Sommerfeld Effect and Bound State Formation for Dark Matter with colored mediators: a Computational Framework

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Technical University of Munich

Based on a work in preparation with

Mathias Becker, Emanuele Copello

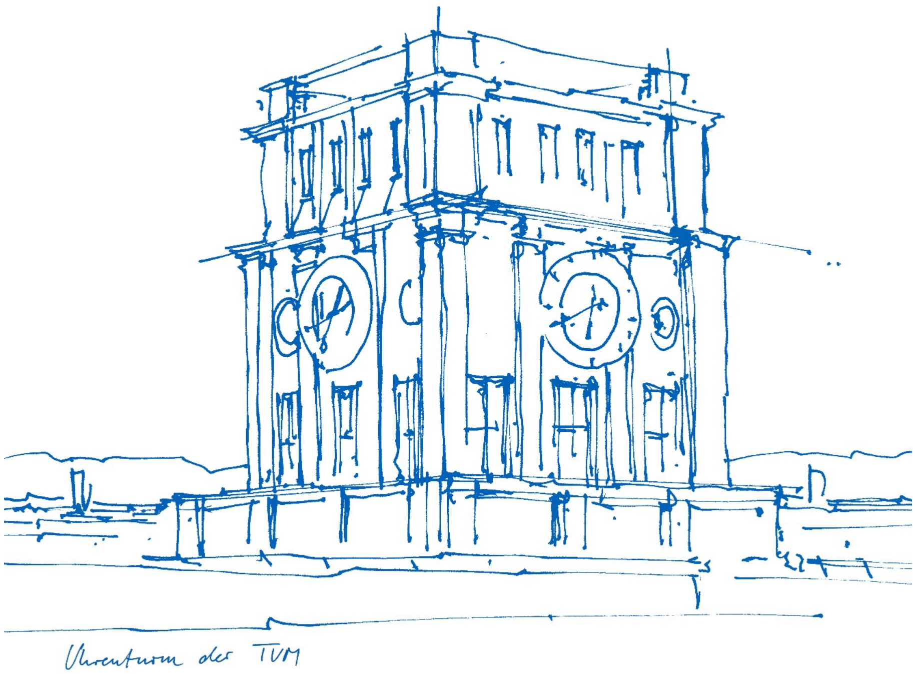
and Julia Harz

MITP workshop:

The Dark Matter Landscape -

From feeble to strong interactions

Thursday, August 29th 2024



Outline

Simplified dark matter models and long-range effects

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Workflow of the code

Showcases of our computational framework

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Classical WIMP evades detection so far.

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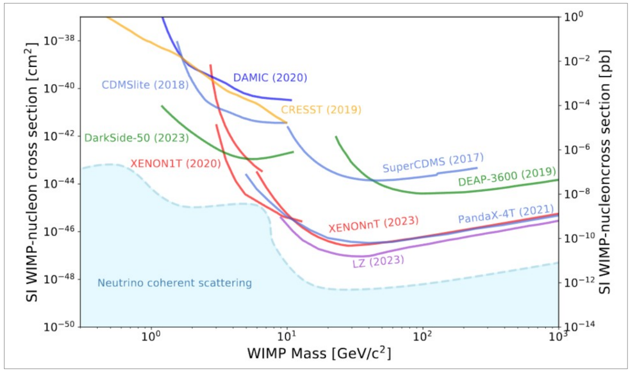

[PDG „Dark Matter“ (2024)]

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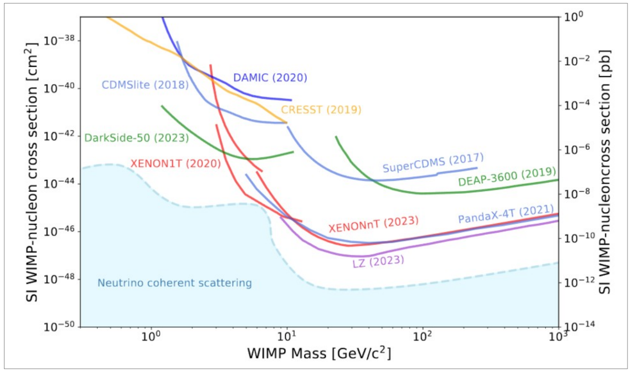

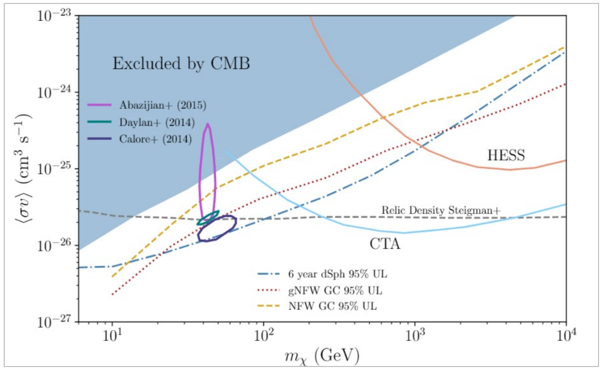

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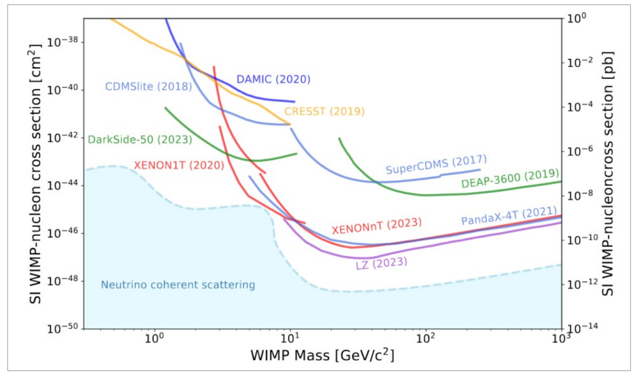
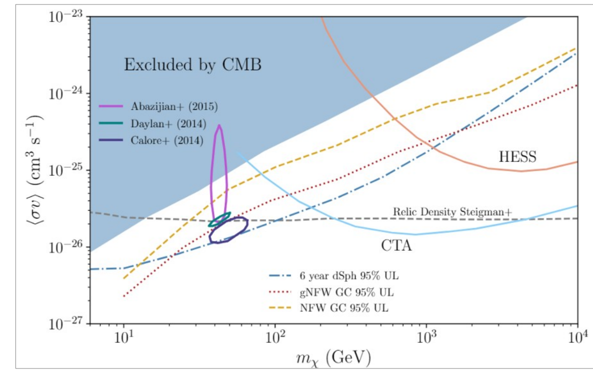
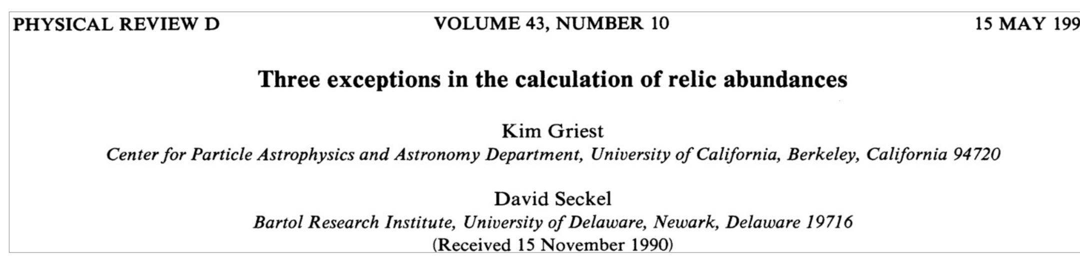




[PDG „Dark Matter“ (2024)]

Heavier, **coannihilating** mediators can be the reason.
Many processes and model parameters render the analysis complicated.


PHYSICAL REVIEW D VOLUME 43, NUMBER 10 15 MAY 1991

Three exceptions in the calculation of relic abundances

Kim Griest
Center for Particle Astrophysics and Astronomy Department, University of California, Berkeley, California 94720

David Seckel
Bartol Research Institute, University of Delaware, Newark, Delaware 19716
(Received 15 November 1990)

$$\langle \sigma_{\text{eff}} v_{\text{rel}} \rangle = \sum_{ij} \langle \sigma_{ij} v_{\text{rel}} \rangle \frac{Y_i^{\text{eq}} Y_j^{\text{eq}}}{\widetilde{Y}_{\text{eq}}^2}$$

Pheno toolbox

Experiment needs **minimal models** (few parameters) -
Theory needs precise and reliable **tools**!

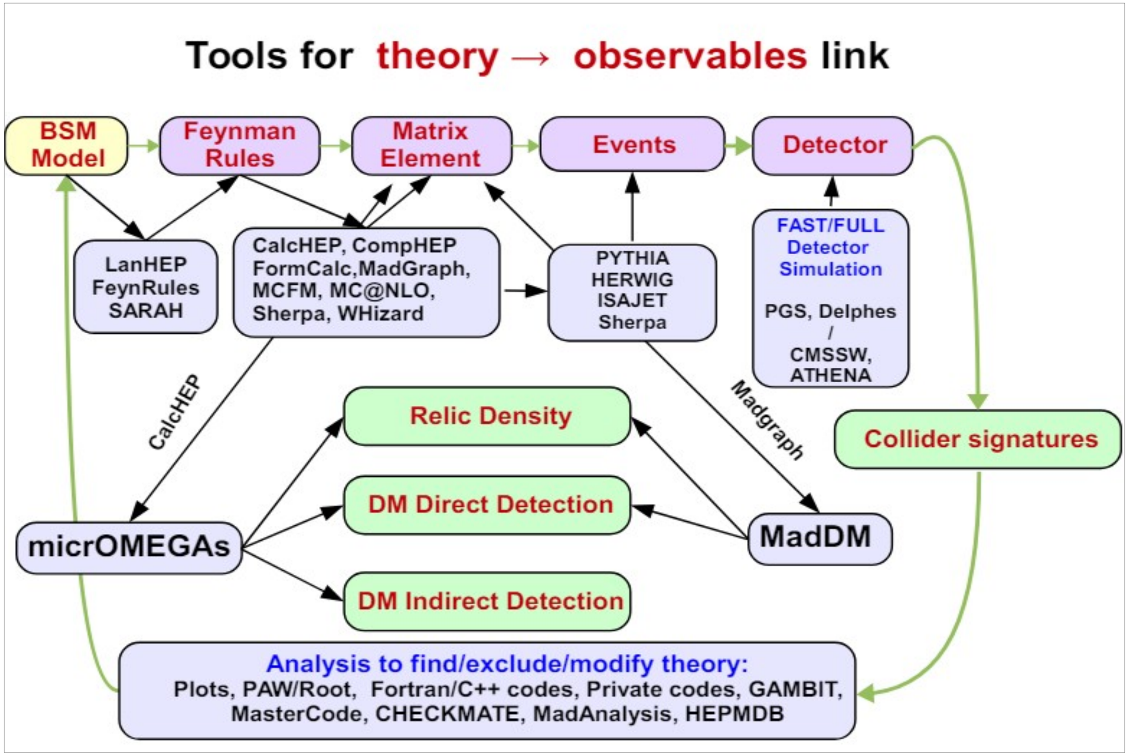
micrOMEGAs

DarkSUSY

MadDM

DM@NLO

...



[A. Belyaev (2018)]

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See talk by Sukanya Sinha

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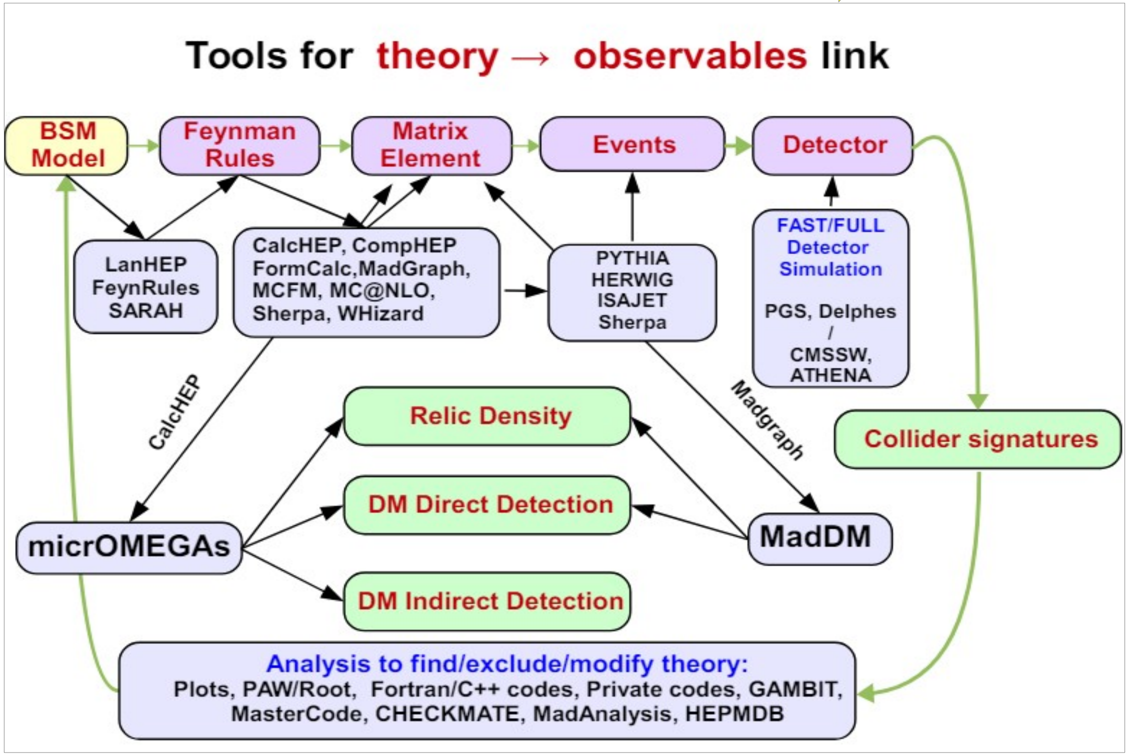

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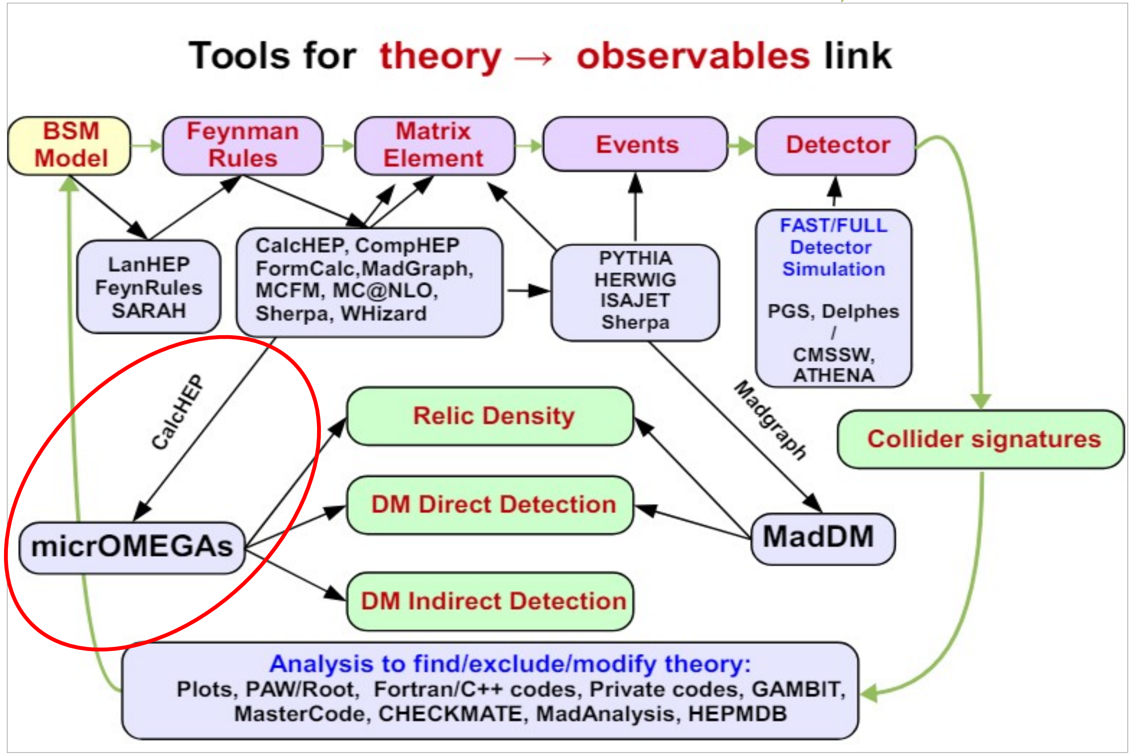

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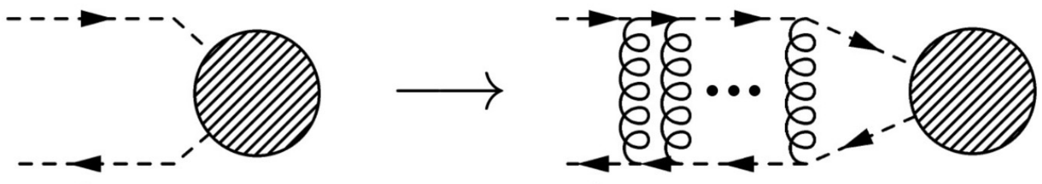
Long-range effects for Dark Matter I: Sommerfeld enhancement

Attractive (repulsive) Coulomb-like potentials significantly enhance (suppress) non-relatvistic scattering processes.



