Quarkonium: a magnifying glass on the gluon TMDs inside protons

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Next generation perturbative QCD

24 October 2024

Recent review on Quarkonium at EIC: 2409.03691

Outline

(transverse momentum dependent parton distribution functions)

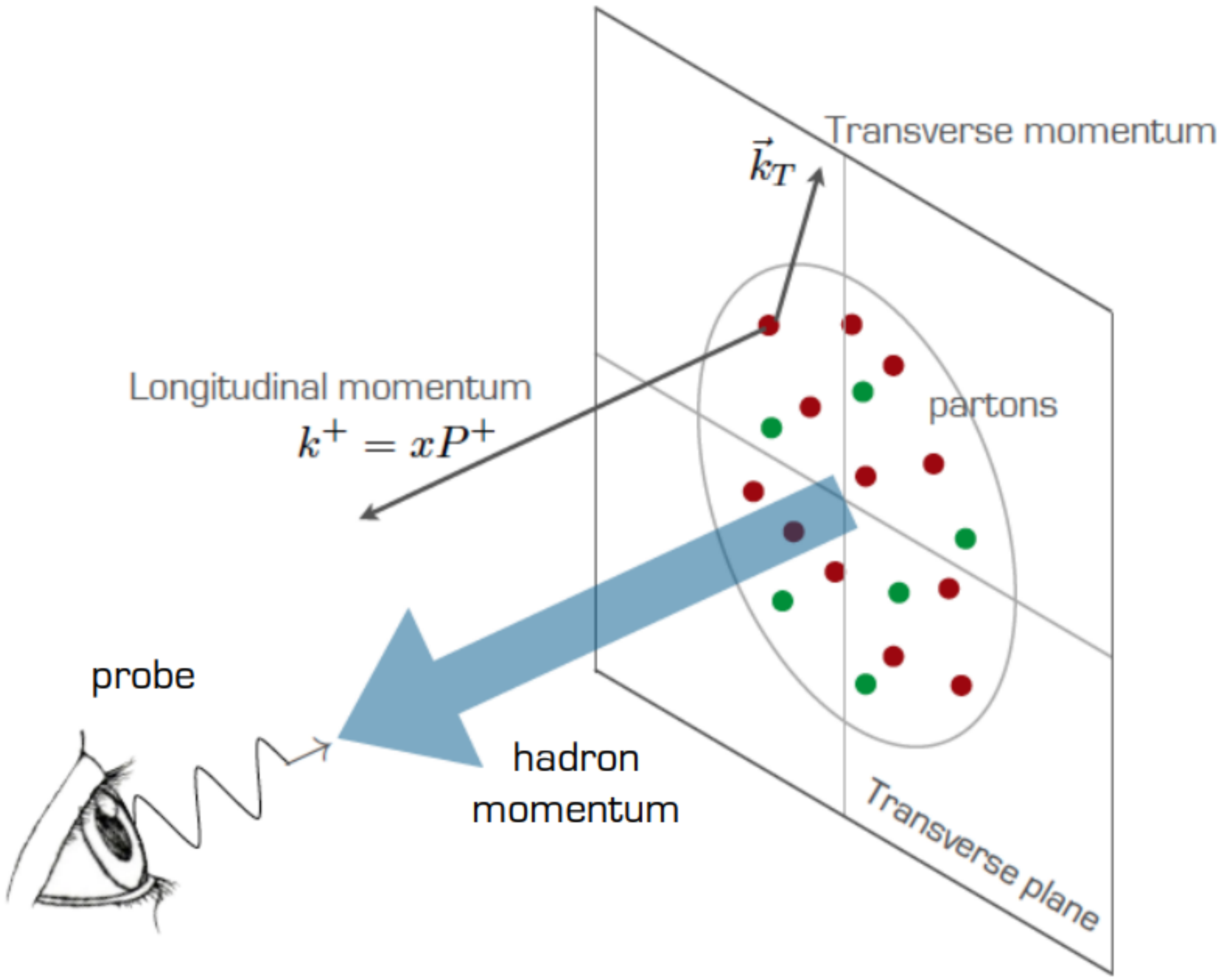
- **Part I:** J/ψ and **TMD-PDFs**
- **Part II: Quarkonium mechanism**
- **Part III:** Accessing **gluon TMDs** at the **EIC**
- **Part IV:** Accessing **gluon TMDs** at the **LHC**
- **Part V:** the **TMDShF**

The gluon TMDs table

TMD-PDFs: $f(x, \boldsymbol{k}_\perp^2; Q^2)$

(transverse momentum dependent PDFs)

Mulders, Rodriguez, *PRD 63* (2001)



picture by A. Bacchetta

gluon polar. / proton polar.	Unpolarized	Circular	Linear
Unpolarized	f_1		$h_1^\perp$
Longitudinal		g_{1L}	$h_{1L}^\perp$
Transverse	$f_{1T}^\perp$	g_{1T}	h_1 , $h_{1T}^\perp$

also collinear   T-Even   T-Odd

Note that the TMD notation is analogous between quarks and gluons!

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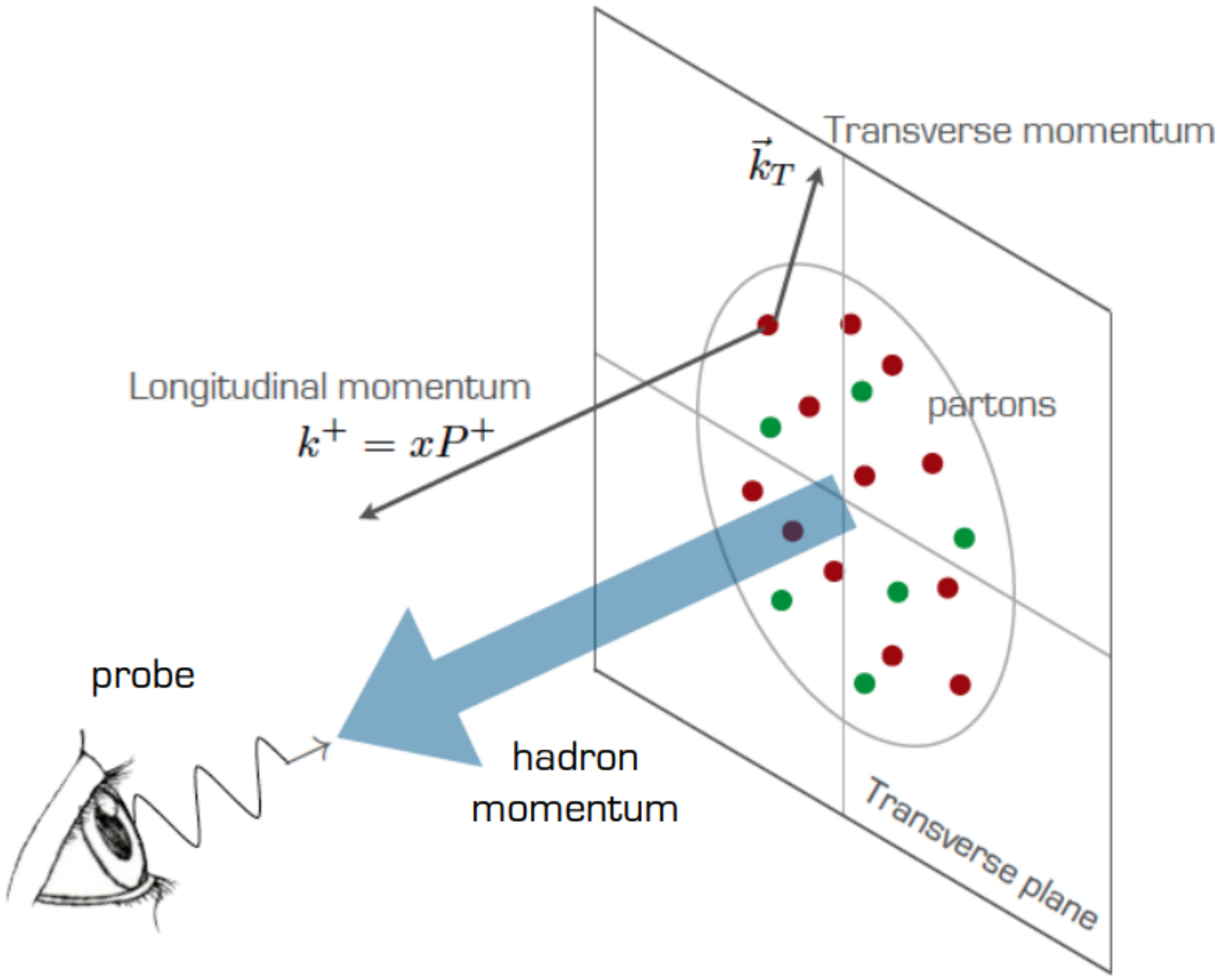

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also collinear — T-Even — T-Odd

Note that even if protons are unpolarised **gluons may be polarised**!

The gluon TMDs positivity bounds

Mulders, Rodriguez, *PRD 63* (2001)

Upper bounds on gluon distributions driven by matrix **positivity** constraints

$$f_1^g \geq 0$$

Matrix in the $|\,\mathrm{gluon};\mathrm{nucleon}\rangle$ spin basis

$$\begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M}\,e^{i\phi}\left(g_{1T} + i\,f_{1T}^{\perp}\right) & \frac{|p_T|}{M}\,e^{-i\phi}\left(h_{1L}^{\perp} + i\,h_1^{\perp}\right) & 2\,h_1 \\ \frac{|p_T|}{M}\,e^{-i\phi}\left(g_{1T} - i\,f_{1T}^{\perp}\right) & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}e^{-2i\phi}\,h_{1T}^{\perp} & -\frac{|p_T|}{M}\,e^{-i\phi}\left(h_{1L}^{\perp} - i\,h_1^{\perp}\right) \\ \frac{|p_T|}{M}\,e^{i\phi}\left(h_{1L}^{\perp} - i\,h_1^{\perp}\right) & \frac{|p_T|^2}{M^2}e^{2i\phi}\,h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\,e^{i\phi}\left(g_{1T} - i\,f_{1T}^{\perp}\right) \\ 2\,h_1 & -\frac{|p_T|}{M}\,e^{i\phi}\left(h_{1L}^{\perp} + i\,h_1^{\perp}\right) & -\frac{|p_T|}{M}\,e^{-i\phi}\left(g_{1T} + i\,f_{1T}^{\perp}\right) & f_1 + g_{1L} \end{pmatrix}$$

$$|\,g_{1L}^g\,| \leq f_1^g \qquad |\,f_{1T}^{\perp\,g}\,|,\,|\,g_{1T}^g\,|,\,|\,h_1^g\,| \leq \frac{M_p}{|\boldsymbol{p}_T|}f_1^g \qquad \frac{1}{2}\,|\,h_1^{\perp\,g}\,| \leq \frac{M_p^2}{\boldsymbol{p}_T^2}f_1^g \qquad \frac{1}{2}\,|\,h_{1T}^{\perp\,g}\,| \leq \frac{M_p^3}{|\boldsymbol{p}_T|^3}f_1^g$$

Positivity bounds are useful to determine asymmetries upper limits

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$$f_1^g \geq 0$$

$$|\, g_{1L}^g \,| \leq f_1^g \qquad |\, f_{1T}^{\perp\, g} \,|, |\, g_{1T}^g \,|, |\, h_1^g \,| \leq \frac{M_p}{|\boldsymbol{p}_T|} f_1^g \qquad \frac{1}{2} |\, h_1^{\perp\, g} \,| \leq \frac{M_p^2}{\boldsymbol{p}_T^2} f_1^g \qquad \frac{1}{2} |\, h_{1T}^{\perp\, g} \,| \leq \frac{M_p^3}{|\boldsymbol{p}_T|^3} f_1^g$$

Positivity bounds are useful to determine asymmetries upper limits

Also used for parameterization, e.g. via a **Gaussian anstaz**

Gaussian with width $\langle p_T^2 \rangle$ — Collinear PDF

$$f_1^g(x, \boldsymbol{p}_T^2) \propto G\left(\boldsymbol{p}_T^2, \langle p_T^2 \rangle\right) f_1^g(x)$$

$\boldsymbol{\rho}$ modifies the broadening — N modifies the x dep.

$$F_1^g(x, \boldsymbol{p}_T^2) \propto G\left(\boldsymbol{p}_T^2, \rho \langle p_T^2 \rangle\right) N(x) f_1^g(x)$$

Quarkonia and gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

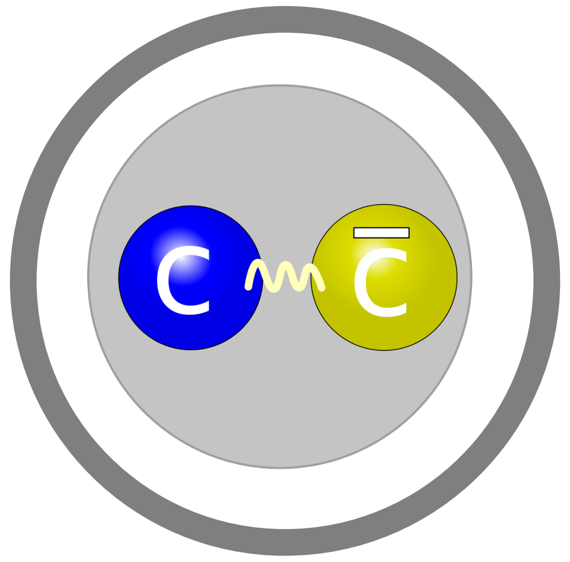
- $p + p \to \eta_Q + X$
- $p + p \to \chi_Q + X$
- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

***ep* collisions**

- $e + p \to e' + J/\psi + X$
- $e + p \to e' + J/\psi + \gamma + X$
- $e + p \to e' + J/\psi + \text{jet} + X$

and more…

J/ψ ID card



J/ψ

$^{2S+1}L_J^{(c)} = {}^3S_1^{(1)}$

$J^{PC} = 1^{--}$

Mass: 3.0969 GeV

Spin: 1

Total ang. mom.: 1

Parity: -1

Charge conjugation: -1

Mean Lifetime: 7.2E-21s

B.R. l^+l^-**:** ~6 % each

Discovered in 1974 at:

BNL

Aubert et al., *PRL 33* (1974)

SLAC

Augustin et al., *PRL 33* (1974)

S (spin) L (orbital a.m.) J (total a.m.) c (color) P (parity) C (charge conj.)

J/ψ formation mechanism

Quarkonia are characterised by:

Bodwin, Braaten, Lepage, *PRD 51* (1994)

- **Large** mass M
- **small** relative velocity v of the heavy-quark pair $(Q\bar{Q})$
  for charmonium $v^2 \approx 0.3\,c^2$

Short-distance scale
perturbative

Long-distance scale
non-perturbative

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Short-distance scale	Long-distance scale
perturbative	non-perturbative

Expanded in power of α_s

Production of the **heavy-quark** pair

(some) Frameworks:

Colour-Singlet Model
CSM

$Q\bar{Q}$ *in a* ***Color-Singlet*** *state*

Non-Relativistic QCD
NRQCD

$Q\bar{Q}$ *in a* ***Color-Singlet***
and **-Octet** states
expansion w.r.t. v
matrix elements (**LDMEs**)

Tests of the underlying J/ψ formation mechanism

Collection of observables (@ **EIC**) that probe the underlying mechanism

- J/ψ **polarization parameters**

  D'Alesio, LM, Murgia, Pisano, Sangem, *PRD 107* (2023)

- **Ratio** Quarkonium/open-quark at **small-**$\boldsymbol{P_T}$

  Boer, Pisano, Taels, *PRD 103* (2021)

- **Azimuthal correlations** in J/ψ plus jet production

  LM, Yuan, *2403.02097* (2024)

Tests of the underlying J/ψ formation mechanism

- J/ψ **polarization parameters**

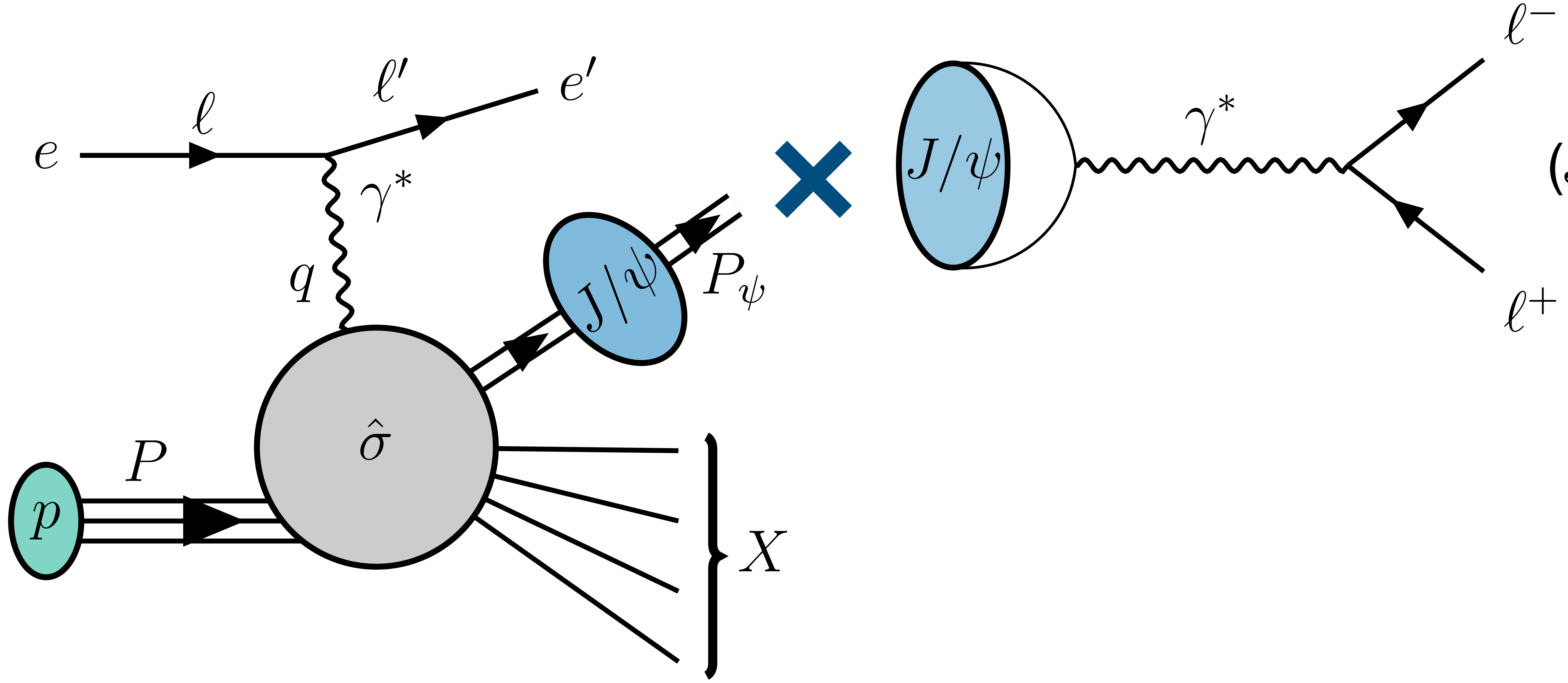
D'Alesio, LM, Murgia, Pisano, Sangem, *PRD 107* (2023)

J/ψ polarization parameters

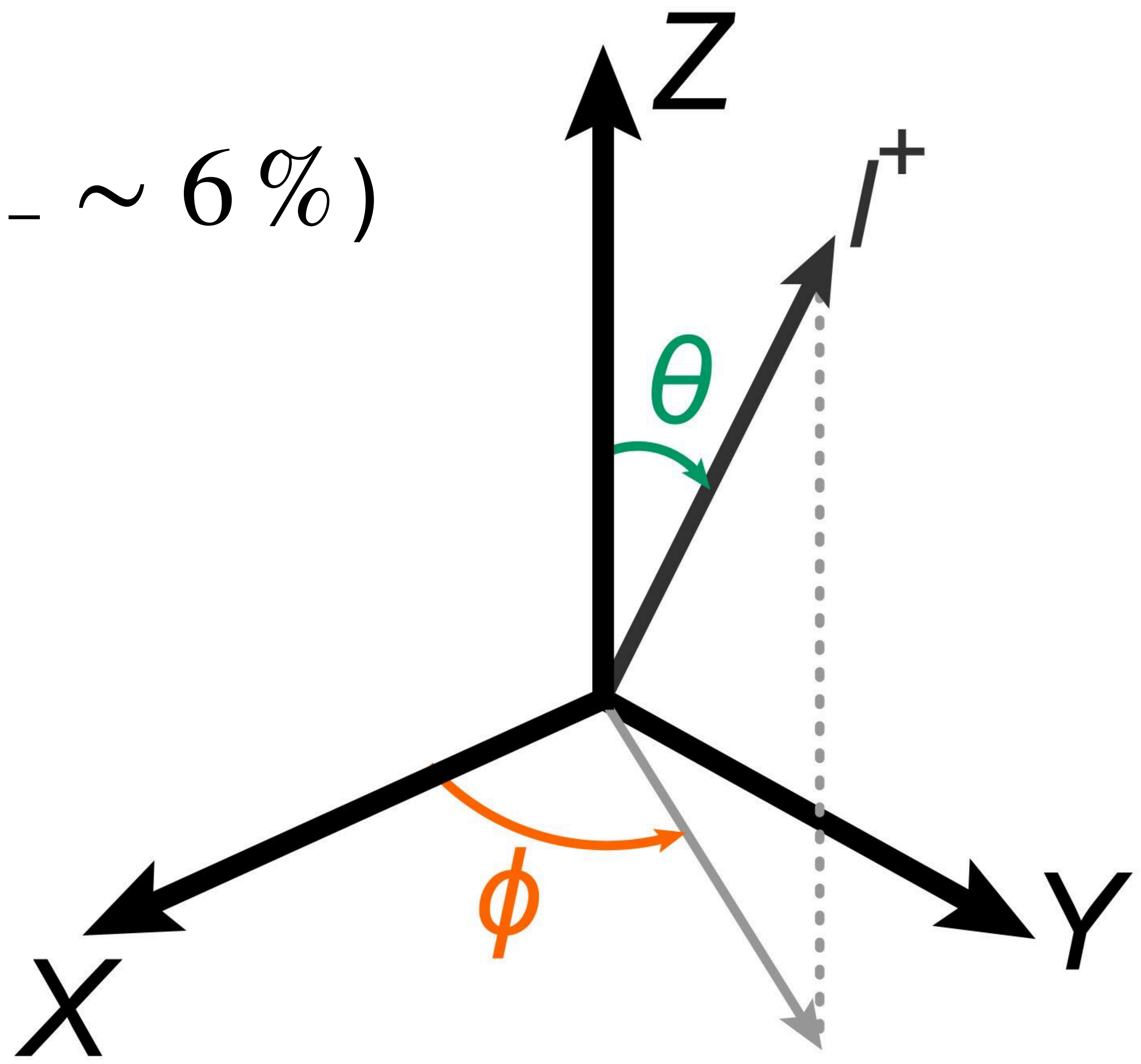
Quarkonium polarization is historically tricky from theoretical pov

We can study the J/ψ polarization by considering its decay into a lepton pair

Faccioli, Lourenço, *Lect.Notes* (2022)



