Spin-orbit coupling in QCD

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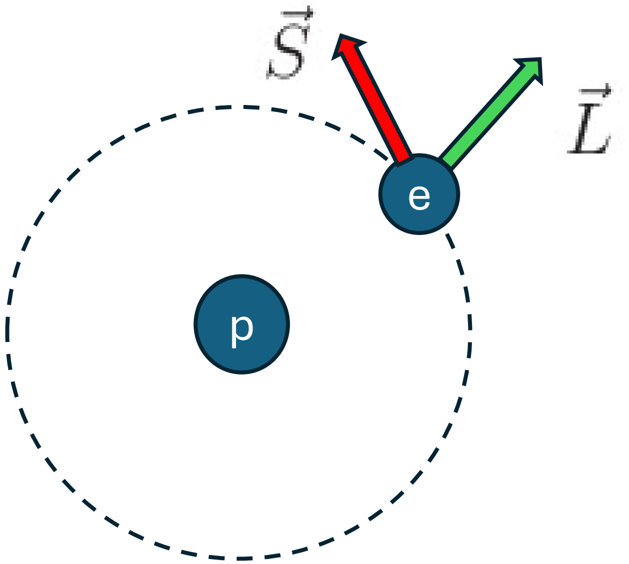
2404.04208; 2404.04209 with Shohini Bhattacharya, Renaud Boussarie,

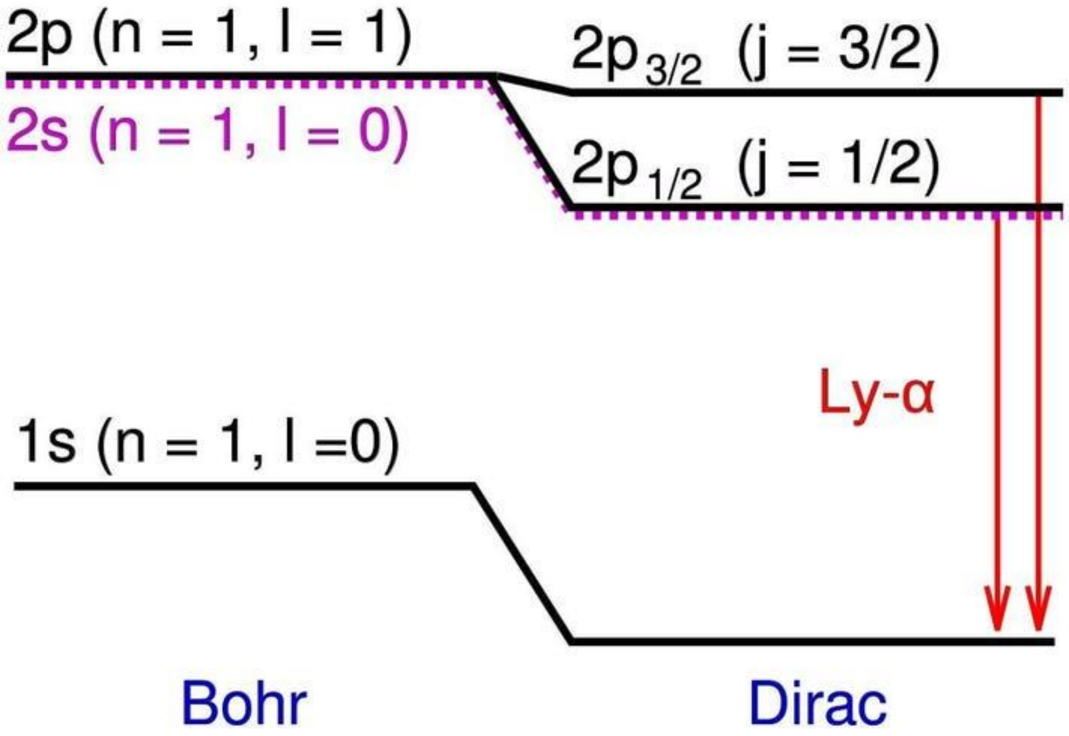
2404.18872 with Jakob Schoenleber

2410.16082 with Jake Montgomery

Spin-orbit coupling in atoms

$$V = -\frac{\mu_B e}{mc^2 r^3}\vec{S}\cdot\vec{L}$$





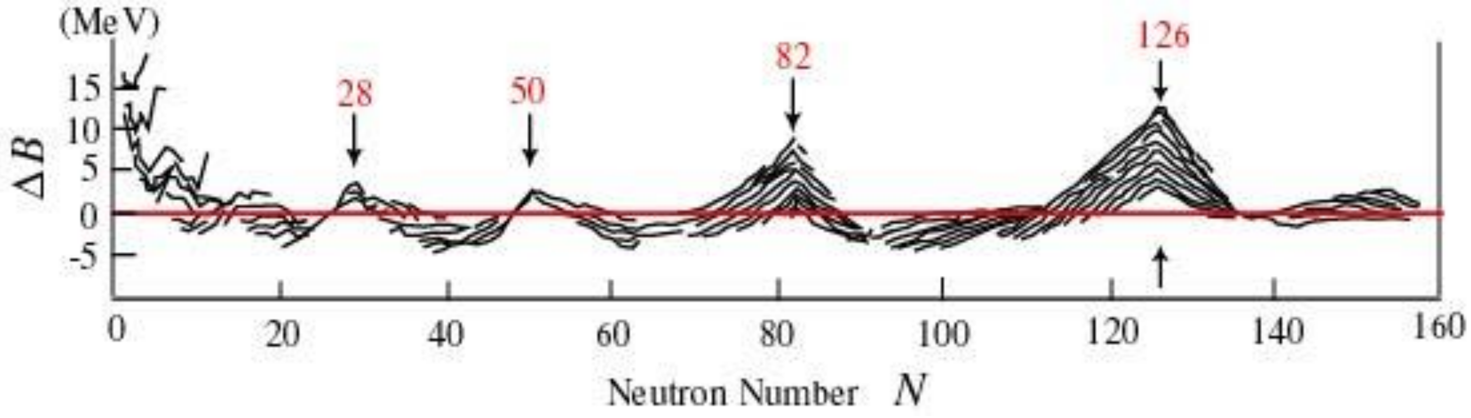
$\vec{\mu}\cdot\vec{B}$ in the electron rest frame + relativistic effects contributes to the fine structure of atoms

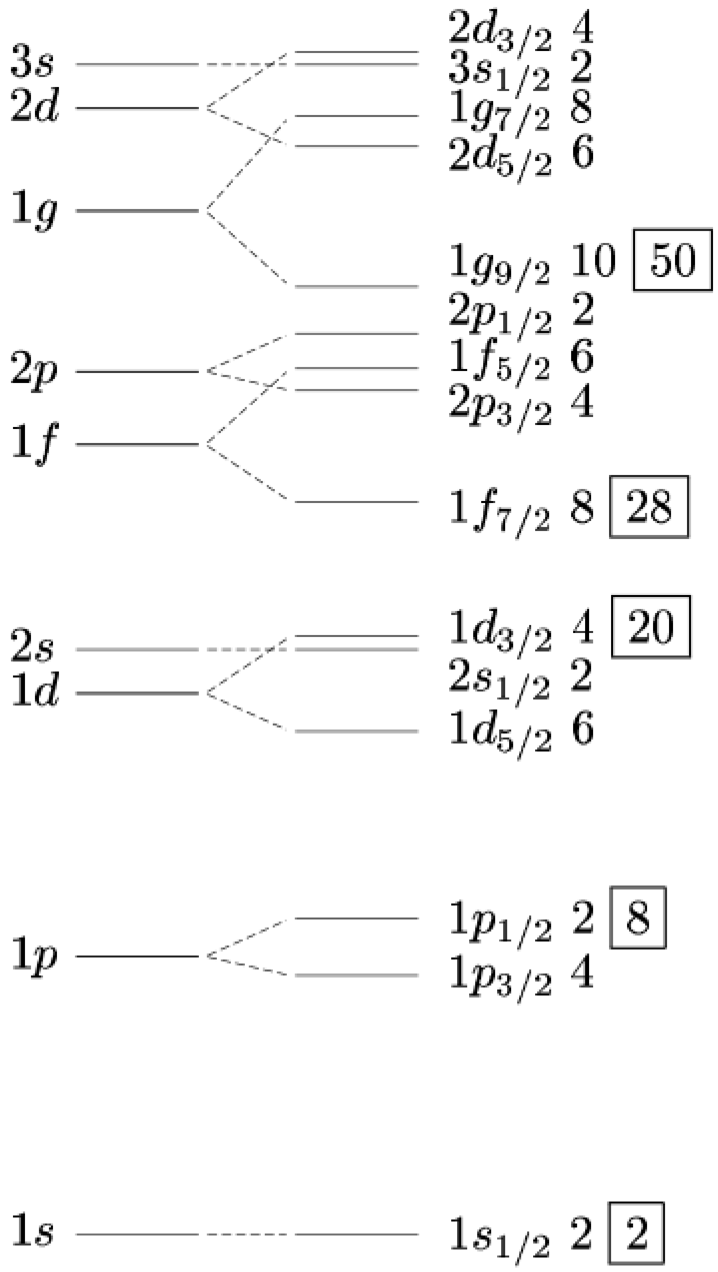
Spin-orbit coupling in nuclei

In the nuclear shell model, nucleons orbiting inside a nucleus feel a spin-orbit force

