RG improved JIMWLK Hamiltonian: running coupling and DGLAP resummation

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A. Kovner, M. Lublinsky, V. V. Skokov and Z. Zhao, arXiv:2308.15545 [hep-ph], in JHEP

T. Altinoluk, G. Beuf, M. Lublinsky and V. V. Skokov, arXiv:2310.10738 [hep-ph], in JHEP

High Energy Scattering

Target ($\rho^t = \rho^-$) **Projectile ($\rho^p = \rho^+$)**

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle \qquad\qquad \rho^+ \sim \int \mathbf{dk^+ a^\dagger\, T\, a}$$

S-matrix:

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T}\,\langle\, \mathbf{P}|\;\; \hat{\mathbf{S}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})\;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle \qquad\qquad \mathbf{Y} \sim \ln(\mathbf{s})$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T}\,\langle\, \mathbf{P}|\;\; \hat{\mathcal{O}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})\;\; |\mathbf{P}\rangle\, \mathbf{T}\rangle$$

How do these averages change with increase in energy of the process?

$$\partial_{\mathbf{Y}} \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \;=\; -\mathcal{H}\, \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \qquad\qquad \mathcal{H} \;\rightarrow\; \text{the HE effective Hamiltonian}$$

$\mathcal{H}$ **defines the high energy limit of QCD and is universal**

Projectile averaged S-matrix:

$$\mathbf{\Sigma(Y)} \equiv \langle\, \mathbf{P}|\, \hat{\mathbf{S}}(\rho^{\mathbf{t}},\, \rho^{\mathbf{p}})\ |\mathbf{P}\rangle$$

evolves with rapidity as

$$\mathbf{\Sigma(Y + \delta Y)} = \mathbf{e}^{-\delta \mathbf{Y}\, \mathcal{H}}\, \mathbf{\Sigma(Y)}$$

Expansion in α_s

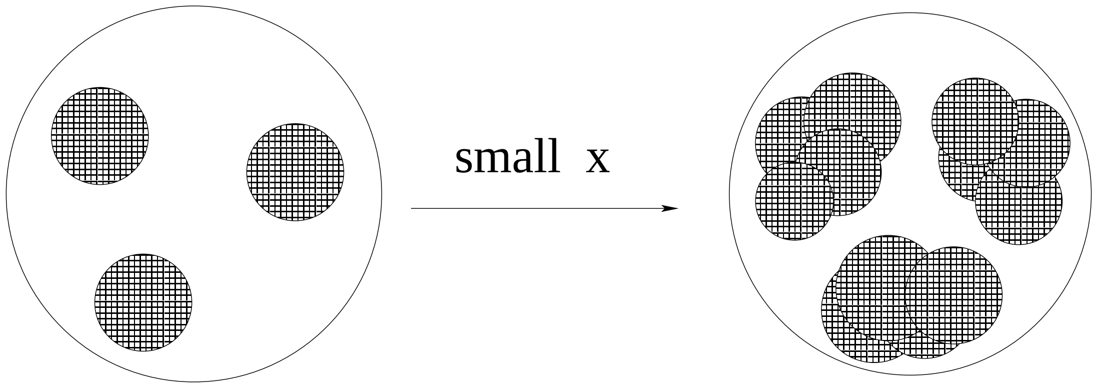
$$\mathcal{H} = \mathcal{H}_{\mathbf{LO}}(\alpha_{\mathrm{s}}) + \mathcal{H}_{\mathbf{NLO}}(\alpha_{\mathrm{s}}^{\mathbf{2}}) + \ldots; \qquad\qquad \mathcal{H} = \mathcal{H}[\rho^{\mathbf{t}},\, \delta/\delta\rho^{\mathbf{t}}]$$

JIMWLK Hamiltonian is a limit of $\mathcal{H}$ for dilute partonic system ($\rho^{\mathbf{p}} \to \mathbf{0}$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{LO}}$ **(1997-2002),** $\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{NLO}}$ **with massless quarks (2007-2016),** $\mathcal{H}^{\mathbf{JIMWLK}}_{\mathbf{NLO}}(\mathrm{m_q})$ **(2022)**

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

Dilute regime: $\delta\rho \sim \rho \quad \rightarrow \quad \rho \simeq \mathbf{e}^{\mathbf{c\,Y}}$ **BFKL** $\quad s = \exp[Y]$



Evolution is generated by boost. Accelerated (color) charged particles radiate

Fast particles emit softer ones

High energy limit = soft gluon emission approximation

Exponential growth of gluon densities leads to unitarity violation.

At high densities the growth should be slowed down due to non-linear effects.

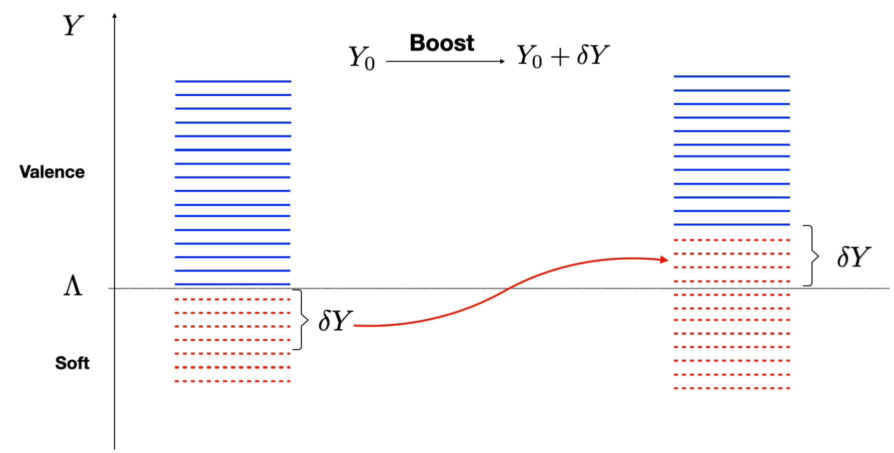
Transition to a non-linear regime is characterized by emergence of a new scale $\mathbf{Q_s}$, known as saturation scale.

