# **Phenomenological aspects of modular symmetry**

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Open Questions and Future Directions in Flavour Physics Mainz Institute for Theoretical Physics 15 November 2024



## **Modular symmetry**

## **Phenomenological aspects**

SMEFT with modular symmetry Kobayashi, Otsuka, Tanimoto, KY [2204.12325](https://arxiv.org/abs/2204.12325)

 **Baryon/Lepton-number violating operator**

Kobayashi, Nishimura,Otsuka, Tanimoto, KY [2207.14014](https://arxiv.org/abs/2207.14014)

**Summary**



### Flavor mixing puzzle



## **Flavor puzzle**

Discrete symmetry has been studied well to describe large mixing angle in neutrino  $T$  Traditional symmetry



### **Superstrip theory Modular symmetry Our universe is 4D**



**Our universe is 4D** Modular group often appears in the superstring theory

Compactification of the superstring theory **The superstring theory The superstring theory**  2D torus  $(T^2)$  is equivalent parallelogram with identifica



superweiget 4D effective Lagrame ans untegralD torus × 3.6D.

**1**  $\alpha'$ 

2D torus  $(T^2)$  is equivalent to  $hat{$  of the  $C = \int d^4x d^6x$ ,  $C = \int d^4x C$ **α1** Two dimensional torus is characterized by modulus

$$
\frac{d^{2} \sin^{2} \theta^{2}}{2} \approx \frac{1}{\sqrt{\frac{\tau}{c^{2}}} \sin \theta \cos \theta} \frac{\sqrt{\frac{\alpha_{2}}{a_{1}}}}{\sqrt{\frac{\alpha_{1}}{a_{1}}}} \tan \theta \cos \theta}
$$
\n
$$
\frac{d^{2} \sin \theta}{2} \tan \theta \cos \theta
$$
\n
$$
\frac{d^{2} \sin \theta}{2} \tan \theta \cos \theta
$$
\n
$$
\frac{d^{2} \sin \theta}{\sqrt{\frac{\alpha_{1}}{a_{1}}}} = \frac{\alpha_{2}}{\alpha_{1}} \frac{b}{\alpha_{2}} \left(\frac{\alpha_{2}}{\alpha_{1}}\right)
$$

### 2.1 Modular flavor symmetry 2.3 LR operators ..................................... 3 **Modular symmetry**



1 *b* ! *s* **Modular symmetry**

$$
S: \tau \to -\frac{1}{\tau} \qquad T: \tau \to \tau + 1
$$

**Modular group** 
$$
\Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I} \}
$$

$$
\Gamma_N \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}, T^N = \mathbb{I}\}
$$



$$
N = 2
$$
\n
$$
N = 3
$$
\n
$$
N = 4
$$
\n
$$
N = 5
$$
\n
$$
N = 5
$$
\n
$$
N = 5
$$
\n
$$
T_1 \approx 5
$$
\n
$$
T_2 \approx 5
$$
\n
$$
T_3 \approx 4
$$
\n
$$
S_4
$$
\n
$$
S_5 \approx A_5
$$

## **Modular symmetry**

10D Superstring theory

Compactification 4D our universe + 2D torus × 3

4D theory (SUSY)  $\qquad \qquad \Gamma_N$  symmetry (modular)

Expectation value of modulus τ breaks the symmetry

Low scale phenomenology

SUSY breaking terms are invariant (covariant) under modular transformation in moduli-mediated SUSY breaking scenario T. Kobayashi, H. Otsuka [2108.02700]

We can consider modular invariant SMEFT  $(\mu_{EW} < \mu < \mu_{NP})$  by supposing modular forms to be *spurion*

 $\rightarrow$  see Ajdin's talk

#### A<sub>4</sub> symmetry doublets are of a triplet of the *A*<sup>4</sup> group. The three right-handed quarks and charged leptons are  $\Delta$ , summatry matrices. Suppose that the suppose that the support of  $\Delta$ doublets are of a triplet of the *A*<sup>4</sup> group. The three right-handed quarks and charged leptons are

 $e_R$  ,  $\mu_R$  ,  $\tau_R$  ,  $(e_L, \mu_L, \tau_L)$ di↵erent singlets of *A*4. On the other hand, the Higgs doublets are supposed to be singlets of  $A$ -Abelian discrete symmetry  $A_4$  group could be adjusted to family symmetry. The minimum group containing triplet **Irreducible** die rent singlets of *A*4. On the other hand, the other hand, the Higgs doublets are supposed to be singlets of the supposed to be supposed to be supposed to be s **A4. The generic assignment of**  $A_4$  **group could be adjusted to failing symmetry**  $A_4$  in the field of the fields and  $\mathbf{r}_1$ The minimum group containing triplet Non-Abelian discrete symmetry  $A_4$  group could be adjusted to family symmetry: The minimum group containing triplet Irreducible representations: 1, 1", 1', 3



#### leads to the summatry matrix matrix matrix matrix  $\mathbf{w}$ doublets are of the three right-handed quarks and charged leptons are right-handed quarks and charged leptons are  $\frac{1}{2}$  $\Delta$  modular symmetry down the triplet of the three right-handed quarks are right-handed quarks and charged leptons are  $\overline{a}$ **A4 modular symmetry**

die die rente synglets of  $A$  and  $A$  are supposed to be singlets for  $A$  to concern the supposed to  $B$  $A_{4}$  and the generic assignment of  $A_{4}$  and  $B_{4}$  could be adjusted to failing symmetry. The minimum group containing triplet **Irreducible** di↵erent singlets of *A*4. On the other hand, the Higgs doublets are supposed to be singlets of  $\mathbf{A}_4$   $\mathbf{B}_1$  and  $\mathbf{B}_2$  are presented in the field  $\mathbf{A}_4$   $\mathbf{B}_1$  and  $\mathbf{B}_2$  are and  $\mathbf{B}_3$  are presented in the fields  $\mathbf{B}_4$ The minimum group containing triplet  $\mathcal{L}(\mathcal{L})$ Non-Abelian discrete symmetry  $A_4$  group could be adjusted to family symmetry: The minimum group containing triplet Irreducible representations: 1, 1", 1', 3  $e_R$ ,  $\mu_R$ ,  $\tau_R$ ,  $(e_L, \mu_L, \tau_L)$ 



Effective theories with  $\Gamma_N$  symmetry

leptons, down-type  $\mathsf{modular}$  forms. leptons, down-type Higgs doublet and the modular forms.

 ${\mathscr L}_{\rm eff}$  Modulz $\phi$  transformation

 $E$ chiral superfield with modular weight

*ρ*(*γ*), *ρ*(*I*)

Holomorphic functions which det*i*sad bombular t

fransform under modular trans., are  $k$  transforms as  $k$  transforms as

[ *E*¯*RE<sup>R</sup>* ][ *D*¯ *<sup>R</sup>D<sup>R</sup>* ] : *Qed , k* transforms as *<sup>I</sup>* called modular for Finaw Pulsy Weight's Antheisen Physelete B2B3,46,148989)

$$
\frac{\partial \mathcal{U}(\mathbf{w})}{\partial \mathbf{w}} = \text{Re}(\text{Im} \mathbf{w}) + \text{Re}(\text{Im} \mathbf{w}) \cdot \text{Im}(\mathbf{w}) \cdot \text{Im}(\mathbf{w
$$

 $f(t) = \int_{0}^{t} f(t) dt$ 

#### leads to the summatry matrix matrix matrix matrix  $\mathbf{w}$ doublets are of the three right-handed quarks and charged leptons are right-handed quarks and charged leptons are  $\frac{1}{2}$  $\Delta$  modular symmetry down the triplet of the three right-handed quarks are right-handed quarks and charged leptons are  $\overline{a}$ **A4 modular symmetry**

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Effective theories with  $\Gamma_N$  symmetry

leptons, down-type  $\mathsf{modular}$  forms. leptons, down-type Higgs doublet and the modular forms.

