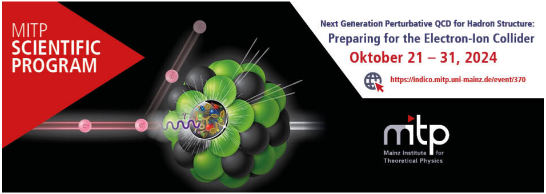
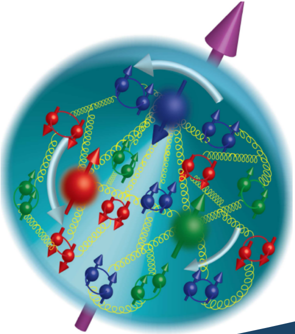
Overview on TMDs

Cristian Pisano

University and INFN Cagliari

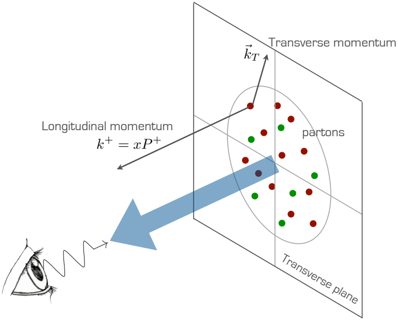




TMD factorization and process dependence

Transverse momentum dependent distributions (TMDs)

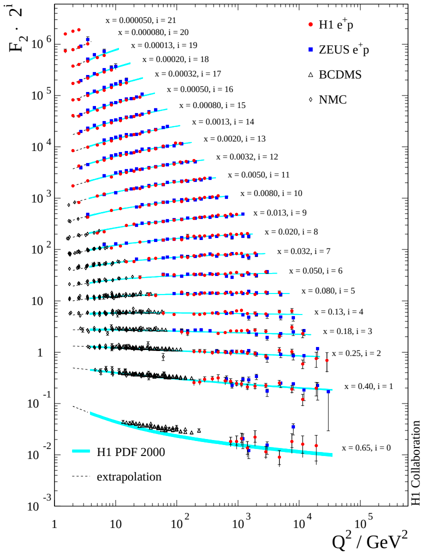
Three-dimensional distributions: provide information on the partonic longitudinal momentum and the two-dimensional transverse momentum



Renormalization scale μ and the Collins-Soper scale ζ not shown explicitly

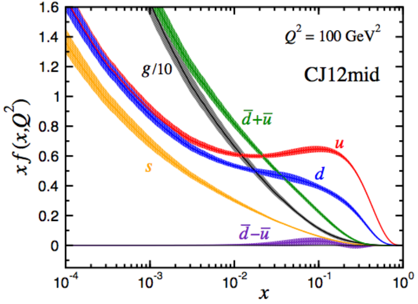
More detailed information on the proton's structure as compared to PDFs: 1D description is not always satisfactory, see i.e. spin effects

Structure Function F_2



$$F_2(x, Q^2) = x \sum_q e_q^2 \, f_1^q(x, Q^2)$$

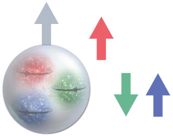
1D collinear parton distributions



CTEQ-JLAB Collaboration, PRD 87 (2013)

Scaling violations: great success of QCD evolution!

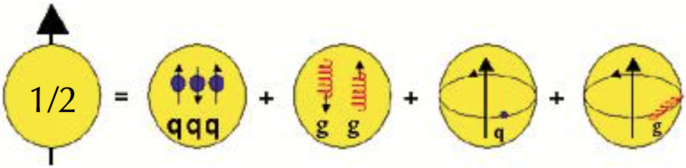
The proton has spin 1/2, the three *valence* quarks have also spin 1/2, we expect:



However: only 30% of the spin of the proton comes from the spin of the quarks

First measurement by the European Muon Collaboration (EMC, CERN 1987)

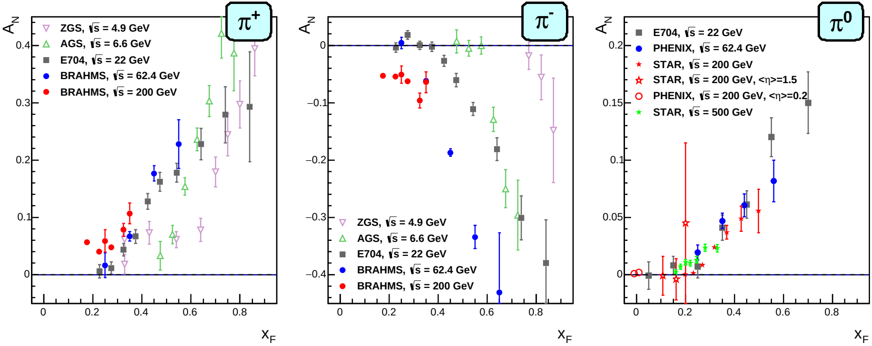
$$\vec{\mu}\vec{p} \to \mu X \qquad\qquad A = \frac{(\vec{\mu}\vec{p}) - (\vec{\mu}\overleftarrow{p})}{(\vec{\mu}\vec{p}) + (\vec{\mu}\overleftarrow{p})}$$



