

# The role of dark matter in supermassive black hole mergers

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Based on [\[2401.14450\]](#) with Caitlyn Dewar & Jim Cline



Physics  
UNIVERSITY OF TORONTO

# Supermassive black holes

## Jets (M87)



Image credit: HST

## AGNs (3C 273)



Image credit: SDSS

## Stellar dynamics (Sgr A\*)

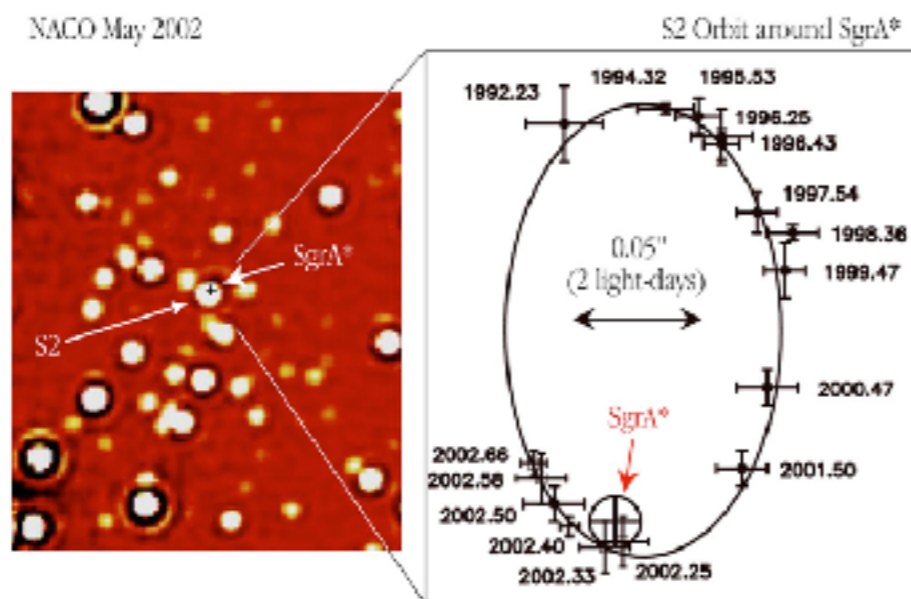


Image credit: ESO

## Direct imaging (Sgr A\*)

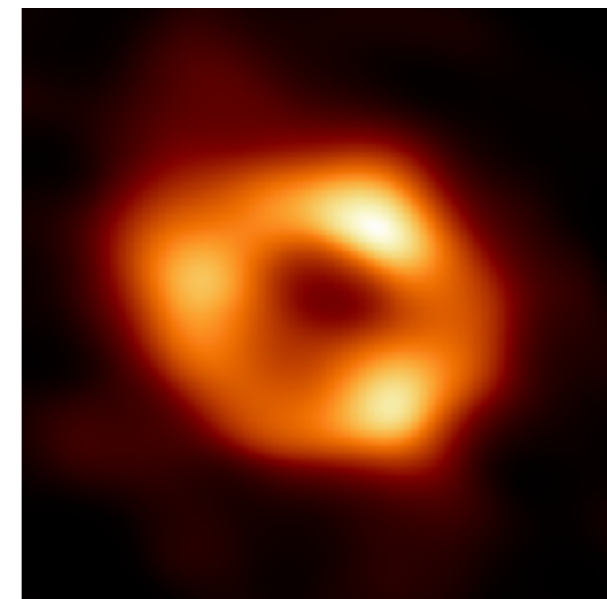


Image credit: EHT

# Galaxies are built via mergers

## Structure formation

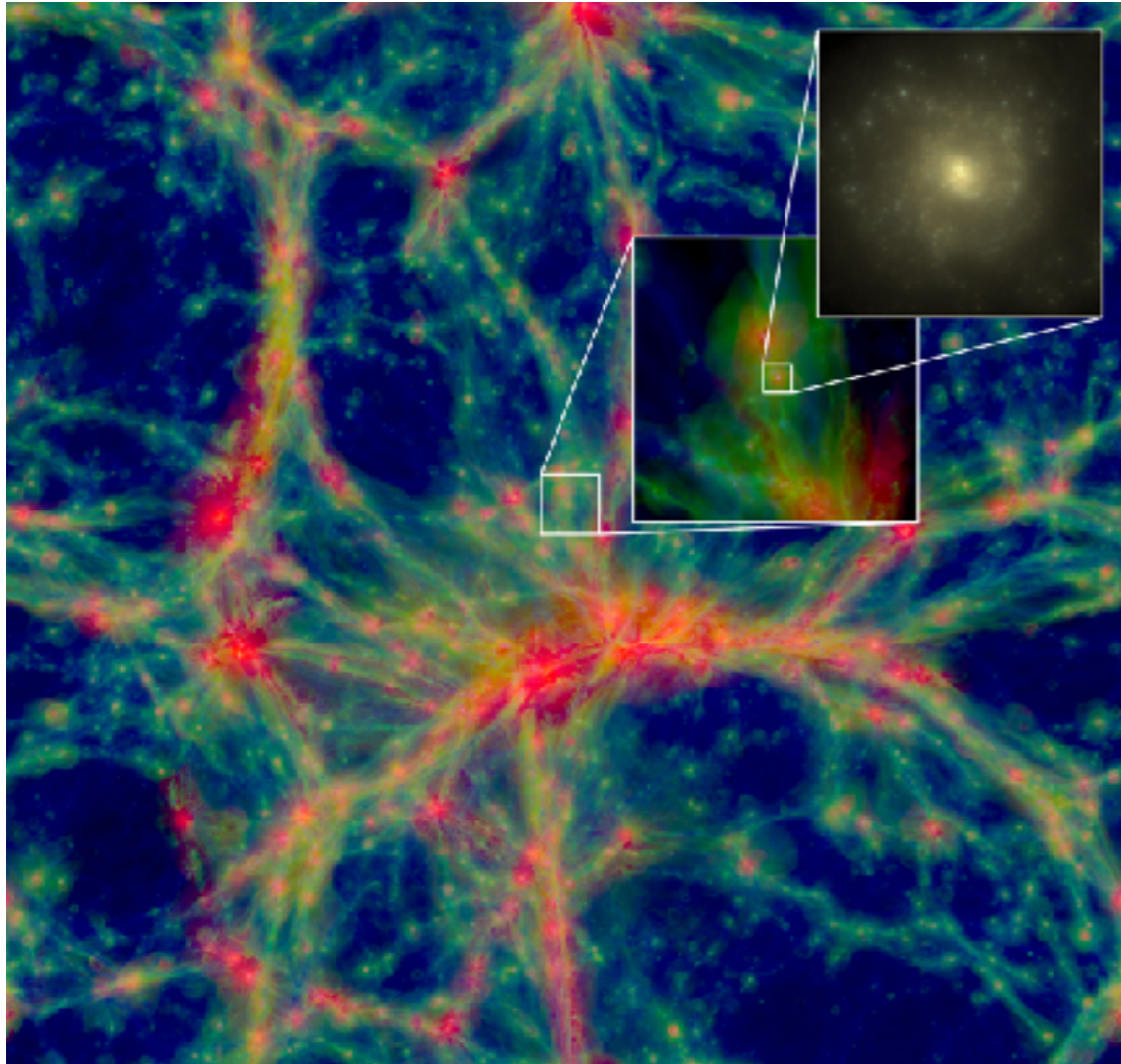


Image credit: EAGLE simulations

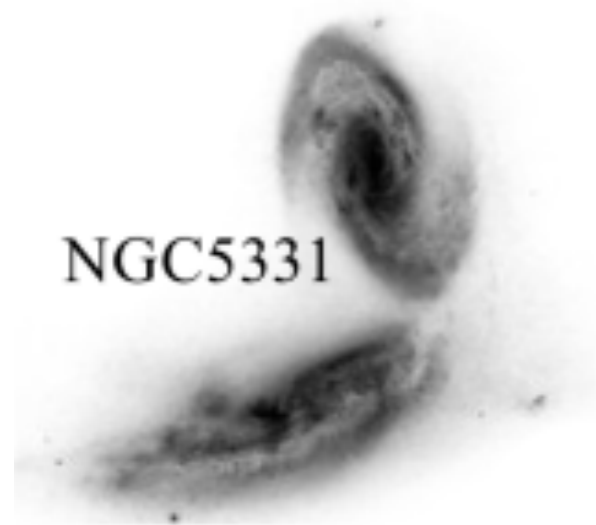
## Galaxy merger



Image credit: HST

# Supermassive black holes mergers

## Galaxy Merger



NGC5331

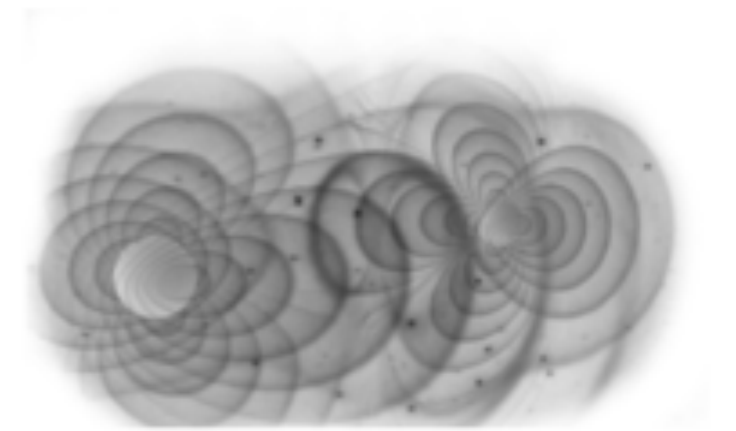
Dynamical friction drives massive objects to central positions

## Binary Formation



Stellar and gas interactions may dominate binary inspiral?

## Continuous GWs



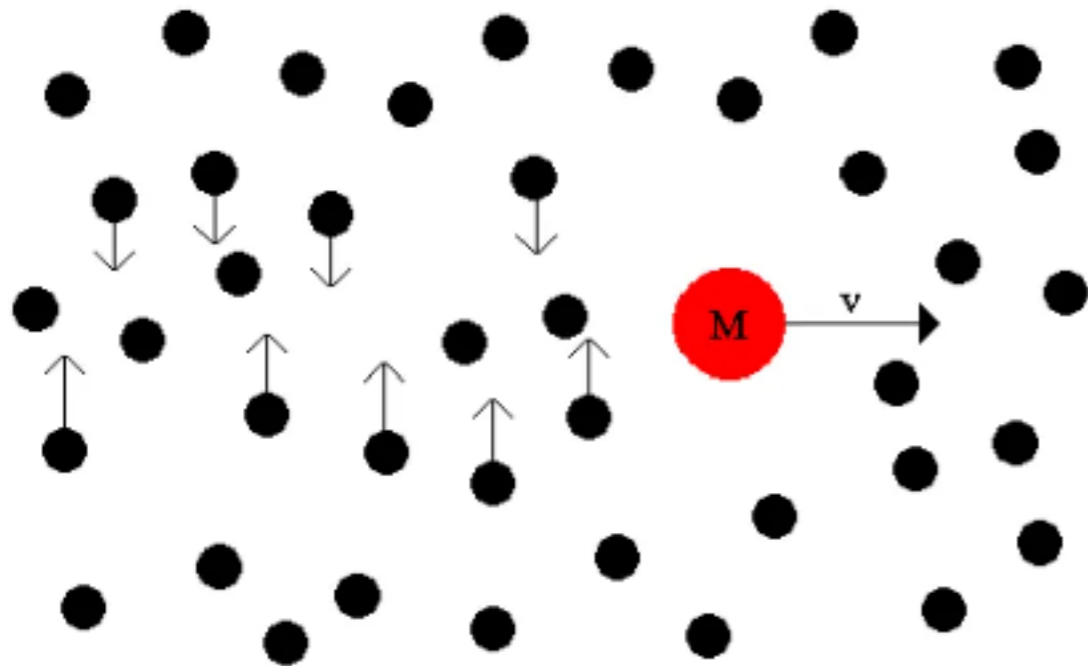
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.

**Adapted from NANOGrav (designed by Sarah Spolaor)**



# Dynamical friction

SMBH creates stellar/gas overdensity behind it



Gravitational pull of the wake slows down SMBH

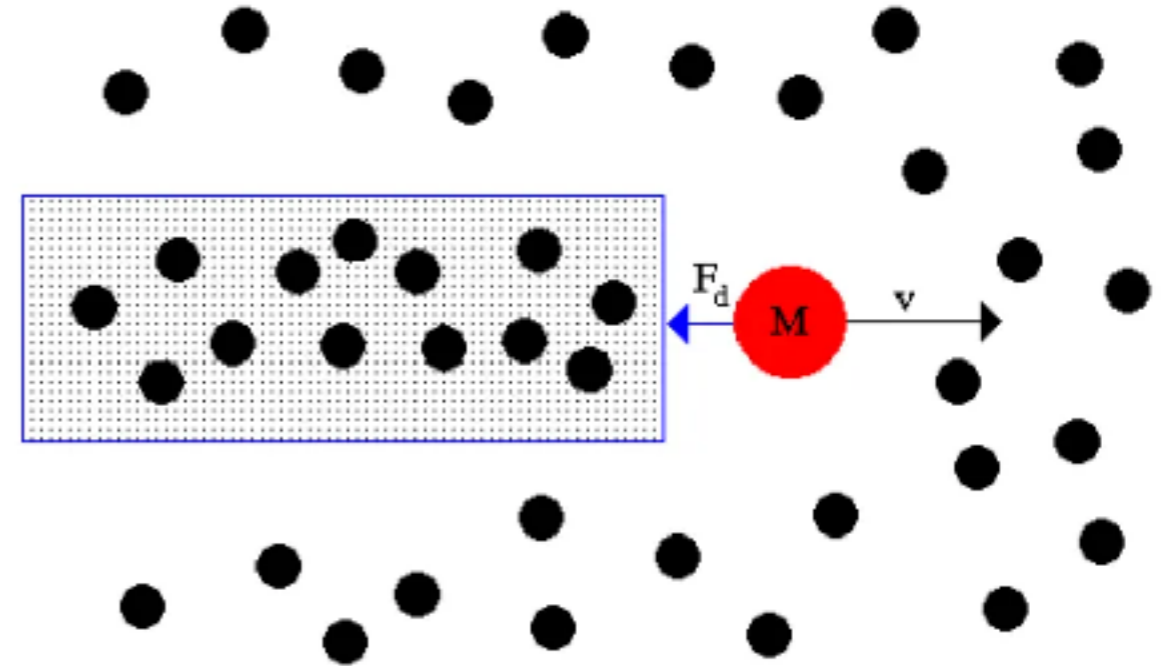
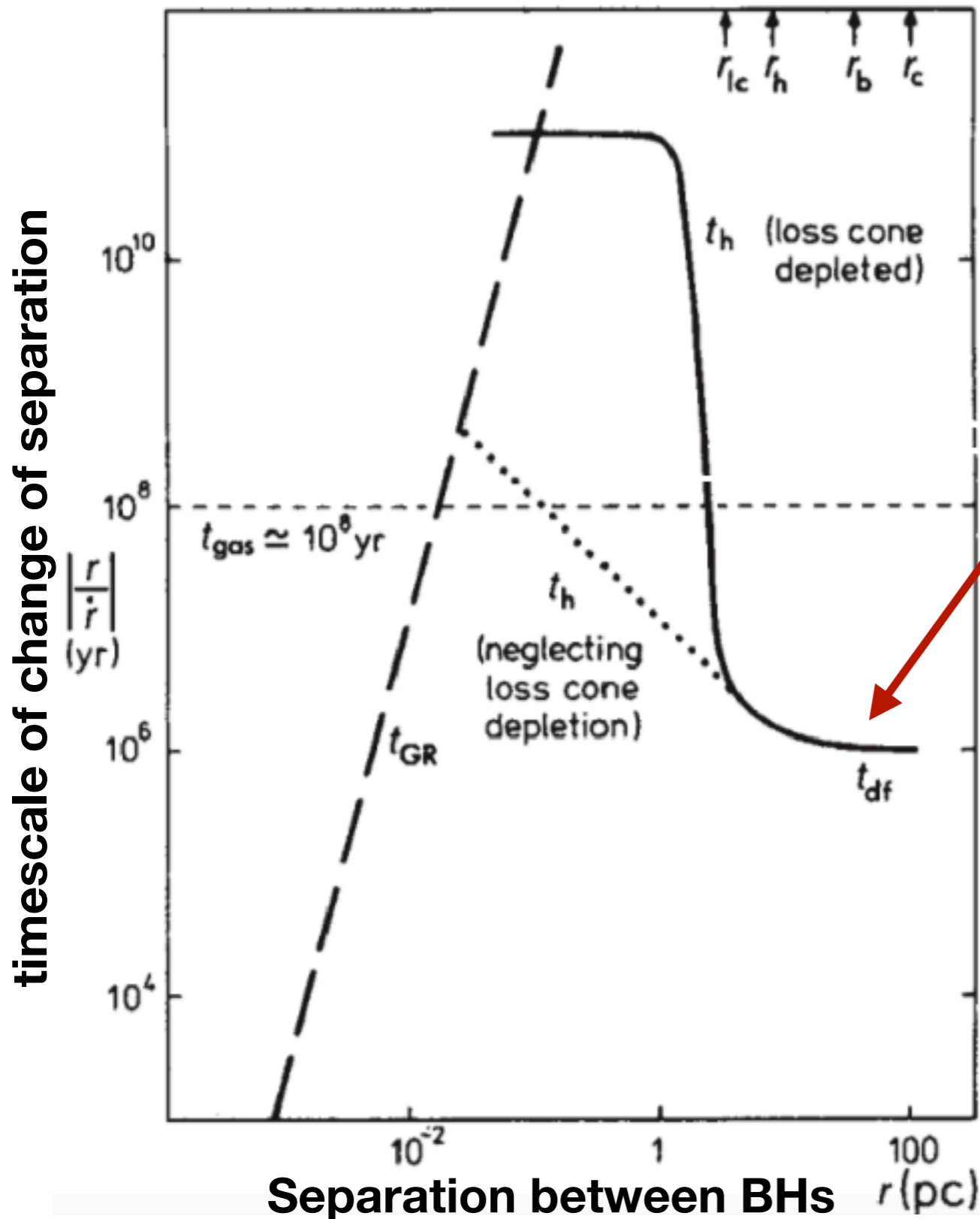


Image credit: J. Schombert (adapted)

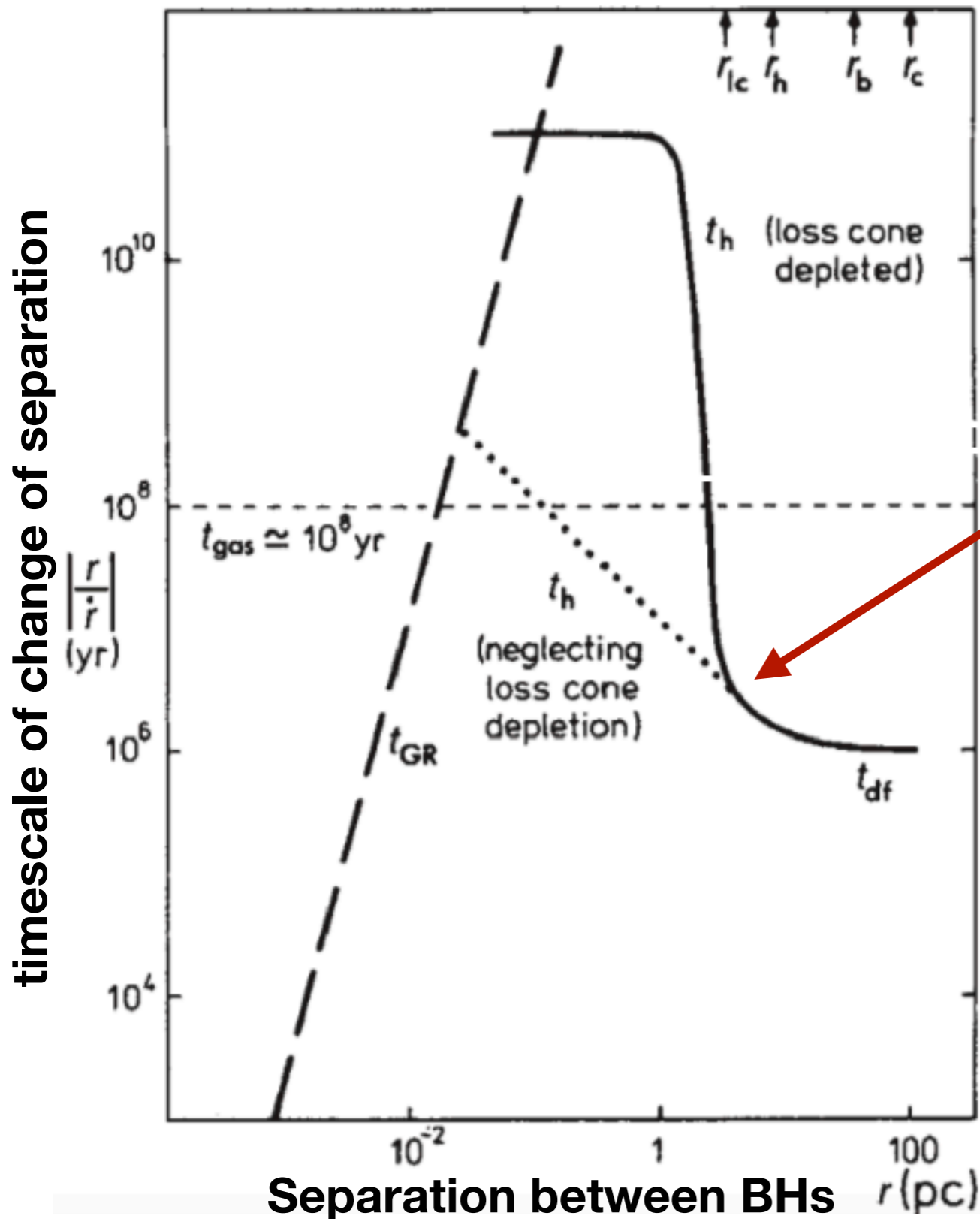
# The final parsec problem



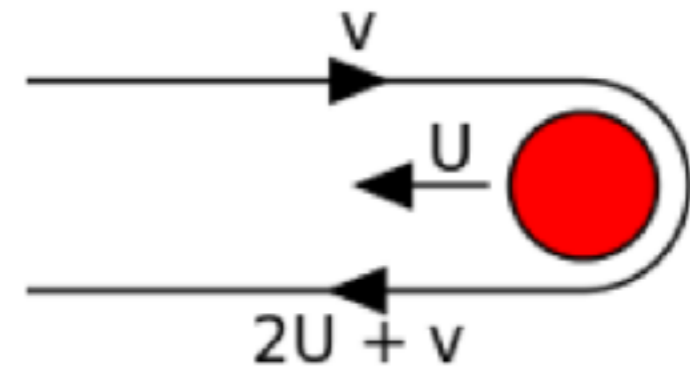
1. Dynamical friction shrinks binary

Begelman, Blandford & Rees (1980)

# The final parsec problem

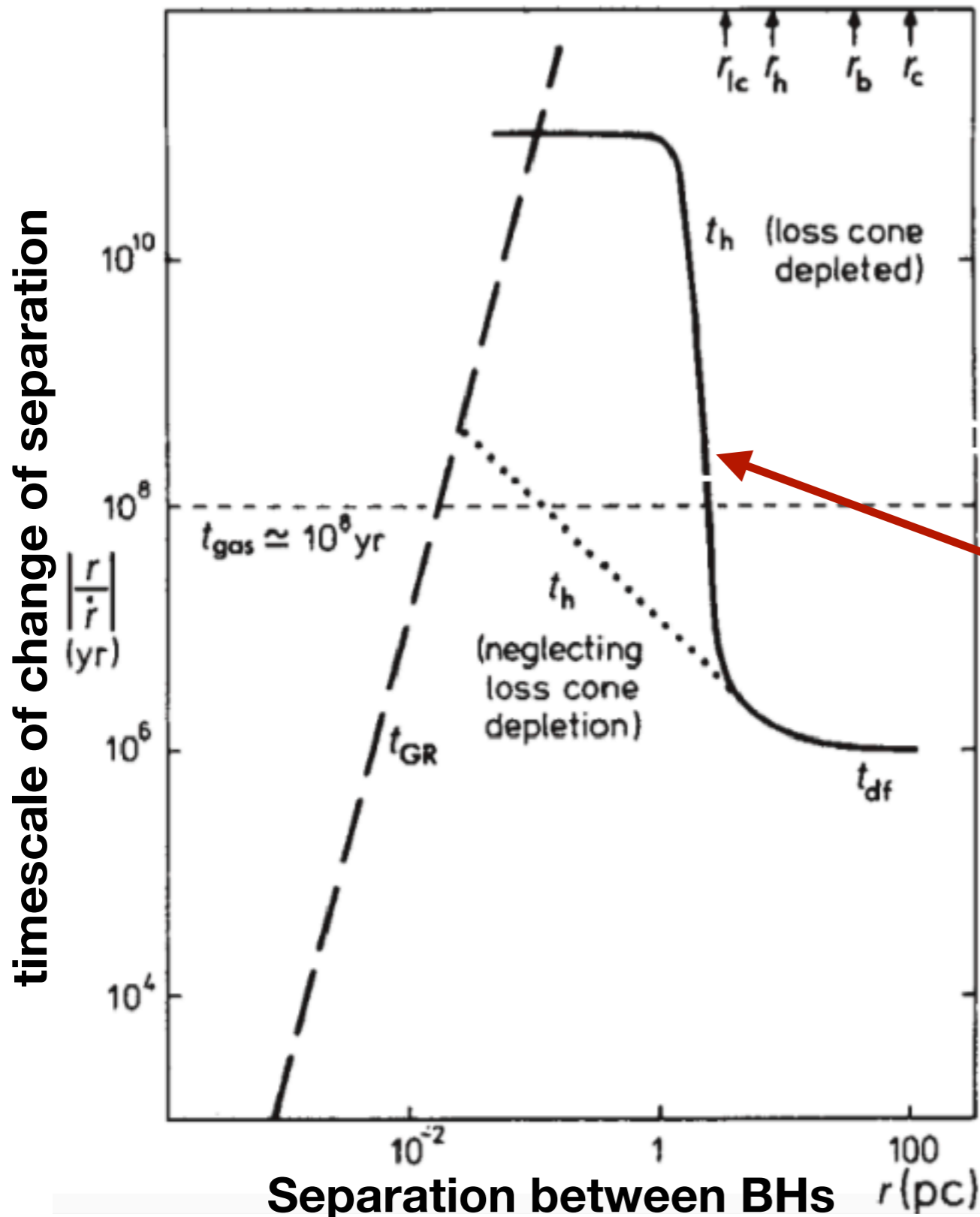


1. Dynamical friction shrinks binary
2. Decay only through close encounters



**Begelman, Blandford & Rees (1980)**

# The final parsec problem



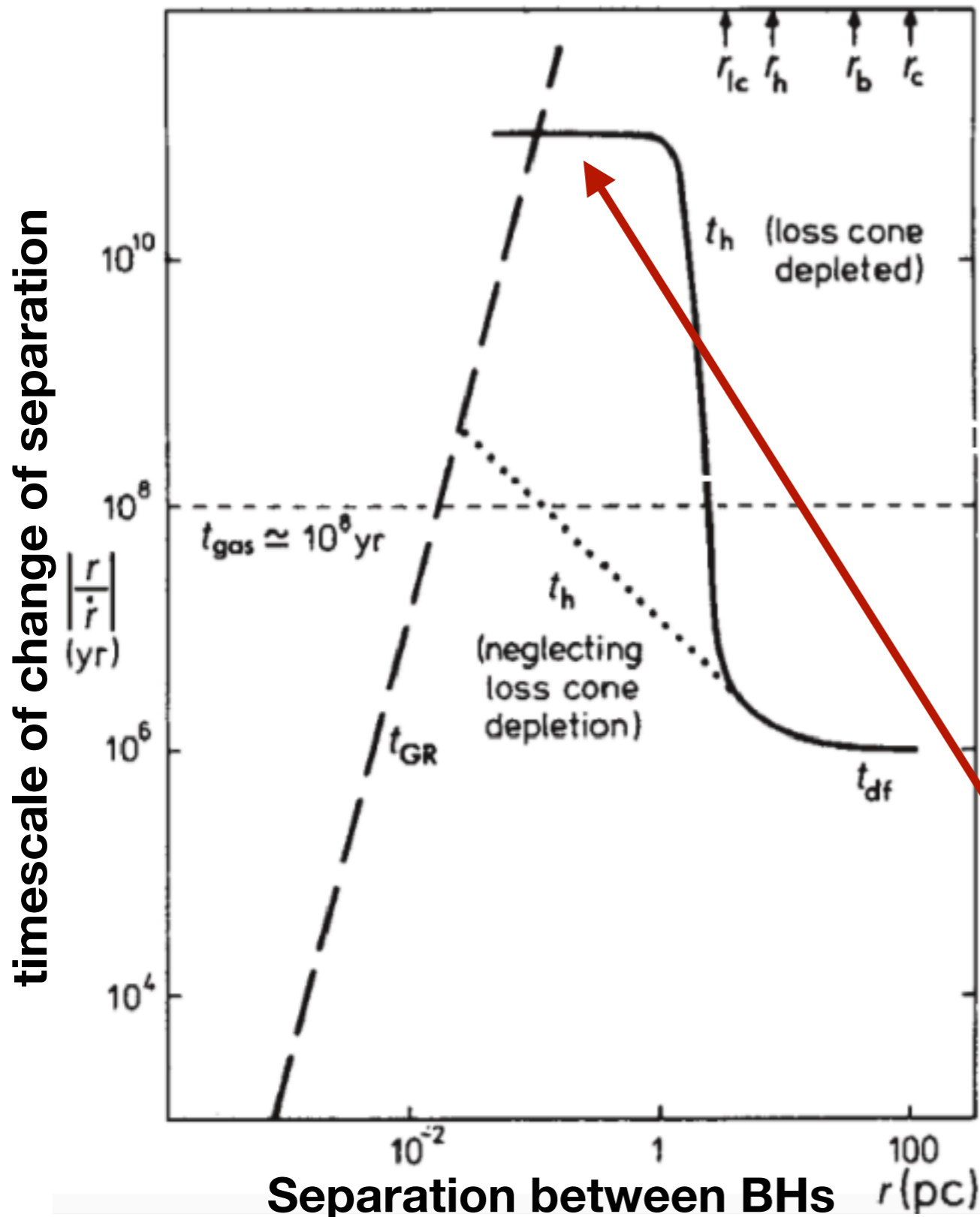
1. Dynamical friction shrinks binary
2. Decay only through close encounters
3. “Loss cone” depleted

