Double parton scattering: a theory overview

M. Diehl

Deutsches Elektronen-Synchrotron DESY

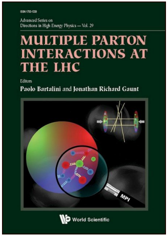
MITP program
"Next Generation Perturbative QCD for Hadron Structure"
21 October 2024

Plan:

- what is double parton scattering?
- main theory results and status
- only a glimpse at experiment
  unfortunately no time for phenomenology, modelling, lattice studies, nuclei

For more information see e.g.

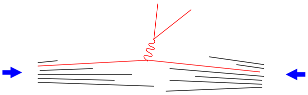
- most recent workshop on MPI@LHC:
  https://indico.cern.ch/event/1281679
- Multiple parton interactions at the LHC
  eds. P Bartalini and J R Gaunt, Dec. 2018
  individual chapters available on arXiv



Hadron-hadron collisions

- standard description based on factorisation formulae

  cross sect = parton distributions × parton-level cross sect

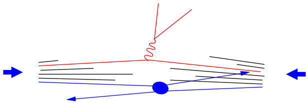


- factorisation formulae are for inclusive cross sections $pp \to A + X$
  where $A =$ produced by parton-level scattering, specified in detail
  $X =$ summed over, no questions asked

Hadron-hadron collisions

- standard description based on factorisation formulae

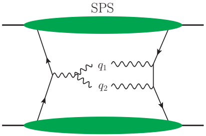
  cross sect = parton distributions × parton-level cross sect

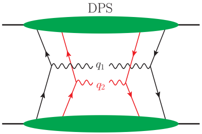


- factorisation formulae are for inclusive cross sections $pp \to A + X$
  where $A =$ produced by parton-level scattering, specified in detail
  $X =$ summed over, no questions asked
- spectator interactions
  - cancel in inclusive cross sections thanks to unitarity
  - can be soft $\rightsquigarrow$ part of underlying event (UE)
  - or hard $\rightsquigarrow$ multiple hard scattering
- double parton scattering $pp \to A_1 + A_2 + X$ with scales $Q_1, Q_2 \gg \Lambda$
  - have factorisation formula with double parton distributions
  - if $Q_1 \gg Q_2 \gg \Lambda$ $\rightsquigarrow$ part of underlying event for $pp \to A_1 + X$

Single vs. double hard scattering

- example: two gauge bosons with transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$





single scattering (SPS)

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$

double scattering (DPS)

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for transverse momenta $\sim \Lambda \ll Q$:

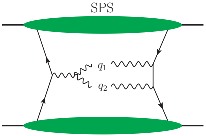
$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but SPS populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double hard scattering

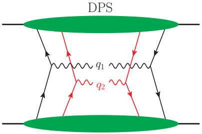
- example: two gauge bosons with transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering (SPS)

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$



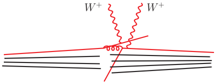
double scattering (DPS)

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

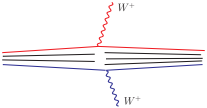
- DPS can be enhanced by
  - small parton mom. fractions x because of parton luminosity
    $\sigma_{\text{SPS}} \sim \text{PDF}^2$ and (roughly) $\sigma_{\text{DPS}} \sim \text{PDF}^4$
  - large rapidity separation ΔY between systems A_1 and A_2
    large invariant mass of overall system $\rightsquigarrow$ large x in SPS
  - coupling constants, etc.

A flagship DPS process: same-sign W pairs

$\sigma_{\text{SPS}} \propto \mathcal{O}(\alpha_s^2)$ with ≥ 2 jets



$\sigma_{\text{DPS}} \propto \mathcal{O}(\alpha_s^0)$



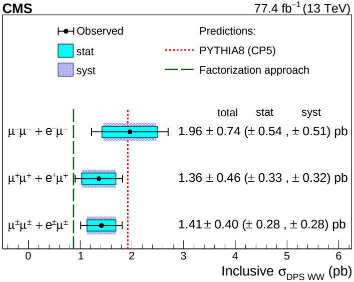
- with leptonic W decays $\rightsquigarrow$ same-sign lepton pairs and missing E_T

  is also a search channel for new physics e.g. SUSY, top partners

- several theory studies

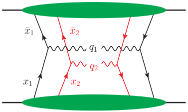
  Kulesza, Stirling 1999; Gaunt et al 2003;
  Ceccopieri, Rinaldi, Scopetta 2017;
  Cotogno, Kasemets, Myska 2018, 2020;
  Cabouat, Gaunt, Ostronlenk 2019

