# (D)DVCS at the Precision Frontier

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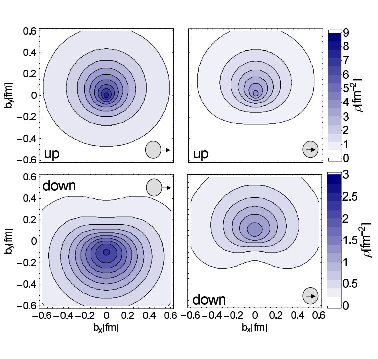
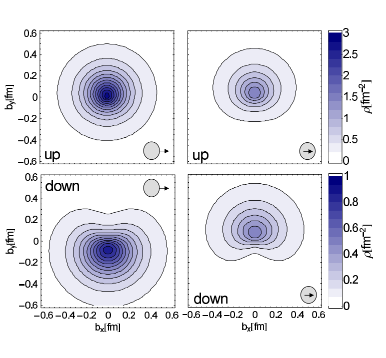
MITP, Mainz, 30.10.2024

## Nucleon Tomography

### access to three-dimensional picture of the nucleon (M. Burkardt)



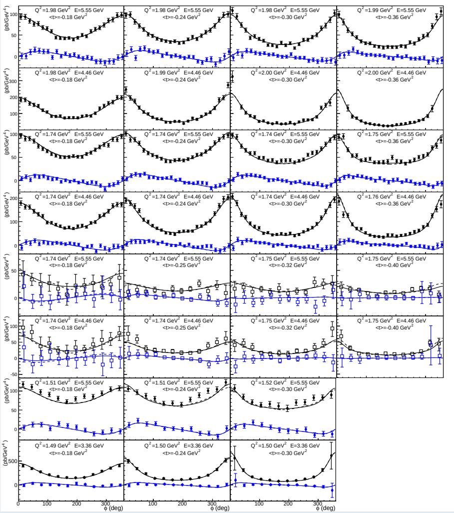




↪ **first two moments of transverse spin parton density**

computer simulations: M. Göckeler *et al.*, PRL 98 (2007) 222001

- **Momentum transfer $t$ defines the resolution of spacial imaging**

# Wealth of new data




2010 data of E07-007 and E08-025 [2109.02076]

- **High statistical accuracy**
- **Several beam energies**
- **Neutron/deuteron**
- **Coherent DVCS from $^4$He**

- ○ **Transverse polarization**

## In this talk, a status update:

1. **Towards NNLO accuracy**
   - Two-loop coefficient functions for DVCS ✓
   - Three-loop evolution equations for GPDS ✓ (flavor nonsinglet)
   - new: Two-loop coefficient functions for DDVCS ✓ (flavor nonsinglet, vector)

2. **Resummation of threshold logarithms in DVCS and (new) DDVCS** ✓

3. **Kinematic power corrections** $(\sqrt{-t}/Q)^k$, $(m/Q)^k$
   - Twist-four corrections, $(\sqrt{-t}/Q)^2$, $(m/Q)^2$ ✓
   - new: Twist-six corrections, $(\sqrt{-t}/Q)^3$, $(m/Q)^4$ ✓

## NNLO coefficient functions

To the leading-twist accuracy

$$\mathcal{A}_{\mu\nu}^{\rm DVCS} = -g_{\mu\nu}^{\perp} V + \epsilon_{\mu\nu}^{\perp} A + \dots$$
$$V(\xi, Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C_V(x/\xi, Q^2/\mu^2)\, F_q(x,\xi,t,\mu)\,.$$

$$F_q(x,\xi) = \frac{1}{2P_+}\left[H_q(x,\xi,t)\bar{u}(p')\gamma_+ u(p) + E_q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2m_N}u(p)\right].$$

— $C_{V(A)}$ are functions of one variable $x/\xi$
— real functions for $|x| < \xi$
— can be continued analytically to $|x/\xi| \geq 1$ using $\xi \to \xi - i\epsilon$ prescription
— $C_V(-x/\xi) = -C_V(x/\xi)$, $\quad C_A(-x/\xi) = +C_A(x/\xi)$

