Exclusive factorization beyond leading twist meets saturation physics

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based on

[R. Boussarie, M. Fucilla, LS, S. Wallon (2407.18203, 2407.18115)]

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Content

Exclusive light vector meson production at the **twist-3** within the shockwave approach

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ii. Both **forward** and **non-forward** results

iii. **Beyond SCHC** amplitudes

iiii. **Coordinate** and **momentum space** representations

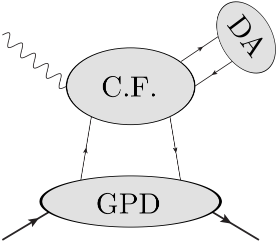
iiiii. Linearization **[Caron-Huot (2013)]** $\Longrightarrow$ **BFKL results**

Deeply virtual meson production (DVMP)

- Exclusive ρ-meson leptoproduction

  $\gamma^{(*)}(p_\gamma) + P(p_0) \to \rho(p_\rho) + P(p_0')$

- Extensively studied at HERA



- NLO corrections to the production of a longitudinally polarized ρ-meson at small-x

[Ivanov, Kotsky, Papa (2004)]

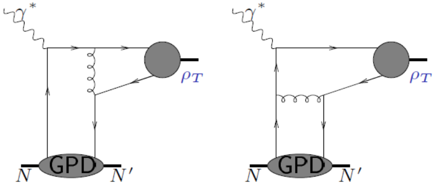
[Boussarie, Grabovsky, Ivanov, LS, Wallon (2017)]

[Mäntysaari, Pentalla (2022)]

The special case of transversally polarized vector meson production

Transversally polarized vector meson production starts at **twist-3**

- the dominant DA of ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- *unfortunately* $\gamma^* \, N \to \rho_T \, N' = 0$
  - This cancellation is true at any order : such a process would require a helicity transfer of 2 from a photon.
  - lowest order diagrammatic argument:



$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha \to 0$$

[Diehl, Gousset, Pire (1999)], [Collins, Diehl (2000)]

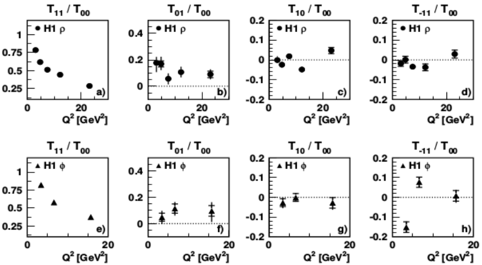
Collinear treatment at twist-3 leads to **end point singularities**

[Mankiewicz, Piller (2000)] [Anikin, Teryaev (2002)]

Transversely polarized vector meson production

- HERA data for the ρ and ϕ meson **[F.D. Aaron et al. (2010)]**

$$\gamma^*(\lambda_\gamma)p \to V(\lambda_V)p \qquad \lambda_\gamma = 0, 1, -1 \quad \text{and} \quad \lambda_V = 0, 1, -1$$



Transversely polarized ρ-meson production

- Momentum space impact factor for the exclusive ρ-meson production at the twist-3 in the dilute limit (BFKL scheme) and forward case

  [Anikin, Ivanov, Pire, LS, Wallon (2009)]

- Phenomenological studies at small-x

  [Besse, LS, Wallon (2013)]

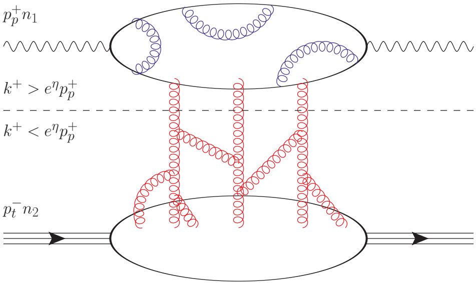
  [Bolognino, Celiberto, Ivanov, Papa (2018)]

  [Bolognino, Szczurek, Schäfer (2019)]

  [Mäntysaari, Pentalla (2022)]

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$n_1^2 = n_2^2 = 0 \quad n_1 \cdot n_2 = 1$$

- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[I. Balitsky (1996-2001)]

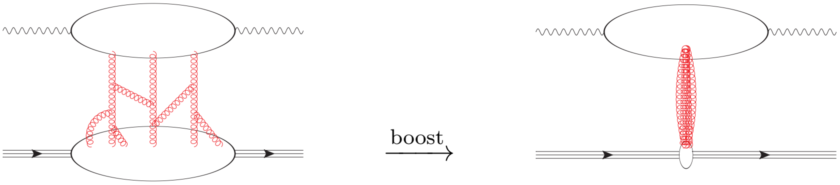
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

Shockwave approach

- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+) \mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0$

$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

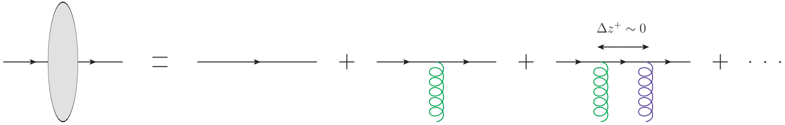
Shockwave approach

- Interactions with the simple shockwave field

    i. Independence from x^- $\Longrightarrow$ conservation of p^+ (**eikonal** approx.)

    ii. $\delta(x^+)$ $\Longrightarrow$ interactions at a **single transverse coordinate**.

