Recent theory results on $b \to s\bar{\ell}\ell$

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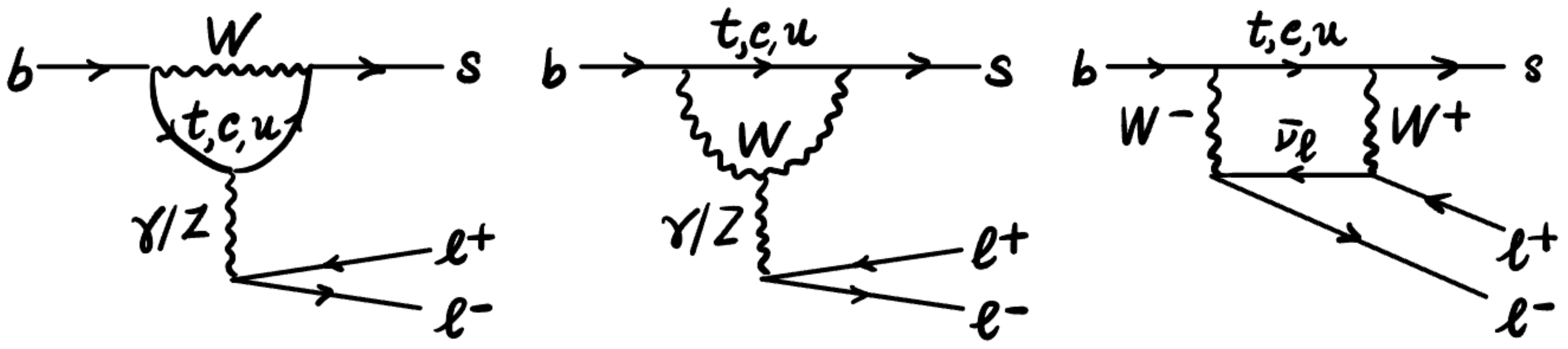
MITP, Open Questions and Future Directions in Flavor Physics
November 7, 2024

Based on 2405.17551, 2401.18007, 2305.03076 (with Gino Isidori, Zachary Polonsky, Marzia Bordone, Sandro Maechler)

Introduction

- $b \to s\bar{\ell}\ell$ decays are very good candidates in the search for BSM.
- Being suppressed in the SM, they are extremely sensitive to a wide range of NP effects.



- **Key decay channels** are $B \to K\bar{\ell}\ell, B \to K^*\bar{\ell}\ell, B_s \to \phi\bar{\ell}\ell, B_s \to \bar{\mu}\mu$.
- Observables: **branching ratios**, (optimized) **angular observables** ($P^{(\prime)}_{1,2,3,4,5,6,8}$), **LFU ratios**.

$b \to s\bar{\mu}\mu$ vs $b \to s\bar{e}e$

Effective Lagrangian

- Effective description of $b \to s\bar{\ell}\ell$ decays below the EW scale:

$$\mathscr{L} = \mathscr{L}_{\text{QCD+QED}}^{[N_f=5]} + \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \mathcal{O}_i$$

$$\mathcal{O}_1 = \frac{4\pi}{\alpha_e} (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \qquad \mathcal{O}_2 = \frac{4\pi}{\alpha_e} (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

$$\mathcal{O}_3 = \frac{4\pi}{\alpha_e} (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q}_L \gamma^\mu q_L) \qquad \mathcal{O}_4 = \frac{4\pi}{\alpha_e} (\bar{s}_L^a \gamma^\mu T^a b_L^b) \sum_q (\bar{q}_L{}^b \gamma^\mu T^a q_L^a)$$

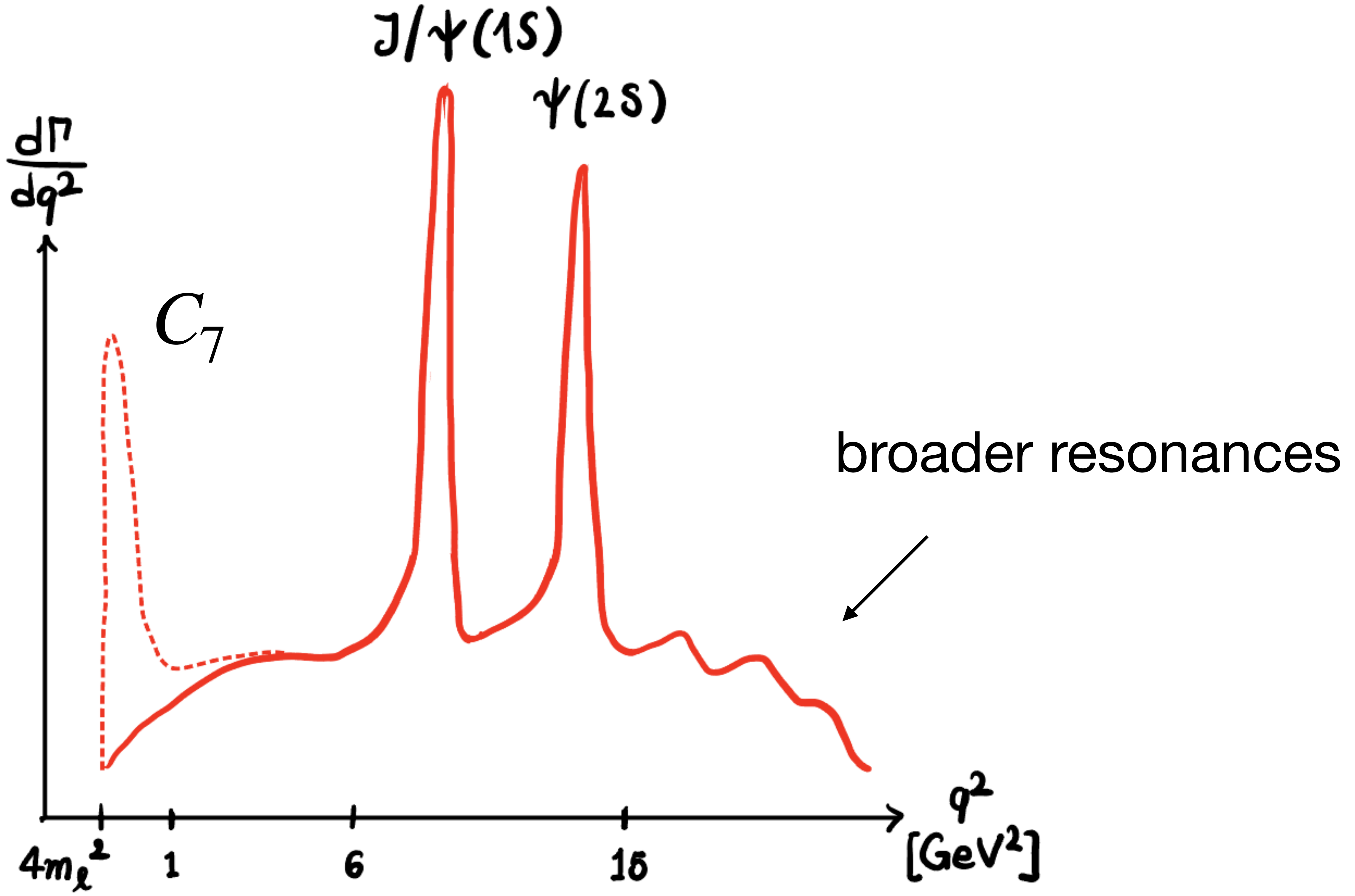
$$\mathcal{O}_5 = \frac{4\pi}{\alpha_e} (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_q (\bar{q}_L \gamma^\mu \gamma^\nu \gamma^\rho q_R) \qquad \mathcal{O}_6 = \frac{4\pi}{\alpha_e} (\bar{s}_L^a \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L^b) \sum_q (\bar{q}_L{}^b \gamma^\mu \gamma^\nu \gamma^\rho T^a q_R^a)$$

$$\mathcal{O}_7 = \frac{m_b}{e} (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{e^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$

$$\mathcal{O}_9 = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell) \qquad \mathcal{O}_{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$b \to s\bar{\ell}\ell$ decays

- General features of $b \to s\bar{\ell}\ell$ branching ratios:



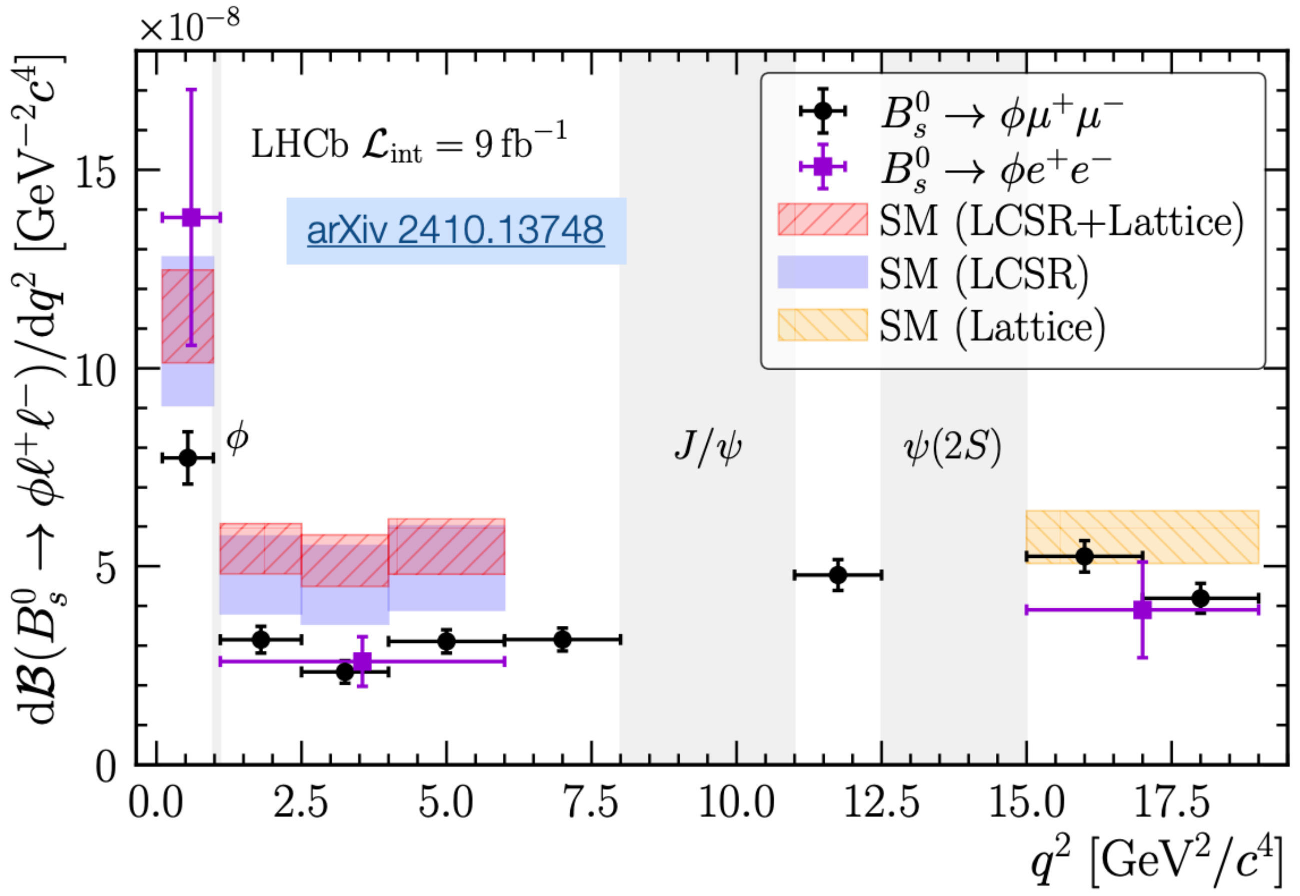
- q^2 is the invariant mass of the lepton pair.
- Separate tests in the low- or high-q^2 region.
- Sensitivity to the WCs C_7, C_9, C_{10}.

Experimental results on $b \to s\bar{\ell}\ell$

Tension in branching ratios

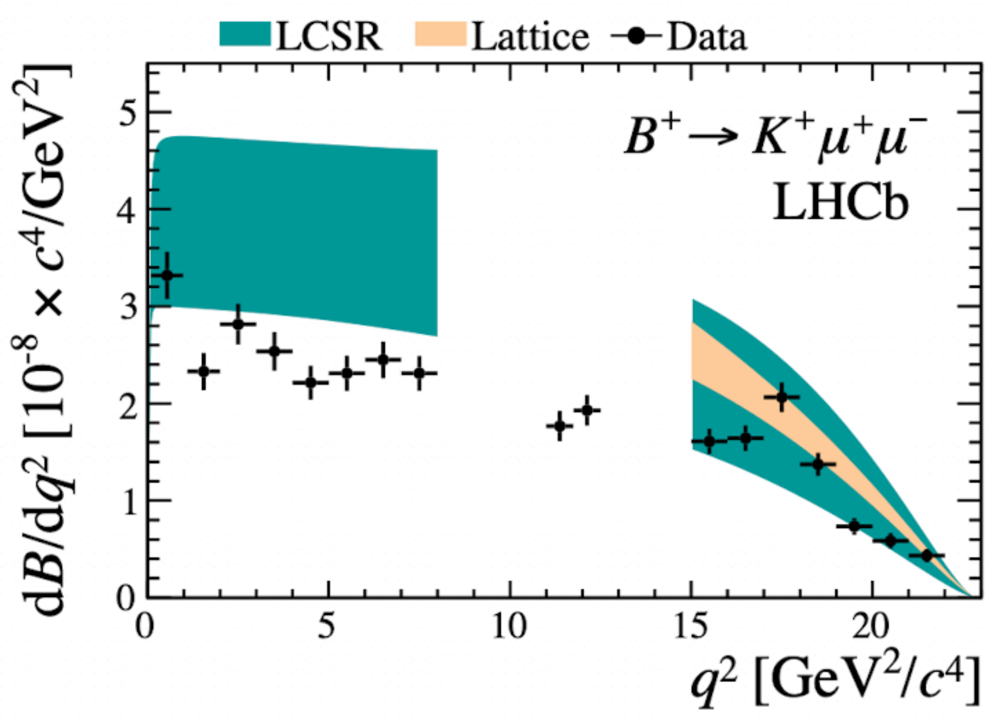
- Long-standing tension in branching ratios:

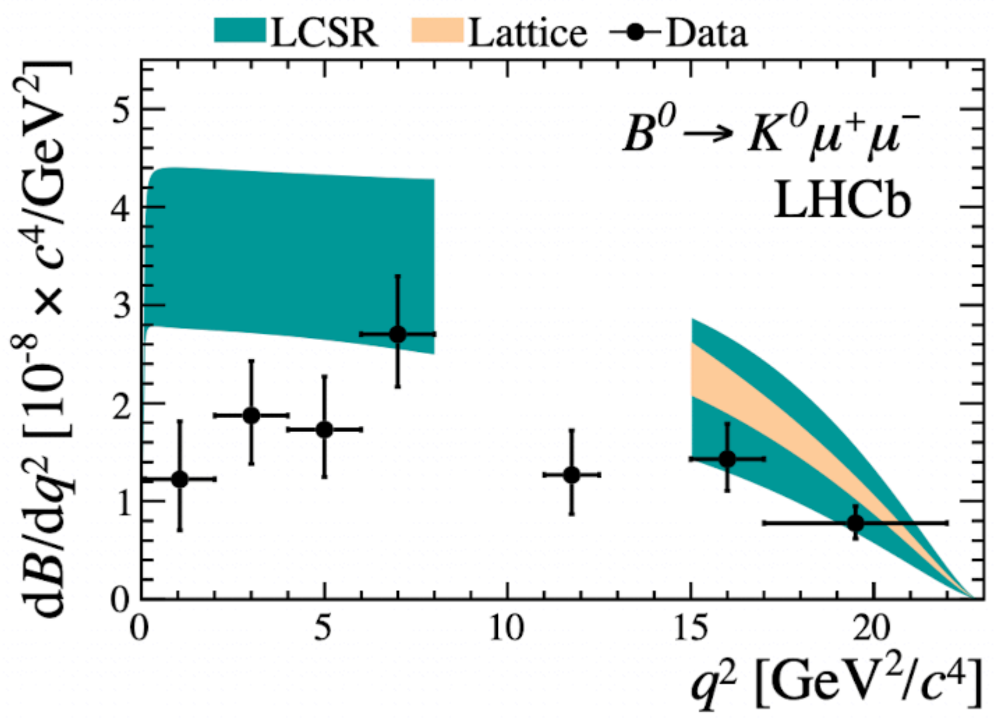
$B_s \to \phi\bar{\mu}\mu$



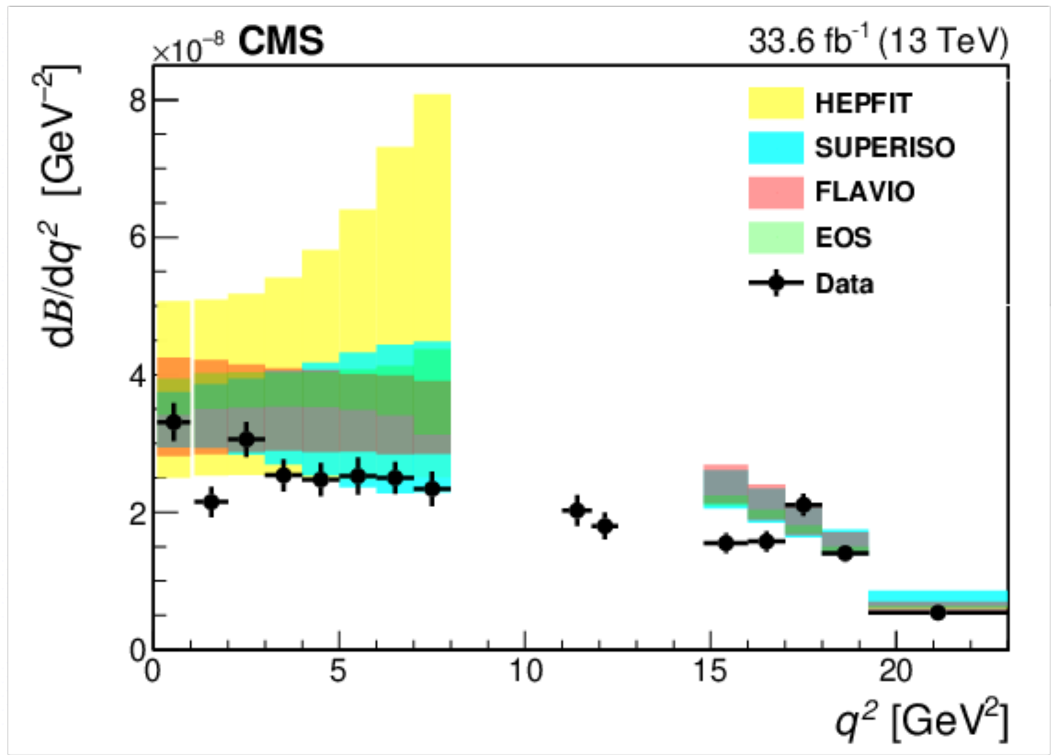
[LHCb, 2410.13748]

$B \to K\bar{\mu}\mu$





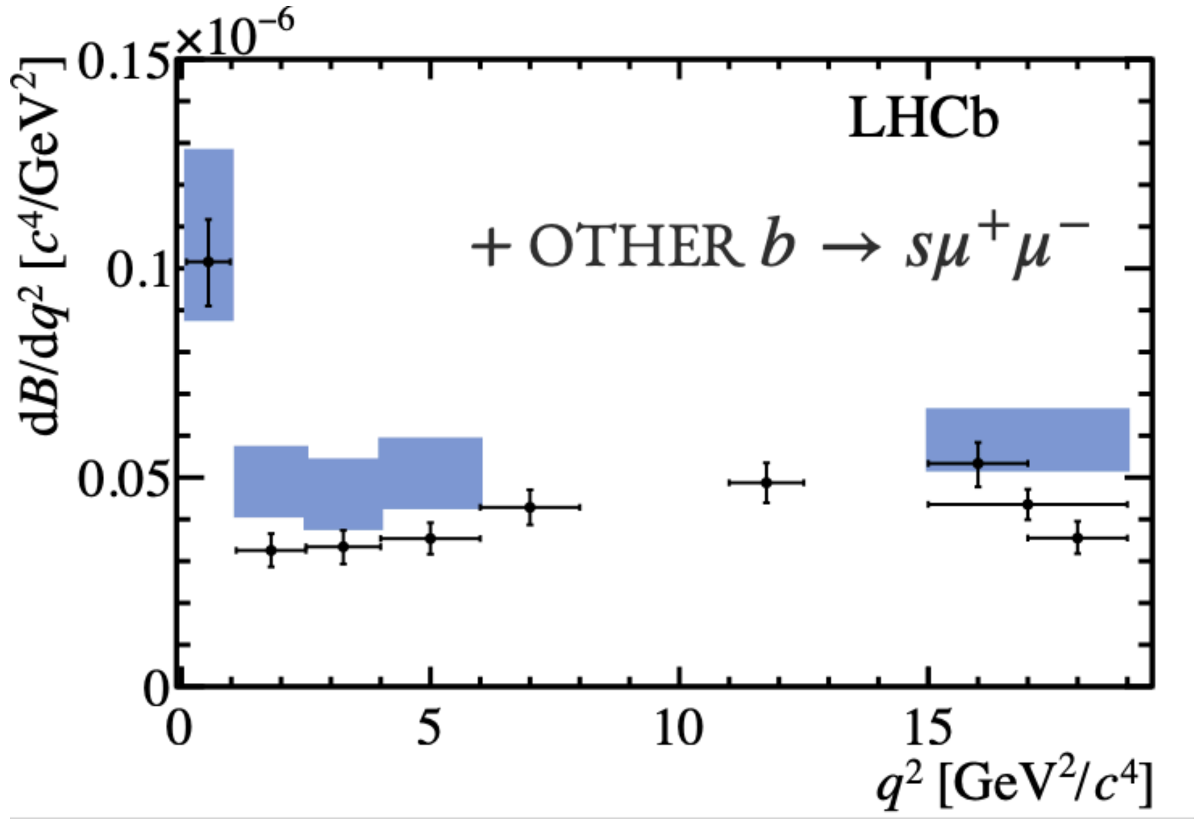
[JHEP 06 (2014) 133]



[CMS: 2401.07090]

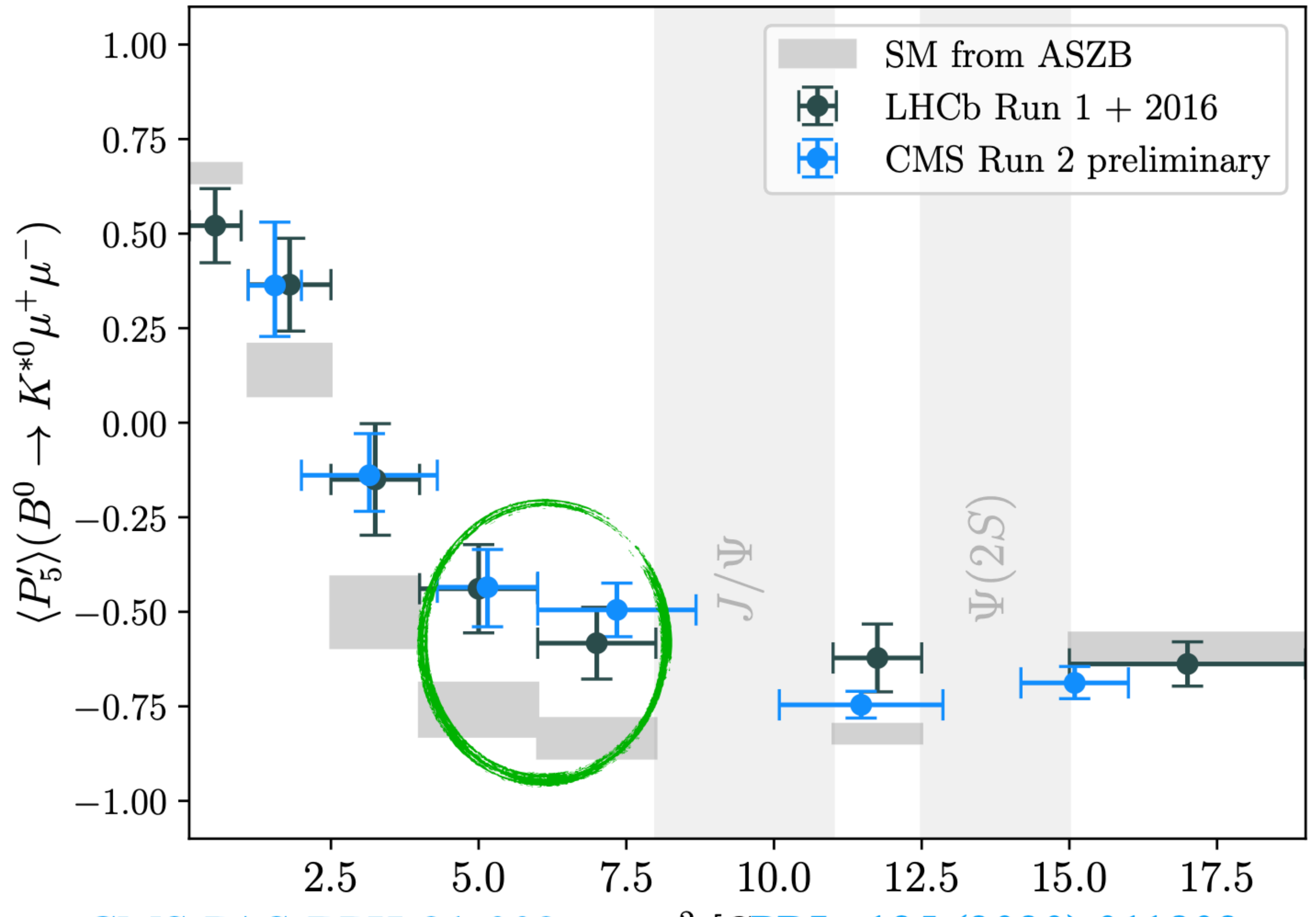
$B \to K^*\bar{\mu}\mu$

[LHCb, JHEP 11 (2016) 047]



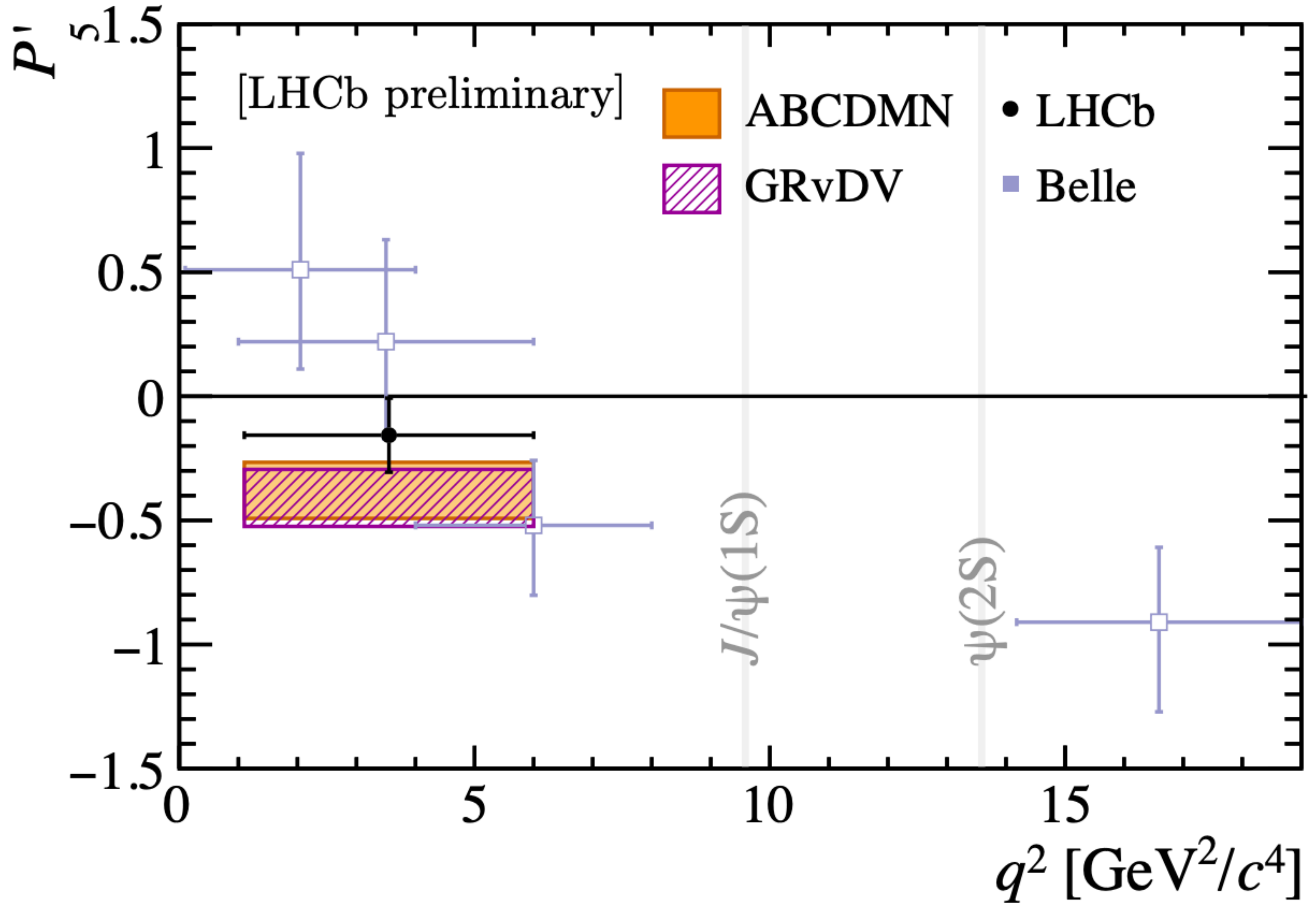
Tension in angular observables

- Long-standing tension in angular observables:



CMS-PAS-BPH-21-002 PRL. 125 (2020) 011802

[Plot by M. Andersson]



[LHCb-PAPER-2024-022, angular analysis of $B \to K^*\bar{e}e$]

- Recent angular analysis by LHCb on $B \to K^*\bar{\mu}\mu$ **[*JHEP* 09 (2024) 026]** **see talk by Andrea & Danny**

Shift in C_9

- The tensions are explainable with a shift in C_9 of around $25\ \%$ wrt the SM value*

From $B \to K^*\bar{\mu}\mu$ by LHCb:

Wilson Coefficient results	
$\mathcal{C}_9$	$3.56 \pm 0.28 \pm 0.18$
$\mathcal{C}_{10}$	$-4.02 \pm 0.18 \pm 0.16$
$\mathcal{C}_9'$	$0.28 \pm 0.41 \pm 0.12$
$\mathcal{C}_{10}'$	$-0.09 \pm 0.21 \pm 0.06$
$\mathcal{C}_{9\tau}$	$(-1.0 \pm 2.6 \pm 1.0) \times 10^2$

[*JHEP* 09 (2024) 026]

$$\Delta\mathcal{C}_9^{\rm NP} = -0.71 \pm 0.33$$

Wilson coefficient	$b \to s\mu\mu$ best fit	$b \to s\mu\mu$ pull	LFU, $B_s \to \mu\mu$ best fit	LFU, $B_s \to \mu\mu$ pull	all rare B decays best fit	all rare B decays pull
$C_9^{bs\mu\mu}$	$-0.77^{+0.21}_{-0.21}$	3.6σ	$-0.21^{+0.17}_{-0.19}$	1.2σ	$-0.42^{+0.13}_{-0.14}$	3.2σ
$C_9'^{bs\mu\mu}$	$+0.29^{+0.25}_{-0.25}$	1.2σ	$-0.22^{+0.17}_{-0.18}$	1.3σ	$-0.04^{+0.13}_{-0.13}$	0.3σ
$C_{10}^{bs\mu\mu}$	$+0.33^{+0.24}_{-0.24}$	1.3σ	$+0.16^{+0.12}_{-0.11}$	1.4σ	$+0.17^{+0.10}_{-0.10}$	1.8σ
$C_{10}'^{bs\mu\mu}$	$-0.05^{+0.16}_{-0.15}$	0.3σ	$+0.04^{+0.11}_{-0.12}$	0.3σ	$+0.02^{+0.09}_{-0.09}$	0.2σ
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$-0.27^{+0.15}_{-0.15}$	1.7σ	$+0.17^{+0.18}_{-0.18}$	1.0σ	$-0.08^{+0.11}_{-0.11}$	0.7σ
$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	$-0.53^{+0.13}_{-0.13}$	3.6σ	$-0.10^{+0.07}_{-0.07}$	1.4σ	$-0.17^{+0.06}_{-0.06}$	2.7σ
$C_9^{bs\ell\ell}$	$-0.77^{+0.21}_{-0.21}$	3.6σ			$-0.78^{+0.21}_{-0.21}$	3.7σ
$C_9'^{bs\ell\ell}$	$+0.29^{+0.25}_{-0.25}$	1.2σ			$+0.30^{+0.25}_{-0.25}$	1.2σ

[*JHEP* 05 (2023) 087, Greljo, Salko, Smolkovic, Stangl]

flavor-universal shifts in C_9
(after R_K, R_{K^*}...)

2D Hyp.	All: Best fit	All: $\mathrm{Pull_{SM}}$	All: p-value
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10\mu}^{\rm NP})$	$(-0.82, -0.17)$	4.4	21.9%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{7'})$	$(-0.68, +0.01)$	4.2	19.4%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9'\mu})$	$(-0.78, +0.21)$	4.3	20.7%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{10'\mu})$	$(-0.76, -0.12)$	4.3	20.5%
$(\mathcal{C}_{9\mu}^{\rm NP}, \mathcal{C}_{9e}^{\rm NP})$	$(-1.17, -0.97)$	5.6	40.3%

**[*Eur.Phys.J.C* 83 (2023) 7, 648
Algueró, Biswas, Capdevila, Descotes-Genon, Matias]**

$$(\mathrm{Re}\, C_9{}^{\rm BSM}, \mathrm{Re}\, C_{10}{}^{\rm BSM}) \simeq (-1.0, +0.4)$$

[Gubernari, Reboud, van Dyk, Virto, 2206.03797]

Other fits: Hurth, Mahmoudi et al (1705.06274), Geng, Grinstein et al (1704.05446), Capdevila, Crivellin et al (1704.05340)

* this assumes we have good theoretical control over the long-distance contributions in the SM

$b \to s\bar{\ell}\ell$ in theory

Theory

- While LFU ratios are theoretically clean, branching fractions and angular observables are **less clean**, being severely affected by hadronic uncertainties.
- It's necessary to look at **complementary observables** (different sensitivity to SD/LD physics and different uncertainties): **inclusive/exclusive level, low/high** q^2
- Having control over hadronic uncertainties is necessary if we want to disentangle possible short-distance physics from long-distance dynamics.

Inclusive rate $B \to X_s \bar{\ell} \ell$ at high q^2

Inclusive $B \to X_s \bar{\ell} \ell$ at high q^2

- The inclusive rate $B \to X_s \bar{\ell} \ell$ is treated with an Operator Product Expansion (OPE) in $1/m_b$

- In the **high-q^2 region**:

  * It is affected by large hadronic uncertainties as it is very sensitive to power corrections in the OPE

  * Breakdown of the OPE → becomes an expansion in $\Lambda_{QCD}/(m_b - \sqrt{q^2})$

- Normalizing $B \to X_s \bar{\ell} \ell$ to $B \to X_u \ell \bar{\nu}$ reduces these uncertainties

[Z. Ligeti and F. J. Tackmann, 0707.1694]

Inclusive $B \to X_s \bar{\ell}\ell$ in the SM

- SM prediction for the **inclusive rate:**

[Z. Ligeti and F. J. Tackmann, 0707.1694]

$$R^{(\ell)}_{\rm incl}(q_0^2) = \frac{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2}\left[\mathcal{R}_L + \Delta\mathcal{R}_{[q_0^2]}\right]$$

$q_0^2 = 15\ \mathrm{GeV}^2$

from Belle, arXiv:2107.13855

[G.Isidori, Z. Polonsky, AT, 2305.03076]

$$\mathcal{R}_L = \frac{\alpha_e^2 C_L^2}{16\pi^2}$$

$$\Delta\mathcal{R}_{[15]} = \frac{\alpha_e^2}{8\pi^2}\Big[C_V^2 + C_V C_L + 0.485 C_L + 0.97 C_V + 0.93 + \Delta_{\rm n.p.} + C_7(1.91 + 2.05 C_L + 4.27 C_7 + 4.1 C_V)\Big]$$

Change of basis: $\{\mathcal{O}_9, \mathcal{O}_{10}\} \to \{\mathcal{O}_V, \mathcal{O}_L\}$

$$\mathcal{O}_V = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) \qquad C_L = -2C_{10}$$

$$\mathcal{O}_L = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_L\gamma^\mu \ell_L) \qquad C_V = C_9 + C_{10}$$

- Significant cancellation of non-perturbative uncertainties since **the hadronic structure is very similar** ($b \to q_{light}$, left-handed current)

Inclusive as sum-over-exclusive

- Agreement in the SM between the inclusive rate and the sum over the leading exclusive modes $B \to K\bar{\ell}\ell, B \to K^*\bar{\ell}\ell, B \to K\pi\bar{\ell}\ell$ (via HHChPT).

$$\sum_i \mathscr{B}(B \to X_s^i \bar{\ell}\ell)^{SM}_{[15]} = (5.07 \pm 0.42) \times 10^{-7}$$

$$\mathscr{B}(B \to X_s \bar{\ell}\ell)^{SM}_{[15]} = (4.10 \pm 0.81) \times 10^{-7}$$

- This compatibility opens up the possibility of comparing the inclusive SM prediction and a sum-over-exclusive experimental result (from LHCb):

$$B \to K\bar{\ell}\ell = (0.85 \pm 0.05) \times 10^{-7}$$

$$B \to K^*\bar{\ell}\ell = (1.58 \pm 0.35) \times 10^{-7}$$

$$B \to K\pi\bar{\ell}\ell = (0.05 \pm 0.09) \times 10^{-7}$$

$$B \to K\pi\pi\bar{\ell}\ell = (0.06 \pm 0.05) \times 10^{-7}$$

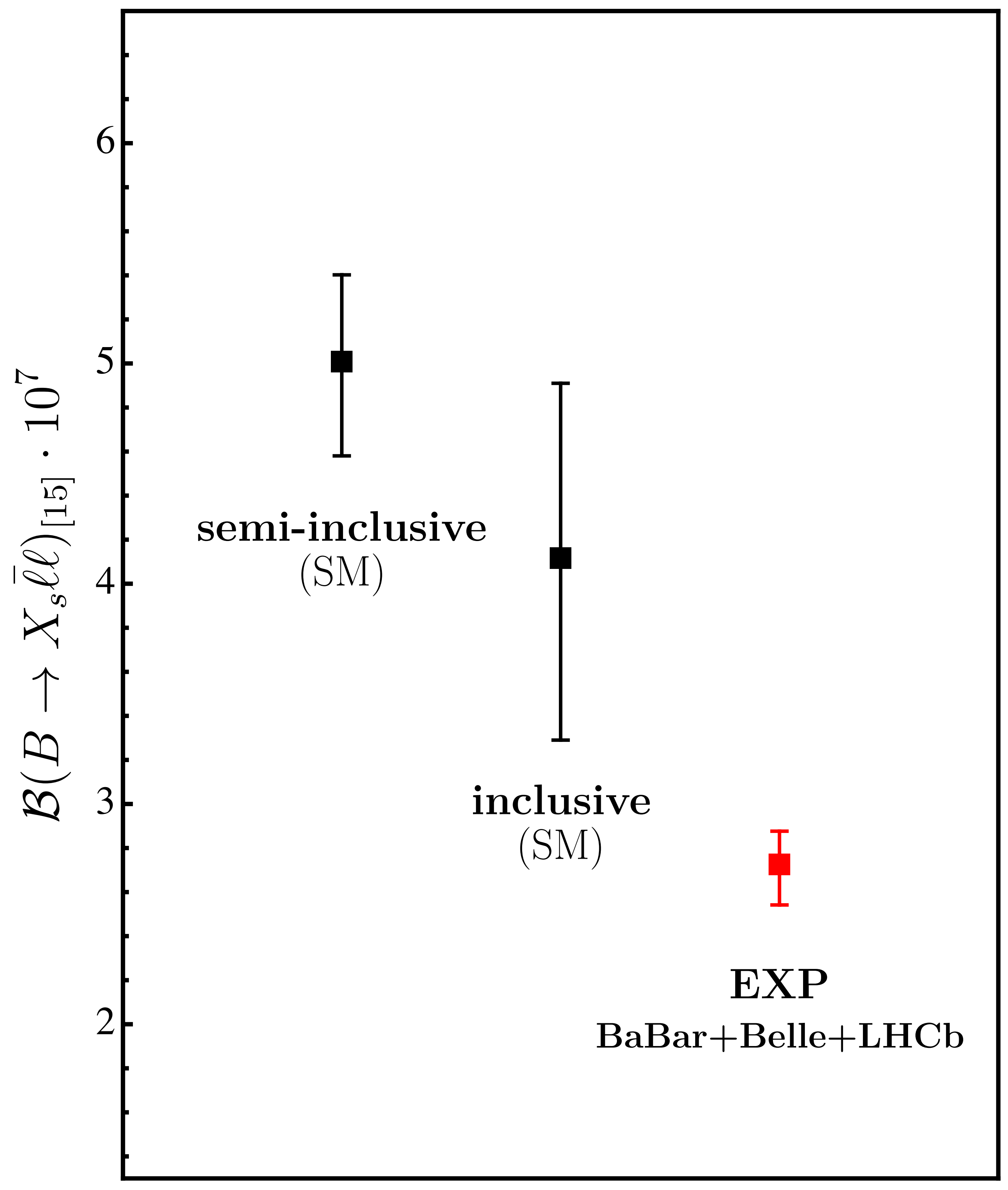
$$B \to K\pi\pi\pi\bar{\ell}\ell = (0.00 \pm 0.04) \times 10^{-7}$$