CMS, arXiv:1909.06265



includes veto $N_{\text{jets}} < 2$

DPS cross section: basic theory

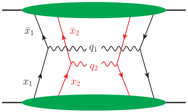


$$\frac{d\sigma_{\text{DPS}}^{A_1 A_2}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{1+\delta_{A_1 A_2}}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})\, F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$$\begin{aligned} \hat{\sigma}_i(x_i, \bar{x}_i) &= \text{ parton-level cross section for } a_i + b_i \to A_i \\ F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) &= \text{ double parton distribution (DPD)} \\ \boldsymbol{y} &= \text{ transverse distance between partons} \end{aligned}$$

- if compute $\hat{\sigma}_i$ at higher orders in α_s
  $\rightsquigarrow$ convolution integrals over x_i for $\hat{\sigma}_i$ and F, as in SPS
- tree-level formula from Feynman graphs and kinematic approximations
  Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2011
- all-order factorisation proof for double Drell-Yan
  Manohar, Waalewijn 2012; Vladimirov 2016, 2017; MD, Buffing, Gaunt, Kasemets, Nagar, Ostermeier, Plößl, Schäfer, Schönwald 2011–2018
  requires modification of above formula (more later)

DPS cross section: basic theory



$$\frac{d\sigma^{A_1 A_2}_{\text{DPS}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{1+\delta_{A_1 A_2}}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})\, F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

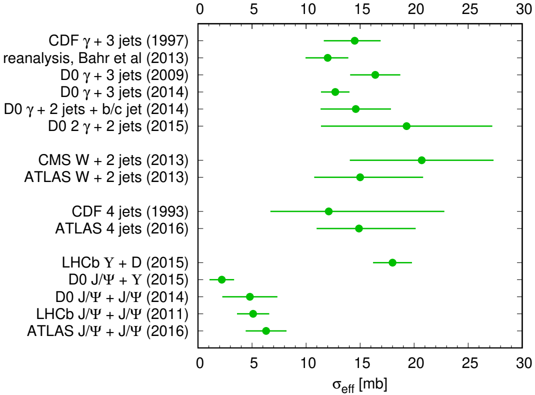
if assume $F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, G(\boldsymbol{y})$ $\Rightarrow$ pocket formula

$$\frac{d\sigma^{A_1 A_2}_{\text{DPS}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{\sigma_{\text{eff}}^{-1}}{1+\delta_{A_1 A_2}}\, \frac{d\sigma^{A_1}_{\text{SPS}}}{dx_1\, d\bar{x}_1}\, \frac{d\sigma^{A_2}_{\text{SPS}}}{dx_2\, d\bar{x}_2} \qquad \text{with} \quad \sigma_{\text{eff}}^{-1} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$$

- straightforward generalisation to N independent scatters
  underlies DPS and UE implementations in `PYTHIA`, `Herwig`, `Sherpa`
  with adjustments for conserving momentum and quark number
- underlies bulk of phenomenological estimates
  exceptions e.g. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013
- fails when the assumption on $F_{a_1 a_2}$ is invalid
  or when cross sect. formula misses important contributions (more later)

Experimental investigations (incomplete)

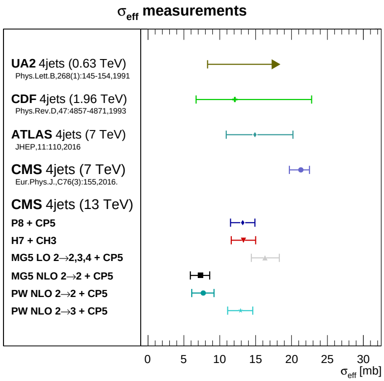
- a compilation of σ_{eff} values see also similar overview in PoS DIS2019 (2019) 258