$\mathbf{Q_s} \gg \mathbf{\Lambda_{QCD}}$ **and perturbative methods are applicable.**

Light Cone Wave Function in Born-Oppenheimer approximation

$$\mathbf{H}^{\mathbf{LC}}_{\mathbf{QCD}}\,|\mathbf{\Psi}\rangle \;=\; \mathbf{E}\,|\mathbf{\Psi}\rangle$$

BO: split the modes into hard and soft.
The hard (valence) modes with $k^+ \;>\; \Lambda$
They act as an external background current
$j^+_a \;=\; \delta(x^-)\,\rho^a$ for the soft modes.



$$\mathbf{H}^{\mathbf{LC}}_{\mathbf{QCD}} \;=\; \mathbf{H}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger] \;=\; \mathbf{H_V}[\rho] \;+\; \mathbf{H_{free}}[\mathbf{a},\,\mathbf{a}^\dagger] \;+\; \mathbf{H_{int}}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger]$$

LCWF with no soft modes

$$\mathbf{H_V}\,|\mathbf{v},\,\mathbf{0_a}\rangle \;=\; \mathbf{E_0}\,|\mathbf{v},\,\mathbf{0_a}\rangle\,; \qquad \mathbf{a}\,|\mathbf{v},\,\mathbf{0_a}\rangle \;=\; \mathbf{0}\,; \qquad \mathbf{E_0} \;=\; \mathbf{0}$$

LCWF with soft gluon/quark dressing

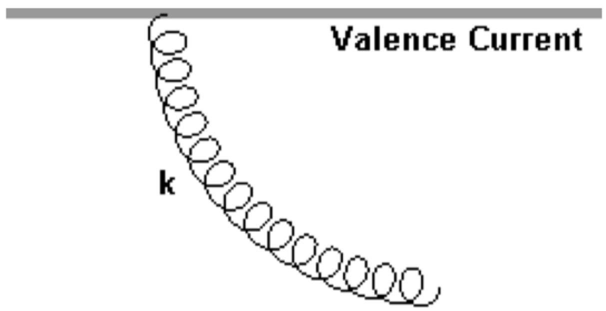
$$|\mathbf{\Psi}\rangle \;=\; \mathbf{\Omega}(\rho,\,\mathbf{a},\,\mathbf{a}^\dagger)\,|\mathbf{v},\,\mathbf{0_a}\rangle\,; \qquad \mathbf{\Omega}^\dagger\,(\mathbf{H_{free}} + \mathbf{H_{int}})\,\mathbf{\Omega} \;=\; \mathbf{H_{diagonal}}$$

Find Ω in perturbation theory

LCWF at LO

Eikonal coupling between valence and soft gluons due to separation of scales

$$\mathbf{H_{int}} = -\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\frac{g\,k_i}{\sqrt{2}\,|k^+|^{3/2}}\left[a_i^{\dagger a}(k^+,\,k_\perp)\;\rho^a(-k_\perp) + a_i^a(k^+,\,-k_\perp)\;\rho^a(k_\perp)\right]$$



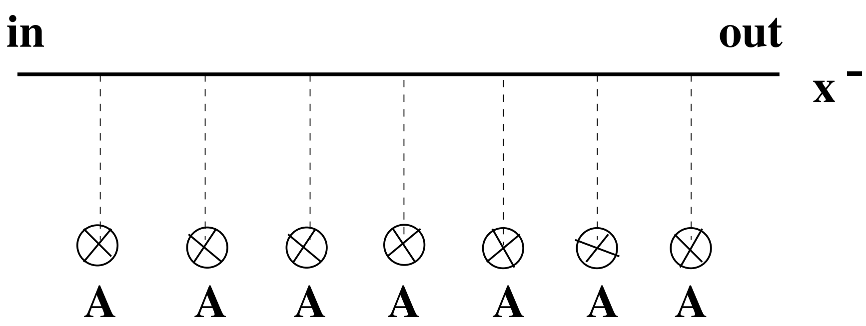
A cloud of classical Weizsaker-Williams gluons dressing the valence ones

$$b_i^a(z) = \frac{g}{2\pi}\int d^2x\frac{(z-x)_i}{(z-x)^2}\;\rho^a(x)$$

$$\Omega_Y(\rho\to 0) \equiv C_Y = \mathrm{Exp}\left\{i\int d^2z\,b_i^a(z)\int_{e^{Y_0}\Lambda}^{e^Y\Lambda}\frac{dk^+}{\pi^{1/2}|k^+|^{1/2}}\left[a_i^a(k^+,z) + a_i^{\dagger a}(k^+,z)\right]\right\}$$

$$\Sigma^{LO} = \langle\Psi_{LO}|\,\hat{S}\,|\Psi_{LO}\rangle = \langle\rho,\,0_a|\;C^\dagger\,\hat{S}\;C\,|\rho,0_a\rangle \to \mathrm{LO\ JIMWLK}$$

Eikonal scattering approximation



Eikonal scattering is a color rotation
Eikonal factor does not depend on rapidity

In the light cone gauge ($A^+ = 0$) the large target field component is $A^- = \alpha^t$.

