Phenomenological aspects of modular symmetry

Kei Yamamoto

Hiroshima Institute of Technology → Iwate University

IWATE UNIVERSITY

Open Questions and Future Directions in Flavour Physics
Mainz Institute for Theoretical Physics
15 November 2024

Outline

Modular symmetry

Phenomenological aspects

SMEFT with modular symmetry

Kobayashi, Otsuka, Tanimoto, KY 2204.12325

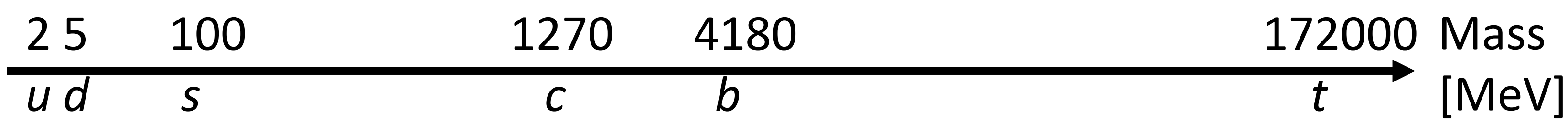
Baryon/Lepton-number violating operator

Kobayashi, Nishimura, Otsuka, Tanimoto, KY 2207.14014

Summary

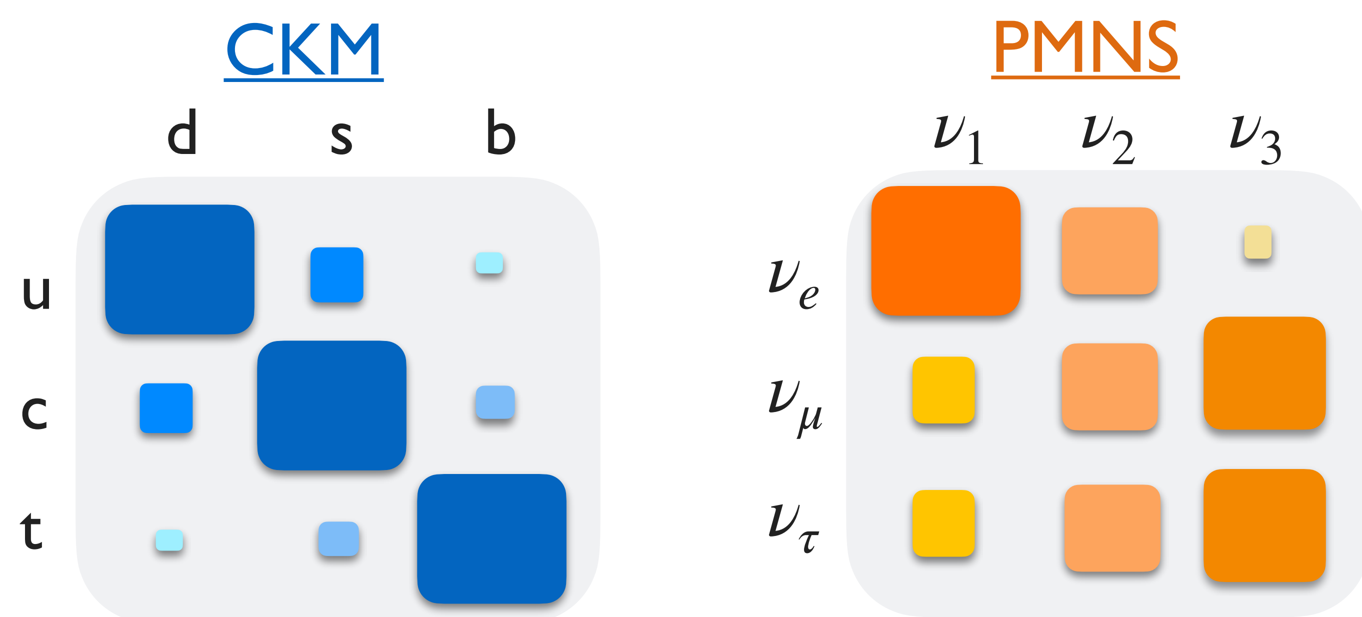
Flavor puzzle

Mass hierarchy puzzle



Flavor mixing puzzle

Small mixing in quarks



large mixing in neutrino

Flavor symmetry ?

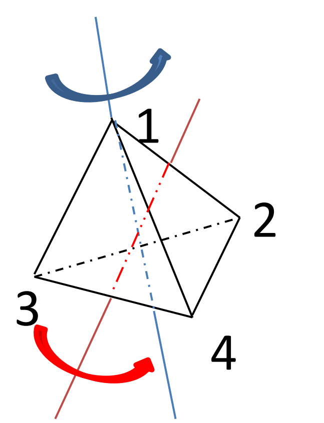
e.g. U(2) flavor symmetry

e.g. Discrete symmetry

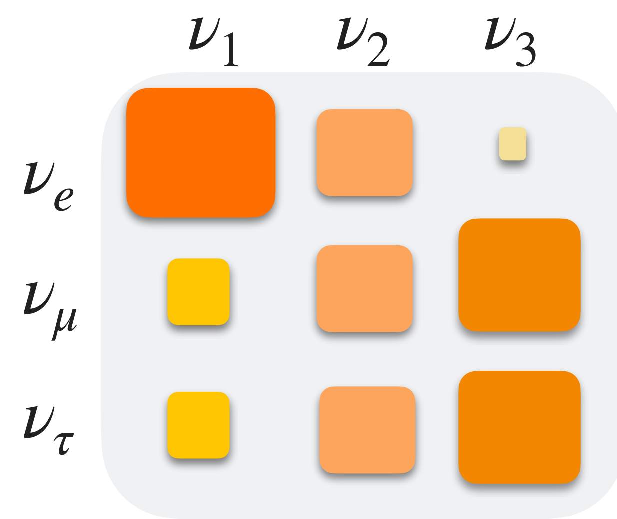
Flavor puzzle

Discrete symmetry has been studied well to describe large mixing angle in neutrino

e.g. A4 discrete symmetry
: Tetrahedral sym.