Relevant for $\alpha \sim v_{\rm rel}$

Resum n gauge boson exchange $\left(\frac{\alpha}{v_{rel}}\right)^n \sim 1$

[A. Sommerfeld (1931)]
[A. D. Sakharov (1948)]
[J. Hisano et al. (2006)]
[S. El Hedri et al. (2017)]

Long-range effects for Dark Matter II: Bound State Formation

Dark sector particles charged under a gauge group form a bound state that subsequently decays → **additional annihilation channel!**

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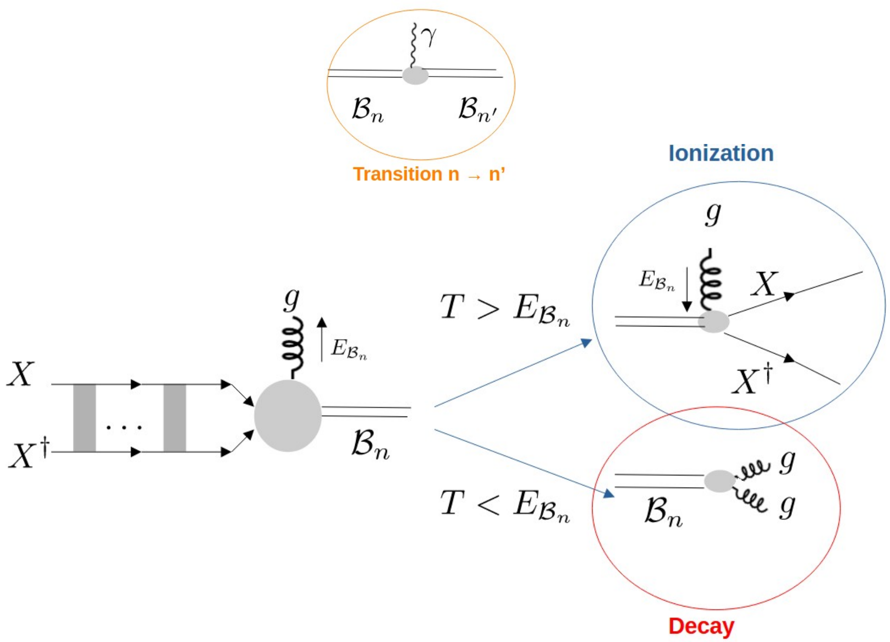

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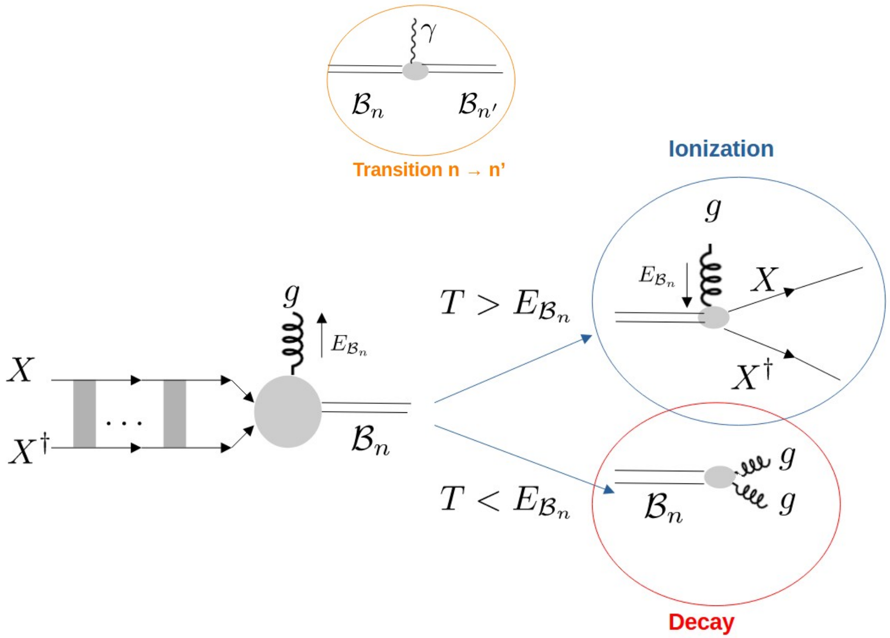

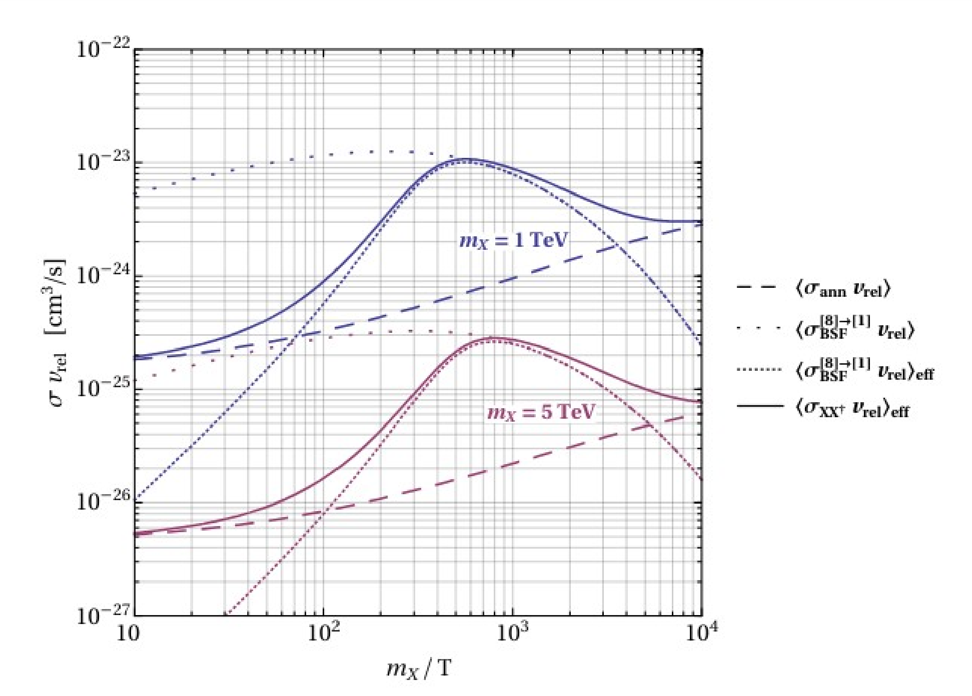

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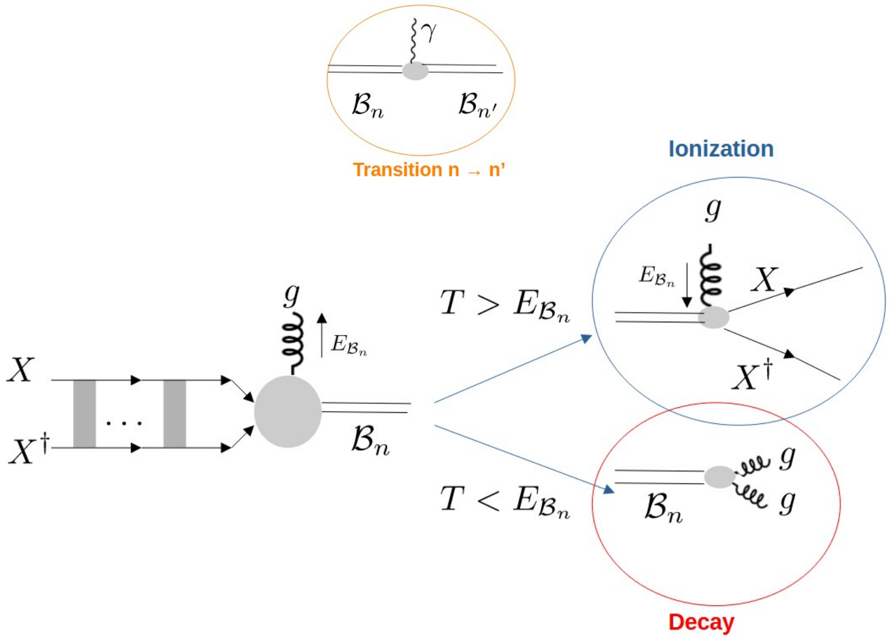

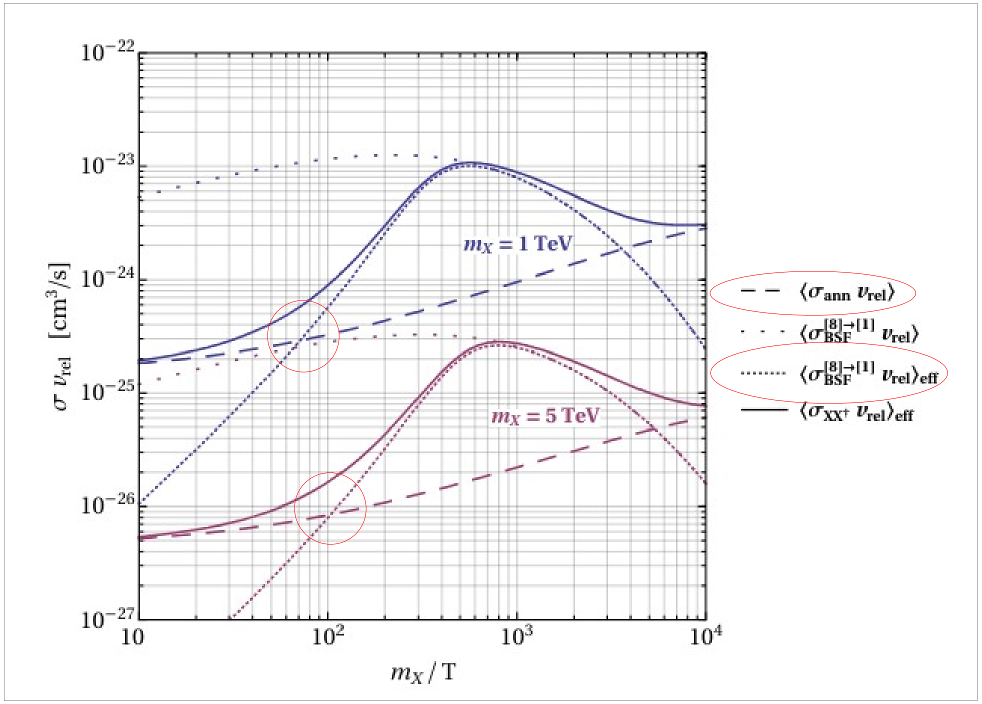

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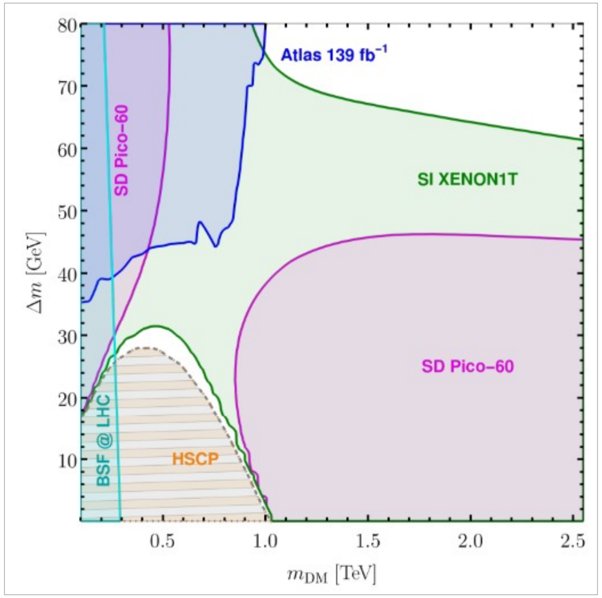
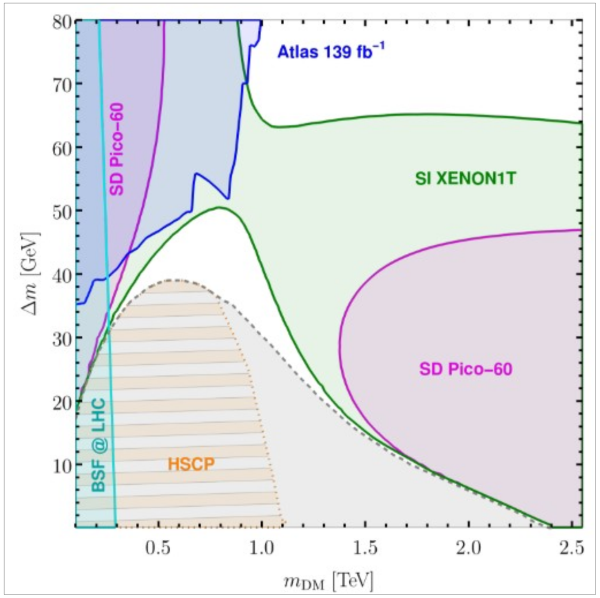
Long-range effects
can relax experimental bounds

Pert. vs non-pert.





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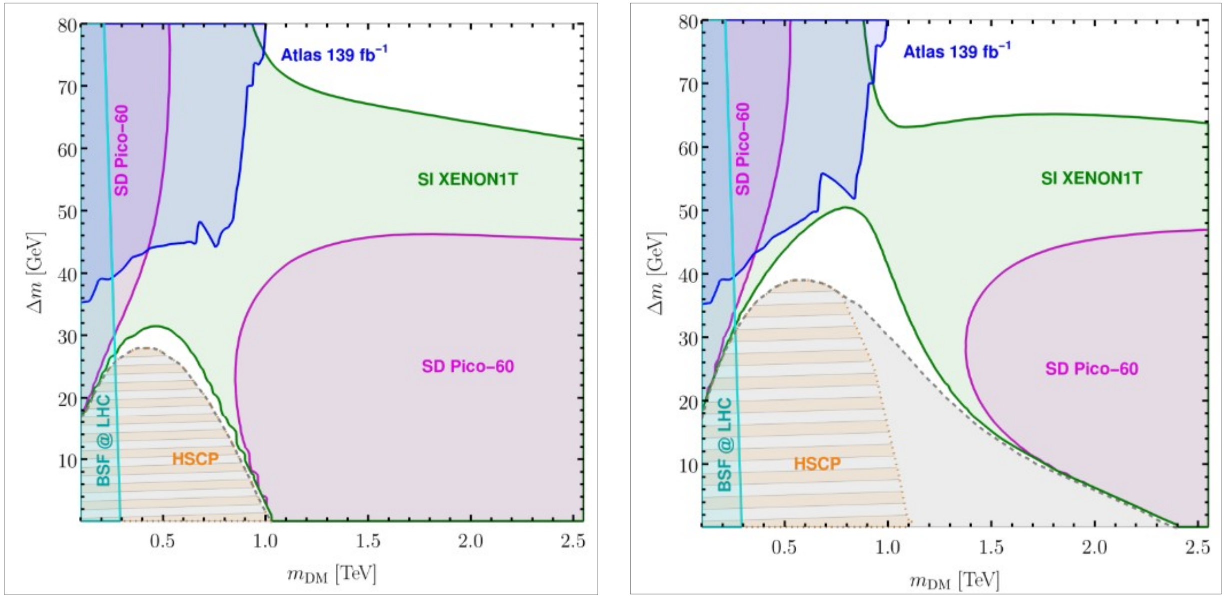

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→ Inhibition threshold for non-experts.

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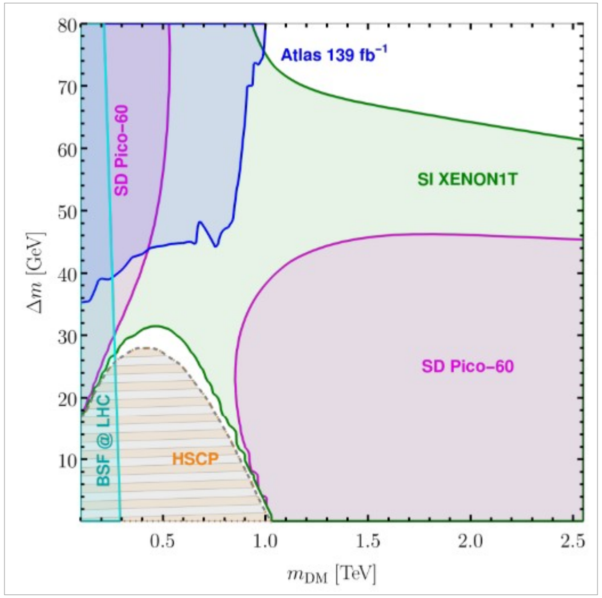

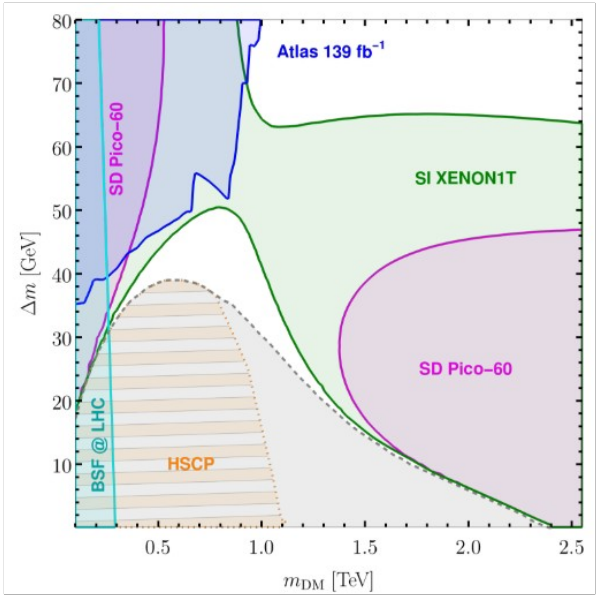

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Previously: Effects need to be added by hand to the relic density calculation.
→ Inhibition threshold for non-experts.

We include excited bound states to a wider class of t-channel models together with a micrOMEGAs add-on package!

Simplified dark matter models and non-perturbative effects

General class of simplified models, studied vastly in the literature. In t-channel models → mediators are colored.