Strong spin-orbit coupling → magic numbers

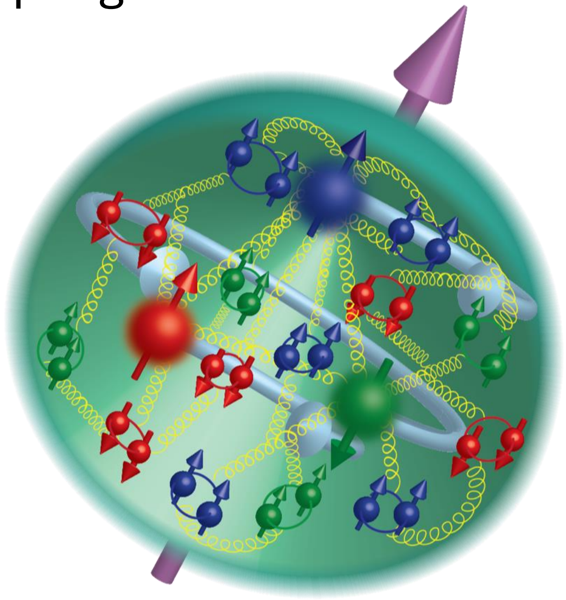
Mayer & Jensen Nobel prize (1963)





Spin-orbit coupling in nucleons?

Quarks and gluons carry spin and OAM. Naturally there should be spin-orbit coupling



The number of quarks and gluons indefinite
Gluon spin and OAM need to be carefully defined

→ Go to infinite momentum frame
→ Gauge invariant canonical OAM

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

spin spin orbit orbit

Consider correlation $S^z L^z$, closest analog of $\vec{S}\cdot\vec{L}$ in nonrelativistic systems

Quark spin-orbit correlation

Polarized quark GTMD

Meissner, Metz, Schlegel (2008)

$$\tilde{f}_q(x, \xi, k_\perp, \Delta_\perp) = \int \frac{d^3z}{2(2\pi)^3} e^{ixP^+z^- - ik_\perp \cdot z_\perp} \langle p's'|\bar{q}(-z/2)W_\pm \gamma^+\gamma_5 q(z/2)|ps\rangle$$

$$= \frac{-i}{2M}\bar{u}(p's')\left[\frac{\epsilon_{ij}k_\perp^i \Delta_\perp^j}{M^2} G_{1,1}^q + \frac{\sigma^{i+}\gamma_5}{P^+}(k_\perp^i G_{1,2}^q + \Delta_\perp^i G_{1,3}^q) + \sigma^{+-}\gamma_5 G_{1,4}^q\right] u(ps)$$

Quark spin-orbit correlation

Lorce, Pasquini (2011)

$$C_q = \int dx \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^q(x, k_\perp, 0) \sim \langle S^z L^z \rangle$$

Associated PDF

$$C_q(x) = \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^q(x, k_\perp, 0)$$

$C_q > 0$ if helicity and OAM are aligned, $C_q < 0$ if they are anti-aligned

Gluon spin-orbit correlation

Polarized gluon GTMD

$$x\tilde{f}_g(x,\xi,k_\perp,\Delta_\perp) \; = \; i\int \frac{d^3z}{(2\pi)^3P^+}e^{ixP^+z^- - ik_\perp\cdot z_\perp}\langle p'|\tilde{F}^{+\mu}(-z/2)\widetilde{W}_\pm F^+_{\ \mu}(z/2)|p\rangle$$

$$= \; \frac{-i}{2M}\bar{u}(p')\left[\frac{\epsilon_{ij}k^i\Delta^j}{M^2}G^g_{1,1} + \frac{\sigma^{i+}\gamma_5}{P^+}(k^iG^g_{1,2} + \Delta^iG^g_{1,3}) + \sigma^{+-}\gamma_5G^g_{1,4}\right]u(p)$$

$$xC_g(x) \; = \; \int d^2k_\perp \frac{k_\perp^2}{M^2}G^g_{1,1}(x,k_\perp,0)$$

$C_g(x)$ is odd. The first moment vanishes $\int dx C_g(x) = 0$

Orbital anguler momentum and spin-orbit correlation

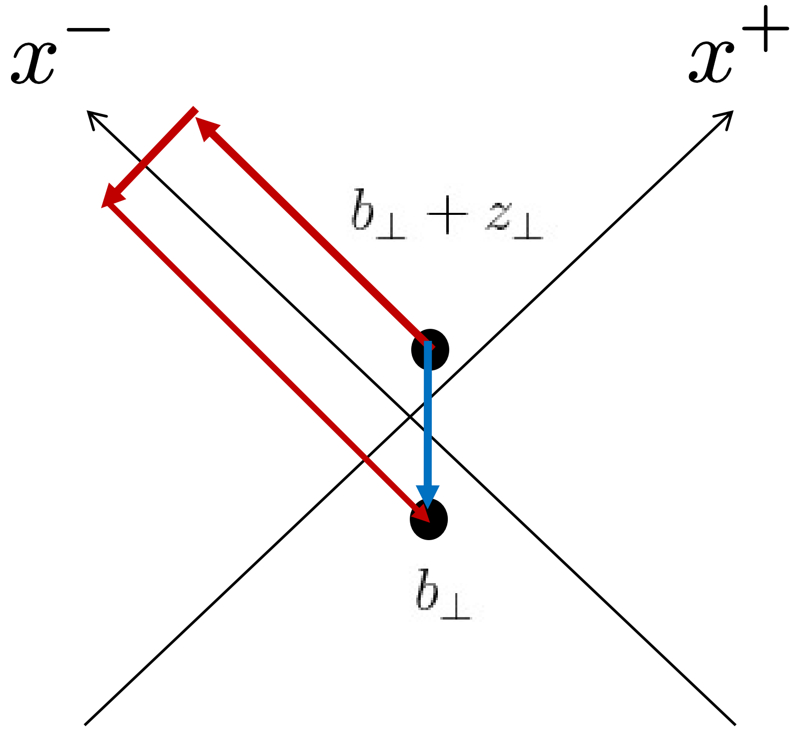
unpol Wigner

$$L_q(x) = \int dk_\perp db_\perp b_\perp \times k_\perp f_q(x, k_\perp, b_\perp)$$

polalized Wigner

$$C_q(x) = \int dk_\perp db_\perp b_\perp \times k_\perp \tilde{f}_q(x, k_\perp, b_\perp)$$

γ_5 rotation



Staple-shaped Wilson line → canonical OAM YH (2011)

Straight Wilson line → kinetic OAM Ji, Xiong, Yuan (2012)

Twist structure of OAM

YH, Yoshida (2012)

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$

Wandzura-Wilczek part

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)}\,.$$

genuine twist-3

$$\Phi_F \sim \langle P'|\bar{\psi}\gamma^+ F^{+i}\psi|P\rangle$$

$$M_F \sim \langle P'|F^{+\mu}F^{+i}F^+_\mu|P\rangle$$

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$

$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX \Phi_F(X, x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^3(x_1 - x_2)}$$

$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1, x_2)\mathcal{P}\frac{2x_1 - x_2}{x_1^3(x_1 - x_2)^2}$$

Twist structure of spin-orbit correlation

YH, Schoenleber (2024)

Unpol PDF

$$C_q(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} x'\Delta q(x') - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \frac{\Psi_{qF}(x_1,x_2)}{x_1-x_2} P\frac{3x_1-x_2}{x_1^2(x_1-x_2)}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Psi}_{qF}(x_1,x_2) P\frac{1}{x_1^2(x_1-x_2)},$$

$$C_g(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} x'\Delta G(x') - 2x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} G(x') - 4x\sum_q \int_x^{\epsilon(x)} \frac{dx'}{x'^3} \int dX \tilde{\Psi}_{qF}(X,x')$$
$$+4x\int_x^{\epsilon(x)} dx_1 \int dx_2 P\frac{\tilde{N}_F(x_1,x_2)}{x_1^3(x_1-x_2)} + 4x\int_x^{\epsilon(x)} dx_1 \int dx_2 \frac{N_F(x_1,x_2)}{x_1^3(x_1-x_2)} P\frac{2x_1-x_2}{x_1-x_2}$$

Unexpected connection to $g_T(x)$

$$C_q(x) = -\frac{\tilde{g}^q(x)}{2} - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} q(x') + \cdots$$

same

$$g_T^q(x) = -\frac{1}{2x}\tilde{g}^q(x) - \frac{1}{2x}\int dx' \frac{\tilde{G}_{Fq}(x,x') + G_{Fq}(x,x')}{x-x'} + \frac{m_q}{M}\frac{h_1^q(x)}{x}$$

` kinematical twist-3 part' of the $g_T(x)$ distribution

2 spin sum rules, 1 momentum sum rule?

