

$b \rightarrow svv$ decays: why, where and how?

Open Questions and Future Directions in
Flavour Physics – MITP – 04/11/2024

Ménil Reboud

Mostly based on:

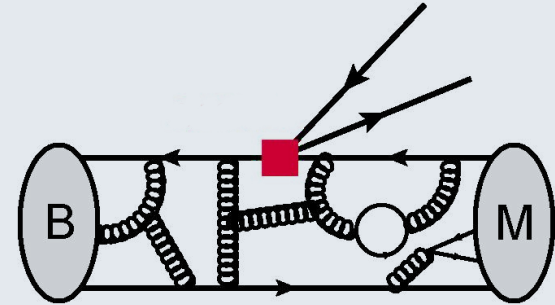
- Amhis, Kenzie, MR, Wiederhold [2309.11353](#)
- Gärtner, MR, *et al* 2402.08417



$$\mathcal{H}_{\text{eff}}^{sb\nu\nu} = -\frac{4G_F}{\sqrt{2}}\lambda_t \sum_i \mathcal{C}_i \mathcal{O}_i + \text{h.c.}$$

$$\mathcal{O}_L^{\nu_i, \nu_j} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j)$$

$$C_L^{\text{SM}} = -6.32(7) \quad \text{@NLO QCD and NNLO EW [Buchalla, Buras '99; Misiak, Urban '99; Brod, Gorbahn, Stamou '10]}$$



- Main message: $b \rightarrow s\bar{\nu}\nu$ is boringly **clean**
- Neutrinos are the only current way of probing 3rd family leptons in FCNC
- I focus on $b \rightarrow s$, but $s \rightarrow d$ has recently been probed via $K \rightarrow \pi\nu\bar{\nu}$ [NA62 '24]

Branching ratios

$$\frac{d\mathcal{B}(B \rightarrow K \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_B |N_B|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 \rho_+^K,$$

$$N_{B_q} = \frac{G_F \alpha_{\text{em}}}{16\pi^2} \sqrt{\frac{m_{B_q}}{3\pi}}$$

$$\frac{d\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_B |N_B|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 (\rho_{A_1}^{K^*} + \rho_{A_{12}}^{K^*} + \rho_V^{K^*}),$$

$$\rho_+^K = \frac{\lambda^{3/2}}{m_B^4} (f_+^K(q^2))^2,$$

$$\rho_V^{K^*} = \frac{2q^2 \lambda^{3/2}}{(m_B + m_{K^*}) m_B^4} \left(V^{K^*}(q^2) \right)^2,$$

$$\rho_{A_1}^{K^*} = \frac{2q^2 \lambda^{1/2} (m_B + m_{K^*})^2}{m_B^4} \left(A_1^{K^*}(q^2) \right)^2,$$

$$\rho_{A_{12}}^{K^*} = \frac{64 m_{K^*}^2 \lambda^{1/2}}{m_B^2} \left(A_{12}^{K^*}(q^2) \right)^2,$$

Dominant sources of uncertainties:

- The CKM element $|\lambda_t|$
- The Wilson coefficient C_L
- The form-factors ρ^M

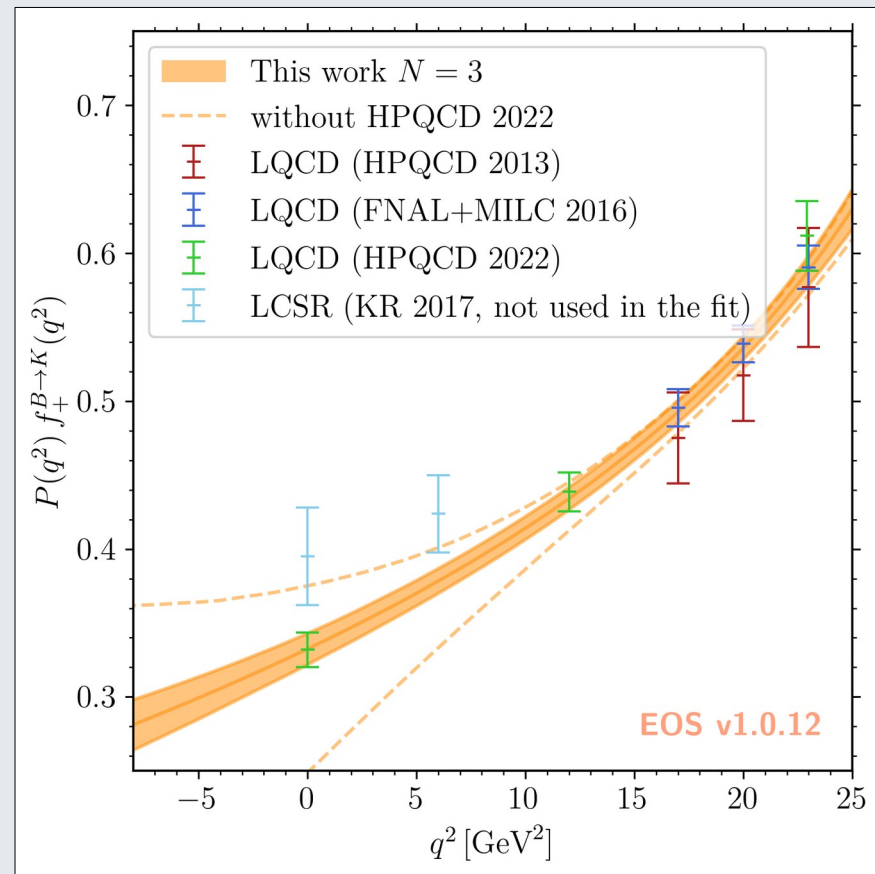
[Buras, Girschbach-Noe *et al* '14]