`*<sup>q</sup> , Q*(3)

`*<sup>q</sup> ,*

 ${\mathscr L}_{\rm eff}$  Modulz $\phi$  transformation

Holomorphic functions which  $\frac{1}{2}$ **E**  $\begin{array}{ccc} \text{called modular form with weight } k \ \text{for } k \end{array}$ 

$$
Y(\tau) \to (c\tau + d)^k, \quad f_i(\tau) \phi^{(I)} \phi^{(J)} H \qquad \phi^{(I)} \qquad \phi^{(I)}
$$

**E**<br> **E** *E E k***l** *Chiral superfield with modular weight*  $k_I$  transforms as  $k$  transforms as  $k$  transforms as

$$
\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}
$$

*ρ*(*γ*), *ρ*(*I*)

[ *E*¯*LE<sup>R</sup>* ][ *D*¯ *<sup>R</sup>D<sup>L</sup>* ] : *Q*`*edq ,* [ *E*¯*LE<sup>R</sup>* ][ *D*¯ *<sup>R</sup>D<sup>L</sup>* ] : *Q*`*edq ,* vanishes if  $k = k_1 + k_2$   $\rightarrow$  Modular invariant  $k$ combination of Dirac matrices, color and *SU*(2)*<sup>L</sup>* generators, which play no role as far as the combination of Dirac matrices, color and *SU*(2)*<sup>L</sup>* generators, which play no role as far as the Automorphy factor Automorphy factor

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Effective theories with  $\Gamma_N$  symmetry

leptons, down-type  $\mathsf{modular}$  forms. leptons, down-type Higgs doublet and the modular forms.

 ${\mathscr L}_{\rm eff}$  Modulz $\phi$  transformation

[ *E*¯*LE<sup>L</sup>* ][ *D*¯*LD<sup>L</sup>* ] : *Q*(1) `*<sup>q</sup> , Q*(3)  $\bm{F}$  and  $\bm{F}$  are  $\bm{F}$  . The  $\bm{F}$  is  $\bm{F}$  is  $\bm{F}$  and  $\bm{F}$  is  $\bm{F}$  and  $\bm{F}$ weight *k* for modular form:

 $\Gamma$  **E**<sup>*l*</sup> **D**</del> *Meight*  $k_I$  *for matter fields:*  $\mathbf{r}$   $\mathbf{I}$   $_N$   $\qquad$   $\qquad$   $\qquad$  no restriction o `*<sup>q</sup> ,* even for  $\Gamma_N$  no restriction on the possible value, a priori S . Ferrara, D. Lust, A

$$
Y(\tau) \to (c\tau + d)^k \int_{\tau}^{f_i(\tau)} \phi^{(I)} \phi^{(J)} H \qquad \qquad \phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}
$$

$$
{}^{(J)}H \qquad \qquad \phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}
$$

*ρ*(*γ*), *ρ*(*I*)

[ *E*¯*LE<sup>R</sup>* ][ *D*¯ *<sup>R</sup>D<sup>L</sup>* ] : *Q*`*edq ,* [ *E*¯*LE<sup>R</sup>* ][ *D*¯ *<sup>R</sup>D<sup>L</sup>* ] : *Q*`*edq ,* vanishes if  $k = k_1 + k_2$   $\rightarrow$  Modular invariant  $k$ combination of Dirac matrices, color and *SU*(2)*<sup>L</sup>* generators, which play no role as far as the combination of Dirac matrices, color and *SU*(2)*<sup>L</sup>* generators, which play no role as far as the Automorphy factor Automorphy factor

### forms have been explicitly given [20] in the symmetric base of the *A*<sup>4</sup> generators *S* and *T* for the A<sub>4</sub> modular symmetry

The holomorphic and anti-holomorphic modular forms with weight 2 compose the *A*<sup>4</sup> triplet as: The holomorphic and anti-holomorphic modular forms with weight 2 compose the  $A_4$  triplet

$$
Y_3^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \overline{Y_3^{(2)}(\tau)} \equiv Y_3^{(2)*}(\tau) = \begin{pmatrix} Y_1^*(\tau) \\ Y_3^*(\tau) \\ Y_2^*(\tau) \end{pmatrix}
$$

 $Y_i$  (i=1.2.3) is a function of the modulus  $\tau$   ${\rm Y}_1$  (*i*=1,2,3) is a function of the modulus  $\tau$ 

$$
\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix} \qquad q = e^{2\pi i \tau}
$$

*V*<sub>1</sub> the Yukawa is fixed *τ* is determined, the Yukawa is fixed  $Q_{\text{RCA}}$   $\tau$  is dotormined the Vulcawe is fixed Once *τ* is determined, the Yukawa is fixed

*Nar* forms with higher weights k=4, 6 8<sup>11</sup> ICS K<sup>-</sup> 7, O ... al **e constructe** *, by dieni*  $M_{\odot}$  different singlets. The light singlets as follows:  $\frac{1}{2}$ Modular forms with higher weights k=4, 6 ... are constructed by them

### **A4 modular group ( <sup>3</sup> ). Taking T3=1, we get A4 modular symmetry**

**Modular symmetry is broken once τ is fixed** 

Fixed point for *τ* **Fixed point for**  $\tau$  from the view point of the vacuum stability study



At exact fixed point, CP is not violated  $\rightarrow$  need small deviation from these point :  $\tau =$  (fixed point)  $+ \epsilon$ 

phenomenologically  $\mathcal{O}(\epsilon) \sim 10^{-2}$ 



## **Modular symmetry**

## **Phenomenological aspects**

 **SMEFT with modular symmetry** Kobayashi, Otsuka, Tanimoto, KY [2204.12325](https://arxiv.org/abs/2204.12325)

 **Baryon/Lepton-number violating operator**

**Summary**

### **Modular symmetry in SMEFT**  $\frac{1}{2}$  *DD* ( $\frac{1}{2}$  *Dd* ( $\frac{1}{2}$  *Dd* ( $\frac{1}{2}$  *dd*<sub> ( $\frac{1}{2}$  *dd*</sub></sub></sub></sub></sub></sub></sub></sub></sub>

### **String Ansatz**

T. Kobayashi, H. Otsuka [2108.02700]  $Q$ <sup>*x*</sup>(*z*) **I***H* (*Q*<sup>tsuk</sup>)<sup>2</sup> <sup>I</sub><sub>2</sub> IO<sub>8</sub> 027001</sup>

*QdB* (¯*qp*σ*<sup>µ</sup>*<sup>ν</sup>*dr*)*HBµ*<sup>ν</sup> n-point couplings y<sup>n</sup> of matter fields are written by products of 3-point couplings



String compactifications leads to 4-dim low energy field theories with the specific structure

 $\frac{\text{Standard model effective field theory (SMEFT) }}{\mu_{EW}} < \mu < \mu_{NP}$  $\mu_{NP}$   $\downarrow$   $\neq$  coefficients of SMEFT operators can be written in terms of 3-point  $\alpha$  $\left[ \begin{array}{cc} Q_{qq}^{(1)} & (\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t) & \propto y_{prX}^{(3)} y_{Nst}^{(3)} & X \end{array} \right]$  $q<sub>r</sub>$  $\mathbf{q}_t$  $\mu_{EW}$  $\mu_{NP}$ coefficients of SMEFT operators can be written in terms of 3-point coupling  $\propto y_{prX}^{(3)}y_{Xst}^{(3)}$ e.g. *String Ansatz qp qr X*  $q_{s}$  $q_t$  $y_{prX}^{(3)}$   $y_{Xst}^{(3)}$  $y_{Xst}^{(3)}$ 

## **Strategy**

 $\bullet \tau = i\infty$  *(T symmetry)* 

 write down fermionic SMEFT operator so as to be invariant at  $A_4$  and modular symmetry

focus on  $(LR)$  bilinear structure in lepton sector

 $\bullet \tau = \omega \quad (ST \text{ symmetry})$  $\bullet \tau = i$  (*S* symmetry) expand modular forms  $Y(\tau)$  at three fixed point, and then include small deviation :  $\tau =$  (fixed point)  $+ \epsilon$ 

focus on  $\tau = i$  case

diagonalize the mass matrix and move to mass eigenstate basis

pheno. study

(*g* − 2)*μ*, Lepton flavor violation, EDM

#### (*L*¯*R*) **structure in the modular symmetry**  $\alpha$  is the modular symmetry  $\overline{\phantom{a}}$ *A*4. The generic assignments of representations and modular weights to the fields are presented in  $T(R)$  structure in the modular symmetry



※  $\gamma_{\mu}$  structure  $\Gamma$  is omitted  $x \ y$  structure  $\Gamma$  is omitted

 $A_4: \{1,1'',1'\} \otimes 3$  $[\bar{L}_R L_L]$  $\overline{I}$   $I$   $I$  $\bar{I}$   $\bar{I}$   $\bar{I}$  $k_1: 0 -2$ 