- plus other studies not giving a σ_{eff} value
- cannot capture the physics of DPS in just one number σ_{eff}

Experimental investigations (incomplete)

- extraction of σ_{eff} can have significant theory uncertainties
- example: 4 jets, CMS, arXiv:2109.13822
  different values for 13 GeV differ by adopted theory description of SPS



Scale dependence

- PDFs and DPDs are matrix elements of twist-two operators $\mathcal{O}_a(\boldsymbol{y}, \mu)$

$$f_a(x;\mu) \sim \langle p|\,\mathcal{O}_a(\mathbf{0};\mu)\,|p\rangle$$

$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\,\mathcal{O}_{a_1}(\mathbf{0};\mu_1)\,\mathcal{O}_{a_2}(\boldsymbol{y};\mu_2)\,|p\rangle$$

$\mathcal{O}_q(\boldsymbol{z};\mu) \sim \bar{q}(\boldsymbol{z},\ldots)\gamma^+ q(\boldsymbol{z},\ldots)$
$\mathcal{O}_g(\boldsymbol{z};\mu) \sim F^{+i}(\boldsymbol{z},\ldots)\,F^{+i}(\boldsymbol{z},\ldots)$
renormalised at scale μ

Scale dependence

- PDFs and DPDs are matrix elements of twist-two operators $\mathcal{O}_a(\boldsymbol{y}, \mu)$

$$f_a(x; \mu) \sim \langle p | \mathcal{O}_a(\mathbf{0}; \mu) | p \rangle$$
$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p | \mathcal{O}_{a_1}(\mathbf{0}; \mu_1) \, \mathcal{O}_{a_2}(\boldsymbol{y}; \mu_2) | p \rangle$$

  $\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

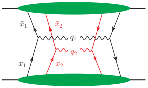
$$\frac{\partial}{\partial \log \mu_i^2} F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) = P \underset{x_i}{\otimes} F \qquad \text{for } i = 1, 2$$

  $P =$ same splitting functions as for usual PDFs

- factorisation scales μ_1, μ_2 can be different
- numerical implementation challenging
  many variables, many parton combinations (a_1, a_2)
  - different codes used in Gaunt, Stirling 2010 and later work and in Elias, Golec-Biernat, Staśto 2017
  - ongoing work on public evolution code ChiliPDF
    MD, Nagar, Plößl, Tackmann 2023
  - first implementation of a parton shower for DPS:
    Cabouat, Gaunt, Ostrolenk 2019; Cabouat, Gaunt 2020

Parton correlations

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{\hat{\sigma}_1\, \hat{\sigma}_2}{1+\delta_{A_1A_2}} \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

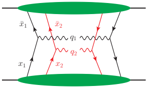


- between x_1 and x_2 — note that $x_1 + x_2 \leq 1$
  - expect $F(x_1, x_2, \boldsymbol{y}) \to 0$ for $x_1 + x_2 \to 1$
  - valence region: large correlations found in constituent quark model
    Rinaldi, Scopetta, Vento 2013
- between x_i and $\boldsymbol{y}$
  for single partons see correlation between x and impact parameter $\boldsymbol{b}$
  - diffraction: HERA results on $\gamma p \to J/\Psi\, p$ give
    $\langle \boldsymbol{b}^2 \rangle \propto \text{const} + 4\alpha' \log(1/x)$ with $4\alpha' \approx (0.16\,\text{fm})^2$
    for gluons with $x \sim 10^{-3}$
  - lattice simulations → strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x above ~ 0.1

  plausible to expect similar correlations in double parton distributions
  impact on DPS: Frankfurt, Strikman, Weiss 2003, Corke, Sjöstrand 2011, Blok, Gunnellini 2015