- Quark line through the shockwave



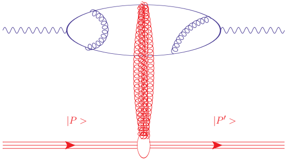
$$V_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_i\right)\theta\left(z_{ij}^+\right) + \cdots$$

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

$$V_{\vec{z}}^{\eta} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}\right)\right]$$

Shockwave approach

- Factorization in the shockwave approximation



$$\mathcal{M}^\eta = N_c \int d^d \boldsymbol{z}_1 d^d \boldsymbol{z}_2 \, \Phi^\eta(\boldsymbol{z}_1, \boldsymbol{z}_2) \langle P' \left| \mathcal{U}^\eta_{12}(\boldsymbol{z}_1, \boldsymbol{z}_2) \right| P \rangle$$

- **Dipole operator** $\mathcal{U}^\eta_{ij} = 1 - \frac{1}{N_c} \mathrm{Tr}\left(V^\eta_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j} \right)$
- Evolution equations
  - **Balitsky-JIMWLK** evolution equations
    [Balitsky (1995)]
    [Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]
  - Large N_c → **Balitky-Kovchegov (BK)** non-linear equation
    [Balitsky (1995)] [Kovchegov (1999)]
  - Evolution at the NLO
    [Balitsky, Chirilli (2007)] [Kovner, Lublinsky, Mulian (2013)]

Theoretical framework

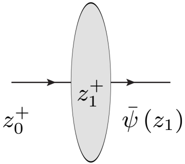
- **Effective background field operator** formalism of small-x physics

$$[\psi_{\text{eff}}\ (z_0)]_{z_0^+<0} = \psi\,(z_0) - \int \mathrm{d}^D z_2 G_0\,(z_{02}) \left(V^\dagger_{\boldsymbol{z}_2} - 1\right) \gamma^+ \psi\,(z_2)\, \delta(z_2^+)$$

$$\left[\bar{\psi}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \bar{\psi}\,(z_0) + \int \mathrm{d}^D z_1 \bar{\psi}\,(z_1)\, \gamma^+\, (V_{\boldsymbol{z}_1} - 1)\, G_0\,(z_{10})\, \delta(z_1^+)$$

$$\left[A^{\mu a}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = A^{\mu a}\,(z_0) + 2i \int \mathrm{d}^D z_3 \delta(z_3^+) F^b_{-\sigma}\,(z_3)\, G^{\mu\sigma_\perp}\,(z_{30}) \left(U^{ab}_{\boldsymbol{z}_3} - \delta^{ab}\right)$$

e.g. of antiquark effective operator:



free quark propagator: $G_0(z) = \int \frac{d^D l}{(2\pi)^D} e^{-il\cdot z} \frac{i\not{k}}{k^2+i0}$

- A fermionic line starts at the light-cone time $z_0^+ < 0$
- freely propagates to z_1^+
- it interacts eikonally at z_1^+ with the background shockwave field.

Theoretical framework

Effective background field operators cntd

- Such operators serve to construct amplitudes involving non-perturbative matrix elements of general off light-cone correlators, i.e. **without any reference to the twist-expansion**
- Proof by induction
- Shockwave effective Feynman rules are reproduced

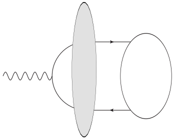
$$\left[v_{\alpha}^{ij}\left(p_{\bar{q}}, z_0\right)\right]_{z_0^+<0} \equiv \left[\psi_{\text{eff},\alpha}^{j}\left(z_0\right)\right]_{z_0^+<0} |i, p_{\bar{q}}\rangle = -\frac{(-i)^{d/2}}{2(2\pi)^{d/2}}\left(\frac{p_{\bar{q}}^+}{-z_0^+}\right)^{d/2} \theta(p_{\bar{q}}^+)\theta(-z_0^+)$$

