OPEN QUESTIONS IN INCLUSIVE B DECAYS

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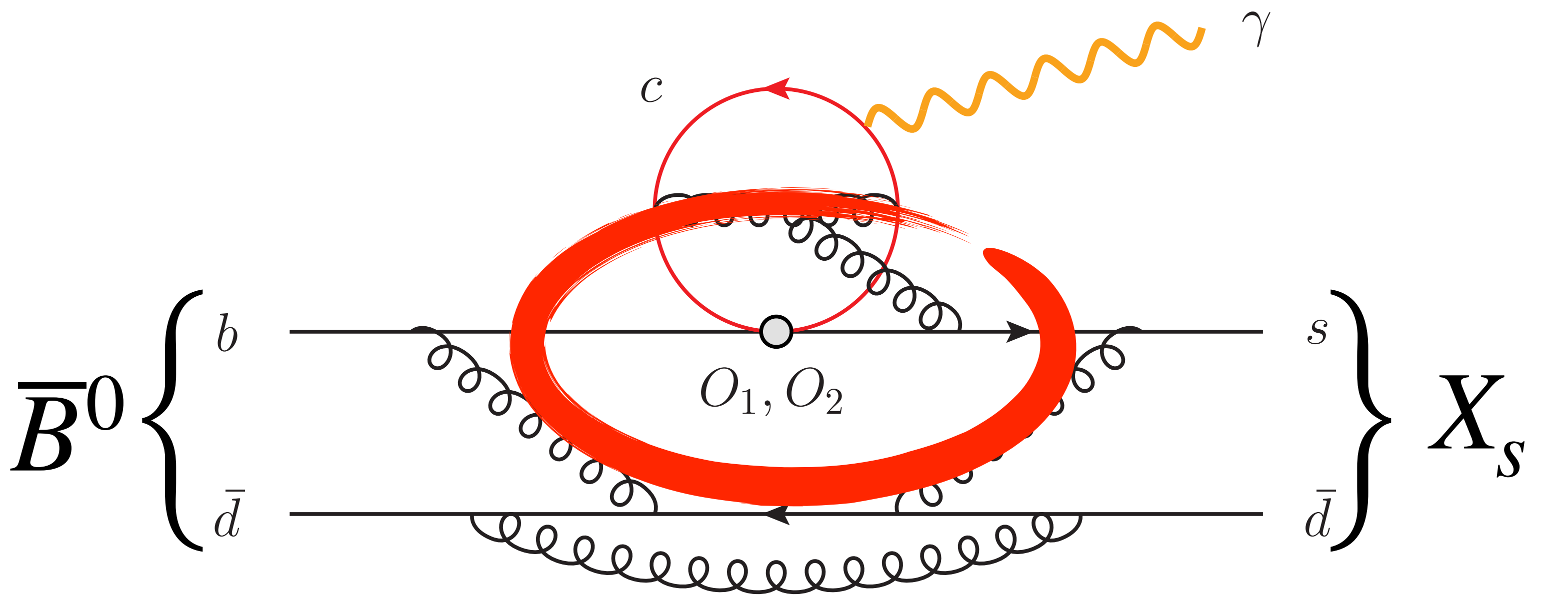
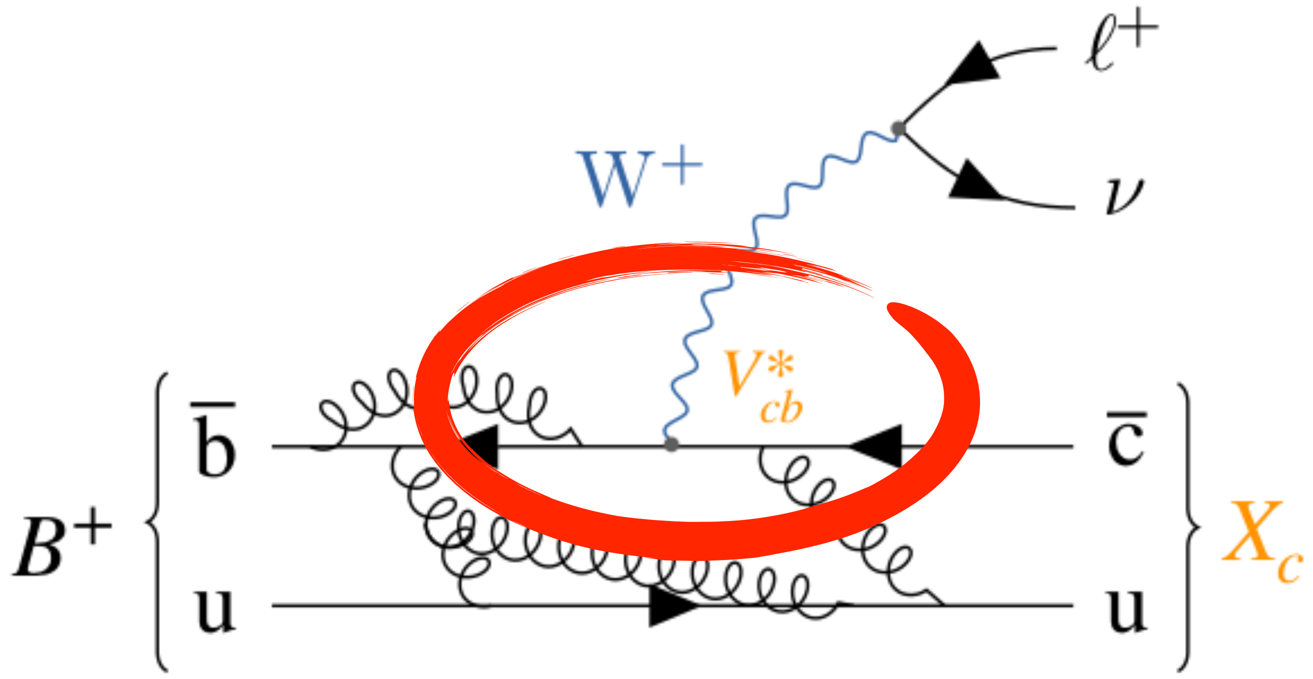
Open directions and future questions in flavour physics

MITP MAINZ - NOVEMBER 14TH, 2024

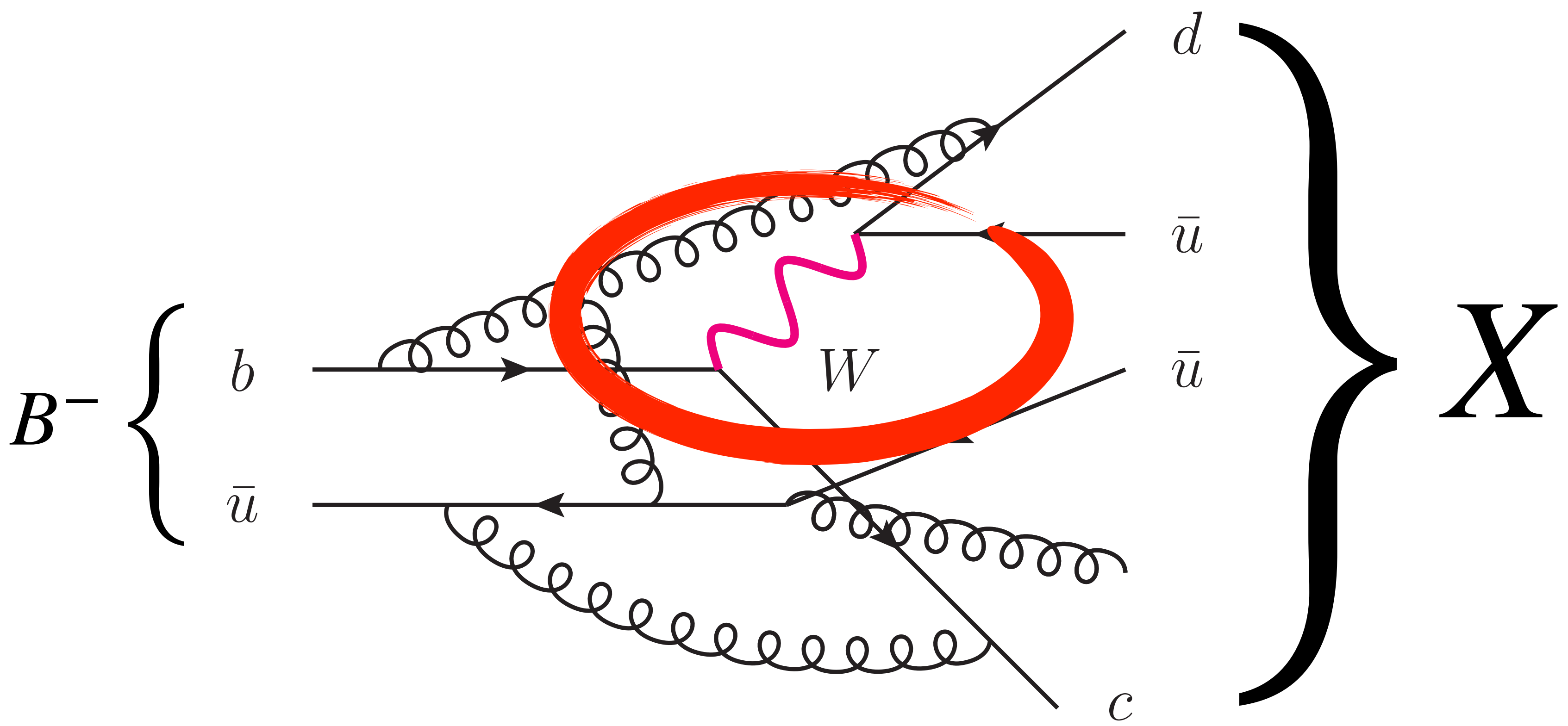
INCLUSIVE DECAYS OF B MESONS



Rare decay $B \to X_s \gamma$



Semileptonic $B \to X_c l \bar{\nu}_l$



Nonleptonic decays

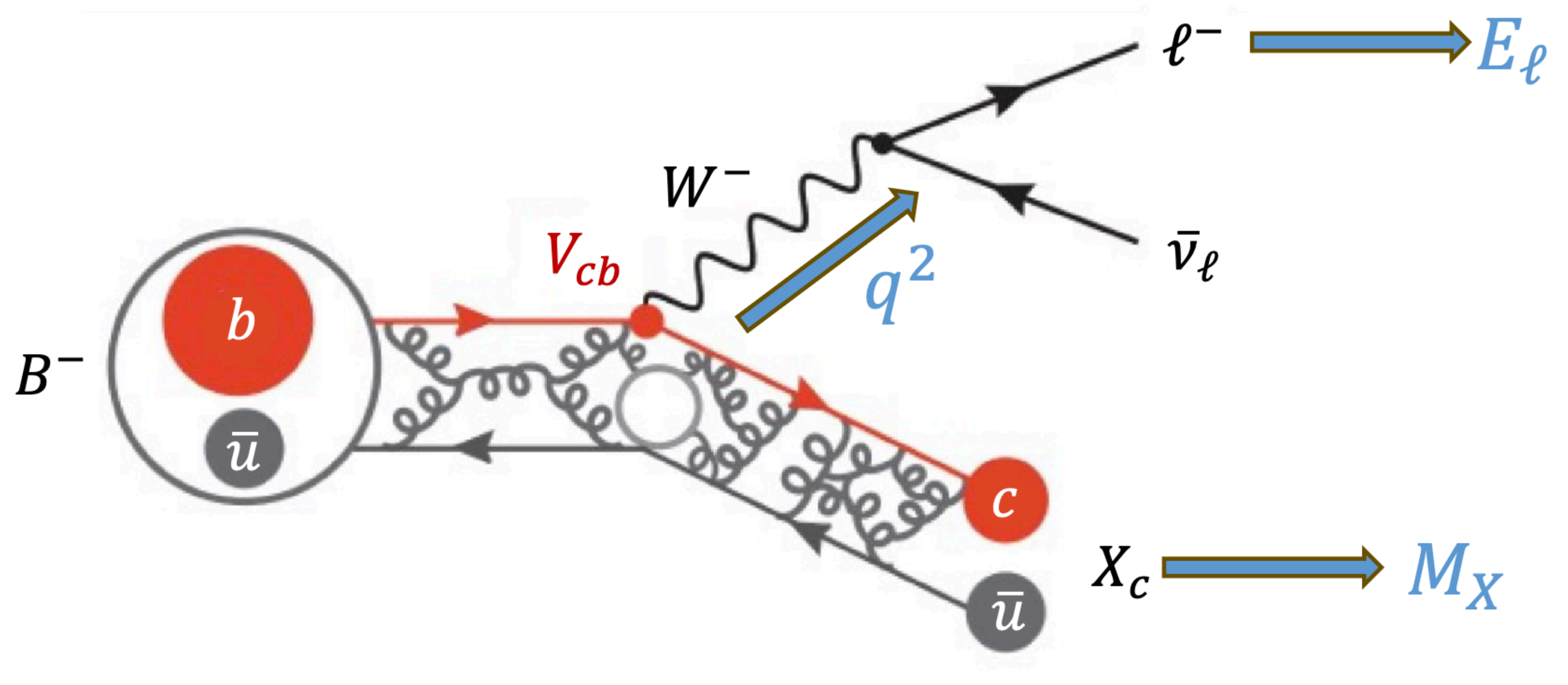
Lifetime of B mesons

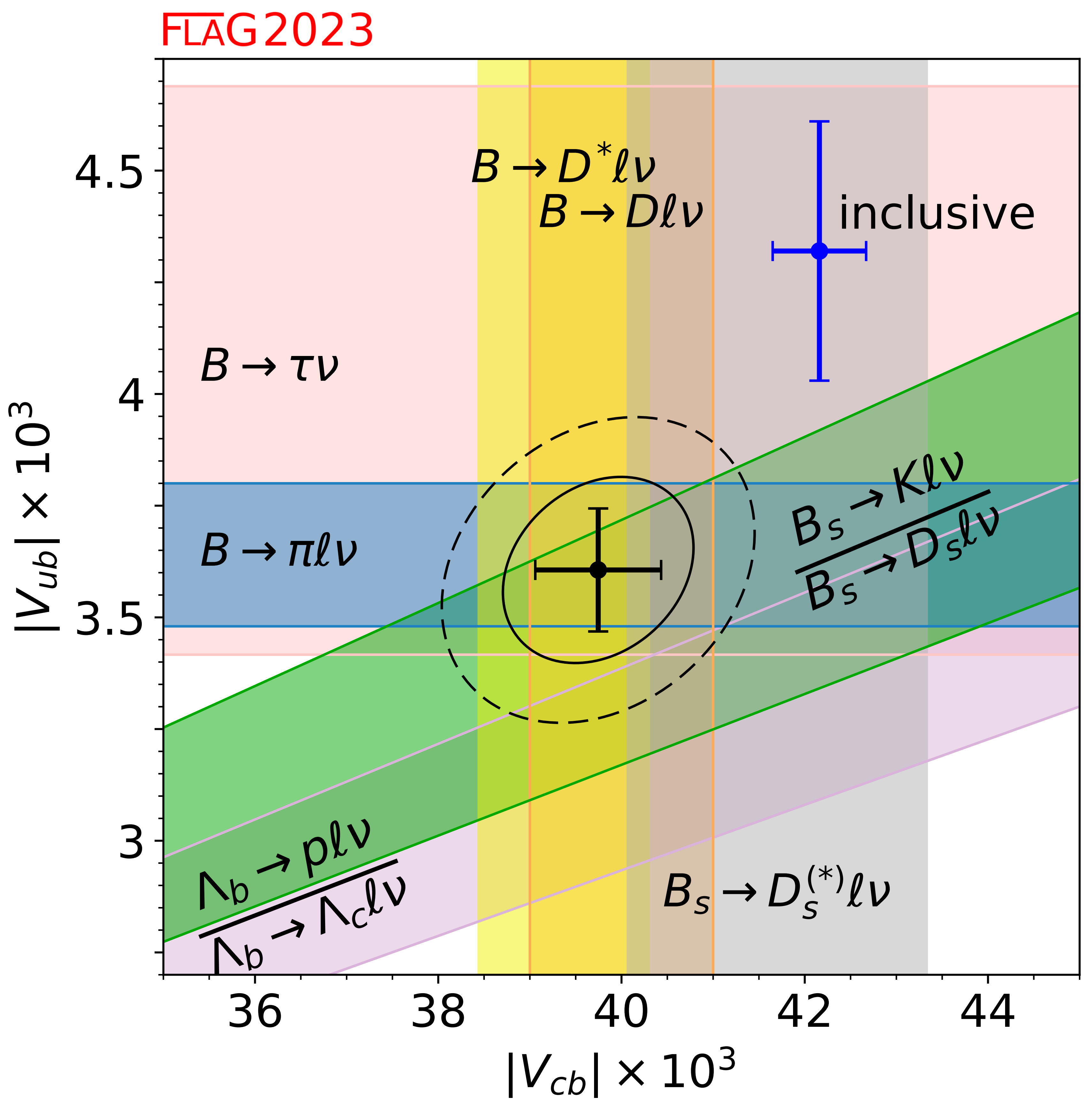
We need precise predictions in the SM, often at the 1-2% level!

OUTLOOK

- New results in semileptonic inclusive $B \to X_c l \bar{\nu}_l$ decays
- The B-meson lifetimes to NNLO

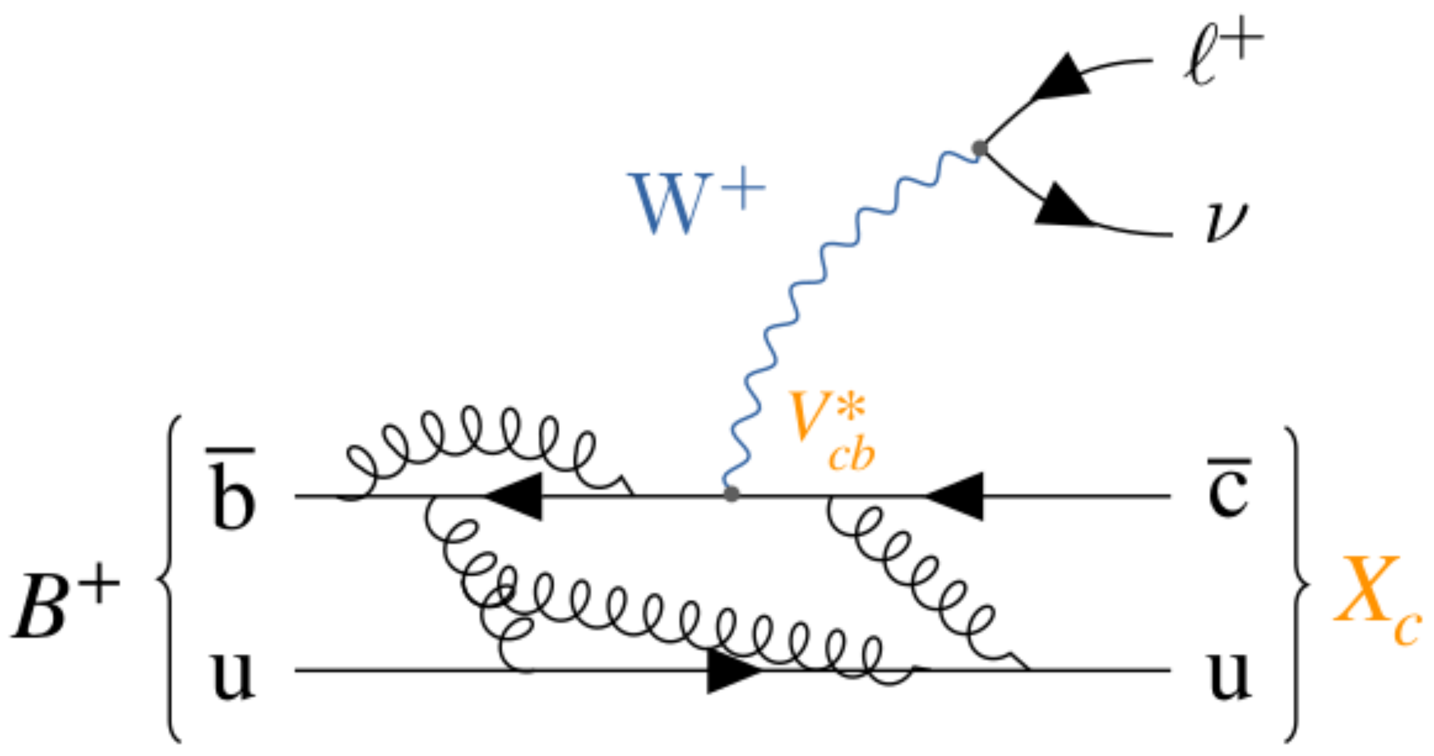
SEMILEPTONIC B DECAYS

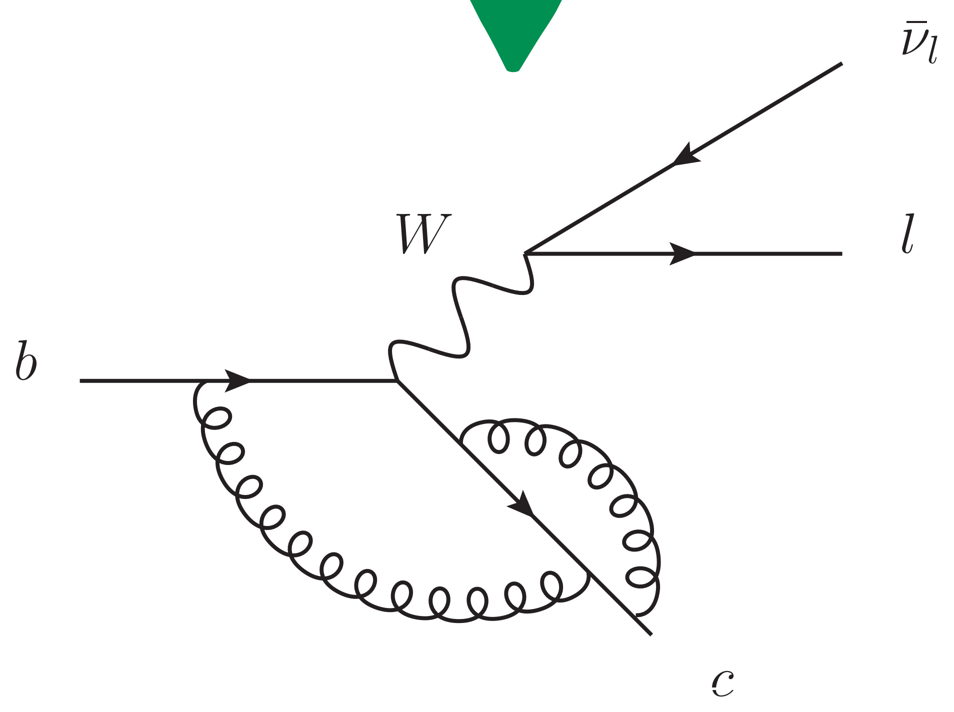




- Extraction of the CKM element $|V_{cb}|$.
- Determination of the non-perturbative matrix elements from experimental data.
- Predictions for processes with FCNC crucially depend on these SM inputs.
  - $|V_{tb}V_{ts}^\star| \simeq |V_{cb}|^2(1 + O(\lambda^2))$
  - $\epsilon_K \simeq |V_{cb}|^4 x$

THE HEAVY QUARK EXPANSION





Use optical theorem

$$\Gamma_{\mathrm{sl}} = \frac{1}{2m_B}\sum_X \left|\langle X|\mathscr{H}_{\mathrm{eff}}|B\rangle\right|^2$$

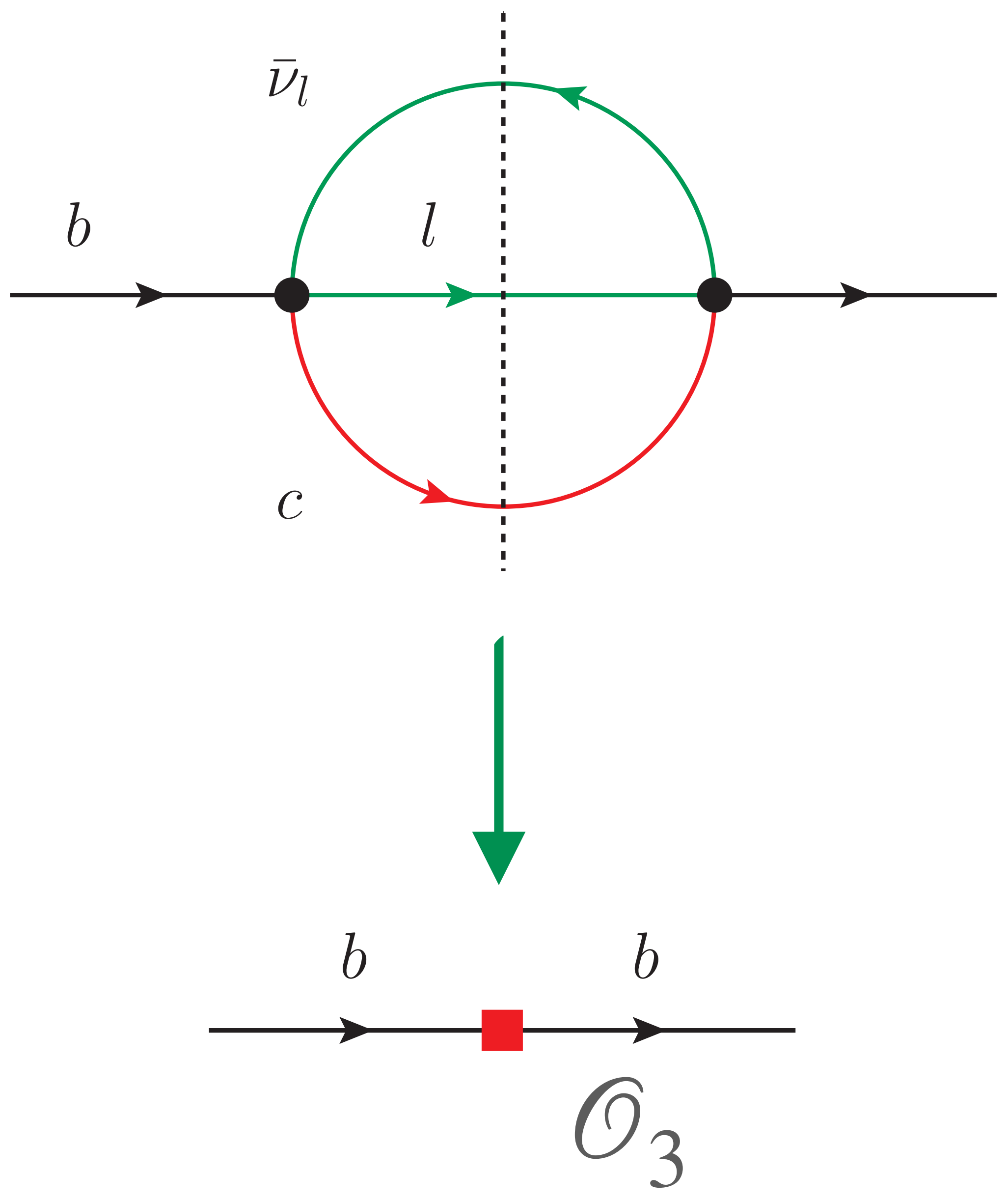
$$= C_3 + \frac{C_5}{m_b^2}\langle B|O_5|B\rangle + \frac{C_6}{m_b^3}\langle B|O_6|B\rangle + \ldots$$

Calculable in perturbative QCD

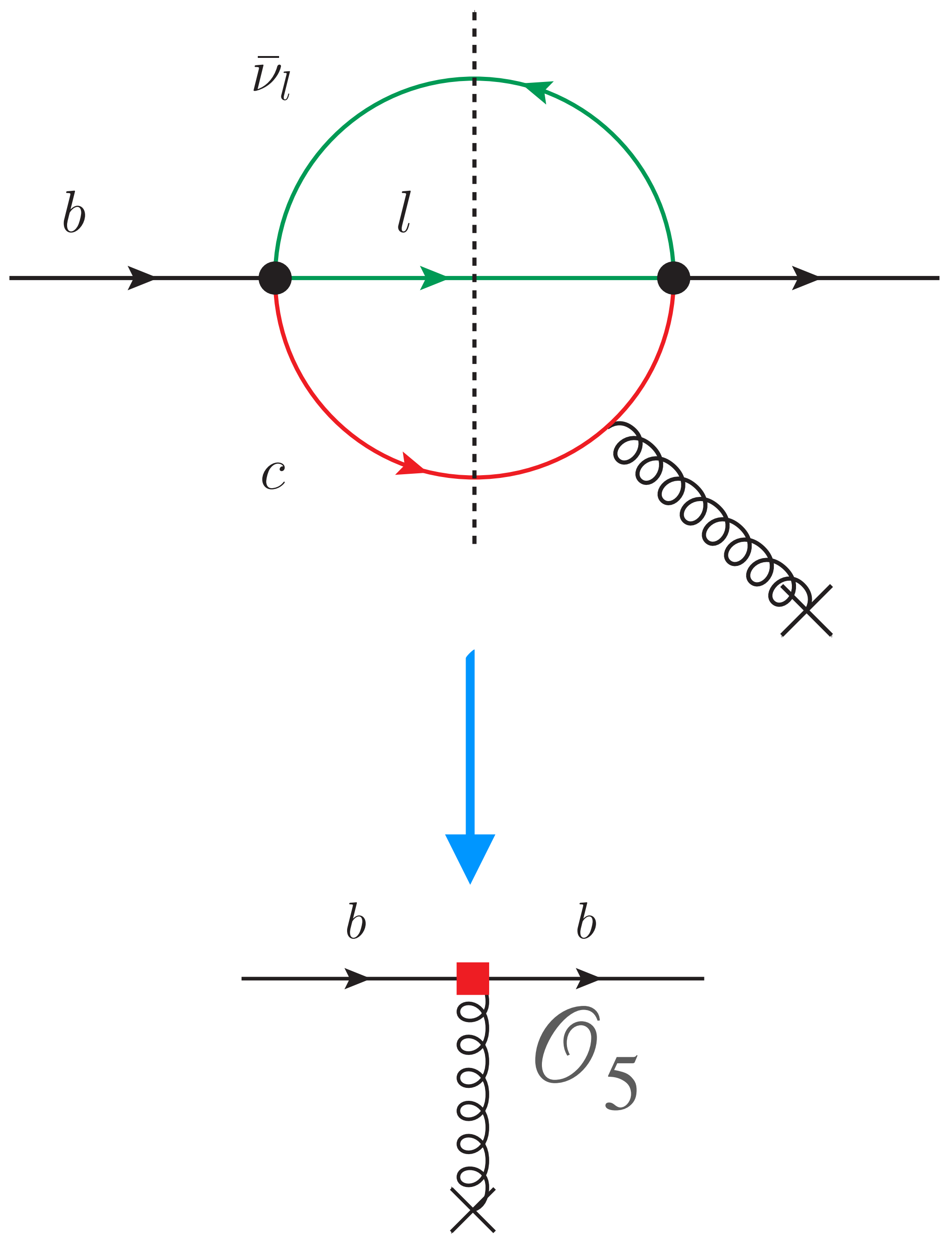
No non-perturbative matrix element at leading power!
In a first approximation we can consider the **decay of a free bottom quark**