Form factors

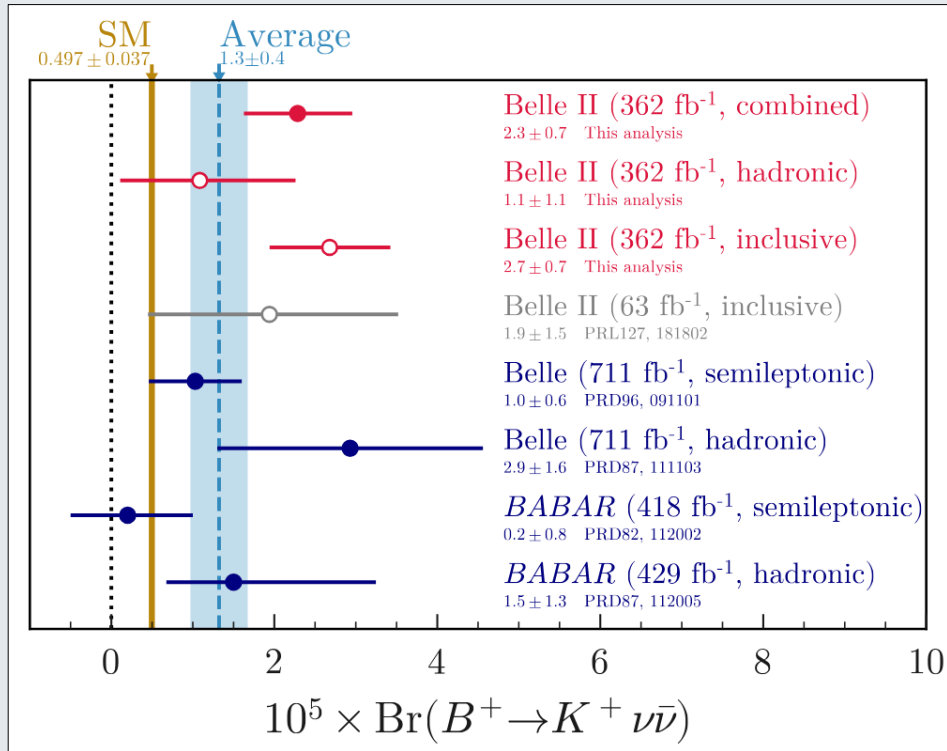
- State-of-the-art form-factor predictions [Boyd, Grinstein, Lebed '94; '97; Gubernari, MR *et al* '23]
 - Lattice QCD and LCSR estimates
 - Analyticity constraints
 - Dispersive bounds
 - Multi-channel analyses
- Investigate **tension between LQCD and LCSR**

$$r_{\text{lh}} = \frac{\mathcal{B}(B \rightarrow K \nu \bar{\nu})_{\text{low-}q^2}}{\mathcal{B}(B \rightarrow K \nu \bar{\nu})_{\text{high-}q^2}}$$

→ Cancellation of normalization, CKM, WC (i.e. heavy NP!), experimental uncertainties, ...
[Bečirević, Piazza, Sumensari '23]



Experimental status



[Belle II '23]

- Combined measurement shows
3.5 σ evidence over the background
2.7 σ 'tension' with SM prediction
- See [Sally's talk](#) for all the details
- This measurement is model-dependent, the signal is assumed to follow a **SM shape** (keep this in mind for later)

$$\mathcal{B}(B \rightarrow K^* \nu \bar{\nu}) < 2.7 \times 10^{-5},$$

@90% CL [Belle '17]

Slightly beyond the SM (only SM-like neutrinos)

- There is only one additional dim-6 operator that can be written with the SM fields

$$\mathcal{O}_R^{\nu_i \nu_j} = \frac{e^2}{(4\pi)^2} (\bar{s}_R \gamma_\mu b_R) (\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j)$$

- No additional theory uncertainties

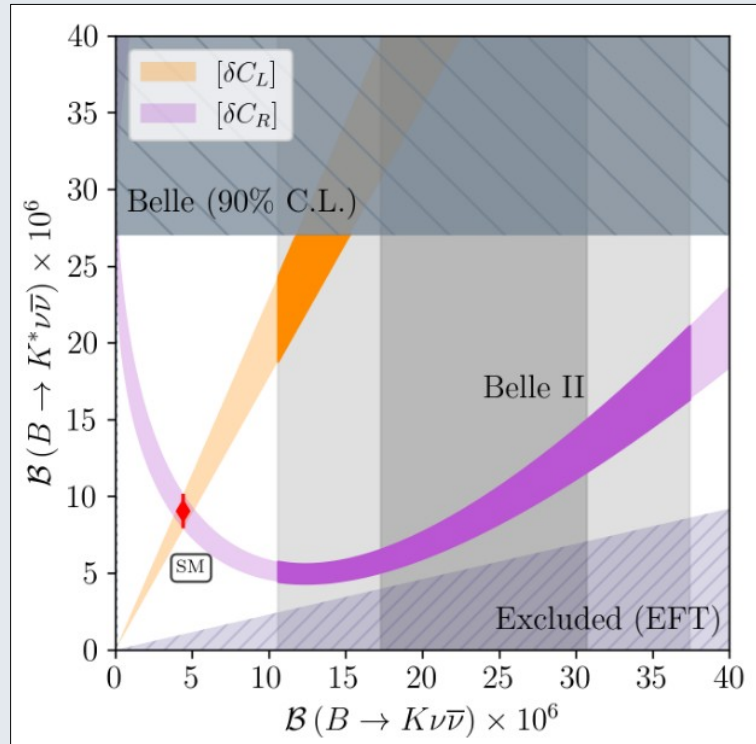
$$\frac{d\mathcal{B}(B \rightarrow K \nu \bar{\nu})}{dq^2} = 3 \tau_B |N_B|^2 |C_L + C_R|^2 |\lambda_t|^2 \rho_+^K,$$

$$\frac{d\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})}{dq^2} = 3 \tau_B |N_B|^2 |\lambda_t|^2 \left(|C_L - C_R|^2 (\rho_{A_1}^{K^*} + \rho_{A_{12}}^{K^*}) + |C_L + C_R|^2 \rho_V^{K^*} \right)$$

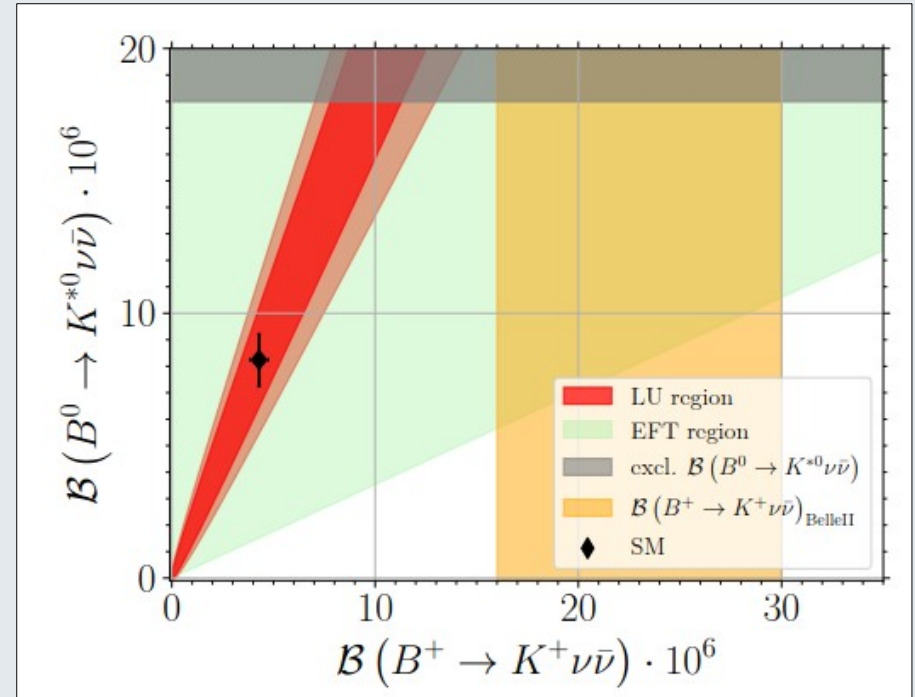
- Clear **blind direction** for pseudo-scalar kaon