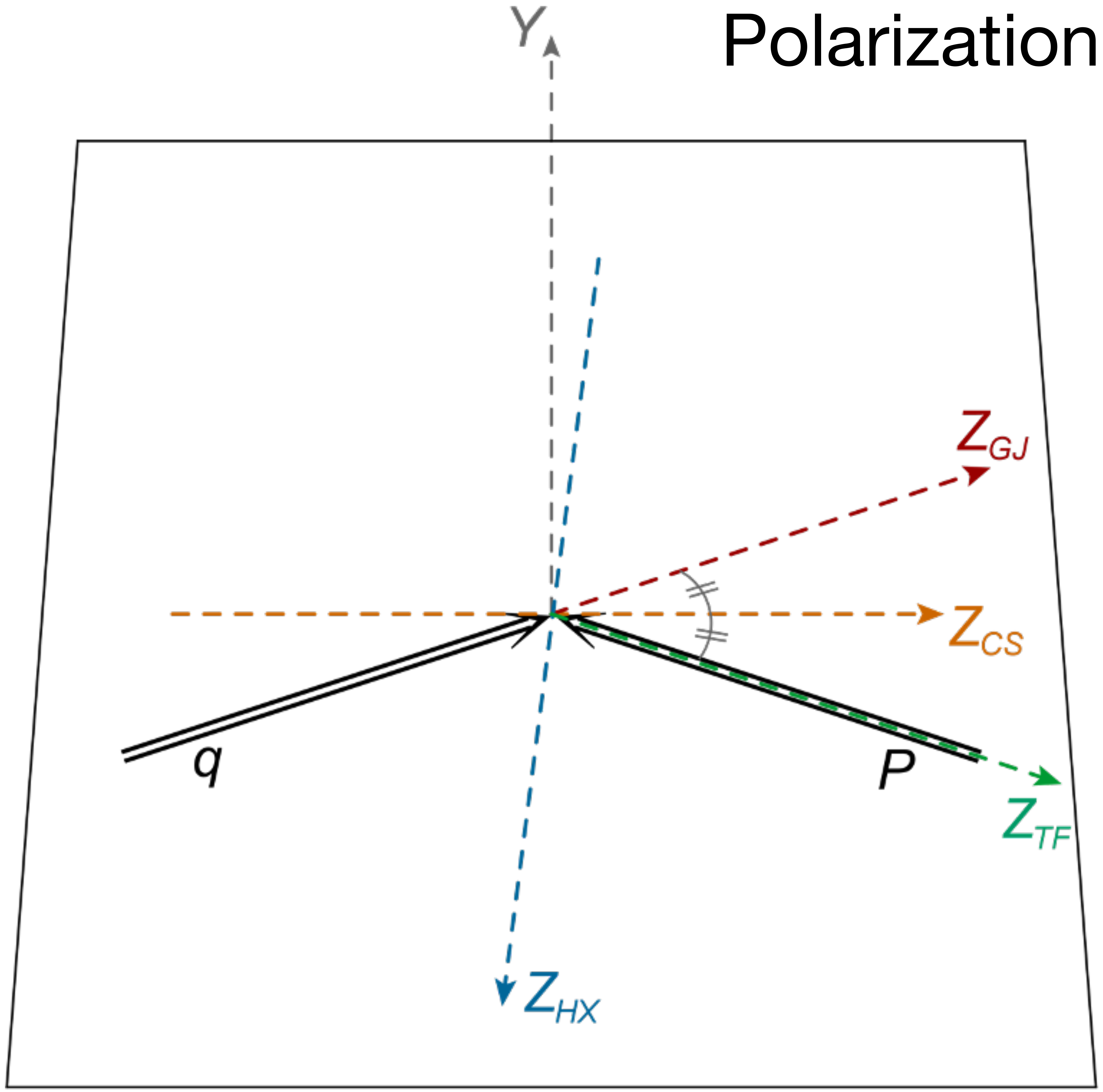
$(\mathscr{B}_{e^+e^-} \sim \mathscr{B}_{\mu^+\mu^-} \sim 6\,\%)$



$$\frac{d\sigma}{dx_B\, dy\, d^4P_\psi\, d\Omega} \propto 1 + \lambda \cos^2\theta + \mu \cos 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi$$

Beneke, Krämer, Vänttinen, *PRD 57* (1998)

Frame choice



Polarization parameters are not frame independent

Different choices for the reference frame

- **GJ** *Gottfried-Jackson* frame
- **CS** *Collins-Soper* frame
- **HX** *Helicity* frame
- **TF** *Target frame*

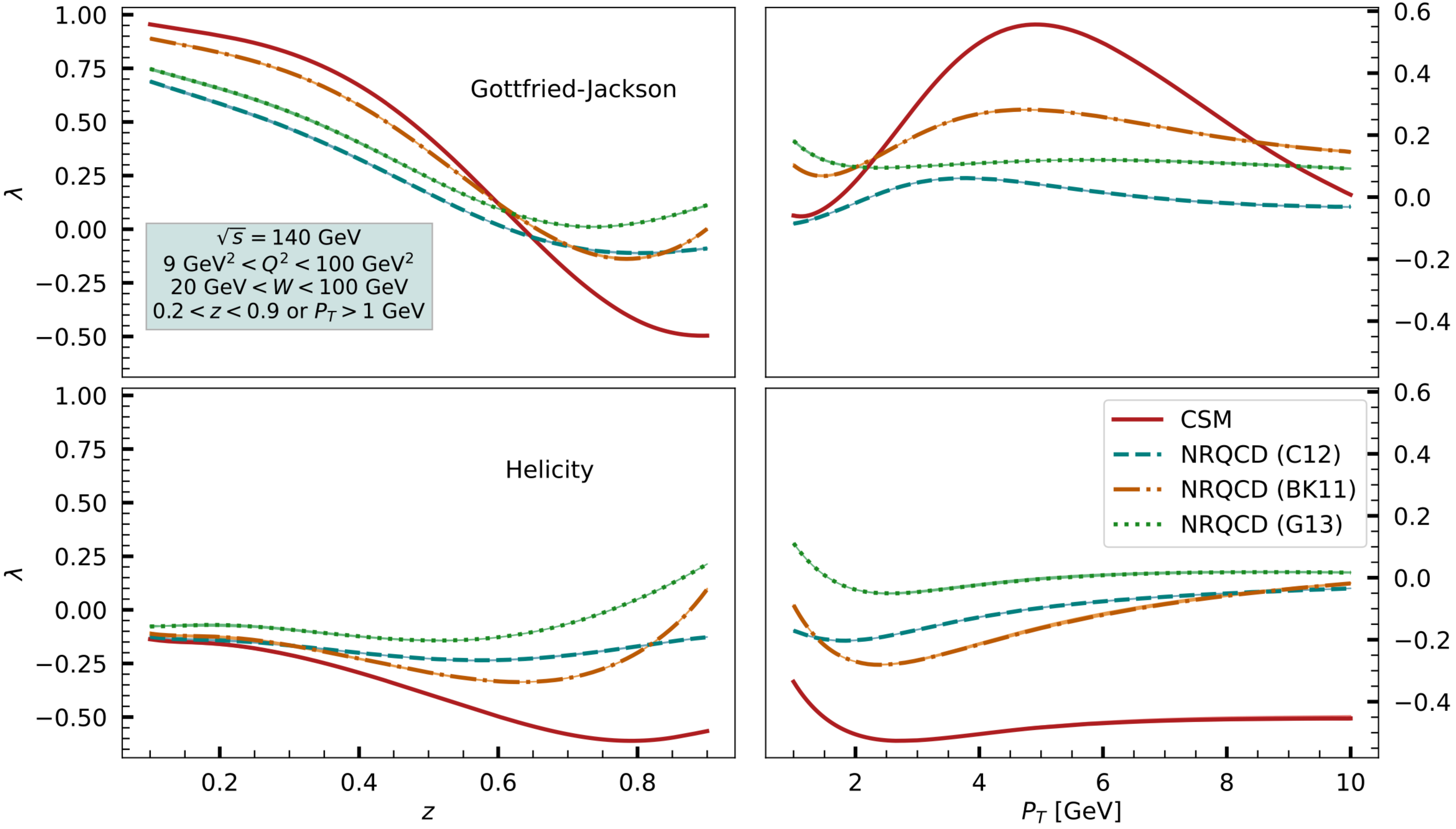
Frames are related by a rotation around the Y-axis

J/ψ pol. parameters at high-P_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD 107* (2023)

P_T distribution varies more on the CO/CS ratio

(especially **shape** and **magnitude**)



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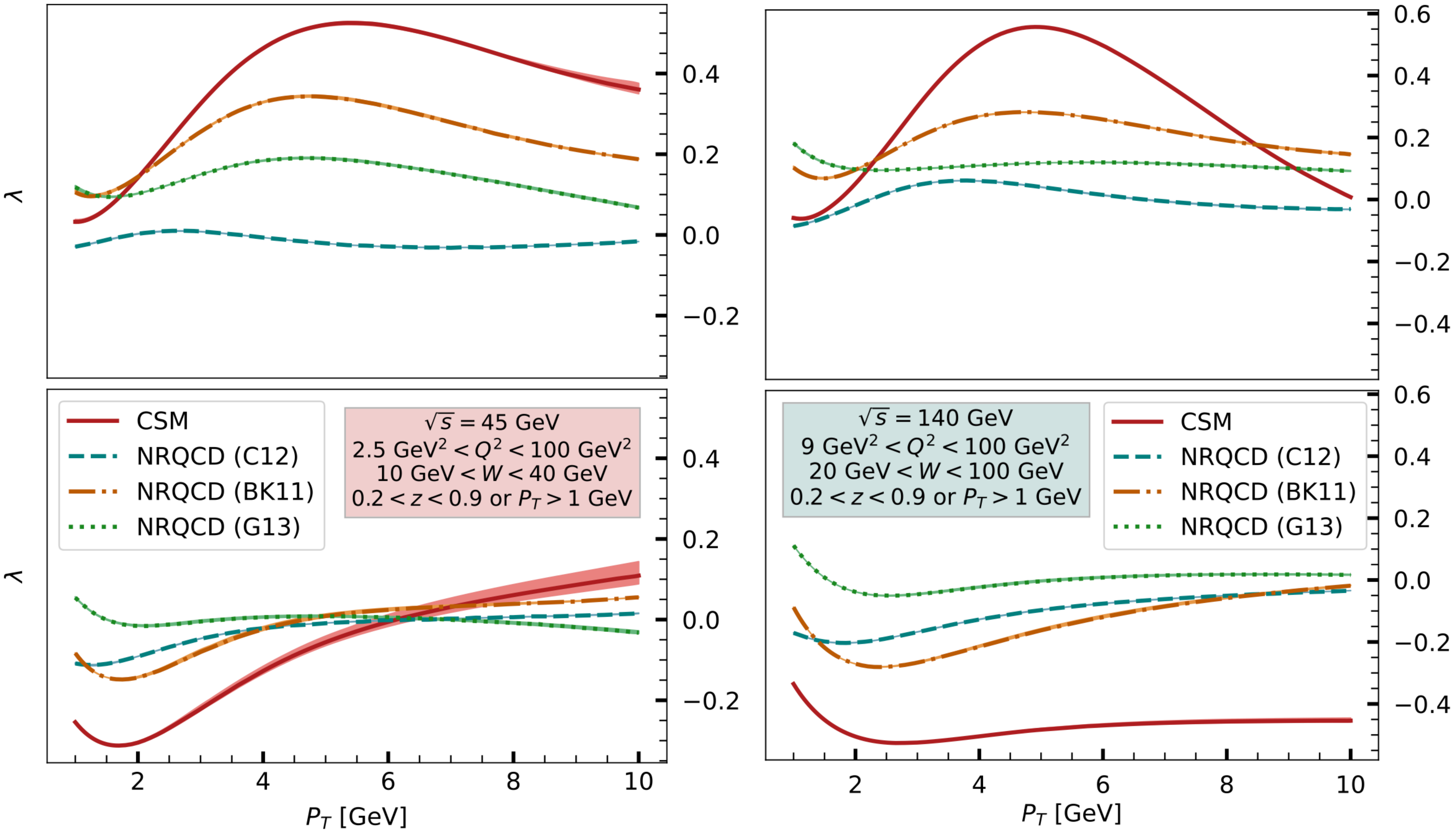
P_T distribution also depends on Q bins

(**low** vs **high** virtualities)



Tests of the underlying J/ψ formation mechanism

- **Ratio** Quarkonium/open-quark at small-$\boldsymbol{P_T}$

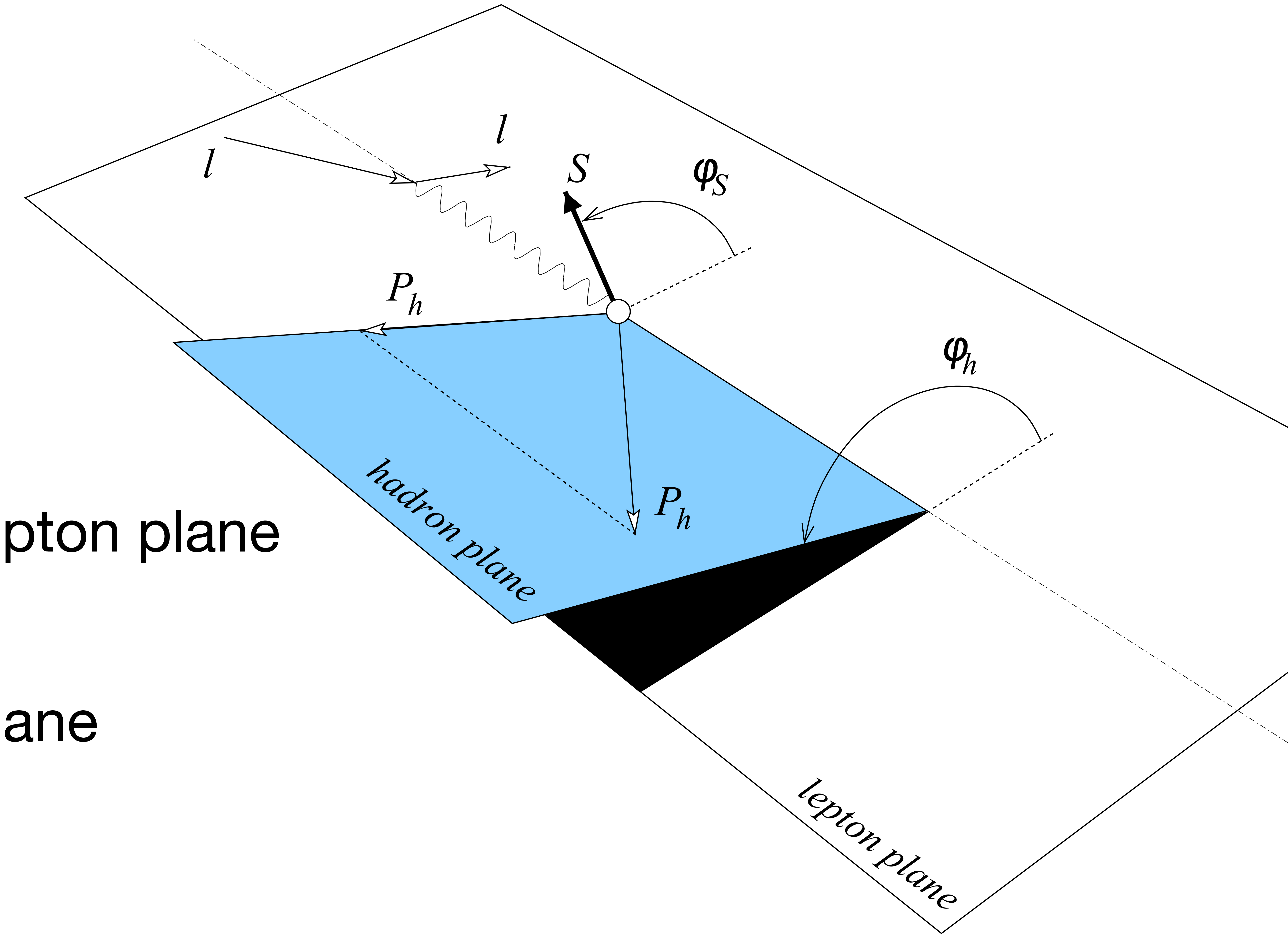
Boer, Pisano, Taels, *PRD 103* (2021)

J/ψ low-P_T production at the EIC

J/ψ electroproduction at small-P_T (and within NRQCD) probes CO LDME ratios

Bachetta, Boer, Pisano, Taels, *EPJ C 80* (2020)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^2\boldsymbol{P}_T} \equiv \mathrm{d}\sigma^U(\phi_T) + \mathrm{d}\sigma^T(\phi_T, \phi_S)$$



$\phi_T \equiv \phi_h$ Quarkonium azimuth. angle w.r.t. lepton plane

ϕ_S proton spin azimuth. angle w.r.t. lepton plane

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$$\mathrm{d}\sigma^U(\phi_T) \propto A^U f_1^g(x, \boldsymbol{q}_T^2) + B^U \frac{\boldsymbol{q}_T^2}{M_p^2} h_1^{\perp g}(x, \boldsymbol{q}_T^2)\cos(2\phi_T)$$

Probes $R^{(8)} = \frac{\langle \mathcal{O}_\psi[^3P_0^{(8)}]\rangle}{\langle \mathcal{O}_\psi[^1S_0^{(8)}]\rangle} = \frac{\mathcal{O}_P^{(8)}}{\mathcal{O}_S^{(8)}}$

$$A^{U\mathcal{Q}_U} = \left[1-(1-y)^2\right]\left[\mathcal{O}_S^{(8)} + 4\,\frac{7M_\psi^4 + 2M_\psi^2Q^2 + 3Q^4}{M_\psi^2(M_\psi^2+Q^2)^2}\mathcal{O}_P^{(8)}\right] + 64\,(1-y)\,\frac{Q^2}{(M_\psi^2+Q^2)^2}\mathcal{O}_P^{(8)}$$

$$B^{U\mathcal{Q}_U} = (1-y)\left[-\mathcal{O}_S^{(8)} + 4\,\frac{3M_\psi^2 - Q^4}{M_\psi^2(M_\psi^2+Q^2)}\mathcal{O}_P^{(8)}\right]$$

Unpolarized J/ψ

Accessing TMDs at the EIC

Boer, Pisano, Taels, *PRD 103* (2021)

To **single out** the ratio of **LDMEs** we need to compare **different final states**

- Comparison of different quarkonium polarization states
- Quarkonium vs open heavy-quark production

$$D^{\mathcal{Q}_P} = \int \mathrm{d}\phi_T \, \frac{\mathrm{d}\sigma^{U\mathcal{Q}_P}}{\mathrm{d}x_B \, \mathrm{d}y \, \mathrm{d}^2\boldsymbol{P}_T} \qquad N^{\mathcal{Q}_P} = \int \mathrm{d}\phi_T \, \cos(2\phi_T) \frac{\mathrm{d}\sigma^{U\mathcal{Q}_P}}{\mathrm{d}x_B \, \mathrm{d}y \, \mathrm{d}^2\boldsymbol{P}_T}$$

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$$\frac{D^{J/\psi_L}}{D^{J/\psi_U}} = \frac{M_\psi^2 \left(q^2+1\right)^2 \mathcal{O}_S^{(8)} + 12\left(q^4 + 2q^2 + 12\right)\mathcal{O}_P^{(8)}}{3\left(M_\psi^2\left(q^2+1\right)^2 \mathcal{O}_S^{(8)} + 4\left(3q^4 + 2q^2 + 7\right)\right)\mathcal{O}_P^{(8)}}$$

where $q = \dfrac{Q}{M_\psi}$

Note that for $M_\psi^2 \, \mathcal{O}_S^{(8)} \gg \mathcal{O}_P^{(8)}$ ratio is constant

Accessing TMDs at the EIC

Boer, Pisano, Taels, *PRD 103* (2021)

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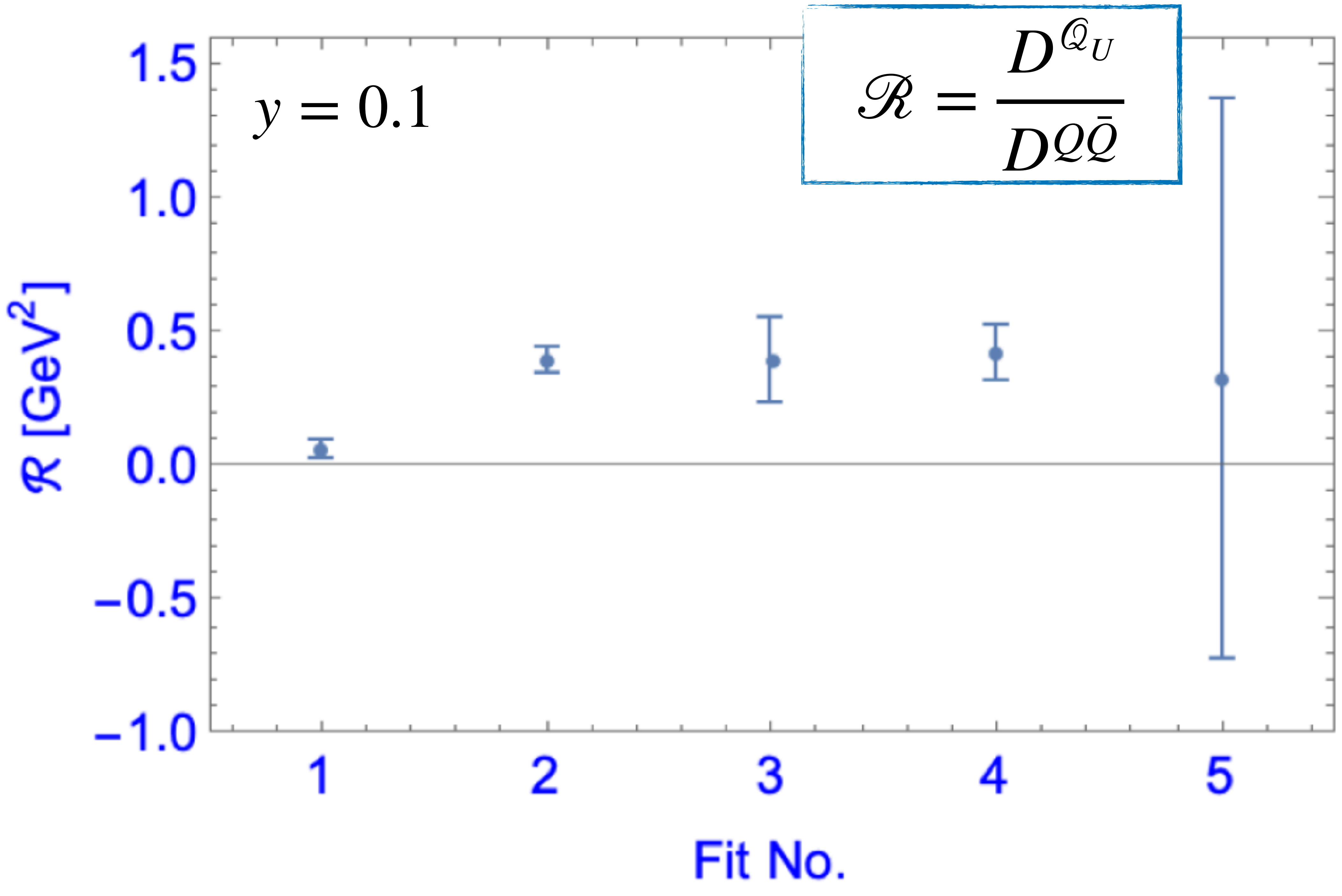
$$D^{Q\bar{Q}} = \int \mathrm{d}\phi_T \, \frac{\mathrm{d}\sigma^{Q\bar{Q}}}{\mathrm{d}x_B \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}^2\boldsymbol{K}_T \, \mathrm{d}^2\boldsymbol{P}_T} \qquad N^{Q\bar{Q}} = \int \mathrm{d}\phi_T \, \cos(2\phi_T) \frac{\mathrm{d}\sigma^{Q\bar{Q}}}{\mathrm{d}x_B \, \mathrm{d}y \, \mathrm{d}z \, \mathrm{d}^2\boldsymbol{K}_T \, \mathrm{d}^2\boldsymbol{P}_T}$$