PMNS



large mixing
in neutrino

Recent hot topics:

Discrete symmetries $\simeq$ Modular symmetry

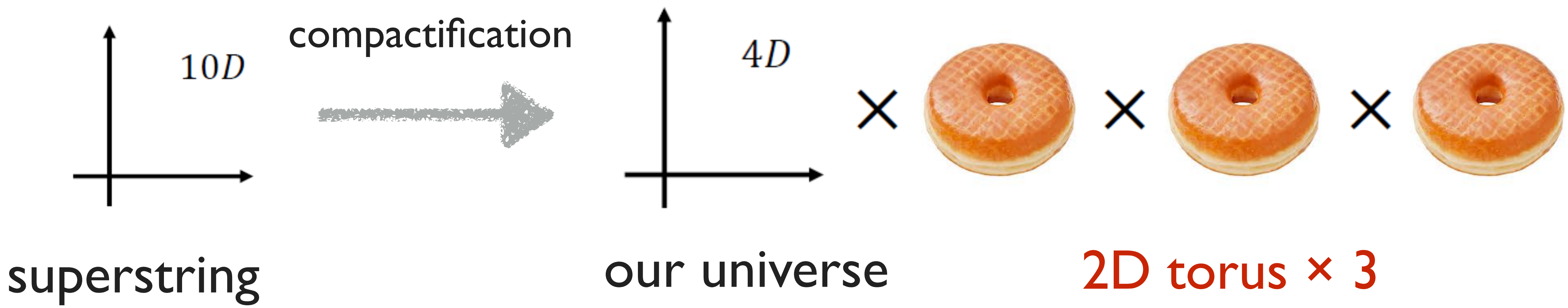
e.g. Discrete symmetry

recent progress → see Modular workshop@Mainz, May 2024

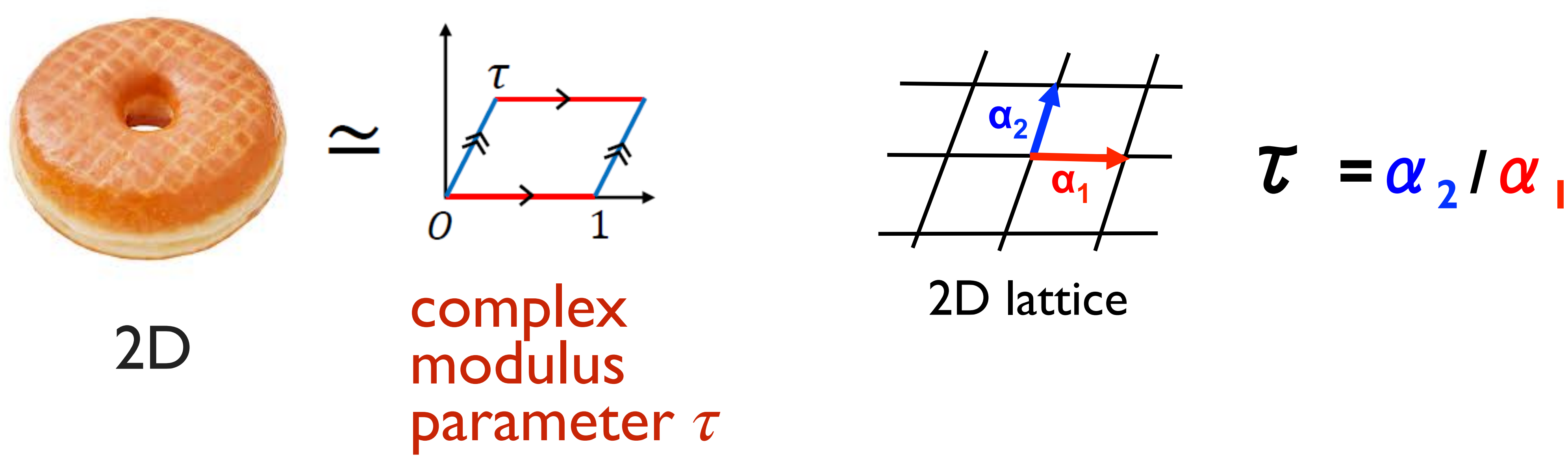
Modular symmetry

Modular group often appears in the superstring theory

Compactification of the superstring theory



Two dimensional torus is characterized by modulus τ



Modular symmetry

Modular transformation does not change the lattice $\tau = \alpha_2 / \alpha_1$

$$\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

modular transformation

(2D lattice)' (2D lattice)

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

$$SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a,b,c,d \in \mathbb{Z}, \quad ad - bc = 1 \right\}$$

The modular group is defined as the transformation group γ, generated by S and T

$$S : \tau \rightarrow -\frac{1}{\tau}$$

duality

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$T : \tau \rightarrow \tau + 1$$

Discrete shift symmetry

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Modular group Γ $\quad \Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}\}$

Modular symmetry

$$S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

Modular group $\Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}\}$

$$\Gamma_N \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}, T^N = \mathbb{I}\}$$

isomorphic
Modular symmetry $\simeq$ Discrete symmetry

$N = 2$ $\qquad \Gamma_2 \simeq S_3$

$N = 3$ $\qquad \Gamma_3 \simeq A_4$ ← focus on in this work

$N = 4$ $\qquad \Gamma_4 \simeq S_4$

$N = 5$ $\qquad \Gamma_5 \simeq A_5$

Modular symmetry

10D Superstring theory

↓ Compactification 4D our universe + 2D torus × 3

4D theory (SUSY) Γ_N symmetry (modular)

↓ Expectation value of modulus τ breaks the symmetry

Low scale phenomenology

SUSY breaking terms are invariant (covariant) under modular transformation in moduli-mediated SUSY breaking scenario T. Kobayashi, H. Otsuka [2108.02700]

We can consider modular invariant SMEFT $(\mu_{EW} < \mu < \mu_{NP})$ by supposing modular forms to be ***spurion***

→ see Ajdin's talk

A_4 symmetry

Non-Abelian discrete symmetry A_4 group could be adjusted to family symmetry:

The minimum group containing triplet

Irreducible representations: 1, 1”, 1’, 3 ← e_R , μ_R , τ_R , (e_L, μ_L, τ_L)

	L_L	$(e^c_R,\ \mu^c_R,\ \tau^c_R)$	H_d
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$

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k	2	(0, 0, 0)	0	2

modular form

k_i : modular weights

Effective theories with Γ_N symmetry

modular form

$$\mathscr{L}_{\text{eff}} \supset Y(\tau) H \phi^{(I)} \phi^{(J)}$$

Holomorphic functions which transform under modular trans., are called modular form with weight k

$$Y(\tau) \to (c\tau + d)^k \rho(\gamma) Y(\tau)$$

chiral superfield with modular weight k_I transforms as

$$\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

$\phi^{(I)}, f(\tau)$:representation of Γ_N

$\rho(\gamma), \rho^{(I)}(\gamma)$: unitary rep. matrix

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Automorphy factor $(c\tau + d)^k (c\tau + d)^{-k_I} (c\tau + d)^{-k_J} = (c\tau + d)^{k - k_I - k_J}$

vanishes if $k = k_I + k_J$ → Modular invariant

A_4 modular symmetry

Non-Abelian discrete symmetry A_4 group could be adjusted to family symmetry:

The minimum group containing triplet

Irreducible representations: 1, 1'', 1', 3 ← e_R , μ_R , τ_R , (e_L, μ_L, τ_L)

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modular form

k_i : modular weights

Effective theories with Γ_N symmetry

modular form

$$\mathscr{L}_{\rm eff} \supset Y(\tau) H \phi^{(I)} \phi^{(J)}$$

weight k for modular form: even for Γ_N

$$Y(\tau) \to (c\tau + d)^k \rho(\gamma) Y(\tau)$$

weight k_I for matter fields:

no restriction on the possible value, a priori

$$\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}$$

Automorphy factor $(c\tau + d)^k (c\tau + d)^{-k_I} (c\tau + d)^{-k_J} = (c\tau + d)^{k - k_I - k_J}$

vanishes if $k = k_I + k_J$ → Modular invariant

A_4 modular symmetry

F. Feruglio [1706.08749]

The holomorphic and anti-holomorphic modular forms with weight 2 compose the A_4 triplet

$$Y_{\mathbf{3}}^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \overline{Y_{\mathbf{3}}^{(2)}(\tau)} \equiv Y_{\mathbf{3}}^{(2)*}(\tau) = \begin{pmatrix} Y_1^*(\tau) \\ Y_3^*(\tau) \\ Y_2^*(\tau) \end{pmatrix}$$

Y_i (i=1,2,3) is a function of the modulus τ

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix} \qquad q = e^{2\pi i \tau}$$

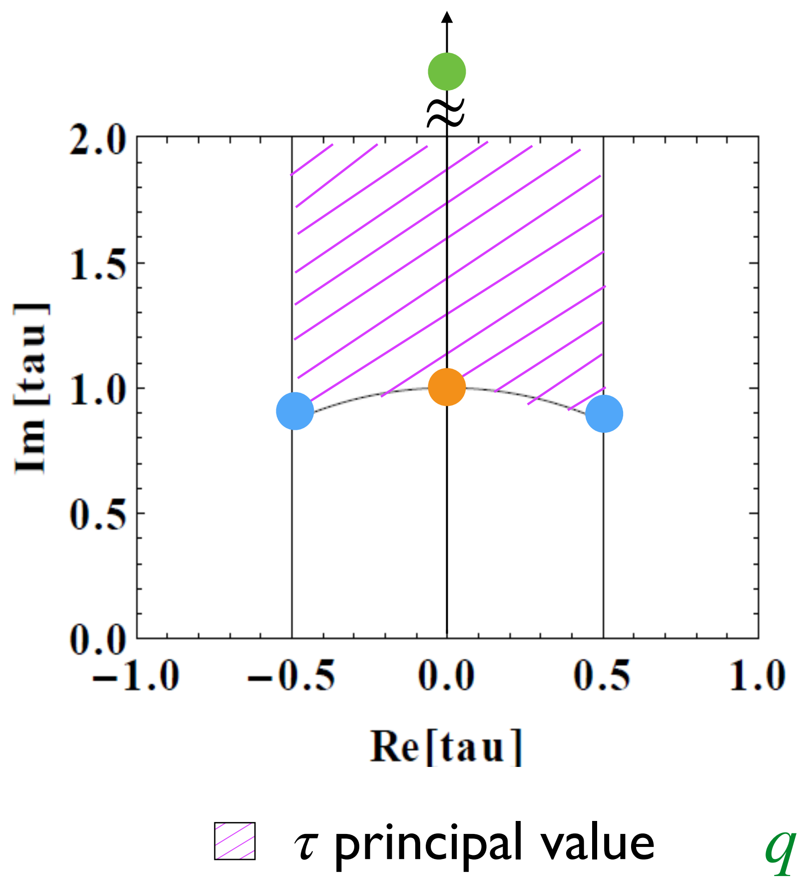
Once τ is determined, the Yukawa is fixed

Modular forms with higher weights k=4, 6 ... are constructed by them

A_4 modular symmetry

Modular symmetry is broken once τ is fixed

<u>Fixed point for τ</u> from the view point of the vacuum stability study



$q = e^{2\pi i\tau}$

- $\tau = \omega$ (ST symmetry) $\omega = \dfrac{-1 + i\sqrt{3}}{2}$
- $\tau = i$ (S symmetry) ← focus on in this talk
  (the successful lepton & quark mass matrix has been reproduced)
- $\tau = i\infty$ (T symmetry)

$$S : \tau \to -\frac{1}{\tau}$$

$$T : \tau \to \tau + 1$$

At exact fixed point, CP is not violated

→ need small deviation from these point : $\tau = \text{(fixed point)} + \epsilon$

phenomenologically $\mathcal{O}(\epsilon) \sim 10^{-2}$

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Summary

Modular symmetry in SMEFT

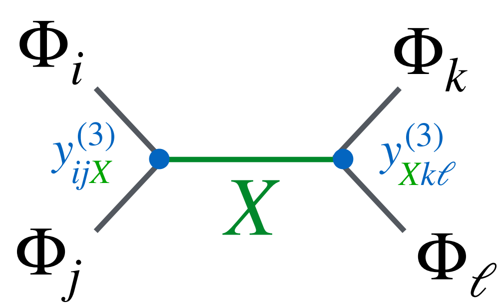
String Ansatz

T. Kobayashi, H. Otsuka [2108.02700]

n-point couplings y^n of matter fields are written by products of 3-point couplings

e.g. (4-point coupling)=(3-point coupling)×(3-point coupling)

$$y^{(4)}_{ijk\ell} = \sum_X y^{(3)}_{ijX} y^{(3)}_{Xk\ell}$$

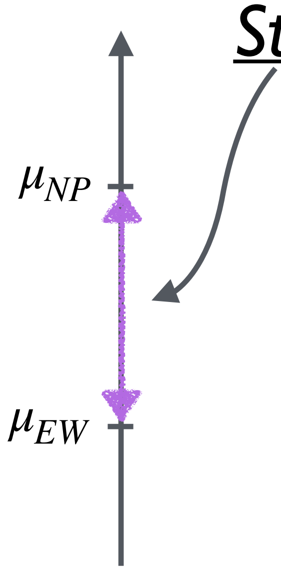


Φ : fermion

$y^{(3)}_{ijX}$: 3-point coupling

X : scalar or gauge

String compactifications leads to 4-dim low energy field theories with the specific structure