The partonic orbital angular momentum needs to be considered

A_N in $p^\uparrow p \to \pi X$ is a long standing puzzle, only a few % in twist-2 collinear QCD

$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \qquad x_F = \frac{2p_L}{\sqrt{2}}$$

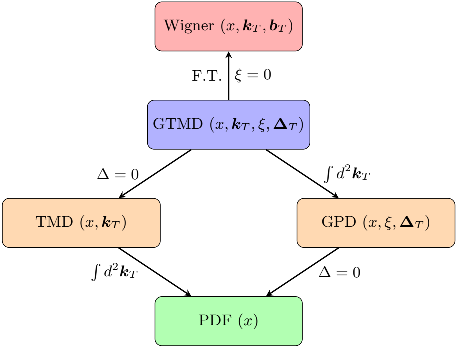


Aschenauer, D'Alesio, Murgia, EPJA52 (2016)

Almost energy independent

TMD factorization does not hold, collinear twist-3 approach more popular

Proton's image in 3D: in momentum (TMDs) and in configuration space (GPDs)



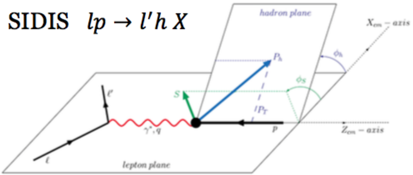
Courtesy of G. Deplano

Richer information as compared to collinear PDFs, especially on parton spin

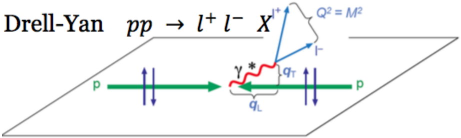
TMD factorization

Two scale processes $Q^2 \gg q_T^2$

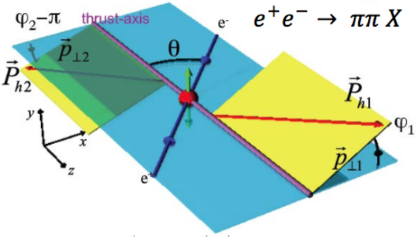
SIDIS $lp \to l'h\,X$



Drell-Yan $pp \to l^+ l^- \, X$



$e^+e^- \to \pi\pi\,X$



Factorization proven

All orders in α_s

Leading order in powers of $1/Q$ (twist)

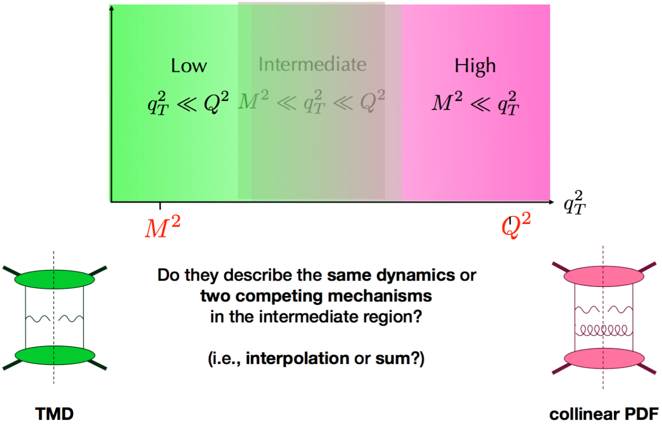
Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

Attempts to establish factorization at one-loop and next-to-leading power

Rodini, Vladimirov, PRD 110 (2024)
Gamberg, Kang, Shao, Terry, Zhao, 2211.13209

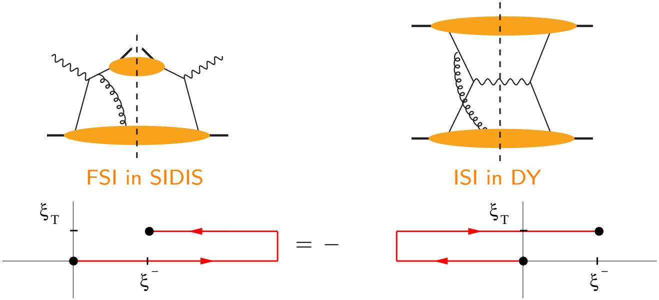
Three physical scales, two theoretical tools

Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)



Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0)\, \mathcal{U}^{\mathcal{C}}_{[0,\xi]}\, \psi(\xi) \right| P, S \right\rangle$$



Sign change of T-odd distributions: fundamental test, still under experimental scrutiny

ISI/FSI lead to process dependence of TMDs, could even break factorization

Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)

Quark TMDs

Quark TMDs

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

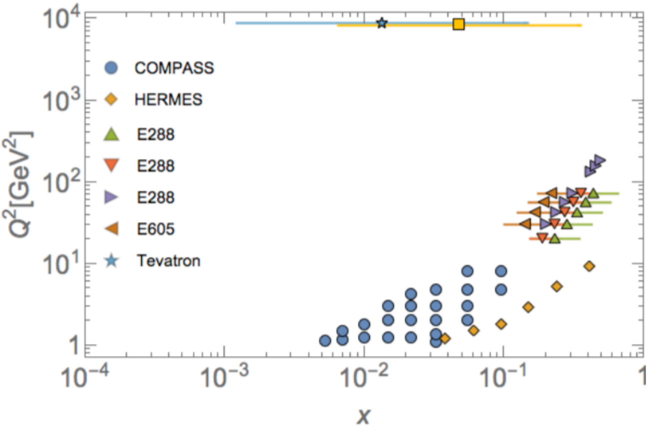
Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Tangerman, NPB 461 (1996)
Boer, Mulders, PRD 57 (1998)