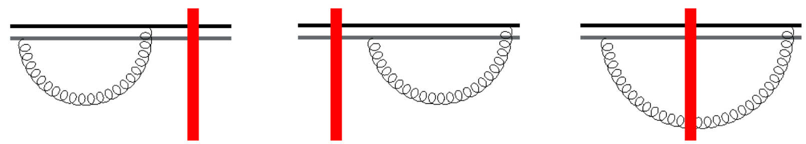
$$S(x) \;=\; \mathcal{P}\,\exp\left\{ i \int dx^+\, T^a\, \alpha_t^a(x, x^+) \right\}\,. \qquad\qquad \text{"}\Delta\text{"}\,\alpha^t \;=\; \rho^t \quad (YM)$$

$$|in\rangle \;=\; |z, b\rangle\,; \qquad |out\rangle \;=\; |z, a\rangle\,; \qquad |out\rangle \;=\; S\,|in\rangle$$

LO JIMWLK Hamiltonian

$$\mathcal{H}^{\mathrm{JIMWLK}}_{\mathrm{LO}} = \int_{x,y,z} K_{\mathrm{LO}} \left\{ J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}_A(z) J^b_R(y) \right\}$$

$$K_{\mathrm{LO}}(x, y, z) = \frac{\alpha_s}{2\,\pi^2} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2} \equiv \frac{\alpha_s}{2\,\pi^2} \frac{X_i Y_i}{X^2 Y^2}$$



$$S_A{}^{cd}(z) = \mathcal{P} \exp \left\{ i \int dx^+ \, T^a \, \alpha^a_t(z, x^+) \right\}^{cd} .$$

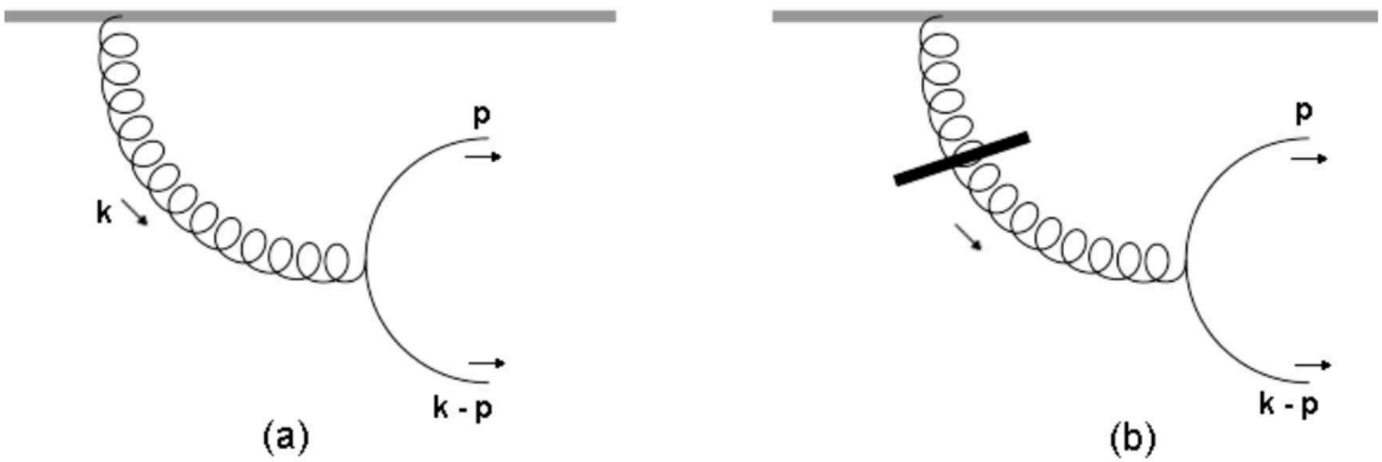
Here $\rho^p \to J_L$ and $\hat{S} \rho^p \to J_R$ are left and right SU(N) generators:

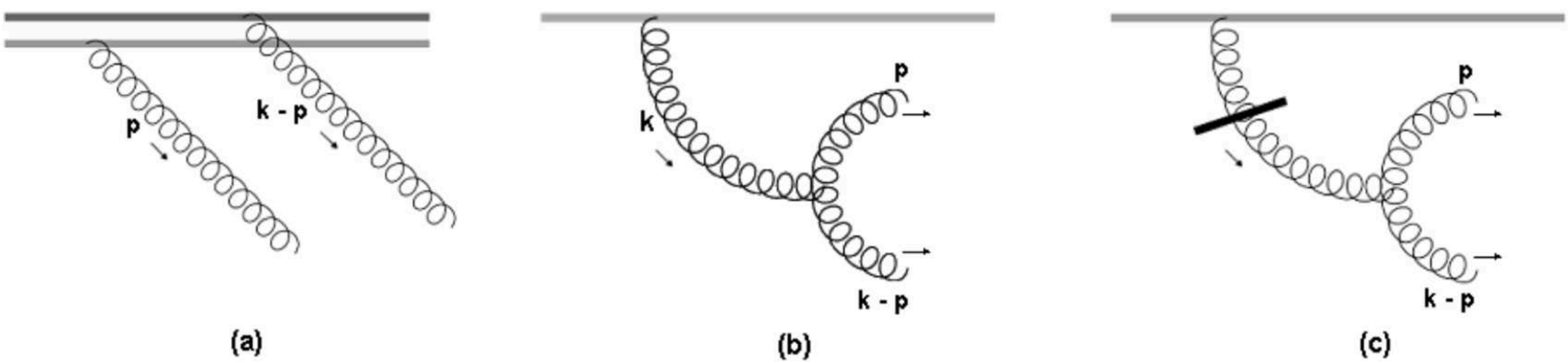
$$J^a_L(x) S^{ij}_A(z) = (T^a S_A(z))^{ij} \, \delta^2(x-z) \qquad J^a_R(x) S^{ij}_A(z) = (S_A(z) T^a)^{ij} \, \delta^2(x-z)$$