> not invariant both  $A_4$  and modular

#### (*L*¯*R*) **structure in the modular symmetry**  $\alpha$  is the modular symmetry  $\overline{\phantom{a}}$ *A*4. The generic assignments of representations and modular weights to the fields are presented in  $T(R)$  structure in the modular symmetry



 $A_4: \{1,1'',1'\}\otimes 3 \{1,1'',1'\}\otimes 3 \otimes 3$  $k_1: 0 -2$  $[\bar{L}_R L_L]$   $\longrightarrow$   $[\bar{L}_R Y(\tau_q) L_L]$  modular form Table 1: The assignment of *A4 v* structure  $\Gamma$  is omitted [ *E*¯*LE<sup>L</sup>* ][ *D*¯*LD<sup>L</sup>* ] : *Q*(1)  $0 \t 2 \t -2$ `*<sup>q</sup> , Q*(3)

※  $\gamma_{\mu}$  structure  $\Gamma$  is omitted

not invariant both  $A_4$  and modular

invariant  $\frac{1}{2}$  invariant  $\mathbb{E}\left[\mathbf{E}\left(\mathbf{E}\right)\mathbf{E}\right]$  :  $\mathbf{E}\left(\mathbf{E}\right)$  :  $\mathbf{E}\$ 

#### (*L*¯*R*) **structure in the modular symmetry**  $\alpha$  is the modular symmetry  $\overline{\phantom{a}}$ *A*4. The generic assignments of representations and modular weights to the fields are presented in  $T(R)$  structure in the modular symmetry



## (*L*¯*R*) **structure in the modular symmetry**

 $[\bar{L}_R \otimes Y(\tau) \otimes L_L]_1 = \alpha_e \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_1 + \beta_e \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_1$  $+ \gamma_e \bar{\tau}_R \otimes (Y_3 \tau_I + Y_1 \mu_I + Y_2 e_I)_{1'}$  $=(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R)$ *α<sup>e</sup>* 0 0 0 *β<sup>e</sup>* 0 0 0 *γ<sup>e</sup> Y*<sub>1</sub>(*τ*) *Y*<sub>3</sub>(*τ*) *Y*<sub>2</sub>(*τ*) *Y*<sub>2</sub>(*τ*) *Y*<sub>1</sub>(*τ*) *Y*<sub>3</sub>(*τ*)  $Y_3(\tau)$   $Y_2(\tau)$   $Y_1(\tau)$  (*Y eL μL τL*  $\int$ 

Same structure with mass matrix :

$$
M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}
$$

if virtual mode is only higgs

 $\rightarrow$  no flavor changing like  $\mu \rightarrow e$  $\Phi_{i}$  $\Phi_j$ *h*  $y_{ijm}^{(3)}$  $\alpha_e$ <sup>(3)</sup>  $\alpha$   $\alpha$   $\alpha$   $\alpha$   $\alpha$  $= c\alpha_{e(m)}, \ \beta_e = c\beta_{e(m)}, \ \gamma_e = c\gamma_{e(m)}$ 

 $\overline{a}$ 

## (*L*¯*R*) **structure in the modular symmetry**

 $[\bar{L}_R \otimes Y(\tau) \otimes L_L]_1 = \alpha_e \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_1 + \beta_e \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_1$  $+ \gamma_e \bar{\tau}_R \otimes (Y_3 \tau_I + Y_1 \mu_I + Y_2 e_I)_{1'}$  $=(\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R)$ *α<sup>e</sup>* 0 0 0 *β<sup>e</sup>* 0 0 0 *γ<sup>e</sup> Y*<sub>1</sub>(*τ*) *Y*<sub>3</sub>(*τ*) *Y*<sub>2</sub>(*τ*) *Y*<sub>2</sub>(*τ*) *Y*<sub>1</sub>(*τ*) *Y*<sub>3</sub>(*τ*)  $Y_3(\tau)$   $Y_2(\tau)$   $Y_1(\tau)$  (*Y eL μL τL*  $\int$ 

Same structure with mass matrix :

$$
M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}
$$

if there are additional unknown modes (e.g. multi Higgs modes)

$$
\Phi_{i}_{ijm}^{(3)} \leftarrow \mathcal{L}^2
$$
\n
$$
\Phi_{j}^{(3)} \leftarrow \mathcal{L}^2
$$
\n
$$
\Phi_{j}
$$

*y*(*n*) = (*y*(3))*<sup>n</sup>*−<sup>2</sup>. Thus, the symmetries in 3-point couplings are still symmetries even for higher-Suppose unknown mode contribution being small and couplings are Higgs-like

> $\sim$   $\gamma$  $s$  is the string scale, but it holds at the low-energy scale. This is because new operators appear- $\alpha_e - \alpha_{e(m)} \ll \alpha_e, \quad \beta_e - \beta_{e(m)} \ll \beta_e, \quad \gamma_e - \gamma_{e(m)} \ll \gamma_e$

3-point couplings. This structure in the string-derived low-energy effective field theory meets the

#### (*L*¯*R*) **structure in the modular symmetry**  $\overline{\phantom{a}}$  *C*0 *e* 1  $\overline{\phantom{a}}$ in the contract of the contrac ˜ *e* i<br>L *e* in the m  $\overline{\mathsf{K}}$  $\mathbf{r}$

$$
[\bar{L}_R \otimes Y(\tau) \otimes L_L]_1 = \alpha_e \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_1 + \beta_e \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_1
$$
  
 
$$
+ \gamma_e \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_1
$$

$$
\text{Same} \left( \begin{array}{c} \tilde{\beta}_e \\ \tilde{\beta}_{e(m)} = \frac{\tilde{\beta}_{e(m)} + c_{\beta}}{\tilde{\beta}_{e(m)}} = 1 + \frac{c_{\beta}}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_{\beta} \,, \\ \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)} + c_{\alpha}}{\tilde{\alpha}_{e(m)}} = 1 + \frac{c_{\alpha}}{\tilde{\alpha}_{e(m)}} \equiv 1 + \delta_{\alpha} \,, \qquad \text{for a very small} \\ \frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} = \frac{\tilde{\gamma}_{e(m)} + c_{\gamma}}{\tilde{\gamma}_{e(m)}} = 1 + \frac{c_{\gamma}}{\tilde{\gamma}_{e(m)}} \equiv 1 + \delta_{\gamma} \,, \end{array} \right)
$$

if there are additional unknown modes (e.g. multi Higgs modes)

$$
\Phi_{ij}^{(3)}(x)
$$
\n
$$
\Phi_{ij}^{(3)}(x)
$$
\n
$$
\Phi_{ij}^{(4)}(x)
$$
\n
$$
\Phi_{ij}^{(5)}(x)
$$
\n
$$
\Phi_{ij}^{(5)}(x)
$$
\n
$$
\Phi_{ij}^{(6)}(x)
$$

*y*(*n*) = (*y*(3))*<sup>n</sup>*−<sup>2</sup>. Thus, the symmetries in 3-point couplings are still symmetries even for higher-Suppose unknown mode contribution being small and couplings are Higgs-like

$$
\alpha_e - \alpha_{e(m)} \ll \alpha_e, \quad \beta_e - \beta_{e(m)} \ll \beta_e, \quad \gamma_e - \gamma_{e(m)} \ll \gamma_e
$$

3-point couplings. This structure in the string-derived low-energy effective field theory meets the

## **Strategy**

 $\bullet \tau = i\infty \ \ (T \text{ symmetry})$ 

 write down fermionic SMEFT operator so as to be invariant at  $A_4$  and modular symmetry

 $\bullet \tau = \omega \quad (ST \text{ symmetry})$  $\bullet \tau = i$  (*S* symmetry) expand modular forms  $Y(\tau)$  at three fixed point, and then include small deviation :  $\tau =$  (fixed point)  $+ \epsilon$ 

focus on  $\tau = i$  case

diagonalize the mass matrix and move to mass eigenstate basis

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O pheno. study
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#### **at** *τ* = *i* **(***S* **symmetry); Diagonalization** where *MRL* denotes the mass matrix of charged leptons and quarks, *M<sup>E</sup>* and *M<sup>q</sup>* (*q* = *u, d*). Then the mass matrices *M† <sup>E</sup>M<sup>E</sup>* and *M† <sup>q</sup>M<sup>q</sup>* could be diagonal in the diagonal basis of G at the

