Running in the Hidden Valley

Simulating dark showers for near-conformal confining Hidden Valleys

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Modelling dark sector signatures

- Running coupling, α, governs the properties of the dark shower, including event shape and multiplicity.

Renormalisation Group Equation (RGE) at two-loop

$$\mu^2 \frac{d\alpha}{d\mu^2} = -\alpha^2 \left(\beta_0 + \beta_1 \alpha\right)$$

Integrate assuming $\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right) \gg -\frac{\beta_1}{\beta_0}$ → Approximate running coupling implemented in Pythia.

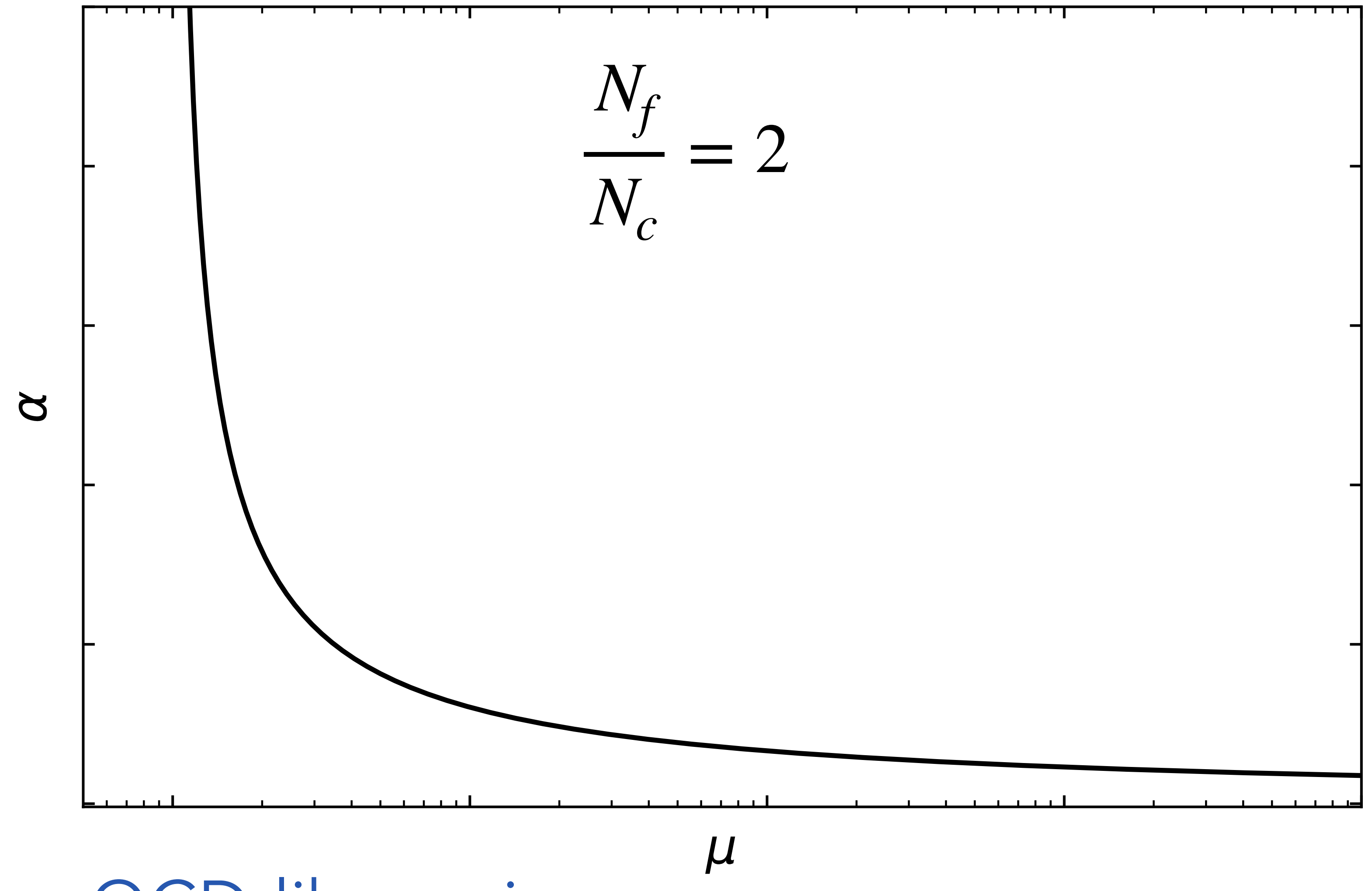
↕ Equivalent at one-loop order, but not higher orders.

Integrate to obtain implicit solution → "Exact" running coupling needed to understand infra-red properties.

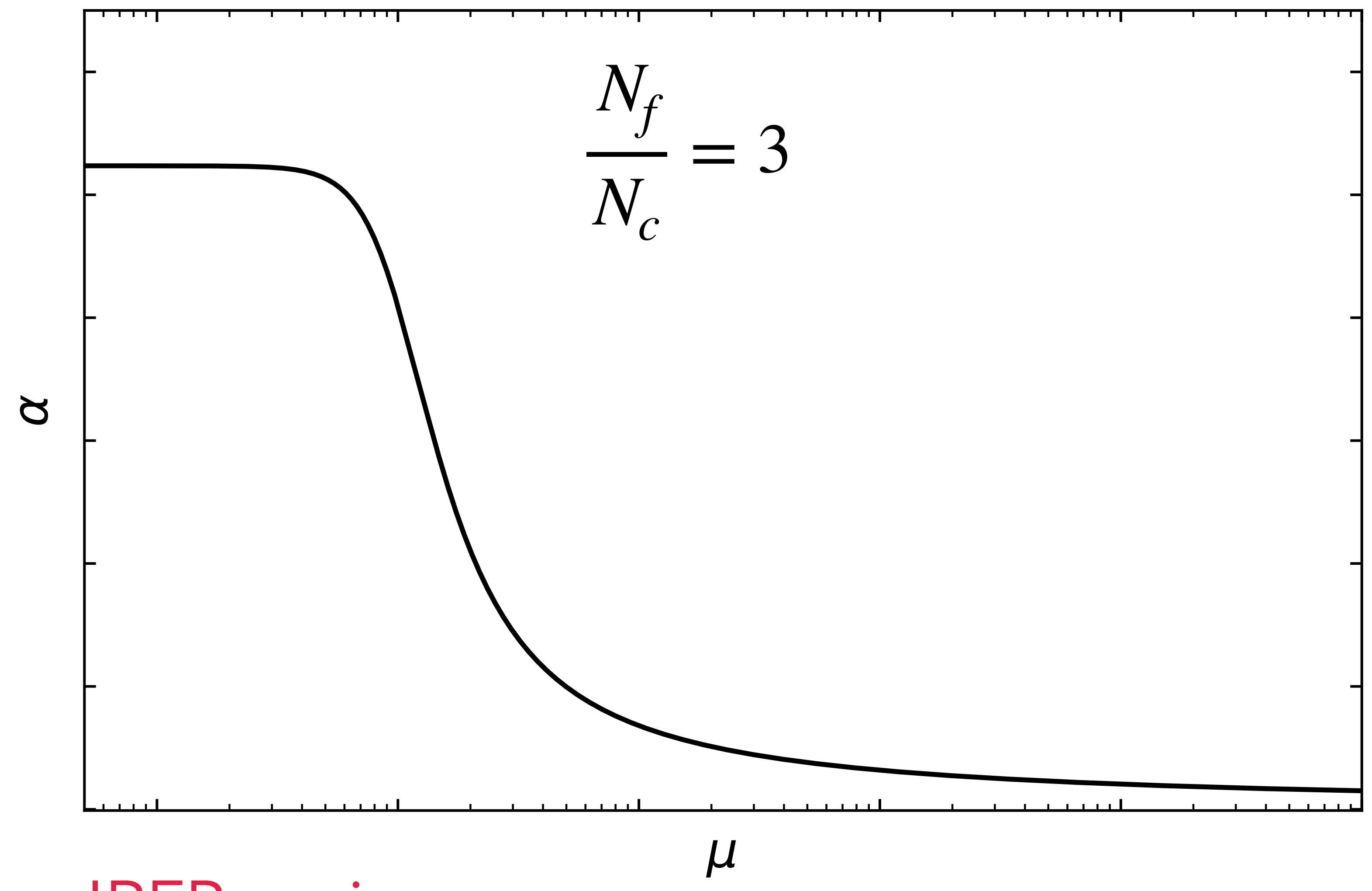
- Λ is defined to be the scale where the α diverges; below this scale the perturbative expansion is not reliable. This scale provides a sensible proxy for the scale of the QCD-like theory.

- We need a framework to define both α and Λ if we are to correctly model the signatures of dark showers within event generators.