$$\times \int d^d z_2 V_{\vec{z}_2}^{ij\dagger} \frac{-z_0^+\gamma^- + \hat{z}_{20\perp}}{-z_0^+}\gamma^+ \frac{v(p_{\bar{q}})}{\sqrt{2p_{\bar{q}}^+}} \exp\left\{ip_{\bar{q}}^+\left(z_0^- - \frac{\vec{z}_{20}^{\,2}}{2z_0^+} + i0\right) - i\vec{p}_{\bar{q}}\cdot\vec{z}_{20}\right\}$$

$$G_{ij}\left(z_2, z_0\right)\big|_{z_2^+>0>z_0^+} \equiv \overline{\psi_i(z_2)\left[\overline{\psi}\right.}\left._{\text{eff},j}\left(z_0\right)\right]_{z_0^+<0}$$

$$= \frac{i\Gamma(d+1)}{4(2\pi)^{d+1}}\int d^d\vec{z}_1 V_{ij}\left(\vec{z}_1\right)\frac{\left(z_2^+\gamma^- + \hat{z}_{21\perp}\right)\gamma^+\left(-z_0^+\gamma^- + \hat{z}_{10\perp}\right)}{\left(-z_0^+z_2^+\right)^{\frac{D}{2}}\left(-z_{20}^- + \frac{\vec{z}_{21}^2}{2z_2^+} - \frac{\vec{z}_{10}^2}{2z_0^+} + i\varepsilon\right)^{d+1}}\theta(z_2^+)\theta(-z_0^+)$$

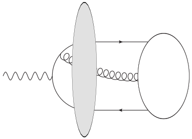
- **Two-body contribution**



i. Dependence of the leading Fock state wave function – with a minimal number of (valence) partons – on **transverse momentum**

$$\mathcal{A}_2 = -ie_f \int \mathrm{d}^D z_0 \theta(-z_0^+) \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\rm eff}\left(z_0\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\rm eff}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

- **Three-body contribution**



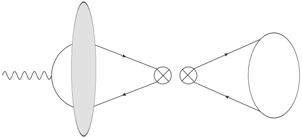
i. Distribution with a **non-minimal parton configuration**

$$\mathcal{A}_{3,q} = \left(-ie_q\right)\left(ig\right) \int \mathrm{d}^D z_4 \mathrm{d}^D z_0 \theta(-z_4^+) \theta(-z_0^+)$$

$$\times \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\rm eff}\left(z_4\right) \gamma_\mu A^{\mu a}_{\rm eff}(z_4) t^a G(z_{40}) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\rm eff}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

ρ-meson production: factorization

- **Two-body contribution**



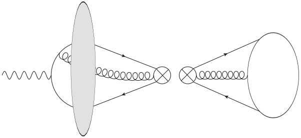
$$\mathcal{A}_2 = ie_f \int \mathrm{d}^D z_0 \int \mathrm{d}^D z_1 \int \mathrm{d}^D z_2 \theta(-z_0^+)\delta(z_1^+)\delta(z_2^+) \left\langle M\left(p_M\right) \left| \overline{\psi}\left(z_1\right) \Gamma^\lambda \psi\left(z_2\right) \right| 0 \right\rangle$$

$$\times \left\langle P\left(p'\right) \left| 1 - \frac{1}{N_c} \mathrm{tr}\left(V_{\boldsymbol{z}_1} V_{\boldsymbol{z}_2}^\dagger\right) \right| P\left(p\right) \right\rangle \frac{1}{4} \mathrm{tr}_D \left[\gamma^+ G_0\left(z_{10}\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} G_0\left(z_{02}\right) \gamma^+ \Gamma_\lambda\right]$$

hard part

ρ-meson production: factorization

- **Three-body contribution**



$$\mathcal{A}_{q3} = -ie_q \int \mathrm{d}^D z_4 \mathrm{d}^D z_3 \mathrm{d}^D z_2 \mathrm{d}^D z_1 \mathrm{d}^D z_0 \theta(-z_4^+)\delta(z_3^+)\delta(z_2^+)\delta(z_1^+)\theta\left(-z_0^+\right) \mathrm{e}^{-i(q\cdot z_0)}$$
$$\times \left\langle P\left(p'\right)\left|\mathrm{tr}\left(V_{\boldsymbol{z}_1} t^a V^\dagger_{\boldsymbol{z}_2} t^b U^{ab}_{\boldsymbol{z}_3}\right)\right| P\left(p\right)\right\rangle \left\langle M\left(p_M\right)\left|\overline{\psi}\left(z_1\right)\Gamma^\lambda g F_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\right|0\right\rangle$$
$$\times \frac{1}{N_c^2-1}\mathrm{tr}_D\left[\gamma^+ G_0\left(z_{14}\right)\gamma_\mu G^{\mu\sigma_\perp}\left(z_{34}\right) G_0\left(z_{40}\right)\hat{\varepsilon}_q G_0\left(z_{02}\right)\gamma^+\Gamma_\lambda\right] - \text{n.i.}$$

