New developments on PDFs & GPDs from lattice QCD and potential synergies

Martha Constantinou

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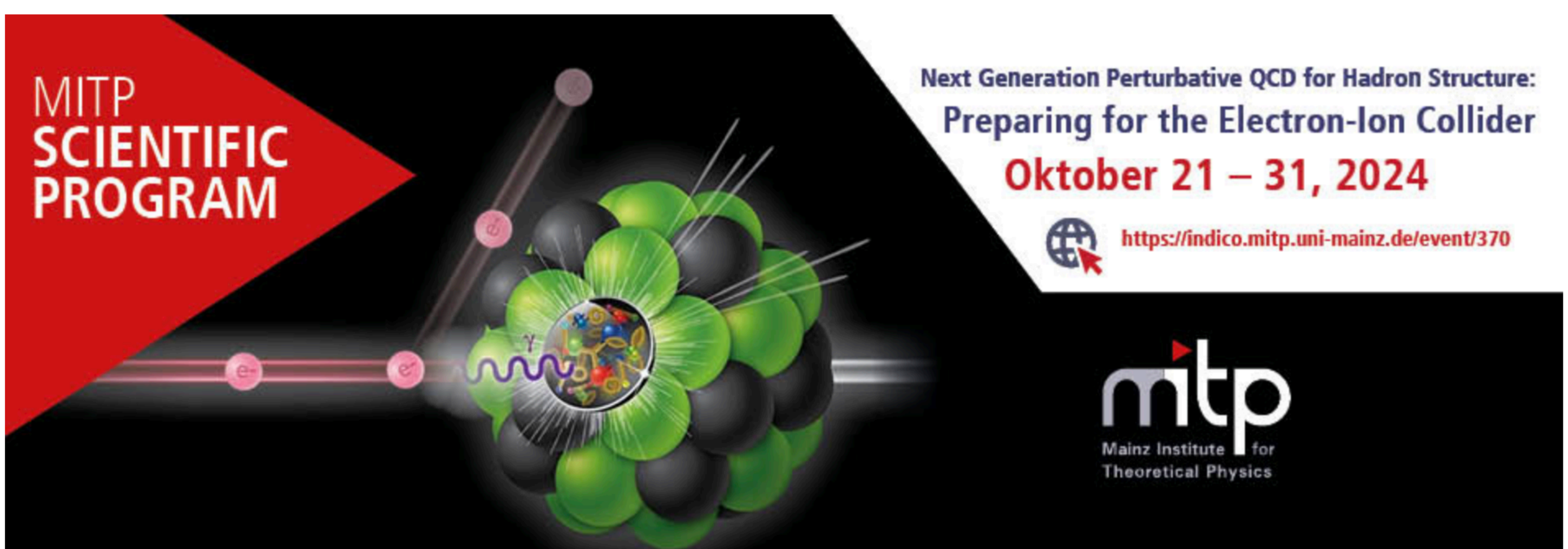

Next Generation Perturbative QCD for Hadron Structure: Preparing for the Electron-Ion Collider

Disclaimer

The field of GPDs is still developing and sources of systematic uncertainties have not been fully addressed

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- Discretization effects
- physical pion mass
- Volume effects
- inverse problem
- matching formalism
- connection to light-cone, higher twist contaminations, …
- …

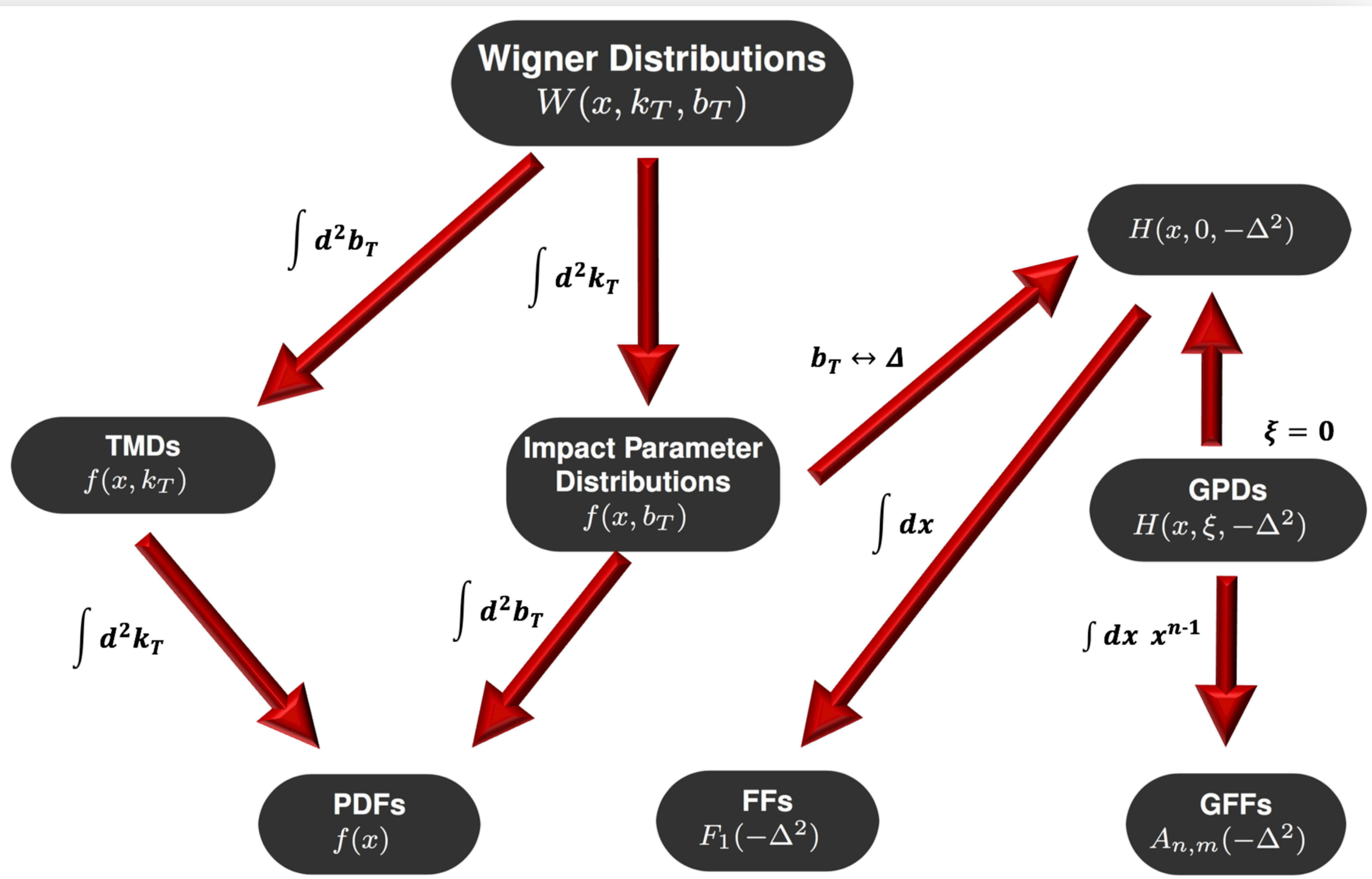
Accessing PDFs/GPDs from lattice QCD

Nucleon Characterization

Wigner distributions

★ Fully characterize partonic structure of hadrons

★ Provide multi-dim images of the parton distributions in phase space



★ Correlations between momenta, positions, spins

★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

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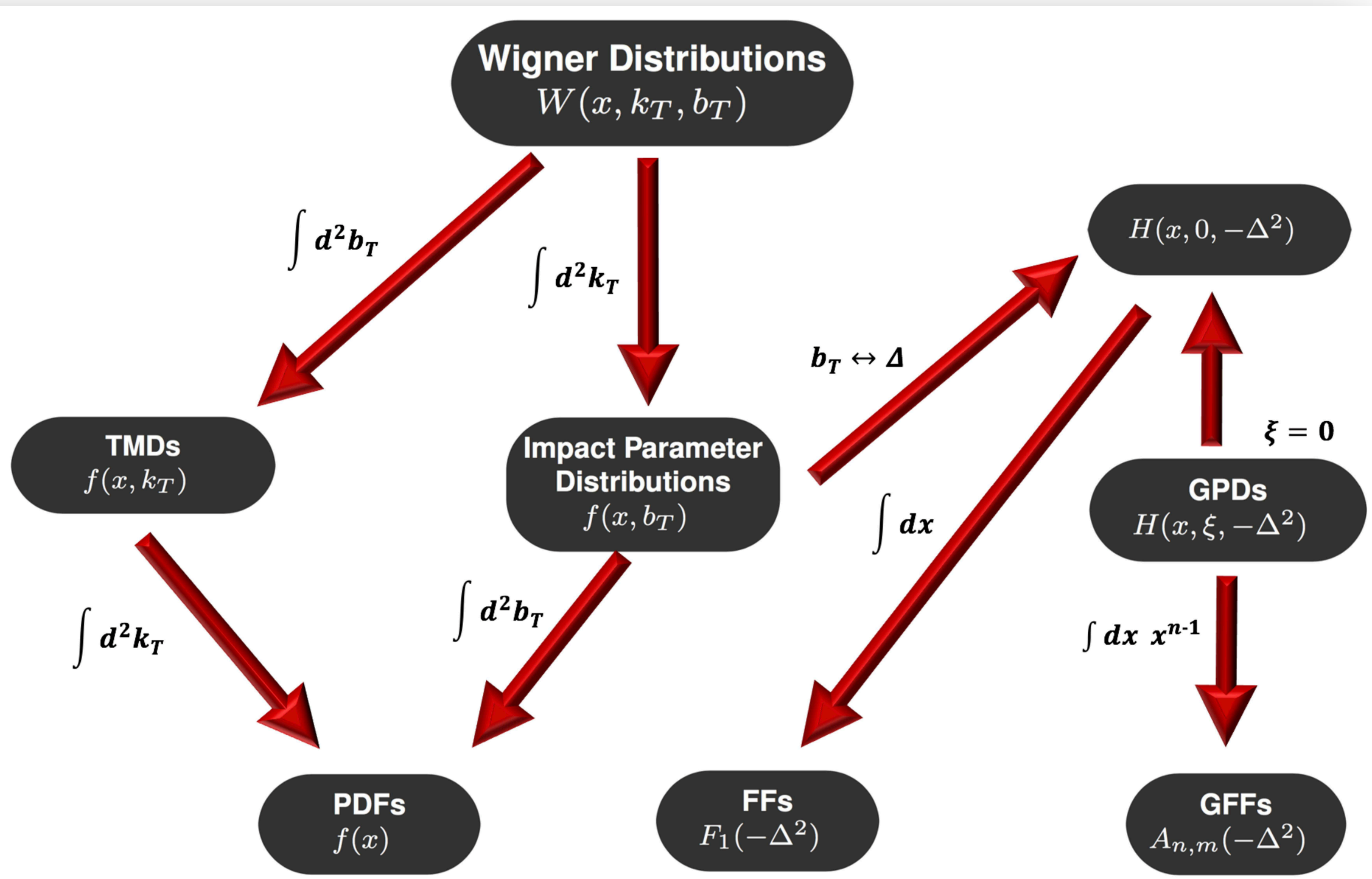
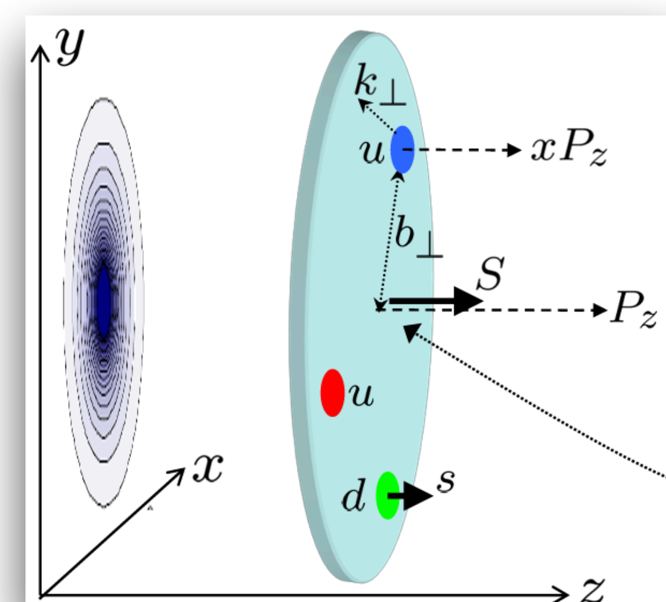


★ Partons contain information on
x: longitudinal momentum fraction
k_T: transverse momentum
$b_\perp$: impact parameter



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Accessing information on PDFs/GPDs

★ In parton model, physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

★ PDFs parameterized via matrix elements of nonlocal light-cone operators

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

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★ **Mellin moments**

(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z, \tfrac{1}{2}z]\,q(\tfrac{1}{2}z) = \sum_{n=0}^{\infty} \frac{1}{n!}\, z_{\alpha_1} \ldots z_{\alpha_n} \Big[\underbrace{\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q}_{\text{local operators}}\Big]$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1} \left\{ \gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} A_{n,i}(t) - i\frac{\Delta_\alpha \sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}} B_{n,i}(t) \right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big) \Big\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2)\Big|_{n\ \text{even}} \Big] U(P)$$

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local operators

Reconstruction of PDFs/GPDs very challenging

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(quasi-GPDs, pseudo-GPDs, …)

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Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

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This talk

Novel Approaches

★ Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
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Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
PDFs without Wilson line	[Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
Moments of PDFs of any order	[A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

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★ Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248

- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543

- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Novel Approaches

Good lattice cross sections

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

x-dependent distribution functions

quasi-PDFs

[X. Ji, PRL 110 (2013) 262002]

pseudo-PDFs

[A. Radyushkin, PRD 96, 034025 (2017)]

current-current correlators

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

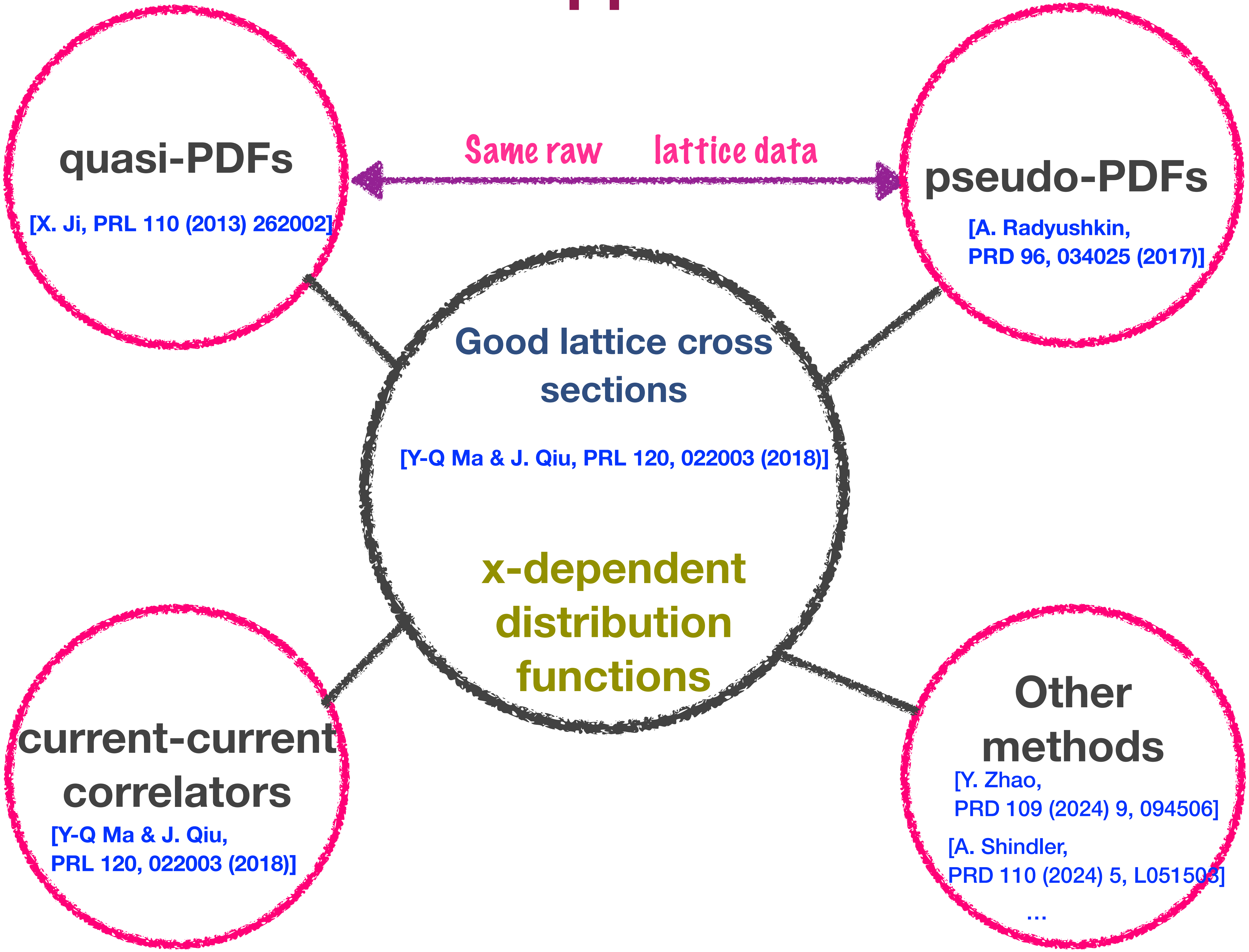
Other methods

[Y. Zhao, PRD 109 (2024) 9, 094506]

[A. Shindler, PRD 110 (2024) 5, L051503]

...

Novel Approaches



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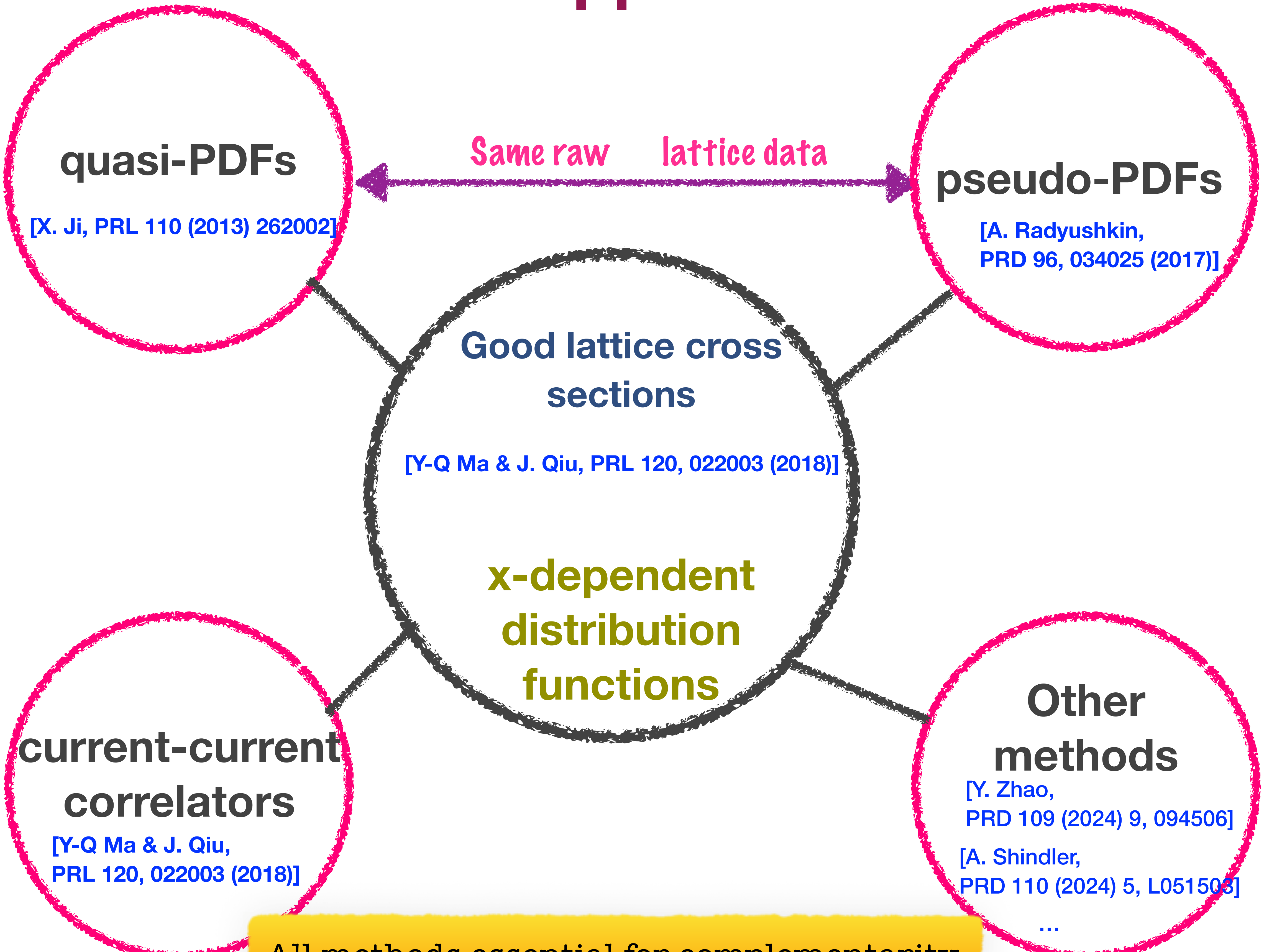

All methods essential for complementarity

Well-studied "novel" methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with boosted hadrons

$$\mathscr{M}(P_f, P_i, z) = \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

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quasi-PDFs

$$\tilde{q}_\Gamma^{\rm GPD}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\mathcal{M}(P_f, P_i, z)$$

pseudo-ITD

[A. Radyushkin, PRD 96, 034025 (2017)]

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathcal{M}(\nu, \xi, t; z_3^2)}{\mathcal{M}(0,0,0; z^2)} \qquad (\nu = z \cdot p)$$

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Matching in momentum space
(Large Momentum
Effective Theory)

Matching in ν space

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2)$$

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- length of the Wilson line (z)
- nucleon momentum boost (P_3)
- momentum transfer (t)
- skewness (ξ)

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} PDFs, GPDs

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} GPDs

PDFs:

The unpolarized case

See also QCD@LHC 2024

PDFs:

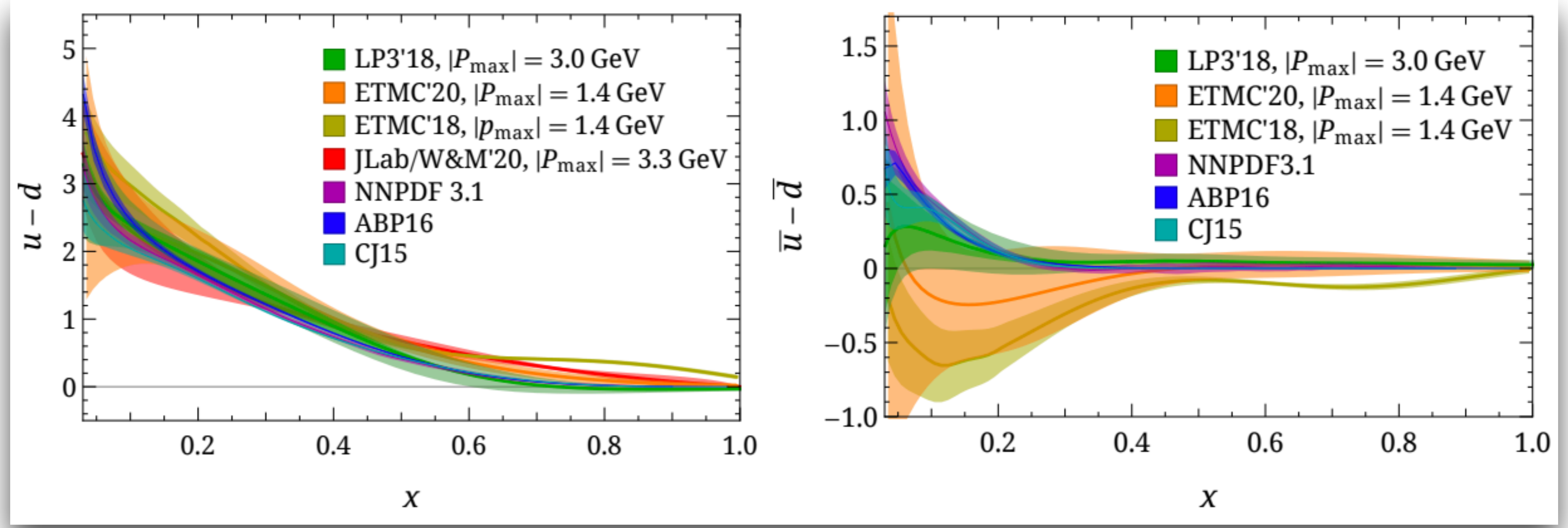
The unpolarized case

See also QCD@LHC 2024

- Results presented in MSbar scheme at 2 GeV

- Errors reported are statistical (in majority of cases)

Collection of results



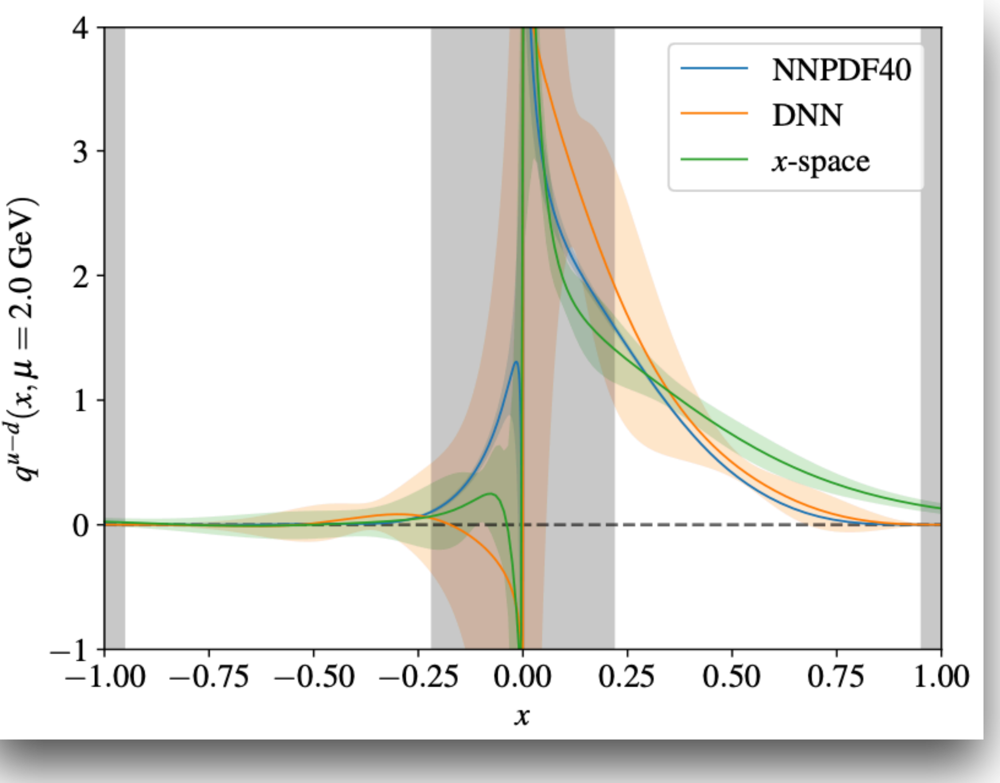
[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908]

★ Several improvements:
- More calculations at physical quark masses
- Ensembles at various lattice spacings
- Addressing systematic uncertainties due to methodologies

Refining the unpolarized proton PDF (u-d)

★ Physical quark masses

- HISQ, a=0.076 fm, P~1.5 GeV
- Deep Neural Network for inverse problem
- NNLO for matching



[X. Gao et al., PRD 107 (2023) 7, 074509]

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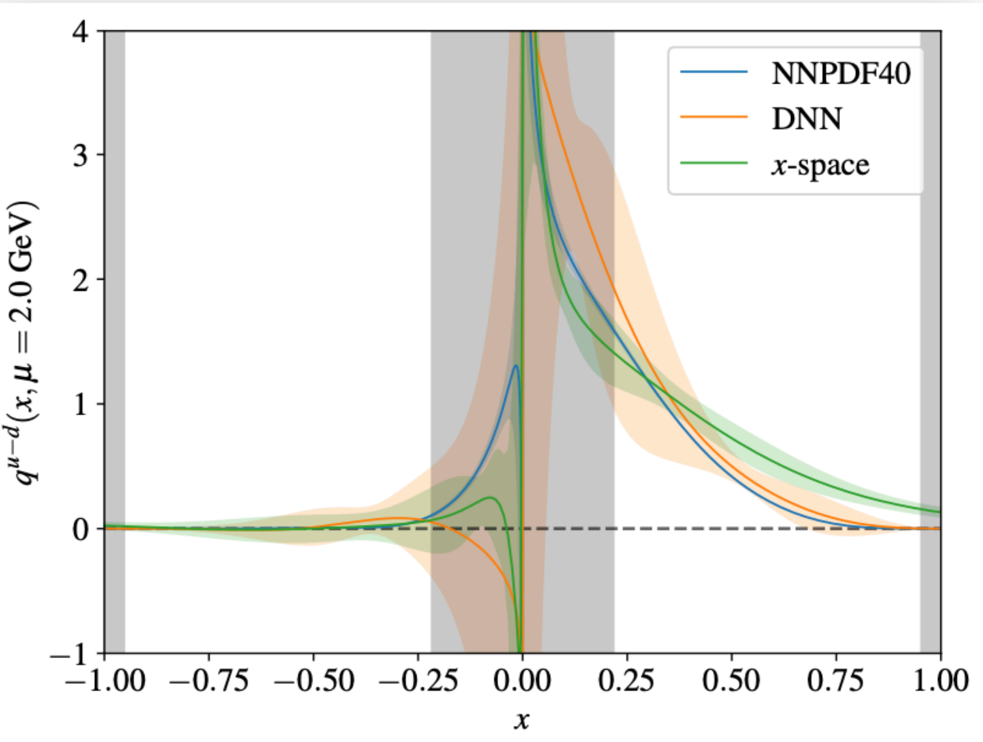
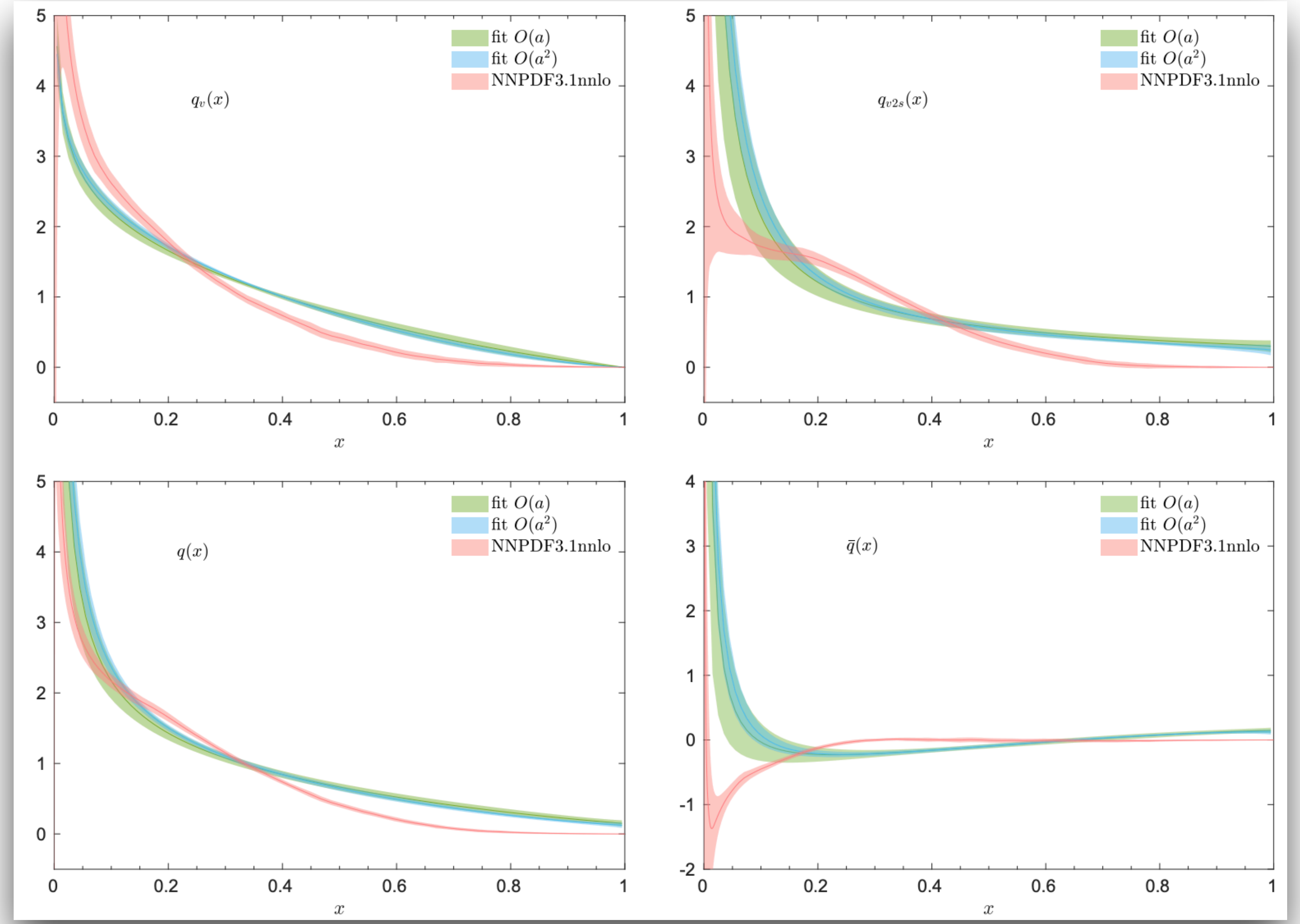


[X. Gao et al., PRD 107 (2023) 7, 074509]

★ **Continuum limit**

- TM&clover, a=0.09 fm, m_π=350 MeV
- P~1.8 GeV
- NNLO for matching

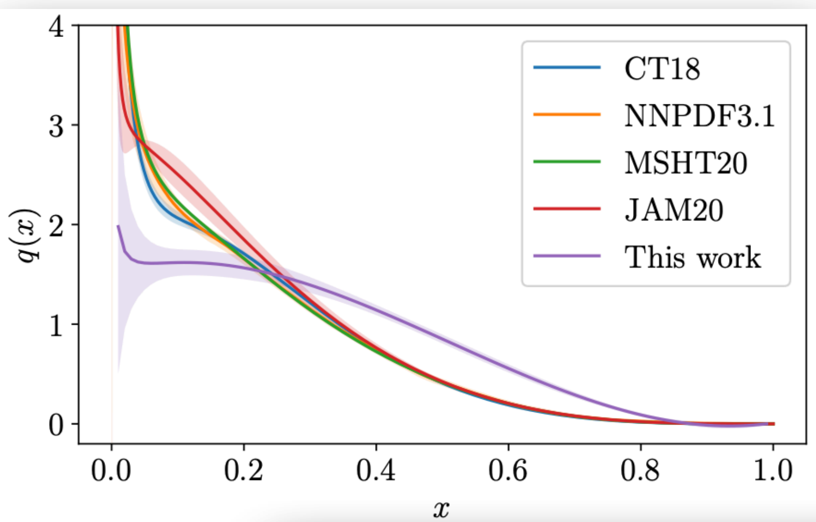


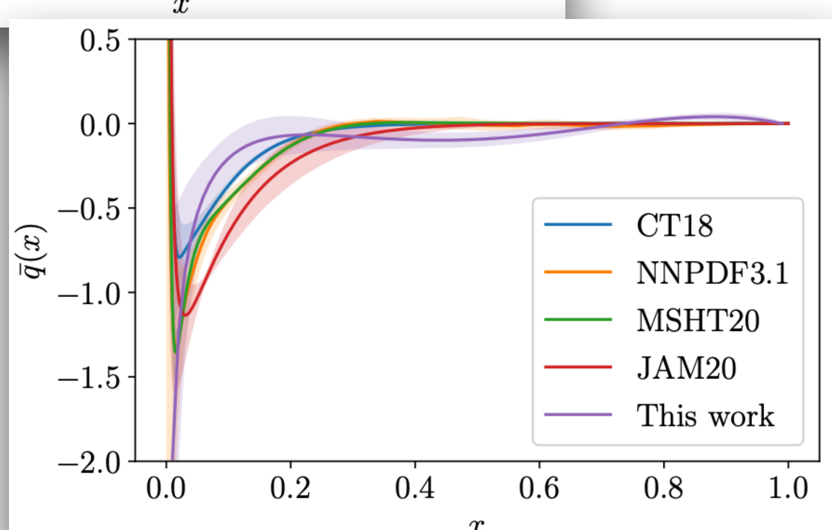
[Bhat et al., PRD 106 (2022) 5, 054504]

Improving evolution of PDFs

★ Continuum limit - higher twist effects

- Clover, a=0.075, 0.065, 0.048 fm
- m_π=440 MeV
- Jacobi polynomials for controlling finite-a & higher twist



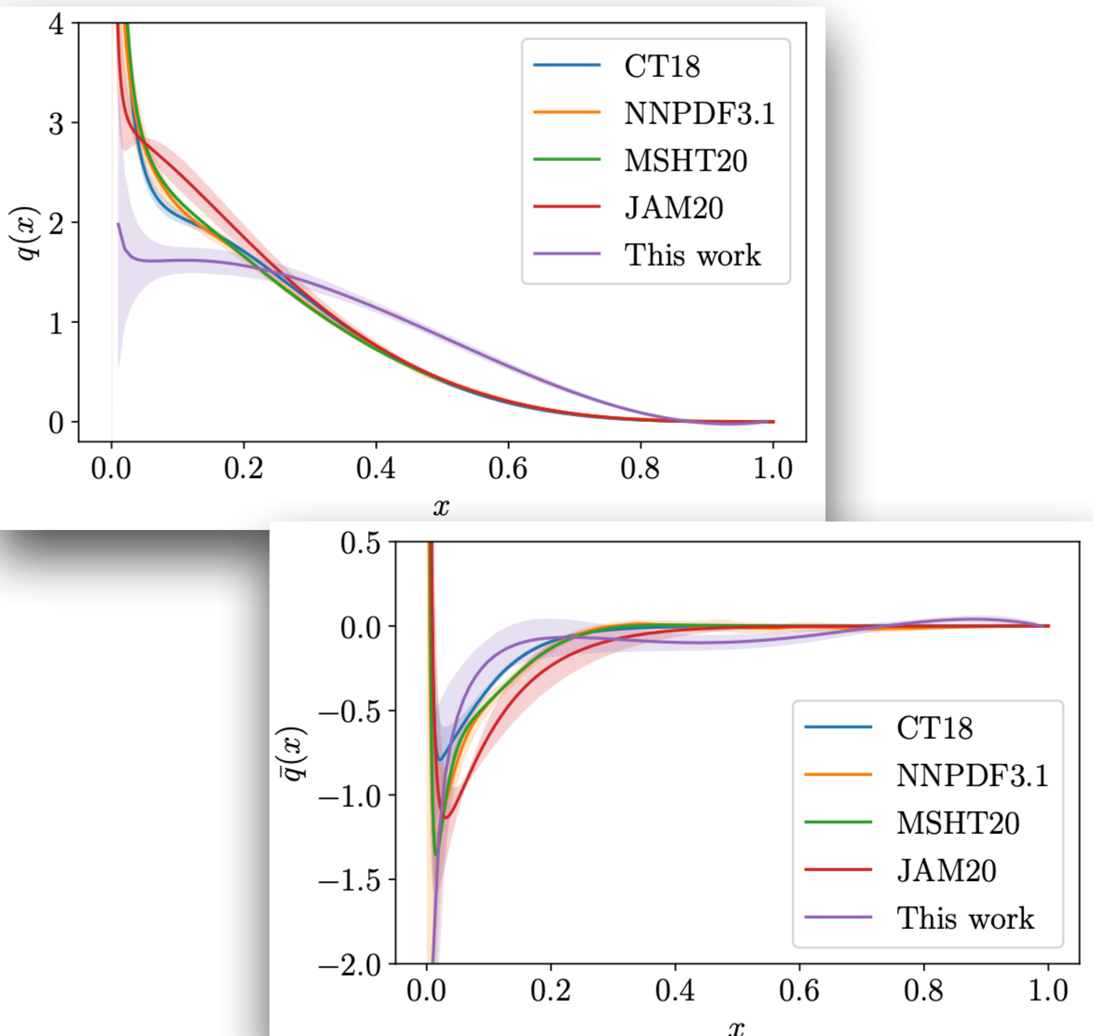


[Karpie et al., JHEP 11 (2021) 024]