$m_{K\pi}$ region limited **(LHCb, 1606.04731)**, there will be an update soon

LHCb, 1408.1137

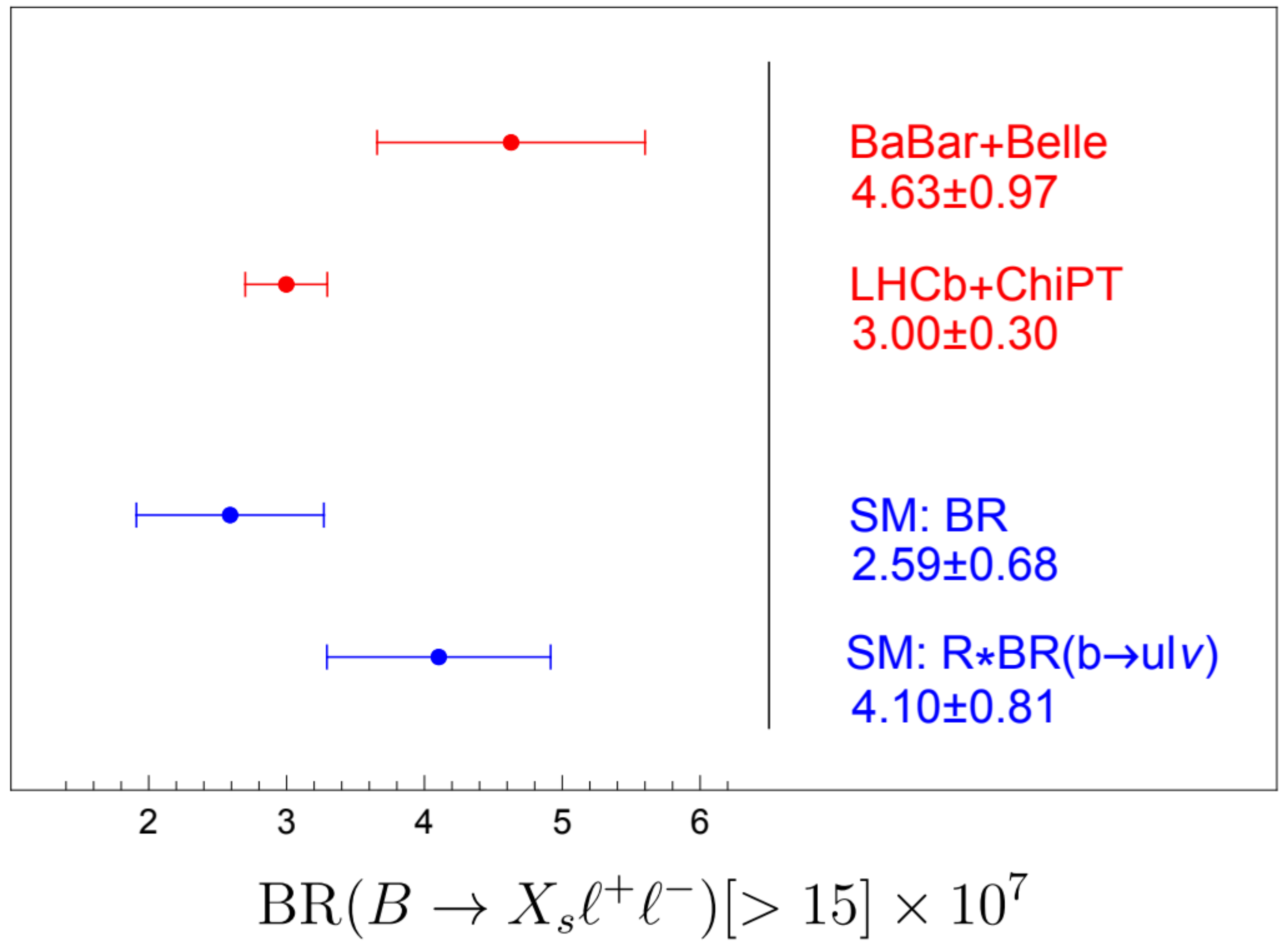
$$\to \boxed{\mathscr{B}(B \to X_s \bar{\ell}\ell)^{exp}_{[15]} = (2.65 \pm 0.17) \times 10^{-7}}$$

Comparison with data



- **Confirmation of sizable suppression** on the $b \to s\bar{\mu}\mu$ rates at low q^2 compared to SM predictions
- Independent verification **not sensitive to uncertainties on the form factors**
- Sizable uncertainty but mainly **experimental** on $B \to X_u \ell \bar{\nu}$
- Modification of C_9 of around $25\ \%$ as well

Quite good agreement with Huber, Hurth, Jenkins, Lunghi, Qin (2404.03517) →



Charm rescattering

Exclusive modes

* Matrix element for exclusive modes:

$$\mathcal{A}(B \to M\ell^+\ell^-) = \frac{G_F \alpha V_{ts}^* V_{tb}}{\sqrt{2}\pi}\left[(C_9\ \ell\gamma^\mu\ell + C_{10}\ \ell\gamma^\mu\gamma_5\ell)\langle M|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle\ - \frac{1}{q^2}\ell\gamma^\mu\ell\ (2im_b C_7\langle M|\bar{s}\sigma_{\mu\nu}q^\nu P_R b|B\rangle + \mathcal{H}_\mu)\right]$$

Local form factors	**Lattice QCD + Light-Cone Sum Rules**	**Bharucha, Straub, Zwicky, 1503.05534** **Gubernari, Reboud, van Dyk, Virto, 2305.06301**

Non-local form factors

matrix elements of the four-quark operators:

$$\mathcal{M}(B \to H_\lambda \ell\ell)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell\int d^4x e^{iqx}\langle H_\lambda|T\{j_\mu^{\text{em}}(x), \sum_{i=1,6} C_i\mathcal{O}_i(0)\}|B\rangle$$

only $\mathcal{O}_1, \mathcal{O}_2$ give a significant contribution

$$\mathcal{O}_1 = (\bar{s}_L^\alpha\gamma_\mu c_L^\beta)(\bar{c}_L^\beta\gamma^\mu b_L^\alpha) \qquad \mathcal{O}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

Matrix elements of four-quark operators

* The non-local form factors contain the matrix elements of the **four-quark operators** $\mathcal{O}_{1-6}$.

* Note that to all orders in α_s, and to first order in α_{em}, **these matrix elements have the same structure as the matrix elements of $\mathcal{O}_7$ and $\mathcal{O}_9$:**

$$\mathcal{M}(B \to H_\lambda \ell\ell)|_{C_{1-6}} = -i\frac{32\pi^2\mathcal{N}}{q^2}\bar{\ell}\gamma^\mu\ell \int d^4x e^{iqx}\langle H_\lambda | T\{j_\mu^{\text{em}}(x), \sum_{i=1,6} C_i\mathcal{O}_i(0)\} | B\rangle = \left(\Delta_9^\lambda(q^2) + \frac{m_B^2}{q^2}\Delta_7^\lambda\right)\langle H_\lambda\, \ell^+\ell^- | \mathcal{O}_9 | B\rangle$$

* The (regular for $q^2 \to 0$) contributions of the non-local matrix elements of the four-quark operators can be effectively taken into account by a **shift in C_9**:

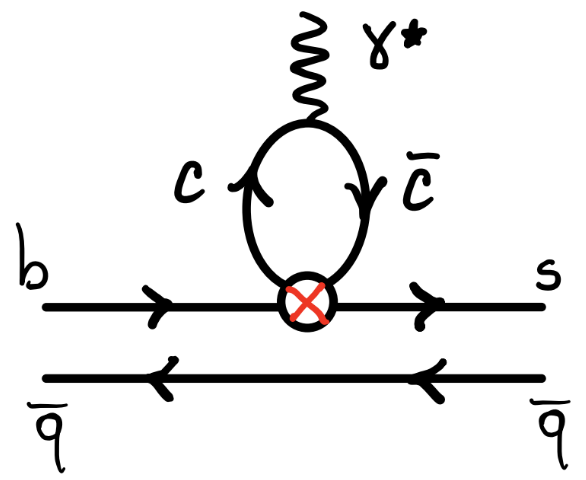
$$C_9 \to C_9^\lambda(q^2) = C_9^{\text{SM}} + \Delta_9^\lambda(q^2) + C_9^{\text{SD}}$$

LD + NP ?

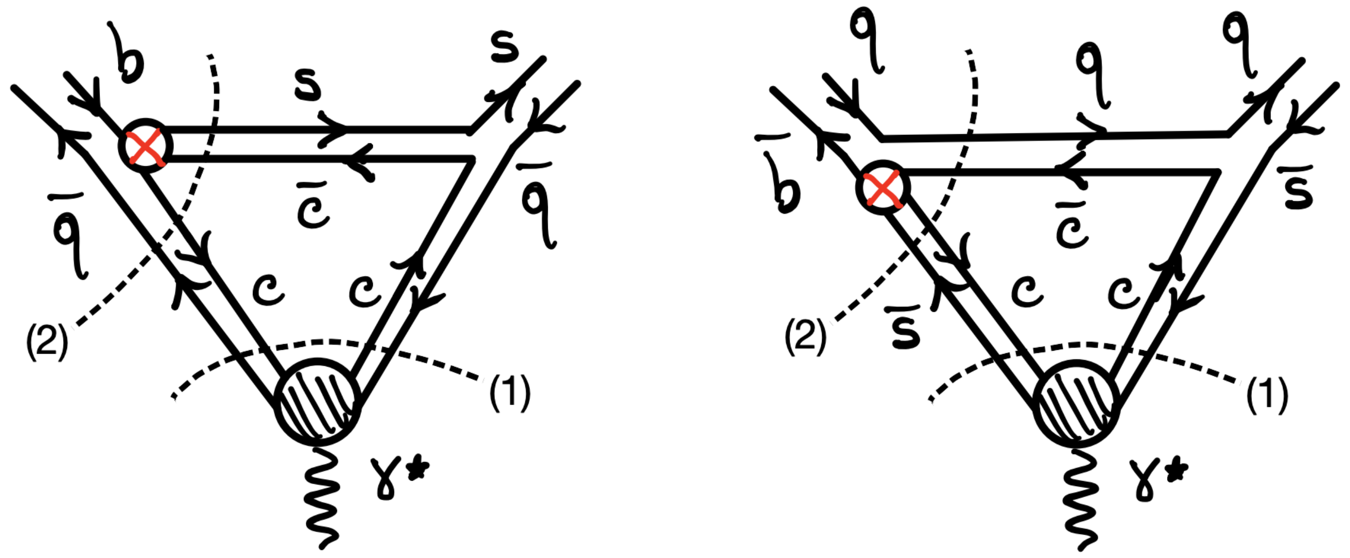
* Therefore, even though the tension with the data could be well described by a shift in C_9 of $\mathcal{O}(25\%)$ with respect to the SM value, **this shift could come from an inaccurate description of the non-local matrix elements.**

Non-local contributions

The correlator in $\int d^4x\, e^{iqx} \langle H_\lambda | T\{j_\mu^{\rm em}(x), \sum_{i=1,6} C_i \mathcal{O}_i(0)\} | B \rangle$ receives two kinds of contributions:



(a)

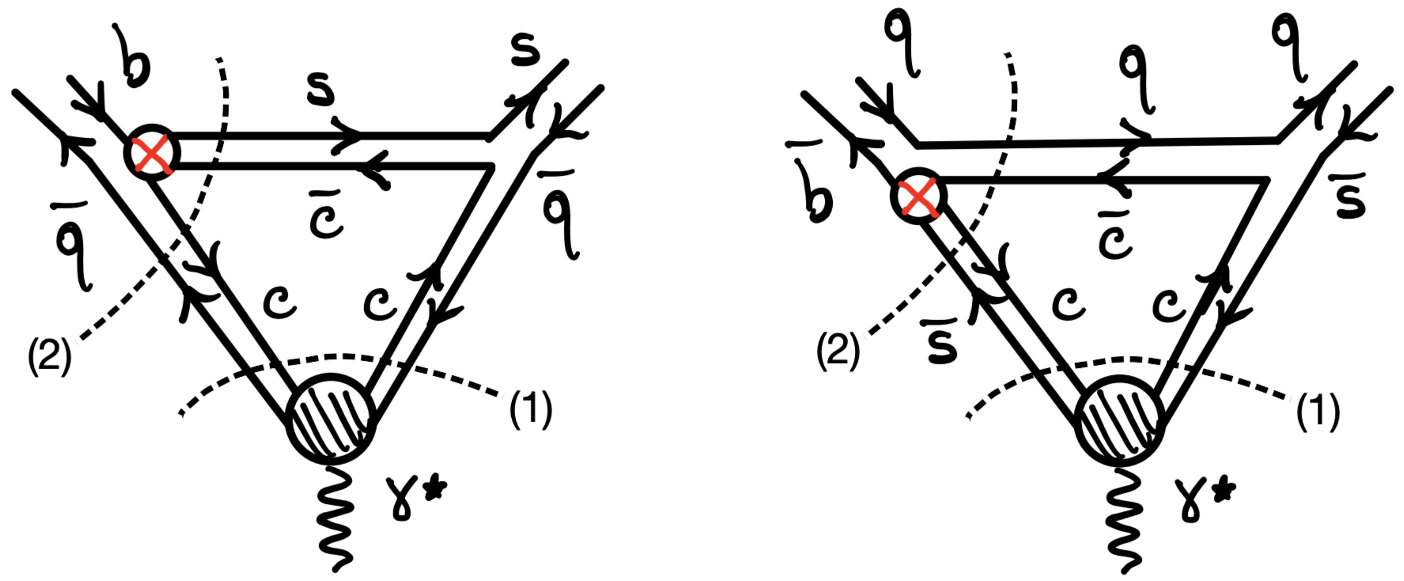


(b)

Pictures from [Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli, 2212.10516]

- Studied with light-cone sum rules for $q^2 \ll 4m_c^2$ + dispersion relations to extend to larger values of q^2 **[Khodjamirian, Mannel, Pivovarov, Wang, 1006.4945]**
- Also using negative q^2 region to further constrain **[Bobeth, Chrzaszcz, van Dyk, Virto, 1707.07305]**
- Unitarity bounds **[Gubernari, van Dyk, Virto, 2011.09813]**
- Small effect in the large-recoil region **[Gubernari, Reboud, van Dyk, Virto, 2206.03797]**

Charm rescattering

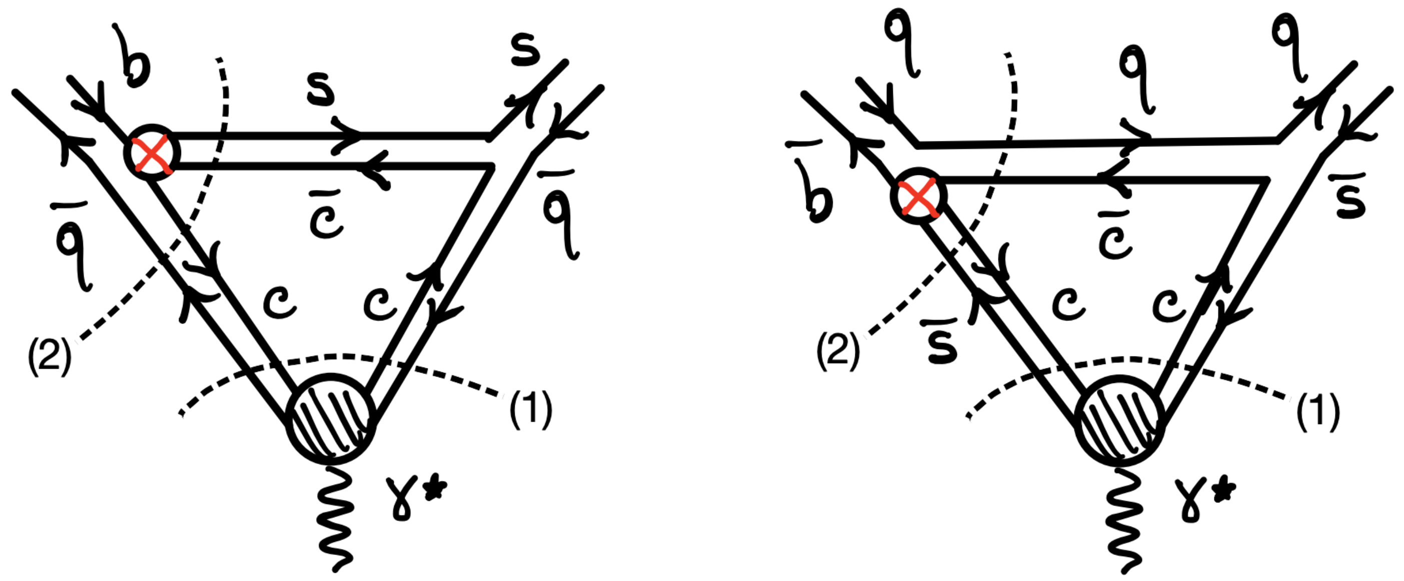


- As pointed out by **Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli (2212.10516)**, applying dispersive methods could be tricky because the analytic structure is quite involved depending on the external momenta and internal masses.