Spin

$$\frac{1}{2} = \frac{1}{2}\sum_q (A_{q+\bar{q}} + B_{q+\bar{q}}) + \frac{1}{2}(A_g + B_g)$$

Ji (1996)

$$= \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Jaffe, Manohar (1990)

Momentum

$$1 = \sum_q A_{q+\bar{q}} + A_g$$

Feynman (1969)

2 spin sum rules, 2 momentum sum rules!

Spin

$$\frac{1}{2} = \frac{1}{2}\sum_q (A_{q+\bar{q}} + B_{q+\bar{q}}) + \frac{1}{2}(A_g + B_g)$$

Ji (1996)

$$= \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Jaffe, Manohar (1990)

Momentum

$$1 = \sum_q A_{q+\bar{q}} + A_g$$

Feynman (1969)

$$= -3C_q^{(2)} - \frac{3}{2}C_g^{(2)} + \frac{3}{2}\int_{-1}^{1} dx dx' \left[\Lambda_q(x,x') + \frac{2x\tilde{\Lambda}_q(x,x') + \tilde{\Lambda}_G(x,x')}{x - x'}\right]$$

YH, Schoenleber (2024)

Physical meaning of the sum rule

2404.18872

$$1 = \underbrace{-3C_q^{(2)} - \frac{3}{2}C_g^{(2)}}_{\text{kinetic energy}} + \underbrace{\frac{3}{2}\int_{-1}^{1} dx dx' \left[\Lambda_q(x,x') + \frac{2x\tilde{\Lambda}_q(x,x') + \tilde{\Lambda}_G(x,x')}{x - x'}\right]}_{\text{potential energy}}$$

$$\langle p'|\bar{q}\gamma^+ F^{+i} q|p\rangle \approx i\Delta^i \int dx dx' \Lambda_q(x,x')$$

Transverse force

$$F_a^{+i} = \frac{1}{\sqrt{2}}(\vec{E} + \vec{v} \times \vec{B})_a^i$$

color Lorentz force

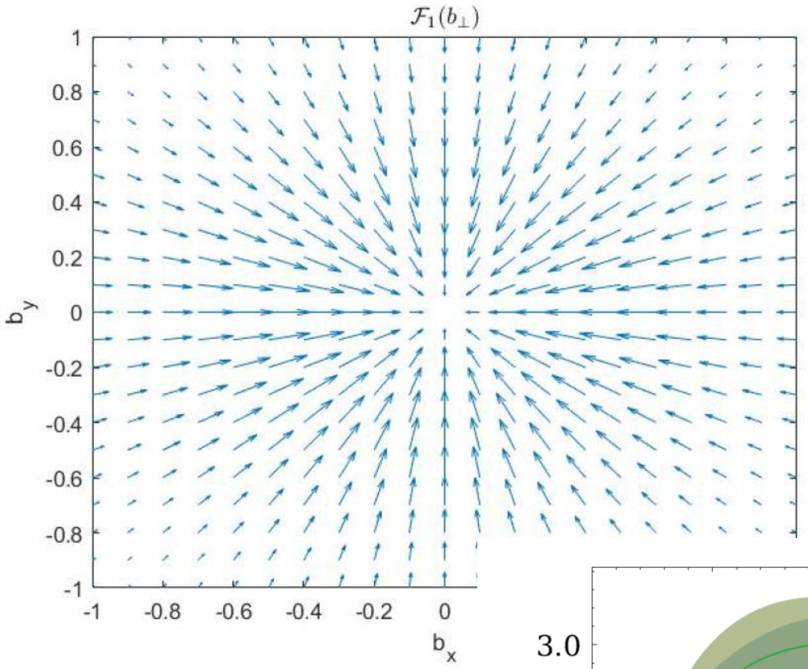
Burkardt (2008)

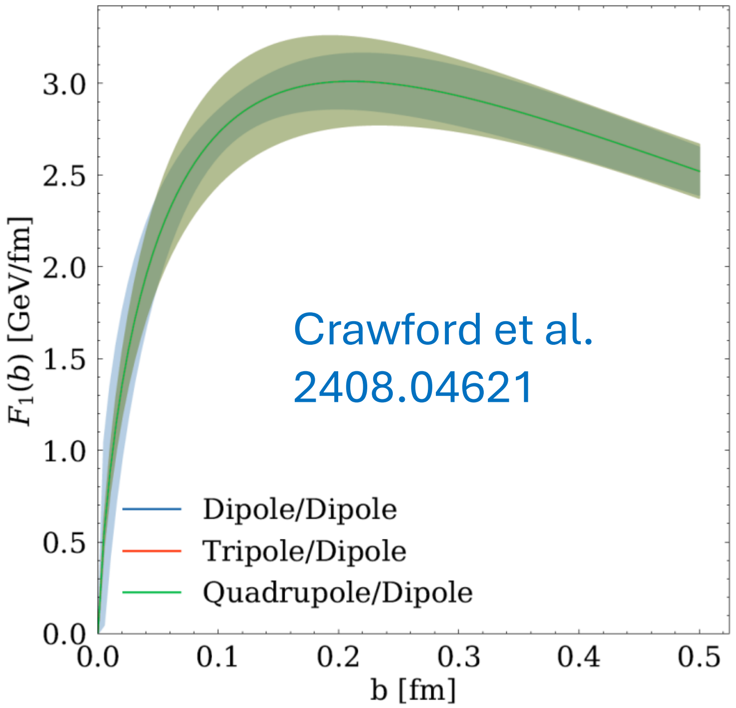
Force → gradient of a potential

$$\mathcal{F}_q^i(b_\perp) \equiv -\frac{\partial}{\partial b^i} V_q(b_\perp)$$

$$\frac{3}{2}\int dx dx' \Lambda_q(x, x') = \int d^2 b_\perp V_q(b_\perp)$$

Aslan, Burkardt, Schlegel (2019)





Crawford et al.
2408.04621

Dual transverse force

$$\tilde{F}_a^{+i} = -\frac{1}{\sqrt{2}}(\vec{B} - \vec{v} \times \vec{E})_a^i$$

Would-be Lorentz force acting on a magnetic monopole

$$\langle p'|\bar{q}(0)\not{n}\gamma_5 g\left(t^b i\overrightarrow{D}^n - i\overleftarrow{D}^n t^b\right) q(0)\int_0^{\pm\infty} d\tau \mathcal{W}_{0,\tau n}^{ba}\tilde{F}_a^{ni}(\tau n)|p\rangle \overset{\Delta}{\Rightarrow} i\Delta^i \int dx dx' \frac{2x\tilde{\Lambda}_q(x,x')}{x-x'}$$

final state interaction

Associated potential

$$\frac{3}{2}\int dx dx' \frac{2x\tilde{\Lambda}_q(x,x')}{x-x'} = \int d^2b_\perp \tilde{V}_q(b_\perp) = \tilde{V}_q$$

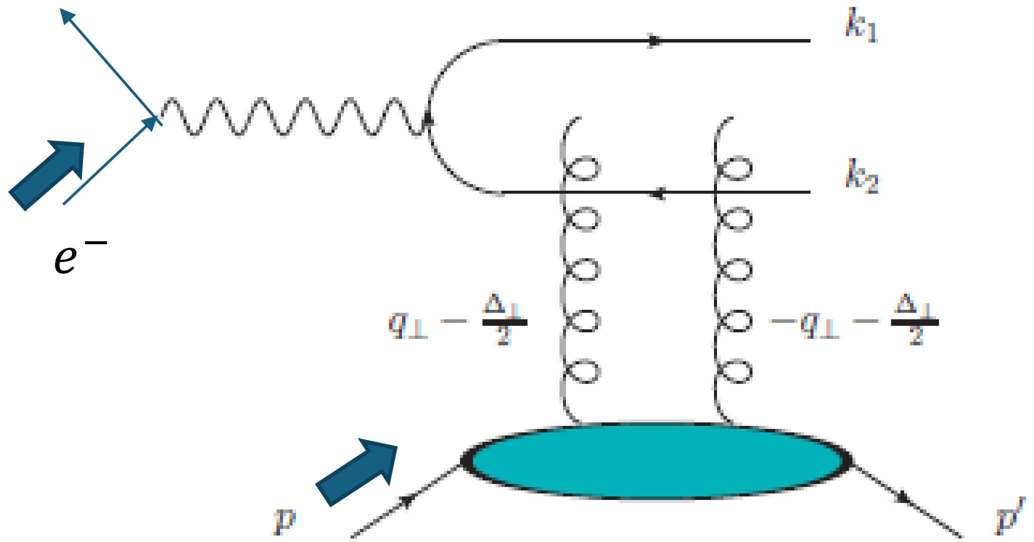
$$1 = T_q + T_G + V_q + \tilde{V}_q + \tilde{V}_G$$

Measuring spin-orbit correlation at the EIC

Quark and gluon GTMDs $G_{1,1}$ appeared in certain exclusive reactions,
e.g., Bhattcharya, Metz, Zhou (2017)
but no quantitative estimate made.