In perturbation theory

$$C(x/\xi, Q^2/\mu^2) = C^{(0)}(x/\xi) + a_s C^{(1)}(x/\xi, Q^2/\mu^2) + a_s^2 C^{(2)}(x/\xi, Q^2/\mu^2) + \dots \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}$$

with, e.g., flavor-nonsinglet — X. D. Ji and J. Osborne, PRD 57, 1337 (1998)

$$C_V^{(0)}(x/\xi) = \frac{\xi}{\xi - x} - \frac{\xi}{\xi + x},$$

$$C_V^{(1)}(x/\xi, 1) = \frac{2C_F\xi}{\xi - x}\left[-\frac{9}{2} - \frac{1}{2}\ln^2 2 + \left[\frac{1}{2}\ln\left(1 - \frac{x}{\xi}\right) - \frac{3}{2}\frac{\xi - x}{\xi + x}\right]\ln\left(1 - \frac{x}{\xi}\right)\right] - (x \leftrightarrow -x).$$

$C_A^{(1)}$ known from — E. Braaten, PRD28, 524 (1983)

## Recent: two-loop CFs

- Flavor-nonsinglet calculated using two different techniques

$C_V^{(2)}$ : V.Braun, A.Manashov, S.Moch, J.Schönleber, JHEP **09**, 117 (2020)
J. Schönleber, unpublished

$C_A^{(2)}$ : V.Braun, Manashov, Moch, Schönleber, 2106.01437
J.Gao, T.Huber, Y.Ji and Y.M.Wang, 2106.01390

- Flavor-singlet CFs:

$C_V^{(2)}$ : V.Braun, Y. Ji, J. Schönleber, PRL **129** 172001 (2022)

$C_A^{(2)}$ : Y. Ji, J. Schönleber, JHEP **01** (2024) 053

- Heavy-quark contrubutions only known to one loop accuracy

J.D. Noritzsch, PRD **69** 094016 (2004)

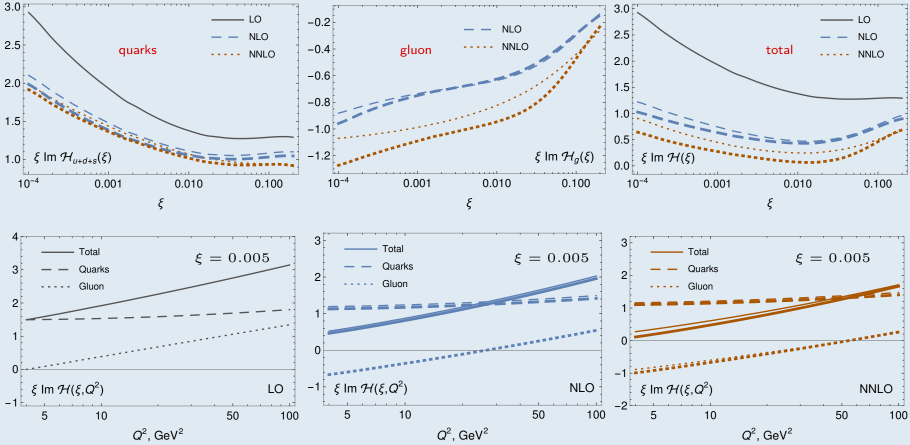
## Example (flavor-nonsinglet)

$$C_V^{(2)}(x) = C_F^2 C_P^{(2)}(x) + \frac{C_F}{N_c} C_{NP}^{(2)}(x) + \beta_0 C_F C_\beta^{(2)}(x)$$