Modelling dark sector signatures



QCD-like region



IRFP region

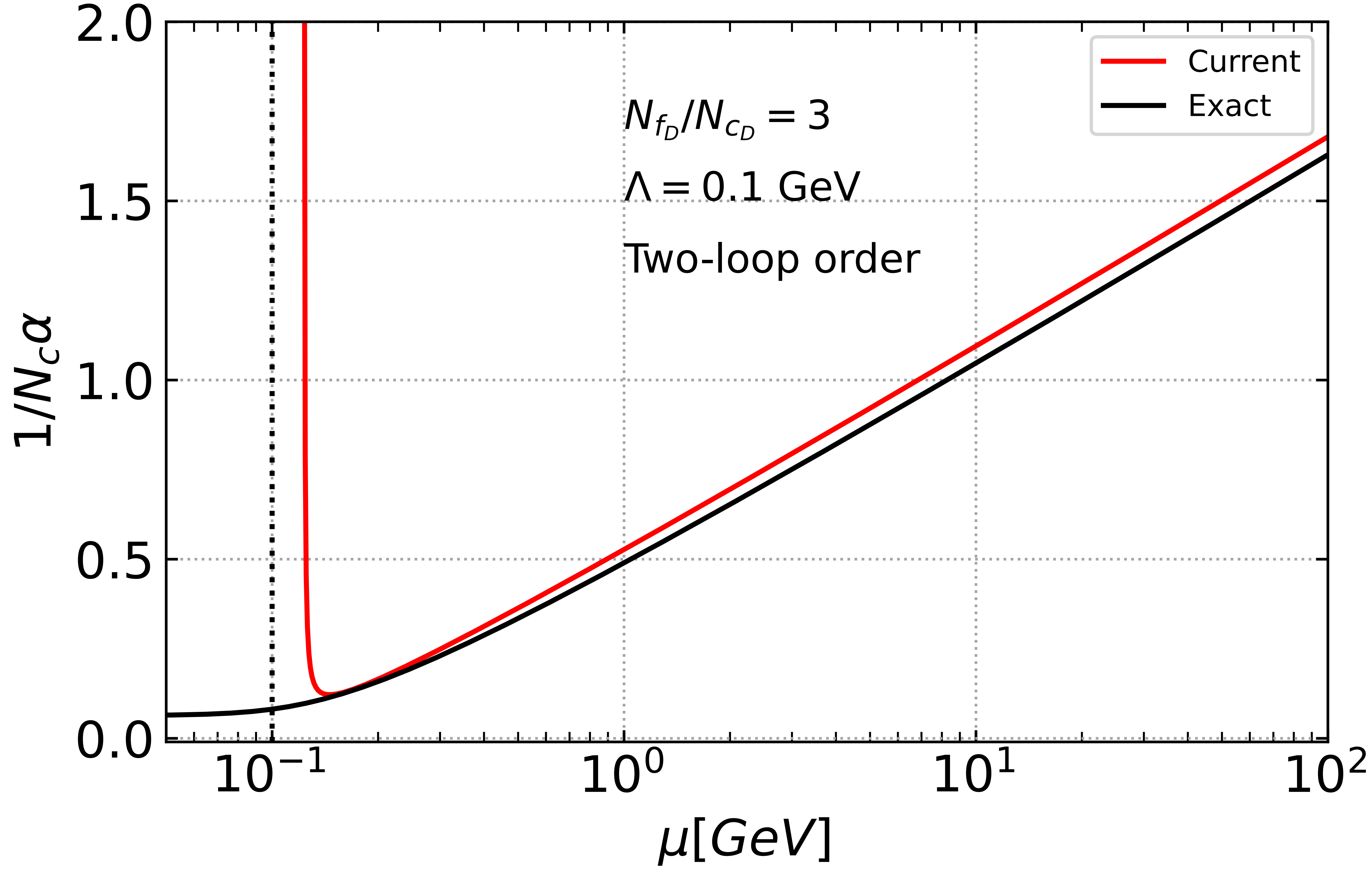
W.-M. Yao et al., Review of Particle Physics (2006) , arXiv:0607209

- To implement a form of two-loop running coupling within Pythia, one must make a series of approximations of the exact running coupling. The current approximation is known to work in the QCD-like region.

- At two-loop order, the exact running coupling α flows to an infra-red fixed point (IRFP), for $\frac{N_f}{N_c} \gtrsim 2.7$. Are the current approximations within Pythia still valid due of this qualitative change of behaviour?

Running coupling - current procedure

- The approximation breaks down due to its inability to handle the presence of IRFPs, manifesting in the unphysical turning of the running coupling.



W.-M. Yao et al., Review of Particle Physics (2006) , arXiv:0607209

arxiv:9602385,

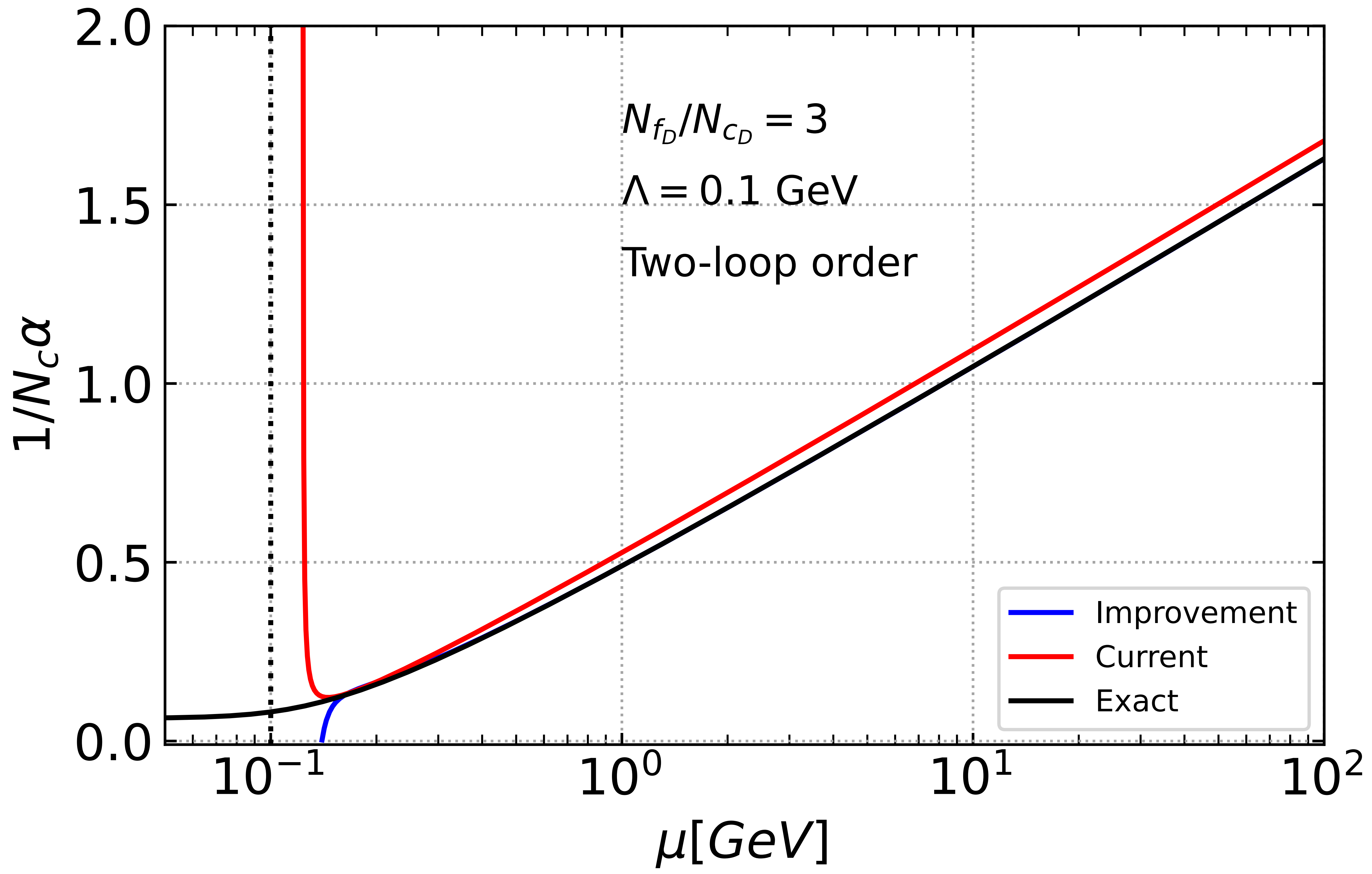
arxiv:9806409 - T. Appelquist et al.

arxiv:9810192 - E. Gardi et al.

- Exact running coupling in the IRFP region does not diverge in the IR but instead takes on a power-law form.

An improved procedure for Pythia

- In the IRFP region, Λ can be defined as the transition to power-law behaviour. For our purposes this gives a sensible framework by which to approximate IRFP region running coupling within Pythia.



- Provides an excellent match to the exact RGE solution across a wide range of energies and $\frac{N_f}{N_c}$ values.

Conclusion

- Current implementation of running coupling within Pythia is insufficient to describe the expected behaviour at high number of flavours due to the presence of IRFPs. Our recommendation would be not to use Pythia to simulate any confining Hidden Valley with high number of flavours.

- We have made significant progress in finding a suitable framework to describe running coupling within this region across a wide range of energies, flavours and colours.

- Endeavour to fill in the gaps of $\frac{N_f}{N_c}$ and $\frac{\mu}{\Lambda}$ space where our current approximations can not reach. A unified framework within Pythia for this is in the works!

Thank you! Any questions?