Loss cone: region of phase space in which stars have close encounters with the binary black hole

Begelman, Blandford & Rees (1980)



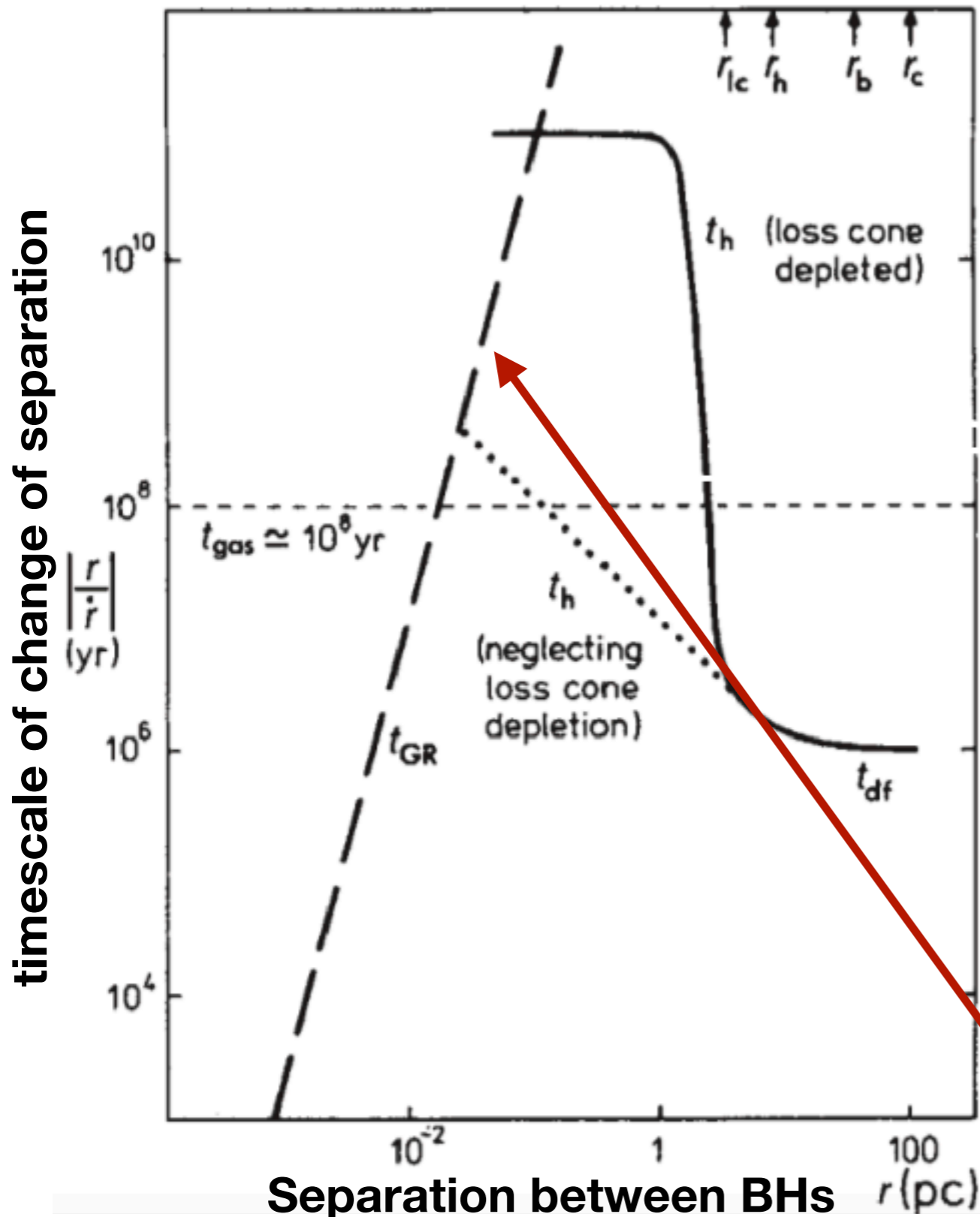
# The final parsec problem



1. Dynamical friction shrinks binary
2. Decay only through close encounters
3. “Loss cone” depleted
4. Bottleneck: slow refilling of loss cone

**Begelman, Blandford & Rees (1980)**

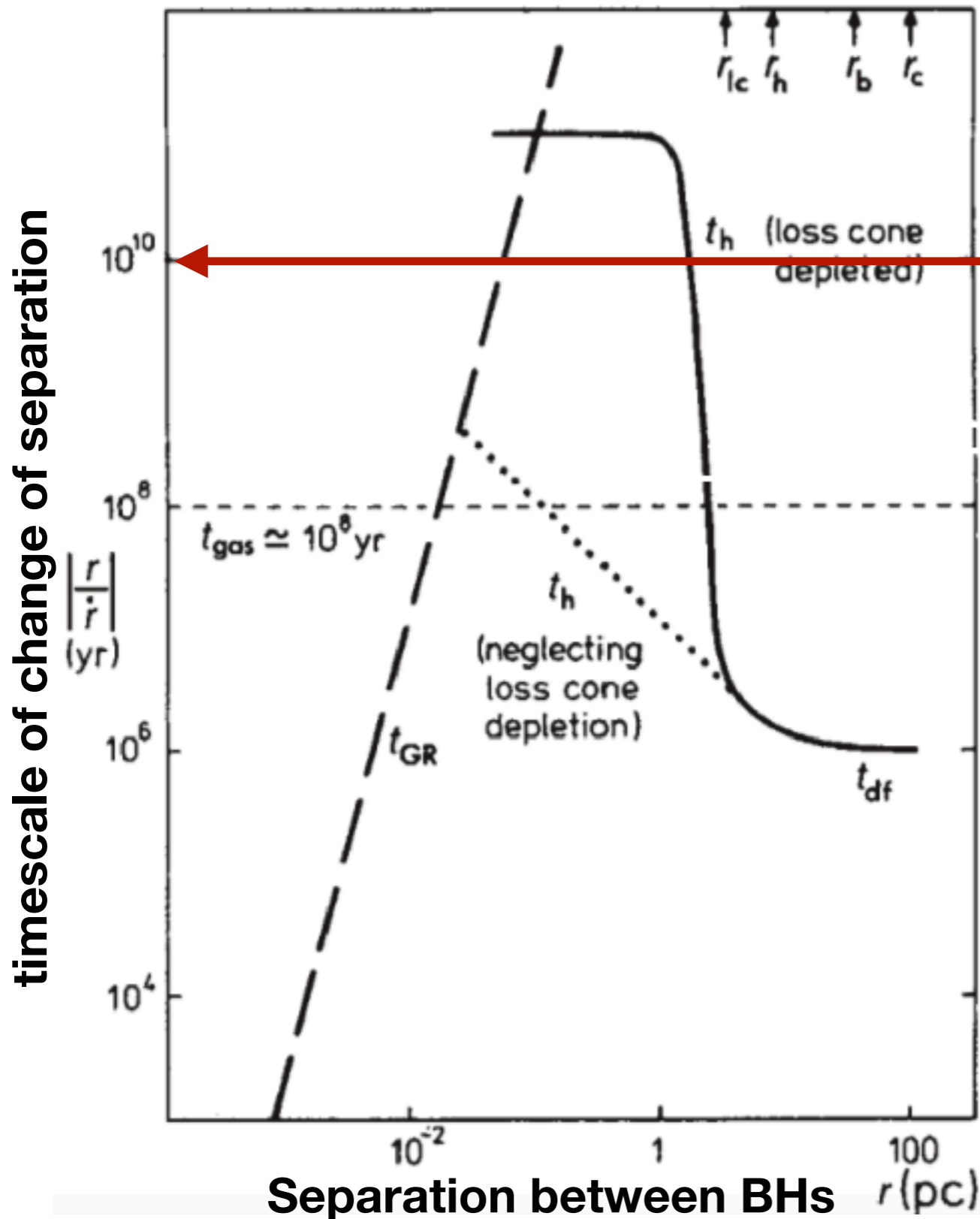
# The final parsec problem



1. Dynamical friction shrinks binary
2. Decay only through close encounters
3. “Loss cone” depleted
4. Bottleneck: slow refilling of loss cone
5. GW radiation

Begelman, Blandford & Rees (1980)

# The final parsec problem



Hubble time

Binary does not merge within lifetime of Universe

Maybe ok?

Begelman, Blandford & Rees (1980)

# Gravitational wave astronomy

## THE SPECTRUM OF GRAVITATIONAL WAVES



Observatories & experiments

Ground-based experiment



Space-based observatory



Pulsar timing array



Cosmic microwave background polarisation



Timescales

milliseconds

seconds

hours

years

billions of years

Frequency (Hz)

100

1

$10^{-2}$

$10^{-4}$

$10^{-6}$

$10^{-8}$

$10^{-15}$

Cosmic fluctuations in the early Universe

Cosmic sources



Supernova



Pulsar



Compact object falling onto a supermassive black hole



Merging supermassive black holes



Merging neutron stars in other galaxies



Merging stellar-mass black holes in other galaxies



Merging white dwarfs in our Galaxy

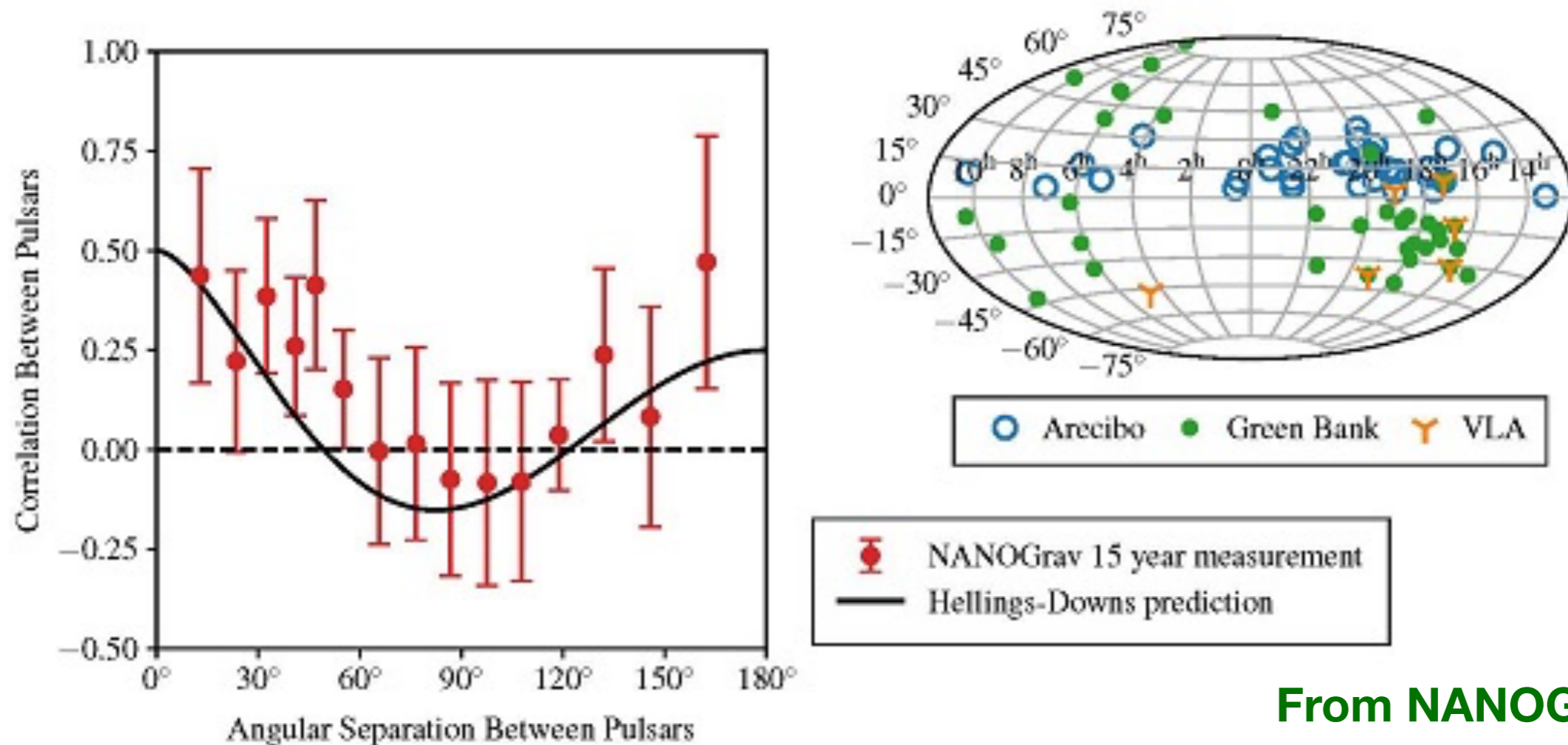
#lisa



Image credit: ESA



# June 2023: Evidence for GW signal



Agreement between 4 main PTA collaborations:

- NANOGrav
- Parkes Pulsar Timing Array
- European + Indian Pulsar Timing Array
- Chinese Pulsar Timing Array



# Astrophysical explanation: SMBH mergers

## Supermassive black holes DO merge

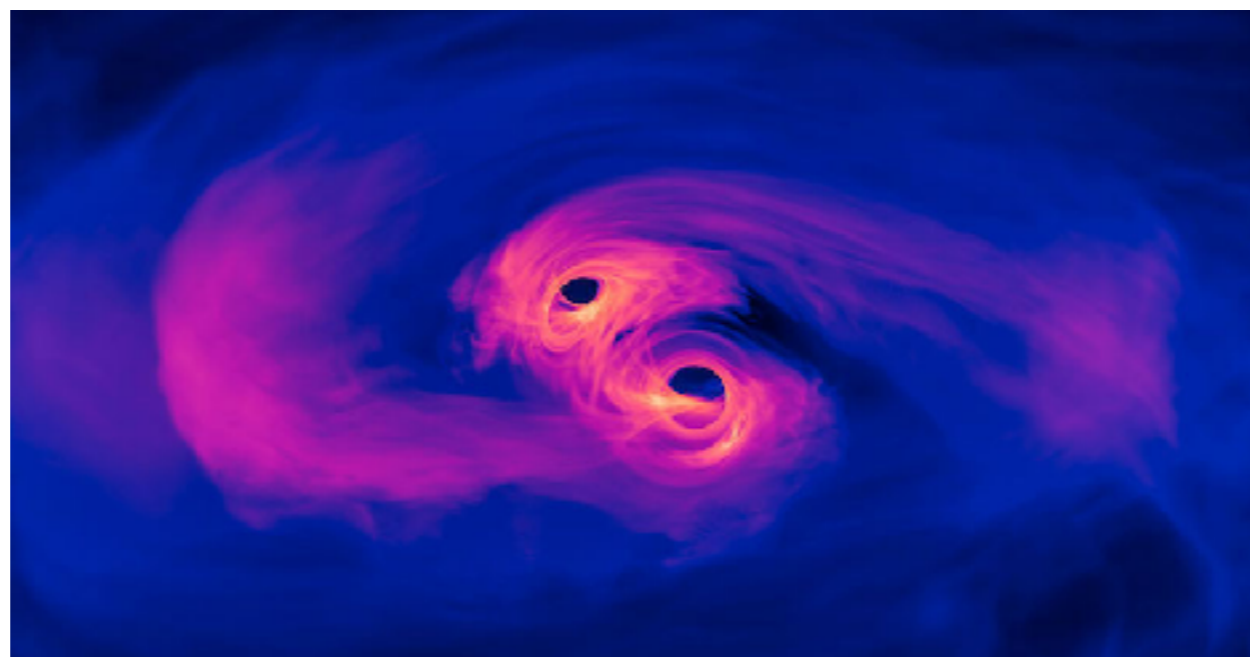


Image credit: NASA

### Final parsec problem must be addressed

- Axisymmetry of galactic halo
- Accretion disk
- Multiple black holes
- **... what else is in the BHs environment?**

# Dark matter spikes around SMBH

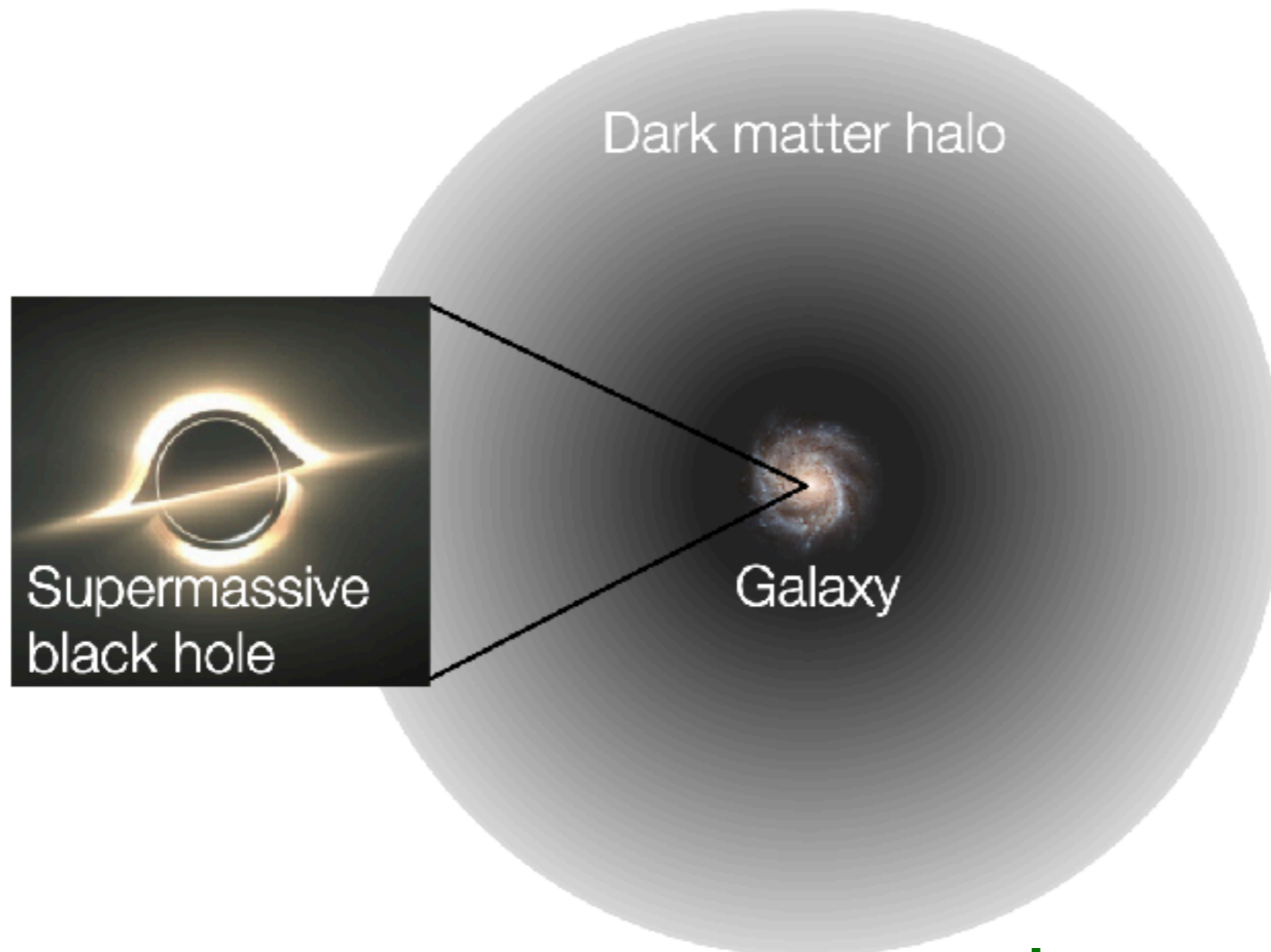
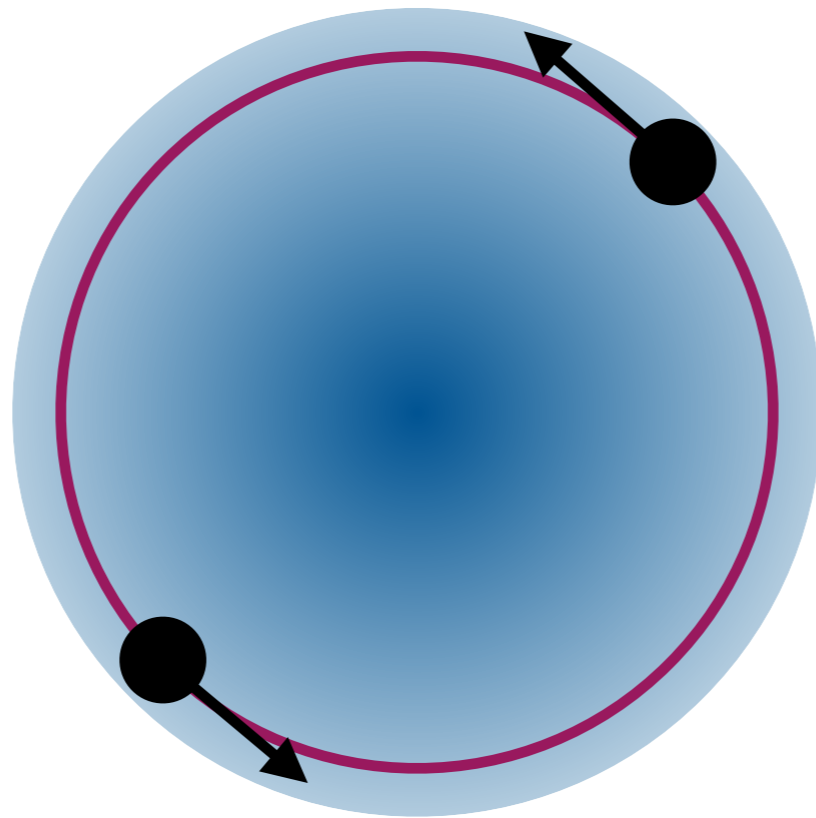


Image credit: M. Powell

DM density enhanced by gravity of SMBH

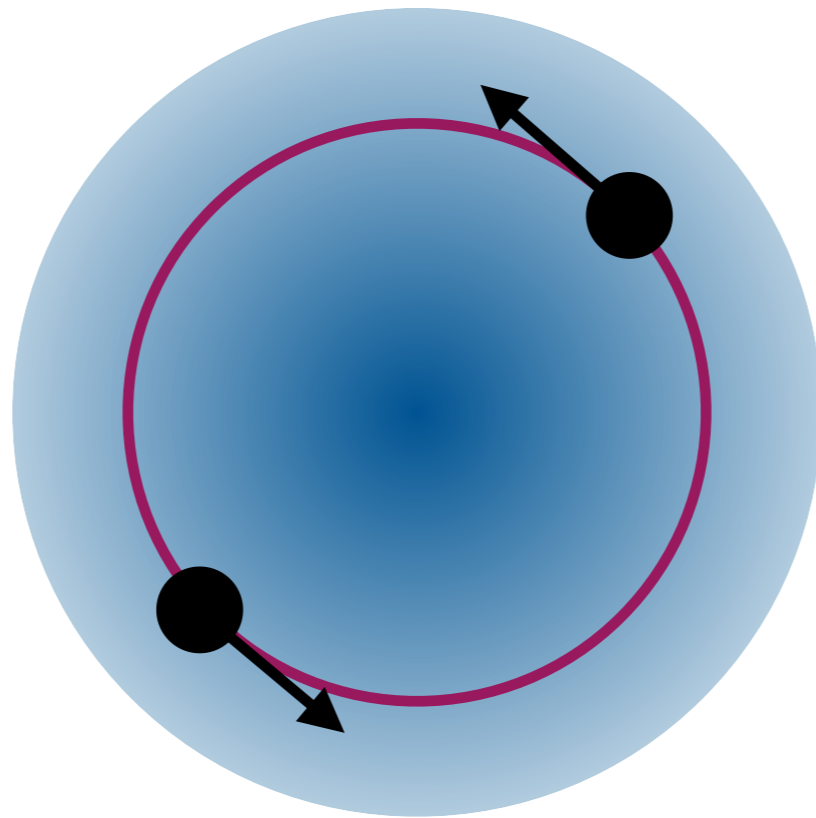
# SMBH orbit decay & Cold Dark Matter

DM-BH dynamical friction shrinks binary



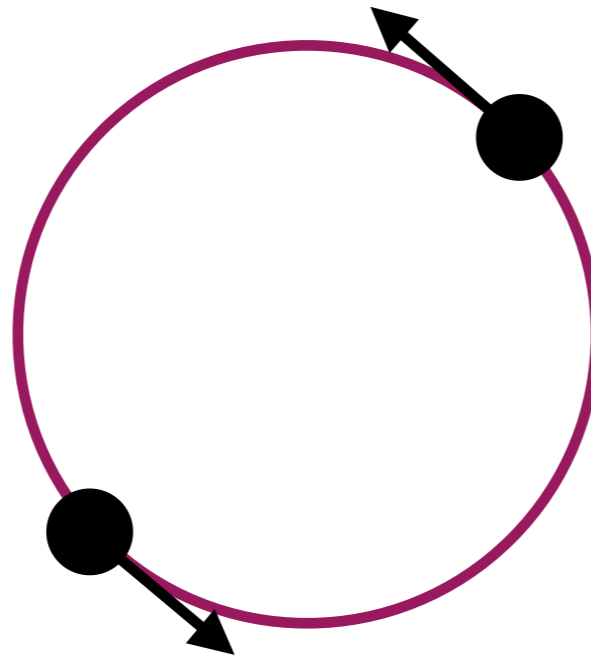
# SMBH orbit decay & Cold Dark Matter

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# SMBH orbit decay & Cold Dark Matter