Results of  $(\bar{L}R)$  structure in interaction basis *Results of*  $(\bar{L}R)$  *structure in interaction basis* 

**EXECUTE:** The flavor structure of the FC biline<sup>Xt</sup> of 
$$
\frac{E\sum T\tau_L}{Y_1(t)} = \frac{E\sum T\tau_L}{Y_1(t)} = \frac{E\sum T\mu_L}{Y_2(t)} = \frac{E\sum T\mu_R}{Y_2(t)} = \frac{E\sum T\mu
$$

Mass eigenstate basis at  $\tau = i$  and  $\tau = i + \epsilon$ Since the modulus τ is different ones for the quark and lepton sectors each other, we use the notation Mass eigenstate basis at  $\tau = i$  and  $\tau = i+\epsilon$ 

#### **at** *τ* = *i* **(***S* **symmetry); Diagonalization**  $\tau = i$ <sup>*(C*</sup> even motion), Diogenelization  $\tau = i$  (*S* symmetry); Diagonalization  $\frac{1}{2}$   $\$  $\overline{6}$ **nmetry):** *s<sup>e</sup> <sup>L</sup>*<sup>12</sup> 1 *s<sup>e</sup>* **іг** A *, URme* ' **14** *r*ation *sR***<sub>2</sub> 1 <b>s**  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$ ↵˜*e*(*m*) *, s<sup>e</sup> <sup>R</sup>*<sup>23</sup> ' 2 ˜*e*(*m*) *, s<sup>e</sup> <sup>R</sup>*<sup>13</sup> ' <sup>p</sup>3)*Y*1(*i*)↵*e*(*m*), ˜ <sup>p</sup>3)*Y*1(*i*)*e*(*m*) and ˜*e*(*m*) = (6 <sup>3</sup>

Mass eigenstate basis at  $\tau = i$  and  $\tau = i + \epsilon$ eigenstate, the *A*<sup>4</sup> flavor coecients of charged lepton bilinear operators are given in terms of  $\frac{1}{2}$  **b**  $\frac{1}{2}$  **c** at  $\tau = \iota$  and  $\tau = \iota$  $\frac{1}{2}$ <br> $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$ ss eigenstate basis at  $\tau = i$  and  $\tau = i+\epsilon$ 

(

<sup>23</sup>*<sup>L</sup>* + *s<sup>e</sup>*

12*L|*✏⇤

<sup>1</sup>*|*)˜*<sup>e</sup>* <sup>3</sup>

2 *se*

*<sup>R</sup>*23↵˜*<sup>e</sup>*



 $E(m)$  is easily  $E(m)$ ,  $E(m)$  and  $E(m)$  and  $E(m)$  are much suppressed to  $E(m)$  and  $E(m)$  $\tilde{\alpha}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\alpha_{e(m)}, \ \tilde{\beta}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\beta_{e(m)} \text{ and } \tilde{\gamma}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\gamma_{e(m)}$  $\alpha e(m)$  (*e*  $\alpha$   $\gamma$   $\beta$ )  $\alpha$ <sub>1</sub>(*i*</sup>) $\alpha$ <sub>*e*(*m*)  $\alpha$   $\beta$   $\gamma$ <sup>2</sup>)  $\alpha$ <sub>1</sub>(*i*</sup>) $\alpha$ <sub>1</sub>(*i*<sub>1</sub>). I<sub>1</sub>(*i*<sub>1</sub>)<sub> $\beta$ </sub>(*n*)</sub>  $\tilde{\alpha}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\alpha_{e(m)}, \tilde{\beta}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\beta_{e(m)} \text{ and } \tilde{\gamma}_{e(m)} = (6 - 3\sqrt{3})Y_1(i)\gamma_{e(m)}$ *<sup>e</sup>*(*m*) ˜<sup>2</sup>  $\zeta$ ...

in spite of ↵*<sup>e</sup>* 6= ↵*e*(*m*)*, <sup>e</sup>* 6= *e*(*m*)*, <sup>e</sup>* 6= *e*(*m*), by inputting mixing angles of Eq.(3.16) into them.  $\mathsf{parameters:}\>$  modulus  $\tau,$  small deviation from fixed point  $\epsilon_1$ , coefficients  $\alpha_e,\beta_e,\gamma_e$ mixing angles *s<sup>e</sup>* <sup>12</sup>, *s<sup>e</sup>* <sup>13</sup> and ✏<sup>1</sup> at ⌧ = *i* + ✏ in Table 2 <sup>5</sup>. eigenstate, the *A*<sup>4</sup> flavor coecients of charged lepton bilinear operators are given in terms of  $\mathsf{parameters:}\>$  modulus  $\tau,$  small deviation from fix  $\Omega$  $\int_{0}^{\infty} e^{i\theta} f^{e} e^{i\theta} f^{e}$  ${\sf parameters:}\;$  *modulus*  $\tau,$  *small deviation from fixed point*  $\epsilon_1$ *, coefficients*  $\alpha_e, \beta_e, \gamma_e$ 

Best fit values of parameters in A4 modular invariant model to realize lepton mass matrix, neutrino data *Okada and Tanimoto* [2012.01688] *e*¯*Lµ<sup>R</sup>* after calculations of the next-to-leading terms. Numerical values of these parameter are  $\frac{1}{2}$  in values of parameters in  $\frac{1}{2}$  in  $\frac{1}{2}$ ↵˜*e*(*m*) ↵˜*e* and Tanımotd<br>——————————————————— *i* parameters in A4 modular invariant model to real parameters in A4 modular invariant model to realize ( <sup>p</sup>3*s<sup>e</sup>* <sup>23</sup>*<sup>L</sup>* + *s<sup>e</sup>* 12*L|*✏⇤ Dkada and Tanimoto  $\mathbf{f}$ <sub>R</sub> $\mathbf{f}$ *µ*¯*L*⌧*<sup>R</sup> e*¯*R*⌧*<sup>L</sup> e*¯*L*⌧*<sup>R</sup> e*¯*Rµ<sup>L</sup>* **Fix, neutrino data**<br>*Reading terms. Numerical 2016881* st fit values of parameters in A4 modular invariant model to realize lepton mass <sup>p</sup>3*s<sup>e</sup>* <sup>p</sup>3*s<sup>e</sup>* 3 <sup>2</sup> (˜*<sup>e</sup>* <sup>+</sup> *<sup>s</sup><sup>e</sup>* <sup>12</sup>*R*↵˜*e*) <sup>12</sup>*<sup>L</sup>* <sup>p</sup>3*s<sup>e</sup>* given at the best fit point as follows  $\mathcal{G}(\mathcal{G})$  . The best fit point as follows  $\mathcal{G}(\mathcal{G})$ 

<sup>13</sup>*<sup>L</sup>* + *|*✏⇤

(

$$
\tau = -0.080 + 1.007 i, \quad |\epsilon_1| = 0.165, \quad \frac{\tilde{\alpha}_{e(m)}}{\tilde{\gamma}_{e(m)}} \simeq \frac{\tilde{\alpha}_e}{\tilde{\gamma}_e} = 6.82 \times 10^{-2}, \quad \frac{\beta_{e(m)}}{\tilde{\alpha}_{e(m)}} \simeq \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} = 1.50 \times 10^{-2}
$$

<sup>1</sup>*|*)˜*<sup>e</sup>*

 $5$ These results are di $4$ erent from ones in the previous our works  $\mathcal{S}(\mathcal{S})$ 

*e*(*m*)

<sup>13</sup>*<sup>L</sup>* + 2*|*✏⇤

<sup>2</sup> (3*s<sup>e</sup>*

*e*

<sup>1</sup>*|*)˜↵*<sup>e</sup>*

 $\overline{\phantom{a}}$ 

→ predict flavor observables approximately due to the di↵erent condition from Eq.(3.10), such as ↵*<sup>e</sup>* ↵*e*(*m*) ⇠ ↵*e*, etc.. in spite of ↵*<sup>e</sup>* 6= ↵*e*(*m*)*, <sup>e</sup>* 6= *e*(*m*)*, <sup>e</sup>* 6= *e*(*m*), by inputting mixing angles of Eq.(3.16) into them.  $\rightarrow$  predict flavor observables in the previous result was our was obtained in a flavor was obta

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### pheno. study

(*g* − 2)*μ*, Lepton flavor violation, EDM

### **Lepton dipole operator** We take the assumption that NP is heavy and can be given by the SMEFT Lagrangian by the SMEFT Lagrangian. Let