THE HEAVY QUARK EXPANSION

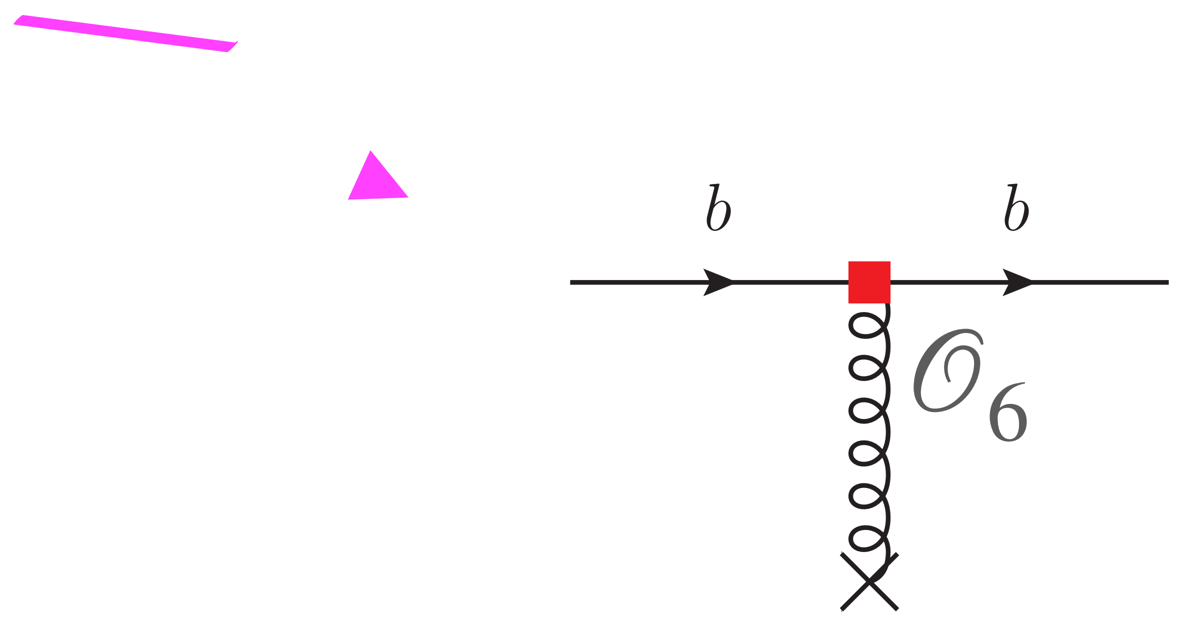
$$\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B | \mathcal{O}_5 | B \rangle}{m_b^2} + \Gamma_6 \frac{\langle B | \mathcal{O}_6 | B \rangle}{m_b^3} + \ldots$$



Free quark decay



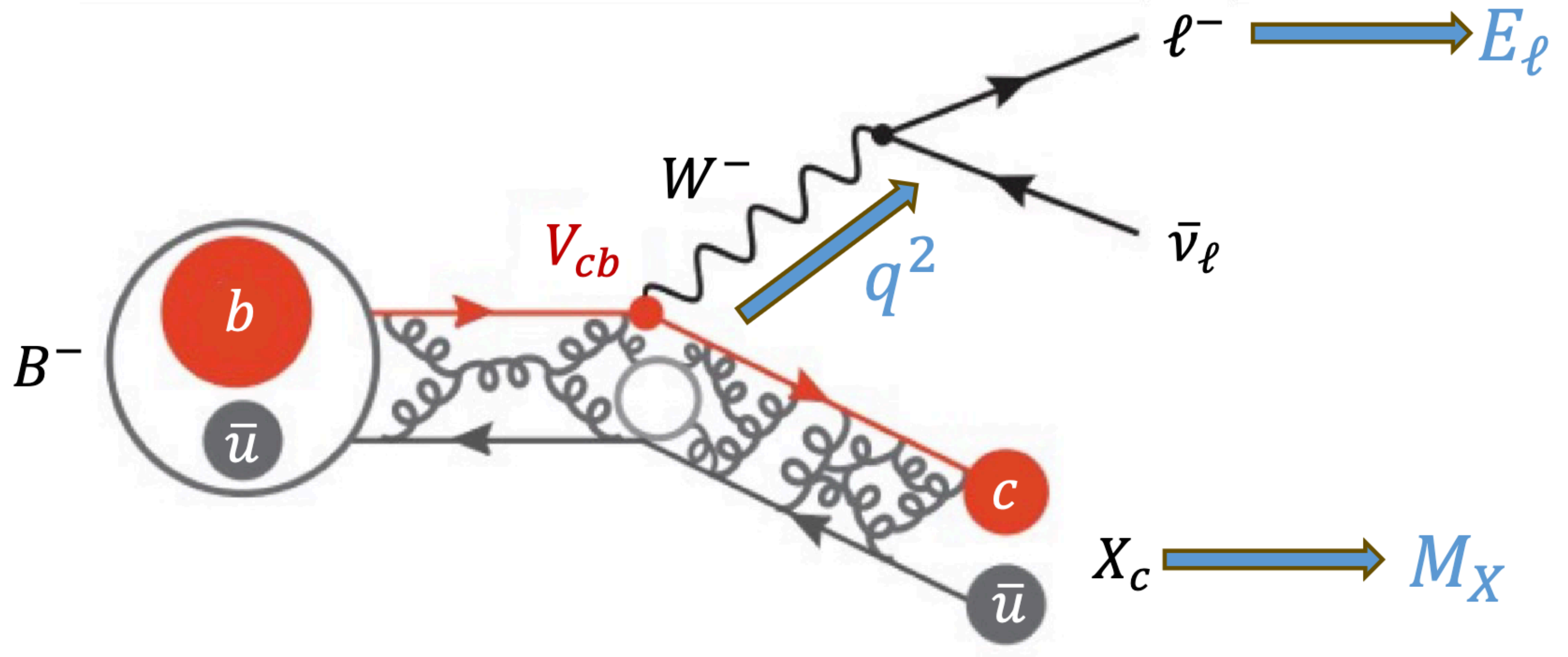
Kinetic term μ_π^2, chromomagnetic term μ_G^2



Darwin term ρ_D^3

Spin-Orbit term ρ_{LS}^3

SPECTRAL MOMENTS



$$\langle O^n \rangle_{\text{cut}} = \int_{\text{cut}} (O)^n \frac{d\Gamma}{d\Phi} d\Phi \Big/ \int_{\text{cut}} \frac{d\Gamma}{d\Phi} d\Phi$$

Cut: moments are measured with progressive cuts in E_l or q^2

$\langle E_l \rangle, \langle M_X^2 \rangle, \langle q^2 \rangle$

↓

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}, \ldots$

↓

$$\Gamma = \Gamma_3 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \ldots$$ → $|V_{cb}|$

$O = (p_l + p_\nu)^2 = q^2$	leptonic invariant mass
$O = (p_B - q)^2 = M_X^2$	hadronic invariant mass
$O = v_B \cdot p_l = E_l$	lepton energy

Q2 MOMENTS

$$\frac{\partial O}{\partial v_B} = 0$$

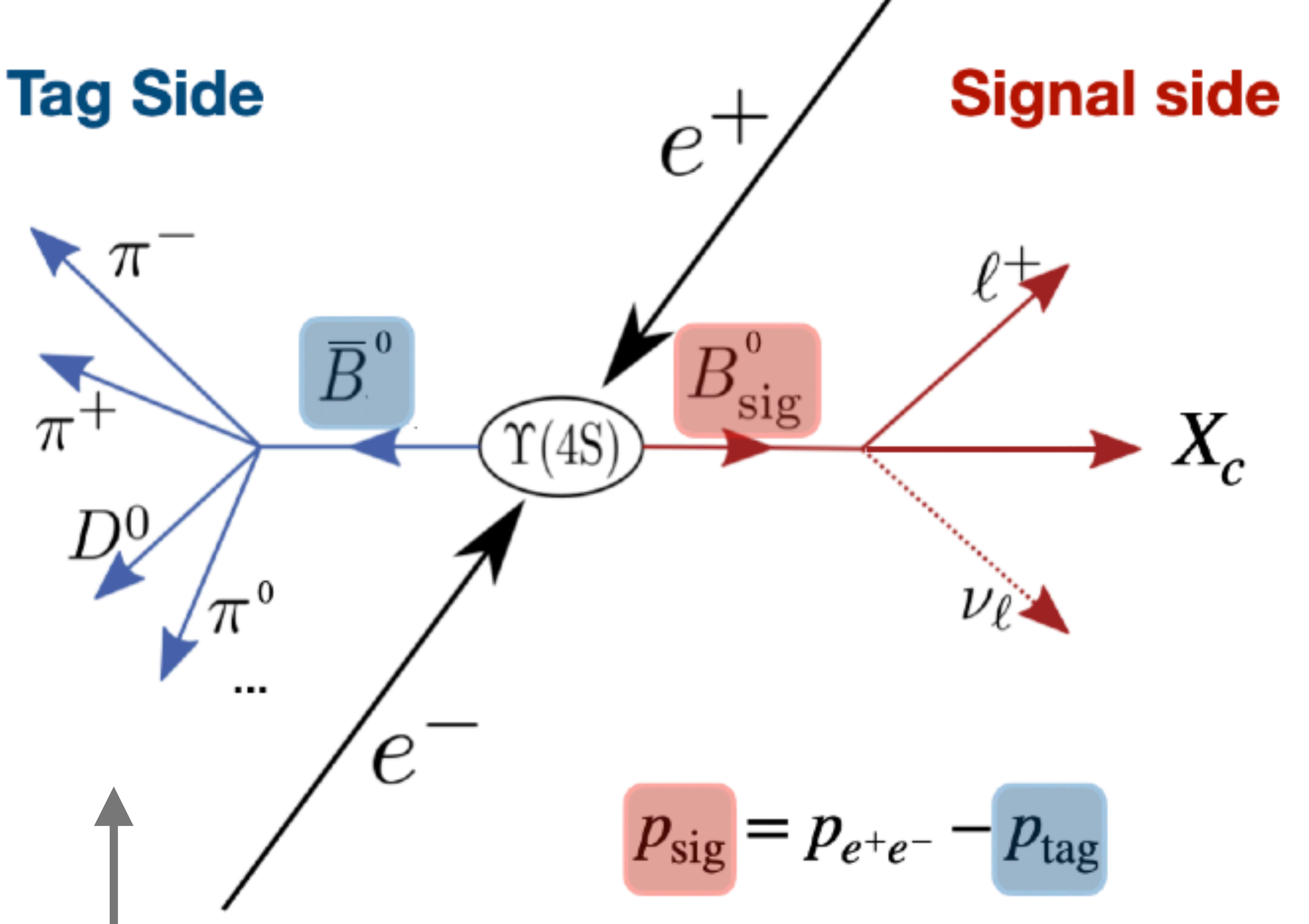
$$O = (p_l + p_\nu)^2 = q^2$$

$$O = (m_B v_B - q)^2 = M_X^2$$

$$O = v_B \cdot p_l = E_l$$

- $\Gamma_{\rm sl}$ and $\langle q^{2n} \rangle$ are invariant under reparametrization
- HQE parameters: 8 instead of 13 up to $1/m_b^4$
- NEW METHOD: extract $|V_{cb}|$ from q^2 moments

MF, Mannel, Vos, JHEP 02 (2019) 177



Hadronic Tagging with Belle II algorithm

Keck et al, Comput.Softw. Big Sci. 3 (2019) 1, 6

First new data since 2010!

New: Inclusive semileptonic $b \to c l \bar{\nu}_l$ decays to order $1/m_b^5$

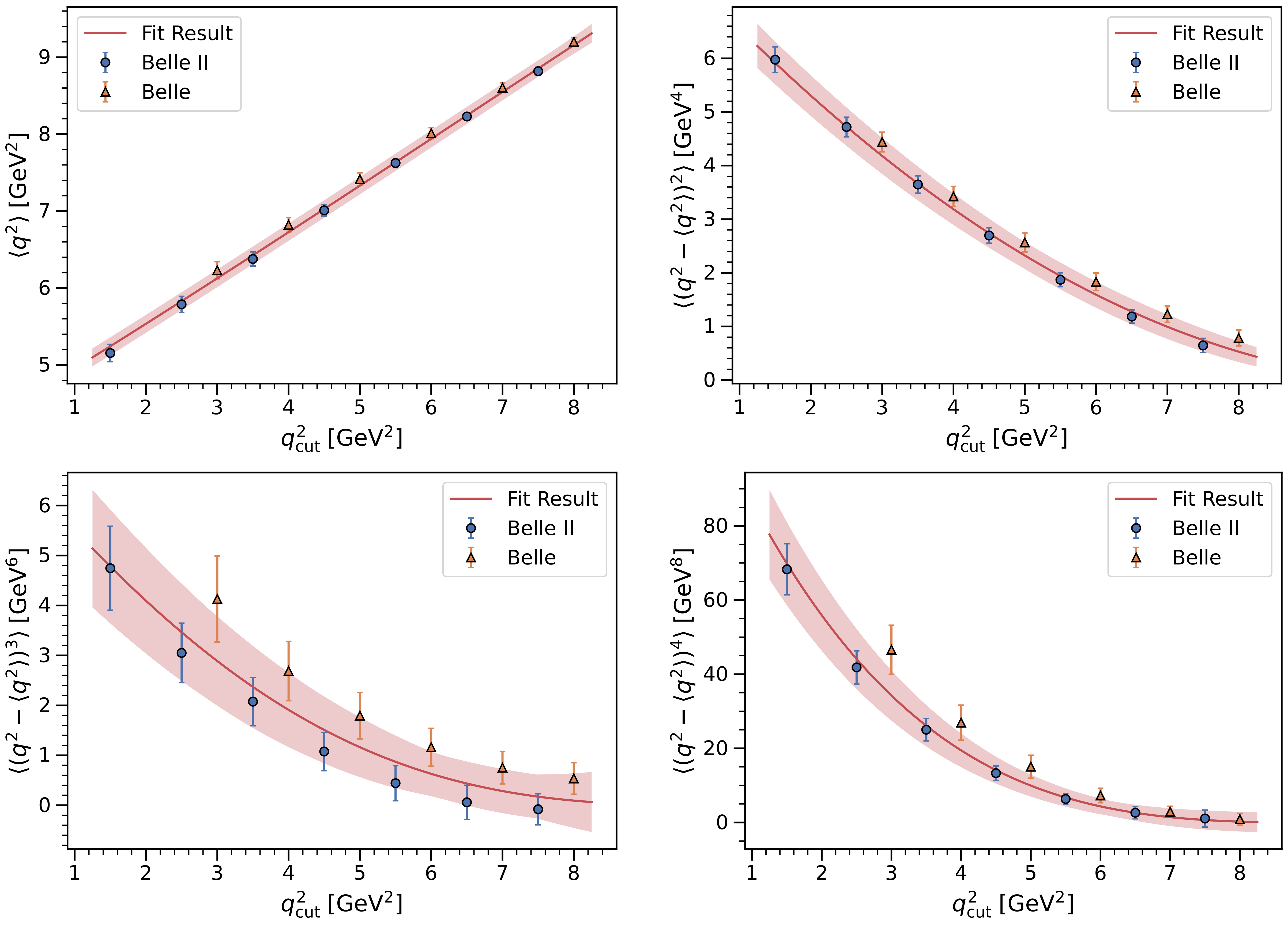
Mannel, Milutin, Vos, hep-ph/2311.12002

- 10 instead of 19 HQE parameters

Measurements of q^2 moments of inclusive $B \to X_c l^+ \nu_l$ decays with hadronic tagging

Belle, Phys. Rev. D 104, 112011 (2022)
Belle II, Phys. Rev. D 107, 072002 (2023)

$|V_{cb}|$ FROM q^2 MOMENTS



$$|V_{cb}| = (41.69 \pm 0.59_{\mathrm{fit}} \pm 0.23_{\mathrm{h.o.}}) \times 10^{-3}$$
$$= (41.69 \pm 0.63) \times 10^{-3}$$

Bernlochner, **MF**, Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068

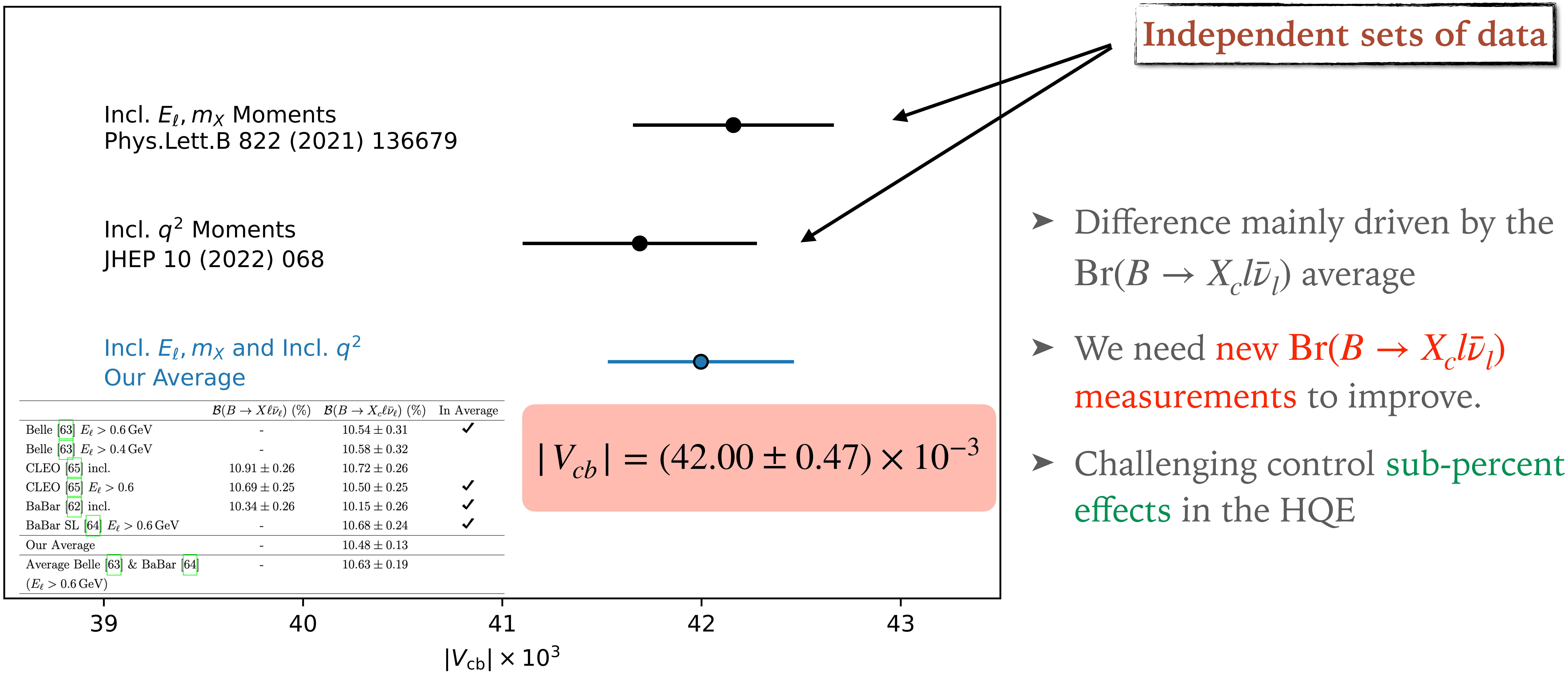
Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			
$m_b^{\mathrm{kin}}/\overline{m}_c$		✓	✓	✓

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			

NNLO corrections missing!

N3LO corrections to the total rate!

MF, Schönwald, Steinhauser, Phys.Rev.Lett. 125 (2020) 5, 052003
Phys.Rev.D 103 (2021) 1, 014005, Phys.Rev.D 104 (2021) 1, 016003



	$\mathcal{B}(B \to X\ell\bar{\nu}_\ell)$ (%)	$\mathcal{B}(B \to X_c\ell\bar{\nu}_\ell)$ (%)	In Average
Belle [63] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.54 ± 0.31	✓
Belle [63] $E_\ell > 0.4\,\mathrm{GeV}$	-	10.58 ± 0.32	
CLEO [65] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [65] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [62] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [64] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [63] & BaBar [64] ($E_\ell > 0.6\,\mathrm{GeV}$)	-	10.63 ± 0.19	

MF, Prim, Vos, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

- Difference mainly driven by the $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ average
- We need new $\mathrm{Br}(B \to X_c l \bar{\nu}_l)$ measurements to improve.
- Challenging control sub-percent effects in the HQE

NNLO CORRECTIONS q^2 SPECTRUM

MF, Herren, JHEP 05 (2024) 287

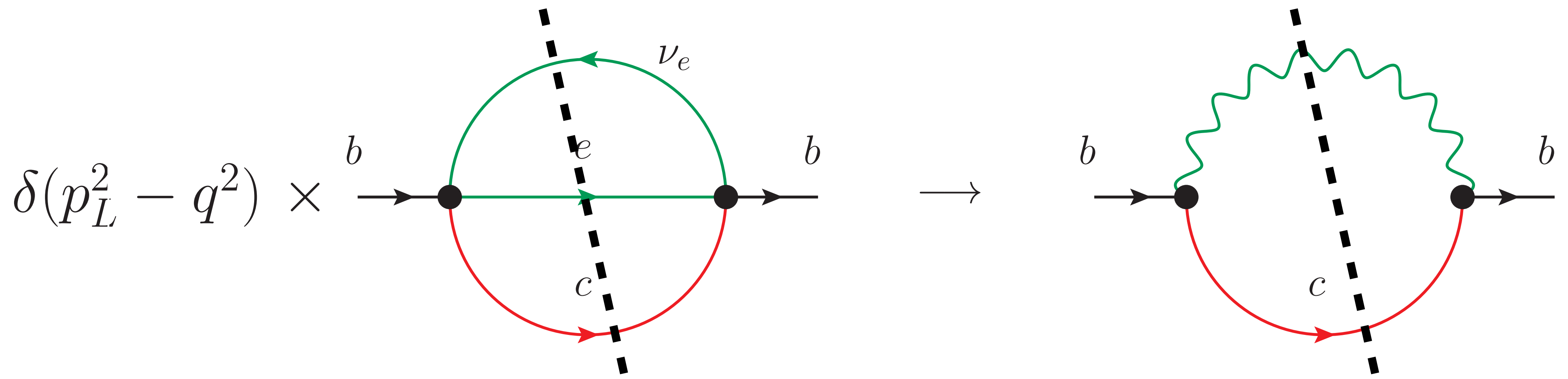
$$\frac{d\Gamma}{d\hat{q}^2} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[F_0(\rho,\hat{q}^2) + \frac{\alpha_s}{\pi}F_1(\rho,\hat{q}^2) + \left(\frac{\alpha_s}{\pi}\right)^2 F_2(\rho,\hat{q}^2)\right] + O\left(\frac{1}{m_b^2}\right)$$

NEW: Analytic expressions at NNLO!

with $\rho = m_c/m_b$

Integration w.r.t. neutrino-electron phase space

$$\mathscr{L}^{\mu\nu}(p_L) = \int L^{\mu\nu} d\Phi_2(p_L; p_l, p_\nu) = \frac{1}{384\pi^5}\left(1-\frac{m_\ell^2}{p_L^2}\right)^2\left[\left(1+\frac{2m_\ell^2}{p_L^2}\right)p_L^\mu p_L^\nu - g^{\mu\nu}p_L^2\left(1+\frac{m_\ell^2}{2p_L^2}\right)\right]$$

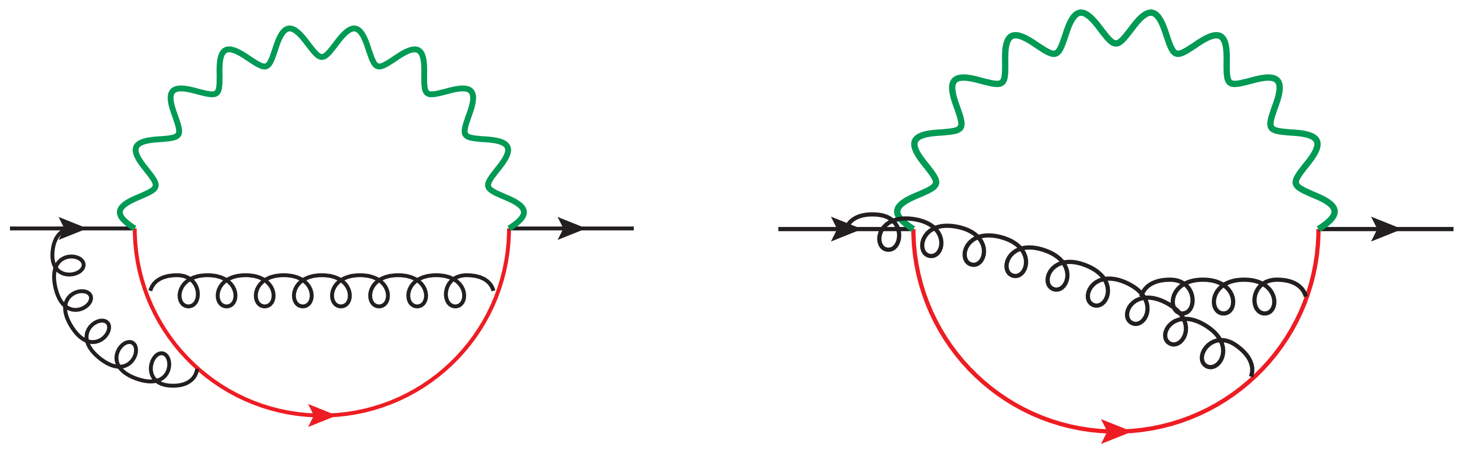


Inverse unitarity

$$\delta(p_L^2 - q^2) \to \frac{1}{2\pi i}\left[\frac{1}{p_L^2 - q^2 - i0} - \frac{1}{p_L^2 - q^2 + i0}\right]$$

NNLO calculation

- Three-loop diagrams
- Three different masses: m_b^2, m_c^2, q^2

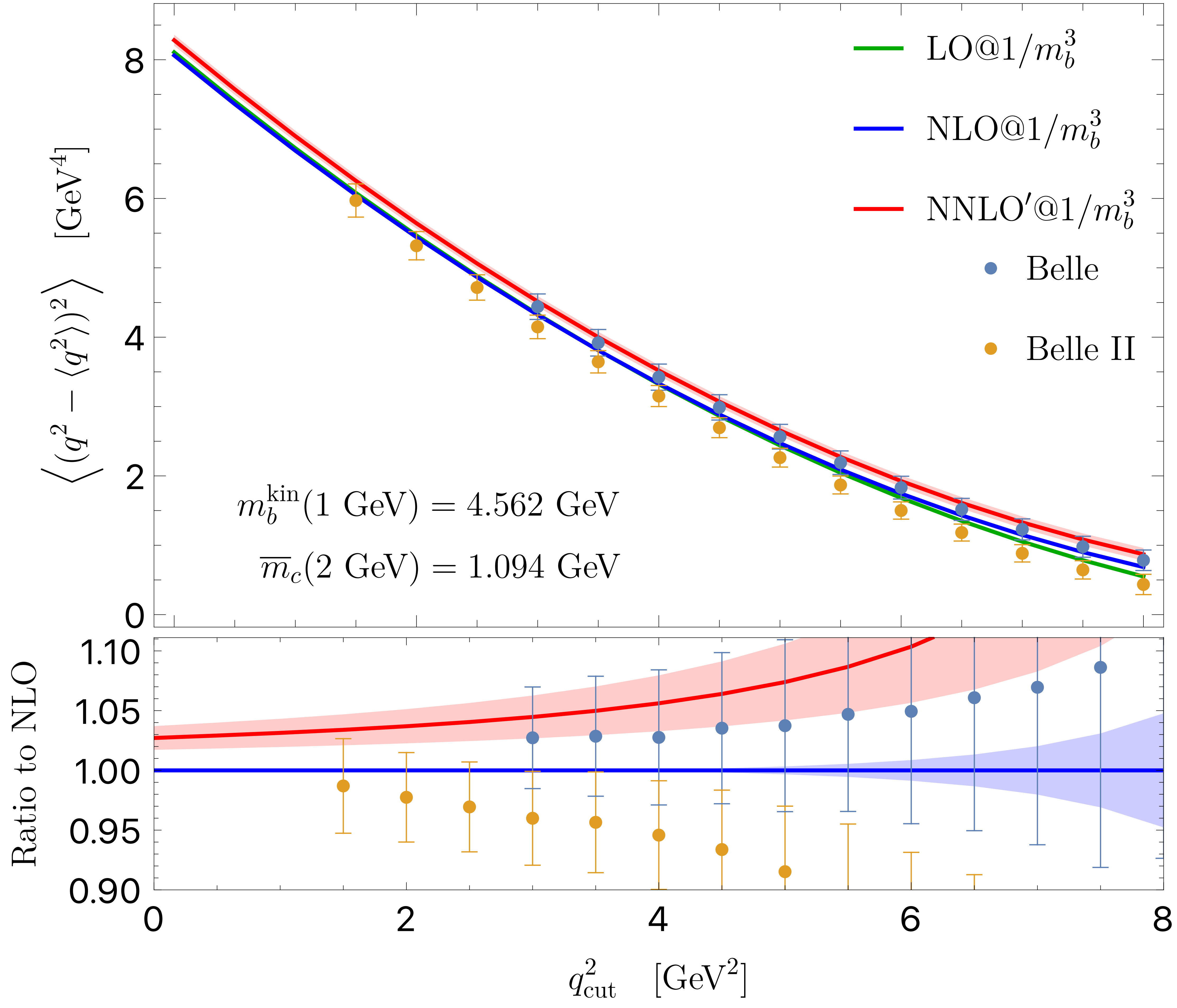


NEW: NNLO CORRECTIONS Q2 SPECTRUM

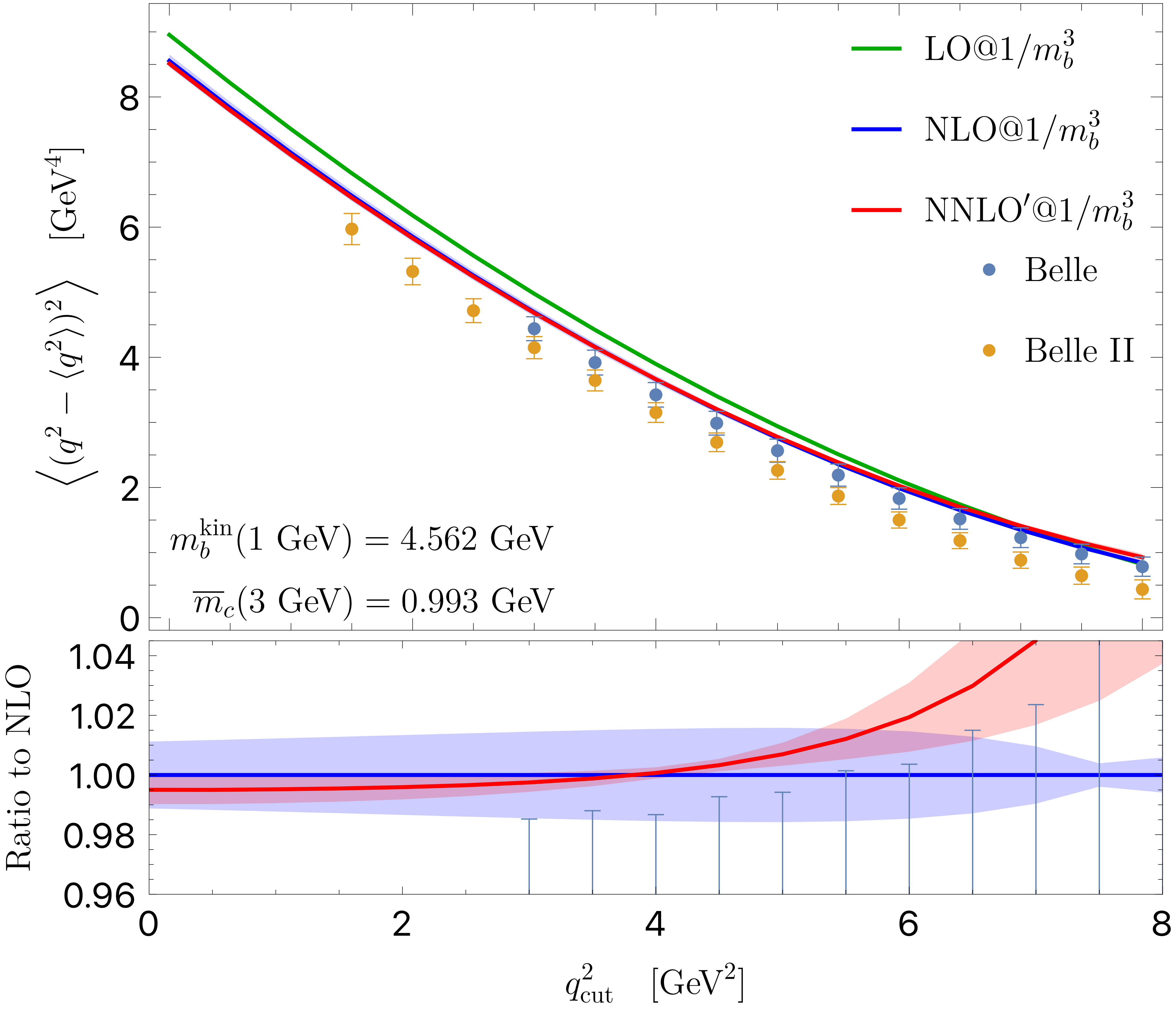
MF, Herren, JHEP 05 (2024) 287

NNLO effects mainly re-absorbed in the fit into a shift of ρ_D, r_E and r_G with reduced uncertainty. No major shift in $|V_{cb}|$.