Slightly beyond the SM (only SM-like neutrinos)

- Left-handed currents alone cannot account for the current tension



[Bečirević, Piazza, Sumensari '23]



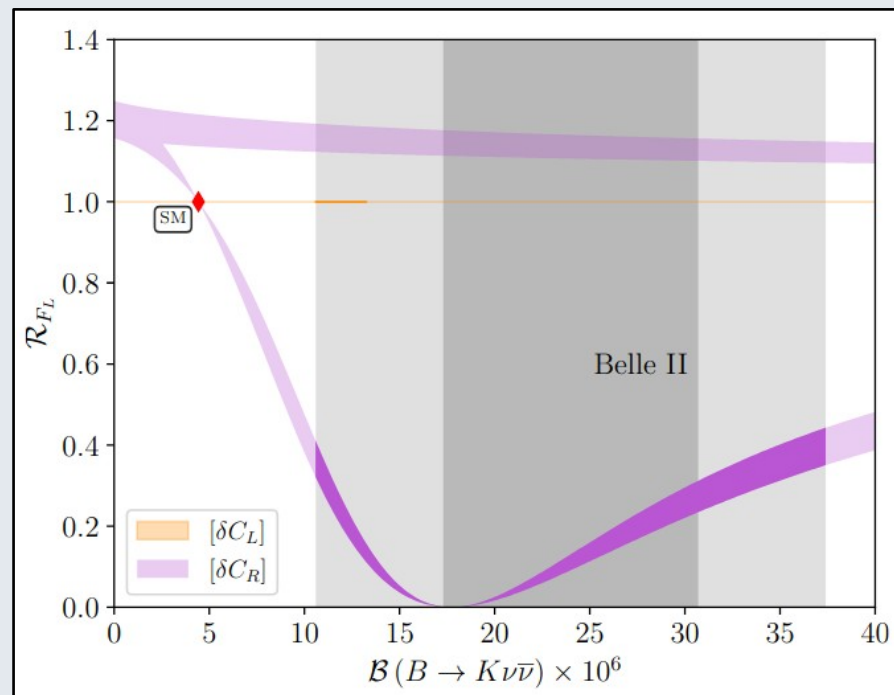
[Bause, Gisbert, Hiller '23]

Other observables (1)

- With a bit more data, one can measure more involved observables such as:
 - Longitudinal fraction [Buras, Girschbach-Noe *et al* '14; Altmannshofer, Buras *et al* '09...]

$$F_L(B \rightarrow K^* \nu \bar{\nu})_{\text{SM}} = \frac{\rho_{A_{12}}^{K^*}}{\rho_{A_1}^{K^*} + \rho_{A_{12}}^{K^*} + \rho_V^{K^*}}$$

$$F_L(B \rightarrow K^* \nu \bar{\nu}) = \frac{|C_L - C_R|^2 \rho_{A_{12}}^{K^*}}{|C_L - C_R|^2 (\rho_{A_1}^{K^*} + \rho_{A_{12}}^{K^*}) + |C_L + C_R|^2 \rho_V^{K^*}}$$

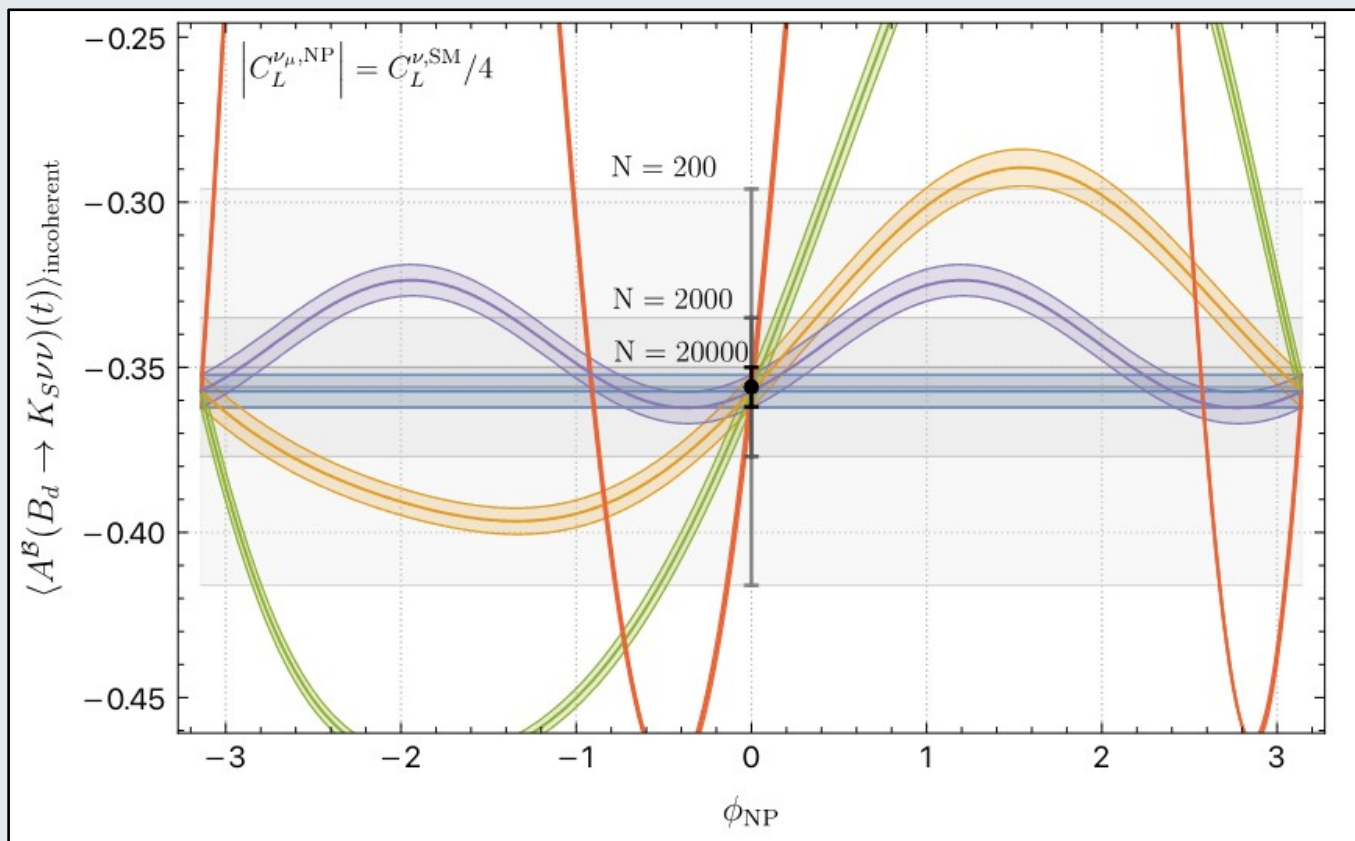


Other observables (2)