Longitudinal double spin asymmetry in diffractive dijets
→ previously proposed as a signal of gluon OAM

Bhattacharya, Boussarie, YH, (2022)



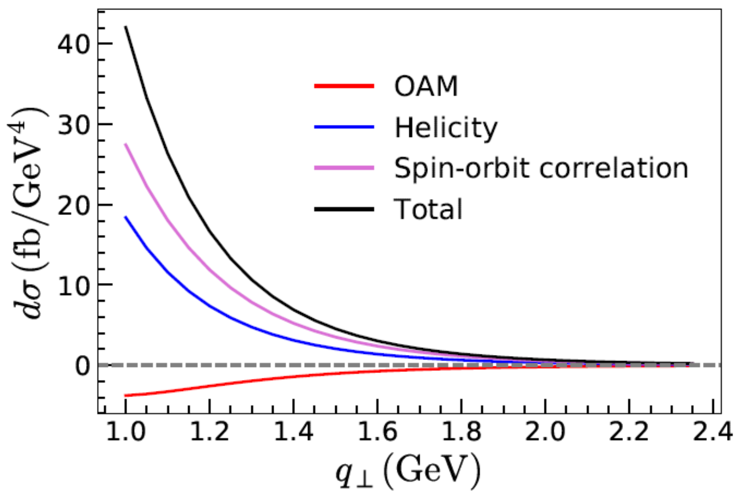
$$L^z \sim b_\perp \times k_\perp$$

conjugate to $\Delta_\perp$
proton recoil momentum

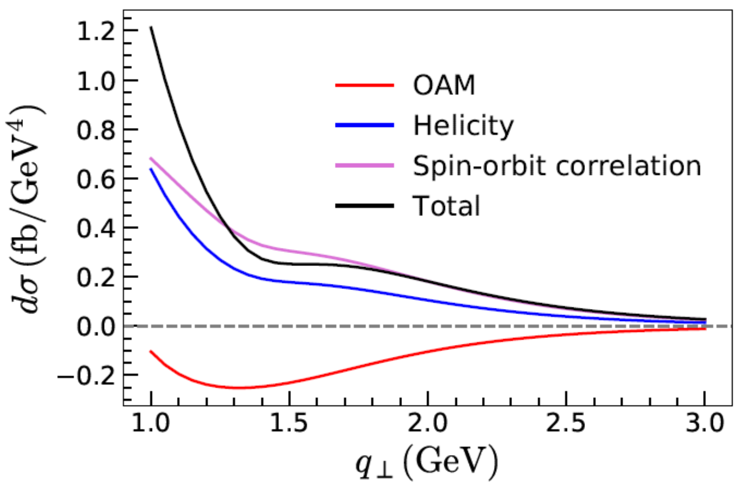
correlated with jet
transverse momentum

Cross section at the EIC (revised)

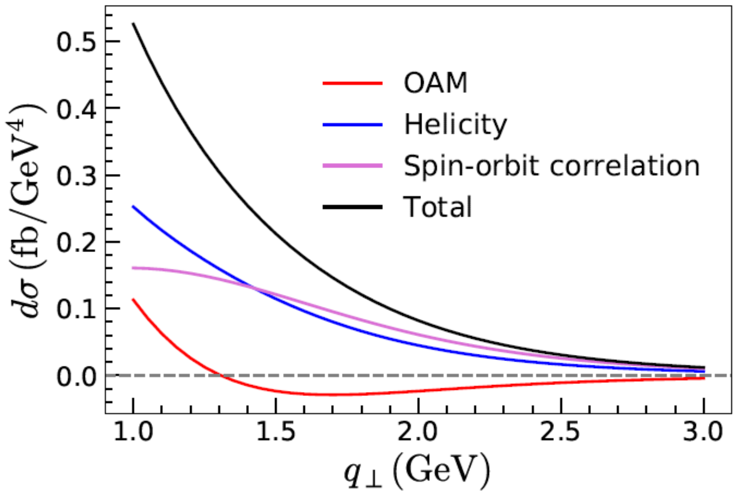
Bhattacharya, Boussarie, YH (2024)