setup from: Bernlochner, **MF,** et al, JHEP 10 (2022) 068



Unfortunate choice of $\overline{m}_c(2\,{\rm GeV})$



Much better $\overline{m}_c(3\,{\rm GeV})$

COMBINED FIT: q^2, E_l AND M_X^2 MOMENTS

Finauri, Gambino, JHEP 02 (2024) 206 + Sept. 2024 Update (G. Finauri talk in Vienna)

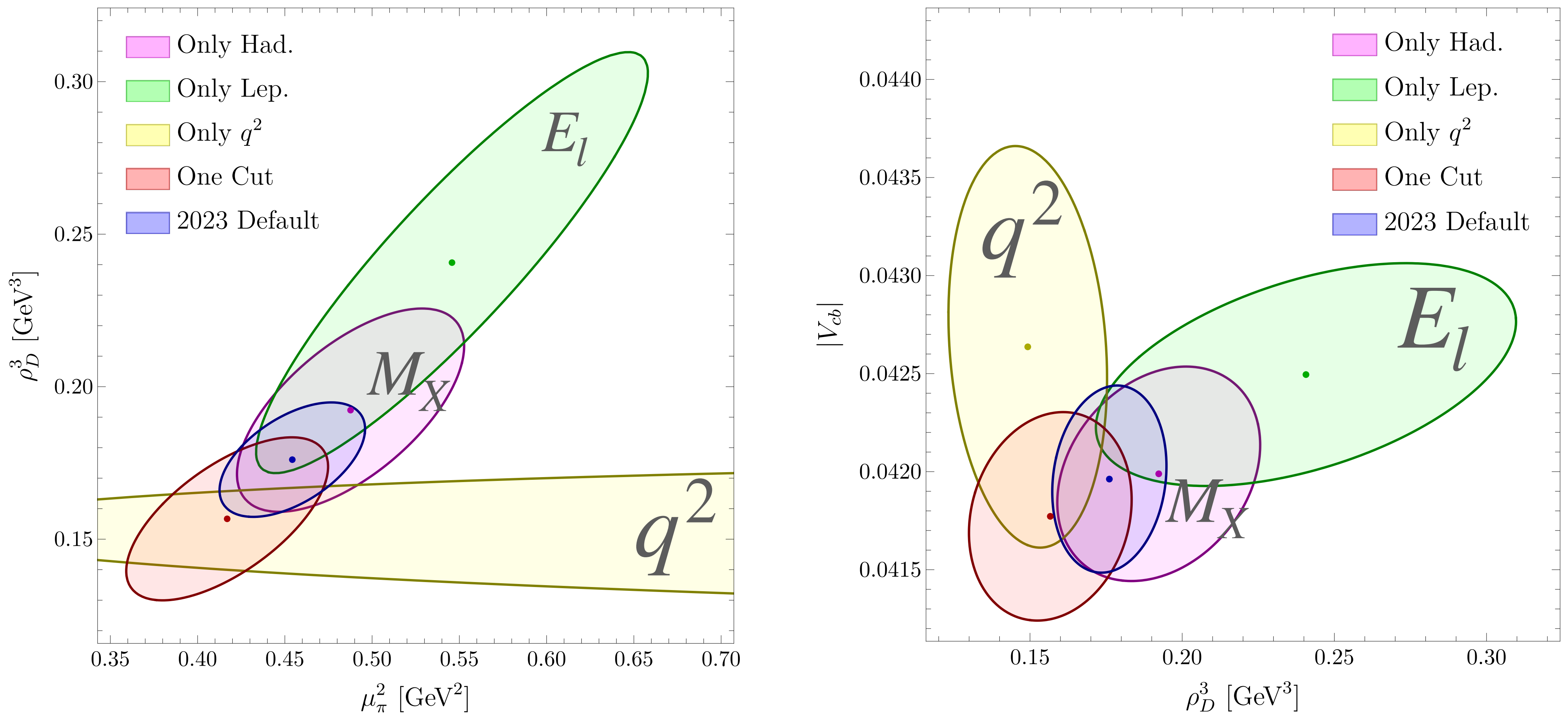
- Old DELPHI, CDF, BaBar, Belle data:
  $\langle E_l \rangle_{E_{\rm cut}}$, $\langle M_X^2 \rangle_{E_{\rm cut}}$, $\Delta {\rm Br}_{E_{\rm cut}}$
- New Belle & Belle II: $\langle q^2 \rangle_{q^2_{\rm cut}}$

$$|V_{cb}| = (41.83 \pm 0.47) \times 10^{-3}$$

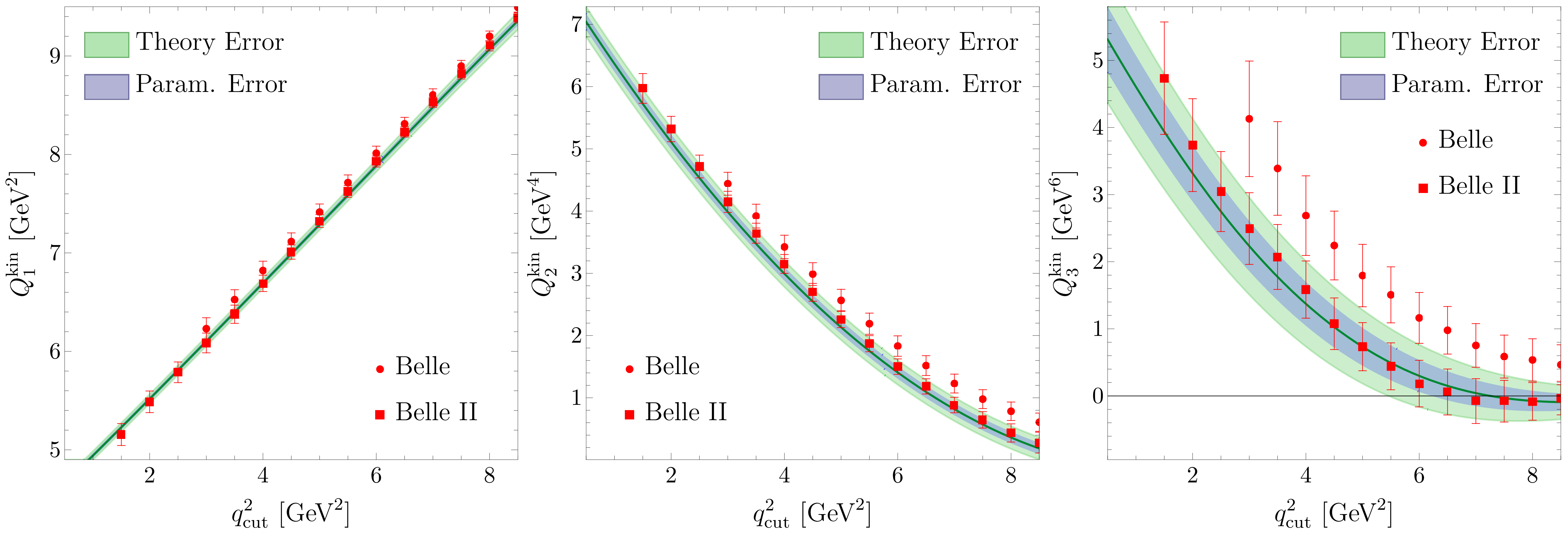
Compared with 2021 fit: 0.51 → 0.47 reduction

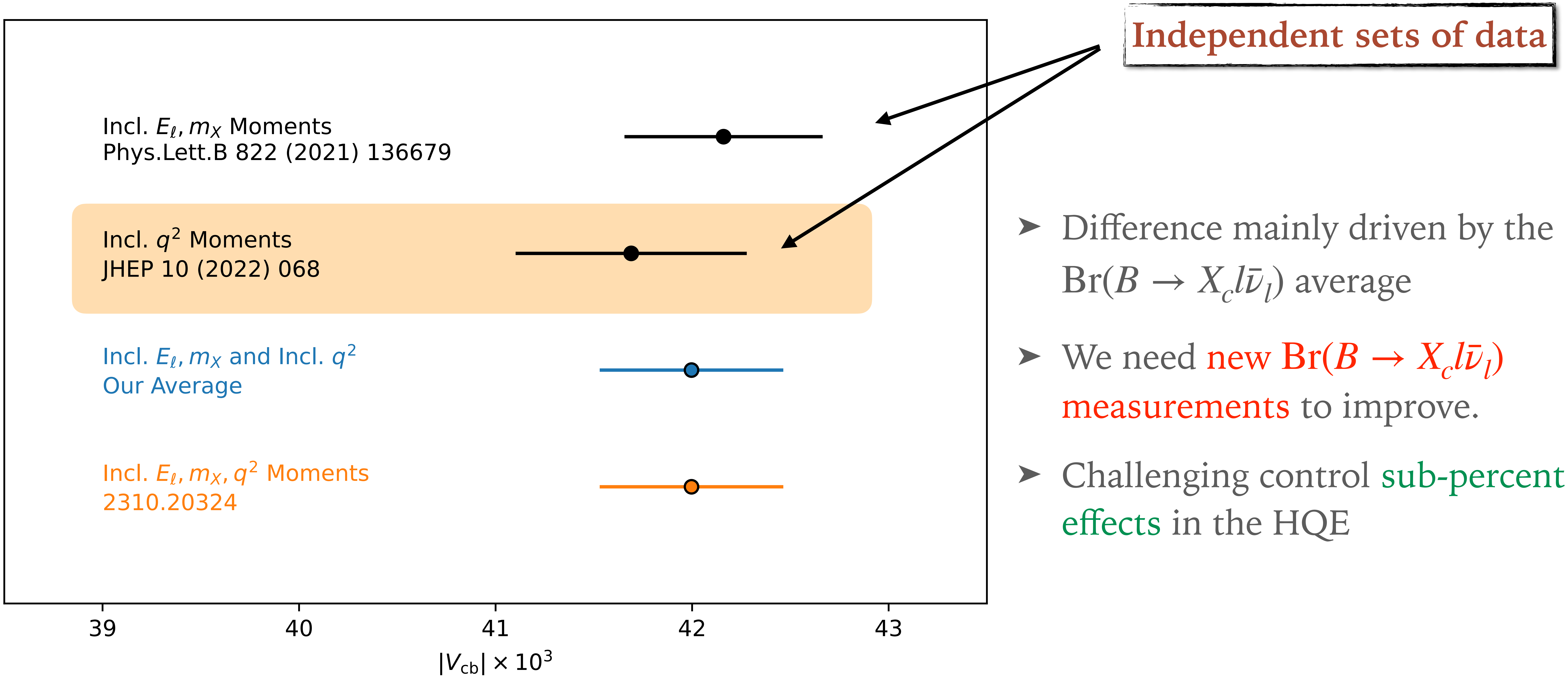
0.031 → 0.018 reduction

$m_b^{\rm kin}$	$\overline{m}_c(2\,{\rm GeV})$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	${\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.572	1.090	0.430	0.282	0.161	−0.091	10.61	41.83
0.012	0.010	0.040	0.048	0.018	0.089	0.15	0.47
1	0.389	−0.229	0.561	−0.025	−0.181	−0.062	−0.422
	1	0.019	−0.238	−0.030	0.083	0.033	0.076
		1	−0.097	0.536	0.262	0.142	0.334
			1	−0.261	0.006	0.006	−0.260
				1	−0.019	0.022	0.139
					1	−0.011	0.067
						1	0.697
							1



New α_s^2 corrections now included for $\langle q^2 \rangle$





MF, Prim, Vos, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w

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BELLE II MEASUREMENT OF R(X)

$$R(X_{\ell_1/\ell_2}) = \frac{\Gamma_{B\to X\ell_1\bar{\nu}_1}}{\Gamma_{B\to X\ell_2\bar{\nu}_{l_2}}}$$

$$R^{\text{exp}}(X_{e/\mu}) = 1.007 \pm 0.009(\text{stat}) \pm 0.019(\text{syst})$$

Belle II, Phys.Rev.Lett. 131 (2023) 5, 051804

$$R^{\text{exp}}(X_{\tau/l}) = 0.228 \pm 0.016(\text{stat}) \pm 0.036(\text{syst})$$

Belle II, hep-ex/2311.07248

$$R^{\text{SM}}(X_{\tau/l}) = 0.225 \pm 0.005$$

Rahimi, Vos, JHEP 11 (2022) 007
Ligeti, Luke, Tackmann, Phys. Rev. D 105, 073009 (2022)

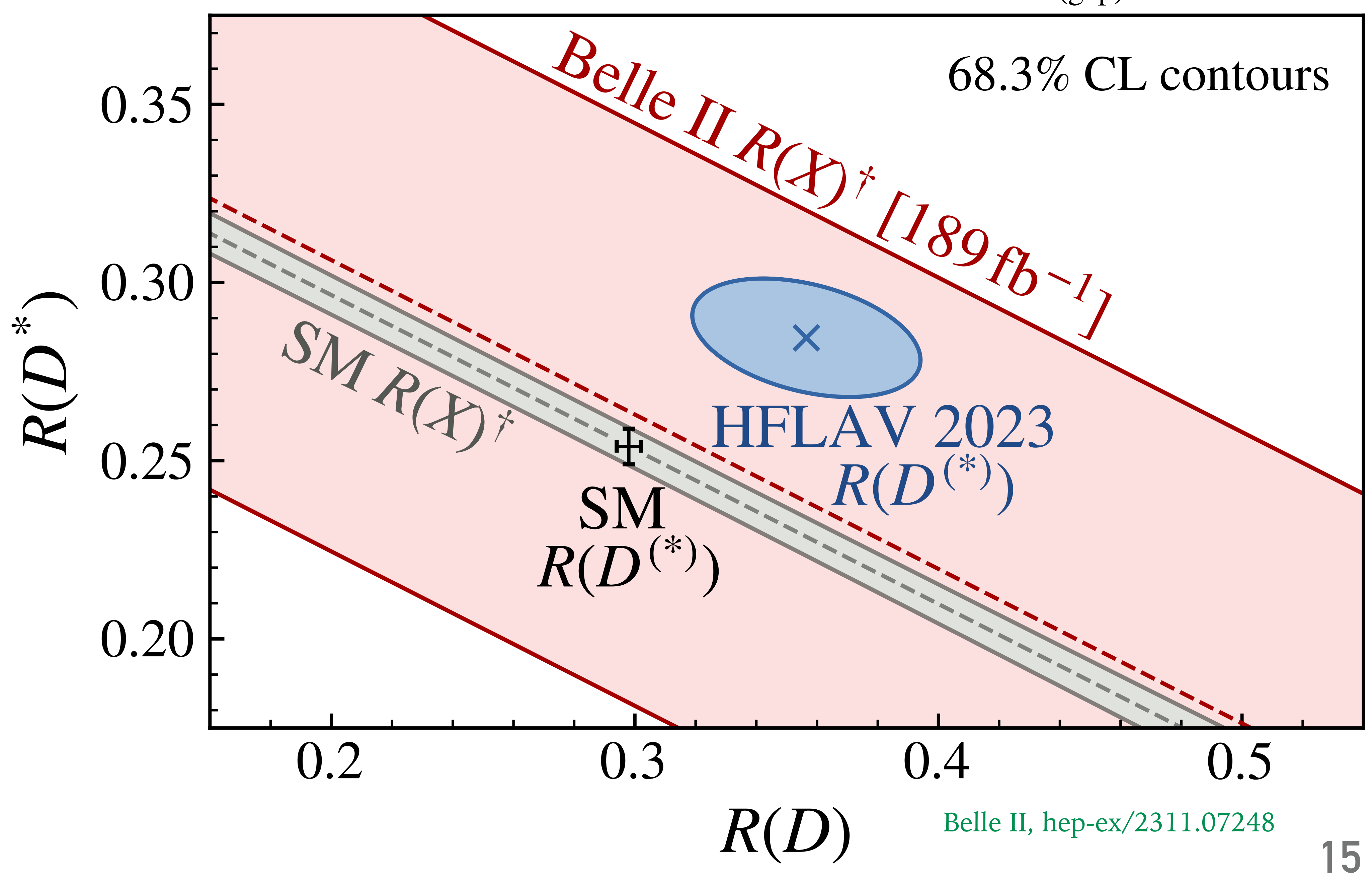
Enrichment with q^2 selection cut

$$R(X_c) = 0.241\left[1 - 0.156\frac{\alpha_s}{\pi} - 1.766\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

$$R(X_c)\Big|_{q^2>6\,\text{GeV}^2} = 0.350\left[1 - 0.782\frac{\alpha_s}{\pi} - 8.355\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

MF, Herren, JHEP 05 (2024) 287

† = with expected SM contributions of $D^{**}_{(\text{gap})}, X_u$ removed



Belle II, hep-ex/2311.07248

INCLUSIVE DECAYS: OPEN-SOURCE LIBRARY

MF, Milutin, Vos, hep-ph/2409.15007

Nikolai Uraltsev 1957 - 2013

Open-source library in python: **KOLYA**

https://gitlab.com/vcb-inclusive/kolya

- Prediction in the HQE for
  - $\Gamma_{\rm sl}$ and $\Delta\Gamma_{\rm sl}(E_{\rm cut})$
  - Centralised moments $\langle E_\ell \rangle_{E_{\rm cut}}$, $\langle M_X^2 \rangle_{E_{\rm cut}}$
  - Centralised moments $\langle q^2 \rangle_{q^2_{\rm cut}}$

- Use the kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017
Czarnecki, Melnikov, Uraltsev, Phys.Rev.Lett. 80 (1998) 3189
MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003

- Interface to CRunDec for automatic α_s, $m_b^{\rm kin}$ and $\overline{m}_c$ RGE evolution

Chetyrkin, Kuhn, Steinhauser, Comput. Phys. Commun. 133 (2000) 43
Schmidt, Steinhauser, Comput. Phys. Commun. 183 (2012) 1845
Herren, Steinhauser, Comput. Phys. Commun. 224 (2018) 333

HEAVY QUARK EXPANSION

Double series expansion in the strong coupling constant α_s and power suppressed terms $\Lambda_{\mathrm{QCD}}/m_b$

- Total rate

$$\Gamma_{\mathrm{sl}} = \frac{G_F^2 m_b^5 A_{\mathrm{ew}}}{192\pi^3}|V_{cb}|^2 \left[\left(1-\frac{\mu_\pi^2}{2m_b^2}\right)\left(X_0(\rho)+\frac{\alpha_s}{\pi}X_1(\rho)+\left(\frac{\alpha_s}{\pi}\right)^2 X_2(\rho)+\left(\frac{\alpha_s}{\pi}\right)^3 X_3(\rho)+\ldots\right)\right.$$
$$\left.+\left(\frac{\mu_G^2}{m_b^2}-\frac{\rho_D^3}{m_b^3}\right)\left(g_0(\rho)+\frac{\alpha_s}{\pi}g_1(\rho)+\ldots\right)+\frac{\rho_D^3}{m_b^3}\left(d_0(\rho)+\frac{\alpha_s}{\pi}d_1(\rho)+\ldots\right)+O\left(\frac{1}{m_b^4}\right)\right]$$

- Moments of differential distribution

$$\langle O^n\rangle_{\mathrm{cut}} = (m_b)^{mn}\left[X_0^{(O,n)}+\frac{\alpha_s}{\pi}X_1^{(O,n)}+\left(\frac{\alpha_s}{\pi}\right)^2 X_2^{(O,n)}+\frac{\mu_\pi^2}{m_b^2}\left(p_0^{(O,n)}+\frac{\alpha_s}{\pi}p_1^{(O,n)}+\ldots\right)\right.$$
$$\left.+\frac{\mu_G^2}{m_b^2}\left(g_0^{(O,n)}+\frac{\alpha_s}{\pi}g_1^{(O,n)}+\ldots\right)+\frac{\rho_D^3}{m_b^3}\left(d_0^{(O,n)}+\frac{\alpha_s}{\pi}d_1^{(O,n)}+\ldots\right)+\frac{\rho_{LS}}{m_b^2}\left(l_0^{(O,n)}+\frac{\alpha_s}{\pi}l_1^{(O,n)}+\ldots\right)+O\left(\frac{1}{m_b^4}\right)\right]$$

BUILDING BLOCKS IN THE HQE

$\Gamma_{\rm sl}$	tree	α_s	α_s^2	α_s^3
Partonic		[1]	[2–5]	[6]
μ_π^2, μ_G^2	[7, 8]	[9–12]		
ρ_D^3, ρ_{LS}^3	[13]	[14]		
$1/m_b^4, 1/m_b^5$	[15–18]			
$q_n(q^2_{\rm cut})$	tree	α_s	α_s^2	
Partonic		[14, 19]	[20]	
μ_G^2, μ_π^2	[7, 8]	[10, 11]		
ρ_D^3, ρ_{LS}	[13]	[14]		
$1/m_b^4, 1/m_b^5$	[17, 18]			
$\ell_n(E_{\rm cut}), h_n(E_{\rm cut})$	tree	α_s	$\alpha_s^2\beta_0$	α_s^2
Partonic		[19, 21, 22]	[19, 22]	[23]
μ_G^2	[7, 8]	[10, 11]		
ρ_D^3	[13]			
$1/m_b^4, 1/m_b^5$	[15, 16, 18]			

- Power up to $1/m_b^5$

  Mannel, Milutin, Vos, hep-ph/2311.12002

- Perturbative corrections to $\Gamma_{\rm sl}$ up to $O(\alpha_s^3)$

  MF, Schönwald, Steinhauser, Phys.Rev.D 104 (2021) 016003, JHEP 08 (2022) 039

- NLO corrections to power suppressed terms for q^2 moments

  Mannel, Moreno, Pivovarov, JHEP 08 (2020) 089

- NNLO corrections to q^2 moments

  MF, Herren, JHEP 05 (2024) 287

complete references in backup slides

IMPLEMENTATION

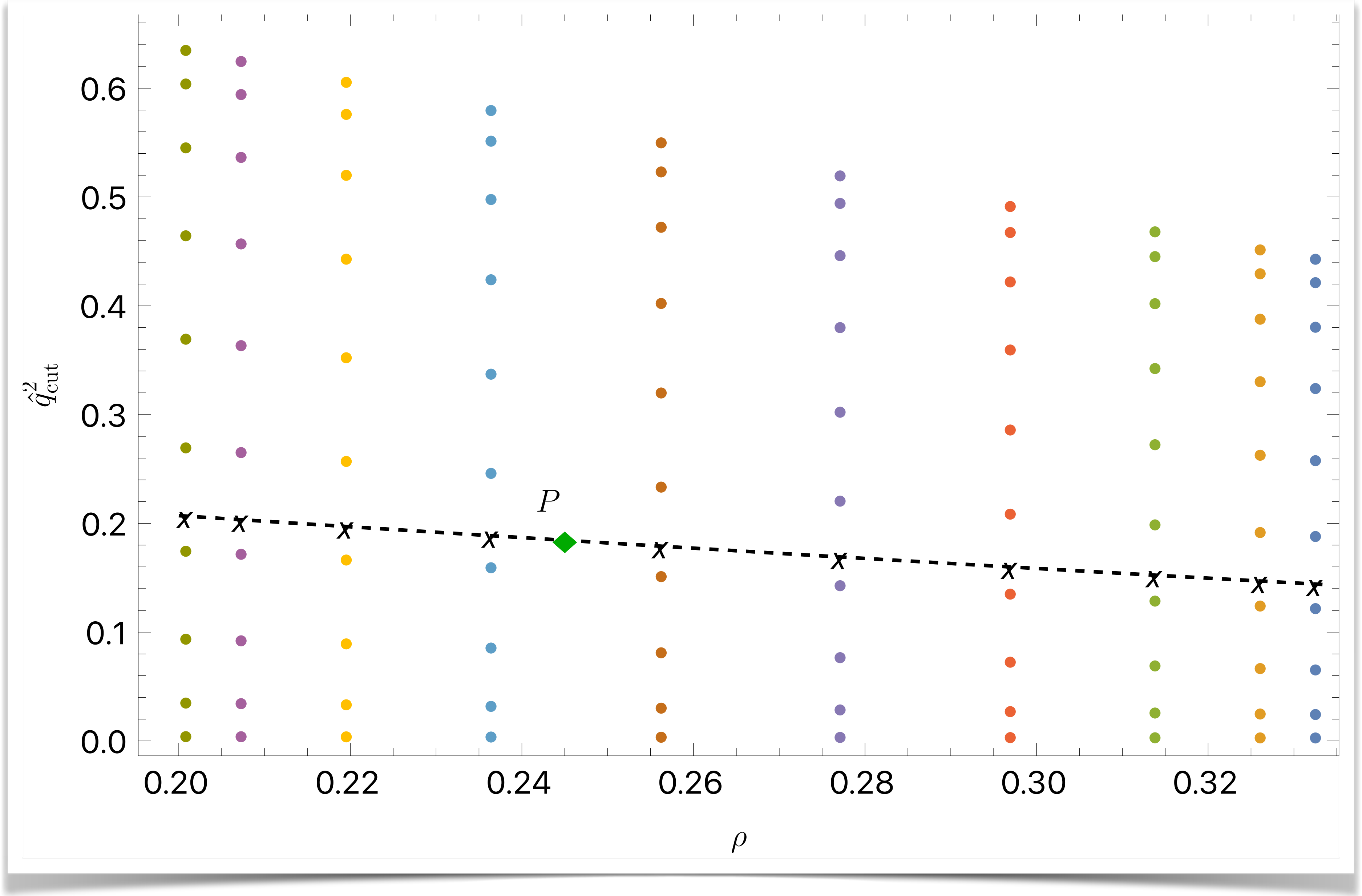
$\rho = m_c/m_b \quad \hat{q}^2 = q^2/m_b^2$

- Tree level implemented in exact form
- We implement analytic results for higher QCD corrections for $\Gamma_{\rm sl}$
  - Exact results at NLO
  - Asymptotic expansions at NNLO and N3LO
- Use Numba for fast numerical evaluation

https://numba.pydata.org

- Chebyshev interpolation grids for QCD corrections to the moments

$$f(\rho, \hat{q}^2_{\rm cut}) = \int_{q^2 > q^2_{\rm cut}} (q^2)^i \, (q_0)^j \, \frac{d^3\Gamma^{\rm NLO}}{dq^2\, dq_0\, dE_l} dq^2\, dq_0\, dE_l$$

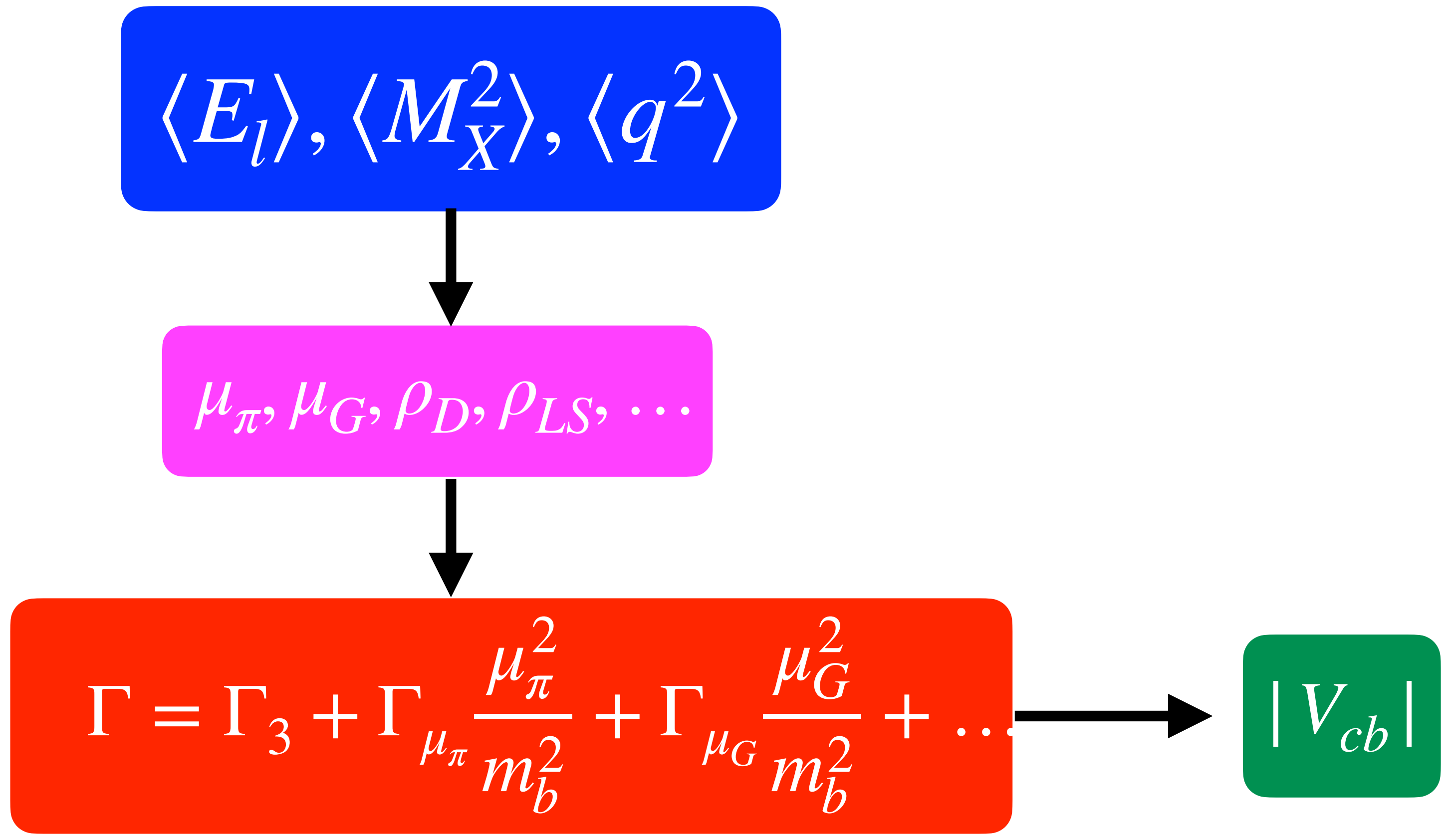


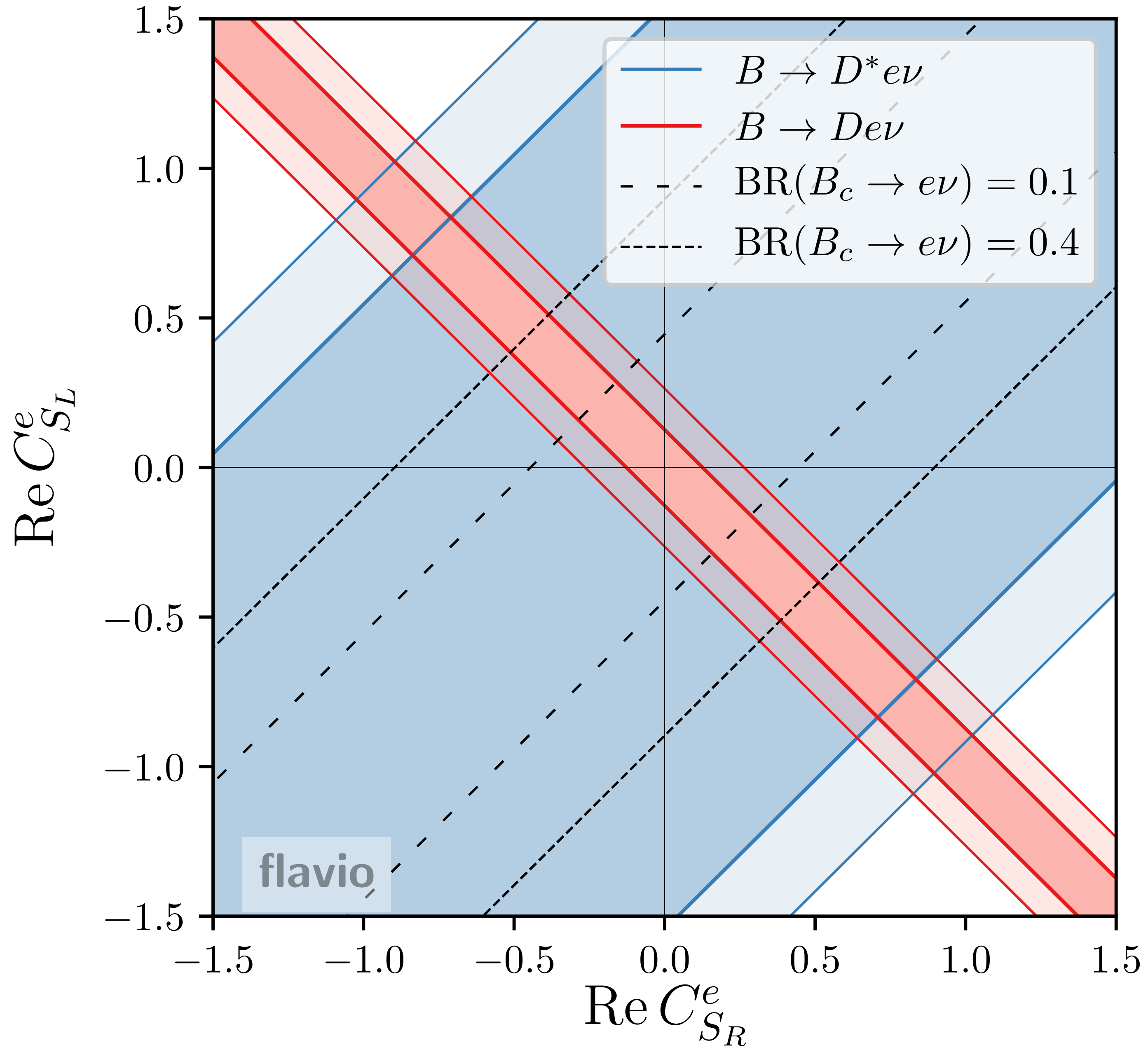
EFFECTIVE HAMILTONIAN FOR SEMILEPTONIC DECAYS

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[\left(1 + C_{V_L}\right) O_{V_L} + \sum_{i=V_R,S_L,S_R,T} C_i\, O_i \right]$$

$$O_{V_{L(R)}} = \left(\bar{c}\gamma_\mu P_{L(R)} b\right)\left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right) \qquad O_{S_{L(R)}} = \left(\bar{c} P_{L(R)} b\right)\left(\bar{\ell} P_L \nu_\ell\right) \qquad O_T = \left(\bar{c}\,\sigma_{\mu\nu} P_L b\right)\left(\bar{\ell}\,\sigma^{\mu\nu} P_L \nu_\ell\right)$$