Parton correlations: spin

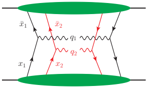
$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{\hat{\sigma}_1\, \hat{\sigma}_2}{1 + \delta_{A_1 A_2}} \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- polarisations of two partons can be correlated even in unpolarised proton
  - quarks: longitudinal and transverse pol.
  - gluons: longitudinal and linear pol.
- can be included in factorisation formula
  $\rightsquigarrow$ extra terms with polarised DPDs and partonic cross sections
- parton spin correlations can affect final state distributions in DPS
  gauge boson pairs: Manohar, Waalewijn 2011; Kasemets, MD 2012
  double charm: Echevarria, Kasemets, Mulders, Pisano arXiv:1501.07291
- large correlations seen in MIT bag model Chang, Manohar, Waalewijn 2012
  most correlations found small in lattice study Bali et al 2021
  for low x_1, x_2 size of correlations unknown
- evolution to high scales tends to wash out spin correlations
  unpol. densities evolve faster than polarised ones MD, Kasemets 2014

Parton correlations: colour

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{\hat{\sigma}_1\, \hat{\sigma}_2}{1+\delta_{A_1 A_2}} \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

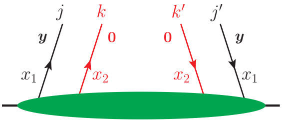


- colour of two quarks and gluons can be correlated
- can again be included in factorisation formula
  $\rightsquigarrow$ extra terms with colour-dependent DPDs and partonic cross sections
- colour correlated DPDs evolve differently than ordinary PDFs
  - evolution to higher scales $\rightsquigarrow$ Sudakov logarithms $\rightsquigarrow$ suppression
- colour correlated DPDs depend on rapidity parameter ζ
  - physically: $\log\sqrt{\zeta} \leftrightarrow$ rapidity separation when including soft gluons in one or the other DPD
  - same construction as for TMDs

Mekhfi 1988; Manohar, Waalewijn 2012
Diehl, Fabry, Vladimirov 2022; Diehl, Fabry, Plößl 2023

Colour structure of DPDs

- project colour of partons on SU(N_c) representations



- singlet: $P_1^{jj',kk'} = \delta^{jj'}\delta^{kk'}/N_c$
  as in usual PDFs
- octet: $P_8^{jj',kk'} = 2t_a^{jj'}t_a^{kk'}$
- for gluons: $8^A, 8^S, 10, \overline{10}, 27$

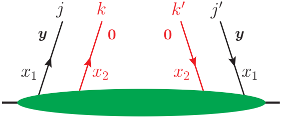
- notation: ${}^{R_1R_2}F_{a_1a_2}$ for parton a_i in colour representation R_i
- DGLAP evolution in μ_1 and μ_2
  - kernels depend on R
    equal to PDF kernels for $R = 1$
  - does not mix ${}^{R_1R_2}F_{a_1a_2}$ with different dimensions $\dim(R_i)$
  - flavour mixing:

$$ {}^{SR_2}F_{ga_2} \longleftrightarrow {}^{8R_2}F_{qa_2} + {}^{8R_2}F_{\bar{q}a_2} $$

$$ {}^{AR_2}F_{ga_2} \longleftrightarrow {}^{8R_2}F_{qa_2} - {}^{8R_2}F_{\bar{q}a_2} $$

Colour structure of DPDs

- project colour of partons on SU(N_c) representations

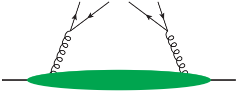


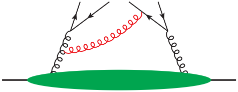
- singlet: $P_1^{jj',kk'} = \delta^{jj'}\delta^{kk'}/N_c$ as in usual PDFs
- octet: $P_8^{jj',kk'} = 2t_a^{jj'}t_a^{kk'}$
- for gluons: $8^A, 8^S, 10, \overline{10}, 27$

- notation: ${}^{R_1 R_2}F_{a_1 a_2}$ for parton a_i in colour representation R_i
- Collins-Soper evolution in ζ
  - same form as for TMDs
  - evolution kernel depends on transverse distance
  - (kernel for $R = 8$ DPDs) = (kernel for gluon TMDs)