Improving evolution of PDFs

★ Continuum limit - higher twist effects

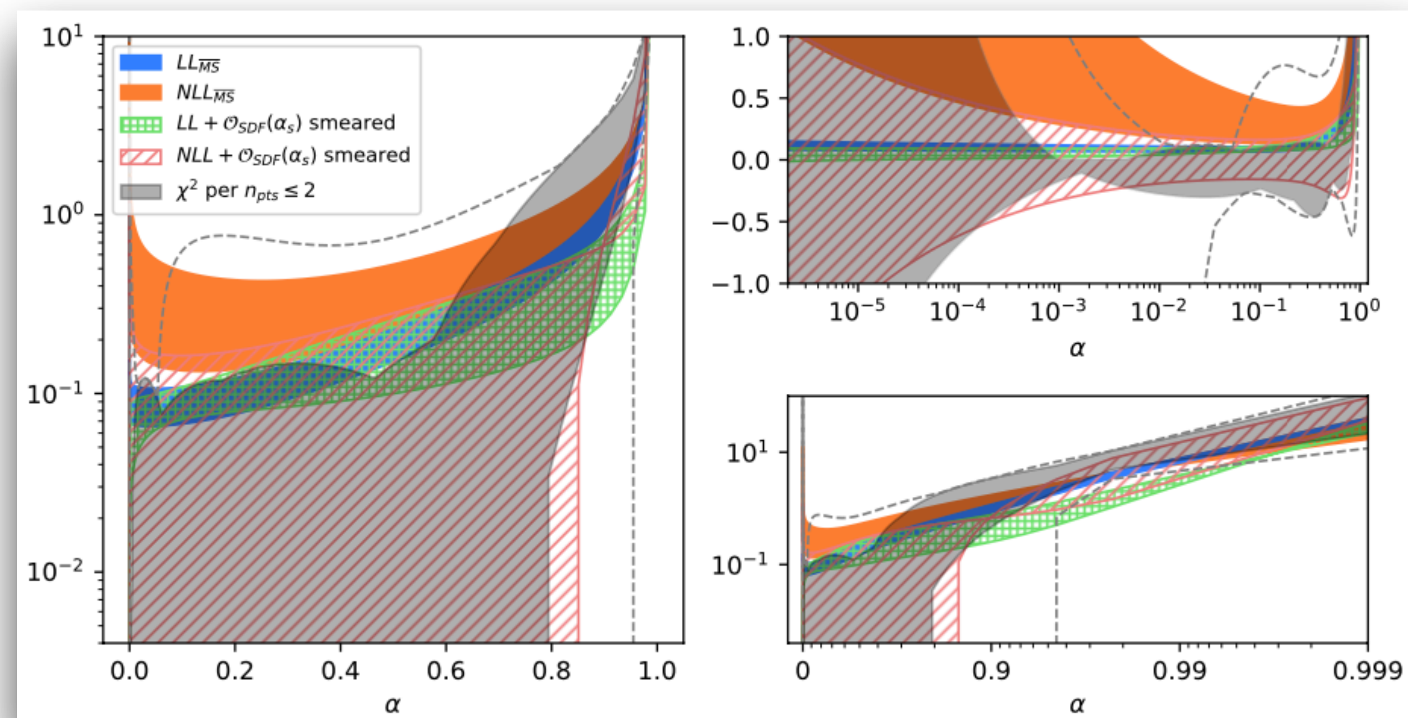
- Clover, a=0.075, 0.065, 0.048 fm
- m_π=440 MeV
- Jacobi polynomials for controlling finite-a & higher twist



[Karpie et al., JHEP 11 (2021) 024]

★ Non-perturbative scale evolution of pseudo distributions:

- lattice scale much different than scale for light-cone PDFs
- addresses the subtle z^2 behavior of matrix elements
- reduces fluctuation of lattice data

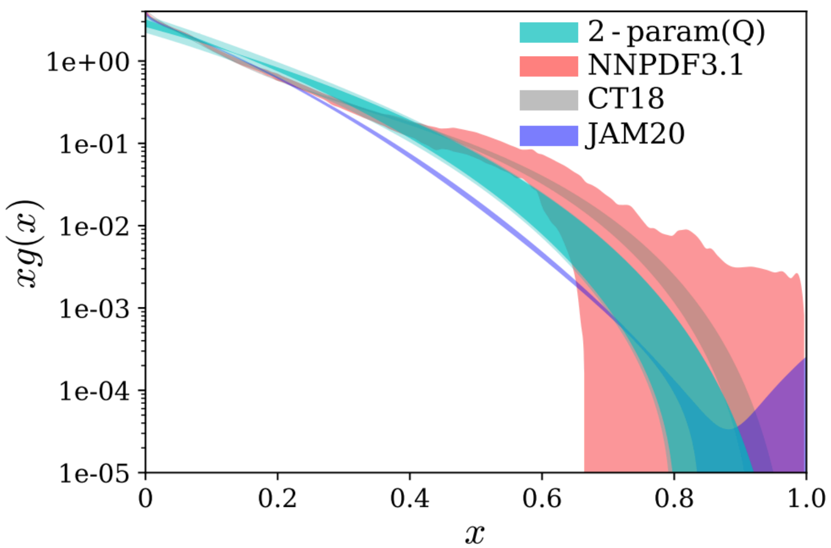


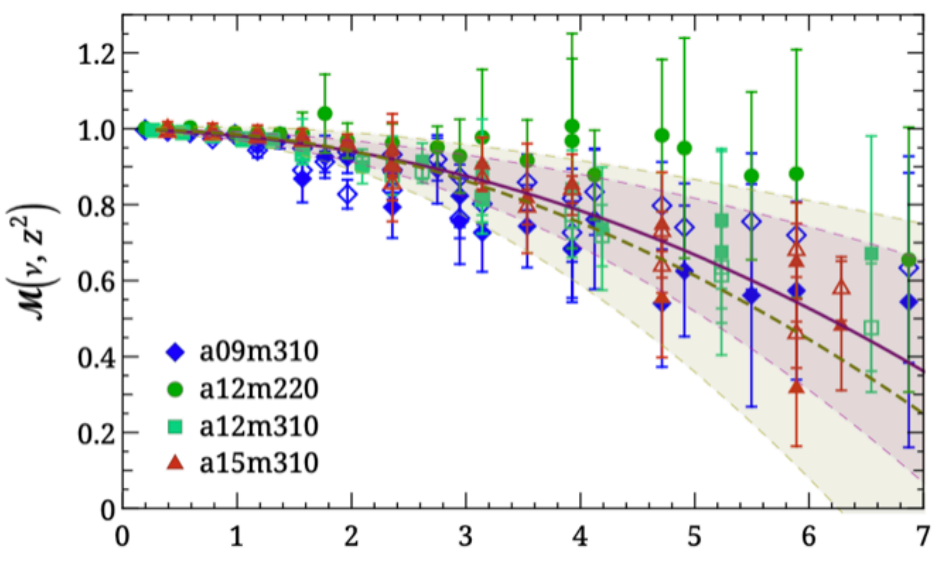
[H. Dutrieux et al. (HadStruc), JHEP 04 (2024) 061]

Evolution of vector operator much larger than anticipated

Gluon PDF

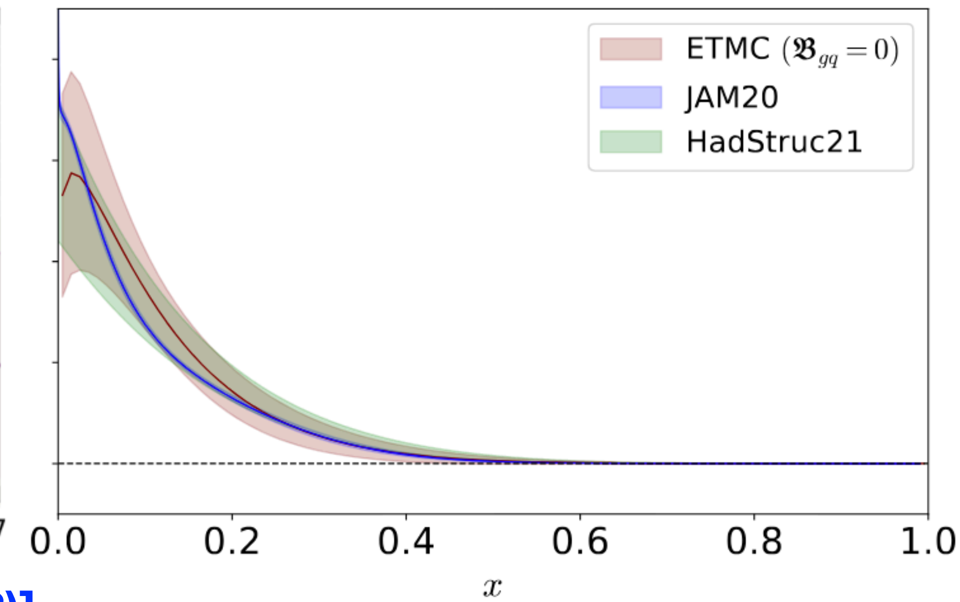
[Khan et al., JHEP 11, 148 (2021)]





[Fan et al., Phys. Rev. D 108, 014508 (2023)]

[Delmar et al., PRD 108 (2023) 9, 094515]



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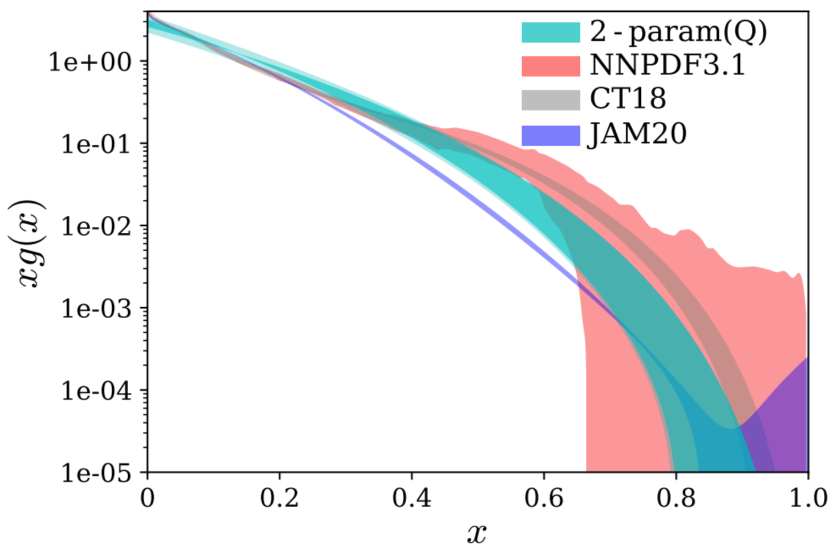

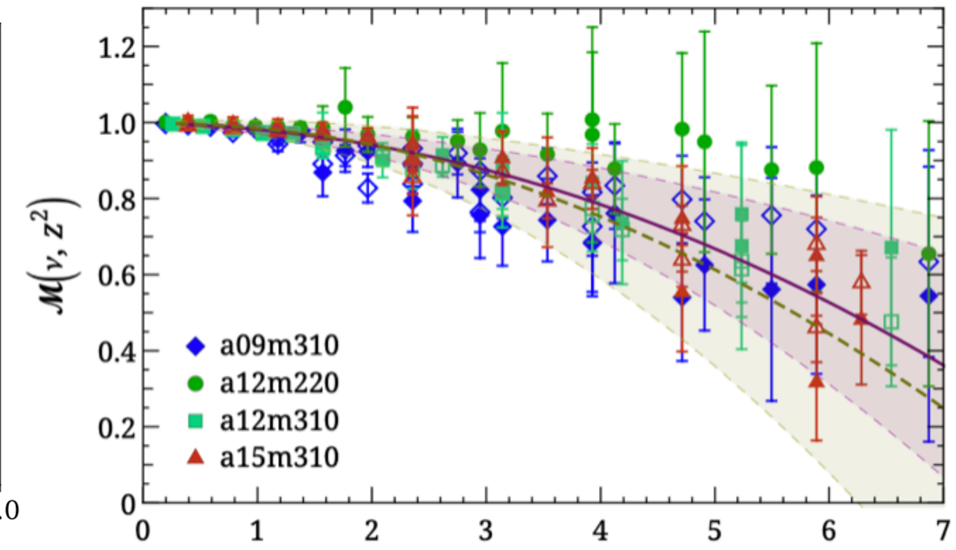

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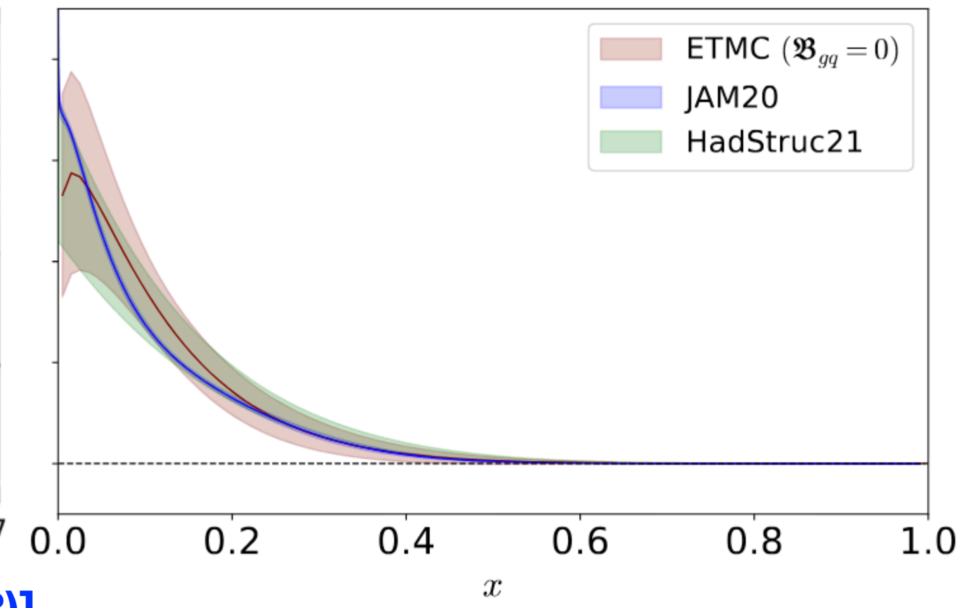
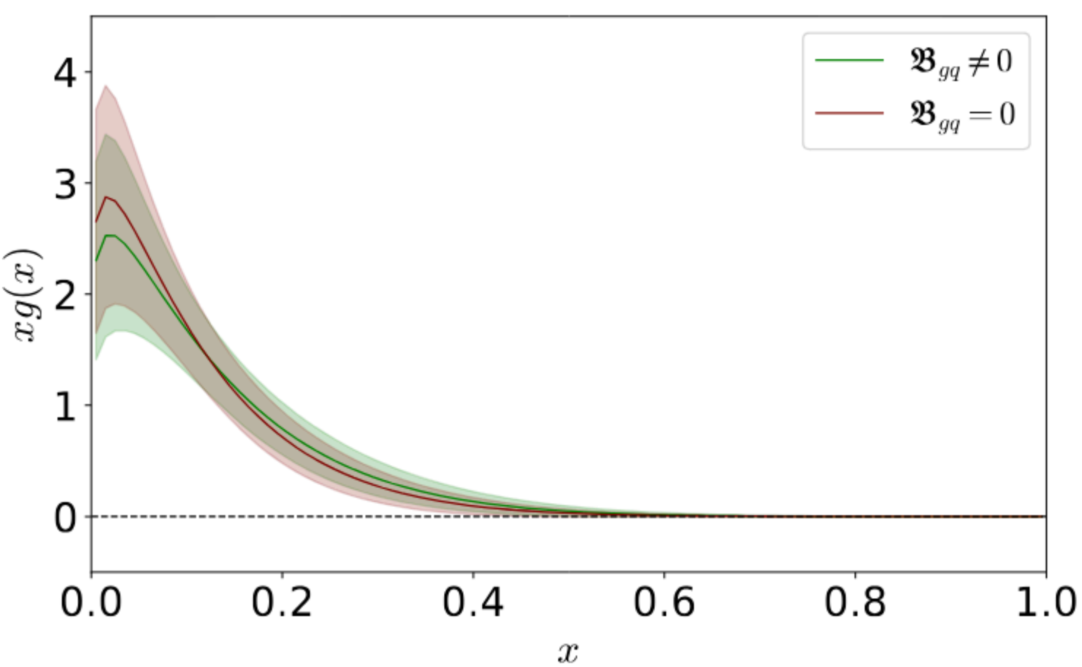
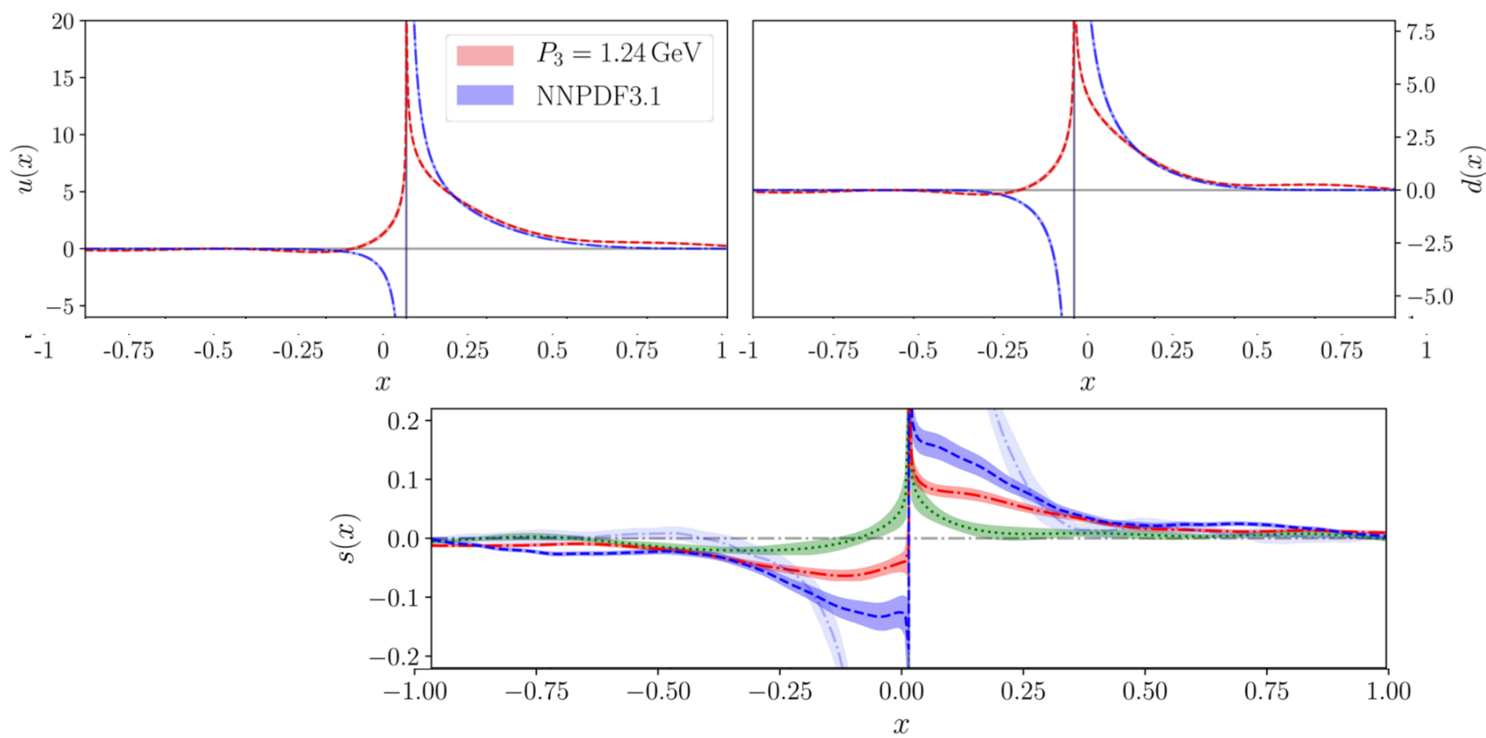


★ Elimination of Mixing with Quark Singlet PDFs

[J. Delmar et al., PRD 108 (2023) 9, 094515]



Dedicated calculation of quark-single PDFs



[C. Alexandrou, PRD 104 (2021) 5, 054503]

Lattice QCD can provide key information

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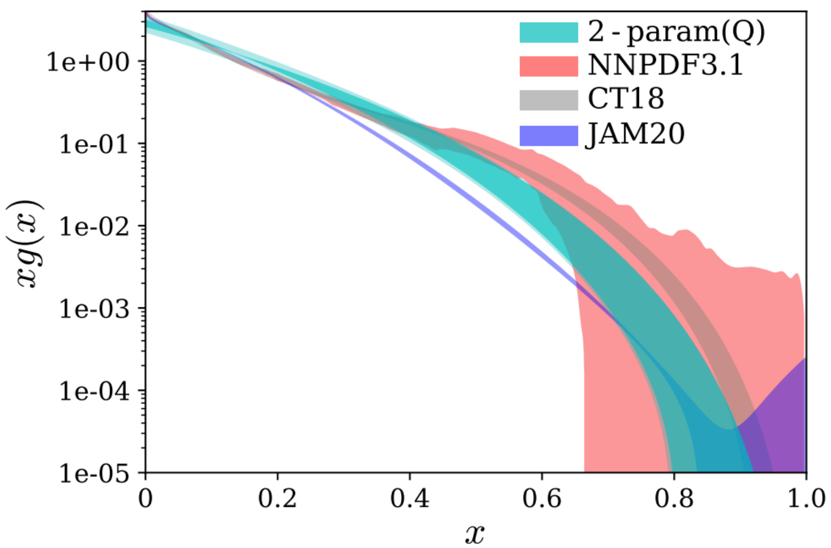

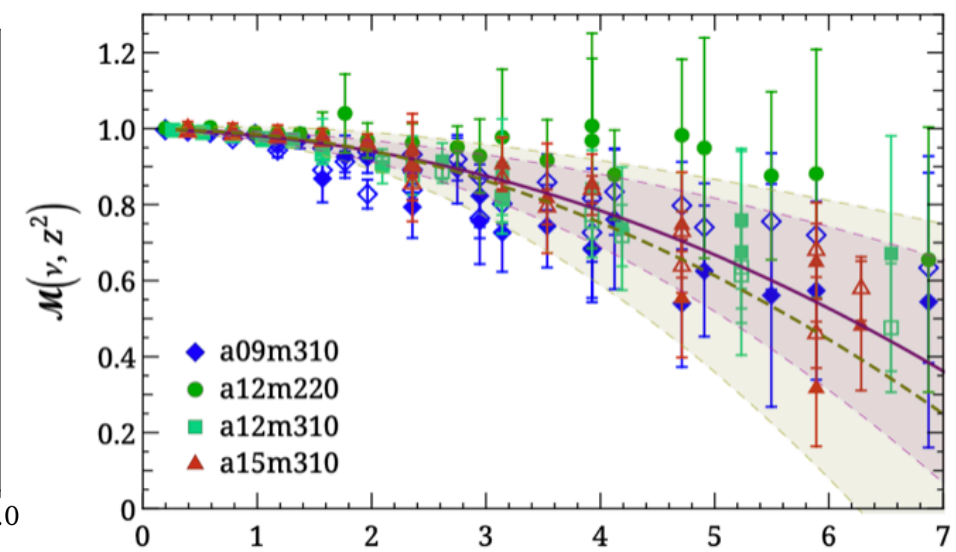

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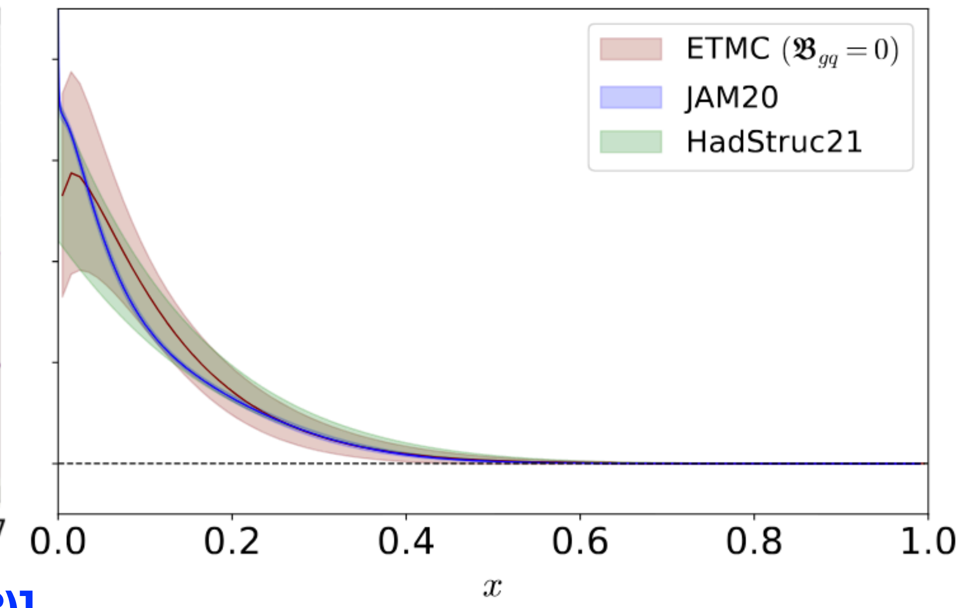

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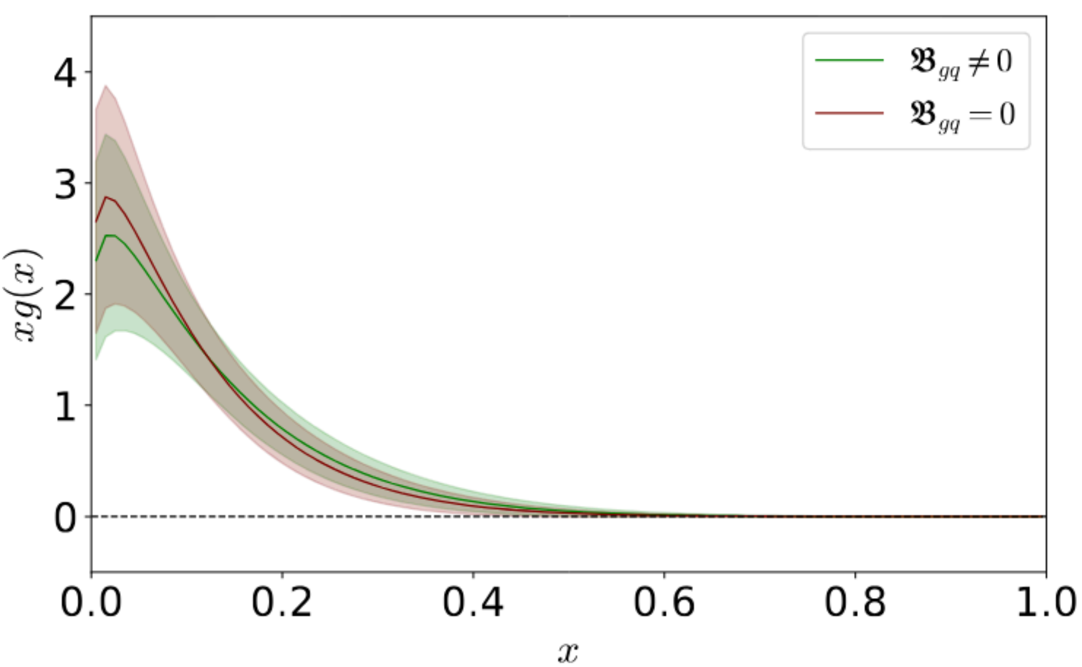

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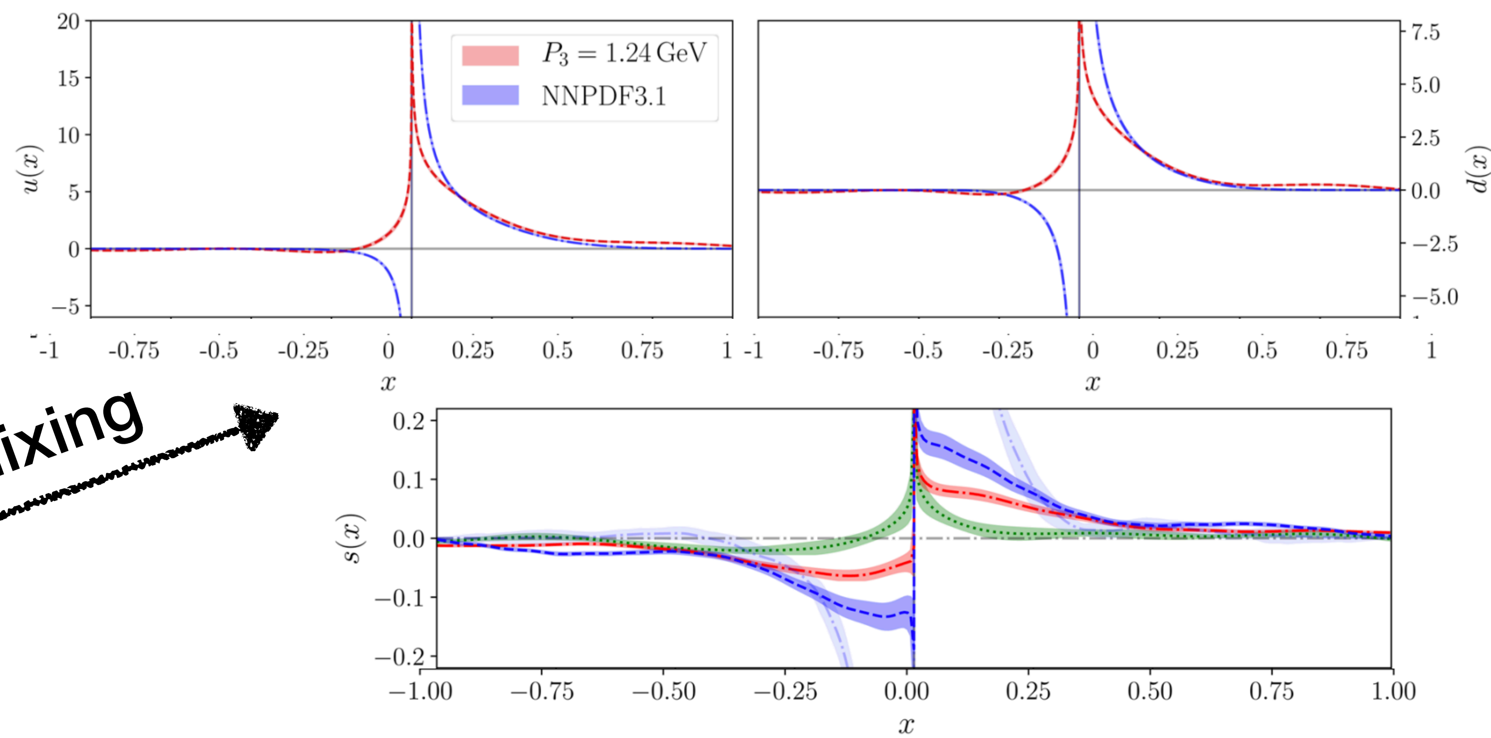

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GPDs

leading twist

GPDs on the lattice

★ Off-forward matrix elements of non-local light-cone operators

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})\,|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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γ^0 parametrization is prohibitively expensive

Definition of GPDs in Euclidean lattice

★ Calculation expected to be performed in symmetric frame to extract "standard" GPDs

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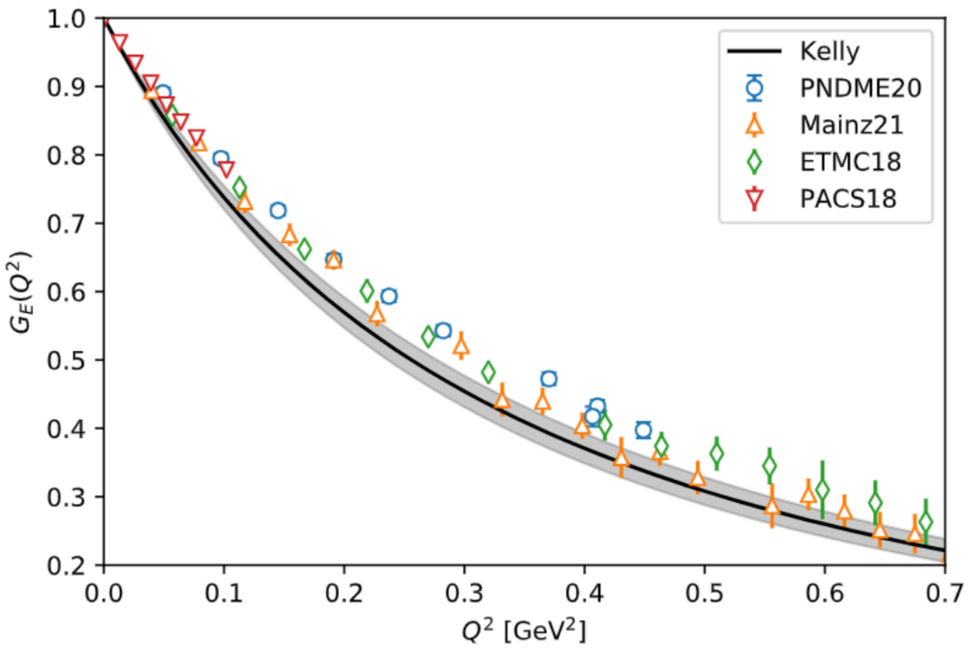

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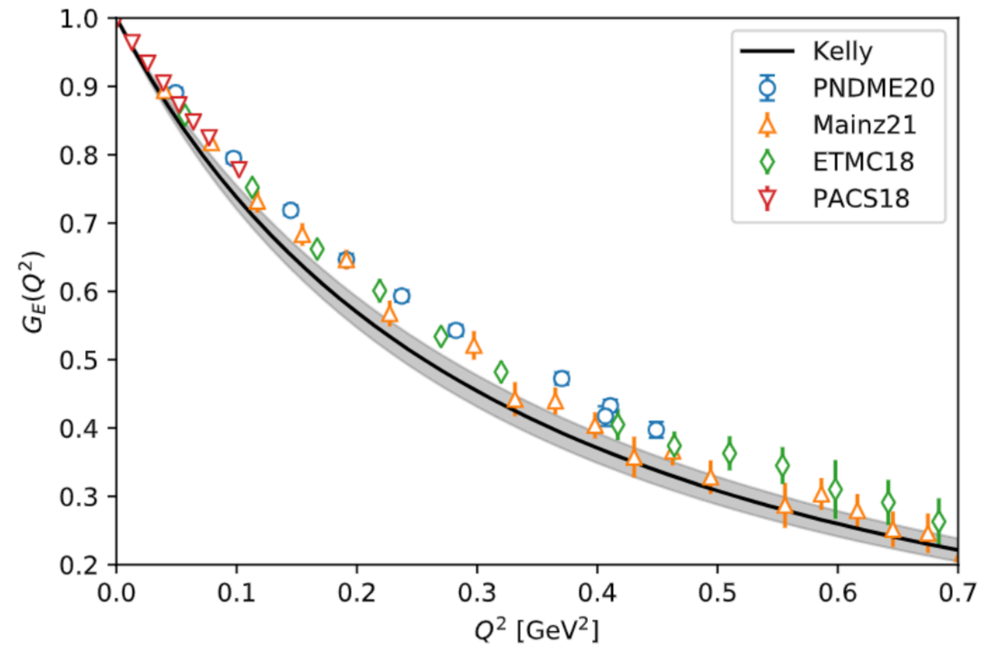

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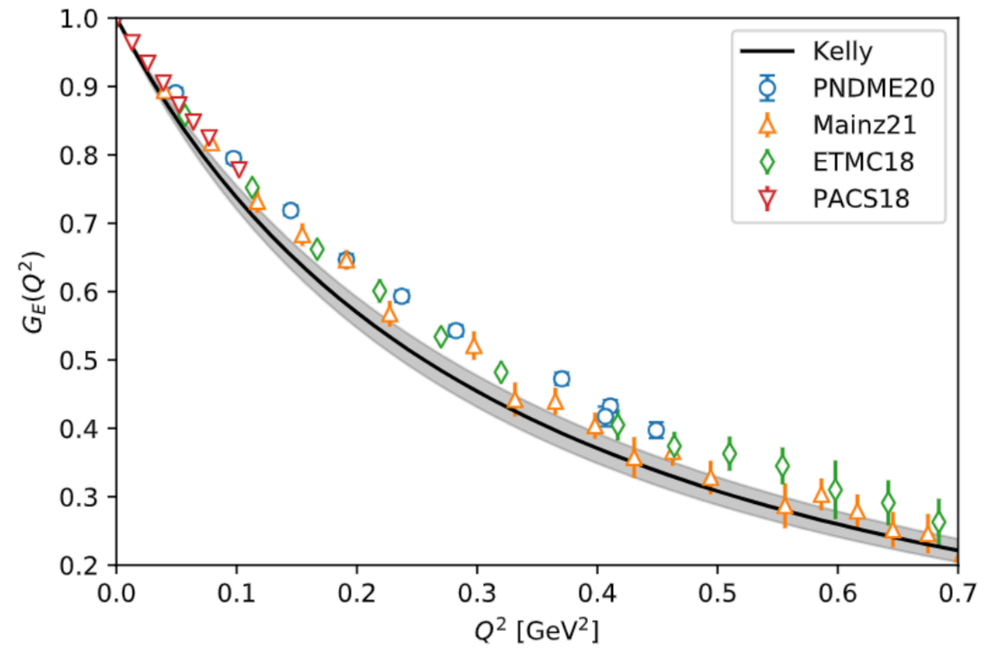

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[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

Advantages

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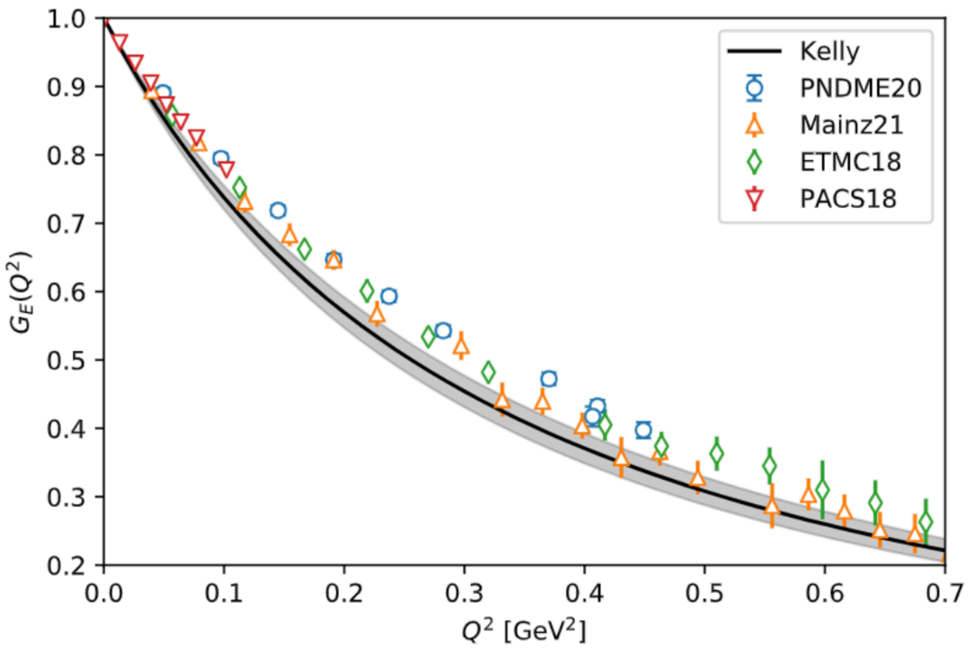

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➡ Proof-of-concept calculation ($\xi = 0$):

- symmetric frame: $\vec{p}^{\,s}_f = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}^{\,s}_i = \vec{P} - \frac{\vec{Q}}{2} \qquad -t^s = \vec{Q}^2 = 0.69\,GeV^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \vec{P}, \qquad \vec{p}^{\,a}_i = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

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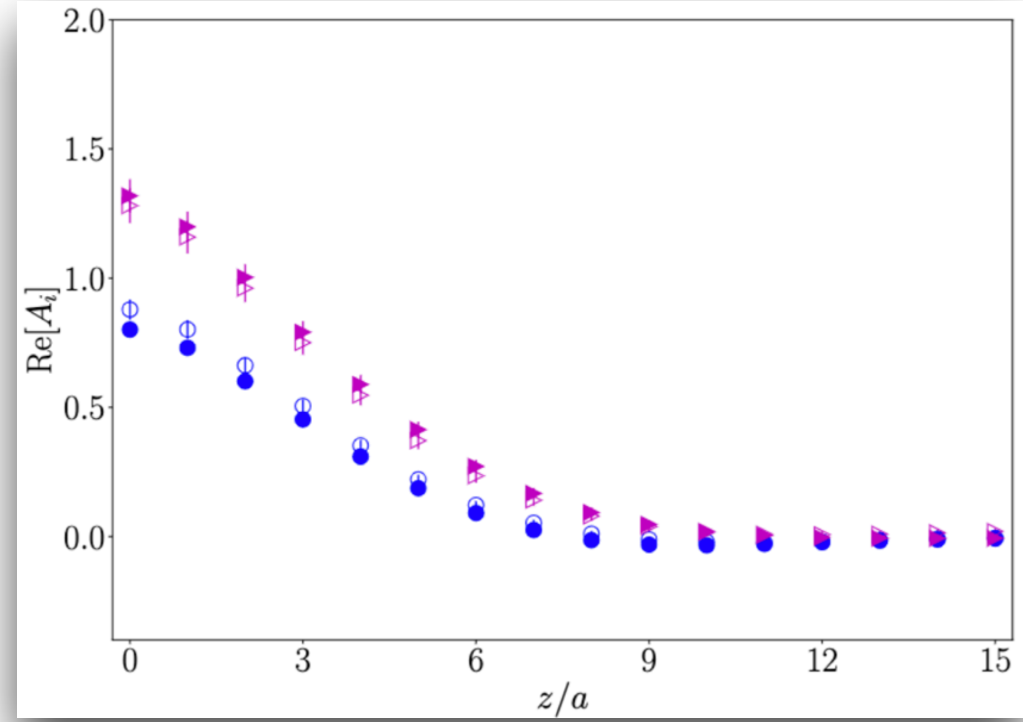

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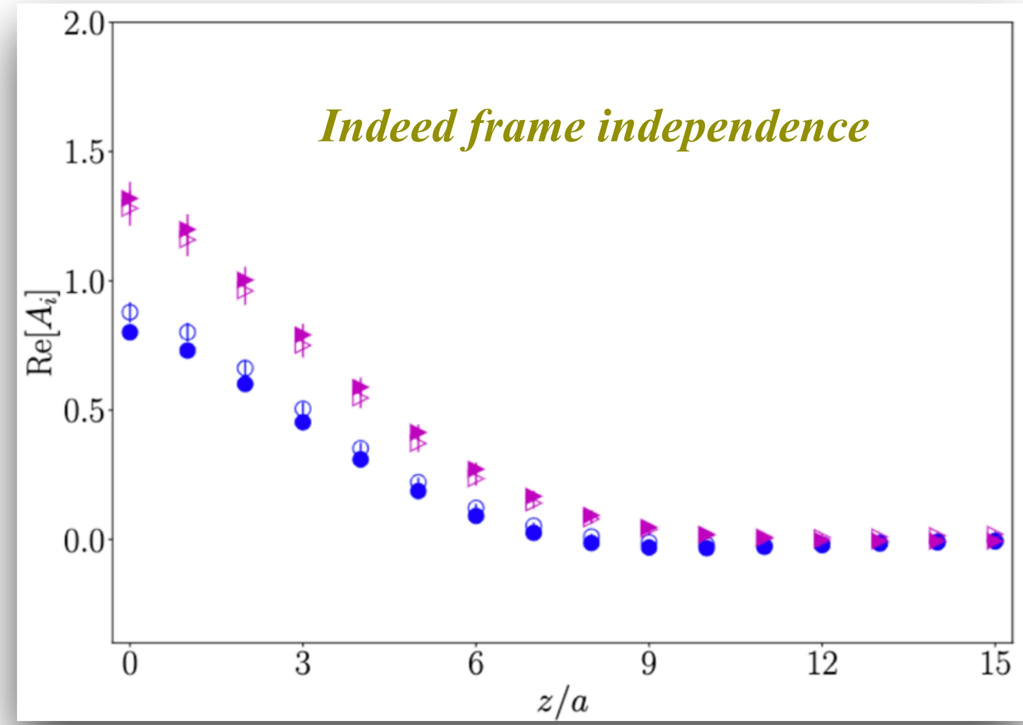