Back-up

Beta functions

- Beta coefficients for arbitrary gauge group and representation.

$$4\pi\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F N_f$$

$$(4\pi)^2\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F N_f - \frac{20}{3}C_A T_F N_f$$

arxiv:9701390

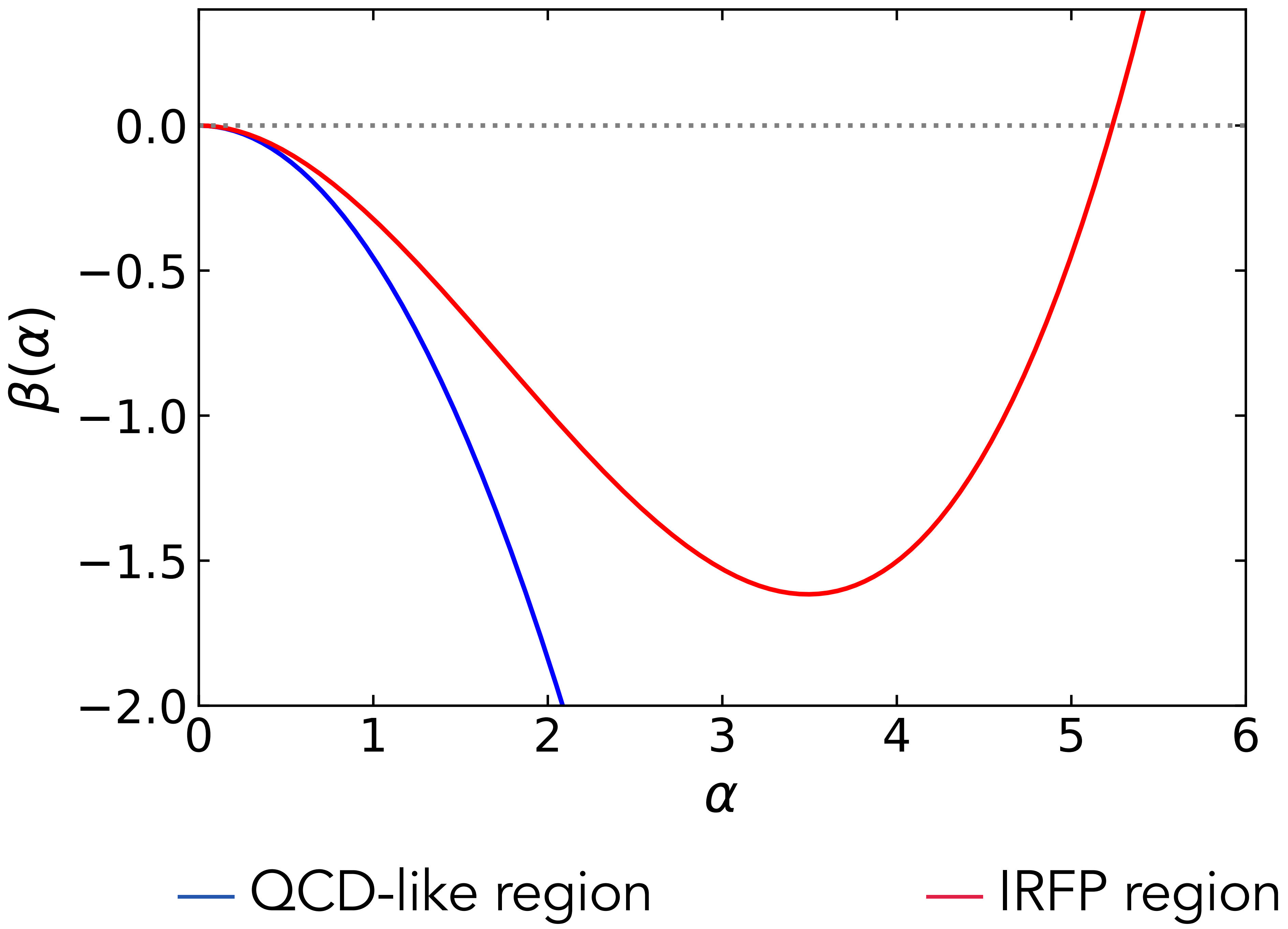
$$(4\pi)^3\beta_2 = \frac{2857}{54}C_A^3 + 2C_F^2 T_F N_f - \frac{205}{9}C_F C_A T_F N_f - \frac{1415}{27}C_A^2 T_F N_f + \frac{44}{9}C_F T_F^2 N_f^2 + \frac{158}{27}C_A T_F^2 N_f^2$$

- $SU(N_c)$ with N_f fundamental fermions.

$$4\pi\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$(4\pi)^2\beta_1 = \frac{34}{3}N_c^2 - 2\frac{N_c^2 - 1}{2N_c}N_f - \frac{10}{3}N_c N_f$$

What can we see perturbatively?



- Two-loop running coupling flows to a perturbative IR fixed-point (IRFP) when $\alpha_* > 0$, the Banks-Zaks fixed point. This is the first non-trivial zero of $\beta(\alpha)$.

$$\beta(\alpha) = \mu^2 \frac{d\alpha}{d\mu^2} = -\alpha^2 \left(\beta_0 + \beta_1 \alpha\right)$$

- Appearance of two-loop IRFPs at x_f^{FP} provides an approximation of the true IRFPs appearance at x_f^c. Two-loop running coupling with IRFPs provides a perturbative approximation of behaviour near and around the conformal window.

Veneziano limit, $\mu = 0$, $T = 0$

arXiv:2008.12223 - J.W. Lee

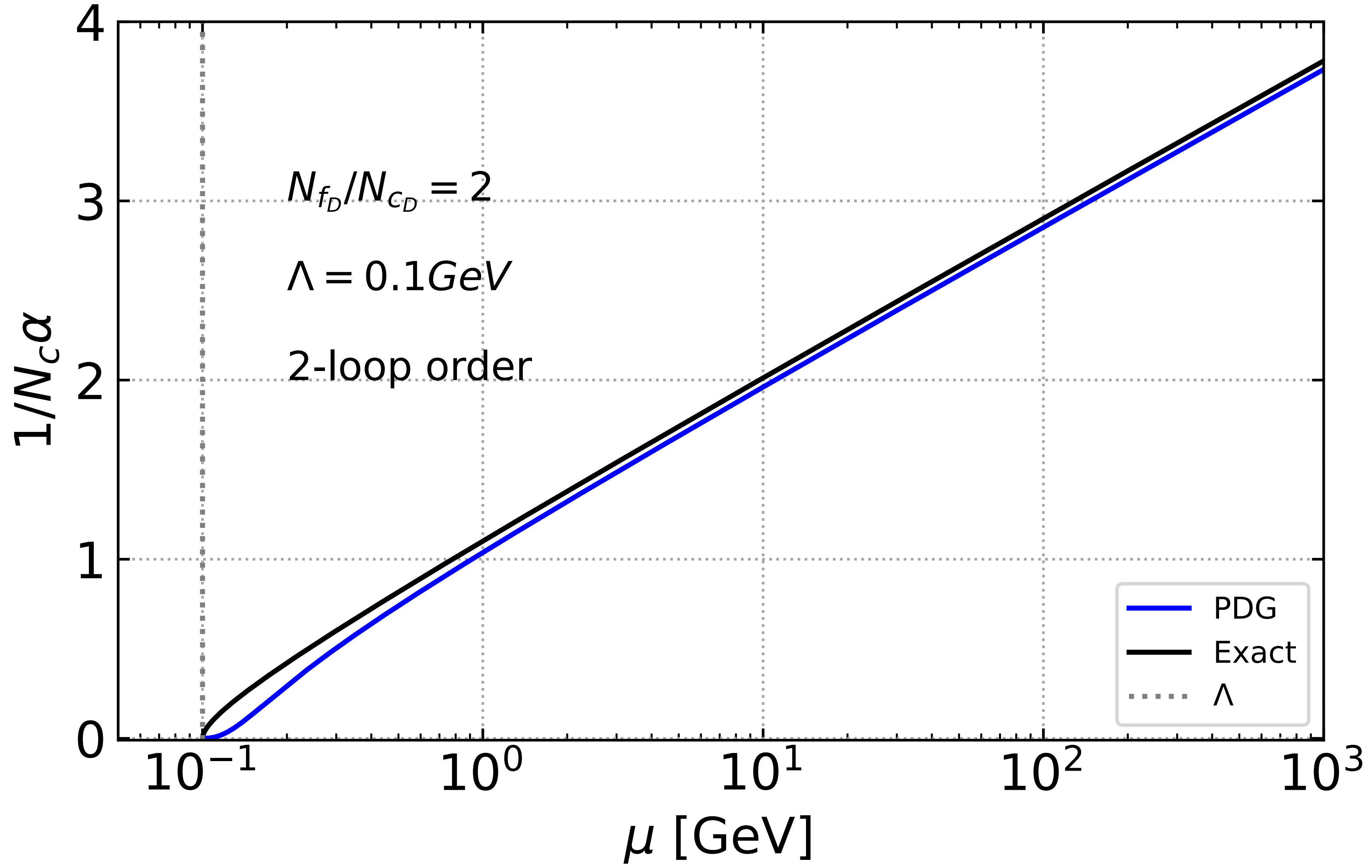
Current Pythia implementation at low Nf/Nc

- The current implementation of running coupling is given by the following asymptotic form:

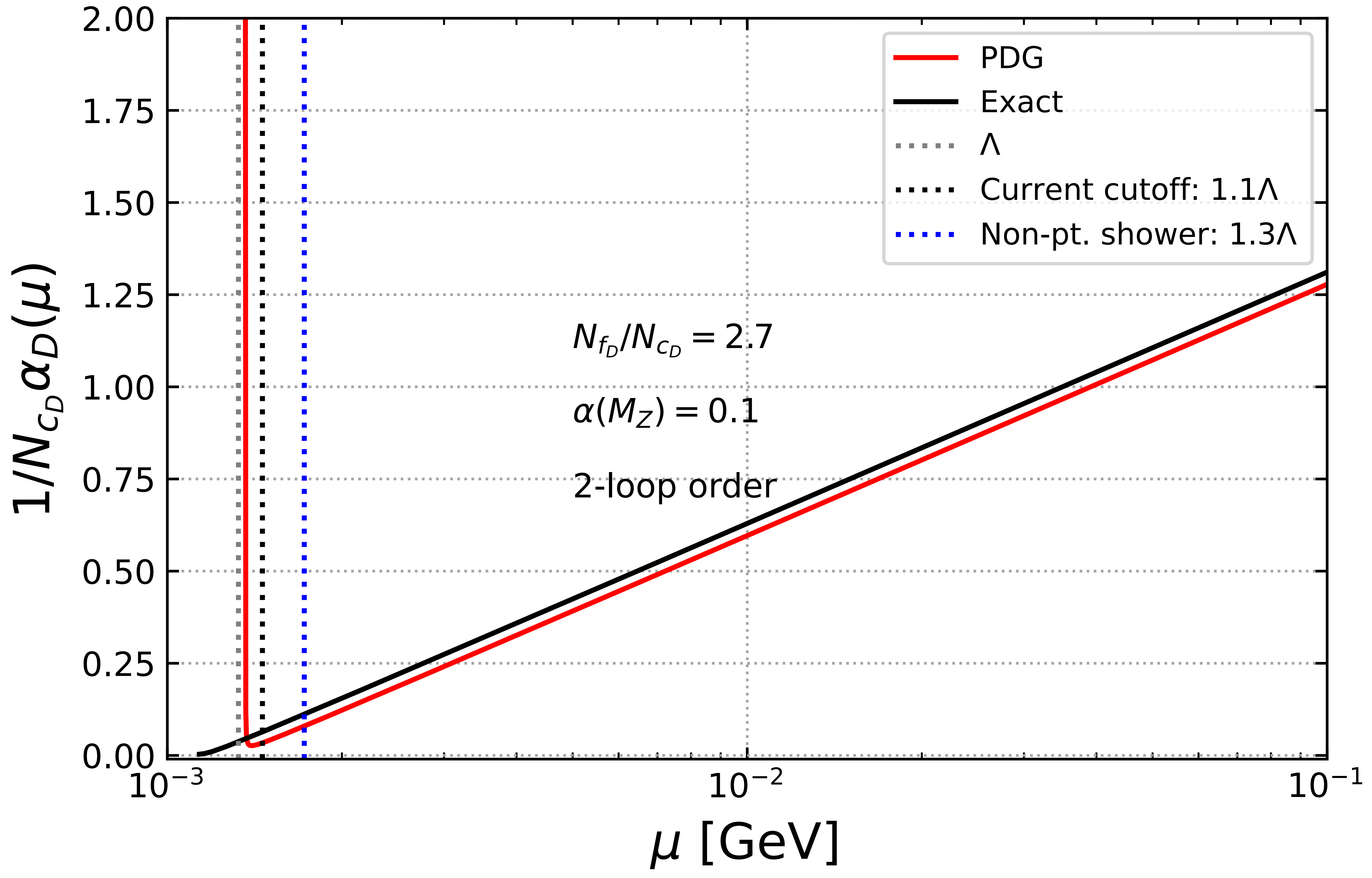
$$\alpha = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}\left[1 + \frac{1}{\alpha_*}\frac{\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0 \ln(\mu^2/\Lambda^2)}\right]$$

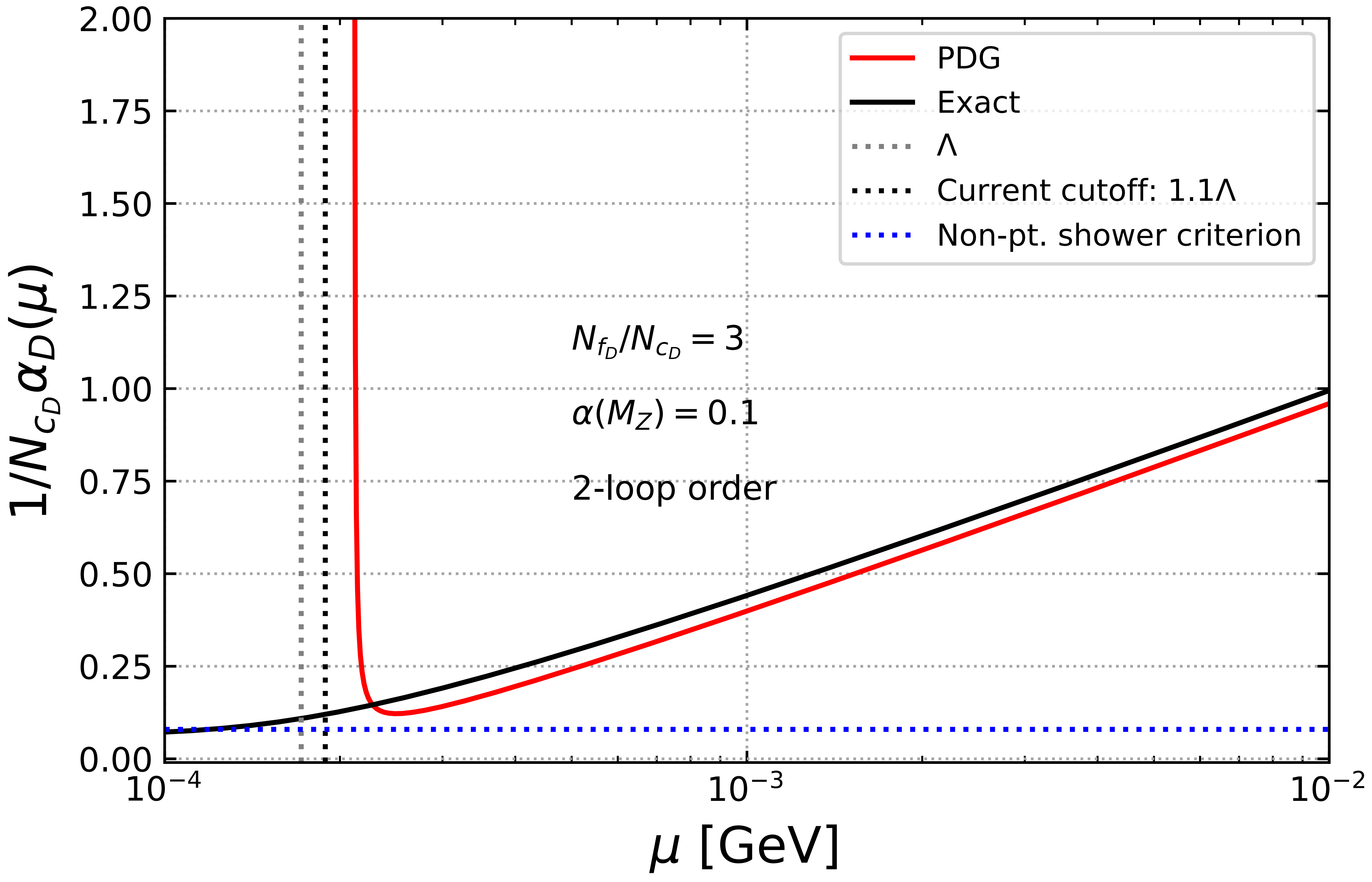
W.-M. Yao et al., Review of Particle Physics (2006) , arXiv:0607209