hard part

Expression of $U^{ab}_{\boldsymbol{z_3}}$ in adjoint through fundamental representation:

$$\mathrm{tr}\left(V_{\boldsymbol{z}_1} t^a U^{ab}_{\boldsymbol{z}_3} V^\dagger_{\boldsymbol{z}_2} t^b\right) = \frac{1}{2}\left[\mathrm{tr}\left(V_{\boldsymbol{z}_1} V^\dagger_{\boldsymbol{z}_3}\right)\mathrm{tr}\left(V_{\boldsymbol{z}_3} V^\dagger_{\boldsymbol{z}_2}\right) - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{z}_1} V^\dagger_{\boldsymbol{z}_2}\right)\right].$$

Results: two-body contribution

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\Psi_2(x,\boldsymbol{r}) \int \mathrm{d}^d\boldsymbol{b}\, \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P(p') \left| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \right| P(p) \right\rangle$$

- **Coordinate-space impact factor**

$$\Psi_2(x,\boldsymbol{r}) = e_q \delta \left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+} q_\mu\right)$$
$$\times \left[\phi_{\gamma^+}(x,\boldsymbol{r})\left(2x\bar{x}q^\mu - i(x-\bar{x})\frac{\partial}{\partial r_{\perp\mu}}\right) + \epsilon^{\mu\nu+-}\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r})\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Two-body vacuum to meson matrix elements

$$\phi_{\gamma^+}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M(p_M) \left| \overline{\psi}(r)\gamma^+\psi(0) \right| 0 \right\rangle_{r^+=0}$$

$$\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M(p_M) \left| \overline{\psi}(r)\gamma^+\gamma^5\psi(0) \right| 0 \right\rangle_{r^+=0}$$

at this stage, r^2 is arbitrary, in principle off the light-cone.

Results: three-body contribution

- Three-body amplitude: involves dipole and double dipole contributions

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3}\int dx_i\theta(x_i)\right)\delta(1-x_1-x_2-x_3)\int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1+x_2\boldsymbol{z}_2+x_3\boldsymbol{z}_3)}$$

$$\times\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)\left\langle P\left(p'\right)\middle|\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}-\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}-\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}+\frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2}\middle|P\left(p\right)\right\rangle$$

- **Coordinate-space impact factor** (with $Z=\sqrt{x_1x_2\boldsymbol{z}_{12}^2+x_1x_3\boldsymbol{z}_{13}^2+x_2x_3\boldsymbol{z}_{23}^2}$)

$$\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)=\frac{e_q q^+}{2(4\pi)}\frac{N_c^2}{N_c^2-1}\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)$$

$$\times\left\{\chi_{\gamma^+\sigma}\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]\right.$$

$$\left.-\chi_{\gamma^+\gamma^5\sigma}\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)+(1\leftrightarrow 2)\right]\right\}$$

- Three-body vacuum to meson non-perturbative matrix elements

$$\chi_{\Gamma^\lambda,\sigma}\equiv\chi_{\Gamma^\lambda,\sigma}(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3)=$$

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}z_1^-}{2\pi}\frac{\mathrm{d}z_2^-}{2\pi}\frac{\mathrm{d}z_3^-}{2\pi}\mathrm{e}^{-ix_1q^+z_1^--ix_2q^+z_2^--ix_3q^+z_3^-}\left\langle M\left(p_M\right)\middle|\overline{\psi}\left(z_1\right)\Gamma^\lambda gF_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\middle|0\right\rangle_{z_{1,2,3}^+=0}$$

Again, at this stage, z_1^2, z_2^2, z_3^2 are arbitrary, in principle off the light-cone

Light-cone collinear factorization

- *Light-cone collinear factorization*

  [Ellis, Furmanski, Petronzio (1982)] [Anikin, Teryaev (2002)]

- Factorization around the dominant **light-cone direction** is naturally implemented in **momentum space**
- **Overcomplete set of distributions** must be reduced exploiting QCD equations of motion

$$\langle i(\hat{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad \langle i\psi_\alpha(0)(\bar{\psi}(z)\overleftarrow{\hat{D}}(z))_\beta\rangle = 0$$

- Invariance of the amplitude under **rotation on the light-cone**

  [Anikin, Ivanov, Pire, LS, Wallon (2009)]

  i. Independence of the amplitude from the choice of n

  ii. Given a "natural" choice n_0, we can define

  $$n^\mu = \alpha p^\mu + \beta n_0^\mu + n_\perp^\mu$$

  iii. Imposing $p \cdot n = 1$ and $n^2 = 0 \longrightarrow \beta = 1\,, \alpha = -n_\perp^2/2$

  iiii. The freedom is parametrized in terms of the transverse component

  $$\frac{\partial \mathcal{A}}{\partial n_\perp^\mu} = 0$$

Covariant collinear factorization = non-local OPE (1)