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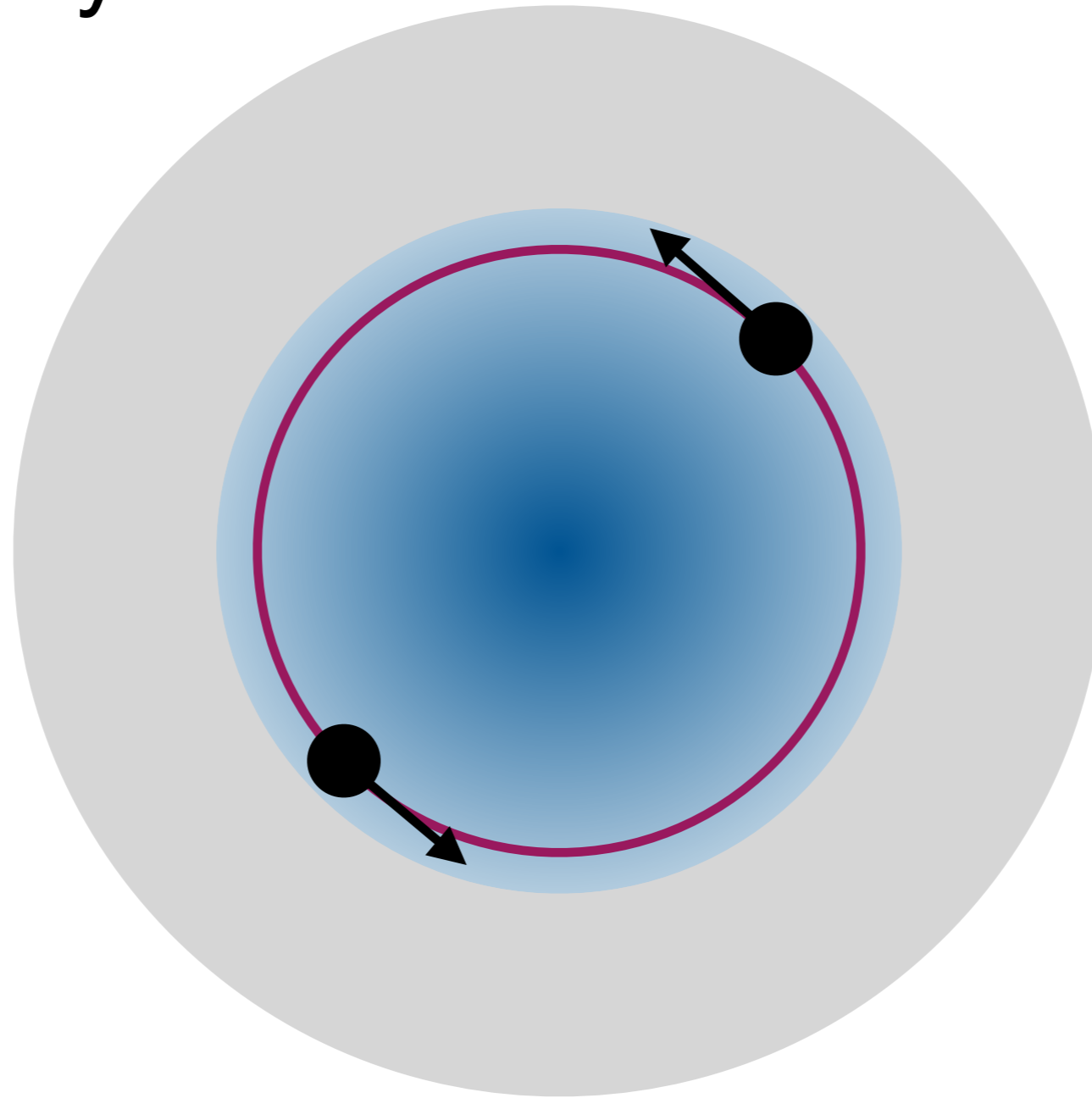


Injected heat evaporates DM spike: binary stalls



# Self-interacting Dark Matter

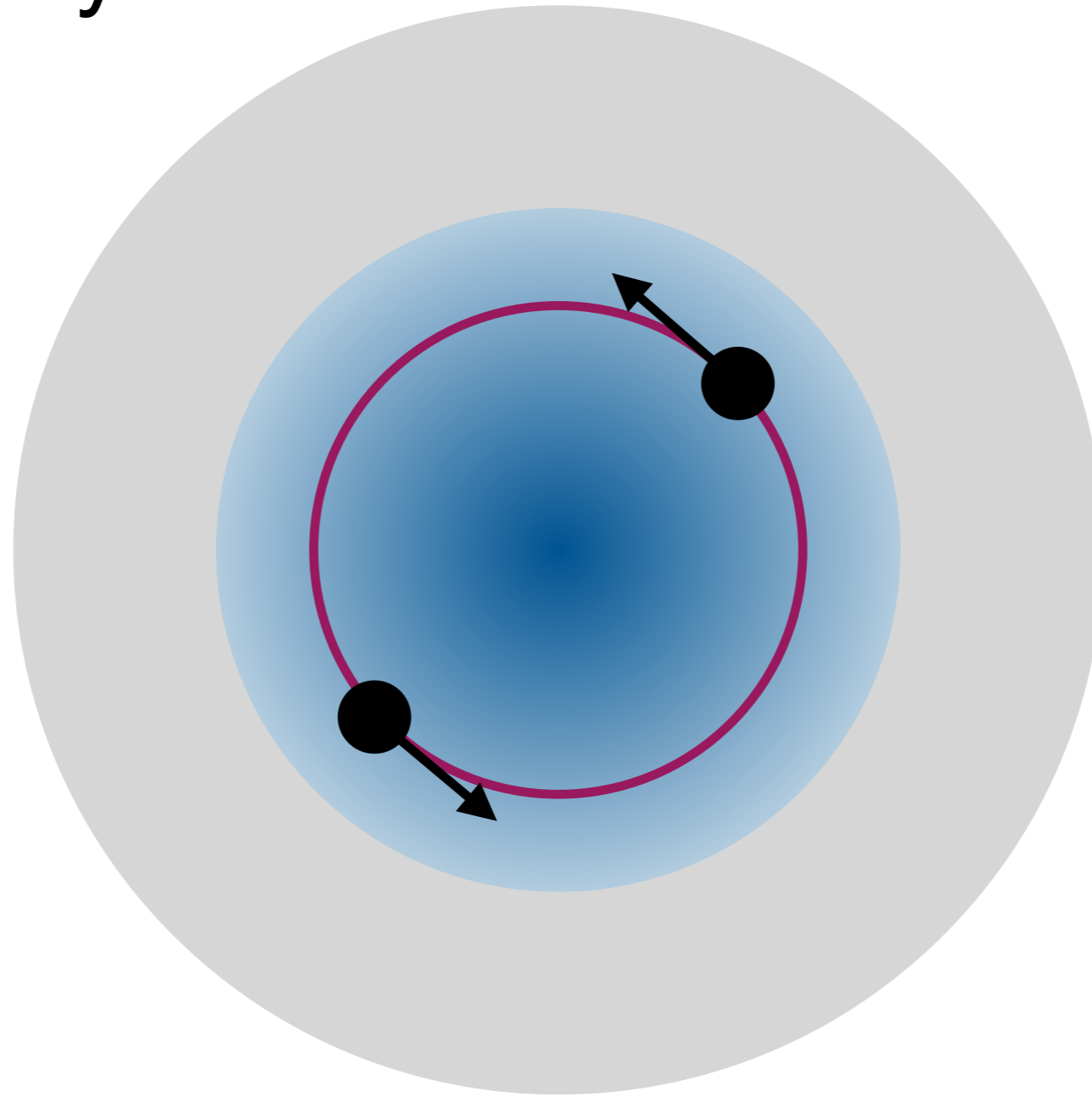
DM-BH dynamical friction shrinks binary



Self-interactions prevent dispersion: BHs coalesce

# Self-interacting Dark Matter

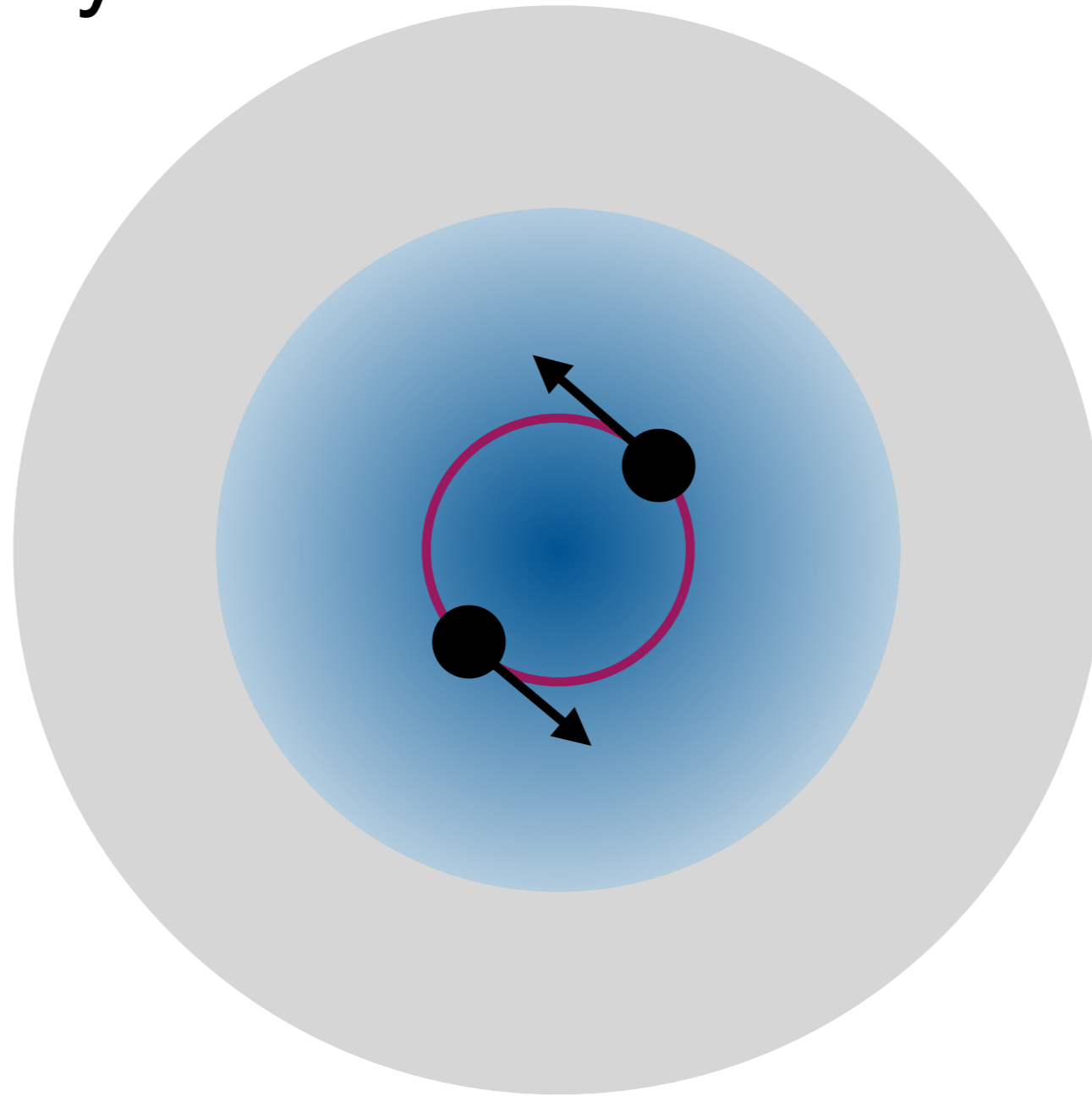
DM-BH dynamical friction shrinks binary



Self-interactions prevent dispersion: BHs coalesce

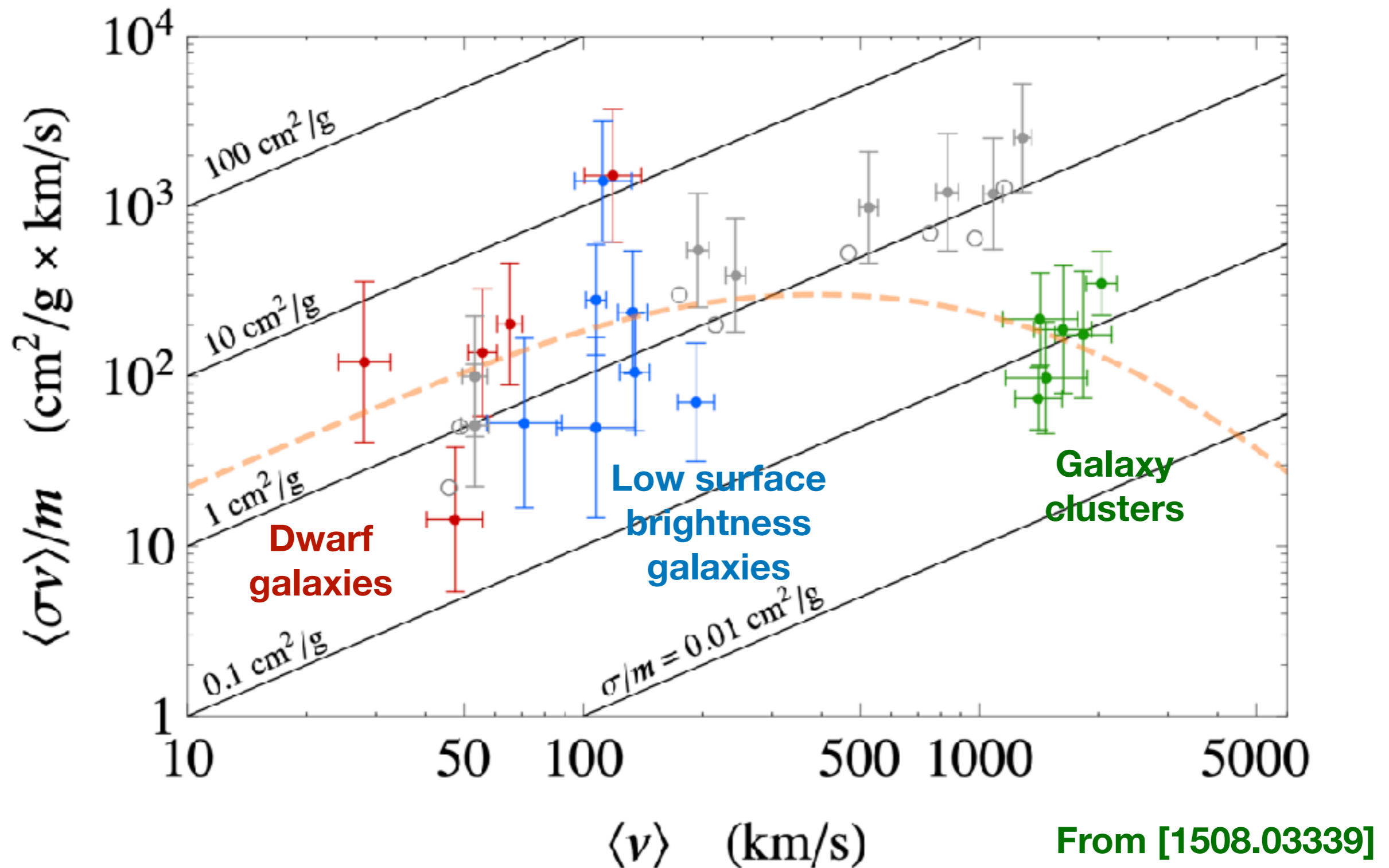
# Self-interacting Dark Matter

DM-BH dynamical friction shrinks binary

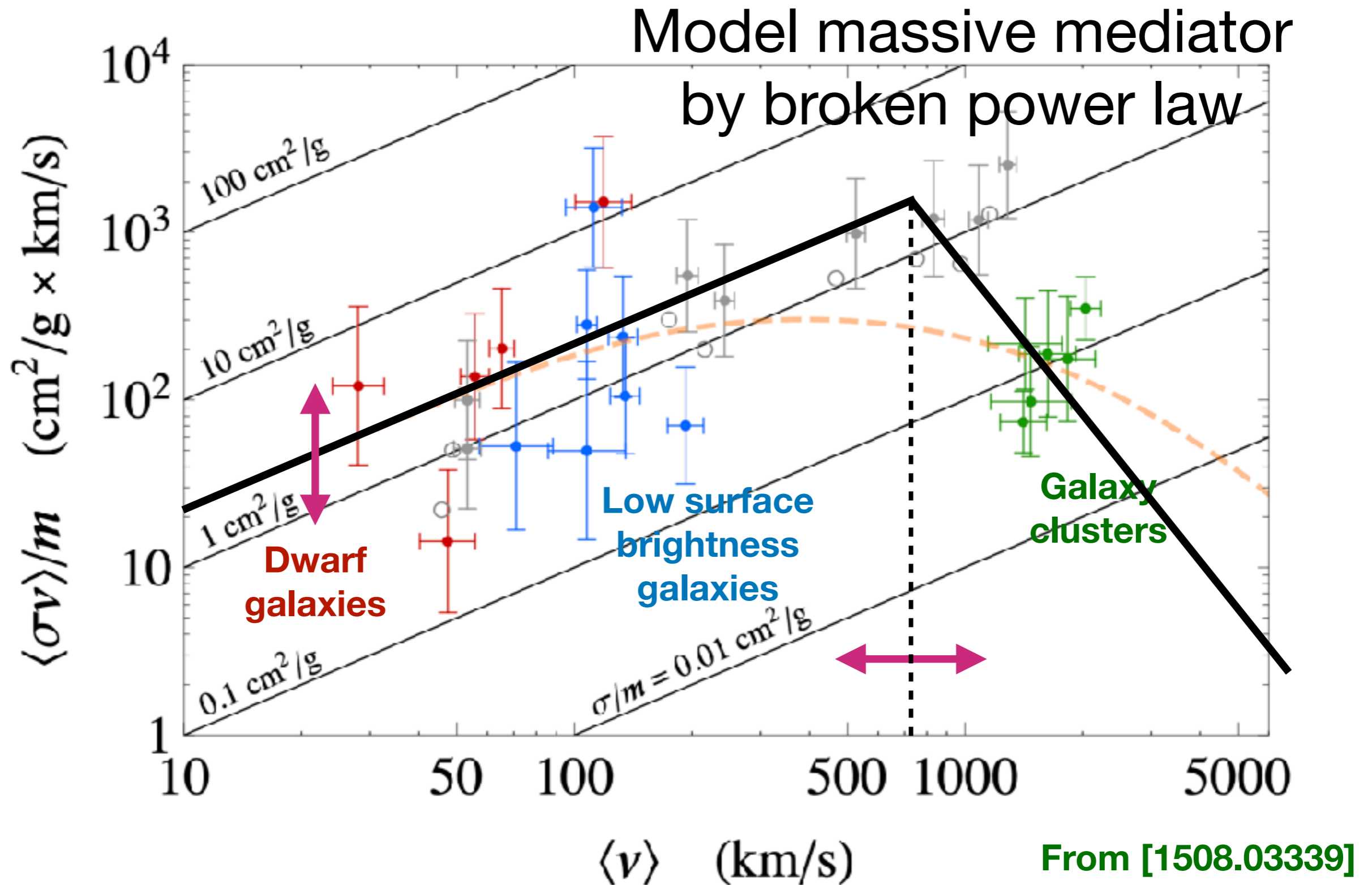


Self-interactions prevent dispersion: BHs coalesce

# Velocity-dependent self interactions



# Velocity-dependent self interactions





# OUTLINE

1. Reconstruct the spike profile
2. Calculate BH merger dynamics
3. Predict the GW spectrum

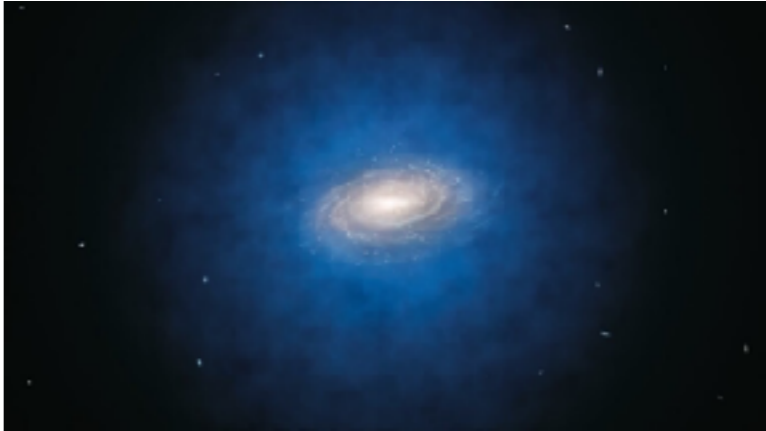
# OUTLINE

- 1. Reconstruct the spike profile**
2. Calculate BH merger dynamics
3. Predict the GW spectrum

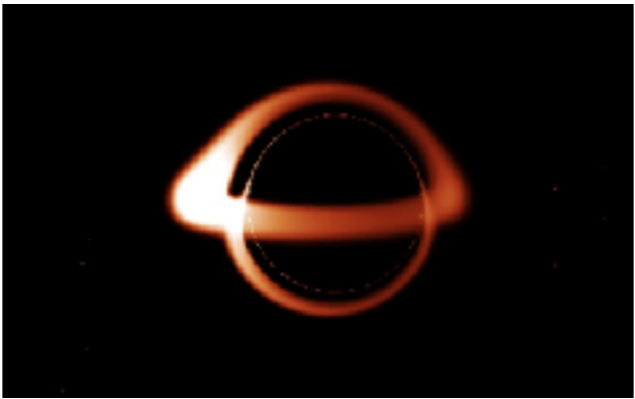
# Observational & semi-analytical correlations