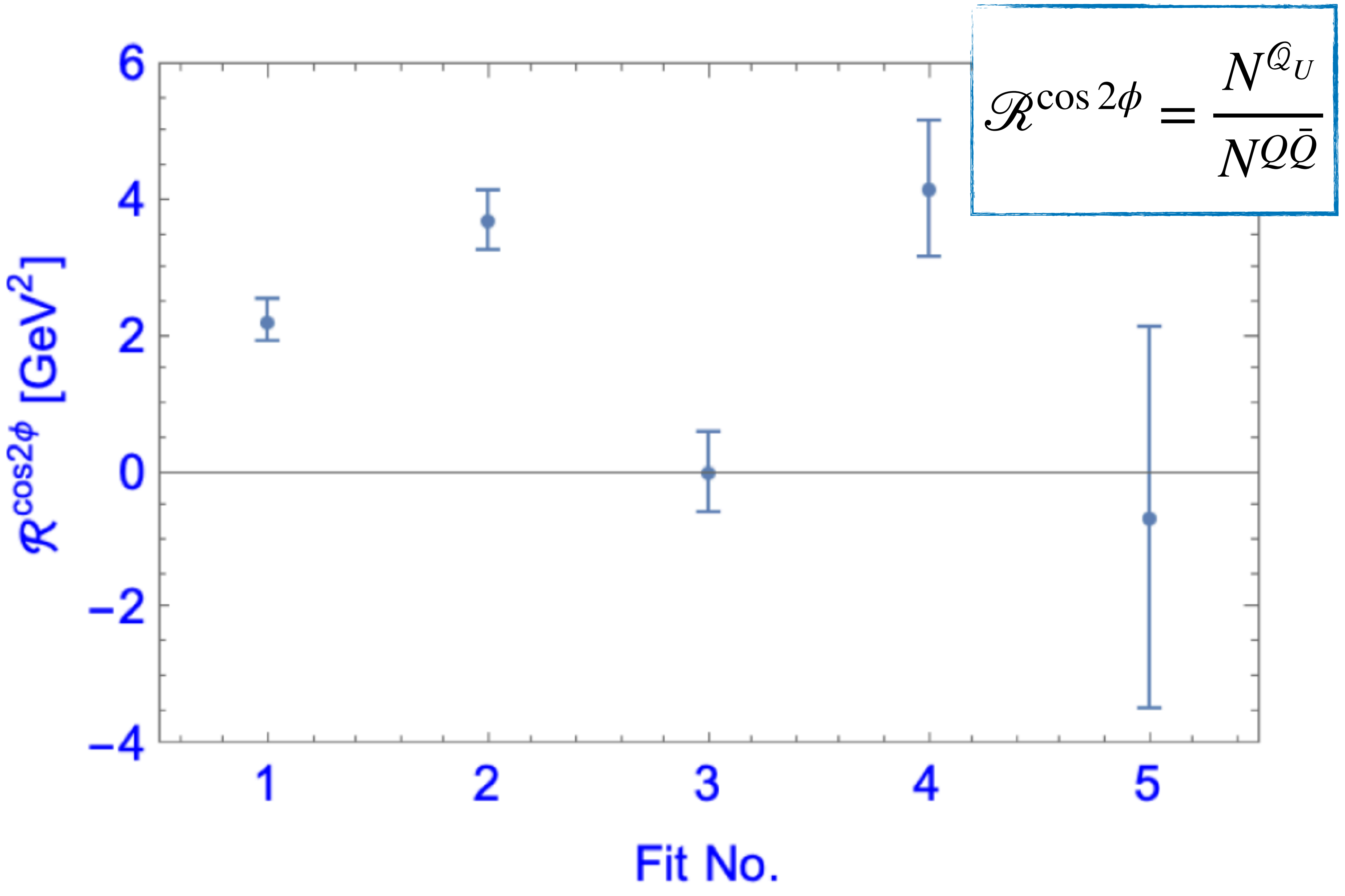
where $z = \dfrac{K_Q \cdot P}{q \cdot P}$ is the outgoing quark en. fraction, $\boldsymbol{K}_T = (\boldsymbol{K}_{QT} - \boldsymbol{K}_{\bar{Q}T})/2$ is the transv. difference

Ratio Onia/open-quark

[Boer, Pisano, Taels, *PRD 103* (2021)](#)

To avoid contributions from TMD evol. $\boldsymbol{K}_T = Q\,(\equiv M_\psi)$, whereas $z = 1/2$





1. Butenschoen, Kniehl, *PRL 106* (2011)
2. Chao, Ma, Shao, Wang, Zhang, *PRL 108* (2012)
3. Sharma, Vitev, *PRC 87* (2013) - J/ψ
4. Bodwin, Chung, Kim, Lee, *PRL 113* (2014)
5. Sharma, Vitev, *PRC 87* (2013) - Υ(1S)

Tests of the underlying J/ψ formation mechanism

Next, I will present a collection of suggested observables at the **EIC**

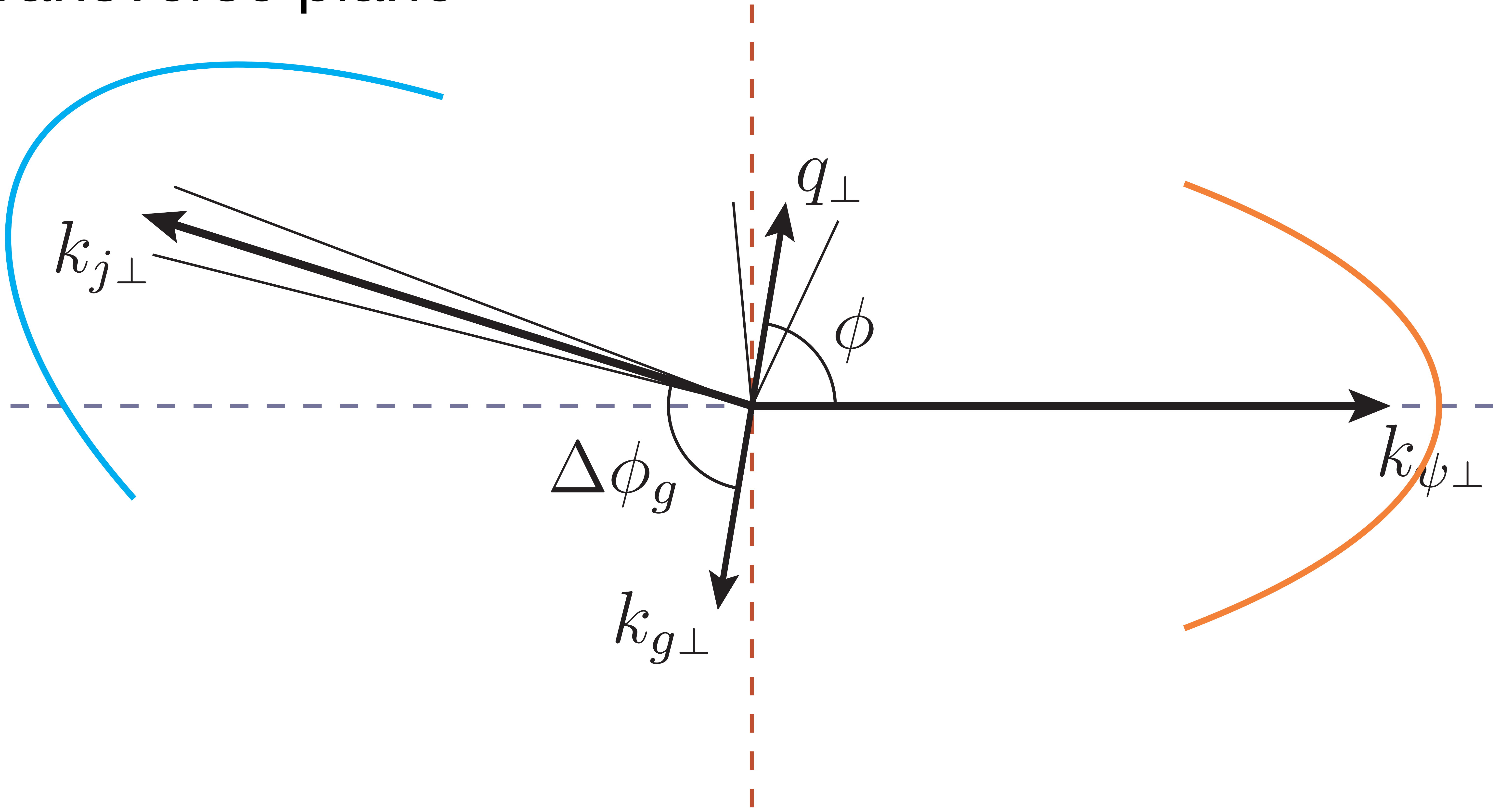
- **Azimuthal correlations** in J/ψ plus jet production

  LM, Yuan, *2403.02097* (2024)

Quarkonium plus jet

LM, Yuan, *2403.02097* (2024)

transverse plane



Correlation limit

(J/ψ and jet back-to-back)

$$q_\perp \ll P_\perp$$

$$\vec{q}_\perp = \vec{k}_{\psi\perp} + \vec{k}_{j\perp}$$

$$\vec{P}_\perp = \frac{\vec{k}_{\psi\perp} - \vec{k}_{j\perp}}{2}$$

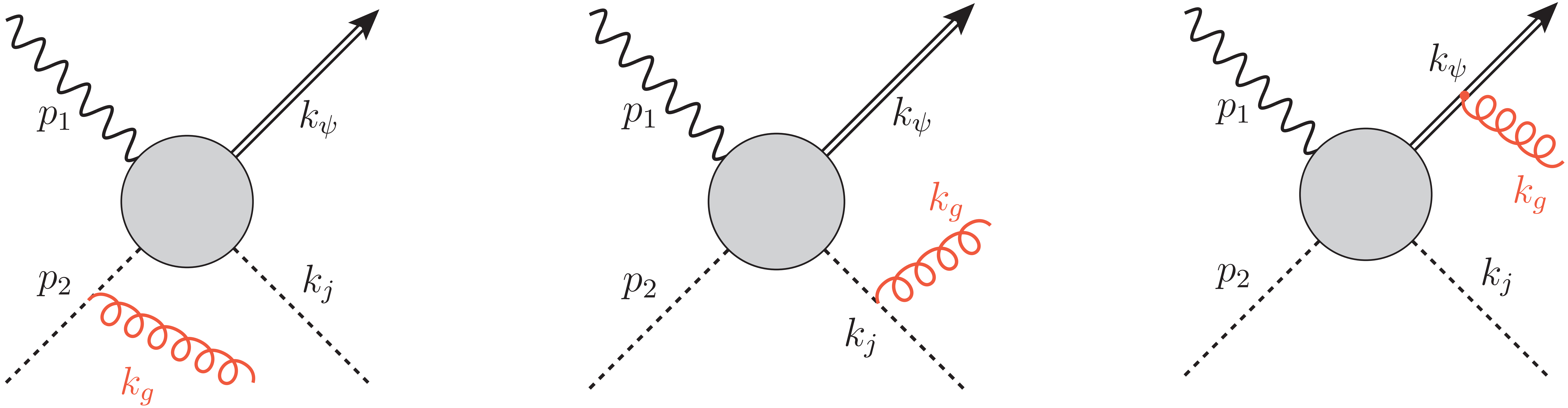
$\Delta\phi_g \approx \phi$ ➡ Azimuthal Imbalance

A diagramatic view of soft gluon emissions

LM, Yuan, *2403.02097* (2024)

We consider **photoproduction**

Three possibilities to emit a soft gluon from the Born amplitude



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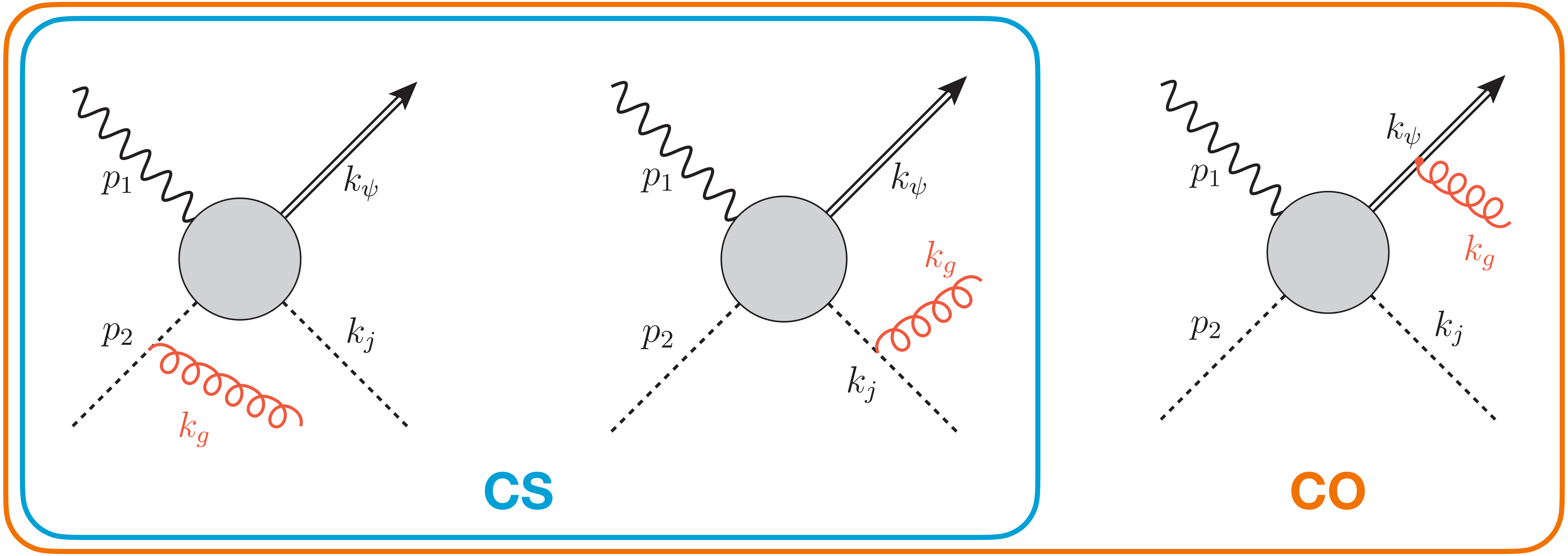
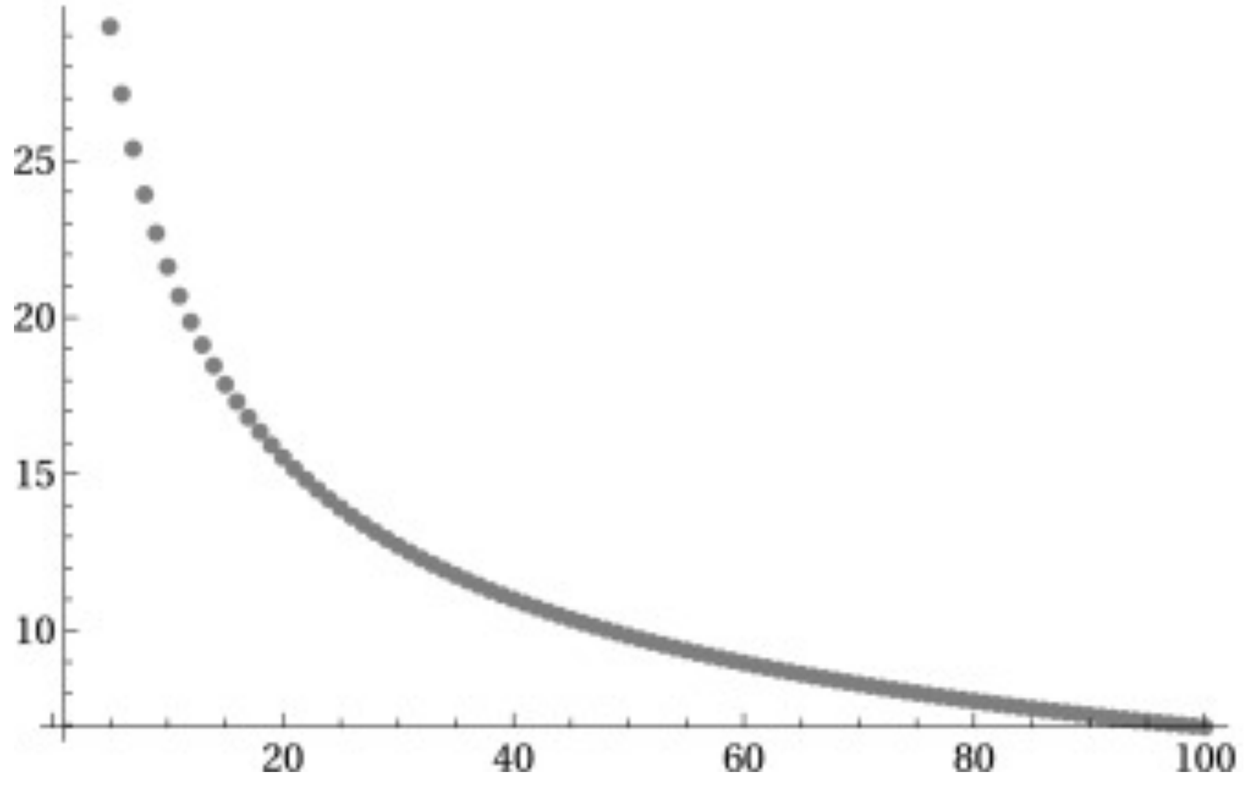


Single and double logarithms: CS

$$\int \frac{\mathrm{d}^3 k_g}{(2\pi)^3 2E_{k_g}} |\overline{\mathscr{A}_1^{(1)}}|^2 \delta^{(2)}(q_\perp + k_{g\perp}) = \frac{\alpha_s C_A}{2\pi^2 q_\perp^2} |\overline{\mathscr{A}_0^{(1)}}|^2 \left[\ln \frac{\hat{s}}{q_\perp^2} + \ln \frac{\hat{t}}{\hat{u}} + I_j(R, \phi) \right]$$



- $\ln \frac{\hat{s}}{q_\perp^2}$: dominant behavior at low $q_\perp$
- $\ln \frac{\hat{t}}{\hat{u}}$: related to jet rapidity $y_j = \frac{1}{2} \ln \frac{k_j^+}{k_j^-}$
- $I_j(R, \phi)$: under investigation

can be foud in

Hatta, Xiao, Yuan, Zhou, *PRL 126* (2021)

Hatta, Xiao, Yuan, Zhou, *PRD 104* (2021)

Sun, Yuan, Yuan, *PRL 113* (2014)

Liu, Ringer, Vogelsang, Yuan, *PRL 122* (2019)

Single and double logarithms: CO

LM, Yuan, *2403.02097* (2024)

$$\int \frac{d^3 k_g}{(2\pi)^3 2E_{k_g}} |\overline{\mathcal{A}_1^{(8)}}|^2 \delta^{(2)}(q_\perp + k_{g\perp}) = \frac{\alpha_s C_A}{2\pi^2 q_\perp^2} |\overline{\mathcal{A}_0^{g,(8)}}|^2 \left[\ln\frac{\hat{s}}{q_\perp^2} + \frac{1}{2}\ln\frac{1 - M_\psi^2/\hat{u}}{1 - M_\psi^2/\hat{t}} + I_j(R,\phi) \right.$$

$$\left. + I_\psi(m_{\psi\perp}, \phi) + \frac{1}{2} I_{\psi\text{-}j}(m_{\psi\perp}, \Delta y, \phi) - \frac{1}{2} I_\psi^{\text{jet}}(R, m_{\psi\perp}, \Delta y, \phi) \right]$$

+ quark + LDME evolution

- $\ln\frac{\hat{s}}{q_\perp^2}$ and $I_j(R,\phi)$ do not vary from CS case → number of double-logs is not affected by the J/ψ mass or its production channel
- $\frac{1}{2}\ln\frac{1 - M_\psi^2/\hat{u}}{1 - M_\psi^2/\hat{t}}$: related to jet and J/ψ rapidities
- $I_\psi(m_{\psi\perp}, \phi)$, $I_{\psi\text{-}j}(m_{\psi\perp}, \Delta y, \phi)$, $I_\psi^{\text{jet}}(R, m_{\psi\perp}, \Delta y, \phi)$ under study

from removal of jet rapidity region

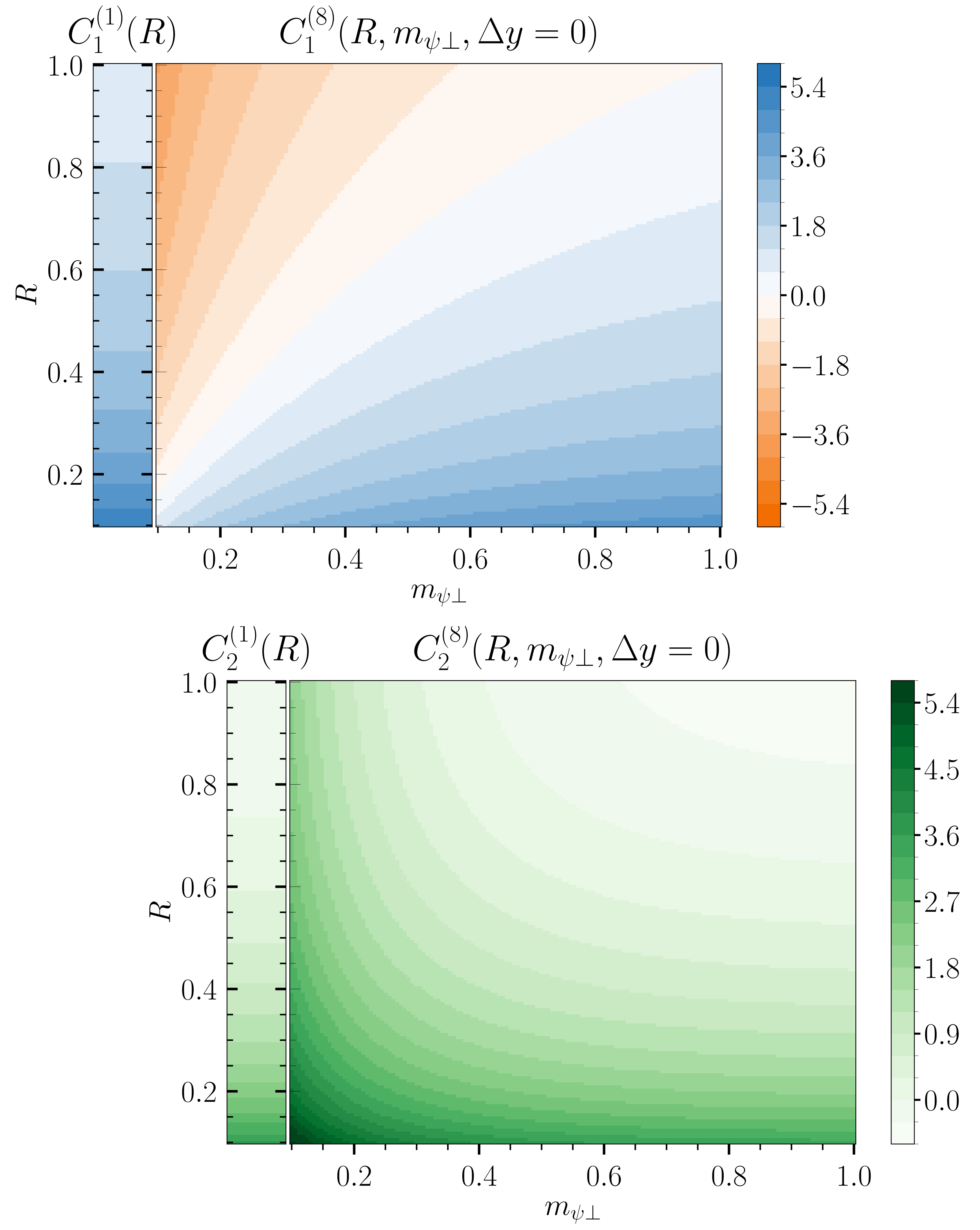
Azimuthal asymmetries: CS vs CO

LM, Yuan, *2403.02097* (2024)