[Arina et al. (2021)]
[Giacchino, Lopez-Honorez et al. (2016)]
[Becker, Harz, Sengupta et al. (2022)]
[Garny et al. (2020)]

A phenomenological toolbox exists (DMSimpt).

[Arina et al. (2020)]

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kin} + \mathcal{L}_F(\chi) + \mathcal{L}_F(\tilde{\chi}) + \mathcal{L}_S(S) + \mathcal{L}_S(\tilde{S}) + \mathcal{L}_V(V) + \mathcal{L}_V(\tilde{V})$$

$$\mathcal{L}_F(X) = \left[\lambda_{\mathbf{Q}} \bar{X} Q \varphi_Q^\dagger + \lambda_{\mathbf{u}} \bar{X} u \varphi_u^\dagger + \lambda_{\mathbf{d}} \bar{X} d \varphi_d^\dagger + \text{h.c.}\right]$$

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DM annihilation | Mediator coannihilation

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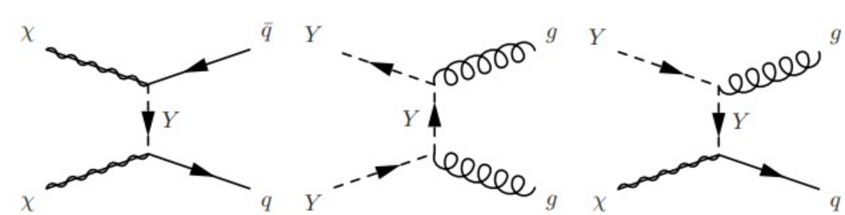

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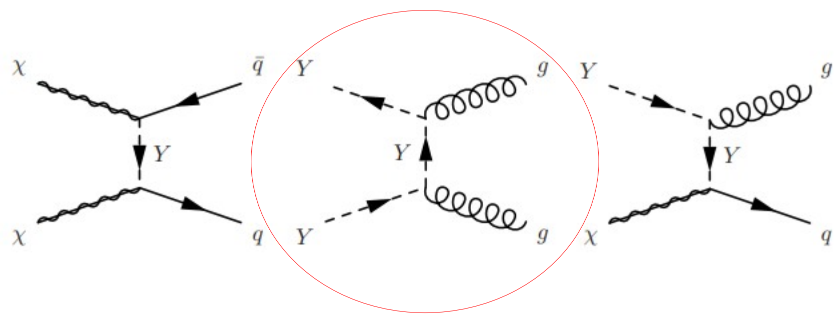

Subject to non-perturbative effects

Simplified dark matter models and non-perturbative effects (II)

Tools for relic density calculation with perturbative cross sections exist abundantly.

→ **Need for an automated framework for the inclusion of non-perturbative effects.**

We provide such a framework for the relic density calculation for colored mediators.

Name	DM	Mediators	Parameters
S3M_uni	$\tilde{\chi}$	$\varphi_{Q_f},\ \varphi_{u_f},\ \varphi_{d_f}$	$M_\varphi,\ M_\chi,\ \lambda_\varphi$
S3D_uni	χ		
S3M_3rd	$\tilde{\chi}$	$\varphi_{Q_3},\ \varphi_{u_3},\ \varphi_{d_3}$	
S3D_3rd	χ		
S3M_uR	$\tilde{\chi}$	φ_{u_1}	
S3D_uR	χ		
F3S_uni	$\tilde{S}$	$\psi_{Q_f},\ \psi_{u_f},\ \psi_{d_f}$	$M_S,\ M_\psi,\ \hat{\lambda}_\psi$
F3C_uni	S		
F3S_3rd	$\tilde{S}$	$\psi_{Q_3},\ \psi_{u_3},\ \psi_{d_3}$	
F3C_3rd	S		
F3S_uR	$\tilde{S}$	ψ_{u_1}	
F3C_uR	S		
F3V_uni	$\tilde{V}_\mu$	$\psi_{Q_f},\ \psi_{u_f},\ \psi_{d_f}$	$M_V,\ M_\psi,\ \hat{\lambda}_\psi$
F3W_uni	V_μ		
F3V_3rd	$\tilde{V}_\mu$	$\psi_{Q_3},\ \psi_{u_3},\ \psi_{d_3}$	
F3W_3rd	V_μ		
F3V_uR	$\tilde{V}_\mu$	ψ_{u_1}	
F3W_uR	V_μ		

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This talk: Four representative examples

	up-quark	top-quark
Scalar mediator	S3MuR: $\mathcal{L}_{\text{int}} = g_{\text{DM}} \left(\overline{\chi}\, X^\dagger\, u_R + \overline{u}_R\, X\, \chi\right)$	S3MtR: $\mathcal{L}_{\text{int}} = g_{\text{DM}} \left(\overline{\chi}\, X^\dagger\, t_R + \overline{t}_R\, X\, \chi\right)$
Fermionic mediator	F3SuR: $\mathcal{L}_{\text{int}} = g_{\text{DM}} \left(\overline{X}\, \chi\, u_R + \overline{u}_R\, \chi\, X\right)$	F3StR: $\mathcal{L}_{\text{int}} = g_{\text{DM}} \left(\overline{X}\, \chi\, t_R + \overline{t}_R\, \chi\, X\right)$

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F3S_uni	$\tilde{S}$	$\psi_{Q_f},\ \psi_{u_f},\ \psi_{d_f}$	$M_S,\ M_\psi,\ \hat{\lambda}_\psi$
F3C_uni	S		
F3S_3rd	$\tilde{S}$	$\psi_{Q_3},\ \psi_{u_3},\ \psi_{d_3}$	
F3C_3rd	S		
F3S_uR	$\tilde{S}$	ψ_{u_1}	
F3C_uR	S		
F3V_uni	$\tilde{V}_\mu$	$\psi_{Q_f},\ \psi_{u_f},\ \psi_{d_f}$	$M_V,\ M_\psi,\ \hat{\lambda}_\psi$
F3W_uni	V_μ		
F3V_3rd	$\tilde{V}_\mu$	$\psi_{Q_3},\ \psi_{u_3},\ \psi_{d_3}$	
F3W_3rd	V_μ		
F3V_uR	$\tilde{V}_\mu$	ψ_{u_1}	
F3W_uR	V_μ		

[Arina et al. (2020)]

Parameter space

Minimal setup contains 3 parameters:
$$\begin{cases} m_{DM} \\ \Delta m = m_X - m_{DM}; \quad \delta = \dfrac{\Delta m}{m_{DM}} \\ g_{DM} \end{cases}$$

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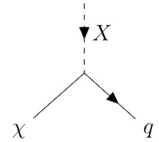

$$\Gamma^{\chi \to X} \gg H(T = m_X); \quad \textbf{coannihilation}$$

$$\Gamma^{\chi \to X} \sim H(T = m_X); \quad \textbf{coscattering/conversion-driven}$$

$$\Gamma^{\chi \to X} \ll H(T = m_X); \quad \textbf{super-WIMP/freeze-in}$$

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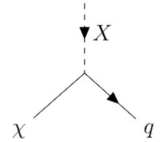

We focus on the coannihilation regime

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Setup of the computation

$$< \sigma v >_{\text{total}} = < \mathcal{S} \sigma v >_{\text{eff}} + < \sigma_{\text{BSF}} v >_{\text{eff}}$$

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All perturbative (co-)annihilations automatically calculated by micrOMEGAs

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Sommerfeld enhancement for s-wave annihilations with the color structure

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

Setup of the computation

All perturbative (co-)annihilations automatically calculated by micrOMEGAs

Bound state effects are covered via an effective cross section.

[Ellis et al. (2015)]
[Petraki et al. (2015)]
[Harz & Petraki (2018)]
[Garny & Heisig (2022)]
[Binder, Petraki et al. (2022)]

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We include bound state formation for processes

$$(X + X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g\}_{[\mathbf{8}]}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

Sommerfeld effect for colored particles

Color decomposition splits the cross section into an enhanced (attractive configuration) and a suppressed part (repulsive configuration).

$$V(r)_{\mathbf{R}_1 \otimes \mathbf{R}_2 \to \hat{\mathbf{R}}} = -\frac{\alpha_s}{2r}\left(C_2(\mathbf{R}_1) + C_2(\mathbf{R}_2) - C_2(\hat{\mathbf{R}})\right)$$

[El Hedri et al. (2017)]

We use explicitly

$$\mathcal{S}\sigma = \left[c_{0,[\mathbf{1}]} S_0\left(\frac{4}{3}\frac{\alpha_s}{v_{\text{rel}}}\right) + c_{0,[\mathbf{8}]} S_0\left(-\frac{1}{6}\frac{\alpha_s}{v_{\text{rel}}}\right) + c_{0,[\bar{\mathbf{3}}]} S_0\left(\frac{2}{3}\frac{\alpha_s}{v_{\text{rel}}}\right) + c_{0,[\mathbf{6}]} S_0\left(-\frac{1}{3}\frac{\alpha_s}{v_{\text{rel}}}\right)\right]\sigma_0 + \dots$$

with

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Sommerfeld effect for colored particles

Color decomposition splits the cross section into an enhanced (attractive configuration) and a suppressed part (repulsive configuration).

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[El Hedri et al. (2017)]

We use explicitly

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Only $\mathcal{O}(v_{\rm rel}^0)$

Sommerfeld enhancement

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Only $\mathcal{O}(v_{\rm rel}^0)$

4 coefficients needed

$\mathcal{O}(v_{\rm rel}^2)$

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Sommerfeld implementation (caveats) I

Coefficients for the color decomposition are not uniquely determined by the inital and final state representations.

[Giacchino, Lopez-Honorez et al. (2016)]
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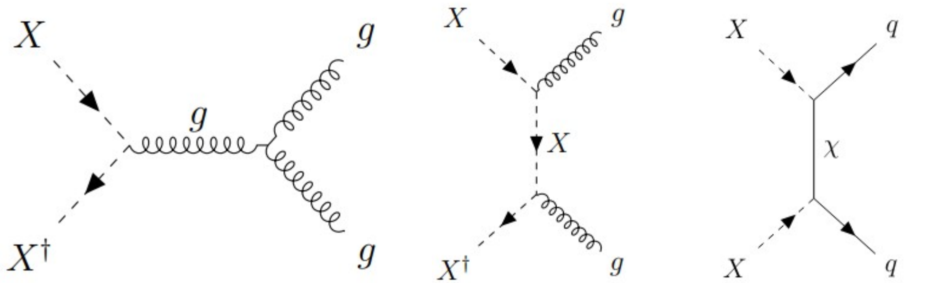

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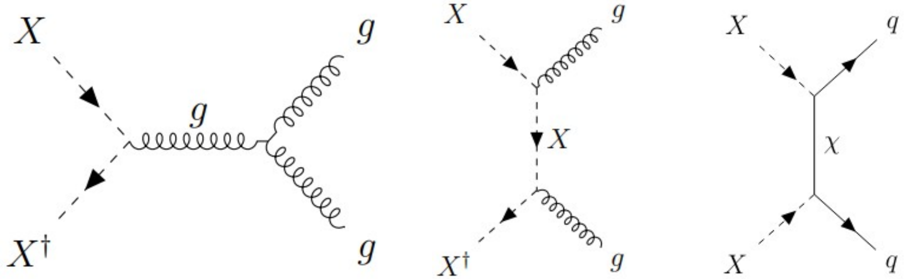

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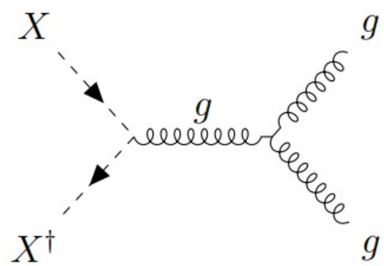

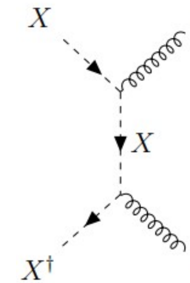

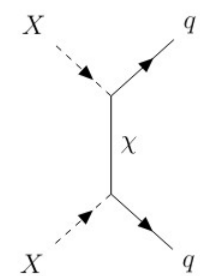

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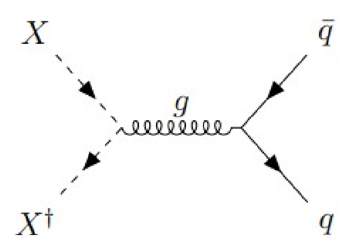
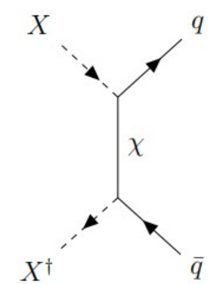


$$[\mathbf{8}]$$



$$[\mathbf{1}] + [\mathbf{8}]$$

Sommerfeld implementation (caveats) II

To order v_{rel}^0 and assuming $g_s \simeq g_{DM} >> g_w, g_Y$, there is a simplified color decomposition for scalar and fermionic mediators → „Default settings“

$$\mathcal{S}\sigma^{gg} \simeq \left(\frac{2}{7} S_0^{[\mathbf{1}]} + \frac{5}{7} S_0^{[\mathbf{8}]}\right) \sigma_0^{gg}$$

Scalar & fermionic mediators

$$\mathcal{S}\sigma^{q\bar{q}} \simeq \left(\frac{1}{9} S_0^{[\mathbf{1}]} + \frac{8}{9} S_0^{[\mathbf{8}]}\right) \sigma_0^{q\bar{q}}$$

Scalar & fermionic mediators

$$\mathcal{S}\sigma^{qq} \simeq \left(0 \cdot S_0^{[\bar{\mathbf{3}}]} + 1 \cdot S_0^{[\mathbf{6}]}\right) \sigma_0^{qq}$$

Scalar mediators

$$\mathcal{S}\sigma^{qq} \simeq \left(\frac{1}{3} S_0^{[\bar{\mathbf{3}}]} + \frac{2}{3} S_0^{[\mathbf{6}]}\right) \sigma_0^{qq}$$

Fermionic mediators

(Excited) bound states

Network of Boltzmann equations for excited states can be **simplified to one** and an **effective bound state formation cross section** can be obtained.

[Garny & Heisig (2022)]
[Binder, Petraki et al. (2022)]
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$$\langle \sigma_{BSF} v \rangle_{\mathrm{eff}} = \sum_i \langle \sigma_{\mathrm{BSF,i}} v \rangle \left(1 - (M)^{-1}_{ij} \frac{\langle \Gamma^j_{\mathrm{ion}} \rangle}{\langle \Gamma^j \rangle}\right)$$

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In the coannihilation regime, including only the **ground state is usually sufficient**.

$$\langle \sigma_{BSF} v \rangle_{\rm eff} = \langle \sigma_{BSF,n=1} v \rangle \frac{\langle \Gamma^{n=1}_{\rm dec} \rangle}{\langle \Gamma^{n=1}_{\rm ion} \rangle + \langle \Gamma^{n=1}_{\rm dec} \rangle} \quad \text{with} \quad v_{\rm rel} \frac{d\sigma_{\mathbf{k} \to \{100\}}}{d\Omega} = \frac{|\mathbf{P}_g|}{64 \pi^2 M^2 \mu} \left(|\boldsymbol{\mathcal{M}}_{\mathbf{k} \to \{100\}}|^2 - |\hat{\mathbf{P}}_g \cdot \boldsymbol{\mathcal{M}}_{\mathbf{k} \to \{100\}}|^2 \right)$$

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Cross sections and rates available in the literature:

Excited bound states and their role in dark matter production

Tobias Binder,[1,*] Mathias Garny,[1,†] Jan Heisig,[2,3,‡] Stefan Lederer,[1,§] and Kai Urban[1,¶]

[1] Physik Department T31, Technische Universität München, James-Franck-Straße 1, D-85748 Garching, Germany
[2] Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, D-52056 Aachen, Germany
[3] Department of Physics, University of Virginia, Charlottesville, Virginia 22904-4714, USA

Limiting scenarios for excited bound states

1) At early times: **Ionization equilibrium**: $\Gamma^{i}_{\text{ion}} >> \Gamma^{i}_{\text{dec}}, \Gamma^{ij}_{\text{trans}}$ [Garny & Heisig (2022)]

$$< \sigma_{BSF} v >_{\text{eff}} = \sum_i \frac{g_{\mathcal{B}_i}}{g_X^2} \left(\frac{2\pi m_{\mathcal{B}_i}}{T m_X^2} \right)^{3/2} e^{E_{\mathcal{B}_i}/T} \Gamma^{i}_{\text{dec}}$$

2) **Efficient transition limit**: $\Gamma^{ij}_{\text{trans}} >> \Gamma^{i}_{\text{dec}}, \Gamma^{i}_{\text{ion}}$

$$< \sigma_{BSF} v >_{\text{eff}} = < \sigma_{BSF} v >_{\text{sum}} \frac{\Gamma^{\text{eff}}_{\text{dec}}}{\Gamma^{\text{eff}}_{\text{ion}} + \Gamma^{\text{eff}}_{\text{dec}}} \qquad \Gamma^{\text{eff}}_{\text{ion/dec}} = \frac{\sum_i \Gamma^{i}_{\text{ion/dec}} Y^{\text{eq}}_{\mathcal{B}_i}}{Y^{\text{eq}}_{\mathcal{B}}}$$

3) **No transition limit**: $\Gamma^{i}_{\text{dec}} >> \Gamma^{i}_{\text{ion}}, \Gamma^{ij}_{\text{trans}}$