**NFW parameters**



**DM halo mass**



**Black hole mass**



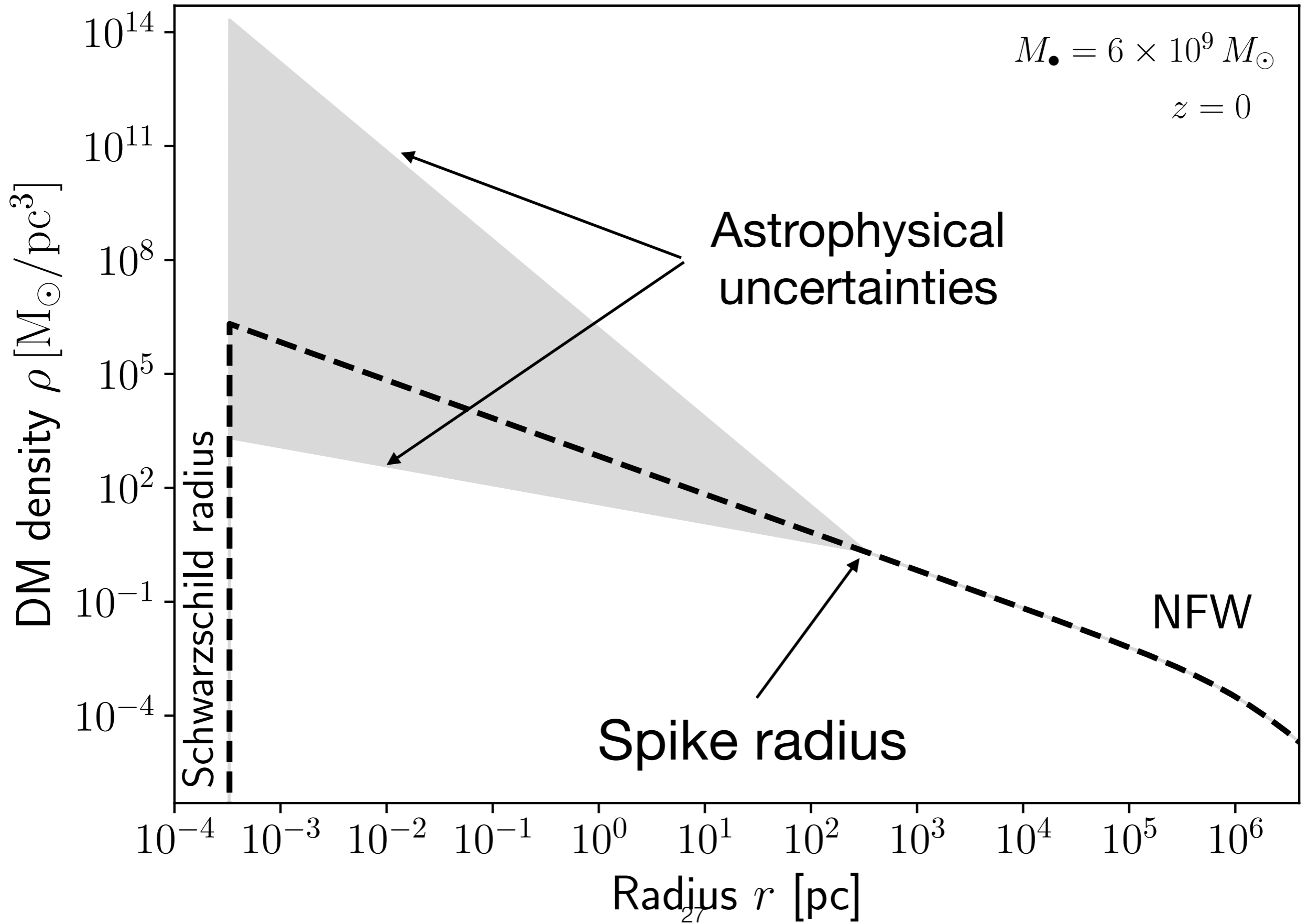
**Stellar bulge mass**



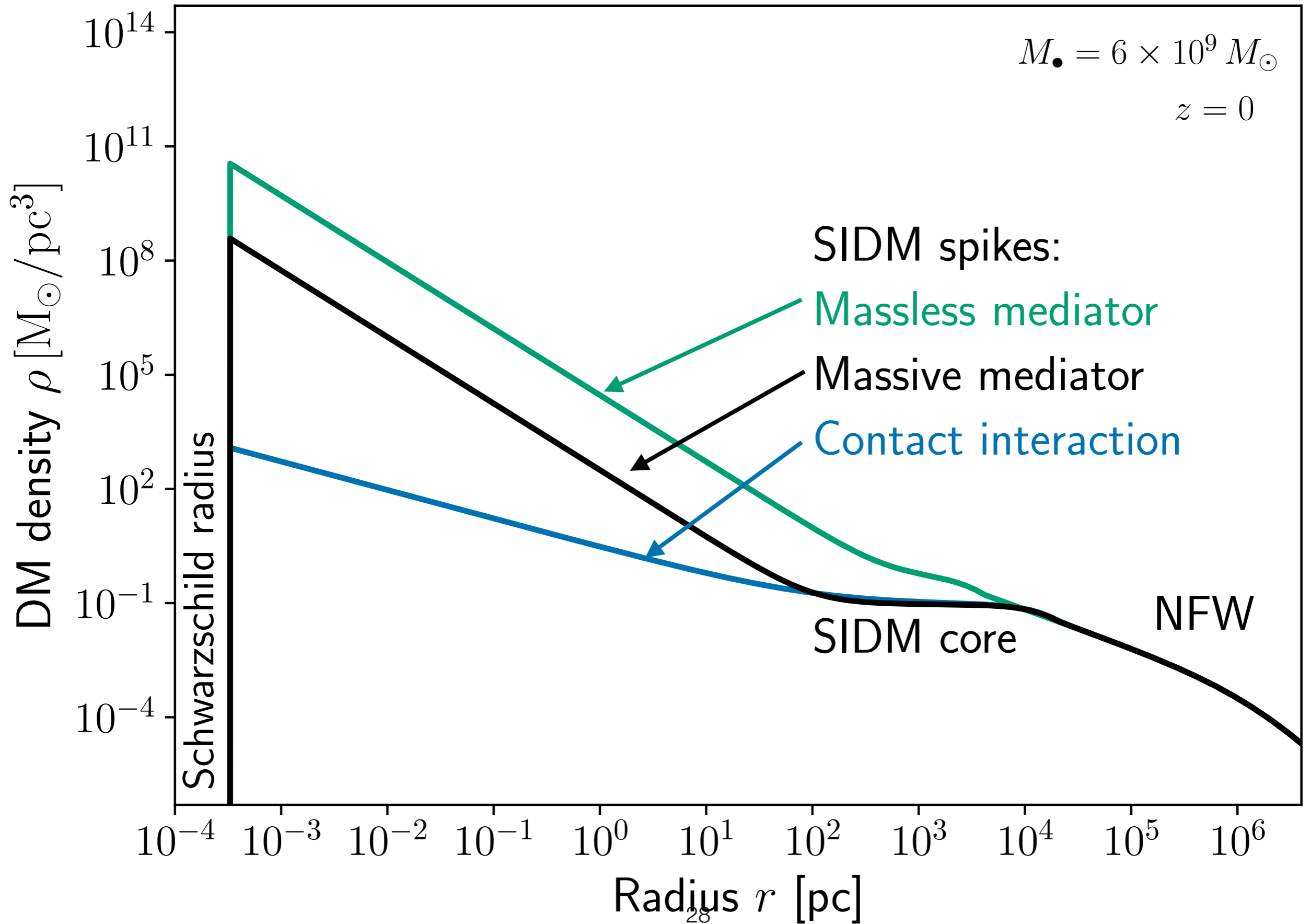
**Stellar mass**



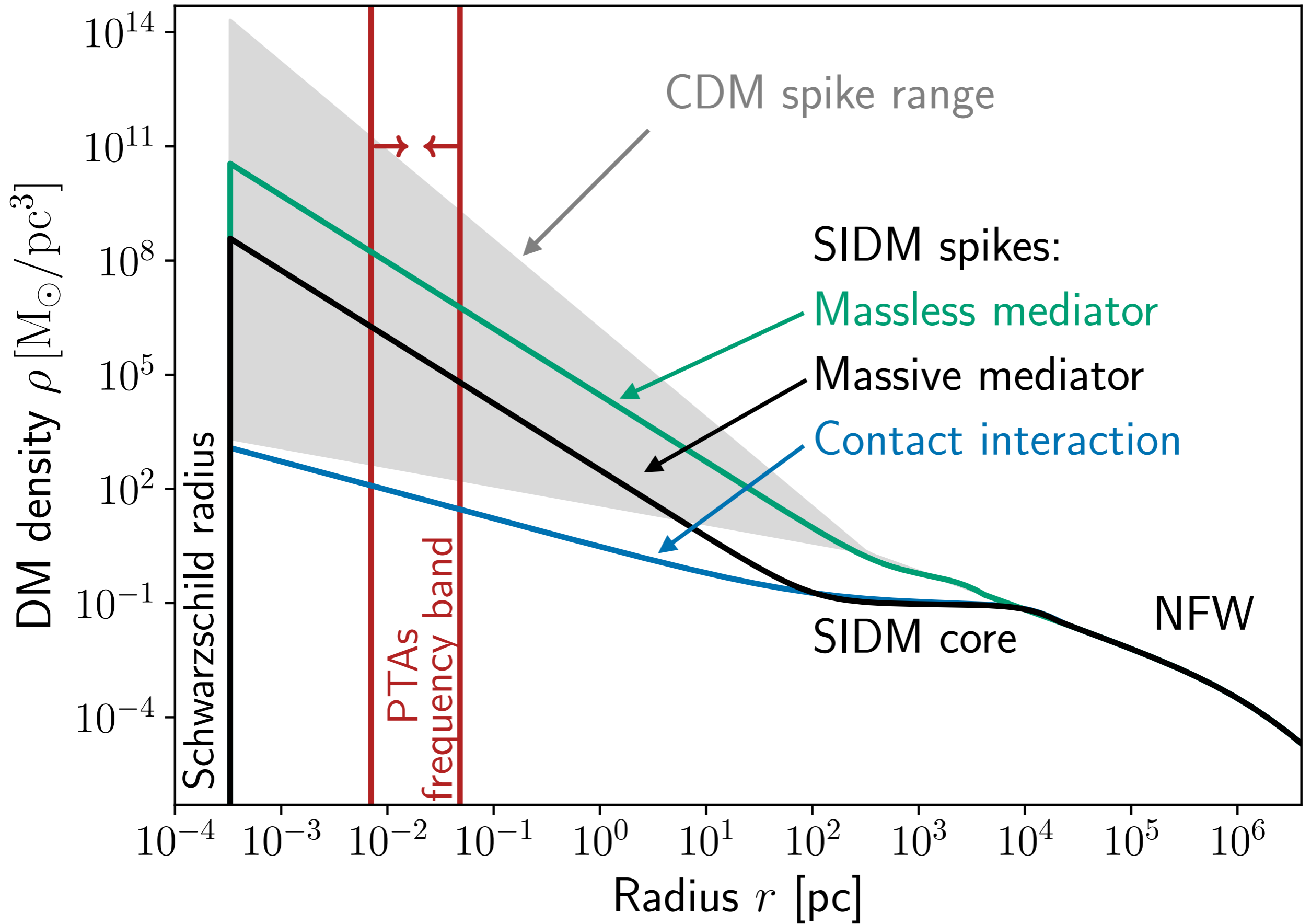
# Cold DM spike



# Self Interacting DM spike



# CDM & SIDM spikes



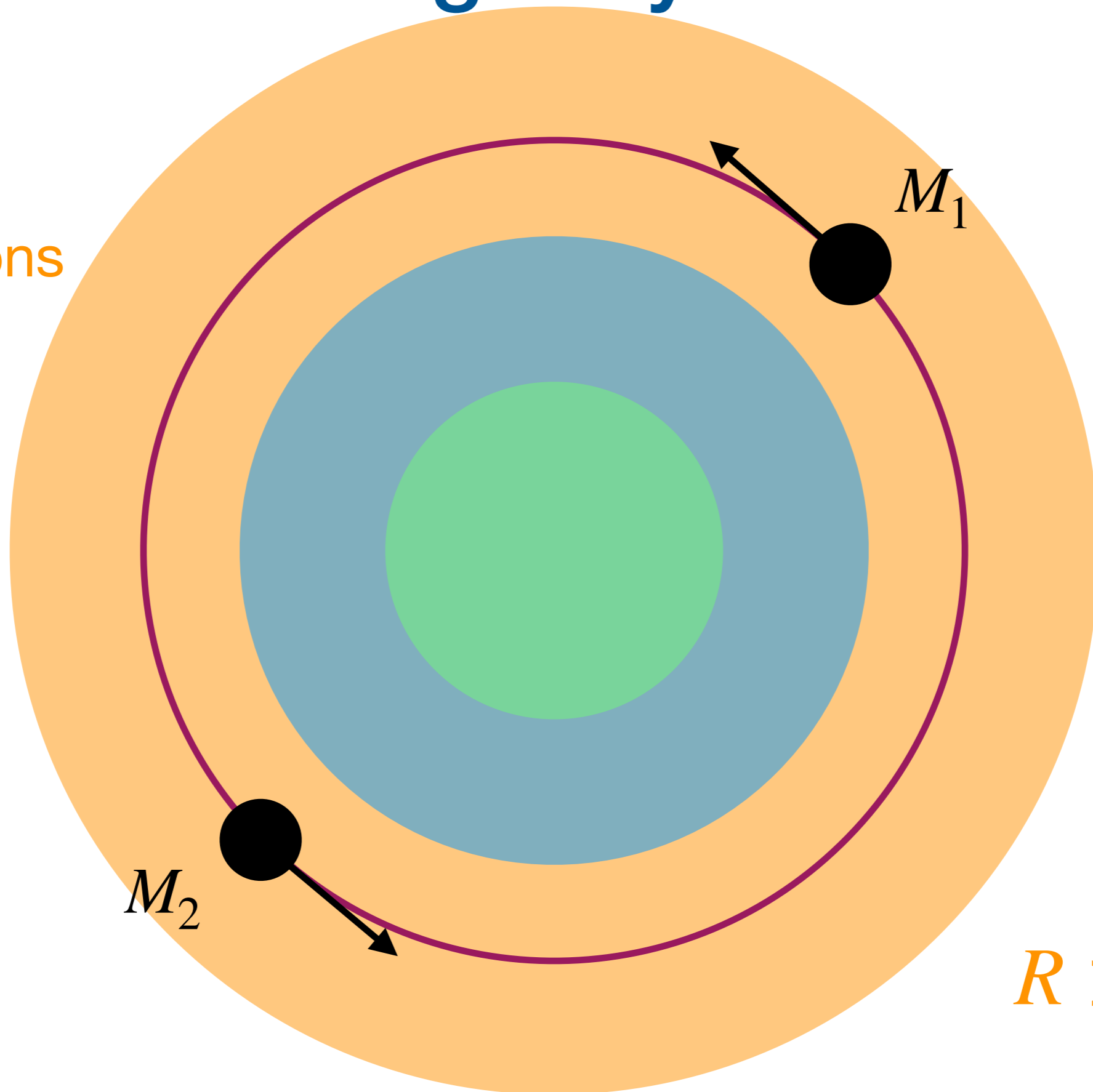


# OUTLINE

1. Reconstruct the spike profile
- 2. Calculate BH merger dynamics**
3. Predict the GW spectrum

# BH merger dynamics

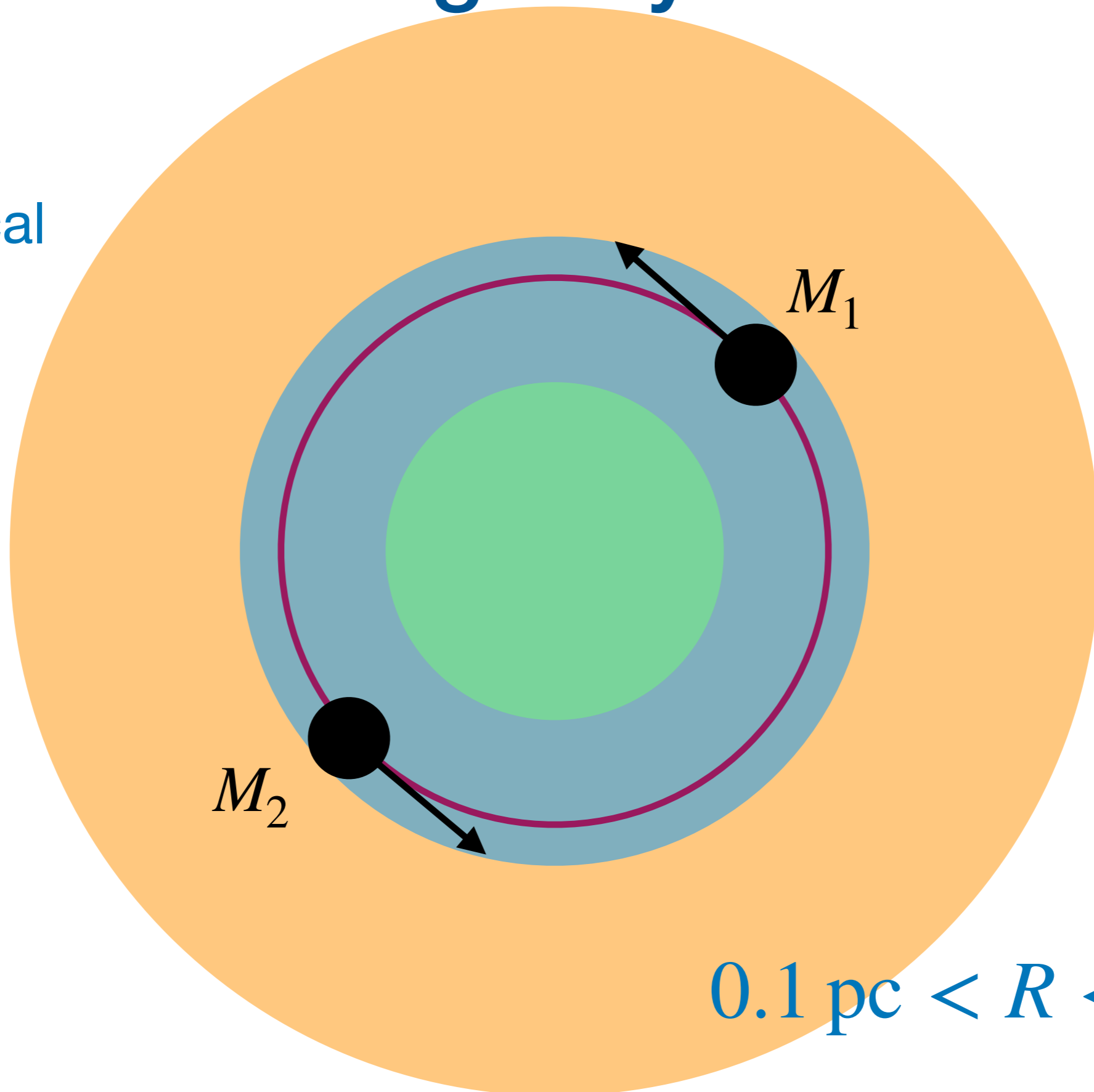
Stellar  
interactions



$R > 10 \text{ pc}$

# BH merger dynamics

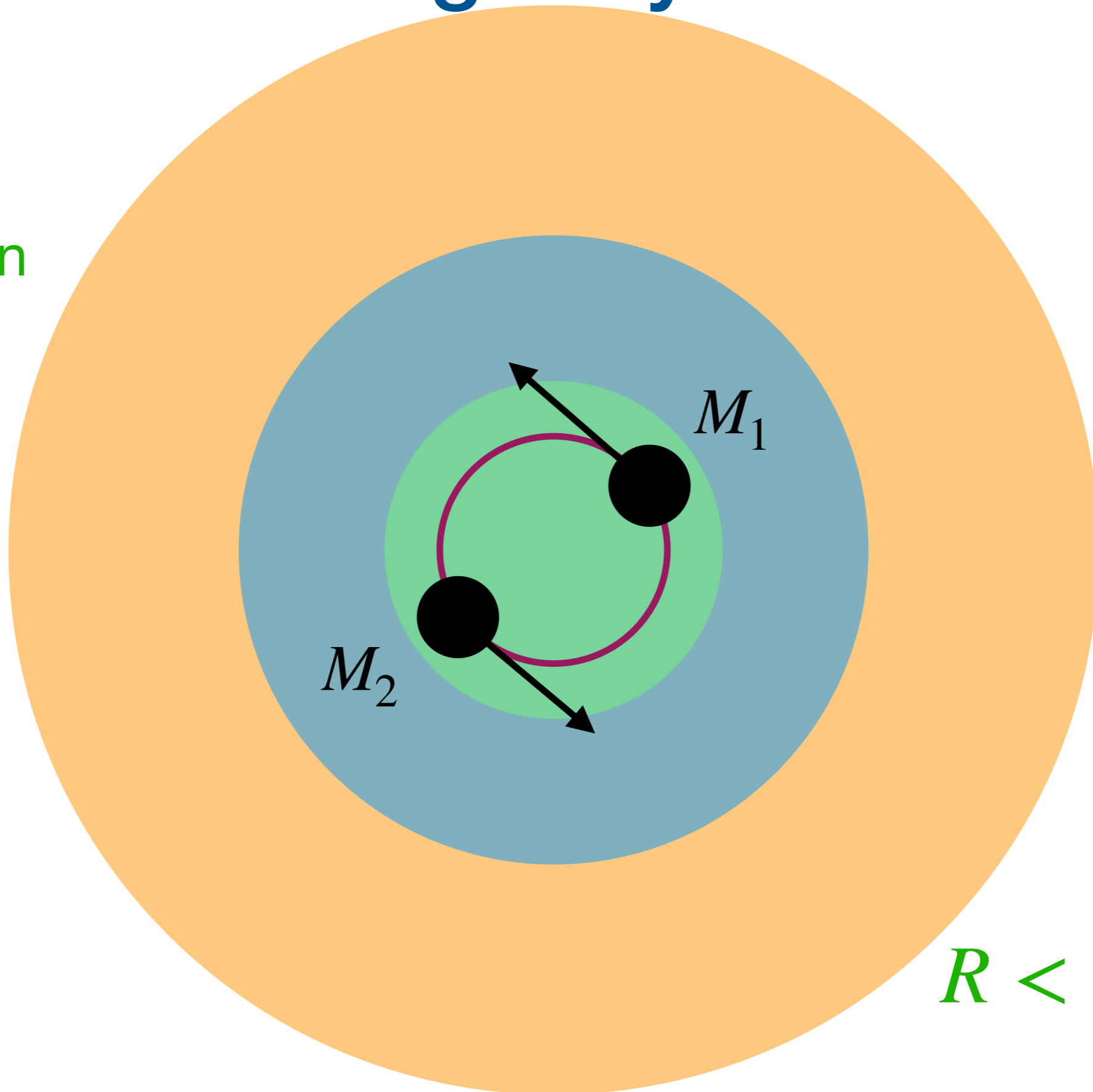
DM  
dynamical  
friction



$0.1 \text{ pc} < R < 10 \text{ pc}$

# BH merger dynamics

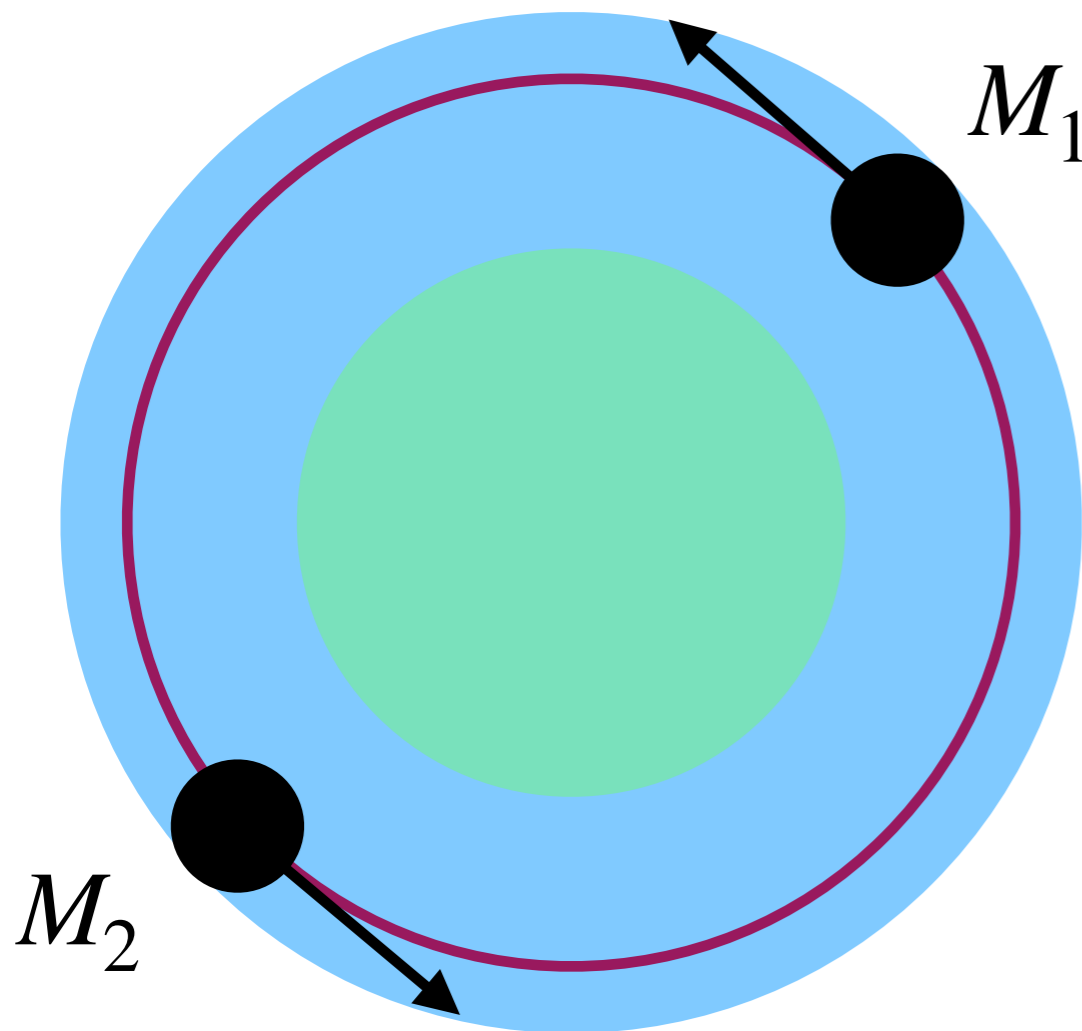
GW  
emission



# Dynamical friction timescale

DM  
dynamical friction + GW  
emission

Binary *hardens* (shrinks)  
due to dynamical friction



$$t_{\text{DF}} = \Delta t (10 \text{ pc} \rightarrow 0.1 \text{ pc})$$

$$\propto 1/\rho_{\text{spike}}$$

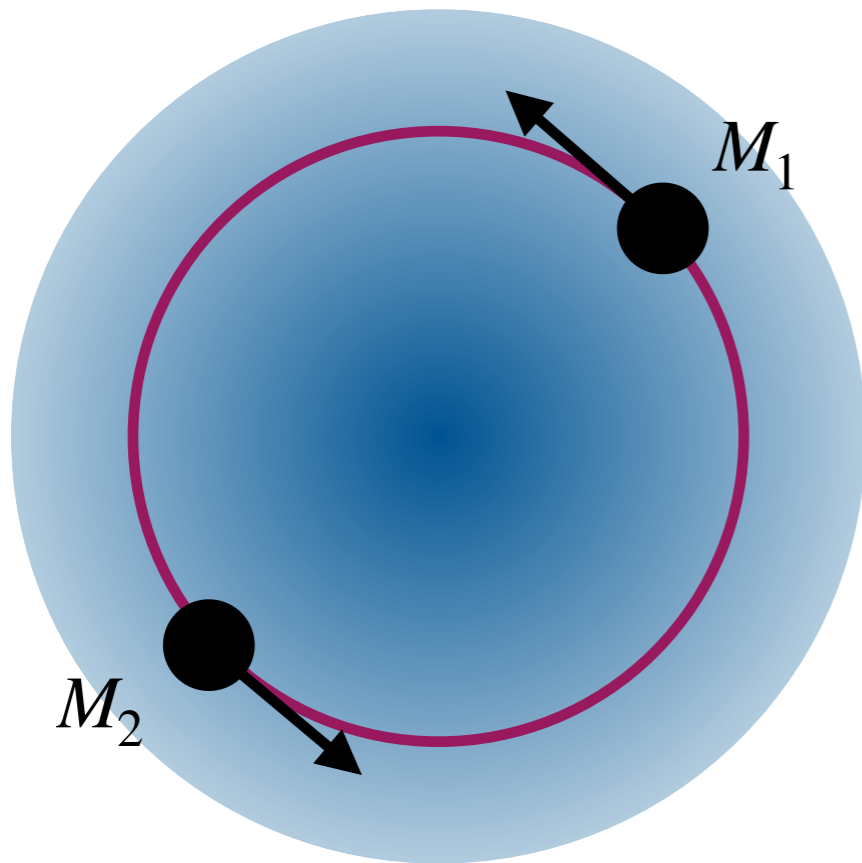
Favours dense spikes

**Demand  $t_{\text{DF}} < 1 \text{ Gyr} \rightarrow$  Upper limit on  $\sigma_0/m$**

# Back-reaction destroys CDM spike

**CDM:**  $\Delta E_{\text{orbit}} \gg U_{\text{spike}}$

Spike evaporates and only  
replenished gravitationally





# Self interactions replenish SIDM spike

**CDM:**  $\Delta E_{\text{orbit}} \gg U_{\text{spike}}$

Spike evaporates and only replenished gravitationally

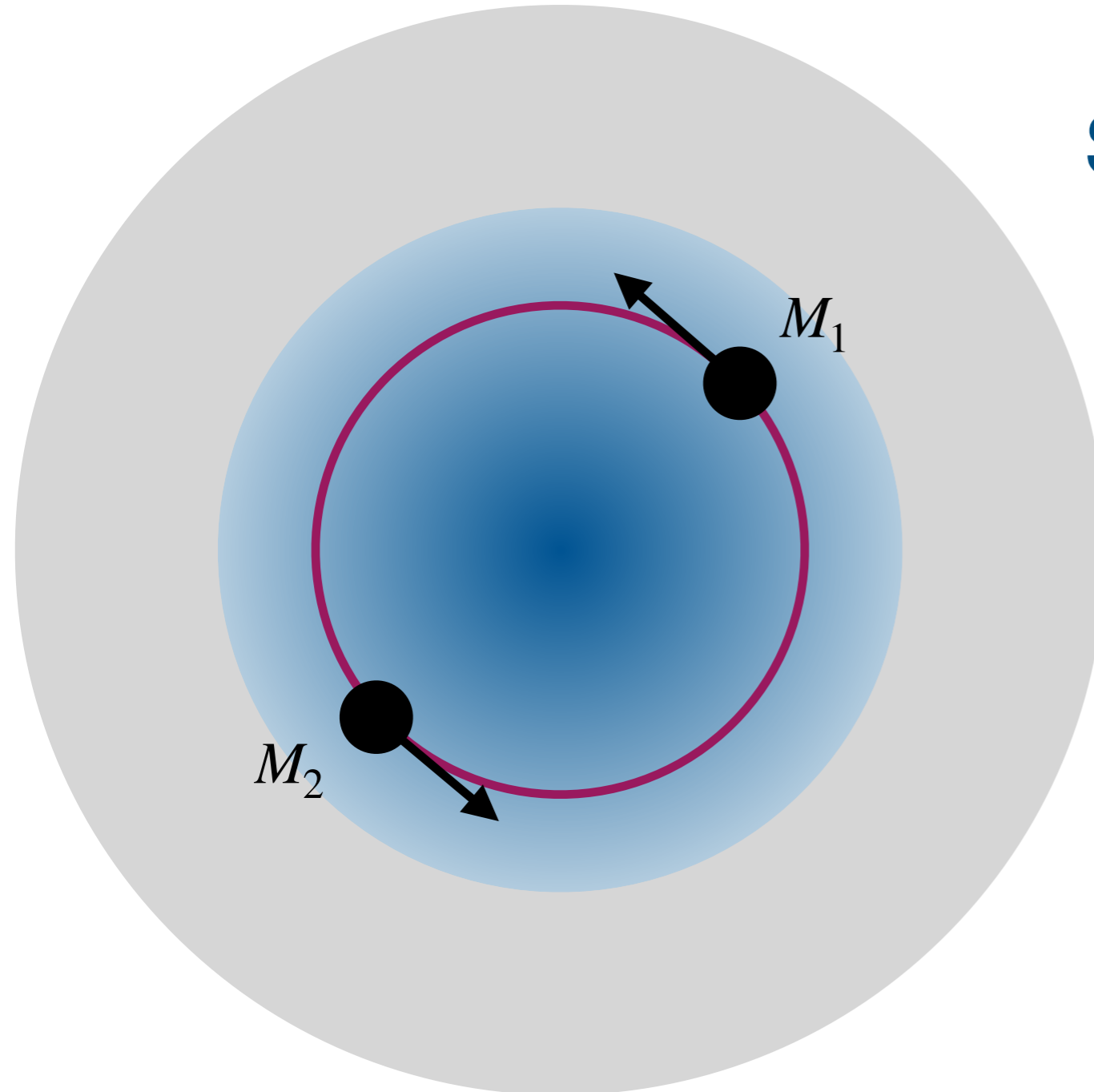
**SIDM:**

Whole core is in equilibrium with spike