 $(g-2)_{\mu}$  &  $\mu \rightarrow e\gamma$  $(e-2)$  as  $u \rightarrow e \nu$  $\overline{3}$  wilson  $\overline{3}$  and  $\overline{3}$ We take the assumption that NP is heavy and can be given by the SMEFT Lagrangian. Let  $\mathcal{L}$  $f(\mathcal{L})$ , so  $\mu \to e\gamma$ *Oe rs*  $\mathbf{I}$ *v*  $\mu \rightarrow e$ *Fµ*⌫ *,* (A.1)  $\frac{18}{4}$  *c*)<sub> $\mu$ </sub> as  $\mu$  / c<sub>)</sub>



$$
(g-2)_{\mu} \& \mu \rightarrow e\gamma
$$

Wilson coefficients in A4 modular symmetry in mass basis  $\int_{\varepsilon}^{\infty}$ Wilson coefficients in A4 modular symmetry in mass basis On the other hand, the diagonal coecients of the bilinear *RL*¯ operators ¯*eReL*, ¯*µRµ<sup>L</sup>* and **<u>RUISON COETTIC</u>** 

$$
\begin{aligned}\n\mathcal{C}'_{\substack{e\gamma\\e\epsilon}} &= 3(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|, \qquad \mathcal{C}'_{\substack{e\gamma\\e\mu}} = \frac{3}{2}(1-\sqrt{3})\tilde{\alpha}_e, \qquad \mathcal{C}'_{\substack{e\gamma\\r\tau}} &= \sqrt{3}(1-\sqrt{3})\tilde{\gamma}_e \qquad \mathcal{C}'_{\substack{e\gamma\\e\mu}} &= \left(\frac{\mathcal{C}'_{\substack{e\gamma}{e\mu}}}{\mu} \frac{\mathcal{C}'_{\substack{e\gamma}{e\mu}}}{\mu} \right) \frac{\mathcal{C}'_{\substack{e\gamma\\e\tau}}}{\mu} \right) \\
\mathcal{C}'_{\substack{e\gamma\\r\epsilon}} &= \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e \left(1-\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} \frac{\tilde{\alpha}_e(m)}{\tilde{\alpha}_e}\right), \\
\mathcal{C}'_{\substack{e\gamma\\r\epsilon}} &= \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e \left(1+\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} \frac{\tilde{\beta}_e(m)}{\tilde{\beta}_e} - 2\frac{\tilde{\beta}_e(m)}{\tilde{\beta}_e} \frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right) \\
\mathcal{C}'_{\substack{e\gamma\\e\tau}} &= \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e \left(1-\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} \frac{\tilde{\beta}_e(m)}{\tilde{\beta}_e}\right),\n\end{aligned}
$$

metry in mass basis  
\n
$$
\tilde{\alpha}_e, \qquad \mathcal{C}'_{e\gamma} = \sqrt{3}(1-\sqrt{3})\tilde{\gamma}_e \qquad \mathcal{C}'_{e\gamma} = \begin{pmatrix} \mathcal{C}'_{e\gamma} & \mathcal{C}'_{e\gamma} & \mathcal{C}'_{e\gamma} \\ \frac{\mathcal{C}'_{e\gamma}}{\mu e} & \frac{\mathcal{C}'_{e\gamma}}{\mu e} & \frac{\mathcal{C}'_{e\gamma}}{\mu \mu} & \frac{\mathcal{C}'_{e\gamma}}{\mu \tau} \\ \frac{\mathcal{C}'_{e\gamma}}{\tau e} & \frac{\mathcal{C}'_{e\gamma}}{\tau \mu} & \frac{\mathcal{C}'_{e\gamma}}{\tau \tau} \end{pmatrix}
$$

*LR*

 $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\mathbf{\mathbf{I}}$  $\vert$  $\mathcal{C}'_{e\gamma}$ *eµ*  $\mathcal{C}'_{e\gamma}$  $\mu\mu$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $\mathsf{I}$  $\overline{\phantom{a}}$  $\vert$ =  $\tilde{\beta}_e$  $\tilde{\alpha}_e$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \end{array}$  $1 - \frac{\tilde{\alpha}_e}{\tilde{\alpha}_e}$  $\tilde{\alpha}_{e(m)}$  $\tilde{\beta}_{e(m)}$  $\tilde{\beta}_e$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\begin{array}{c} \hline \end{array}$  $\vert\, \mathcal{C}'_{\bm{e}\gamma}\,\vert \qquad \tilde{\!\!\sigma}\quad\vert \qquad \text{``a''} \qquad \tilde{\!\!\sigma}\qquad\vert \quad \text{~flavor alignment.}$  $\left|\frac{C_{e\mu}}{E}\right|_{\alpha} = \frac{\rho_e}{\rho_e} \left|1 - \frac{\alpha_e}{\alpha_e} \frac{\rho_e(m)}{m}\right|$  and  $\Omega$  and  $\beta_{e(m)}$  $\left|\frac{C_{\text{ev}}}{\tilde{c}}\right|$  and  $\tilde{c}$  in the string and the stringger and  $\frac{\rho_e}{\tilde{c}} = \frac{\rho_{e(m)} + 1}{\tilde{c}}$  $\left|\frac{-e\mu}{\mu}\right| = \frac{\mu e}{\mu} \left|1 - \frac{\mu e}{\mu}\frac{\mu e(m)}{m}\right|$ 4.1 (*g* 2)*<sup>µ</sup>* and (*g* 2)*<sup>e</sup>* flavor alignment  $\left|\overline{\mathcal{C}'_{\alpha\alpha}}\right| = \frac{1}{\tilde{\alpha}_e} \left|1-\frac{1}{\tilde{\alpha}_e} \frac{1}{\tilde{\beta}_e}\right|$  < 2.1 × 10<sup>-3</sup>  $\hat{\alpha}_e$   $\hat{\alpha}_e$   $\hat{\alpha}_{e(m)} + c_{\alpha-1}$ experimental bound of this ratio in  $\mu\mu$  in Eq.(3.17) with Eq.(3.17), we obtain  $< 2.1 \times 10^{-5}$  $\left[\begin{array}{c} e\gamma \\ e\mu \end{array}\right]$   $\tilde{\alpha}_e$   $\beta_{e(m)}$   $\left[\begin{array}{c} \tilde{\alpha}_e \end{array}\right]$   $\tilde{\beta}_{e(m)}$   $\tilde{\beta}_{e(m)} = \frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_{e(m)}} = 1 + \frac{\tilde{\beta}}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_{\beta}$ ,  $\left|\begin{array}{cc} \mathcal{C}'_{e\gamma} \end{array}\right|$   $\tilde{\alpha}_e$   $\left| \begin{array}{cc} \tilde{\alpha}_{e} & \tilde{\alpha}_{e} \end{array}\right|$  with  $\tilde{\alpha}_e \overline{\alpha}_{e}$   $\frac{\tilde{\alpha}_{e}}{c_{e}} = \frac{\tilde{\alpha}_{e(m)} + c_{\alpha}}{c_{e}} = 1 + \frac{c_{\alpha}}{c_{e}} = 1$ 

**flavor alignment**

\n
$$
\begin{aligned}\n &\left| \begin{array}{r}\n \hat{\beta}_e \\
 & \hat{\beta}_{e(m)}\n \end{array}\right| & \frac{\hat{\beta}_e}{\hat{\beta}_{e(m)}} = \frac{\hat{\beta}_{e(m)} + c_{\beta}}{\hat{\beta}_{e(m)}} = 1 + \frac{c_{\beta}}{\hat{\beta}_{e(m)}} \equiv 1 + \delta_{\beta} \,, \\
 &\left| \begin{array}{r}\n \hat{\alpha}_e \\
 \frac{\hat{\alpha}_e}{\hat{\alpha}_{e(m)}}\n \end{array}\right| & \frac{\hat{\alpha}_e}{\hat{\alpha}_{e(m)}} = \frac{\hat{\alpha}_{e(m)} + c_{\alpha}}{\hat{\alpha}_{e(m)}} = 1 + \frac{c_{\alpha}}{\hat{\alpha}_{e(m)}} \equiv 1 + \delta_{\alpha} \,, \\
 &\left| \begin{array}{r}\n \frac{\hat{\gamma}_e}{\hat{\gamma}_e}\n \end{array}\right| & \frac{\hat{\gamma}_e}{\hat{\gamma}_{e(m)}} = \frac{\hat{\gamma}_{e(m)} + c_{\gamma}}{\hat{\gamma}_{e(m)}} = 1 + \frac{c_{\gamma}}{\hat{\gamma}_{e(m)}} \equiv 1 + \delta_{\gamma} \,, \\
 &\left| \begin{array}{r}\n \frac{\hat{\gamma}_e}{\hat{\gamma}_{e(m)}}\n \end{array}\right| &\n\end{aligned}
$$

$$
\left|1 - \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} \frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right| \simeq |\delta_\beta - \delta_\alpha| < 1.4 \times 10^{-3}
$$