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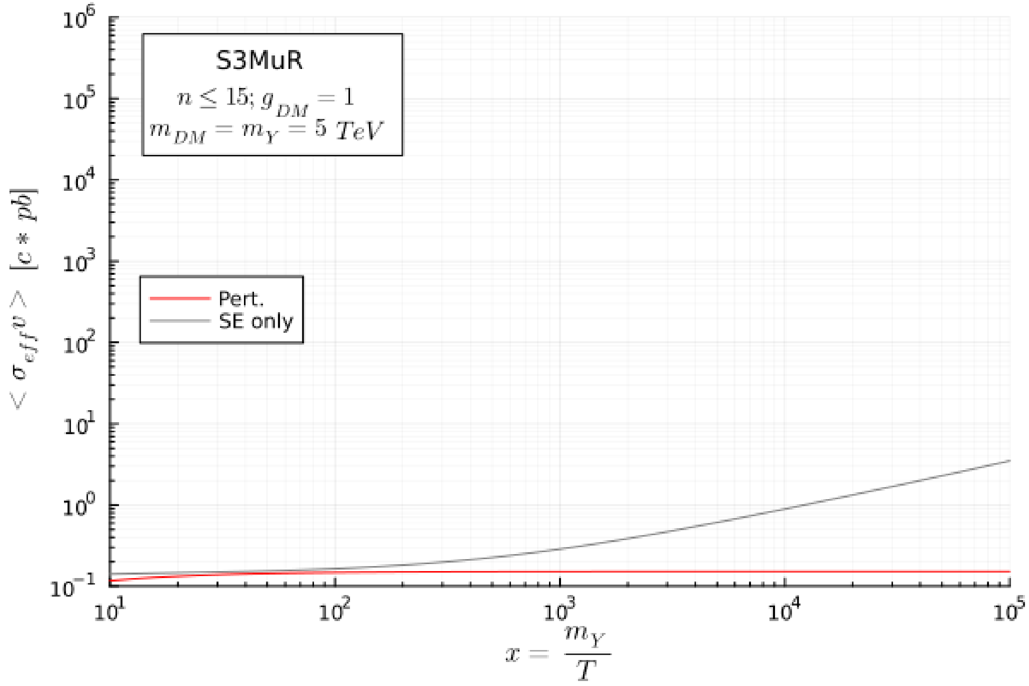
See talk by Tobias Binder

Comparison of non-perturbative effects



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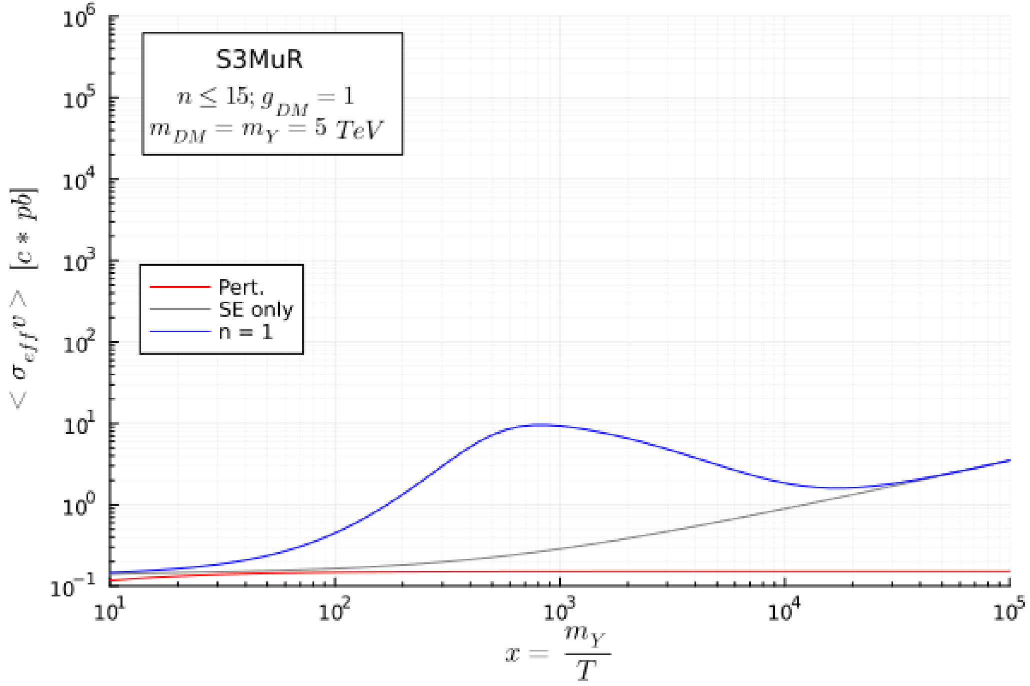

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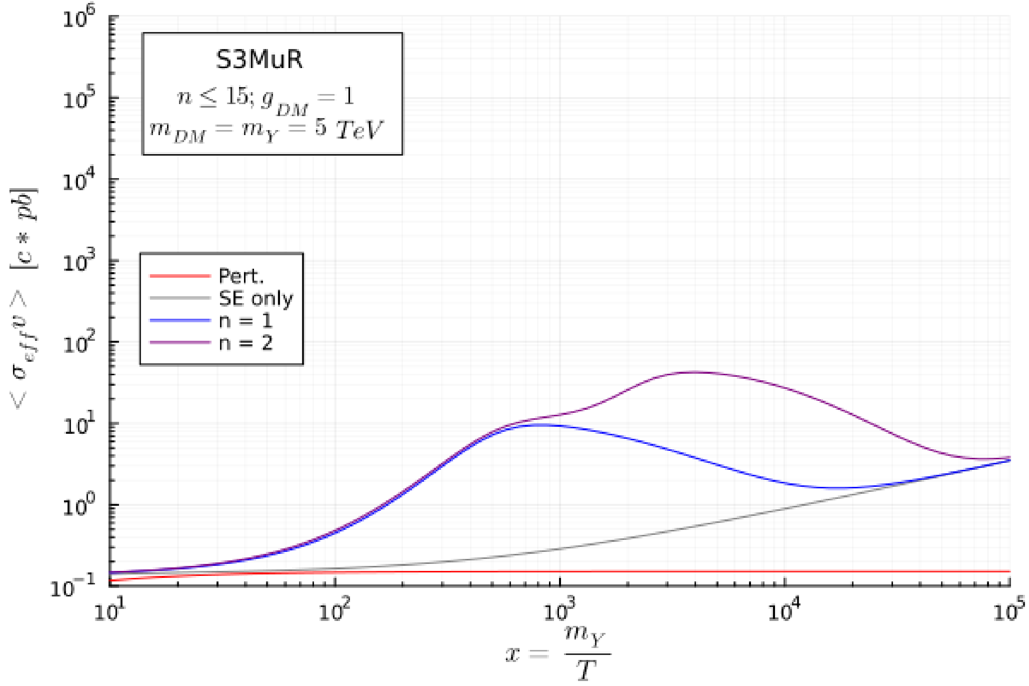

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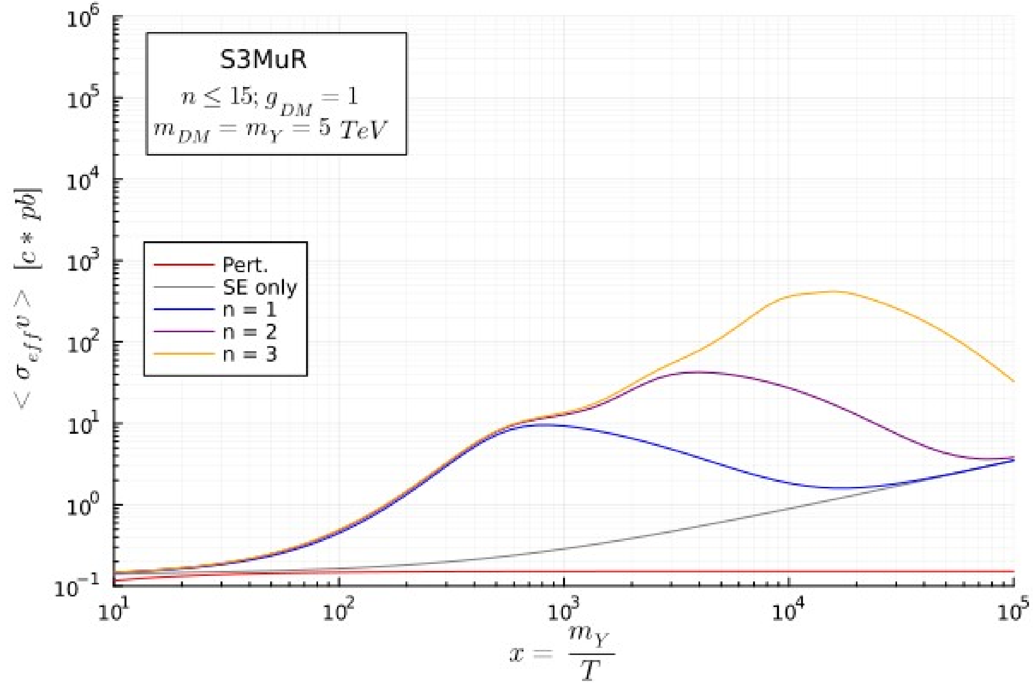

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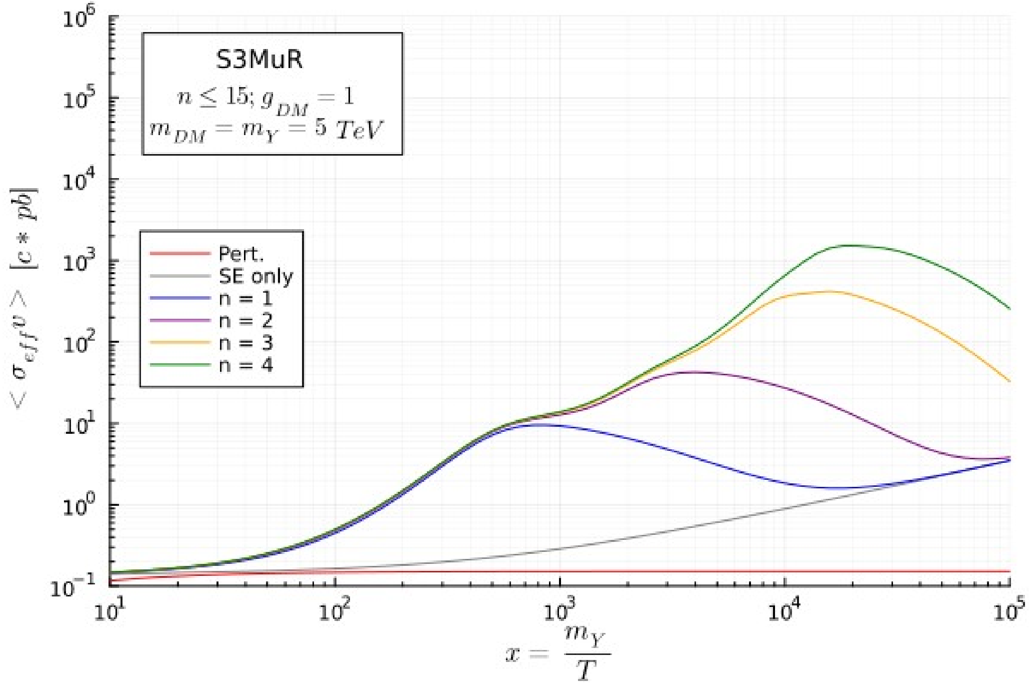

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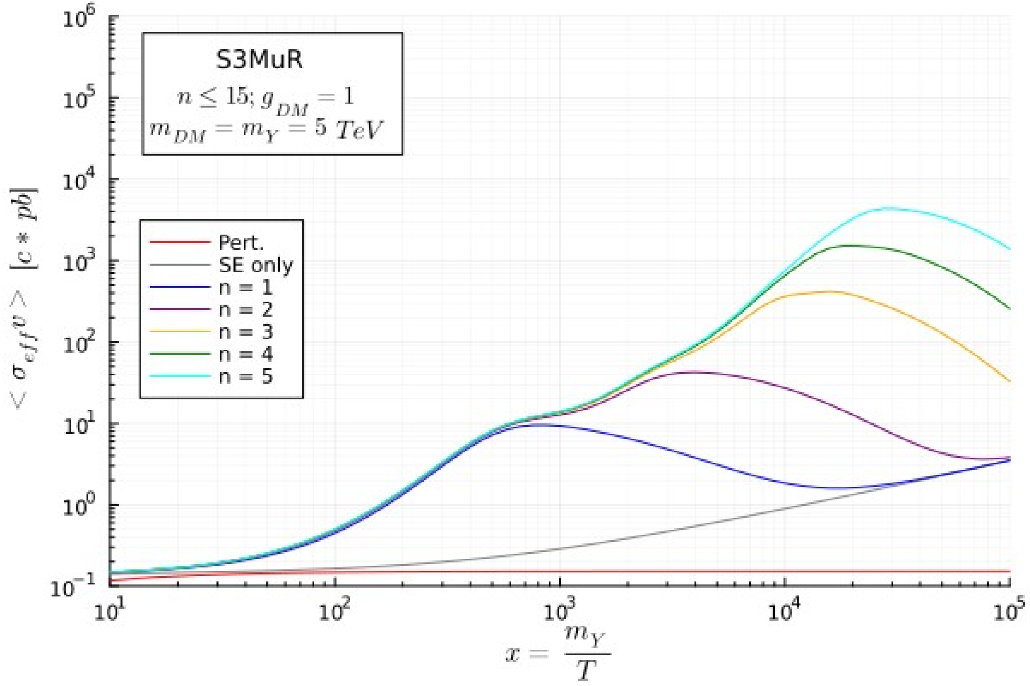

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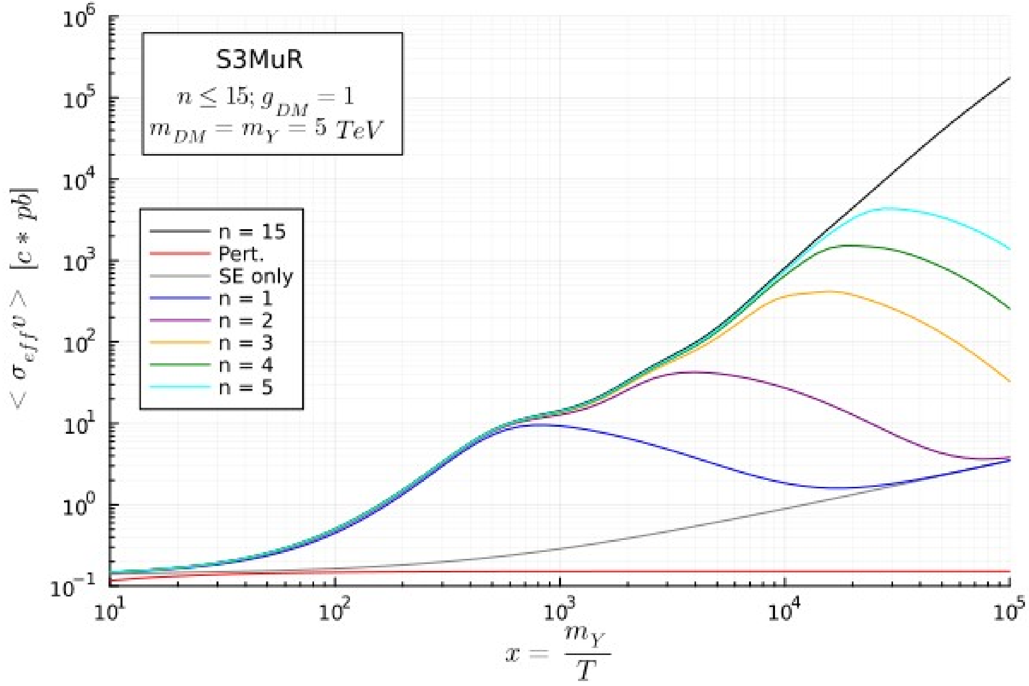

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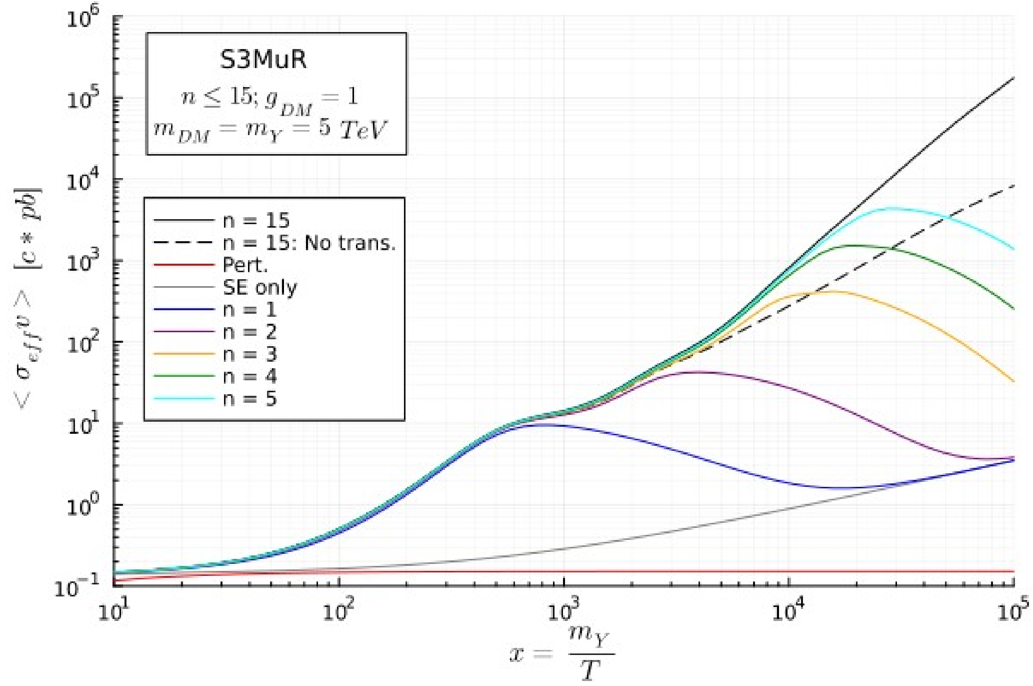

