THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

SIMON KUBERSKI

OPEN QUESTIONS AND FUTURE DIRECTIONS IN FLAVOUR PHYSICS
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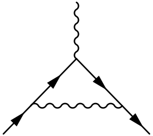
THE MUON $g-2$: A PROBE FOR NEW PHYSICS

- Magnetic moment of charged leptons $l \in \{e, \mu, \tau\}$:

$$\vec{\mu}_l = g_l \cdot \frac{e}{2m_l} \cdot \vec{s}$$

- Quantum corrections lead to deviations from the classical value $g = 2$ (Dirac), the anomalous magnetic moment,

$$a_l = \frac{g_l - 2}{2} = \frac{\alpha}{2\pi} + \mathrm{O}(\alpha^2) \quad \text{(Schwinger)}$$



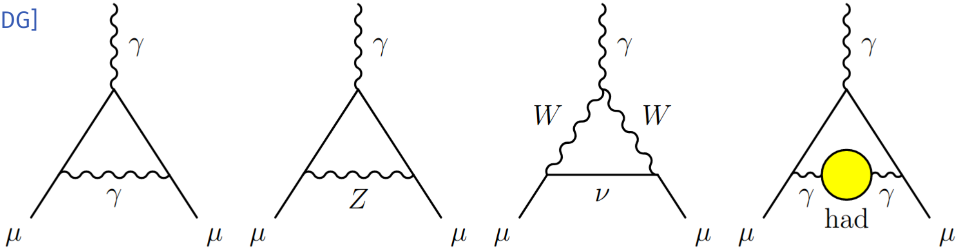
- Contributions from new physics at the scale Λ_{NP} enter a_l via

$$a_l - a_l^{\mathrm{SM}} \propto \frac{m_l^2}{\Lambda_{\mathrm{NP}}^2}$$

with $m_\mu/m_e \approx 207$.

THE MUON $g-2$: A PROBE FOR NEW PHYSICS

Figure: [PDG]



- SM prediction from QED, electroweak and hadronic contributions:

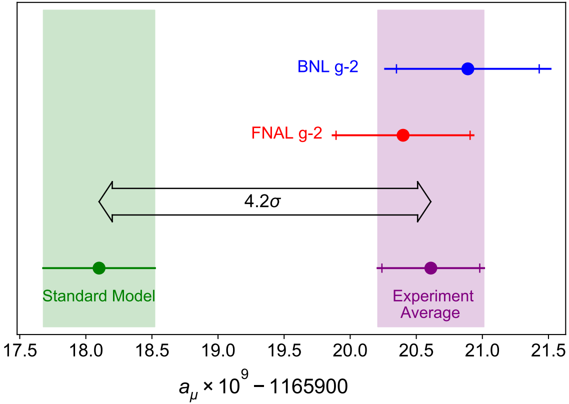
$$a_l^{\mathrm{SM}} = a_l^{\mathrm{QED}} + a_l^{\mathrm{EW}} + a_l^{\mathrm{had}} \quad \text{where} \quad a_l^{\mathrm{had}} = \boxed{a_l^{\mathrm{hvp}}} + a_l^{\mathrm{HLbL}} \,.$$

- Contributions from new physics at the scale Λ_{NP} enter a_l via

$$a_l - a_l^{\mathrm{SM}} \propto \frac{m_l^2}{\Lambda_{\mathrm{NP}}^2}$$

where a_τ is inaccessible for experiment and $m_\mu/m_e \approx 207$.

THE MUON $g-2$: A PROBE FOR NEW PHYSICS



- Comparison of Standard Model prediction and experimental average [Muon $g-2$, 2104.03281]
- After Run-1 results of the Fermilab $g-2$ experiment.
- Standard Model prediction based on the White Paper of the Muon g-2 Theory Initiative [Aoyama et al., 2006.04822]

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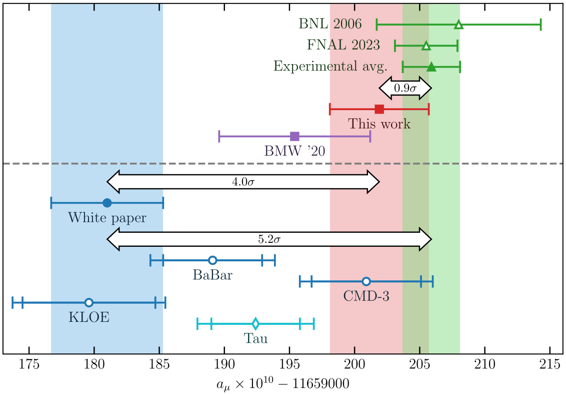

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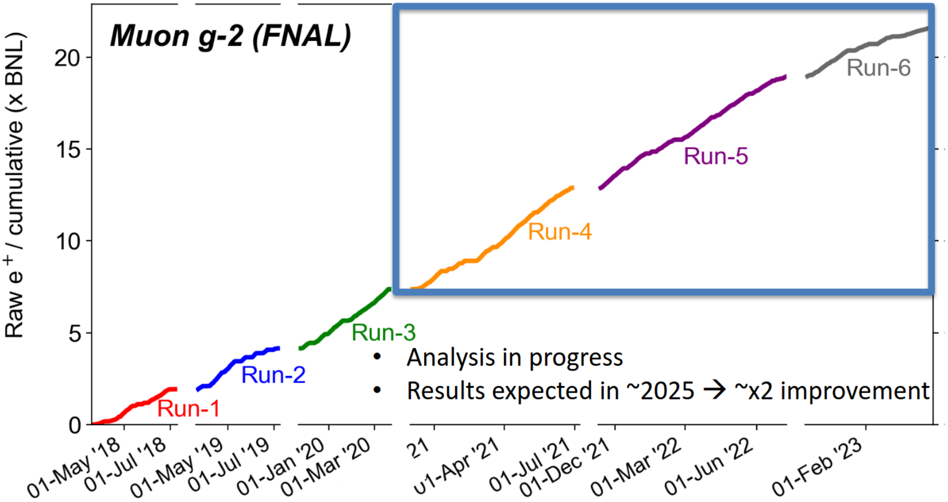
← Everything is much more complicated now [Boccaletti et al., 2407.10913] [Muon $g-2$, 2308.06230].