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Light-cone GPDs

★ Nf=2+1+1 twisted mass fermions with a clover term

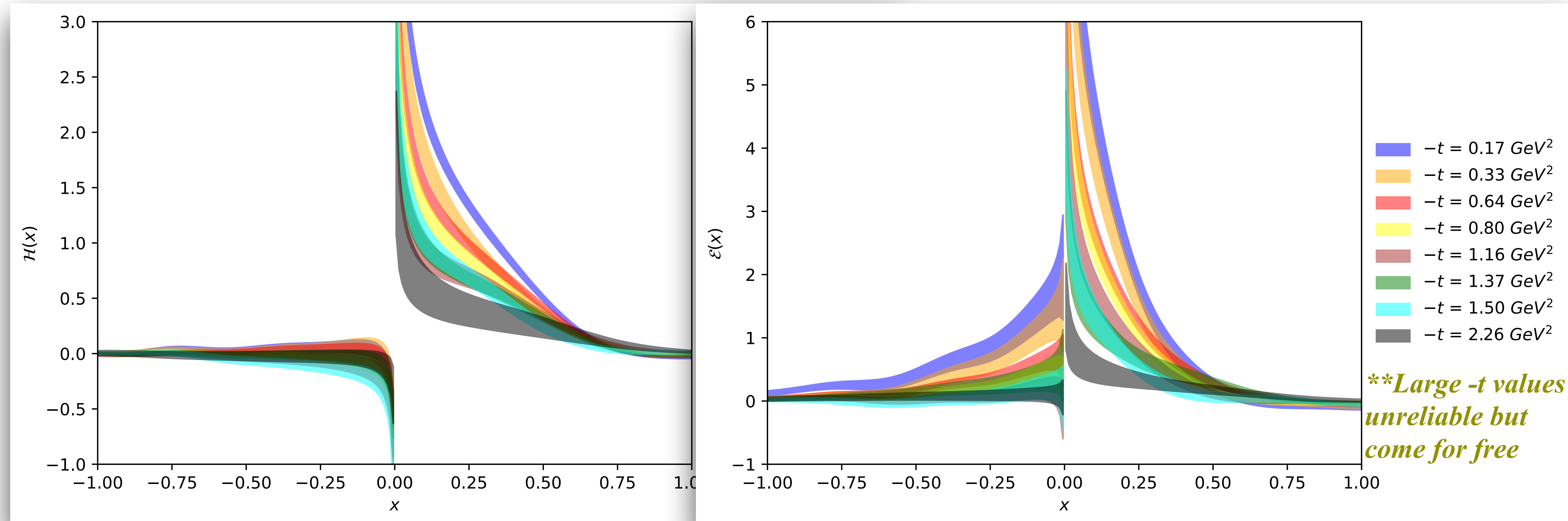
Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

Light-cone GPDs

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

★ Nf=2+1+1 twisted mass fermions with a clover term

$H -$ GPD $\qquad\qquad$ $E -$ GPD



★ +/-x correspond to quark and anti-quark region

★ anti-quark region susceptible to systematic uncertainties

★ small-x region not reliably extracted

★ perturbative matching breaks down at $\xi = x$

Helicity GPDs

$$F^{[\gamma^3\gamma_5]}(x,\Delta;P^3) \;= \frac{1}{2P^0}\bar{u}(p_f,\lambda')\left[\gamma^3\gamma_5\widetilde{\mathcal{H}}_3(x,\xi,t;P^3) + \frac{\Delta^3\gamma_5}{2m}\widetilde{\mathcal{E}}_3(x,\xi,t;P^3)\right]u(p_i,\lambda)$$

$$\widetilde{F}^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z\Delta}}{m}\widetilde{A}_1 + \gamma^\mu\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^\mu}{m}\widetilde{A}_3 + mz^\mu\widetilde{A}_4 + \frac{\Delta^\mu}{m}\widetilde{A}_5\right)\right.$$
$$\left. + m\not{z}\gamma_5\left(\frac{P^\mu}{m}\widetilde{A}_6 + mz^\mu\widetilde{A}_7 + \frac{\Delta^\mu}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)\,,$$

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$$\boxed{\widetilde{\mathcal{H}}_3(\widetilde{A}_i^{s/a};z) = \widetilde{A}_2 + zP_3\widetilde{A}_6 - m^2z^2\widetilde{A}_7}$$

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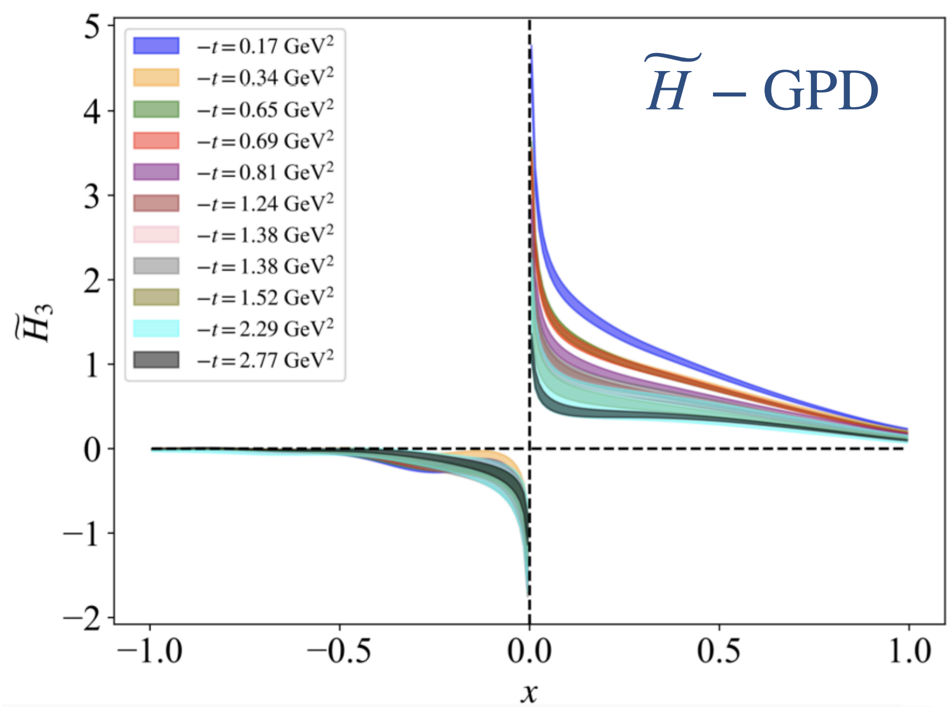

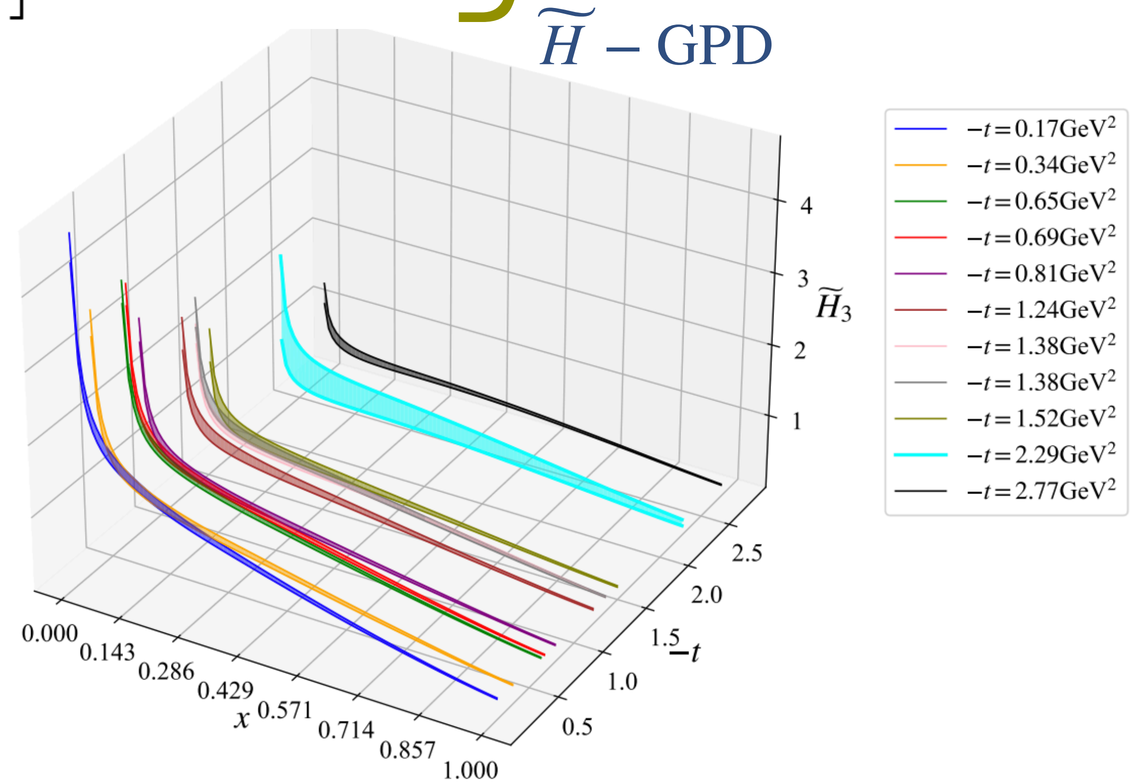

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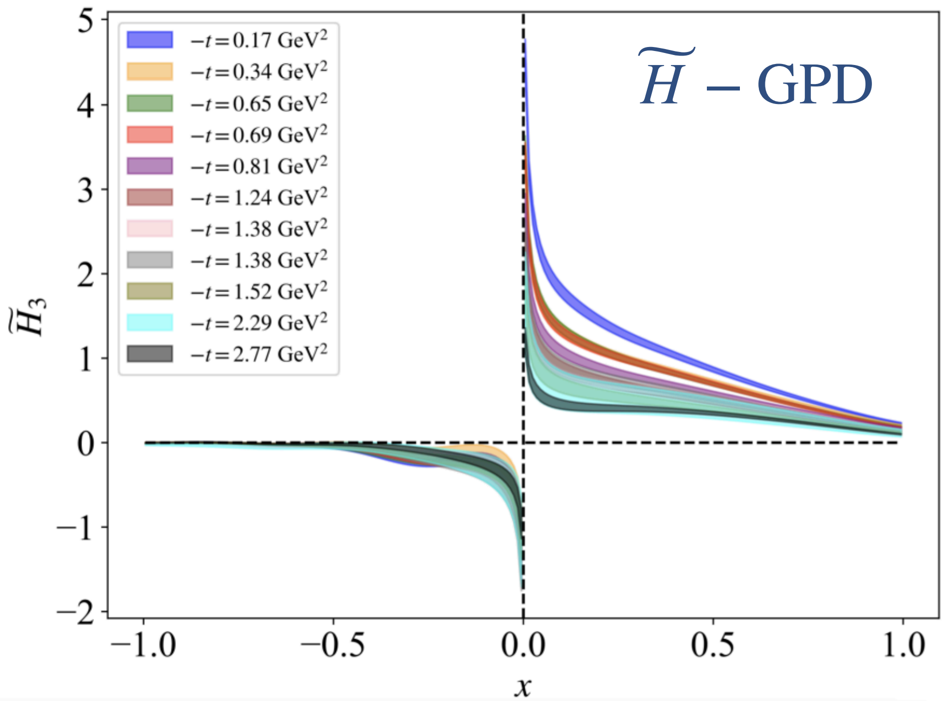

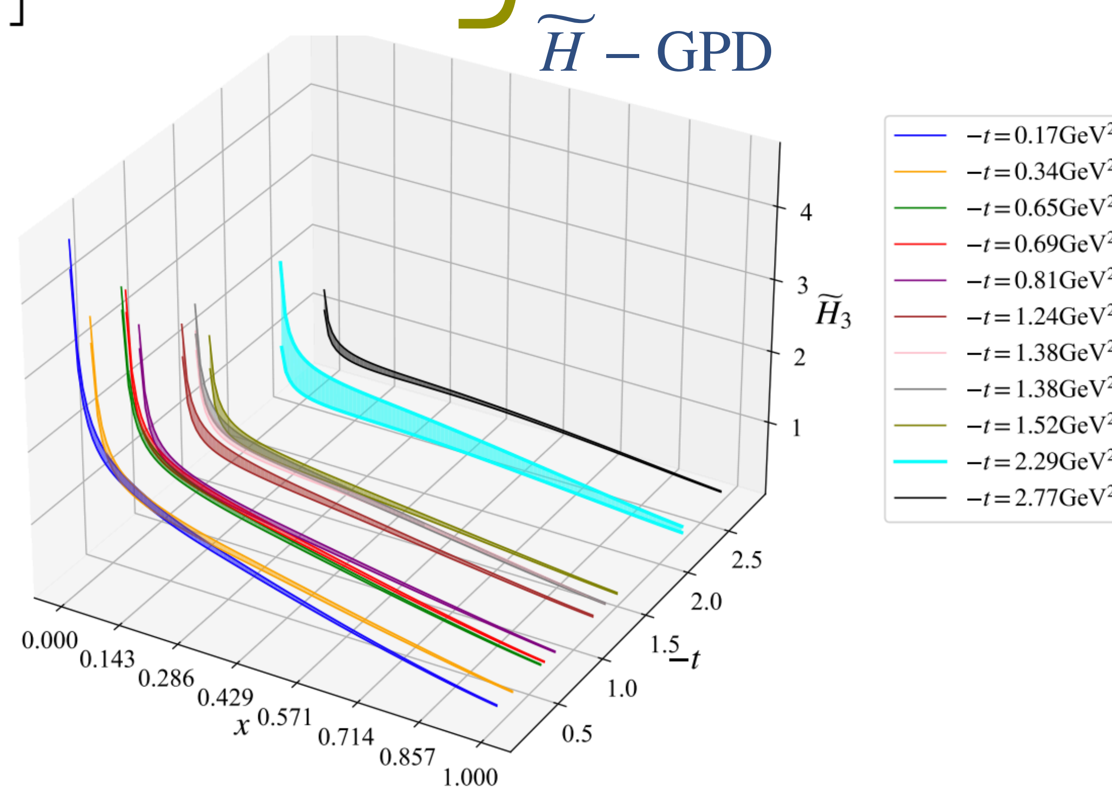

- ★ Large values of $-t$ not reliably extracted
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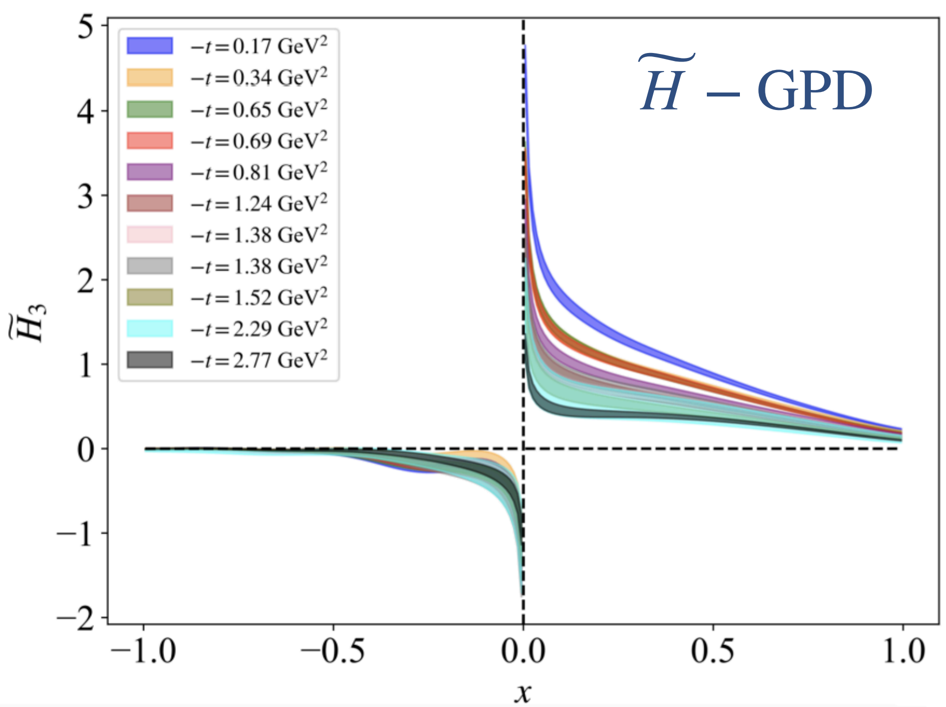

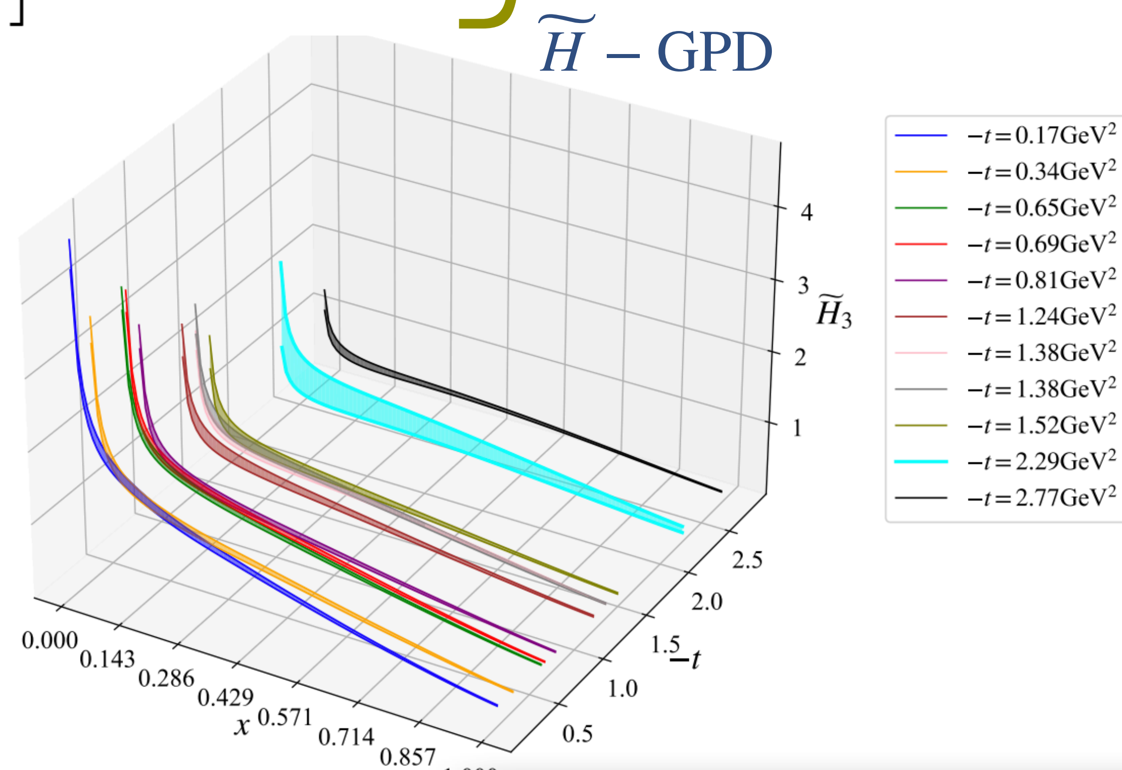

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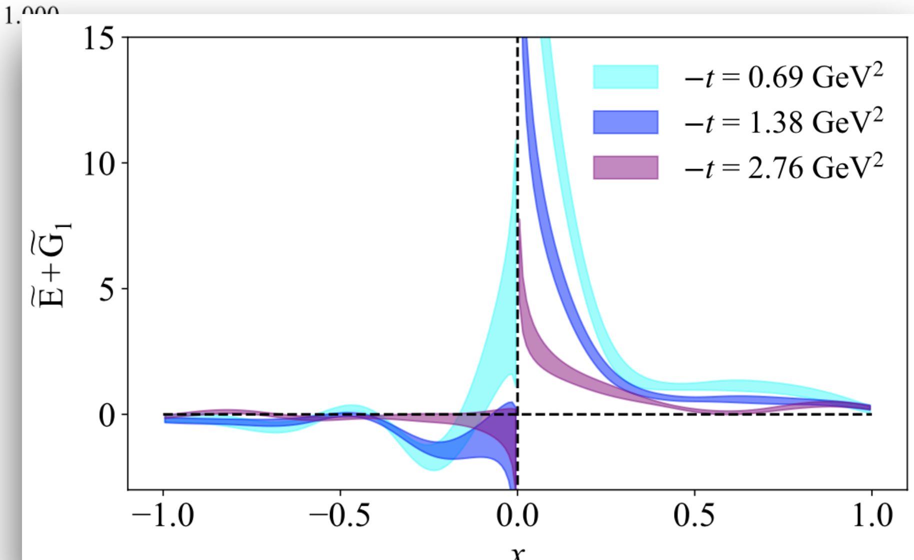
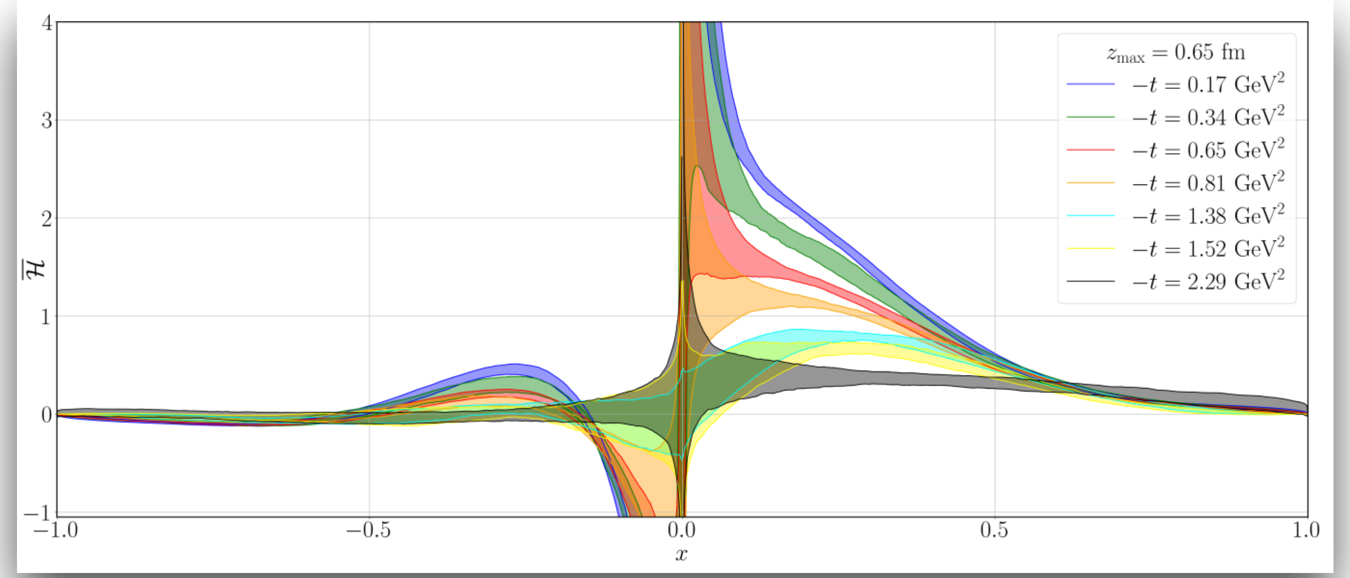
Glimpse of $\widetilde{E}$ from twist-3 GPDs



Alternative approach: pseudo-ITD



[Battacharya et al., PRD 110 (2024) 5, 054502]

Different steps between approaches:

- renormalization

- x-dependence reconstruction

- matching formalism

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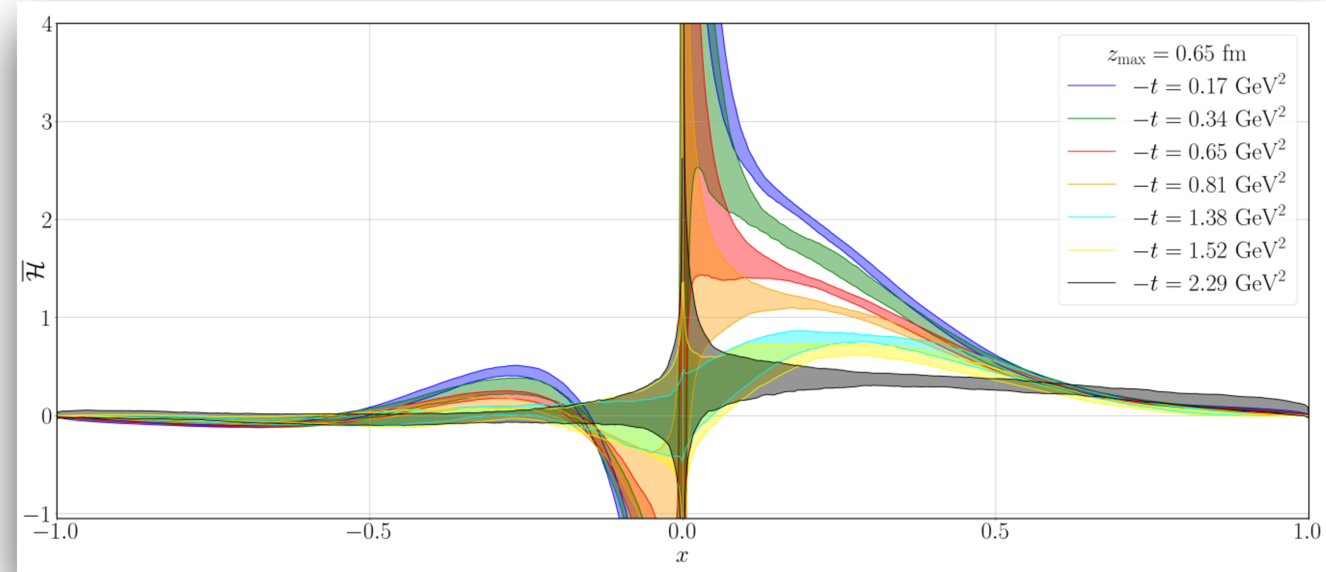

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Different steps between approaches:
- renormalization
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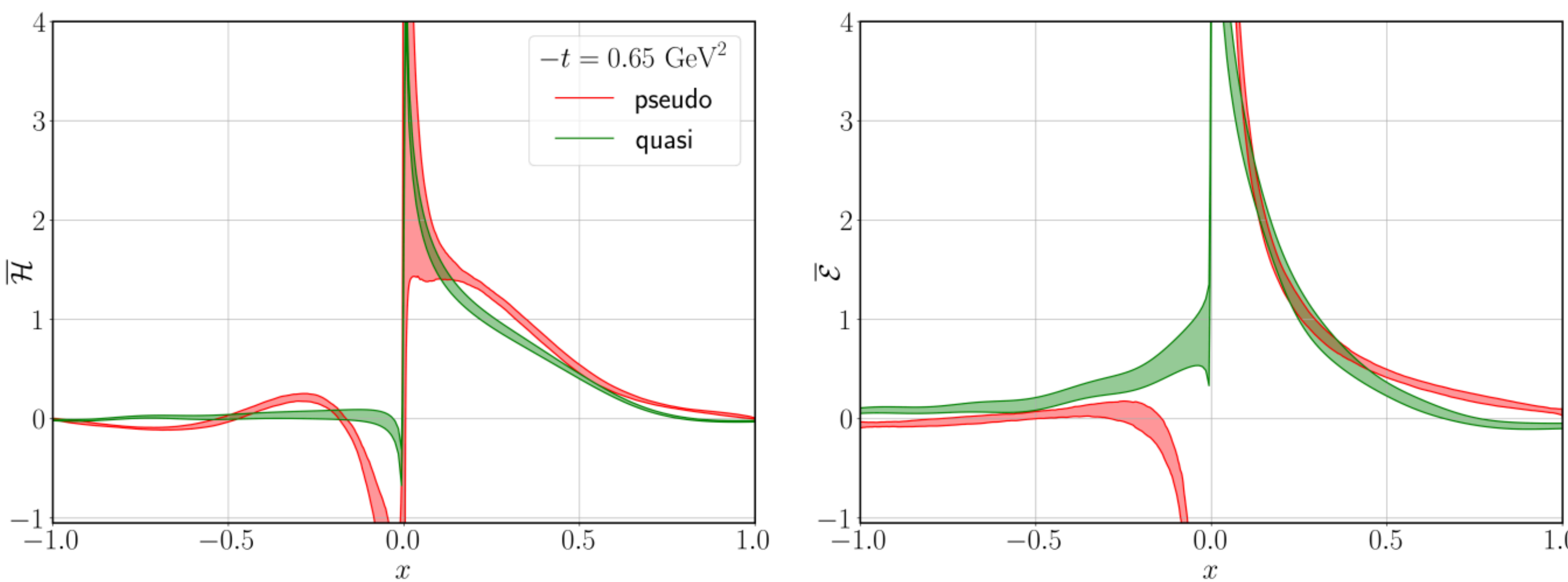
★ Comparison between methods helps assess systematic effects



★ x<0 and small-x regions susceptible to systematic effects

★ Comparison only includes systematic uncertainties

Mellin moments from non-local operators

[S. Bhattacharya et al., PRD 108 (2023) 1, 014507; arXiv:2410.03539]

★ Leading-twist factorization formula

$$\mathscr{M}(z,P,\Delta) \equiv \frac{\mathscr{F}(z,P,\Delta)}{\mathscr{F}(z,P=0,\Delta=0)} = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\overline{\rm MS}}(\mu^2 z^2)}{C_0^{\overline{\rm MS}}(\mu^2 z^2)} \langle x^n \rangle + \mathscr{O}(\Lambda_{\rm QCD}^2 z^2)$$

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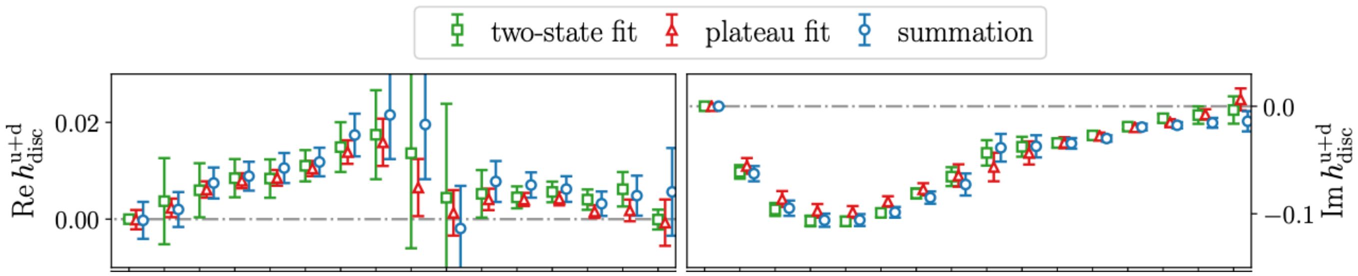

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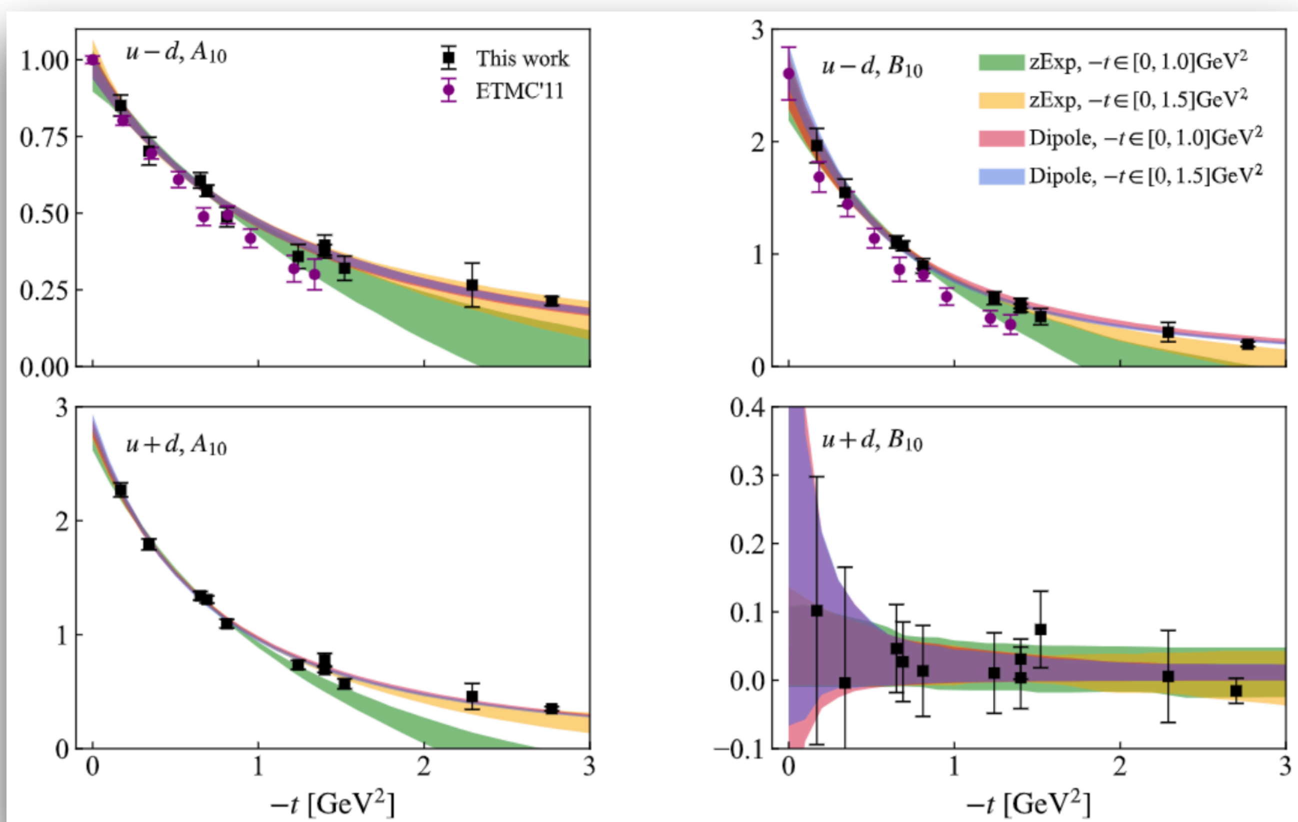

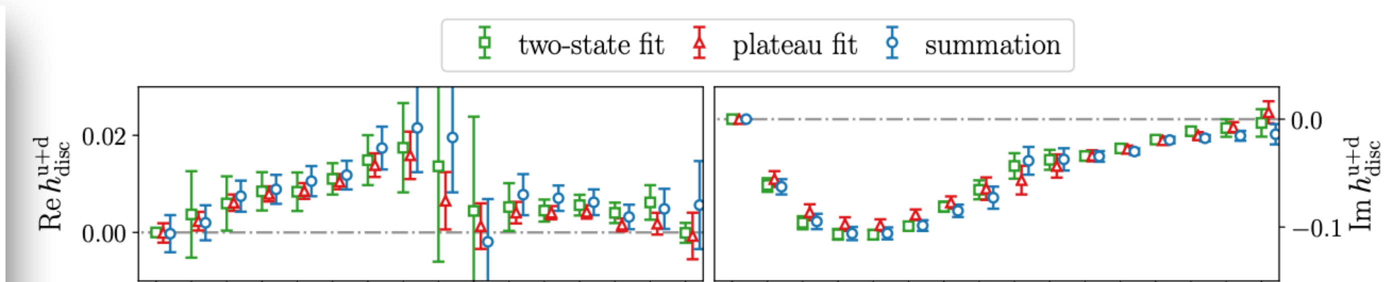

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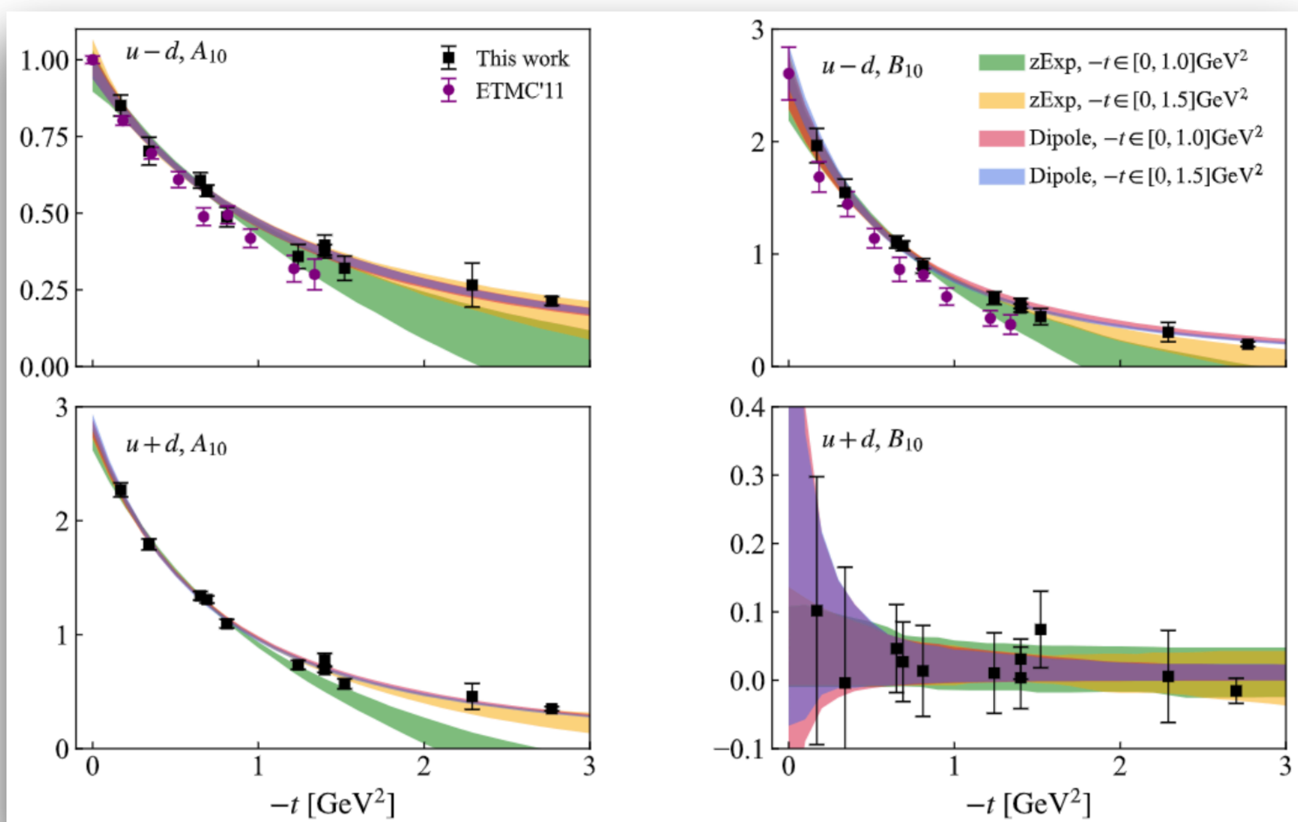