- In the QCD-like region this provides an excellent modelling of showering,



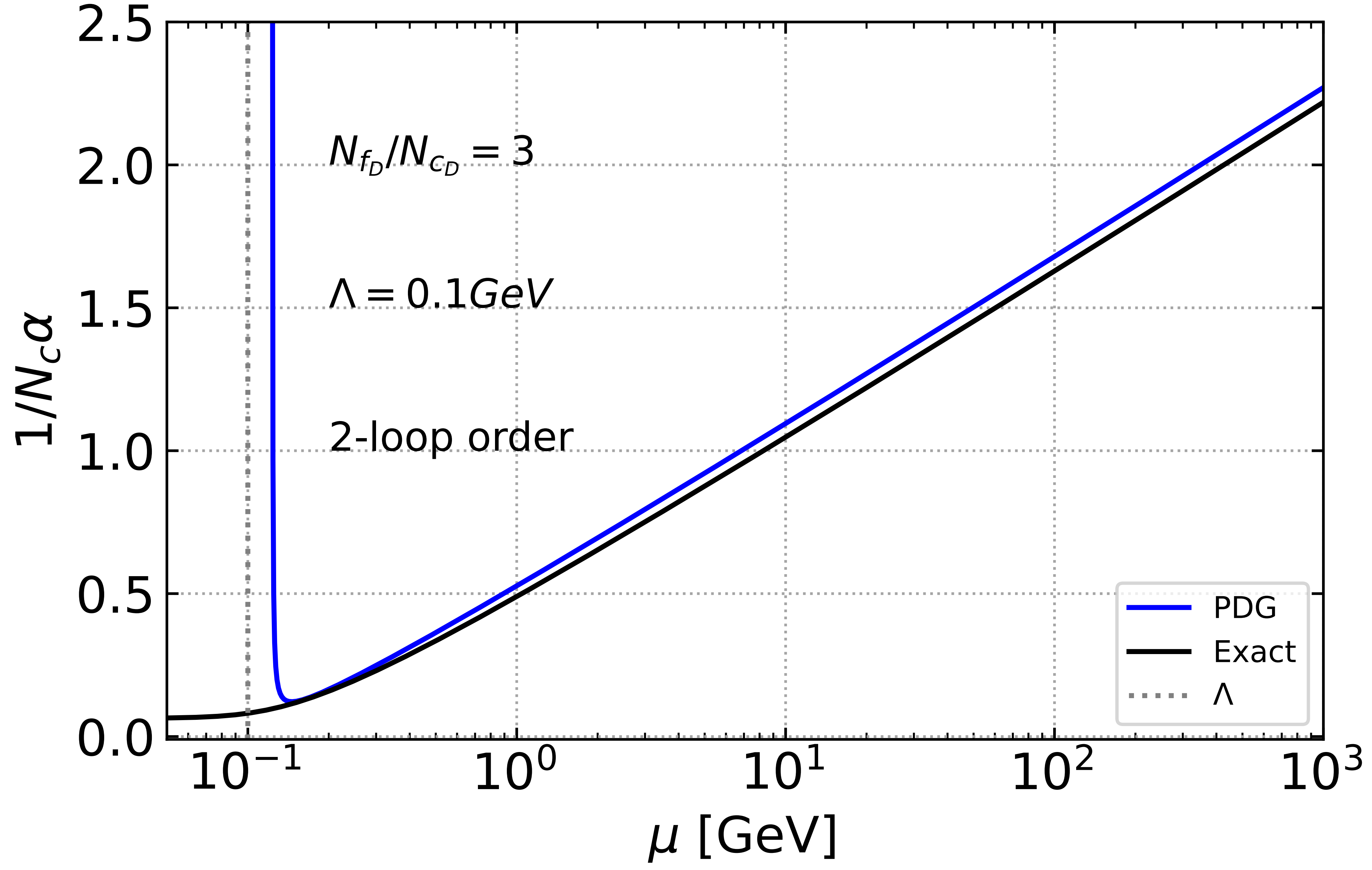
PDG for fixed reference scale

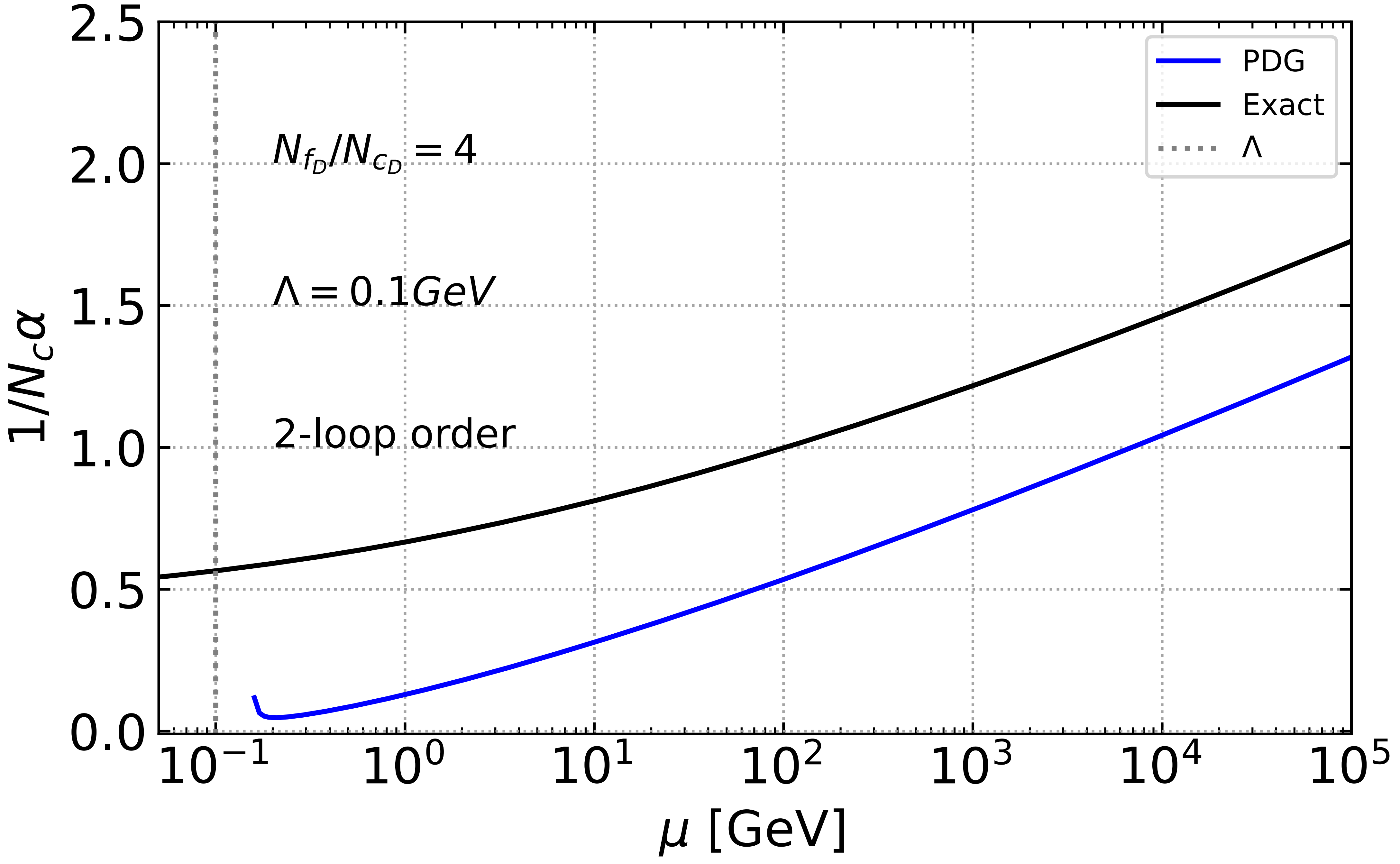




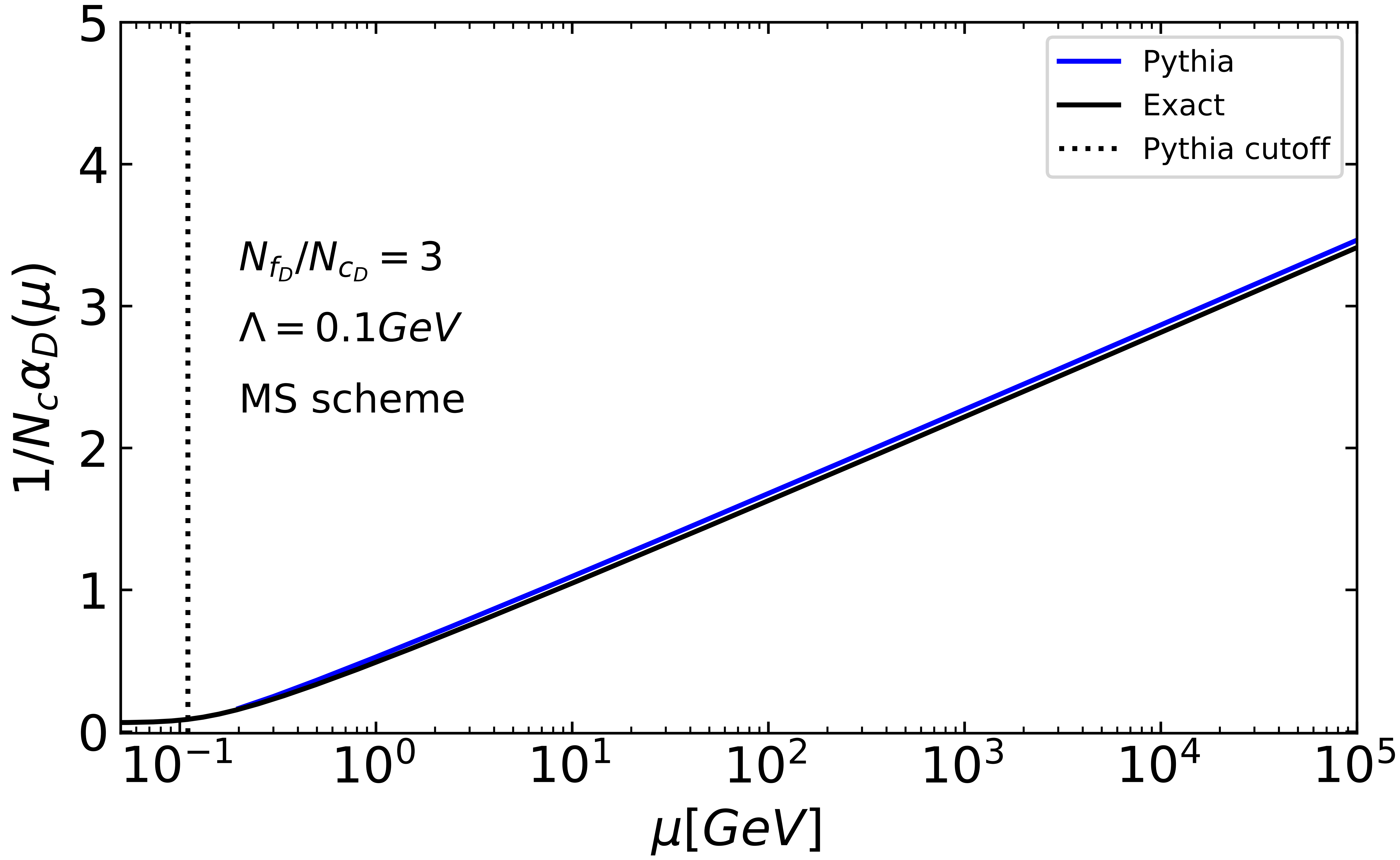
Running coupling - current procedure

- Difference between PDG and two-loop exact solution across a wide energy range underestimates total number of dark hadrons formed in jet.