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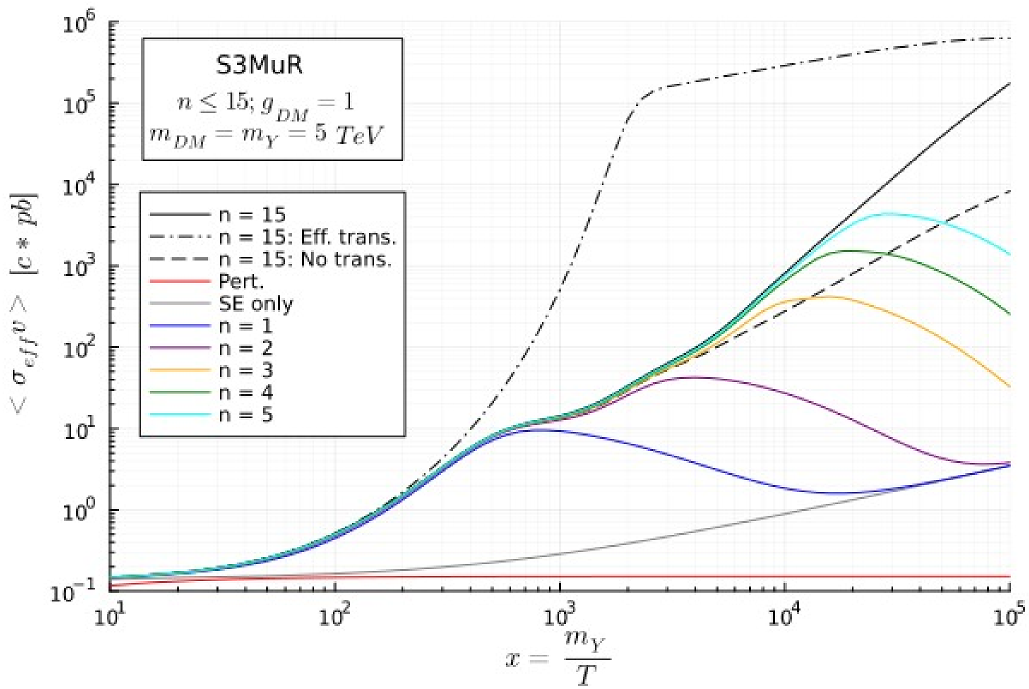

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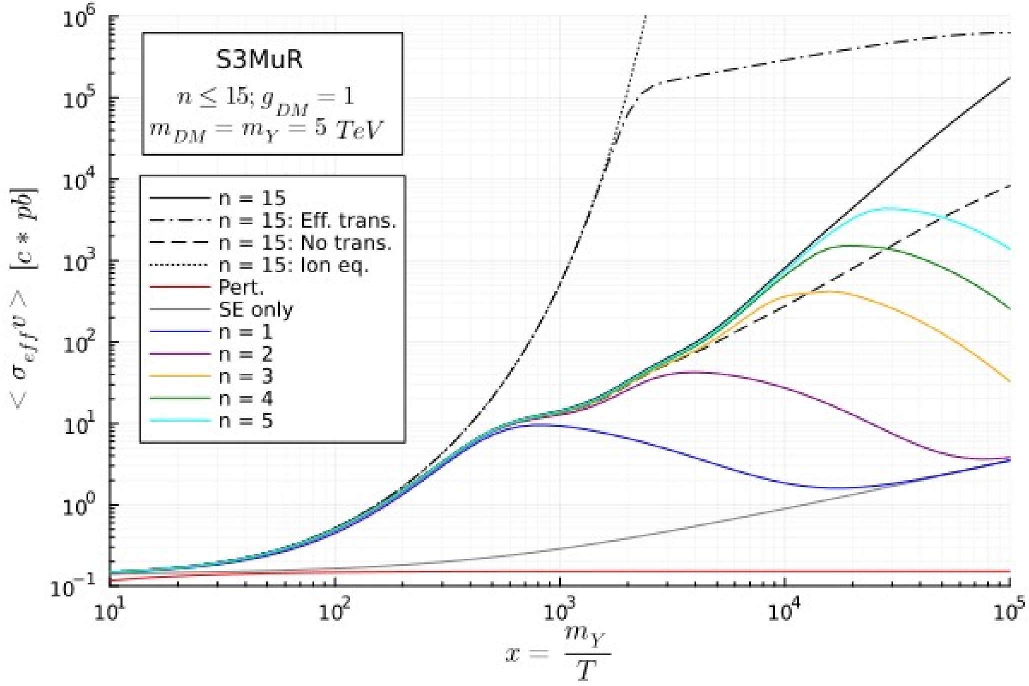
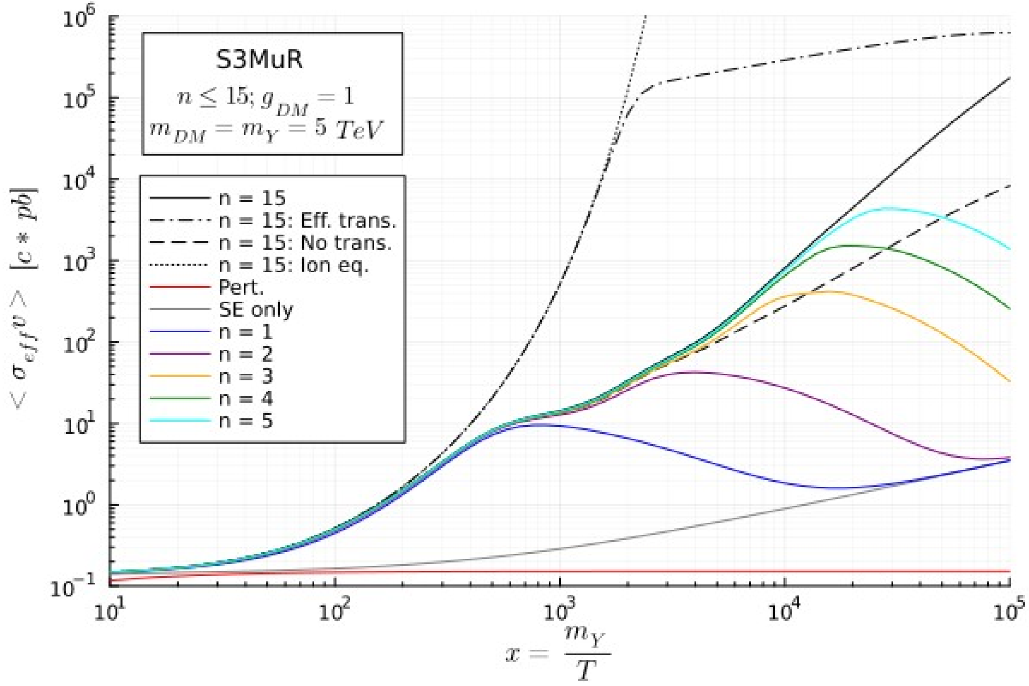


Comparison of non-perturbative effects

Bound state formation cross section **never freezes-out** for colored DM candidates (but they do for coannihilation).

[Binder et al. (2023)]



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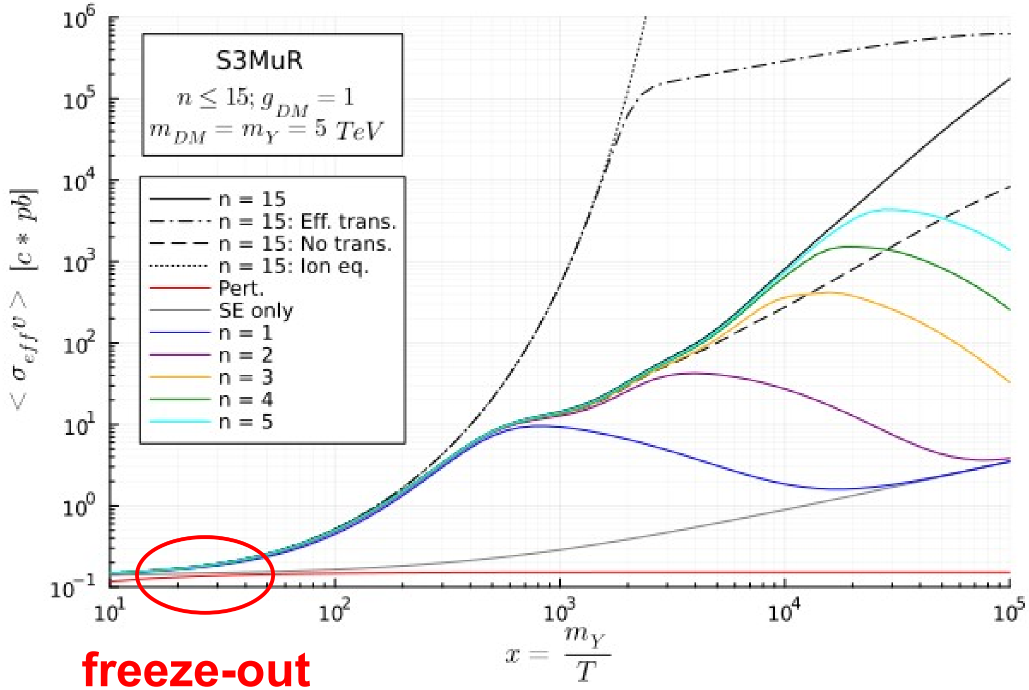

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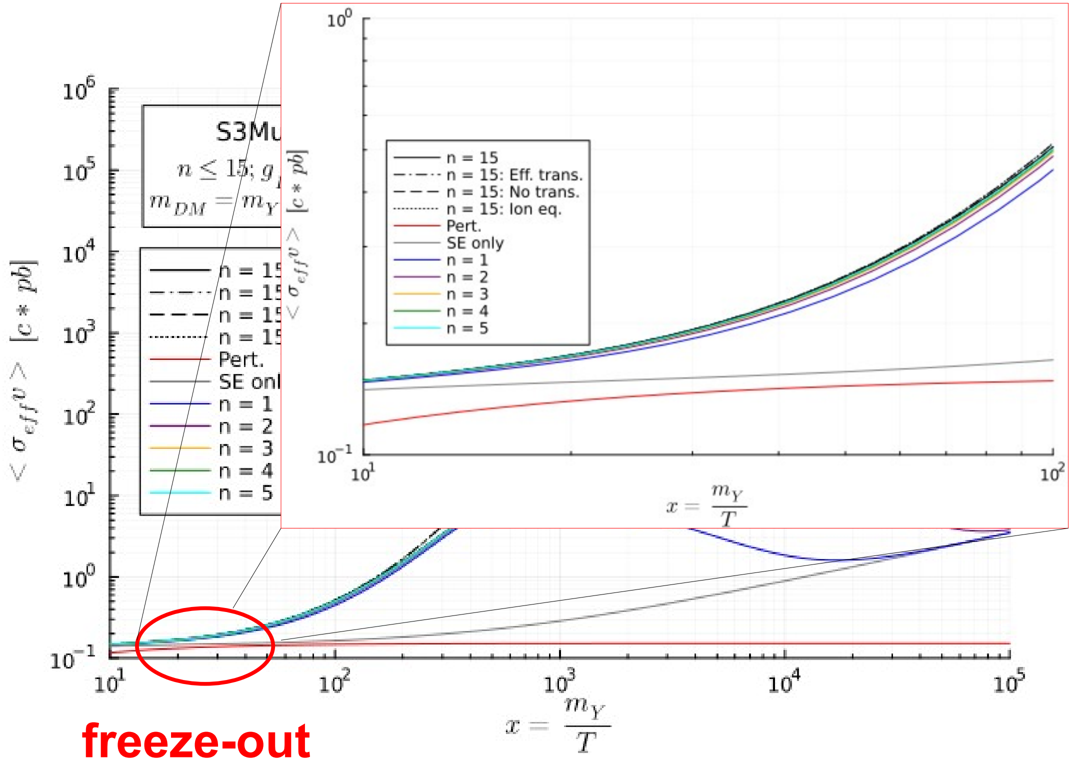

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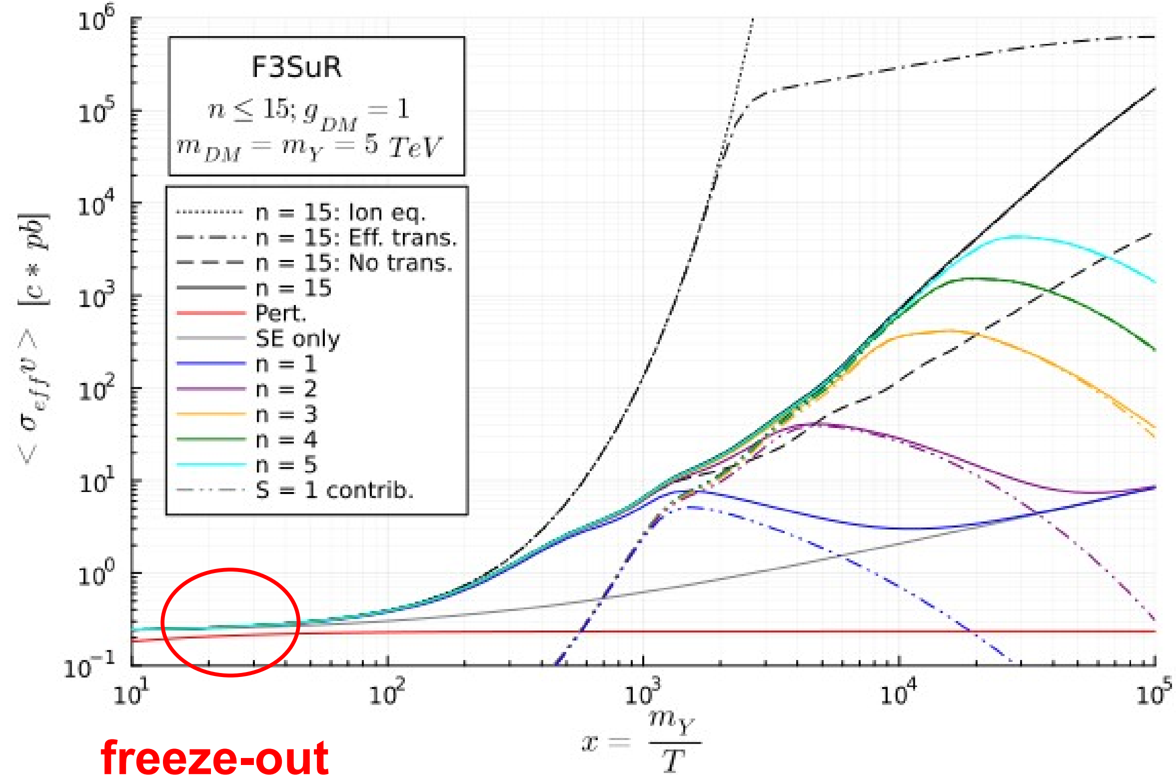
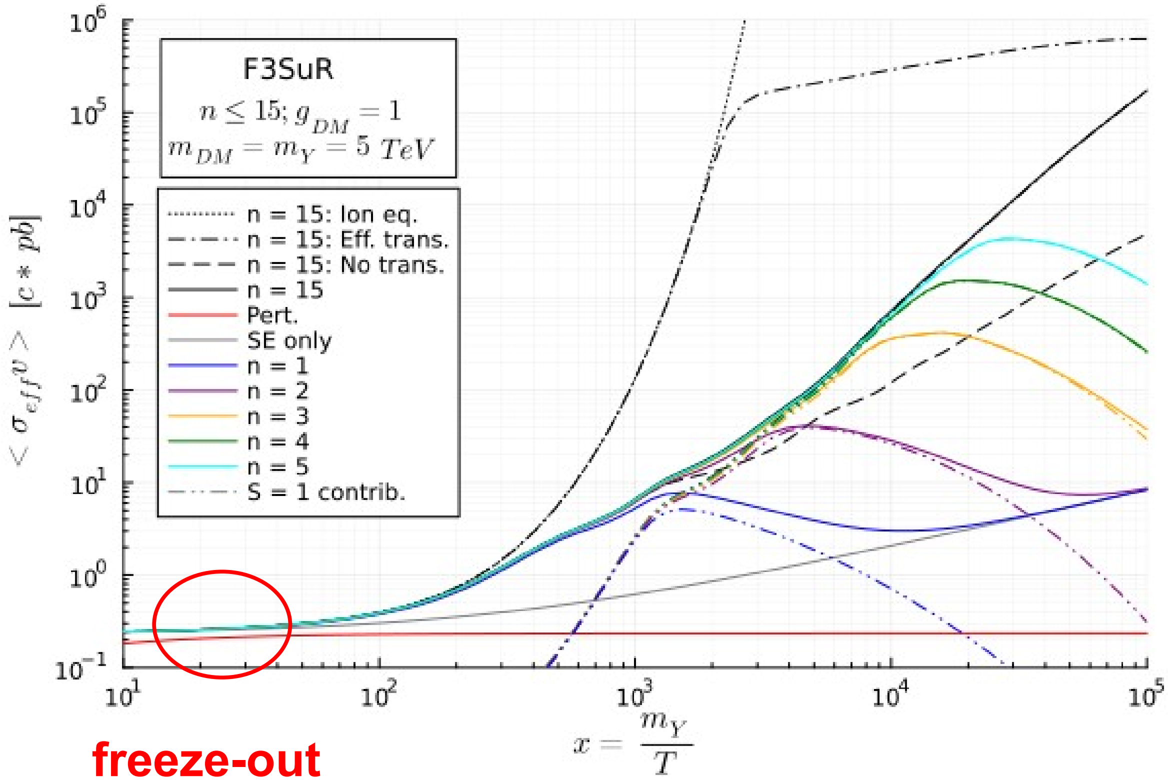
Cross sections for fermionic mediators