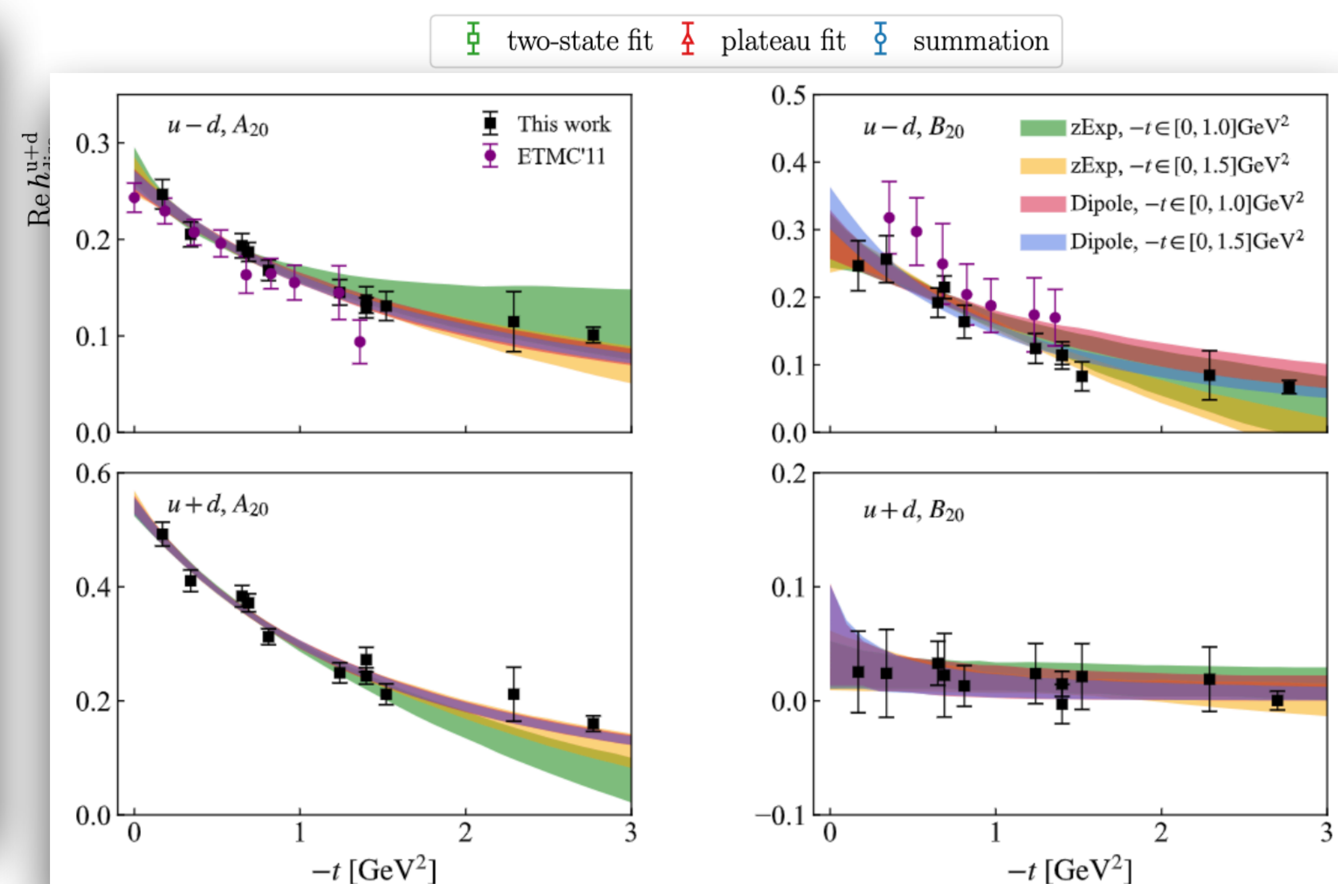

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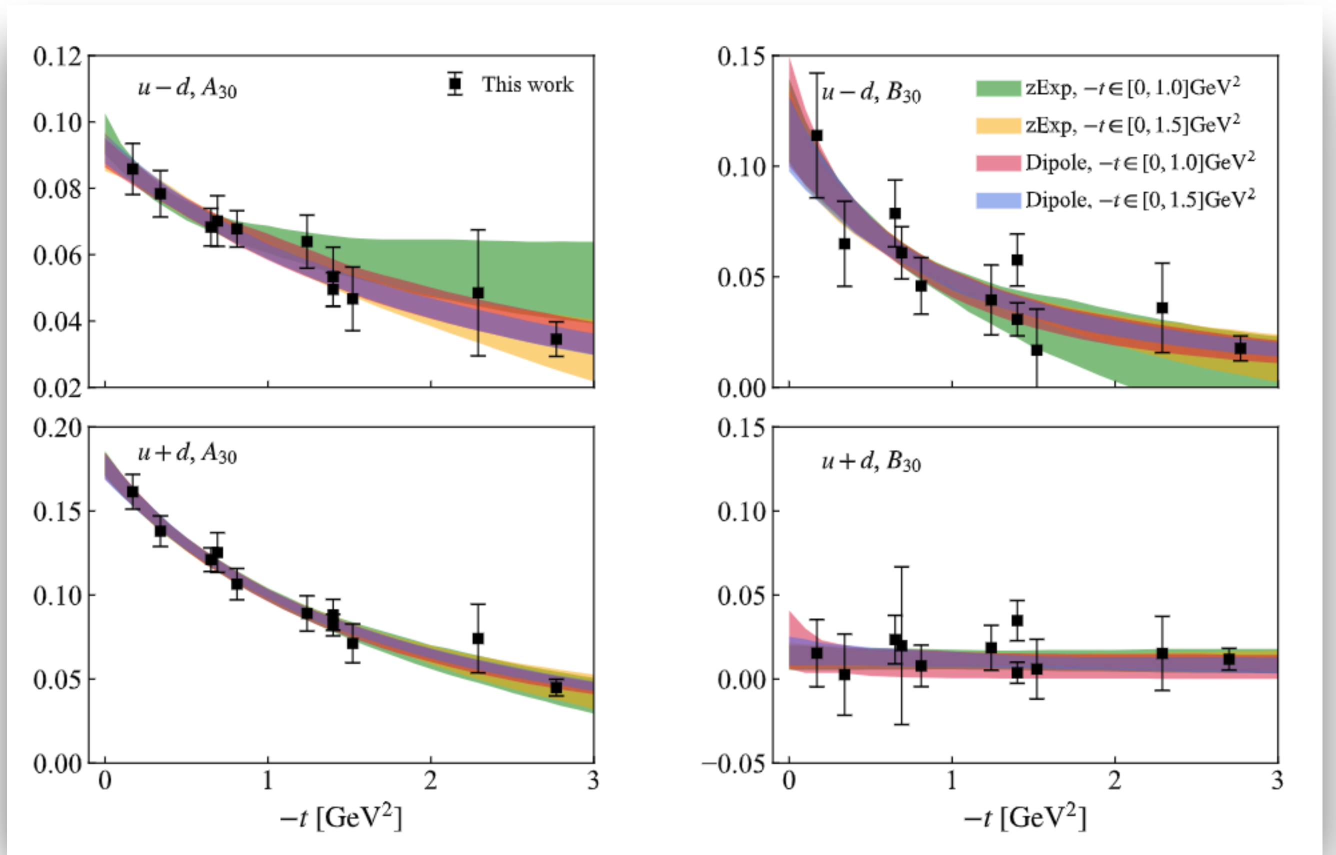

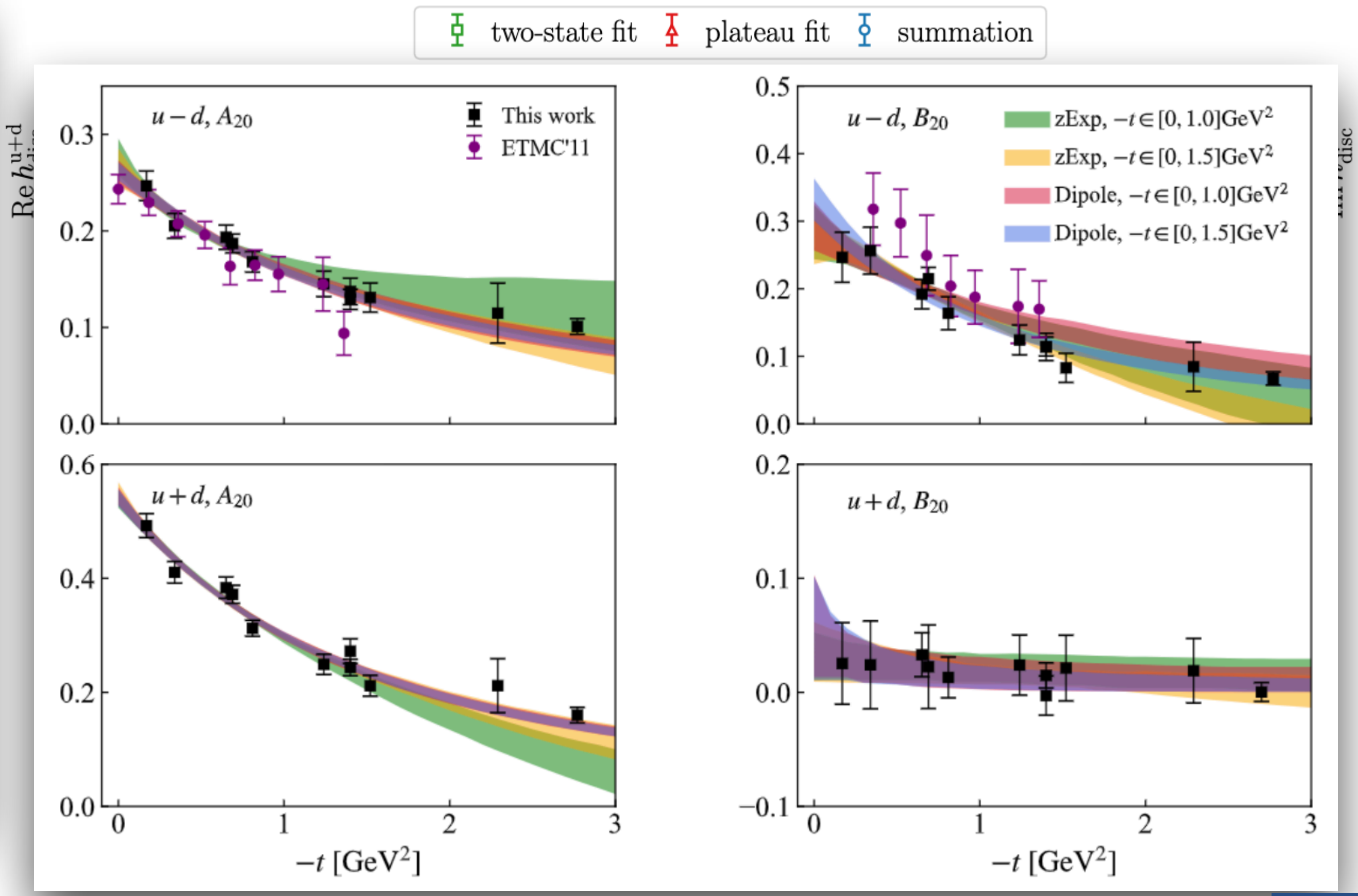

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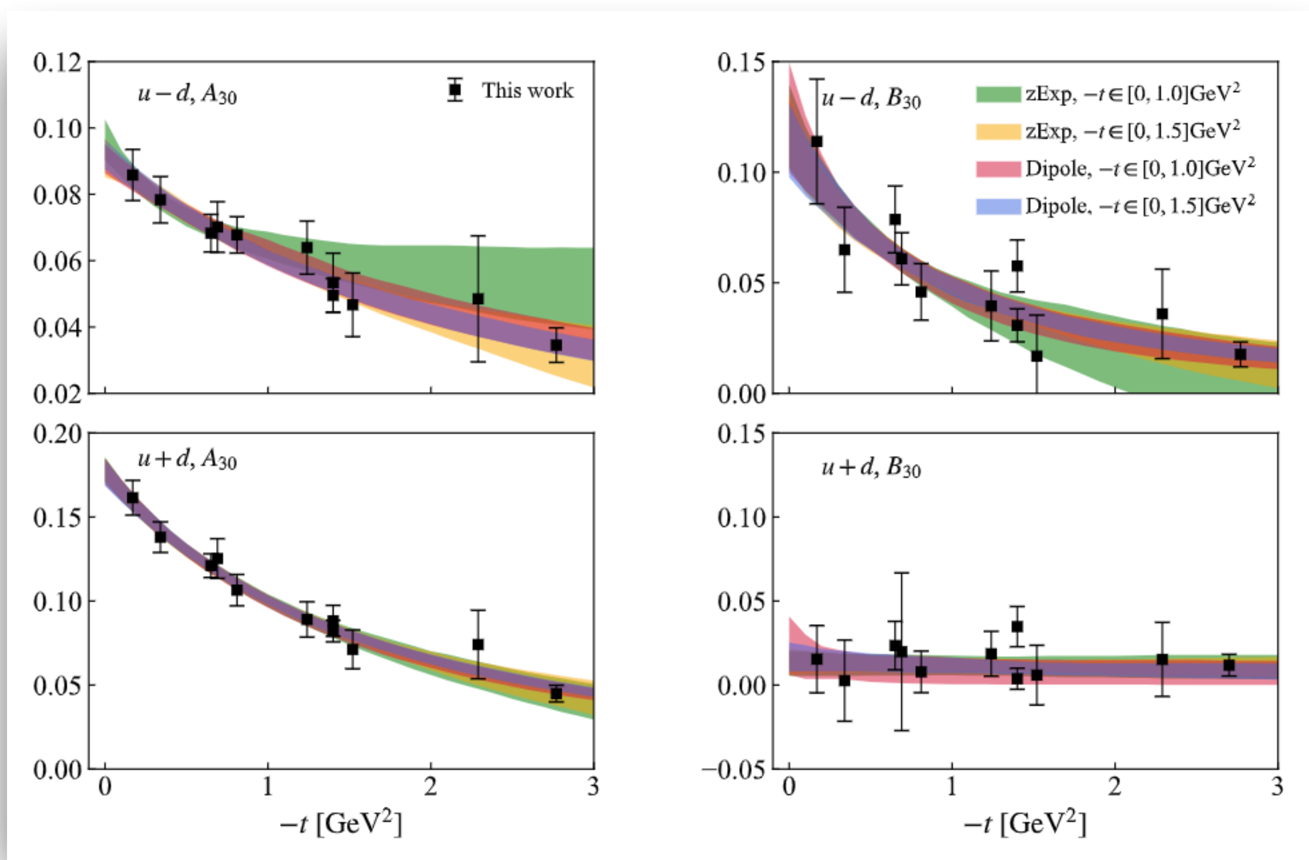

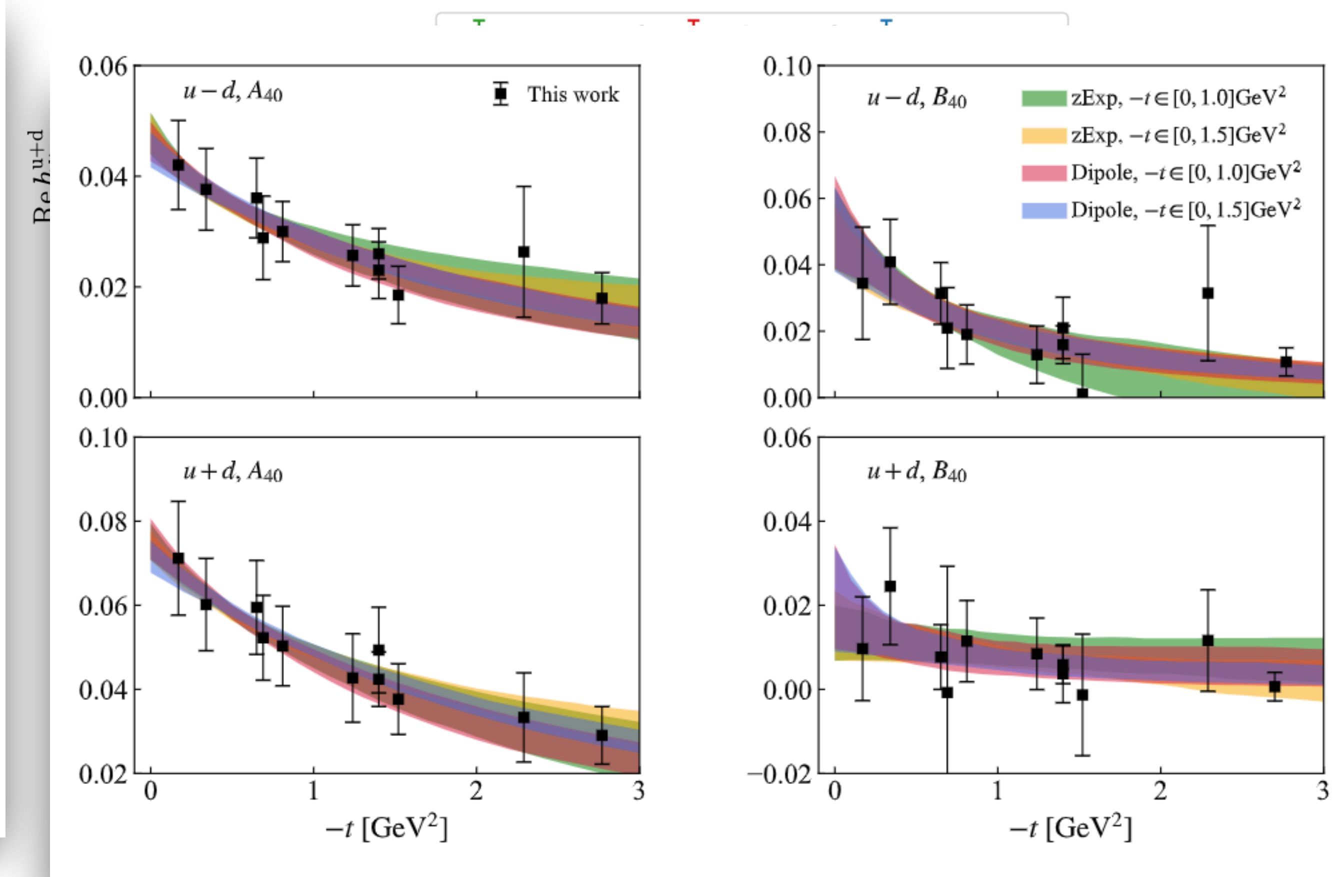

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Access to Mellin moments beyond local operators

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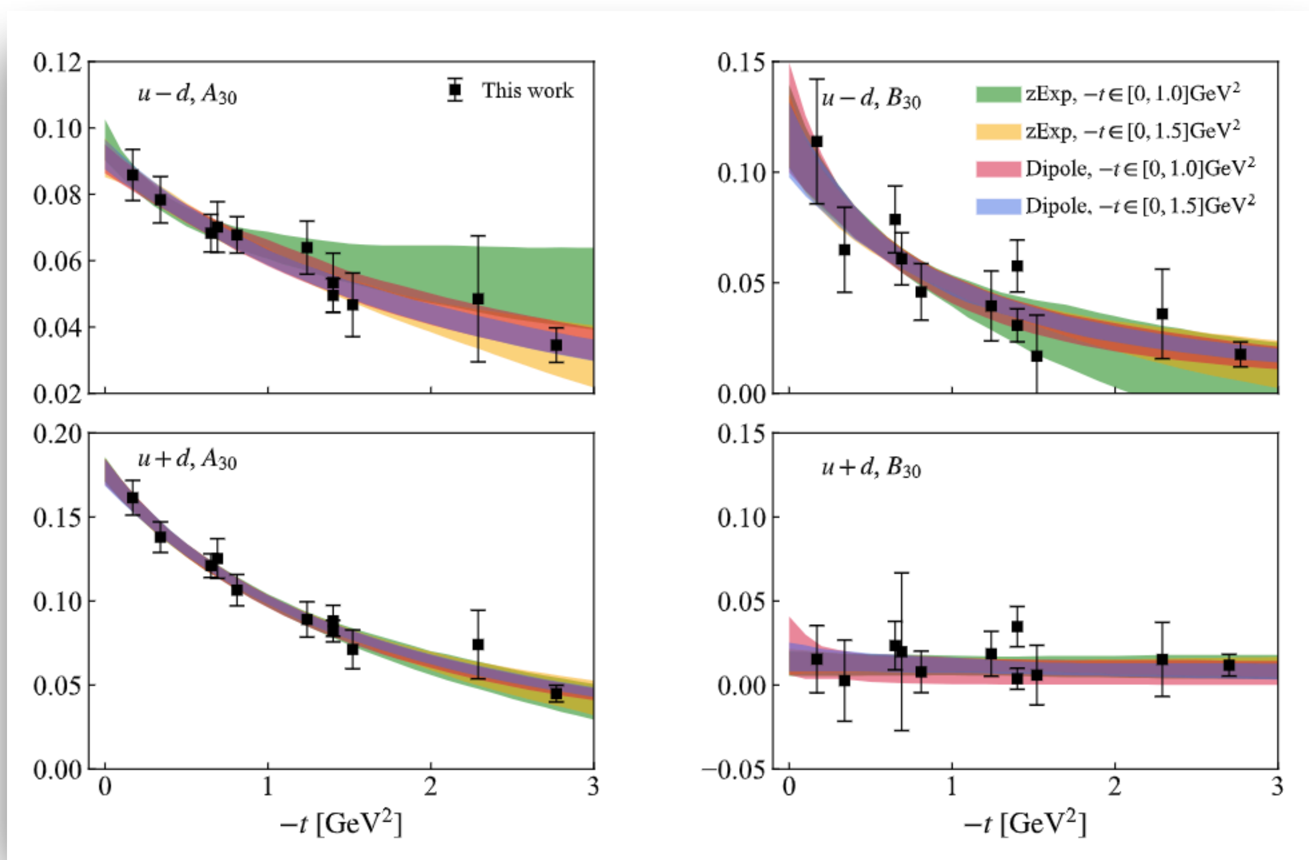

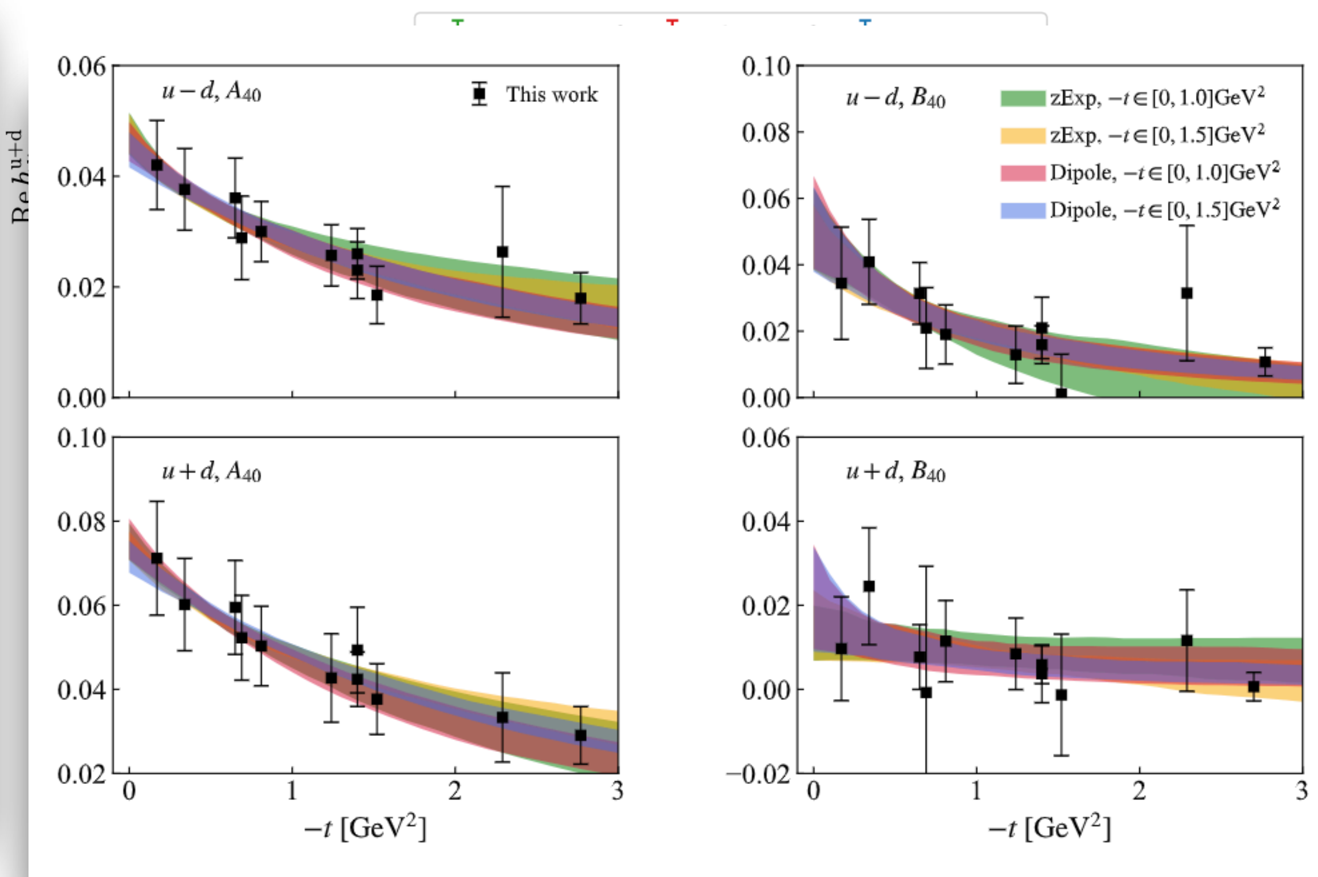

GPDs

beyond leading twist

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[V. Braun et al., JHEP 05 (2021) 086; JHEP 10 (2021) 087]

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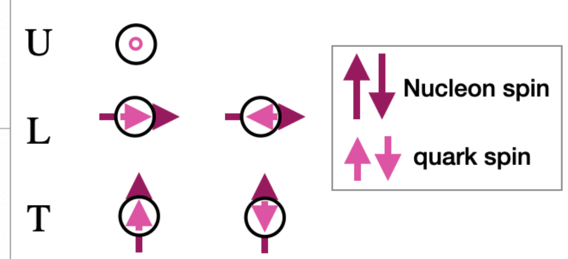
Twist-classification of PDFs, GPDs, TMDs

★ Twist: The order in Q^{-1} entering factorization

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Quark / Nucleon	U (γ^+)	L $(\gamma^+\gamma^5)$	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

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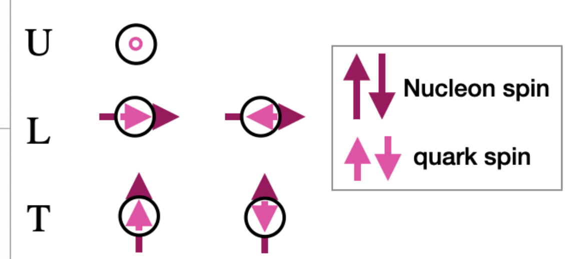

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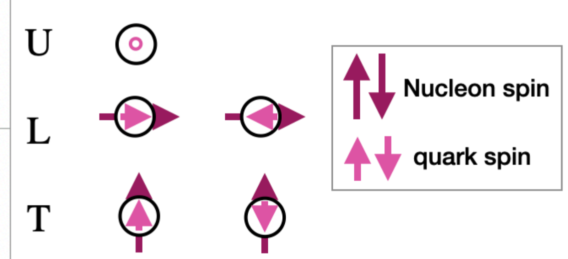

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The extraction of twist-3 is very challenges both experimentally and theoretically

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
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Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

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★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

Theoretical setup

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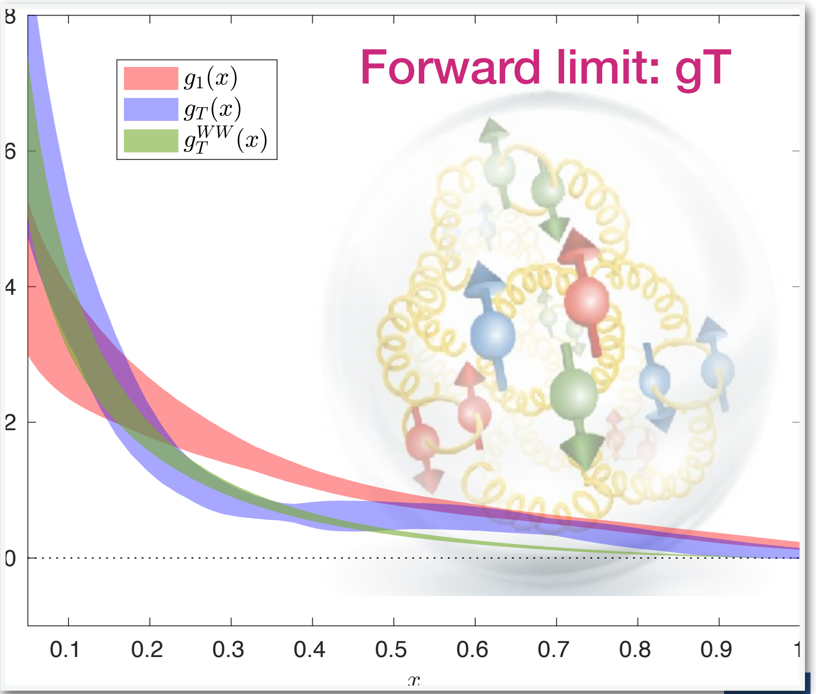

[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Theoretical setup

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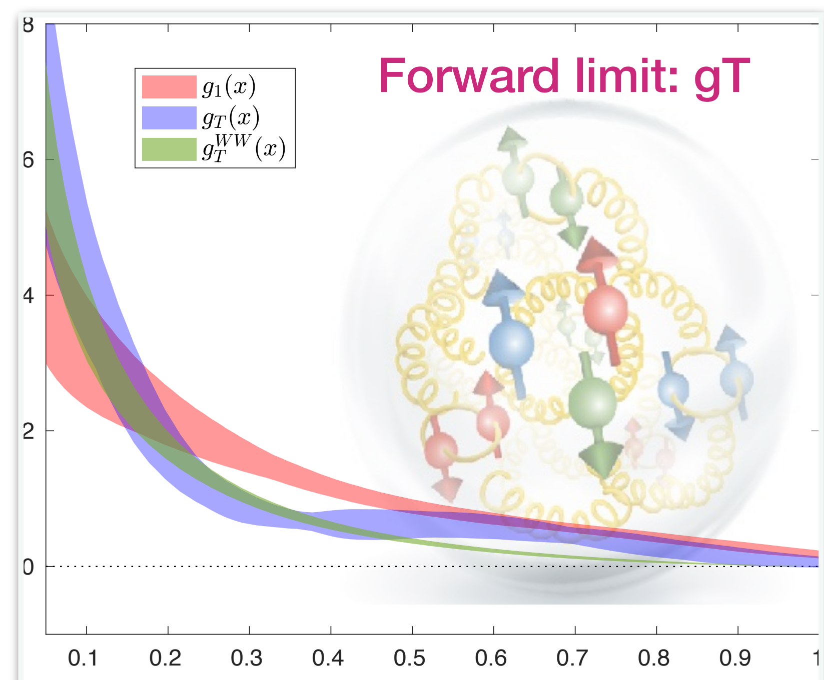
★ Twist-3 contributions to helicity GPDs: $\gamma^{1,2}\gamma_5$

★ Kinematic twist-3 contributions to pseudo- and quasi-GPDs to restore translation invariance

[V. Braun et al., JHEP 10 (2023) 134]



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[S. Bhattacharya et al., 109 (2024) 3, 034508]

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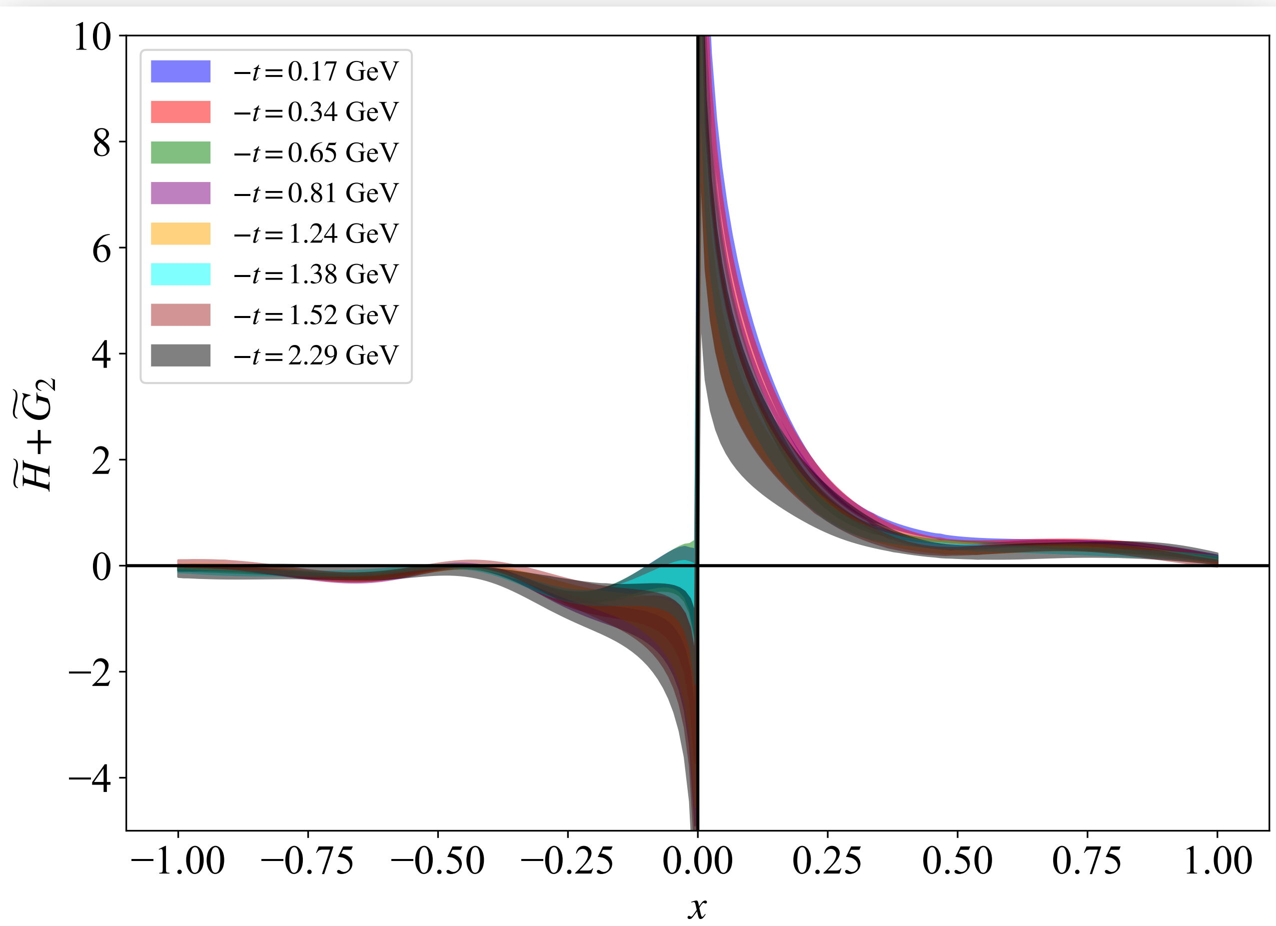
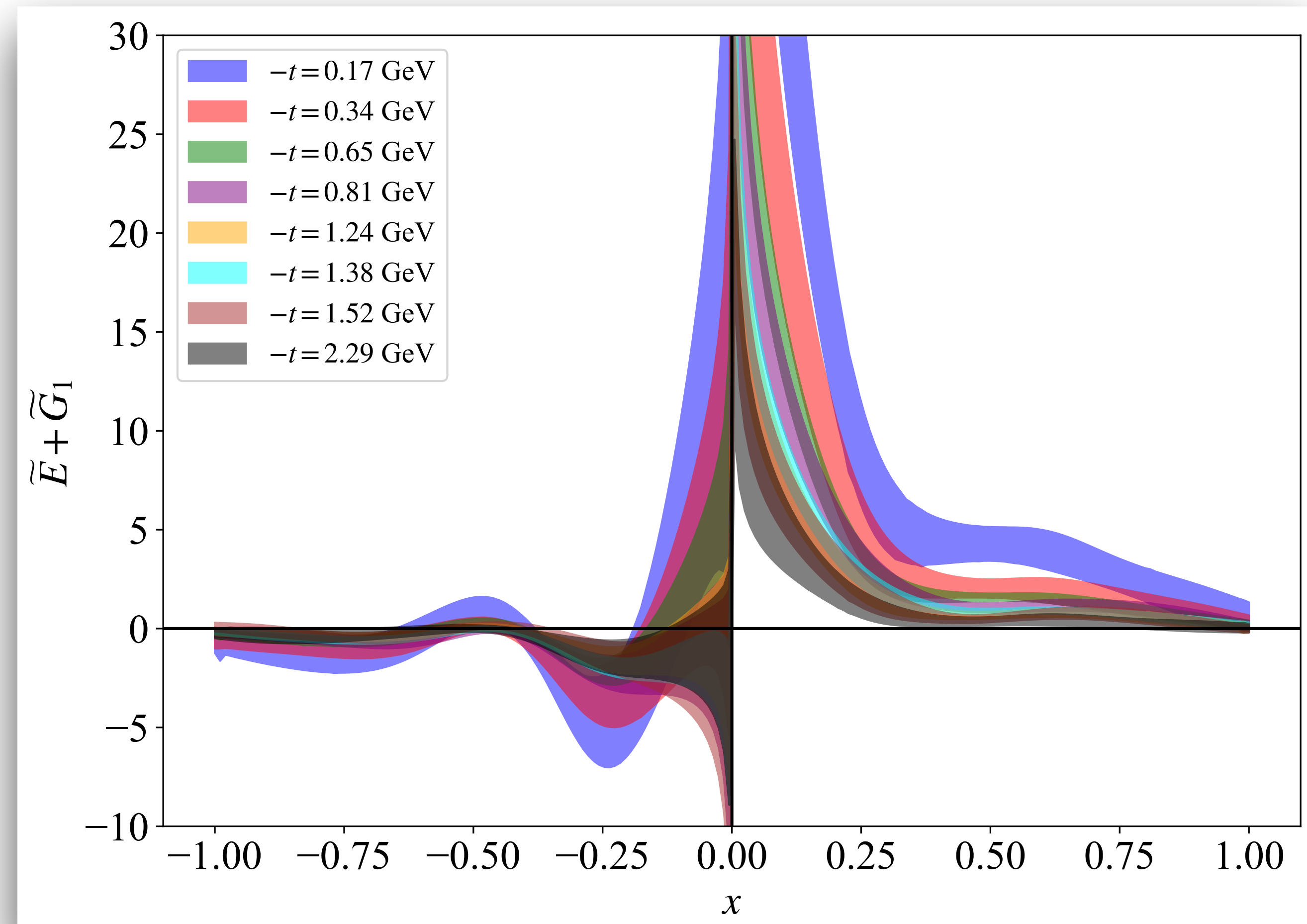
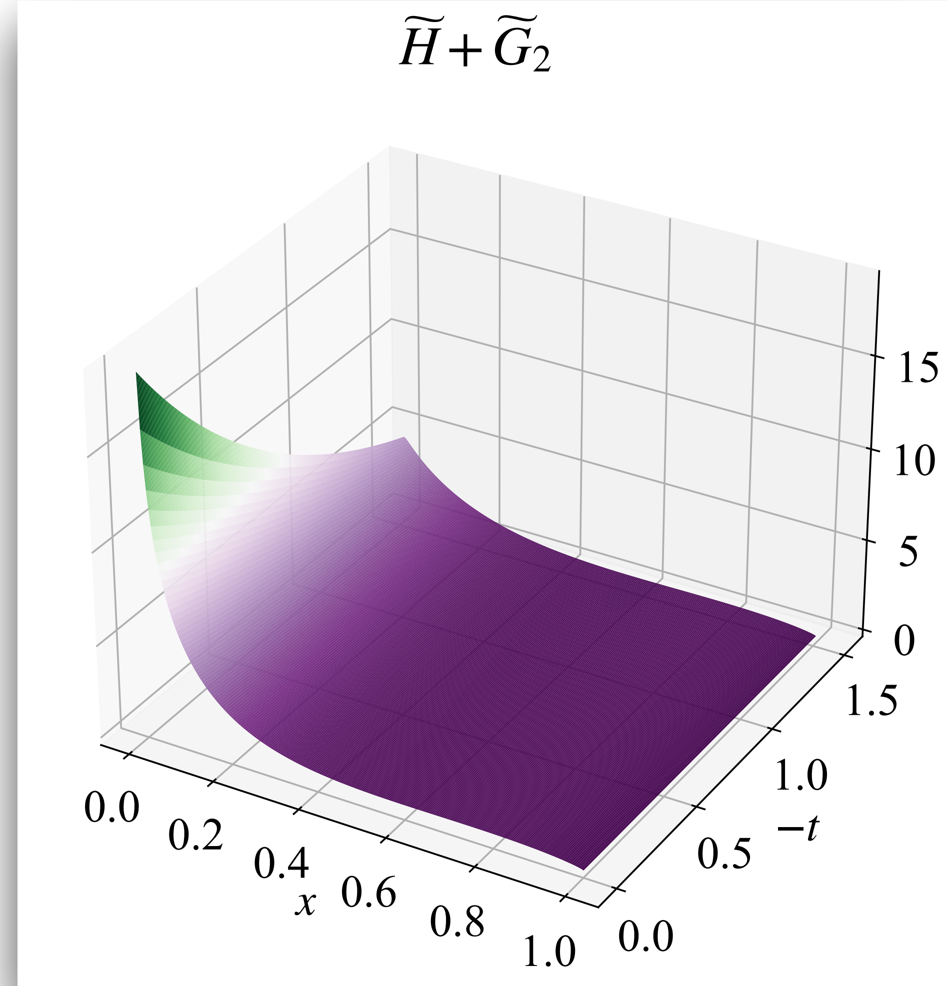


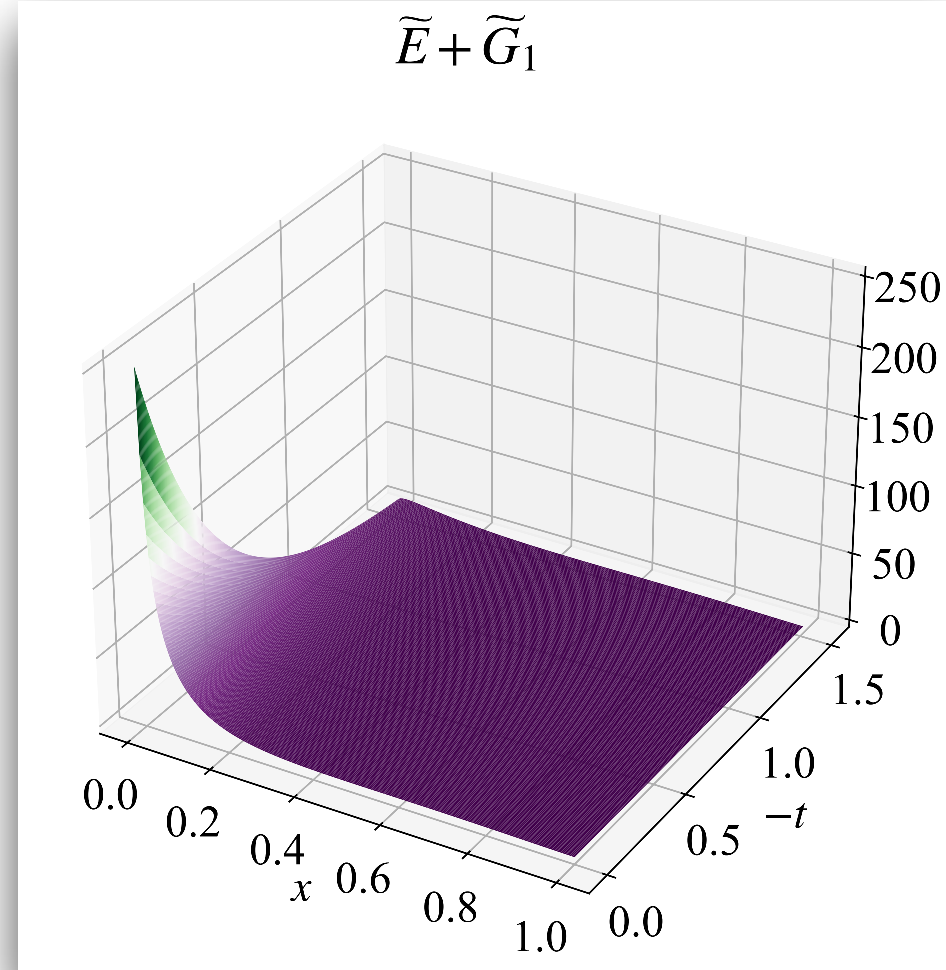


★ **Parametrization of -t dependence**

$$\mathrm{GPD}(x,-t,0) = A x^{\alpha_0-\alpha_1 t}(1-x)^\beta$$

Ademollo & Del Giudice Gatto & Preparata





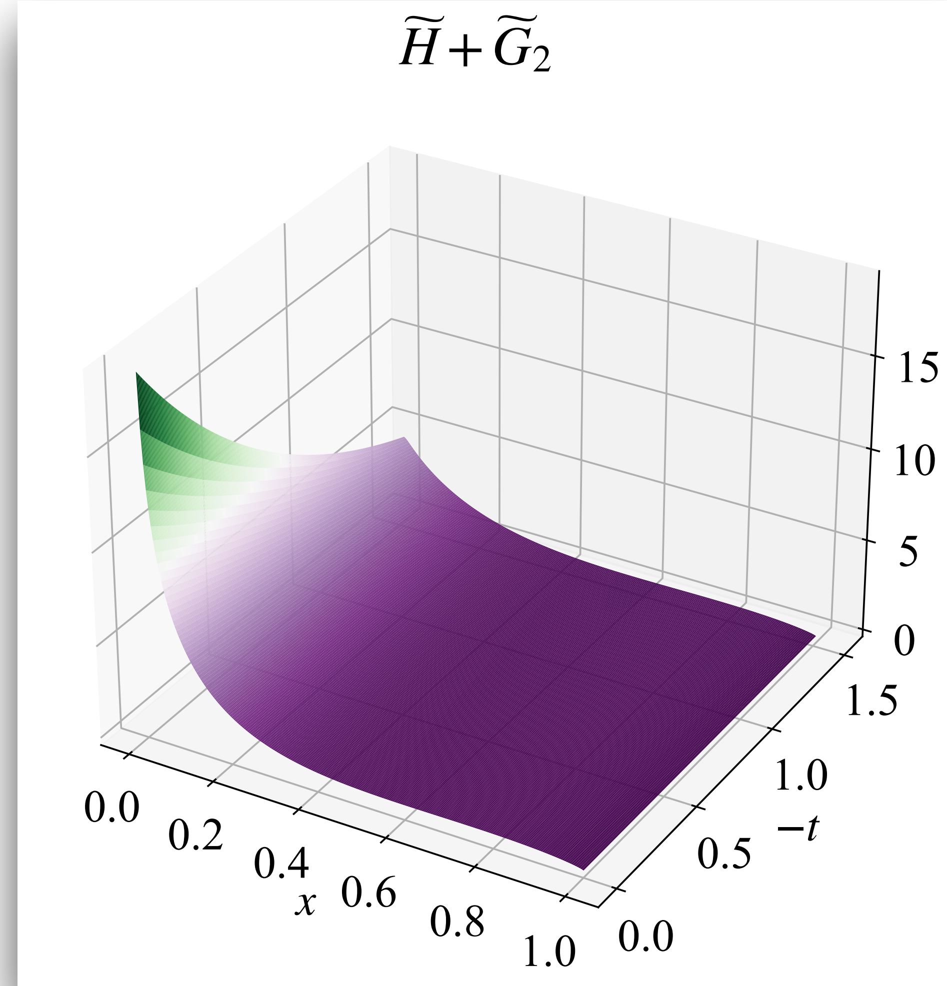
★ Direct access to $\widetilde{E}$ -GPD not possible for zero skewness