- In the SM all $C_i = 0$
- In the WET the expansion parameter is $1/v^2$, i.e. Wilson coefficients are $O(1)$
- NP effects from SMEFT are suppressed by $1/\Lambda_{\text{NP}}^2$. The matching to WET leads to a $(v/\Lambda_{\text{NP}})^2$ suppression.
- In the following we assume $|C_i| \ll 1$





Jung, Straub, *JHEP* 01 (2019) 009

SEMILEPTONIC INCLUSIVE DECAYS: NP EFFECTS

MF, Rahimi, Vos, JHEP 02 (2023) 086

- Contribution to the moments of $B \to X_c l \bar{\nu}_l$

$$\langle O \rangle = \xi_{\mathrm{SM}} + |C_{V_R}|^2\, \xi_{\mathrm{NP}}^{\langle V_R, V_R\rangle} + |C_{S_L}|^2\, \xi_{\mathrm{NP}}^{\langle S_L, S_L\rangle} + |C_{S_R}|^2\, \xi_{\mathrm{NP}}^{\langle S_R, S_R\rangle} + |C_T|^2\, \xi_{\mathrm{NP}}^{\langle T, T\rangle}$$
$$+\mathrm{Re}((C_{V_L}-1)C_{V_R}^*)\, \xi_{\mathrm{NP}}^{\langle V_L, V_R\rangle} + \mathrm{Re}(C_{S_L}C_{S_R}^*)\, \xi_{\mathrm{NP}}^{\langle S_L, S_R\rangle} + \mathrm{Re}(C_{S_L}C_T^*)\, \xi_{\mathrm{NP}}^{\langle S_L, T\rangle}$$
$$+\mathrm{Re}(C_{S_R}C_T^*)\, \xi_{\mathrm{NP}}^{\langle S_R, T\rangle}$$

$$O_{V_{L(R)}} = \left(\bar{c}\gamma_\mu P_{L(R)} b\right)\left(\bar{\ell}\gamma^\mu P_L \nu_\ell\right) \qquad O_{S_{L(R)}} = \left(\bar{c} P_{L(R)} b\right)\left(\bar{\ell} P_L \nu_\ell\right) \qquad O_T = \left(\bar{c}\,\sigma_{\mu\nu} P_L b\right)\left(\bar{\ell}\,\sigma^{\mu\nu} P_L \nu_\ell\right)$$

HEAVY QUARK EXPANSION WITH NP EFFECTS

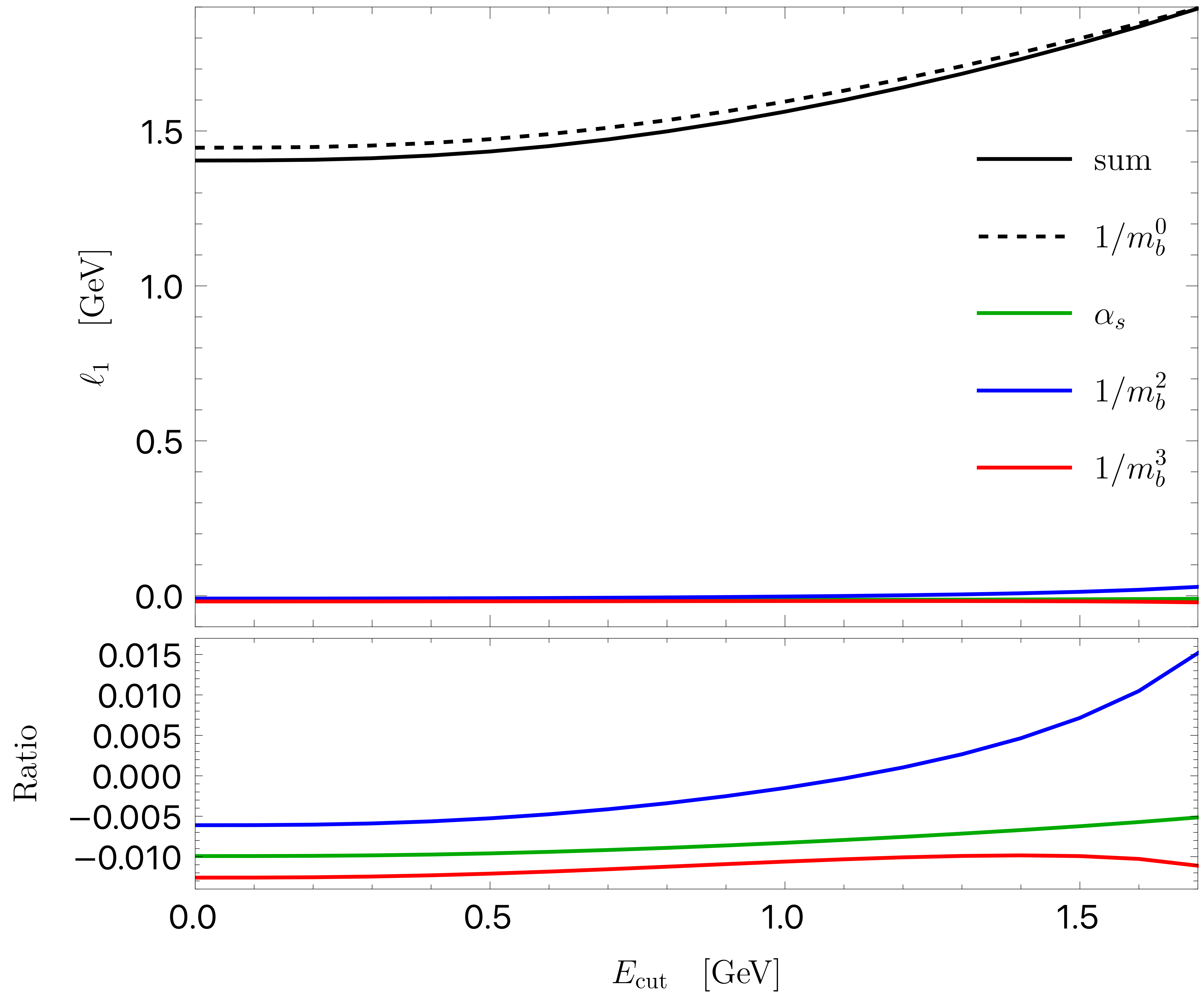
➤ Series expansion in three parameters:

- Λ_{QCD}/m_b
- α_s
- $(v/\Lambda_{\rm NP})^2$

To properly catch the leading effects:

- $(v/\Lambda_{\rm NP})^2 \times \alpha_s^0 \times (1/m_b)^0$: NP at tree level in the free-quark approximation.

$$\ell_1 = \langle E_e \rangle_{E_{\rm cut}}$$



values of HQE param. from
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

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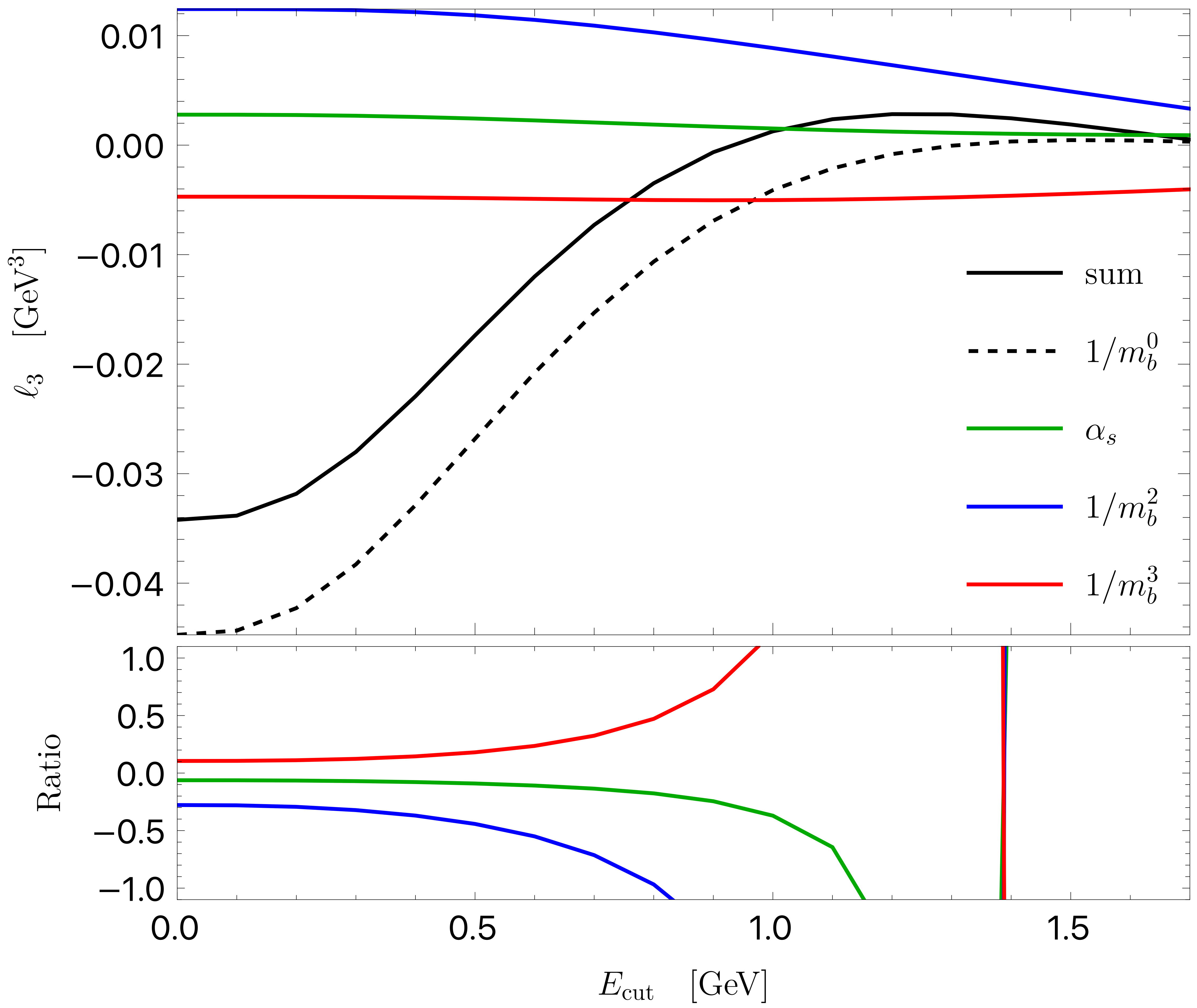
- $(v/\Lambda_{\rm NP})^2 \times \alpha_s^0 \times (1/m_b)^{2,3}$: power-suppressed terms for NP effects

$$\ell_e = \left\langle \left(E_e - \langle E_e \rangle \right)^3 \right\rangle_{E_{\rm cut}}$$



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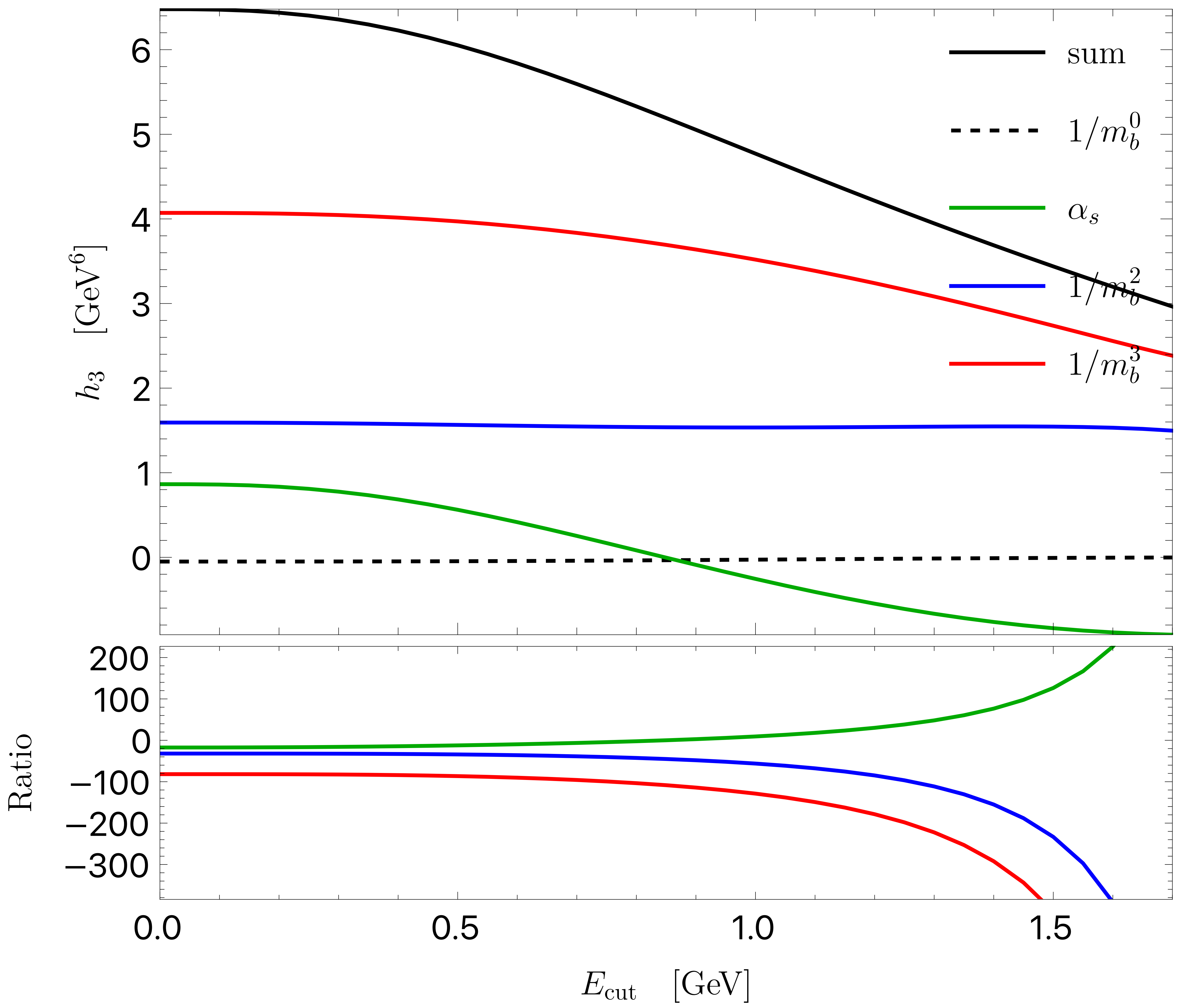
- $(v/\Lambda_{\rm NP})^2 \times \alpha_s^0 \times (1/m_b)^{2,3}$: power-suppressed terms for NP effects
- $(v/\Lambda_{\rm NP})^2 \times \alpha_s^1 \times (1/m_b)^0$: QCD NLO corrections to NP effects

$$h_3 = \left\langle \left(M_X^2 - \langle M_X^2 \rangle \right)^3 \right\rangle_{E_{\rm cut}}$$



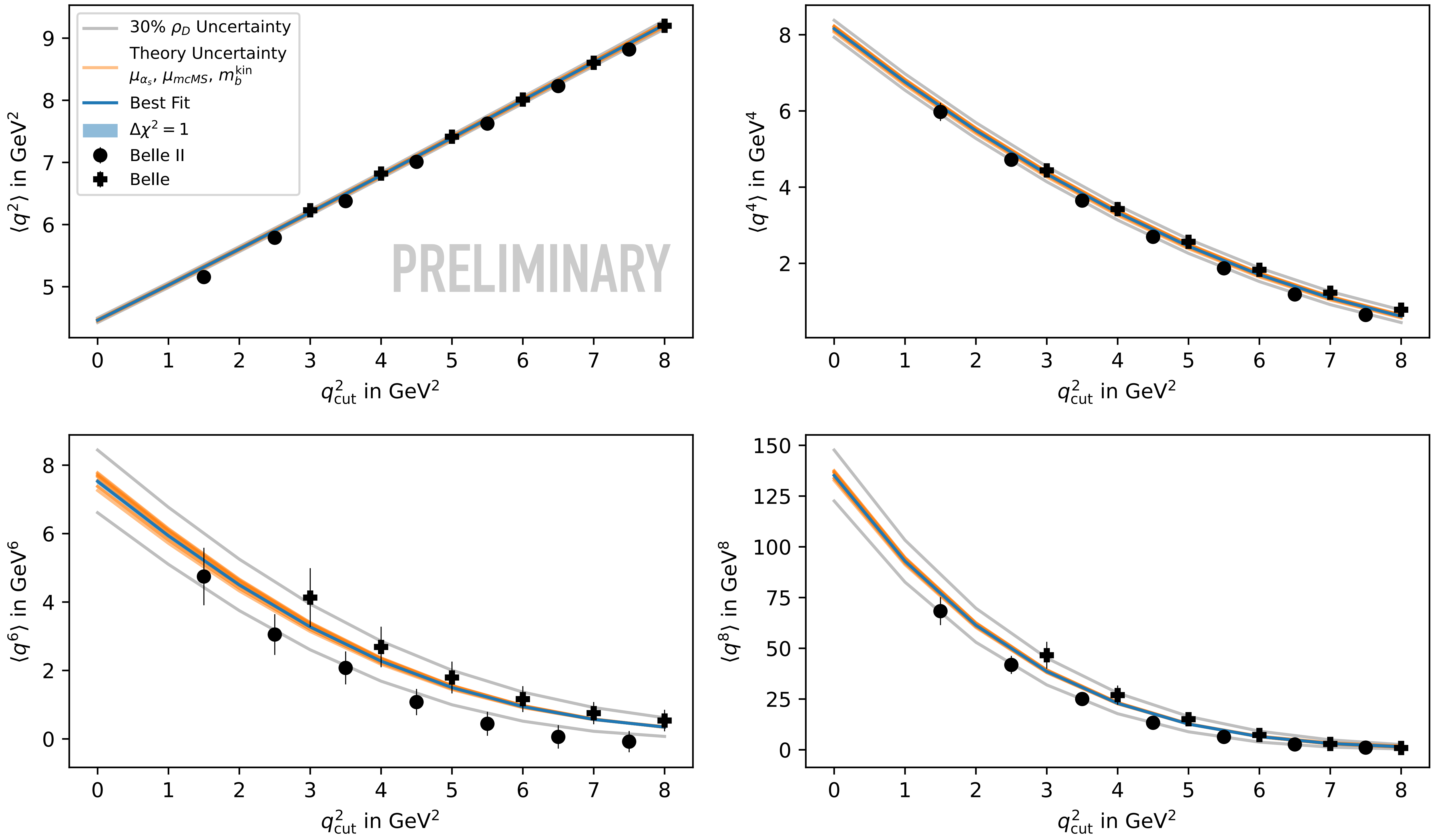
values of HQE param. from
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

GLOBAL FIT OF q^2 MOMENTS

Bernlochner, **MF**, Milutin, Prim, Vos, in preparation

Perturbative QCD uncertainties

- Variation of unphysical scales $\alpha_s(\mu_s), \overline{m}_c(\mu_c), m_b^{\rm kin}(\mu_b)$
- Very low p-Value in the fit



HQE uncertainties

- **Current recipe:** inflate uncertainty on μ_G and ρ_D to cover the truncation of $1/m_b$ expansion.

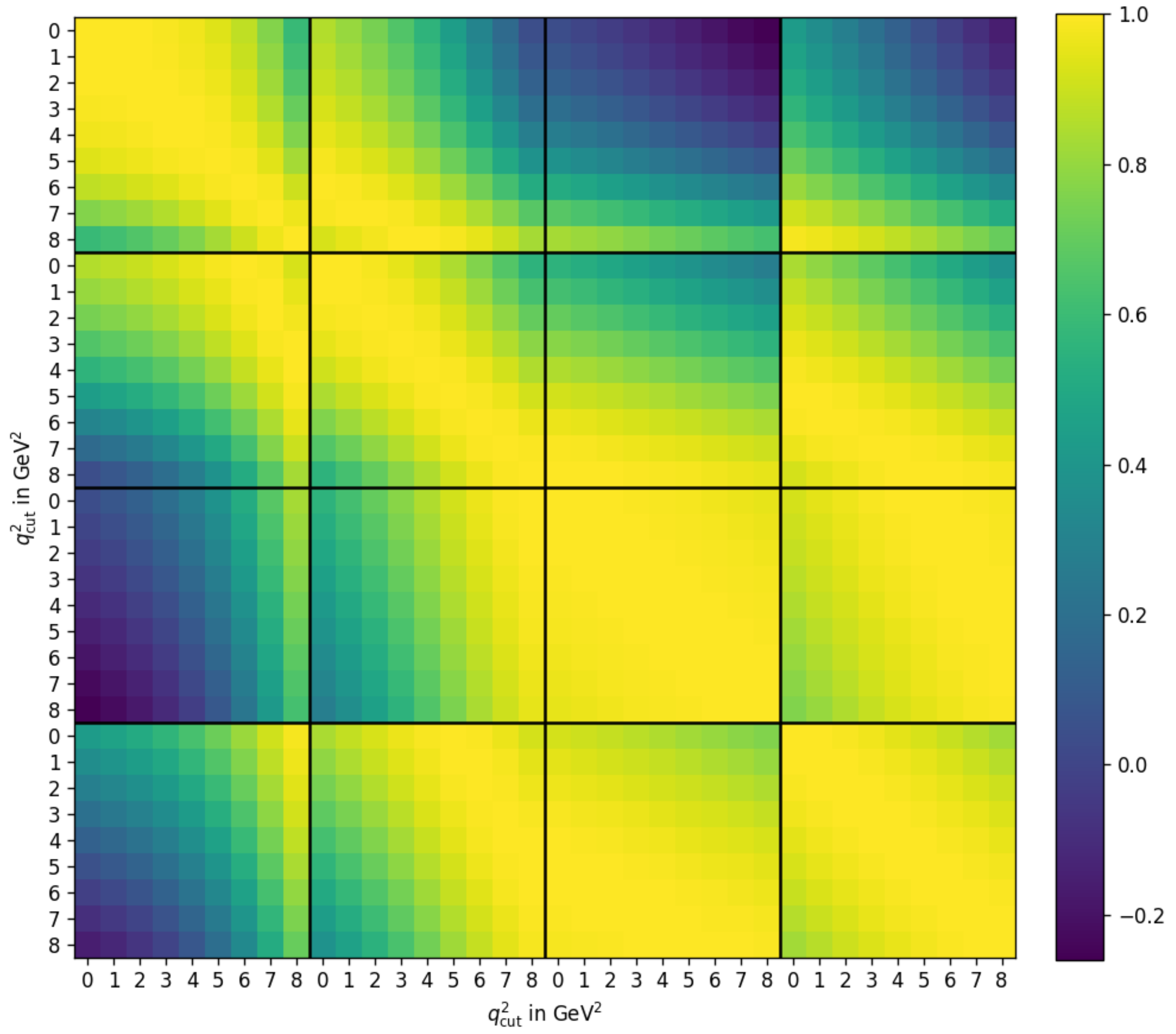
Finauri, Gambino, JHEP 02 (2024) 206

Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

Does the HQE explain the experimental data?
Do we observe a convergence of the HQE?

THEORY CORRELATIONS: CURRENT STATUS

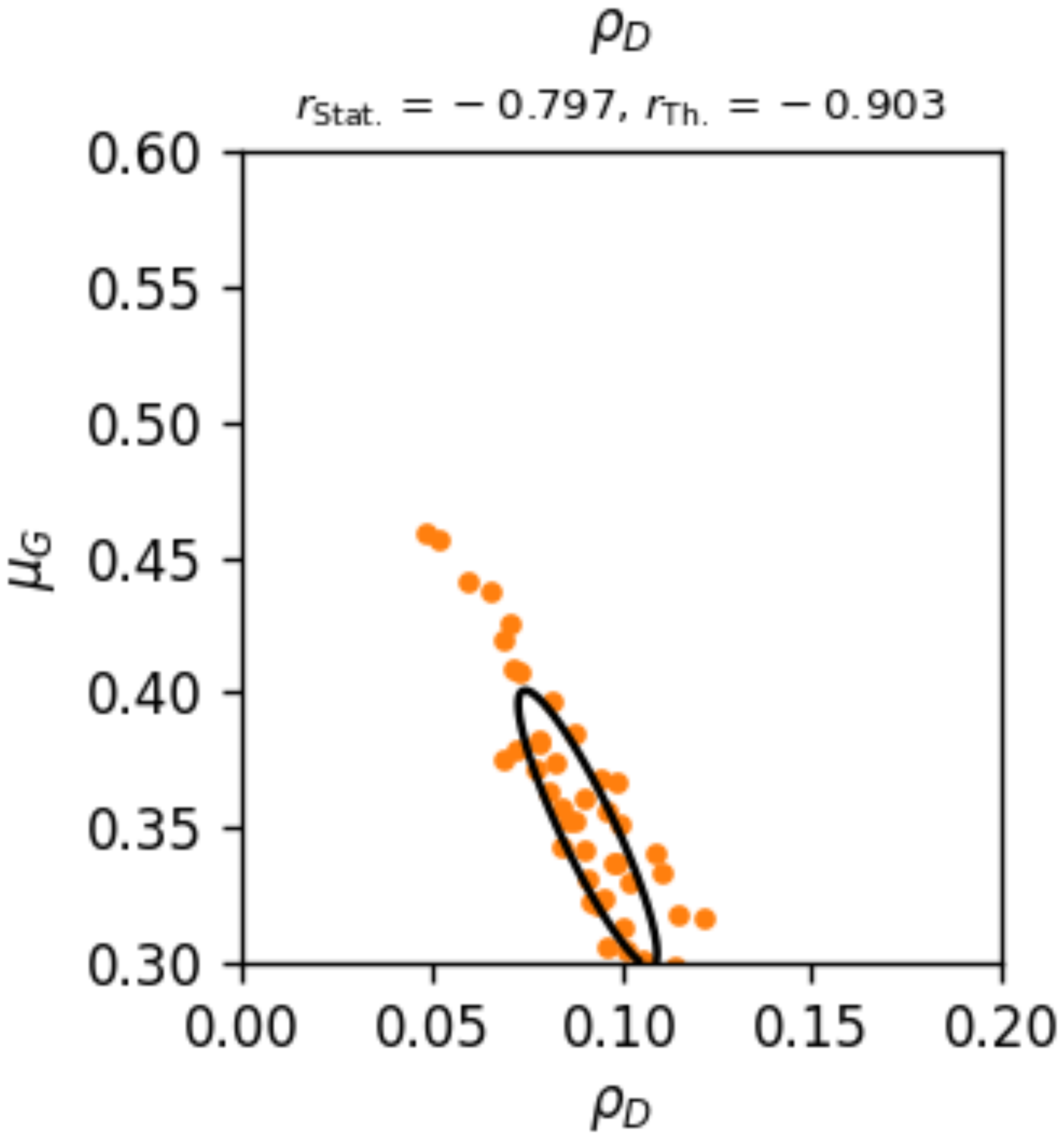
- ρ_D : $\pm\, 30\,\%$ estimate theory uncertainty.
- Observed correlation is meaningless.
- Necessary to model theory correlations:
  - Strong correlations between different cuts, no correlation between different moments
    Gambino, Schwanda, Phys.Rev.D 89 (2014) 1, 014022
  - Flexible correlation via nuisance parameters
    Bernlochner, **MF,** Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068



THEORY CORRELATIONS: EXPLOITING THE $1/m_b^{4,5}$ CORRECTIONS

Bernlochner, **MF**, Milutin, Prim, Vos, in preparation

- Do not include uncertainty on ρ_D and μ_G
- Exploit known expressions for $1/m_b^{4,5}$.
- Check order by order if the fit improves and stabilises.
- Sample μ_s, μ_c and μ_b with uniform distribution and refit.
- LLSA inputs as estimator of the $1/m_b^{4,5}$.
- $1/m_b^{4,5}$ terms determine correlations in the fit.



