meson properties at finite chemical potential





- Quarks/meson wave functions do change !

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

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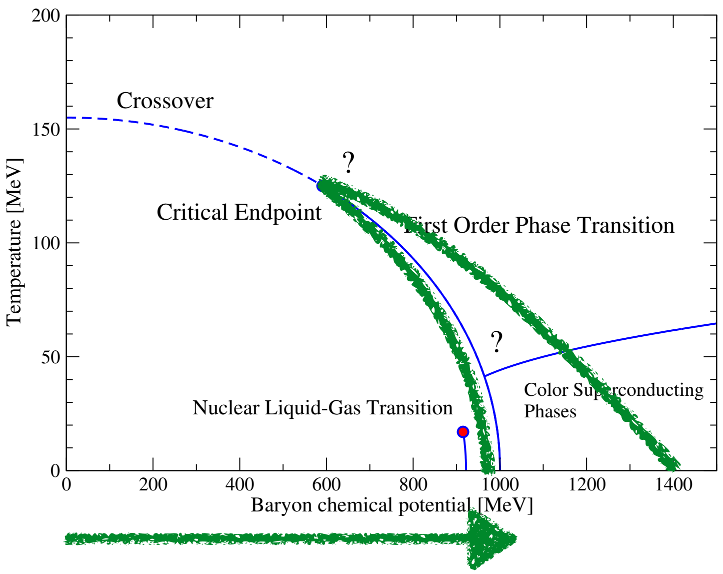