$$Q^2 = 2.7\,\mathrm{GeV}^2$$



$$Q^2 = 4.8\,\mathrm{GeV}^2$$

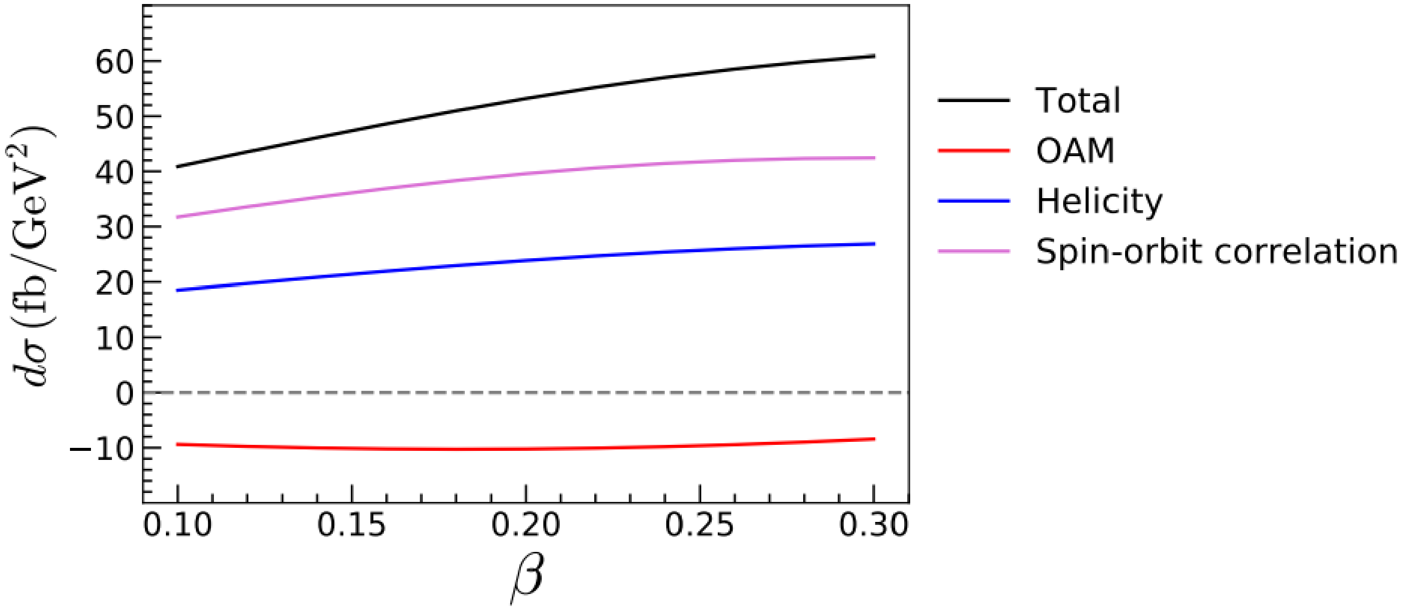


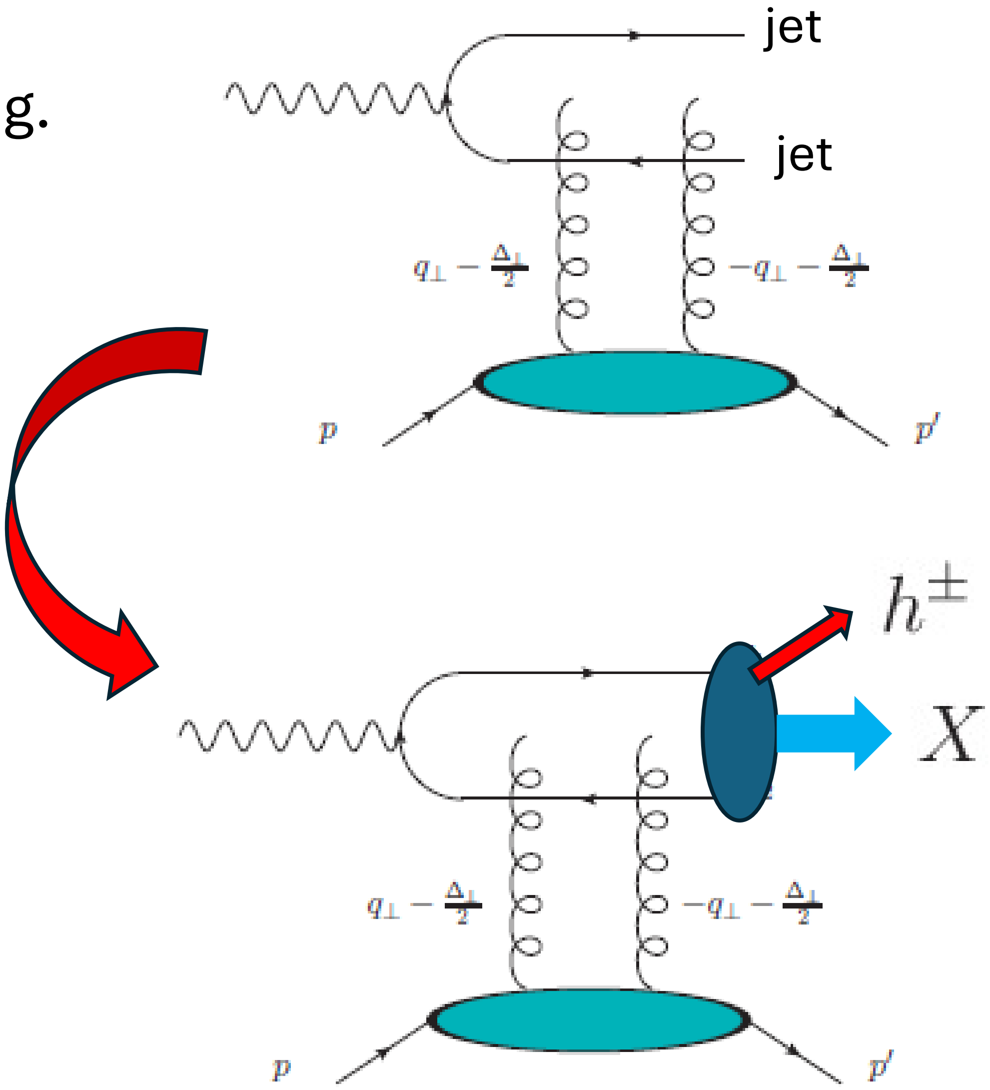
$$Q^2 = 10\,\mathrm{GeV}^2$$

Semi-inclusive diffractive DIS (SIDDIS)

In practice, jet reconstruction at low-Pt is challenging.
Re-interpret as SIDDIS

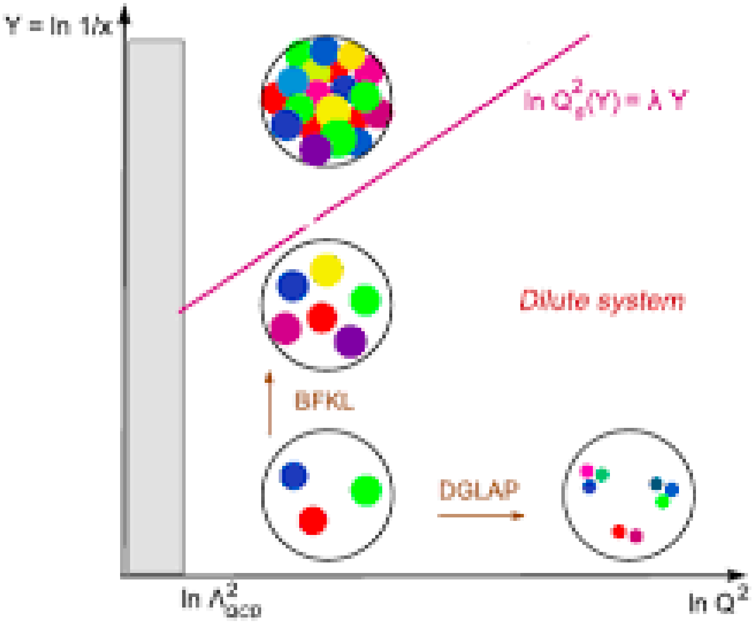
YH, Xiao, Yuan (2022)





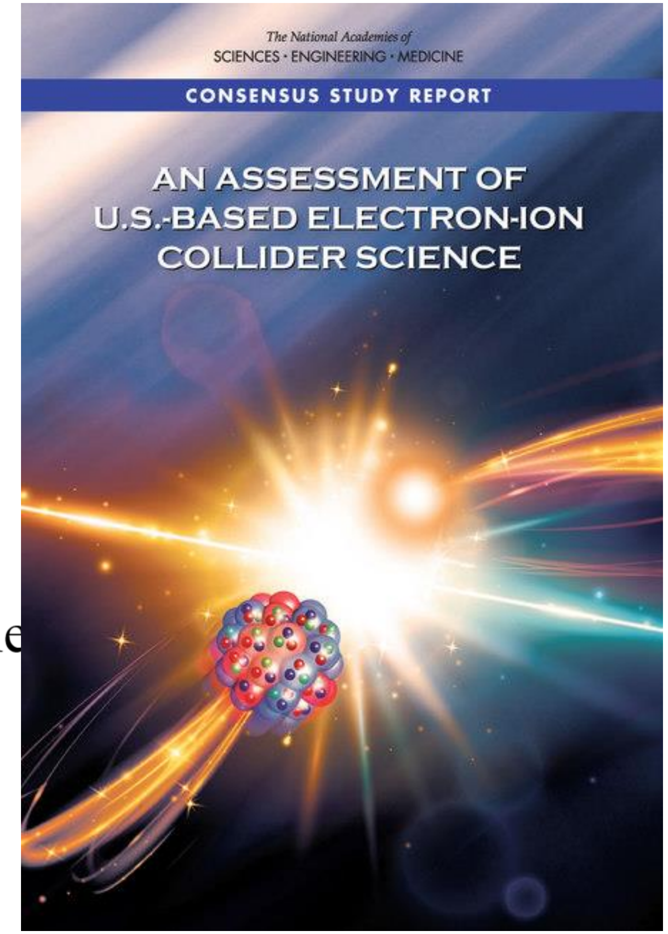
Spin-orbit correlation at small-x

Gluon saturation at small-x:
one of the core topics of EIC



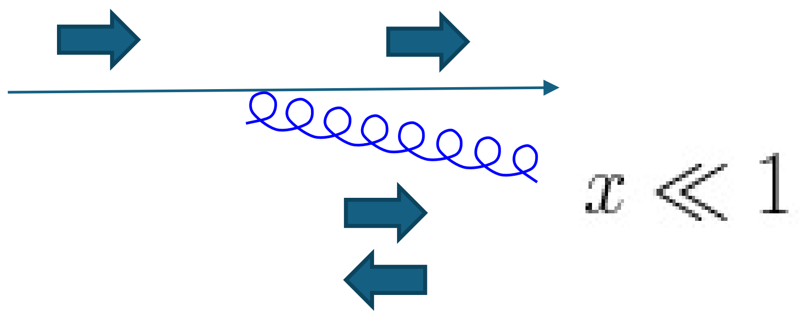
Finding 1: An EIC can uniquely address three profound questions about nucle
protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?



Intuitive argument

Imagine a very energetic quark emits a soft gluon

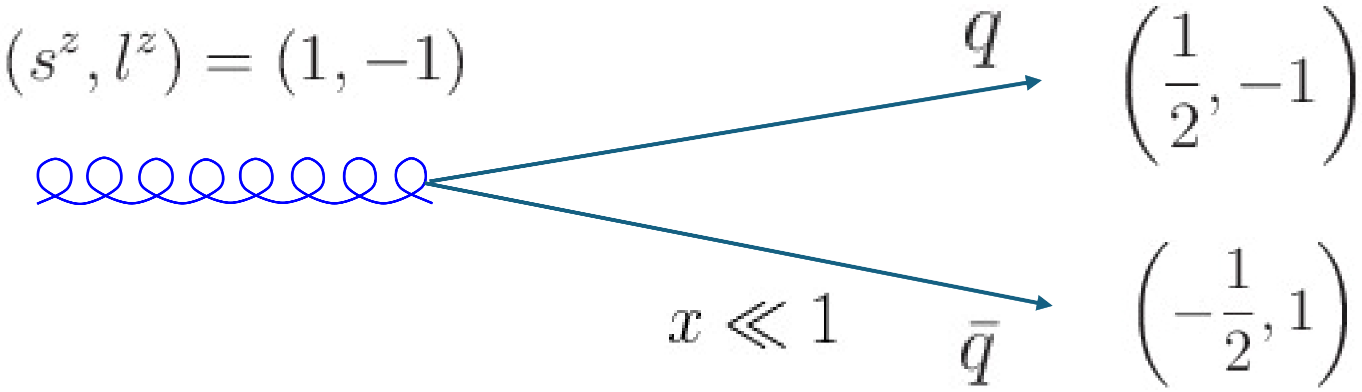


Quark spin and momentum (and OAM) unchanged.