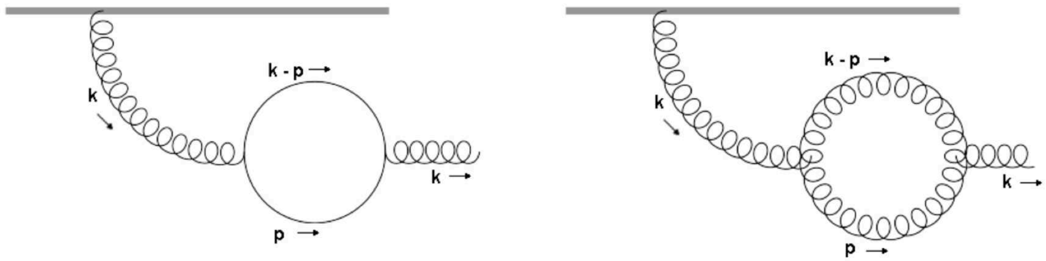
JIMWLK is valid for dilute-on-dense collisions only ($Q_s^P \ll Q_s^T$)

LCWF at NLO

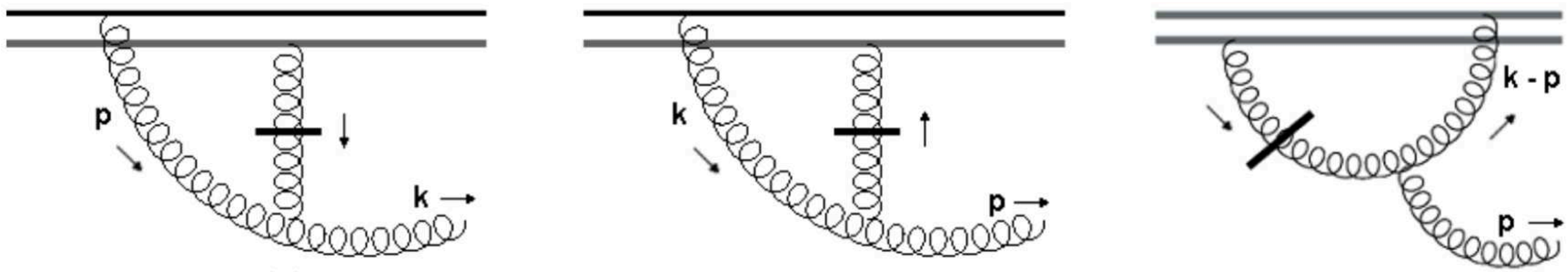
ML and Yair Mulian, arXiv:1610.03453



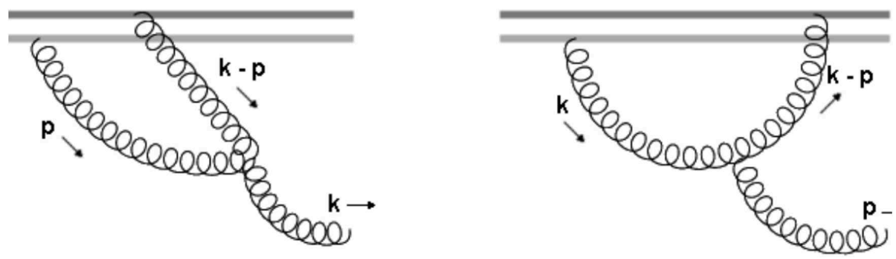




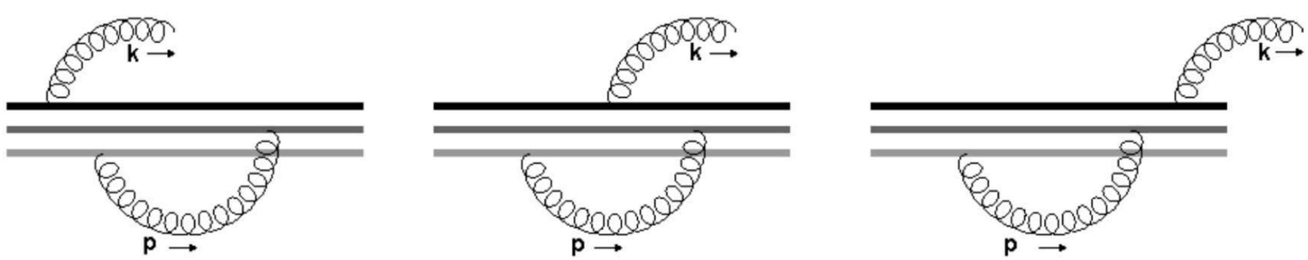
(a) (b)



(a) (b) (c)



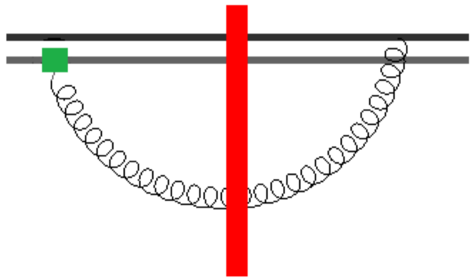
(a) (b)