- expansion in powers of the hard scale
  = expansion of **string operators** in powers of deviation from the light-cone
  [Balitsky, Braun (1989)]

e.g. **up to twist 3:**

- *2-body:* expansion in powers of r^2 $(r^2 \to 0)$ of

$$\left\langle M(p_M)\left|\overline{\psi}(r)\,\gamma^+\psi(0)\right|0\right\rangle_{r^+=0} \quad \text{and} \quad \left\langle M(p_M)\left|\overline{\psi}(r)\,\gamma^+\gamma^5\psi(0)\right|0\right\rangle_{r^+=0}$$

- *3-body:* expansion in powers of $(z_3-z_1)^2, (z_2-z_3)^2$
  $((z_3-z_1)^2, (z_2-z_3)^2 \to 0)$ of

$$\left\langle M(p_M)\left|\overline{\psi}(z_1)\,\Gamma^\lambda g F_{-\sigma}(z_3)\,\psi(z_2)\right|0\right\rangle_{z^+_{1,2,3}=0}$$

- each coefficient of this OPE expansion
  = finite sum of on-light-cone **non-local** correlators
- for each term in this Taylor expansion:
  vacuum-to-meson matrix elements contribute to **different kinematic twist:**
  - matrix element = linear combination of $p_{M\mu}$, r_μ, $\varepsilon^*_{M\mu}$ **(now** $r^2=0$**)**
  - coefficients depend on the available Lorentz inv. $p_M \cdot r$, $\varepsilon_M \cdot r$, m_M^2
  - these quantities have **different scaling** in the $Q \to \infty$ limit

Covariant collinear factorization = non-local OPE (2)

- example: *parametrization of the 2-body vector matrix element* (up to twist 3)

$$\left\langle M(p_M)\left|\overline{\psi}\,(r)\,\gamma^{\mu}[r,0]\psi\,(0)\right|0\right\rangle|_{r^2=0}$$

$$\sim f_M m_M \int_0^1 \mathrm{d}x e^{ix(p_M\cdot r)}\left[\underbrace{p_M^{\mu}\frac{(\varepsilon^*_M\cdot r)}{(p_M\cdot r)}\phi(x)}_{\sim Q} + \underbrace{\varepsilon^{*\mu}_{M,T}\,g_{\perp}^{(v)}(x)}_{\sim 1}\right]$$

twist 2   twist 3

- **up to twist 3:**
  only the first term in the Taylor expansion of the off-light-cone matrix elements survives
  the next one is twist 4, i.e. z^2 suppressed
  we keep twist 2 and twist 3 terms:
  2-body: $\rightarrow$ twist-2 and twist-3 (kinematic) (see above example)
  3-body: $\rightarrow$ twist-3 (genuine)

Covariant collinear factorization

- **Covariant collinear factorization**

[Braun, Filyanov (1990)]

[Ball, Braun, Koike, Tanaka (1998)]

  i. Minimal basis of *independent distributions*
  ii. *Minimal numbers of parameters*
  iii. Easy to perform the calculation directly into coordinate space

- 2 and 3-body operators in gauge invariant form, on the light-cone $z^2 = 0$

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,gF^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,g\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

where

$$[z,0] = \mathcal{P}_{\exp}\left[ig\int_0^1 dt A^\mu\,(tz)\,z_\mu\right]$$

Results: twist 3 expansion

A subtlety: making contact with covariant collinear factorization

- before twist expansion, our result does do not contain gauge links between fields
- this should be taken into account, through:

$$\mathcal{P}\exp\left[ig\int_0^1 dt A^\mu(tz)z_\mu\right] = 1 + ig\int_0^1 dt A^\mu\left(tz\right)z_\mu + \ldots\,,$$

- it does not affect the 3-body twist-3 result
- it does contribute to the 2-body twist-3 result

Reminder:

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\, \Psi_2\left(x, \boldsymbol{r}\right) \int \mathrm{d}^d\boldsymbol{b}\; \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P\left(p'\right) \middle| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \middle| P\left(p\right) \right\rangle$$

- Three-body amplitude: with **dipole** and **double dipole** contributions

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3} \int dx_i \theta(x_i)\right) \delta(1 - x_1 - x_2 - x_3) \int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1 + x_2\boldsymbol{z}_2 + x_3\boldsymbol{z}_3)}$$

$$\times \Psi_3\left(x_1, x_2, x_3, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3\right) \left\langle P\left(p'\right) \middle| \mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} - \mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3} - \mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2} + \frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2} \middle| P\left(p\right) \right\rangle$$