- $h_1^{\perp\, q}$: T-odd distribution of transversely polarized quarks inside an unp. hadron
- $f_{1T}^{\perp\, g}$: T-odd distributions of unp. gluons inside a transversely pol. hadron
- h_{1T}^q, $h_{1T}^{\perp\, q}$: helicity flip distributions: T-even and chiral odd
- Transversity $h_1^q \equiv h_{1T}^q + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, q}$ survives under p_T integration

They are all known and can all be accessed in SIDIS (mostly COMPASS data)

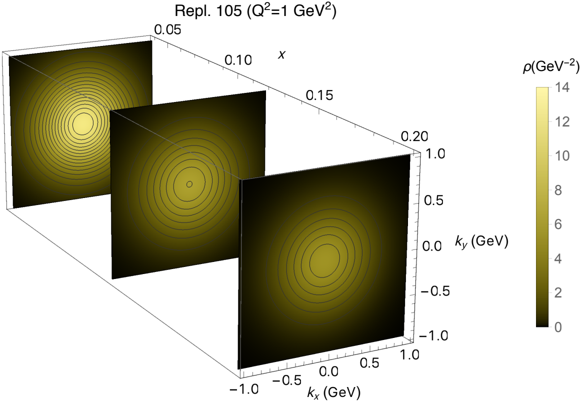
	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06
MAP24 arXiv:2405.13833	N^3LL^-	✓	✓	✓	✓	2031	1.08

Extraction of unpolarized quark TMDs
$x - Q^2$ coverage



Bacchetta, Delcarro, CP, Radici, Signori, JHEP (2017)

Distribution of unpolarized quarks



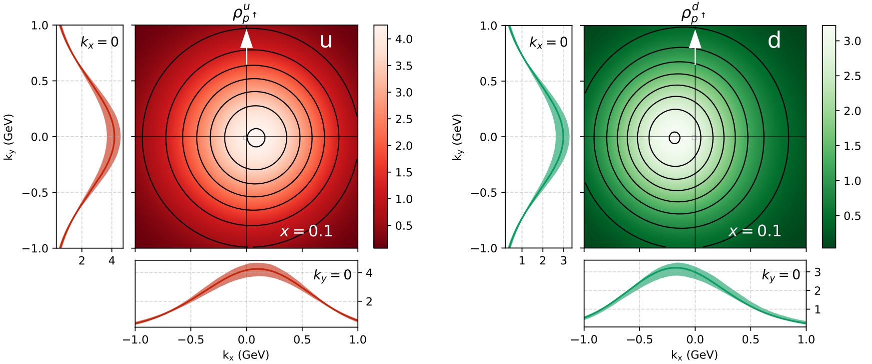
For unpolarized protons, the distribution of unp. quarks is cylindrically symmetric

What happens if the proton is transversely polarized?

Same formalism can be used to have a consistent picture (125 data points)

Distortion in the transverse plane of the TMD quark distribution in a $p^{\uparrow}$

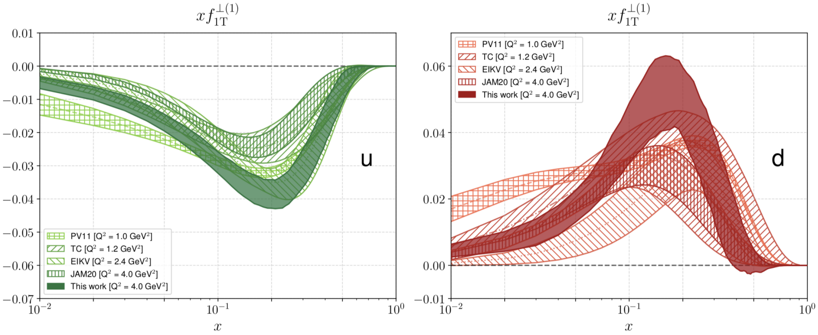
$$\Phi^{[\gamma^+]}_{q/p^{\uparrow}}(x, k_x, k_y) = f_1^q(x, k_T^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2) \qquad [Q^2 = 4\ \mathrm{GeV}^2]$$



Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

Non zero Sivers effect related to parton orbital angular momentum

$$f_{1T}^{\perp(1)\,q}(x) = \int \mathrm{d}^2\boldsymbol{k}_T\, \frac{k_T^2}{2M_p^2}\, f_{1T}^{\perp\,q}(x, \boldsymbol{k}_T^2)$$



Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

More data from CERN, JLab, EIC will help to reduce error bands and extend the ranges in x and Q^2