- Parametrization of hadronic contributions rooted on a phenomenological basis -> interplay between NP and hadronic contributions.

$$
\begin{aligned}
H_V^- \propto \frac{m_B^2}{q^2} & \left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L-} - 16\pi^2 h_-^{(2)}\, q^4\right] \\
& + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L-}\,, \\
H_V^+ \propto \frac{m_B^2}{q^2} & \left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L+} - 16\pi^2 \left(h_+^{(0)}\right.\right. \\
& \left.\left. + h_+^{(1)}\, q^2 + h_+^{(2)}\, q^4\right)\right] + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L+}\,, \\
H_V^0 \propto \frac{m_B^2}{q^2} & \left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L0} - 16\pi^2 \sqrt{q^2}\left(h_0^{(0)}\right.\right. \\
& \left.\left. + h_0^{(1)}\, q^2\right)\right] + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L0}\,.
\end{aligned}
$$

Charm rescattering



- As pointed out by **Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli (2212.10516)**, applying dispersive methods could be tricky because the analytic structure is quite involved depending on the external momenta and internal masses.

$$H_V^- \propto \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L-} - 16\pi^2 h_-^{(2)}\, q^4\right] + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L-}\,,$$

$$H_V^+ \propto \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L+} - 16\pi^2\left(h_+^{(0)} + h_+^{(1)}\, q^2 + h_+^{(2)}\, q^4\right)\right] + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L+}\,,$$

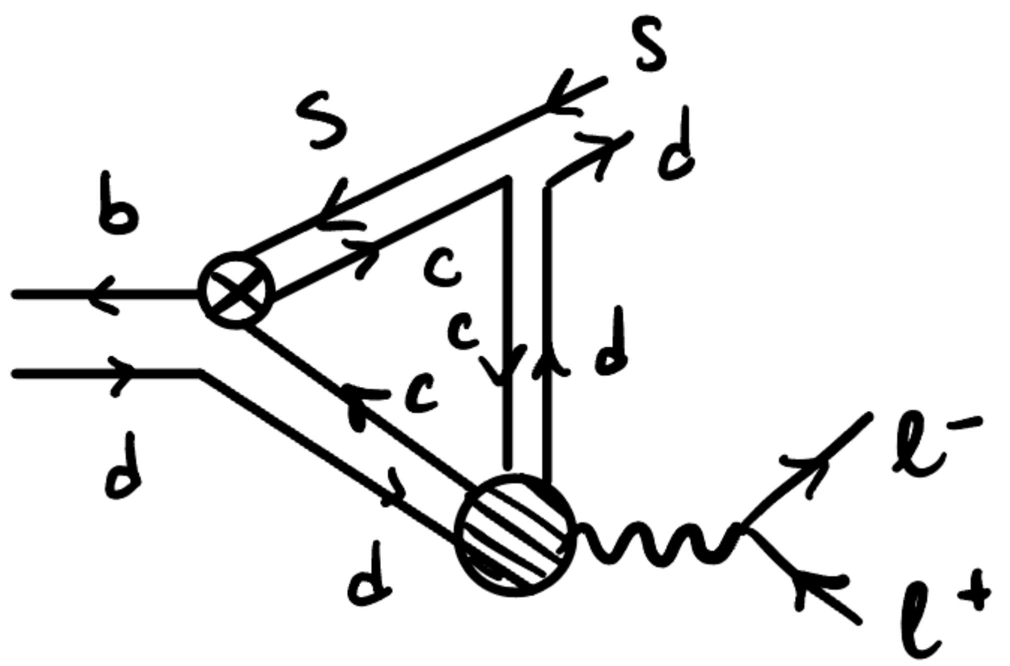
$$H_V^0 \propto \frac{m_B^2}{q^2}\left[\frac{2m_b}{m_B}\left(C_7^{\rm SM}+h_-^{(0)}\right)\widetilde{T}_{L0} - 16\pi^2\sqrt{q^2}\left(h_0^{(0)} + h_0^{(1)}\, q^2\right)\right] + \left(C_9^{\rm SM}+h_-^{(1)}\right)\widetilde{V}_{L0}\,.$$

- Parametrization of hadronic contributions rooted on a phenomenological basis -> interplay between NP and hadronic contributions.
- Analytical structure: an additional singularity in the case of an **anomalous threshold** could move into the q^2 integration domain, requiring a non trivial deformation of the path.
- **Mutke, Hoferichter, Kubis *JHEP* 07 (2024) 276**: classification of anomalous thresholds in all possible mass configurations for light-quark loops -> contribution as large as 10% of the non-local form factors.

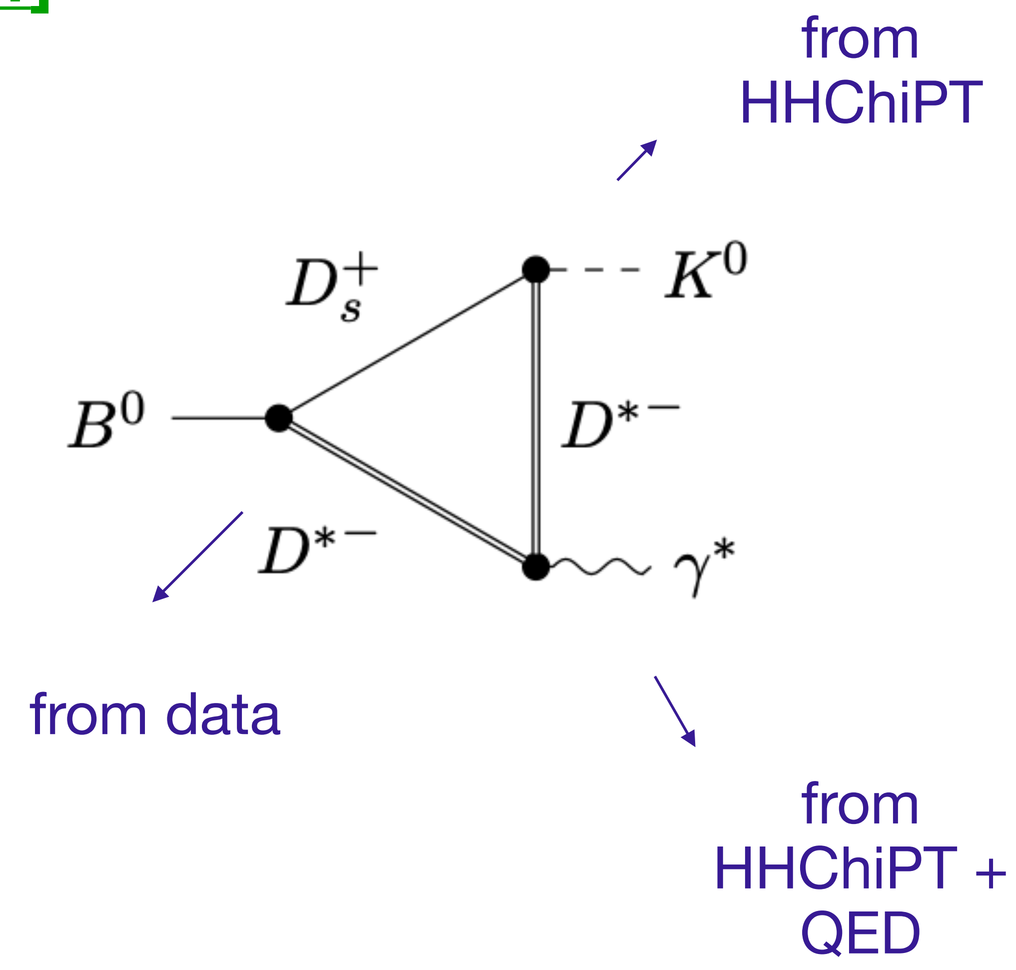
- For charm loop: it seems to be the moderate case yielding smaller corrections.

Charm rescattering

- We give an estimate of long-distance effects associated with the rescattering of a charmed and a charmed-strange mesons.
- We look at the simplest rescattering contribution from the leading two-body intermediate state $D_s D^*$ and $D_s^* D$.



[G.Isidori, Z. Polonsky, AT, 2405.17551]



- We estimate this diagram using an effective description in terms of hadronic degrees of freedom, using **data** on $B \to DD^*$ and **Heavy Hadron Chiral Perturbation Theory** for the $DD_s^*(D_s D^*)K$ vertex.
- We obtain an accurate description in the low recoil (or **high** q^2) limit; we extrapolate to the whole kinematical region introducing appropriate form factors.

Model

- **Dynamics of $D^{(*)}_{(s)}$ mesons close to their mass shell, determined by:**
  - Lorentz invariance
  - Gauge invariance under QED
  - $SU(3)$ light-flavor symmetry
  - Heavy-quark spin symmetry

$$\mathcal{L}_{D,\text{free}} = -\frac{1}{2}\left(\Phi^{\mu\nu}_{D^*}\right)^\dagger \Phi_{D^*\,\mu\nu} - \frac{1}{2}\left(\Phi^{\mu\nu}_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu\nu} + \left(D_\mu \Phi_D\right)^\dagger D^\mu \Phi_D + \left(D_\mu \Phi_{D_s}\right)^\dagger D^\mu \Phi_{D_s} + m_D^2\left[\left(\Phi^\mu_{D^*}\right)^\dagger \Phi_{D^*\,\mu} + \left(\Phi^\mu_{D_s^*}\right)^\dagger \Phi_{D_s^*\,\mu}\right] - m_D^2\left[\Phi^\dagger_D \Phi_D + \Phi^\dagger_{D_s}\Phi_{D_s}\right] + \text{h.c.}\,.$$

- **Weak $B \to DD^*$ transition described by (using heavy-quark spin symmetry + data)**

$$\mathcal{L}_{BD} = g_{DD^*}\left(\Phi^{\mu\dagger}_{D_s^*}\,\Phi_D \partial_\mu \Phi_B + \Phi^\dagger_{D_s}\,\Phi^\mu_{D^*}\partial_\mu \Phi_B\right) + \text{h.c.}$$

$$g_{DD^*} = \sqrt{2} G_F\,|V^*_{tb} V_{ts}| m_B m_D\,\bar{g} \qquad \bar{g} \approx 0.04$$

In principle g_{DD^*} could have a phase

- **From HHChiPT** (valid close to endpoint $q^2 \approx m_B^2$):

$$\mathcal{L}_{DK} = \frac{2 i g_\pi m_D}{f_K}\left(\Phi^{\mu\dagger}_{D^*}\Phi_{D_s}\partial_\mu \Phi^\dagger_K - \Phi^\dagger_D \Phi^\mu_{D_s^*}\partial_\mu \Phi^\dagger_K\right) + \text{h.c.}$$

Form factors

In order to obtain a reliable estimate **over the entire kinematical range,** we introduce the following form factors:

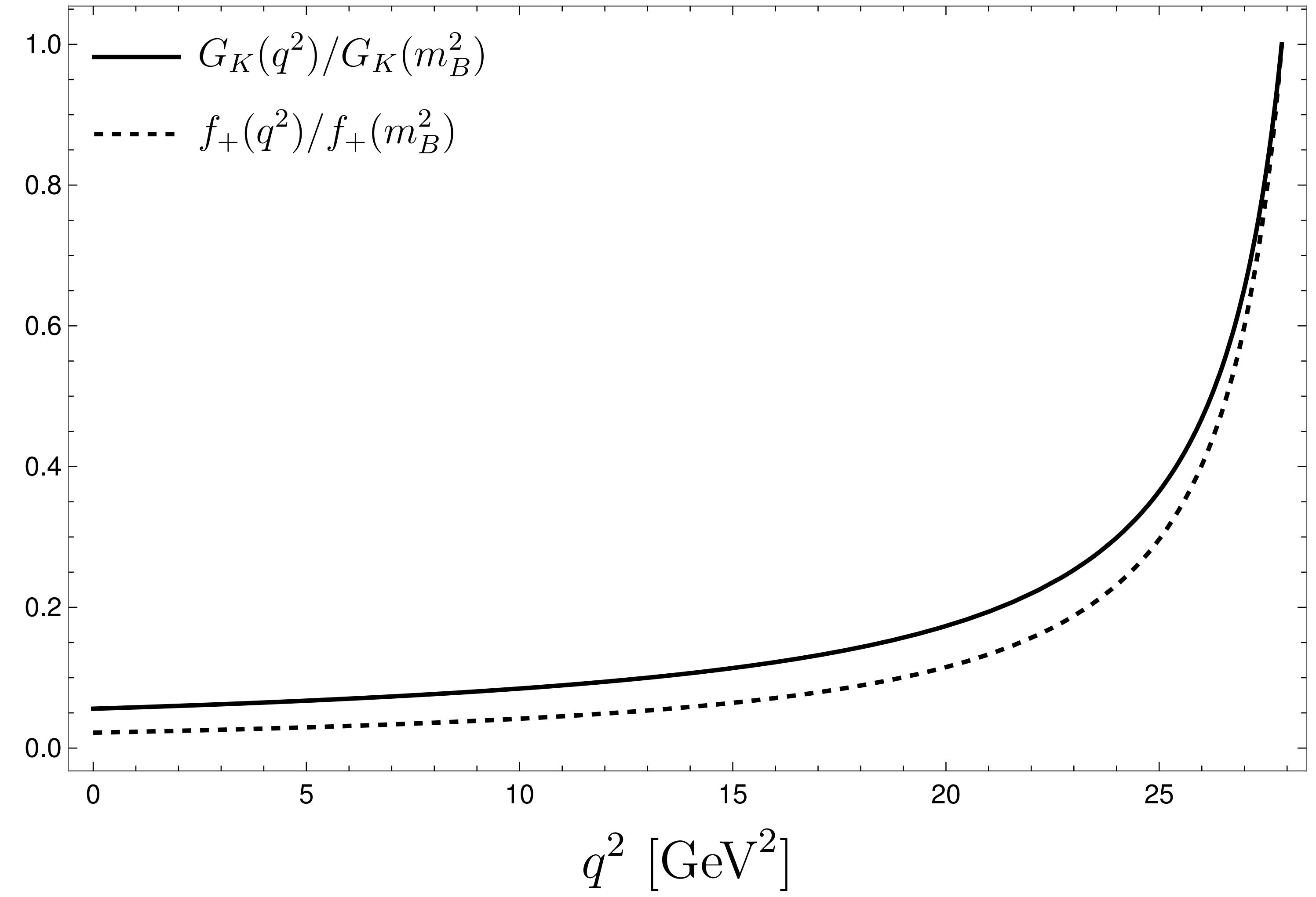
* Correction for **QED vertex** (using Vector Meson Dominance):