- With a bit more data, one can measure more involved observables such as:
 - Longitudinal fraction [Buras, Girschbach-Noe *et al* '14; Altmannshofer, Buras *et al* '09...]
 - (mixing induced) CP-asymmetries [Descotes-Genon, Fajfer *et al* '22]
 - gives a **clean access to the phase of the WC** (many cancellations)
 - e.g. for $B^0 \rightarrow K_S \nu \nu$, this gives direct access to:

$$\text{Im}[e^{-2i\beta} (V_{tb} V_{ts}^*)^2 (C_L^\nu + C_R^\nu)^2]$$

$B^0 \rightarrow K_S \nu \bar{\nu}$ direct CP asymmetry



Rule of thumb:

- Belle II 50 ab^{-1}
→ $N = 200$
- FCC-ee Tera Z
→ $N > 20\text{k}$

Colors:

- Blue (flat) → SM
- Other → benchmark BSM models

[Descotes-Genon, Fajfer *et al* '22]

Other observables (3)

- With a bit more data, one can measure more involved observables such as:
 - Longitudinal fraction [Buras, Girschbach-Noe *et al* '14; Altmannshofer, Buras *et al* '09...]
 - (mixing induced) CP-asymmetries [Descotes-Genon, Fajfer *et al* '22]
 - ν/ℓ ratio [Bečirević, Piazza, Sumensari '23]:

$$\mathcal{R}_K^{(\nu/\ell)}[1.1, 6] \Big|_{\text{SM}} = 7.58 \pm 0.04 \qquad \mathcal{R}_{K^*}^{(\nu/\ell)}[1.1, 6] \Big|_{\text{SM}} = 8.6 \pm 0.3$$

Summed over the three ν and $\ell = e, \mu$
(minimalist implementation of the charm-loops...)

- As discussed, the $B \rightarrow K\nu\nu$ analysis assumes the **SM kinematics**.
- In general, such analyses require theory inputs for the form-factors:
 - For fully reconstructed final-state, the uncertainty assigned to form-factors is **usually small** (but has to be checked!)
 - For partially reconstructed final-state, this can be a **large source of uncertainties**
- In the case of $B \rightarrow K\nu\nu$, switching on **scalar or tensor WC** changes the kinematics completely, as they involve other form-factors!
- Sometimes overlooked in the literature.

$$\frac{d\Gamma}{dq^2} = 3 \left(\frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi} \right)^2 |V_{ts}^* V_{tb}|^2 \frac{\sqrt{\lambda_{BK}} q^2}{(4\pi)^3 M_B^3} \times \left[\frac{\lambda_{BK}}{24q^2} |f_+(q^2)|^2 |C_{VL} + C_{VR}|^2 + \frac{(M_B^2 - M_K^2)^2}{8(m_b - m_s)^2} |f_0(q^2)|^2 |C_{SL} + C_{SR}|^2 + \frac{2\lambda_{BK}}{3(M_B + M_K)^2} |f_T(q^2)|^2 |C_{TL}|^2 \right],$$

- Several techniques have been developed:
 - **Full reinterpretation:** new MC samples are created based on an alternative model [CheckMate; MadAnalysis5; RECAST]
 - **Simplified reinterpretation:** assumes the kinematic distribution to be weakly impacted by BSM physics [SModels]
 - **Reweighting:** Use the existing simulation but reweight the distributions according to a new model [HAMMER; Gärtner, MR *et al* '24]
- The choice of the tool completely depends on the experimental analysis
 - **Compromise** between the needs and the computational cost

Our reinterpretation framework in a nutshell

Updated model
(WCs, resonances...)

Nuisance parameters
(form factors, CKM entries...)

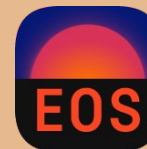
Analysis framework

Efficiency maps

Signal benchmark
MC distributions σ_0

Experimental datapoints
and background shapes

- Analytic implementation
- Plotting framework
- Bayesian analysis (model sampling, ...)
- Discretisation (correlated uncertainties, ...)



Updated signal
or background
distributions σ_1

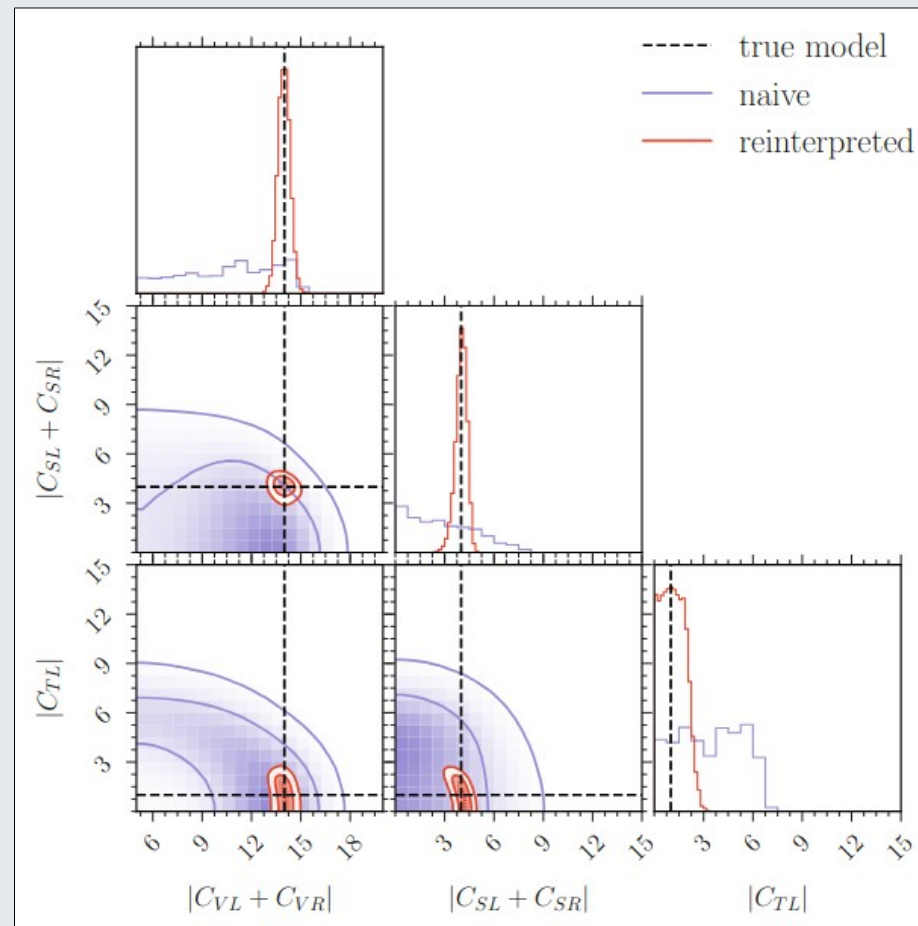
Model
likelihood

pyhf [Feickert, Heinrich *et al* '21; '24]