Behaviour at small interparton distance

- for $\boldsymbol{y} \ll 1/\Lambda$ in perturbative region:
  $F(x_1, x_2, \boldsymbol{y})$ is dominated by graphs with splitting of single parton



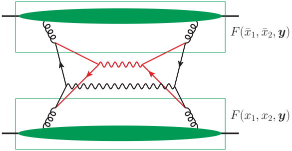


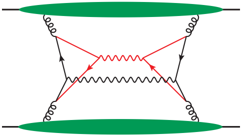
- gives strong correlations in x_1, x_2, spin and colour between two partons
  e.g. -100% correlation for longitudinal pol. of q and $\bar{q}$
- can compute short-distance behaviour:

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

calculated at NLO: MD, J Gaunt, P Plößl 2019, 2021
finite quark mass effects: MD, R Nagar, P Plößl 2022

Problems with the splitting graphs





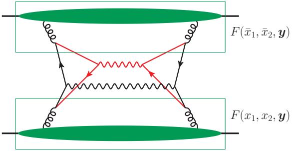
- splitting contribution to DPS cross section ⇒ UV divergent integrals

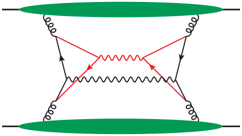
  $$\int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \sim \int d\boldsymbol{y}^2/\boldsymbol{y}^4$$

  in fact, formula is not valid for $|\boldsymbol{y}| \sim 1/Q$
- double counting between DPS with splitting and SPS at higher loops

MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
already noted by Cacciari, Salam, Sapeta 2009

Problems with the splitting graphs





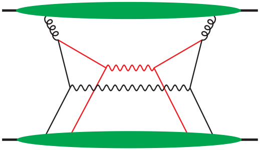
- splitting contribution to DPS cross section $\Rightarrow$ UV divergent integrals

$$\int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \sim \int d\boldsymbol{y}^2/\boldsymbol{y}^4$$

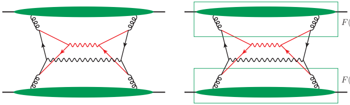
  in fact, formula is not valid for $|\boldsymbol{y}| \sim 1/Q$
- also have graphs with splitting in only one proton

$$\sim \int d\boldsymbol{y}^2/\boldsymbol{y}^2 \;\times F_{\textit{no split}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



Combining DPS with SPS



- how to delineate DPS from SPS is a matter of definition/scheme choice
- intuitively: small $|\boldsymbol{y}| \sim 1/Q$ is SPS, large $|\boldsymbol{y}| \gg 1/Q$ is DPS
- scheme worked out in MD, Gaunt, Schönwald 2017
  - σ_{SPS} defined as usual $\Rightarrow$ no new calculation needed
  - define σ_{DPS} with cutoff $|\boldsymbol{y}| > 1/\nu$ in factorisation formula
    separation scale ν, similar to factorisation scale μ
  - full cross section:

$$\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}}(\nu) - \sigma_{\text{sub}}(\nu)$$

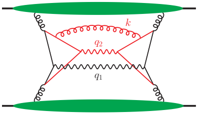
  - double counting removed by σ_{sub}
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in perturb. theory
    ν dependence cancels between σ_{DPS} and σ_{sub}
    up to higher-order terms beyond accuracy of the calculation
- other proposed treatments: Blok, Dokshitzer, Frankfurt, Strikman 2012, 2013; Ryskin, Snigirev 2011, 2012; Manohar, Waalewijn 2012

Parton splitting and evolution

DPD splitting and DGLAP evolution for partons 1 and 2:

$$F^{\text{spl}}(x_i, \boldsymbol{y}; \mu, \mu) = \frac{1}{y^2} \text{ splitting kernel} \times f(x_1 + x_2; \mu)$$

$$\frac{d}{d \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$$



- natural scale to evaluate DPD splitting formula: $\mu_1 = \mu_2 \sim 1/y$
- DGLAP equations resum logarithms / ladder graphs
  - from scales Λ to $1/y$ in PDFs
  - from scales $1/y$ to Q_1 or Q_2 in DPDs