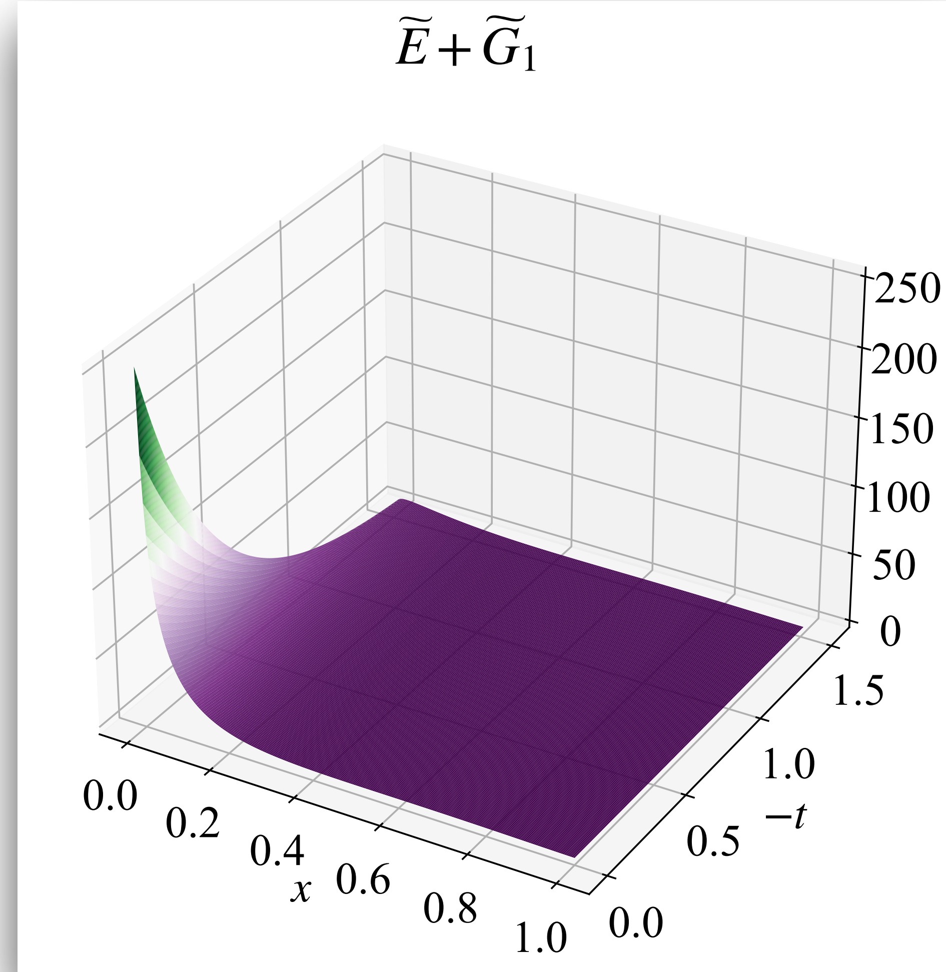
$$P^\mu \frac{\Delta^3 \gamma_5}{2mP^0} F_{\widetilde{E}}(x,\xi,t;P^3)$$

★ Glimpse into $\widetilde{E}$ -GPD through twist-3 :

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$





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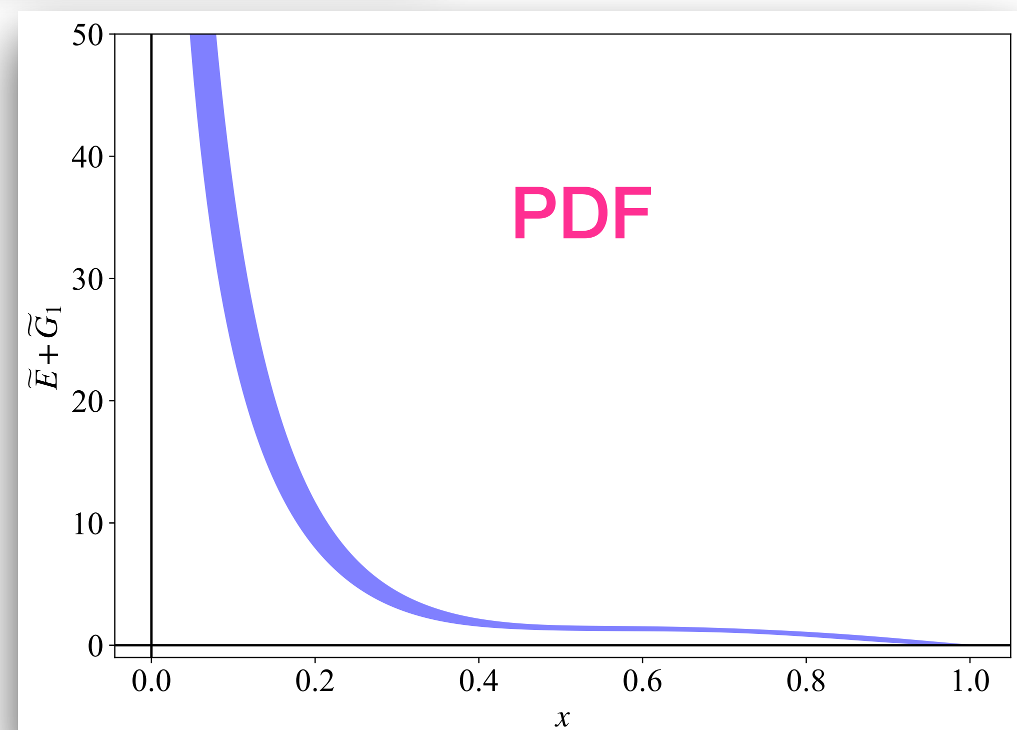
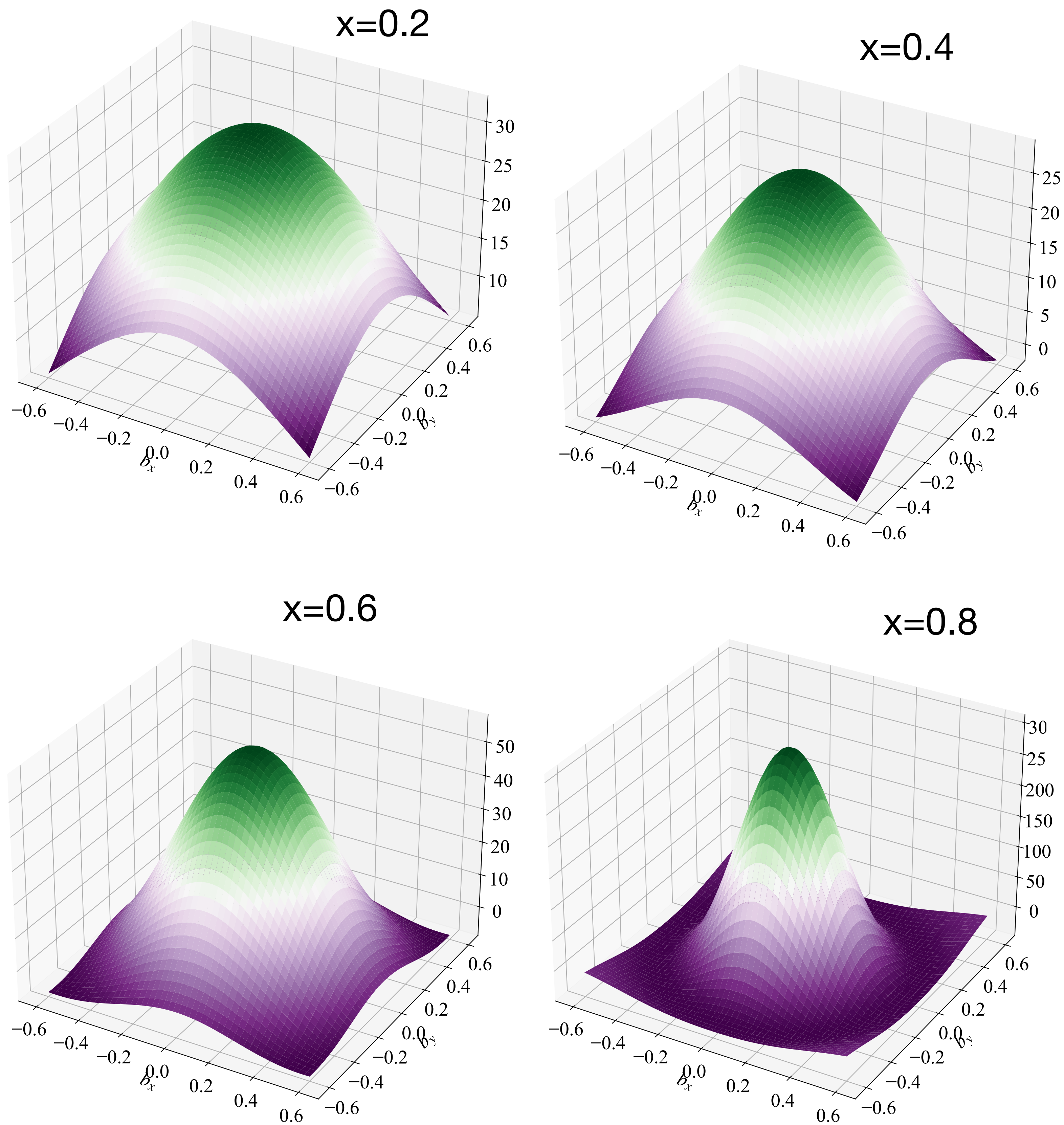
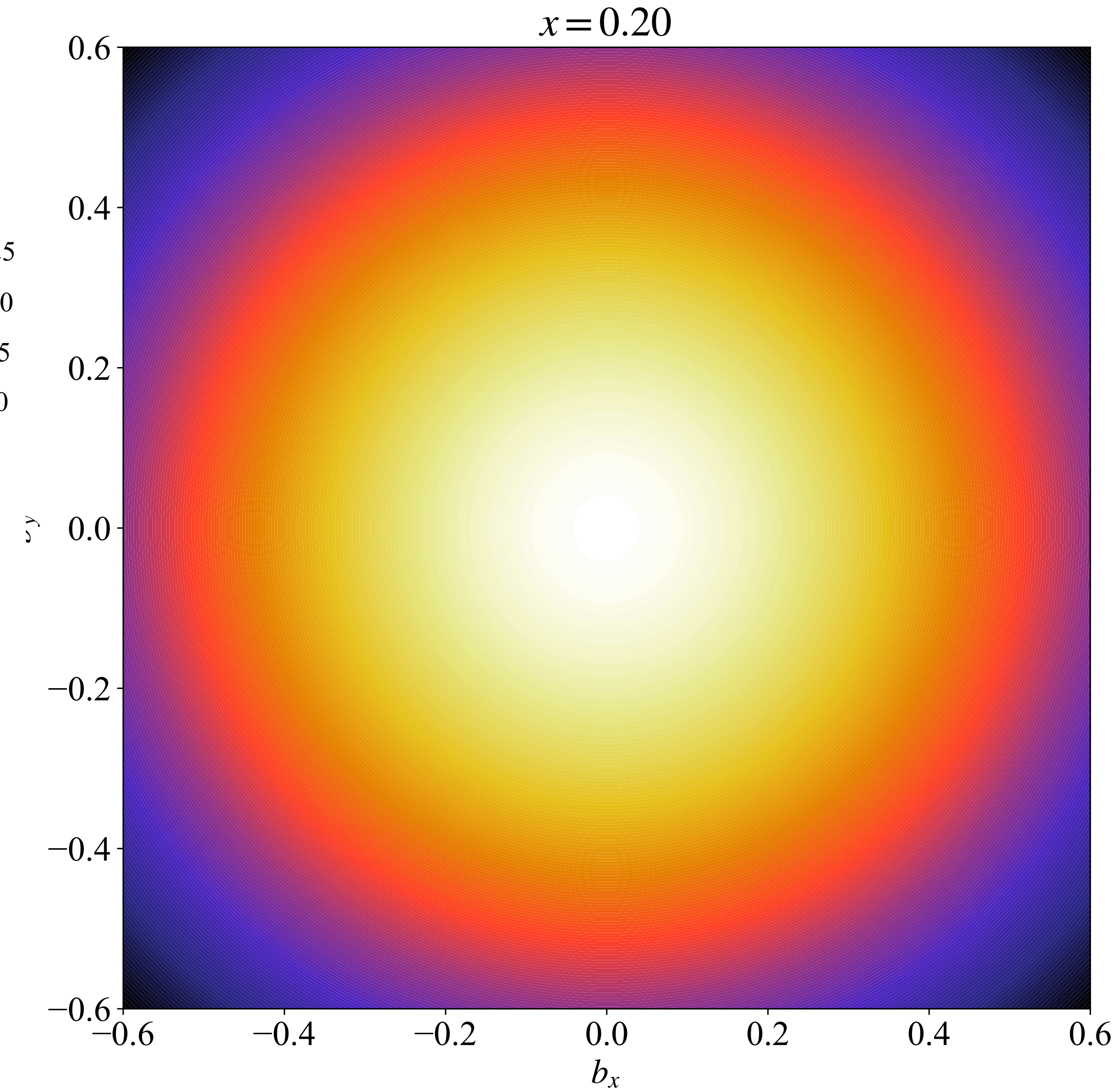


Impact parameter space $\widetilde{H} + \widetilde{G}_2$



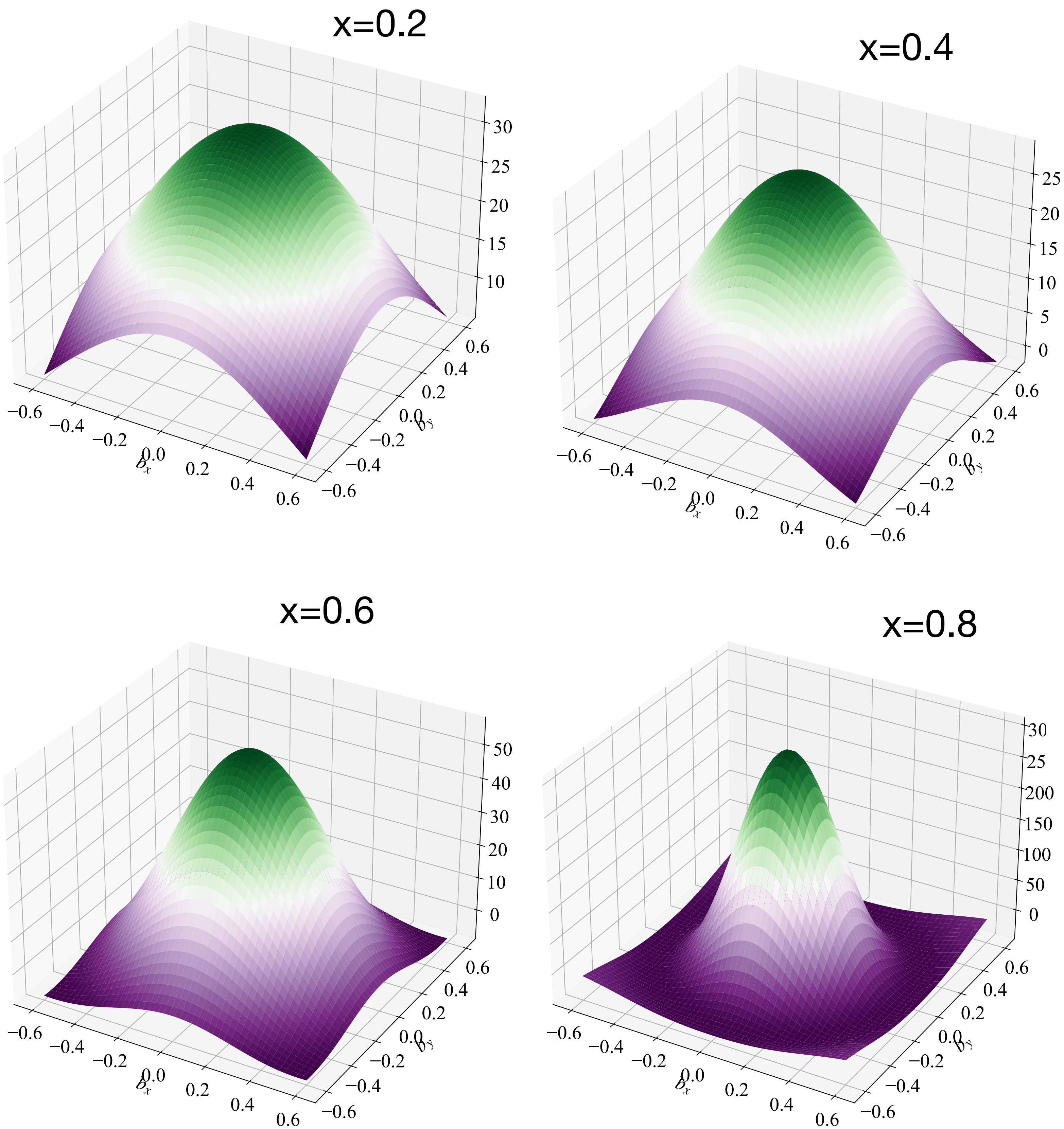


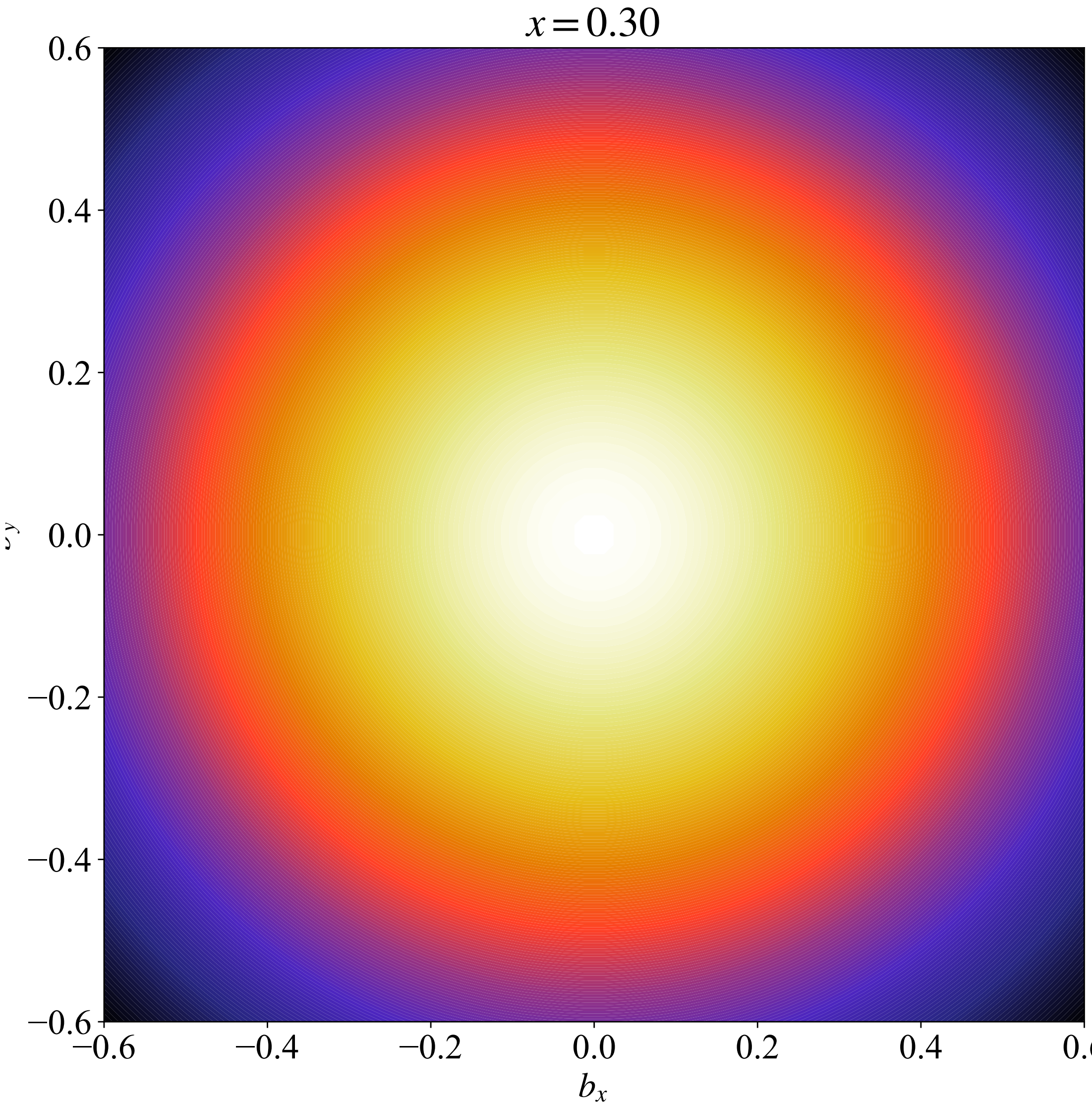
★ GPDs in transverse plane

$$q(x, \mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$
$$= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp},$$

$b_\perp$: transverse distance from the (transverse) center of momentum

Impact parameter space $\widetilde{H} + \widetilde{G}_2$





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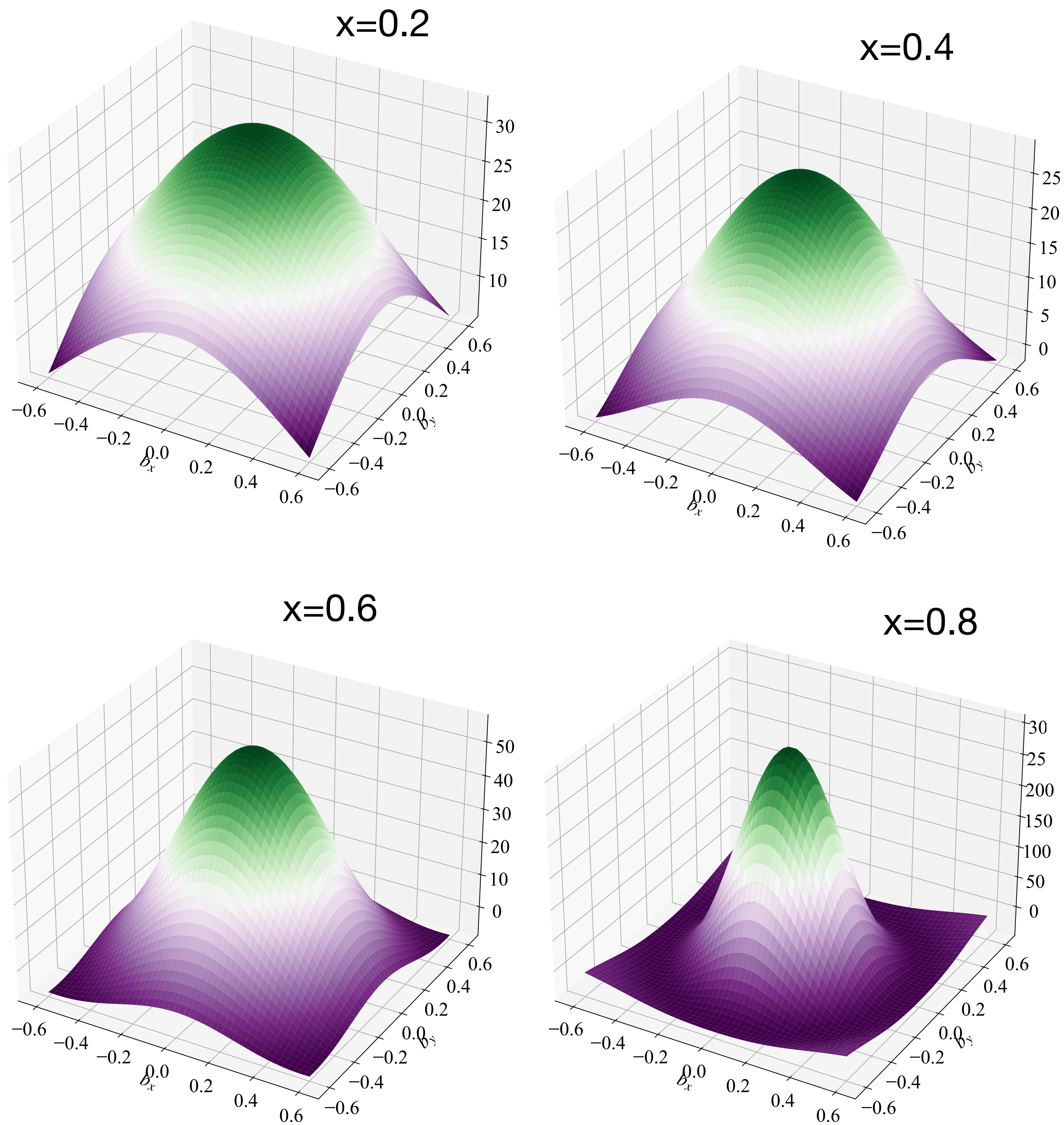

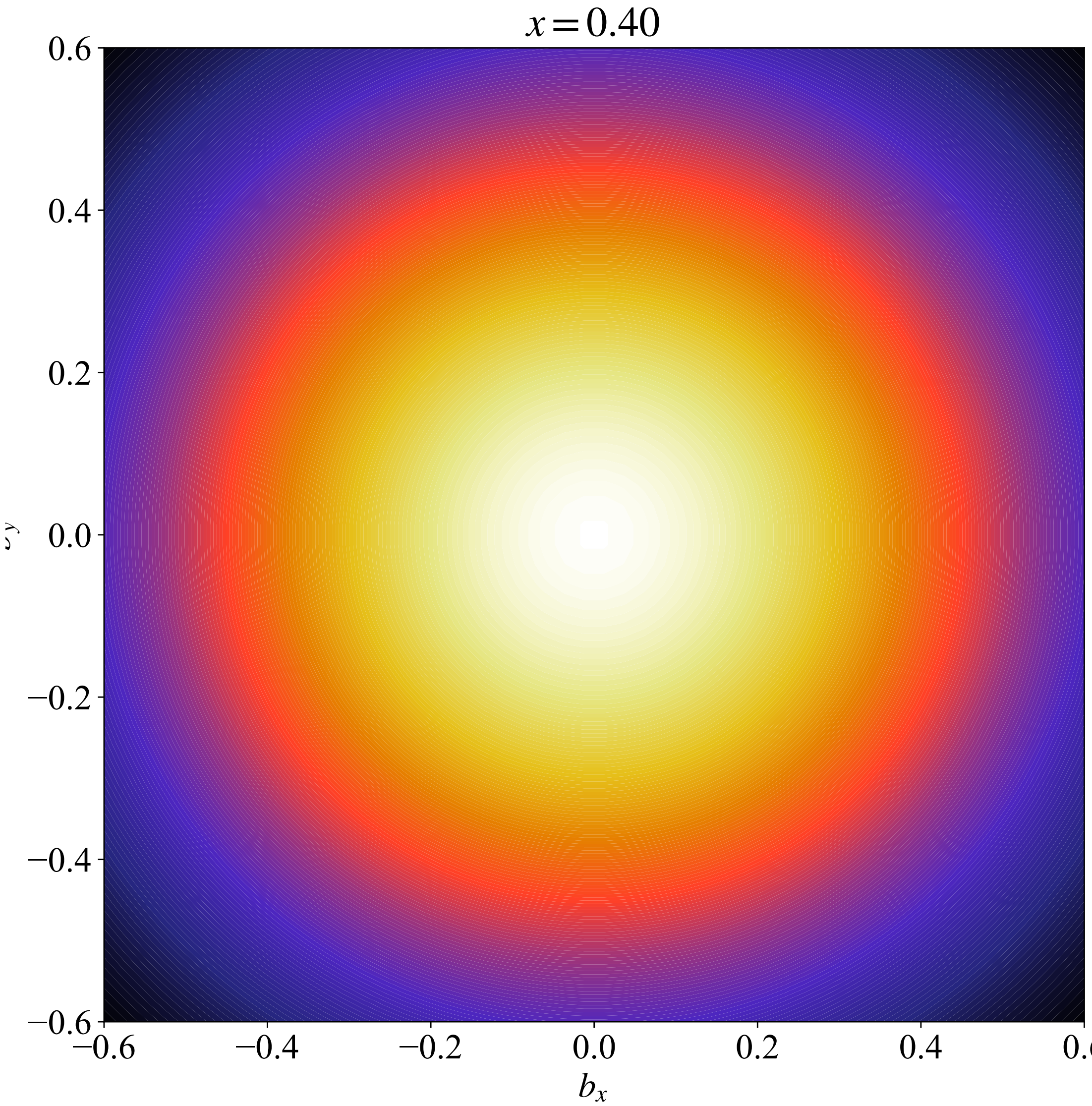

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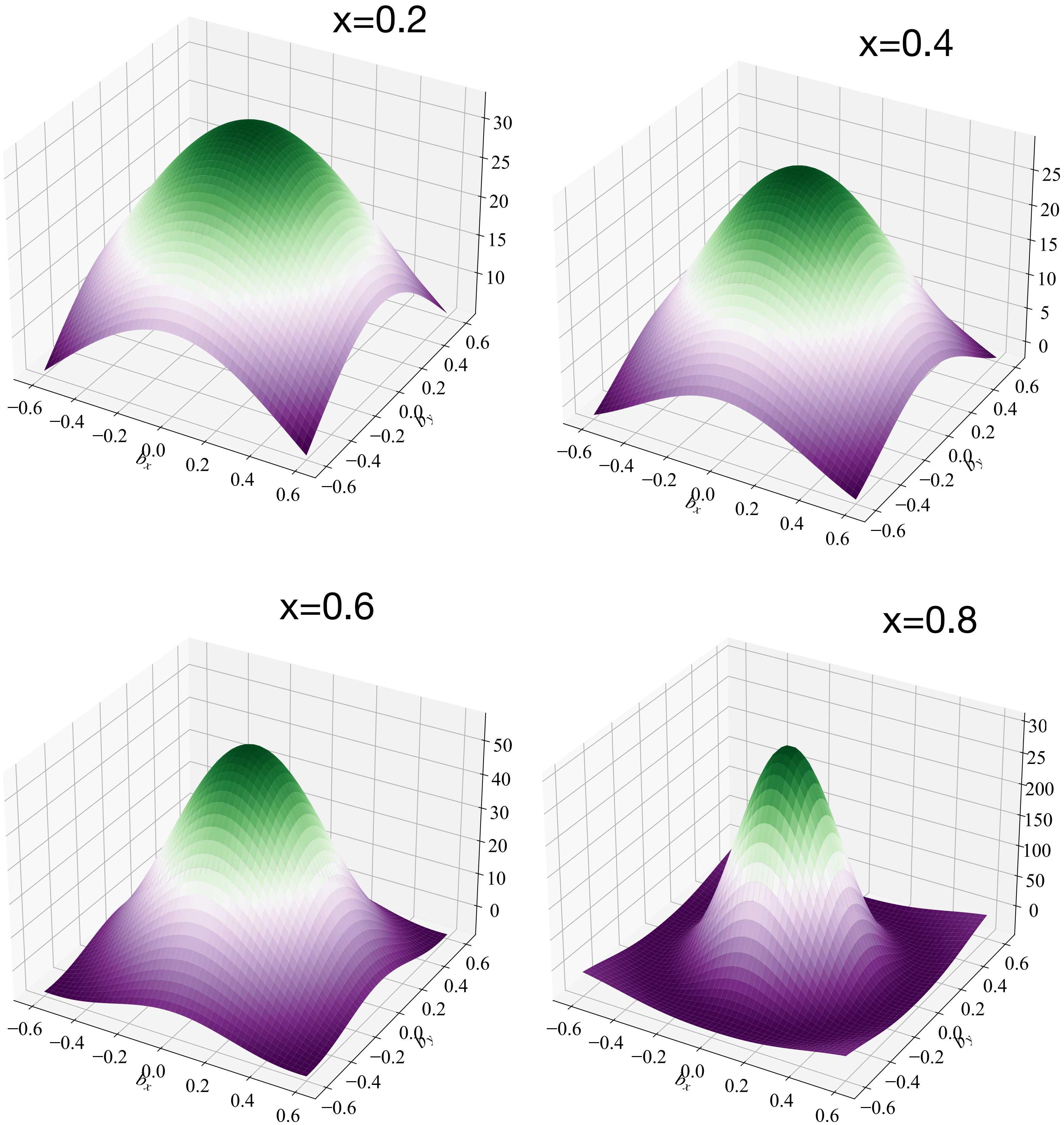

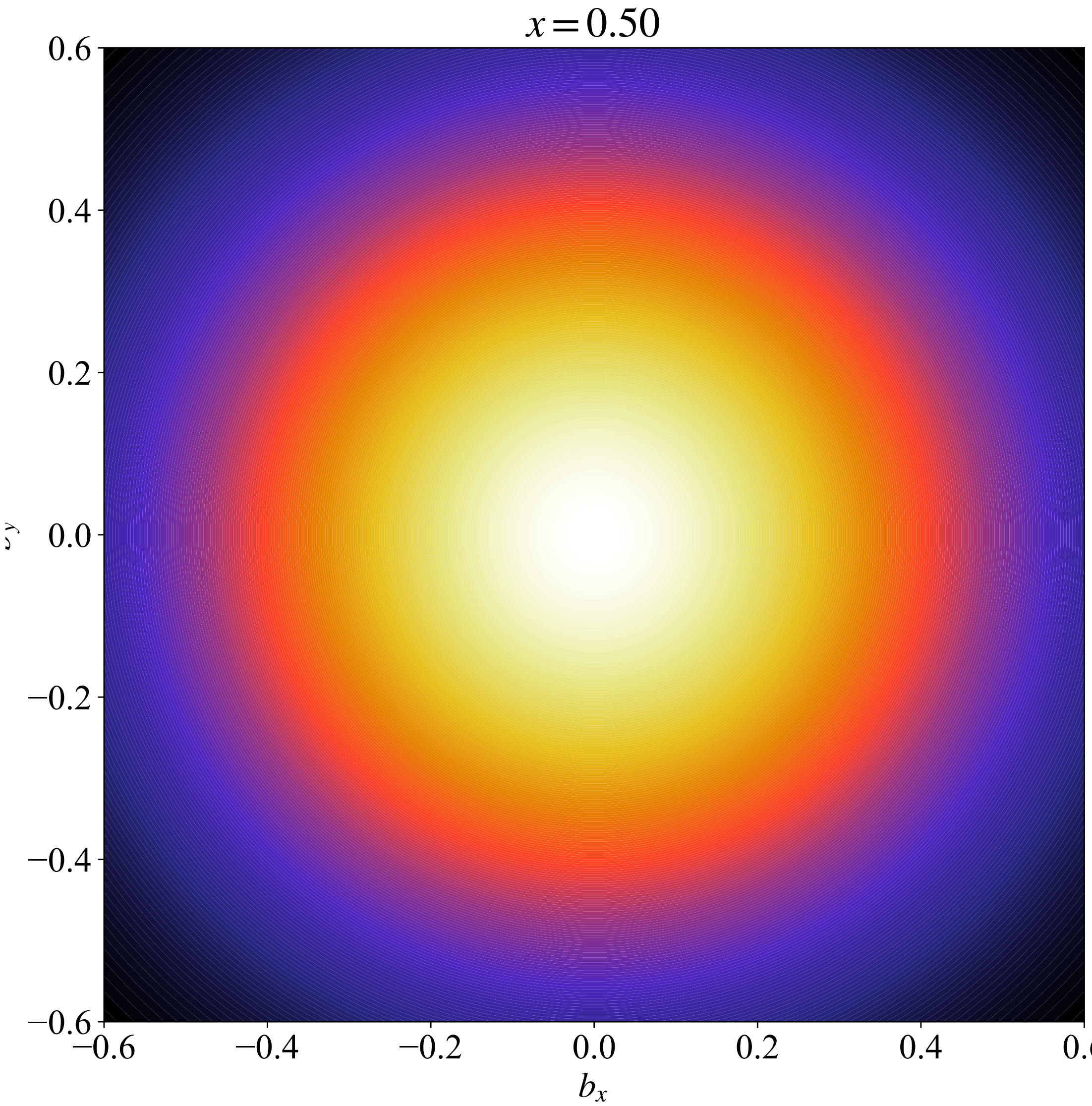

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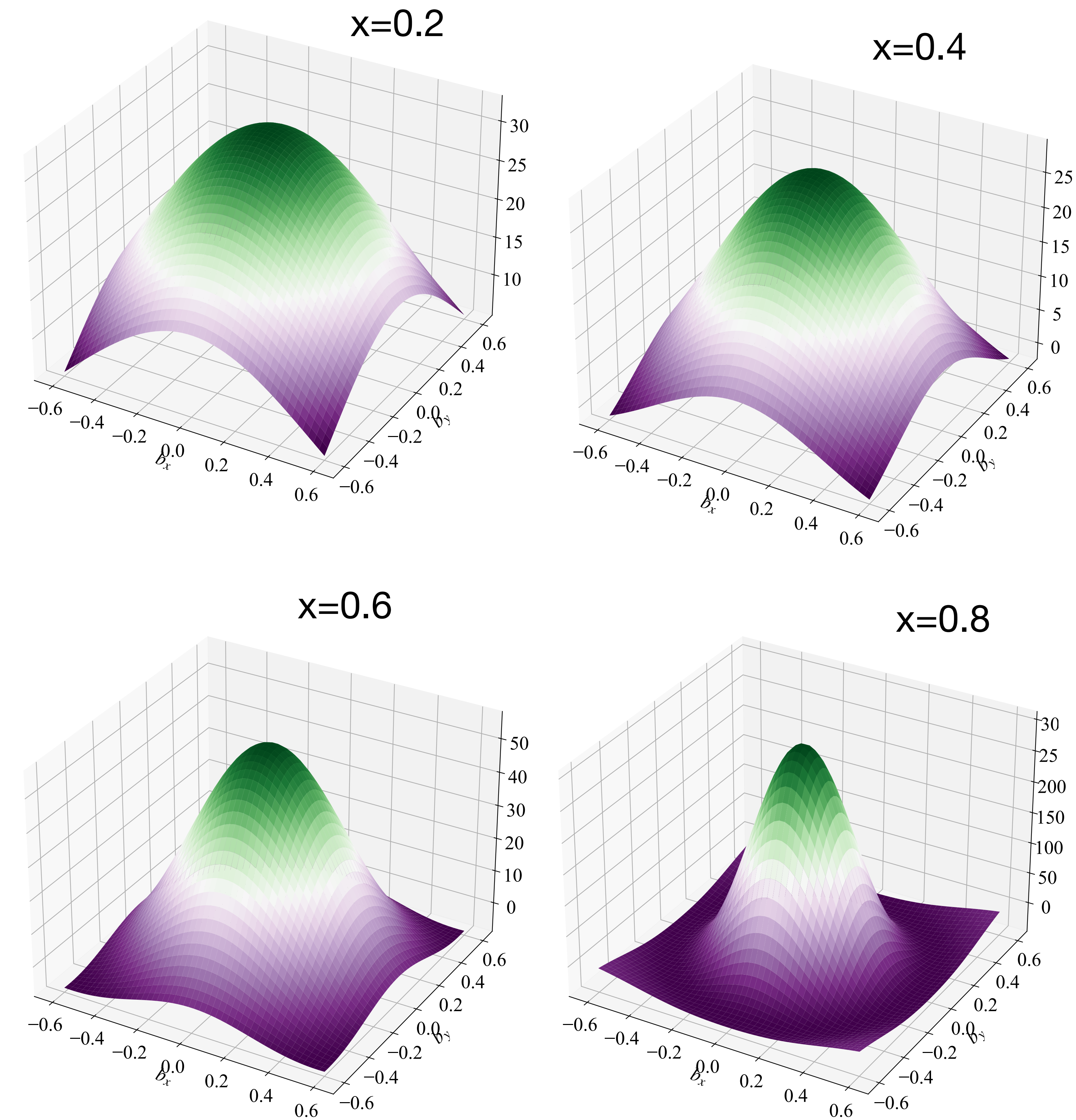
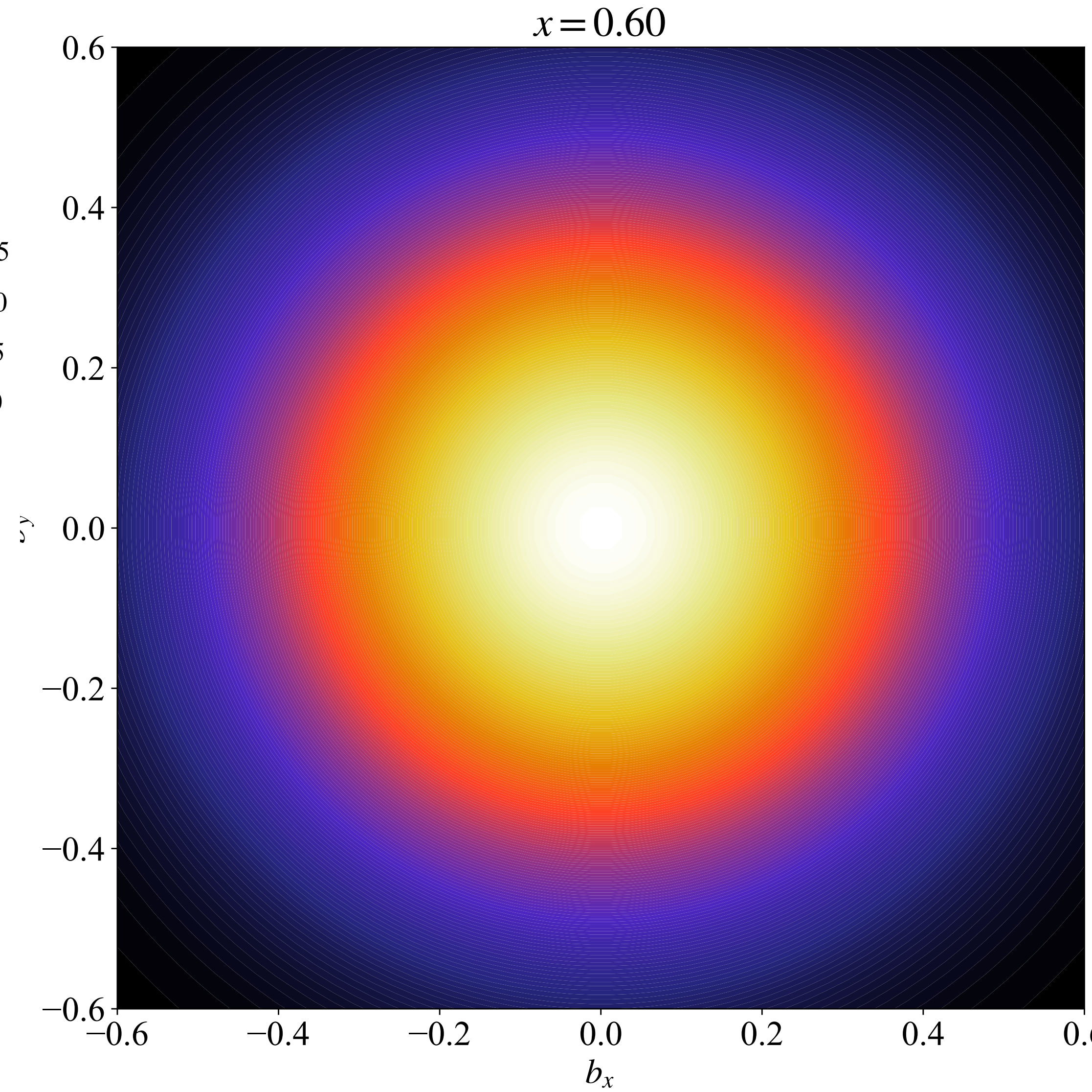
Impact parameter space $\widetilde{H} + \widetilde{G}_2$