$$e \to eF_V(q^2)\,, \qquad F_V(q^2) = \frac{m_{J/\psi}^2}{m_{J/\psi}^2 - q^2}$$

* Correction for DD^*K **vertex**:

$$\frac{1}{f_K} \to \frac{1}{f_K} G_K(q^2)\,,$$

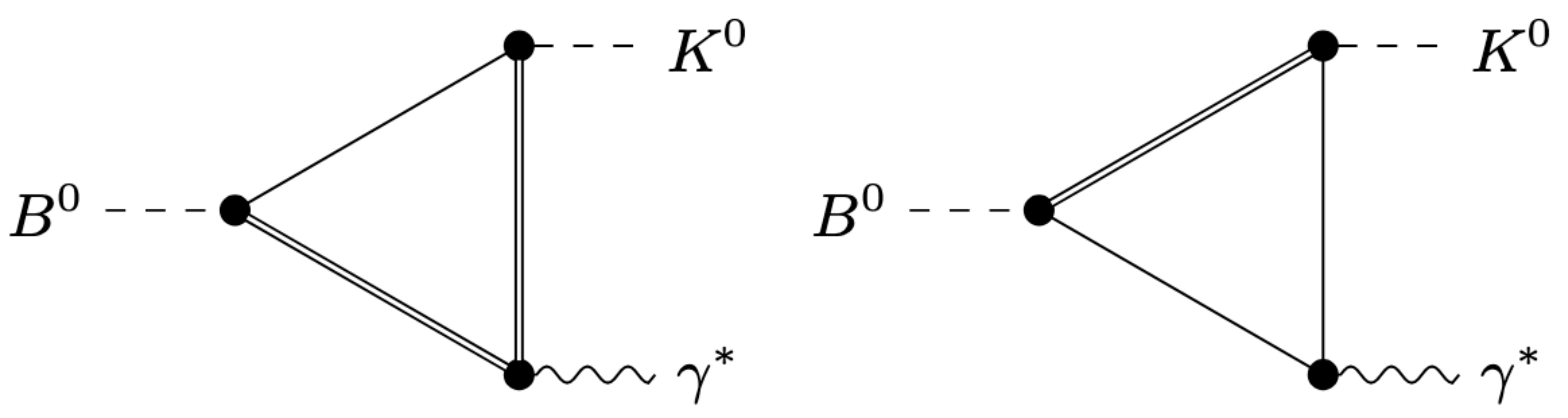
$$G_K(q^2) = \frac{1}{1 + E_K(q^2)/f_K} = \frac{2m_B f_K}{2m_B f_K + m_B^2 - q^2}$$



Useful consistency check: G_K has a similar scaling to the vector form factor $f_+(q^2)$ for $B_0 \to K_0$

Results

- We compute the **one-loop diagrams** appearing in the model presented.
- In the $SU(3)$-symmetric limit, the diagrams obtained by swapping $D_s^{(*)} \leftrightarrow D^{(*)}$ are symmetric.



$$L_\mu = \log(\mu^2/m_D^2)$$

$$\delta L(q^2, m_B^2, m_D^2) = \frac{L(m_B^2, m_D^2) - L(q^2, m_D^2)}{q^2 - m_B^2},$$

$$L(x,y) = \log\left(\frac{2y - x + \sqrt{x(x-4y)}}{2y}\right) \times \left[\sqrt{x(x-4y)} + y \log\left(\frac{2y - x + \sqrt{x(x-4y)}}{2y}\right)\right]$$

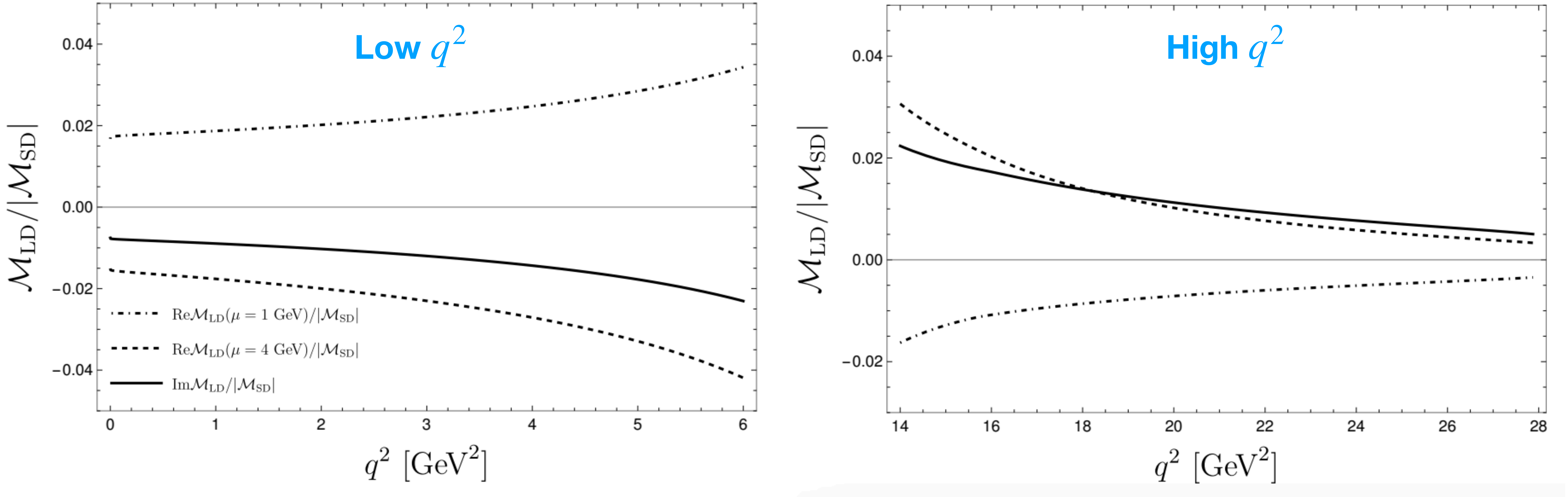
$$\mathcal{M}_{\mathrm{LD}} = -\frac{e g_{DD^*} g_\pi F_V(q^2) G_K(q^2)}{8\pi^2 f_K m_D}(p_B \cdot j_{\mathrm{em}}) \times \left[(2 + L_\mu) - \delta L(q^2, m_B^2, m_D^2)\right],$$

- Compare it to the **short-distance matrix element**:

$$\mathcal{M}_{\mathrm{SD}} = \frac{4G_F}{\sqrt{2}} \frac{e}{16\pi^2} V_{tb}^* V_{ts} (p_B \cdot j_{\mathrm{em}}) f_+(q^2)(2C_9)$$

Results

- Ratios of long-distance vs short-distance matrix elements:



- LD contributions don't exceed a few percent relative to the SD one.
- The absorptive part is finite and corresponds to the discontinuity of the amplitude corresponding to the kinematical regions where the internal mesons go on-shell.

Effective shift in C_9

- We can encode the effect of the $\mathscr{M}_{LD}$ via a q^2–**dependent shift in C_9:**

$$\delta C^{\rm LD}_{9,DD^*}(q^2,\mu) = \bar{g}\,\Delta(q^2)\Big[2 + L_\mu - \delta L(q^2, m_B^2, m_D^2)\Big] \qquad \Delta(q^2) = -\frac{g_\pi m_B F_V(q^2) G_K(q^2)}{2 f_K f_+(q^2)}$$

- Averaging over the low- and high-q^2 regions, we find:

$$\delta\bar{C}^{\rm LD,low}_{9,DD^*}(\mu) = -0.003 - 0.059\,i - 0.156\log\left(\frac{\mu}{m_D}\right)$$

$$\delta\bar{C}^{\rm LD,high}_{9,DD^*}(\mu) = 0.009 + 0.053\,i + 0.063\log\left(\frac{\mu}{m_D}\right).$$

- Varying the renormalization scale μ in the range $[1{,}4]$ GeV:

$$|\delta\bar{C}^{\rm LD}_{9,DD^*}| \leq 0.11 \quad \longrightarrow \quad \boxed{\frac{\delta C_9}{C_9^{SM}} \approx 2.5\,\%}$$

Additional intermediate states

- So far we focused on the D^*D_s or D_s^*D intermediate states, but in principle there are **other states** with $\bar{c}c\bar{s}d$ valence structure.
- Consider all intermediate states the allow parity-conserving strong interactions with the kaon:

- Conservative **multiplicity factor** accounting for all possible intermediate states:

$$\mathcal{N} = \frac{\sum_X \mathcal{M}(B^0 \to X)}{\mathcal{M}(B^0 \to D^*D_s) + \mathcal{M}(B^0 \to DD_s^*)} = \approx \frac{1}{2}\sum_X \sqrt{\frac{\mathcal{B}(B^0 \to X)}{\mathcal{B}(B^0 \to DD_s^*)}} \approx 3$$

B^0 Decay	$\mathcal{B}(B^0 \to X) \times 10^3$
D^*D_s	8.0 ± 1.1
DD_s^*	7.4 ± 1.6
$D^*D_s^*$	17.7 ± 1.4
$DD_{s0}(2317)$	1.06 ± 1.6
$D^*D_{s1}(2457)$	9.3 ± 2.2
$D^*D_{s1}(2536)$	0.50 ± 0.14
$DD_{s2}(2573)$	$(3.4 \pm 1.8) \times 10^{-2}$
$D^*D_{s2}(2573)$	< 0.2
$DD_{s1}(2700)$	0.71 ± 0.12

$$\longrightarrow \quad |\delta C_9^{\rm LD}| \leq \mathcal{N}|\delta \bar{C}_{9,DD^*}^{\rm LD}| \leq 0.33 \quad \longrightarrow \quad \boxed{\frac{\delta C_9}{C_9^{SM}} \approx 8 - 10\,\%}$$

Fit of C_9 from exclusive modes

Sign of δC_9

- The sign of δC_9 is **opposite** in the two cases (regardless of the phase of g_{DD^*}): comparing the extraction of C_9 at low- and high-q^2 provides a useful data-driven check for such long-distance contributions.

Sign of δC_9

- The sign of δC_9 is **opposite** in the two cases (regardless of the phase of g_{DD^*}): comparing the extraction of C_9 at low- and high-q^2 provides a useful data-driven check for such long-distance contributions.

- We perform a fit of C_9 from the branching ratio and angular observables in $B \to K^* \bar{\mu}\mu$, assuming:

2014 LHCb, 2023 CMS

2016 and 2020 LHCb

$$C_9 \to C_9^\lambda(q^2) + Y_{q\bar{q}}^{[0]}(q^2) + Y_{b\bar{b}}^{[0]}(q^2) + Y_{c\bar{c}}^\lambda(q^2)$$

$Y_{q\bar{q}}^{[0]}$, $Y_{b\bar{b}}^{[0]}$: encodes (factorizable) perturbative contributions from 4-quark operators

$Y_{c\bar{c}}^\lambda$: encodes the perturbative charm-loop contributions and $c\bar{c}$ resonances

To estimate the non-perturbative contributions generated by the $c\bar{c}$ resonances, we use dispersive relations in combination with data:

$$Y_{c\bar{c}}^\lambda(q^2) = Y_{c\bar{c}}^\lambda(q_0^2) + \frac{16\pi^2}{\mathcal{F}_\lambda(q^2)} \Delta\mathcal{H}_{c\bar{c}}^\lambda(q^2), \; q_0^2 = 0$$

$$\Delta\mathcal{H}_{c\bar{c}}^{\lambda,1P} = \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{m_V^2} A_V^{\text{res}}(q^2) \qquad A_V^{\text{res}}(q^2) = \frac{m_V \Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}$$

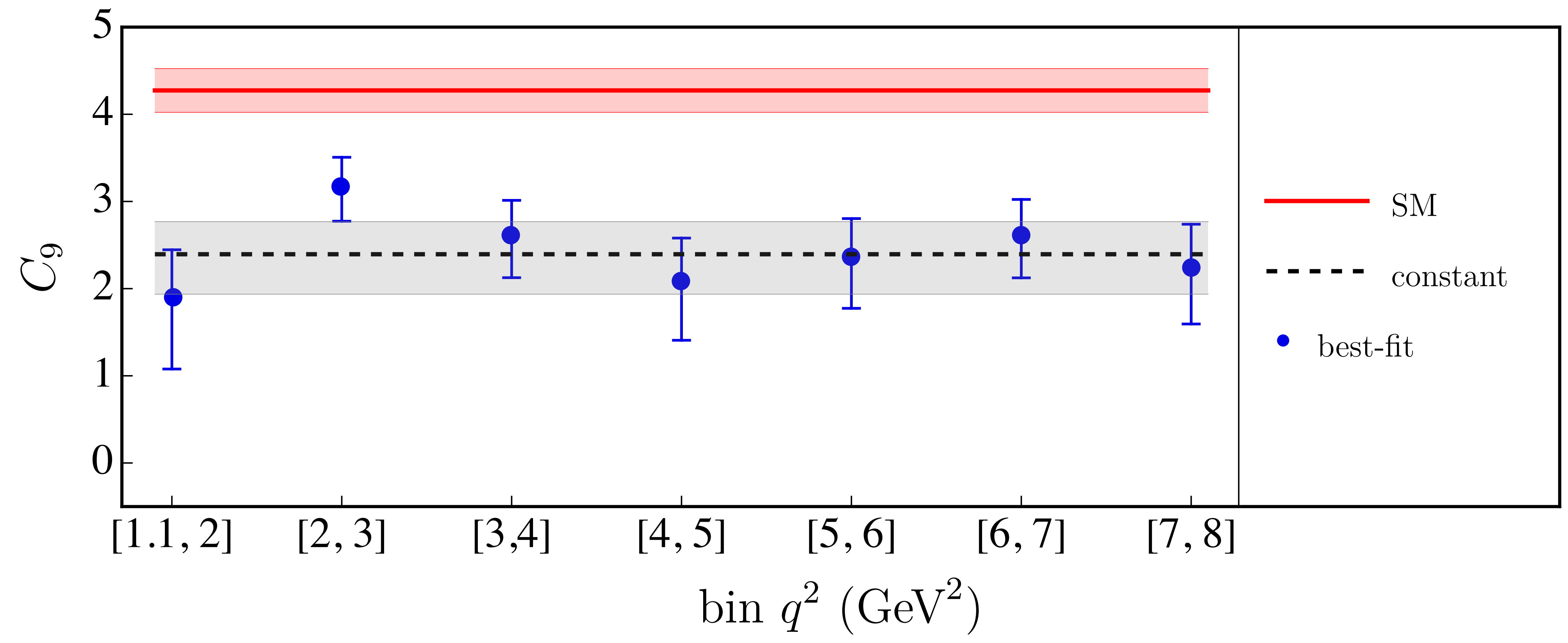
- We extract the residual contribution to C_9 from data:

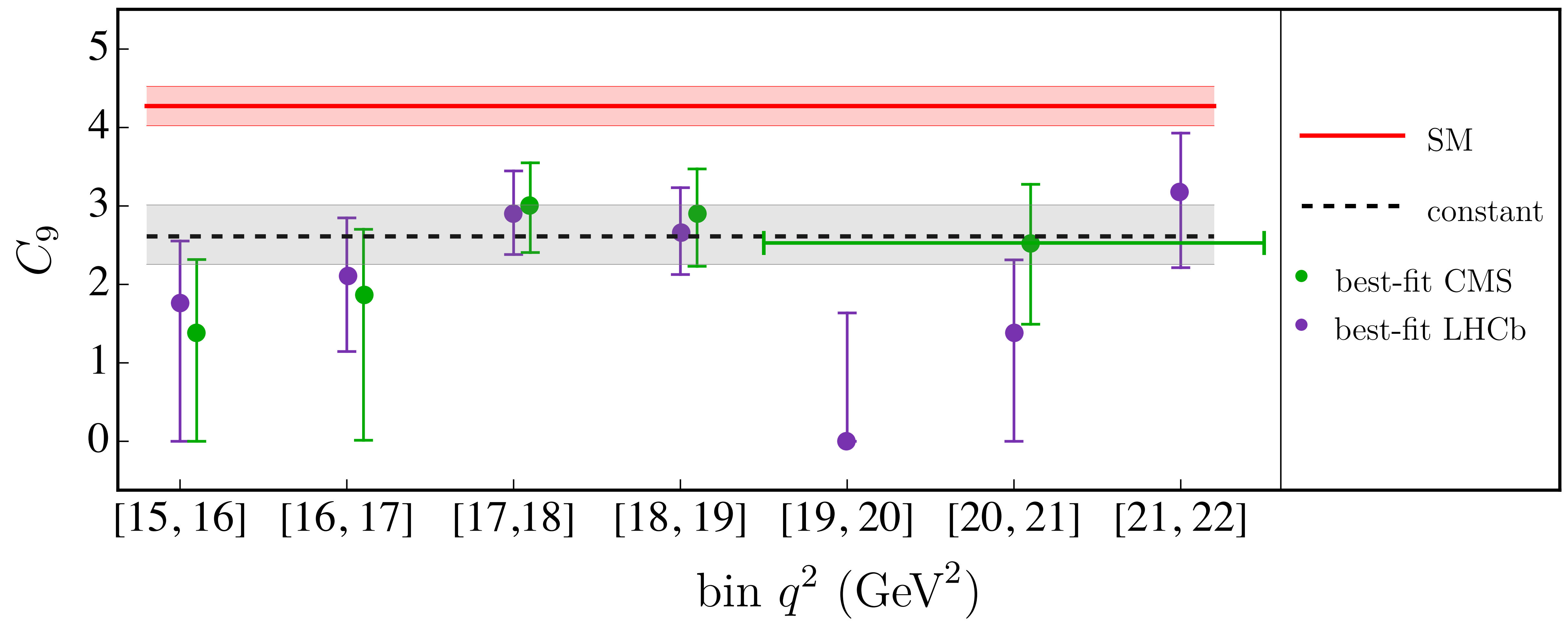
$$C_9^\lambda(q^2) = C_9^{\text{SM}} + C_9^{\text{LD},\lambda}(q^2) + C_9^{\text{SD}}$$

C_9^{SD}: Short-distance, independent of λ and q^2

$C_9^{\text{LD},\lambda}(q^2)$: Long-distance, no reason to assume it is independent of λ or q^2

Results for $B \to K\bar{\mu}\mu$





q^2 (GeV^2)	C_9^K
[1.1, 2]	$1.9^{+0.5}_{-0.8}$
[2, 3]	$3.2^{+0.3}_{-0.4}$
[3, 4]	$2.6^{+0.4}_{-0.5}$
[4, 5]	$2.1^{+0.5}_{-0.7}$
[5, 6]	$2.4^{+0.4}_{-0.6}$
[6, 7]	$2.6^{+0.4}_{-0.5}$
[7, 8]	$2.3^{+0.5}_{-0.7}$
constant	$2.4^{+0.4}_{-0.5}$ ($\chi^2/\text{dof} = 1.35$)

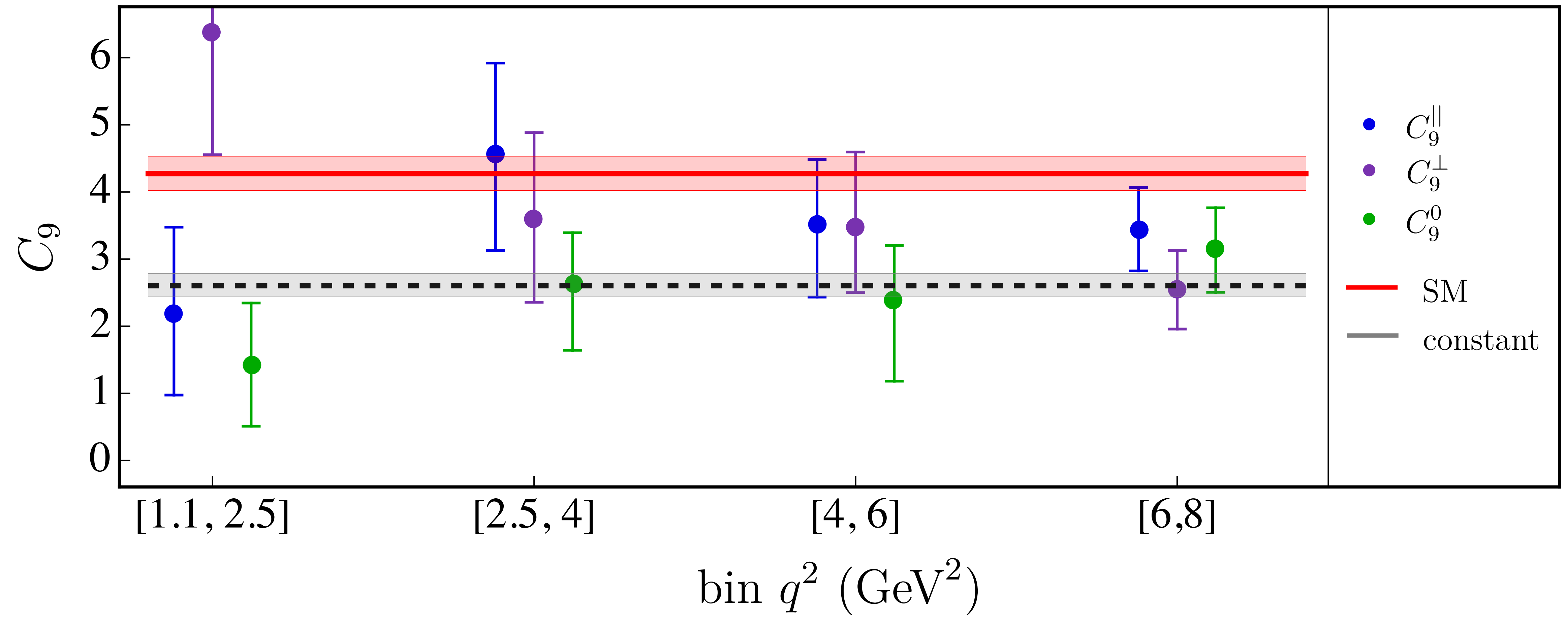
q^2 (GeV^2)	C_9^K (LHCb)	C_9^K (CMS)
[15, 16]	$1.8^{+0.8}_{-1.8}$	$1.4^{+0.9}_{-1.4}$
[16, 17]	$2.1^{+0.7}_{-1.0}$	$1.9^{+0.8}_{-1.9}$
[17, 18]	$2.9^{+0.5}_{-0.5}$	$3.0^{+0.5}_{-0.6}$
[18, 19]	$2.7^{+0.6}_{-0.5}$	
[18, 19.24]		$2.9^{+0.6}_{-0.7}$
[19, 20]	$0^{+1.6}_{-0}$	
[20, 21]	$1.4^{+0.9}_{-1.4}$	
[21, 22]	$3.2^{+0.8}_{-0.9}$	
[19.24, 22.9]		$2.5^{+0.7}_{-1.0}$
constant	2.6 ± 0.4 ($\chi^2/\text{dof} = 1.06$)	

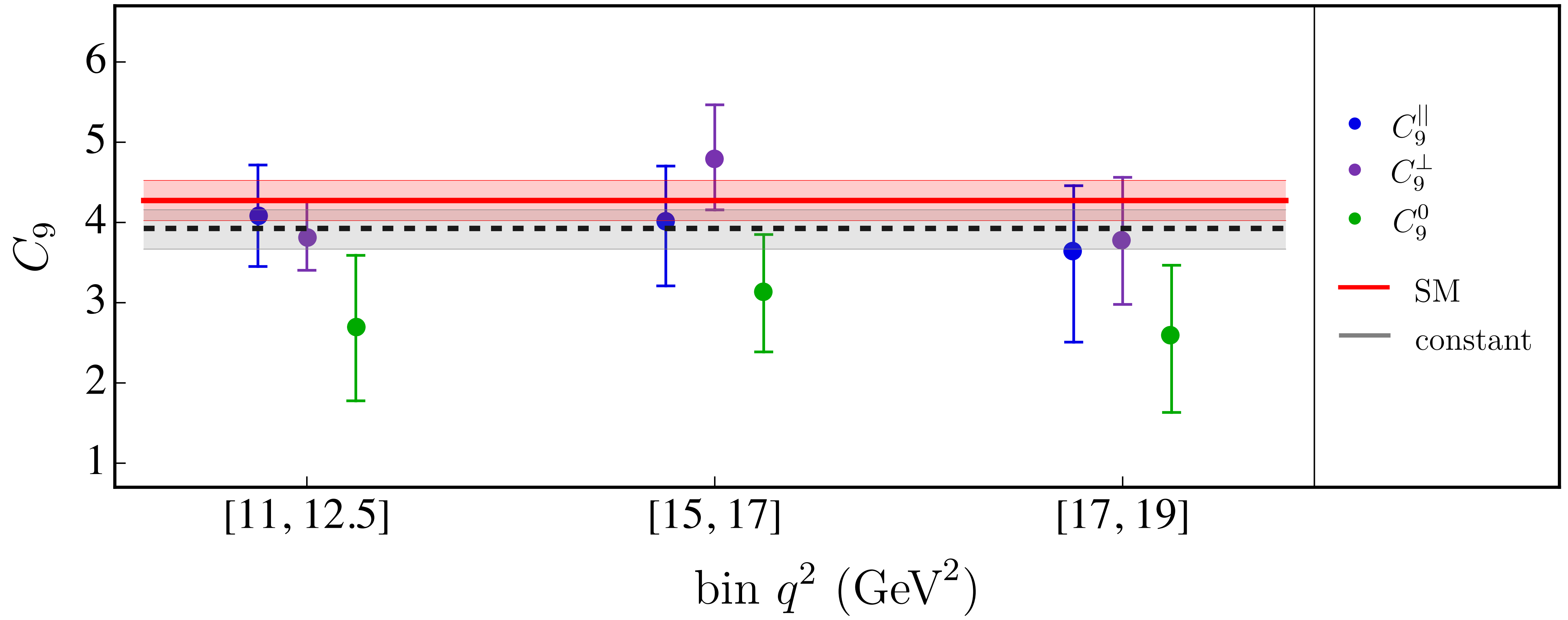
Table 3.3: Determinations of C_9 from $B \to K\mu^+\mu^-$ in the low-q^2 (left) and high-q^2 (right) regions. The p-values for the constant fits are 0.17 (low-q^2) and 0.39 (high-q^2).

[M. Bordone, G.Isidori, S. Mächler, AT, 2401.18007]

Results for $B \to K^* \bar{\mu}\mu$

Using resonance parameters found by LHCb recently (2405.17347)





	constant C_9	$C_9^{\parallel}$	$C_9^{\perp}$	C_9^{0}
Low q^2	$2.60^{+0.18}_{-0.17}$	$2.4^{+0.6}_{-0.6}$	$2.6^{+0.7}_{-0.6}$	$2.8^{+0.7}_{-0.8}$
High q^2	$3.93^{+0.23}_{-0.26}$	$4.0^{+0.5}_{-0.5}$	$4.0^{+0.4}_{-0.4}$	$2.9^{+0.6}_{-0.6}$

$$C_9 = 3.40^{+0.16}_{-0.16} \quad (\chi^2/dof = 1.5)$$

The shift in C_9 we find from charm rescattering + NP shift of ~-1 gives a better global fit than a shift of ~-1

Importance of extracting the value of C_9 at different values of q^2

Summary

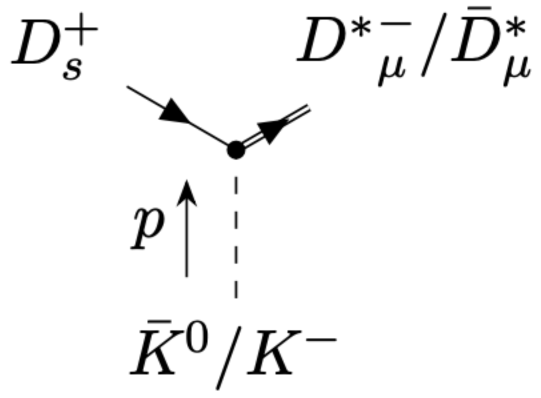
* Non-local contributions in $b \to s\bar{\ell}\ell$ could significantly impact the extraction of C_9.
* We have presented an estimate of $B^0 \to K^0\bar{\ell}\ell$ long-distance contributions induced by the rescattering of a charmed and a charmed-strange meson;
* For the particular intermediate state we considered, charm rescattering contributions don't seem to be very large.
  * The multiplicity factor needs to be better understood;
  * We neglected some effects ($SU(3)$ breaking effects, higher-mass charmonium resonances, higher-multipole photon couplings).
* Going forward:
  * Experimental level: measure $D-$meson form factors (to follow Mutke, Hoferichter, Kubis's approach), $B \to K^{(*)}D\bar{D}$, differential information to disentangle phases and relative importance of decay mechanisms, extraction of C_9 at different values of q^2
  * Theoretical level: extension of known methods, combinations and comparisons, something else? Lattice?

Thank you for your attention!