↵˜*e* = ↵˜*e*(*m*) + *c*↵  $-\alpha,p$ , ⌘ 1 + ↵ *,* 4 Phenomenology of (*g* 2)*µ, e*, LFV and EDM  $T_{\alpha}$  and  $T_{\alpha}$  is a powerful probe between  $\alpha$ , *a*<sub>*n*</sub>  $\alpha$ <sub>*l*</sub>  $\alpha$ <sub>*l*</sub> *C*0 *e*  $\frac{1}{2}$ ↵˜*e*  $\overline{\phantom{a}}$   $\overline{\$  $\partial_\alpha$  $\overline{\mathcal{L}}$  $(10)$  $(1)^{-3}$   $|\delta_{\circ}| < 0$  $\frac{1}{2}$  *Decween O*  $\alpha, \beta$ ,  $|\omega_{\alpha}| \leq C(10)$ ,  $|\omega_{\beta}| \leq C(10)$  $|\delta_{\alpha}| < \mathcal{O}(10^{-3}), \qquad |\delta_{\beta}| < \mathcal{O}(10^{-3})$ )*,* (4.8) without tuning between  $\delta_{\alpha,\beta}$  ,

$$
LFV \tau \rightarrow \mu \gamma \& \tau \rightarrow e \gamma \& \mu \rightarrow e \gamma
$$

*<sup>|</sup>*↵*<sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> )*, <sup>|</sup><sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> Wilson coefficients in A4 modular symmetry in mass basis On the other hand, the diagonal coecients of the bilinear *RL*¯ operators ¯*eReL*, ¯*µRµ<sup>L</sup>* and *<u>Ruison coefficie</u> <sup>|</sup>*↵*<sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> )*, <sup>|</sup><sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup>

$$
\mathcal{C}'_{\substack{e\gamma\\ee}'} = 3(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|, \qquad \mathcal{C}'_{\substack{e\gamma\\mu}} = \frac{3}{2}(1-\sqrt{3})\tilde{\alpha}_e, \qquad \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} = \sqrt{3}(1-\sqrt{3})\tilde{\gamma}_e \qquad \mathcal{C}'_{\substack{e\gamma\\E\tau}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma} \\E} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma} \\E} \\ \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} \\E'_{\substack{e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\\tau\tau}} \\E''_{\substack{e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\
$$

 $\left(\begin{matrix} C' & C' & C' \end{matrix}\right)$ 

 $\mathcal{C}'_{e\gamma}$  $e\tau$ 

 $\setminus$ 

 $\mathcal{C}'_{e\gamma}$ *eµ*

 $\left\{\begin{array}{ccc} e\gamma & e\gamma & e\gamma \\ e e & e\mu & e\tau \end{array}\right\}$ 

 $\sqrt{2}$ 

 $\mathcal{C}'_{e\gamma}$ *ee*

that the additional unknown mode of *m* is the Higgs-like mode, that is, ↵ ⇠ ⇠ . Then, we the case that the additional unknown mode is the Higgs-like *ehe case that the additional unknown mode is the Hi* On the other hand, the diagonal coecients of the bilinear *RL*¯ operators ¯*eReL*, ¯*µRµ<sup>L</sup>* and processes that the additional unknown mode is the Higgs-like mode  $\beta$ symmetry. We study the correlation them in this section the correlation of  $\sim$ the case that the additional unknown mode is the Higgs-like mode  $(\delta \sim \delta_e \sim \delta)$ one case chac che addicional dinviown mode is che i nggs n the case that the additional unknown mode is the Higgs-like mode  $(\delta_\alpha \thicksim \delta_\beta \thicksim \delta_\gamma)$ 

$$
\frac{\mathcal{C}'_{\substack{e\gamma\\ \mathcal{C}'_{\substack{e\gamma\\ \mu e}}}=\frac{1}{\sqrt{3}}\times\mathcal{O}(1)\qquad \qquad \frac{\mathcal{C}'_{\substack{e\gamma\\ \tau e}}}{\mathcal{C}'_{\substack{e\gamma\\ \tau\mu}}}=\frac{\tilde{\beta}_e}{\tilde{\alpha}_e}\times\mathcal{O}(1)\sim 10^{-2}
$$

$$
BR(\tau \to \mu \gamma) \gg BR(\mu \to e \gamma) \sim BR(\tau \to e \gamma)
$$

e.g.  $U(2)$  case  $BR(\tau \to \mu \gamma) \gg BR(\mu \to e \gamma) \gg BR(\tau \to e \gamma)$  $\mathcal{L}_{\mathcal{S}}\colon \mathcal{L}_{\mathcal{S}}$ , and discrepancy with the  $\mathcal{L}_{\mathcal{S}}\colon \mathcal{L}_{\mathcal{S}}$ e.g.  $U(2)$  case  $BR(\tau \to \mu \gamma) \gg BR(\mu \to e \gamma) \gg BR(\tau \to e \gamma)$ almost agree with the charged lepton mass ratios *me/m<sup>µ</sup>* = 4*.*84⇥10<sup>3</sup> and *mµ/m*⌧ = 5*.*95⇥10<sup>2</sup>.

$$
LFV \tau \rightarrow \mu \gamma \& \tau \rightarrow e \gamma \& \mu \rightarrow e \gamma
$$

*<sup>|</sup>*↵*<sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> )*, <sup>|</sup><sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> Wilson coefficients in A4 modular symmetry in mass basis On the other hand, the diagonal coecients of the bilinear *RL*¯ operators ¯*eReL*, ¯*µRµ<sup>L</sup>* and *<u>Ruison coefficie</u> <sup>|</sup>*↵*<sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup> )*, <sup>|</sup><sup>|</sup> <sup>&</sup>lt; <sup>O</sup>*(10<sup>3</sup>

$$
\mathcal{C}'_{\substack{e\gamma\\ee}'} = 3(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|, \qquad \mathcal{C}'_{\substack{e\gamma\\mu}} = \frac{3}{2}(1-\sqrt{3})\tilde{\alpha}_e, \qquad \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} = \sqrt{3}(1-\sqrt{3})\tilde{\gamma}_e \qquad \mathcal{C}'_{\substack{e\gamma\\E\tau}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma} \\E} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma} \\E} \\ \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} \\E'_{\substack{e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\\tau\tau}} \\E''_{\substack{e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\gamma}} & \mathcal{C}'_{\substack{e\gamma\\e\gamma\\e\
$$

 $\left(\begin{matrix} C' & C' & C' \end{matrix}\right)$ 

 $\mathcal{C}'_{e\gamma}$  $e\tau$ 

 $\setminus$ 

 $\mathcal{C}'_{e\gamma}$ *eµ*

 $\left\{\begin{array}{ccc} e\gamma & e\gamma & e\gamma \\ e e & e\mu & e\tau \end{array}\right\}$ 

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$$
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$$

$$
BR(\tau \to \mu \gamma) \gg BR(\mu \to e \gamma) \sim BR(\tau \to e \gamma)
$$

10<sup>4</sup> : 1 : 10, where we take account of the kinematical factor. Since the present upper bounds The anomalous magnetic moment of the muon, *a<sup>µ</sup>* = (*g* 2)*µ/*2, is a powerful probe beyond the  $\frac{B}{B}$  ance the present upper bounds of  $B(1 + B)$  and  $B(1 + B)$  are 3.3  $\sim 10^{-6}$  and 4.4  $\sim 10^{-6}$ ,  $\sim$ experiment test of the support of the superimental test of this prediction for Since the present upper bounds of  $B(\tau \to e\gamma)$  and  $B(\tau \to \mu\gamma)$  are 3.3  $\times$  10<sup>-8</sup> and 4.4  $\times$  10<sup>-8</sup>, respectively, we expect the experimental test of this prediction for  $\tau \rightarrow \mu\gamma$ in Ref. [3]. If this result is evidence of NP, we can relate it with other phenomena, (*g* 2)*e*, LFV respectively, we expect the experimental test of this prediction for  $\tau \to \mu \gamma$  in the future

 $f d \mathbf{R}$  ( $\sigma = 2$ ) . ٦  $FDM \, d \, 2$  (a )  $EDM d_e$  &  $(g - 2)_\mu$ 3 Wilson Coecients of dipole operator in mass basis



**EDM** 
$$
d_e
$$
 **&**  $(g - 2)_{\mu}$  **&**  $\mu \rightarrow e_{\gamma}$ 

\n**electron EDM**<sub>e</sub>  $(g - 2)_{\mu}$  with coupling relation in A4 modular sym.