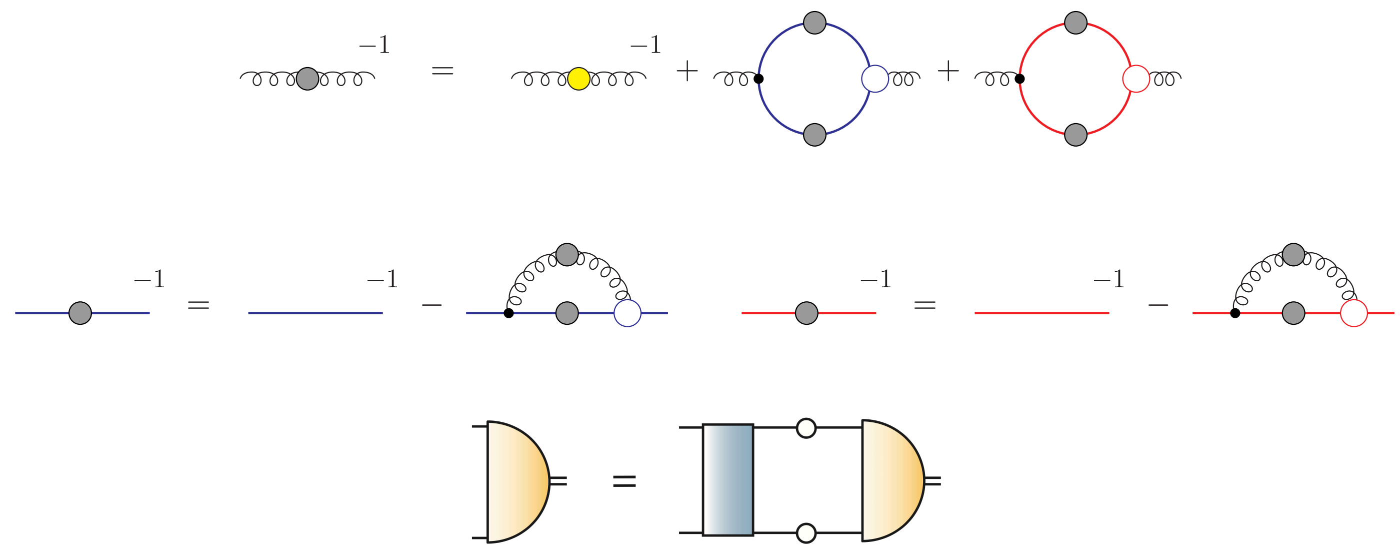

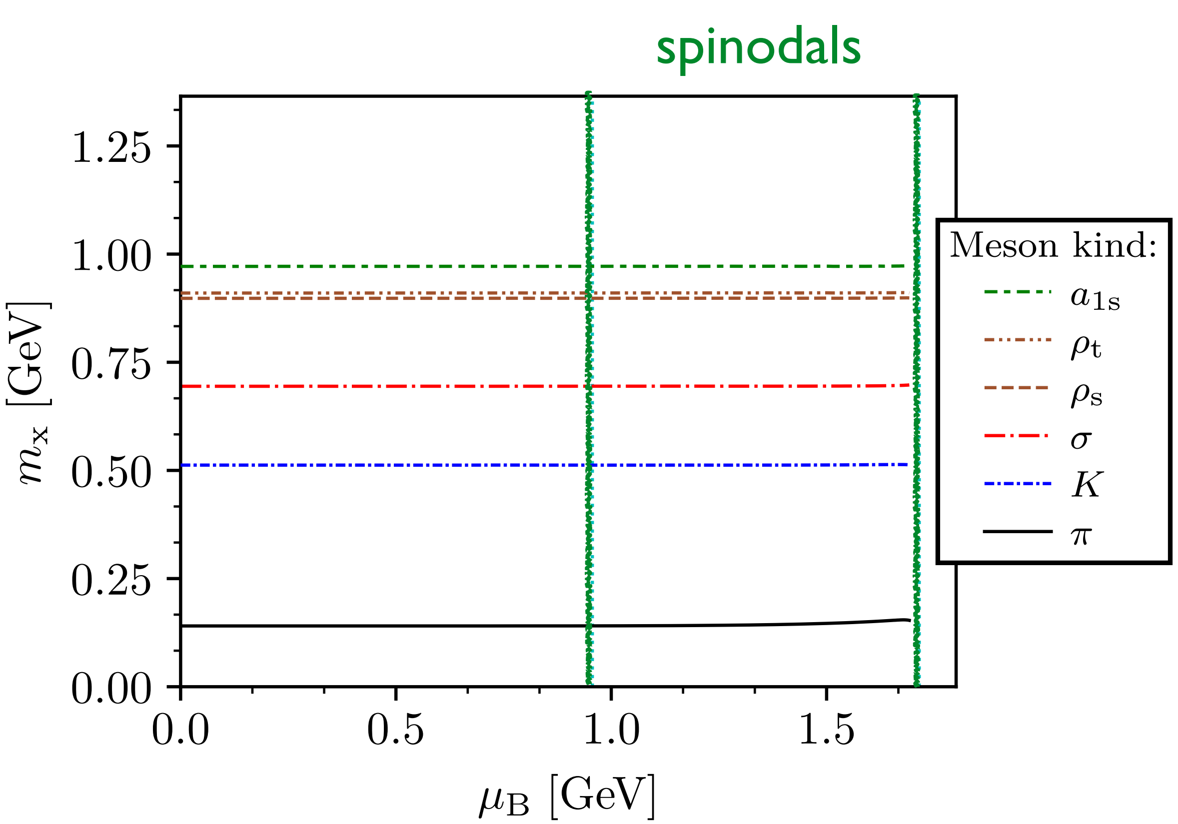

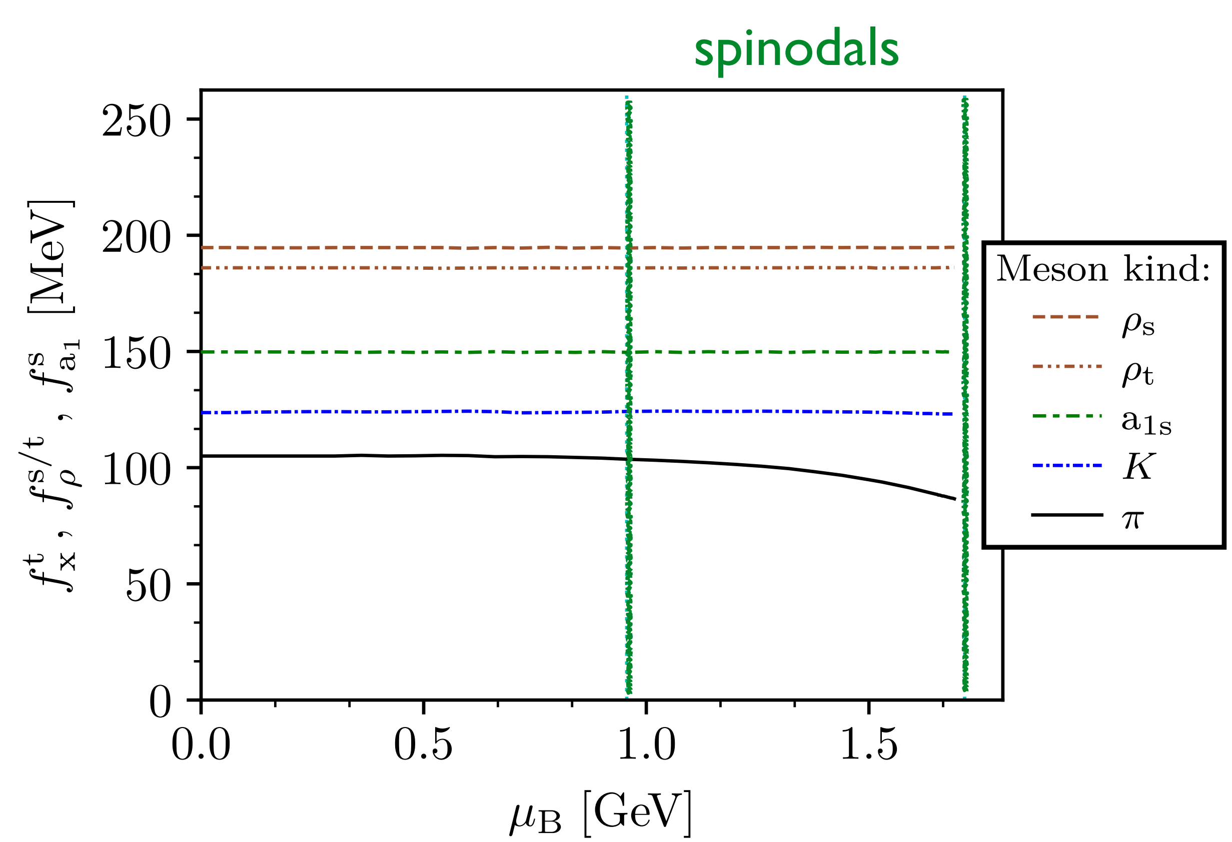

- Quarks/meson wave functions do change !
- But: Silver blaze satisfied

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147
T. D. Cohen, PRL 91 , 222001 (2003)