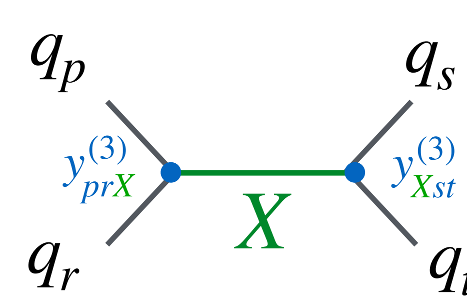


Standard model effective field theory (SMEFT) $\mu_{EW} < \mu < \mu_{NP}$

coefficients of SMEFT operators can be written in terms of 3-point coupling

e.g. *String Ansatz*

$$Q^{(1)}_{qq} \qquad (\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t) \propto y^{(3)}_{prX} y^{(3)}_{Xst}$$



Strategy

In this talk

- write down fermionic SMEFT operator so as to be invariant at A_4 and modular symmetry

  focus on $(\bar{L}R)$ bilinear structure in lepton sector

- expand modular forms $Y(\tau)$ at three fixed point, and then include small deviation : $\tau =$ (fixed point) $+\epsilon$

  - $\tau = \omega$ (ST symmetry)
  - $\tau = i$ (S symmetry)
  - $\tau = i\infty$ (T symmetry)

  focus on $\tau = i$ case

- diagonalize the mass matrix and move to mass eigenstate basis

- pheno. study

  $(g-2)_\mu$, Lepton flavor violation, EDM

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	(0, 0, 0)	0	2

※ γ_μ structure Γ is omitted

$$[\bar{L}_R L_L]$$

$$A_4: \quad \{1, 1'', 1'\} \otimes 3$$

$$k_I: \quad 0 \quad -2$$

not invariant both A_4 and modular

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
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modular form

※ γ_μ structure Γ is omitted

$$[\bar{L}_R L_L] \longrightarrow [\bar{L}_R Y(\tau_q) L_L]$$

$A_4 :\quad \{1,1'',1'\} \otimes 3 \qquad \{1,1'',1'\} \otimes 3 \otimes 3$

$k_I :\quad 0 \quad -2 \qquad 0 \quad 2 \quad -2$

not invariant both A_4 and modular

invariant

$(\bar{L}R)$ structure in the modular symmetry

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_e)$
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k	2	(0, 0, 0)	0	2

modular form

※ γ_μ structure Γ is omitted

$$[\bar{L}_R L_L] \rightarrow [\bar{L}_R Y(\tau_q) L_L]$$

decomposition

$$\{1,1'',1'\} \otimes 3 \otimes 3$$

$$= 1 \oplus 1'' \oplus 1' \oplus 3s \oplus 3a \quad \text{(i)}$$

$$1 \otimes 1 = 1 \text{ and } 1' \otimes 1'' = 1 \quad \text{(ii)}$$

The generators of A4 triplet

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},$$

$$\omega = e^{i\frac{2}{3}\pi}$$

(i)

$$Y(\tau) \otimes L_L = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}_{\mathbf{3}}$$

A_4 multiplication rule

$$= (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'} + (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$
$$+ (\cdots)_{\mathbf{3s}} + (\cdots)_{\mathbf{3a}}$$

(ii) $\bar{L}_R \otimes (Y(\tau) \otimes L_L)$

$$= \bar{e}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1 \otimes (1 \oplus 1' \oplus 1'') \to 1 \otimes 1} = \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}}$$

$$+\bar{\mu}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1' \otimes (1 \oplus 1' \oplus 1'') \to 1' \otimes 1''} +\bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

$$+\bar{\tau}_R \otimes (Y(\tau) \otimes L_L) \xrightarrow{1'' \otimes (1 \oplus 1' \oplus 1'') \to 1'' \otimes 1'} +\bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

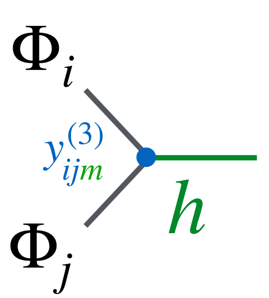
$$+ \gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

Same structure with mass matrix :

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

if virtual mode is only higgs



$\propto$ *Yukawa*

$$\alpha_e = c\alpha_{e(m)}, \quad \beta_e = c\beta_{e(m)}, \quad \gamma_e = c\gamma_{e(m)}$$

→ no flavor changing like $\mu \to e$

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

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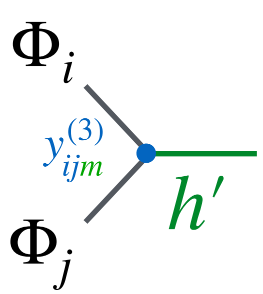
Same structure with mass matrix :

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

if there are additional unknown modes (e.g. multi Higgs modes)



~~$\propto$~~ Yukawa → it causes flavor violations

Suppose unknown mode contribution being small and couplings are Higgs-like

$$\alpha_e - \alpha_{e(m)} \ll \alpha_e, \quad \beta_e - \beta_{e(m)} \ll \beta_e, \quad \gamma_e - \gamma_{e(m)} \ll \gamma_e$$

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$$+ \gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

Same

$$\frac{\tilde{\beta}_e}{\tilde{\beta}_{e(m)}} = \frac{\tilde{\beta}_{e(m)} + c_\beta}{\tilde{\beta}_{e(m)}} = 1 + \frac{c_\beta}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_\beta \,,$$

$$\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)} + c_\alpha}{\tilde{\alpha}_{e(m)}} = 1 + \frac{c_\alpha}{\tilde{\alpha}_{e(m)}} \equiv 1 + \delta_\alpha \,,$$

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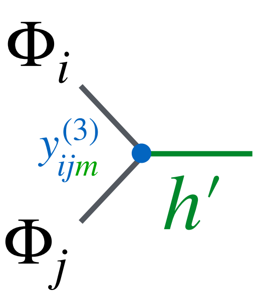
δ's are very small

if there are additional unknown modes (e.g. multi Higgs modes)



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at $\tau = i$ (S symmetry); Diagonalization

Results of $(\bar{L}R)$ structure in interaction basis

$\bar{R}L$ $\bar{L}R$	$\bar{\mu}_R\Gamma\tau_L$ $\bar{\mu}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\tau_L$ $\bar{e}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\mu_L$ $\bar{e}_L\Gamma\mu_R$
Coeff.	$\beta_e\, Y_3(\tau_e)$ $\gamma_e\, Y_2^*(\tau_e)$	$\alpha_e\, Y_2(\tau_e)$ $\gamma_e\, Y_3^*(\tau_e)$	$\alpha_e\, Y_3(\tau_e)$ $\beta_e\, Y_2^*(\tau_e)$