Extend PDG to higher energies



Finding a scale in the IRFP region

- In general, the scale Λ describes a cross-over between two regions, below which perturbative expansion is invalid. Unlike the QCD-like region, the low energy behaviour of running in the IRFP region takes on a power-law form,

$$\alpha - \alpha_* \sim \left(\frac{\mu^2}{\mu_0^2}\right)^{\beta_0 \alpha_*}$$

- Then we can define Λ_{FP} as the transition between the asymptotic free $\sim \frac{1}{log}$ and power-law behaviour. The exact scale below which the power-law dominates can be found to be,

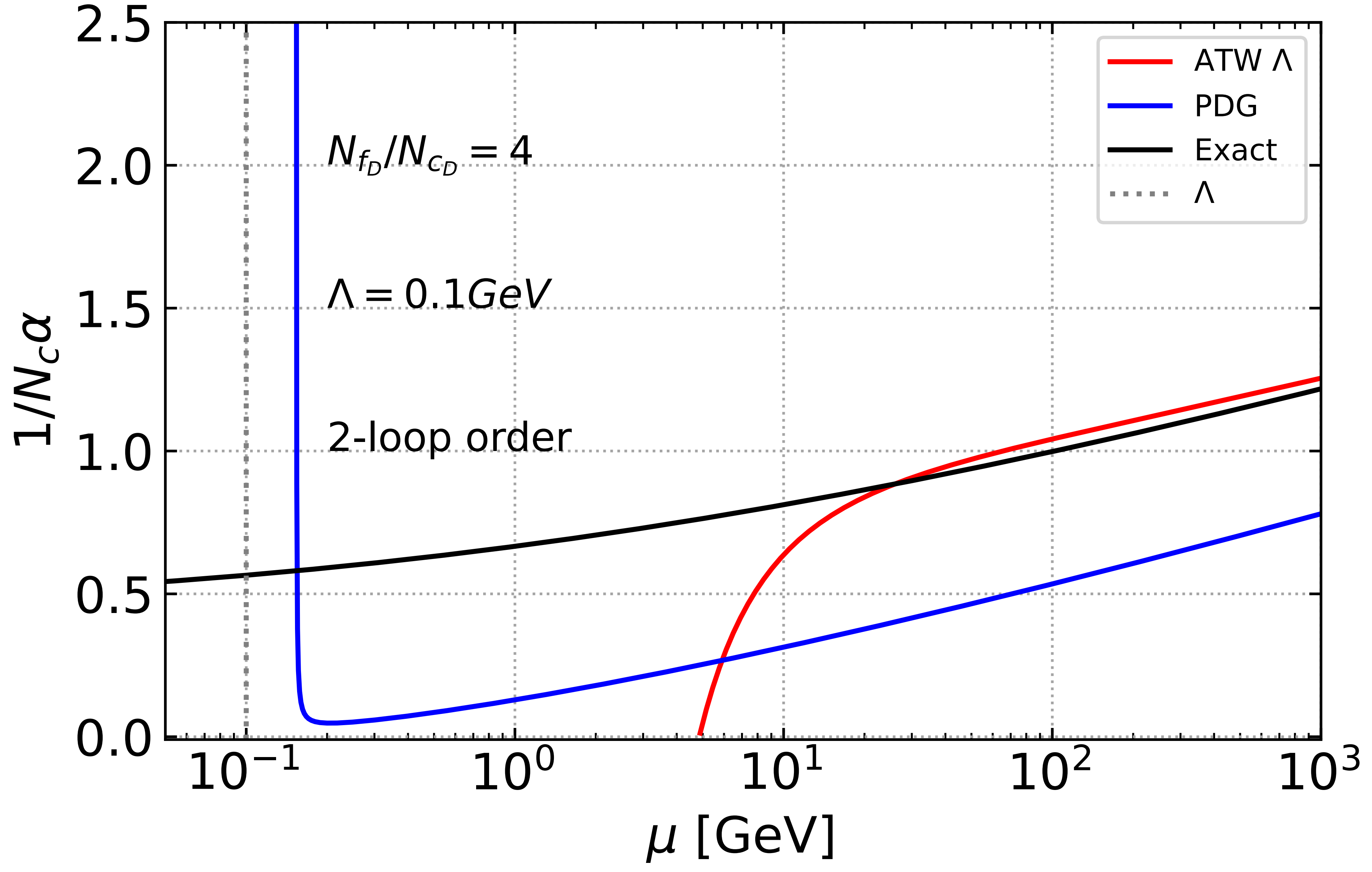
$$\beta_0 \ln\left(\frac{\Lambda_{FP}^2}{\mu_0^2}\right) = -\frac{1}{\alpha_0} - \frac{1}{\alpha_*} \ln\left(\frac{\alpha_*}{\alpha_0} - 1\right)$$

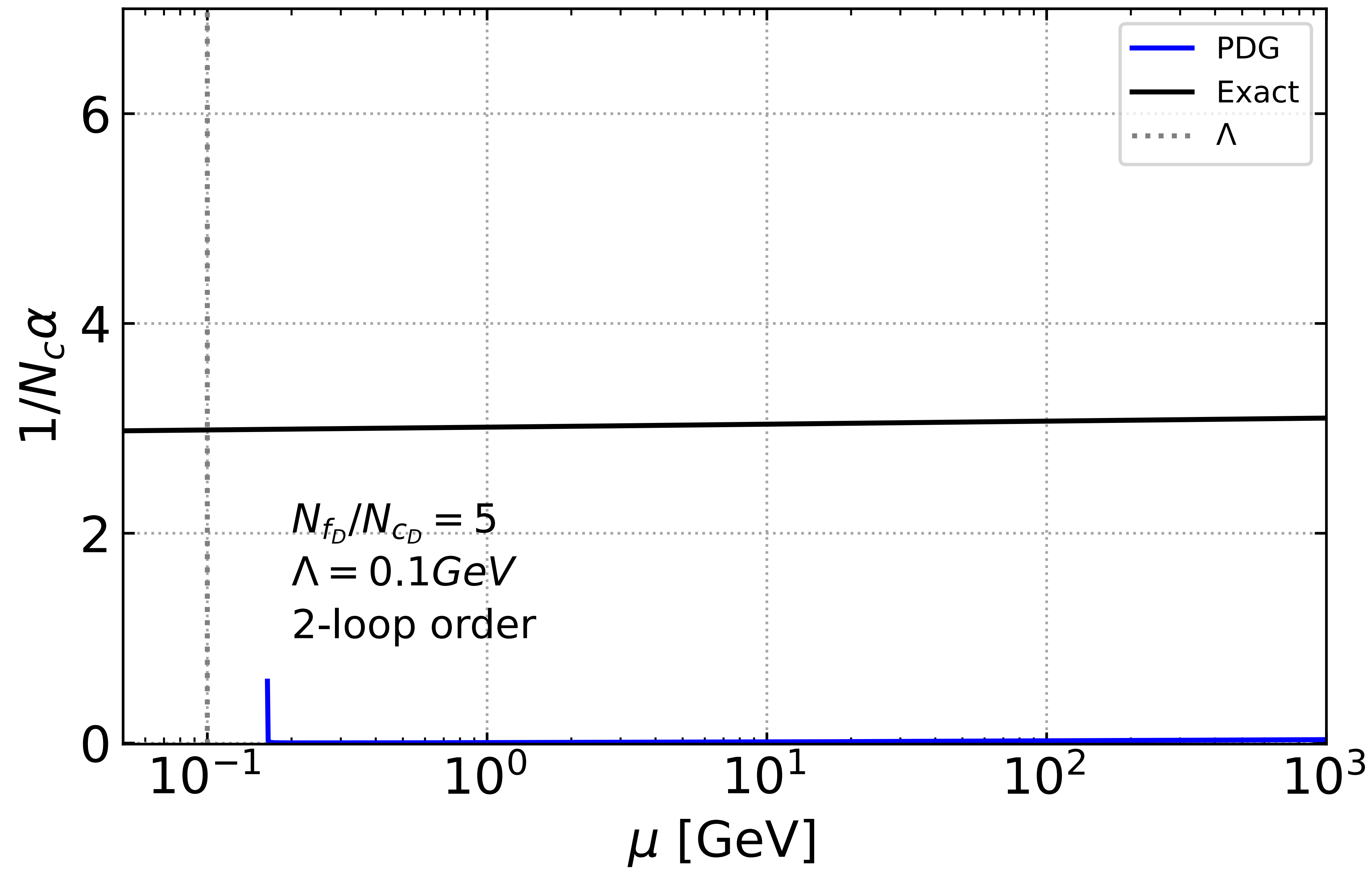
arxiv:9602385,
arxiv:9806409 - T. Appelquist et al.
arxiv:9810192 - E. Gardi et al.

- This can be seen as an analytic continuation of the QCD-like definition of Λ.