Gluon TMDs

Gluon TMDs

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

- $h_1^{\perp\, g}$: T-even distribution of linearly polarized gluons inside an unp. hadron
- h_{1T}^g, $h_{1T}^{\perp\, g}$: helicity flip distributions like h_{1T}^q, $h_{1T}^{\perp\, q}$, but T-odd, chiral even!
- $h_1^g \equiv h_{1T}^g + \frac{\boldsymbol{p}_T^2}{2M_p^2}\, h_{1T}^{\perp\, g}$ does not survive under p_T integration, unlike transversity

In contrast to quark TMDs, gluon TMDs are almost unknown
However models have been proposed:

Bacchetta, Celiberto, Radici, Taels, EPJC 80 (2020)
Chakrabarti, Choudhary, Gurjar, Kishore, Maji, Mondal, Mukherjee, PRD 108 (2023)

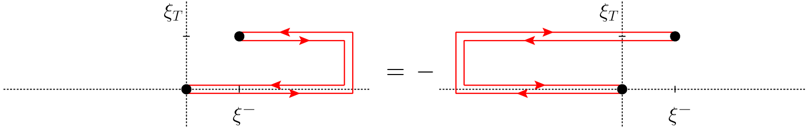
Related Processes

$ep^\uparrow \to e' Q\overline{Q}X$, $ep^\uparrow \to e'$ jet jet X probe GSF with $[++]$ gauge links (WW)

$p^\uparrow p \to \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

Analogue of the sign change of $f_{1T}^{\perp\,q}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp\,g}$)

$$f_{1T}^{\perp\,g\,[e\,p^\uparrow \to e'\,Q\overline{Q}\,X]} = -f_{1T}^{\perp\,g\,[p^\uparrow\,p \to \gamma\,\gamma\,X]}$$



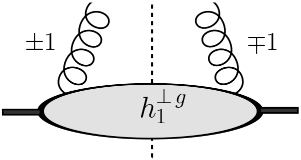
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

Linearly polarized gluons

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum



Interference between ± 1 gluon helicity states

Like the unpolarized gluon TMD, it is T-even and exists in different versions:

- $[++] = [--]$ (WW) (SIDIS and DY-like process)

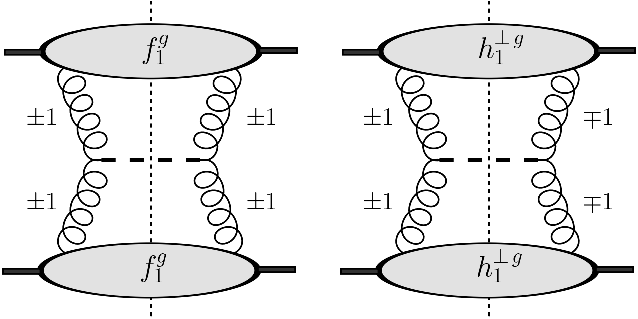
Gluons can be probed in heavy quark production in both *ep* and *pp* scattering

Mukherjee, Rajesh, EPJC 77 (2017)
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
Rajesh, Kishore, Mukherjee, PRD 98 (2018)
Bacchetta, Boer, CP, Taels, EPJC 80 (2020)

Higgs boson production happens mainly via $gg \to H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)



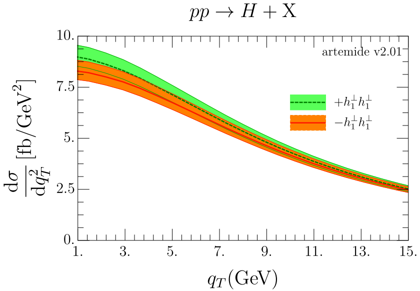
The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low q_T

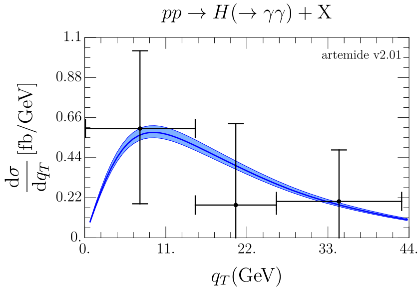
Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012)
Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)
Echevarria, Kasemets, Mulders, CP, JHEP 1507 (2015)

q_T-distribution of the Higgs boson

$$\frac{1}{\sigma}\frac{d\sigma}{d\boldsymbol{q}_T^2} \propto 1 + R(\boldsymbol{q}_T^2) \qquad R = \frac{h_1^{\perp\,g} \otimes h_1^{\perp\,g}}{f_1^g \otimes f_1^g} \qquad |h_1^{\perp\,g}(x,\boldsymbol{p}_T^2)| \leq \frac{2M_p^2}{\boldsymbol{p}_T^2} f_1^g(x,\boldsymbol{p}_T^2)$$

The perturbative tails of f_1^g and $h_1^{\perp\,g}$ (matching coefficients to collinear PDFs) are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO); g_{1L} up to $\mathcal{O}(\alpha_s)$ (NLO)





Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, JHEP 11 (2019) 121

The matching coefficients for the other gluon TMDs are still unkown

$C = +1$ quarkonium production

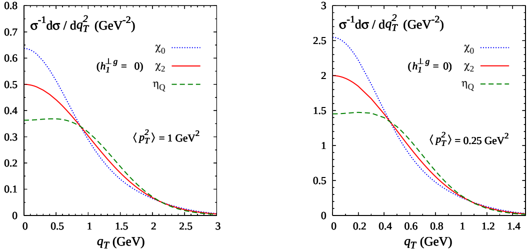
q_T-distribution of η_Q and χ_{QJ} $(Q = c, b)$ in the kinematic region $q_T \ll 2M_Q$

$$\frac{1}{\sigma(\eta_Q)}\frac{d\sigma(\eta_Q)}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g\,[1 - R(\boldsymbol{q}_T^2)] \qquad \text{[pseudoscalar]} \qquad R(\boldsymbol{q}_T^2) = \frac{h_1^{\perp\,g} \otimes h_1^{\perp\,g}}{f_1^g \otimes f_1^g}$$

$$\frac{1}{\sigma(\chi_{Q0})}\frac{d\sigma(\chi_{Q0})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g\,[1 + R(\boldsymbol{q}_T^2)] \qquad \text{[scalar]}$$

$$\frac{1}{\sigma(\chi_{Q2})}\frac{d\sigma(\chi_{Q2})}{d\boldsymbol{q}_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012)



Proof of factorization at NLO for $p\,p \to \eta_Q\,X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013); PLB 737 (2014)
Echevarria, JHEP 1910 (2019)

Future fixed target experiments at LHC

Structure of the cross section for the doubly polarized process $p(S_A)+p(S_B)\to\mathcal{Q}\,X$

$$
\begin{aligned}
\frac{\mathrm{d}\sigma[\mathcal{Q}]}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T} = {} & F_{UU}^{\mathcal{Q}} + F_{UL}^{\mathcal{Q}}\,S_{BL} + F_{LU}^{\mathcal{Q}}\,S_{AL} + F_{UT}^{\mathcal{Q},\sin\phi_{S_B}}\,|\boldsymbol{S}_{BT}|\sin\phi_{S_B} + F_{TU}^{\mathcal{Q},\sin\phi_{S_A}}\,|\boldsymbol{S}_{AT}|\sin\phi_{S_A} \\
& + F_{LL}^{\mathcal{Q}}\,S_{AL}\,S_{BL} + F_{LT}^{\mathcal{Q},\cos\phi_{S_B}}\,S_{AL}\,|\boldsymbol{S}_{BT}|\cos\phi_{S_B} + F_{TL}^{\mathcal{Q},\cos\phi_{S_A}}\,|\boldsymbol{S}_{AT}|\,S_{BL}\cos\phi_{S_A} \\
& + |\boldsymbol{S}_{AT}||\boldsymbol{S}_{BT}|\left[F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}-\phi_{S_B})}\cos(\phi_{S_A}-\phi_{S_B}) + F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}+\phi_{S_B})}\cos(\phi_{S_A}+\phi_{S_B})\right]
\end{aligned}
$$

Kato, Maxia, CP, PRD 110 (2024)

Single spin asymmetries for different quarkonia are sensitive to different TMDs

$$
\begin{aligned}
F_{UT}^{\eta_Q,\sin\phi_{S_B}} &\propto -f_1^g\otimes f_{1T}^{\perp g} + h_1^{\perp g}\otimes h_1^g - h_1^{\perp g}\otimes h_{1T}^{\perp g} \\
F_{UT}^{\chi_{Q0},\sin\phi_{S_B}} &\propto -f_1^g\otimes f_{1T}^{\perp g} - h_1^{\perp g}\otimes h_1^g + h_1^{\perp g}\otimes h_{1T}^{\perp g} \\
F_{UT}^{\chi_{Q2},\sin\phi_{S_B}} &\propto -f_1^g\otimes f_{1T}^{\perp g}
\end{aligned}
$$

Such observables are in principle measurable at the planned LHCspin experiment

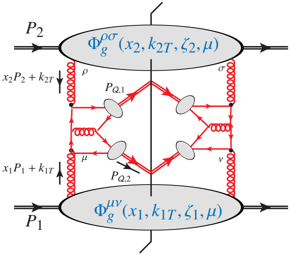
J/ψ-pair production at the LHC

J/ψ's are relatively easy to detect. Accessible at the LHC: already studied by LHCb, CMS & ATLAS

LHCb PLB 707 (2012)
CMS JHEP 1409 (2014)
ATLAS EPJC 77 (2017)

gg fusion dominant, negligible $q\bar{q}$ contributions even at fixed target energies

Lansberg, Shao, NPB 900 (2015)

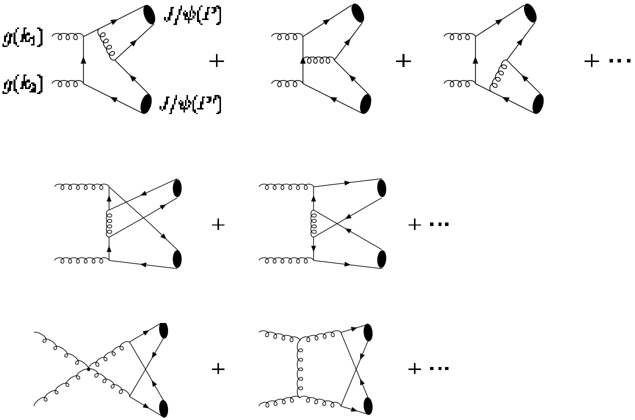


No final state gluon needed for the Born contribution in the Color Singlet Model. Pure colorless final state, hence simple color structure because one has only ISI