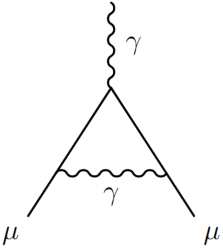
THE MUON $g-2$: EXPERIMENT AT FNAL

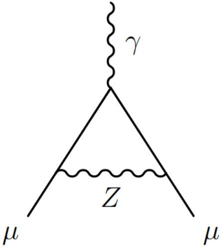


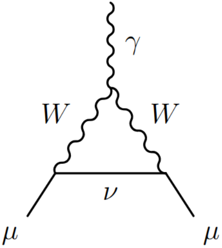
- Analysis in progress
- Results expected in ~2025 → ~x2 improvement

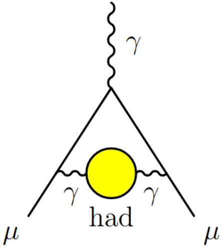
The $g-2$ experiment will surpass their initial target precision! [Venanzoni]

THE STANDARD MODEL PREDICTION FOR a_μ

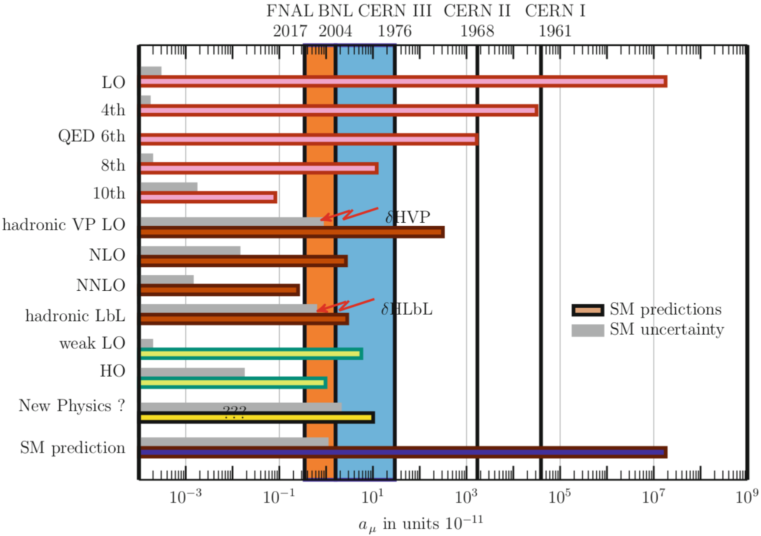








THE MUON $g-2$: STANDARD MODEL PREDICTION



Muon $g-2$ experiments and the sensitivity to various contributions. [Jegerlehner, 2017]

THE MUON $g-2$

Contribution	Value $\times 10^{11}$
Experiment (E821 + E989)	$116\,592\,059(22)$
HVP LO (e^+e^-)	$6931(40)$
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	$12.4(1)$
HVP LO (lattice, $udsc$)	$7116(184)$
HLbL (phenomenology)	$92(19)$
HLbL NLO (phenomenology)	$2(1)$
HLbL (lattice, uds)	$79(35)$
HLbL (phenomenology + lattice)	$90(17)$
QED	$116\,584\,718.931(104)$
Electroweak	$153.6(1.0)$
HVP (e^+e^-, LO + NLO + NNLO)	$6845(40)$
HLbL (phenomenology + lattice + NLO)	$92(18)$
Total SM Value	$116\,591\,810(43)$
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	$249(48)$

- Theory initiative: Status for a_μ [Colangelo et al., 2203.15810], updated with Run-2/3.

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- $5.1\,\sigma$ discrepancy between experiment and prediction?
- Uncertainty from $a_\mu^{\text{hvp,LO}}$ dominates a_μ.

THE MUON $g - 2$

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- $5.1\,\sigma$ discrepancy between experiment and prediction?
- Uncertainty from $a_\mu^{\text{hvp,LO}}$ dominates a_μ.
- A number of new inputs for the **upcoming second White Paper**…

QED AND ELECTROWEAK CONTRIBUTIONS

- The QED contribution completely dominates the SM prediction.

- Recent developments [Makiko Nio at KEK]:
  - Small inconsistencies in the 10th order contributions are resolved.
  - Uncertainty halved due to improved measurement of the fine-structure constant.

- Improved precision for the electroweak contribution [Martin Hoferichter at KEK].

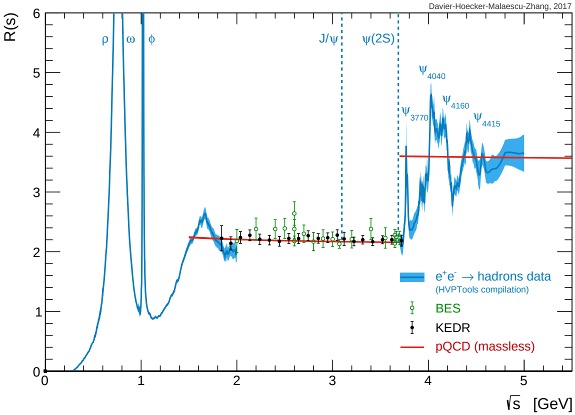
- Both changes are completely irrelevant for the final uncertainty.

THE STANDARD MODEL PREDICTION FOR a_μ

HADRONIC CONTRIBUTIONS TO a_μ

a_μ^{hvp}: THE DISPERSIVE APPROACH

R-ratio: $$R(s) = \frac{\sigma^0(e^+e^- \to \mathrm{hadrons}(+\gamma))}{\sigma_{\mathrm{pt}}}\,, \qquad \sigma_{\mathrm{pt}} = \frac{4\pi\alpha^2}{3s}$$



- Data-driven extraction of the HVP contribution via dispersion integral

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{K(s)}{s}R(s)\mathrm{d}s$$

[Davier et al., 1706.09436]

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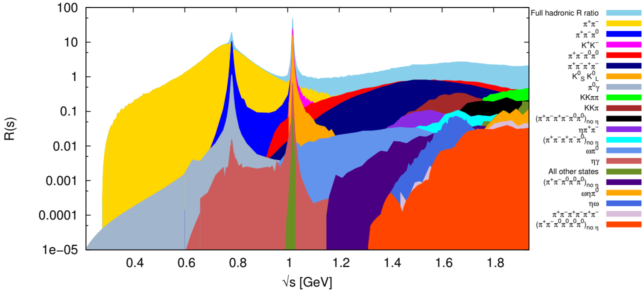

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- R-ratio constructed from exclusive channels
  $\to$ source of systematic uncertainty.

[Davier et al., 1706.09436] [Keshavarzi et al., 1802.06229]

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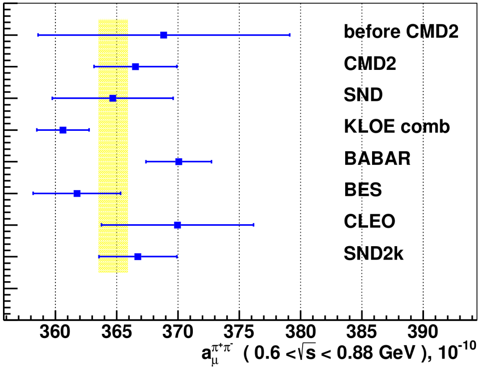

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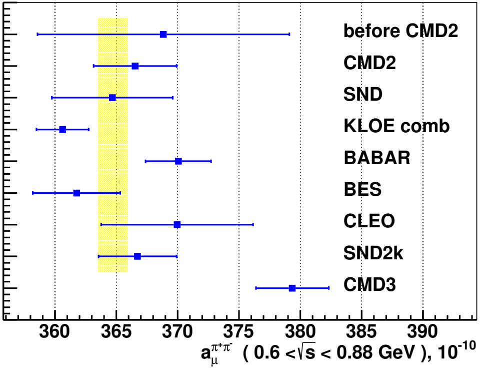

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- R-ratio constructed from exclusive channels
  → source of systematic uncertainty.