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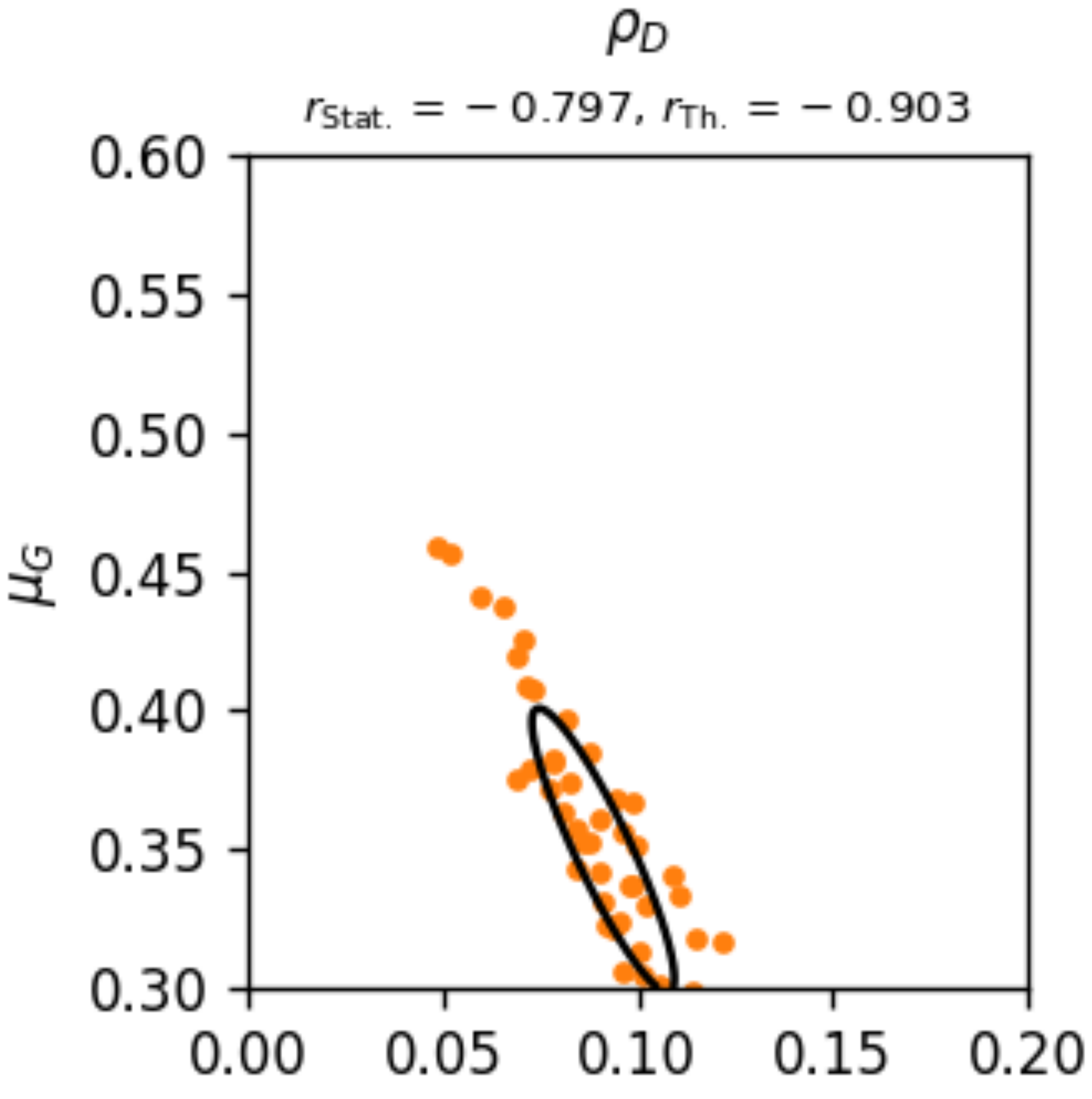


THE LIFETIMES OF B MESONS TO NNLO

LIFETIMES

Total width

$$\frac{1}{\tau(B_q)} = \Gamma_b + \delta\Gamma_{B_q}$$
$$= \Gamma_{\text{non leptonic}} + \sum_{l=e,\mu,\tau} \Gamma(B \to X l \bar{\nu}_l) + \dots$$

- Nonleptonic decays (dominant)
  - $b \to c\bar{u}d$
  - $b \to c\bar{c}s$

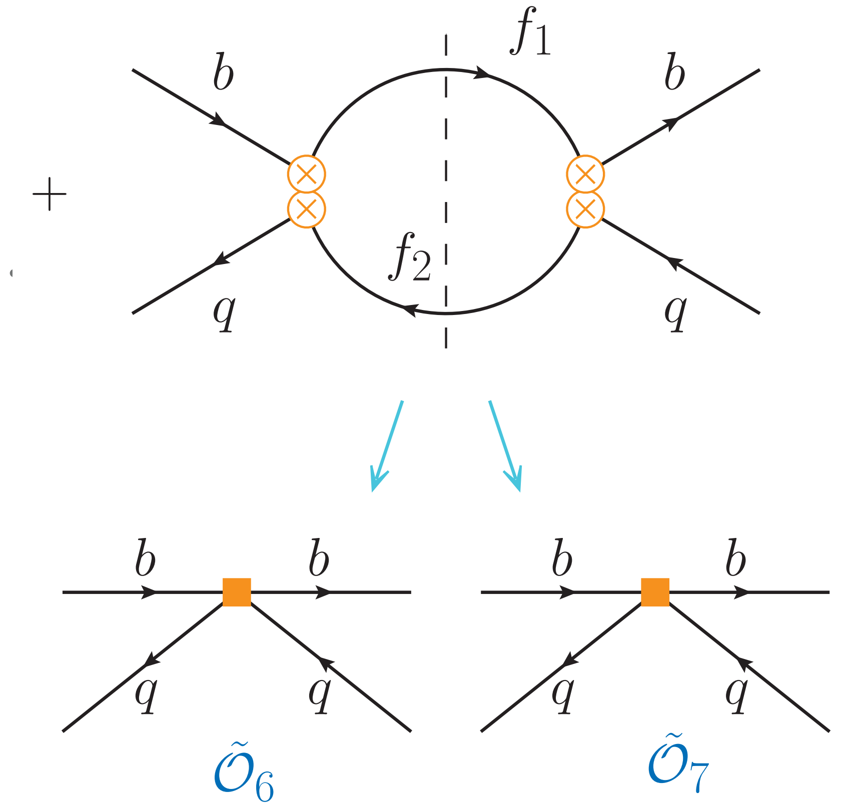
Lifetime ratios

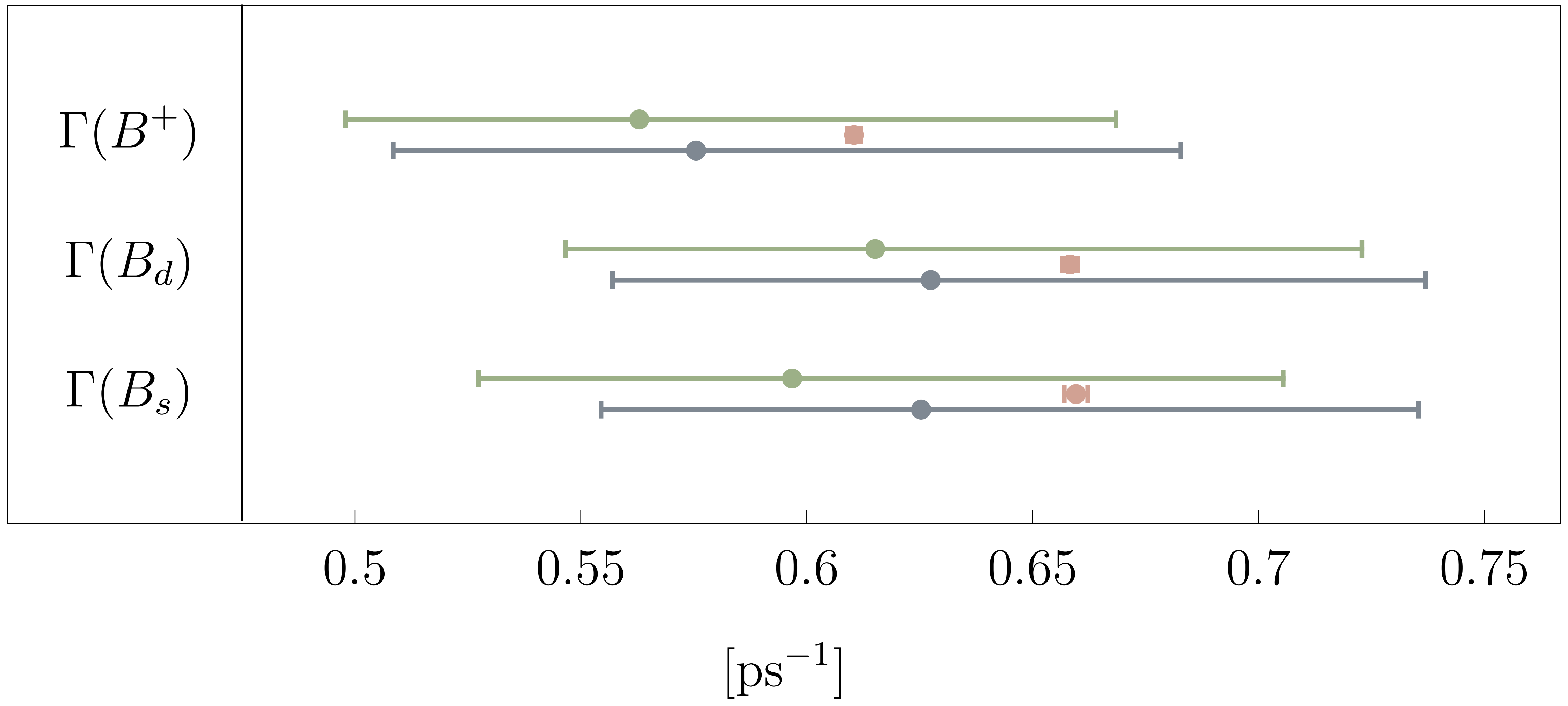
$$\frac{\tau(B_q)}{\tau(B_{q'})} = 1 + (\delta\Gamma_{B_q} - \delta\Gamma_{B'_q})\,\tau(B_q)$$

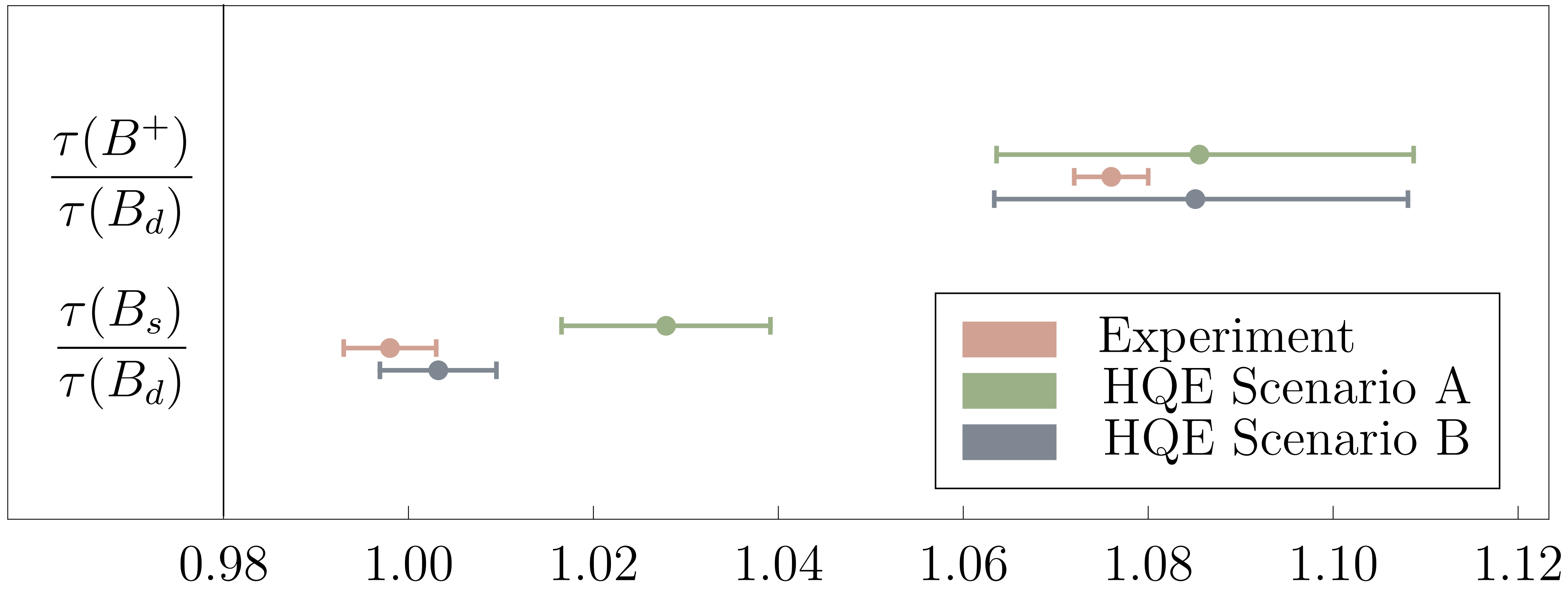
- Test the SM and framework used
- Perform indirect BSM searches

THE HEAVY QUARK EXPANSION

$$\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B | \mathcal{O}_5 | B \rangle}{m_b^2} + \Gamma_6 \frac{\langle B | \mathcal{O}_6 | B \rangle}{m_b^3} + 16\pi^2 \frac{\langle B | \tilde{\mathcal{O}}_6 | B \rangle}{m_b^3} + \ldots$$

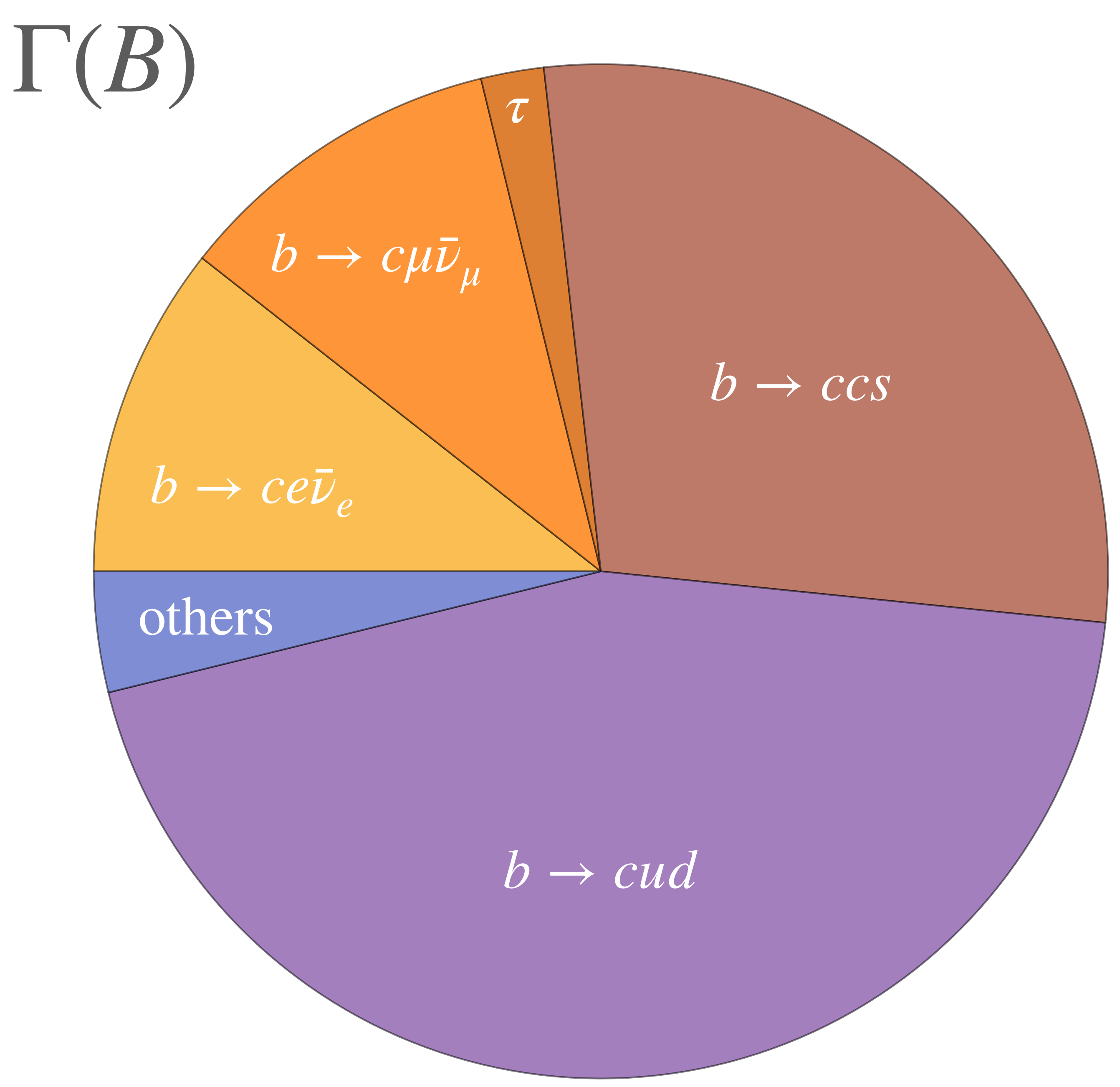


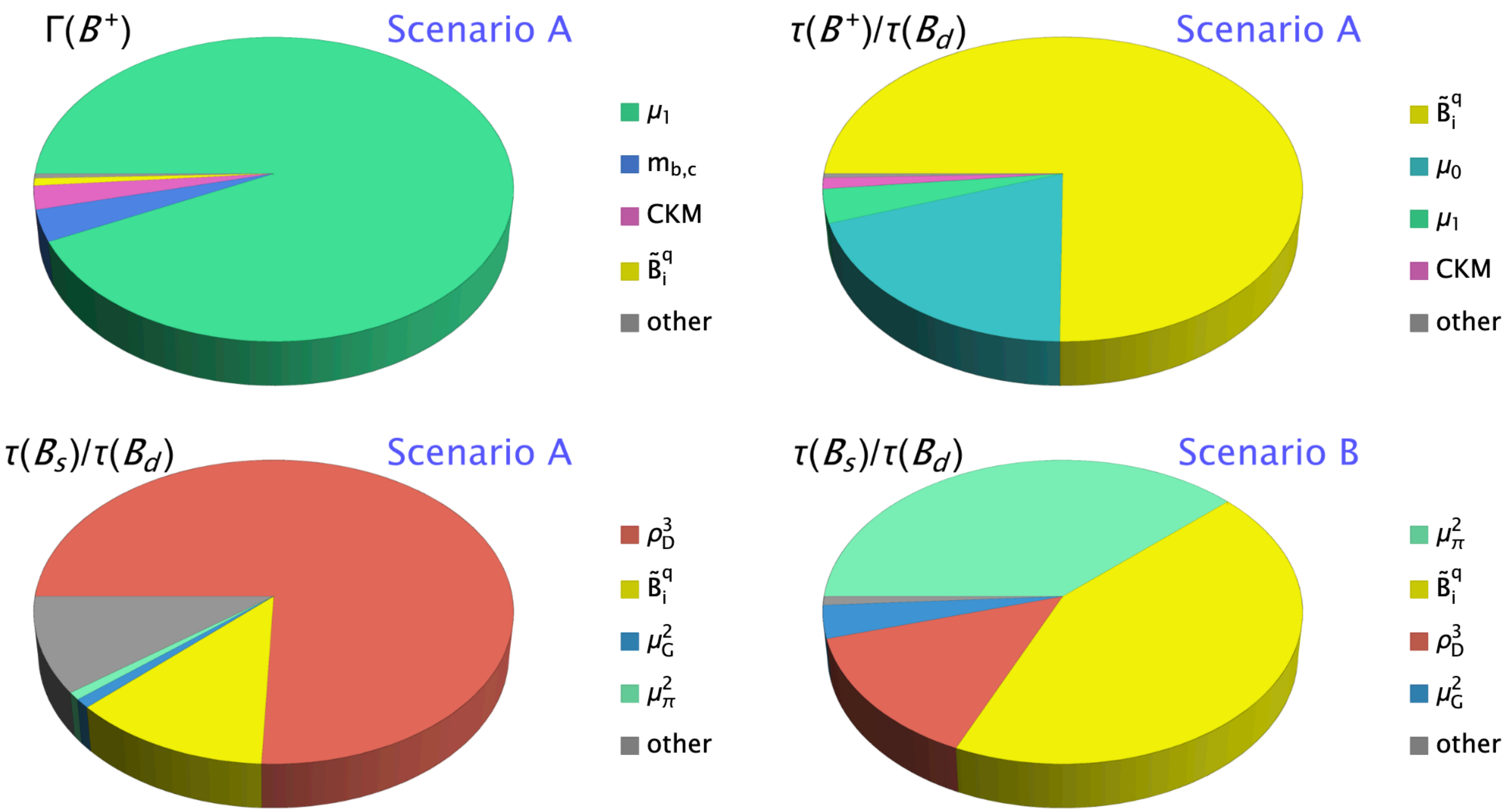




Lenz, Piscopo, Rusov, JHEP 01 (2023) 004

Error budget

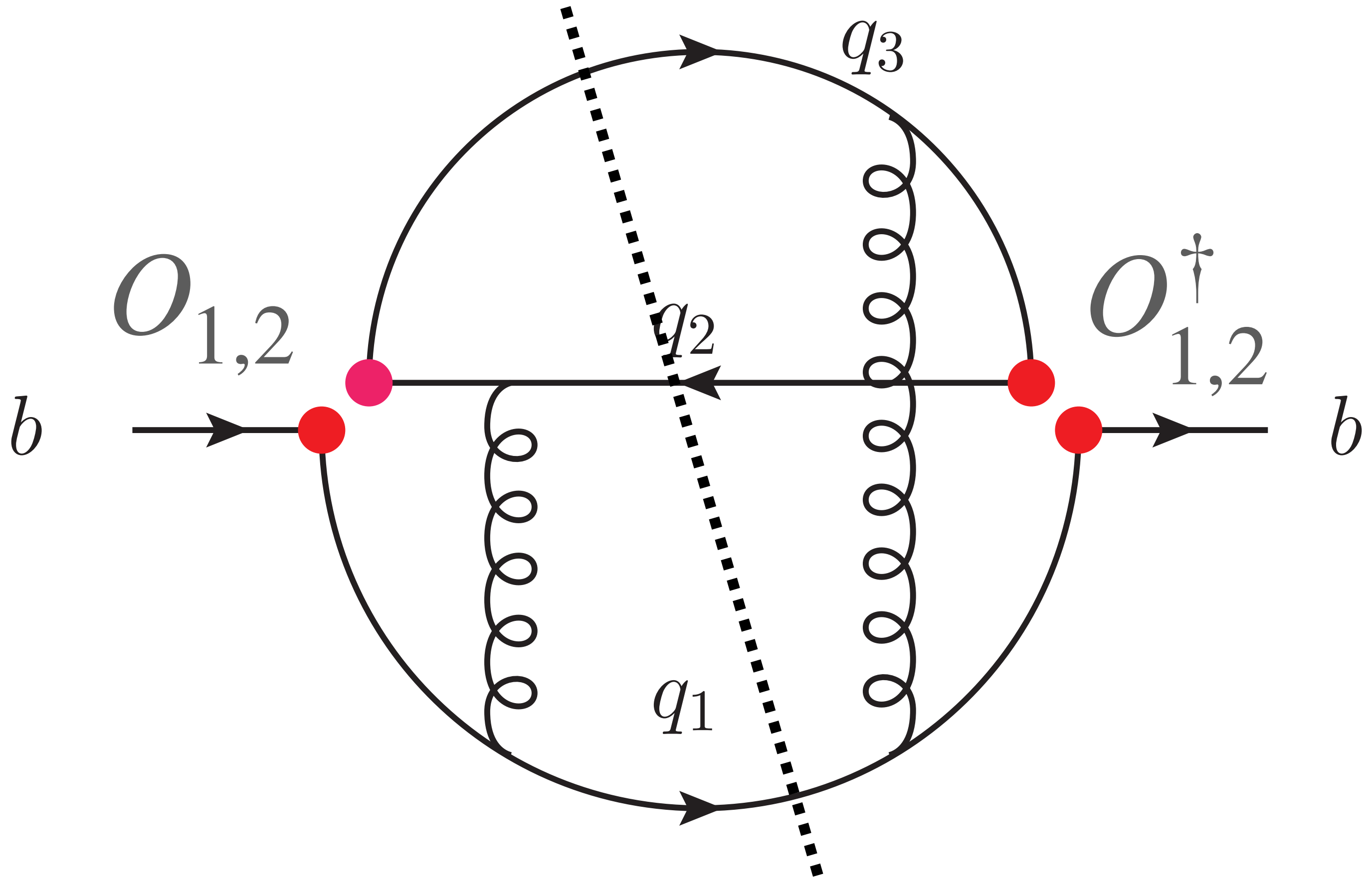




Lenz, Piscopo, Ruov, JHEP 01 (2023) 004

- Error on $\Gamma(B_q)$ dominated by theoretical uncertainties on Γ_3!
- GOAL: push accuracy for $\Gamma_3^{\text{non leptonic}}$ at NNLO

NONLEPTONIC DECAYS AT NNLO: CHALLENGES



$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} \lambda_{q_1q_2q_2}\Big(C_1(\mu_b) O_1^{q_1q_2q_3} + C_2(\mu_b) O_2^{q_1q_2q_3} \Big) + \text{h.c.}$$

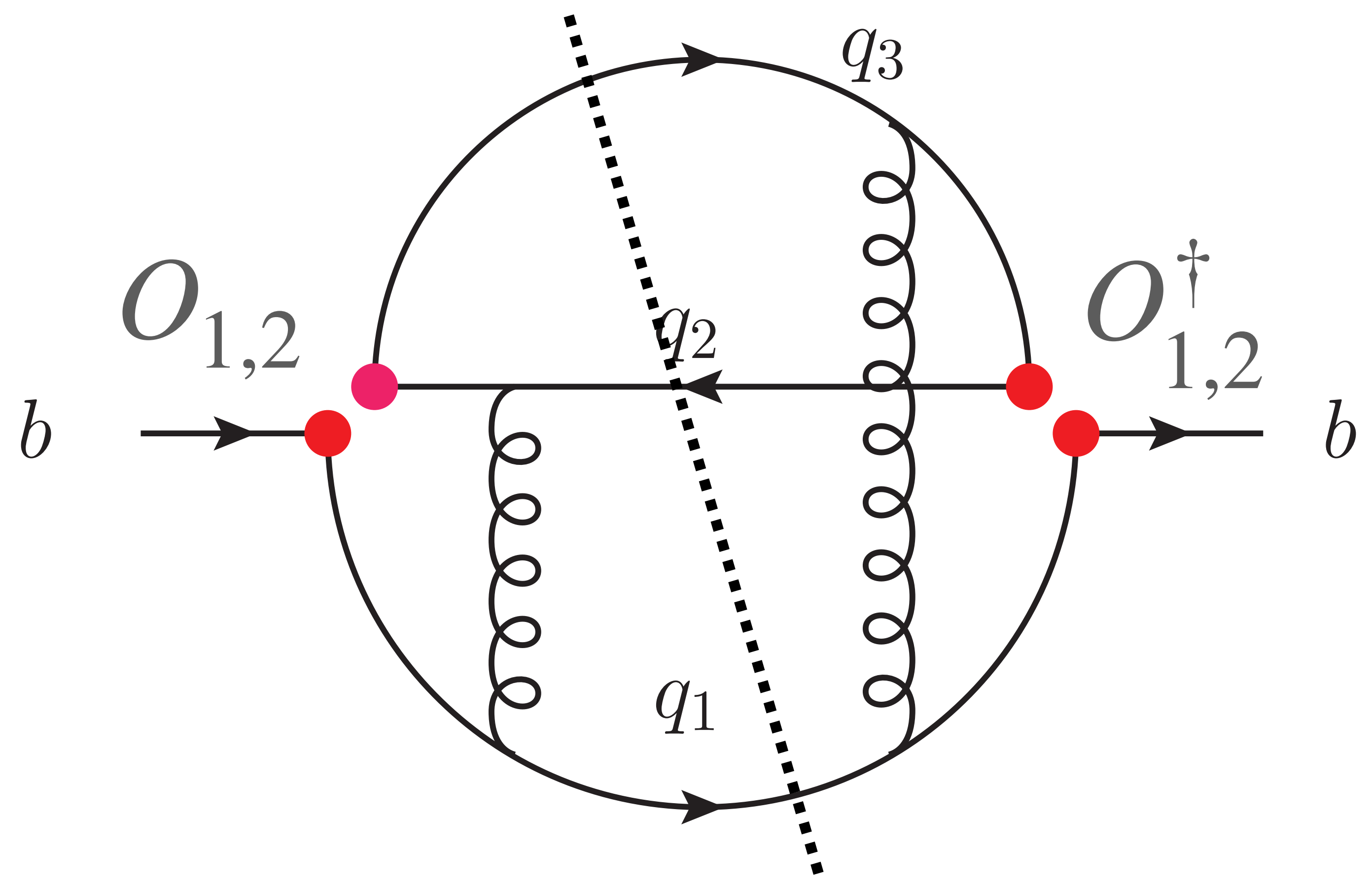
$$O_1^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha) \qquad O_2^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\alpha)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\beta)$$

↓ Four loop master integrals depending on $\rho = m_c/m_b$

↓ Non-trivial renormalization of effective operators

Issues with γ_5 in dimensional regularisation

NONLEPTONIC DECAYS AT NNLO: CHALLENGES



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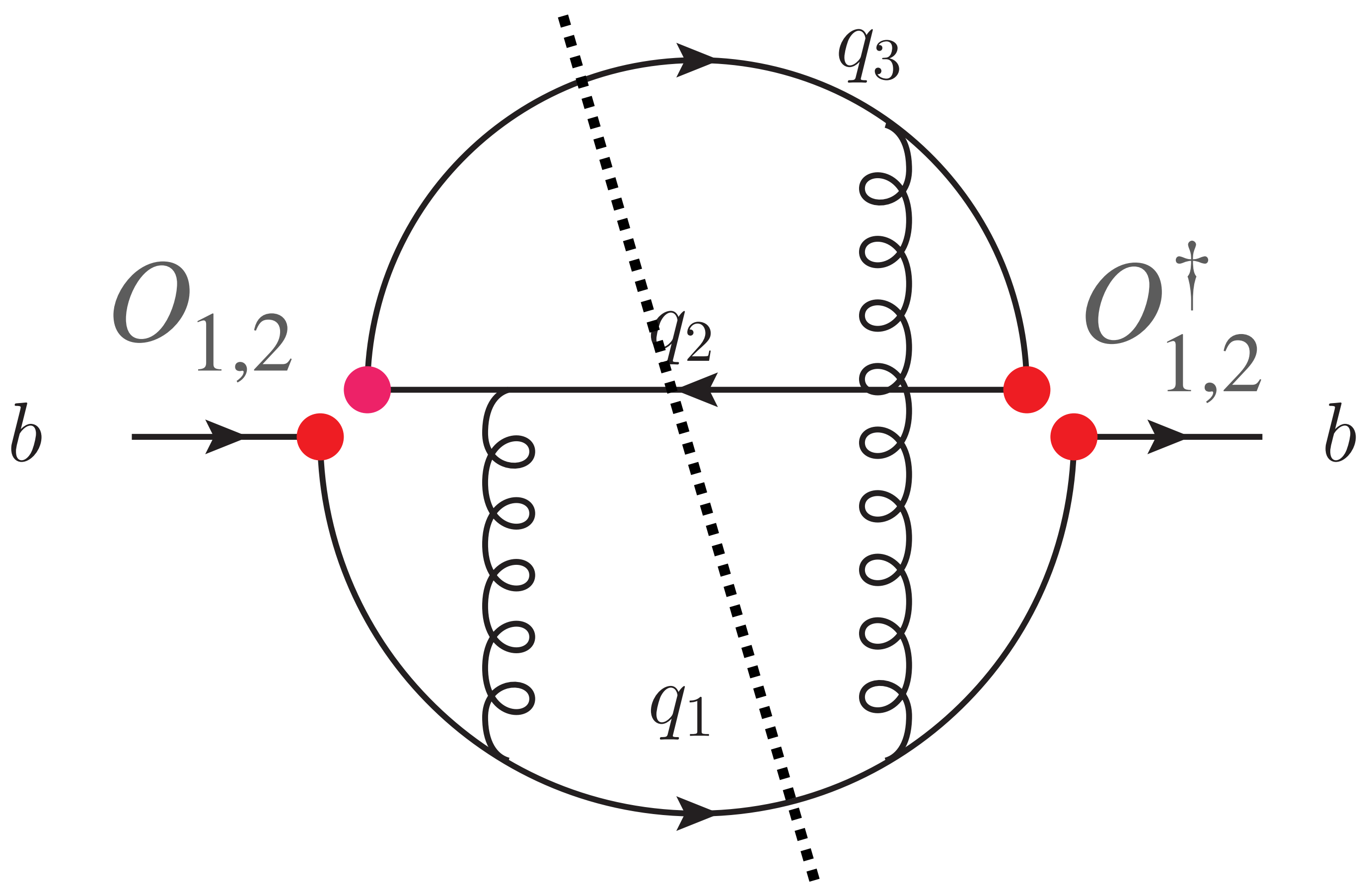
Four loop master integrals depending on $\rho = m_c/m_b$