Lansberg, Shao, PRL 111 (2013)

Negligible Color Octet contributions, in particular at low $P_T^{\Psi\Psi}$

At LO pQCD in the Color Singlet Model, one needs to consider 36 diagrams



Qiao, Sun, Sun, JPG 37 (2010)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega} \approx A\, f_1^g \otimes f_1^g + B\, f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C\, h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

- valid up to corrections $\mathcal{O}(q_T/Q)$
- Y: rapidity of the J/ψ-pair, along the beam in the hadronic c.m. frame
- $\mathrm{d}\Omega = \mathrm{d}\cos\theta_{CS}\, \mathrm{d}\phi_{CS}$: solid angle for J/ψ-pair in the Collins-Soper frame

Analysis similar to the one for $pp \to \gamma\gamma X$, $pp \to J\psi\, \gamma^{(*)}\, X$, $pp \to H\, \mathrm{jet}\, X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)
den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)
Lansberg, CP, Schlegel, NPB 920 (2017)
Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

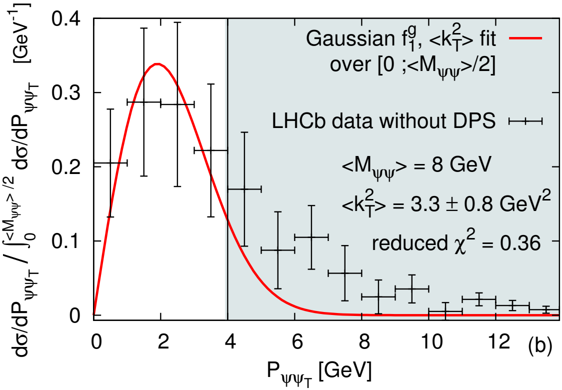
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \cos(n\,\phi_{CS})\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}}{\int_0^{2\pi} \mathrm{d}\phi_{CS}\, \frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2 q_T \mathrm{d}\Omega}} \qquad (n = 2, 4)$$

$$\int \mathrm{d}\phi_{CS}\, \mathrm{d}\sigma \implies f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \implies h_1^{\perp g} \otimes h_1^{\perp g}$$

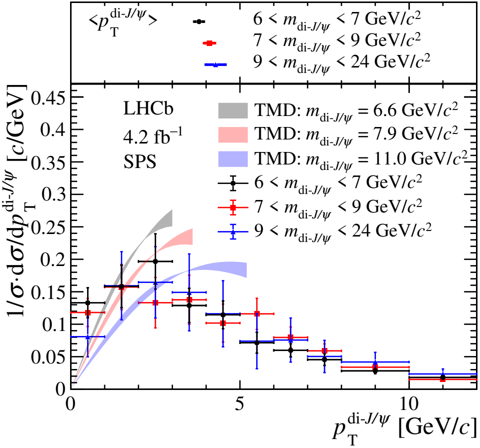
We consider $q_T = P_T^{\Psi\Psi} \leq M_{\Psi\Psi}/2$ in order to have two different scales



Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model: $$f_1^g(x, \boldsymbol{k}_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{\boldsymbol{k}_T^2}{\langle k_T^2 \rangle} \right)$$

No obvious broadening can be seen due to the large uncertainties



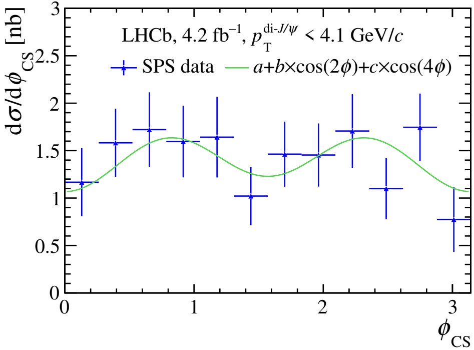
Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., 2311.14085

The average values of the p_T distributions slightly increase with mass

$$\langle \cos 2\phi \rangle = -0.029 \pm 0.050\,(\text{stat}) \pm 0.009\,(\text{syst})$$
$$\langle \cos 4\phi \rangle = -0.087 \pm 0.052\,(\text{stat}) \pm 0.013\,(\text{syst})$$

Theoretical predictions consistent with measureaments

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)



LHCb Coll., 2311.14085

The results are consistent with zero, but the presence of an azimuthal asymmetry at a few percent level is allowed

$e\,p \to e\,J/\psi\,X$ (with the inclusion of TMD shape functions)

Mukherjee, Rajesh, EPJ.C 77 (2017)
Kishore, Mukherjee, PRD 99 (2019)
Bacchetta, Boer, CP, Taels, EPJ.C 80 (2020)
Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

$e\,p \to e\,J/\psi\,\mathrm{jet}\,X$

D'Alesio, Murgia, CP, Taels, PRD 100 (2019)
Kishore, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

$e\,p \to e\,J/\psi\,\gamma\,X$

Chakrabarti, Kishore, Mukherjee, Rajesh, PRD 107 (2023)