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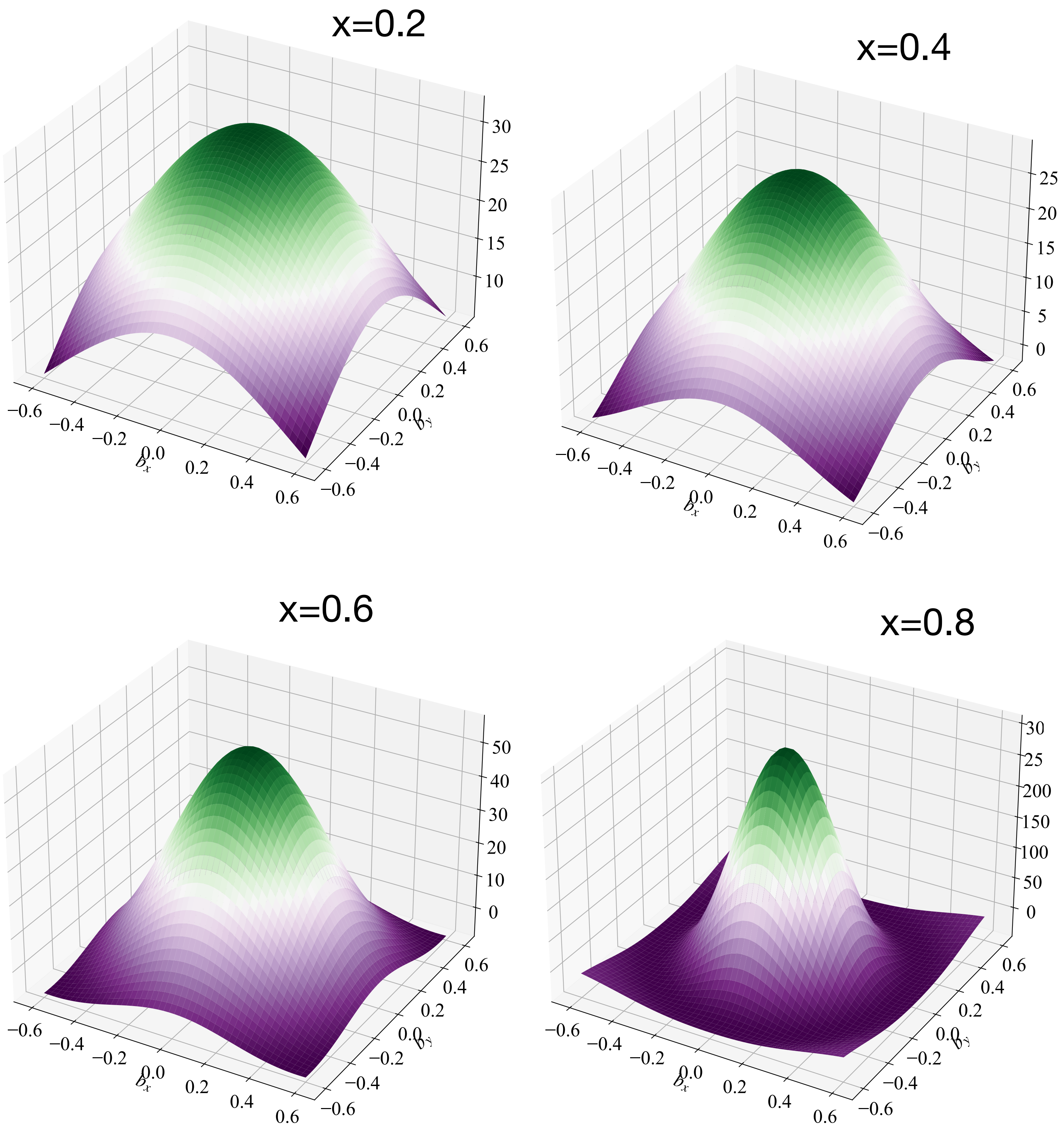
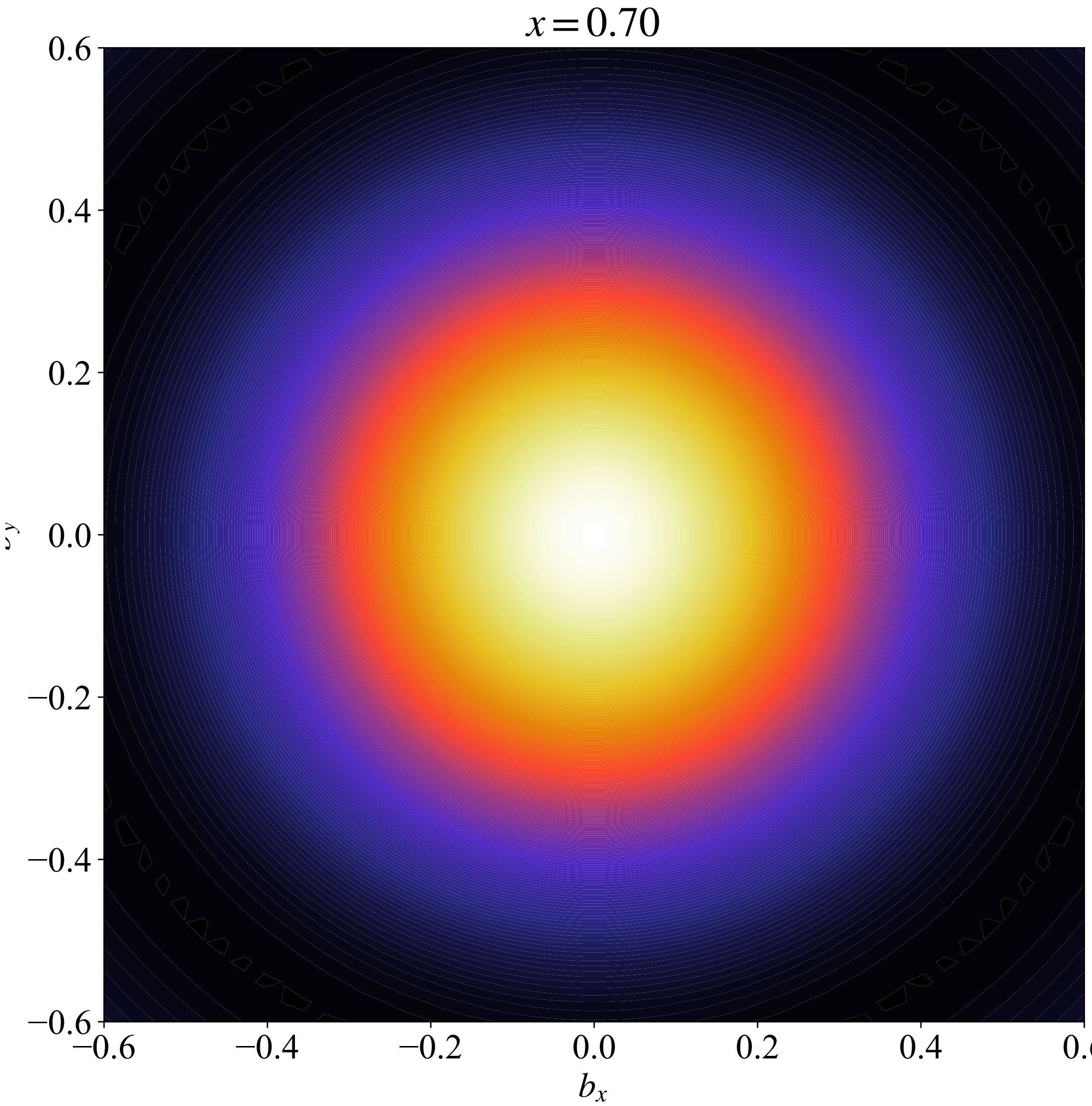
Impact parameter space $\widetilde{H} + \widetilde{G}_2$





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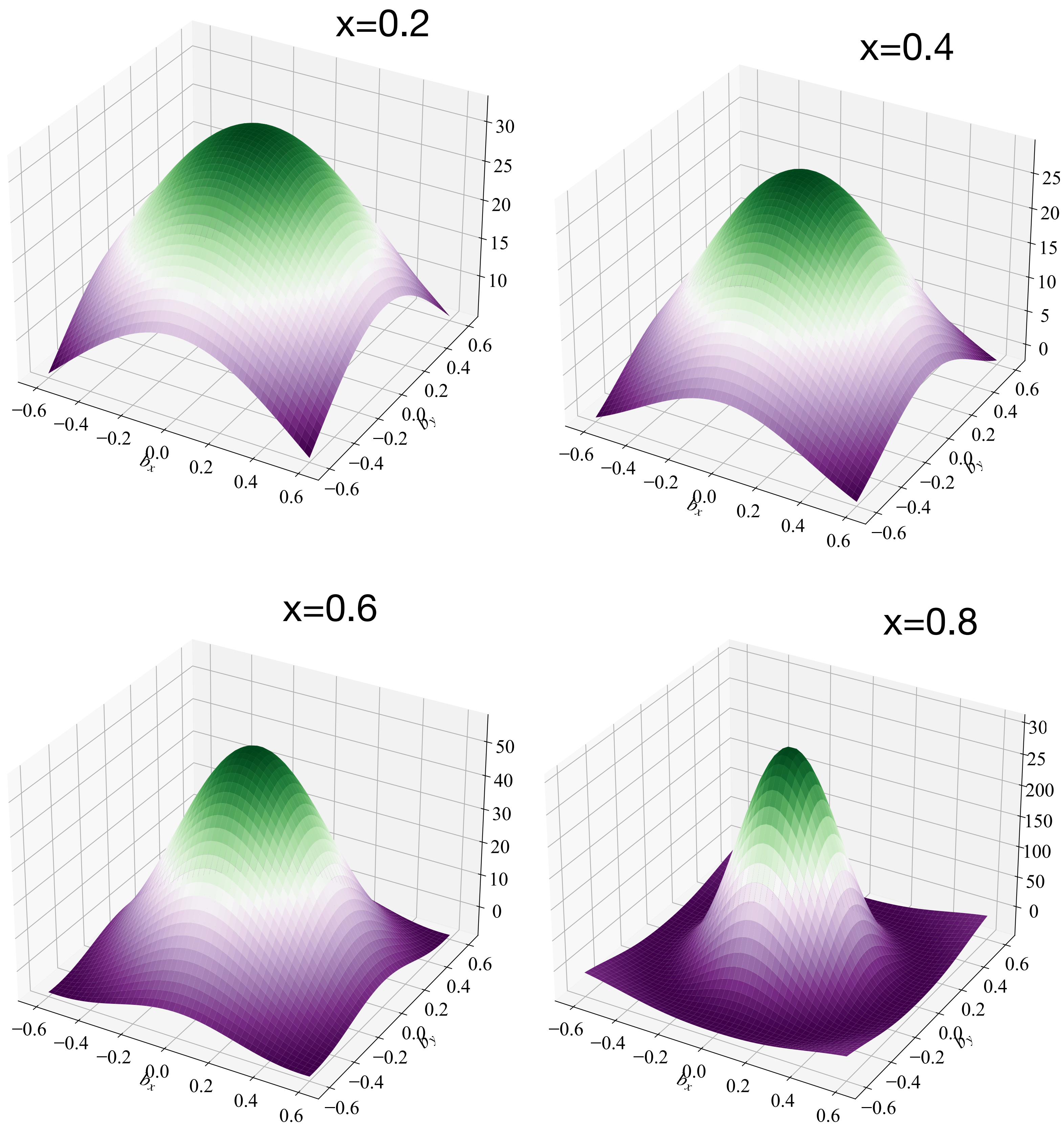
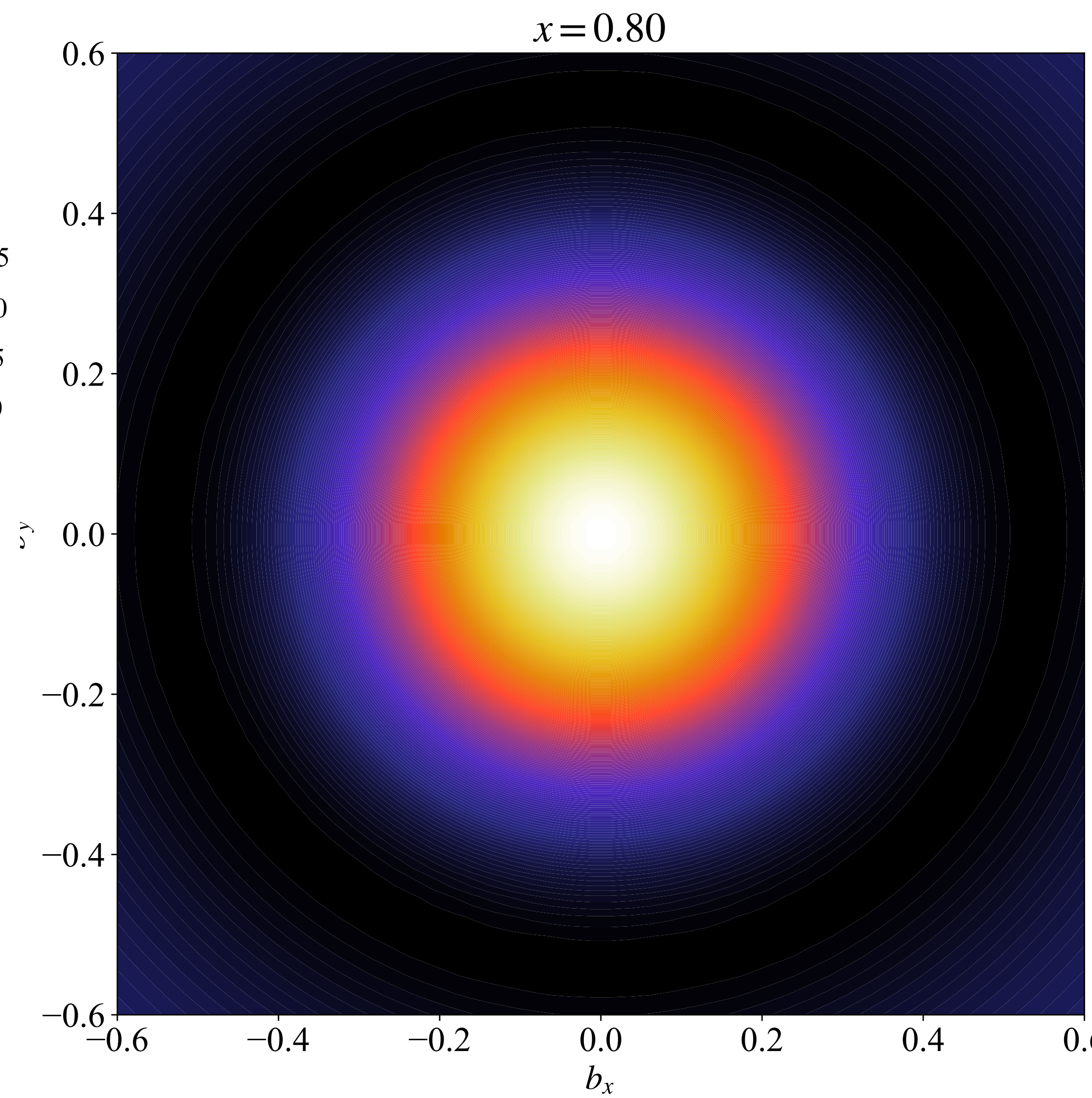
Impact parameter space $\widetilde{H} + \widetilde{G}_2$





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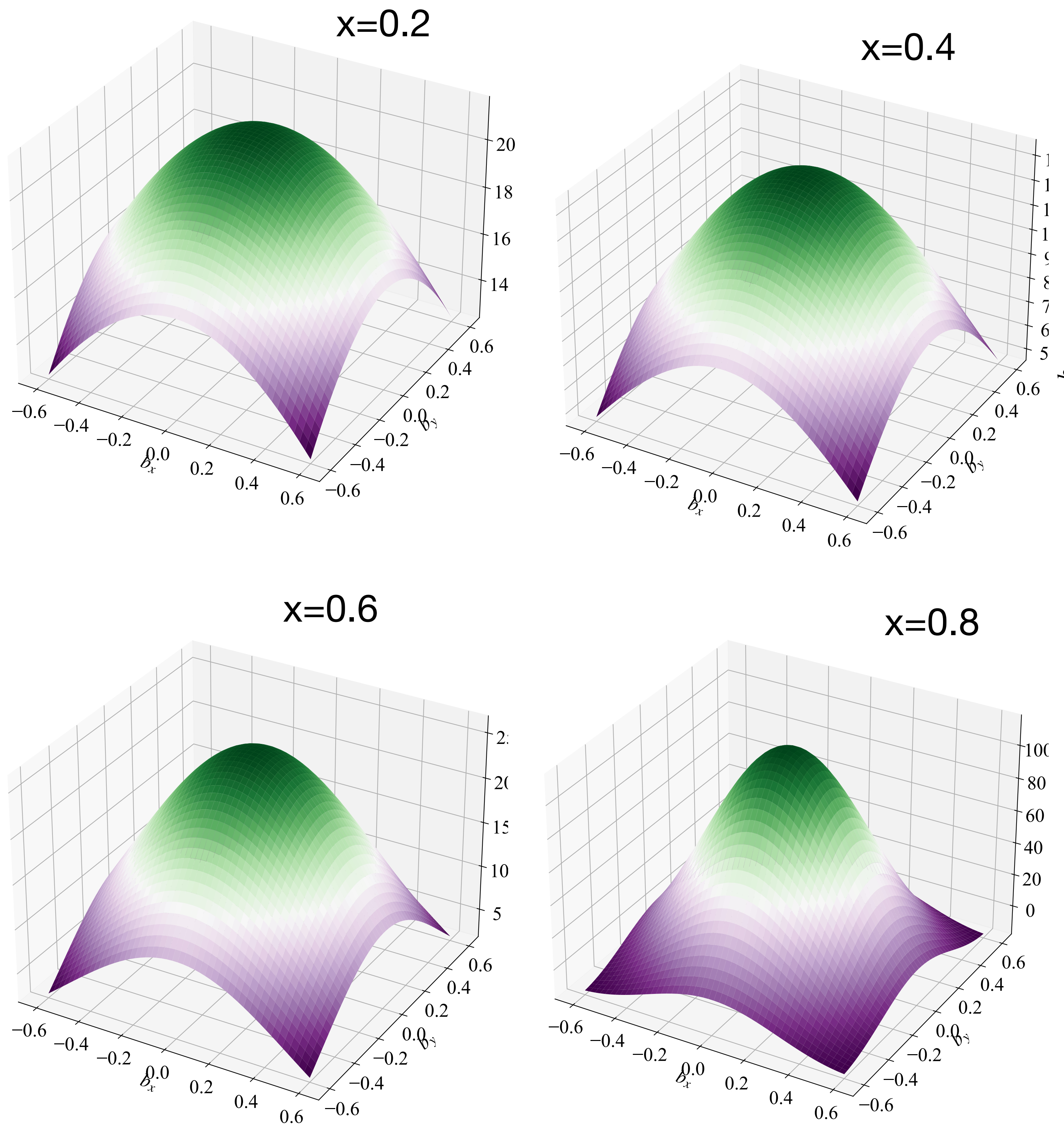
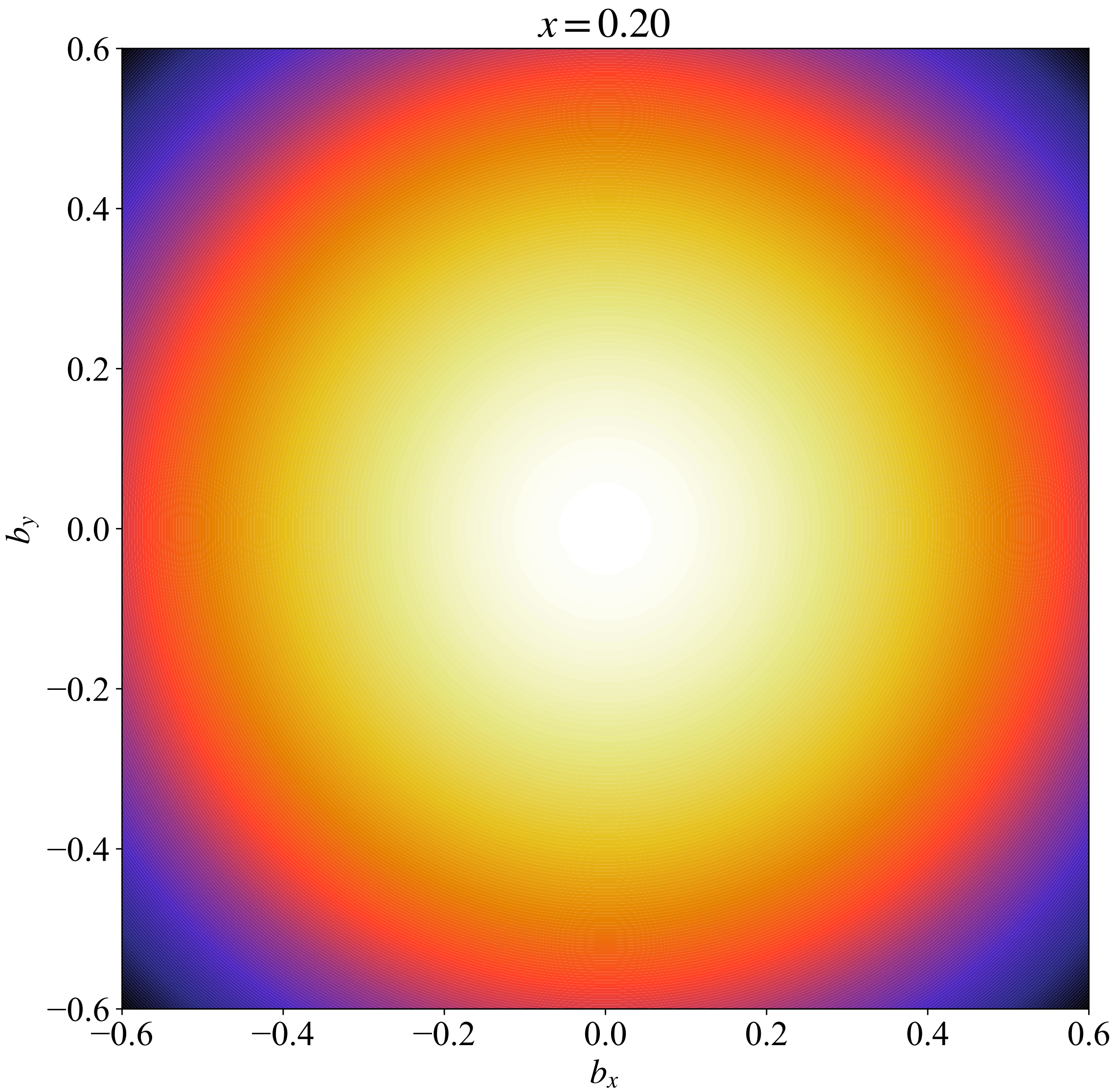
Impact parameter space $\widetilde{E} + \widetilde{G}_1$





★ GPDs in transverse plane

$$q(x, \mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$
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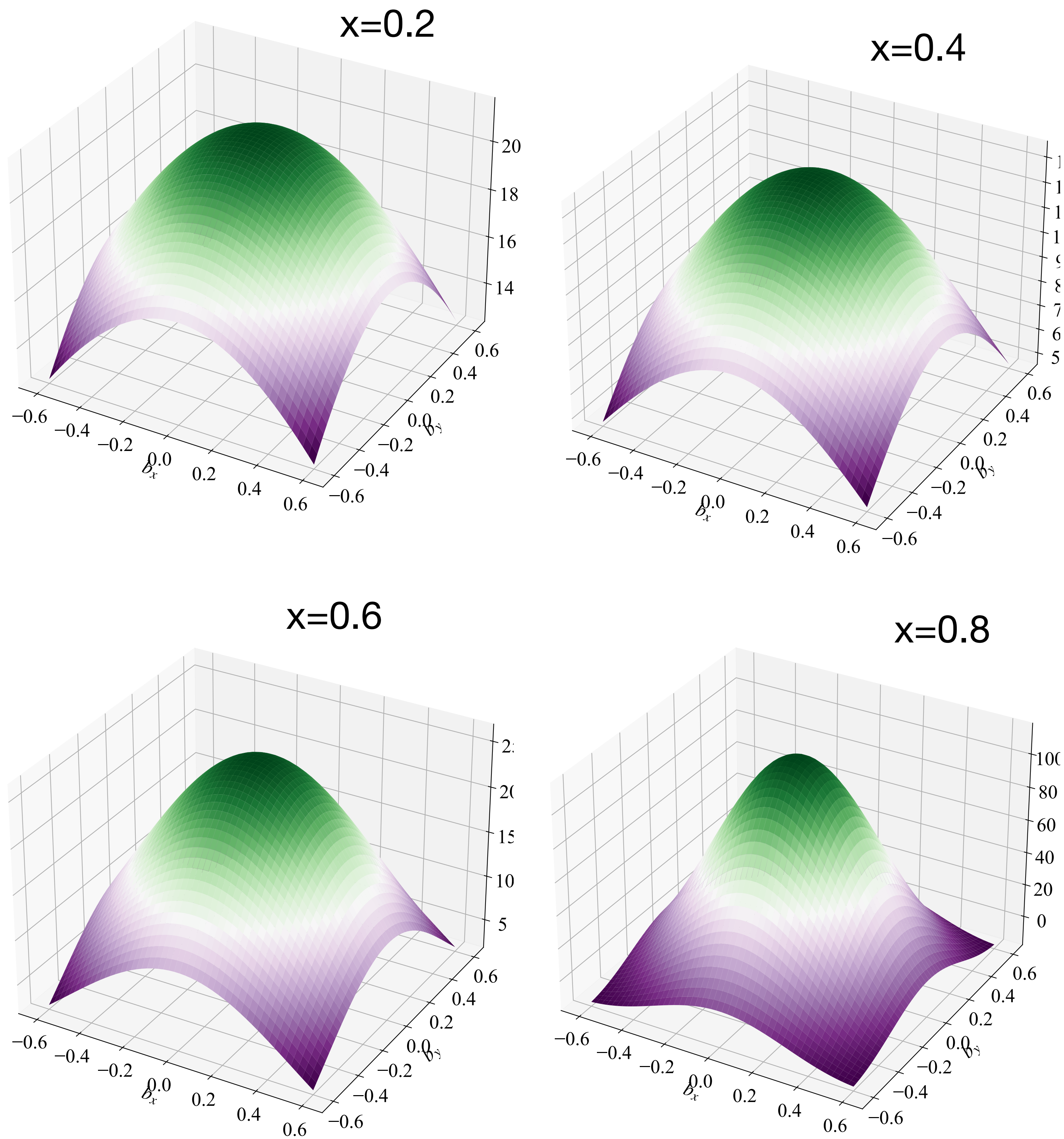
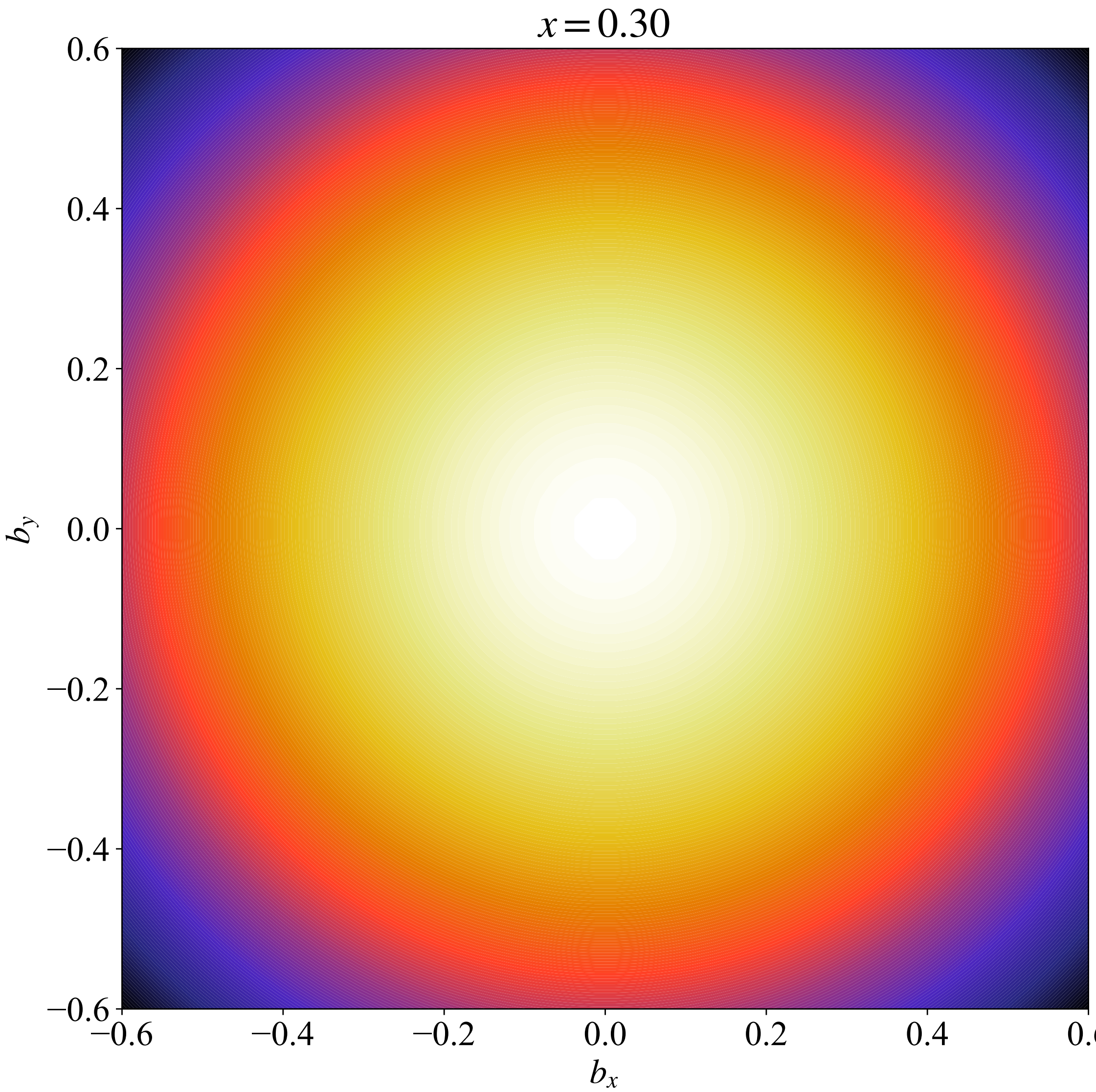
$b_\perp$: transverse distance from the (transverse) center of momentum

Impact parameter space $\widetilde{E} + \widetilde{G}_1$





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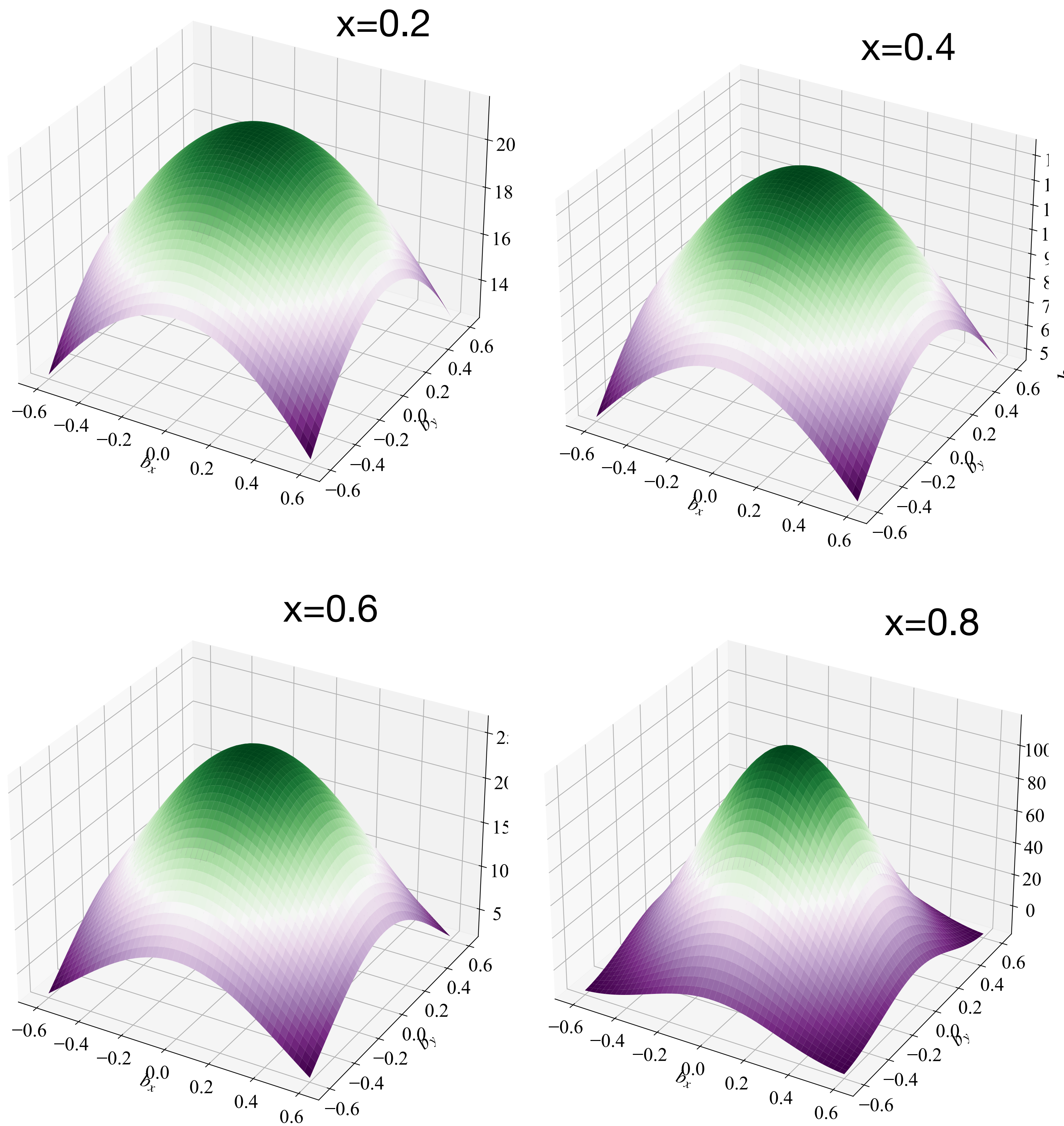

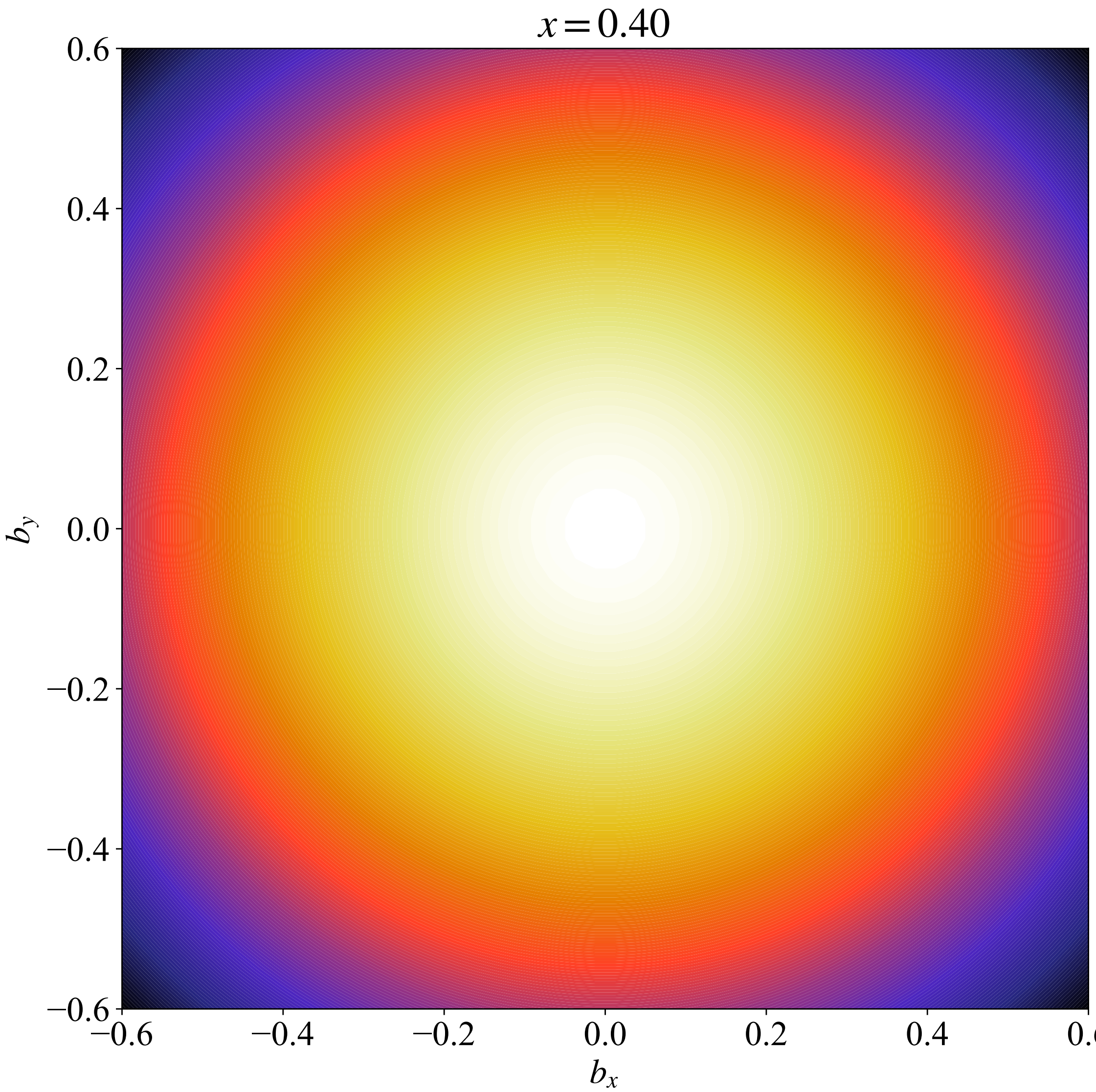

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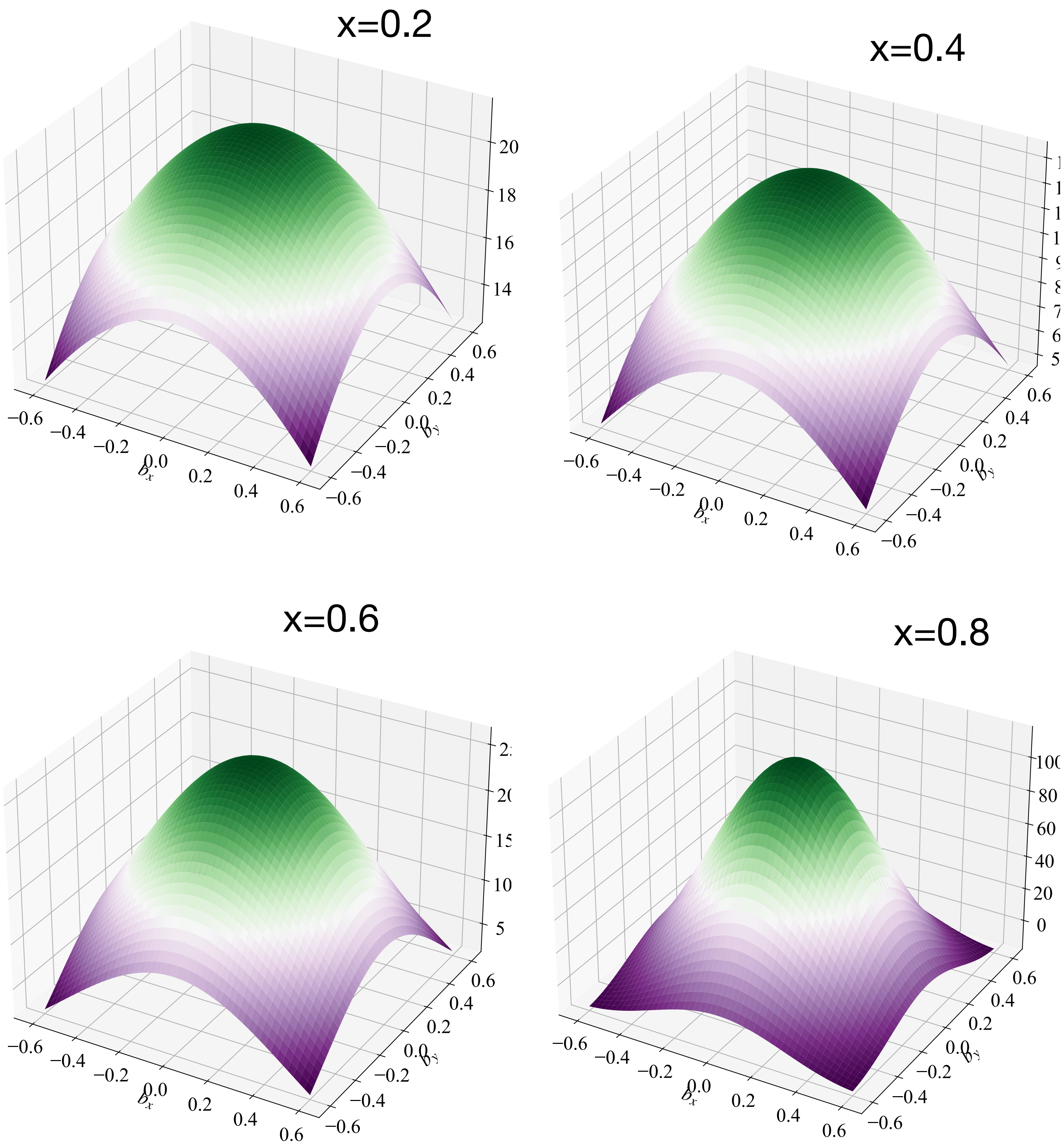