2-body twist-3 expanded result

coordinate space

$$\Psi_2(x,\boldsymbol{r}) = e_q m_M f_M \delta \left(1-\frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu}-\frac{\varepsilon_q^+}{q^+}q_\mu\right)\left(\varepsilon^*_{M\alpha}-\frac{\varepsilon_M^{*+}}{p_M^+}p_{M\alpha}\right)$$
$$\times\Bigg[-ir_\perp^\alpha\left(h(x)-\tilde{h}(x)\right)\left(2x\bar{x}q^\mu+(x-\bar{x})\frac{-i\partial}{\partial r_{\perp\mu}}\right)$$
$$+\epsilon^{\mu\nu+-}\epsilon^{+\alpha-\delta}r_{\perp\delta}\left(\frac{g_\perp^{(a)}(x)-\tilde{g}_\perp^{(a)}(x)}{4}\right)\frac{\partial}{\partial r_\perp^\nu}\Bigg]K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right),$$

with

$$h(x)=\int_0^x \mathrm{d}u\left(\phi(u)-g_\perp^{(v)}(u)\right),$$

$$\widetilde{h}(x)=\frac{f_{3M}^V}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{V\left(x_q,x_{\overline{q}}\right)}{\left(1-x_q-x_{\overline{q}}\right)^2},$$

$$\widetilde{g}_\perp^{(a)}(x)=4\frac{f_{3M}^A}{f_M}\int_0^x \mathrm{d}x_q\int_0^{1-x}\mathrm{d}x_{\overline{q}}\,\frac{A\left(x_q,x_{\overline{q}}\right)}{(1-x_q-x_{\overline{q}}+i\epsilon)^2}.$$

Results: twist 3 expansion

3-body twist-3 expanded result

coordinate space

$$\Psi_3\left(x_1, x_2, x_3, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3\right)$$

$$= \frac{e_q m_M c_f}{8\pi} \delta\left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+} q_\rho\right)\left(\varepsilon_M^{*\mu} - \frac{p_M^\mu}{p_M^+}\varepsilon_M^{*+}\right)\left(\prod_{j=1}^{3}\theta(x_j)\theta(1-x_j)e^{-ix_j \boldsymbol{p}_M \boldsymbol{z}_j}\right)$$

$$\times\Bigg\{ - i f_{3M}^V g_{\sigma\mu} V(x_1, x_2)\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1 x_2}{1-x_2}\frac{Q}{Z}K_1(QZ) + T_1^{\sigma\rho\nu}(\{x_i\})\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right) - (1 \leftrightarrow 2)\right]$$

$$-\epsilon_{-+\sigma\beta} f_{3M}^A g_{\perp\perp\mu}^{\beta} A(x_1, x_2)\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1 x_2}{1-x_2}\frac{Q}{Z}K_1(QZ) + T_2^{\sigma\rho\nu}(\{x_i\})\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right) + (1 \leftrightarrow 2)\right]$$

$V(x_1, x_2)$ = genuine twist-3 vector DAs

$A(x_1, x_2)$ = genuine twist-3 axial DAs

f_M^V and f_M^A = normalization constants

Results: twist 3 expansion

3-body twist-3 expanded result

momentum space

- Fourier transform

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- **Momentum space impact factor** (after twist expansion)

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \frac{e_q m_M}{4} c_f \left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+} q_\rho\right)\left(\varepsilon_M^{*\beta} - \frac{p_M^\beta}{p_M^+}\varepsilon_M^{*+}\right)\delta\left(1 - \frac{p_M^+}{q^+}\right)$$
$$\times\left(\prod_{j=1}^{3}\frac{\theta(1-x_j)\theta(x_j)}{x_j}\right)\frac{(2\pi)^3\delta^{(2)}\left(\sum_{i=1}^3 \boldsymbol{p}_i + x_i\boldsymbol{p}_M\right)}{\left[Q^2 + \sum_{i=1}^3(\boldsymbol{p}_i + x_i\boldsymbol{p}_M)^2/x_i\right]}\left\{g_{\beta\sigma}f_{3M}^V V(x_1,x_2)\left(4g_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\right.\right.$$
$$\left.+\tilde{T}_1^{\sigma\rho\nu}(\{x\})\Big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2 + x_1(1-x_1)Q^2}\right) - \epsilon_{-+\sigma\beta}f_{3M}^A A(x_1,x_2)$$
$$\left.\times\left(4\frac{x_1x_2}{1-x_2}\epsilon^{\sigma\rho+-} + i\tilde{T}_2^{\sigma\rho\nu}(\{x\})\Big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu} - x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2 + x_1(1-x_1)Q^2}\right)\right\}$$
$$+(1\leftrightarrow 2)$$