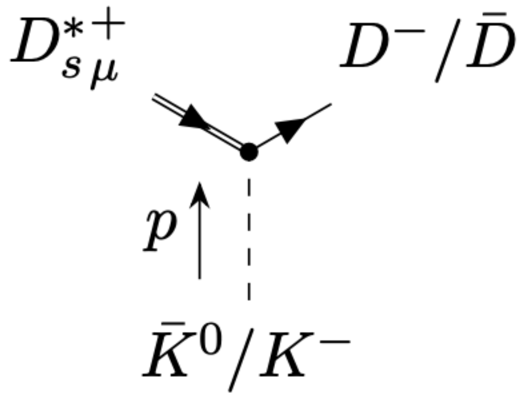
Backup Slides

Model

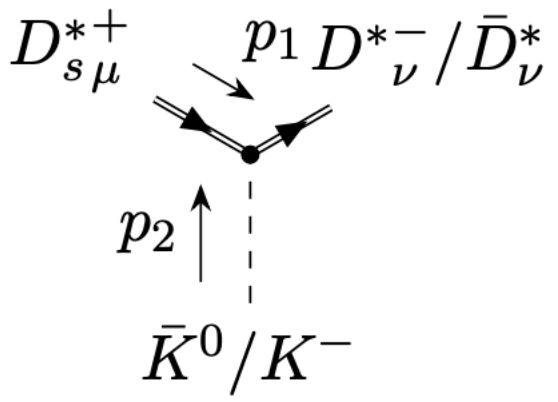
KDD^*



$$= \frac{2ig_\pi m_{D^* D_s}}{f} p_\mu$$

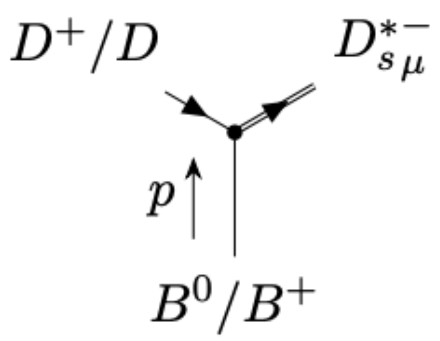


$$= -\frac{2ig_\pi m_{DD_s^*}}{f} p_\mu$$

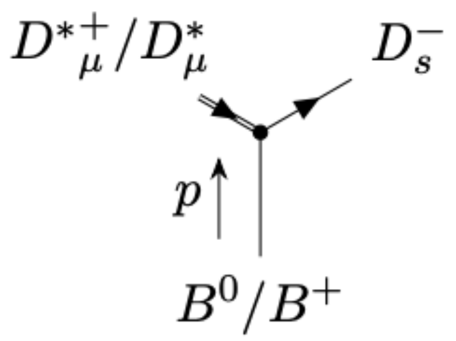


$$= -\frac{2ig_\pi}{f}\sqrt{\frac{m_{D^*}}{m_{D_s^*}}} p_1^\alpha p_2^\beta \epsilon_{\alpha\beta\mu\nu}$$

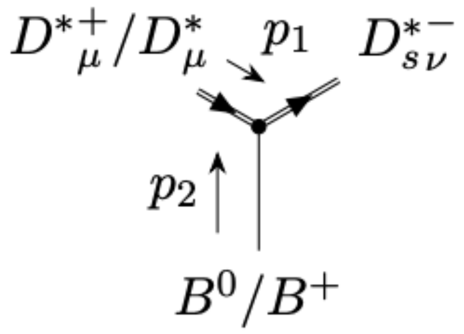
BDD^*



$$= e^{i\varphi_*} g_{DD^*} p_\mu$$

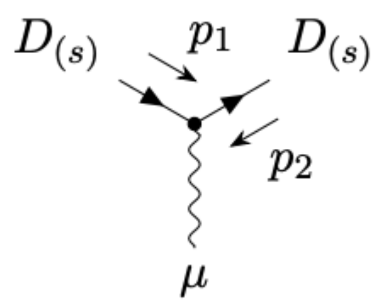


$$= e^{-i\varphi_*} g_{DD^*} p_\mu$$

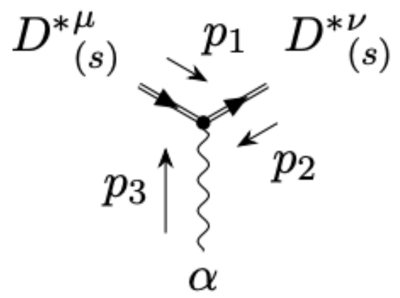


$$= -e^{i\varphi_{**}} g_{D^*D^*} p_1^\alpha p_2^\beta \epsilon_{\alpha\beta\mu\nu}$$

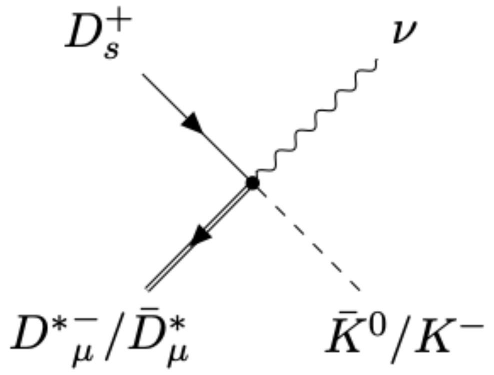
$DD\gamma$ and $DD\gamma K$



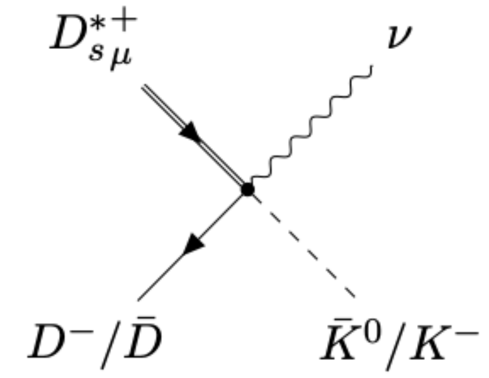
$$= ieQ_D(p_{1\mu} - p_{2\mu})$$



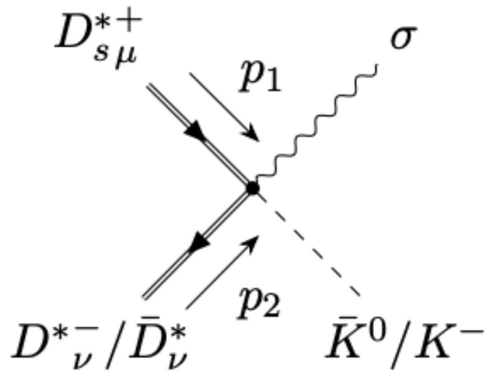
$$= ieQ_{D^*}\left[p_{2\nu}\eta_{\mu\alpha} - p_{1\mu}\eta_{\nu\alpha} + (p_{1\alpha} - p_{2\alpha})\eta_{\mu\nu}\right]$$



$$= \frac{2ig_\pi\sqrt{m_D m_{D^*}}}{f} e\, Q_K\, \eta_{\mu\nu}$$



$$= -\frac{2ig_\pi\sqrt{m_D m_{D^*}}}{f} e\, Q_K\, \eta_{\mu\nu}$$



$$= \frac{2ig_\pi}{f}\sqrt{\frac{m_{D^*}}{m_{D_s^*}}} e\left(p_2{}^\alpha + p_1{}^\alpha Q_{D^*}\right)\epsilon_{\alpha\mu\nu\sigma}$$

Decay rate

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi} = \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K \; +$$
$$I_2^s \sin^2\theta_K \cos 2\theta_\ell + I_2^c \cos^2\theta_K \cos 2\theta_\ell \; +$$
$$I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi$$
$$I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + I_6 \sin^2\theta_K \cos\theta_\ell \; +$$
$$I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi \; +$$
$$I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi \;\Big] \;,$$

Y functions

$$Y^{\lambda}(q^2)\big|_{\alpha_s^0} = Y^{[0]}_{\mathrm{q\bar{q}}}(q^2) + Y^{[0]}_{\mathrm{c\bar{c}}}(q^2) + Y^{[0]}_{\mathrm{b\bar{b}}}(q^2)\,, \tag{2.14}$$

where

$$\begin{aligned}
Y^{[0]}_{\mathrm{q\bar{q}}}(q^2) &= \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 - \frac{1}{2}h(q^2,0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right)\,,\\
Y^{[0]}_{\mathrm{c\bar{c}}}(q^2) &= h(q^2,m_c)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right)\,,\\
Y^{[0]}_{\mathrm{b\bar{b}}}(q^2) &= -\frac{1}{2}h(q^2,m_b)\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right)\,,
\end{aligned}$$

with

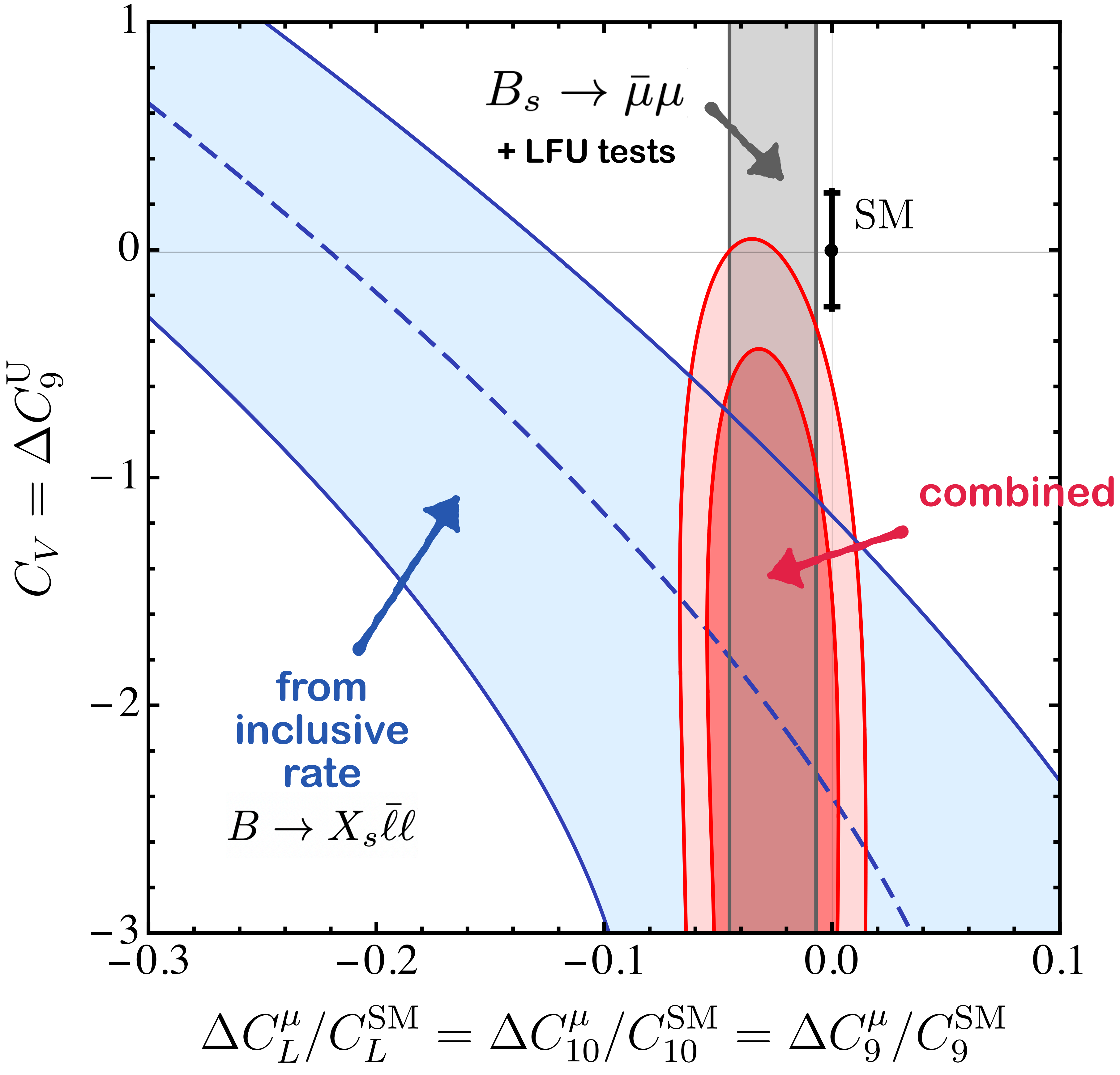
$$h(q^2,m) = -\frac{4}{9}\left(\ln\frac{m^2}{\mu^2} - \frac{2}{3} - x\right) - \frac{4}{9}(2+x)\begin{cases}\sqrt{x-1}\,\arctan\frac{1}{\sqrt{x-1}}\,, & x = \frac{4m^2}{q^2} > 1\,,\\ \sqrt{1-x}\left(\ln\frac{1+\sqrt{1-x}}{\sqrt{x}} - \frac{i\pi}{2}\right), & x = \frac{4m^2}{q^2} \le 1\,.\end{cases}$$

Matrix elements

$$\mathcal{M}\left(B\to K\ell^+\ell^-\right)\big|_{C_{7,9}} = \mathcal{N}\left[C_9 + \frac{2m_b}{m_B+m_K}\frac{f_T(q^2)}{f_+(q^2)}C_7\right] f_+(q^2)(p_B+p_K)_\mu\,\bar\ell\gamma^\mu\ell$$

$$\begin{aligned}\mathcal{M}\left(B\to K^*\ell^+\ell^-\right)\big|_{C_{7,9}} = \mathcal{N}\Bigg\{ & \\ -\left[C_9+\frac{2m_b(m_B+m_{K^*})}{q^2}\frac{T_1(q^2)}{V(q^2)}C_7\right] & \frac{2V(q^2)}{m_B+m_{K^*}} i\epsilon_{\mu\nu\rho\sigma}(\epsilon^*)^\nu p_B^\rho p_{K^*}^\sigma \\ -\left[C_9+\frac{2m_b(m_B+m_{K^*})}{q^2}\frac{T_2(q^2)}{A_2(q^2)}C_7\left(1+O\left(\frac{q^2}{m_B^2}\right)\right)\right] & \frac{A_2(q^2)}{m_B+m_{K^*}}(\epsilon^*\cdot q)(p_B+p_{K^*})_\mu \\ +\left[C_9+\frac{2m_b(m_B^2-m_{K^*})}{q^2}\frac{T_2(q^2)}{A_1(q^2)}C_7\right] & A_1(q^2)\,(m_B+m_{K^*})\,\epsilon^*_\mu\Bigg\}\,\bar\ell\gamma^\mu\ell\,, \end{aligned} \qquad (2.8)$$

Comparison inclusive with data



$C_V = C_9 + C_{10}$ $\quad C_L = -2C_{10}$

- Fit of C_V, C_L from SM prediction on inclusive rate to experimental semi-inclusive determination
- Perturbative and non-perturbative corrections due to charm-rescattering can be accounted for via a modification of C_V
- If $C_L = C_L^{\mathrm{SM}}$, C_V needs a large correction ($\sim 25\,\%$) to explain the data, and it is unlikely that charm re-scattering effects are so large in the high-q^2 region
- Modification of both C_V and C_L could explain well the data ➞ possible small LFU-violating amplitude (assuming LF non-universal modification to C_L)
- SM point not included within 2σ