\n
$$
\frac{1}{\Lambda^2} \text{Im} \left[ \mathcal{C}'_{\frac{e}{\lambda}} \right] < 1.6 \times 10^{-12} \text{ TeV}^{-2}
$$
\n
$$
\frac{1}{\Lambda^2} \text{Re} \left[ \mathcal{C}'_{\frac{e}{\lambda}} \right] = 4.9 \times 10^{-8} \text{ TeV}^{-2}
$$
\n
$$
\frac{\text{Im} \left[ \mathcal{C}'_{\frac{e}{\lambda}} \right]}{\text{Re} \left[ \mathcal{C}'_{\frac{e}{\lambda}} \right]} \simeq (\text{Im} \delta_{\beta}) < \frac{1.6 \times 10^{-12}}{4.9 \times 10^{-8}} = 3.3 \times 10^{-5}
$$
\n
$$
C'_{\frac{e}{\lambda}} = 3(1 - \sqrt{3})\tilde{\beta}_e |\epsilon_1^*| = 3(1 - \sqrt{3})\tilde{\beta}_e(m)(1 + \delta_{\beta}) |\epsilon_1^*|
$$
\n
$$
\text{Im} \left[ C'_{\frac{e}{\lambda}} \right] \simeq 3(1 - \sqrt{3})\tilde{\beta}_{e(m)}(\text{Im} \delta_{\beta}) |\epsilon_1^*|, \quad C'_{\frac{e}{\lambda^*}} \simeq \frac{3}{2}(1 - \sqrt{3})\tilde{\beta}_{e(m)}(\delta_{\beta} - \delta_{\alpha})
$$
\n
$$
\text{suppose}
$$
\n
$$
|\text{Im} \delta_{\beta}| \simeq |\delta_{\beta}| \text{ and } |\delta_{\alpha}| \simeq |\delta_{\beta}| \text{ (or } |\delta_{\alpha}| \ll |\delta_{\beta}|)
$$
\n
$$
M(\pm \pm \pm \sqrt{3}) \text{ Re} \left[ \frac{1}{2} \text{ Im} \left[ \frac{1}{2} \right] \right]
$$

 $\mathcal{B}(\mu^+ \to e^+ \gamma) < 2.3 \times 10^{-16} \, ,$   $\mathcal{B}(\mu^+ \to e^+ \gamma) < 2.3 \times 10^{-16} \, ,$ *ee*  $\mathcal{L}$  $\overline{2}$ Upper bound for B(μ→eγ)  $\mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 2.3 \times 10^{-16} \ \mathrm{Jpc}$  $P(x+h) = P(x+h)$ Id for B( $\mu$   $\rightarrow$   $\alpha$  in high-energy model building in high-energy model building  $\alpha$  and  $\beta$ 



## **Modular symmetry**

## **Phenomenological aspects**

SMEFT with modular symmetry Kobayashi, Otsuka, Tanimoto, KY [2204.12325](https://arxiv.org/abs/2204.12325)

### **Baryon/Lepton-number violating operator**

Kobayashi, Nishimura,Otsuka, Tanimoto, KY [2207.14014](https://arxiv.org/abs/2207.14014)

**Summary**





 $\rightarrow$  assume R-parity  $\rightarrow$  assume R

B/L num. violating ope. can be forbidden by modular symmetry

### **Baryon/Lepton-number violating operators** *<sup>k</sup>*⇢*ij* ()*f<sup>j</sup>* (⌧ )*,* (2.7)

Yukawa/Higher-dim. ops. : even-modular weights  $\left(k_Y\!\in 2Z\right)$ 2.2.2. Thigher-dim ans Yukawa/Higher-dim. ops. :  $\epsilon$ ven-modular weights  $(k_Y$  $\in$  2Z)  $f$ <sup>*i*</sup>( $f$ )  $f$  ( $f$ )  $f$  ( $f$ )  $f$  ( $f$ )  $f$  ( $f$ )  $f$ )  $f$ )  $f$ 

modular form  $f_i(\gamma \tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)$  $\text{modular form}$   $\int i(\gamma U) - (\gamma U + \gamma U) \int f(\gamma U) f(U) \quad \text{for all } X$  $\rho$  : unitaly matrix modular form  $j_i(y_i) = (c_i + a_j)^T p_{ij}(y_j) j(i) - p$  annualy matrix

for *S* transformation  $f_i(S^2\tau) = (-1)^k \rho_{ij}(S^2) f_j(\tau)$ 

$$
(-1)^{k} \rho(S^{2}) = \mathbb{I}.
$$
\n
$$
\text{(i) } k = \text{even}, \qquad \rho(S^{2}) = \mathbb{I}, \text{ i.e., } \rho \in \Gamma_{N},
$$
\n
$$
\text{(ii) } k = \text{odd}, \qquad \rho(S^{2}) = -\mathbb{I}, \text{ i.e., } \rho \in \Gamma_{N}'
$$

focus on Γ<sub>N</sub> rather than its double cover Γ'<sub>N</sub>

### **Baryon/Lepton-number violating operators**  iryon/Lepton-number violating operators



 $\frac{1}{2}$ Modular invariance requires oquiar invariance requires

$$
\text{modular weight} \quad k_Y = \sum_i k_i
$$

### **Baryon/Lepton-number violating operators**   $h_{\text{max}}$  is the dimensional operators in the effective action.

#### In this section, we are modelled the modular symmetric superpotential in the MSSM: we are  $\frac{1}{\sqrt{2}}$ Superpotential in MSSM matial in MCC

 $W = y_{ij}^u Q_i H_u \bar{U}_j + y_{ij}^d Q_i H_d \bar{D}_j + y_{ij}^\ell L_i H_d \bar{E}_j + y_{ij}^n L_i H_u N_j + m_{ij}^n N_i N_j + \mu H_u H_d$  $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$  → + −()  $\ddots$  :  $\ddots$ 

where you have you have you have you have you have the holomorphic modular forms of over modular woight philosophic modular forms of even modular weight under the supermultiplets under the supermultiplets of the **C** Chiral supermultiplets (Q,O,D,L,E,IN,H) have integer modular weight **k** corresponding to the Z<sub>2</sub> charge of the Z<sub>2</sub> charge chiral supermultiplets (Q,U,D,L,E,N,H) have integer modular weight k<br>**waxay** chiral supermultiplets (Q,U,D,L,E,N,H) have integer modular weight k erwise, the *<sup>µ</sup>*-term has an odd modular weight. The same is true for *{U,* ¯ *<sup>D</sup>*¯*}* and *{E,N* ¯ *}*. holomorphic modular forms of even modular weight  $\sum_{i=1}^{n}$  modular invariance  $\sum_{i=1}^{n}$  $\blacksquare$ 