- Weights $w_z = \frac{\sigma_{1,z}}{\sigma_{0,z}} = \frac{\int_{\text{bin } z} dz' \sigma_1(z')}{\int_{\text{bin } z} dz' \sigma_0(z')}$
- Fit framework (MCMC sampling with **Bayesian pyhf** [Feickert, Heinrich, Horstmann '23])

Concrete examples (1)

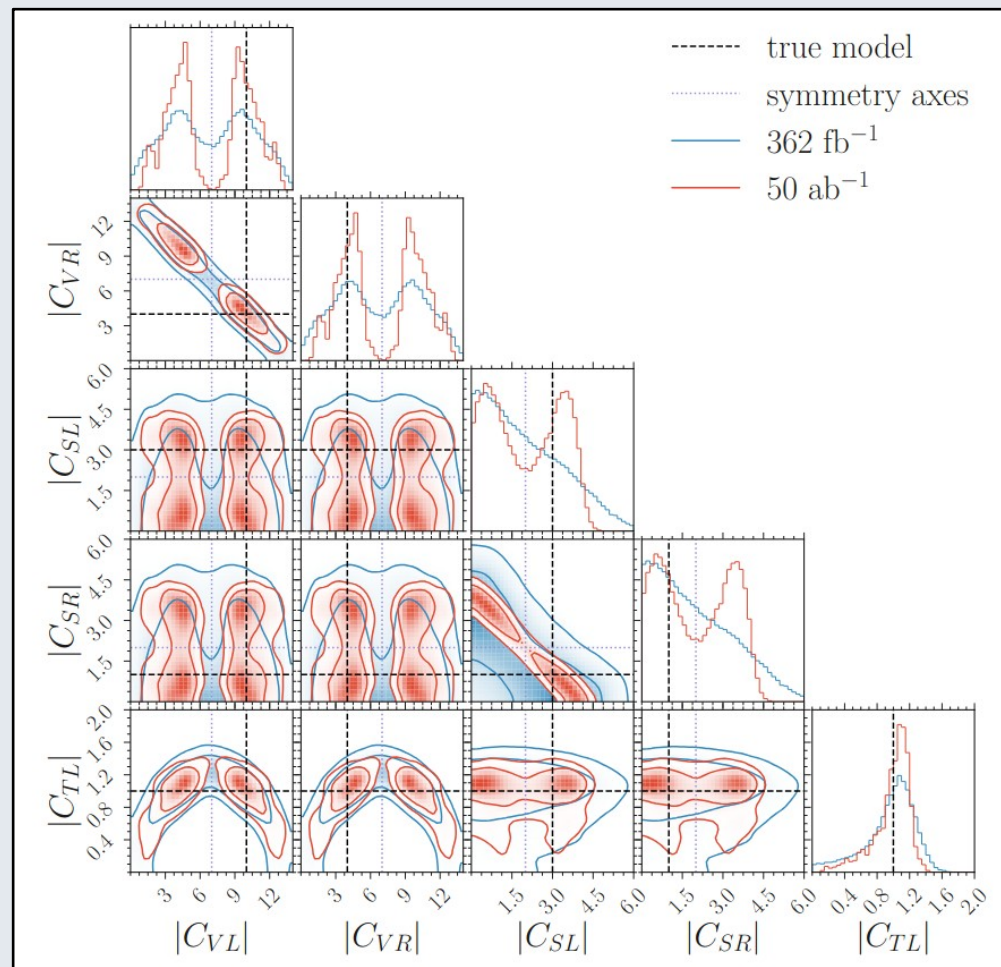
- Comparison between the posterior of a WC analysis **with** and **without** reinterpreting the data:
 - 2 blind directions are due to the decay ($C_{VL} - C_{VR}$, $C_{SL} - C_{SR}$)
 - Reinterpreting the data increases the sensitivity drastically
- The plot is a 50 ab^{-1} projection of the $B \rightarrow K_{\nu\nu}$ analysis [Gärtner, MR *et al* '24]



Concrete examples (2)

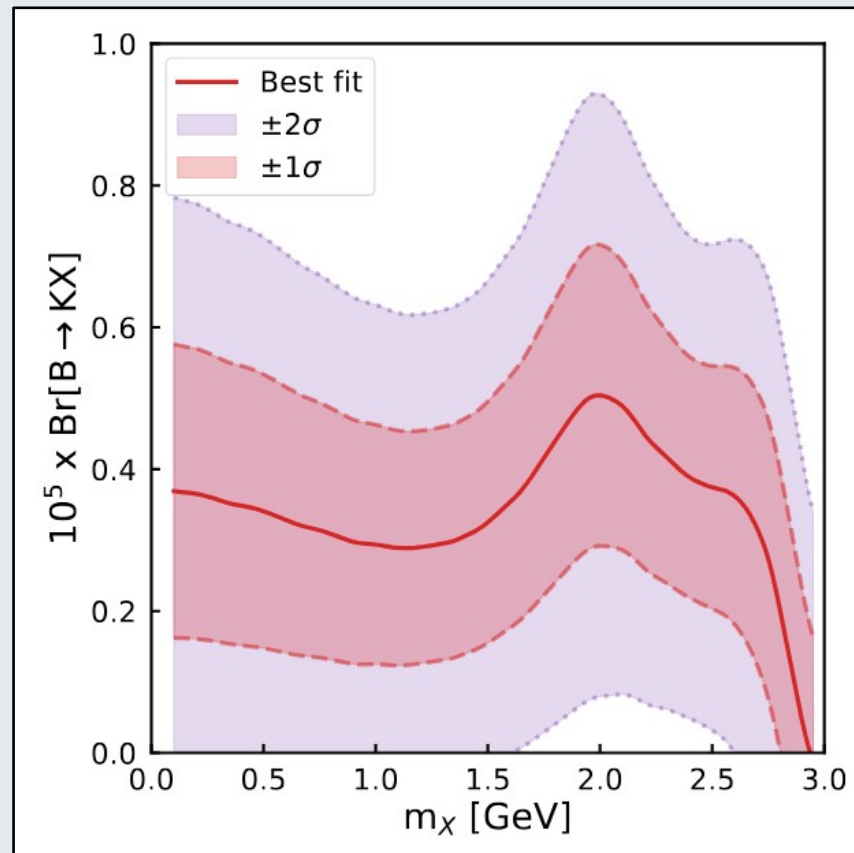
- This framework can easily be generalized to a **combined analysis** of $B \rightarrow K \bar{\nu}\nu$ and $B \rightarrow K^* \bar{\nu}\nu$ branching ratios
- Symmetry axes will prevent from an unambiguous WC determination
→ angular analyses will be needed
- Current luminosity vs full Belle II dataset differs mostly in the scalar sector