- Numerical methods for master integrals
- Auxiliary mass flow (AMFlow)

Non-trivial renormalization of effective operators

Issues with γ_5 in dimensional regularisation

NONLEPTONIC DECAYS AT NNLO: CHALLENGES



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Four loop master integrals depending on $\rho = m_c/m_b$

- Numerical methods for master integrals
- Auxiliary mass flow (AMFlow)

Non-trivial renormalization of effective operators

Issues with γ_5 in dimensional regularisation

- Specific choice of evanescent operators which preserves Fierz identities in $d \neq 4$

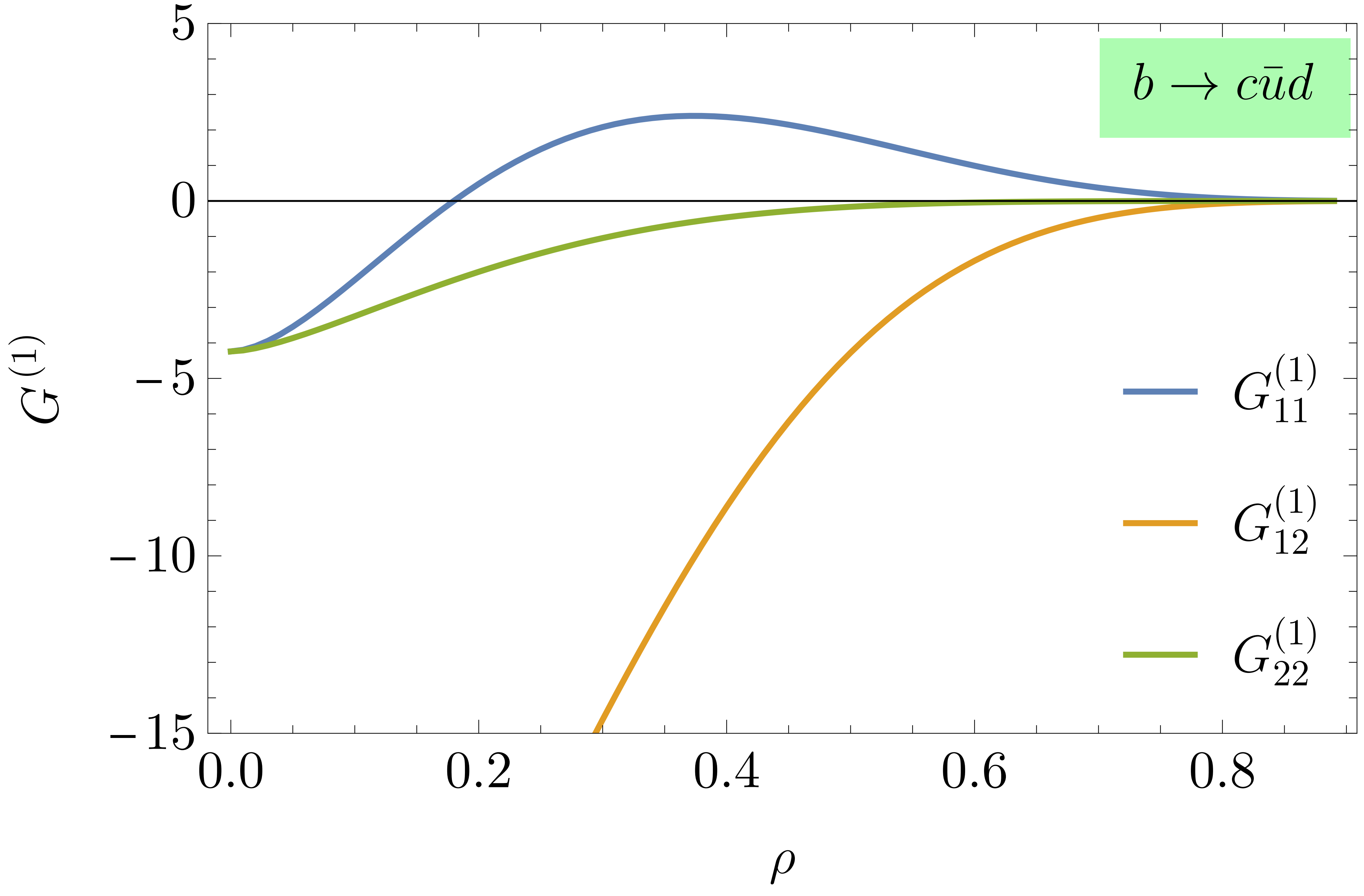
RESULTS IN THE ON SHELL SCHEME

Egner, **MF**, Schönwald, Steinhauser, JHEP10(2024)144

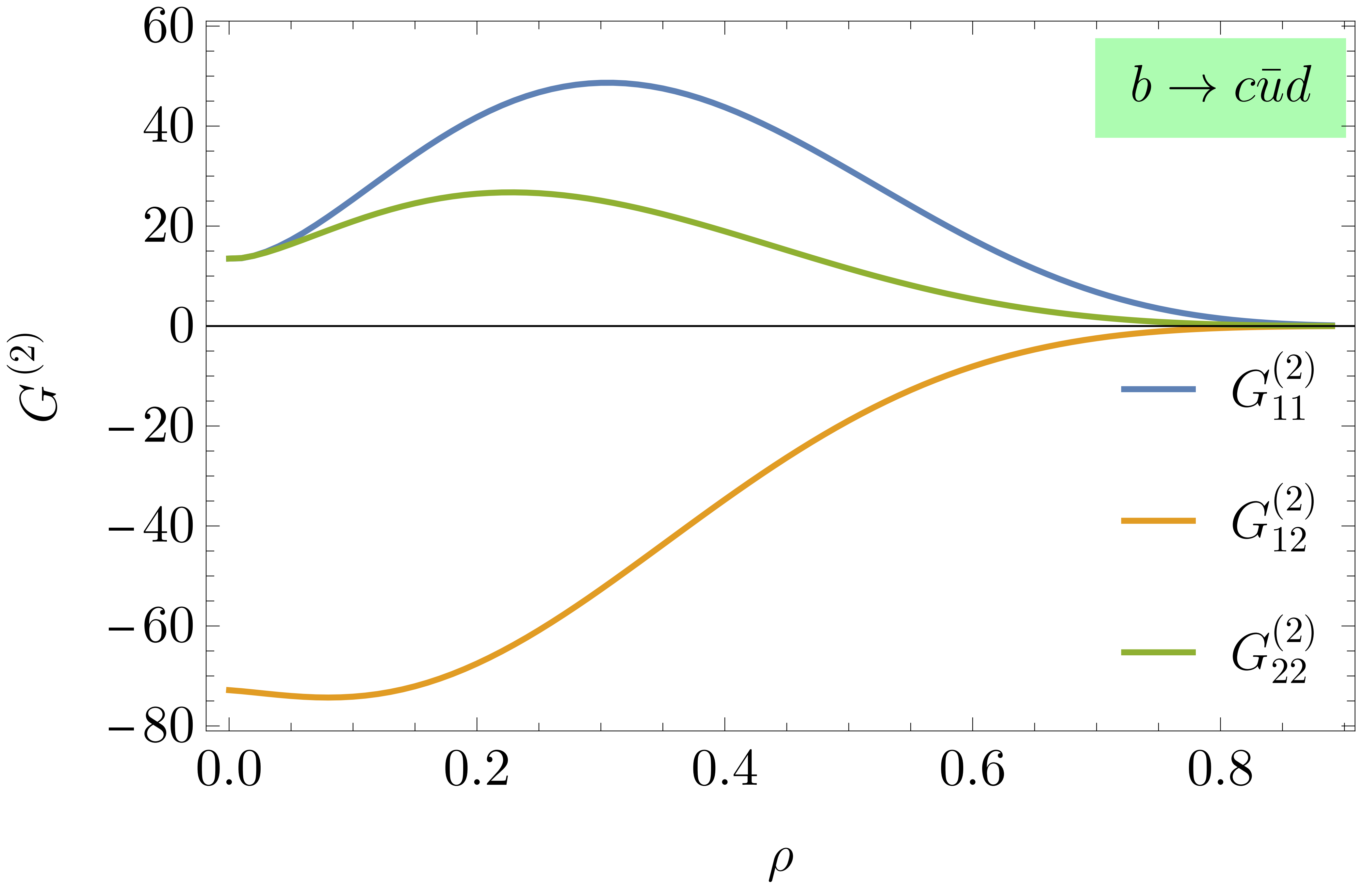
Note: the functions G_{ij} are scheme dependent!

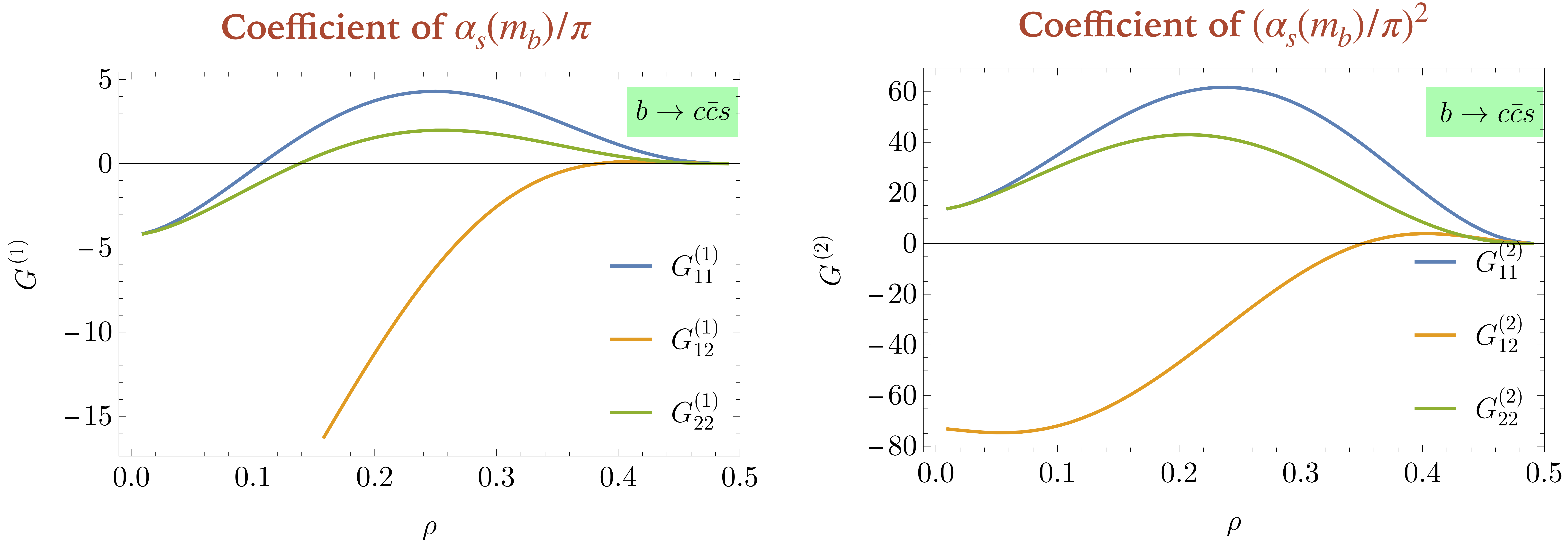
$$\Gamma^{q_1q_2q_3} = \frac{G_F^2 m_b^5 \lambda_{q_1q_2q_3}}{192\pi^3}\left[C_1^2(\mu_b)G_{11}^{q_1q_2q_3} + C_1(\mu_b)C_2(\mu_b)G_{12}^{q_1q_2q_3} + C_2^2(\mu_b)G_{22}^{q_1q_2q_3}\right]$$

Coefficient of $\alpha_s(m_b)/\pi$



Coefficient of $(\alpha_s(m_b)/\pi)^2$

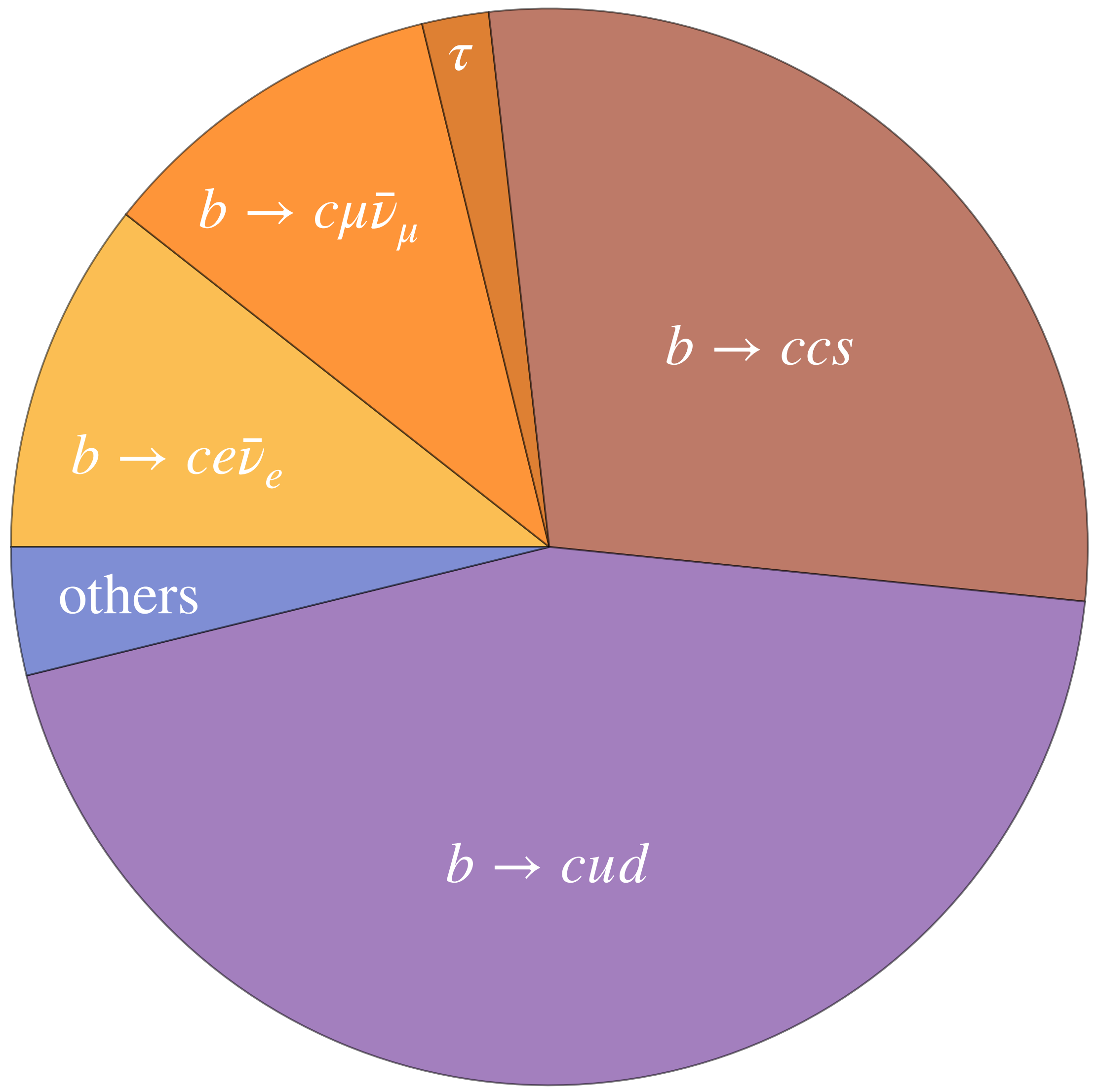


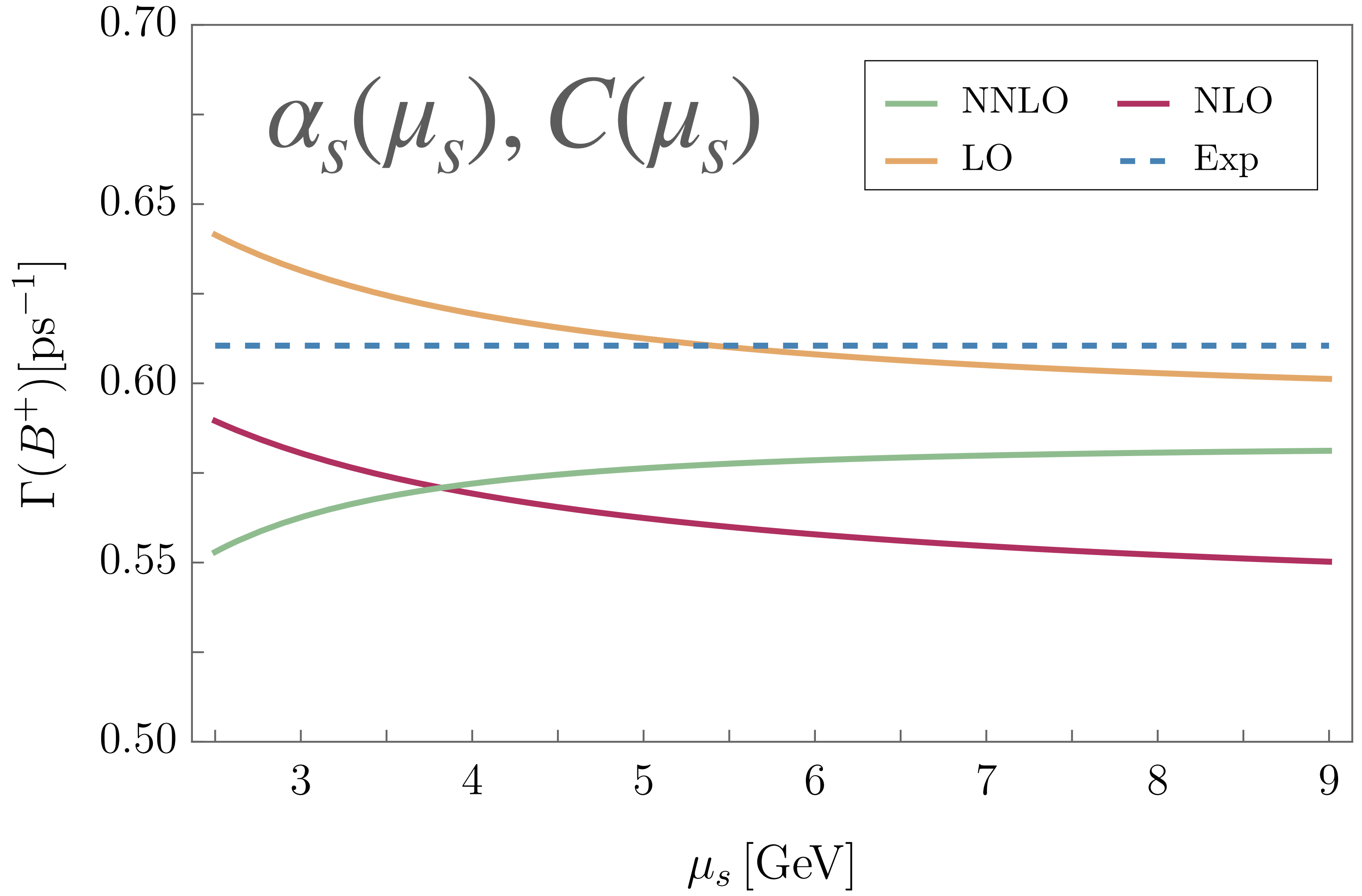


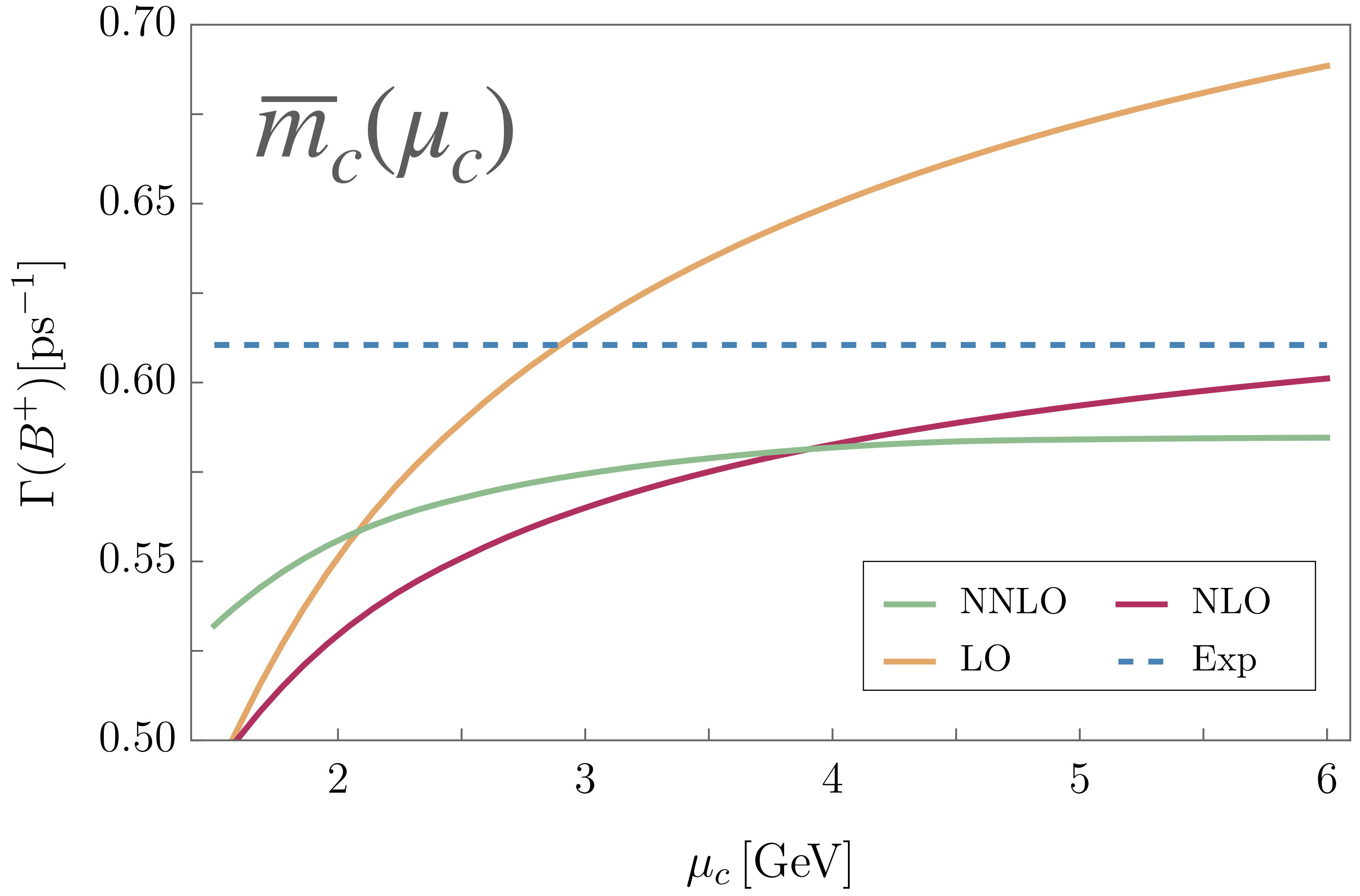
Egner, **MF,** Schönwald, Steinhauser, JHEP10(2024)144

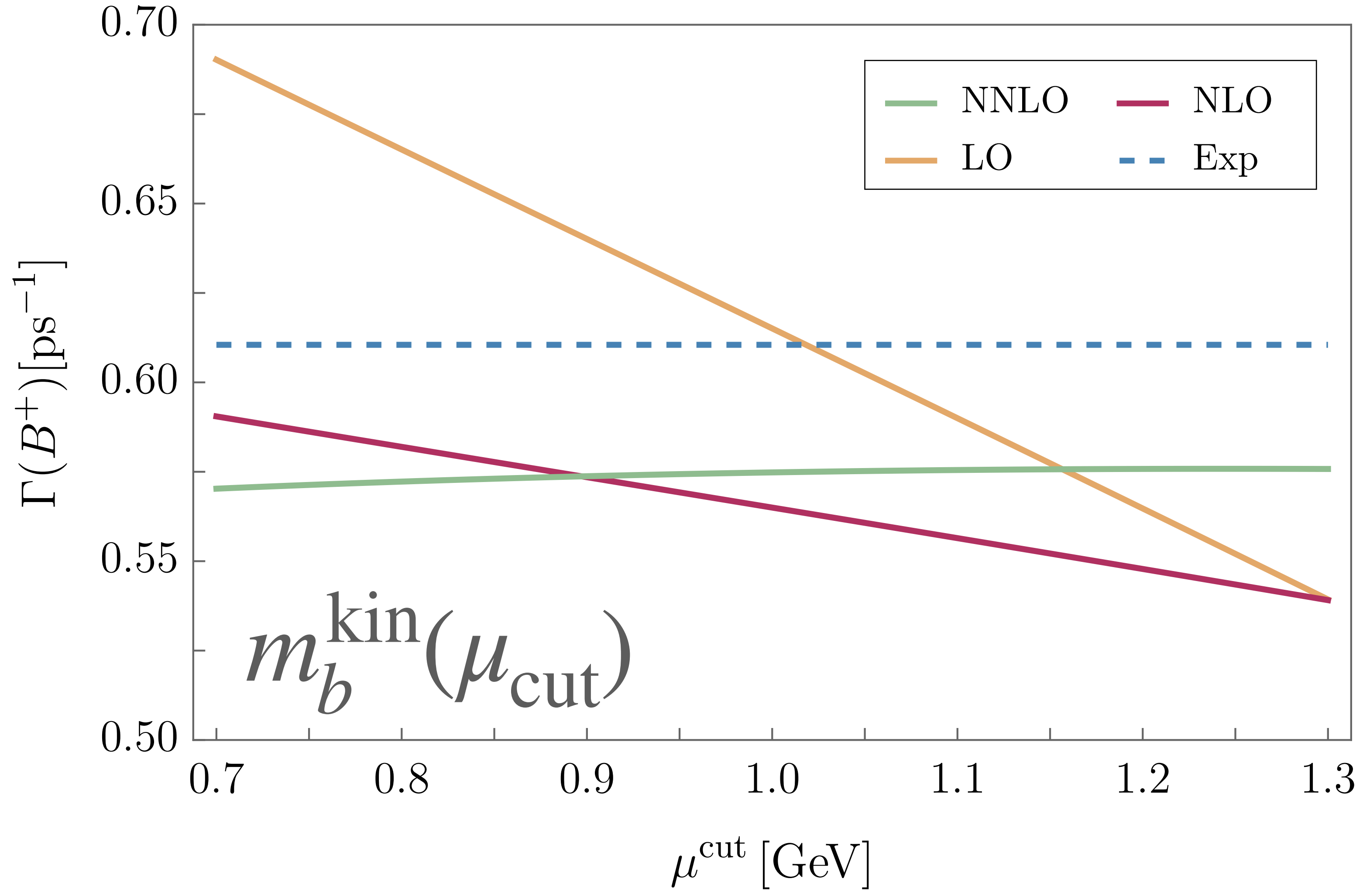
THEORETICAL UNCERTAINTIES [PRELIMINARY!]