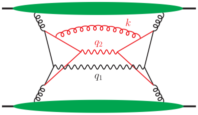
  SPS region: $y \sim 1/Q_i$, DGLAP logarithms only in PDFs
- colour singlet DPDs:
  - evolution tends to flatten $1/y^2$ behaviour of F^{spl}
    $\rightsquigarrow$ can enhance DPS region $y \gg 1/Q_i$ in cross section

Parton splitting and evolution

DPD splitting and DGLAP evolution for partons 1 and 2:

$$F^{\text{spl}}(x_i, \boldsymbol{y}; \mu, \mu) = \frac{1}{y^2} \text{ splitting kernel} \times f(x_1 + x_2; \mu)$$

$$\frac{d}{d \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$$

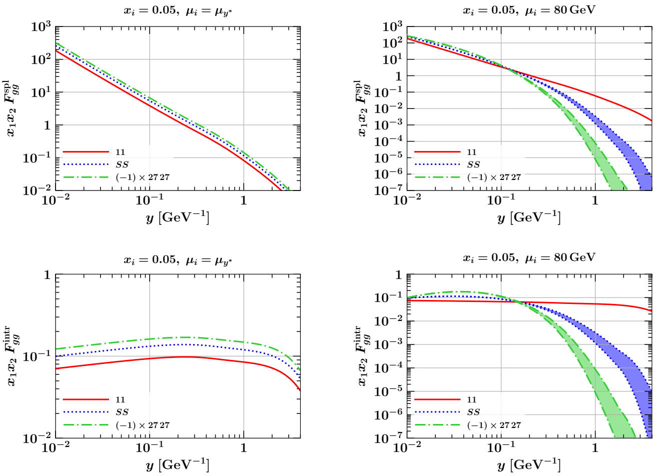


- natural scale to evaluate DPD splitting formula: $\mu_1 = \mu_2 \sim 1/y$
- DGLAP equations resum logarithms / ladder graphs
  - from scales Λ to $1/y$ in PDFs
  - from scales $1/y$ to Q_1 or Q_2 in DPDs

  SPS region: $y \sim 1/Q_i$, DGLAP logarithms only in PDFs
- colour non-singlet DPDs:
  - Sudakov suppression from evolution avoided for y not too far away from $1/Q_i$

Numerical study

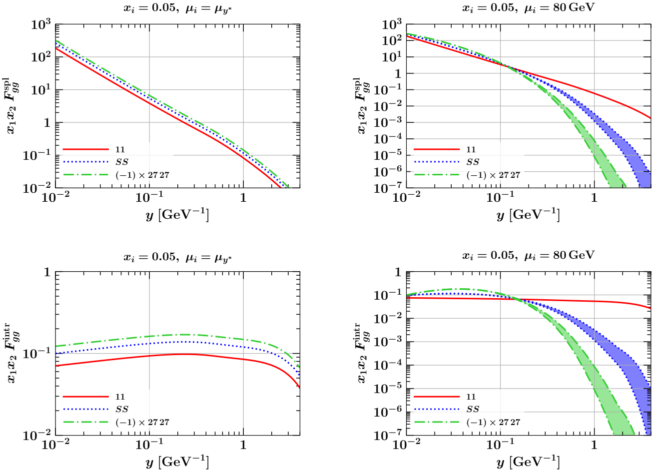
MD, F Fabry, P Plößl arXiv:2310.16432



- plot DPDs for $\mu_1 = \mu_2$ and $\zeta = \mu_1 \mu_2 / (x_1 x_2)$
- initial scale μ_{y^*} is $\sim 1/y$ at small y, saturates at large y