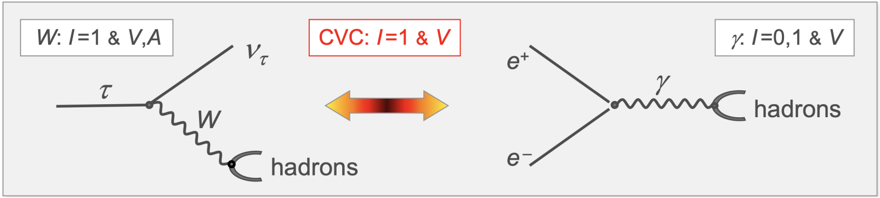
- The discrepancies are **not understood**.

[Davier et al., 1706.09436] [Keshavarzi et al., 1802.06229] [Ignatov et al., 2302.08834]

$e^+e^- \to \pi^+\pi^-$ AT LOW ENERGIES

- Can missing NNLO terms in the MC generators (e.g. PHOKHARA) that are used to remove radiative corrections affect experimental results significantly?
- First, partly preliminary, analyses: Not the case for BaBar, KLOE, BES III.
- The RadioMonteCarlow 2 Effort assesses different Monte Carlo codes.
- New data (KLOE, BaBar, Belle II, BES III, SND) will come in eventually
- All of this will be too late for the WP update.

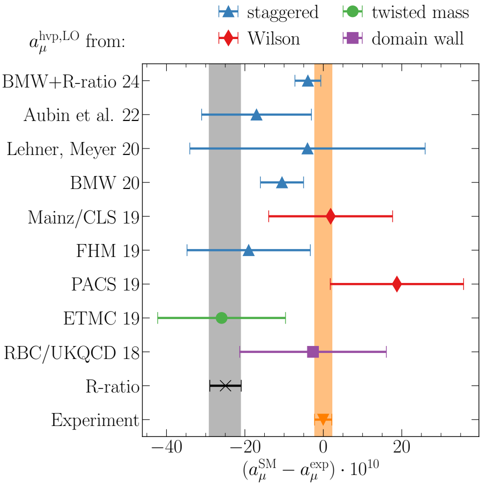
TAU DATA



- Can use τ spectral functions to evaluate the LO HVP [Alemany et al., hep-ph/9703220] [Zhang at KEK].
- Requires (model-dependent) isospin-breaking corrections.
- Model-independent evaluations on the way
  - on the lattice [Bruno et al.]
  - with dispersive methods [Cottini et al.]

  and will allow to include the τ data in a data-driven evaluation.

a_μ^{hvp} FROM LATTICE QCD

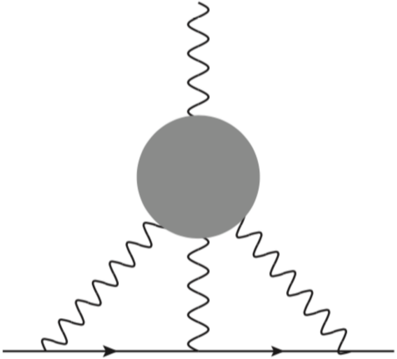


- One sub-percent determination of a_μ^{hvp} from the lattice [BMWc, 2002.12347]: In tension with the dispersive result.
- Several new lattice results at the percent level in the (very) near future to come.
- Consistency would allow to replace the data-driven estimate.

Goal

Several lattice results at $< 0.5\%$ precision.

HADRONIC LIGHT-BY-LIGHT SCATTERING



- Hadronic light-by-light scattering: $\mathrm{O}(\alpha^3)$, target precision: 10%.

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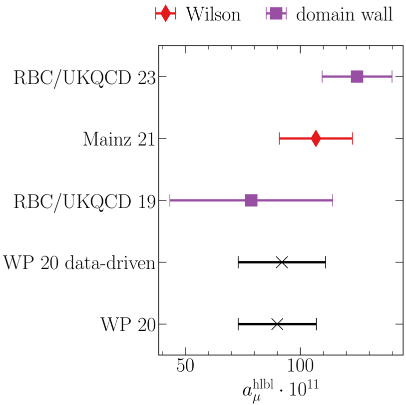

- Hadronic light-by-light scattering: $\mathrm{O}(\alpha^3)$, target precision: 10%.
- White paper recommended value:

$$a_\mu^{\mathrm{hlbl}} = (92 \pm 18) \cdot 10^{-11}$$

- Two lattice calculations since then, [Mainz 21, 2104.02632, 2204.08844] and [RBC/UKQCD 23, 2304.04423].
- Preliminary results by ETMc and BMW.
- Lattice also enters data-driven determination via transition form factors.

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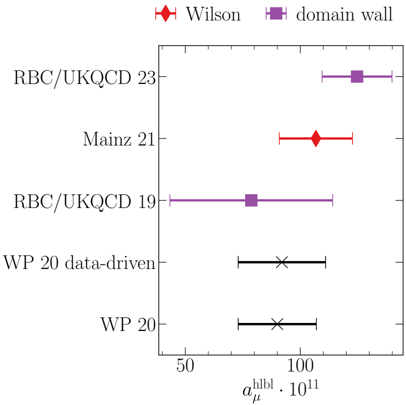

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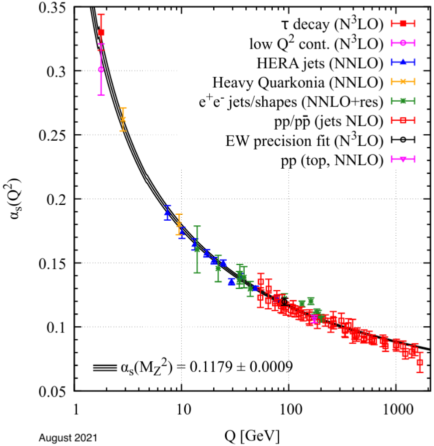
- Preliminary results by ETMc and BMW.
- Lattice also enters data-driven determination via transition form factors.
- Lattice and data-driven computations are an outstanding success.
- No obvious tension at the current level of uncertainty.

a_μ^{hvp} ON THE LATTICE

LATTICE QCD



- QCD is a strongly coupled theory in the hadronic regime at $Q \sim 300\,\mathrm{MeV}$.
- Perturbative expansion fails below $1\,\mathrm{GeV}$.

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[1] [PDG, PTEP **2022** (2022), 083C01]

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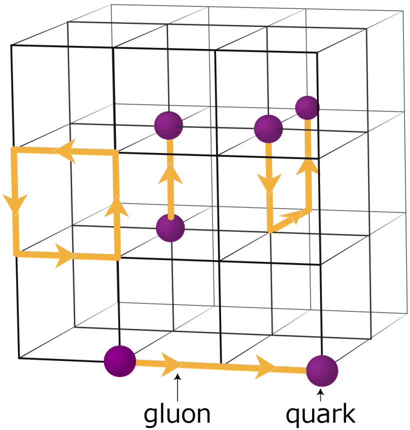

- QCD is a strongly coupled theory in the hadronic regime at $Q \sim 300\,\mathrm{MeV}$.
- Perturbative expansion fails below $1\,\mathrm{GeV}$.
- Formulate the theory
  - on a finite grid → regulator Λ_{UV}.
  - in finite volume → Λ_{IR}.
  - in Euclidean space-time
  - as a Boltzmann distribution
- Compute expectation values $\langle \mathrm{O} \rangle$ by sampling the QCD path integral with Markov Chain Monte Carlo methods.