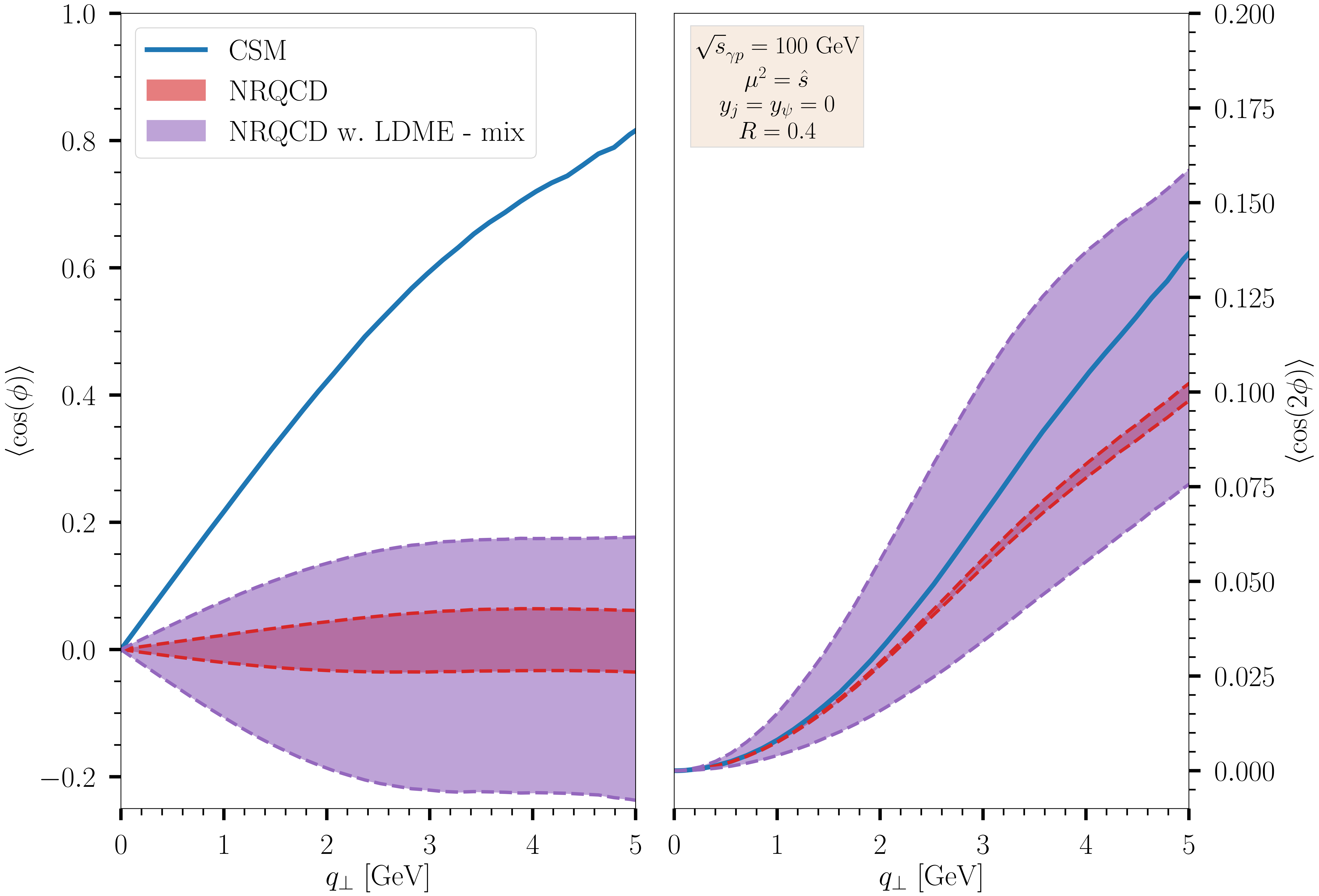
The azimuthal distributions can be expanded in a Fourier series

$$I(R, m_{\psi\perp}) = 2\sum_{n=0}^{\infty} C_n^{(1,8)}(R, m_{\psi\perp}, \Delta y = 0)\,\cos(n\phi)$$

coefficients for gluons



$|\vec{k}_{j\perp}| = 12$ GeV

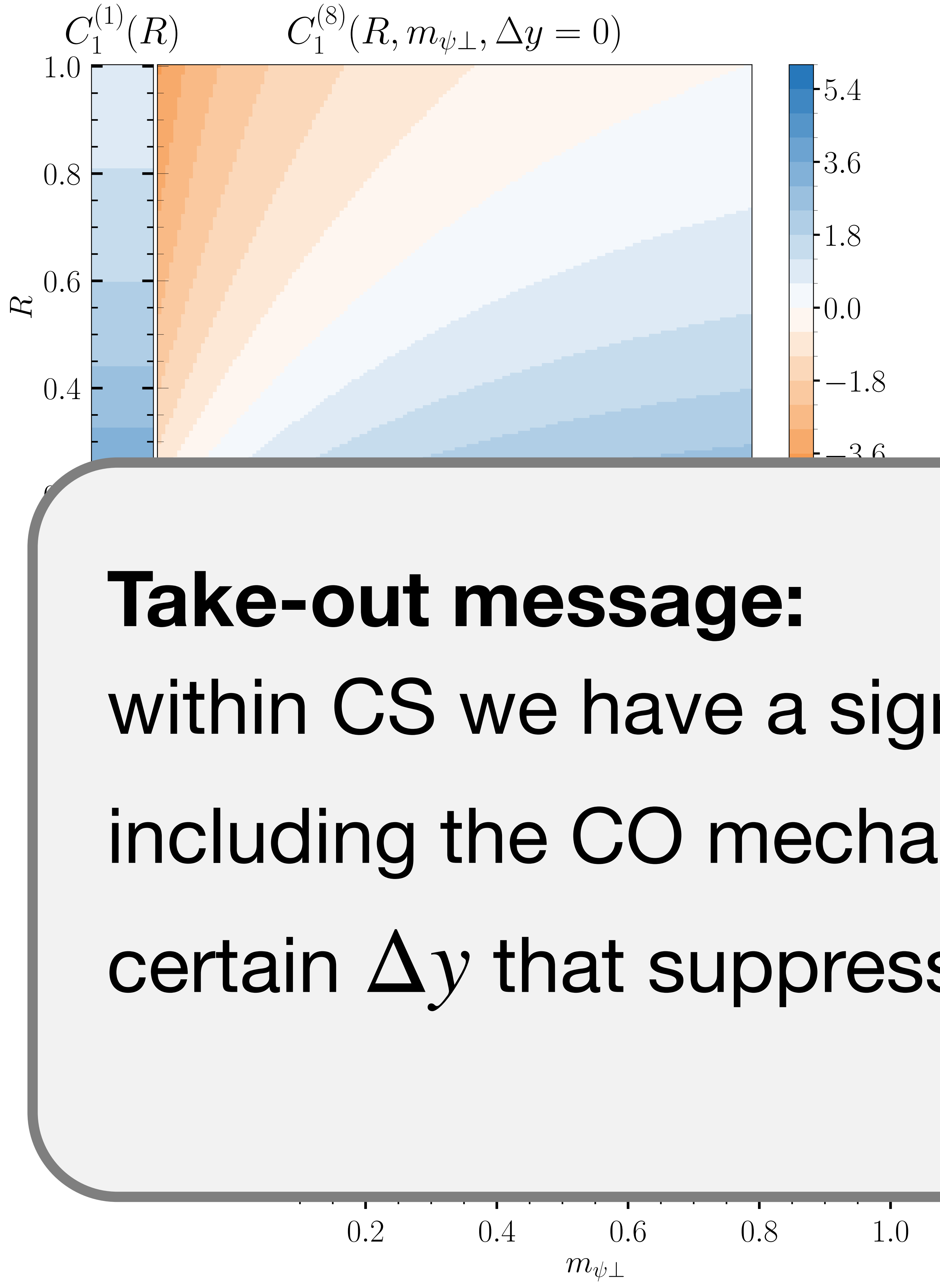


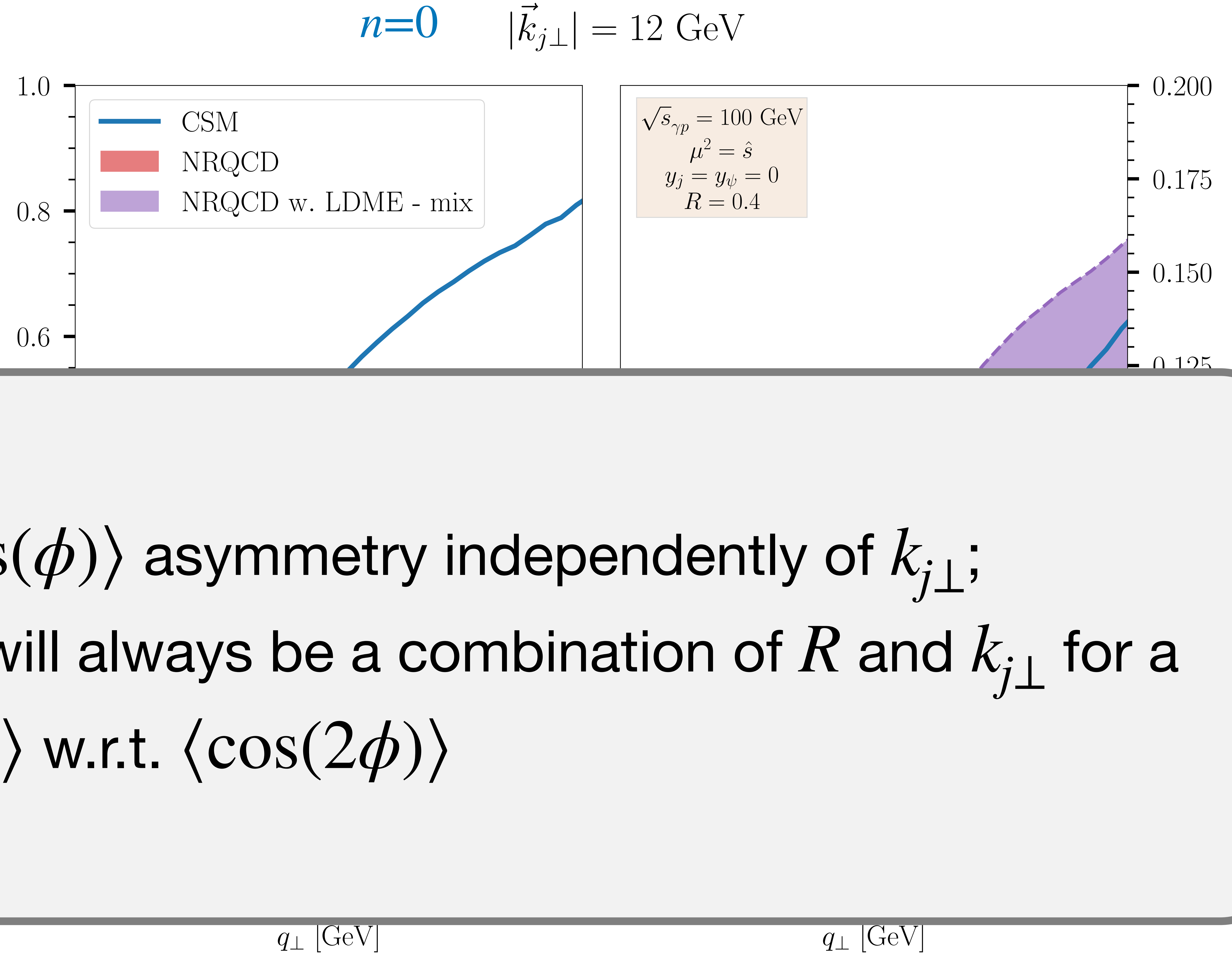
Azimuthal asymmetries: CS vs CO

LM, Yuan, *2403.02097* (2024)

The azimuthal distributions can be expanded in a Fourier series

$$I(R, m_{\psi\perp}) = 2\sum_{n=0}^{\infty} C_n^{(1,8)}(R, m_{\psi\perp}, \Delta y = 0)\cos(n\phi)$$





Take-out message:

within CS we have a significant $\langle\cos(\phi)\rangle$ asymmetry independently of $k_{j\perp}$;

including the CO mechanism there will always be a combination of R and $k_{j\perp}$ for a certain Δy that suppresses $\langle\cos(\phi)\rangle$ w.r.t. $\langle\cos(2\phi)\rangle$

Accessing gluon TMDs at the EIC

Proposed phenomenological studies to probe gluon TMDs at the EIC

(mostly asymmetries)

- **Azimuthal asymmetries** in J/ψ **inclusive** production

  Bacchetta, Boer, Pisano, Taels, *EPJ C 80* (2020)

  Bor, Boer, *PRD 106* (2022)

- **Azimuthal asymmetries** in J/ψ **plus jet** production

  D'Alesio, Murgia, Pisano, Taels, *PRD 100* (2019)

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  Bor, Boer, *PRD 106* (2022)

J/ψ inclusive production

Bacchetta, Boer, Pisano, Taels, *EPJ C 80* (2020)

$$\mathrm{d}\sigma(\phi_T, \phi_S) \equiv \mathrm{d}\sigma^U(\phi_T) + \mathrm{d}\sigma^T(\phi_T, \phi_S)$$

unpolarized

linearly polarized

$$\mathrm{d}\sigma^U(\phi_T) \propto A^U f_1^g(x, \boldsymbol{q}_T^2) + B^U \frac{\boldsymbol{q}_T^2}{M_p^2} h_1^{\perp g}(x, \boldsymbol{q}_T^2) \cos(2\phi_T)$$

$$\mathrm{d}\sigma^T(\phi_T) \propto \frac{|\boldsymbol{q}_T|}{M_p} \left[A^T f_{1T}^{\perp g}(x, \boldsymbol{q}_T^2) \sin(\phi_S - \phi_T) + B^T \left(h_1^g(x, \boldsymbol{q}_T^2) \sin(\phi_S + \phi_T) - \frac{\boldsymbol{q}_T^2}{2M_p^2} h_{1T}^{\perp g}(x, \boldsymbol{q}_T^2) \sin(\phi_S - 3\,\phi_T) \right) \right]$$

Sivers

linearly polarized

where $A^U = A^T$ and $B^U = B^T$

!!Leading-twist requires the **color-octet** mechanism!!

Asymmetries in J/ψ inclusive production

Bacchetta, Boer, Pisano, Taels, *EPJ C 80* (2020)

$$A^W = 2\frac{\int \mathrm{d}\phi_T\,\mathrm{d}\phi_S\; W(\phi_T,\phi_S)\;\mathrm{d}\sigma(\phi_T,\phi_S)}{\int \mathrm{d}\phi_T\,\mathrm{d}\phi_S\;\mathrm{d}\sigma(\phi_T,\phi_S)}$$

Four azimuthal moments:

$$A^{\cos(2\phi_T)} \propto \frac{\boldsymbol{q}_T^2}{2M_p^2}\frac{h_1^{\perp\,g}}{f_1^g} \qquad A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{f_{1T}^{\perp\,g}}{f_1^g}$$

$$A^{\sin(\phi_S+\phi_T)} \propto \frac{|\boldsymbol{q}_T|}{M_p}\frac{h_1^g}{f_1^g} \qquad A^{\sin(\phi_S-3\phi_T)} \propto \frac{|\boldsymbol{q}_T|^3}{2M_p^3}\frac{h_{1T}^{\perp\,g}}{f_1^g}$$

Some asymmetries ratios are direct probes of TMDs ratios:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{f_{1T}^{\perp\,g}}{f_1^g} \qquad \frac{A^{\cos(2\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = \frac{|\boldsymbol{q}_T|}{M_p}\frac{h_1^{\perp\,g}}{h_1^g} \qquad \frac{A^{\sin(\phi_S-3\,\phi_T)}}{A^{\sin(\phi_S+\phi_T)}} = -\frac{\boldsymbol{q}_T^2}{2M_p^2}\frac{h_{1T}^{\perp\,g}}{h_1^g}$$

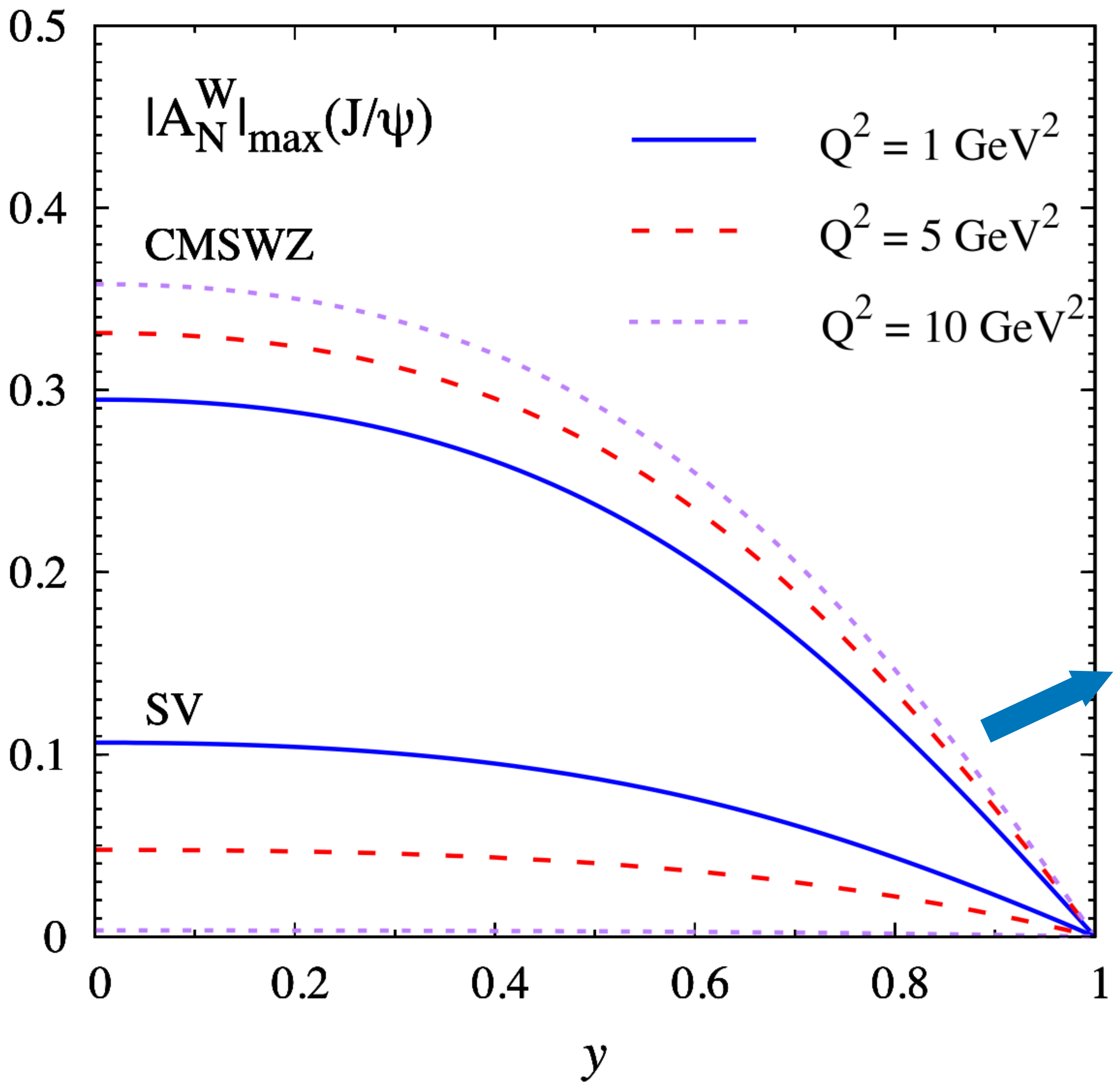
Positivity bounds lead to asymmetries upper limits

$$A^{\sin(\phi_S-\phi_T)} \le 1 \qquad A^{\cos(2\phi_T)},\, A^{\sin(\phi_S+\phi_T)},\, A^{\sin(\phi_S-3\phi_T)} \le A_N^W$$

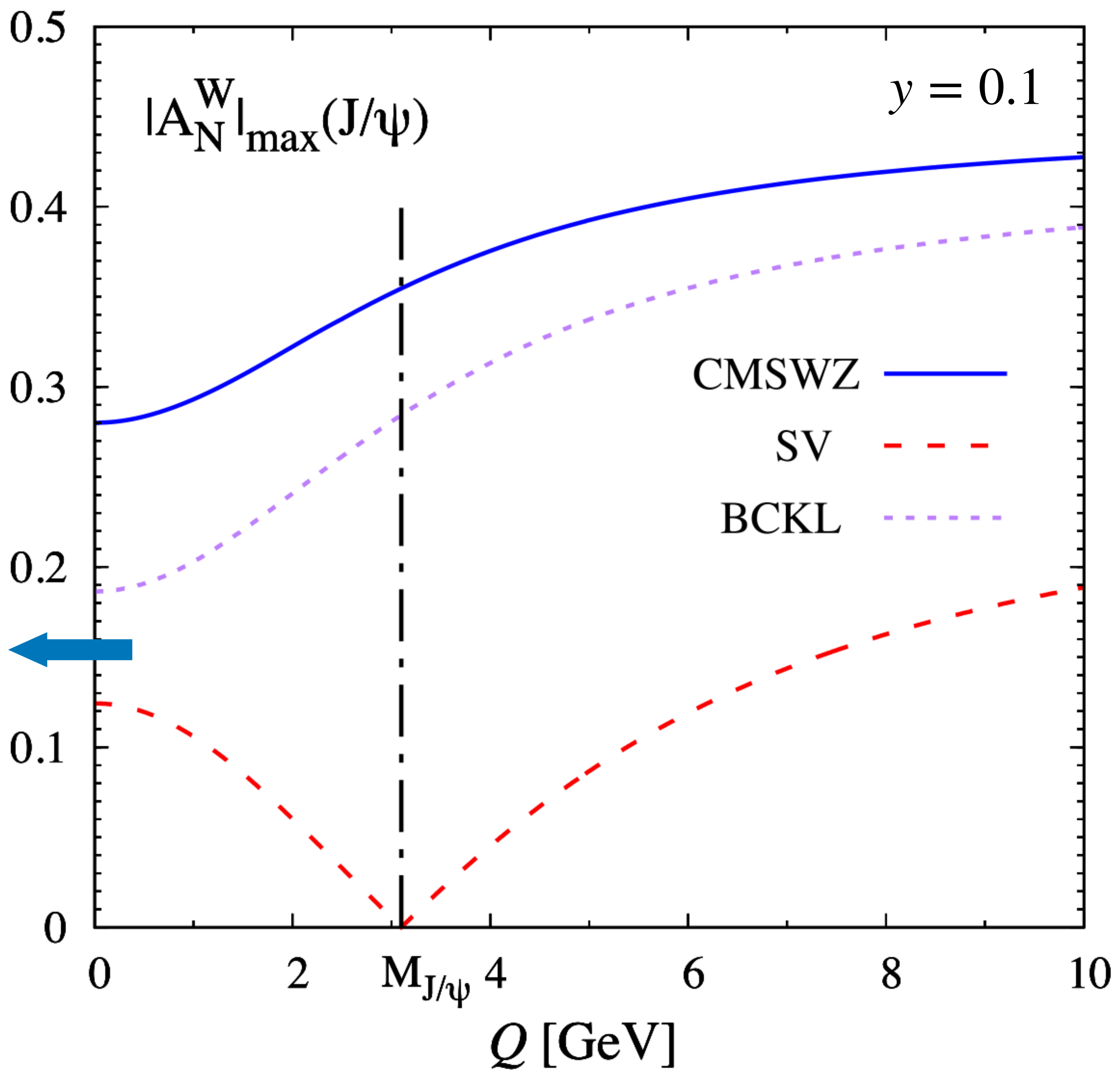
Numerical results

$A^{\sin(\phi_S-\phi_T)} \leq 1$ and $A^{\cos(2\phi_T)}$, $A^{\sin(\phi_S+\phi_T)}$, $A^{\sin(\phi_S-3\phi_T)} \leq A_N^W$

Bacchetta, Boer, Pisano, Taels, *EPJ C 80* (2020)



Q^2 dependence significantly differs among LDME sets



CMSWZ: Chao, Ma, Shao, Wang, Zhang, *PRL 108* (2012)