Pythia at even higher Nf/Nc

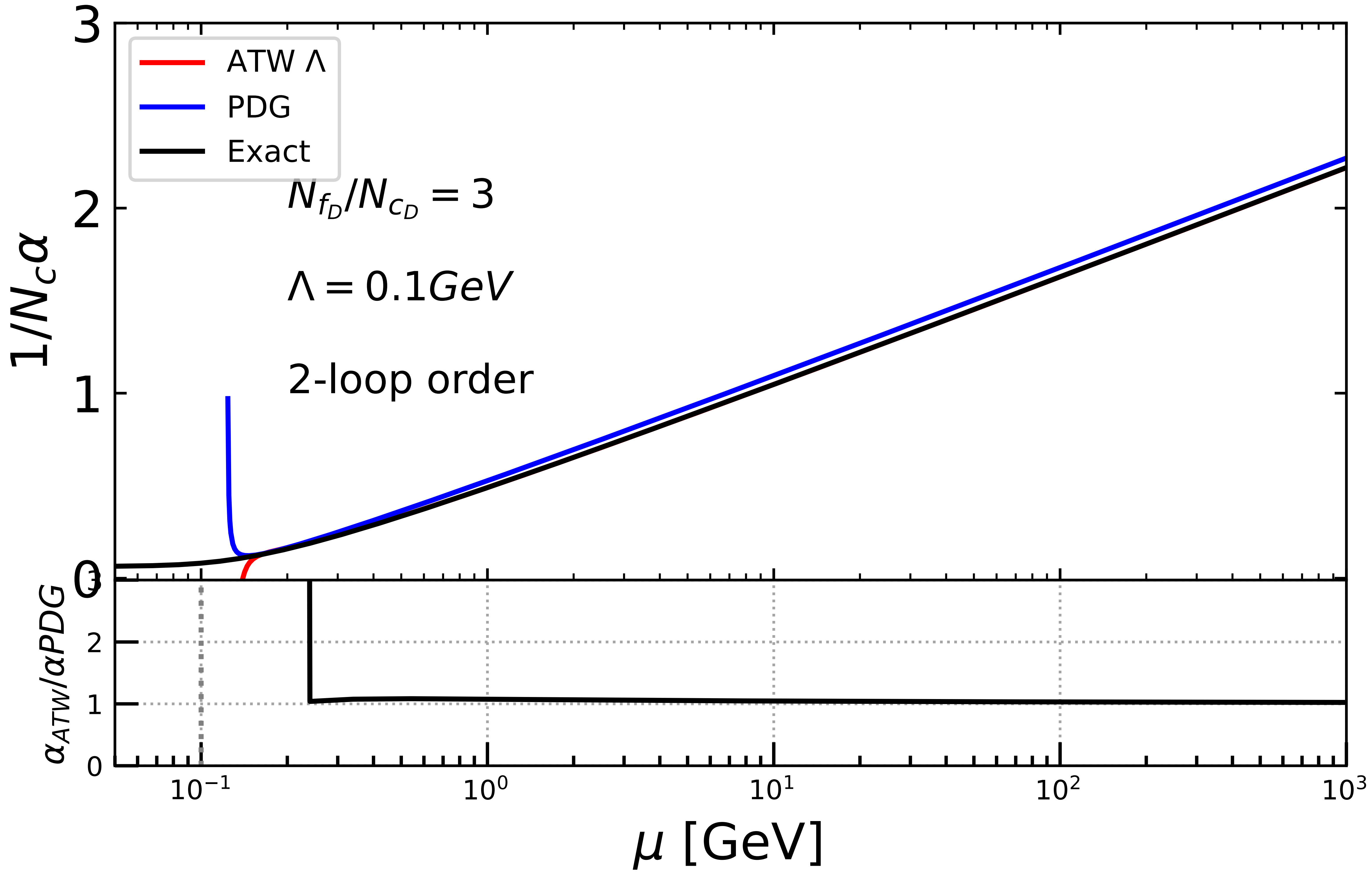
- The PDG approximation was found to not work in the IRFP region, how far can our approximations go?

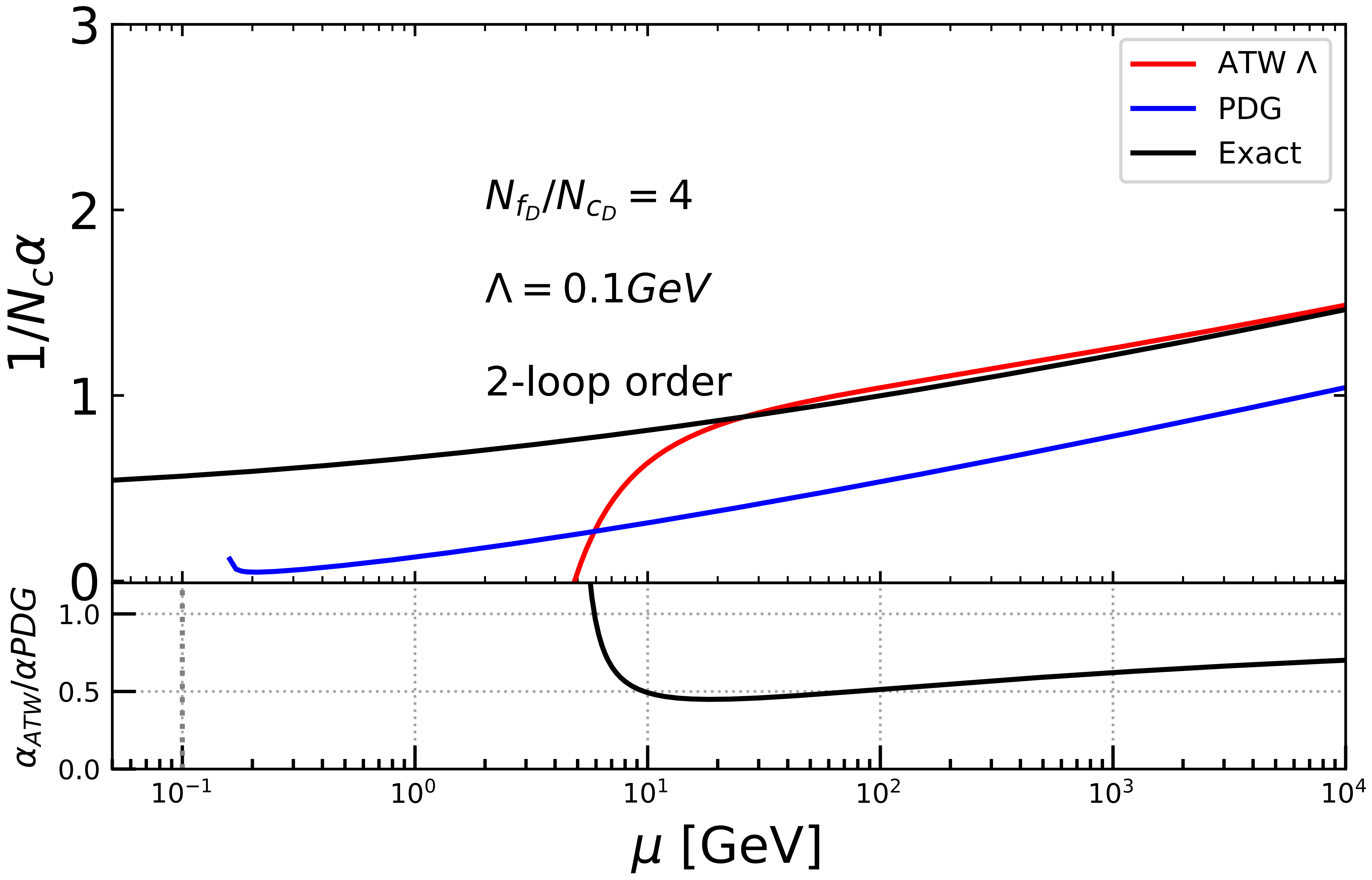




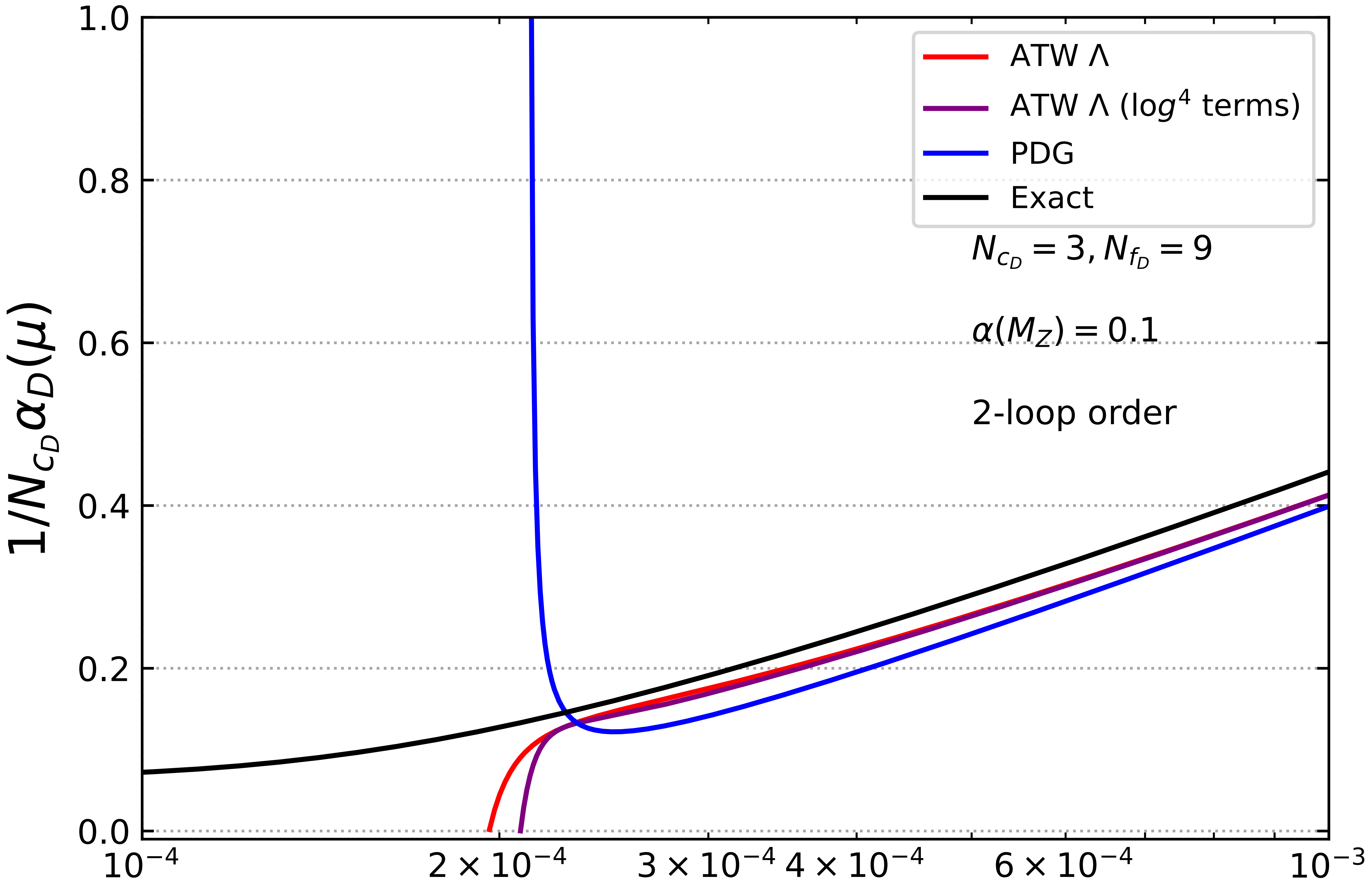
- As $\frac{N_f}{N_c}$ is increased, the energy range over which the ATW approximation proves to be reliable begins to decrease.
- In this region, there are not many cases in which the ATW approximation, even when expanding to higher orders, proves to be reliable. To get consistently reliable results an entirely new method is needed.