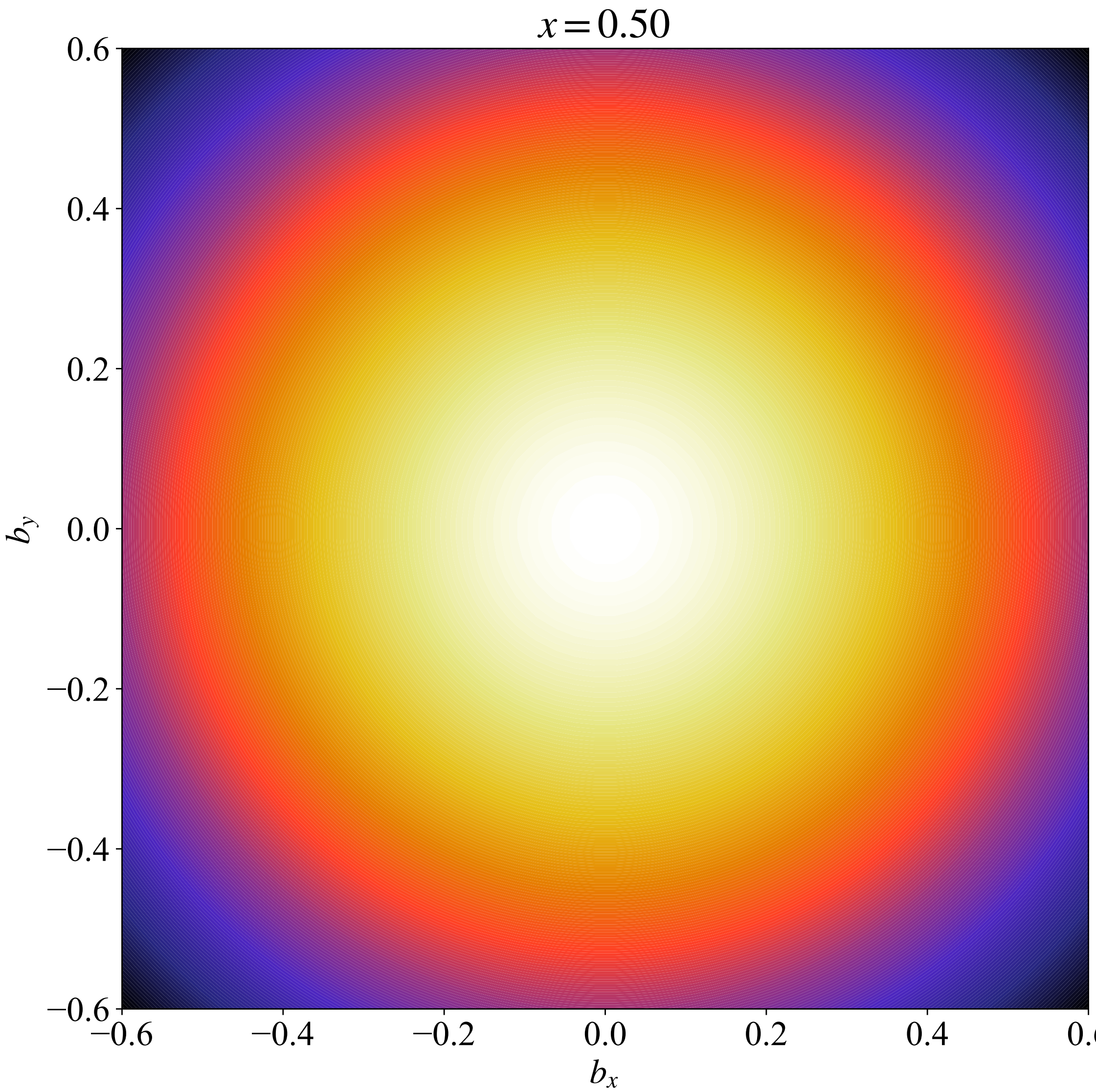

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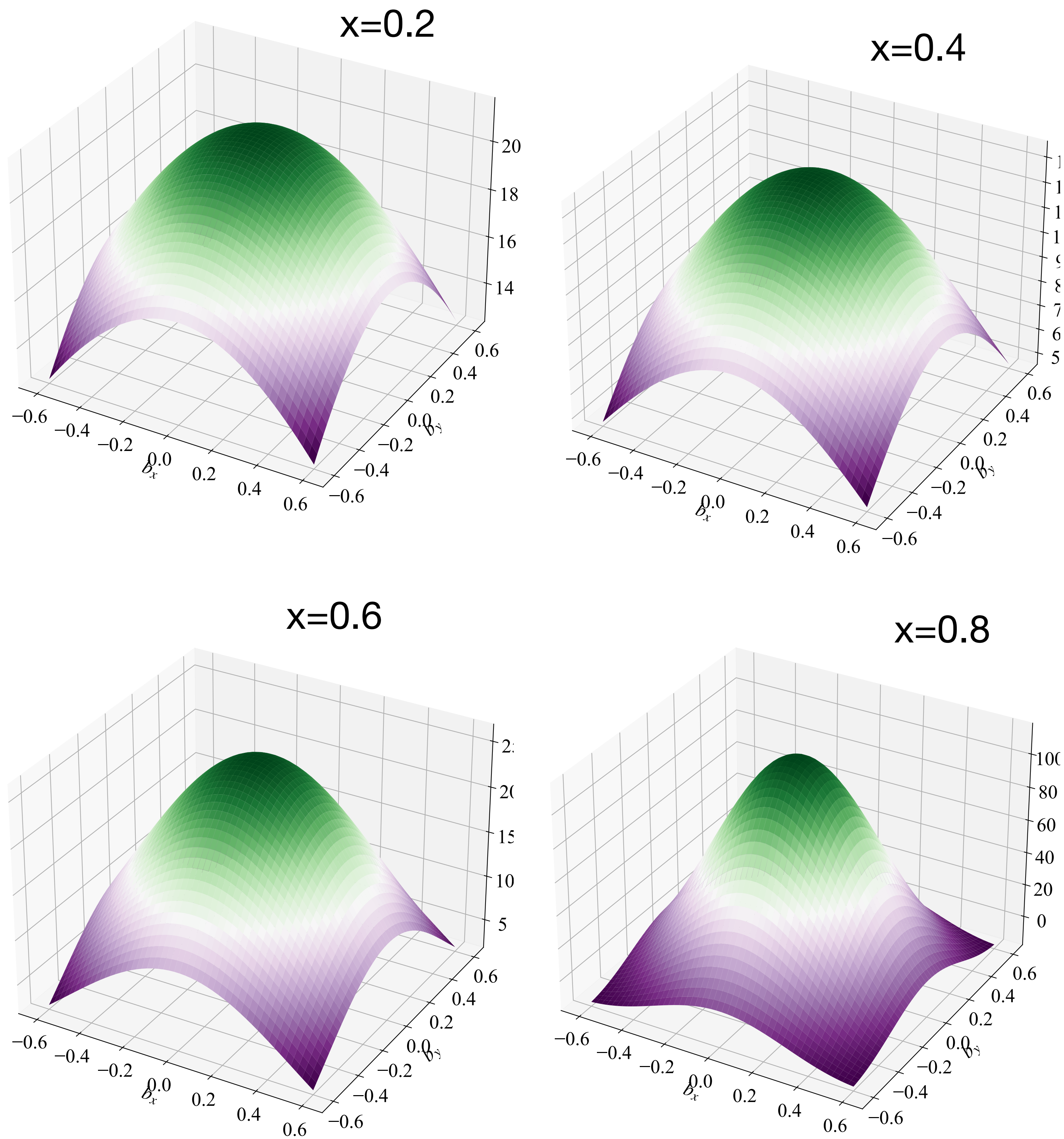

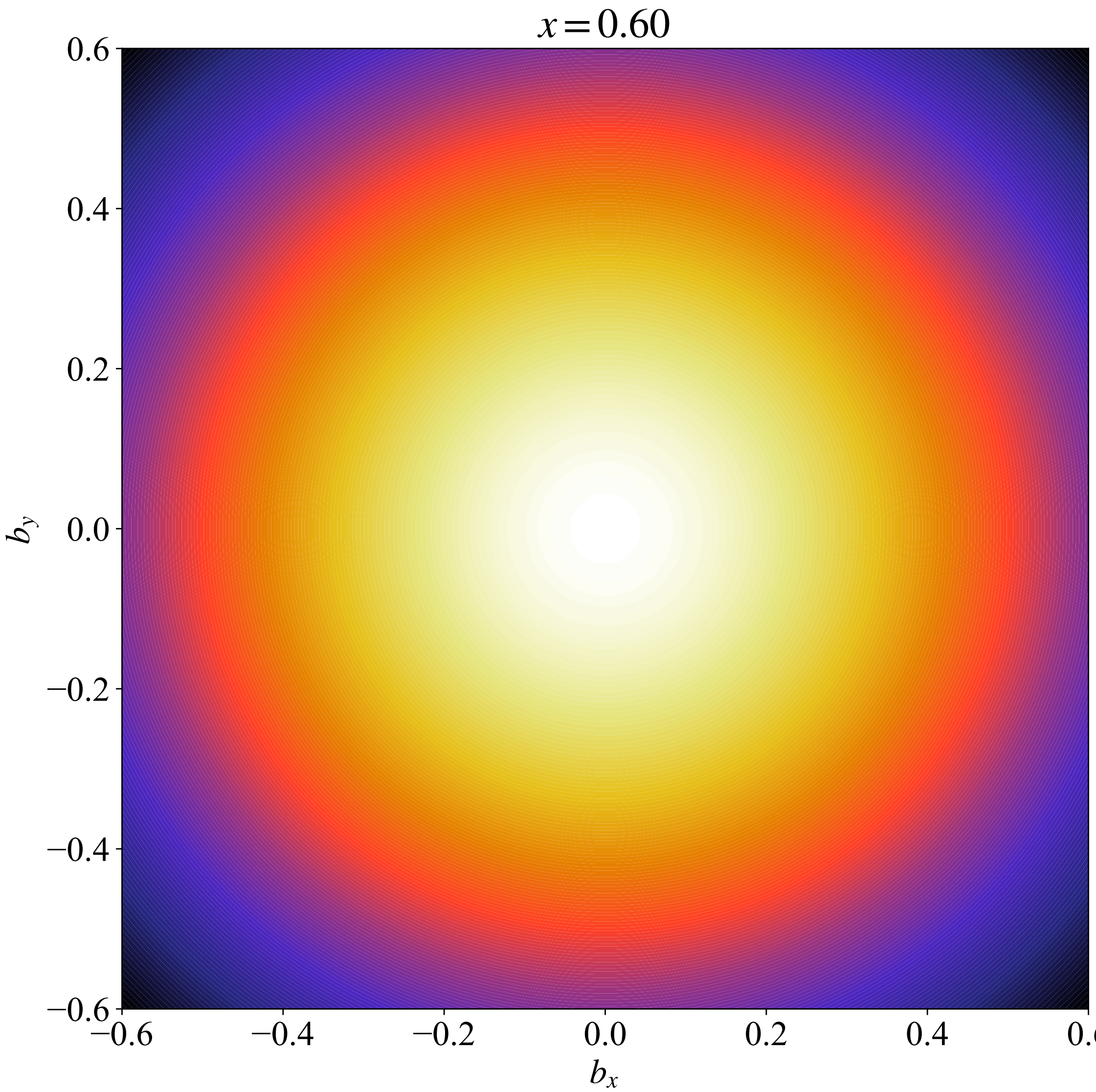

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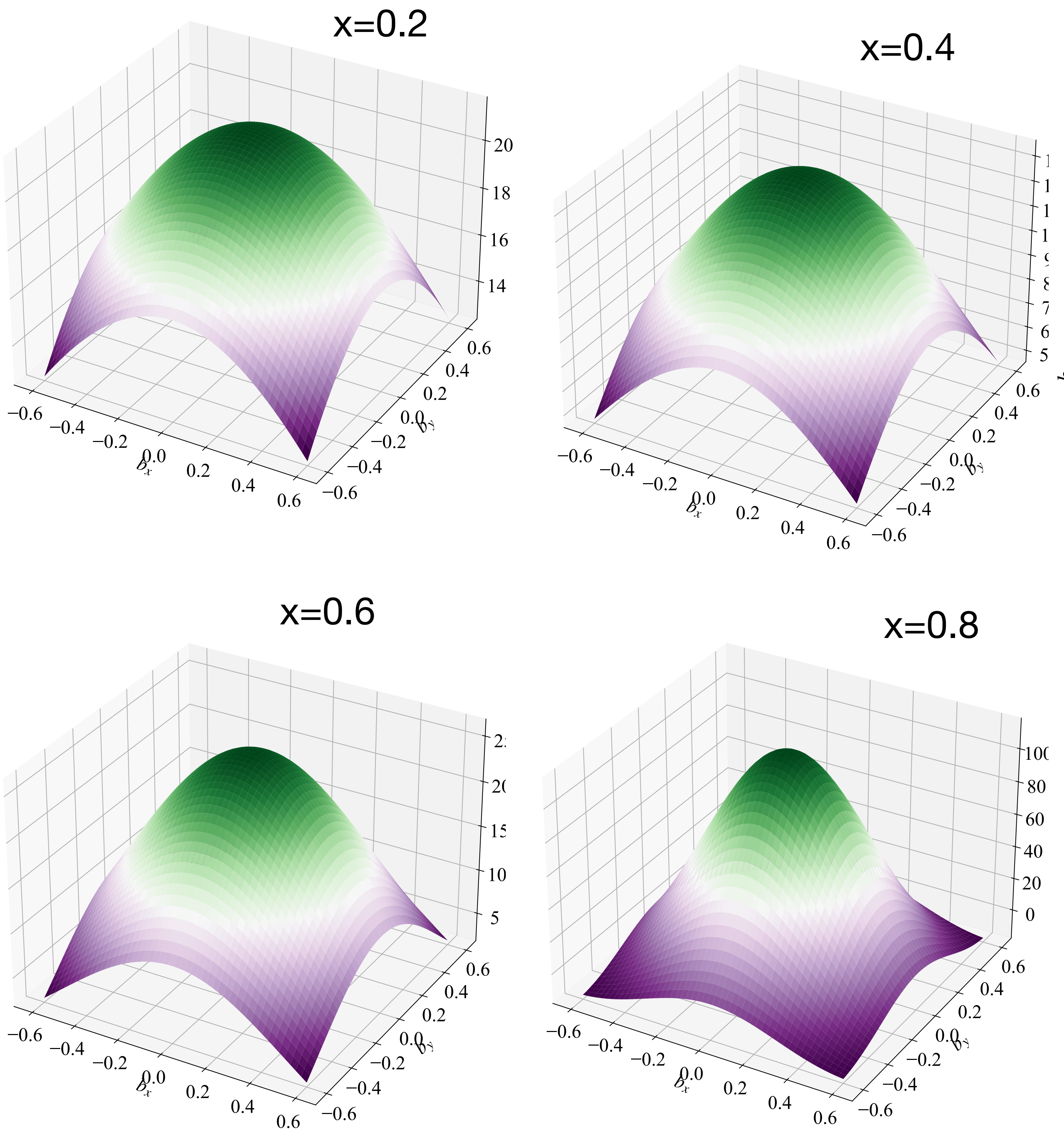

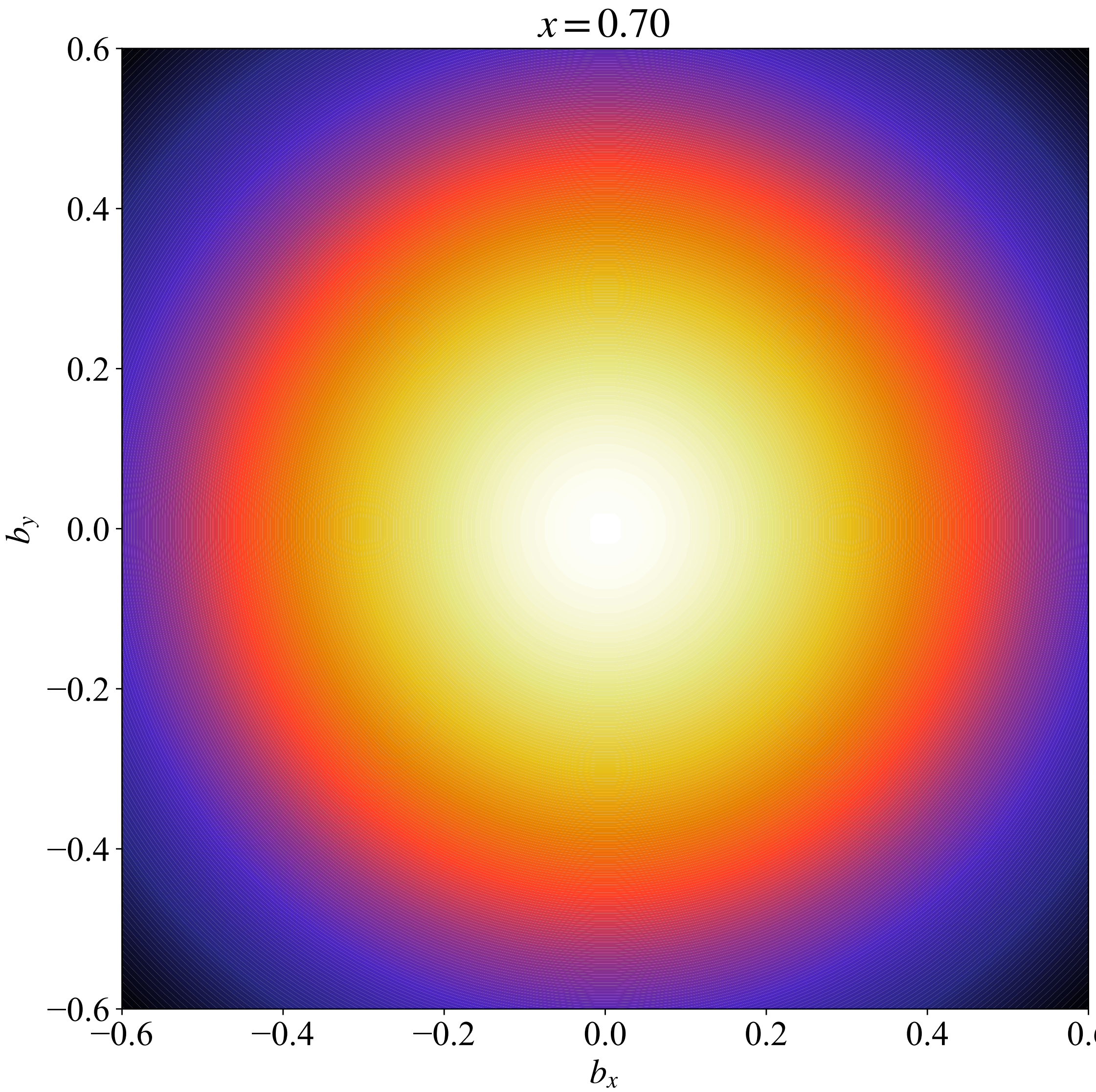

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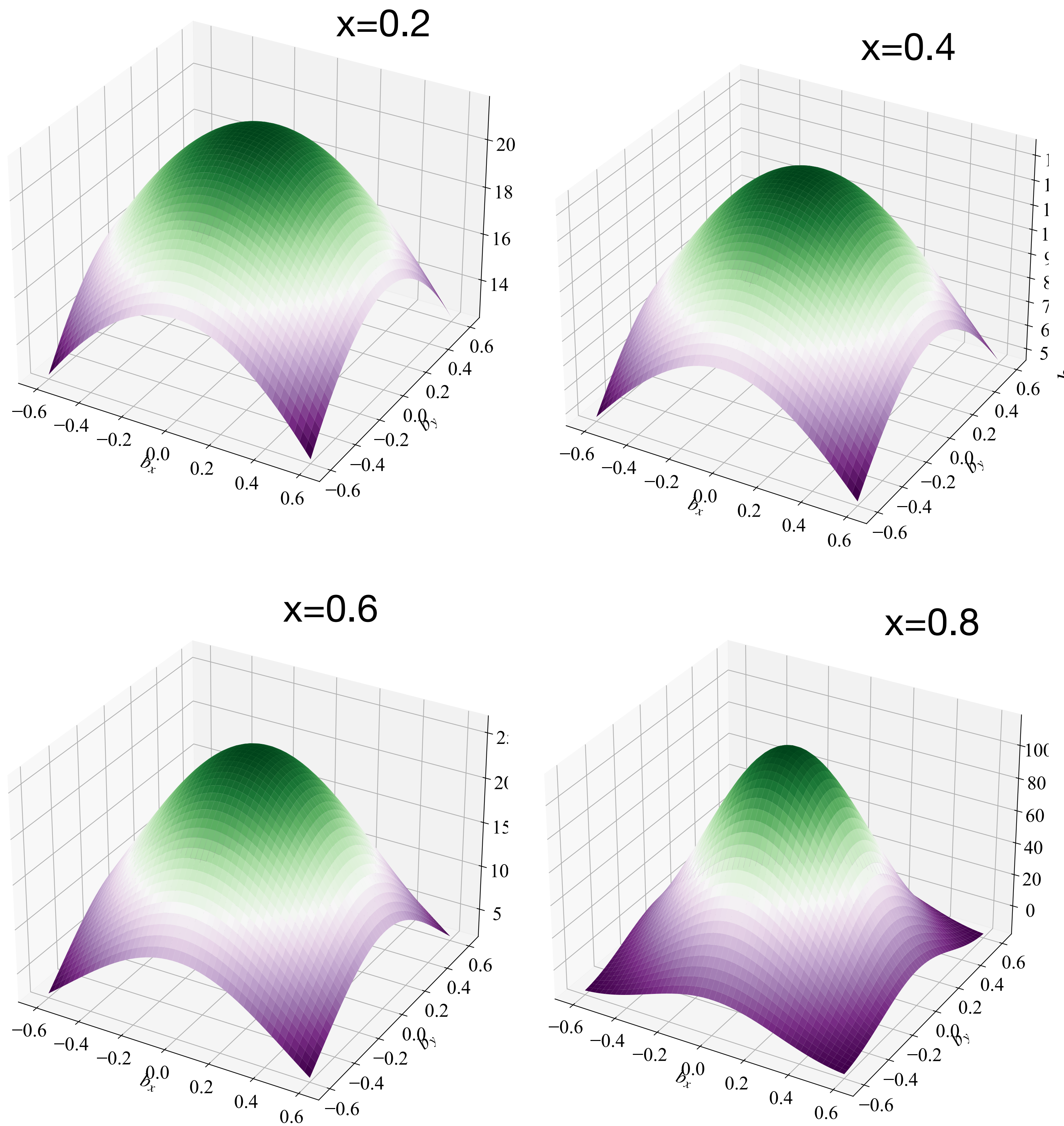

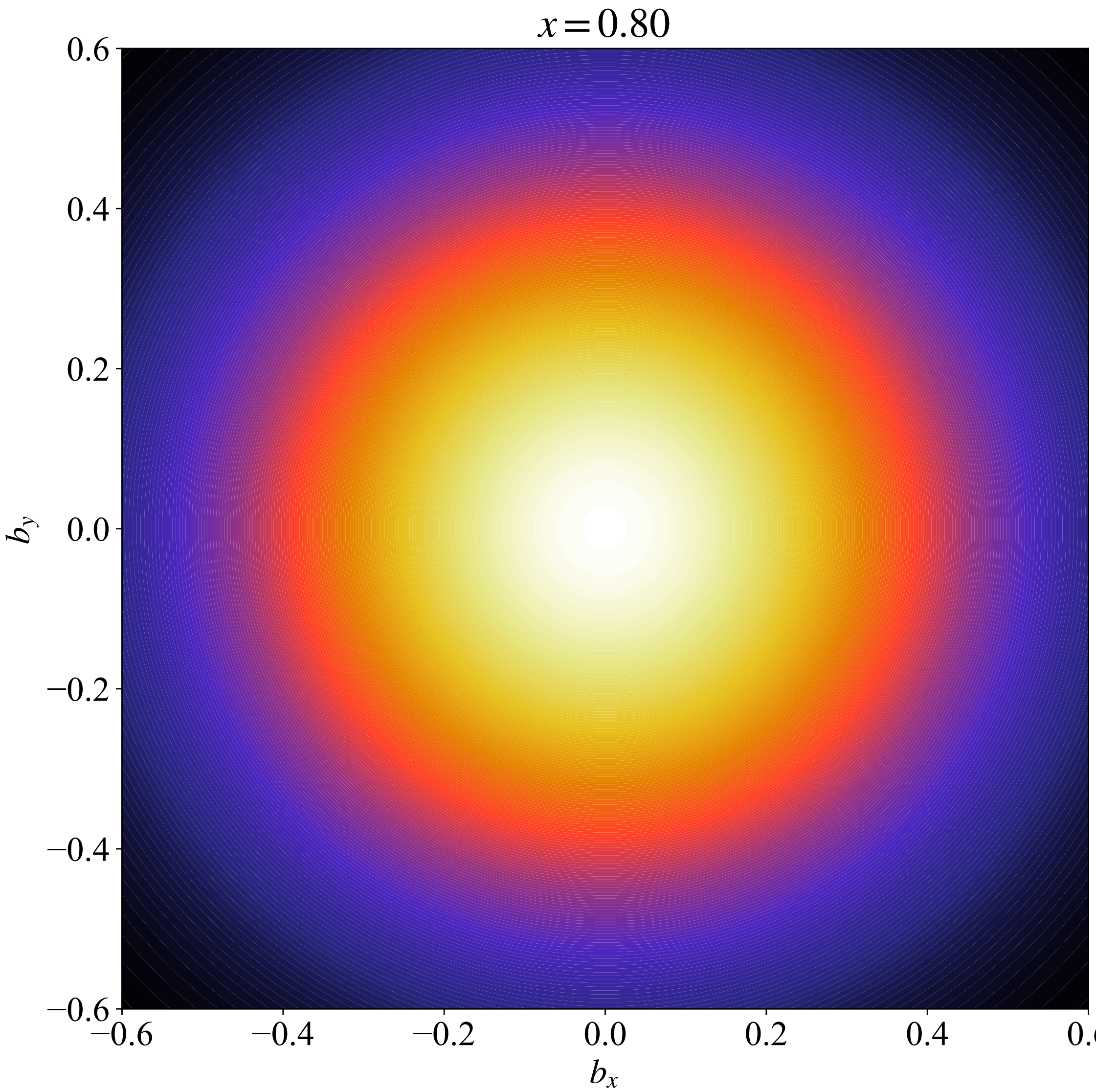

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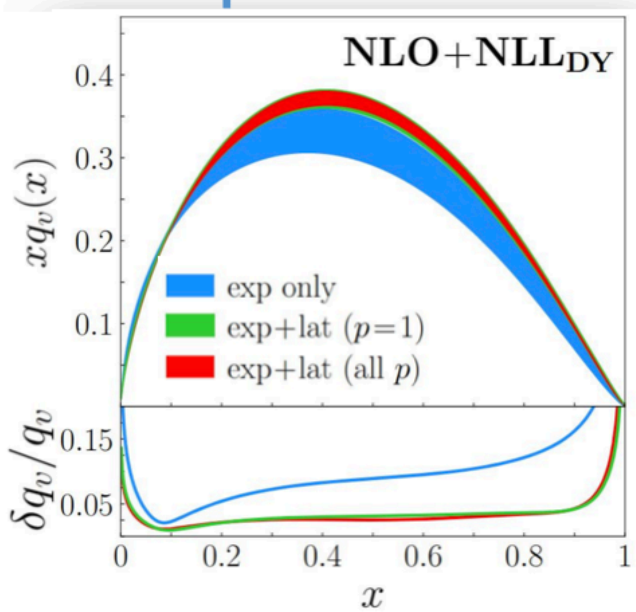
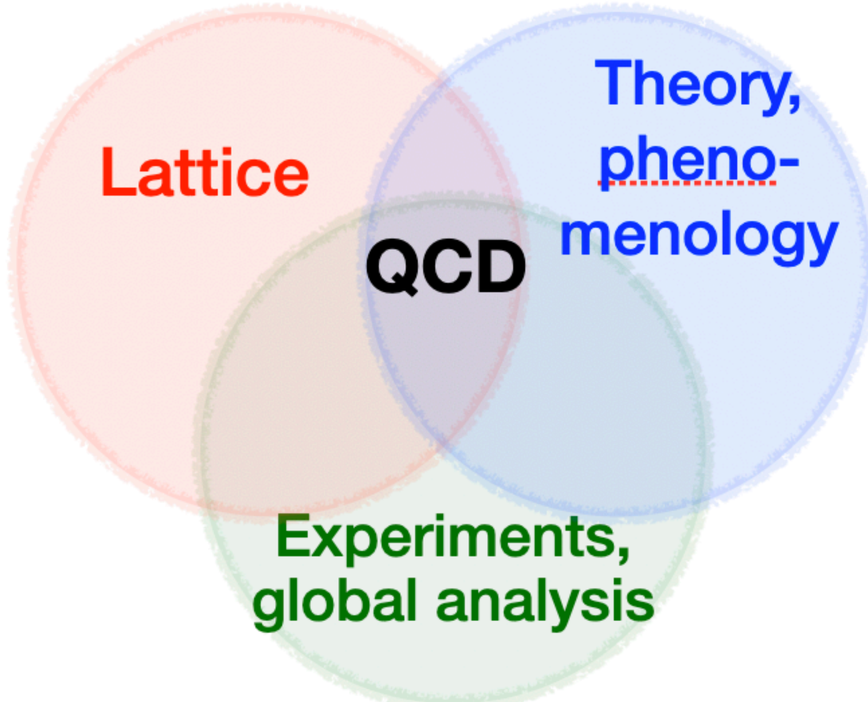
Synergy/Complementarity of lattice and phenomenology

Synergies: constraints & predictive power of lattice QCD

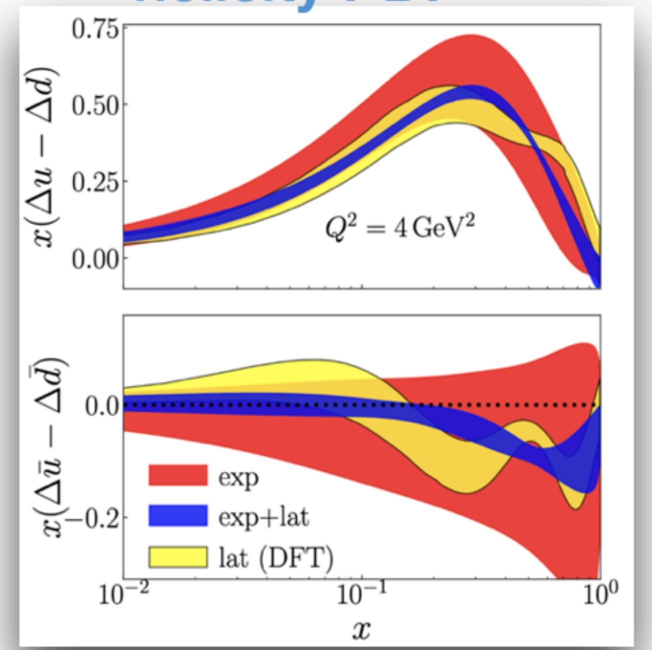
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

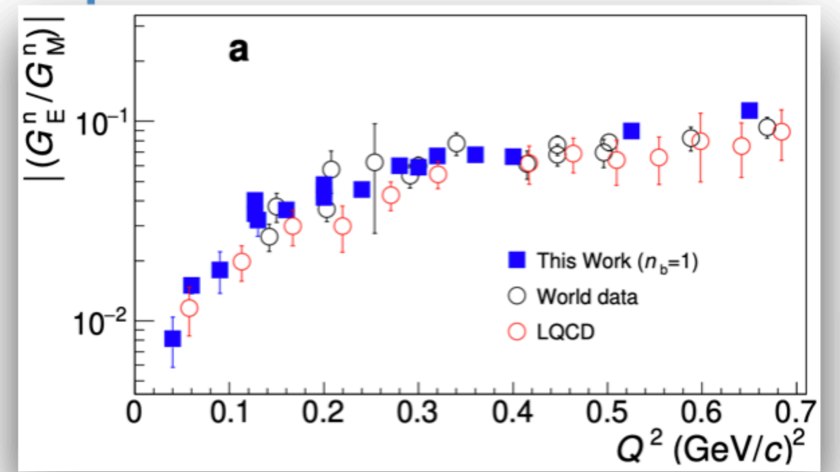


helicity PDF



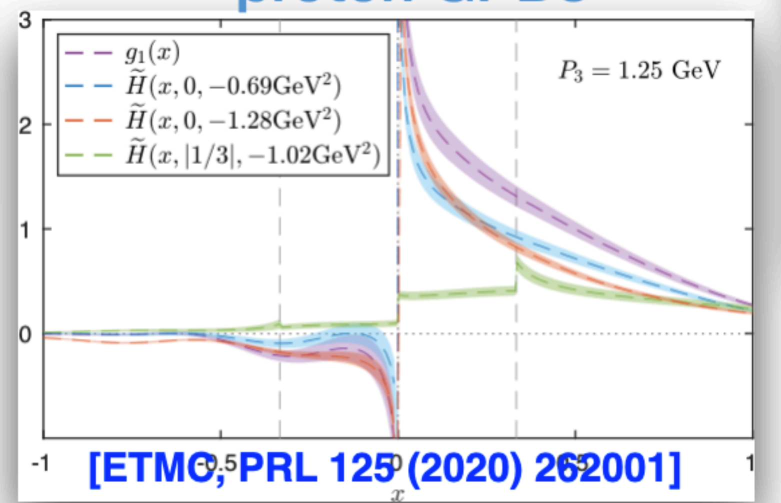
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



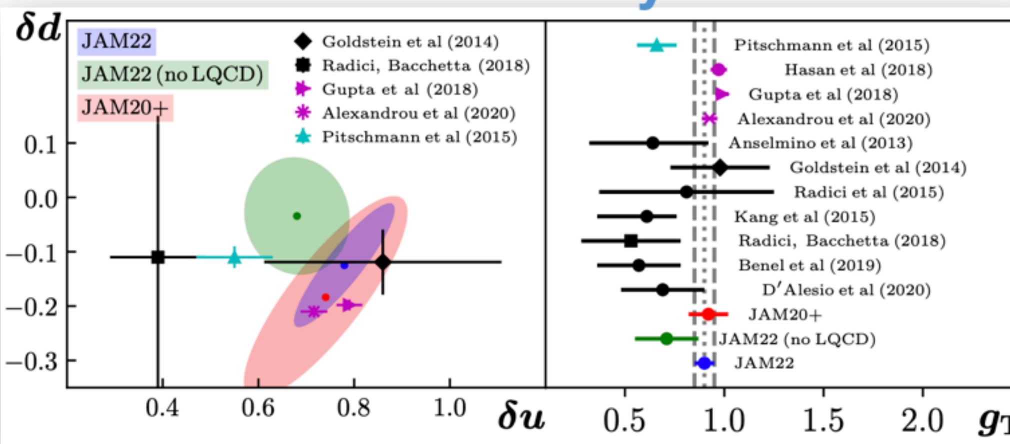
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

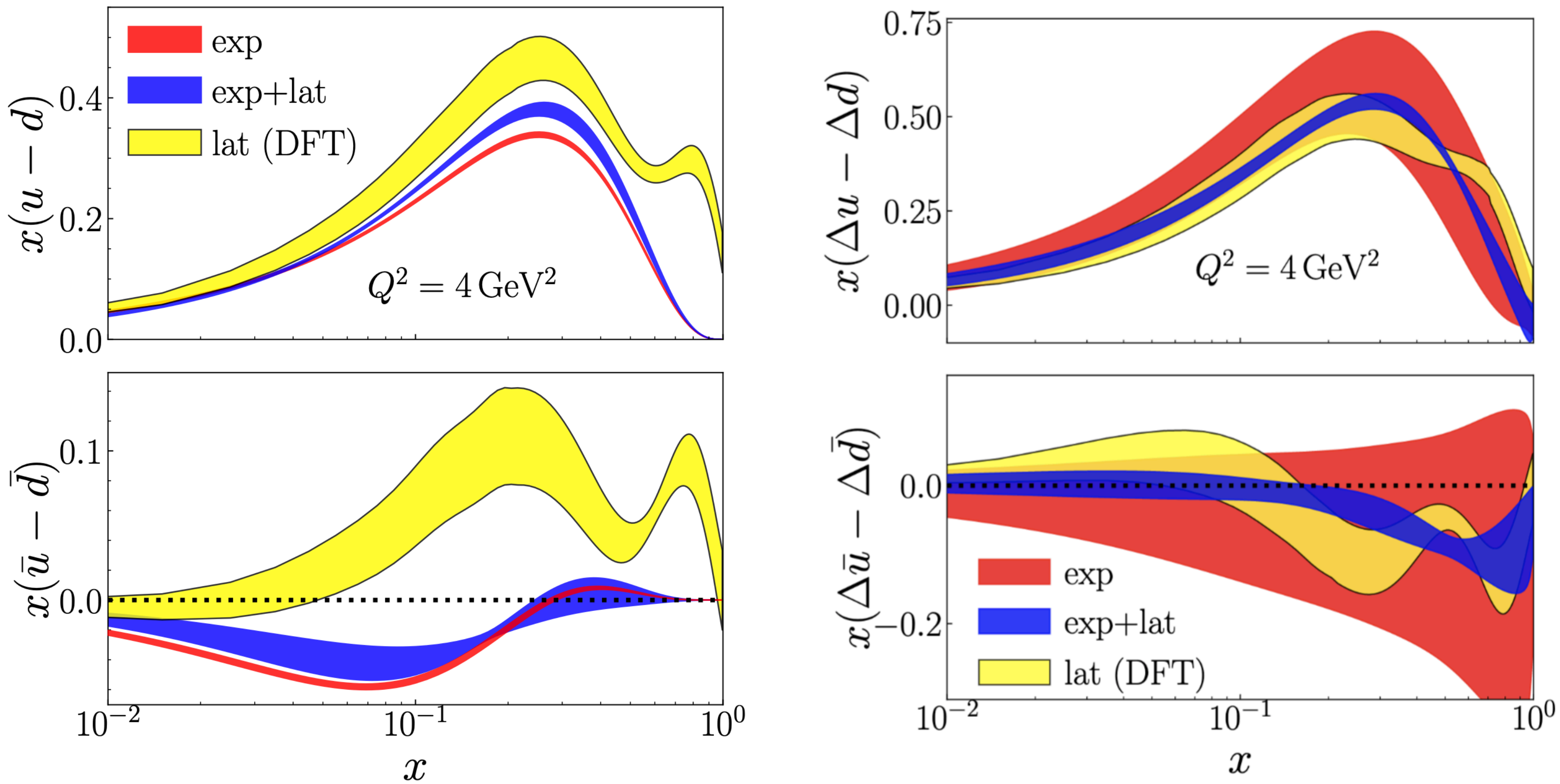
And many more!