Insert holomorphic modular forms of weight 2 at $\tau = i$

$$Y(\tau_e = i) = Y_1(i) \begin{pmatrix} 1 \\ 1-\sqrt{3} \\ -2+\sqrt{3} \end{pmatrix}$$

Transrate

$$D_L \to D_L^S \equiv U_S\, D_L\,, \qquad \bar{D}_L \to \bar{D}_L^S \equiv \bar{D}_L U_S^\dagger\,,$$
$$E_L \to E_L^S \equiv U_S\, E_L\,, \qquad \bar{E}_L \to \bar{E}_L^S \equiv \bar{E}_L\, U_S^\dagger\,,$$

The flavor structure of the FC bilinear operators at $\tau = i$

$\tau = i + \epsilon$, then the left-handed fields are not yet the mass eigenstate, but close to it

using approximate behaviors

$$\frac{Y_2(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_1)\,(1-\sqrt{3})\,, \quad \frac{Y_3(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_2)\,(-2+\sqrt{3})\,, \quad \epsilon_1 = \frac{1}{2}\epsilon_2 \simeq 2.05\, i\,\epsilon$$

Okada and Tanimoto [2009.14242]

These approximate forms are agreement with exact numerical values within $0.1\,\%$ for $|\epsilon| \leq 0.05$

Mass eigenstate basis at $\tau = i$ and $\tau = i + \epsilon$

at $\tau = i$ (S symmetry); Diagonalization

Mass eigenstate basis at $\tau = i$ and $\tau = i + \epsilon$

$\bar\mu_R \Gamma \tau_L$ $\bar\mu_L \Gamma \tau_R$	$\bar e_R \Gamma \tau_L$ $\bar e_L \Gamma \tau_R$	$\bar e_R \Gamma \mu_L$ $\bar e_L \Gamma \mu_R$
$\frac{\sqrt{3}}{2}(\tilde\alpha_e + 2 s^e_{R23}\tilde\gamma_e)$ V $(\sqrt{3}s^e_{23L} + s^e_{12L}\lvert\epsilon_1^*\rvert)\tilde\gamma_e - \frac{3}{2}s^e_{R23}\tilde\alpha_e$	$\frac{\sqrt{3}}{2}(\tilde\beta_e - s^e_{12R}\tilde\alpha_e + 2(s^e_{R13} - s^e_{R12}s^e_{R23})\tilde\gamma_e)$ V $(\sqrt{3}s^e_{13L} + \lvert\epsilon_1^*\rvert)\tilde\gamma_e$	$\frac{3}{2}(\tilde\beta_e + s^e_{12R}\tilde\alpha_e)$ V $\frac{1}{2}(3s^e_{12L} - \sqrt{3}s^e_{13L} + 2\lvert\epsilon_1^*\rvert)\tilde\alpha_e$

$$s^e_{L12} \simeq -|\epsilon_1^*|\,, \qquad s^e_{L23} \simeq -\frac{\sqrt{3}}{4}\frac{\tilde\alpha^2_{e(m)}}{\tilde\gamma^2_{e(m)}}\,, \qquad s^e_{L13} \simeq -\frac{\sqrt{3}}{3}|\epsilon_1^*|\,,$$

$$s^e_{R12} \simeq -\frac{\tilde\beta_{e(m)}}{\tilde\alpha_{e(m)}}\,, \qquad s^e_{R23} \simeq -\frac{1}{2}\frac{\tilde\alpha_{e(m)}}{\tilde\gamma_{e(m)}}\,, \qquad s^e_{R13} \simeq -\frac{1}{2}\frac{\tilde\beta_{e(m)}}{\tilde\gamma_{e(m)}}$$

$$\tilde\alpha_{e(m)} = (6-3\sqrt{3})Y_1(i)\alpha_{e(m)},\ \tilde\beta_{e(m)} = (6-3\sqrt{3})Y_1(i)\beta_{e(m)} \text{ and } \tilde\gamma_{e(m)} = (6-3\sqrt{3})Y_1(i)\gamma_{e(m)}$$

parameters: *modulus τ, small deviation from fixed point ϵ_1, coefficients $\alpha_e, \beta_e, \gamma_e$*

Best fit values of parameters in A4 modular invariant model to realize lepton mass matrix, neutrino data

Okada and Tanimoto[2012.01688]

$$\tau = -0.080 + 1.007\,i\,, \quad |\epsilon_1| = 0.165\,, \quad \frac{\tilde\alpha_{e(m)}}{\tilde\gamma_{e(m)}} \simeq \frac{\tilde\alpha_e}{\tilde\gamma_e} = 6.82\times 10^{-2}\,, \quad \frac{\tilde\beta_{e(m)}}{\tilde\alpha_{e(m)}} \simeq \frac{\tilde\beta_e}{\tilde\alpha_e} = 1.50\times 10^{-2}$$

→ predict flavor observables

Strategy

In this talk

- write down fermionic SMEFT operator so as to be invariant at A_4 and modular symmetry

  focus on $(\bar{L}R)$ bilinear structure in lepton sector

- expand modular forms $Y(\tau)$ at three fixed point, and then include small deviation : $\tau =$ (fixed point) $+\epsilon$

  - $\tau = \omega$ (ST symmetry)
  - $\tau = i$ (S symmetry)
  - $\tau = i\infty$ (T symmetry)

  focus on $\tau = i$ case

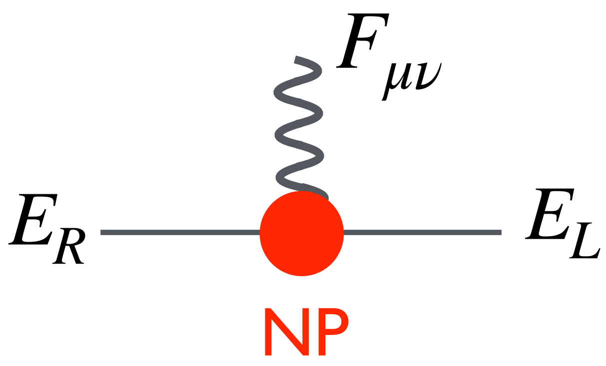
- diagonalize the mass matrix and move to mass eigenstate basis

- pheno. study

  $(g-2)_\mu$, Lepton flavor violation, EDM

Lepton dipole operator

Dipole operator



$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}} \overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu} \qquad \mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(\mathcal{C}'_{\underset{LR}{e\gamma}} \mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}} \mathcal{O}_{\underset{RL}{e\gamma}} \right)$$