Numerical study

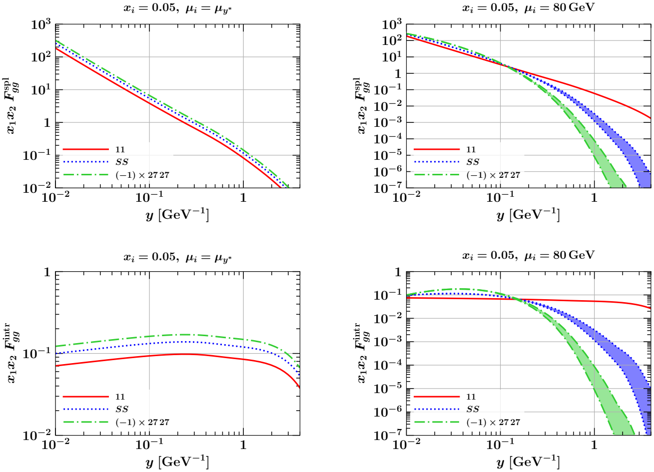
MD, F Fabry, P Plößl arXiv:2310.16432



- at μ_{y^*} use model Ansatz for F^{intr} and for F^{spl} at large y
- bands: range of models for Collins-Soper kernel

Numerical study

MD, F Fabry, P Plößl arXiv:2310.16432



- large y: increasingly strong suppression of colour non-singlets
- small y: little evolution from μ_{y^*} to final μ_i $\rightsquigarrow$ no suppression

NLO effects

MD, P Plößl, in preparation

colour singlet DPDs at $x_{1,2} = 0.01$, evolved from μ_{y^*} to $\mu_{1,2} = 80\,\text{GeV}$

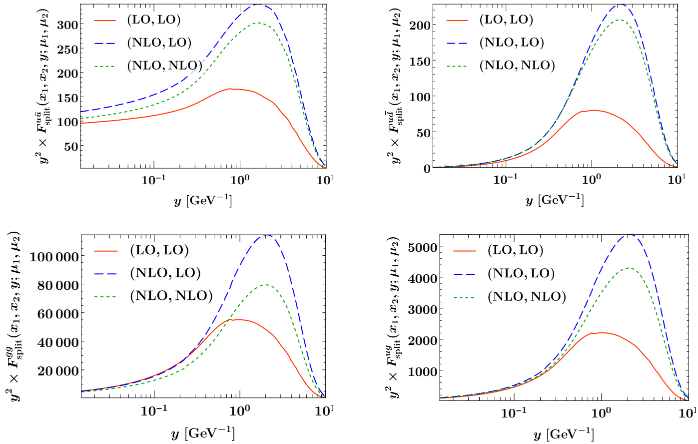


figure keys:

(order of DPD splitting, order of DGLAP evolution for PDFs and DPDs)

Summary

- have a detailed theory for double parton scattering
  largely on a par with single parton scattering
- growing number of NLO calculations available
  and can directly use many N^{k}LO results for SPS
- sensitive to parton structure in ways that cannot easily probe otherwise:
  - transverses distance between two partons
  - parton-parton correlations
- depending on the process, splitting $1 \to 2$ partons can play essential role
  $\rightsquigarrow$ close interplay between DPS and loop corrections to SPS

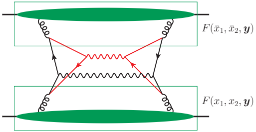
Backup slides

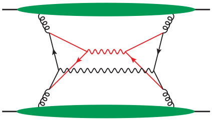
More experimental investigations

LHC studies:

- run 1
  - double open charm $(D^0, D^+, D_s^+, \Lambda_c^+)$ and $J/\Psi+$ open charm — LHCb 2012
  - the same in p-Pb collisions — LHCb 2020
  - $\Upsilon + \Upsilon$ $(\sigma_{\text{eff}} \approx 2.2 \div 6.6\,\text{mb})$ — CMS 2016
  - $W + J/\Psi$ — ATLAS 2014, 2019
  - $Z + J/\Psi$ (limit on σ_{eff}) — ATLAS 2014
  - 4 leptons (limit on σ_{eff}) — ATLAS 2018
  - same-sign WW (limit on σ_{eff}) — CMS 2017
- run 2
  - $J/\Psi + J/\Psi$ $(\sigma_{\text{eff}} \approx 8.8 \div 12.5\,\text{mb})$ — LHCb 2016
  - Z + jets — CMS 2021
  - same-sign WW $(\sigma_{\text{eff}} \approx 12.2^{+2.9}_{-2.2}\,\text{mb})$ — CMS 2019, 2023
  - 4 jets (range of σ_{eff} values) — CMS 2022