SV: Sharma, Vitev, *PRC 87* (2013) - J/ψ

TMD evolution of $\langle \cos 2\phi \rangle$ asymmetry

TMD evolution may play an active role at the EIC

$$\langle \cos(2\phi_T) \rangle = \frac{1}{2} A^{\cos(2\phi_T)} \propto \frac{\mathcal{C}[w\, h_{1T}^{\perp g}\, \Delta_h^{[n]}]}{\mathcal{C}[f_1^g\, \Delta^{[n]}]}$$

→ defined as R

→ $\Delta^{[n]},\ \Delta_h^{[n]} \propto \langle \mathcal{O}_\psi[n] \rangle$

(Term induced by the final state)

The convolution are evaluated within the b_T-space (Fourier conjugate of q_T)

$$\mathcal{C}[f_1^g\, \Delta] = \int_0^\infty \frac{\mathrm{d}b_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \mathrm{e}^{-S_A(b_T,Q^2)}\, \hat{f}_1^g(x, b_T)\, \hat{\Delta}(b_T)$$

$$\mathcal{C}[h_{1T}^{\perp g}\, \Delta_h] = -\int_0^\infty \frac{\mathrm{d}b_T}{2\pi}\, b_T\, J_2(b_T q_T)\, \mathrm{e}^{-S_A(b_T,Q^2)}\, \hat{h}_{1T}^{\perp g}(x, b_T)\, \hat{\Delta}_h(b_T)$$

→ Sudakov S_A resums large logs

The Sudakov in J/ψ electroproduction

Bor, Boer, *PRD 106* (2022)

$$S_A(b_T, Q^2) = -\frac{6}{\beta_0}\left[\log\frac{Q^2}{\mu_b^2} + \log\frac{\log(Q^2/\Lambda^2_{\text{QCD}})}{\log(\mu_b^2/\Lambda^2_{\text{QCD}})}\left(\frac{\beta_0}{6} + B_{\text{CO}} - \log\frac{Q^2}{\Lambda^2_{\text{QCD}}}\right)\right]$$

$$\beta_0 = \frac{23}{3}$$

Term induced by the final state

Determined from $pp \to J/\psi X$ Sun, Yuan, Yuan , *PRD 88* (2013)

To improve convergence the Sudakov b_T dependence has to be modified

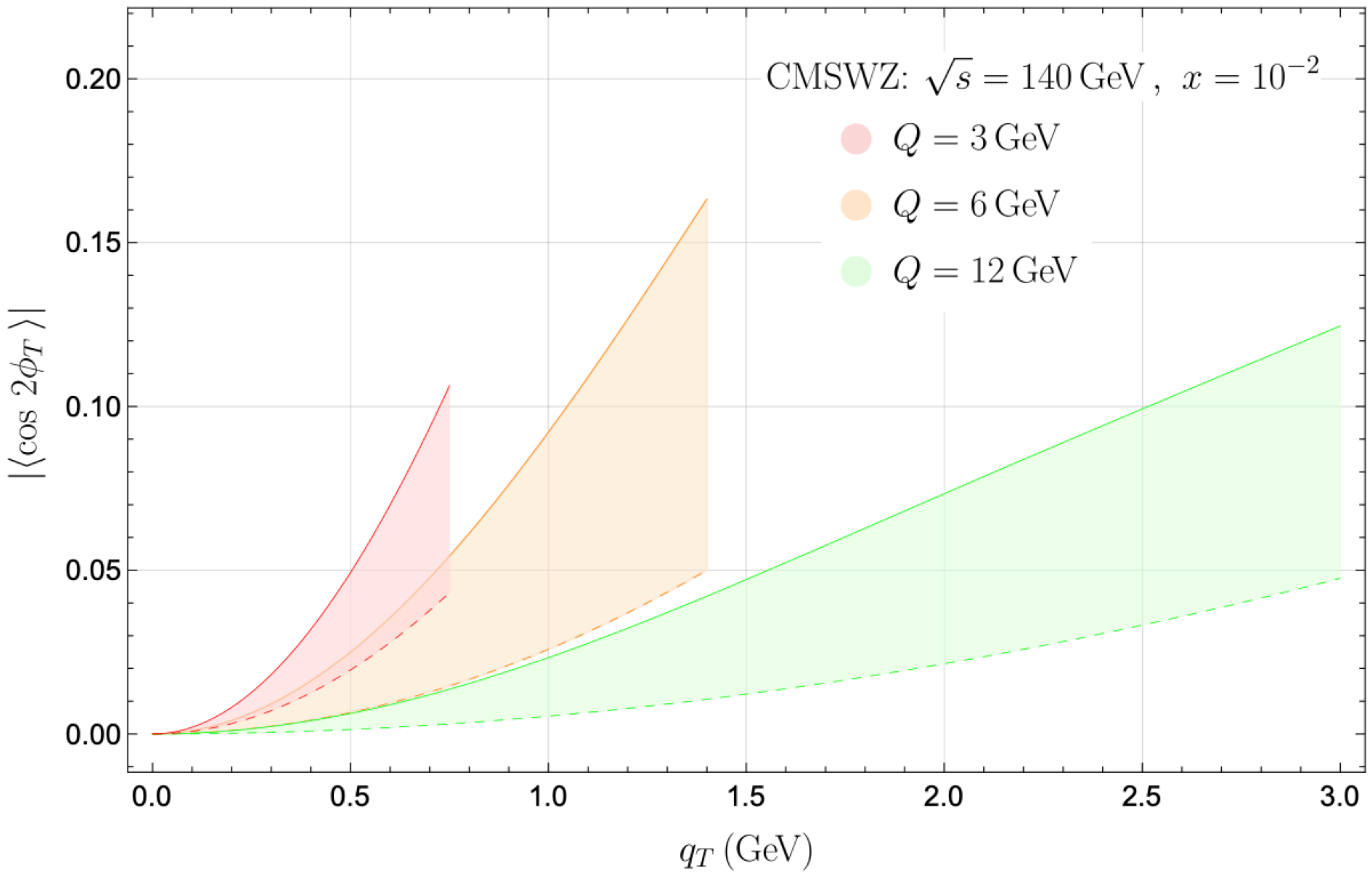
$$\mu_b = \frac{b_0}{b_T} \to \mu'_{b_*} = \frac{b_0}{\sqrt{b_*^2 + (b_0/Q)^2}} \quad \text{where} \quad b_* = \frac{b_T}{\sqrt{1 + \left(b_T/b_{\text{max}}\right)^2}}$$

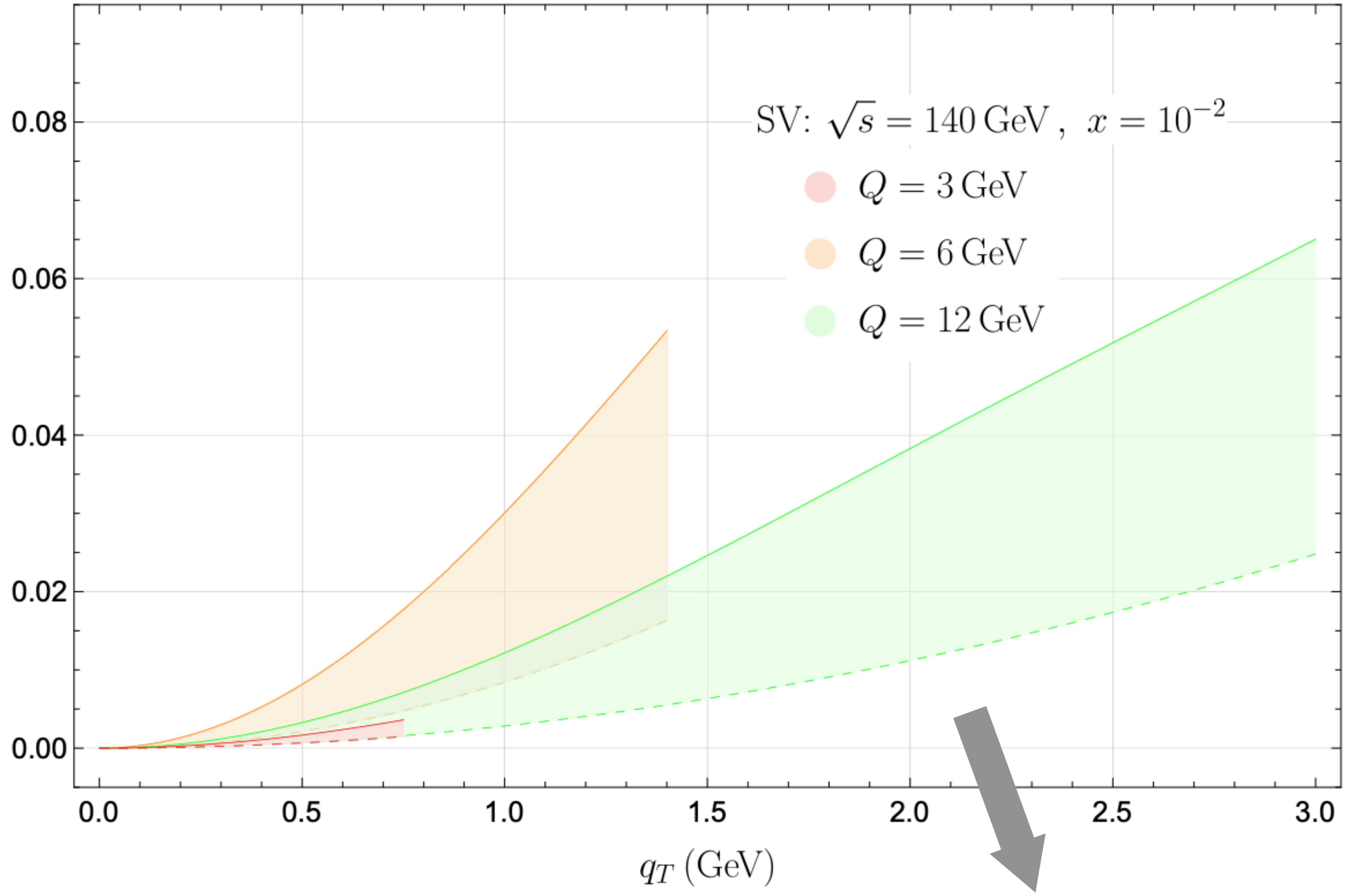
$$\mathrm{e}^{-S_A(Q^2,\mu_b)} \to \mathrm{e}^{-S_A(Q^2,\mu'_{b*})}\,\mathrm{e}^{-S_{\text{NP}}(b_T,Q^2)}$$

$$S_{\text{NP}} = \left[A\log\frac{Q}{Q_{\text{NP}}} + B(x)\right]b_T^2$$

Numerical results (with TMD evolution)

Bor, Boer, *PRD 106* (2022)





Bands driven by nonpertubative Sudakov

Magnitude and Q **dependence** vary with **LDME choice**

Asymmetries **increase monotonically** with q_T

CMSWZ: Chao, Ma, Shao, Wang, Zhang, *PRL 108* (2012)

SV: Sharma, Vitev, *PRC 87* (2013) - J/ψ

Accessing gluon TMDs at the EIC

Proposed phenomenological studies to probe gluon TMDs at the EIC
(mostly asymmetries)

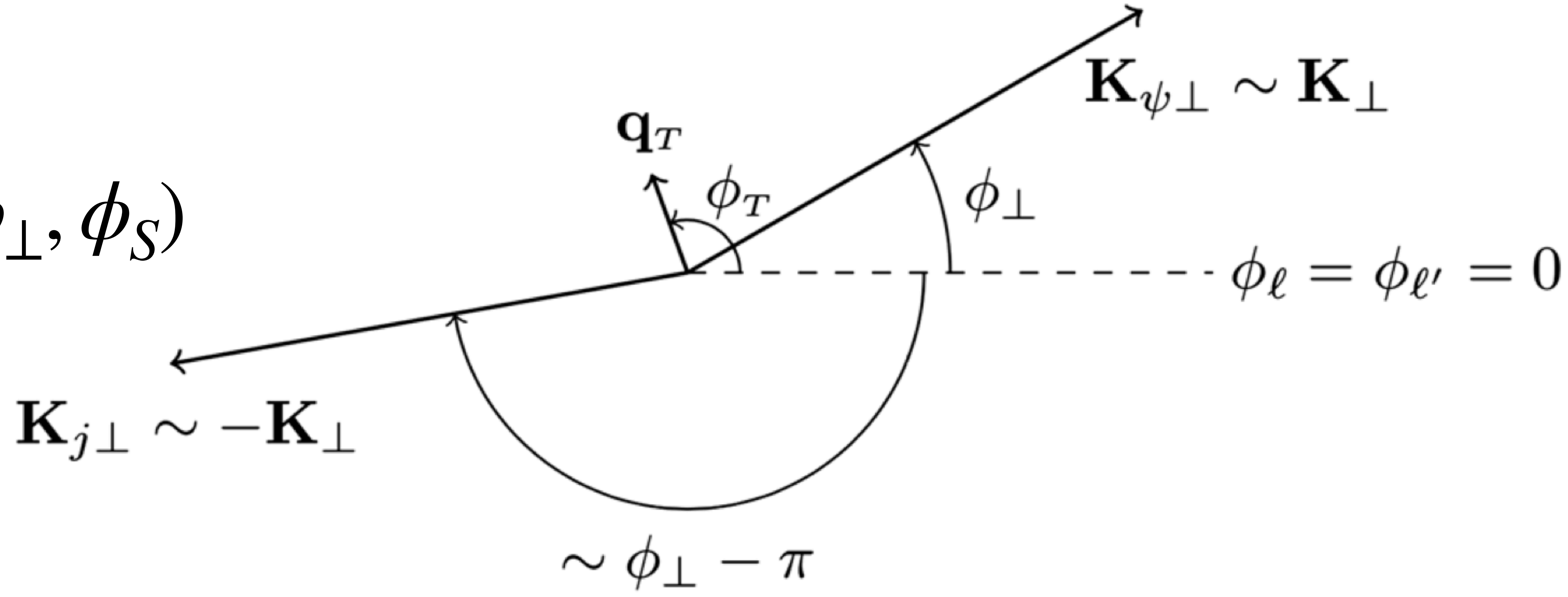
- **Azimuthal asymmetries** in J/ψ **plus jet** production

D'Alesio, Murgia, Pisano, Taels, *PRD 100* (2019)

Asymmetries in J/ψ plus jet

D'Alesio, Murgia, Pisano, Taels, *PRD 100* (2019)

$$\frac{d\sigma}{dz\, dx_B\, dy\, d^2\boldsymbol{q}_T\, d^2\boldsymbol{K}_\perp} \equiv d\sigma(\phi_T, \phi_\perp, \phi_S)$$
$$= d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_T, \phi_\perp, \phi_S)$$



$$d\sigma^U \propto \left(A_0^{eg} + A_1^{eg}\cos\phi_\perp + A_2^{eg}\cos 2\phi_\perp\right) f_1^g$$
$$+\left(B_0^{eg}\cos 2\phi_T + B_1^{eg}\cos(2\phi_T - \phi_\perp) + B_2^{eg}\cos 2(\phi_T - \phi_\perp)\right.$$
$$\left. + B_3^{eg}\cos(2\phi_T - 3\phi_\perp) + B_4^{eg}\cos(2\phi_T - 4\phi_\perp)\right) \frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp g}$$

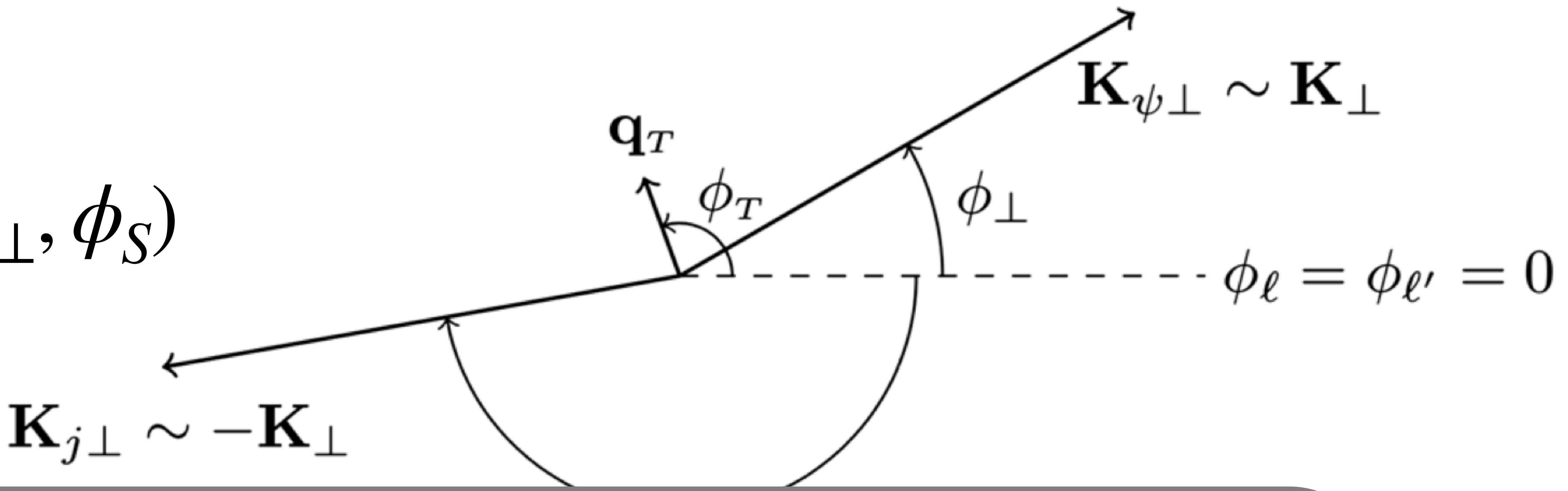
$$d\sigma^T \propto \sin(\phi_S - \phi_T)\left(A_0^{eg} + A_1^{eg}\cos\phi_\perp + A_2^{eg}\cos 2\phi_\perp\right)\frac{|\boldsymbol{q}_T|}{M_p} f_{1T}^{\perp g}$$
$$+\cos(\phi_S - \phi_T)\left(B_0^{eg}\sin 2\phi_T + B_1^{eg}\sin(2\phi_T - \phi_\perp) + B_2^{eg}\sin 2(\phi_T - \phi_\perp) + B_3^{eg}\sin(2\phi_T - 3\phi_\perp) + B_4^{eg}\sin(2\phi_T - 4\phi_\perp)\right)\frac{|\boldsymbol{q}_T|^3}{M_p^3}\, h_{1T}^{\perp g}$$
$$+\left(B_0^{eg}\sin(\phi_S + \phi_T) + B_1^{eg}\sin(\phi_S + \phi_T - \phi_\perp) + B_2^{eg}\sin(\phi_S + \phi_T - 2\phi_\perp) + B_3^{eg}\sin(\phi_S + \phi_T - 3\phi_\perp) + B_4^{eg}\sin(\phi_S + \phi_T - 4\phi_\perp)\right)\frac{|\boldsymbol{q}_T|}{M_p} h_{1T}^g$$

Asymmetries in J/ψ plus jet

D'Alesio, Murgia, Pisano, Taels, *PRD 100* (2019)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T\,\mathrm{d}^2\boldsymbol{K}_\perp} \equiv \mathrm{d}\sigma(\phi_T, \phi_\perp, \phi_S)$$

$$= \mathrm{d}\sigma^U(\phi_T, \phi_\perp) + \mathrm{d}\sigma^T(\phi_T, \phi_\perp, \phi_S)$$



$\mathrm{d}\sigma^U \propto \left(A_0^{eg} + A_1^{eg}\cos\phi_\perp + A_2^{eg}\cos 2\phi_\perp\right) f_1^g$

$\frac{|q_T|}{M_p} h_{1T}^g$

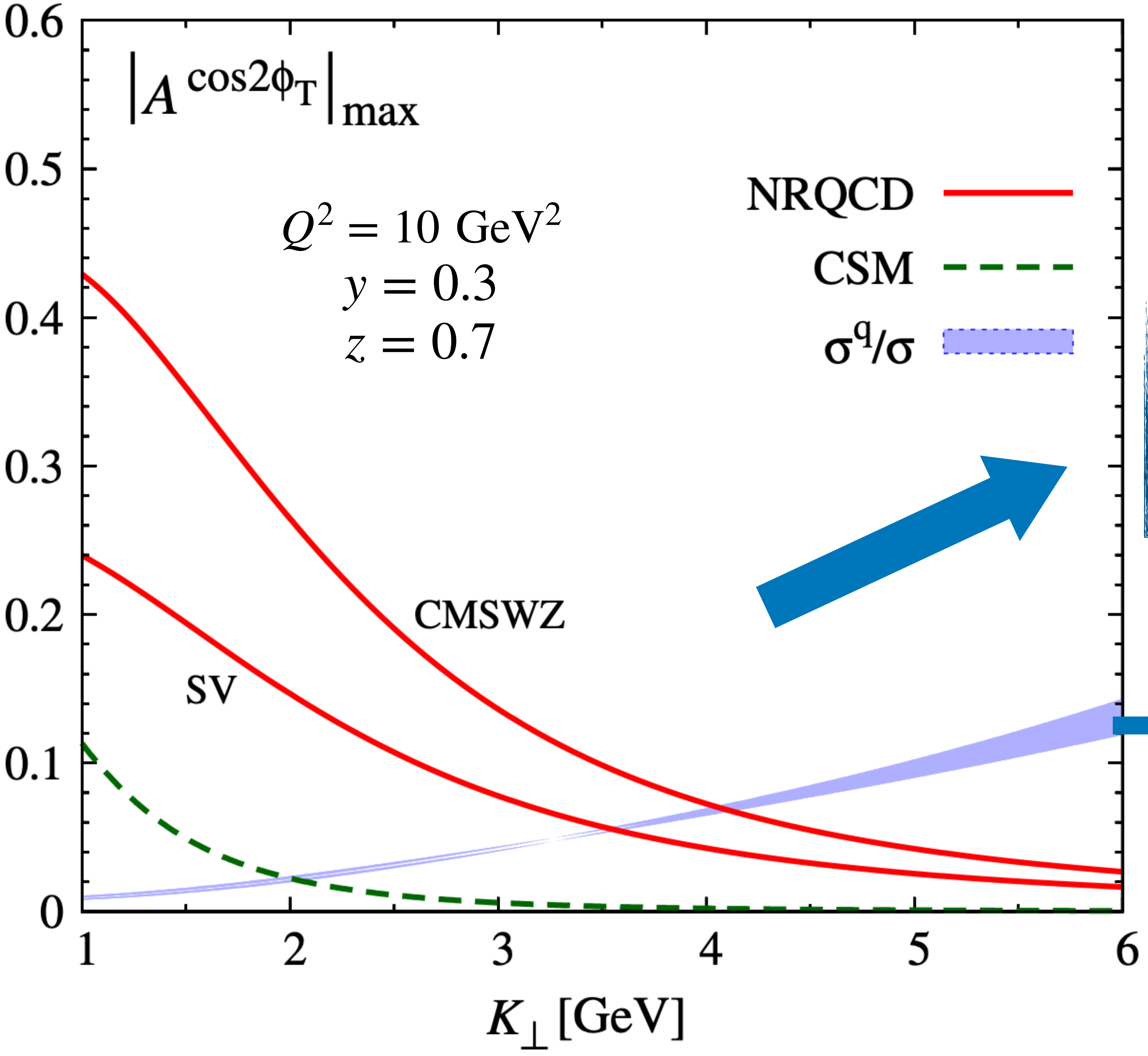
Number of Asymmetries

from ↓ / in →	Inclusive J/ψ	J/ψ plus jet
unpolarized gluons	**1**	**3**
linearly polarized gluons	**1**	**5**