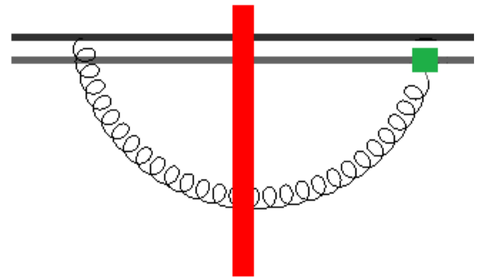
(a) (b) (c)

JIMWLK Hamiltonian @ NLO

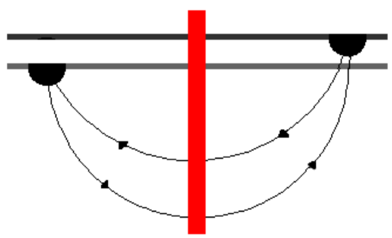
Σ_{JSJ}



Σ_{JSJ}

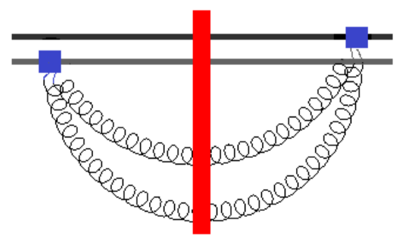


Σ_{qq}



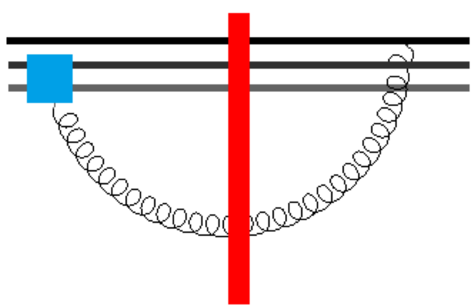
(a)

Σ_{JSSJ}

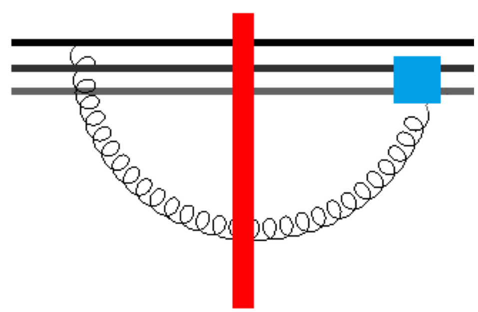


(b)