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[2] http://www.jicfus.jp/en/promotion/pr/mj/guido-cossu/

LATTICE QCD

The QCD Lagrange density

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=1}^{N_f} \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$

- Contains $N_f + 1$ **bare** parameters (gauge coupling and N_f quark masses)
- Renormalize the theory from hadronic input, e.g., m_Ω, m_π, m_K, m_{D_s}, m_{B_s}.
  $\rightarrow$ All other observables are **predictions**.
- Freedom of choice on how to discretize $\mathcal{L}_{\mathrm{QCD}}$: Wilson, twisted mass, staggered, domain wall, overlap, …
- *Ab initio* predictions after lifting the cutoffs:
  - Λ_{IR}: Infinite-volume limit.
  - Λ_{UV}: Continuum limit.

$a_\mu^{\rm hvp}$ ON THE LATTICE

- Compute $a_\mu^{\rm hvp}$ via [Laurup et al.] [Blum, hep-lat/0212018]

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}Q^2 f(Q^2)\hat{\Pi}(Q^2)\,, \qquad \text{with} \quad \hat{\Pi}(Q^2) = 4\pi^2\left[\Pi(Q^2) - \Pi(0)\right]$$

  from a known QED kernel function $f(Q^2)$ and the polarization tensor

$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4x\, \mathrm{e}^{iQ\cdot x}\langle j_\mu^{\rm em}(x)\, j_\nu^{\rm em}(0)\rangle = (Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi(Q^2)\,.$$

- $a_\mu^{\rm hvp}$ in the time-momentum representation (TMR) [Bernecker, Meyer, 1107.4388],

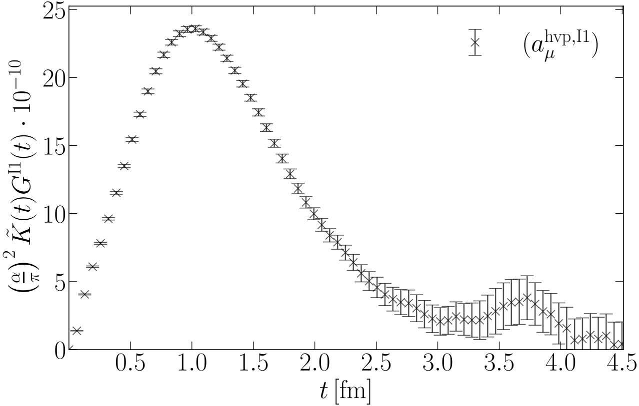
$$a_\mu^{\rm hvp} := \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G(t)\widetilde{K}(t) \quad \text{with the known QED kernel function } \widetilde{K}(t)\,,$$

  in terms of the zero-momentum vector correlator $G(t)$ (de facto standard).

- Alternative: coordinate space method [Meyer, 1706.01139] [Chao et al., 2211.15581].

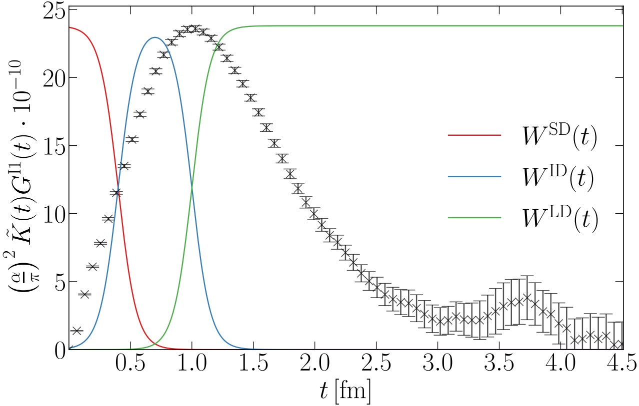
a_μ^{hvp} ON THE LATTICE: EUCLIDEAN TIME WINDOWS

$$(a_\mu^{\text{hvp}}) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G(t)\widetilde{K}(t), \qquad G(t) = -\frac{a^3}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle j_k^{\text{em}}(t,\vec{x})\, j_k^{\text{em}}(0)\rangle$$



a_μ^{hvp} ON THE LATTICE: EUCLIDEAN TIME WINDOWS

$$(a_\mu^{\text{hvp}})^i = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G(t)\widetilde{K}(t)\;\; W^i(t;t_0;t_1)\,, \qquad G(t) = -\frac{a^3}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle j_k^{\text{em}}(t,\vec{x})\, j_k^{\text{em}}(0)\rangle$$



- Windows in the TMR: separate short- from long-distance effects [RBC/UKQCD, 1801.07224].

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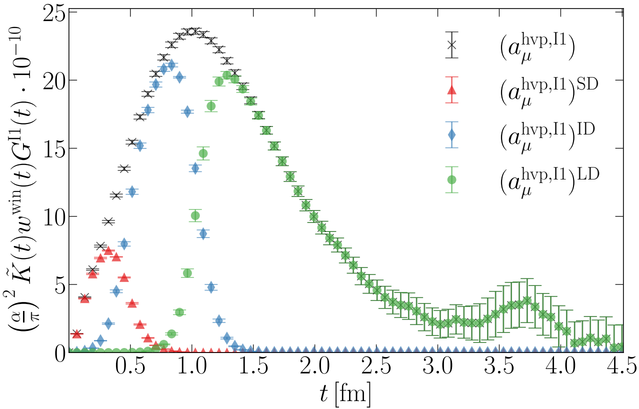


- Windows in the TMR: separate short- from long-distance effects [RBC/UKQCD, 1801.07224].
- Intermediate window a_μ^{win}:
  - Cutoff effects suppressed.
  - No signal-to-noise problem.
  - Finite-volume effects small.

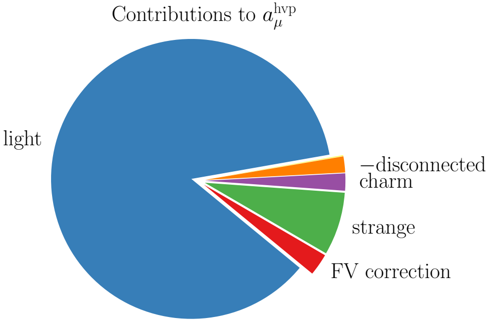
a_μ^{hvp} ON THE LATTICE: CONTRIBUTIONS

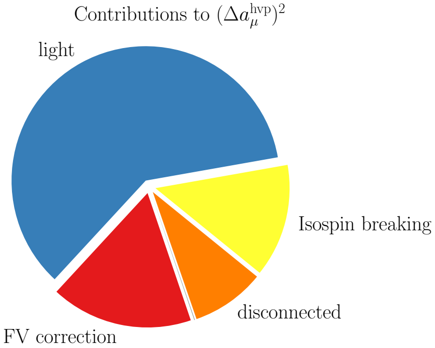
The electromagnetic current

$$j_\mu^{\text{em}} = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \tfrac{2}{3}\bar{c}\gamma_\mu c + \ldots = j_\mu^{I=1} + j_\mu^{I=0}$$

from zero-momentum vector-vector correlation functions

$$G^{\text{isoQCD}}(t) = \tfrac{5}{9}G^{\text{light}}(t) + \tfrac{1}{9}G^{\text{strange}}(t) + \tfrac{4}{9}G^{\text{charm}}(t) + G^{\text{disc}}(t) + \ldots$$