Cross sections for fermionic mediators

Triplet contributions negligible as expected → Only relevant at very **late times**



freeze-out

Cross sections for fermionic mediators

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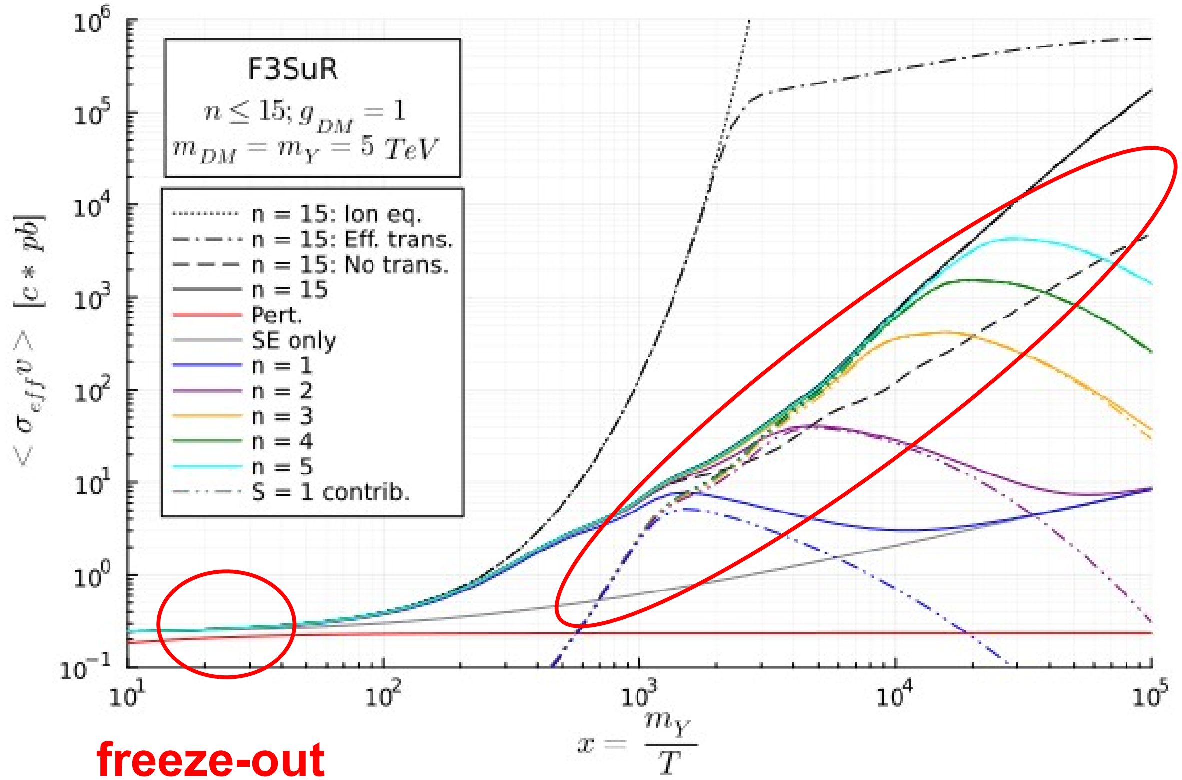
Transitions are **much more efficient** for triplet states



Outline

Simplified dark matter models and long-range effects

Sommerfeld effect and bound state formation for colored mediators

Workflow of the code

Showcases of our computational framework

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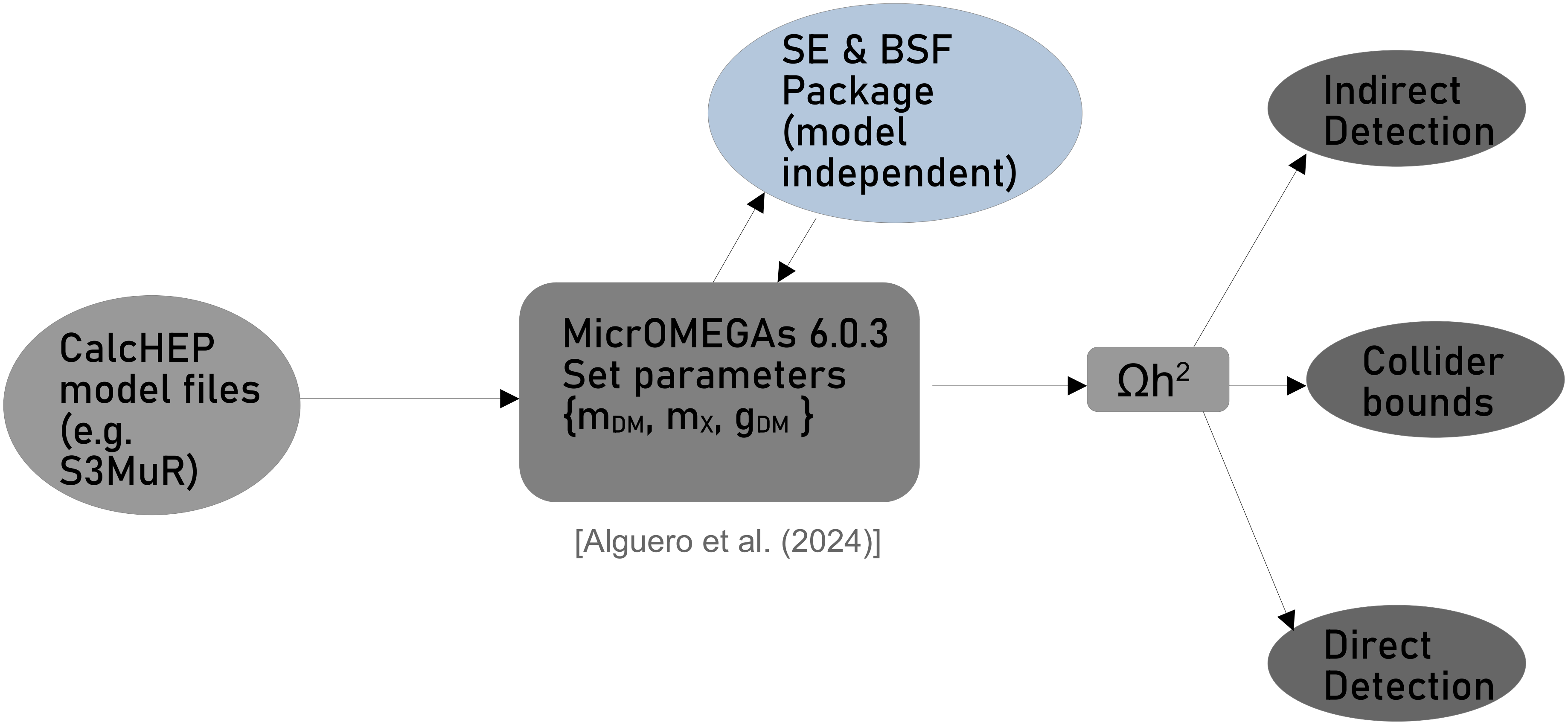
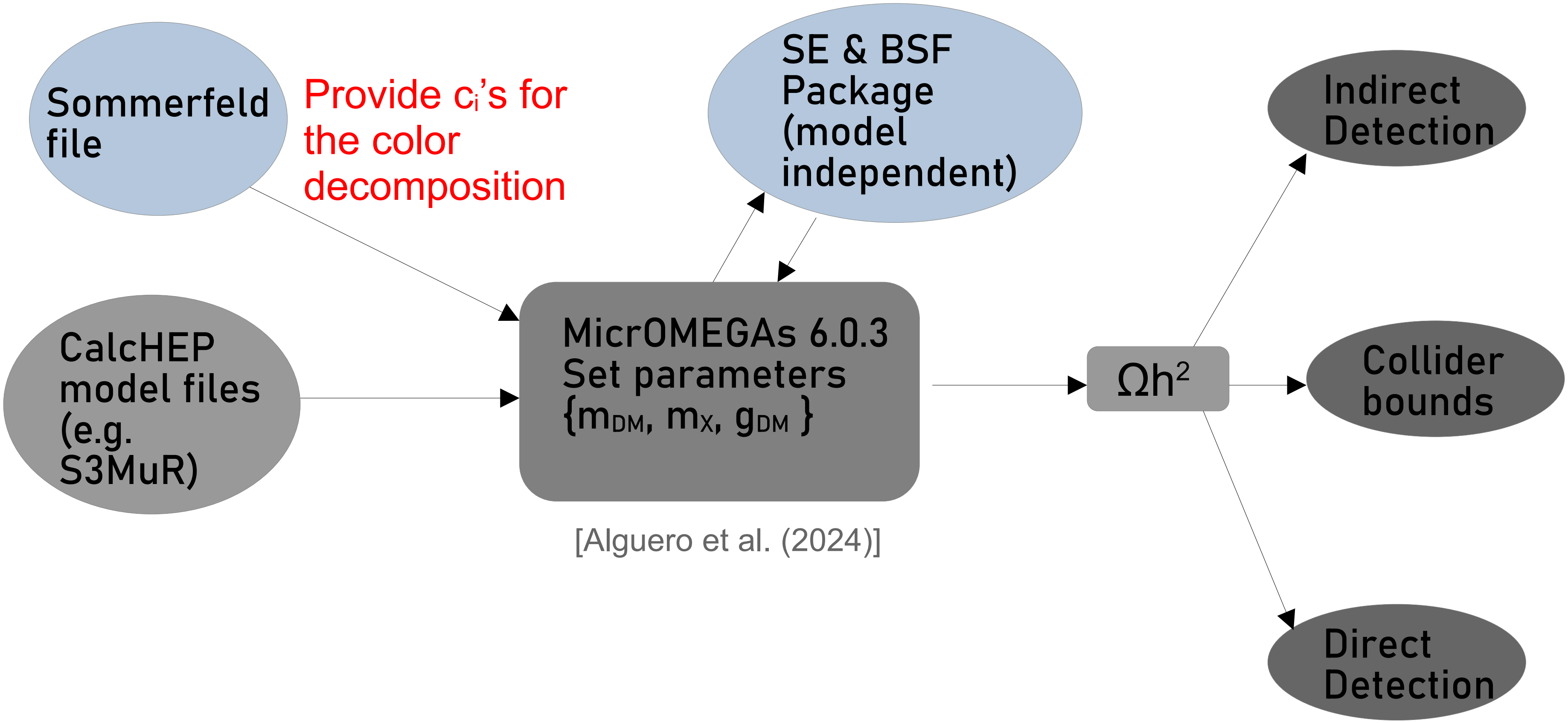


[Alguero et al. (2024)]

Workflow of the code



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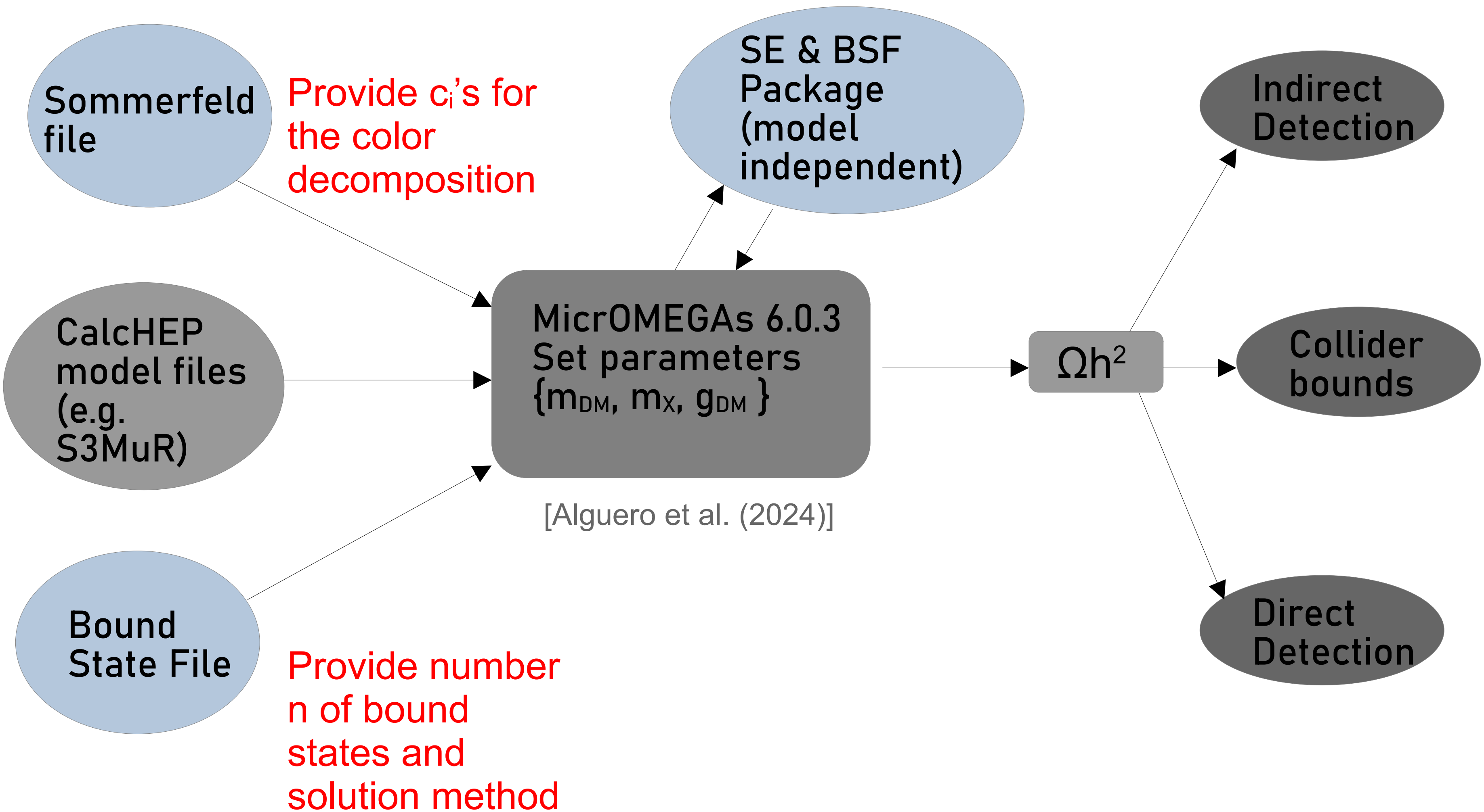

Sommerfeld file

improveCrossSection_Sommerfeld.cpp ●

C: > Users > marti > Documents > t_chann_DM > micromegas_6.0.3 > S3MuR > lib > improveCrossSection_Sommerfeld.cpp

```cpp
#include"../../include/micromegas.h"
#include"../../include/micromegas_aux.h"
#include"../../Packages/SE_BSF/SE_BSF_header.h"

int somm_flag = 0;    // Flag for Sommerfeld effect (0 = not active, 1 = active, else = user defined)

double SommerfeldFactor_BSMmodel(double alphaQCD, double vrel, int c1, int c2, int c3, int c4, long n1, long n2, long n3, long n4)
{ /* This function calculates the Sommerfeld factor for a BSM model
    according to the color decomposition implemented by the user.
    alphaQCD is the value of the strong coupling at the appropriate scale, vrel the relative velocity,
    ci and ni; i = 1,2,3,4; are the dimensions of the SU(N) representation and the names of the particles in the process, respectively.

    Basically, the user has to has to adapt the "if...else" block according to the correct color decomposition. */

  double cfac1=4./3.; double cfac8=-1./6.; double cfac3=2./3.; double cfac6=-1./3.; //cfac is the coefficient of the coupling in the ARGUMENT of the Sommerfeld factor
  double kQfac1=0.; double kQfac8=0.; double kQfac3=0.; double kQfac6=0.; //kQfac is the coefficient IN FRONT OF the Sommerfeld factor coming from the color decomposition
  double zeta1=0.; double zeta3=0.; double zeta8=0.; double zeta6=0.; // zeta = alpha_group/vrel


// **************** BEGIN IF BLOCK OF COLOR DECOMPOSITION (TO BE MODIFIED AT WILL) ****************************************

  if((c1==3&&c2==-3)||(c1==-3&&c2==3)){ // Y Y^\dagger process
    if(c3==1&&c4==1){ // most frequent case: Both final states are colour singlets
      kQfac1=1.; //for all partial waves
      }

    if(c3==8&&c4==8){ //gg final state
      kQfac1=2./7.; kQfac8=5./7.;
      }

    if((c3==3&&c4==-3)||(c3==-3&&c4==3)){
    /* This is the tricky q qbar channel, as elaborated in our publication.
      Interference terms in the color decomposition are neglected for now  */

      kQfac1 = 1./9.;
      kQfac8=8./9.; // for all partial waves in the case gDM >> g_s*vrel
      }

    if((c3!=8&&c4==8)||(c3==8&&c4!=8)){ //g + Z/\gamma. This is purely adjoint for all partial waves.
      kQfac8=1.;
      }
  }

  if((c1==3&&c2==3)||(c1==-3&&c2==-3)){ // XX or X^\dagger X^\dagger process
    if(n1!=n2) {kQfac3=1./3.; kQfac6=2./3.;} //q_i q_j final state
    else {
      kQfac6=1.; //q_i q_i final state.
      }
  }
// **************** END IF BLOCK OF COLOR DECOMPOSITION ****************************************
```

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1} \oplus \mathbf{8}(gg)$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1} \oplus \mathbf{8}(q\bar{q})$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{8}$$

$$\mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}} \oplus \mathbf{6}$$

Sommerfeld file

improveCrossSection_Sommerfeld.cpp

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```
#include"../../include/micromegas.h"
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#include"../../Packages/SE_BSF/SE_BSF_header.h"

int somm_flag = 0;    // Flag for Sommerfeld effect (0 = not active, 1 = active, else = user defined)
```

Turning Sommerfeld on/off

```
double SommerfeldFactor_BSMmodel(double alphaQCD, double vrel, int c1, int c2, int c3, int c4, long n1, long n2, long n3, long n4)
{ /* This function calculates the Sommerfeld factor for a BSM model
     according to the color decomposition implemented by the user.
     alphaQCD is the value of the strong coupling at the appropriate scale, vrel the relative velocity,
     ci and ni; i = 1,2,3,4; are the dimensions of the SU(N) representation and the names of the particles in the process, respectively.