Lepton flavor violation
$\mu \to e\gamma,\ \mu \to 3e,\ \mu N \to eN$

electron $(g-2)_e$, EDM d_e

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$\tau \to e\gamma$

$\tau \to \mu\gamma, \tau \to 3\mu$

tau $(g-2)_\tau$, , EDM d_τ

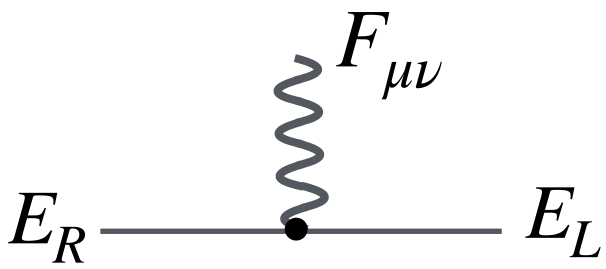
muon $(g-2)_\mu$, EDM d_μ

$[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}]$ Re → $(g-2)_\mu$

$[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}]$ Im → EDM d_μ

$(g-2)_\mu$ & $\mu \to e\gamma$

Dipole operator



$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\underset{LR}{e\gamma}}\mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}}\mathcal{O}_{\underset{RL}{e\gamma}}\right)$$

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L\sigma^{\mu\nu}E_R F_{\mu\nu}$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$(g-2)_\mu$

$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\text{Re}\left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}\right]$$

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \frac{1}{\Lambda^2}\text{Re}\left[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}\right] \approx 1.0 \times 10^{-5}\ \text{TeV}^{-2}$$

Lepton flavor violation $\mu \to e\gamma$

$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}}\frac{1}{\Lambda^4}\left(\left|\mathcal{C}'_{\underset{rs}{e\gamma}}\right|^2 + \left|\mathcal{C}'_{\underset{sr}{e\gamma}}\right|^2\right)$$

$$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}\ (90\%\ \text{C.L.})$$

MEG

$$\longrightarrow \frac{1}{\Lambda^2}\left|\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}\right| < 2.1 \times 10^{-10}\ \text{TeV}^{-2}$$

strong flavor alignment

$$\left|\frac{\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| < 2.1 \times 10^{-5}$$

specific flavor structure

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

Isidori, Pages and Wilsch [2111.13724]

$(g-2)_\mu$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde\beta_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde\alpha_e\,, \qquad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde\gamma_e$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde\alpha_e\left(1-\frac{\tilde\gamma_e}{\tilde\gamma_{e(m)}}\frac{\tilde\alpha_{e(m)}}{\tilde\alpha_e}\right)\,,$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde\beta_e\left(1+\frac{\tilde\alpha_e}{\tilde\alpha_{e(m)}}\frac{\tilde\beta_{e(m)}}{\tilde\beta_e}-2\frac{\tilde\beta_{e(m)}}{\tilde\beta_e}\frac{\tilde\gamma_e}{\tilde\gamma_{e(m)}}\right)$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde\beta_e\left(1-\frac{\tilde\alpha_e}{\tilde\alpha_{e(m)}}\frac{\tilde\beta_{e(m)}}{\tilde\beta_e}\right)\,,$$

$$\left|\frac{\mathcal{C}'_{\underset{e\mu}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| = \frac{\tilde\beta_e}{\tilde\alpha_e}\left|1-\frac{\tilde\alpha_e}{\tilde\alpha_{e(m)}}\frac{\tilde\beta_{e(m)}}{\tilde\beta_e}\right|$$

flavor alignment

$< 2.1\times 10^{-5}$

$$\frac{\tilde\beta_e}{\tilde\beta_{e(m)}} = \frac{\tilde\beta_{e(m)}+c_\beta}{\tilde\beta_{e(m)}} = 1+\frac{c_\beta}{\tilde\beta_{e(m)}} \equiv 1+\delta_\beta\,,$$
$$\frac{\tilde\alpha_e}{\tilde\alpha_{e(m)}} = \frac{\tilde\alpha_{e(m)}+c_\alpha}{\tilde\alpha_{e(m)}} = 1+\frac{c_\alpha}{\tilde\alpha_{e(m)}} \equiv 1+\delta_\alpha\,,$$
$$\frac{\tilde\gamma_e}{\tilde\gamma_{e(m)}} = \frac{\tilde\gamma_{e(m)}+c_\gamma}{\tilde\gamma_{e(m)}} = 1+\frac{c_\gamma}{\tilde\gamma_{e(m)}} \equiv 1+\delta_\gamma\,,$$

$$\longrightarrow \quad \left|1-\frac{\tilde\alpha_e}{\tilde\alpha_{e(m)}}\frac{\tilde\beta_{e(m)}}{\tilde\beta_e}\right| \simeq |\delta_\beta-\delta_\alpha| < 1.4\times 10^{-3}$$

without tuning between $\delta_{\alpha,\beta}$, $\quad |\delta_\alpha| < \mathcal{O}(10^{-3})\,, \quad |\delta_\beta| < \mathcal{O}(10^{-3})$

LFV $\tau \to \mu\gamma$ & $\tau \to e\gamma$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e\left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right)\,,$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}-2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right)$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right)\,,$$

the case that the additional unknown mode is the Higgs-like mode ($\delta_\alpha \sim \delta_\beta \sim \delta_\gamma$)

$$\frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\mu e}{e\gamma}}} = \frac{1}{\sqrt{3}} \times \mathcal{O}(1) \qquad\qquad \frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\tau\mu}{e\gamma}}} = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} \times \mathcal{O}(1) \sim 10^{-2}$$

BR($\tau \to \mu\gamma$) ≫ BR($\mu \to e\gamma$) ~ BR($\tau \to e\gamma$)

e.g. U(2) case BR($\tau \to \mu\gamma$) ≫ BR($\mu \to e\gamma$) ≫ BR($\tau \to e\gamma$)

LFV $\tau \to \mu\gamma$ & $\tau \to e\gamma$ & $\mu \to e\gamma$

Wilson coefficients in A4 modular symmetry in mass basis

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e\left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right)\,,$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}-2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right)$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e\left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right)\,,$$

the case that the additional unknown mode is the Higgs-like mode ($\delta_\alpha \sim \delta_\beta \sim \delta_\gamma$)