$$U_{\text{core}} \gg U_{\text{spike}}$$

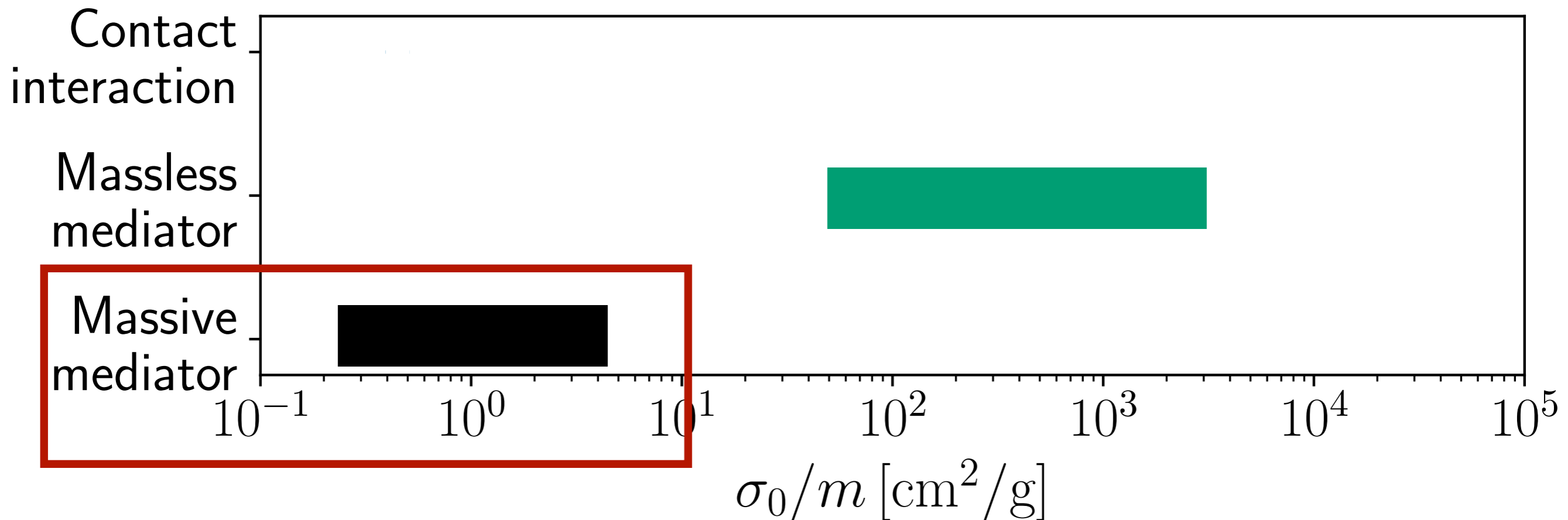
The core acts as a particle & energy reservoir

**Minimum size of core  $\rightarrow$  lower limit on  $\sigma_0/m$**



# Dark matter self-interaction cross section

- Lower limit: large enough core
- Upper limit: fast enough merger

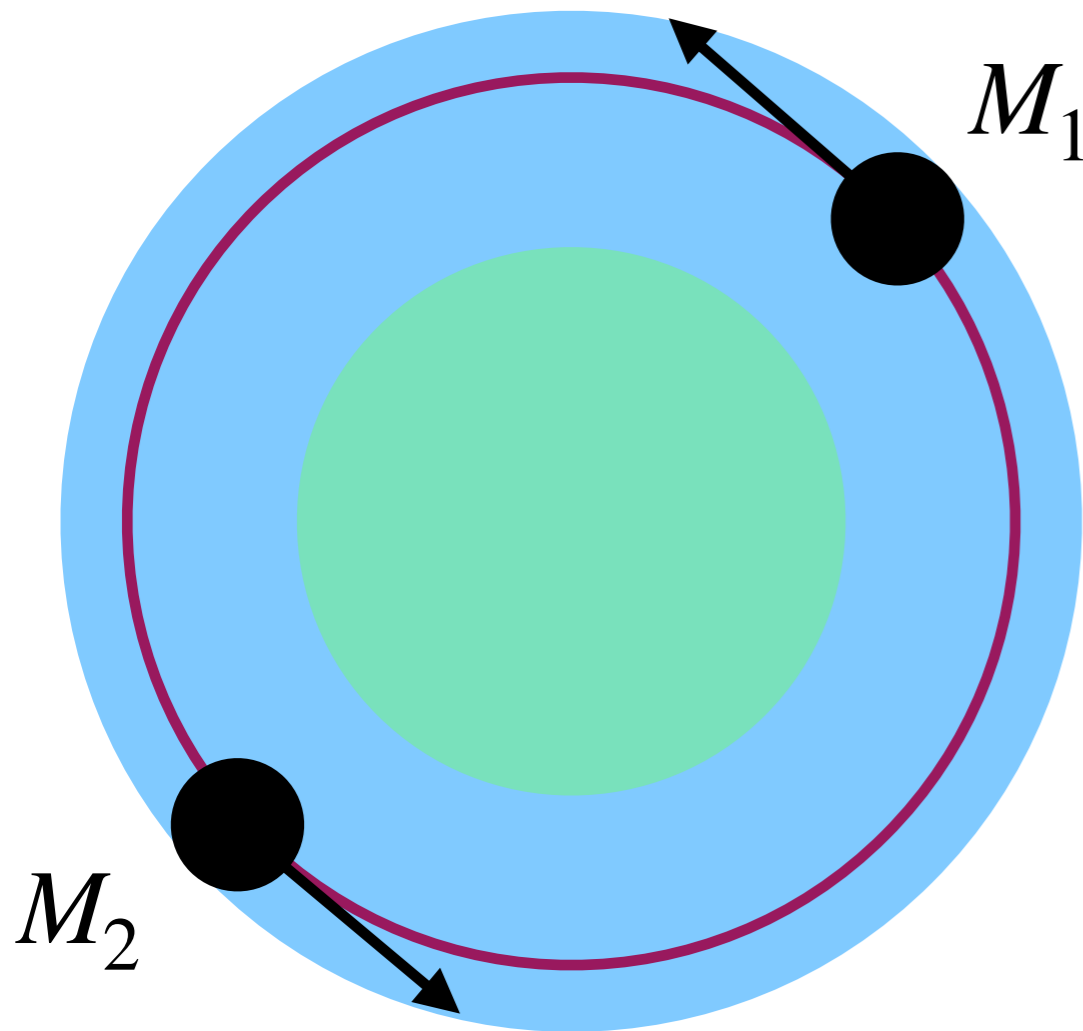


# OUTLINE

1. Reconstruct the spike profile
2. Calculate BH merger dynamics
- 3. Predict the GW spectrum**

# Single-merger GW spectrum

GW frequency is twice the orbital frequency



$$f \propto R^{-3/2}$$

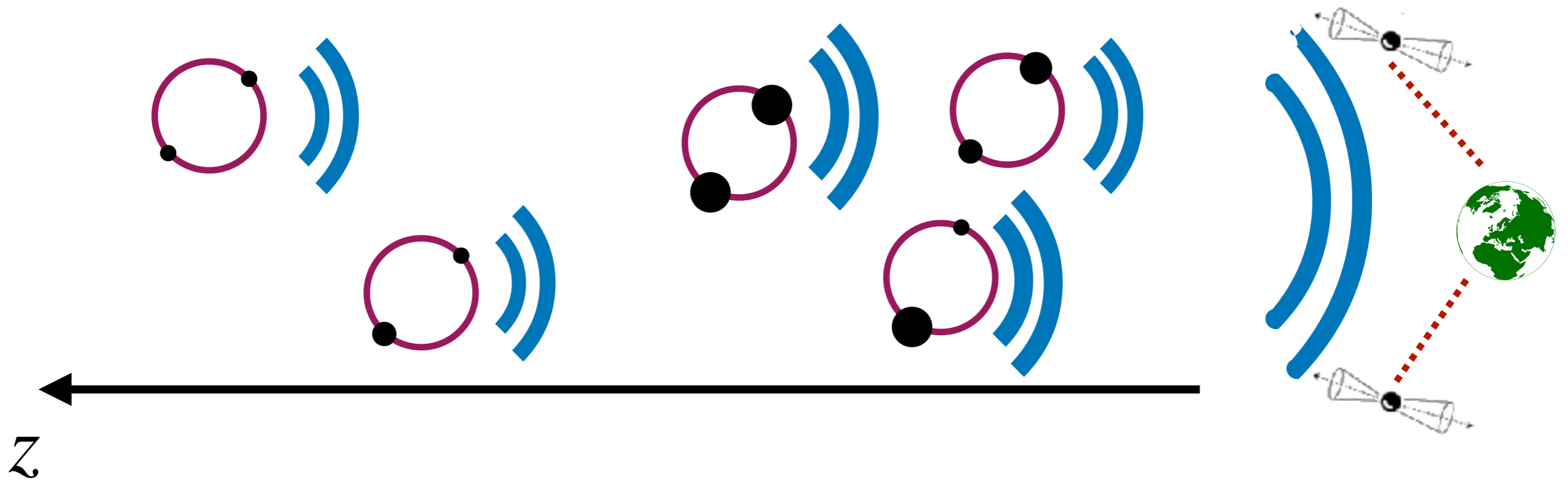
GW energy per unit frequency:

$$\frac{dE_{\text{GW}}}{df} \propto \frac{P_{\text{GW}}(R)}{P_{\text{GW}}(R) + P_{\text{DF}}(R)}$$

**“Branching ratio” of orbital energy into GW / DF**

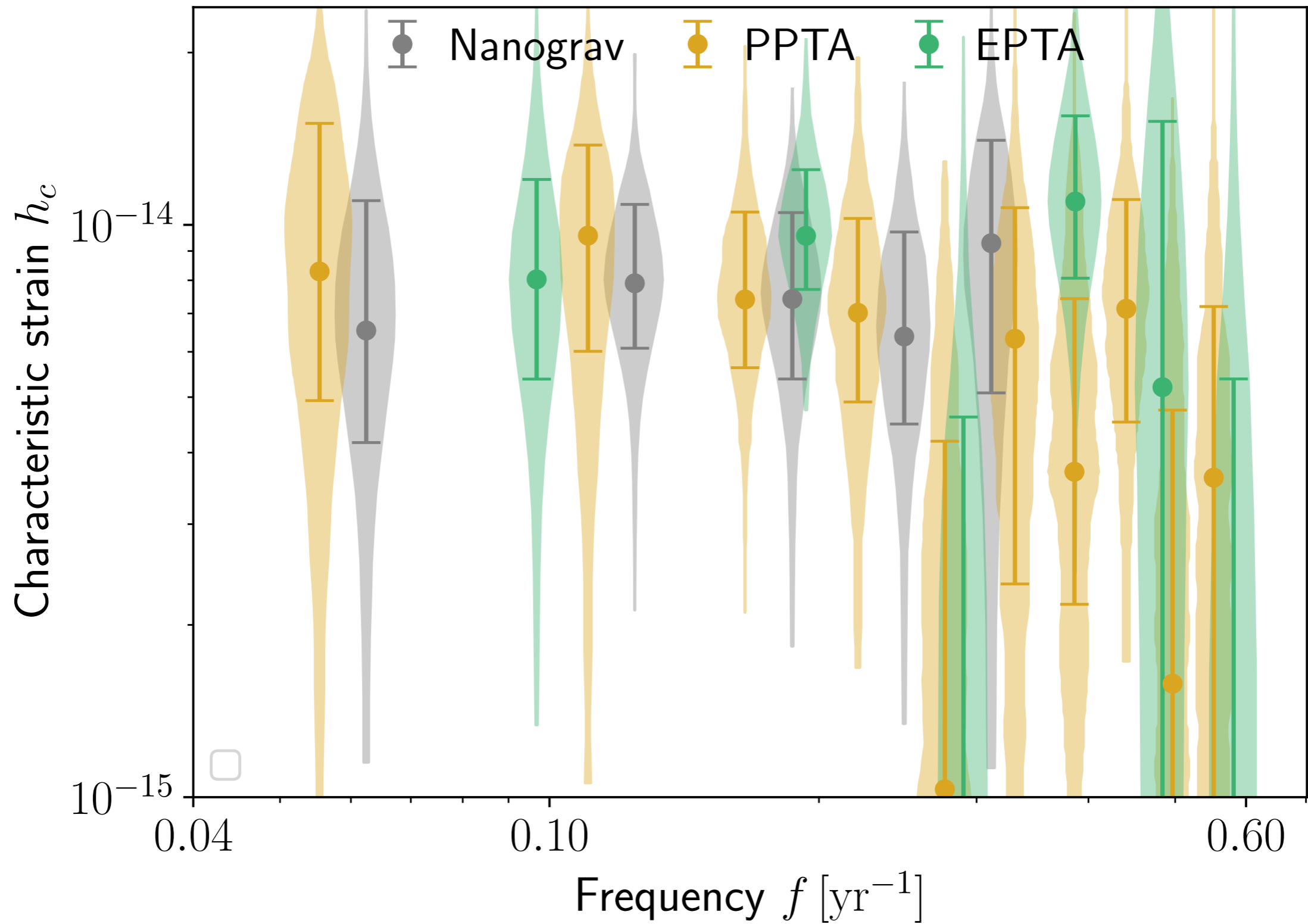
# Stochastic gravitational wave spectrum

Add contributions from all SMBH mergers:



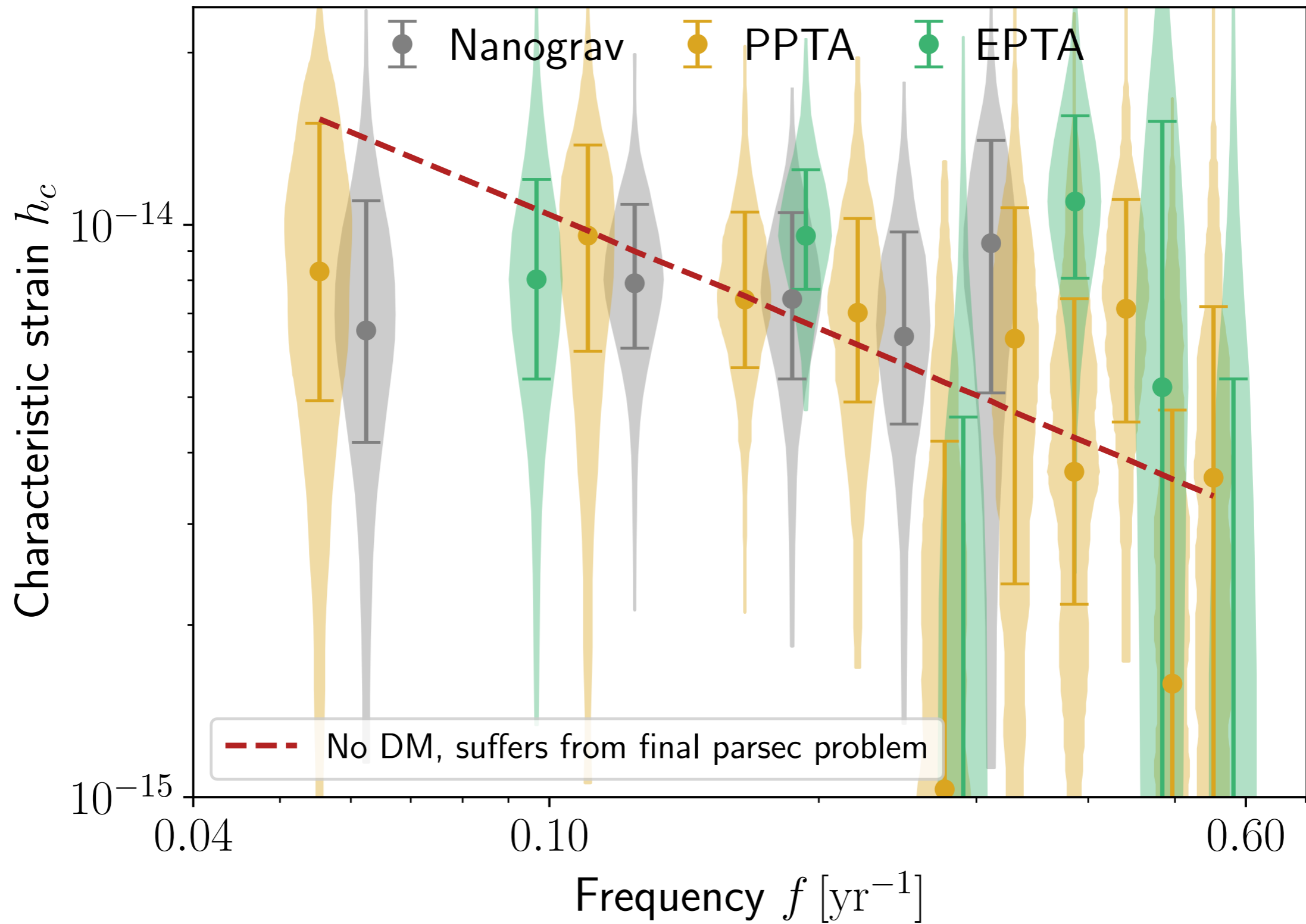
Merger rate: observational + semi analytical models

# GW spectrum at PTAs

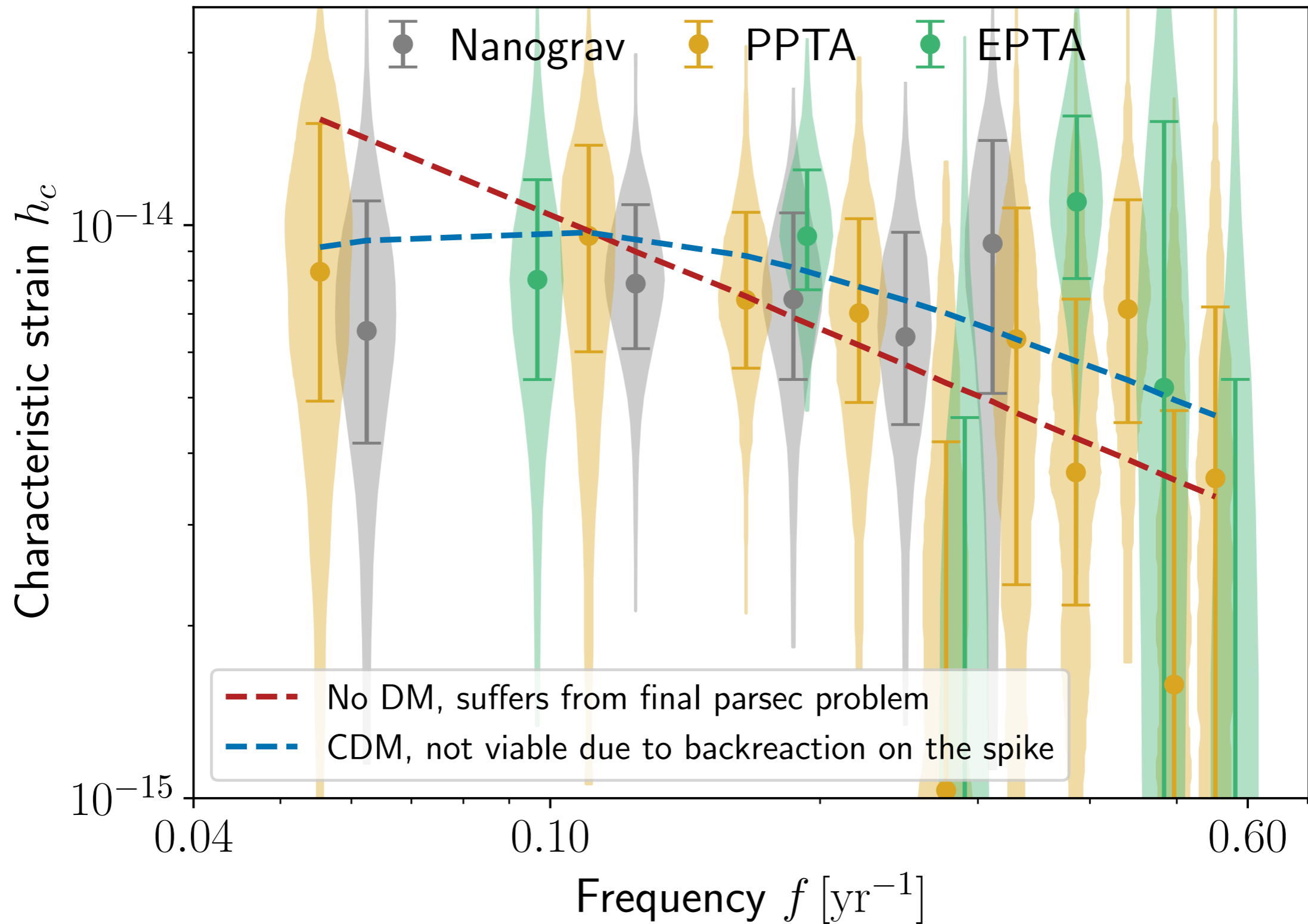




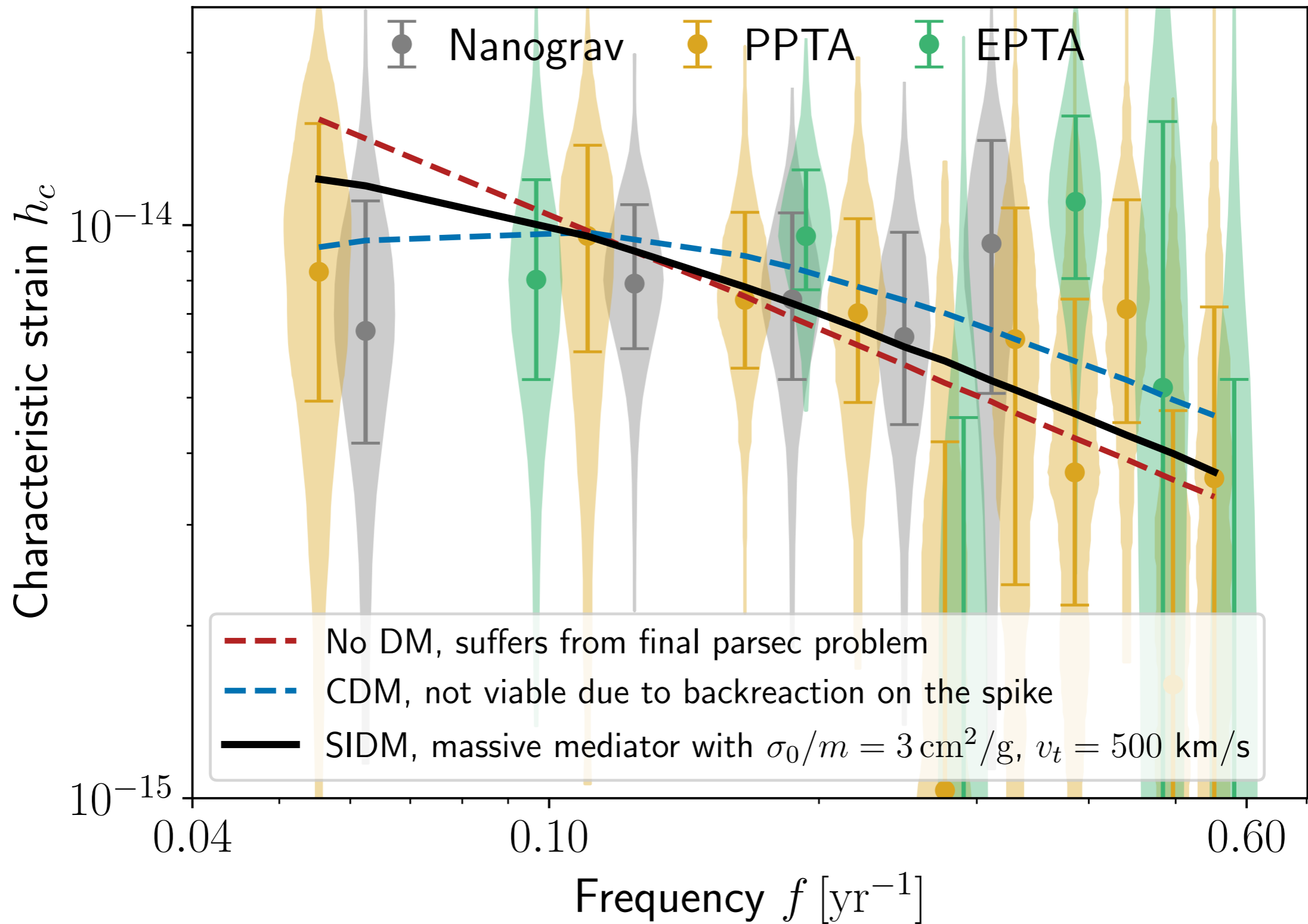
# GW spectrum at PTAs



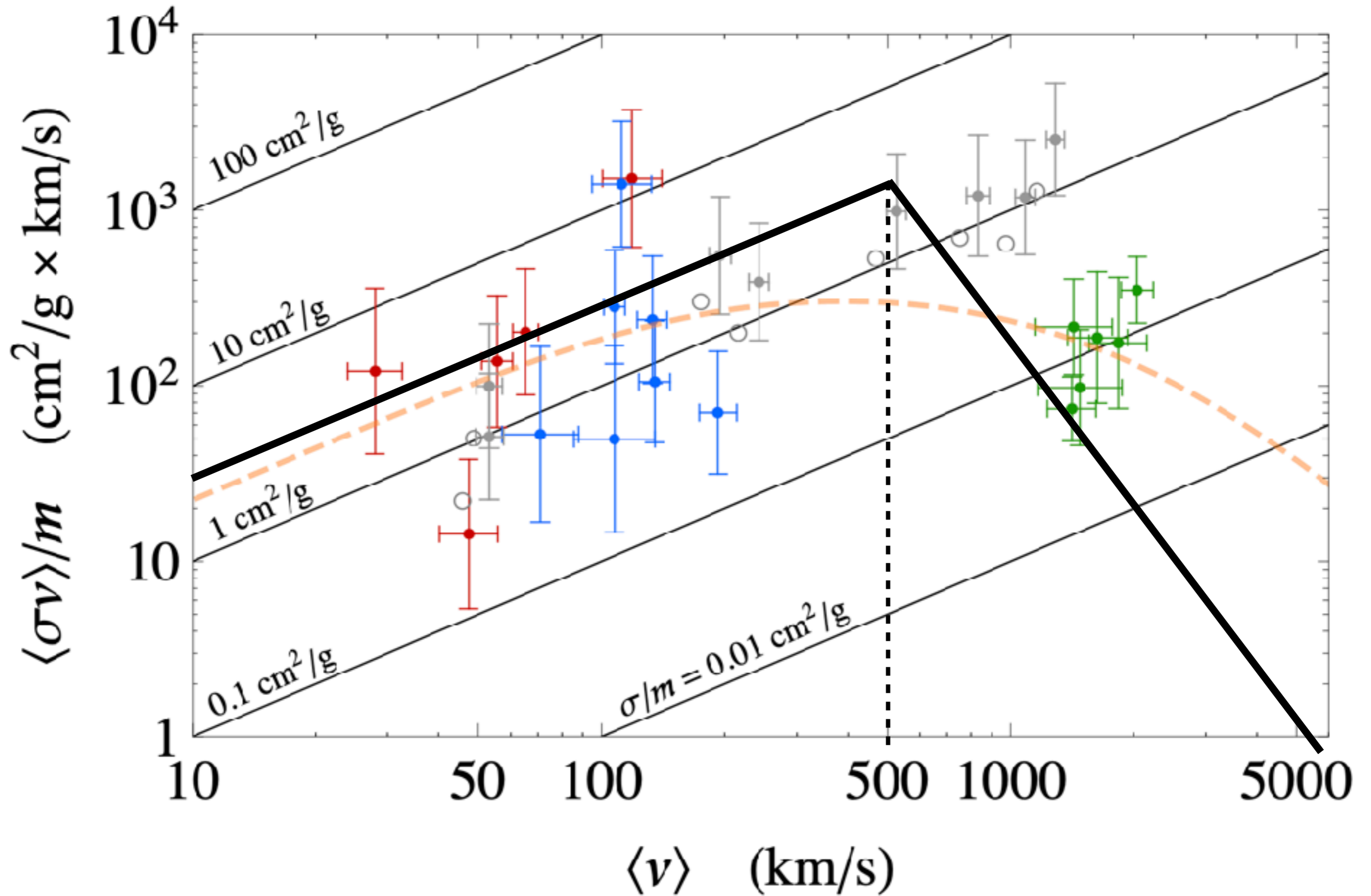
# GW spectrum at PTAs



# GW spectrum at PTAs



# Compatible with small-scale structure



# Conclusions

- Self-interacting dark matter solves the final parsec problem of supermassive black holes.
- Correlated softening of the gravitational wave spectrum at pulsar timing arrays.
- Compatible with small scale structure hints.