From angular momentum conservation, the total angular momentum of the emitted gluon must be zero

$$(s^z, l^z) = (\pm 1, \mp 1)$$

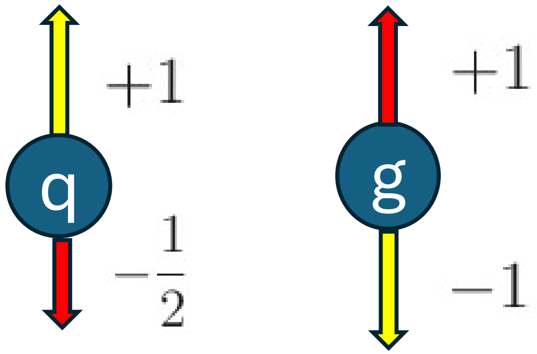
Imagine the emitted soft gluon further splits into a $q\bar{q}$ pair



Same handedness, same OAM

Opposite handedness,
OAM from AM conservation

Helicity and OAM are always in opposite directions
Remarkably, only $L^z = \pm 1$ states appear in this argument



Gluon spin-orbit coupling at small-x

$$\frac{i}{x}\int\frac{d^3z}{(2\pi)^3P^+}e^{ixP^+z^- - ik_\perp\cdot z_\perp}\langle p'|2\mathrm{Tr}[W_+\tilde{F}^{+\mu}(-z/2)W_\pm F^+_\mu(z/2)]|p\rangle = -i\frac{\epsilon_{ij}k^i_\perp\Delta^j_\perp}{M^2}C_g^{[+\pm]}(x,\xi,k_\perp,\Delta_\perp),$$

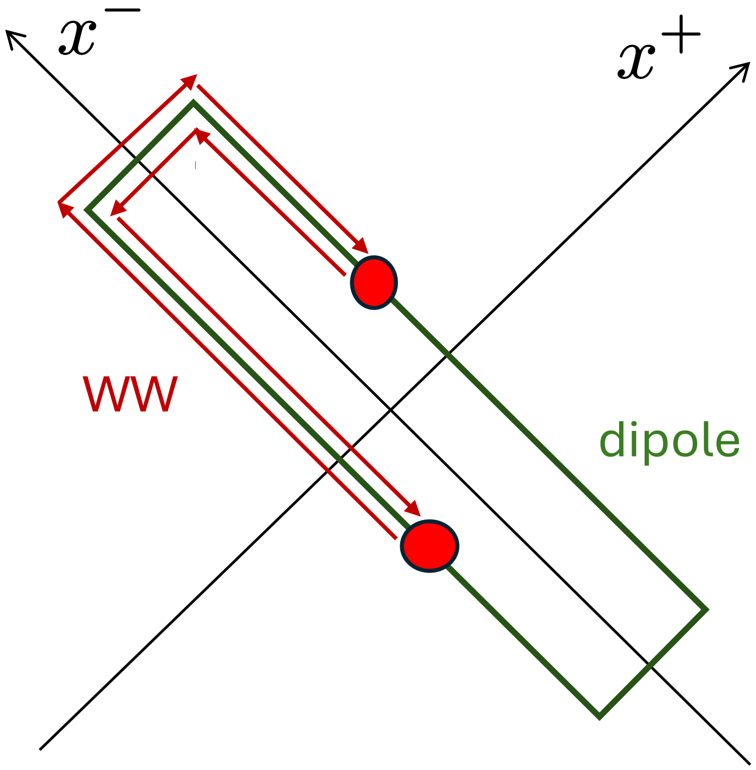
Two gluon GTMDs from different configurations of Wilson lines

Weiszacker-Williams type

Dipole type

Bomhof, Mulders, Pijlman (2006)

Dominguez, Marquet, Xiao, Yuan (2011)



Approximate $e^{ixP^+z^-}\approx 1$ (eikonal approximation)

Dipole gluon

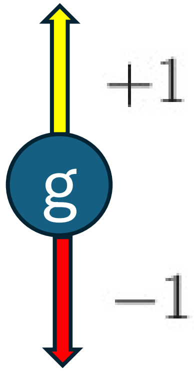
$$\frac{xC_g^{\mathrm{dip}}(x,k_\perp)}{M^2} = -\frac{2N_c}{\alpha_s}\int\frac{d^2w_\perp d^2z_\perp}{(2\pi)^4}e^{-ik_\perp\cdot(z_\perp-w_\perp)}\frac{\langle p|\frac{1}{N_c}\mathrm{Tr}U(w_\perp)U^\dagger(z_\perp)-1|p\rangle}{\langle p|p\rangle}$$

cf. Boer, van Daal, Mulders, Petreska (2018)

WW gluon

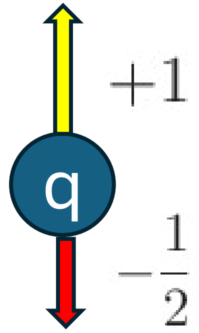
$$k_\perp^2\frac{C_g^{\mathrm{WW}}(x,k_\perp)}{M^2} = -f_g^{\mathrm{WW}}(x,k_\perp) - \frac{C_F}{\pi\alpha_s x}\int\frac{d^2b_\perp d^2r_\perp}{(2\pi)^3}e^{-ik_\perp\cdot r_\perp}\partial_i^r D(r_\perp)\partial_i^r\left(\frac{1-e^{\frac{N_c}{C_F}D(r_\perp)}}{D(r_\perp)}\right)$$

$$C_g^{\mathrm{dip}}(x) = C_g^{\mathrm{WW}}(x) = -G(x)$$



$-1\times 1 = -1$ times the number of gluons

Quark spin-orbit coupling at small-x

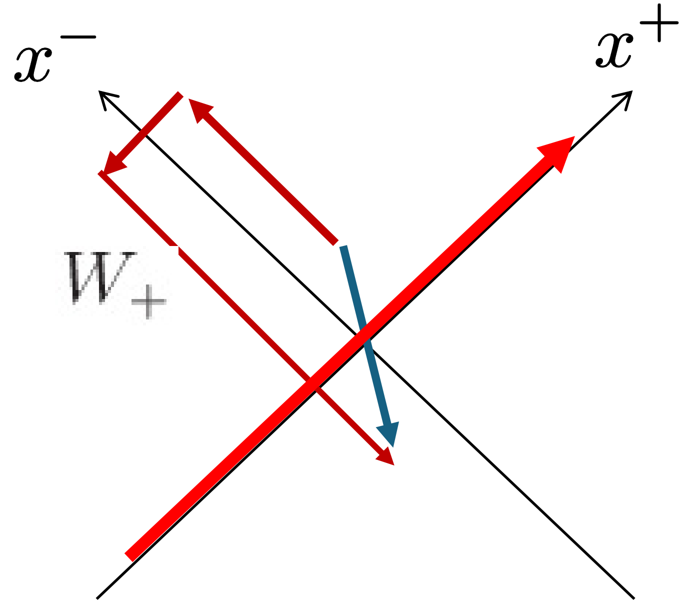


$$\int \frac{d^3z}{2(2\pi)^3} e^{ixP^+z^- - ik_\perp \cdot z_\perp} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5 W_\pm \psi(z/2) | p \rangle = -i \frac{\epsilon_{ij} k_\perp^i \Delta_\perp^j}{M^2} C_q(x, \xi, k_\perp, \Delta_\perp)$$

$$\frac{C_q(x, k_\perp)}{M^2} = \frac{N_c S_\perp}{8\pi^4 x k_\perp^2} \int d^2 k_{g\perp} (k_\perp - k_{g\perp}) \cdot k_\perp \frac{\ln \frac{k_\perp^2}{(k_\perp - k_{g\perp})^2}}{k_\perp^2 - (k_\perp - k_{g\perp})^2} \frac{\langle p | \left(\frac{1}{N_c} \mathrm{Tr} U U^\dagger - 1 \right) (k_{g\perp}) | p \rangle}{\langle p | p \rangle}$$

$$C_q(x) = -\frac{1}{2} q(x)$$

$-\frac{1}{2} \times 1 = -\frac{1}{2}$ times the number of quarks



Quantum entanglement of spin and OAM

Bhattacharya, Boussarie, YH (2024)

$s^z = \pm 1$ Qubit (Alice) $l^z = \pm 1$ Qubit (Bob)

Perfect spin-orbit anti-correlation at small-x → `Bell states'

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_s|-\rangle_l + |-\rangle_s|+\rangle_l\right), \qquad |\Psi^-\rangle = \frac{1}{\sqrt{2}i}\left(|+\rangle_s|-\rangle_l - |-\rangle_s|+\rangle_l\right)$$

Every single quark and gluon at small-x is a maximally entangled Bell state

$\langle S^z \rangle = \langle L^z \rangle = 0$ but $\langle S^z L^z \rangle = -1$