$$\begin{aligned}
C_{NP}^{(2)} = {} & 6(1-2\omega)\Big\{ \mathrm{H}_{20} - \mathrm{H}_3 + \mathrm{H}_{110} - \mathrm{H}_{12} + \zeta_2\Big(\mathrm{H}_0 + \mathrm{H}_1\Big) - 3\zeta_3 \Big\} \\
& + 12\Big(\mathrm{H}_{10} - \mathrm{H}_2 - \mathrm{H}_0 - \mathrm{H}_1 + \zeta_2\Big) + \frac{3}{\bar\omega}\mathrm{H}_0 + \frac{3}{\omega}\mathrm{H}_1 \\
& + \Big\{ \frac{1}{\omega}\Big(12\zeta_3 - \frac{3}{2}\zeta_2^2 - \frac{5}{2}\zeta_2 - \frac{73}{24}\Big) - \frac{3}{\omega}\mathrm{H}_{200} - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{30} + \Big(\frac{4}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_4 \\
& - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{210} + \Big(\frac{3}{\omega} - \frac{2}{\bar\omega}\Big)\mathrm{H}_{22} - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{31} - \frac{5}{\bar\omega}\mathrm{H}_3 + \frac{5}{\bar\omega}\mathrm{H}_{20} \\
& + \Big(\frac{1}{\bar\omega}\Big(\zeta_2 - \frac{9}{2}\Big) + \frac{1}{\omega}\Big(\frac{4}{3} - 2\zeta_2\Big)\Big)\mathrm{H}_{00} - \Big(\frac{2}{\omega}\Big(\zeta_2 - 1\Big) - \frac{1}{\bar\omega}\Big(\zeta_2 + \frac{7}{6}\Big)\Big)\mathrm{H}_2 \\
& + \Big(\frac{1}{\bar\omega}\Big(\frac{19}{6} + 5\zeta_2 - 3\zeta_3\Big) + \frac{1}{\omega}\Big(7\zeta_3 - \frac{16}{9}\Big)\Big)\mathrm{H}_0 - (\omega \leftrightarrow \bar\omega) \Big\}
\end{aligned}$$

where $\omega = (1-x)/2$, $\bar\omega = (1+x)/2$, and $\mathrm{H}_{\vec m} \equiv \mathrm{H}_{\vec m}(\omega)$ are harmonic polylogarithms

## Numerical estimates: Imaginary part of the Compton form factor $\mathcal{H}$, $t = -0.1$ GeV$^2$

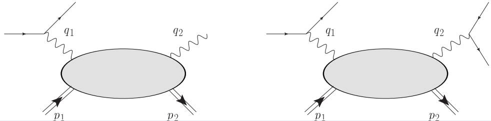



GK-model, normalized at input scale $\mu^2 = 4$ GeV$^2$ to HERAPDF20 (thin lines) and ABMP16 (thick)

— the gluon contribution is large and negative, enhanced at NNLO

## New: DDVCS

- Why DDVCS?




$$V(\xi,\eta,Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\eta - x - i\epsilon} H_q(x,\xi,t) \qquad \frac{\xi}{\eta} = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}$$

- Direct access to GPDs at $x \neq \xi$, e.g.,

$$q_2^2 = 2.5\,\text{GeV}^2 : \qquad \begin{cases} q_1^2 = -0.3\,\text{GeV}^2 \to \frac{\xi}{\eta} = 1.27 \\ q_1^2 = -0.6\,\text{GeV}^2 \to \frac{\xi}{\eta} = 1.67 \end{cases}$$

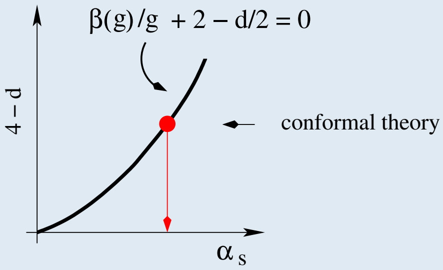
## Conformal symmetry in QCD?

QCD is not a conformal theory, but

$$\mathcal{A}_{\rm QCD} = \mathcal{A}_{\rm QCD}^{\rm conf} + O(\beta(\alpha_s))$$

"Conformal QCD": QCD in $d-2\epsilon$ at Wilson-Fischer critical point $\beta(\alpha_S) = 0$

V.B., A. Manashov, Eur.Phys.J.C 73 (2013) 2544



## Two-loop coefficient function in DDVCS

In conformal QCD

$$C_{\perp(L)}(\omega x,\omega) = \int dx' \left\{ \frac{\omega}{(1-\omega x)^{1(2)+\frac{1}{2}\gamma_N}} + (\omega \to -\omega) \right\} K_{\perp(L)}(x',x)\,, \qquad \omega = \frac{\xi}{\eta}$$

where $K_i(x,x')$ are $\mathrm{SL}(2)$-invariant operators that do not depend on $\omega$

$$\int dx'\, K_i(x',x)\, P_{N-1}^{(\lambda_N)}(x') = K_i\,(N)\; P_{N-1}^{(\lambda_N)}(x)\,.$$