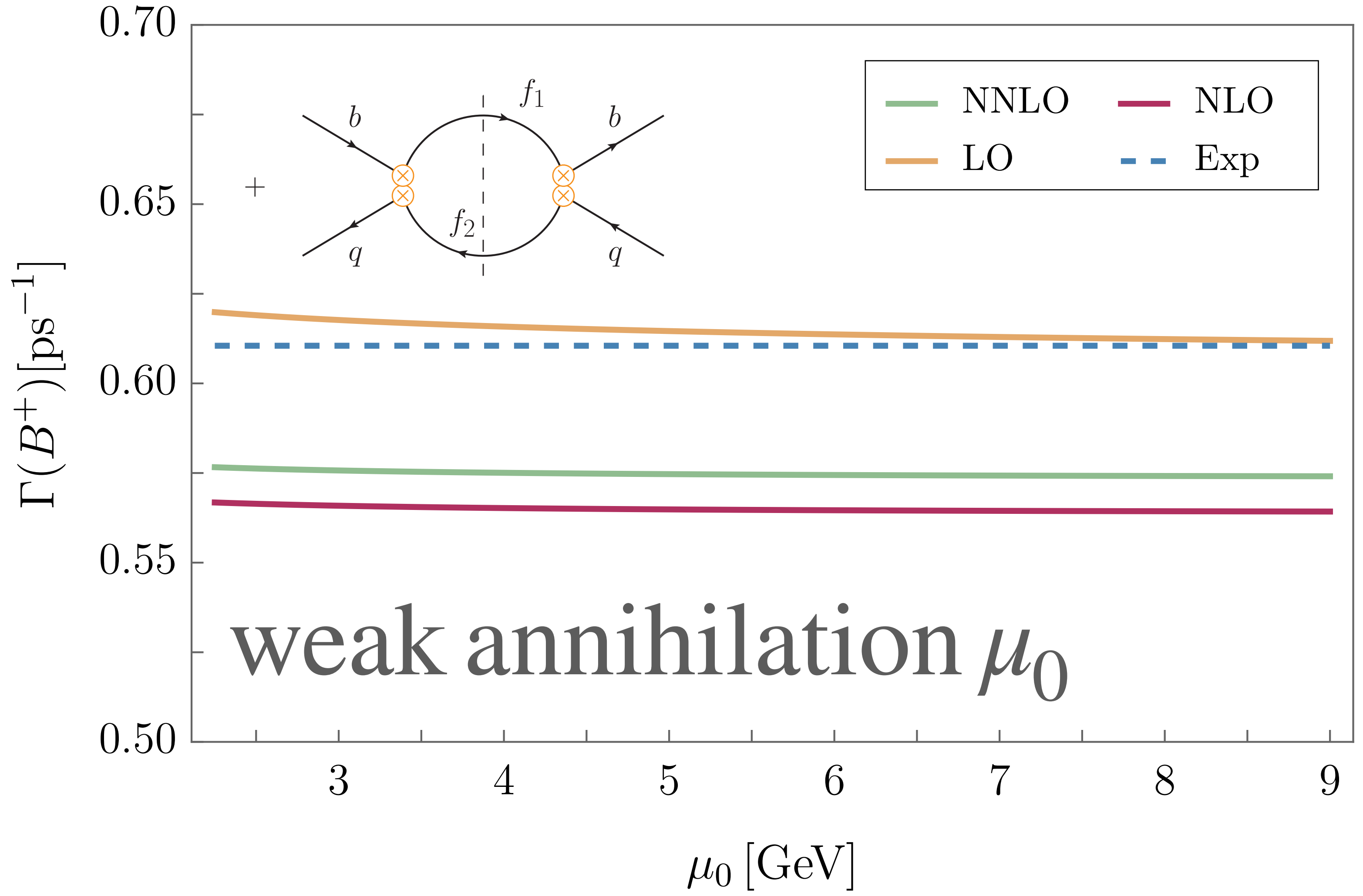
Egner, **MF**, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, in preparation





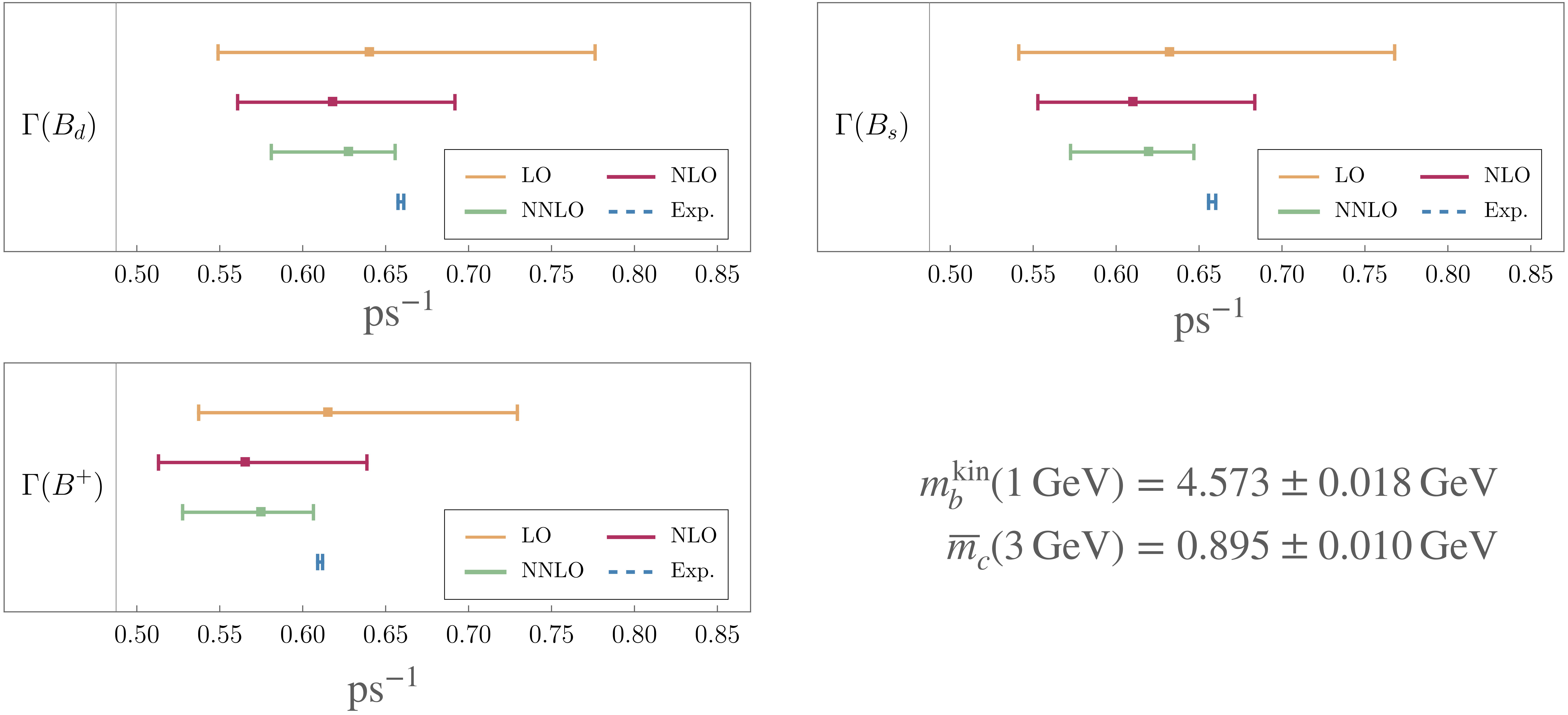






UPDATING THE LIFETIMES OF B MESONS [PRELIMINARY!]

Egner, **MF**, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, in preparation



$$m_b^{\text{kin}}(1\,\text{GeV}) = 4.573 \pm 0.018\,\text{GeV}$$

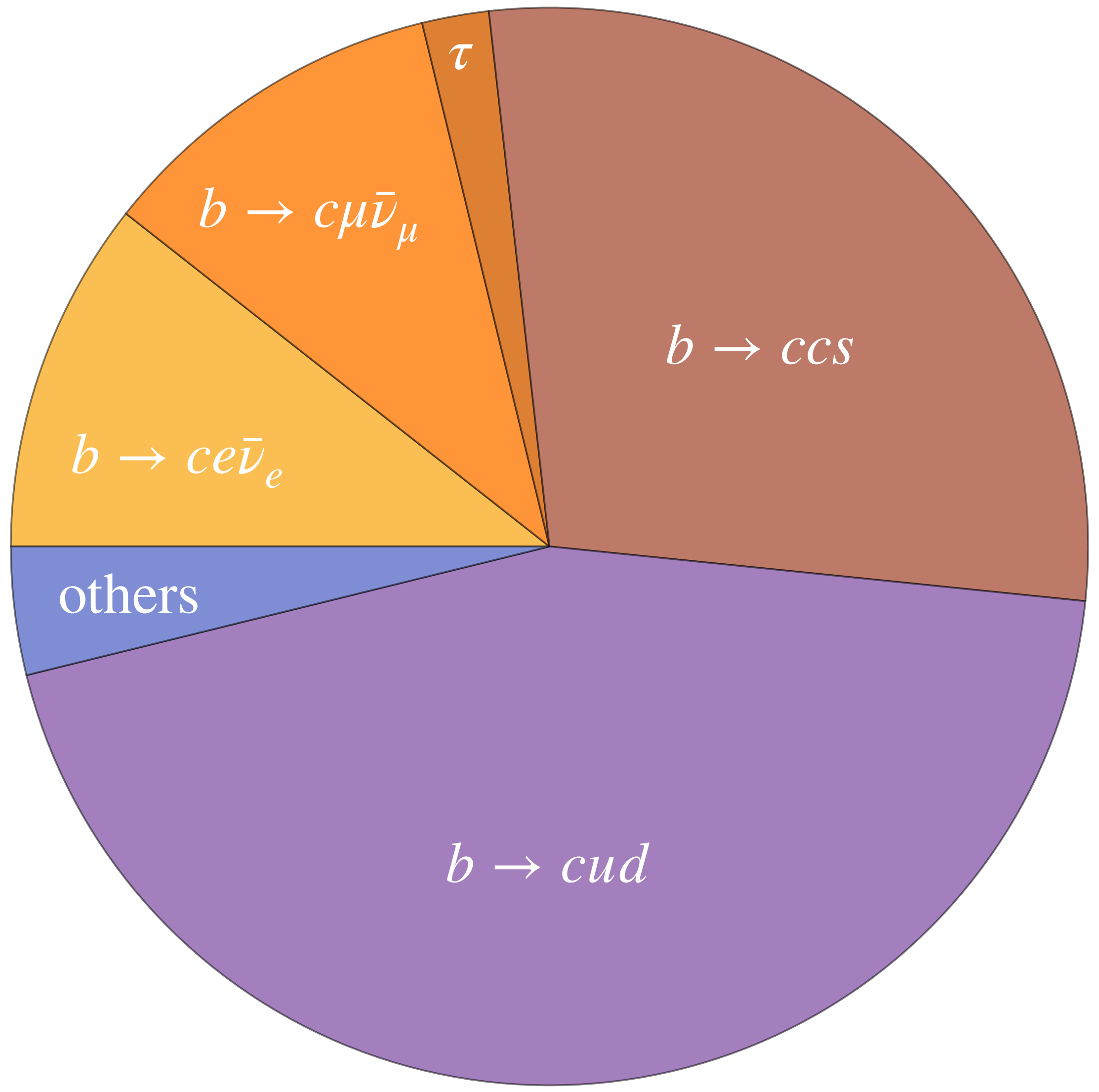
$$\overline{m}_c(3\,\text{GeV}) = 0.895 \pm 0.010\,\text{GeV}$$

SEMILEPTONIC BRANCHING FRACTION PURELY FROM THEORY [PRELIMINARY!]

$$\mathcal{B}_{\mathrm{sl}}(B_q) = \frac{\Gamma(B_q \to X_c l \nu)}{\Gamma(B_q)}$$

The ratio is independent on V_{cb}.

This is a test of the HQE and QCD!



$$\mathcal{B}_{\mathrm{sl}}(B^+) = (11.62^{+0.xx}_{-0.xx})\ \%$$

$$\mathcal{B}_{\mathrm{sl}}(B_d) = (10.64^{+0.xx}_{-0.xx})\ \%$$

$$\mathcal{B}_{\mathrm{sl}}(B_s) = (10.58^{+0.xx}_{-0.xx})\ \%$$

Egner, **MF**, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, in preparation

$$\mathcal{B}_{\mathrm{exp,\,avg}}(B \to X_c l \bar{\nu}_l) = (10.48 \pm 0.13)\ \%$$

Bernlochner, **MF**, Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068

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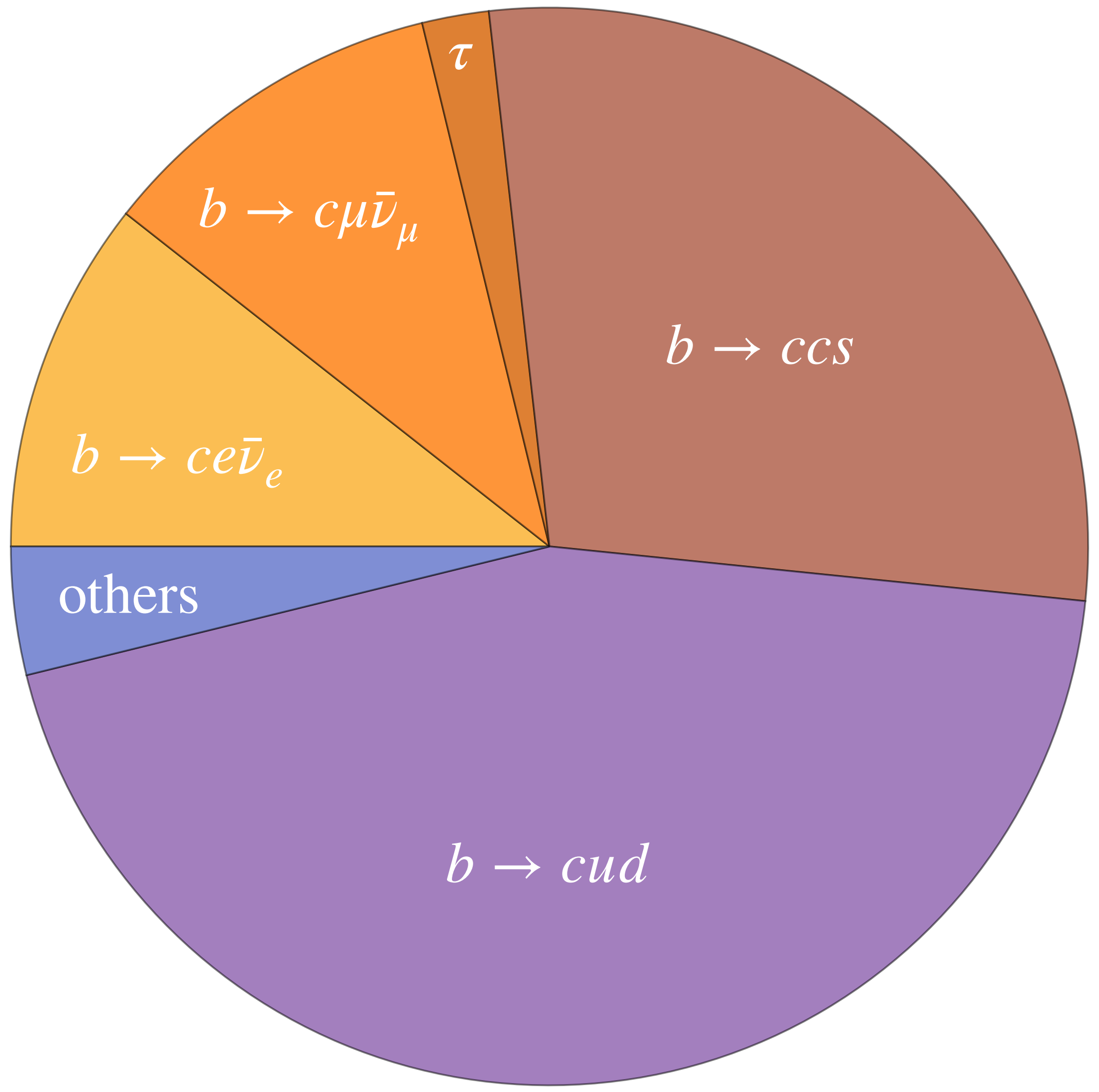

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Egner, **MF**, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, in preparation

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Bernlochner, **MF**, Olschwesky, Person, van Tonder, Vos, Welsch, JHEP 10 (2022) 068

STAY TUNED!

CONCLUSIONS

- New calculations made possible by recent developments in multi-loop techniques:
  - Numerical methods for solving master integrals
  - Auxiliary mass flow (AMFlow)
- First open-source code Kolya with complete predictions for $B \to X_c l \bar{\nu}_l$.
- Inclusive V_{cb} fit: updated estimate of th. uncertainties and use $1/m_b^{4,5}$ corrections.
- $\Gamma(B_q)$: significant reduction of the theoretical unc. after inclusion of NNLO corrections to O_1 and O_2.
- In progress: Update of the lifetime predictions.
- Improved accuracy opens the possibility to use $\tau(B_q)$ in the global fits for V_{cb}.

BACKUP

NUMERICAL EVALUATION OF MASTER INTEGRALS

- Solving master integrals: method of differential equations

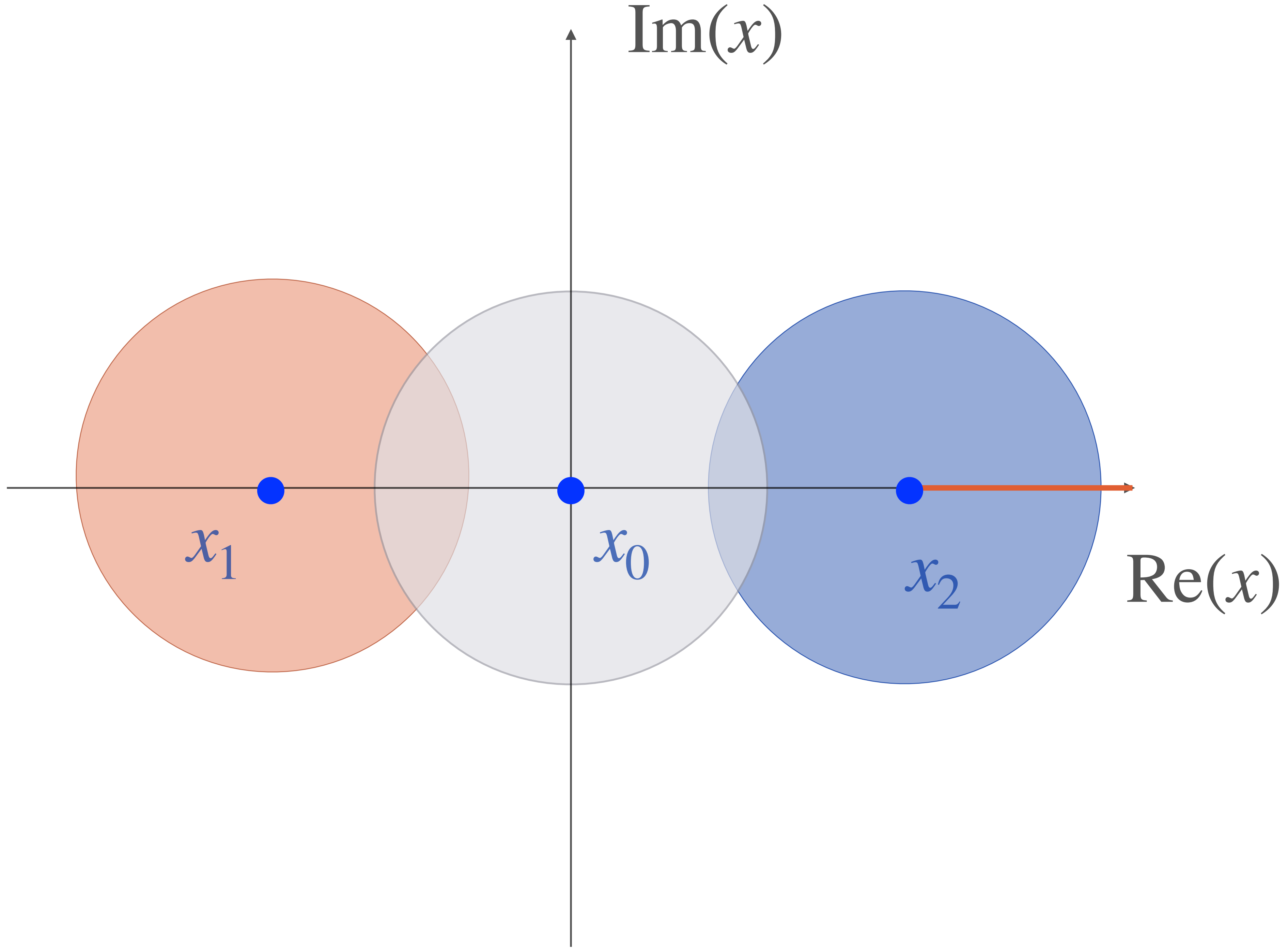
Kotikov, Phys. Lett. B 254 (1991) 158;
Gehrmann, Remiddi, Nucl. Phys. B 580 (2000) 485

$$\frac{\partial \vec{I}}{\partial x} = M(x,\epsilon)\vec{I}$$

Master integrals

- Construct a series expansion around some point x_0 [and $\epsilon = (d-4)/2$]

$$I_a(x,\epsilon) = \sum_{m=m_{\min}}^{m_{\max}} \sum_{n=0}^{n_{\max}} c_{a,mn}\, \epsilon^m (x - x_0)^n$$



S. Pozzorini and E. Remiddi, Comput. Phys. Commun. 175, 381 (2006).
X. Liu, Y.-Q. Ma, and C.-Y. Wang, Phys. Lett. B 779, 353 (2018).
R. N. Lee, A. V. Smirnov, and V. A. Smirnov, JHEP 03, 008 (2018).
M. K. Mandal and X. Zhao, JHEP 03, 190 (2019).
M. L. Czakon and M. Niggetiedt, JHEP 05, 149 (2020)..
F. Moriello, JHEP 01, 150 (2020).
MF, Lange, Schönwald, Steinhauser JHEP 09 (2021) 152
Hidding, Comput.Phys.Commun. 269 (2021) 108125
Armadillo, Bonciani, Devoto, Rana, Vicini, Comput.Phys.Commun. 282 (2023) 108545

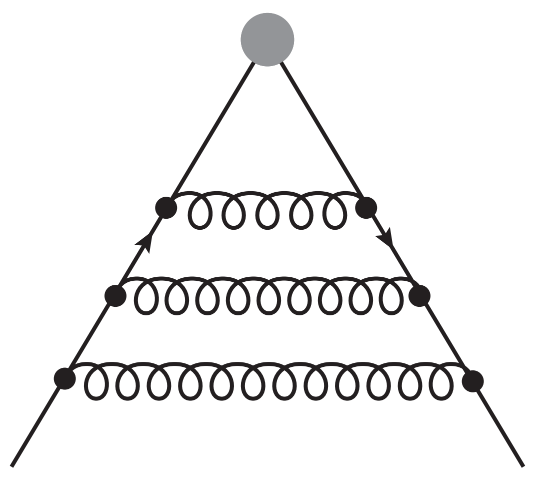
APPLICATIONS

Fix all external kinematics to numerical values
$s = 2, t = 1/10, m = 1$, etc

Several approaches

- DESS
  Lee, Smirnov, Smirnov, JHEP 03 (2018) 008
- DiffExp
  Hidding, Comput.Phys.Commun. 269 (2021) 108125
- SeaSide
  Armadillo, Bonciani, Devoto, Rana, Vicini, Comput.Phys.Commun. 282 (2023) 108545
- Expand and match
  MF, Lange, Schönwald, Steinhauser JHEP 09 (2021) 152

Heavy-quark form factors at $O(\alpha_s^3)$



MF, Lange, Schönwald, Steinhauser Phys.Rev.Lett. 128 (2022) 17; Phys.Rev.D 106 (2022) 3, 034029; Phys.Rev.D 107 (2023), 094017

also application to NRQCD

Egner, **MF,** Lange, Piclum, Schönwald, Steinhauser, Phys.Rev.D 104 (2021) 5, 054033, Phys.Rev.D 105 (2022) 11, 114007

- Auxiliary mass method
  Xiao Liu, Yan-Qing Ma, Comput.Phys.Commun. 283 (2023) 108565

$$I(\vec{n}) = \int \prod_{i=1}^{L} d^D \ell_i \frac{1}{D_1^{n_1} \dots D_N^{n_N}}$$
$$= \lim_{\eta \to i0^-} I_{\text{aux}}(\vec{n}, \eta)$$

- Precise numerical evaluation of boundary conditions

INSTALLATION

```
$: git clone https://gitlab.com/vcb-inclusive/kolya.git
$: cd kolya
$: pip3 install .
```

```
[1]: import kolya
     import numpy as np
```

Physical parameters

Physical parameters like quark masses like $m_b^{\rm kin}(\mu_{WC})$, $\overline{m}_c(\mu_c)$ and $\alpha_s(\mu_s)$ are declared in the class `parameters.physical_parameters`. Initialization set default values

```
[2]: par = kolya.parameters.physical_parameters()
     par.show()
```

```
bottom mass:       mbkin( 1.0  GeV)     =  4.563  GeV
charm mass:        mcMS( 3.0  GeV)      =  0.989  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.2182
```

In order to set the quark masses at scales different from the default ones in a consistent way, we include the method FLAG2023 which internally use CRunDec. For instance, we set the quark masses at a scale $\mu_{WC} = \mu_c = 2$ GeV in the following way:

```
[3]: par = kolya.parameters.physical_parameters()
     par.FLAG2023(scale_mcMS=2.0, scale_mbkin=2.0)
     par.show()
```

```
bottom mass:       mbkin( 2.0  GeV)     =  4.295730717092438  GeV
charm mass:        mcMS( 2.0  GeV)      =  1.0940623249384822  GeV
coupling constant: alpha_s( 4.563  GeV) =  0.21815198098622618
```

HQE parameters

Non-perturbative matrix elements in the HQE are declared in the class `parameters.HQE_parameters`. This class is defined in the historical basis of hep-ph/1307.4551. By default they are initalized to zero. We can set their values in the following way

[4]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
hqe.show()
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3
```

[5]:
```
hqe.show(flagmb4=1)
```

```
mupi  =  0.477  GeV^2
muG   =  0.306  GeV^2
rhoD  =  0.185  GeV^3
rhoLS =  -0.13  GeV^3

m1 =  0  GeV^4
m2 =  0  GeV^4
m3 =  0  GeV^4
m4 =  0  GeV^4
m5 =  0  GeV^4
m6 =  0  GeV^4
m7 =  0  GeV^4
m8 =  0  GeV^4
m9 =  0  GeV^4
```

Wilson coefficients

The Wilson coefficients in the effective Hamiltonian are declared in the class `parameters.WCoefficients`. They are initialized to zero and can be set in the following way

[6]:
```
wc = kolya.parameters.WCoefficients(
    VL = 0,
    VR = 0,
    SL = 0.1,
    SR = 0.1,
    T = 0,
)
wc.show()
```