     Basically, the user has to has to adapt the "if...else" block according to the correct color decomposition. */

  double cfac1=4./3.; double cfac8=-1./6.; double cfac3=2./3.; double cfac6=-1./3.; //cfac is the coefficient of the coupling in the ARGUMENT of the Sommerfeld factor
  double kQfac1=0.; double kQfac8=0.; double kQfac3=0.; double kQfac6=0.; //kQfac is the coefficient IN FRONT OF the Sommerfeld factor coming from the color decomposition
  double zeta1=0.; double zeta3=0.; double zeta8=0.; double zeta6=0.; // zeta = alpha_group/vrel


// **************** BEGIN IF BLOCK OF COLOR DECOMPOSITION (TO BE MODIFIED AT WILL) *****************************************

  if((c1==3&&c2==-3)||(c1==-3&&c2==3)){ // Y Y^\dagger process
    if(c3==1&&c4==1){ // most frequent case: Both final states are colour singlets
      kQfac1=1.; //for all partial waves
      }
```

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1}$$

```
    if(c3==8&&c4==8){ //gg final state
      kQfac1=2./7.; kQfac8=5./7.;
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$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1} \oplus \mathbf{8}(gg)$$

```
    if((c3==3&&c4==-3)||(c3==-3&&c4==3)){
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      Interference terms in the color decomposition are neglected for now  */

      kQfac1 = 1./9.;
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      }
```

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1} \oplus \mathbf{8}(q\bar{q})$$

```
    if((c3!=8&&c4==8)||(c3==8&&c4!=8)){ //g + Z/\gamma. This is purely adjoint for all partial waves.
      kQfac8=1.;
      }
  }
```

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{8}$$

```
  if((c1==3&&c2==3)||(c1==-3&&c2==-3)){ // XX or X^\dagger X^\dagger process
    if(n1!=n2) {kQfac3=1./3.; kQfac6=2./3.;} //q_i q_j final state
    else {
      kQfac6=1.; //q_i q_i final state.
      }
  }
// **************** END IF BLOCK OF COLOR DECOMPOSITION *****************************************
```

$$\mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}} \oplus \mathbf{6}$$

Sommerfeld file

improveCrossSection_Sommerfeld.cpp

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      Interference terms in the color decomposition are neglected for now  */

      kQfac1 = 1./9.;
      kQfac8=8./9.; // for all partial waves in the case gDM >> g_s*vrel
      }

    if((c3!=8&&c4==8)||(c3==8&&c4!=8)){ //g + Z/\gamma. This is purely adjoint for all partial waves.
      kQfac8=1.;
      }
  }

  if((c1==3&&c2==3)||(c1==-3&&c2==-3)){ // XX or X^\dagger X^\dagger process
    if(n1!=n2) {kQfac3=1./3.; kQfac6=2./3.;} //q_i q_j final state
    else {
      kQfac6=1.; //q_i q_i final state.
      }
  }
// **************** END IF BLOCK OF COLOR DECOMPOSITION ****************************************
```

Turning Sommerfeld on/off

$$\mathbf{3} \otimes \bar{\mathbf{3}} \to \mathbf{1}$$

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Coefficients of color decomposition

Bound State file

User input: 4 items

improveCrossSection_Sommerfeld.cpp ● BoundStateFormation.cpp ●

C: > Users > marti > Documents > t_chann_DM > micromegas_6.0.3 > S3MuR > lib > BoundStateFormation.cpp

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 1) The BSF scenario/limit

 2) The number of excited states to be considered

 3) The number of dark sector particles and anti-particles.

 4) The PDG code(s) (integers) of the distinguishable mediators/particles undergoing BSF in an array having the number of binding mediators as size.
 */

int bsf_scenario = 0; //Flag for BSF (0 = no BSF, 1 = no transition limit, 2 = efficient transition limit, 3 = ionization equilibrium, 4 = Full matrix solution)

int num_excited_states = 0; //Number of n states included in the calculation (n = 0 -> no bound states, n = 1 -> ground state etc.)

const int num_of_mediators = 7; //Number of DIFFERENT=DISTINGUISHABLE particles in the dark sector

int pdg_nums_mediators[num_of_mediators] = {52, 2000002, -2000002, 2000004, -2000004, 2000006, -2000006}; //define the PDG number(s) of dark sector particles

// ************************** END USER DEFINITION *********************************************

double BoundStateCoannihilation(double T){ /* This function can be modified by the user according to the theory at hand.
It calculates the (co-)annihilation terms for bound state formation in the spirit of eqns. (2.3) - (2.10) of arXiv:2203.04326v2.
If left unchanged, it calculates the BSF in all possible coannihilation pairings.    */···
}
```