True nature of the system encoded in correlations

QED example

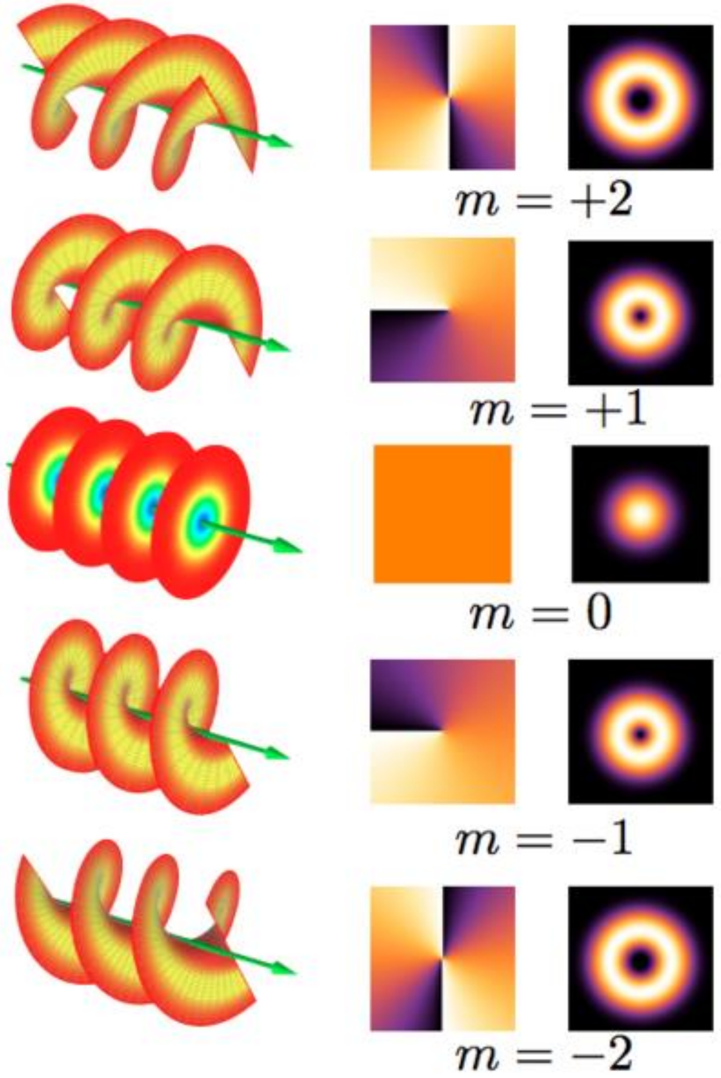
Photon OAM

$$|\pm\rangle_l \sim e^{\pm i\phi}$$

e.g., Laguerre-Gaussian beam

$$|\Psi^+\rangle \sim (1,i)e^{-i\phi} + (1,-i)e^{i\phi} \sim (\cos\phi, \sin\phi)$$

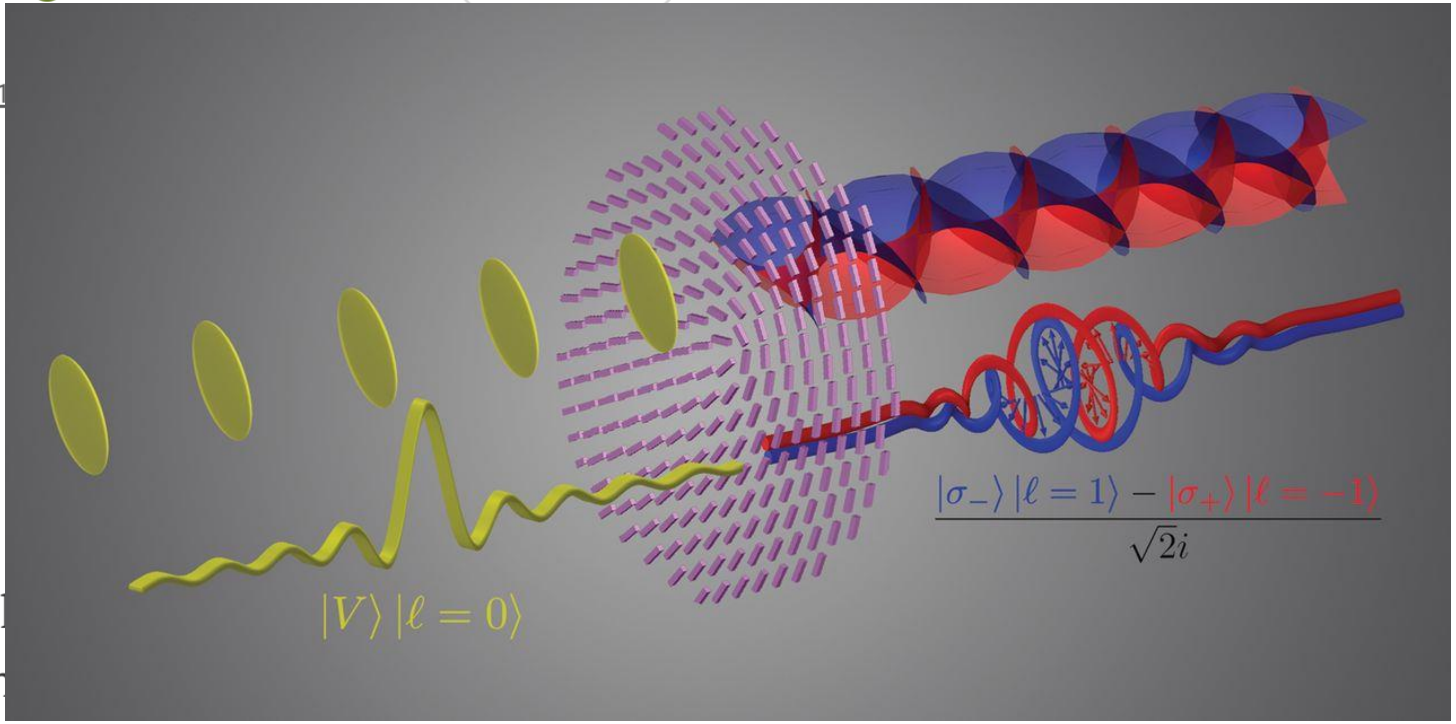
$$|\Psi^-\rangle \sim -i\left((1,i)e^{-i\phi} - (1,-i)e^{i\phi}\right) \sim (-\sin\phi, \cos\phi)$$



Quantum entanglement of the spin and orbital angular momentum of photons using metamaterials

TOMER STAV, ARKADY FAERMAN, ELHANAN MAGUID, DIKLA OREN, [...], AND MORDECHAI SEGEV +2 authors Authors Info & Affiliations

SCIENCE 14 Sep 2018 Vol 361, Issue 6407 pp. 1101-1104 DOI: 10.11



Abstract

Metamaterials constructed from deep subwavel
phenomena ranging from negative refractive in
eral relativity, and superresolution imaging. More recently, metamaterials have been suggested as a new platform for quantum optics. We present the use of a dielectric metasurface to generate entanglement between the spin and orbital angular momentum of photons. We demonstrate the genera-

In QCD, spin-orbit entanglement is a default property of soft partons!

Extension to arbitrary $0 < x < 1$

YH, Montgomery, 2410.16082

Bell states in the limit $x \to 0$, both quarks and gluons

$$|\Phi\rangle \approx |\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}\Big(|+\rangle|-1\rangle \pm |-\rangle|1\rangle\Big)$$

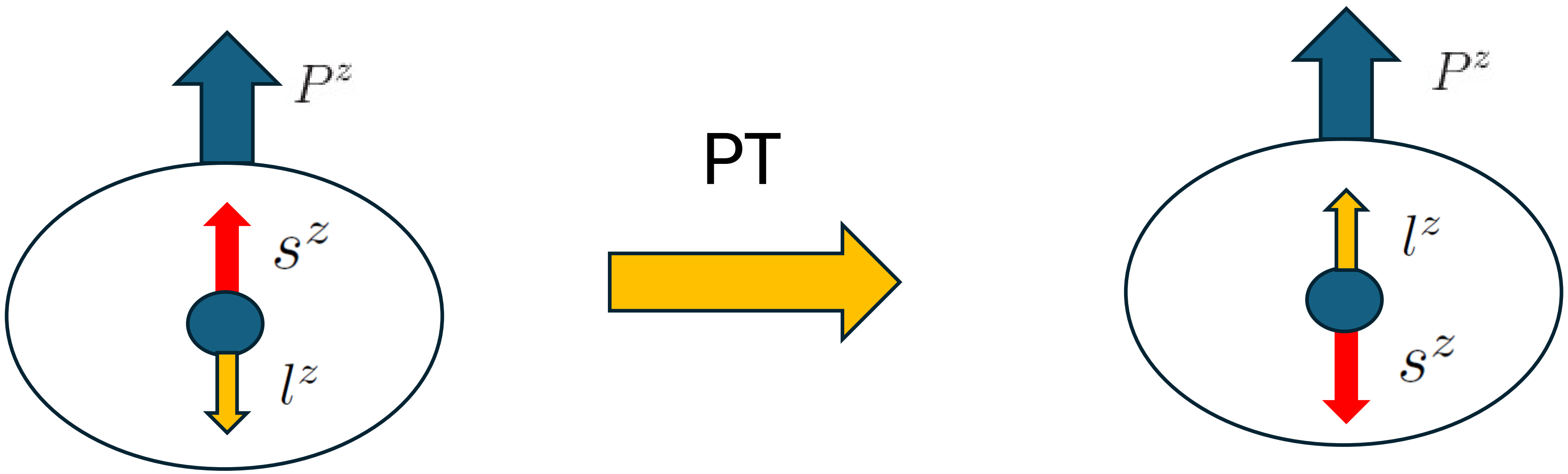
What happens when $x = \mathcal{O}(1)$?
One can no longer argue that only $l^z = \pm 1$ are relevant