[Gärtner, MR *et al* '24]



Light new physics

- The Belle II $B \rightarrow K\nu\bar{\nu}$ results shows a slight excess for $q^2 \sim 4 \text{ GeV}^2$, motivating a **light new physics** interpretation, $B \rightarrow KX$ [Altmannshofer, Crivellin *et al* '23]
 - A **bump search** is performed assuming a Gaussian signal with experimental width only
 - **pyhf** is used with the maximal amount of experimental information \rightarrow would require a fully reinterpreted analysis [Belle II (Gärtner), w.i.p.]
 - Current data favors $m_X \sim 2 \text{ GeV}$



Future of $b \rightarrow sv\bar{v}$

- $q_i \rightarrow q_j \bar{v}\bar{v}$ are very promising transitions but remain an experimental challenge
- As far as $b \rightarrow sv\bar{v}$ is concerned, only $B \rightarrow K^{(*)} \bar{v}\bar{v}$ decays are currently measurable
- A *tera-Z* run at FCC-ee, if it is build, would however open **many possibilities**:
 - $(10^{12} \text{ Z bosons}) \times (\text{Br}(Z \rightarrow b\bar{b}) = 0.15) = \text{lot of } b \text{ hadrons}^*!$ (“LEP in a minute”)
 - All of this in a **clean environment**
 - With many interaction points (as opposed to a linear design).

* But also c hadrons: $\text{Br}(Z \rightarrow c\bar{c}) = 0.12$

Future of $b \rightarrow svv$

- Future e^+e^- (CEPC, FCCee) will give access to many $b \rightarrow svv$ modes
- Let's focus on **charged 4-body modes** (for the tracking)

Decay mode	$\mathcal{B}/ \lambda_t ^2 [10^{-3}]$	$\mathcal{B} [10^{-6}]$
$B^0 \rightarrow K_S^0 \nu \bar{\nu}$	1.33 ± 0.04	2.02 ± 0.12
$B^0 \rightarrow K^{*0} \nu \bar{\nu}$	5.13 ± 0.51	7.93 ± 0.89
$B_s^0 \rightarrow \phi \nu \bar{\nu}$	6.31 ± 0.67	9.74 ± 1.15
$\Lambda_b^0 \rightarrow \Lambda \nu \bar{\nu}$	5.55 ± 0.56	8.57 ± 0.97

[Amhis, MR *et al* '23]



With current form-factor
uncertainties

FCCee analysis (briefly)

- **We generated** 4 signal MC samples as well as background (inclusive $Z \rightarrow b\bar{b}$, $Z \rightarrow c\bar{c}$, $Z \rightarrow q\bar{q}$) samples
 - We assumed an IDEA detector design
 - The kinematic is generated via weights from the generated phase-space-only events and EOS predictions (LQCD + LCSR)
- **We developed** 4 dedicated analyses
 - Assumption: perfect vertex seeding and perfect PID
 - 2-step BDT optimization
- **We studied** few (inclusive) backgrounds

[Amhis, MR *et al* '23]

Results

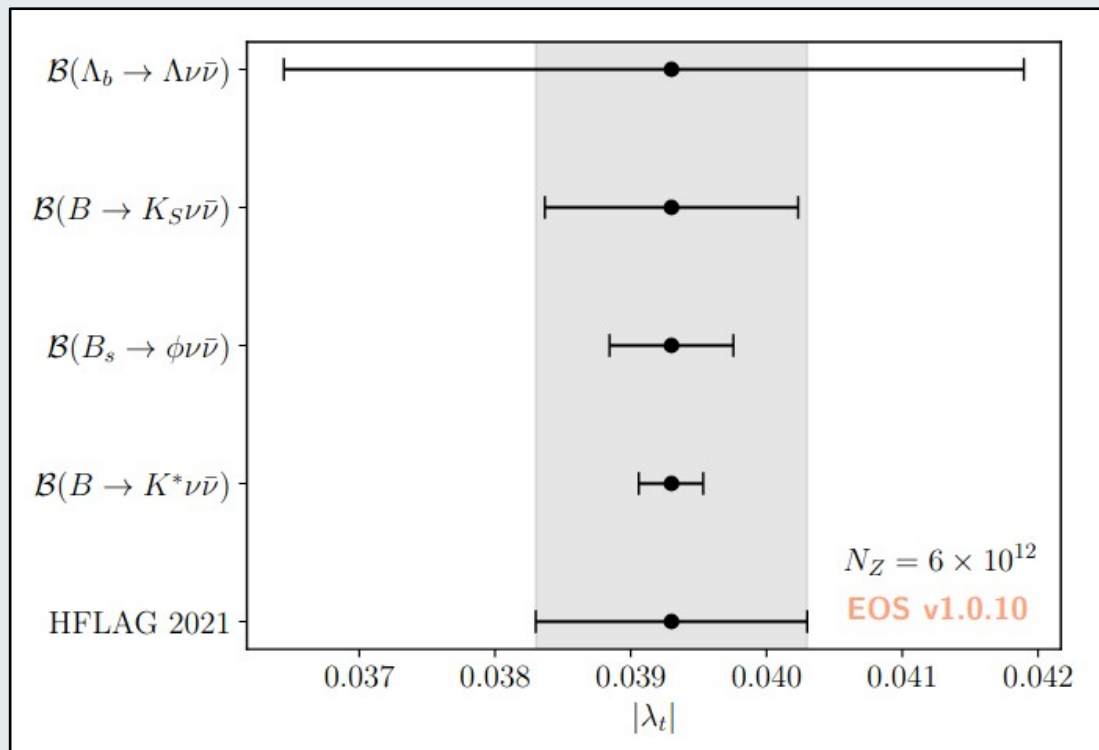
- Final results for the K^* and ϕ modes:

Mode	N_S	N_B	ϵ^s	$\epsilon^{b\bar{b}}$	$\epsilon^{c\bar{c}}$	$\epsilon^{q\bar{q}}$	S/B	$\sqrt{S+B}/S$
$B^0 \rightarrow K^{*0} \nu \bar{\nu}$	231 K	1.27 M	3.7%	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-9})$	$\mathcal{O}(10^{-9})$	0.17	0.53%
$B_s^0 \rightarrow \phi \nu \bar{\nu}$	61 K	0.48 M	7.4%	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-9})$	$\mathcal{O}(10^{-9})$	0.13	1.20%

- The reconstruction of K_S and Λ were not fully available and the results come from extrapolations: $\sigma(K_S) = 3.37\%$, $\sigma(\Lambda) = 9.86\%$, with purities of 4% and 1.5% respectively
- Rough comparison to the current Belle II sensitivity for the $B \rightarrow K \nu \bar{\nu}$ (different analyses), $\epsilon(\text{ITA}) \sim 5 - 10\%$, $\epsilon(\text{HTA}) \sim 0.3 - 0.5\%$, with a purity of 5% in the signal region