$e\,p \to e\,D\,\mathrm{jet}\,X$

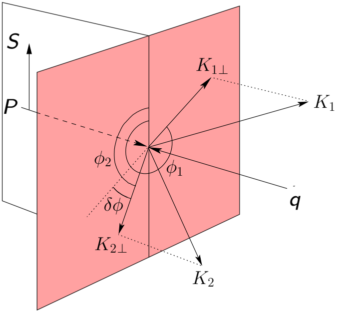
Banu, Mukherjee, Pawar, Rajesh, PRD 108 (2023)

Heavy quark pair production at an EIC

Gluon TMDs probed directly in $e(\ell) + p(P,S) \rightarrow e(\ell') + Q(K_1) + \overline{Q}(K_2) + X$

Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

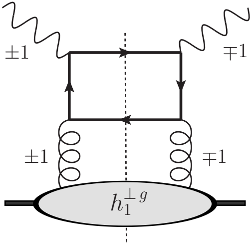
- the $Q\overline{Q}$ pair is almost back to back in the plane $\perp$ to q and P
- $q \equiv \ell - \ell'$: four-momentum of the exchanged virtual photon γ^*



$$\boldsymbol{q}_T \equiv \boldsymbol{K}_{1\perp} + \boldsymbol{K}_{2\perp}$$

$$\boldsymbol{K}_\perp \equiv (\boldsymbol{K}_{1\perp} - \boldsymbol{K}_{2\perp})/2$$

$\Longrightarrow$ Correlation limit: $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$, $\quad |\boldsymbol{K}_\perp| \approx |\boldsymbol{K}_{1\perp}| \approx |\boldsymbol{K}_{2\perp}|$



$\phi_T, \phi_\perp, \phi_S$ azimuthal angles of $q_T, K_\perp, S_T$

At LO in pQCD: only $\gamma^* g \to Q\overline{Q}$ contributes

$$\mathrm{d}\sigma(\phi_S, \phi_T, \phi_\perp) = \mathrm{d}\sigma^U(\phi_T, \phi_\perp) + \mathrm{d}\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

Angular structure of the unpolarized cross section for $ep \to e' Q\overline{Q}X$, $|\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$\frac{\mathrm{d}\sigma^U}{\mathrm{d}^2\boldsymbol{q}_T\, \mathrm{d}^2\boldsymbol{K}_\perp} \propto \left\{A_0^U + A_1^U \cos\phi_\perp + A_2^U \cos 2\phi_\perp\right\} f_1^g(x, \boldsymbol{q}_T^2) + \frac{\boldsymbol{q}_T^2}{M_p^2}\, h_1^{\perp\, g}(x, \boldsymbol{q}_T^2)$$
$$\times \left\{B_0^U \cos 2\phi_T + B_1^U \cos(2\phi_T - \phi_\perp) + B_2^U \cos 2(\phi_T - \phi_\perp) + B_3^U \cos(2\phi_T - 3\phi_\perp) + B_4^U \cos 2(\phi_T - 2\phi_\perp)\right\}$$

The different contributions can be isolated by defining

$$\langle W(\phi_\perp, \phi_T)\rangle = \frac{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, W(\phi_\perp, \phi_T)\, \mathrm{d}\sigma}{\int \mathrm{d}\phi_\perp \mathrm{d}\phi_T\, \mathrm{d}\sigma}, \qquad W = \cos 2\phi_T,\ \cos 2(\phi_\perp - \phi_T), \ldots$$

$h_1^{\perp\,g}$ in $ep \to e' Q \overline{Q} X$

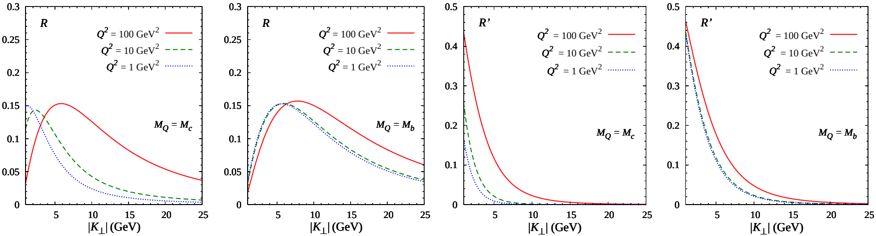
Maximal asymmetries

Positivity bound for $h_1^{\perp\,g}$: $\quad |h_1^{\perp\,g}(x, \boldsymbol{p}_T^2)| \leq \frac{2M_p^2}{\boldsymbol{p}_T^2}\, f_1^g(x, \boldsymbol{p}_T^2)$

It can be used to estimate maximal values of the asymmetries

Asymmetries usually larger when Q and $\overline{Q}$ have same rapidities

Upper bounds on $R \equiv |\langle \cos 2(\phi_T - \phi_\perp)\rangle|$ and $R' \equiv |\langle \cos 2\phi_T \rangle|$ at $y = 0.01$



CP, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013)
Boer, Brodsky, Mulders, CP, PRL 106 (2011)