*k* (mod 2). Specifically, we represent the modular weights of matter multiplets, Yukawa  $\mu_Y - \mu_Y - \mu_Y$  $\kappa_y - \kappa_Q - \kappa_{U,D} - \kappa_{Higgs} = 0$  $k_{u}^{\ell} - k_{L} - k_{E,N} - k_{\text{Higgs}} = 0$ <br>  $k_{u}^{\ell} - 2k_{N} = 0$ <br>  $k_{u}^{\ell} - 2k_{W} = 0$  $\sum_{\mu} m_{\mu}$  independent modular weights in what follows. The invariant under  $\mu$  is  $\mu$  invariant under  $\frac{1}{\sqrt{2}}$ e for the baryon-number violating operators in an explicit model  $\frac{1}{\sqrt{2}}$ where  $\frac{1}{2}$  arrive at two constraints : <sup>2</sup>The flavor structure of *n*-point couplings in the string EFT was discussed from the viewpoint of the  $k_i^{}$  : modular weight  $\frac{1}{h}$  modular invariance  $\frac{1}{h}$   $\frac{1}{h}$   $\frac{1}{h}$ ; modular woisht **e** indicate invariance  $K_Y = \sum_i K_i$  is the same indicate for  $K_i$ : modular weight.  $k^u - k_0 - k_{\text{II}} = k_{\text{II}} = 0$  $\sum_{i=1}^n w_i Q_i + w_i Q_i D_i$  is the modular superpotential requires  $\sum_{i=1}^n w_i Q_i$  $\kappa_y$   $\kappa_Q$   $\kappa_{U,D}$   $\kappa_{Higgs}$  $k^m - 2k_N = 0,$ *<sup>y</sup> k<sup>L</sup> kE,N k*Higgs = 0 *,*  $k_{y}^{x} - k_{L} - k_{E,N} - k_{\text{Higgs}} = 0$  ,  $k_{\mu} - 2k_{\text{Higgs}} = 0$  $\kappa_y - \kappa_L - \kappa_{E,N} - \kappa_{\text{Higgs}} = 0$ arrive at two constraints : *k<sup>Q</sup>* + *kU,D* + *k*Higgs = even *, k<sup>L</sup>* + *kE,N* + *k*Higgs = even *.* (2.18) erwise, the *<sup>µ</sup>*-term has an odd modular weight. The same is true for *{U,* ¯ *<sup>D</sup>*¯*}* and *{E,N* ¯ *}*. nvariance  $\quad_ Y = \sum_i k_i \qquad k_i$  : modular weight  $V_{\varphi}$   $h = k \cos \theta$   $\theta$ *kn*  $k^m - 2k_N = 0$ , *k*<sup>m</sup> – 2*k*<sub>N</sub> = 0  $k_{\mu} - 2k_{\text{Higgs}} = 0$ *<sup>y</sup> }* 2 2Z, we arrive at two constraints:  $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{B}$  $\mathbf{b}$  invariance  $\mathbf{b} = \nabla \mathbf{b}$  is modular weight iai invariance  $ny = \sum_{i} n_i$   $x_i$ . Modular weight  $k_y^u - k_Q - k_{U,D} - k_{\text{Higgs}} = 0$  $k_y^d - k_Q - k_{U,D} - k_{\text{Higgs}} = 0$   $k^m - 2k_N = 0$  $k_y^{\ell} - k_L - k_{E,N} - k_{\text{Higgs}} = 0$   $k_y - 2k_{\text{Higgs}} = 0$  $k^n - k_I - k_{I\!\!N} - k_{I\!\!N}$  $\mu$  2.2 $\mu$  2.2 $\mu$ *k<sup>Q</sup>* + *kU,D* + *k*Higgs = even *, k<sup>L</sup>* + *kE,N* + *k*Higgs = even *.* (2.18) the *<sup>N</sup>* symmetry, the Higgs doublets *{Hu, Hd}* have the same <sup>Z</sup><sup>2</sup> parity under (1)*k*; otherginal modular invariance  $k_Y = \sum_i k_i$  and  $k_i$  : modular weight Therefore, we denote *k*Higgs = *{kH<sup>u</sup> , kH<sup>d</sup> }*, *kU,D* = *{k<sup>U</sup> , kD}* and *kE,N* = *{kE, k<sup>N</sup> }* since our  $k_y^d$  $k^m - 2k_M = 0$  $k_y^n - k_L - k_{E,N} - k_{\text{Higgs}} = 0$ *y, k<sup>n</sup> <sup>y</sup> }* 2 2Z, we arrive at two constraints: *k<sup>Q</sup>* + *kU,D* + *k*Higgs = even *, k<sup>L</sup>* + *kE,N* + *k*Higgs = even *.* (2.18) **Example 19 and 19**  $\mathbf{L}^u = \mathbf{L} - \mathbf{L} - \mathbf{L} - \mathbf{L}$  $\kappa_y - \kappa_Q - \kappa_{U,D} - \kappa_{\text{Higgs}} = 0$  $k_y^\ell$  $k_y^\ell - k_L - k_{E,N} - k_{\rm Higgs} = 0$   $k_\mu - 2k_{\rm Higgs} = 0$ *<sup>y</sup> }* 2 2Z, we arrive at two constraints:  $k^{u}-k_{O}-k_{U,D}-k_{\text{Higgs}}=0$  $\int$  $k_{\mu} - 2k_{\text{Higgs}} = 0$ and the modular invariance  $\boxed{k_Y = \sum_l k_i}$  and  $k_i$  : modular weight erwise, the *<sup>µ</sup>*-term has an odd modular weight. The same is true for *{U,* ¯ *<sup>D</sup>*¯*}* and *{E,N* ¯ *}*. interest is the  $\mathcal{L}^2$  charge. Then,  $y_{\rm ss} = 0$  *km*  $\Omega_k$  0 al the *<sup>N</sup>* symmetry, the Higgs doublets *{Hu, Hd}* have the same <sup>Z</sup><sup>2</sup> parity under (1)*k*; oth $k^u - k_0 - k_{\text{UL}} - k_{\text{UL}} = 0$  $\iiota^{reg}$   $\iota^{reg}$   $\iota^{reg}$   $\iota^{reg}$   $\iota^{reg}$   $\iota^{reg}$  $f_{\text{ggs}} = 0$  *,*  $k_{\text{L}} - 2k_{\text{Higgs}} = 0$  $k_y^n - k_L - k_{E,N} - k_{\text{Higgs}} = 0$ 

$$
k_Q + k_{U,D} + k_{\text{Higgs}} = \text{even}
$$
  

$$
k_L + k_{E,N} + k_{\text{Higgs}} = \text{even}
$$

### **Baryon/Lepton-number violating operators**  *k<sup>I</sup>* + *k<sup>J</sup> k<sup>K</sup>* = even *.* (2.25) Note that *{kYIJ , kY<sup>K</sup> }* are even numbers, and we call modular weights of operators, including

Higher-dim. ops. : even-modular weights  $\sqrt{\frac{1}{\binom{1}{i}}\binom{1}{i}}$ analysis.<br>C  $\rightarrow$  operator with even-modular weights :  $\sqrt{ }$ 

 $\textbf{When } k_{\textbf{Higgs}} \textbf{ is even weight } \quad (k_Q, \, k_{U,D}, \, k_L, \, k_{E,N}, \, k_{\textbf{Higgs}}) \quad \textcolor{red}{\left\| \begin{array}{c|c} \textit{LH}_u & \textbf{ } & \textbf{ } & \textbf{ } & \textbf{ } \\ \textbf{LH}_u & \textbf{LH}_$ 

(i) Others are even  $\rightarrow$  B/L breaking

 $\bigvee$ :  $k_L$  + $k_Q$  + $k_D$  =even+odd+odd=even  $\times$ :  $k_U$  + $k_D$  + $k_D$  =odd+odd+odd=odd (ii)  $k_Q, k_{U,D}$  is odd,  $k_L, k_{E,N}$  is even  $\rightarrow$  L breaking  $\sqrt{\frac{v_1 v_2}{U U D E}}$  $id = even$  confirmed operators in  $OOOH$ , since  $\Box$  $\overline{\text{vol}}$ d $\overline{\text{vol}}$ 

(iii)  $k_Q, k_{U,D}$  is even,  $k_L, k_{E,N}$  is odd  $\rightarrow$  B breaking  $\sqrt{\frac{LH_u LH_u}{\sqrt{2\pi}}}$ 

(iv) Others are odd  $\rightarrow$ B/L breaking are prohibited  $\frac{UD^*E}{H^*H \cdot \bar{E}}$ 

the baryon-number violating operators are allowed, some proton decay operators are →Realization of R-parity(Z2 sym) Possible in SU(5), SO(10) GUTs



B and/or L violating operator (iv)  $\alpha$ *D* and/or L violating operator

## **Summary**

Flavor puzzle (mass and mixing in quark and lepton) might be controlled by flavor symmetry

→ Modular flavor symmetry

Discrete symmetry  $\simeq$  Modular symmetry

We discuss pheno aspects from Modular flavor symmetry

- with SMEFT, predict Lepton flavor observables
- Baryon/Lepton-number violating operator can be controlled with modular weight

Approach to other models with  $S_4, A_5, \ldots$ 

 to other flavor phenomena in the quark sector  $b \rightarrow s \gamma$  ...