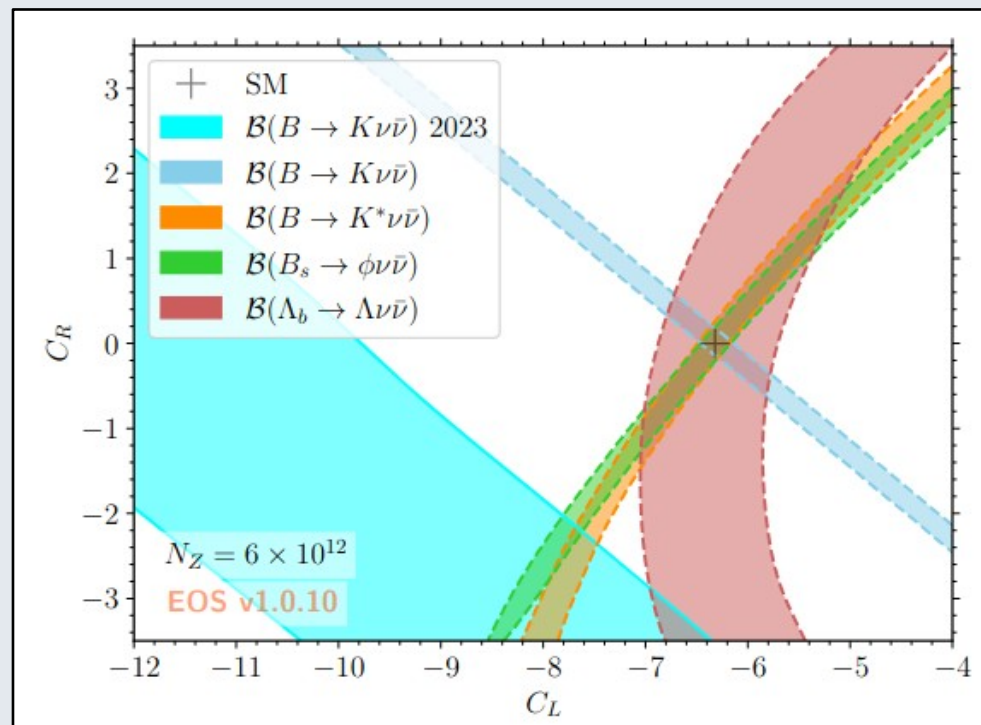
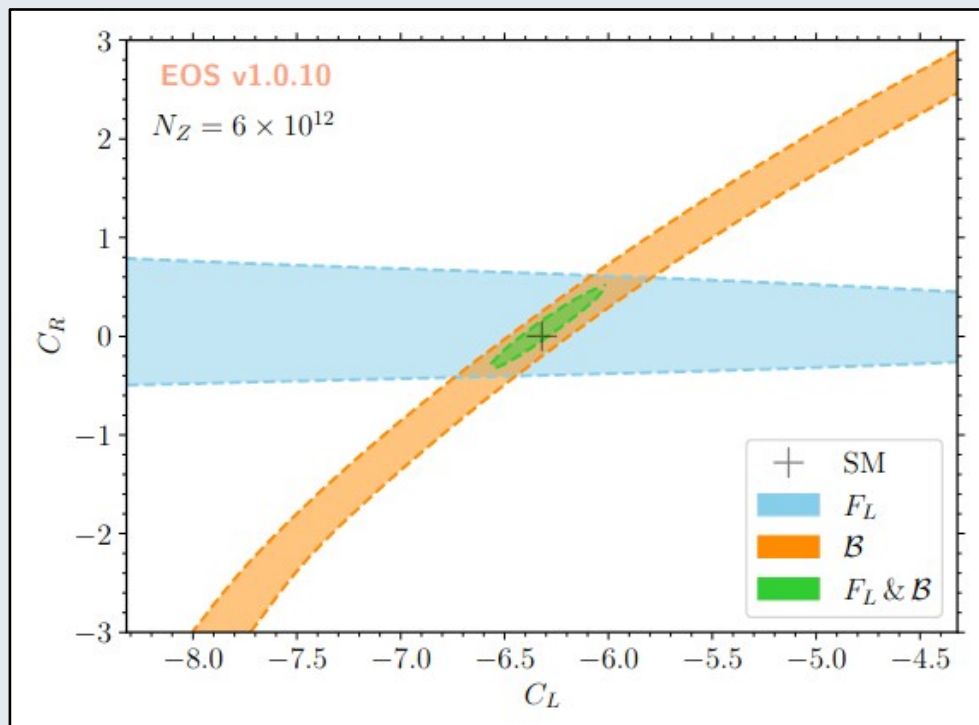
Future phenomenology of $b \rightarrow s\nu\bar{\nu}$

- Assuming these efficiency, we would get clean access to $|\lambda_t|$:



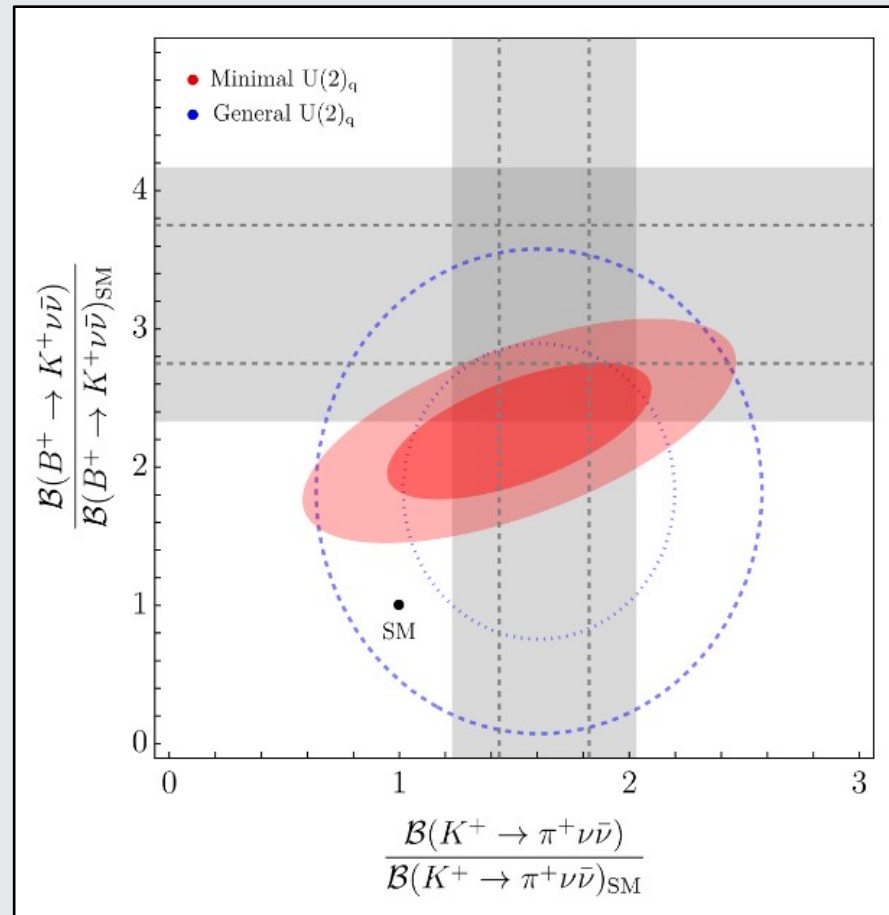
Future phenomenology of $b \rightarrow sv$