Numerical results - I

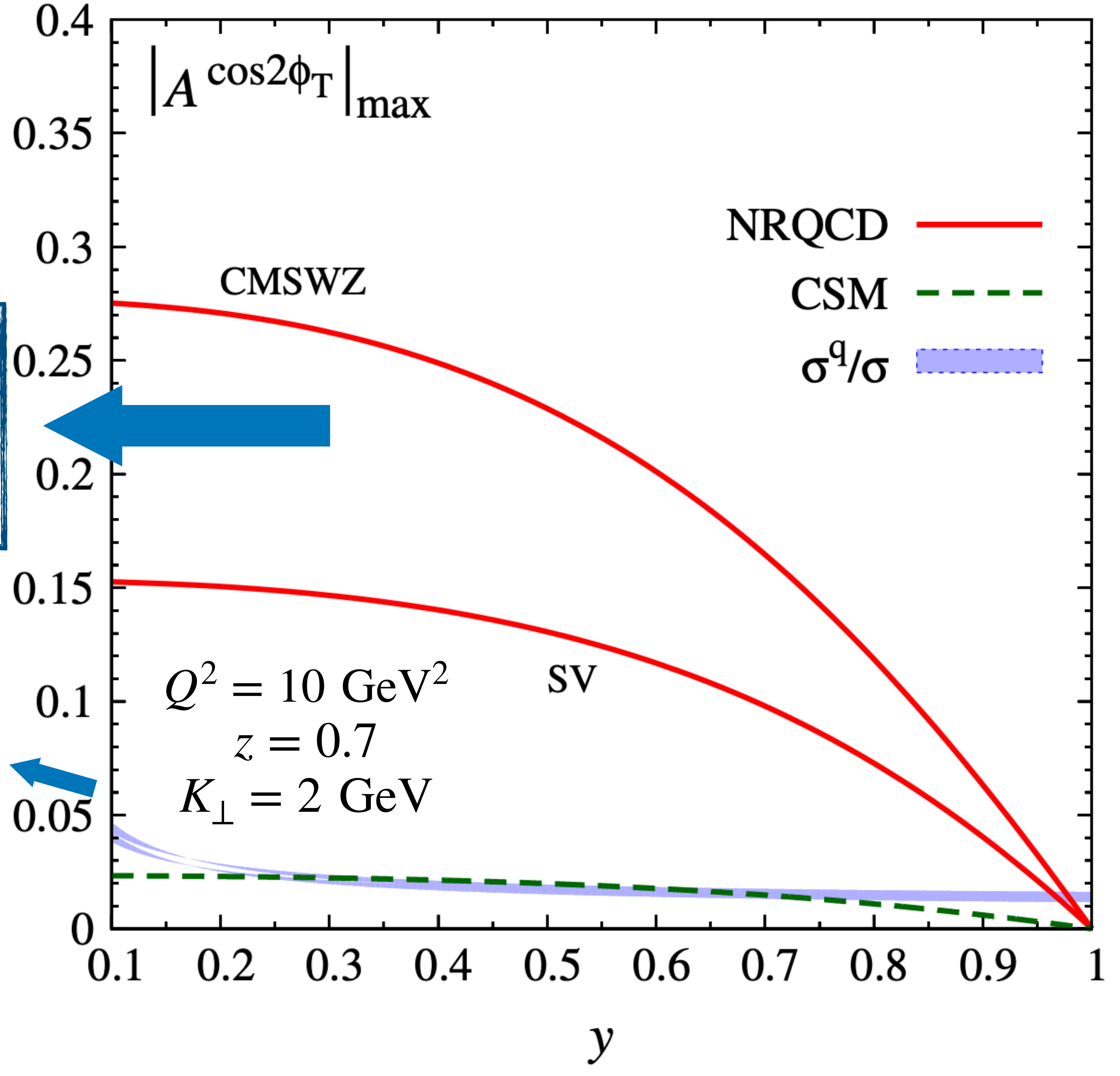
D'Alesio, Murgia, Pisano, Taels, *PRD 100* (2019)

Positivity bounds lead to upper bounds for the asymmetries



CSM underneath NRQCD predicitions

quark channel negligible



CMSWZ: Chao, Ma, Shao, Wang, Zhang, *PRL 108* (2012)

SV: Sharma, Vitev, *PRC 87* (2013) - J/ψ

Accessing gluon TMDs at the LHC

Proposed phenomenological studies to probe gluon TMDs at the LHC

- **Double J/ψ production**

  Lansberg, Pisano, Scarpa, Schlegel, *PLB 784* (2018)

  Scarpa, Boer, Echevarría, Lansberg, Pisano, *EPJ C 80* (2020)

  Bor, Colpani-Serri, Lansberg, *in preparation*

- **Azimuthal asymmetries in C-even quarkonia productions**

  Kato, LM, Pisano, *2403.20017* (2024)

Inclusive single J/ψ production in pp collisions may be accompanied with factorization-breaking effects

Nonetheless, one can use phenomenological, TMD-based approaches such as GPM and CGI-GPM

D'Alesio, LM, Murgia, Pisano, Rajesh, *PRD 102* (2020)

Accessing gluon TMDs at the LHC

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  Lansberg, Pisano, Scarpa, Schlegel, *PLB 784* (2018)

  Scarpa, Boer, Echevarría, Lansberg, Pisano, *EPJ C 80* (2020)

  Bor, Colpani-Serri, Lansberg, *in preparation*

Asymmetries in double-J/ψ production

Double J/ψ production is dominated by the CS mechanism

→ absence of color-octet final states that can lead to breaking effects

Two asymmetries generated by the angular distribution of the J/ψ-J/ψ system

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mathcal{QQ}}\,\mathrm{d}Y_{\mathcal{QQ}}\,\mathrm{d}^2\boldsymbol{P}_{\mathcal{QQ}T}\,\mathrm{d}\Omega} \propto 1 + \langle\cos 2\phi_{\mathrm{CS}}\rangle + \langle\cos 4\phi_{\mathrm{CS}}\rangle$$

Distribution measured in the **Collins-Soper** frame

Associated to the double-J/ψ system

Asymmetries in double-J/ψ production

Double J/ψ production is dominated by the CS mechanism

→ absence of color-octet final states that can lead to breaking effects

Two asymmetries generated by the angular distribution of the J/ψ-J/ψ system

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mathcal{Q}\mathcal{Q}}\,\mathrm{d}Y_{\mathcal{Q}\mathcal{Q}}\,\mathrm{d}^2\boldsymbol{P}_{\mathcal{Q}\mathcal{Q}T}\,\mathrm{d}\Omega} \propto 1 + \langle\cos 2\phi_{\mathrm{CS}}\rangle + \langle\cos 4\phi_{\mathrm{CS}}\rangle$$

$$\langle\cos 2\phi_{\mathrm{CS}}\rangle = \frac{1}{2}\frac{F_3\,\mathcal{C}[w_3\,f_1^g\,h_1^{\perp g}] + F_3'\,\mathcal{C}[w_3'\,h_1^{\perp g}\,f_1^g]}{F_1\,\mathcal{C}[f_1^g\,f_1^g] + F_2\,\mathcal{C}[w_2\,h_1^{\perp g}\,h_1^{\perp g}]}$$

→ $F_i(\theta_{\mathrm{CS}}, M_{\mathcal{Q}\mathcal{Q}})$

hard-scattering coefficients

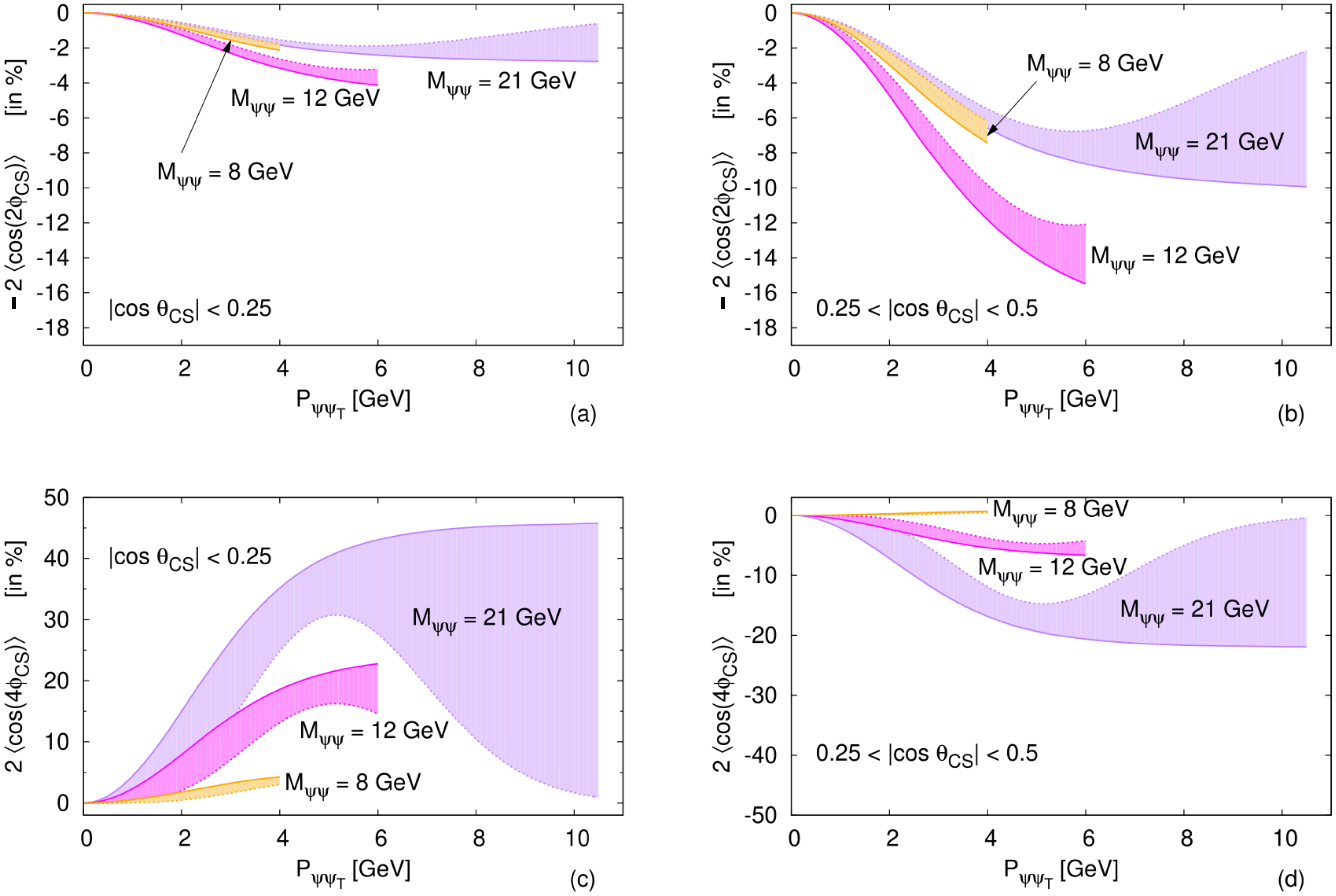
$$\langle\cos 2\phi_{\mathrm{CS}}\rangle = \frac{1}{2}\frac{F_4\,\mathcal{C}[w_4\,h_1^{\perp g}\,h_1^{\perp g}]}{F_1\,\mathcal{C}[f_1^g\,f_1^g] + F_2\,\mathcal{C}[w_2\,h_1^{\perp g}\,h_1^{\perp g}]}$$

Double J/ψ anno 2018

Lansberg, Pisano, Scarpa, Schlegel, *PLB 784* (2018)

Band are constructed combining **“positivity-bound”** and **“gaussian-like”** predictions

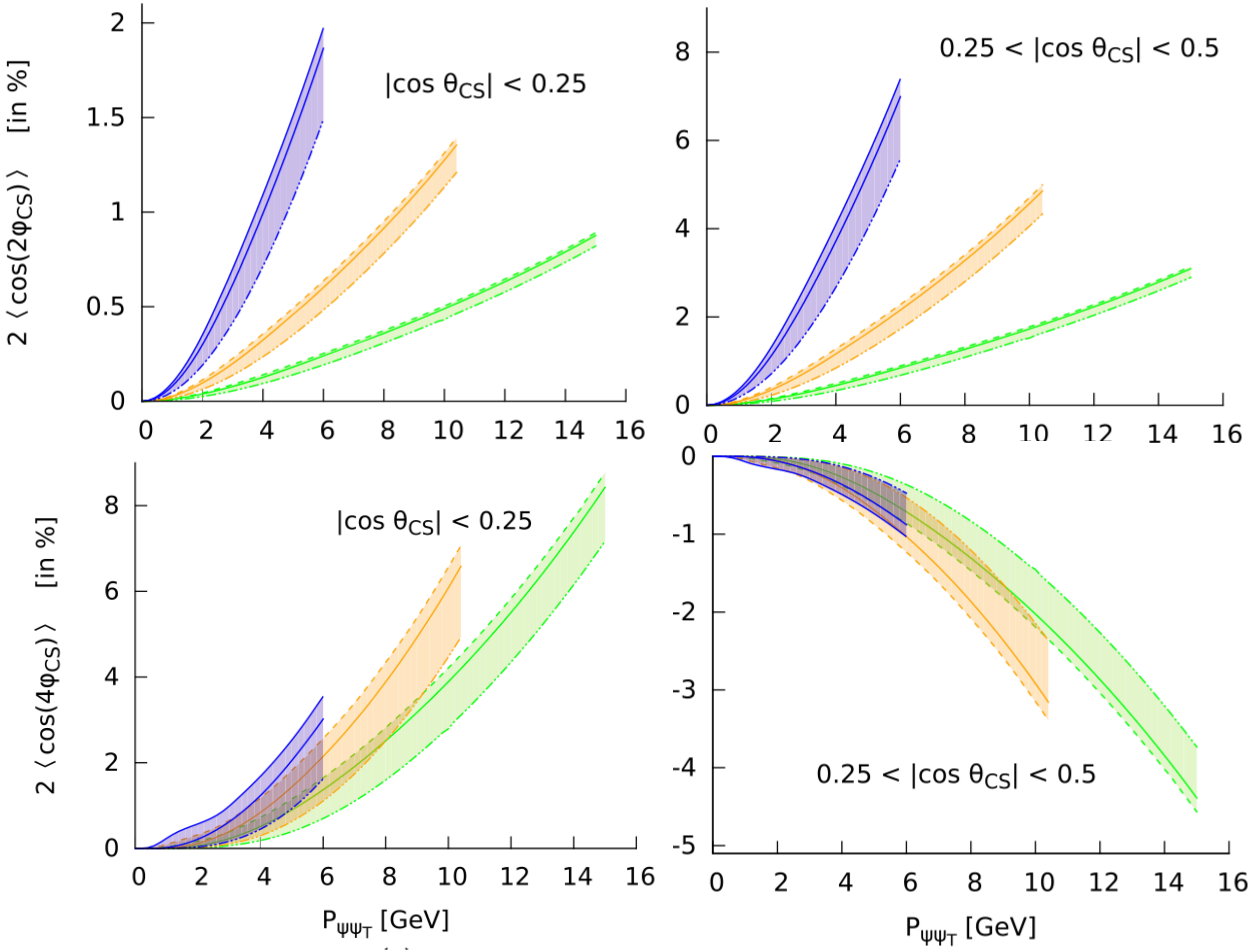
Discrepancy on the asymmetries dependence of $M_{\psi\psi}$



Double J/ψ anno 2020

Scarpa, Boer, Echevarría, Lansberg, Pisano, *EPJ C 80* (2020)

b_T-range of nonpert. physics



(in absolute)

Asymmetries **increase monotonically** with q_T

$\langle\cos 2\phi_{CS}\rangle$ **peak** is due to the ratio F_3/F_1 at $M_{\psi\psi} = 12$ GeV

Double J/ψ anno 2024

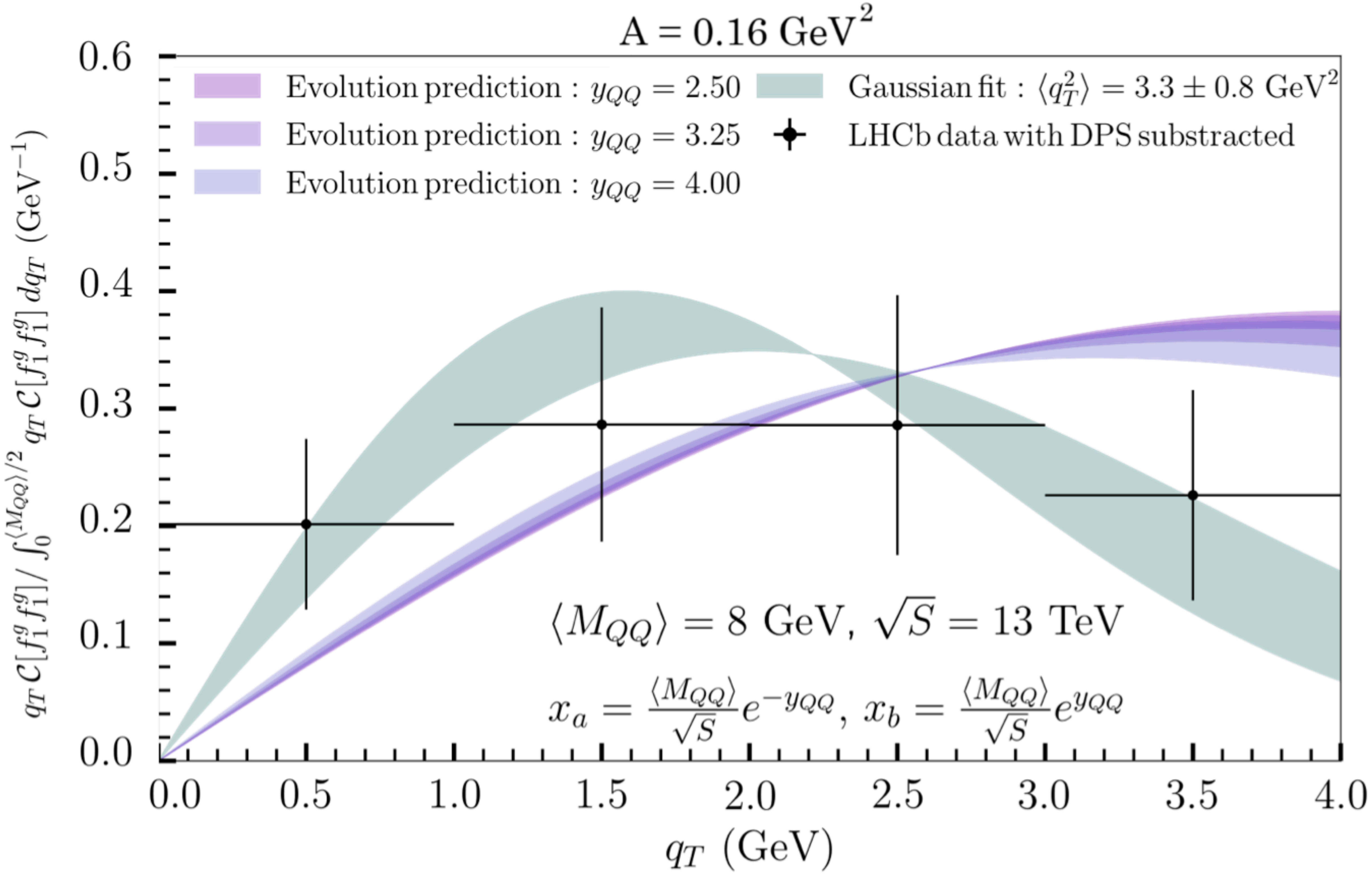
Bor, Colpani-Serri, Lansberg, *in preparation*

Cross-section data are not well reproduced by predictions employing TMD evolution

→ Hints to modify the non-perturbative component of the Sudakov

(Work in progress)

Normalized $P_{\mathcal{Q}\mathcal{Q}} = q_T$ spectrum



Accessing gluon TMDs at the LHC

- **Azimuthal asymmetries** in **C-even** quarkonia productions

Kato, LM, Pisano, *2403.20017* (2024)

Other Quarkonia states at the LHC

Inclusive $C = +$ quarkonia are also (effectively) described by the CSM

$\eta_Q \; ({}^1S_0)$ $\chi_{Q0} \; ({}^3P_0)$ $\chi_{Q2} \; ({}^3P_2)$

At LHC we can have unpolarized and transversely polarized protons

$$\frac{\mathrm{d}\sigma[\mathcal{Q}]}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T} = F_{UU}^{\mathcal{Q}} + F_{UT}^{\mathcal{Q}} |\boldsymbol{S}_{BT}| \sin\phi_{S_B} + F_{UL}^{\mathcal{Q}} S_{BL}$$

Excluded by parity conservation

We are considering solely the polarization of the target proton

Convolutions for C-even quarkonia

Kato, LM, Pisano, *2403.20017* (2024)

Combining different $C = +$ states, we can single out different convolutions

$$F_{UU}^{\chi_{Q2}} \propto \mathscr{C}\left[f_1^g f_1^g\right]$$

$$F_{UU}^{\chi_{Q0}} \propto \mathscr{C}\left[f_1^g f_1^g\right] + \mathscr{C}\left[w_{UU} h_1^{\perp\, g} h_1^{\perp\, g}\right]$$

$$F_{UU}^{\eta_0} \propto \mathscr{C}\left[f_1^g f_1^g\right] - \mathscr{C}\left[w_{UU} h_1^{\perp\, g} h_1^{\perp\, g}\right]$$

$$F_{UU}^{\chi_{Q2}} \propto \mathscr{C}\left[f_1^g f_{1T}^{\perp\, g}\right]$$

$$F_{UU}^{\chi_{Q0}} \propto \mathscr{C}\left[f_1^g f_{1T}^{\perp\, g}\right] + \mathscr{C}\left[w_{UT}^{h} h_1^{\perp\, g} h_1^{g}\right] - \mathscr{C}\left[w_{UT}^{h^\perp} h_1^{\perp\, g} h_{1T}^{\perp\, g}\right]$$

$$F_{UU}^{\eta_0} \propto \mathscr{C}\left[f_1^g f_{1T}^{\perp\, g}\right] - \mathscr{C}\left[w_{UT}^{h} h_1^{\perp\, g} h_1^{g}\right] + \mathscr{C}\left[w_{UT}^{h^\perp} h_1^{\perp\, g} h_{1T}^{\perp\, g}\right]$$

Numerical results within a Gaussian model - I

Boer, Pisano, *PRD 86* (2012)

Gluon TMDs parameterization:

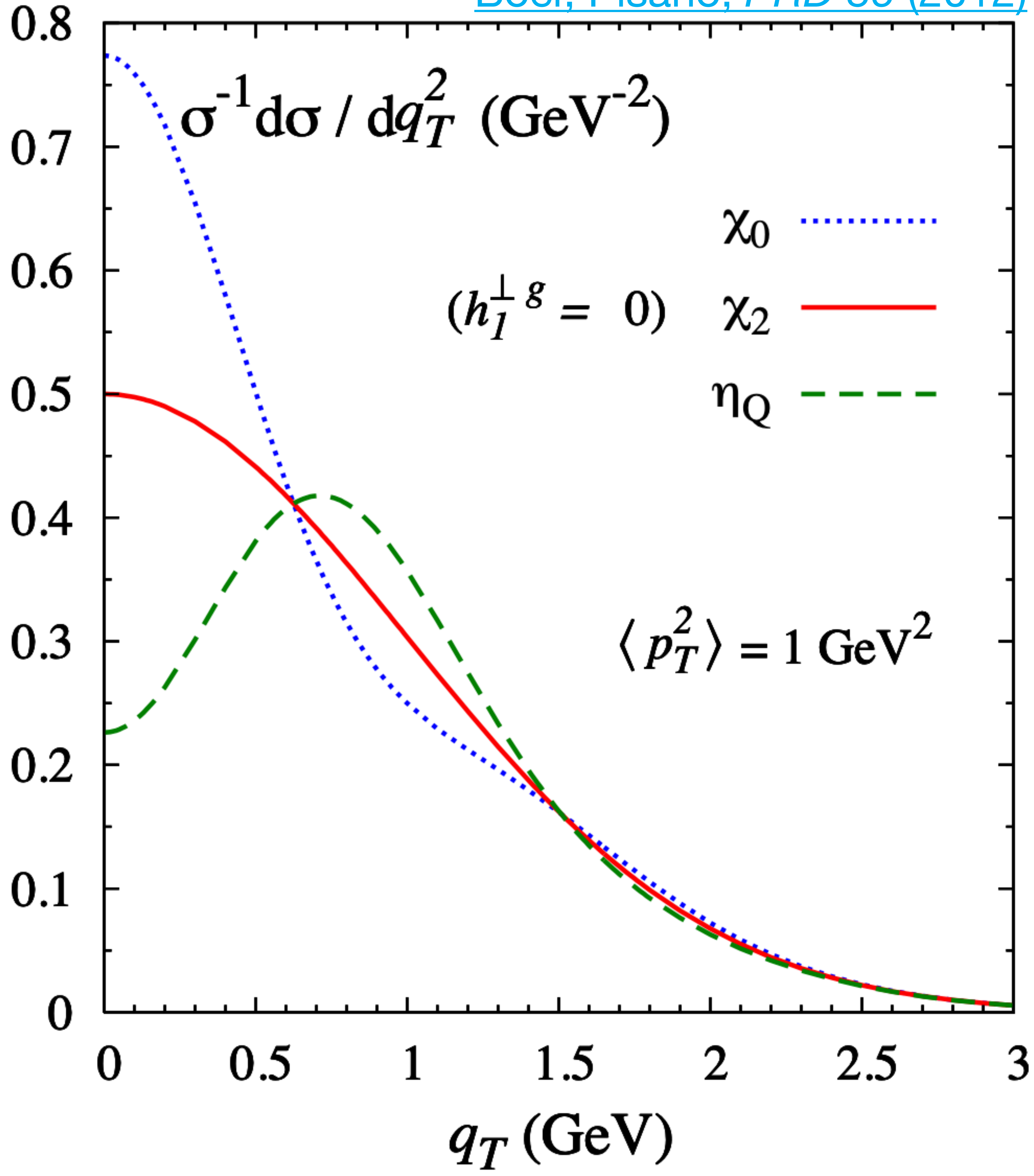
$$f_1^g(x, \boldsymbol{p}_T^2) = G\big(\boldsymbol{p}_T^2, \langle p_T^2 \rangle\big)\, f_1^g(x)$$

$$F_1^g(x, \boldsymbol{p}_T^2) \propto G\big(\boldsymbol{p}_T^2, \rho\langle p_T^2 \rangle\big)\; N(x) f_1^g(x)$$

$$N = +1 \qquad \rho = 1/3$$

$\langle p_T^2 \rangle$ determines magnitude and broadening of the distribution

The double-node feature of the convolution is observed in the comparison



Numerical results within a Gaussian model - II

Kato, LM, Pisano, *2403.20017* (2024)

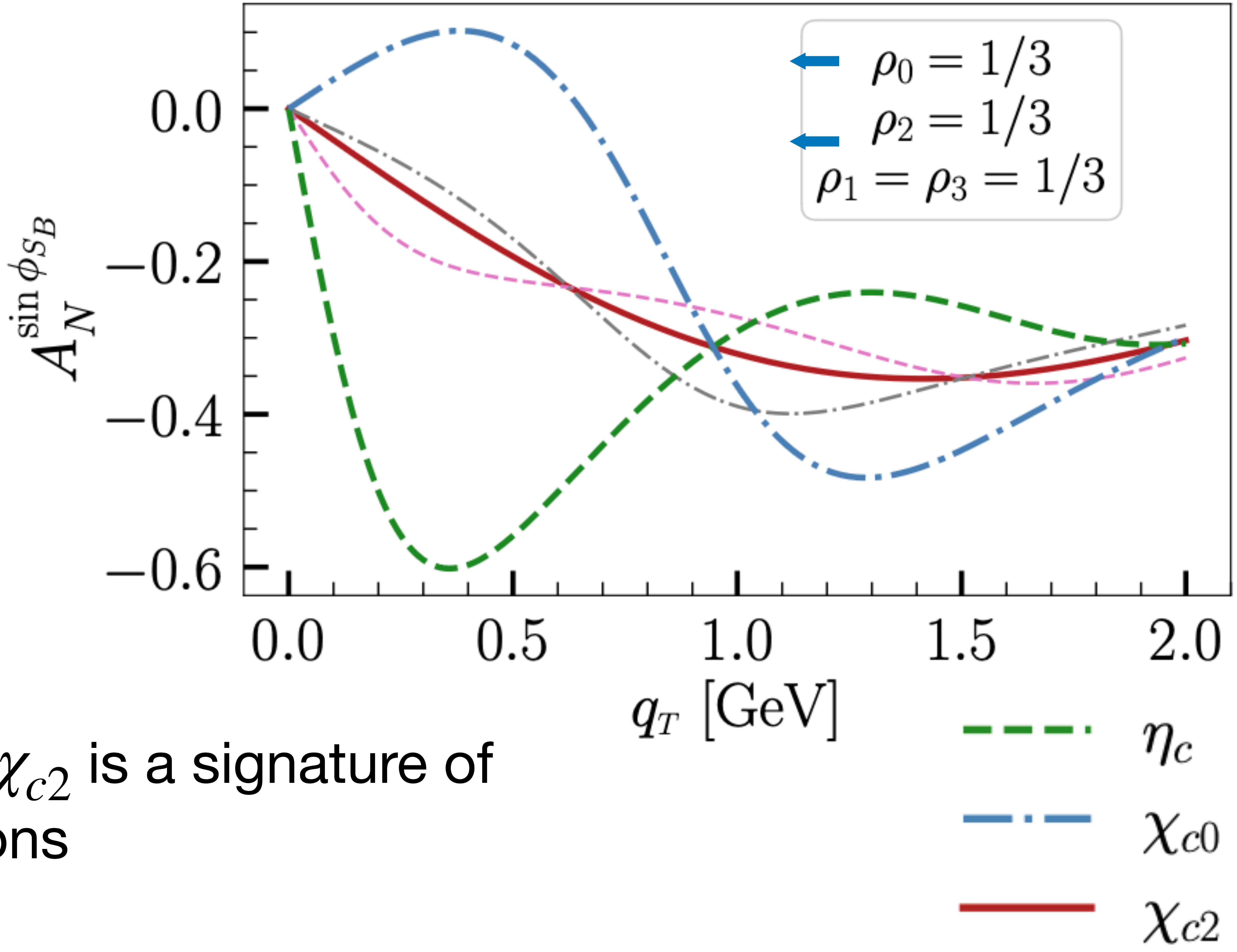
Gluon TMDs parameterization:

$$f_1^g(x,\boldsymbol{p}_T^2) = G\big(\boldsymbol{p}_T^2, \langle p_T^2\rangle\big)\, f_1^g(x)$$

$$F_1^g(x,\boldsymbol{p}_T^2) \propto G\big(\boldsymbol{p}_T^2, \rho\langle p_T^2\rangle\big)\; N(x) f_1^g(x)$$

$$N = +1 \qquad \langle p_T^2\rangle = 1\ \mathrm{GeV}^2$$

$\langle p_T^2\rangle$ determines broadening of the distribution



The presence of oscillations around χ_{c2} is a signature of the Sivers and linearly polarized gluons

The TMD shape function

(properly)

Previous studies presented do not include transverse momentum effect from quarkonium formation

$$\Delta_{\mathcal{Q}}^{[n]}(z, \boldsymbol{k}_T^2) = \sum_{n'} C_{nn'}(z, \boldsymbol{k}_T^2) \otimes \langle \mathcal{O}_{\mathcal{Q}}[n'] \rangle$$

(prod. in pp collisions)

Echevarría, *JHEP 144* (2019)

Fleming, Makris, Mehen, *JHEP 112* (2020)

(decays to light quarks)

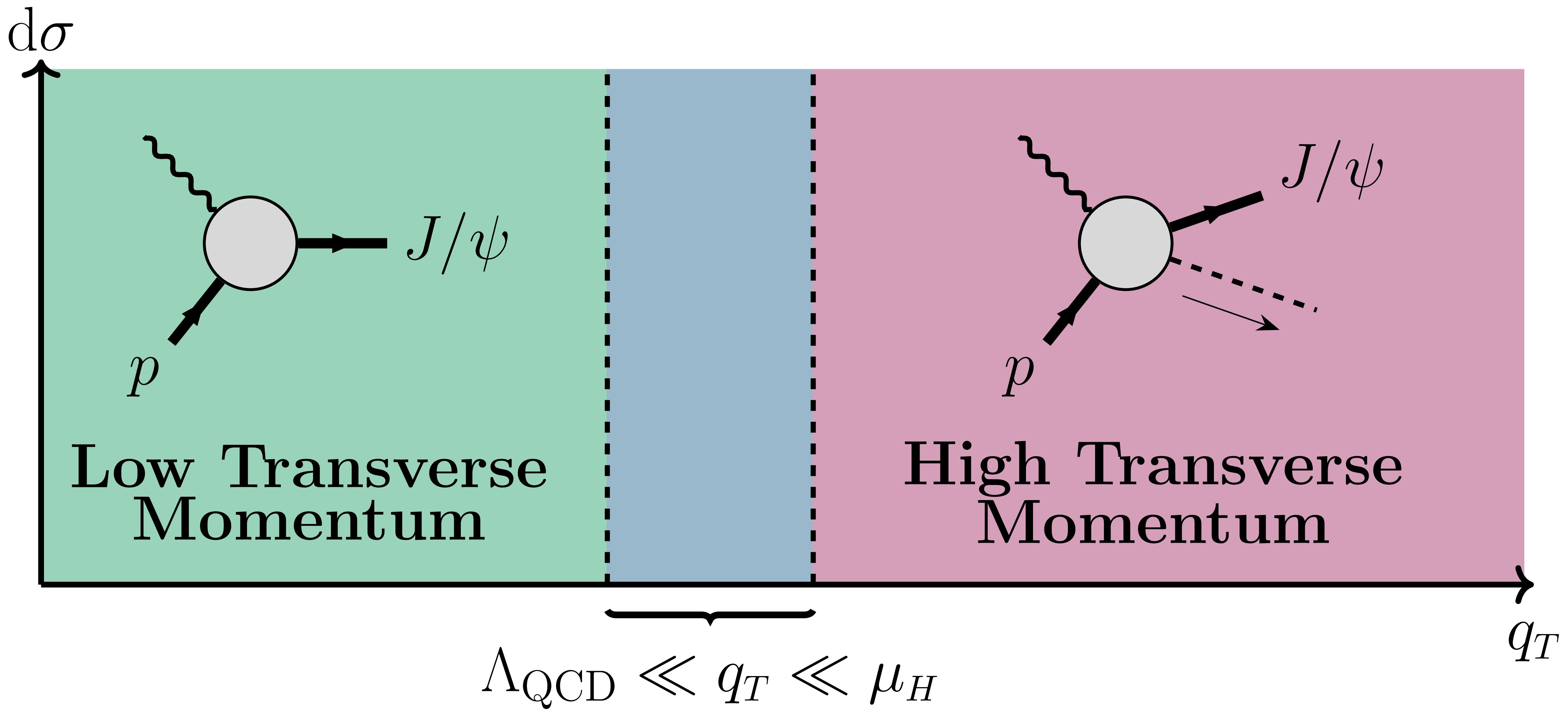
The TMD shape function

Previous studies presented do not (properly) include transverse momentum effect from quarkonium formation

$$\Delta^{[n]}_{\mathcal{Q}}(z, \boldsymbol{k}_T^2) = \sum_{n'} C_{nn'}(z, \boldsymbol{k}_T^2) \otimes \langle \mathcal{O}_{\mathcal{Q}}[n'] \rangle$$

(prod. in pp collisions)
Echevarría, *JHEP 144* (2019)
Fleming, Makris, Mehen, *JHEP 112* (2020)
(decays to light quarks)

Perturbative tail in **SIDIS** achieved by matching TMD and collinear frameworks



Boer, Bor, LM, Pisano, Yuan, *JHEP 08* (2023)

$$\int \mathrm{d}\phi_T \, \mathrm{d}\sigma^U(\phi_T)\,|_{\mathrm{TMD}} \neq \int \mathrm{d}\phi_T \, \mathrm{d}\sigma^U(\phi_T)\,|_{\mathrm{coll.}}$$

Mismatch solved by employing

$$\Delta^{[n]}_{\psi}(z, \boldsymbol{k}_T^2; \mu^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log \frac{M_\psi^2 \, \mu^2}{(M_\psi^2 + Q^2)^2}\right) \langle \mathcal{O}_\psi[n] \rangle \, \delta(1-z)$$

The TMD shape function process dependence

The perturbative tail presents a process-induced dependence on Q

Boer, Bor, LM, Pisano, Yuan, *JHEP 08* (2023)

$$\Delta_{\psi}^{[n]}(z, \boldsymbol{k}_T^2; \mu^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log \frac{M_\psi^2\, \mu^2}{(M_\psi^2 + Q^2)^2}\right) \langle \mathcal{O}_\psi[n] \rangle\, \delta(1-z)$$

Consequence:

The **TMDShF depends on a process-induced quantity** (photon virtuality Q) **unrelated to** neither a specific **hard scale** or **rapidity regulator choice**, as it usually happens for other TMDs!

This suggests to split up this quantity

$$\Delta_{ep}^{[n]} = \Delta_{\psi}^{[n]} \times S_{ep}$$

Universal ($\Delta_{\psi}^{[n]}$)

Process dependent (S_{ep})

The TMD shape function process dependence

The perturbative tail presents a process-induced dependence on Q

Boer, Bor, LM, Pisano, Yuan, *JHEP 08* (2023)

$$\Delta_\psi^{[n]}(z, \boldsymbol{k}_T^2; \mu^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log \frac{M_\psi^2 \mu^2}{(M_\psi^2 + Q^2)^2}\right) \langle \mathcal{O}_\psi[n] \rangle \, \delta(1-z)$$

$$\boldsymbol{\Delta_{ep}^{[n]} = \Delta_\psi^{[n]} \times S_{ep}}$$

Universal ($\Delta_\psi^{[n]}$)

Process dependent (S_{ep})

Phenomenological test of the separation:

pp **collisions (1 hard scale)**	$S_{pp}(M_\psi) \approx 0$ →	Easier extraction of $\Delta_\psi^{[n]}(M_\psi)$ →	$\Delta_\psi^{[n]}(M_\psi)$ can be evolved to other scales
SIDIS (2 hard scales)	$\Delta_{ep}^{[n]}(M_\psi + Q^2) = (\Delta_\psi^{[n]} \times S_{ep})(M_\psi + Q^2)$ →	Tested at other scales, e.g. in Υ production	

Summary of the talk

- **Gluon TMDs** are still vastly unkown objects

- **Quarkonia** (and particular J/ψ) allow to access gluon TMDs at **low Q**

- Several observables have been proposed to discriminate and understand the importance of the **color-octet mechanism** in J/ψ **formation**

- Proposal of observables at both **EIC** and **LHC** to probe **gluon TMDs**

- To properly describe **quarkonium** observable at **low q_T** we need to adopt the **correct factorization**

→ **TMD shape function**

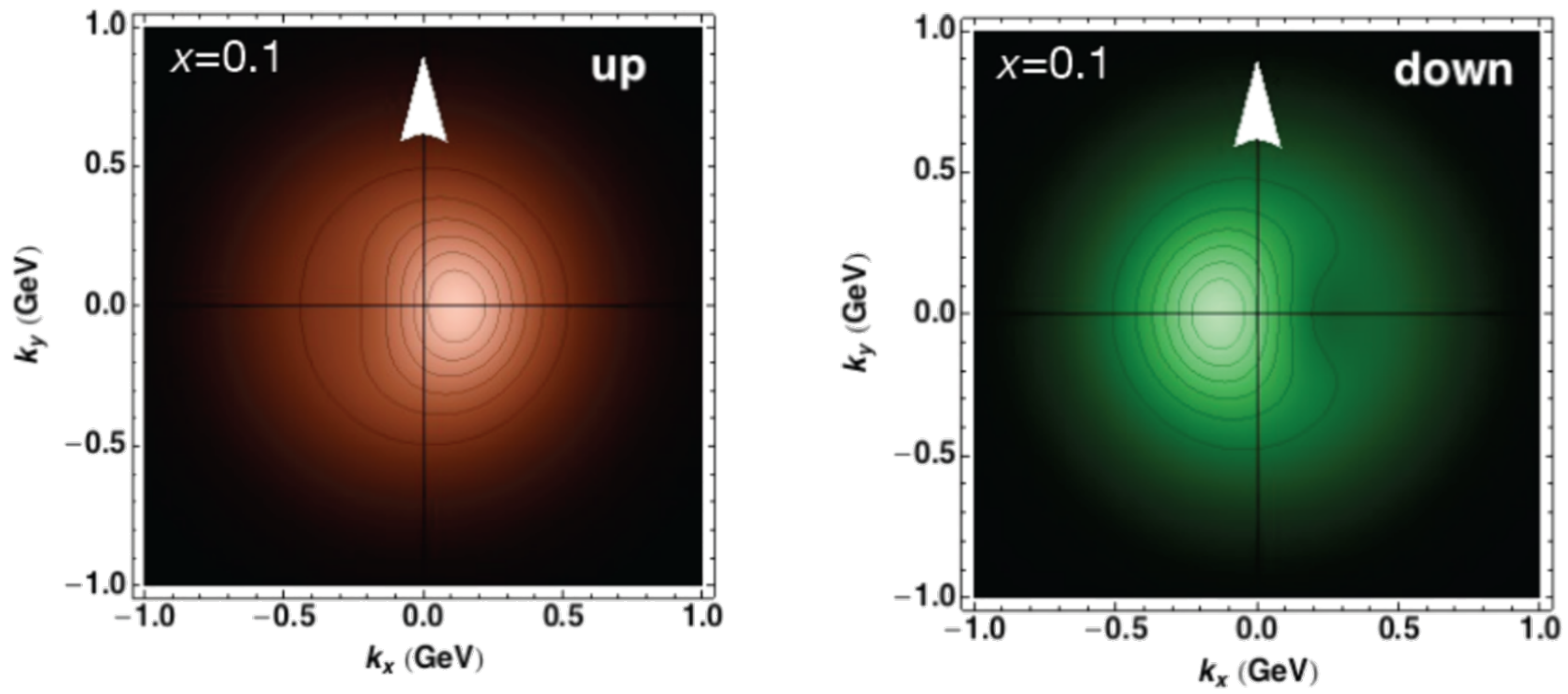
See also: Echevarria, Romera, Taels, *JHEP 09* (2024)

Accessing gluon TMDs through Quarkonium observables

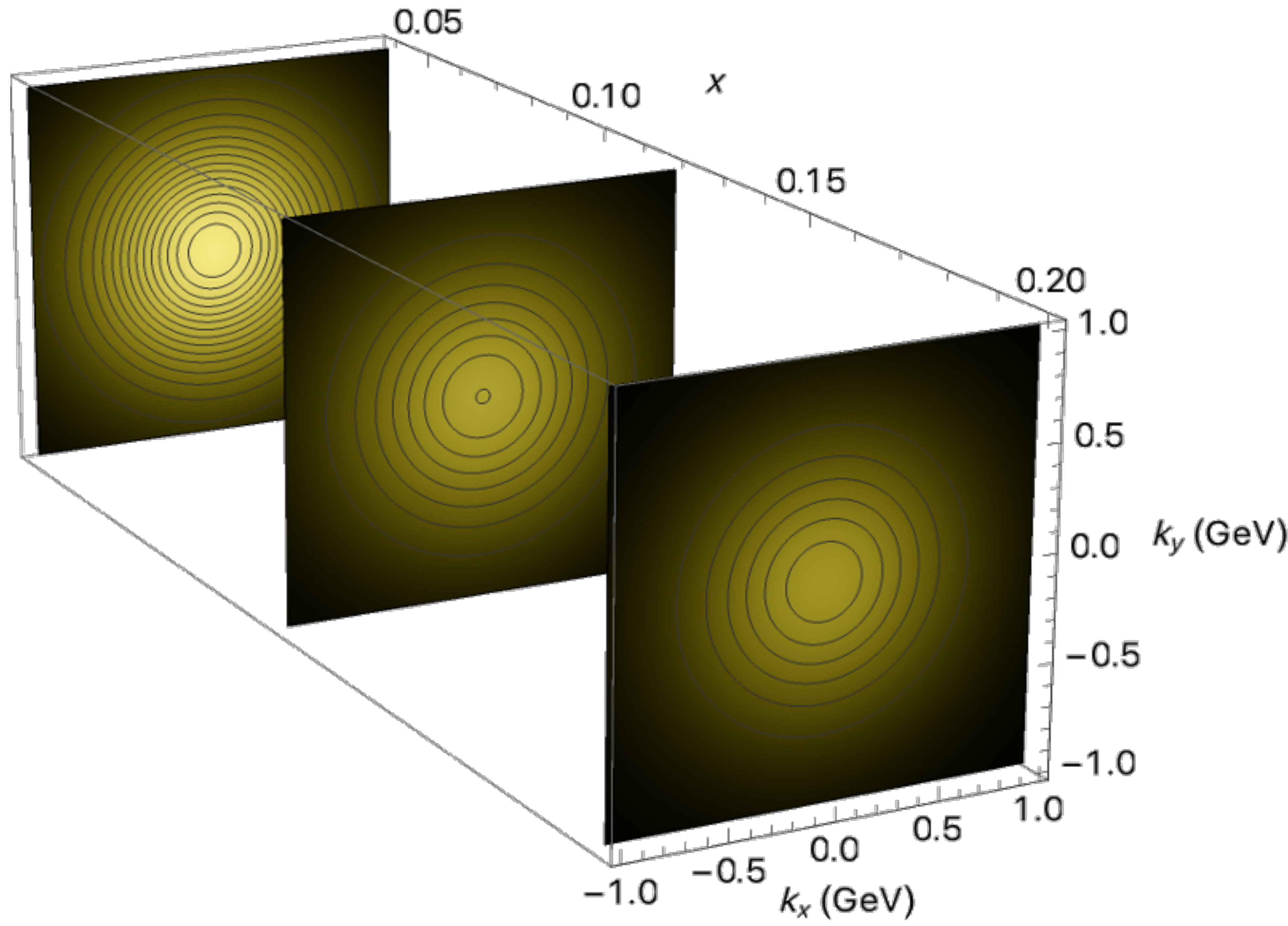
Back-up slides

TMD progress in the quark sector

Quark TMDs are entering the **precision era**



Sivers effect



Unpolarized

Bacchetta et al., *JHEP 81 (2017)*

MAP collab, *JHEP 127 (2022)*

Global fits of unpolarized TMDs combining Drell-Yan (DY) and semi-inclusive deep-inelastic scattering (SIDIS) data

J/ψ polarization within NRQCD

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD 107* (2023)

$$\frac{d\sigma}{dx_B\, dy\, d^4P_\psi\, d\Omega} \propto 1 + \lambda \cos^2\theta + \mu \cos 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi$$

Angular parameters are connected to **helicity amplitudes** $\mathscr{W}_{\Lambda\Lambda'}$

with $\Lambda = -1, 0, +1$

Parameterization is in accordance to **model-independent** arguments!