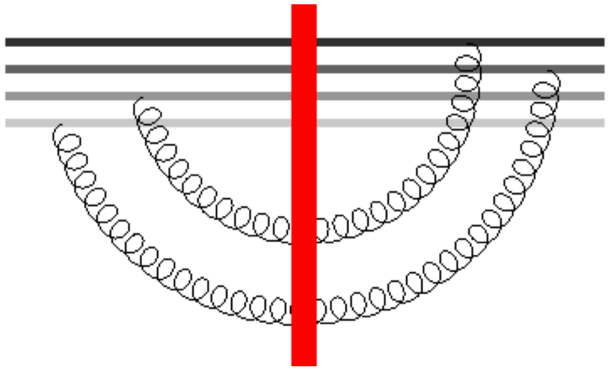
Σ_{JJSJ}



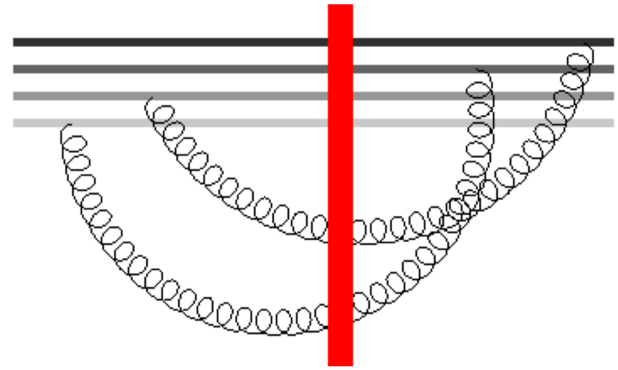
Σ_{JJSJ}



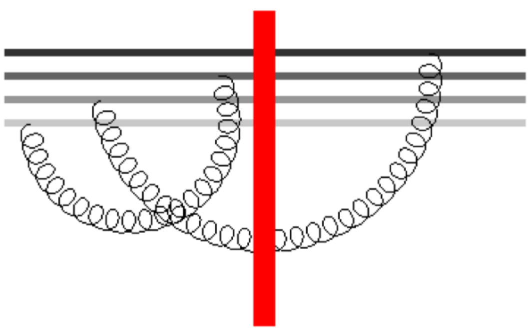
$\mathbf{\Sigma_{JJSSJJ}}$

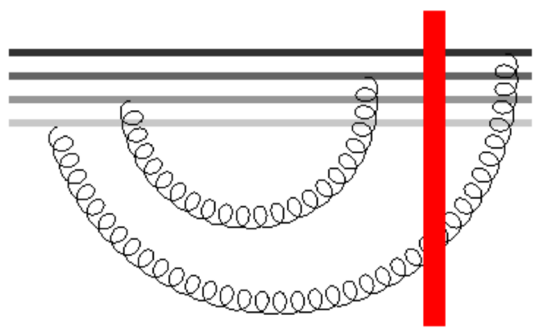


$\mathbf{\Sigma_{JJSSJJ}}$

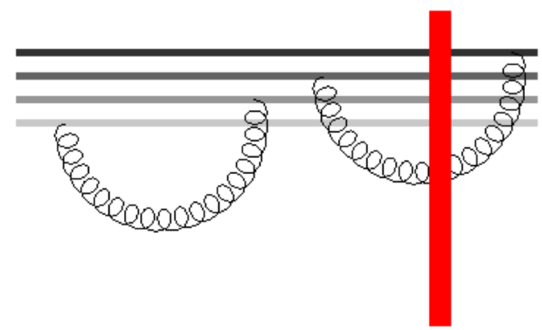


$\mathbf{\Sigma_{JJJSJ}}$



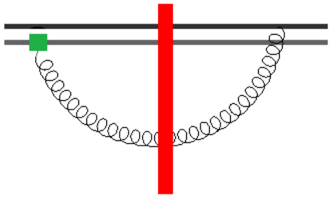


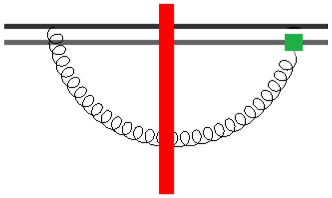
$\mathbf{\Sigma_{JJJSJ}}$

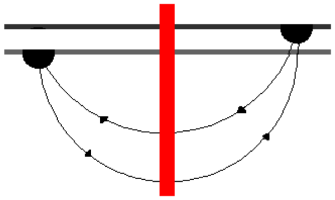


JIMWLK Hamiltonian @ NLO

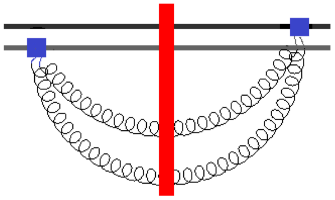
Kovner, ML & Mulian (2013) based on Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)







(a)



(b)

$$\mathcal{H}^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

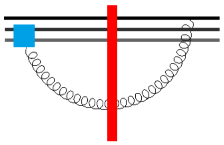
$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

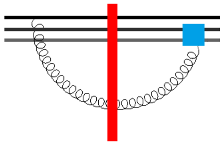
$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S_F^\dagger(z)\,t^a\,S_F(z')t^b]\,J_R^b(y) - J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\left[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\right]$$

$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$

$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w) - J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$





Motivation and Objectives

Precise saturation physics phenomenology at NLO is badly needed.

The JIMWLK Hamiltonian at NLO is known for some years, but there are problems there.

- **No known recipe for numerical evaluation**
- **Large transverse logarithms emerge:** $\mathcal{H} \sim \alpha_s(\# + \alpha_s(\# + \mathbf{Log}))$,
  If the Log is large, then $\alpha_s \, \mathbf{Log} \sim 1$ **– not a small correction to LO**
  There are various types of the large Logs there:
  running coupling effects, (Ioffe) time ordering, DGLAP logs.
  All have to be identified, clearly separated, and independently resummed.

LO JIMWLK kernel beyond LO

$$\mathcal{H} = \int_{x,y,z} \mathbf{K(x,y;z)} \left[\mathbf{J^a_L(x)\, J^a_L(y) \; + \; J^a_R(x) J^a_R(y) \; - \; 2J^a_L(x)\, S^{ab}_A(z)\, J^b_R(y)}\right]$$

An effective kernel $\mathbf{K} \; = \; \mathbf{K_{LO}} \; + \; \mathbf{K_{NLO}} \; + \; .... \sim \; \alpha_s(\# \; + \; \alpha_s\,(\# \, + \, \mathbf{Logs}) \, + \, \cdots)$

Large transverse logarithms emerge at NLO. There are various types of large Logs - all have to be identified, clearly separated, and independently resummed.
Proper resummation requires understanding of physics beyond NLO!

- Running coupling effects (UV divergent) – rcJIMWLK:

$$\mathbf{K_{LO}} \; = \; \frac{\alpha_s}{2\pi^2}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}} \; \rightarrow \; \mathbf{K_{rc}} \; = \; \frac{\alpha_s[\mathbf{running}]}{2\pi^2}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}$$

- DGLAP logs: Large transverse logs of the $\log(Q_s^T/Q_s^P)$ type (dilute-on-dense).

NLO Kernels (Large UV Logs only)

$$\mathbf{X} = \mathbf{x} - \mathbf{z}$$
$$Y = y - z$$

$$K_{JSJ}(\text{b terms}) = \frac{\alpha_s^2}{16\pi^3}\left\{-b\frac{(x-y)^2}{X^2Y^2}\ln(x-y)^2\mu^2 + \frac{b}{X^2}\ln Y^2\mu^2 + \frac{b}{Y^2}\ln X^2\mu^2\right\} + \cdots$$

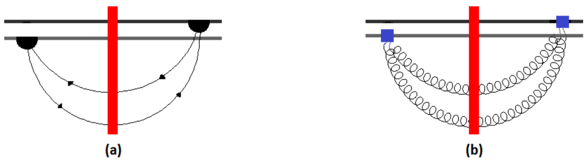
Here μ is the normalization point, $b = \frac{11}{3}N_c - \frac{2}{3}n_f$, $\quad b\,\ln Q^2/\mu^2 \rightarrow \alpha_s(Q^2)$

Huge ambiguity in identifying Q

Resum large Logs into an effective kernel $\quad K = K_{LO} + K_{JSJ} + \ldots.$

$$\int_{x\,y\,z,z'} K_{JSSJ}(x, y; z, z')\, J_L^a(x) J_R^b(y)\, \left[D^{ab}(z, z')\right] \quad \sim \quad b \times (\text{UV divergent Log})$$

$$D^{ab}(z, z') \equiv Tr[T^a S_A(z) T^b S_A^{+}(z')]$$



The UV divergence in $JSSJ$ is trivial: when the two gluons are too close to each other ($z \sim z'$), they cannot be resolved by the target and hence should be counted as a single gluon scattering. We are thus prompted to introduce a "resolution scale" Q