3-body twist-3 expanded result cntd

momentum space

where

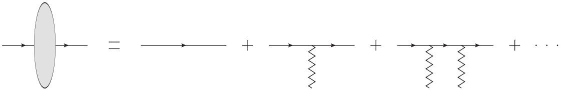
$$\tilde{T}_1^{\sigma\rho\nu}(\{x\}) = 4\left[2\frac{x_1(\bar{x}_1+x_2)}{x_3}g_\perp^{\sigma\nu}q^\rho - (k_1-p_1)_{\perp\mu}\right.$$
$$\left.\times\left(\frac{(\bar{x}_1+x_2)(\bar{x}_1-x_1)}{\bar{x}_1 x_3}g_\perp^{\sigma\nu}g_\perp^{\rho\mu} - \frac{1}{\bar{x}_1}\left(g_\perp^{\nu\rho}g_\perp^{\sigma\mu} - g_\perp^{\rho\sigma}g_\perp^{\nu\mu}\right)\right)\right]$$

$$\tilde{T}_2^{\sigma\rho\nu}(\{x\}) = \frac{4i}{\bar{x}_1}\left[2x_1\bar{x}_1 q^\rho\epsilon^{\nu\sigma+-} + (k_1-p_1)_{\perp\mu}\right.$$
$$\left.\times\left(\left(1+\frac{2x_2}{x_3}\right)\left(g_\perp^{\sigma\mu}\epsilon^{\nu\rho+-} - g_\perp^{\rho\sigma}\epsilon^{\nu\mu+-}\right) + (x_1-\bar{x}_1)g_\perp^{\rho\mu}\epsilon^{\nu\sigma+-}\right)\right]\,,$$

Dilute regime: two-body contribution

- **Reggeon** definition **[Caron-Huot (2013)]** $\quad R^a(\boldsymbol{z}) \equiv \frac{f^{abc}}{gC_A} \ln\left(U_{\boldsymbol{z}}^{bc}\right)$
- Expansion of the *Wilson line* in Reggeized gluons

$$V_{\boldsymbol{z}_1} = 1 + ig\boldsymbol{t}^a R^a(\boldsymbol{z}_1) - \frac{1}{2} g^2 \boldsymbol{t}^a \boldsymbol{t}^b R^a(\boldsymbol{z}_1) R^b(\boldsymbol{z}_1) + O\left(g^3\right)$$



- **BFKL $\boldsymbol{k}_{\mathrm{T}}$-factorization**

$$\mathcal{A}_2^{\text{dilute}} = \frac{g^2}{4N_c}(2\pi)^d \delta^d(\boldsymbol{q} - \boldsymbol{p}_M - \boldsymbol{\Delta}) \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathcal{U}(\ell) \int_0^1 \mathrm{d}x$$

$$\times \underbrace{\left[\Phi_2\left(x, \boldsymbol{\ell} - \frac{x-\bar{x}}{2}\boldsymbol{\Delta}\right) + \Phi_2\left(x, -\boldsymbol{\ell} - \frac{x-\bar{x}}{2}\boldsymbol{\Delta}\right) - \Phi_2(x, \bar{x}\boldsymbol{\Delta}) - \Phi_2(x, -x\boldsymbol{\Delta})\right]}_{\Phi_{2,\mathrm{BFKL}}(x, \boldsymbol{l}, \boldsymbol{\Delta})}$$

- $\mathcal{U}(\boldsymbol{l}) \to k_T$-unintegrated gluon density (UGD) in the BFKL sense

$$\mathcal{U}(\boldsymbol{\ell}) \equiv \int \mathrm{d}^d \boldsymbol{v} \mathrm{e}^{-i(\boldsymbol{\ell}\cdot\boldsymbol{v})} \left\langle P(p') \left| R^a\left(\frac{\boldsymbol{v}}{2}\right) R^a\left(-\frac{\boldsymbol{v}}{2}\right) \right| P(p) \right\rangle ,$$

- Φ_2 is the Fourier transform of Ψ_2

Explicit two-body term in the dilute and $\Delta = 0$ limit

- **BK impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) = 2\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{2,\text{f.}}(x) - \frac{x\bar{x}Q^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{2,\text{n.f.}}(x) \right]$$