Hermeticity **Parity** **Gauge Invariance**

D'Alesio, LM, Murgia, Pisano, Sangem, *JHEP 03* (2022)

Within **NRQCD** helicity amplitudes involve interferences among waves!

up to v^4 order

$$\mathscr{W}_{\Lambda\Lambda'} = \mathscr{W}_{\Lambda\Lambda'}[{}^3S_1^{(1)}] + \mathscr{W}_{\Lambda\Lambda'}[{}^1S_0^{(8)}] + \mathscr{W}_{\Lambda\Lambda'}[{}^3S_1^{(8)}] + \mathscr{W}_{\Lambda\Lambda'}[\{S=1, L=1\}^{(8)}]$$

Benele, Krämer, Vänttinen, *PRD 57* (1998)

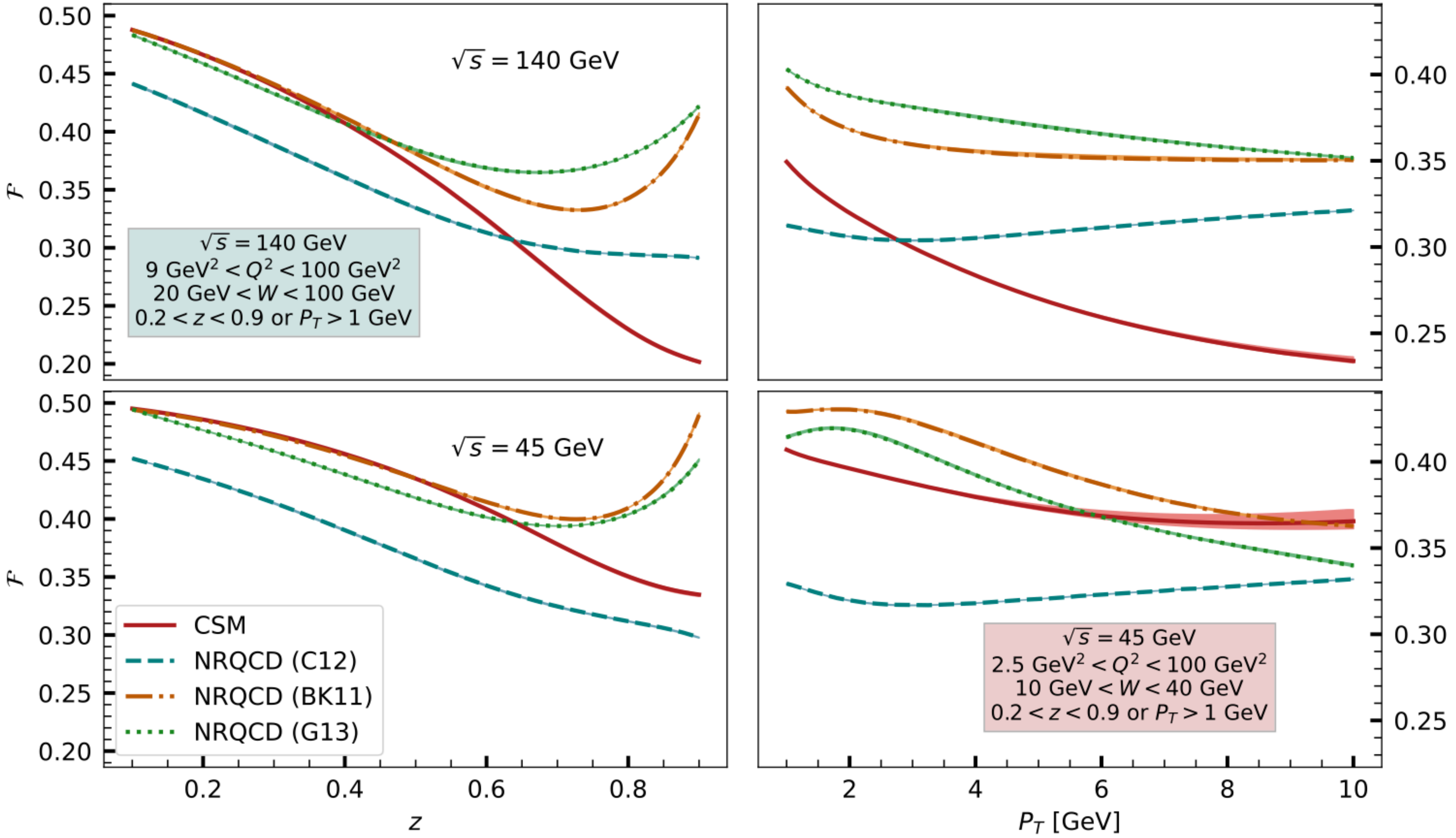
J/ψ invariants at high-P_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD 107* (2023)

Combinations of λ, μ, and ν can provide frame-invariant quantities

e.g. $\mathscr{F} = \dfrac{1+\lambda+\nu}{3+\lambda}$

Validity of a Lam-Tung like relation?



J/ψ polarization at low P_T

D'Alesio, LM, Murgia, Pisano, Sangem, *JHEP* 03 (2022)

At low transverse momentum all frames coincide

$$\lambda = \frac{\mathscr{W}_T - \mathscr{W}_L}{\mathscr{W}_T + \mathscr{W}_L} \rightarrow \begin{cases} \mathscr{C}\left[f_1^g \Delta_T^{[n]}\right] \\ \mathscr{C}\left[f_1^g \Delta_L^{[n]}\right] \end{cases}$$

$$\mu = \frac{\mathscr{W}_\Delta}{\mathscr{W}_T + \mathscr{W}_L}$$ (crossed out)

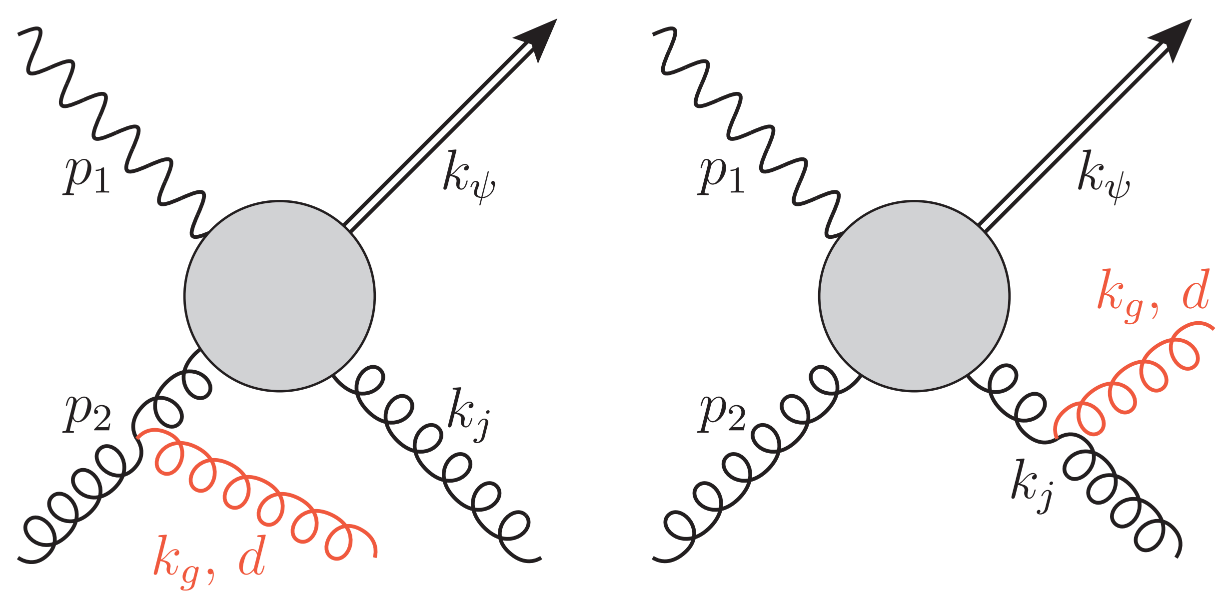
$$\nu = \frac{2\mathscr{W}_{\Delta\Delta}}{\mathscr{W}_T + \mathscr{W}_L} \rightarrow \mathscr{C}\left[w\, h_1^{\perp g} \Delta_{\Delta\Delta}^{[n]}\right] \rightarrow \Delta_{\Delta\Delta}^{[n]} = ??$$

Matching procedure works in the same way!

$(\Lambda_\psi = T,\, L)$

$$\Delta_{\Lambda_\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathscr{O}_\psi[n] \rangle\, \delta(1-z)$$

Soft gluon radiation: CS



: from the interference $|\overline{\mathscr{A}_1^{(1)}}|^2 = g_s^2 C_A S_g(p_2, k_j) |\overline{\mathscr{A}_0^{(1)}}|^2$

The extra dof has to be integrated out, leading to

$$\int \frac{\mathrm{d}^3 k_g}{(2\pi)^3 2E_{k_g}} |\overline{\mathscr{A}_1^{(1)}}|^2 \delta^{(2)}(q_\perp + k_{g\perp}) = \frac{\alpha_s C_A}{2\pi^2 q_\perp^2} |\overline{\mathscr{A}_0^{(1)}}|^2 \left[\ln\frac{\hat{s}}{q_\perp^2} + \ln\frac{\hat{t}}{\hat{u}} + I_j(R,\phi) \right]$$

for covenience, integration over rapidity difference: $\Delta y_{gj} = y_g - y_j$

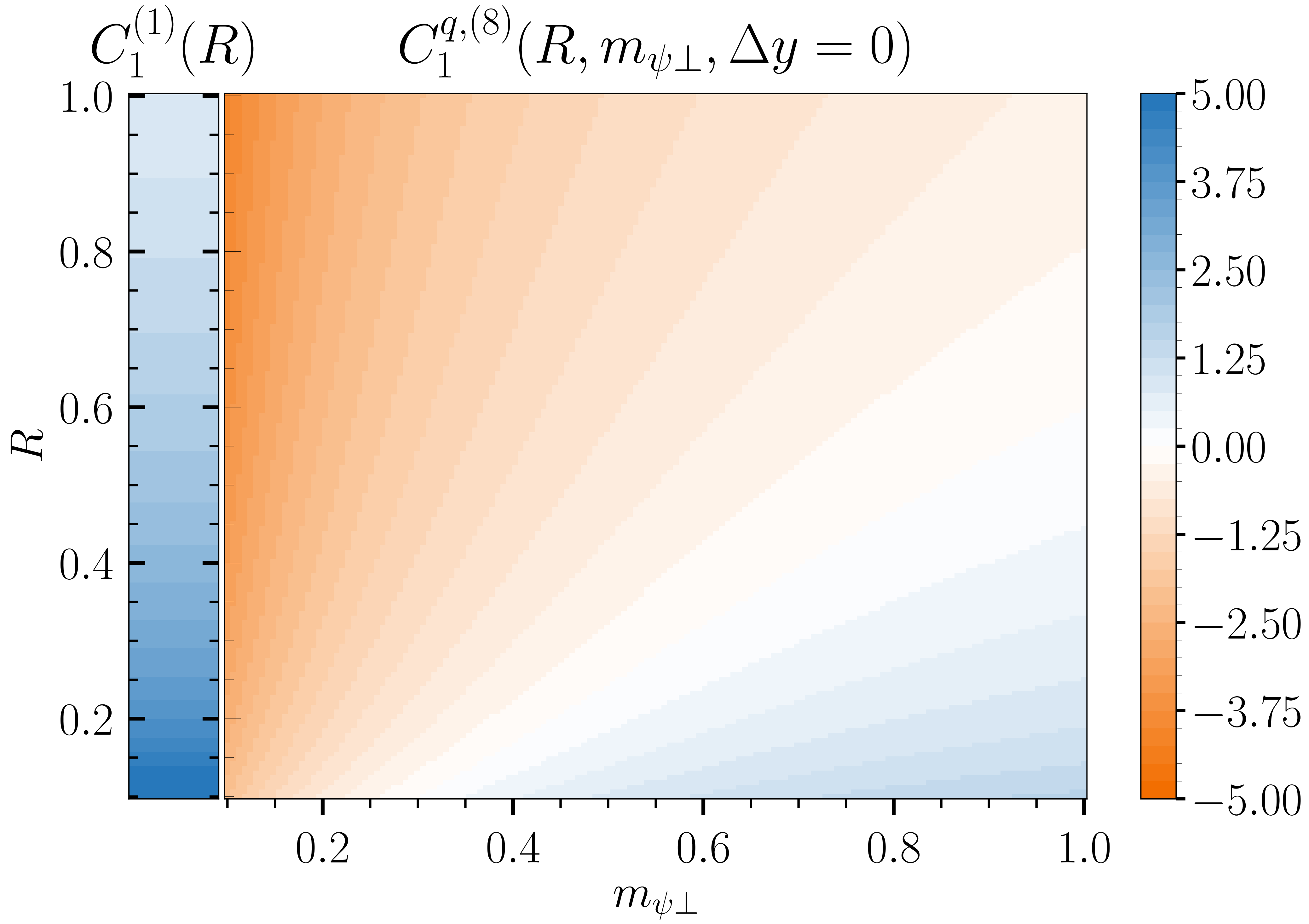
$$\int \mathrm{d}\Delta y_{gj}\, S_g(p_2, k_j)\, \Theta(\Delta_{k_g k_j} > R^2)$$

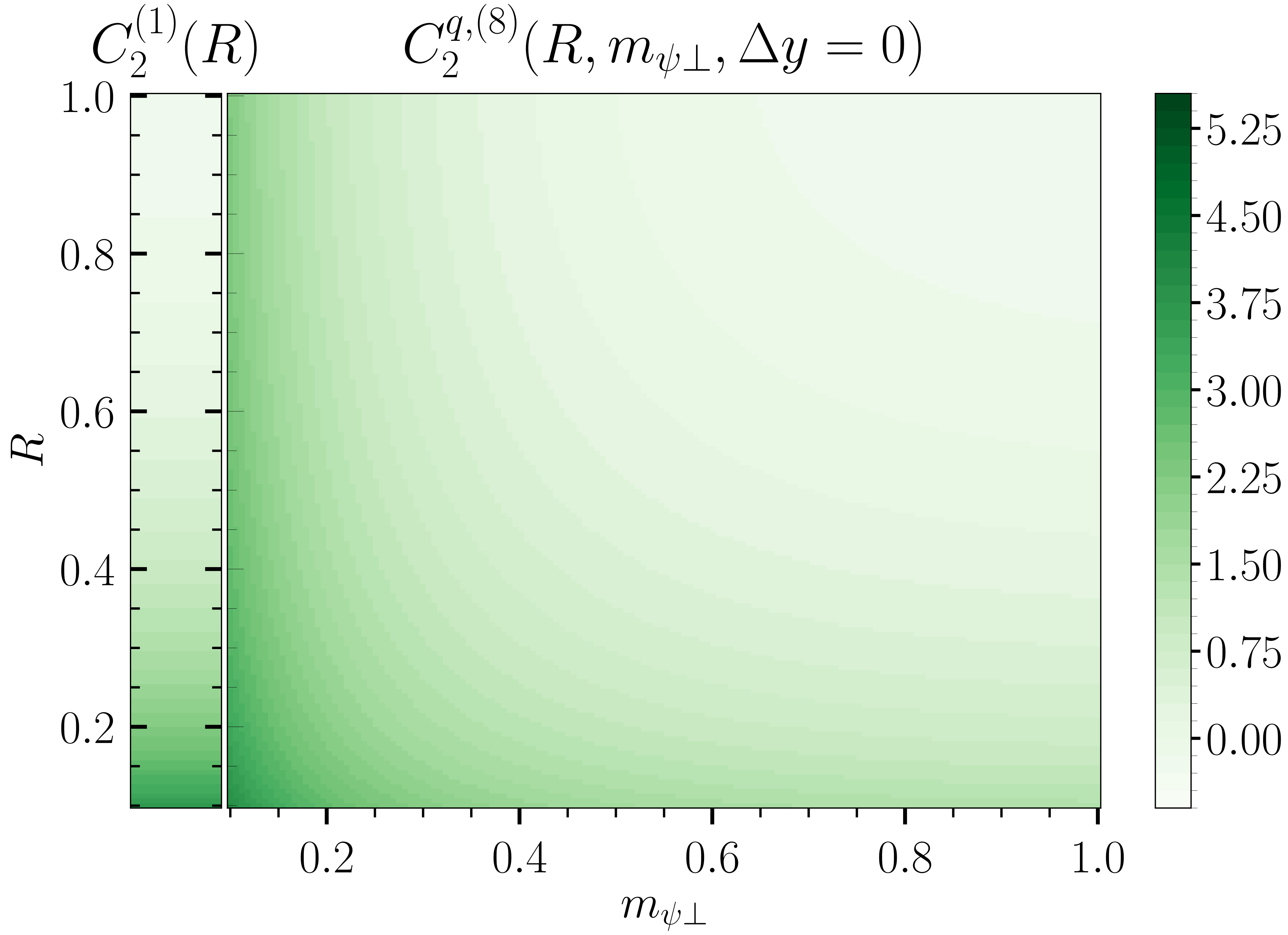
Excludes the jet rapidity region

Soft gluon radiation: CO quark

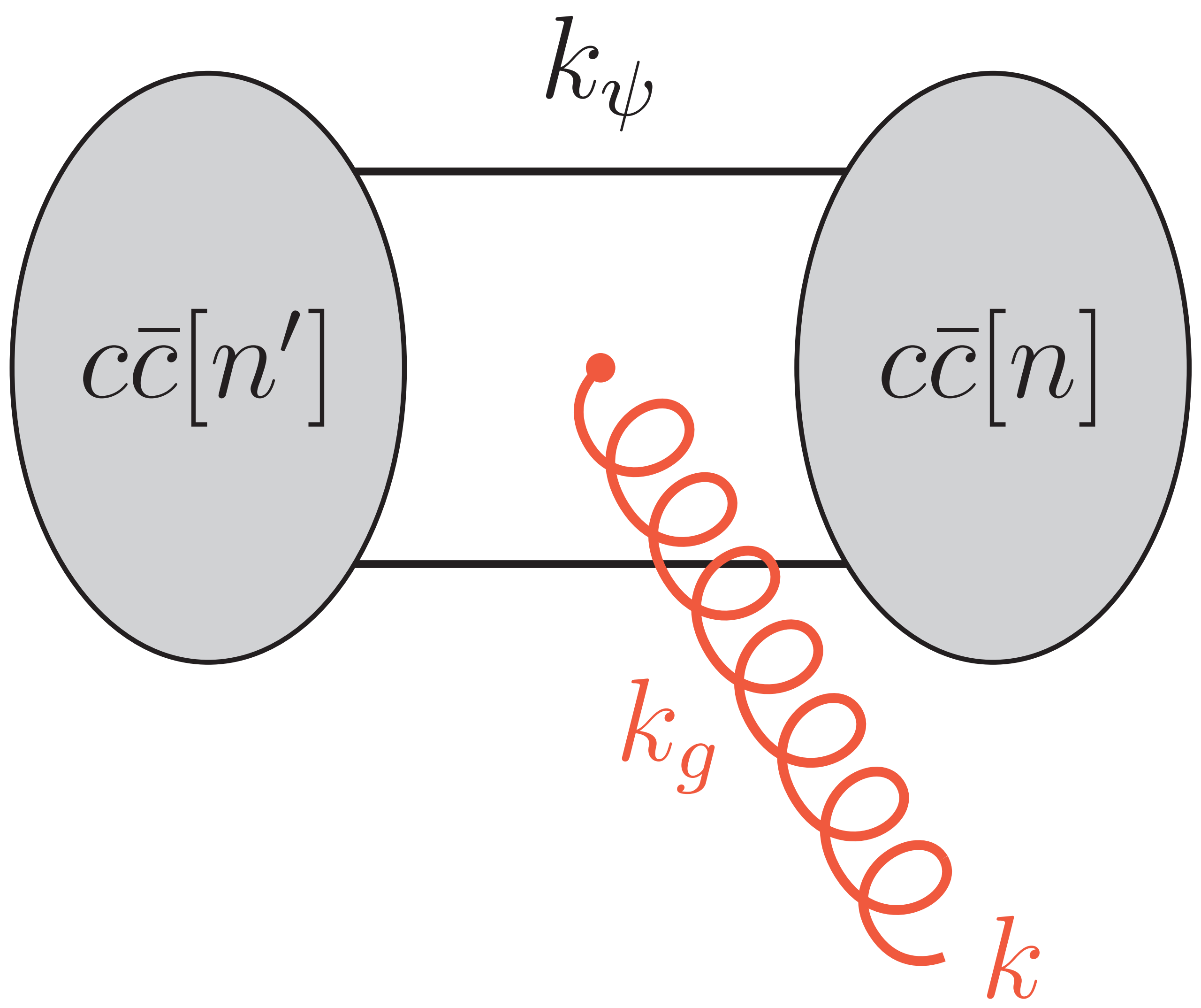
$$\int \frac{\mathrm{d}^3 k_g}{(2\pi)^3 2E_{k_g}} |\overline{\mathscr{A}_1^{q,(8)}}|^2 \delta^{(2)}(q_\perp + k_{g\perp}) = \frac{\alpha_s C_F}{2\pi^2 q_\perp^2} |\overline{\mathscr{A}_0^{q,(8)}}|^2 \Bigg[\ln\frac{\hat{s}}{q_\perp^2} + \frac{C_F - C_A}{C_F}\log\frac{\hat{u}}{\hat{t}} + \frac{C_A}{2C_F}\ln\frac{1-M_\psi^2/\hat{u}}{1-M_\psi^2/\hat{t}} + I_j(R,\phi)$$

$$+\frac{C_A}{C_F}\left(I_\psi(m_{\psi\perp},\phi) + \frac{1}{2} I_{\psi\text{-}j}(m_{\psi\perp},\Delta y,\phi) - \frac{1}{2} I_\psi^{\mathrm{jet}}(R, m_{\psi\perp},\Delta y,\phi)\right)\Bigg]$$





Soft gluon radiation: LDME evolution



Contribution of the evolution from $S \to P$ waves

$$A_{1f}^{(^3P_J^{(8)})} = (i\, g_s\, f_{dd'k}) \frac{k_\psi \cdot \epsilon_{\lambda_g}}{k_\psi \cdot k_g} A_0^{(^3P_J^{(8)})}$$

$$A_{1d}^{(^3P_J^{(8)})} = \left(-4\sqrt{3}\, i\, g_s \frac{R_1'}{R_0}\right) \frac{\epsilon_{L_z} \cdot \epsilon_{\lambda_g}}{k_\psi \cdot k_g} \left(\sqrt{\frac{2}{N}}\, \delta_{dk}\, A_0^{(^3S_1^{(1)})} + d_{dd'k}\, A_0^{(^3S_1^{(8)})}\right)$$

The two contributions never mix because of $k_\psi \cdot \epsilon_\psi = 0$

$$\int |\overline{A_{1d}^{a,(^3P_J^{(8)})}}|^2\, \mathrm{d}k_g \sim \frac{\alpha_s}{2\pi^2 |\vec{q}_\perp|^2} \frac{96}{M_\psi^2} \left(\frac{|\overline{A_0^{a,(^3S_1^{(1)})}}|^2}{\langle \mathcal{O}_1(^3S_1)\rangle} + B_F \frac{|\overline{A_0^{a,(^3S_1^{(8)})}}|^2}{\langle \mathcal{O}_8(^3S_1)\rangle} \right) \langle \mathcal{O}_8(^3P_0)\rangle \frac{I_{\psi\text{-}\psi}}{2}$$

agreement with: Butenschoen, Knieh, *Nucl. Phys. B 950* (2020)

Motivations to split up the TMDShF

[Boer, Bor, LM, Pisano Yuan, *JHEP 08* (2023)](#)

$$\tilde{\Delta}^{[n]}_{ep}(z, b_T; Q, \mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log \frac{M_\psi^2 \mu_H^2}{(M_\psi^2 + Q^2)^2}\right) \log \frac{\mu_H^2}{\mu_b^2}\right] \langle \mathcal{O}[n] \rangle \, \delta(1-z)$$

Reasons to split-up this term:

1. A purely quarkonium quantity should depend on M_ψ solely

2. In open-quark production the soft-factor may produce azimuthal dependeces

[Catani, Grazzini, Torre, *Nucl.Phys. B 890* (2014)](#)

Figure taken from Ferrera's talk @ Heavy-Quark Hadroproduction from Collider to Astroparticle Physics (2019)

$$\frac{d\sigma^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{s} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c}}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q_T}}$$
$$\times\, S_c(M,b) \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2} \left[(\mathbf{H}\boldsymbol{\Delta}) C_1 C_2\right]_{c\bar{c};a_1 a_2}$$
$$\times\, f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)\,,$$