Subtraction formalism at work





$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \sim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \quad \sigma \approx \sigma_{\text{SPS}}$
  ν dependence cancels between σ_{DPS} and σ_{sub} up to higher order terms
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \quad \sigma \approx \sigma_{\text{DPS}}$
- same argument for 2v1 term and $\sigma_{\text{tw2}\times\text{tw4}}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory

J Collins, Foundations of Perturbative QCD, Chapt. 10

Evolution of colour dependent DPDs

- Collins-Soper equation:

$$\frac{d}{d\ln\sqrt{\zeta}}\,{}^{R_1 R_2}F_{a_1 a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)$$
$$= {}^{R_1}J(y;\mu_1,\mu_2)\,{}^{R_1 R_2}F_{a_1 a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)\,.$$

with RGE $\quad \frac{d}{d\ln\mu_1}\,{}^{R}J(y;\mu_1,\mu_2) = -\,{}^{R}\gamma_J(\mu_1)$

- Collins-Soper kernel = derivative of soft factor w.r.t. rapidity
- for colour singlet: ${}^{1}J = 0 \;\rightsquigarrow\;$ no ζ dependence
- same form for TMDs

$$ {}^{R_1 R_2}F_{a_1 a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta) \to f_a(x,b;\mu,\zeta)$$
$$ {}^{R}J(y;\mu_1,\mu_2) \to K_a(b;\mu)$$
$$ {}^{R}\gamma_J(\mu) \to \gamma_{K,a}(\mu)$$

remarkably ${}^{8}J(b;\mu,\mu) = K_g(b;\mu)$ A Vladimirov 2018

- can be solved analytically

Evolution of colour dependent DPDs

- DGLAP equations:

$$\frac{d}{d\ln\mu_1}\, {}^{R_1R_2}F_{a_1a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)$$
$$= -\, {}^{R_1}\gamma_J(\mu_1)\, \ln\left(\frac{x_1\sqrt{\zeta}}{\mu_1}\right)\, {}^{R_1R_2}F_{a_1a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)$$
$$+ 2\sum_{b_1,R_1'}\int_{x_1}^{1}\frac{dz}{z}\, {}^{R_1\overline{R}_1'}P_{a_1b_1}\left(\frac{x_1}{z};\mu_1\right)\, {}^{R_1'R_2}F_{b_1a_2}(z,x_2,y;\mu_1,\mu_2,\zeta)$$

likewise for μ_2 dependence

- kernels ${}^{RR'}P$ known at NLO — F Fabry, MD, A Vladimirov 2022
- similar form for TMDs:

$$\frac{d}{d\ln\mu_1}\, f_a(x,b;\mu,\zeta)$$
$$= -\,\gamma_{K,a}(\mu)\ln\left(\frac{x\sqrt{\zeta}}{\mu}\right)\, f_a(x,b;\mu,\zeta)$$
$$+\,\gamma_a(\mu)\, f_a(x;\mu,\zeta)$$

Evolution of colour dependent DPDs

- DGLAP equations:

$$\frac{d}{d\ln\mu_1}\, {}^{R_1 R_2}F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta)$$
$$= -\, {}^{R_1}\gamma_J(\mu_1)\, \ln\!\left(\frac{x_1\sqrt{\zeta}}{\mu_1}\right)\, {}^{R_1 R_2}F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta)$$
$$+\, 2\sum_{b_1, R_1'} \int_{x_1}^{1} \frac{dz}{z}\, {}^{R_1 \overline{R}_1'}P_{a_1 b_1}\!\left(\frac{x_1}{z}; \mu_1\right)\, {}^{R_1' R_2}F_{b_1 a_2}(z, x_2, y; \mu_1, \mu_2, \zeta)$$

likewise for μ_2 dependence

- kernels ${}^{RR'}P$ known at NLO — F Fabry, MD, A Vladimirov 2022
- γ_J term $\rightsquigarrow$ Sudakov logarithms