# Work in progress

- Gravo-thermal simulation of merger & back-reaction on spike.
- Upgraded characteristic strain calculation including finite inspiral duration.
- Improved statistical analysis of PTA data.



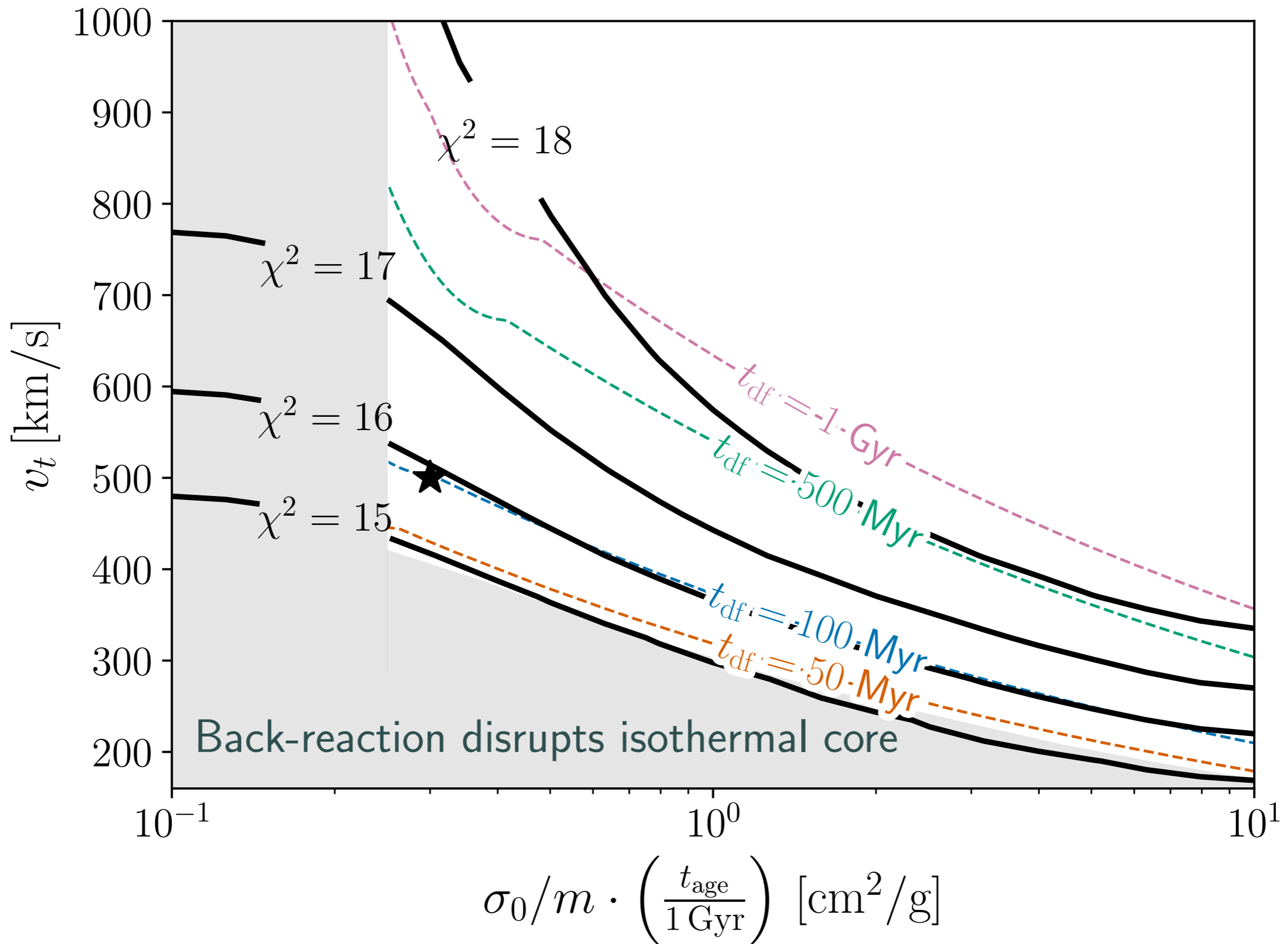
# Work in progress

- Gravo-thermal simulation of merger & back-reaction on spike.
- Upgraded characteristic strain calculation including finite inspiral duration.
- Improved statistical analysis of PTA data.

**Thanks!**

# Backup material

# SIDM massive mediator



# Pulsar timing arrays

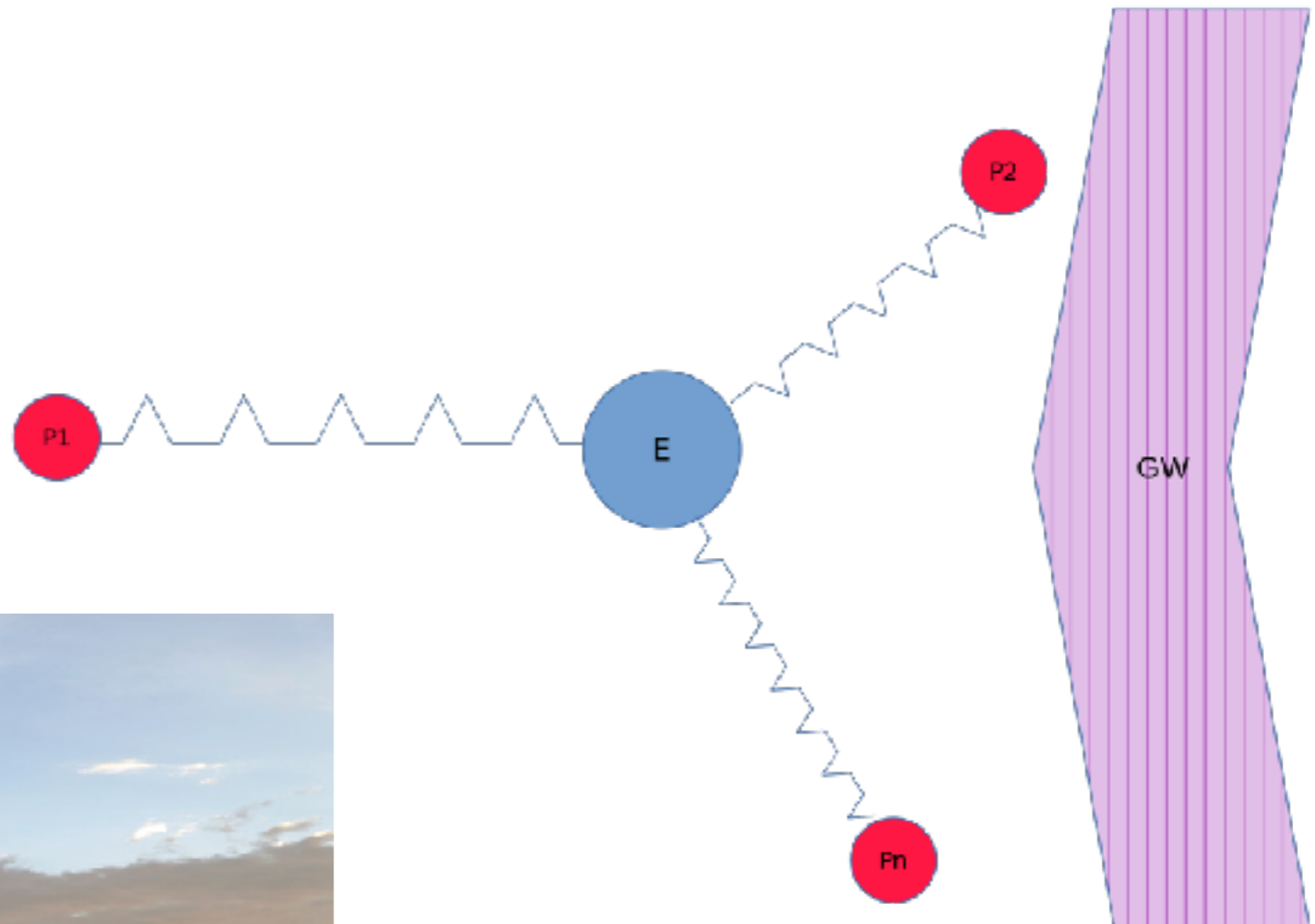


Image credit: Wikipedia



Very Large Array radio telescope

# New physics explanations

## Inflationary GW

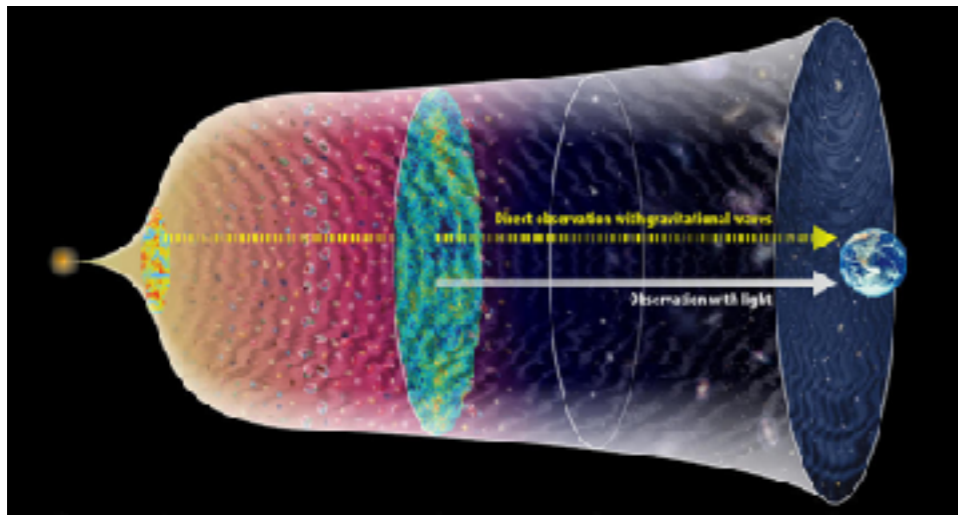


Image credit: NAOJ

## Phase transitions

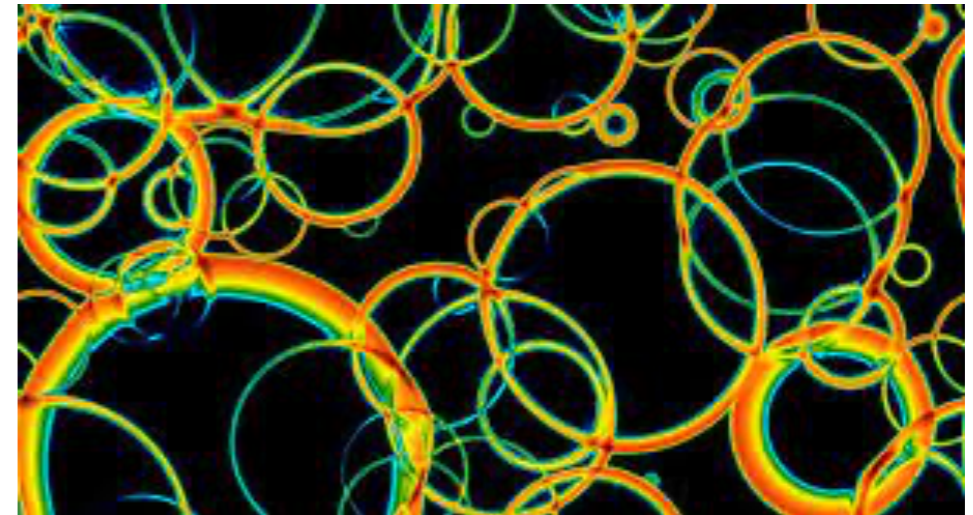


Image credit: Weir et al (2016)

## Cosmic strings

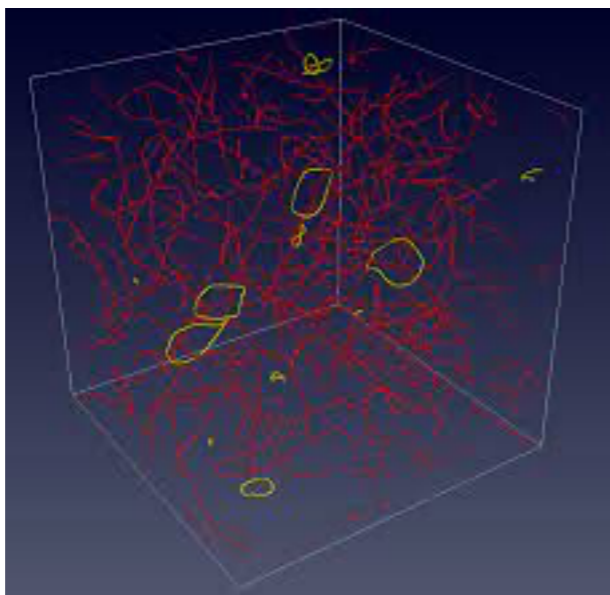


Image credit: Kitajima et al (2023)

## Domain Walls

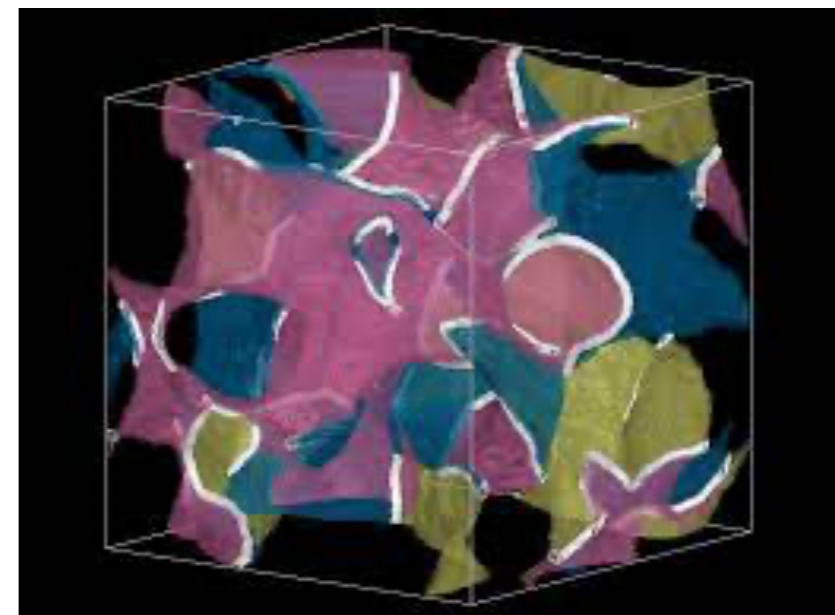
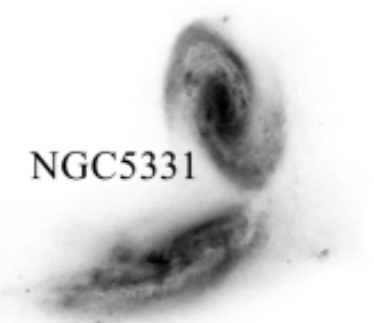


Image credit: Hiramatsu et al (2013)



# SMBH mergers

## Galaxy Merger



NGC 5331  
Dynamical friction drives massive objects to central positions

## Stellar Core Merger



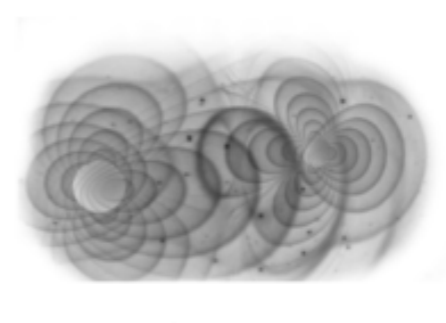
NGC 17  
Dynamical friction less efficient as SMBHs form a binary.

## Binary Formation



4C 37.11  
Stellar and gas interactions may dominate binary inspiral?

## Continuous GWs



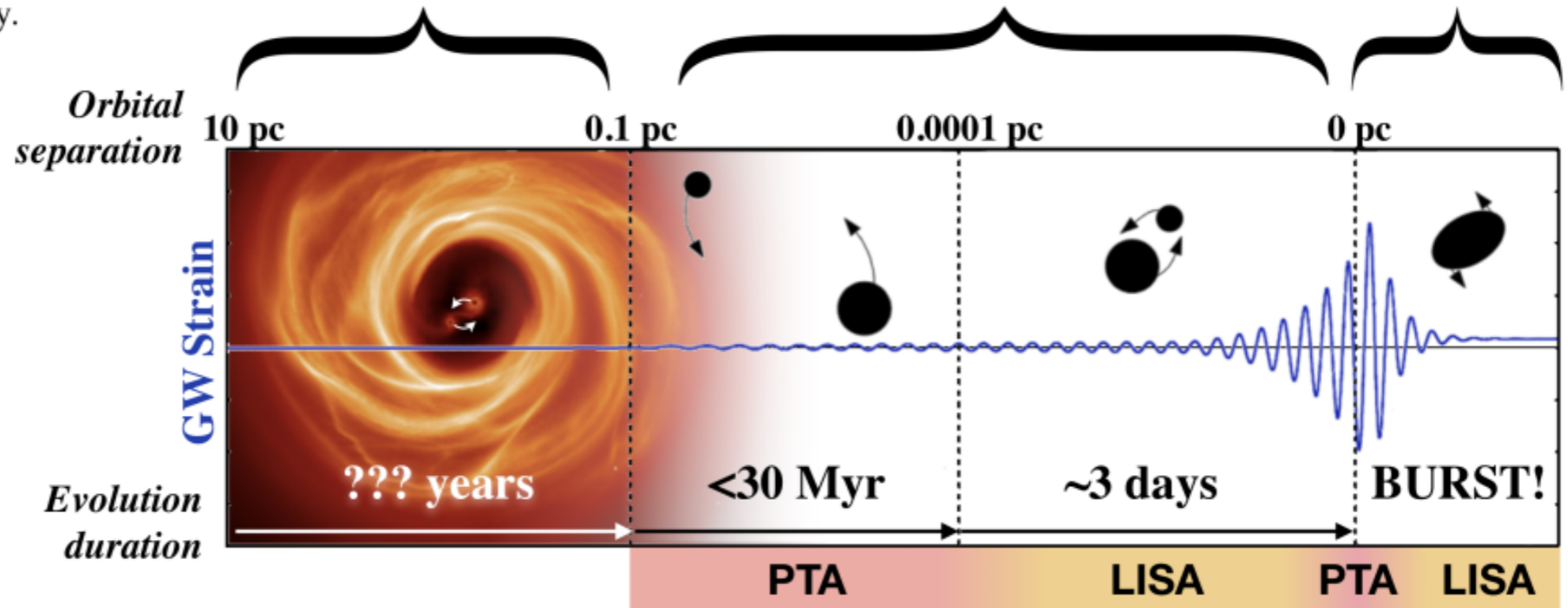
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.

## Coalescence, Memory & Recoil



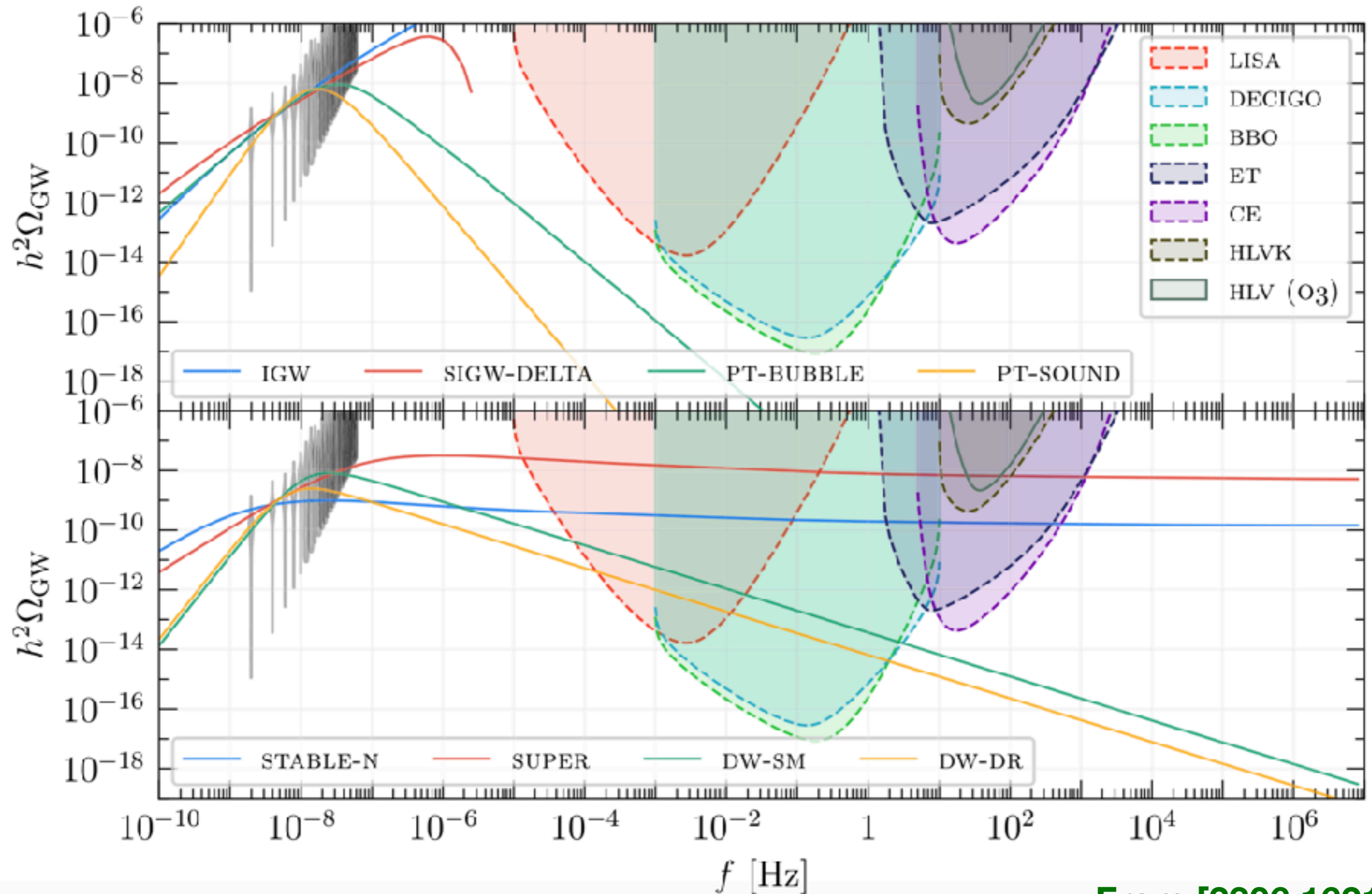
Post-coalescence system may experience gravitational recoil.