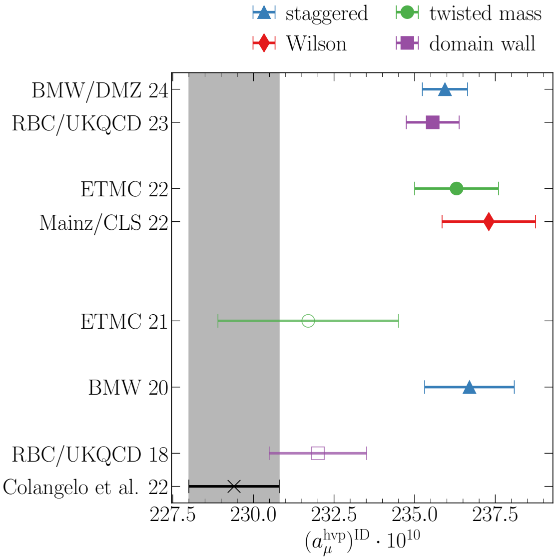


Based on [BMWc, 2002.12347]: $a_\mu^{\text{hvp}} = 707.5\,(5.5)\cdot 10^{-10}$

a_μ^{hvp} ON THE LATTICE

WINDOW OBSERVABLES

THE INTERMEDIATE-DISTANCE WINDOW



- $3.8\,\sigma$ tension between lattice QCD and data-driven evaluation [Colangelo et al., 2205.12963].
- This accounts for 50% **of the difference** between BMW 20 and the White Paper average for a_μ^{hvp}.

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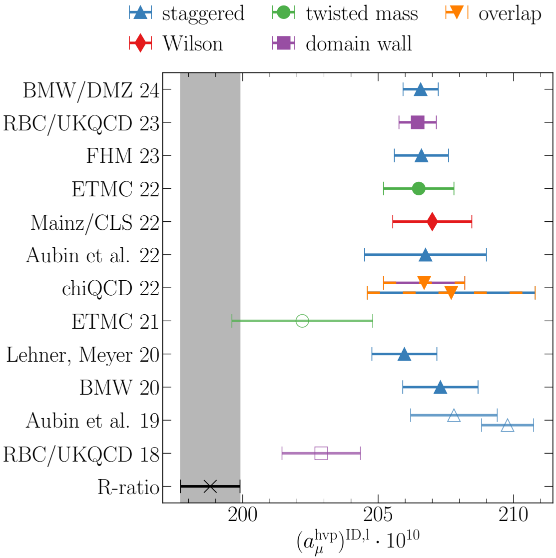
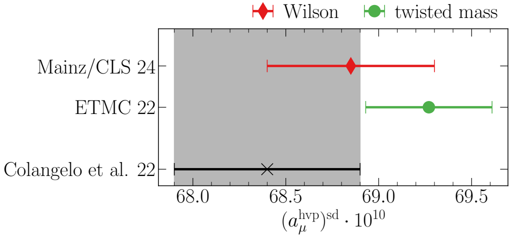
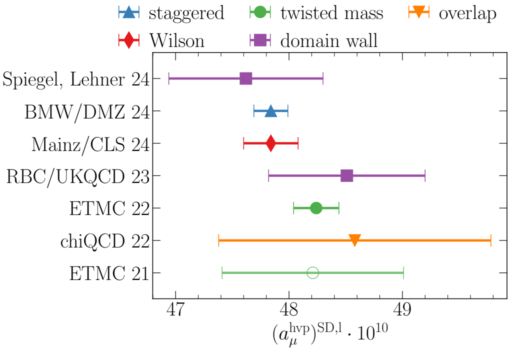


- $3.8\,\sigma$ tension between lattice QCD and data-driven evaluation [Colangelo et al., 2205.12963].
- This accounts for 50% **of the difference** between BMW 20 and the White Paper average for a_μ^{hvp}.
- Agreement across many actions for the light-connected contribution (87% of $(a_\mu^{\mathrm{hvp}})^{\mathrm{ID}}$).
- Data-driven estimate: [Benton et al., 2306.16808]

THE SHORT-DISTANCE WINDOW



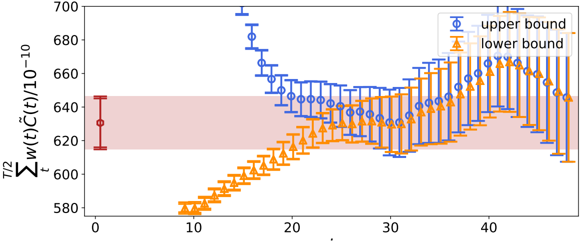


- Continuum extrapolation is the major difficulty for the short-distance window.
- However: Small uncertainties w.r.t. the full HVP.
- No significant difference between lattice and R-ratio - could expect about 1 unit (1.44%) based on what is seen in the intermediate window [SK at al., 2401.11895].

a_μ^{hvp} ON THE LATTICE

DOMINANT SOURCES OF UNCERTAINTY FOR a_μ^{hvp}

CONTROLLING THE LONG-DISTANCE TAIL



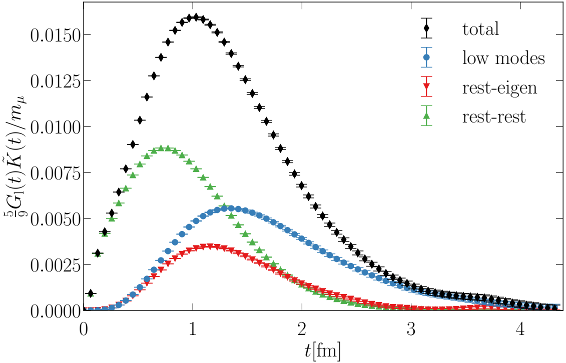
[RBC/UKQCD, 1910.11745]

Exponential deterioration of the signal-to-noise ratio.

Improve the signal at large t via:

- Bounds on the correlator.
- Noise reduction methods:
  - Truncated Solver Method
  - Low Mode Averaging
  - All Mode Averaging
- Spectral reconstruction of the $\pi\pi$ contributions.
- Multi-level integration. [Dalla Brida et al., 2007.02973]

CONTROLLING THE LONG-DISTANCE TAIL



[Mainz, Lattice 2022]

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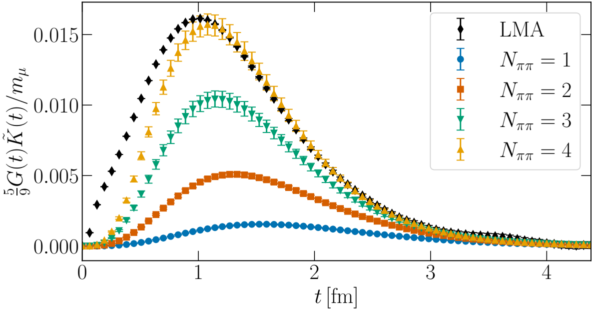

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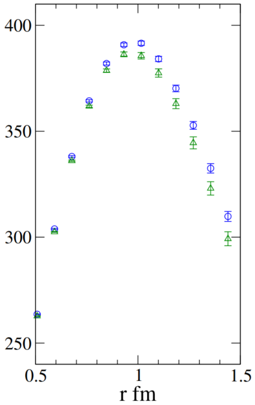
[Mainz]

FINITE-VOLUME EFFECTS

3% finite-L corrections for a_μ^{hvp} at $m_\pi L = 4$, mostly in the **isovector channel**.