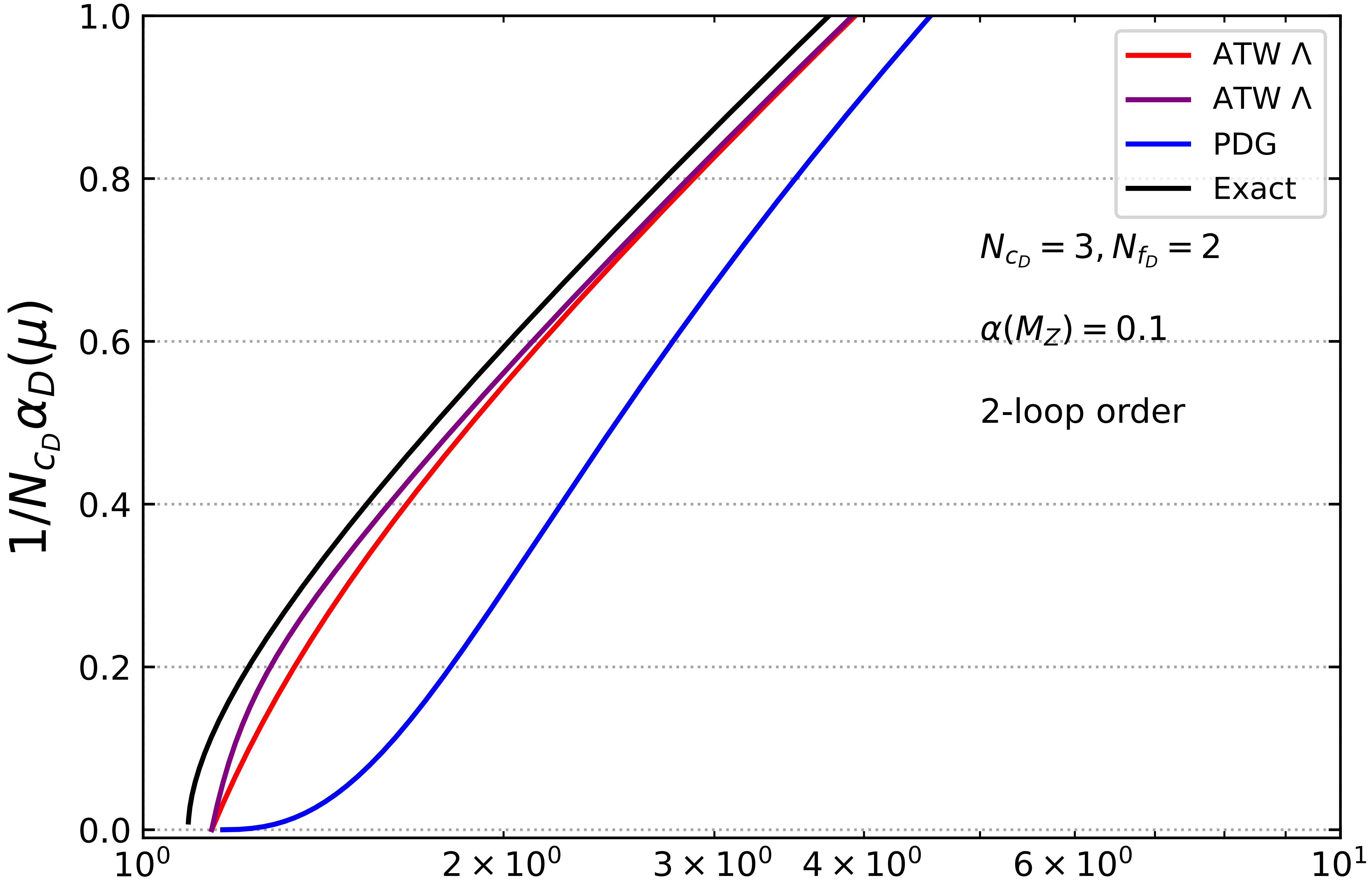
High Nf/Nc investigations - ratio plots



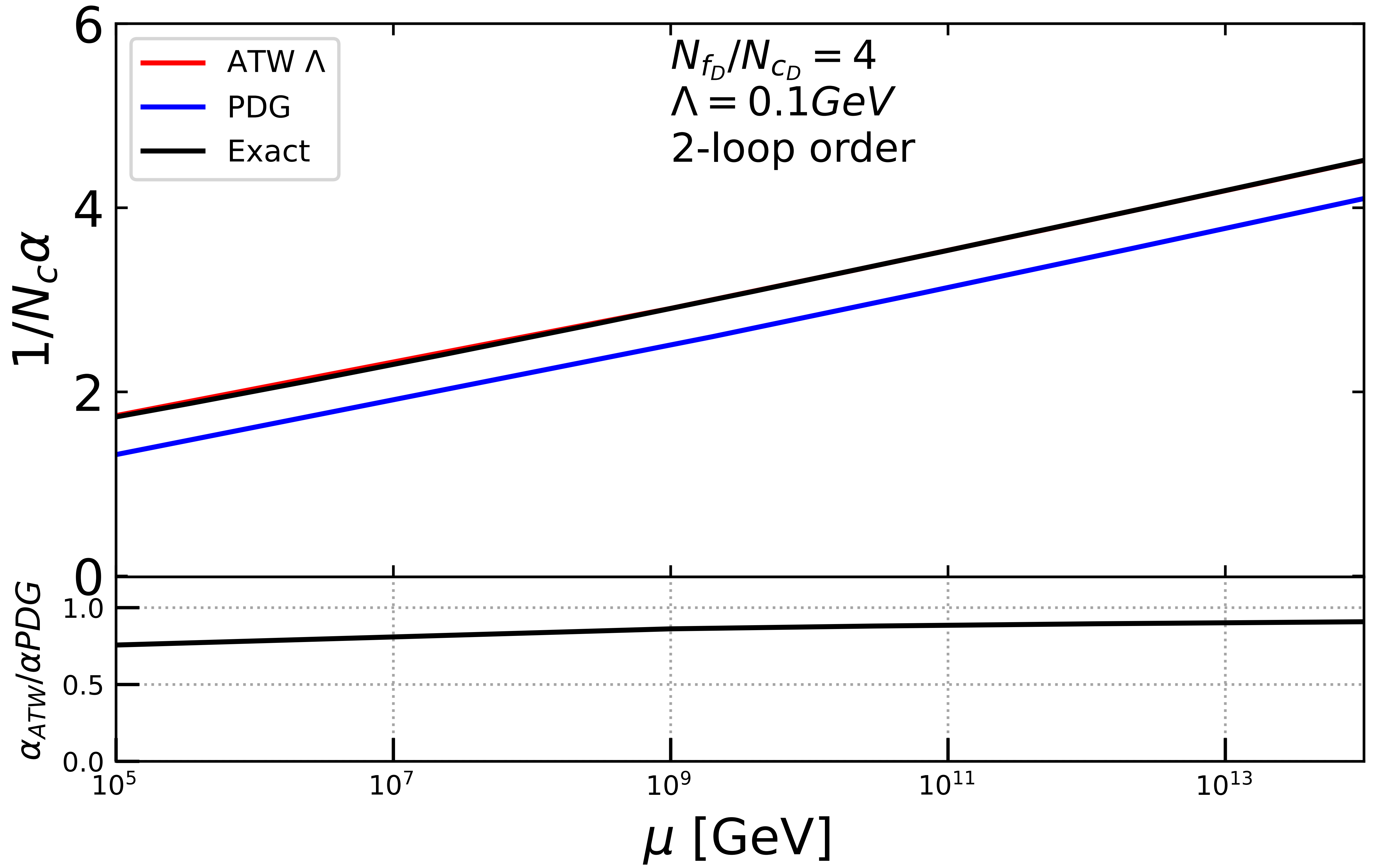


ATW - Higher order expansion





High Nf investigations - higher scales



- Extending our investigation to higher energy scales; PDG approximation never approaches ATW approximation.