**The Lifecycle of Binary Supermassive Black Holes**



From NANOGrav (designed by Sarah Spolaor)

# New physics explanations

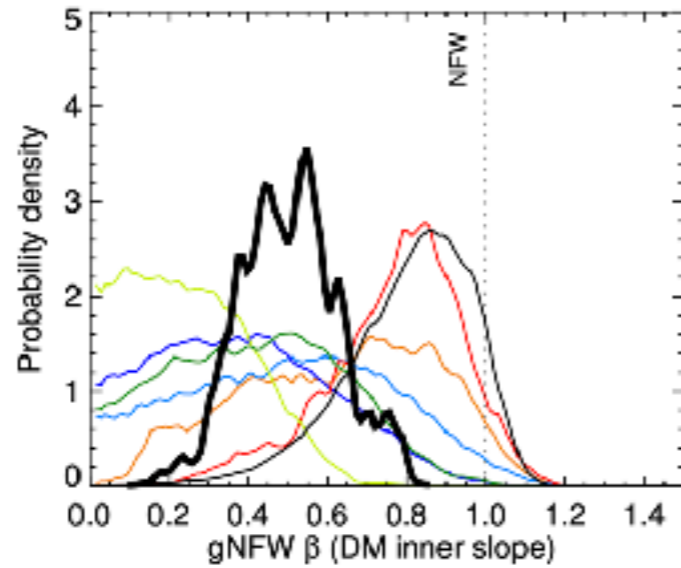


From [2306.16219]

# Self-interacting dark matter

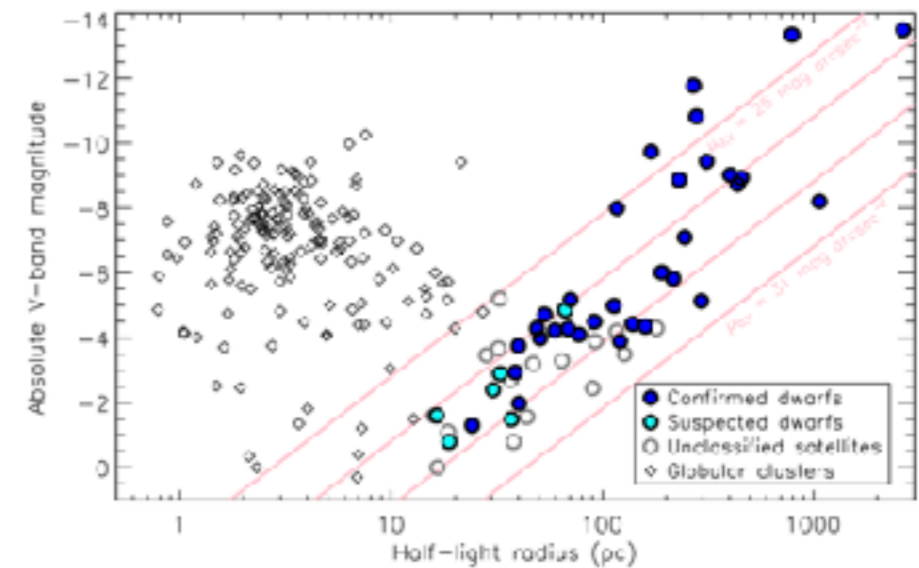
- **Hints**

## Core vs cusp



[1209.1392]

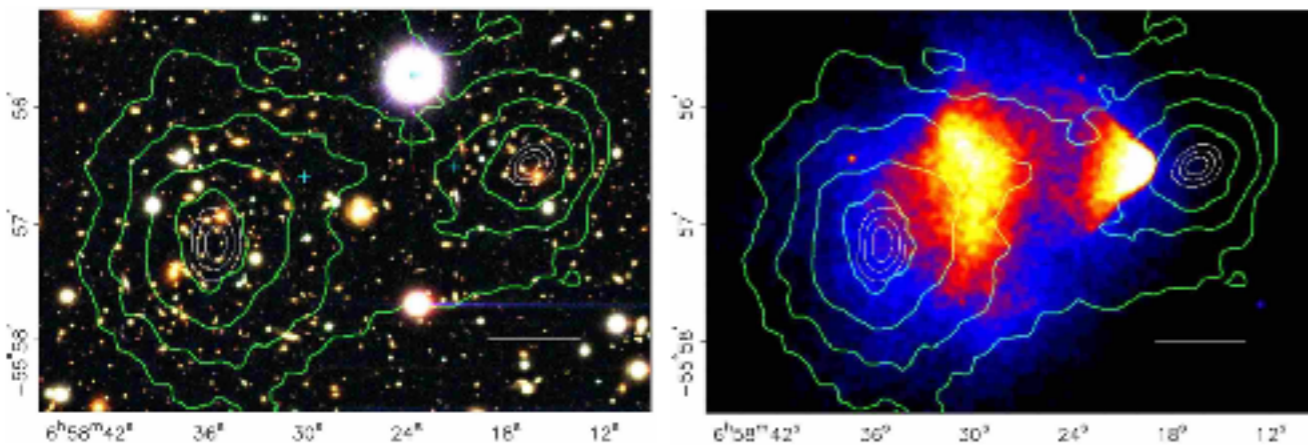
## Missing satellites (?)



[1901.05465]

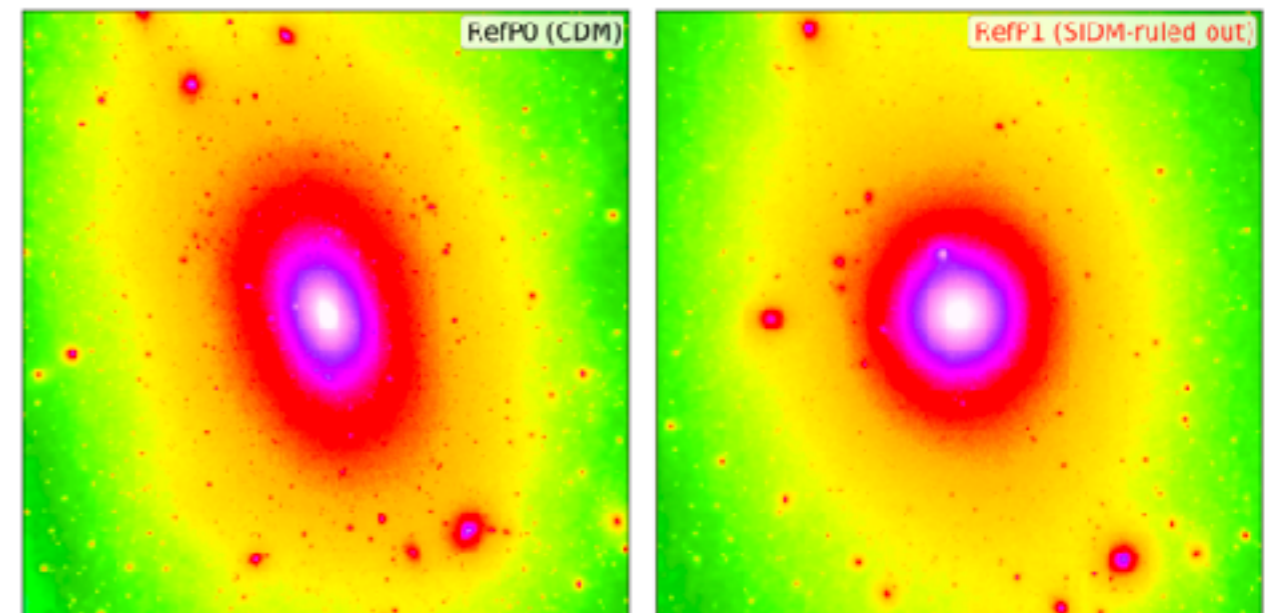
- **Constraints**

## Galaxy cluster mergers



[astro-ph/0608407]

## Halo shape

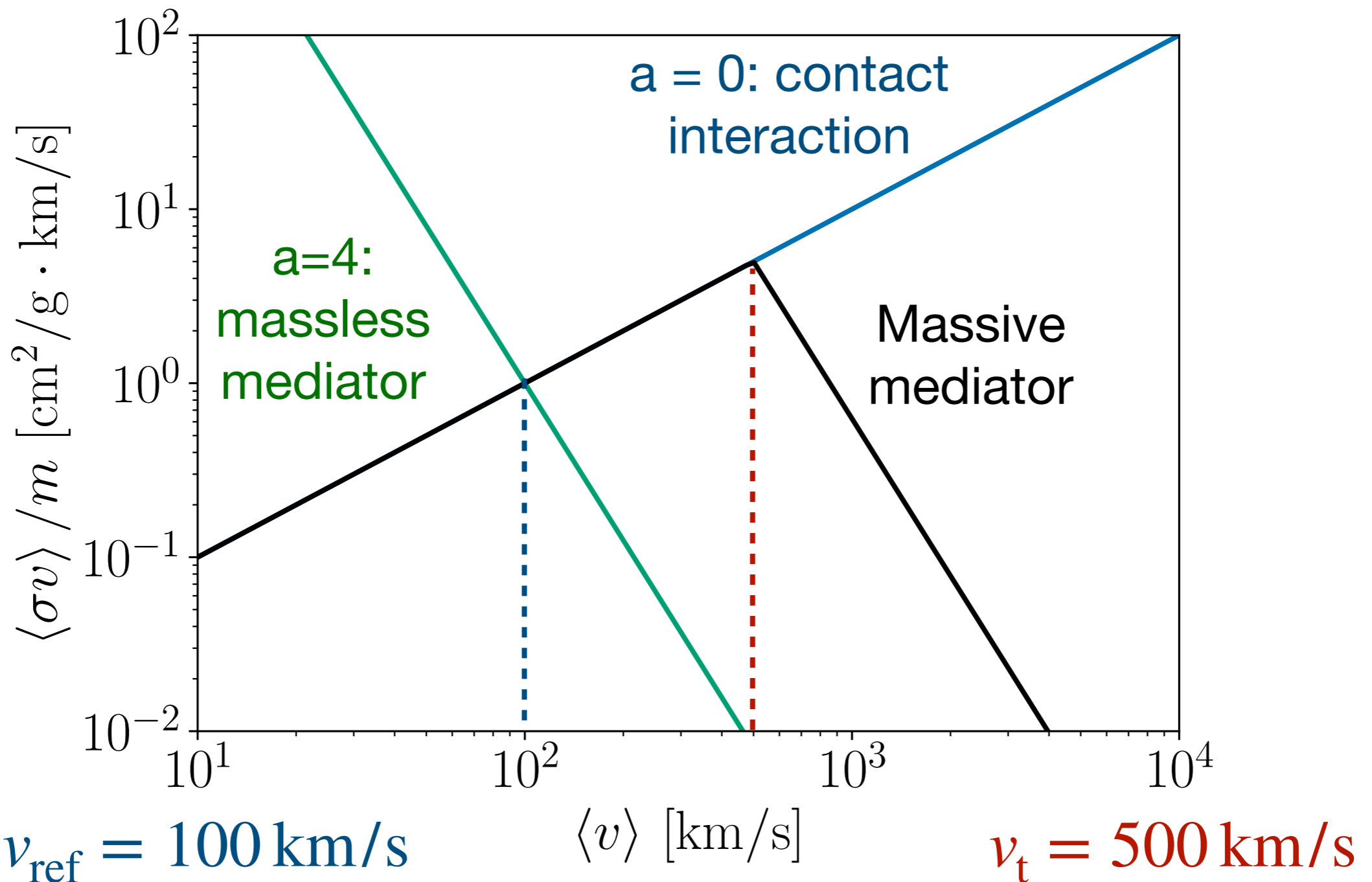


[1201.5892]

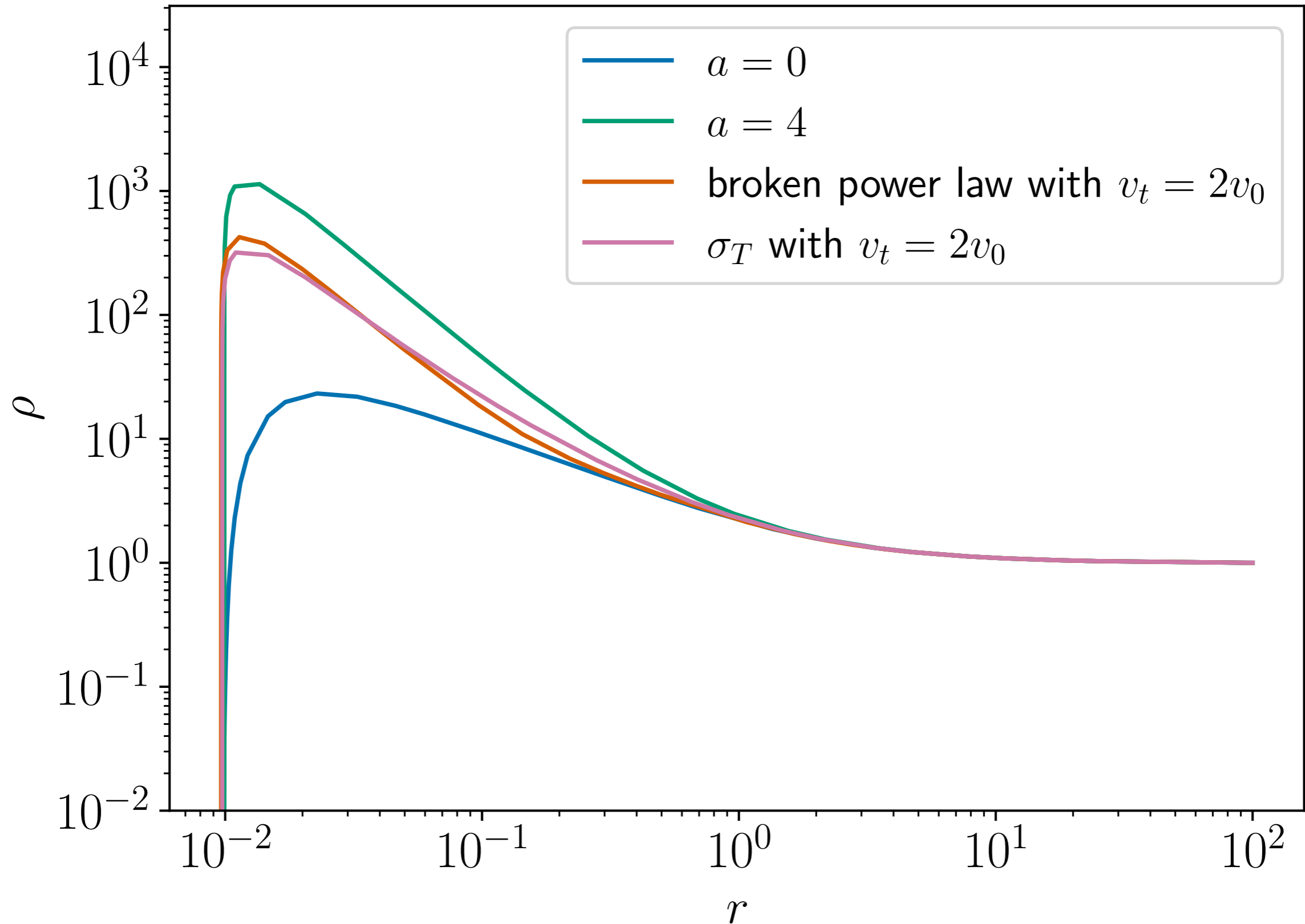


# SIDM parametrization

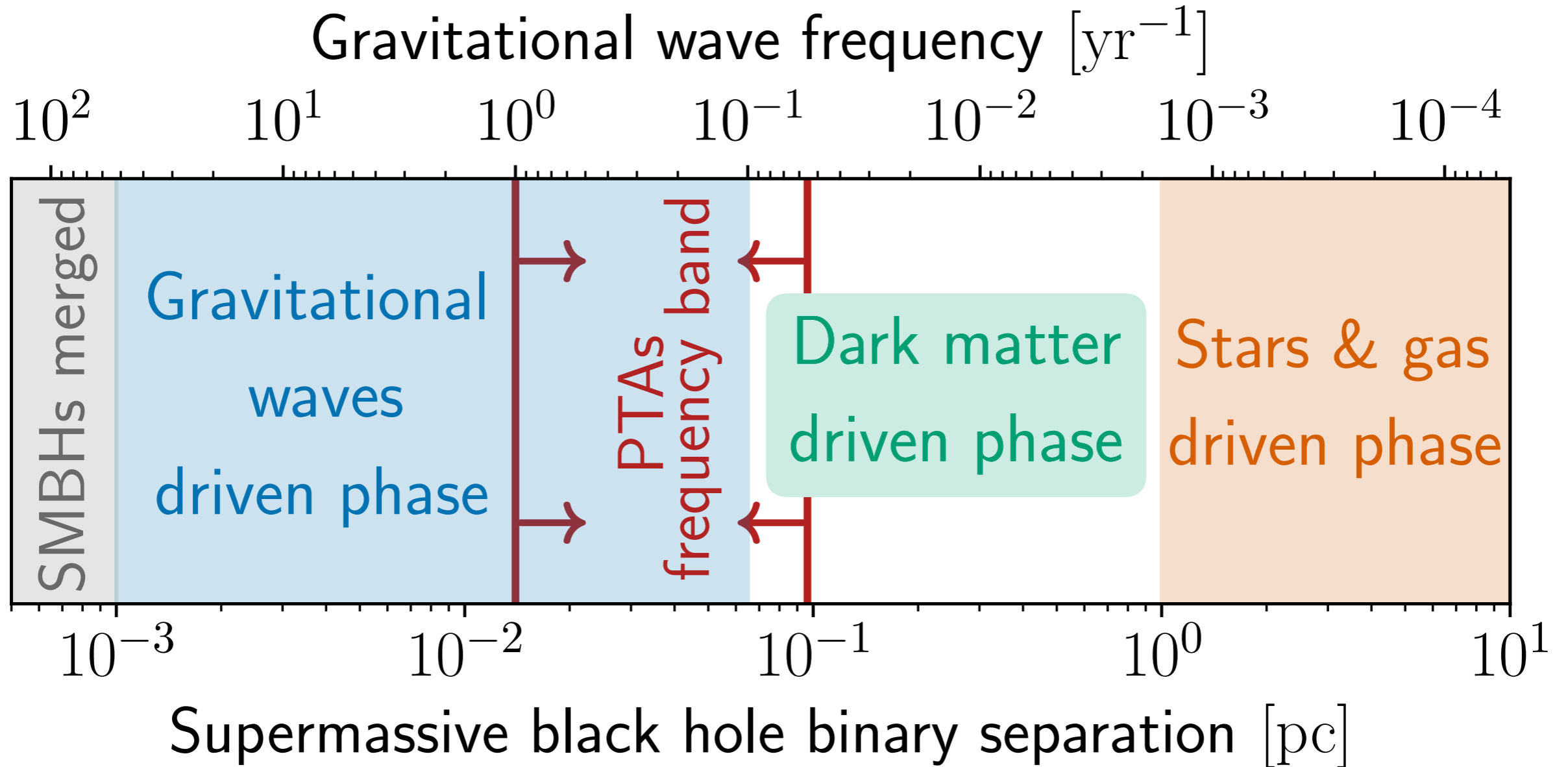
$$\frac{\langle \sigma v \rangle}{m} = \frac{\sigma_0}{m} \left( \frac{v_{\text{ref}}}{v} \right)^a v$$