Qubit $l^z = \pm 1$ → Qudit $l^z = 0, \pm 1, \pm 2, \cdots$

$$|\Phi\rangle = |+\rangle\Big\{a_1|1\rangle + a_0|0\rangle + a_{-1}|-1\rangle + \cdots\Big\} + |-\rangle\Big\{b_1|1\rangle + b_0|0\rangle + b_{-1}|-1\rangle + \cdots\Big\}$$

Parton as an entangled system of a qubit and a qudit

Parity & time-reversal



$$|\Phi\rangle = |+\rangle\left\{a_1|1\rangle + a_0|0\rangle + a_{-1}|-1\rangle + \cdots\right\} + |-\rangle\left\{b_1|1\rangle + b_0|0\rangle + b_{-1}|-1\rangle + \cdots\right\}$$

$$PT|\Phi\rangle = e^{i\varphi}|\Phi\rangle \quad \Longrightarrow \quad b_l = e^{-i\varphi}(-1)^l a^*_{-l}$$

Caveat: The argument works only for gluons, not quarks (Kramers degeneracy)

Qutrit

Restrict to $l^z = 0, \pm 1$ ⟶ Qutrit

$$|l^z| \sim |\vec{k} \times \vec{b}| \sim 1 \quad \Longleftarrow \quad |\vec{k}| \lesssim \Lambda_{QCD} \sim 200 \text{ MeV}, \quad |\vec{b}| \lesssim 1 \text{ fm}$$

Gluon as an entangled state between a qubit and a qutrit

$$|a_1|^2 + |a_0|^2 + |a_{-1}|^2 = \frac{1}{2} \qquad |a_1|^2 - |a_{-1}|^2 = \frac{1}{2}\frac{C_g(x)}{G(x)}$$

Maximal entanglement

Most general evolution of a qubit-qutrit system → U(6) $(a_1, a_0, a_{-1}, b_1, b_0, b_{-1})$

PT & norm conservation $|a_1|^2 + |a_0|^2 + |a_{-1}|^2 = \frac{1}{2}$ → U(3) x U(2)

Local unitary transformations conserve entanglement entropy

Gluons maximally entangled for any $0 < x < 1$

$$\epsilon = \frac{1 + \frac{C_g(x)}{G(x)}}{2}$$

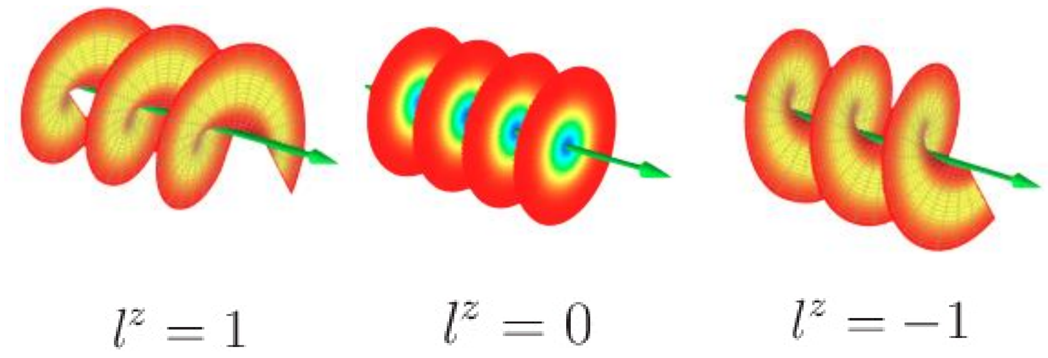
$$\begin{pmatrix} a_1 \\ a_0 \\ a_{-1} \end{pmatrix} = e^{i\theta} \begin{pmatrix} 1-\epsilon & e^{i\chi}\sqrt{2\epsilon(1-\epsilon)} & e^{2i\chi}\epsilon \\ -e^{-i\chi}\sqrt{2\epsilon(1-\epsilon)} & 1-2\epsilon & e^{i\chi}\sqrt{2\epsilon(1-\epsilon)} \\ e^{-2i\chi}\epsilon & -e^{-i\chi}\sqrt{2\epsilon(1-\epsilon)} & 1-\epsilon \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

OAM conditional probability

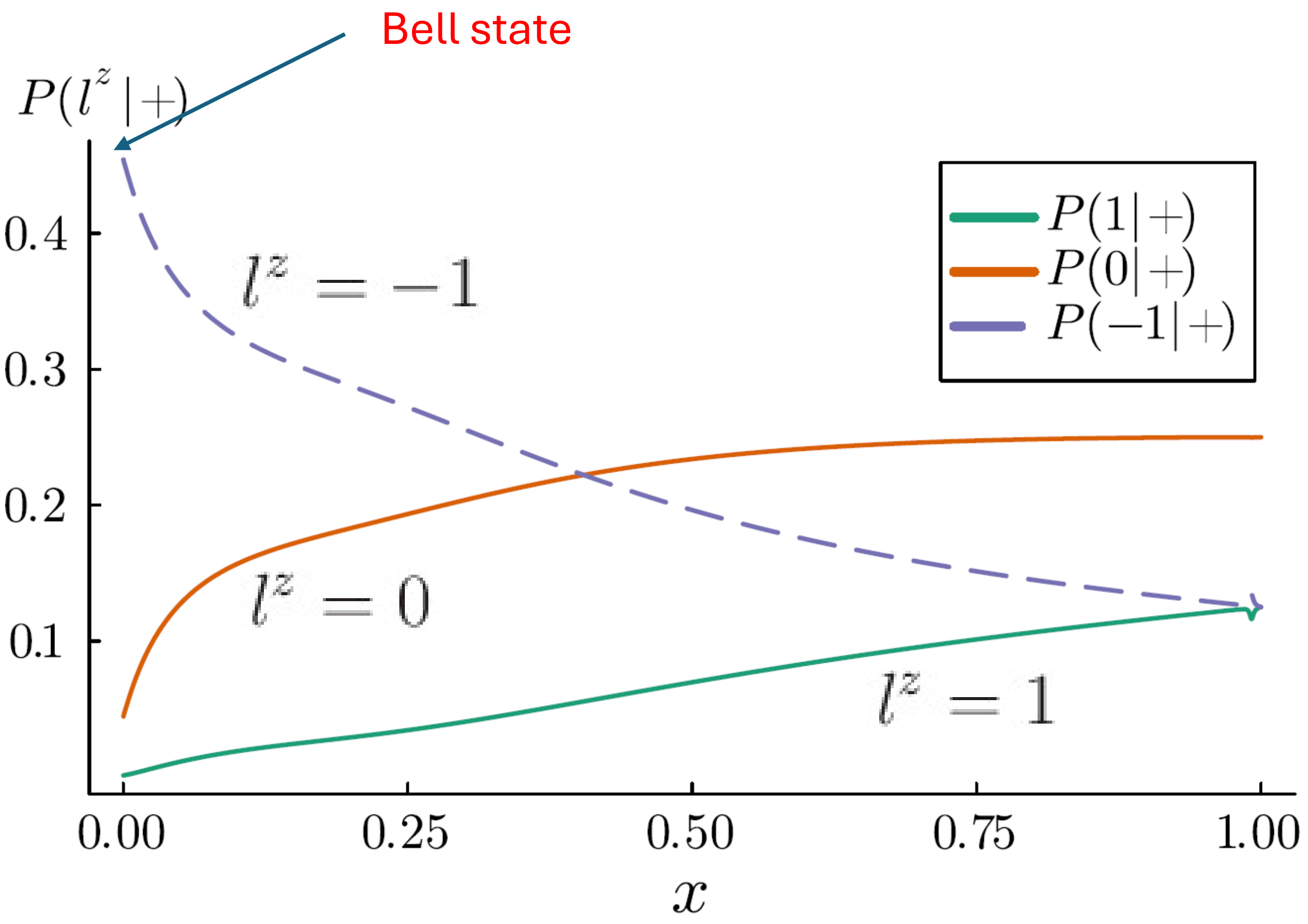
2410.16082

Pick a gluon with $s^z = 1$

What are the probabilities that different values of l^z are realized?



$$C_g(x) \approx -2x \int_x^1 \frac{dz}{z^2} g(z)$$



Conclusions

Spin-orbit coupling: ubiquitous phenomena in atomic physics, chemistry, and QCD

New momentum sum rule: momentum version of Jaffe-Manohar

New QCD-QIS connection:
Maximal entanglement between spin and OAM for quarks and gluons

Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?