- EFT and model calculations.
  - NNLO χPT
  - Two-pion spectrum in finite-volume and the timelike pion form factor [Meyer, 1105.1892] [Lellouch and Lüscher, hep-lat/0003023] [Giusti et al., 1808.00887].
  - Pions winding around the torus and the electromagnetic pion form factor [Hansen, Patella, 1904.10010, 2004.03935].
  - Rho-pion-gamma model [Sakurai] [Jegerlehner, Szafron, 1101.2872] [HPQCD, 1601.03071].
- Simulations at $L > 10\,\mathrm{fm}$ [PACS, 1902.00885] [BMWc, 2002.12347].
  - Uncertainty statistics dominated.
  - Show good consistency with models.



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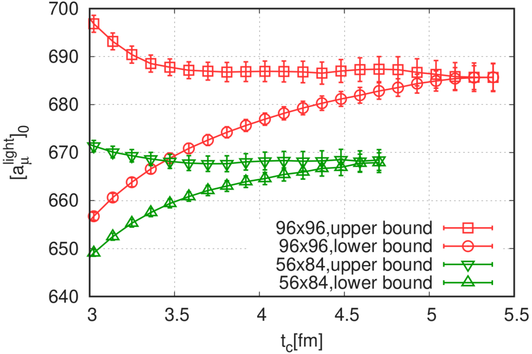
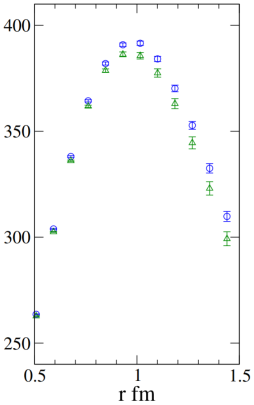


].



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  - Uncertainty statistics dominated.
  - Show good consistency with models.

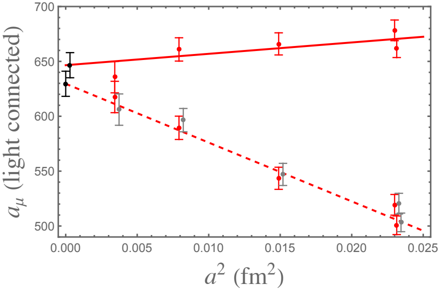
CUTOFF EFFECTS

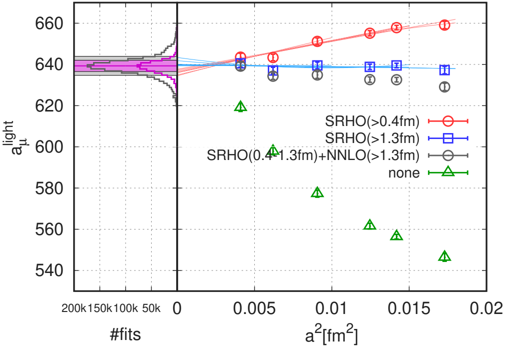
Systematic uncertainties from the continuum extrapolation may be dominant.

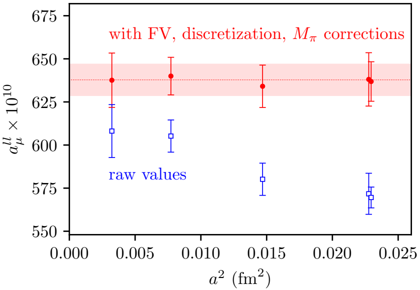
- Extrapolation to the continuum limit guided by Symanzik effective theory.
- Cutoff effects start at $\mathrm{O}(a^2)$ in modern lattice calculations.
- Mandatory to
  - include ≥ 4 resolutions to constrain higher order cutoff effects.
  - include fine resolutions $a \leq 0.05\,\mathrm{fm}$ for per-mil uncertainties.
- Staggered quarks: taste violations distort the pion spectrum.
  - This is a cutoff effect: Vanishes in the continuum limit.
  - Taste breaking may introduce non-linear effects (in a^2).

  → Corrections applied at finite lattice spacing.

THE CONTINUUM LIMIT: STAGGERED QUARKS





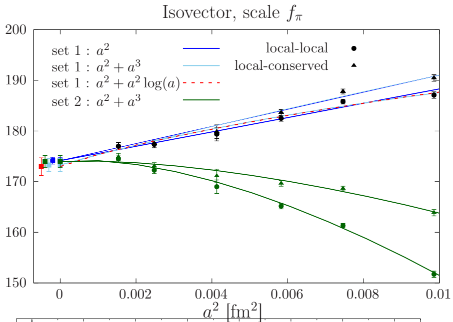


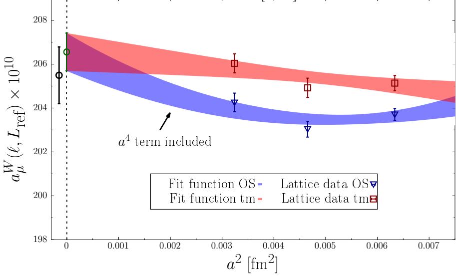
- Continuum extrapolations of a_μ^{hvp} computed with staggered quarks.

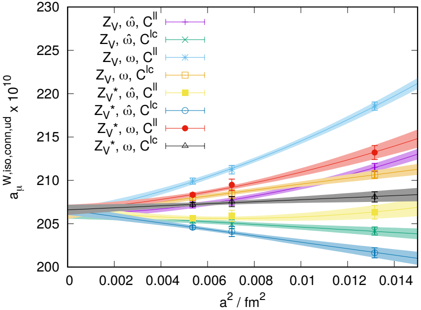
- Compare raw and corrected data.

[Aubin et al., 2204.12256] [BMWc, 2002.12347]
[Fermilab, HPQCD, MILC, 1902.04223]

THE CONTINUUM LIMIT: INTERMEDIATE WINDOW







- Different discretization prescriptions have to agree in the continuum.
- Strong cross-check for **valence** cutoff effects.