- Assuming these efficiency, we would get clean access to the WET WCs:



Beyond $b \rightarrow sv$

- $K \rightarrow \pi \bar{\nu} \nu$ recently measured [NA62 '24]
 - Allows to disentangle NP scenarios [Buras, Harz, Mojahed '24]
 - Possibility of combined analysis with a flavour structure [Allwicher, Bordone *et al* '24]
- Combined analysis in the (v)SMEFT framework and impact for $b \rightarrow c \ell \nu$ [Allwicher, Bečirević *et al*; Leal, Rosauero-Alcaraz; Bernlochner, Fedele, *et al*; Datta, Kumar *et al*; Bečirević, Fajfer, *et al*, Marzocca, Nardecchia *et al*; Hou, Li *et al*; Chen, Xu *et al*] (All '24, I hope I didn't forget any groups)



Conclusions

- $b \rightarrow s\bar{\nu}\nu$ decays offer plenty of **extremely clean observables**, opening many opportunities for future phenomenology analyses of
 - (B)SM parameters: CKM elements, WET/SMEFT coefficients...
 - QCD effects: form-factors, QCD penguins...
- This comes with the price of **high experimental challenges**
 - At the level of the measurements: missing energy, vertexing...
 - At the level of the interpretation: model-dependent analyses that need to be reinterpreted
- Belle II will already offer a **first set of measurements**, the rest will have to wait for **future colliders**

Back-up slides

More observables (for FCC-ee)

$$\frac{d\mathcal{B}(B^0 \rightarrow K_S^0 \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_{B^0} |N_{B^0}|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 \rho_+^{K_S^0},$$

$$\rho_+^{K_S^0} = \frac{\lambda^{3/2}}{2m_{B^0}^4} (f_+^K(q^2))^2, \quad \frac{d\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_{B^0} |N_{B^0}|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 (\rho_{A_1}^{K^{*0}} + \rho_{A_{12}}^{K^{*0}} + \rho_V^{K^{*0}}),$$

$$\rho_V^{K^{*0}} = \frac{2q^2 \lambda^{3/2}}{(m_{B^0} + m_{K^{*0}}) m_{B^0}^4} (V^{K^*}(q^2))^2, \quad \frac{d\mathcal{B}(B_s^0 \rightarrow \phi \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_{B_s^0} |N_{B_s^0}|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 (\rho_{A_1}^\phi + \rho_{A_{12}}^\phi), \quad \rightarrow \Lambda \nu \bar{\nu})_{\text{SM}} = \frac{\rho_{f_0^A}^{f_0^V} + \rho_{f_0^A}^{f_0^A}}{\rho_{f_\perp^A}^{f_\perp^V} + \rho_{f_\perp^A}^{f_\perp^A} + \rho_{f_0^A}^{f_0^V} + \rho_{f_0^A}^{f_0^A}},$$

$$\rho_{A_1}^{K^{*0}} = \frac{2q^2 \lambda^{1/2} (m_{B^0} + m_{K^{*0}})^2}{m_{B^0}^4} (A_1^{K^*}(q^2))^2, \quad \frac{d\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda \nu \bar{\nu})_{\text{SM}}}{dq^2} = 3 \tau_{\Lambda_b^0} |N_{\Lambda_b^0}|^2 |C_L^{\text{SM}}|^2 |\lambda_t|^2 (\rho_{f_\perp^A}^\Lambda + \rho_{f_\perp^A}^{f_\perp^A} + \rho_{f_0^A}^\Lambda + \rho_{f_0^A}^{f_0^A}),$$

$$A_{\text{FB}}^\Lambda(\Lambda_b^0 \rightarrow \Lambda \nu \bar{\nu})_{\text{SM}} = \frac{\alpha}{2} \frac{\tilde{\rho}_\perp^\Lambda + \tilde{\rho}_0^\Lambda}{\rho_{f_\perp^A}^\Lambda + \rho_{f_\perp^A}^{f_\perp^A} + \rho_{f_0^A}^\Lambda + \rho_{f_0^A}^{f_0^A}},$$

$$\rho_{A_{12}}^{K^{*0}} = \frac{64 m_{K^{*0}}^2 \lambda^{1/2}}{m_{B^0}^2} (A_{12}^{K^*}(q^2))^2,$$

$$\rho_{f_\perp^A}^{f_0^A} = \frac{32 q^2 \lambda^{1/2} ((m_{\Lambda_b^0} \mp m_\Lambda)^2 - q^2)}{m_{\Lambda_b^0}^4} (f_\perp^{V/A}(q^2))^2,$$

$$\rho_{f_0^A}^{f_0^A} = \frac{16 \lambda^{1/2} (m_{\Lambda_b^0} \pm m_\Lambda)^2 ((m_{\Lambda_b^0} \mp m_\Lambda)^2 - q^2)}{m_{\Lambda_b^0}^4} (f_0^{V/A}(q^2))^2,$$

$$\tilde{\rho}_\perp^\Lambda = \frac{32 q^2 \lambda^{1/2} ((m_{\Lambda_b^0} \mp m_\Lambda)^2 - q^2)}{m_{\Lambda_b^0}^4} f_\perp^V(q^2) f_\perp^A(q^2),$$

$$\tilde{\rho}_0^\Lambda = \frac{16 \lambda^{1/2} (m_{\Lambda_b^0} \pm m_\Lambda)^2 ((m_{\Lambda_b^0} \mp m_\Lambda)^2 - q^2)}{m_{\Lambda_b^0}^4} f_0^V(q^2) f_0^A(q^2).$$

$$F_L(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}} = \frac{\rho_{A_{12}}^{K^{*0}}}{\rho_{A_1}^{K^{*0}} + \rho_{A_{12}}^{K^{*0}} + \rho_V^{K^{*0}}},$$

$$F_L(B_s^0 \rightarrow \phi \nu \bar{\nu})_{\text{SM}} = \frac{\rho_{A_{12}}^\phi}{\rho_{A_1}^\phi + \rho_{A_{12}}^\phi + \rho_V^\phi}.$$

$$N_{B_q} = \frac{G_F \alpha_{\text{em}}}{16\pi^2} \sqrt{\frac{m_{B_q}}{3\pi}}$$