Incorporating lattice PDFs in global analyses

Synergy between lattice and phenomenology

★ Lattice and experimental data sets data within the same global analysis (JAM framework)

[J. Bringewatt et al., PRD 103 (2021) 016003, arXiv:2010.00548]



- Consistent picture with JAM for unpolarized PDF

- Significant impact for helicity PDF

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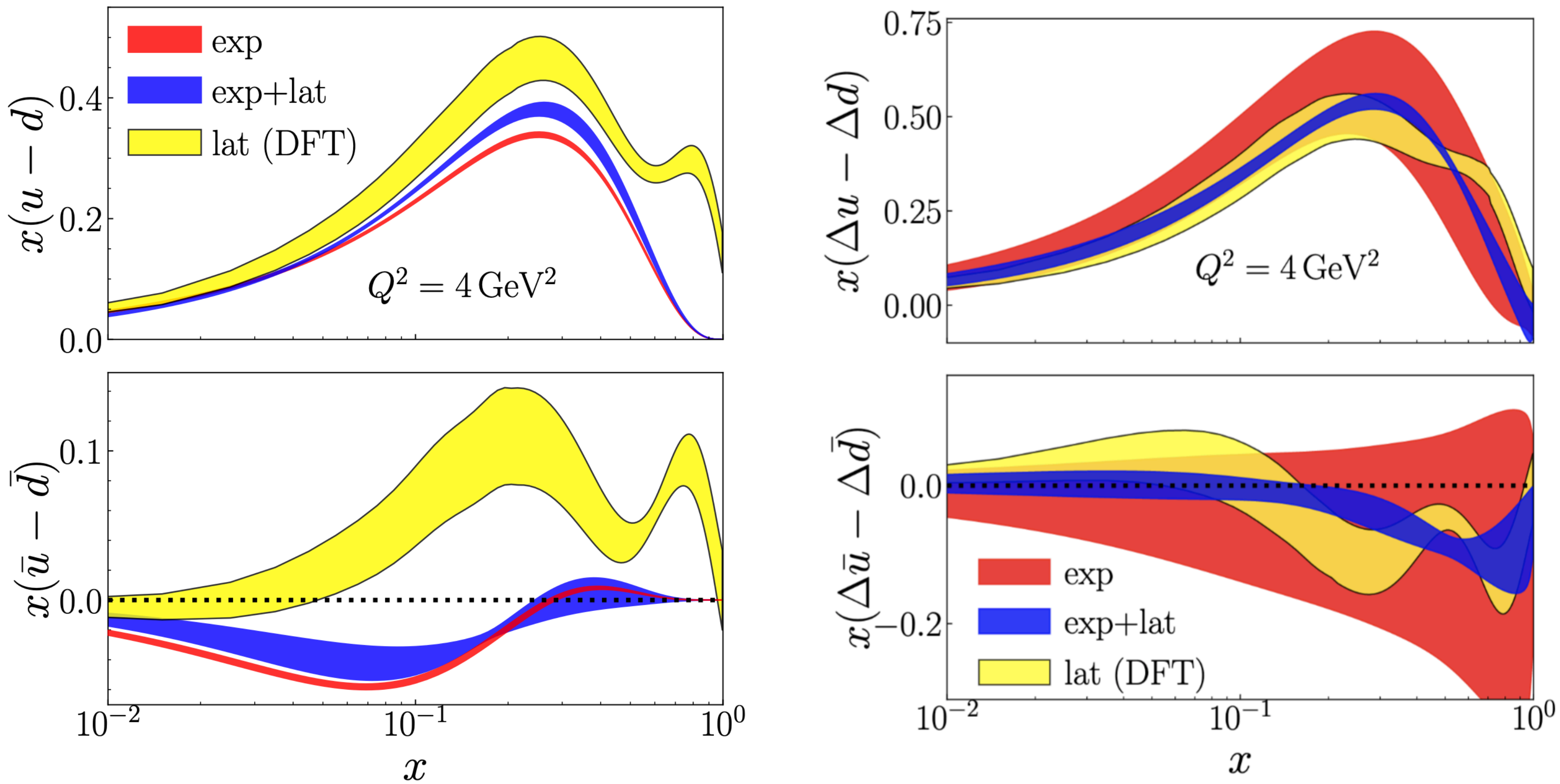

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★ Other efforts within NNPDF framework

[K. Cichy et al., JHEP 10 (2019) 137, arXiv:1907.06037]
[L. Del Debbio et al., JHEP 02 (2021) 138, 2010.03996]

★ Interest in applying similar approach to quantities that are more challenging to extract experimentally (GPDs, twist-3 distributions, …)

Toward synergy for GPDs

- Forming ratios of GPDs seems to suppress systematic uncertainties

[K. Cichy et al., arXiv:2409.17955]

$$\mathrm{DR}^{\hat{H}^q}_{\mathrm{Re}}(\nu,t) = \frac{\mathrm{Re}\hat{H}^q(\nu,t)}{\mathrm{Re}\hat{H}^q(\nu,0)}\,\frac{\mathrm{Re}\hat{H}^q(0,0)}{\mathrm{Re}\hat{H}^q(0,t)}\,,$$

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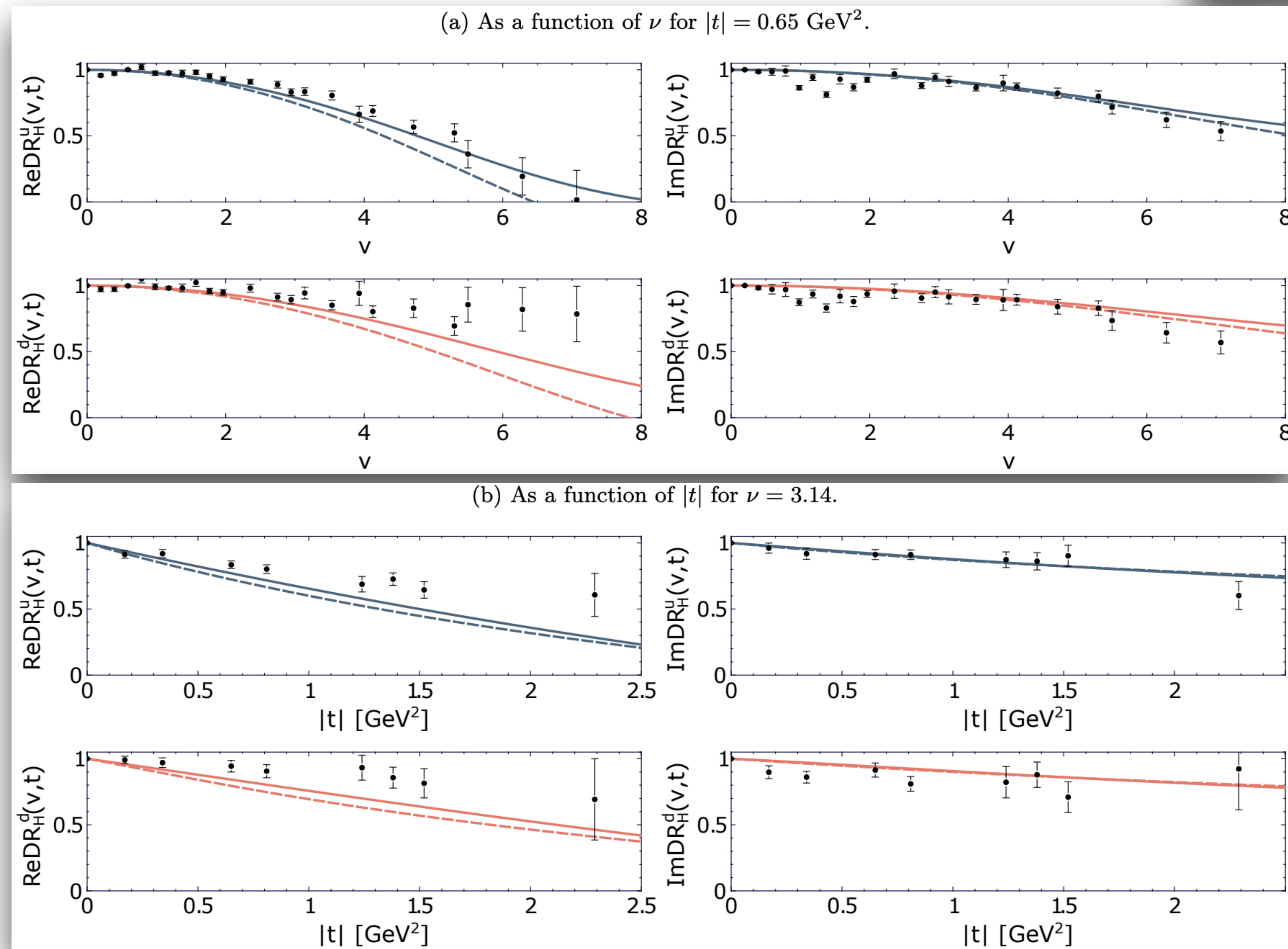

- GK (solid curve)
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

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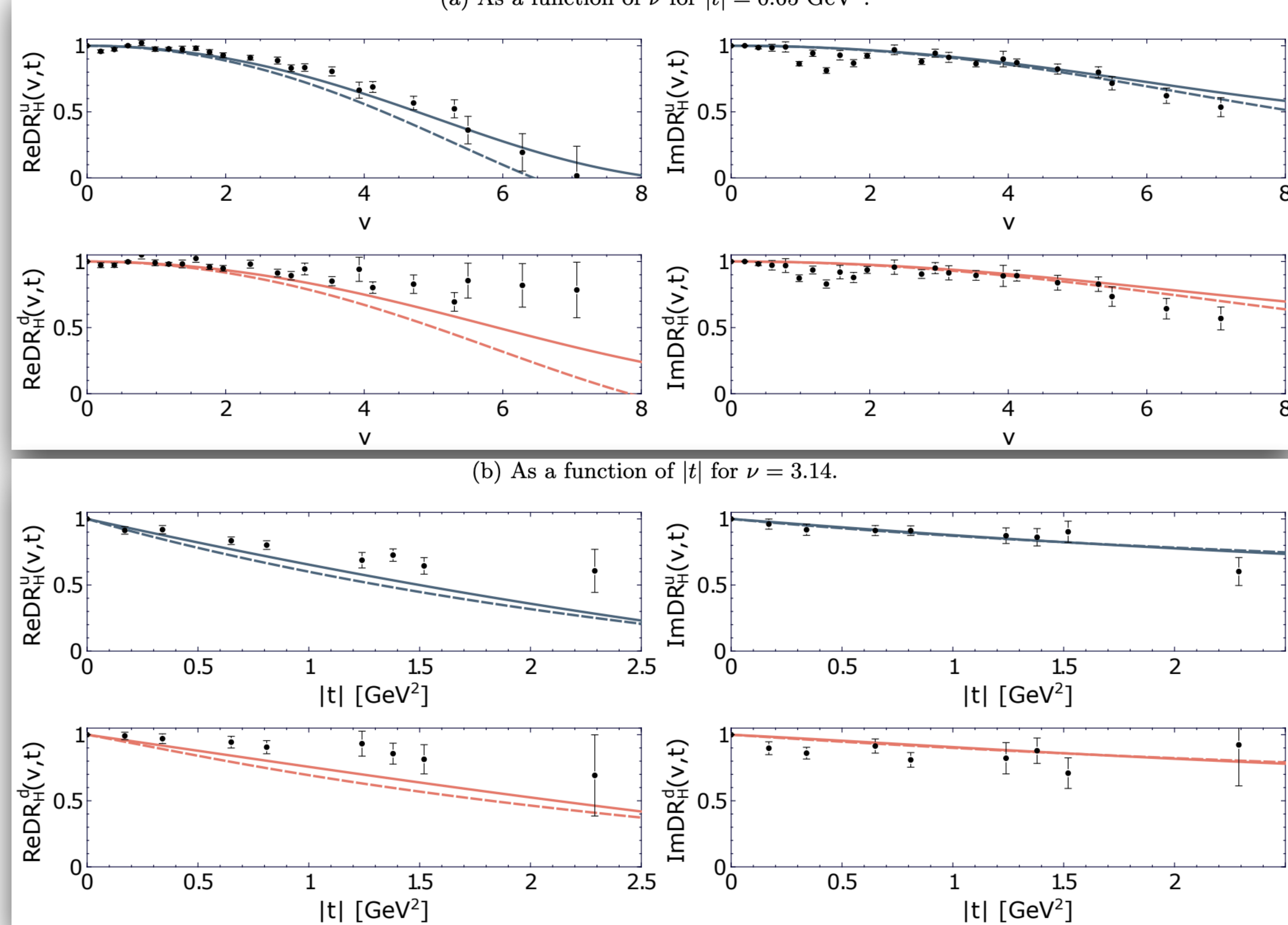


(a) As a function of ν for $|t| = 0.65$ GeV^2.

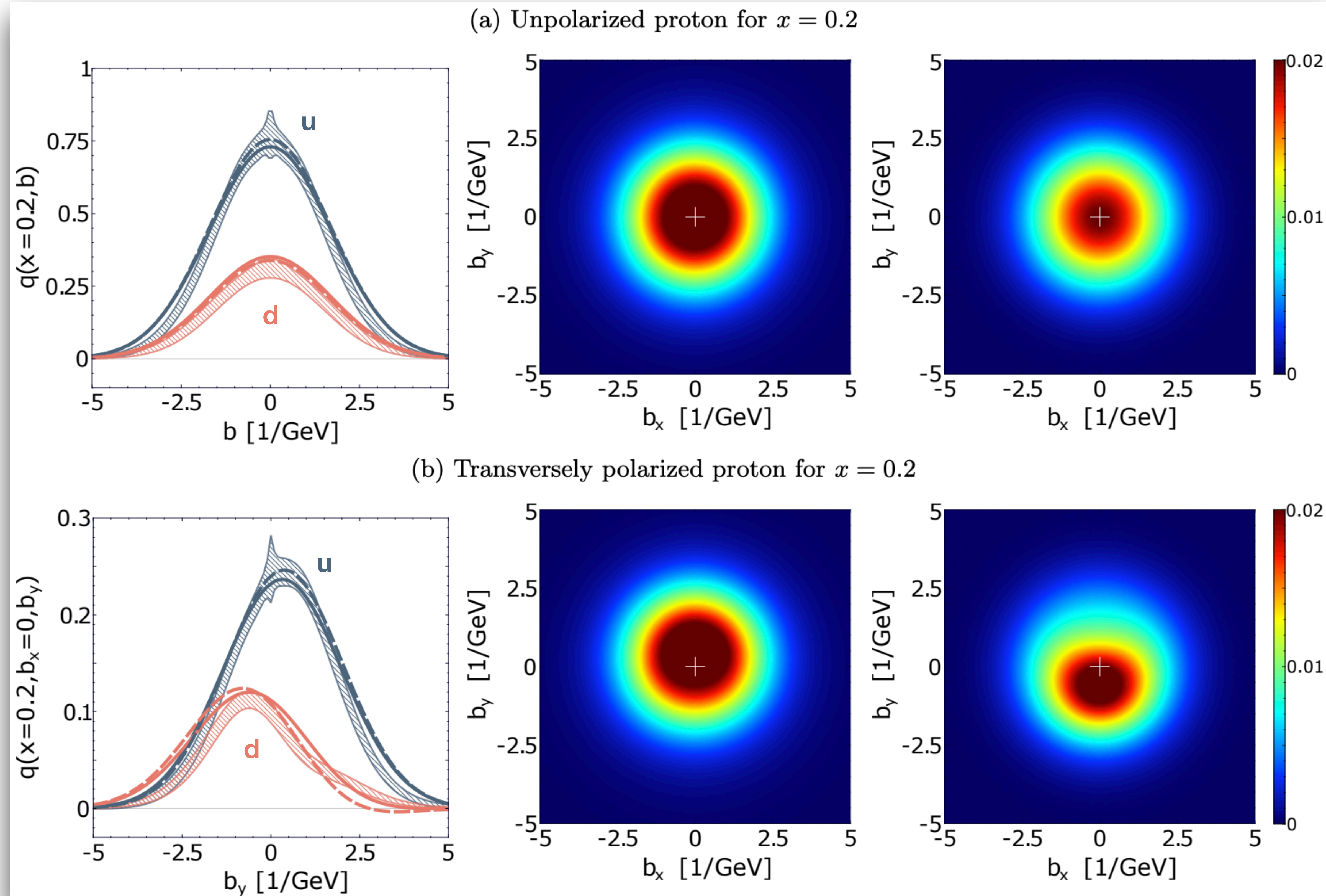
(b) As a function of $|t|$ for $\nu = 3.14$.

- GK (solid curve)
- VGG (dashed curve)

- Good agreement for up quark
- Reasonable agreement for down quark

- Further study needed on how to combine lattice results with data

Tomographic Images



- GK (solid line), VGG (dashed line)

[K. Cichy et al., arXiv:2409.17955]

How to lattice QCD data fit into the overall effort for hadron tomography

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. Lattice QCD calculations of GPDs and related structures
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

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Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Concluding remarks

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★ Impressive progress in the extraction of PDFs from Lattice QCD

★ Extensive programs in Gluon PDFs

★ New Developments in several promising directions:
DA, GPDs, TMDs

★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

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Thank you

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U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

NERSC

ASCR Leadership Computing Challenge ALCC

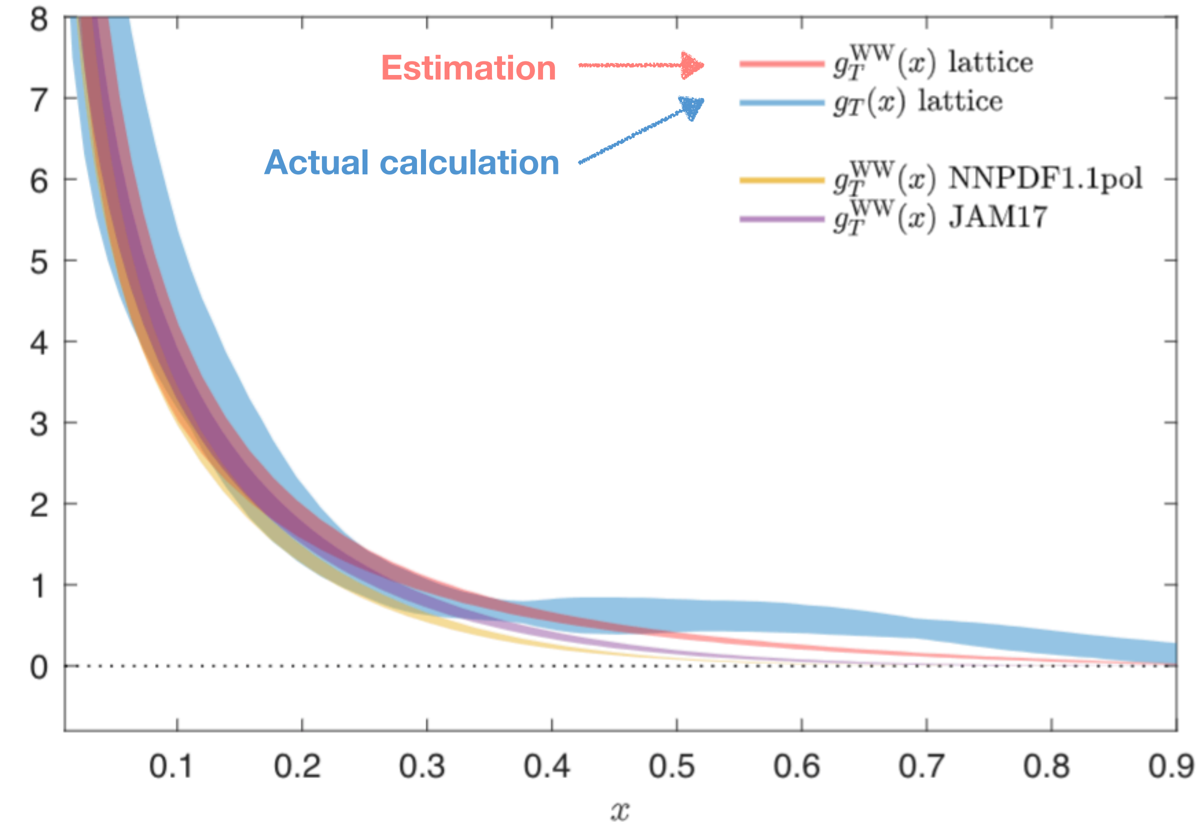
WW approximation

[S. Bhattacharya et al., PRD 102 (2020) 11 (Editors Selection)]

WW approximation: $g_T^{\rm WW}(x) = \int_x^1 \frac{dy}{y} g_1(y)$

twist-3 $g_T(x)$ determined by the twist-2 $g_1(x)$

- $g_T(x)$ agrees with $g_T^{\rm WW}(x)$ for $x < 0.5$ (violations up to 30-40% possible)
- Violations of 15-40% expected from experimental data [A. Accardi et al., JHEP 11 (2009) 093]



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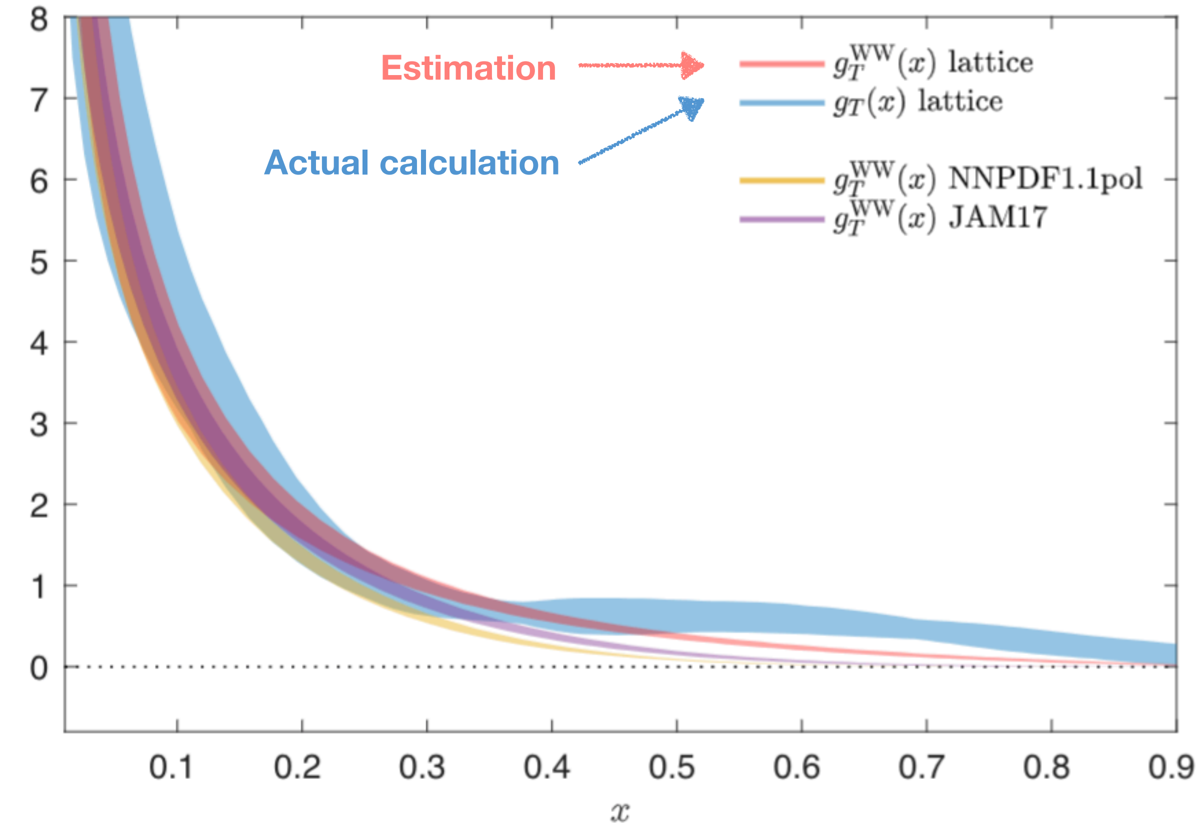
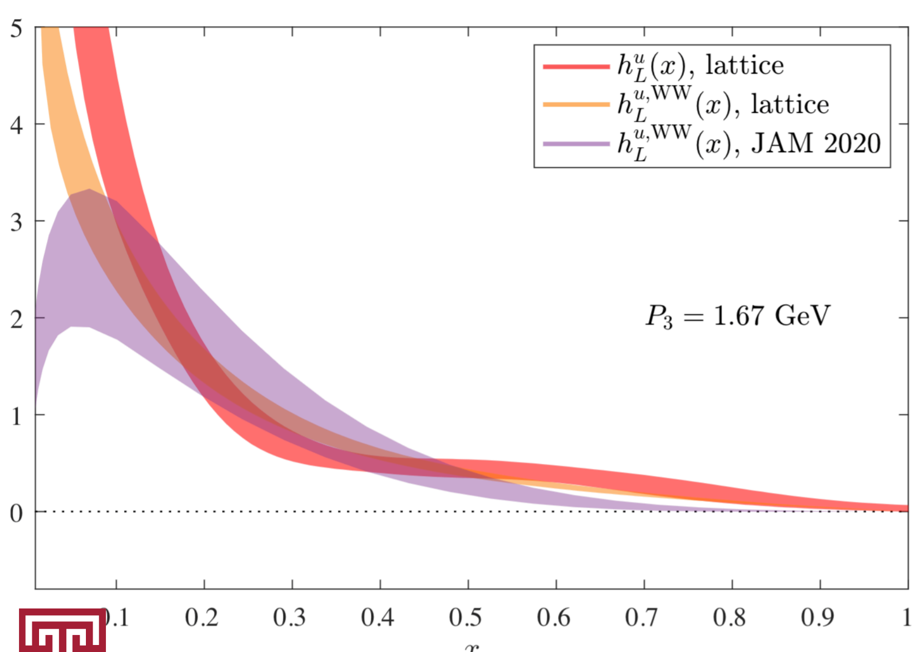
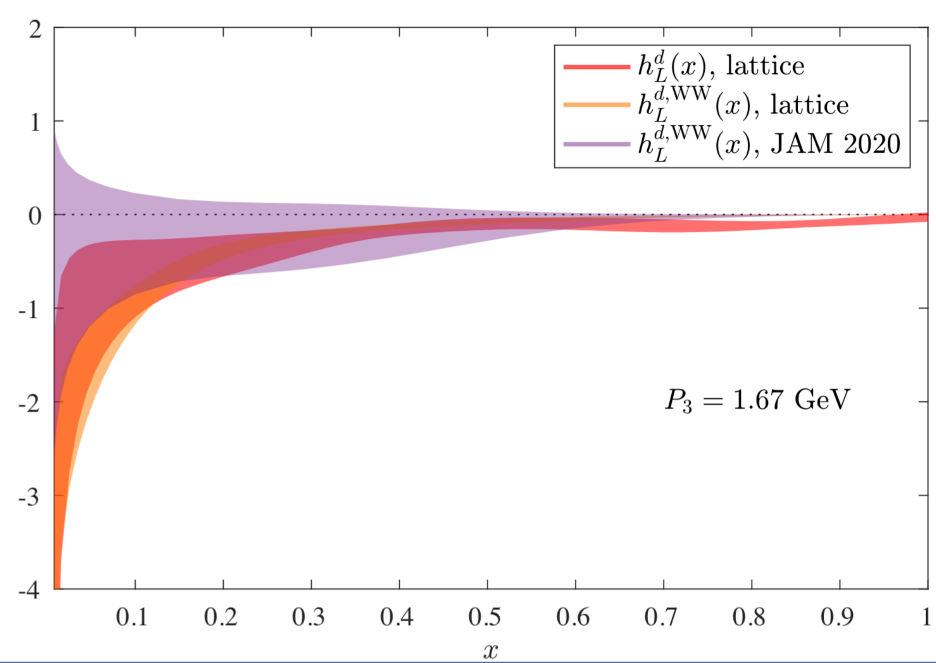


Twist-3 $h_L(x)$ PDF

[S. Bhattacharya et al., PRD 104 (2021) 11, 114510]

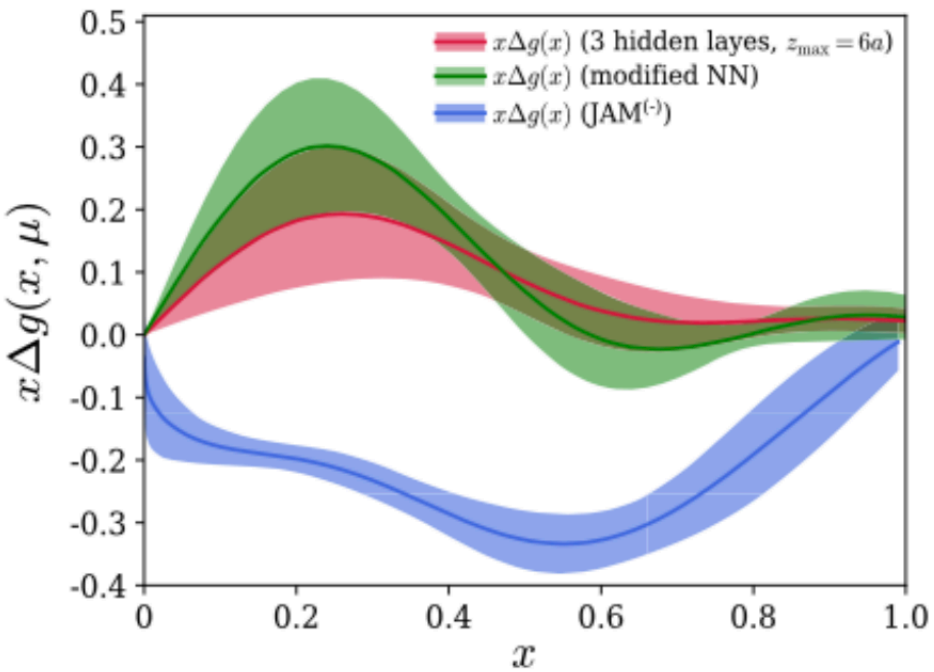




- h_L^u dominant - tension between h_L & $h_L^{\rm WW}$
- h_L^d <0 and decays faster then h_L^u

Gluon Helicity PDF

- Neural network analysis of lattice calculation disfavors negative gluon polarizability



[T. Khan et al., PRD 108, 074502]

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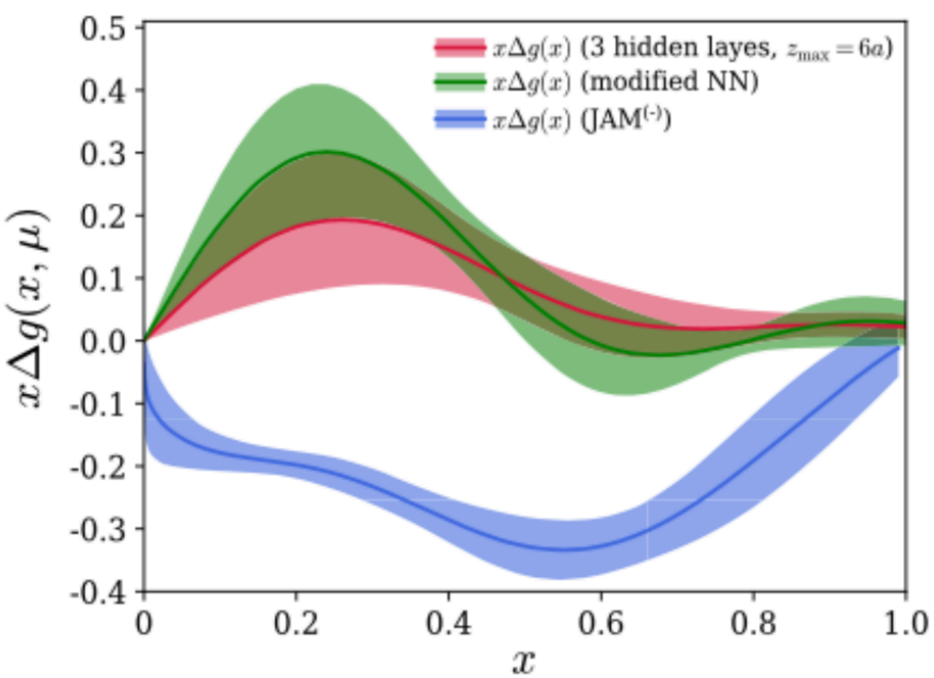
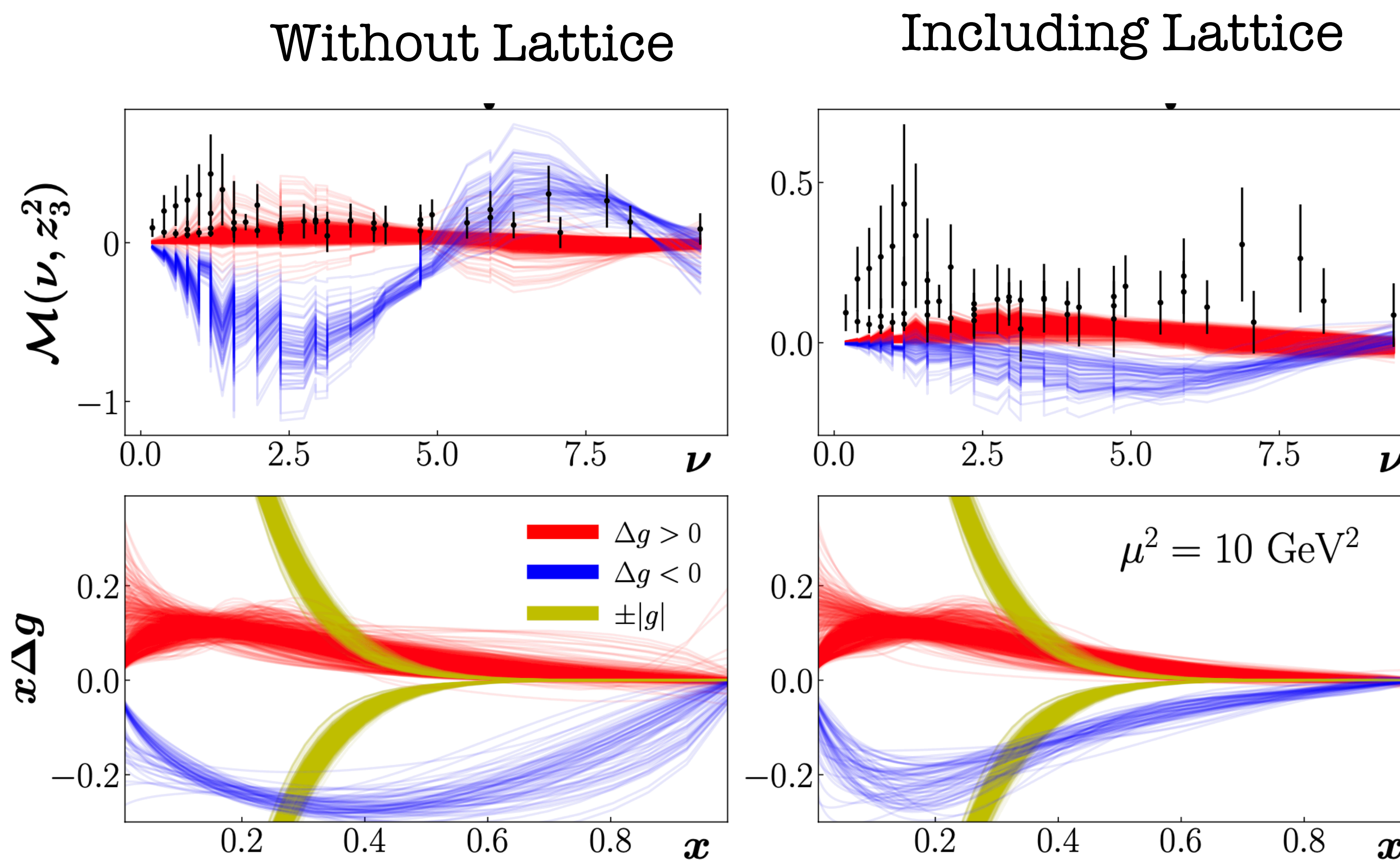


[T. Khan et al., PRD 108, 074502]

[J. Karpie et al., PRD 109 (2024) 3, 036031]



LQCD: Hint for a nonzero gluon spin (proton)
JAM analysis: No positivity constraint
($\Delta g > |g|$ for some regions of x)

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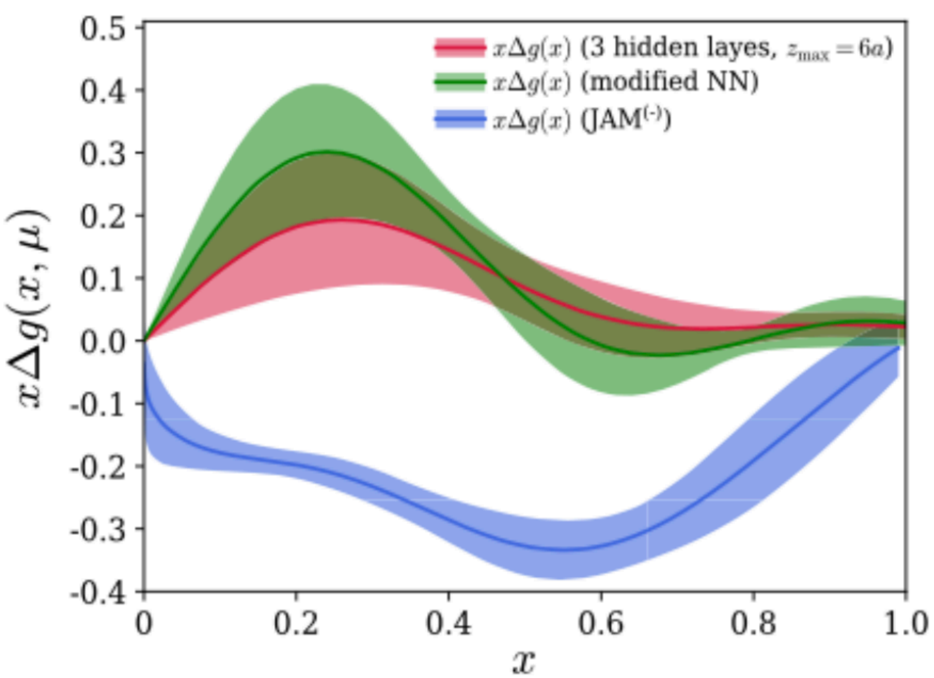

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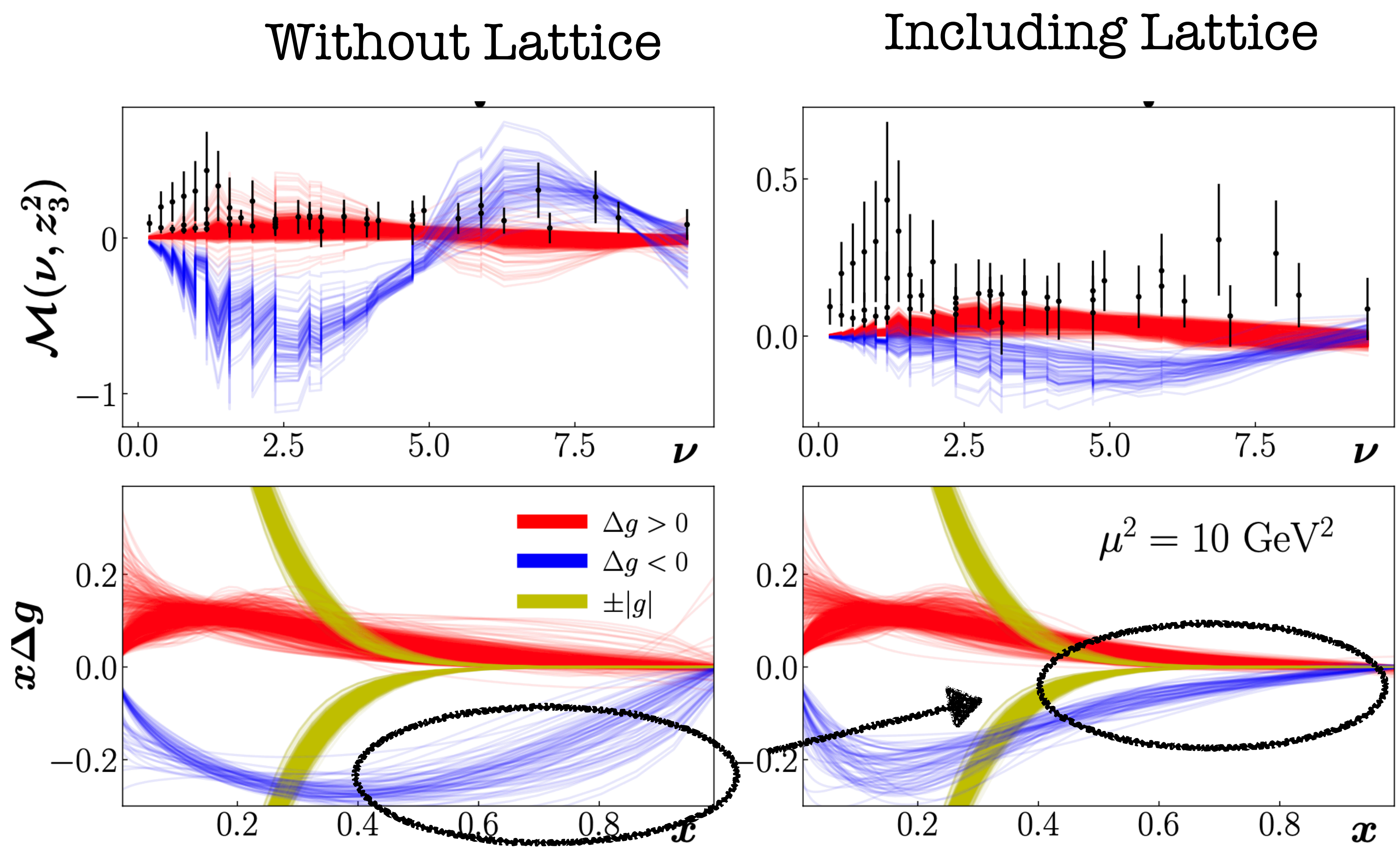

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