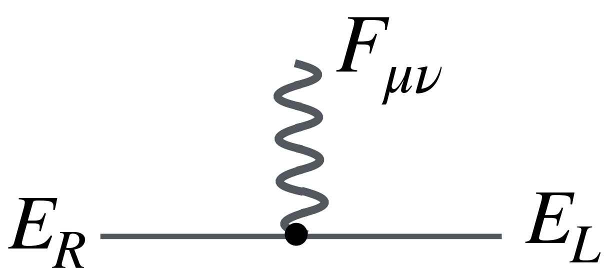
$$\frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\mu e}{e\gamma}}} = \frac{1}{\sqrt{3}} \times \mathcal{O}(1) \qquad\qquad \frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\tau\mu}{e\gamma}}} = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} \times \mathcal{O}(1) \sim 10^{-2}$$

BR($\tau \to \mu\gamma$) ≫ BR($\mu \to e\gamma$) ~ BR($\tau \to e\gamma$)

Since the present upper bounds of B($\tau \to e\gamma$) and B($\tau \to \mu\gamma$) are 3.3×10^{-8} and 4.4×10^{-8}, respectively, we expect the experimental test of this prediction for $\tau \to \mu\gamma$ in the future

EDM d_e & $(g-2)_\mu$

Dipole operator



$$\mathcal{L}_{\rm dipole} = \frac{1}{\Lambda^2}\left(\mathcal{C}'_{\underset{LR}{e\gamma}}\mathcal{O}_{\underset{LR}{e\gamma}} + \mathcal{C}'_{\underset{RL}{e\gamma}}\mathcal{O}_{\underset{RL}{e\gamma}}\right)$$

$$\mathcal{C}'_{\underset{LR}{e\gamma}} = \begin{pmatrix} \mathcal{C}'_{\underset{ee}{e\gamma}} & \mathcal{C}'_{\underset{e\mu}{e\gamma}} & \mathcal{C}'_{\underset{e\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\mu e}{e\gamma}} & \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} & \mathcal{C}'_{\underset{\mu\tau}{e\gamma}} \\ \mathcal{C}'_{\underset{\tau e}{e\gamma}} & \mathcal{C}'_{\underset{\tau\mu}{e\gamma}} & \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} \end{pmatrix}$$

$$\mathcal{O}_{\underset{LR}{e\gamma}} = \frac{v}{\sqrt{2}}\overline{E}_L\sigma^{\mu\nu}E_R F_{\mu\nu}$$

$$\mathcal{O}_{\rm edm} = -\frac{i}{2}\,d_e(\mu)\,\bar{e}\sigma^{\mu\nu}\gamma_5 e F_{\mu\nu}$$

electron EDM d_e

$$d_e = -\sqrt{2}\,\frac{v}{\Lambda^2}\,{\rm Im}\left[\mathcal{C}'_{\underset{ee}{e\gamma}}\right]$$

$$|d_e/e| \lesssim 1.1\times 10^{-29}\,{\rm cm} = 5.6\times 10^{-13}\,{\rm TeV}^{-1}$$

ACME

$$\rightarrow \quad \frac{1}{\Lambda^2}{\rm Im}\left[\mathscr{C}'_{\underset{ee}{e\gamma}}\right] < 1.6\times 10^{-12}\ {\rm TeV}^{-2}$$

Re vs. Im

$(g-2)_\mu$

$$\frac{1}{\Lambda^2}{\rm Re}\left[\mathscr{C}'_{\underset{\mu\mu}{e\gamma}}\right] \approx 1.0\times 10^{-5}\ {\rm TeV}^{-2}$$

with

coupling relation in A4 modular sym.

$$\frac{\mathcal{C}'_{\underset{ee}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}} = 2\frac{\tilde{\beta}_e}{\tilde{\alpha}_e}|\epsilon_1^*| \simeq 4.9\times 10^{-3} \rightarrow$$

$$\frac{1}{\Lambda^2}{\rm Re}\left[\mathscr{C}'_{\underset{ee}{e\gamma}}\right] = 4.9\times 10^{-8}\ {\rm TeV}^{-2}$$

EDM d_e & $(g-2)_\mu$ & $\mu \to e\gamma$

electron EDM d_e

$$\frac{1}{\Lambda^2}\mathrm{Im}\,[\mathscr{C}'_{\underset{ee}{e\gamma}}] < 1.6 \times 10^{-12}\ \mathrm{TeV}^{-2}$$

$(g-2)_\mu$ with coupling relation in A4 modular sym.

$$\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathscr{C}'_{\underset{ee}{e\gamma}}] = 4.9 \times 10^{-8}\ \mathrm{TeV}^{-2}$$

$$\frac{\mathrm{Im}\,[\mathscr{C}'_{\underset{ee}{e\gamma}}]}{\mathrm{Re}\,[\mathscr{C}'_{\underset{ee}{e\gamma}}]} \simeq (\mathrm{Im}\,\delta_\beta) < \frac{1.6\times 10^{-12}}{4.9\times 10^{-8}} = 3.3\times 10^{-5}$$

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*| = 3\,(1-\sqrt{3})\tilde{\beta}_{e(m)}(1+\delta_\beta)|\epsilon_1^*|$$

$$\mathrm{Im}\,[\mathcal{C}'_{\underset{ee}{e\gamma}}] \simeq 3\,(1-\sqrt{3})\tilde{\beta}_{e(m)}(\mathrm{Im}\,\delta_\beta)|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu e}{e\gamma}} \simeq \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_{e(m)}(\delta_\beta-\delta_\alpha)$$

suppose

$$|\mathrm{Im}\,\delta_\beta| \simeq |\delta_\beta| \text{ and } |\delta_\alpha| \simeq |\delta_\beta| \text{ (or } |\delta_\alpha| \ll |\delta_\beta| \text{)}$$