[Mainz, 2206.06582] [RBC/UKQCD, 2301.08696] [ETMC, 2206.15084]

a_μ^{hvp} ON THE LATTICE

THE LONG-DISTANCE CONTRIBUTION

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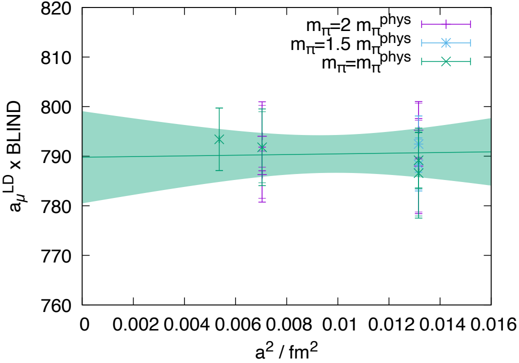

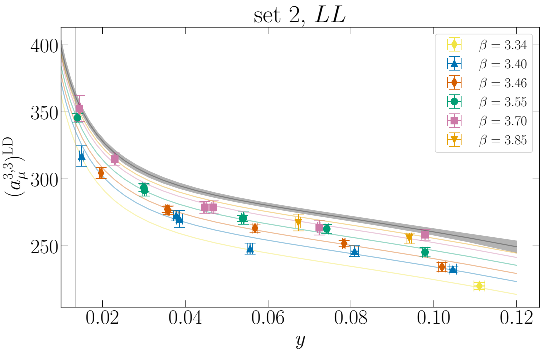

- Two new high-precision evaluations of the long-distance light-connected contribution [RBC/UKQCD, 2410.20590][Mainz, 2411.07969][SK at KEK].
- The main stepping stone towards a_μ^{hvp} at sub-percent precision.

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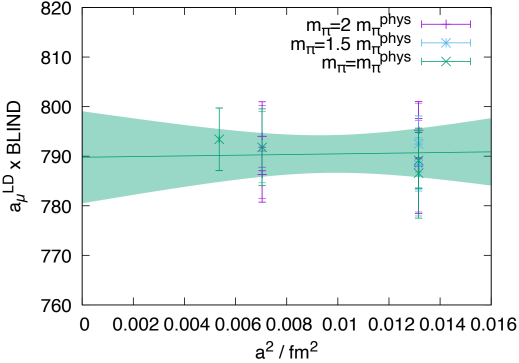

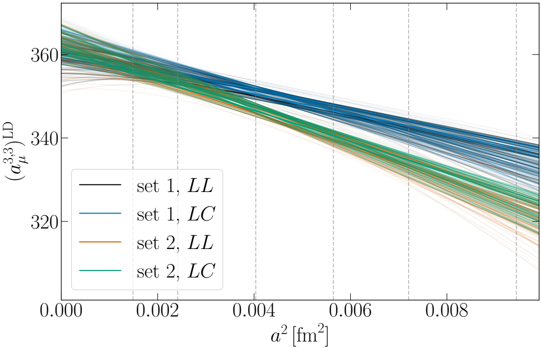

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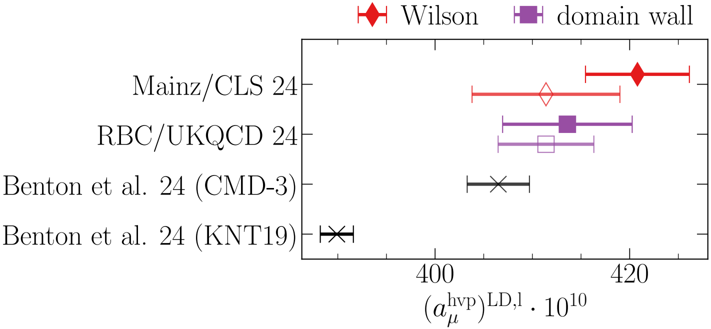

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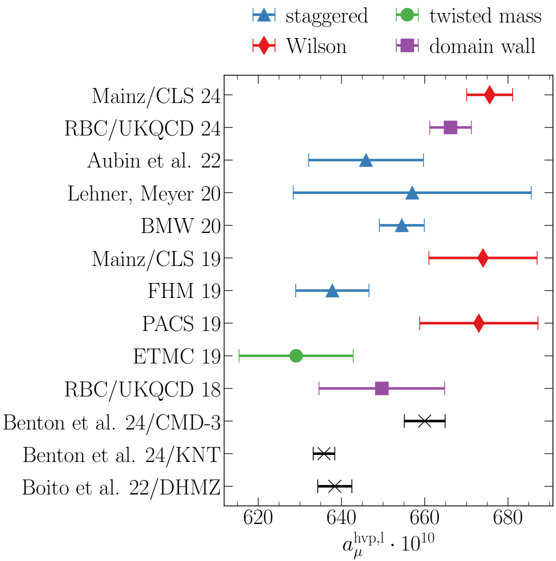
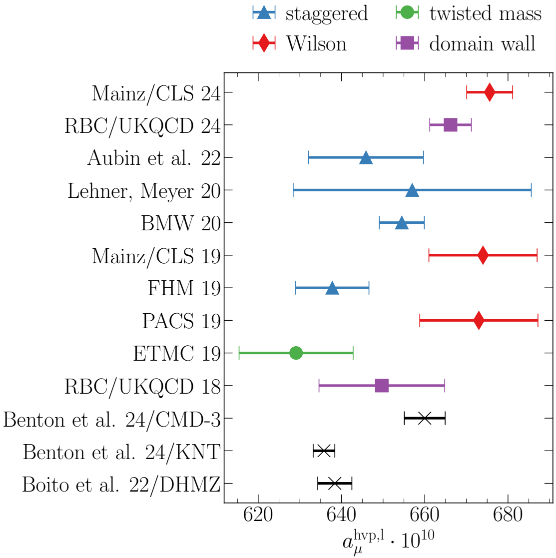
- The main stepping stone towards a_μ^{hvp} at sub-percent precision.
- Significant scheme dependence! Agreement in the same scheme for isoQCD.

THE LIGHT-CONNECTED CONTRIBUTION



- The light-connected contribution makes up for $> 90\%$ of a_μ^{hvp}.
- No consistent isoQCD scheme in this comparison!
- Unfortunately no update by BMW in [BMWc, 2407.10913]...

THE LIGHT-CONNECTED CONTRIBUTION



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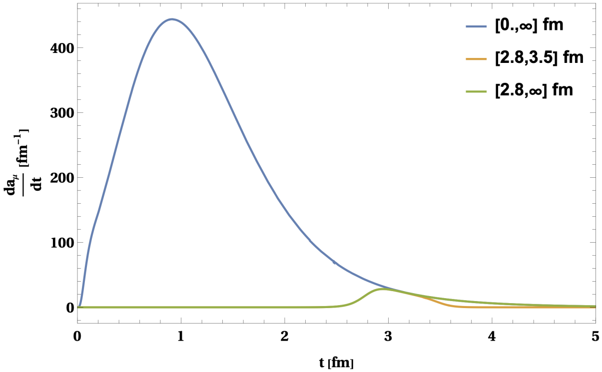
- Unfortunately no update by BMW in [BMWc, 2407.10913]...
- No clear tension when using CMD-3 for the data-driven estimate [Benton et al., 2411.06637].

THE LIGHT-CONNECTED CONTRIBUTION



[Lellouch at KEK]

- [BMWc, 2407.10913] uses a data-driven of the contribution beyond $2.8\,\mathrm{fm}$.
  - About 4% of a_μ^{hvp}.
  - Smaller statistical uncertainties.
  - Smaller finite-volume effects.
  - Smaller cutoff effects with staggered fermions.