- Helicity (flip and non-flip) structures and DAs combinations

$$T_{\text{n.f.}} = \boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^* \qquad \phi_{2,\text{n.f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) + \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

$$T_{\text{f.}} = \frac{(\boldsymbol{\varepsilon}_q \cdot \boldsymbol{l})(\boldsymbol{\varepsilon}_M^* \cdot \boldsymbol{l})}{\boldsymbol{l}^2} - \frac{\boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^*}{2} \qquad \phi_{2,\text{f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) - \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

- **Forward limit matching**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 2\left(\Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{l}) - \Phi_{2,\boldsymbol{\Delta}=0}(x,\boldsymbol{0})\right)$$

- **BFKL impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}(x,\boldsymbol{l}) = 4\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{\text{f.}}(x) + \frac{\boldsymbol{l}^2(\boldsymbol{l}^2 + 2x\bar{x}Q^2)}{x\bar{x}Q^2\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{\text{n.f.}}(x) \right]$$

Explicit three-body term in the dilute and $\Delta = 0$ limit

- The 3-body BFKL impact factor is a combination of 12 BK impact factors

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- Transverse to transverse transition in the **forward** and **dilute** limit

$$c_f = N_c^2/(N_c^2-1)$$

$$\mathcal{A}_{3T,\boldsymbol{\Delta}=\mathbf{0}}^{\text{dilute}} = e_q m_M \frac{g^2}{N_c}(2\pi)\delta\left(1-\frac{p_M^+}{q^+}\right)(2\pi)^2\delta^2\left(\boldsymbol{q}-\boldsymbol{p}_M\right)\int\frac{\mathrm{d}^d\boldsymbol{\ell}}{(2\pi)^d}\mathcal{U}(\boldsymbol{\ell})$$
$$\times\left(\prod_{i=1}^{3}\int_0^1\frac{dx_i}{x_i}\right)\frac{\delta(1-x_1-x_2-x_3)}{x_3}\frac{\boldsymbol{\ell}^2}{Q^2}\Big\{T_{\text{f.}}\left[f_{3M}^V V\left(x_1,x_2\right)-f_{3M}^A A\left(x_1,x_2\right)\right]$$
$$\times 2x_1\left(\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_2x_3}{x_2+x_3}Q^2}+\frac{x_3c_f}{\boldsymbol{\ell}^2+\frac{x_1x_3}{x_1+x_3}Q^2}-\frac{\bar{x}_3\left(1-c_f\right)}{\boldsymbol{\ell}^2+\frac{x_1x_2}{x_1+x_2}Q^2}+\frac{x_2-\bar{x}_1c_f}{\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2}+\frac{x_1-\bar{x}_2c_f}{\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2}\right)$$
$$-T_{\text{n.f.}}\left[f_{3M}^V V\left(x_1,x_2\right)+f_{3M}^A A\left(x_1,x_2\right)\right]$$
$$\times\left(\frac{\left(1-c_f\right)x_1\bar{x}_3}{\bar{x}_3\boldsymbol{\ell}^2+x_1x_2Q^2}-\frac{c_f x_3^2}{\bar{x}_1\boldsymbol{\ell}^2+x_2x_3Q^2}-\frac{\left(x_2-\bar{x}_1c_f\right)x_1x_2}{\bar{x}_1\left(\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2\right)}-\frac{\left(x_1-\bar{x}_2c_f\right)\bar{x}_2}{\left(\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2\right)}\right)\Big\}$$

- **The forward and dilute limit matches our previous result**

  [Anikin, Ivanov, Pire, LS, Wallon (2009)]

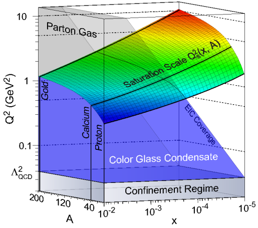
  BFKL approach + twist-expansion via light-cone collinear factorization

Summary

- **Transversally polarized light vector meson production**
- DVMP in the **non-linear** regime in the transversely polarized case
- Both **forward** and **non-forward** results and **s-channel non-conserving helicity amplitudes**
- **Coordinate** and **momentum space** representations
- Reggeized gluon expansion **[Caron-Huot (2013)]** $\Longrightarrow$ **BFKL results**
- To be used for a complete description of HERA and future EIC data
- Higher-twist corrections are essential to describe medium energy data of exclusive processes:

  data for $ep \to e\pi^0 p$ need a twist 3 π^0 DA **[M. Defurne et al. (2016)]**

- Method to deal with twist corrections at small-x including saturation
- what's next? what about the NLO frontier?
  - Wandzura-Wilczek approximation: no genuine twist-3, i.e. no $q\bar{q}g$ 3-body in principle, "straightforward"
  - Full NLO? Out of reach for the moment without a full automatization of the calculations...



THANK YOU FOR ATTENTION !!