Upper bound for B(μ→eγ) $\mathcal{B}(\mu^+ \to e^+\gamma) < 2.3\times 10^{-16}$

Outline

Modular symmetry

Phenomenological aspects

SMEFT with modular symmetry

Kobayashi, Otsuka, Tanimoto, KY 2204.12325

Baryon/Lepton-number violating operator

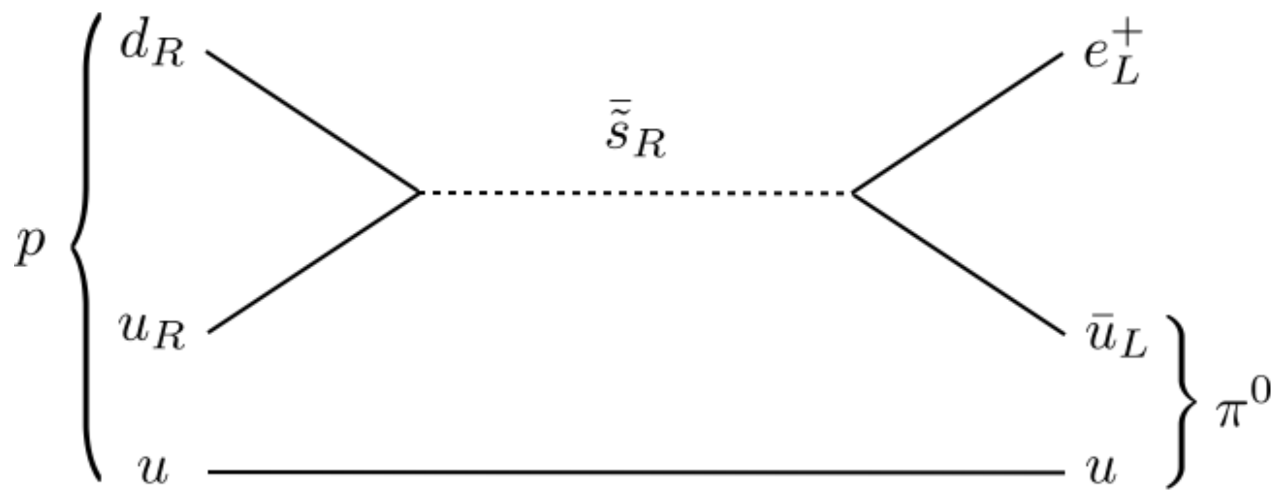
Kobayashi, Nishimura, Otsuka, Tanimoto, KY 2207.14014

Summary

Baryon/Lepton-number violating operators

SUSY extensions of the SM have Baryon/Lepton numbers breaking interactions →Proton decay

$$p \to \pi^0 e^+$$



→ assume R-parity

B/L num. violating ope. can be forbidden by modular symmetry

Baryon/Lepton-number violating operators

Yukawa/Higher-dim. ops. : even-modular weights ($k_Y \in 2Z$)

modular form $f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)$ ρ : unitaly matrix

for S transformation $f_i(S^2\tau) = (-1)^k \rho_{ij}(S^2) f_j(\tau)$

$(-1)^k \rho(S^2) = \mathbb{I}.$

(i) $k = \text{even}, \quad \rho(S^2) = \mathbb{I}, \quad \text{i.e.,} \quad \rho \in \Gamma_N,$

(ii) $k = \text{odd}, \quad \rho(S^2) = -\mathbb{I}, \quad \text{i.e.,} \quad \rho \in \Gamma'_N$

focus on Γ_N rather than its double cover Γ'_N

Baryon/Lepton-number violating operators

Superpotential

$$W = \sum_n Y_{I_1 \cdots I_n}(\tau)\, \Phi_{I_1} \cdots \Phi_{I_N}$$

Modular trf. : $\tau \to \gamma\tau = \dfrac{p\tau + q}{s\tau + t}$

Yukawa/Higher-dim. : $Y(\tau) \to (s\tau + t)^{k_Y} \rho(\gamma) Y(\tau)$

Matters ϕ_i :
（Modular wegiths (k_i)） $\phi_i \to (s\tau + t)^{-k_i} \rho_i(\gamma) \phi_i$

Modular invariance requires

modular weight $k_Y = \sum_i k_i$

Baryon/Lepton-number violating operators

Superpotential in MSSM

$$W = y^u_{ij} Q_i H_u \bar{U}_j + y^d_{ij} Q_i H_d \bar{D}_j + y^\ell_{ij} L_i H_d \bar{E}_j + y^n_{ij} L_i H_u N_j + m^n_{ij} N_i N_j + \mu H_u H_d$$

- holomorphic modular forms of even modular weight
- chiral supermultiplets (Q,U,D,L,E,N,H) have integer modular weight k

↓ modular invariance $k_Y = \sum_i k_i$ $\quad k_i$: modular weight

$$k^u_y - k_Q - k_{U,D} - k_{\text{Higgs}} = 0$$
$$k^d_y - k_Q - k_{U,D} - k_{\text{Higgs}} = 0$$
$$k^\ell_y - k_L - k_{E,N} - k_{\text{Higgs}} = 0$$
$$k^n_y - k_L - k_{E,N} - k_{\text{Higgs}} = 0$$

$$k^m - 2k_N = 0$$
$$k_\mu - 2k_{\text{Higgs}} = 0$$

↓ arrive at two constraints :

$$k_Q + k_{U,D} + k_{\text{Higgs}} = \text{even}$$
$$k_L + k_{E,N} + k_{\text{Higgs}} = \text{even}$$

Baryon/Lepton-number violating operators

Higher-dim. ops. : even-modular weights
→ operator with even-modular weights : ✓

When k_{Higgs} is even weight $(k_Q,\ k_{U,D},\ k_L,\ k_{E,N},\ k_{\text{Higgs}})$

(i) Others are even →B/L breaking

(ii) $k_Q, k_{U,D}$ is odd, $k_L, k_{E,N}$ is even →L breaking

✓: $k_L + k_Q + k_D = even + odd + odd = even$

×: $k_U + k_D + k_D = odd + odd + odd = odd$

(iii) $k_Q, k_{U,D}$ is even, $k_L, k_{E,N}$ is odd →B breaking

(iv) Others are odd →B/L breaking are prohibited

→Realization of R-parity(Z2 sym)
Possible in SU(5), SO(10) GUTs

	(i)	(ii)	(iii)	(iv)
Yukawa	✓	✓	✓	✓
$H_u H_d$	✓	✓	✓	✓
LH_u	✓	✓		
$LL\bar{E}$	✓	✓		
$LQ\bar{D}$	✓	✓		
$\bar{U}\bar{D}\bar{D}$	✓		✓	
$QQQL$	✓			✓
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓			✓
$QQQH_d$	✓		✓	
$Q\bar{U}\bar{E}H_d$	✓	✓		
LH_uLH_u	✓	✓	✓	✓
$LH_uH_dH_u$	✓	✓		
$\bar{U}\bar{D}^*\bar{E}$	✓	✓		
$H_u{}^*H_d\bar{E}$	✓	✓		
$Q\bar{U}L^*$	✓	✓		
$QQ\bar{D}^*$	✓		✓	

B and/or L violating operator

Summary

Flavor puzzle (mass and mixing in quark and lepton) might be controlled by flavor symmetry

→ Modular flavor symmetry

Discrete symmetry ≃ Modular symmetry

We discuss pheno aspects from Modular flavor symmetry

- with SMEFT, predict Lepton flavor observables
- Baryon/Lepton-number violating operator can be controlled with modular weight

Approach to other models with S_4, A_5 ...

to other flavor phenomena in the quark sector

$b \to s\gamma$ …