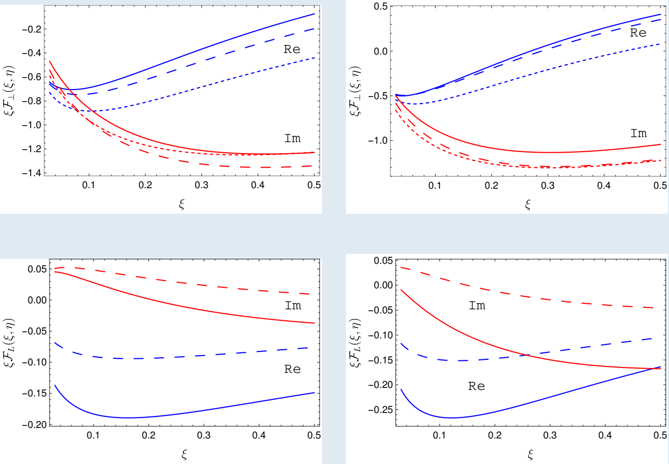
with the spectrum

$$K_\perp(N) = \frac{\Gamma(N)\Gamma(1+\frac{1}{2}\gamma_N)}{\sigma_N \Gamma(N+\frac{1}{2}\gamma_N)} C_1^{DIS}(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*)$$

$$K_L(N) = \frac{\Gamma(N)\Gamma(2+\frac{1}{2}\gamma_N)}{\sigma_N \Gamma(N+1+\frac{1}{2}\gamma_N)} C_L^{DIS}(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*)$$

# Two-loop coefficient function in DDVCS

$\xi/\eta = 1.67$ $\qquad\qquad$ $\xi/\eta = 1.27$




LO: short dashes; NLO: long dashes; NNLO: solid curves

V.B., Hua-Yu Jiang, A.N. Manashov, A. von Manteuffel, paper in preparation

## Resummation of threshold logarithms

Sudakov-type double logarithms in the CFs:

$$C_V(x/\xi, a_s) \sim \frac{1}{1-x/\xi}\left[1 + a_s C_F \ln^2\left(1-\frac{x}{\xi}\right) + \frac{1}{2}(a_s C_F)^2 \ln^4\left(1-\frac{x}{\xi}\right) + \dots\right]$$

Resummation to the NNLL accuracy J. Schoenleber, JHEP **02** (2023), 207

$$C_V(x/\xi, a_s) \sim \frac{1}{1-\frac{x}{\xi}} \exp\left\{\frac{1}{2}\int_{Q^2(1-\frac{x}{\xi})}^{Q^2} \frac{d\mu^2}{\mu^2}\left[-\Gamma_{\rm cusp}(\alpha_s(\mu)) \ln\frac{Q^2(1-\frac{x}{\xi})}{\mu^2} + \gamma_f(\alpha_s(\mu))\right]\right\}$$
$$\times\, H(\alpha_s(Q))\, F(\alpha_s(\sqrt{1-\tfrac{x}{\xi}}\,Q))$$

$\leftarrow \gamma_f$, $H$ and $F$ are known to $\mathcal{O}(\alpha_s^2)$

new: Threshold logarithms in DDVCS (J. Schoenleber, paper in preparation)

## Evolution equations for GPDs

- Two loops (NLO): singlet + nonsiglet

A. Belitsky, A. Freund, D. Müller, NPB 574, 347 (2000)

— checked by an independent calculation

— evolution code available but not general enough

- Three loops much more difficult:

Conformal symmetry:

- Make use of the NNLO results for anomalous dimensions
- One loop less compared to direct calculation

## Evolution equations for GPDs

Methods:

- Two-loop conformal anomaly V.B., A.Manashov, S. Moch, M. Strohmaier, JHEP **03** (2016), 142

  $\Rightarrow$ Three-loop evolution equations for flavor-nonsinglet light-ray operators