Spin asymmetries in $ep^\uparrow \to e' Q\overline{Q} X$

Angular structure of the single polarized cross section for $ep^\uparrow \to e' Q\overline{Q} X$, $\quad |\boldsymbol{q}_T| \ll |\boldsymbol{K}_\perp|$

$$\begin{aligned}
\mathrm{d}\sigma^T \propto\ & \sin(\phi_S - \phi_T)\Big[A_0^T + A_1^T \cos\phi_\perp + A_2^T \cos 2\phi_\perp\Big] f_{1T}^{\perp\,g} + \cos(\phi_S - \phi_T)\Big[B_0^T \sin 2\phi_T \\
& + B_1^T \sin(2\phi_T - \phi_\perp) + B_2^T \sin 2(\phi_T - \phi_\perp) + B_3^T \sin(2\phi_T - 3\phi_\perp) + B_4^T \sin(2\phi_T - 4\phi_\perp)\Big] h_{1T}^{\perp\,g} \\
& + \Big[B_0'^{\,T} \sin(\phi_S + \phi_T) + B_1'^{\,T} \sin(\phi_S + \phi_T - \phi_\perp) + B_2'^{\,T} \sin(\phi_S + \phi_T - 2\phi_\perp) \\
& \quad + B_3'^{\,T} \sin(\phi_S + \phi_T - 3\phi_\perp) + B_4'^{\,T} \sin(\phi_S + \phi_T - 4\phi_\perp)\Big] h_{1T}^{g}
\end{aligned}$$

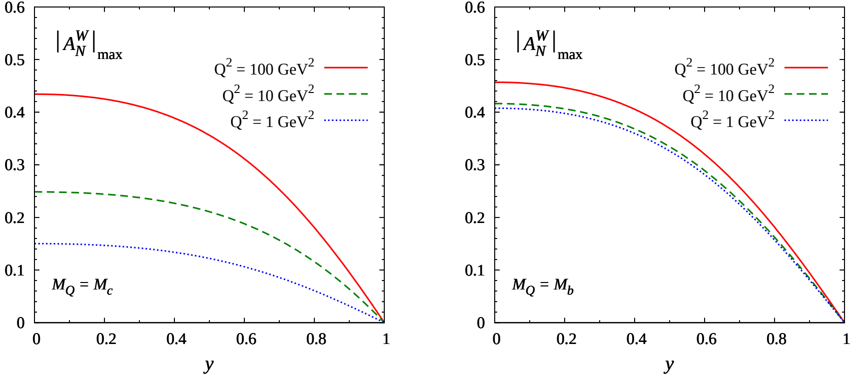
The ϕ_S dependent terms can be singled out by means of azimuthal moments A_N^W

$$A_N^{W(\phi_S,\phi_T)} \equiv 2\, \frac{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, W(\phi_S,\phi_T)\, \mathrm{d}\sigma_T(\phi_S,\, \phi_T,\, \phi_\perp)}{\int \mathrm{d}\phi_T\, \mathrm{d}\phi_\perp\, \mathrm{d}\sigma_U(\phi_T,\phi_\perp)}$$

$$A_N^{\sin(\phi_S-\phi_T)} \propto \frac{f_{1T}^{\perp\,g}}{f_1^g} \qquad A_N^{\sin(\phi_S+\phi_T)} \propto \frac{h_1^g}{f_1^g} \qquad A_N^{\sin(\phi_S-3\phi_T)} \propto \frac{h_{1T}^{\perp\,g}}{f_1^g}$$

Same modulations as in SIDIS for quark TMDs ($\phi_T \to \phi_h$)

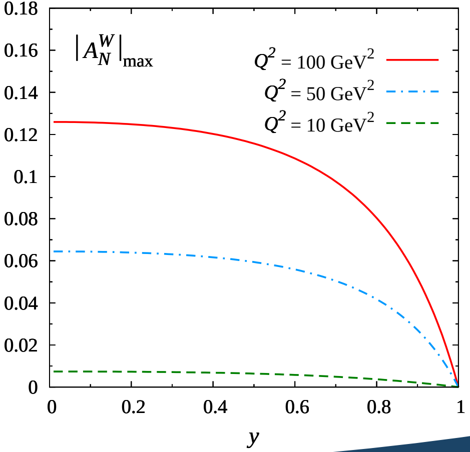
Maximal values for $|A_N^W|$, $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$ $\quad$ ($|\boldsymbol{K}_\perp| = 1$ GeV)



Contribution to the denominator also from $\gamma^* q \to gq$, negligible at small-x

Asymmetries much smaller than in $c\bar{c}$ case for $Q^2 \leq 10$ GeV^2

Upper bounds for A_N^W for $K_\perp \geq 4$ GeV



Conclusions

- Unpolarized quark TMDs of the proton are quite well-known, theoretical analysis can be improved (by looking at Y-term, flavor dependence, ...)
- Azimuthal asymmetries in heavy quark pair and dijet production in DIS could probe WW-type gluon TMDs (similar to SIDIS for quark TMDs)
- Quarkonia as well are good probes for gluon TMDs: first extraction of unpolarized gluon TMD from LHC data on di-J/ψ production

  Talk by L. Maxia

- Different behavior of WW and dipole gluon TMDs accessible at RHIC, LHCspin and at EIC, overlap of both *spin* and *small-x* programs