# Power law vs. velocity transfer cross section

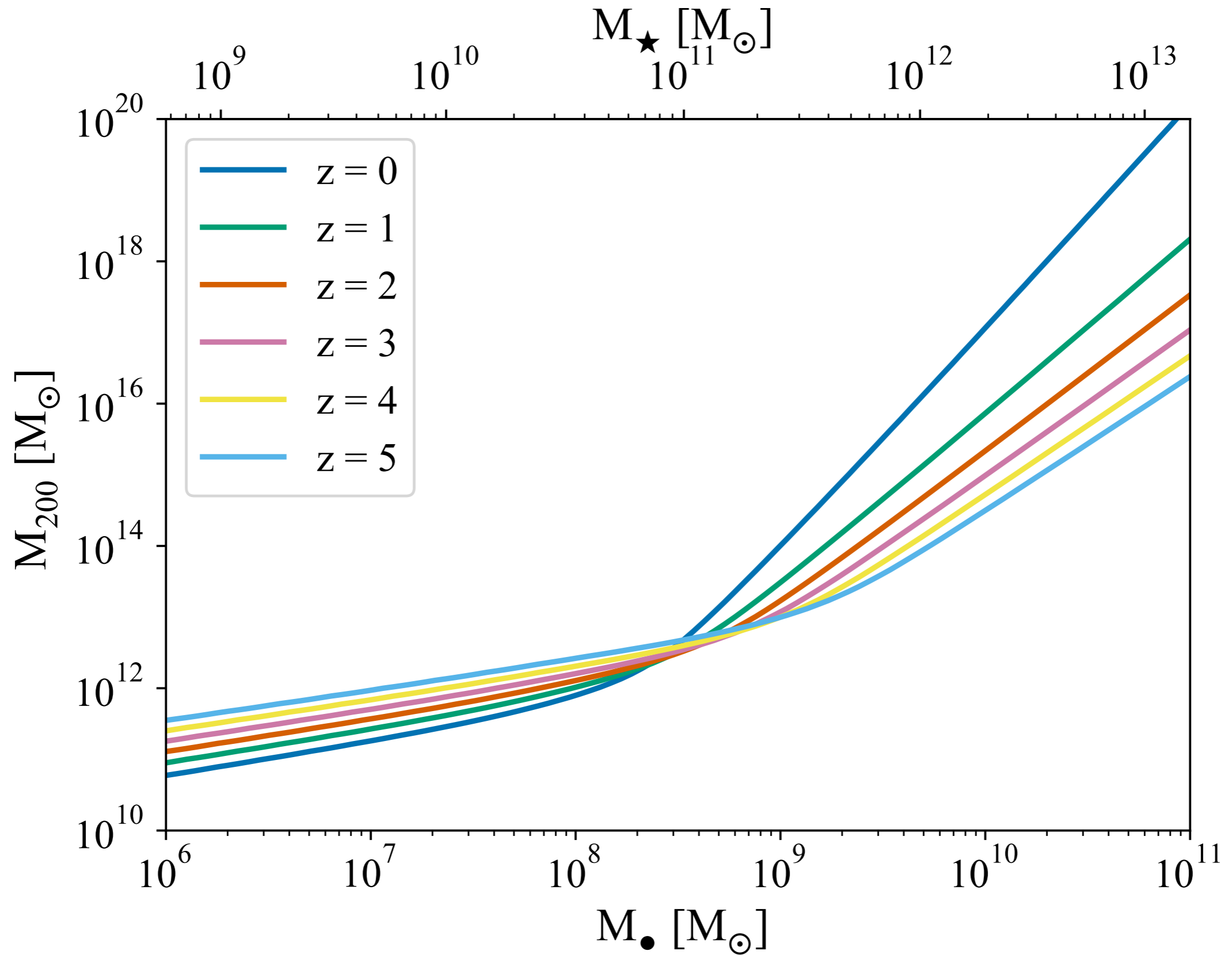


# DM spike dynamical friction

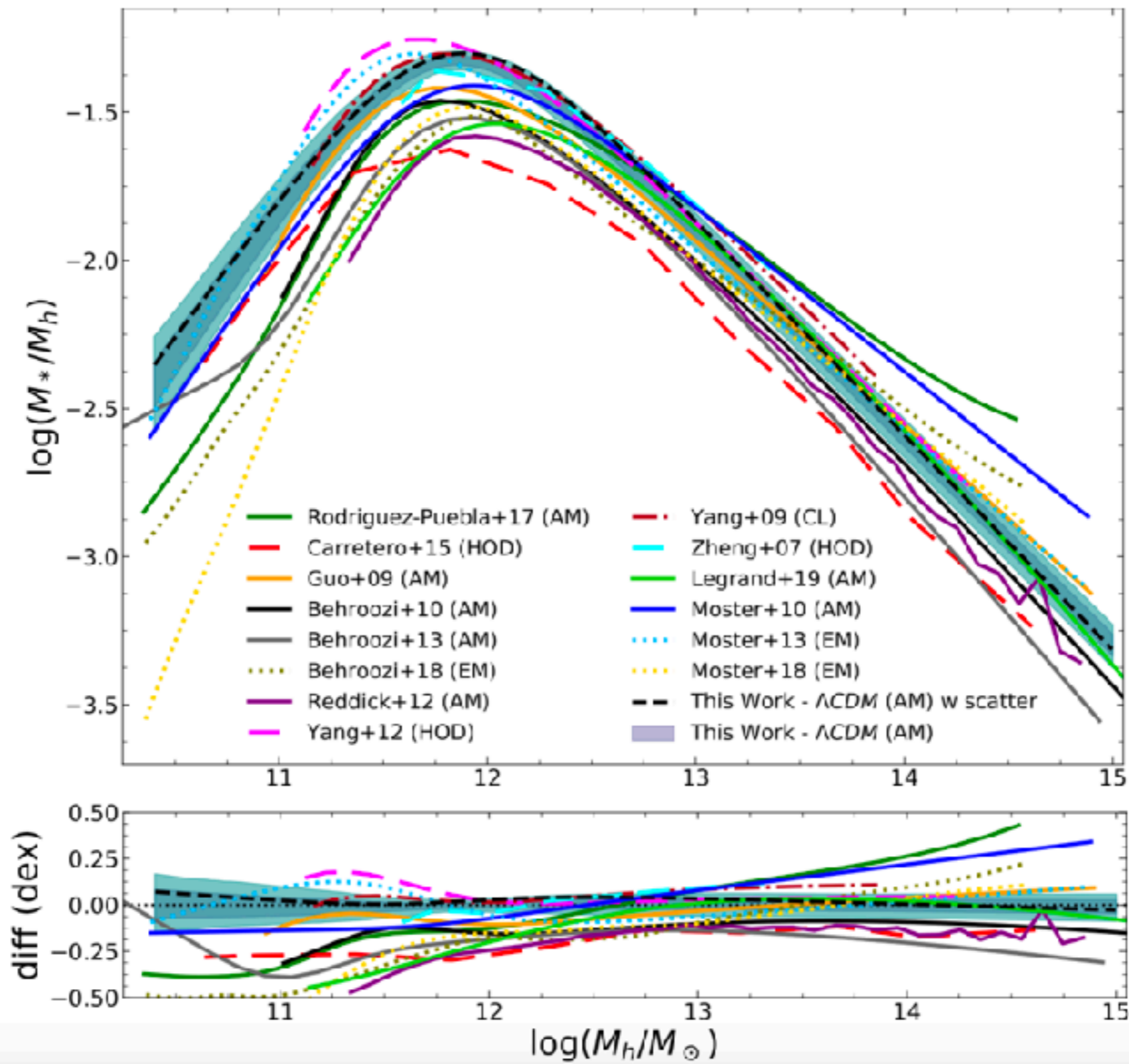


Crucial impact on SMBH binary evolution

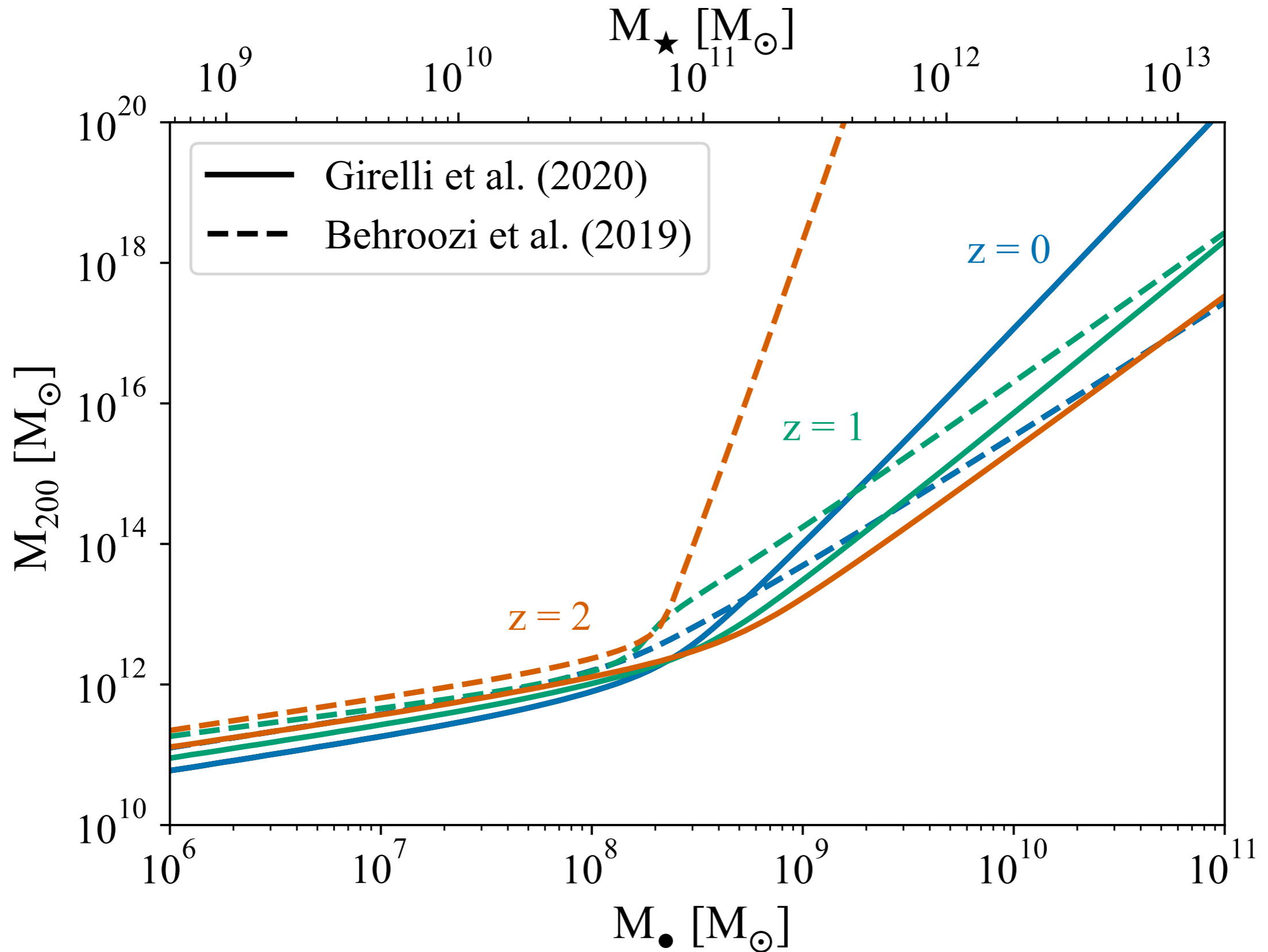
# BH-to-halo mass relation



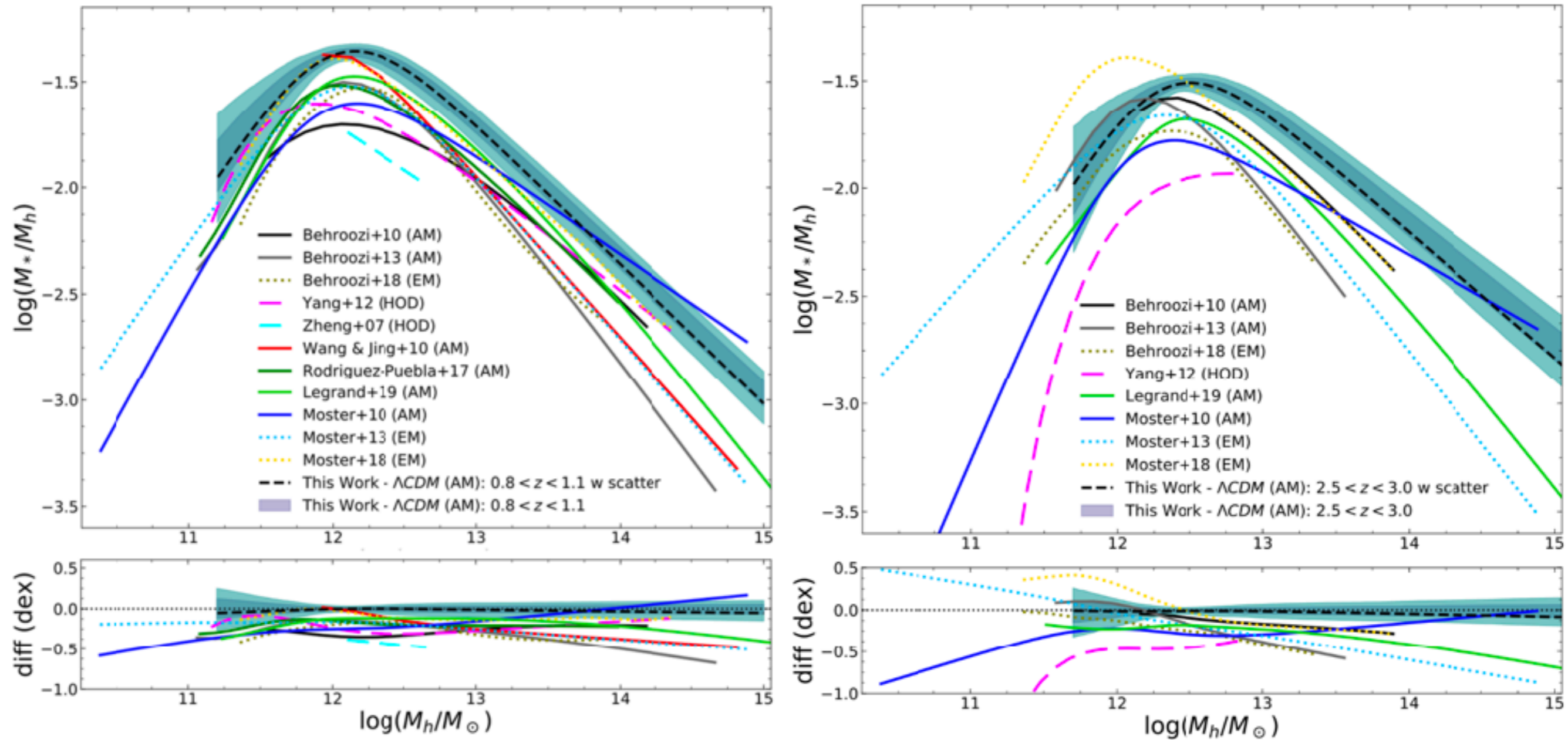
# Stellar-to-halo mass relations



# Stellar-to-halo mass relations



# Stellar-to-halo mass relations

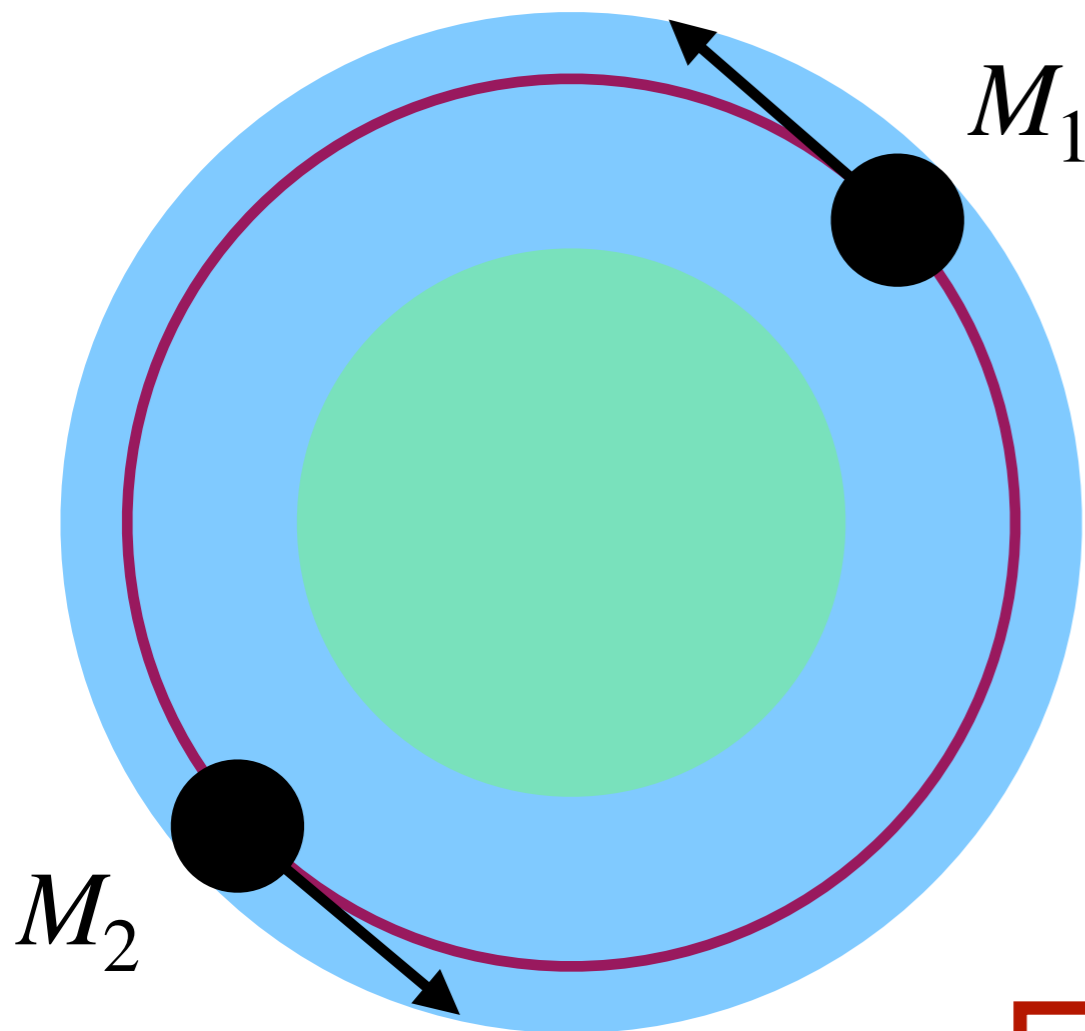


**Fig. 10.** Best-fit SHMR compared to previous results at  $z \sim 1$  (left) and  $z \sim 3$  (right). Symbols are the same as in Fig. 9.



# Orbital radius evolution

DM dynamical friction + GW emission



$R < 10 \text{ pc}$

Power lost by the binary

$$P_{\text{gw}} \propto R^{-5}$$

$$P_{\text{df}} \propto R^{1/2} \rho_{\text{spike}}(R)$$

Relate to orbital energy loss

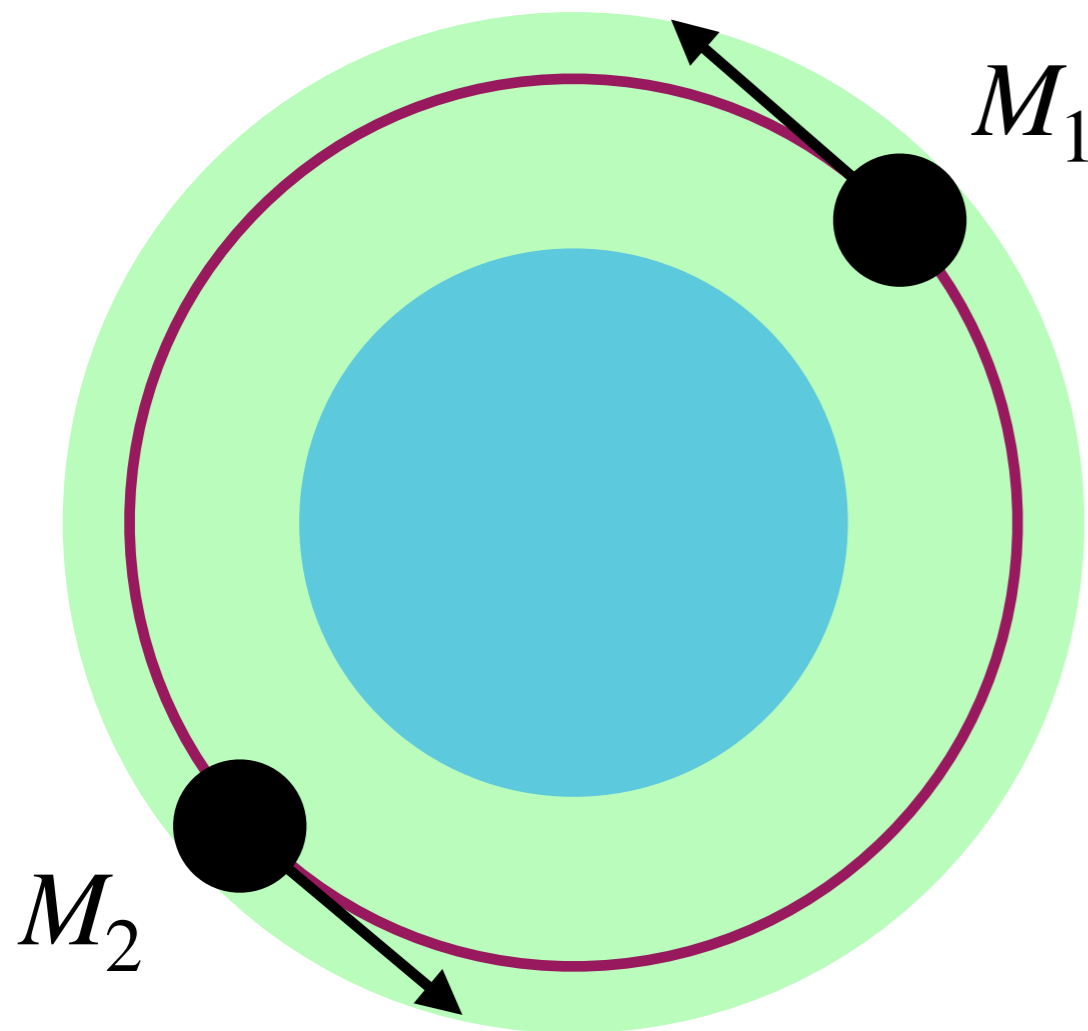
$$\dot{E}_{\text{orbit}} \propto \frac{\dot{R}}{R^2}$$

$$\dot{E}_{\text{orbit}} = P_{\text{gw}} + P_{\text{df}} \rightarrow R(t)$$



# Orbital radius evolution

DM dynamical friction + GW emission



$R < 10 \text{ pc}$

$$P_{\text{gw}} = \frac{32}{5} q^2 (1 + q) G^4 \frac{M_1^5}{R^5}$$

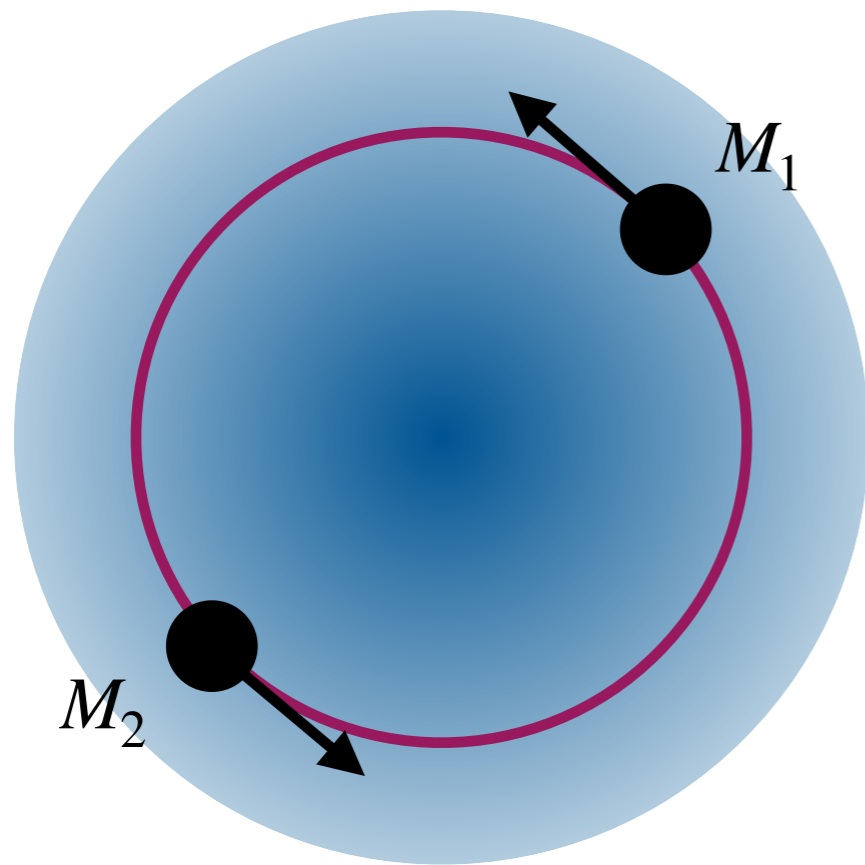
$$P_{\text{df}} = 12\pi^2 q^2 \sqrt{(1 + q)} (GM_1)^{3/2} \times R^{1/2} \left( N_2 \rho(r_2) + N_1 \frac{\rho(r_1)}{q^3} \right)$$

$$\dot{E}_{\text{orb}} = q G M_1 \frac{\dot{R}}{2R^2}$$

$$\dot{E}_{\text{orb}} = P_{\text{gw}} + P_{\text{df}} \rightarrow R(t)$$

# Back-reaction on the spike

$$M_{\text{spike}} \sim M_1 + M_2$$



Orbital energy lost:

$$\Delta E_{\text{orbit}} \sim \frac{G M_1^2}{R_{\text{gw}}}$$

Binding energy of spike:

$$U_{\text{spike}} \sim \frac{G M_1^2}{r_{\text{spike}}}$$

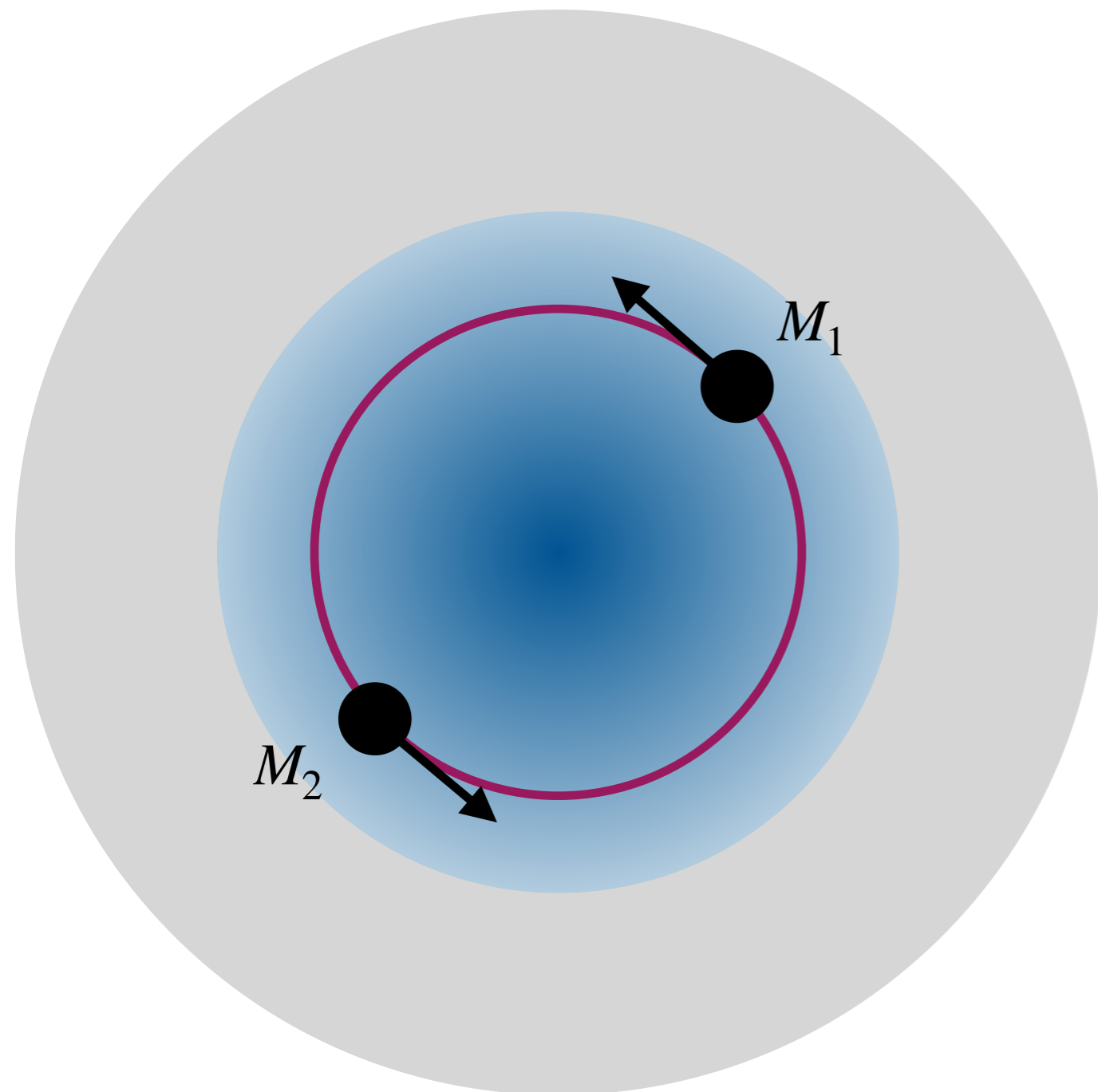
Since  $r_{\text{spike}} \gg R_{\text{gw}}$ :

$$\Delta E_{\text{orbit}} \gg U_{\text{spike}}$$

**Spike is quickly disrupted**

# SIDM core absorbs orbital energy

$$M_{\text{core}} \gg M_1 + M_2$$



Orbital energy lost:

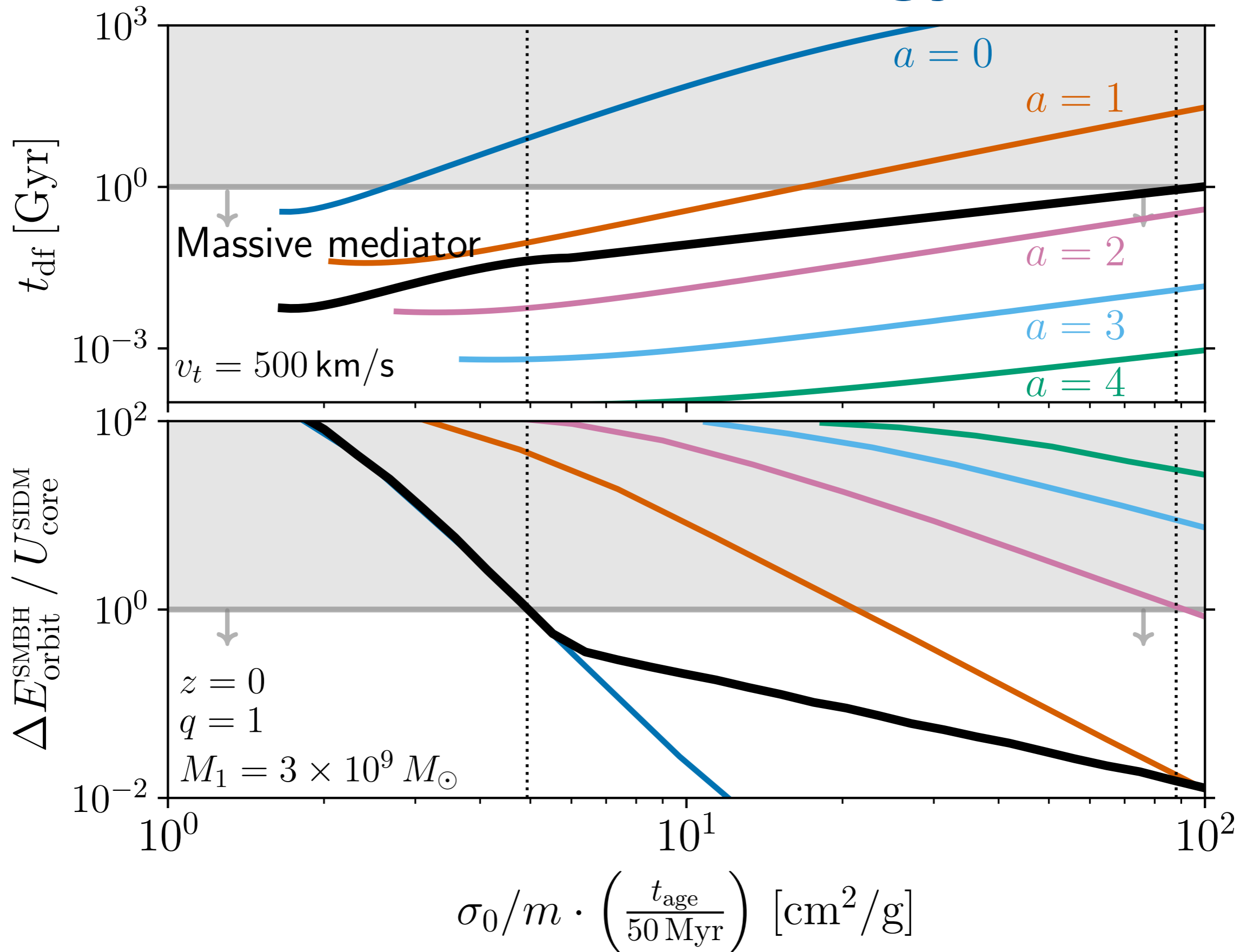
$$\Delta E_{\text{orbit}} \sim \frac{G M_1^2}{R_{\text{gw}}}$$

Binding energy of the core:

$$U_{\text{core}} \sim \frac{G M_{\text{core}}^2}{r_{\text{core}}}$$

**Minimum size of core  $\rightarrow$  lower limit on  $\sigma_0/m$**

# DF timescale & energy ratio



# Core relaxation timescale

Timescale at which the SIDM spike is replenished

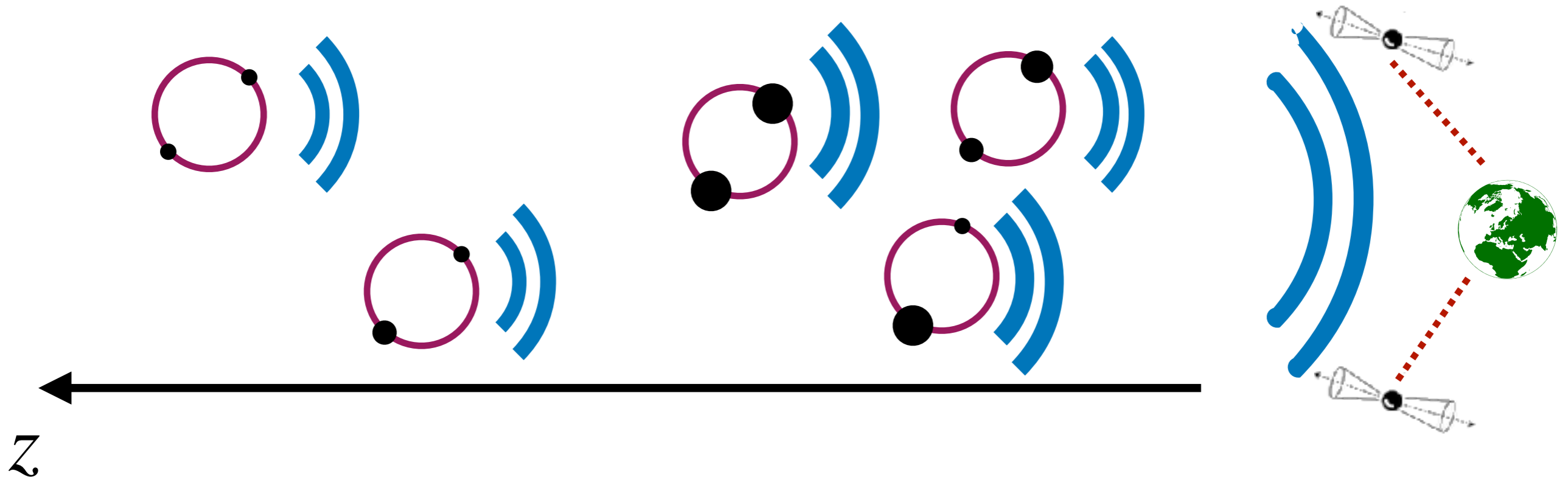
$$t_r \sim \left( \rho_c \frac{\sigma v_0}{m} \right)^{-1} \cong t_{\text{age}}$$

- Must be  $t_r \lesssim t_{\text{df}}$  for spike in equilibrium
- The core is at least that old:  $t_{\text{age}} \gtrsim t_{\text{df}}$

**Demand  $t_{\text{age}} = t_{\text{df}}$**

# Characteristic strain

Add contributions from all SMBH mergers:



**Characteristic strain at detection**

$$h_c^2(f) = \frac{4G}{\pi f} \int dz dM_1 dq \frac{d^3n}{dz dM_1 dq} \frac{dE}{df_s}$$

$$q = \frac{M_2}{M_1}$$

$$f = \frac{f_s}{1+z}$$

# SMBH merger rate

Relate to galaxy merger rate

$$\frac{d^3 n}{dz dM dq} = \frac{d^3 n_g}{dz dM_\star dq_\star} \frac{dM_\star}{dM} \frac{dq_\star}{dq}$$

Input: observational + semi analytical models