  V.B., A.Manashov, S. Moch, M. Strohmaier, JHEP **06** (2017), 037
  Y. Ji, A. Manashov, S. Moch, PRD **108** (2023) 054009

- Orthogonality of conformal operators

  $\Rightarrow$ Three-loop mixing matrices for flavor-singlet operators with $N \leq 8$

  vector: V.B., K. Chetyrkin, A. Manashov, PLB **834** (2022) 137409

  axial-vector: V.B., K. Chetyrkin, A. Manashov, in progress

- Numerical impact expected to be moderate because of limited $Q^2$ range

# Kinematic power corrections $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$

- Ambiguity in the choice of collinear directions makes "leading-twist" calculations ambiguous. In addition, electromagnetic Ward identities are violated.
  - Repaired by power-suppressed corrections, $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$
  - "Kinematic" — do not involve new nonperturbative input apart from usual GPDs
  - Factorizable

- Twist-four completed

  V.B., A. Manashov, JHEP **01** (2012), 085 ← method
  V.B., A. Manashov, D. Müller, B. Pirnay, PRD **89** (2014) 074022

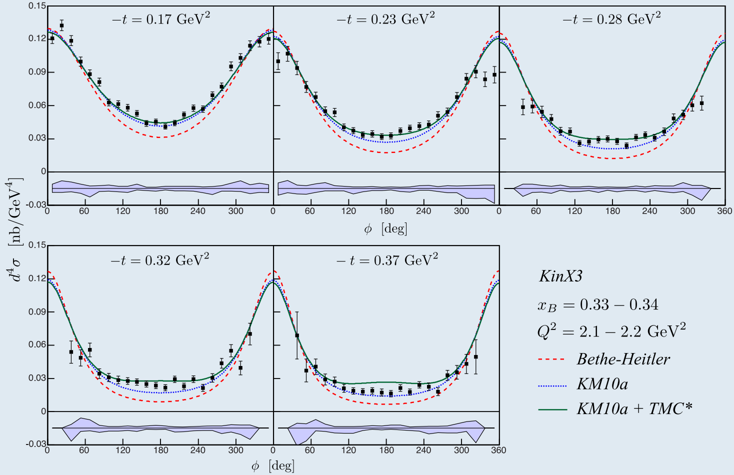
  - Large effects in certain regions of phase space

- Twist-six in progress

  V.B., Y. Ji, A. Manashov, JHEP **03** (2021), 051 ← method
  V.B., Y. Ji, A. Manashov, JHEP **01** (2023), 078 ← scalar target
  new: V.B., Y. Ji, A. Manashov, in preparation ← nucleon

# Large kinematic corrections for the total cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453




GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

## Operator Product Expansion

schematically

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\ldots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$
$$+ C_N^{\mu_1\ldots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\ldots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\}$$
$$+ \text{ quark-gluon operators}$$

"kinematic" corrections that repair the frame dependence and Ward identities come from

(1) corrections $m/Q$ and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)

(2) higher-twist operators that are obtained from twist-two by adding total derivatives

Problem: matrix elements of some descendant operators over free quarks vanish

Ferrara, Grillo, Parisi, Gatto, '71-'73

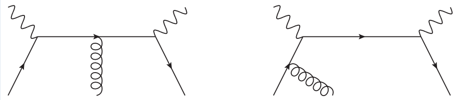
**Example**

$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,, \qquad\qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

- Usual procedure to calculate the coefficient functions does not work

VB, A. Manashov, D. Müller, B. Pirnay '11-'14

- Consider quark-gluon matrix elements



- Use hermiticity of evolution equations for twist-4 operators to separate "kinematic" terms

## New approach: all twists

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\cdots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\cdots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\cdots\mu_N}}_{\text{descendants}} + \ldots \Big\} + \ldots$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\, \mathcal{O}^N_{\mu_1\cdots\mu_N} + \text{ quark-gluon operators}$$

S. Ferrara, A. F. Grillo and R. Gatto, 1971-1973: "Conformally covariant OPE"

In conformal field theories, the CFs of descendants are related to the CFs of twist-2 operators by symmetry and do not need to be calculated directly