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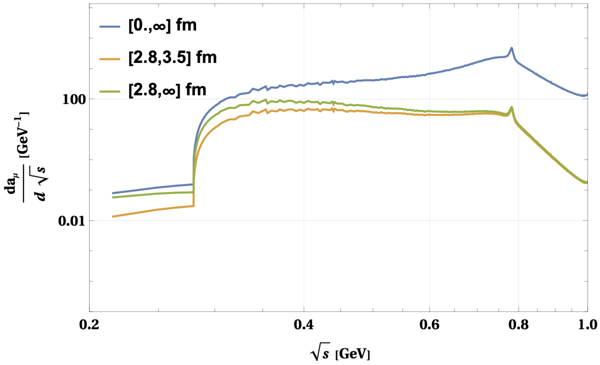

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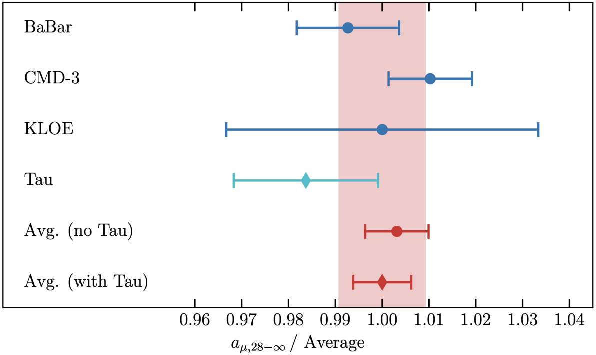

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  - About 4% of a_μ^{hvp}.
  - Smaller statistical uncertainties.
  - Smaller finite-volume effects.
  - Smaller cutoff effects with staggered fermions.
- Contribution below the ρ peak seems to be compatible across experiments.
- No updated pure lattice result for a_μ^{hvp} or $(a_\mu^{\mathrm{hvp}})^{(\mathrm{LD})}$.

a_μ^{hvp} ON THE LATTICE

ISOSPIN BREAKING EFFECTS

QED AND STRONG ISOSPIN BREAKING

Need to include $\mathrm{O}(\frac{m_\mathrm{u}-m_\mathrm{d}}{\Lambda_\mathrm{QCD}})$ and $\mathrm{O}(\alpha)$ effects for per-mil precision.

- Various ways to compute isospin breaking corrections:
  - **Perturbative expansion around isospin symmetric QCD** [RM123, 1303.4896].
  - Simulation of dynamical QCD+QED [CSSM/QCDSF/UKQCD] [RC*, 2212.11551].
  - Infinite volume QED [RBC/UKQCD, 1801.07224] [Biloshytskyi et al., 2209.02149]

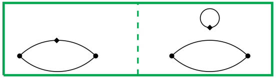
- Major challenge: Formulation of QED in a finite box.

- QED_L: Finite-volume corrections scale as $\mathrm{O}(1/L^3)$ [Bijnens et al., 1903.10591]
  $\rightarrow$ sufficient for the precision goal.

QED AND STRONG ISOSPIN BREAKING: RESULTS

Overview of published results - contributions to $a_\mu \times 10^{10}$

- Strong isospin breaking:
  Five groups agree within $1\,\sigma$.



6.60(63)(53)		BMW
10.6(4.3)(6.8)		RBC/UKQCD
6.0(2.3)		ETM
7.7(3.7)	**9.0(2.3)**	FHM
9.0(0.8)(1.2)		LM

BMW [Nature 593 (2021) 7857, 51-55]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
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Adapted from [V. Gülpers @ Lattice HVP workshop 2020]

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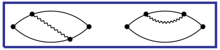
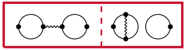


BMW	**$-1.23(40)(31)$**
RBC/UKQCD	**$5.9(5.7)(1.7)$**
ETM	**$1.1(1.0)$**



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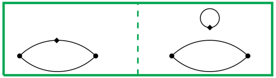
$-6.9(2.1)(2.0)$	RBC/UKQCD



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- QED: agreement on the total valence contribution.

Adapted from [V. Gülpers @ Lattice HVP workshop 2020]

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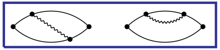
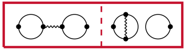
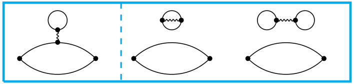


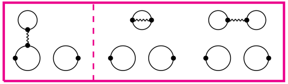
BMW	−1.23(40)(31)
RBC/UKQCD	5.9(5.7)(1.7)
ETM	1.1(1.0)



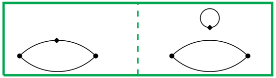
	−0.55(15)(10)	BMW
−6.9(2.1)(2.0)		RBC/UKQCD



−0.0093(86)(95)	0.37(21)(24)	BMW



0.011(24)(14)	−0.040(33)(21)	BMW



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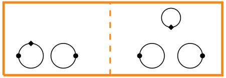

−4.67(54)(69)	BMW

BMW [Nature 593 (2021) 7857, 51-55]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
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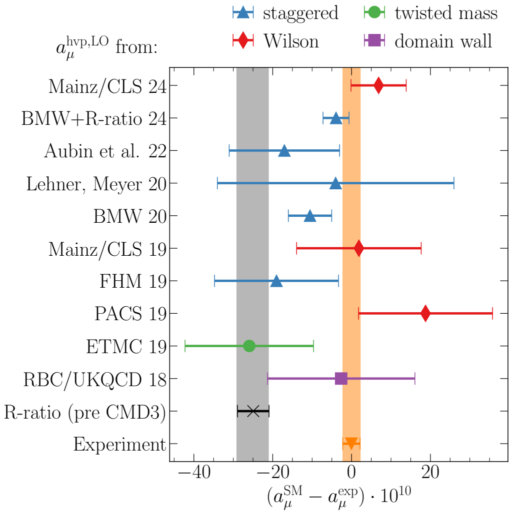
LM [Phys.Rev.D 101 (2020) 074515]

Adapted from [V. Gülpers @ Lattice HVP workshop 2020]

- Strong isospin breaking: Five groups agree within $1\,\sigma$.
- QED: agreement on the total valence contribution.
- One complete calculation [BMWc, 2002.12347]: $\delta a_\mu^{\text{hvp}} = 0.5(1.4) \cdot 10^{-10}$
- Work in progress:
  [Mainz, 2206.06582]
  [RBC/UKQCD, Lattice 2022]
  [BMWc, Lattice 2022]
  [FHM, 2212.12031]
  [Harris et al., 2301.03995]
  [Mainz, 2411.07969]

HADRONIC VACUUM POLARIZATION CONTRIBUTION TO THE MUON $g-2$



- Adding further contributions and isospin breaking corrections allows to add Mainz/CLS 24 to the overview for a_μ^{hvp} [Mainz, 2411.07969].
- More independent data points to follow.
- Going significantly below 1% uncertainty remains a challenge.
- Combination with data-driven estimates and MUonE in a few years?

CONCLUSIONS

- Final result of the FNAL $g-2$ experiment in spring 2025.
- New White Paper will provide an updated SM prediction before that.
- The main difference compared to 2020 will be a_μ^{hvp}
  - Currently no reliable estimate from data-driven dispersive methods.
  - Multiple high-precision lattice QCD results.
  - An *ab initio* lattice QCD result for a_μ^{hvp} can replace the old estimate.
- There is little hope for the anomaly in $(g-2)_\mu$ to survive.
- More work on the SM prediction is required to match the expected experimental precision.