Dressed Wilson line

Within the finite resolution Q, bare gluons $\to$ dressed gluons,
bare Wilson lines $\to$ *dressed Wilson lines*, $\mathbf{S} \,\to\, \mathbf{S_Q}$

$$\mathbf{S^{ab}_Q(z) = S^{ab}_A(z) + \frac{\alpha_s}{2\pi^2}\int_0^1 d\xi\,\sigma(\xi)\int^{Q^{-1}}\frac{d^2Z}{Z^2}\Big(D^{ab}(z+(1-\xi)Z, z-\xi Z)\,-\,N_c\,S^{ab}_A(z)\Big)}$$

ξ is the fraction of longitudinal momentum carried by one of the gluons.

$$\sigma(\xi) = \left[\frac{1}{\xi(1-\xi)}\left(\xi^2+(1-\xi)^2+\xi^2(1-\xi)^2\right)\right]_+ ;\quad \mathbf{2N_c}\int_0^1 d\xi\sigma(\xi) \,=\, -\frac{\mathbf{11N_c}}{\mathbf{3}} \to -\,\mathrm{b}$$

This is the P_{gg} splitting function except that we introduce the "+" prescription both for $\xi=1$ and $\xi=0$ poles The "+" prescription emerges from the $1/\xi$ subtraction absorbed into (LO)^2 part of the evolution.

The sign is negative – correcting for the over-subtraction in the LO.

We go beyond the usual DGLAP: we allow simultaneous scattering of all gluons.

For $\mathrm{Q} > \mathrm{Q^T_s}$, $\mathrm{S_Q} \simeq \mathrm{S_A}$ - the target does not resolve gluon splitting
at distances smaller than $1/\mathrm{Q^T_s}$.

Resolution scale and the running coupling

Express S in terms of $\mathrm{S_Q}$ and substitute it into the LO+NLO JIMWLK Hamiltonian. $\mathcal{H}[\mathrm{S}] \to \mathcal{H}[\mathrm{S_Q}]$. The Hamiltonian will feature $\ln \mathbf{Q}^2$ terms such as $\ln(\mathbf{Q}^2\mathbf{X}^2)$.

$$\mathbf{K} \;=\; \mathbf{K_{LO}}\left(\mathbf{1}+\frac{\alpha_s}{\mathbf{4}\pi}\mathbf{b}\left(\ln \mathbf{X}^2\mu^2+\ln \mathbf{Y}^2\mu^2-\ln \mathbf{Q}^{-2}\mu^2\right)\right)\;+\;\mathrm{other}\,\mathrm{O}(\alpha_s^2)\,\mathrm{terms}$$

We assume existence of a typical scale $\mathrm{Q_s^P} \ll \mathrm{Q_s^T}$ associated with the projectile, such that $\ln(\mathrm{Q_s^P X^2})$ are small. The UV finite parts of the Hamiltonian proportional to b do not have any large Logs

$$\mathbf{K_{in}} \;=\; \mathbf{K}(\mathbf{Q}=\mathbf{Q_s^P}) \;=\; \frac{\sqrt{\alpha_s(\mathbf{X})\,\alpha_s(\mathbf{Y})}}{\mathbf{2}\pi^2}\,\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}\left[\mathbf{1}+\frac{\alpha_s}{\mathbf{8}\pi}\mathbf{b}\,(\mathrm{small\ logs})\right]$$

However, at $Q = Q_s^P$, $\mathrm{S_Q}$ is very different from $\mathrm{S_A}$, $\mathbf{S_Q} \sim \mathbf{S_A}\,[\mathbf{1} + \alpha_\mathrm{s}\,\#\,\mathbf{Log}(\mathbf{Q^2}/\mathbf{Q_s^T})]$. This large Log has to be resummed via inclusion of multiple consecutive DGLAP splittings:

$$\frac{\partial \mathbf{S_Q}(\mathbf{z})}{\partial \ln \mathbf{Q}} = -\frac{\alpha_\mathrm{s}}{\mathbf{2}\pi^\mathbf{2}} \int_\xi \sigma(\xi) \int_{\phi_\mathbf{Q}} [\mathbf{D_Q}(\mathbf{z}) \, - \, \mathbf{N_c}\, \mathbf{S_Q}(\mathbf{z})]$$

$$\mathbf{D_Q}(\mathbf{z_1}, \mathbf{z_2}) \equiv \mathbf{Tr}[\mathbf{T^a} \mathbf{S_Q}(\mathbf{z_1}) \mathbf{T^b} \mathbf{S^+_Q}(\mathbf{z_2})]$$

If we were to take $\mathrm{Q} = \mathrm{Q_s^T}$ then $\mathrm{S_Q} \simeq \mathrm{S_A}$ but the $\ln \mathrm{Q^2}$ terms in the Hamiltonian would be large and have to be resummed.

Either way, we have to resum large logs of the order $\log Q_s^T/Q_s^P$.