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

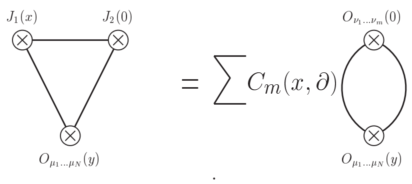
## Conformal triangles

A.M. Polyakov, 1970:

$$\langle O_1(x_1)\, O_2(x_2)\rangle = \frac{\text{const}}{|x_1-x_2|^{2\Delta_1}}\,\delta_{\Delta_1\Delta_2}$$

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}\,|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}\,|x_2-x_3|^{\Delta_3+\Delta_2-\Delta_1}}$$

- $\leftarrow$ $\Delta_k$ is a scaling dimension (canonical + anomalous)




- $\leftarrow$ exact to all orders of perturbation theory

- Done:

$$\mathcal{A}^{(\pm\pm)} \sim 1 + \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(\pm 0)} \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots \qquad \checkmark$$

$$\mathcal{A}^{(\pm\mp)} \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots \qquad \checkmark$$

- further terms can be calculated if necessary

- Observe:
  - — factorization valid at twist 6 (IR divergences cancel)
  - — target mass corrections absorbed in the dependence on $t_{min} = -\frac{\xi^2 m^2}{1-\xi^2}$

Compare DIS, Nachtmann variable

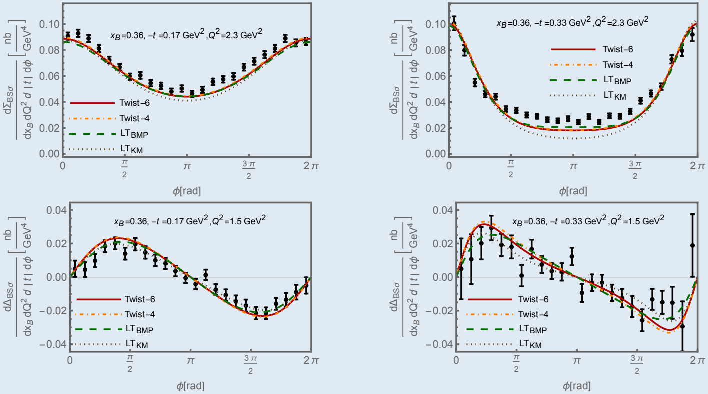
$$\xi_N = \frac{2x_B}{1+\sqrt{1+\frac{4x_B^2 m^2}{Q^2}}} = x_B\left(1 - \frac{x_B^2 m^2}{Q^2} + \dots\right)$$

- On a nucleus $m \mapsto Am$, $x_B \mapsto x_B/A$, $\xi \mapsto \xi/A$, hence TMCs are the same
  $\rightarrow$ factorization not in danger

## Cross sections

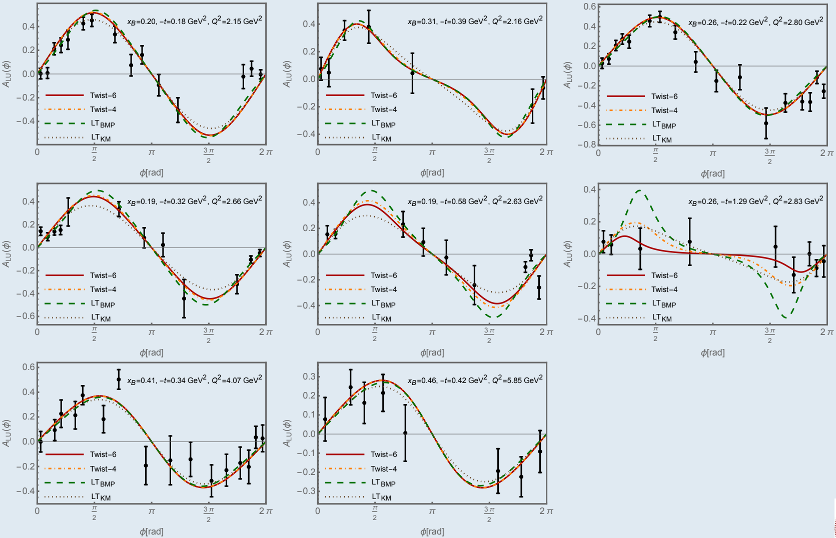
Hall A, nucl-ex/0607029, vs. KM12 PRELIMINARY