```
C_{V_L} =  0
C_{V_R} =  0
C_{S_L} =  0.1
C_{S_R} =  0.1
C_{T} =  0
```

Total Rate

We define the total rate as

$$\Gamma_{\rm sl} = \frac{G_F^2 (m_b^{\rm kin})^5}{192\pi^3} |V_{cb}|^2 X$$

The coefficients X is a function of the quark masses, α_s, the HQE parameters and the Wilson coefficients. It is evaluated by the function `X_Gamma_KIN_MS(par, hqe, wc)`

[5]:
```
hqe = kolya.parameters.HQE_parameters(
    muG = 0.306,
    rhoD = 0.185,
    rhoLS = -0.13,
    mupi = 0.477,
)
wc  = kolya.parameters.WCoefficients()
kolya.TotalRate.X_Gamma_KIN_MS(par,hqe,wc)
```

[5]: 0.539225163728085

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb,par,hqe,wc)`

[6]:
```
Vcb = 42.2e-2
kolya.TotalRate.BranchingRatio_KIN_MS(Vcb,par,hqe,wc)
```

[6]: 10.555834162102016

Centralized q^2 moments

Q2 moments are evaluated with `Q2moments.moment_n_KIN_MS(q2cut, par, hqe, wc)`, where $q^2_{\rm cut}$ must be provided in GeV^2. The first centralized moment is calculated as follows:

[6]:
```
q2cut = 8.0 # GeV^2
kolya.Q2moments.moment_1_KIN_MS(q2cut,par,hqe,wc)
```

[6]:
```
8.996406491856465
```

The result for the moment $\langle q^{2n} \rangle$ is in GeV^{2n}

Centralized electron energy moments

E_l moments are evaluated with `Elmoments.moment_n_KIN_MS(Elcut, par, hqe, wc)`, where $E_{\rm cut}$ must be provided in GeV. The first centralized moment is calculated as follows:

[9]:
```
elcut = 0.5 # GeV
kolya.Elmoments.moment_1_KIN_MS(elcut,par,hqe,wc)
```

[9]:
```
1.4192938891883413
```

The result for $\langle E_l^n \rangle$ is in GeV^n

Centralized M_X^2 moments

M_X^2 moments are evaluated with `MXmoments.moment_n_KIN_MS(El_cut, par, hqe, wc)`, where $E_{\rm cut}$ must be provided in GeV. The first centralized moment is calculated as follows:

[13]:
```
elcut = 0.5 #GeV
kolya.MXmoments.moment_1_KIN_MS(elcut,par,hqe,wc)
```

[13]:
```
4.492408891792521
```

The result for $\langle M_X^{2n} \rangle$ is in GeV^{2n}

NONLEPTONIC DECAYS AND γ_5

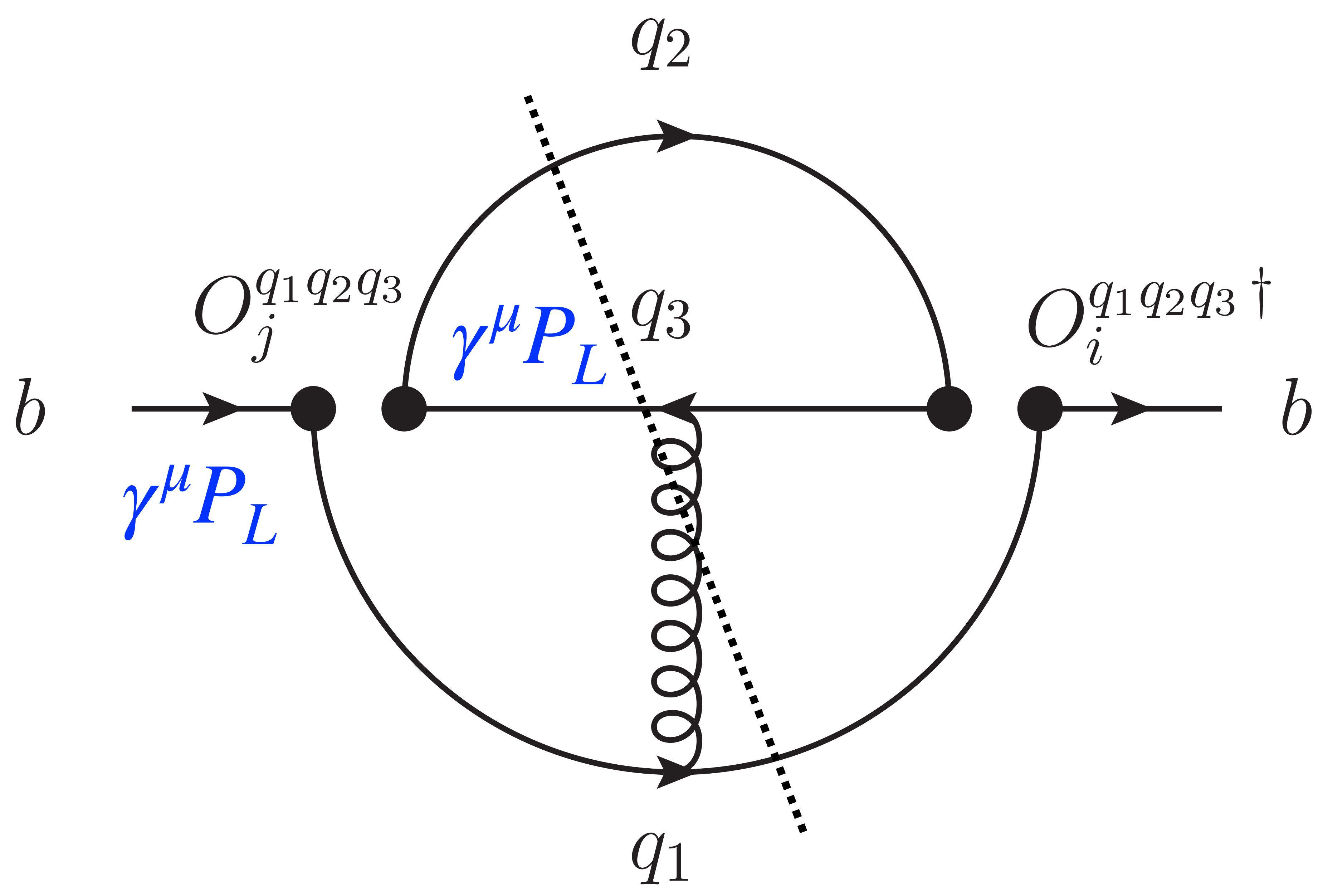
$$\mathscr{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} \lambda_{q_1q_2q_2} \Big(C_1(\mu_b) O_1^{q_1q_2q_3} + C_2(\mu_b) O_2^{q_1q_2q_3} \Big) + \text{h.c.}$$

***Traditional* basis**

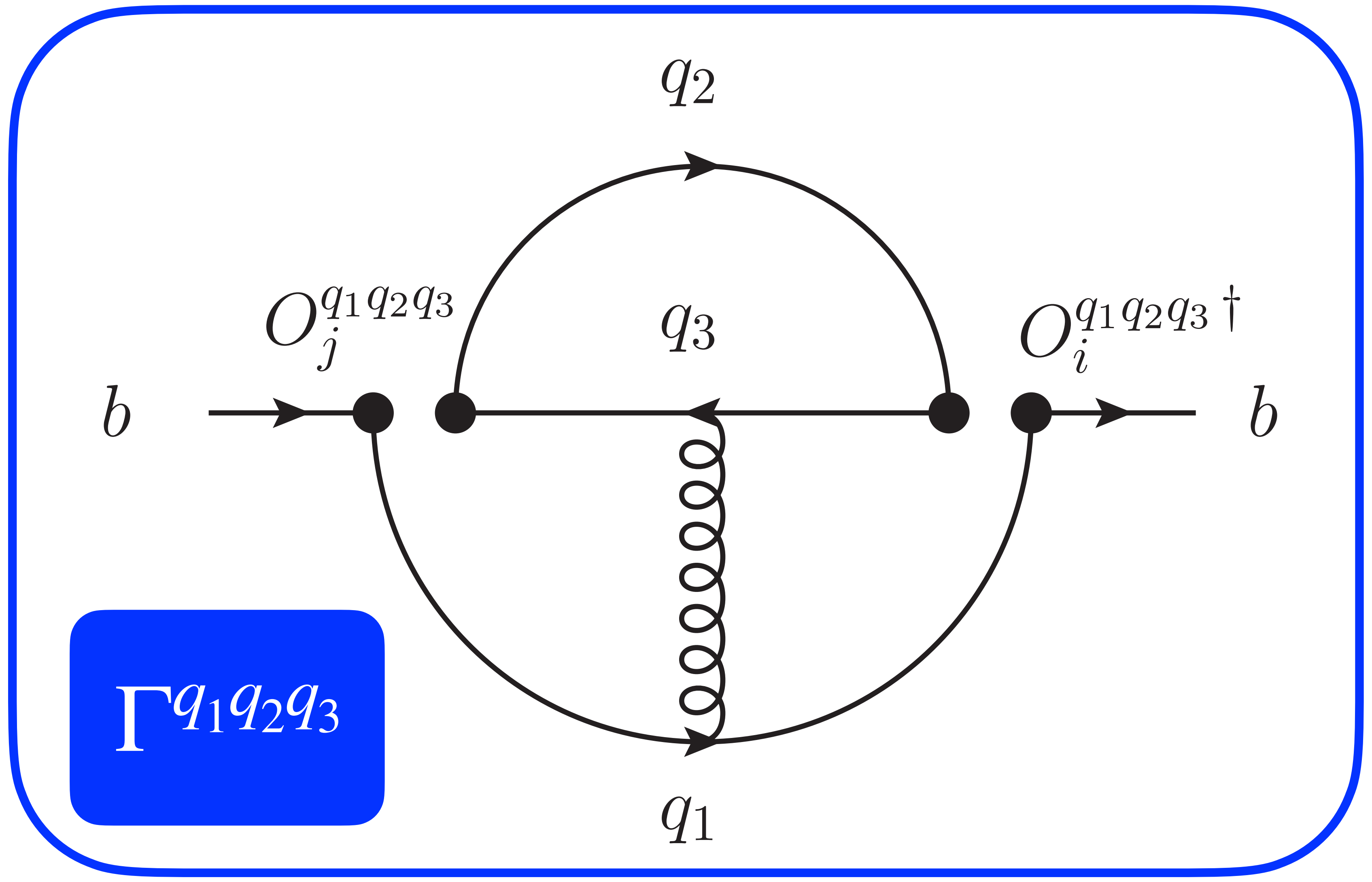
Buras, Weisz, NPB 333 (1990) 66

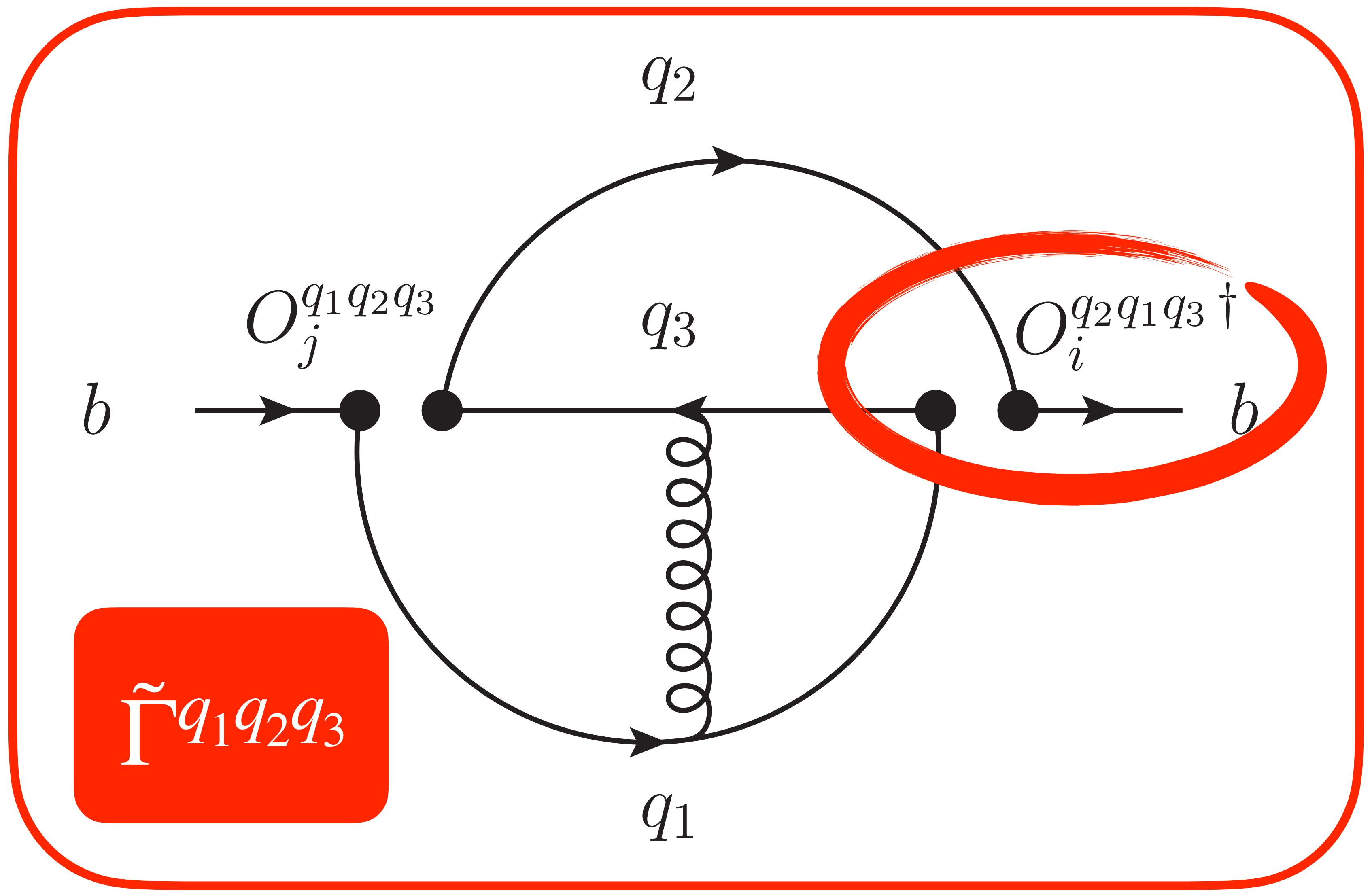
$$O_1^{q_1q_2q_3} = (\bar{q}_1^{\alpha} \gamma^\mu P_L b^\beta)(\bar{q}_2^{\beta} \gamma_\mu P_L q_3^\alpha),$$

$$O_2^{q_1q_2q_3} = (\bar{q}_1^{\alpha} \gamma^\mu P_L b^\alpha)(\bar{q}_2^{\beta} \gamma_\mu P_L q_3^\beta),$$



$$\simeq \text{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) \text{Tr}(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5)$$

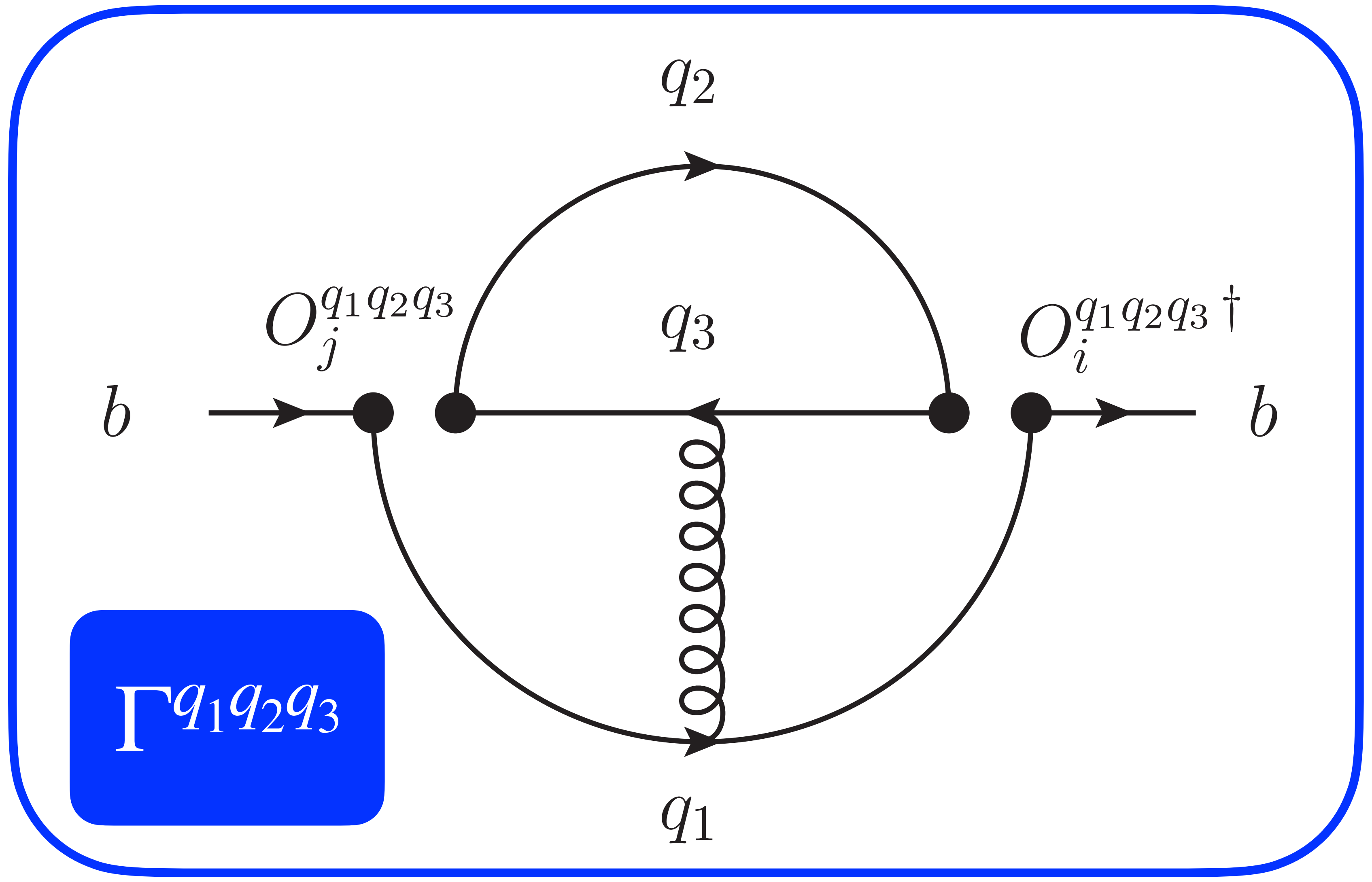




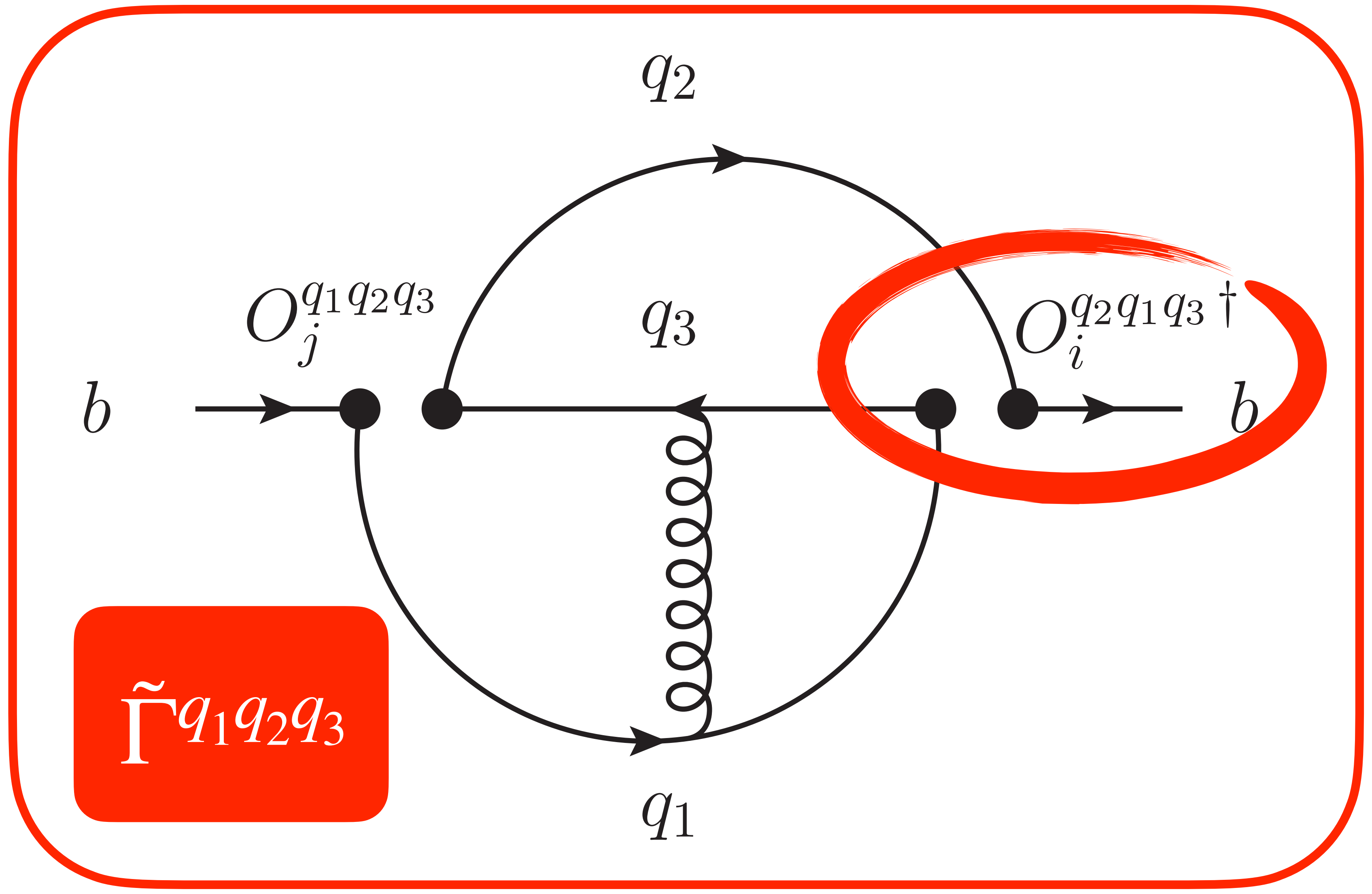
Fierz identity in $d = 4$

$$O_1^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha)$$
$$= (\bar{q}_2^\alpha \gamma^\mu P_L b^\alpha)(\bar{q}_1^\beta \gamma_\mu P_L q_3^\beta) = O_2^{q_2q_1q_3}$$

$$\Gamma^{q_1q_2q_3}(\rho) = \tilde{\Gamma}^{q_1q_2q_3}(\rho)\Big|_{\tilde{C}_1 \to C_2, \tilde{C}_2 \to C_1}$$



$\longrightarrow$



Fierz identity in $d = 4$

$$O_1^{q_1q_2q_3} = (\bar{q}_1^{\alpha}\gamma^{\mu}P_L b^{\beta})(\bar{q}_2^{\beta}\gamma_{\mu}P_L q_3^{\alpha})$$

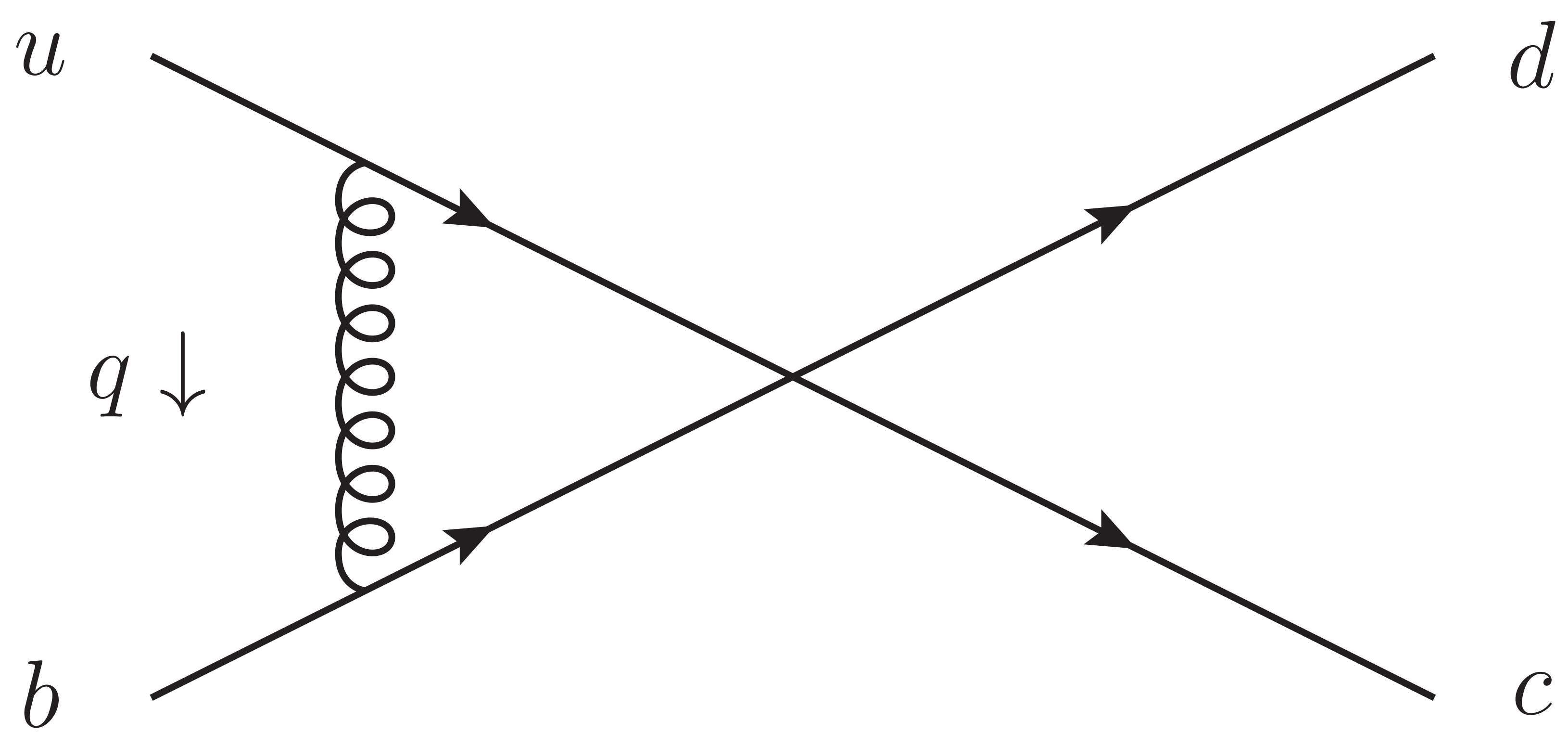
$$= (\bar{q}_2^{\alpha}\gamma^{\mu}P_L b^{\alpha})(\bar{q}_1^{\beta}\gamma_{\mu}P_L q_3^{\beta}) = O_2^{q_2q_1q_3}$$

$$\Gamma^{q_1q_2q_3}(\rho) = \tilde{\Gamma}^{q_1q_2q_3}(\rho)\Big|_{\tilde{C}_1\to C_2,\tilde{C}_2\to C_1}$$

How to preserve Fierz symmetries in dimensional regularisation?

[INTERLUDE] EVANESCENT OPERATORS

NDR : $\{\gamma^\mu, \gamma_5\} = 0$



This contraction cannot be reduced in $d \neq 4$

$$\Big(\bar{u}(p_c)\gamma^\mu\gamma^\nu\gamma^\rho P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\rho\gamma_\nu\gamma_\mu P_L u(p_u)\Big)$$

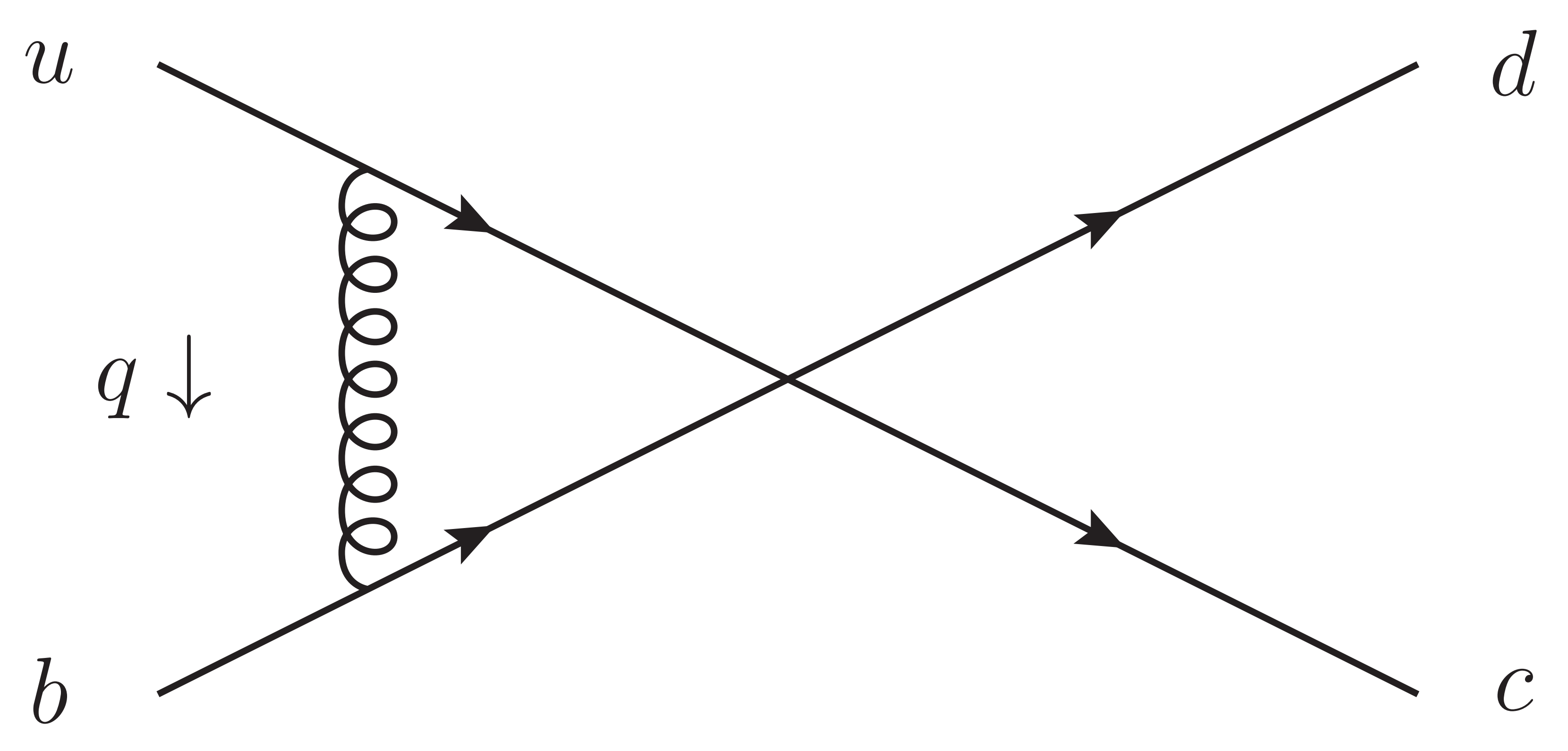
Add and subtract its $d = 4$ version:

$$\Big[\Big(\bar{u}(p_c)\gamma^\mu\gamma^\nu\gamma^\rho P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\rho\gamma_\nu\gamma_\mu P_L u(p_u)\Big) - 4\Big(\bar{u}(p_c)\gamma^\mu P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\mu P_L u(p_u)\Big)\Big] + 4\Big(\bar{u}(p_c)\gamma^\mu P_L u(p_b)\Big)$$

O_E $\qquad$ O_P

INTERLUDE: EVANESCENT OPERATORS

$\text{NDR}: \ \{\gamma^\mu, \gamma_5\} = 0$



This contraction cannot be reduced in $d \neq 4$

$$\Big(\bar{u}(p_c)\gamma^\mu\gamma^\nu\gamma^\rho P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\rho\gamma_\nu\gamma_\mu P_L u(p_u)\Big)$$

Add and subtract its $d = 4$ version:

$$\Big[\Big(\bar{u}(p_c)\gamma^\mu\gamma^\nu\gamma^\rho P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\rho\gamma_\nu\gamma_\mu P_L u(p_u)\Big) - (4 + A_0\epsilon)\Big(\bar{u}(p_c)\gamma^\mu P_L u(p_b)\Big)\Big(\bar{u}(p_d)\gamma_\mu P_L u(p_u)\Big)\Big] + (4 + A_0\epsilon)\Big(\bar{u}(p_c)\gamma^\mu P_L u(p_b)\Big)$$

O_E $\qquad$ O_P

SCHEME DEPENDENCE

Matching

$$C(M_W, A_0) = C^{(0)}(M_W) + \frac{\alpha_s}{4\pi} C^{(1)}(M_W, A_0) + \dots$$

RGE

$$\hat{\gamma} = \gamma^{(0)} + \frac{\alpha_s}{4\pi} \gamma^{(1)}(A_0) + \dots$$

$$C(\mu_b, A_0) = C^{(0)}(\mu_b) + \frac{\alpha_s}{4\pi} C^{(1)}(\mu_b, A_0) + \dots$$

Energy

Matrix element

$$\left| \langle f | O_i | i \rangle \right|^2 = G^{(0)}(\mu_b) + \frac{\alpha_s}{4\pi} G^{(1)}(\mu_b, A_0) + \dots$$

SCHEME DEPENDENCE

$$\Gamma \simeq C(\mu_b, A_0) G(\mu_b, A_0)$$

Only a proper combination of Wilson coefficients and the matrix element is scheme independent!

PRESERVING FIERZ IDENTITY IN $d \neq 4$

- Fierz identity can be restored order by order in perturbation theory
- Use definition of evanescent operator which preserves a symmetric ADM

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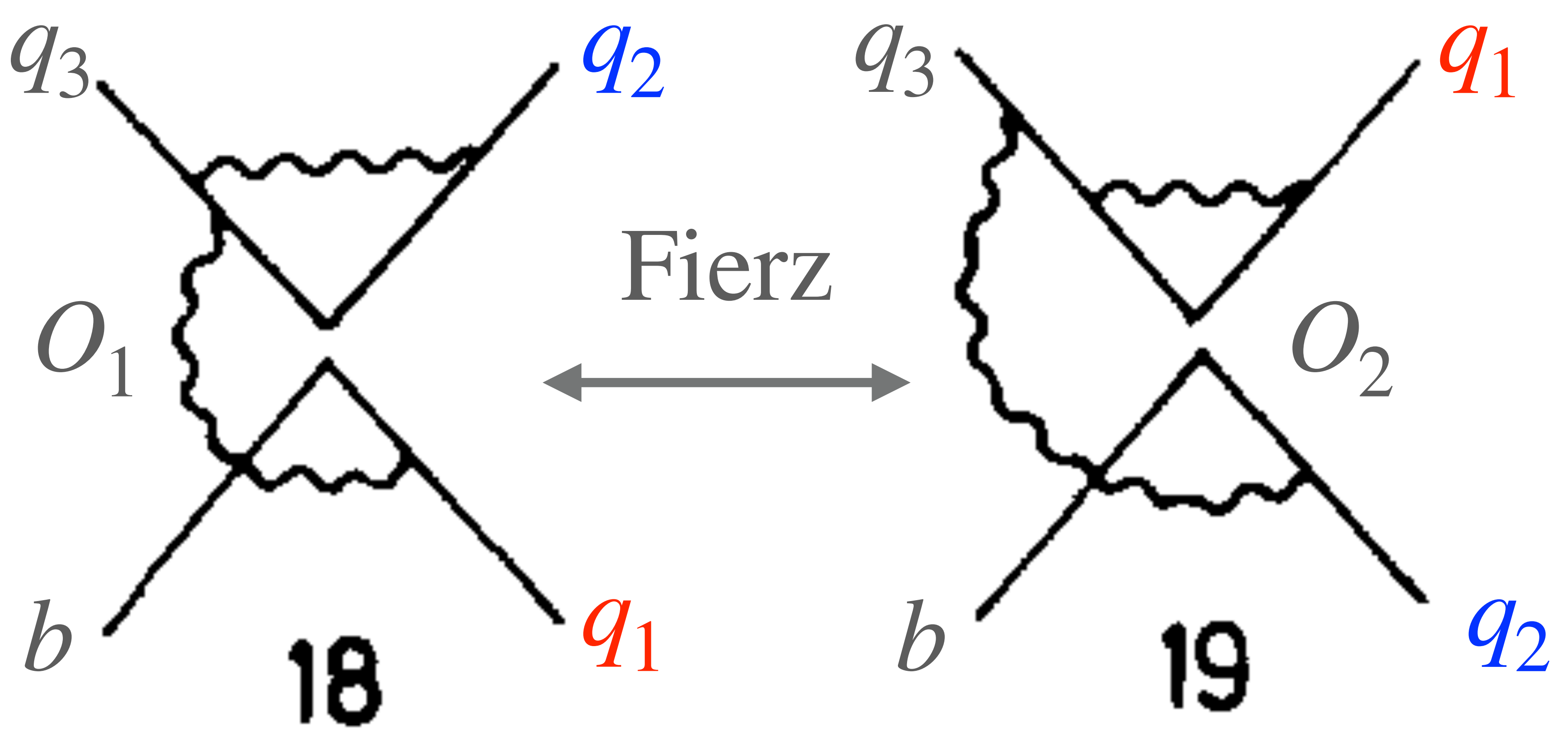
$$\gamma_{11} = \gamma_{22} \qquad \gamma_{12} = \gamma_{21}$$

- Equivalent to require that $O_\pm = (O_1 \pm O_2)/2$ do not mix under renormalization.

$$\hat{\gamma}_\pm = \begin{pmatrix} \gamma_+ & 0 \\ 0 & \gamma_- \end{pmatrix}$$



EVANESCENT OPERATORS

$$E_1^{(1),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\alpha) - (16 - 4\epsilon + A_2\epsilon^2) O_1^{q_1q_2q_3}$$

$$E_2^{(1),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3} P_L q_3^\beta) - (16 - 4\epsilon + A_2\epsilon^2) O_2^{q_1q_2q_3}$$

$$E_1^{(2),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\beta)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\alpha) - (256 - 224\epsilon + B_1\epsilon^2) O_1^{q_1q_2q_3}$$

$$E_2^{(2),q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L b^\alpha)(\bar{q}_2^\beta \gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5} P_L q_3^\beta) - (256 - 224\epsilon + B_2\epsilon^2) O_2^{q_1q_2q_3}$$

$\hat{\gamma}^{(2)}$ in the CMM basis → $\hat{\gamma}^{(2)}$ in the Traditional basis → Impose $\gamma_{11} = \gamma_{22}, \gamma_{12} = \gamma_{21}$

Chetyrkin, Misiak, Munz, hep-ph/9711280;
Gorbahn, Heisch, hep-ph/0411071

$$B_1 = -\frac{4384}{115} - \frac{32}{5}n_f + A_2\left(\frac{10388}{115} - \frac{8}{5}n_f\right)$$

$$B_2 = -\frac{38944}{115} - \frac{32}{5}n_f + A_2\left(\frac{19028}{115} - \frac{8}{5}n_f\right)$$