Functional RG

The resummed Hamiltonian should be Q-independent:

$$\frac{\mathbf{d}\mathcal{H}}{\mathbf{d}\ln\mathbf{Q}} = \frac{\partial\mathcal{H}}{\partial\ln\mathbf{Q}} + \int_{\mathbf{u}}\left[\frac{\delta\mathcal{H}}{\delta\mathbf{S}_{\mathbf{Q}}(\mathbf{u})}\frac{\partial\mathbf{S}_{\mathbf{Q}}(\mathbf{u})}{\partial\ln\mathbf{Q}}\right] = \mathbf{0}$$

DGLAP-like evolution for the Hamiltonian (evolution in the space of Hamiltonians):

$$\mathcal{H}[\mathbf{Q_s^P}] \;=\; \mathbf{Exp}\left[\int_{\mathbf{Q_s^P}}^{\mathbf{Q_s^T}}\frac{\mathbf{dQ}}{\mathbf{Q}}\mathbf{H}_{\mathbf{DGLAP}}\right]\mathcal{H}_{\mathbf{in}}$$

$$\mathbf{H}_{\mathbf{DGLAP}} \;=\; \frac{\alpha_{\mathbf{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{u}}\int_{\xi}\sigma(\xi)\int_{\phi_{\mathbf{Q}}}\mathbf{Tr}\left([\mathbf{D_Q}(\mathbf{u}) \;-\; \mathbf{N_c}\,\mathbf{S_Q}(\mathbf{u})]\,\frac{\delta}{\delta\mathbf{S_Q}(\mathbf{u})}\right)$$

$\mathbf{Q_s^P} = \mathbf{Q_s^P}(\eta)$ **– Q_s^P is dynamical (rapidity dependent); hence the resummed Hamiltonian is too.**

Weak target field approximation – linearization

$$\mathbf{S^{ab}_Q} = \delta^{ab} + \mathbf{f^{abc}}\alpha^{c}_{Q}\,; \qquad \mathbf{D^{ab}_Q(z_1, z_2) = N_c}\left(\delta^{ab} + \frac{1}{2}\mathbf{f^{abc}}\left[\alpha^{c}_{Q}(\mathbf{z_1}) + (\alpha^{c}_{Q}(\mathbf{z_2}))^{*}\right]\right)$$

Expand the Hamiltonian (BFKL-like)

$$\mathbf{H_{DGLAP}} \sim \alpha_{Q}\frac{\delta}{\delta\alpha_{Q}}$$

$\mathbf{H_{DGLAP}}$ is homogeneous and hence solvable

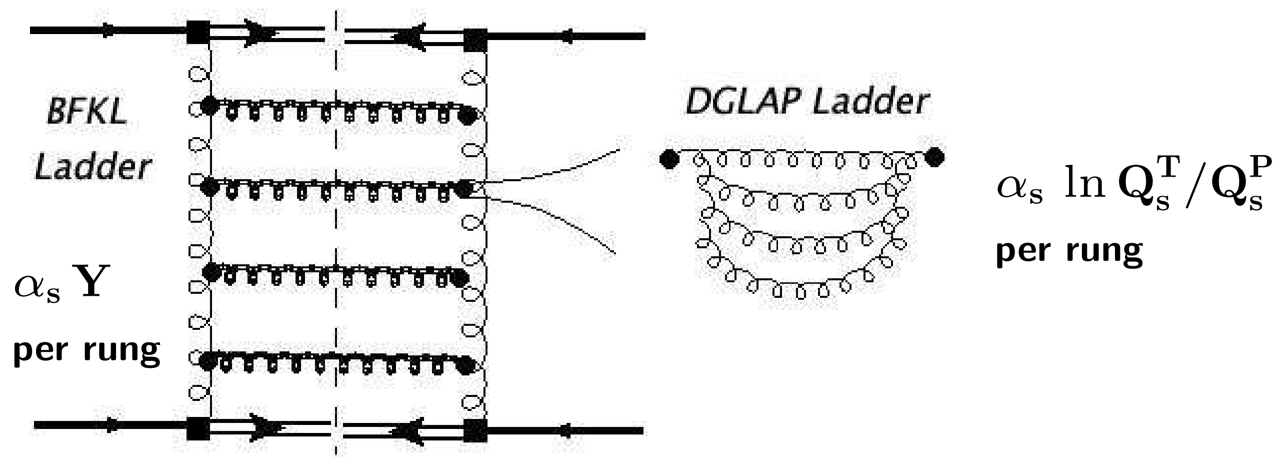
Saturation region

$$\mathbf{H_{DGLAP}} = \frac{\alpha_s}{2\pi^2}\int_{\mathbf{u}}\int_{\xi}\sigma(\xi)\int_{\phi_{Q}}\mathbf{Tr}\left(\left[\mathbf{Tr}[\mathbf{T^a S_Q(z_1) T^b S^{+}_Q(z_2)}]_{\mathbf{u}} - \mathbf{N_c\,S_Q(u)}\right]\frac{\delta}{\delta\mathbf{S_Q(u)}}\right)$$

Since $|\mathbf{z_1} - \mathbf{z_2}| = 1/\mathbf{Q} > 1/\mathbf{Q^T_s}$, the two gluons are well separated and outside the correlation region in the target (in a sense of averaging over the target). Neglect the first term. H_{DGLAP} is again homogeneous

Summary/Outlook

- DGLAP-like resummation inside the JIMWLK Hamiltonian has been performed. These DGLAP corrections are large whenever there is a large disparity between the correlation lengths (or saturation momenta) in the projectile and the target. This is precisely JIMWLK's regime of validity.
  The result is a smearing of the WW fields within the $1/Q_s^T$ distance



- rcJIMWLK emerges with the scale choice for the running coupling:
  $\mathbf{K} \sim \sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}$