!!! Expansion parameter $1/Q^2 \to 1/(qq') = 2/(Q^2 + t)$

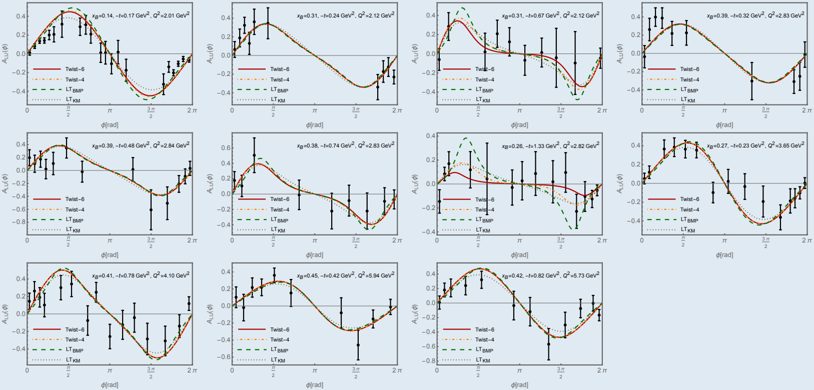



- red solid — twist 6
- orange dash-dotted — twist 4
- green dashed — BMP twist 2
- black dots — KM twist 2

# CLAS12 DVCS beam asymmetries, PRL. **130** (2023) 211902 (10.3 GeV)



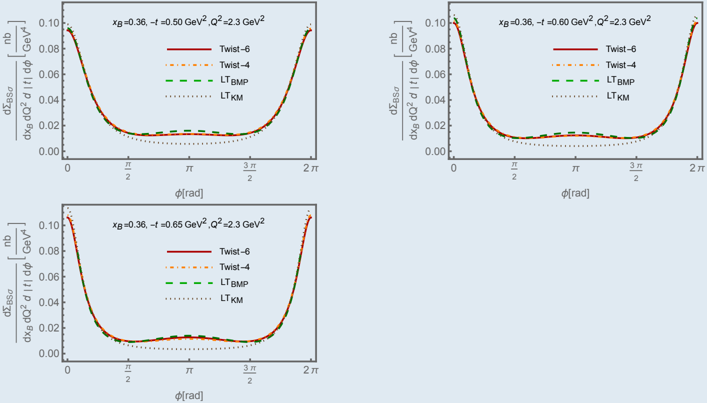
# CLAS12 DVCS beam asymmetries, PRL. **130** (2023) 211902 (10.6 GeV)



## Cross sections (2)

Increasing $t$

PRELIMINARY




!!! Strong cancellations in

$$\mathcal{F}_{0+}^{DIS} = -(1+\varkappa)\mathcal{F}_{0+}^{phot} + \varkappa_0\left[\mathcal{F}_{++}^{phot} + \mathcal{F}_{-+}^{phot}\right]$$

## Summary

1. **Towards NNLO accuracy**
   - Two-loop coefficient functions for DVCS
     — sizeable corrections, completed for light quarks
     — new: DDVCS, flavor-nonsinglet only
   - Three-loop evolution equations for GPDS
     — flavor-nonsiglet in position space, singlet for the first few moments
     — pressing issue: numerical implementation, also in NLO
   - Threshold resummations at $x \to \xi$
     — completed to NNLL; new: DDVCS,

2. **Kinematic power corrections**
   - new: Twist-six accuracy, $(\sqrt{-t}/Q)^3$, $(m/Q)^3$
     — complete results available, numerical code (B.Pirnay + ...)
     — good convergence if expansion organized in $1/(Q^2+t)$
     — large effects for parts of phase space and in collider kinematics
     — coherent DVCS from nuclei: Target mass corrections do not spoil factorization

3. **Further issues**
   — establishing NLO accuracy (at minimum) as standard of the field
   — GPDs from Compton form factors; Neural networks or ansätze?
   — $t$-dependence of “genuine” higher-twist contributions; models