Bound State file

User input: 4 items

improveCrossSection_Sommerfeld.cpp ● BoundStateFormation.cpp ●

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```

Bound State file

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If left unchanged, it calculates the BSF in all possible coannihilation pairings.    */⋯
}
```

Outline

Simplified dark matter models and long-range effects

Sommerfeld effect and bound state formation for colored mediators

Workflow of the code

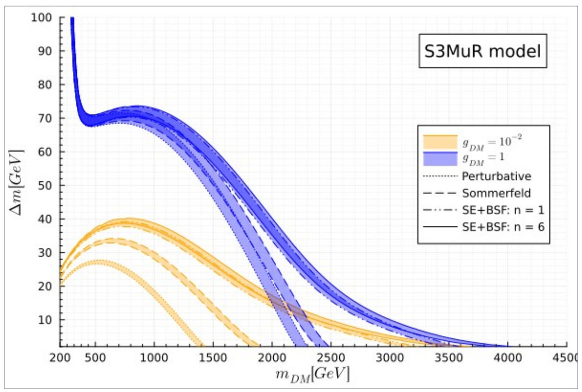
Showcases of our computational framework

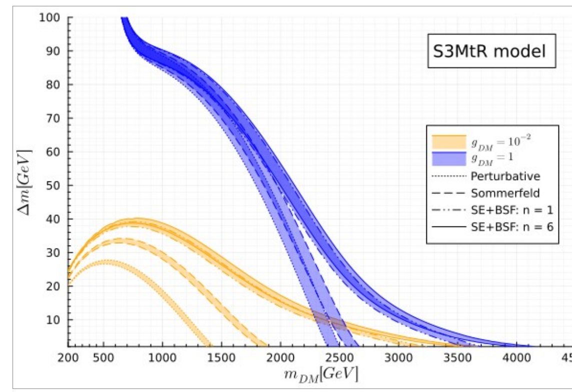
Impact of long-range effects: SE & BSF$_{n=6}$

Allowed bands for $\Omega_{DM} = 0.1200 \pm 0.0050$ (5σ)

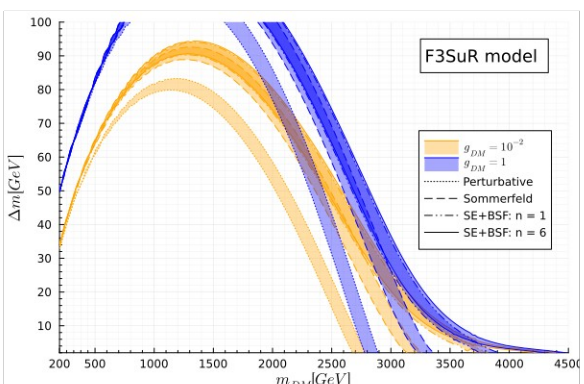
up-quark | **top-quark**

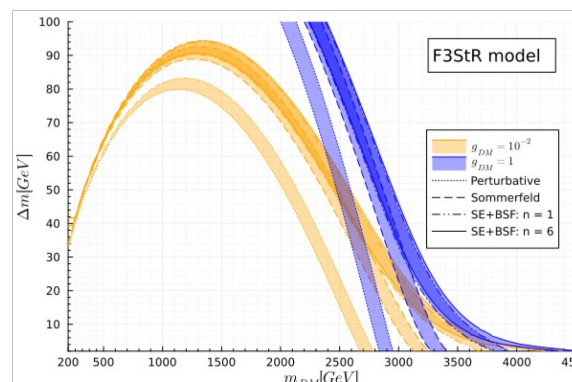
Scalar mediators





Fermionic mediators

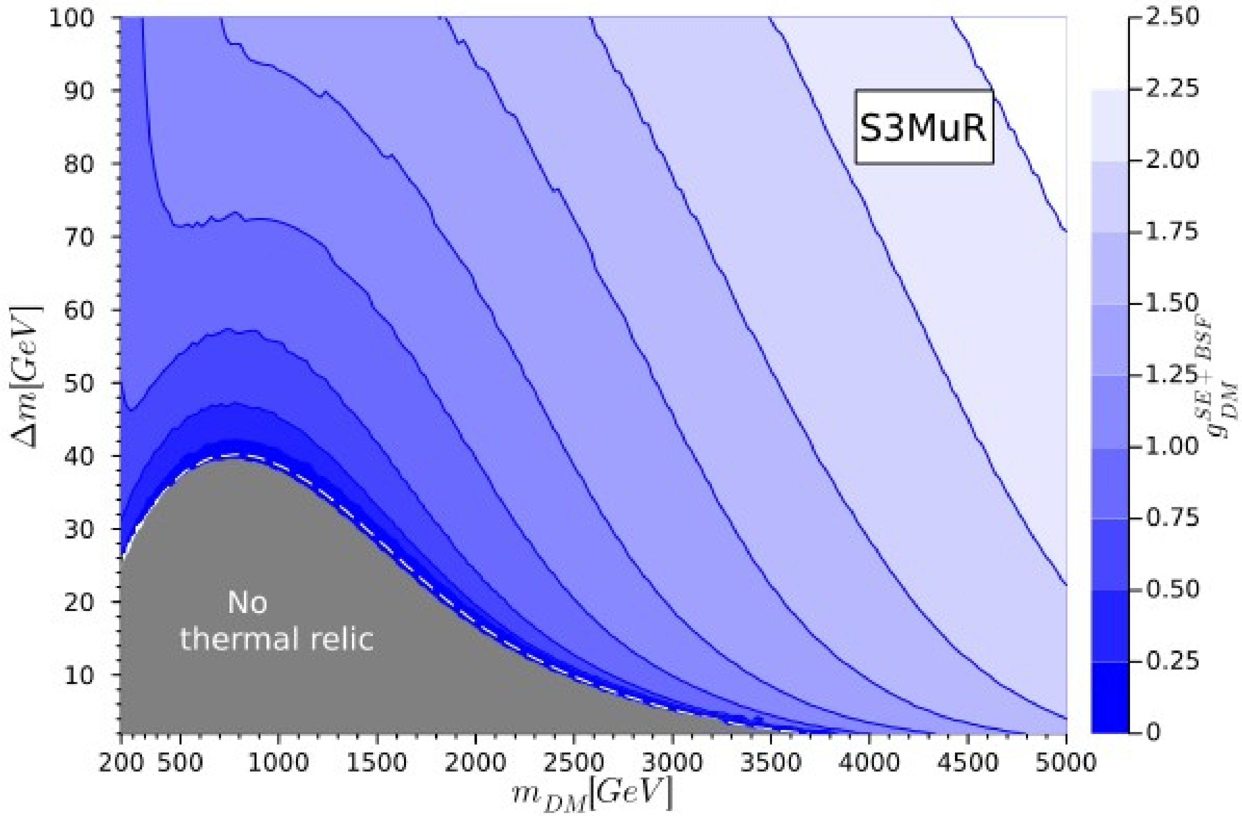




Preliminary!

Scan for S3MuR: SE + $BSF_{n=6}$

Upper limit on g_{DM} for Ω_{DM} = 0.125 (+5σ)



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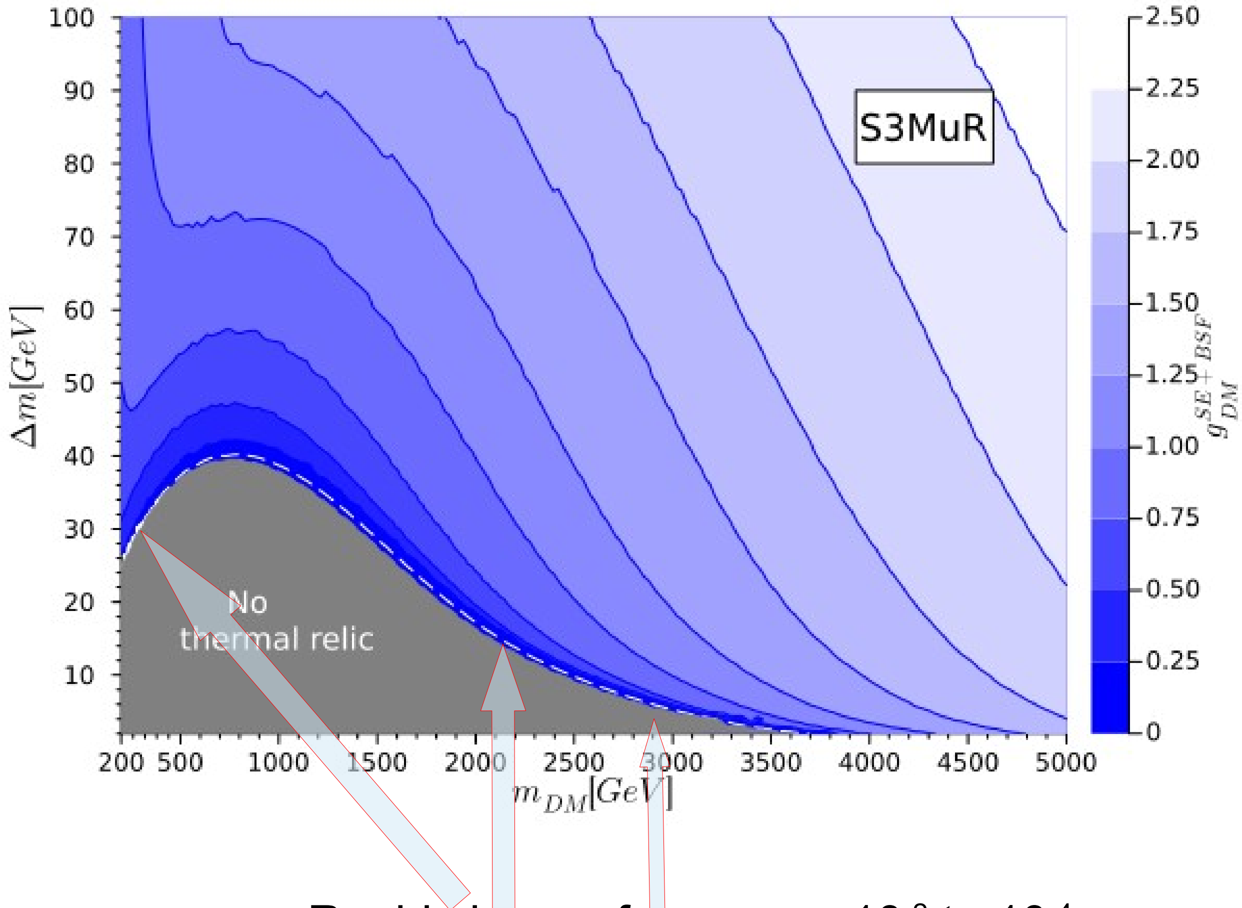
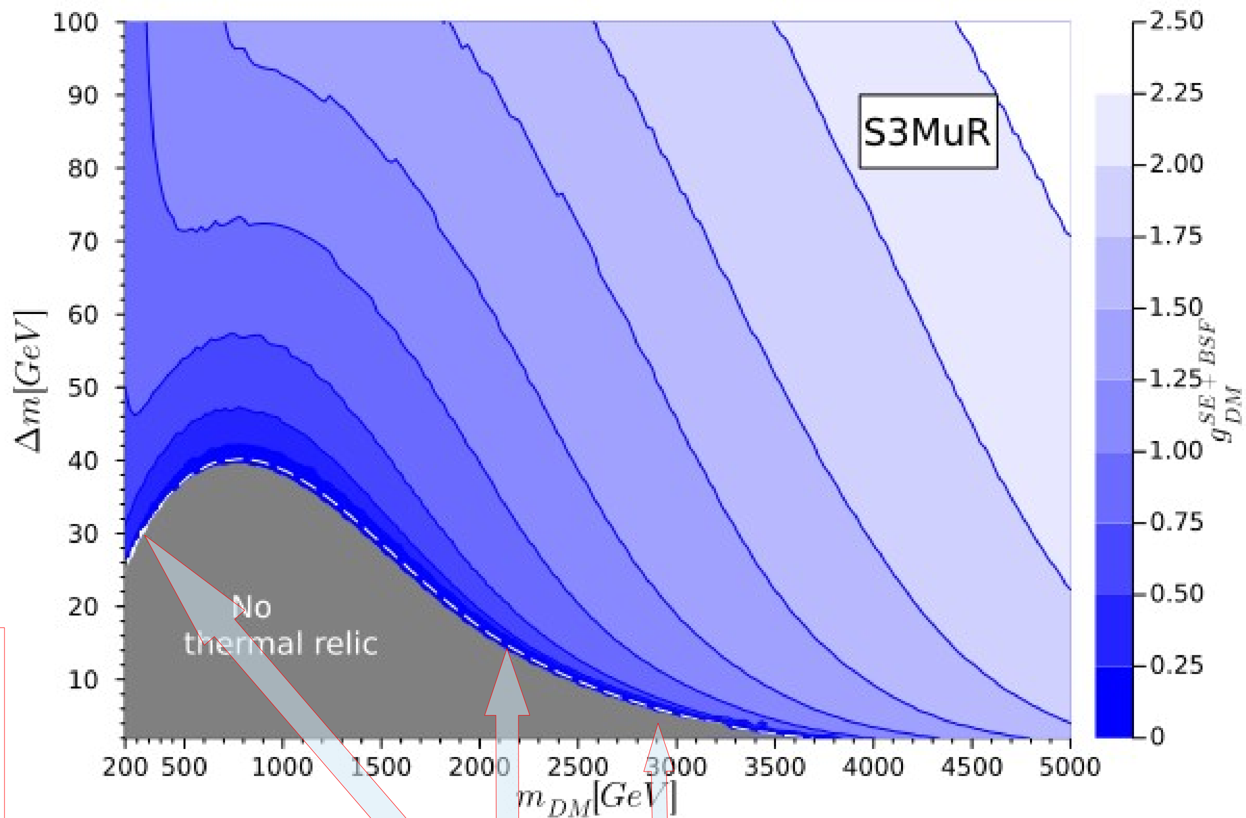


Rapid change from g_{DM} = 10^{-2} to 10^{-4}

Scan for S3MuR: SE + $BSF_{n=6}$

Upper limit on g_{DM} for $\Omega_{DM} = 0.125$ (+5σ)



Gray region:

$$\Gamma_X \frac{Y_X^{eq}}{Y_\chi^{eq}} < H$$

No freeze-out

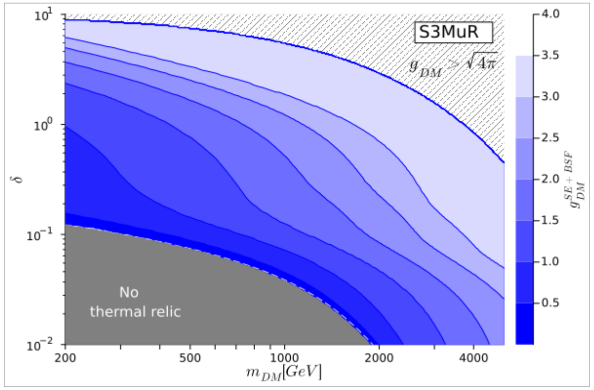
Rapid change from $g_{DM} = 10^{-2}$ to 10^{-4}

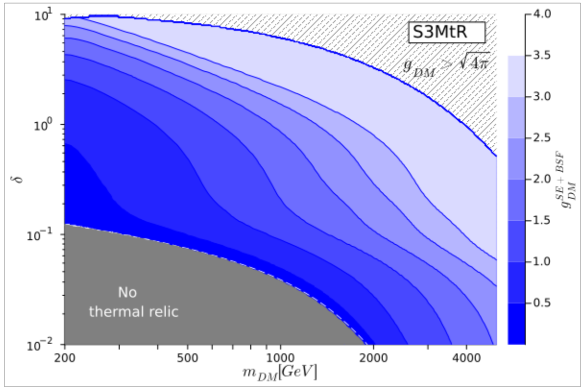
Comparison of models

Upper limit on g_{DM} for $\Omega_{DM} = 0.125$ (+5σ)

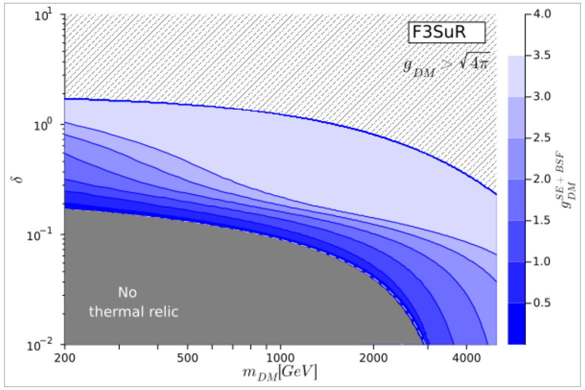
up-quark | **top-quark**

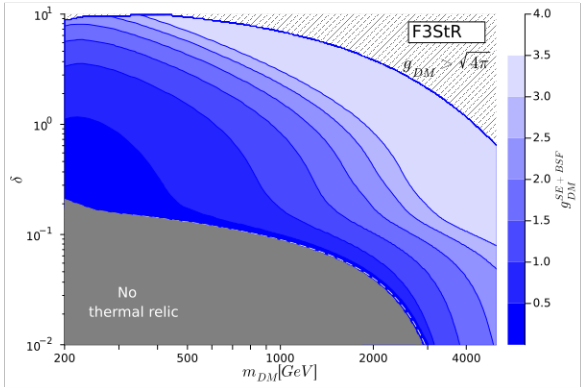
Scalar mediators





Fermionic mediators





Preliminary!

Conclusions & Outlook

Non-perturbative long range effects have **a sizeable impact** on the predicted relic abundance and **soften experimental constraints**.

Simplified dark matter models allow for a universal treatment of these effects, which can be **efficiently incorporated by our framework**.

Impact of Sommerfeld enhancement depends on the dominant annihilation channels and **spin of the mediator**. However, for reasonable approximations, the color decomposition is simple for both types of mediators.

The inclusion of bound state formation **lifts the predicted DM mass and (re-)opens parameter space**.
In the coannihilation regime, excited bound states amount to a correction of (at most) 20%.

Future plans: Work out direct detection and collider limits.

Conclusions & Outlook

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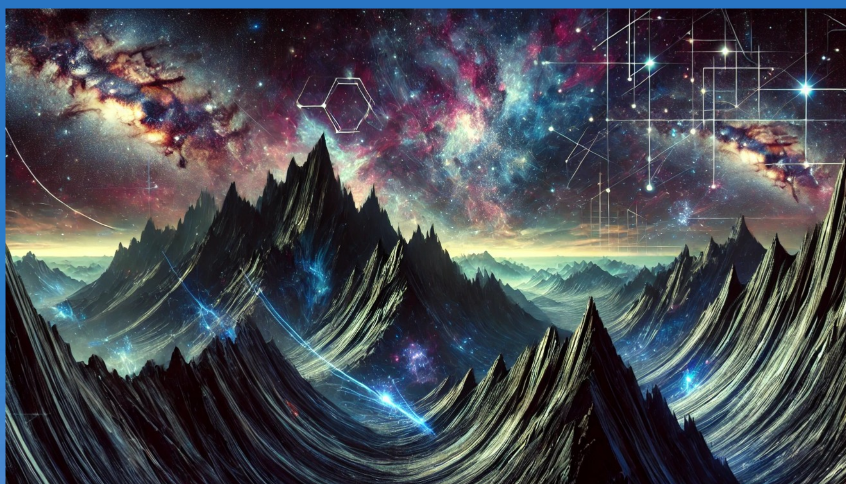
In the coannihilation regime, excited bound states amount to a correction of (at most) 20%.

Future plans: Work out direct detection and collider limits.

Our paper (and the code) will be publicly available soon!

Thank you for your attention!



DALL-E interpretation of
„The Dark Matter Landscape“

Backup

Partial waves and velocity dependence

For the Sommerfeld effect, the partial wave is important, NOT the power of the velocity!

$$\sigma v_{\text{rel}} = \underbrace{\left(a + b\, v_{\text{rel}}^2 + \ldots\right)}_{\text{s-wave}} + \underbrace{\left(c\, v_{\text{rel}}^2 + \ldots\right)}_{\text{p-wave}} + \mathcal{O}(v_{\text{rel}}^4)$$

Partial waves and velocity dependence

For the Sommerfeld effect, the partial wave is important, NOT the power of the velocity!

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Running coupling at different scales

Vertices	α_s	α_g	Average momentum transfer Q
Wavefunction (ladder diagrams) of scattering state in colour rep. $\hat{\mathbf{R}}$	α_s^S	$\alpha_{g,[\hat{\mathbf{R}}]}^S = (\alpha_s^S/2)\times \times [C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\hat{\mathbf{R}})]$	$k \equiv \mu\, v_{\rm rel}$
Wavefunction (ladder diagrams) of bound state in colour rep. $\hat{\mathbf{R}}$	$\alpha_{s,[\hat{\mathbf{R}}]}^B$	$\alpha_{g,[\hat{\mathbf{R}}]}^B = (\alpha_{s,[\hat{\mathbf{R}}]}^B/2)\times \times [C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\hat{\mathbf{R}})]$	$\kappa_{\hat{\mathbf{R}}} \equiv \mu\, \alpha_{g,[\hat{\mathbf{R}}]}^B$
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[J. Harz and K. Petraki (2018)]

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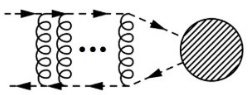
Sommerfeld effect



[J. Harz and K. Petraki (2018)]

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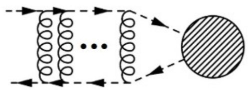
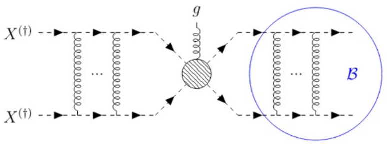


Bound state formation



[J. Harz and K. Petraki (2018)]

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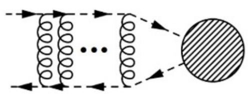

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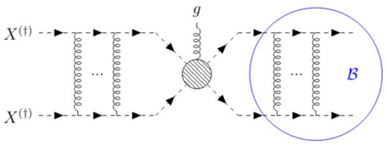


[J. Harz and K. Petraki (2018)]

Numerically solve for given running of QCD coupling $\alpha_B = \alpha_s\left(Q = \mu\frac{\alpha_B}{n}\right)$

Numerical bottleneck: diagonalization of transition matrix

$$\langle\sigma_{BSF}v\rangle_{\text{eff}} = \sum_i \langle\sigma_{\text{BSF,i}}v\rangle \left(1-(M)^{-1}_{ij}\frac{\langle\Gamma^j_{\text{ion}}\rangle}{\langle\Gamma^j\rangle}\right)$$

$$M_{ij} = \delta_{ij} - \frac{\langle\Gamma^{i\to j}_{\text{trans}}\rangle}{\langle\Gamma^i\rangle} \qquad \Gamma^i = \langle\Gamma^i_{\text{dec}}\rangle + \langle\Gamma^i_{\text{ion}}\rangle + \sum_{j\neq i}\langle\Gamma^{i\to j}_{\text{trans}}\rangle$$

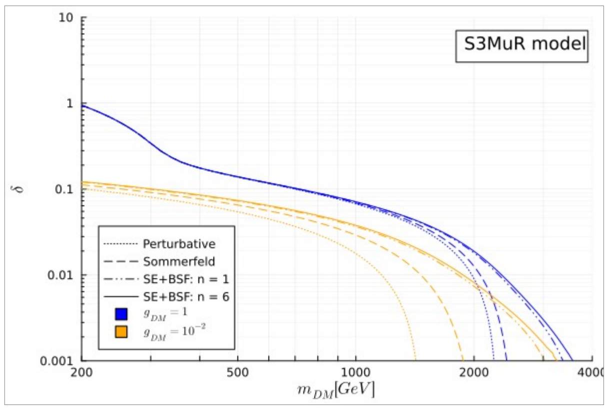
Inversion of matrix M with micrOMEGAs internal Jacobi routine (intended for diagonalizing mass matrices) → room for improvement.

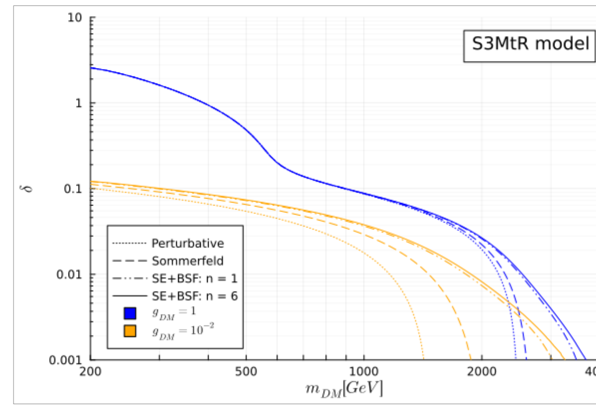
Scaling: $n^2 \times n^2 \sim \mathcal{O}$ (n^4)

Bandscans on a logarithmic scale

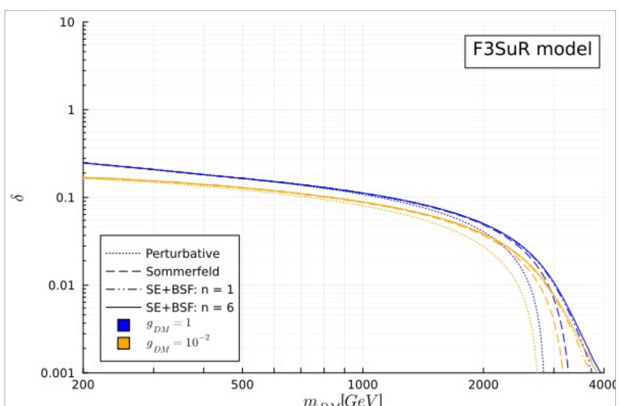
up-quark | **top-quark**

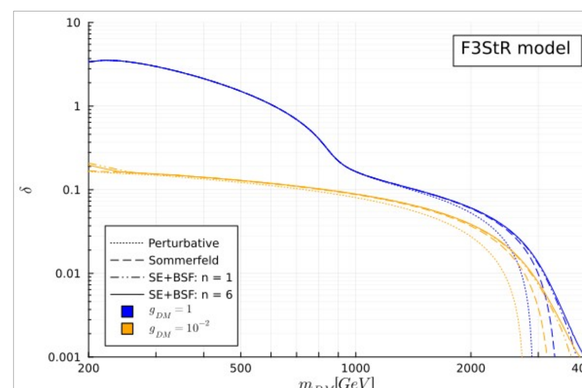
Scalar mediators





Fermionic mediators





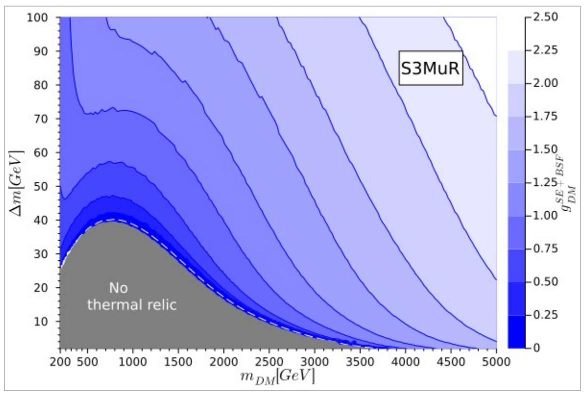
Preliminary!

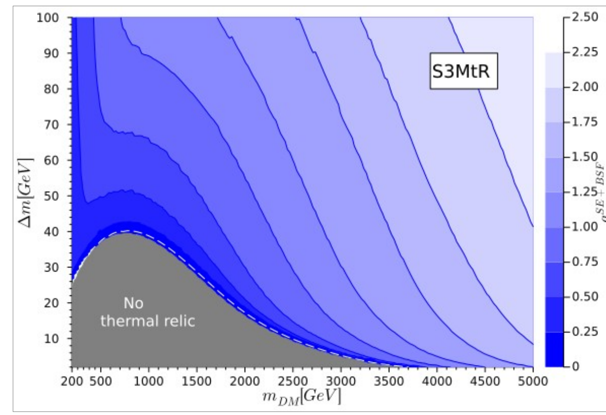
Comparison of models (linear scale)

Upper limit on g_{DM} for $\Omega_{DM} = 0.125$ (+5σ)

up-quark | **top-quark**

Scalar mediators





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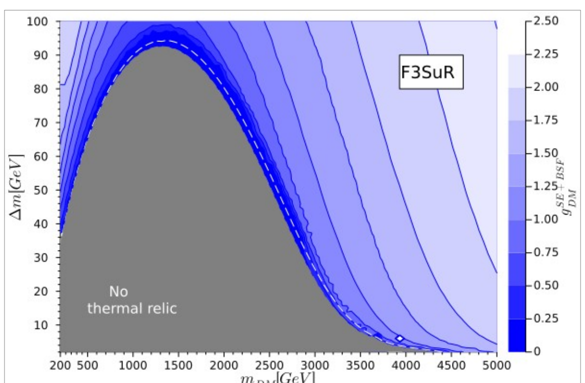
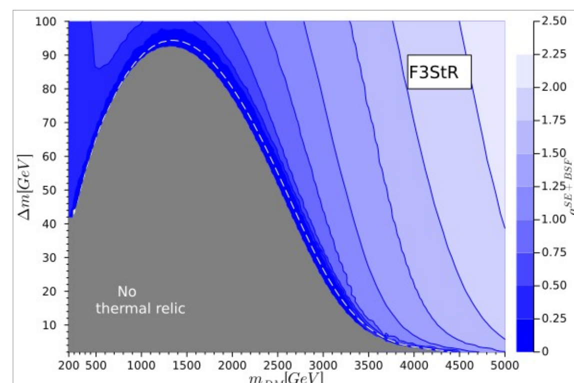


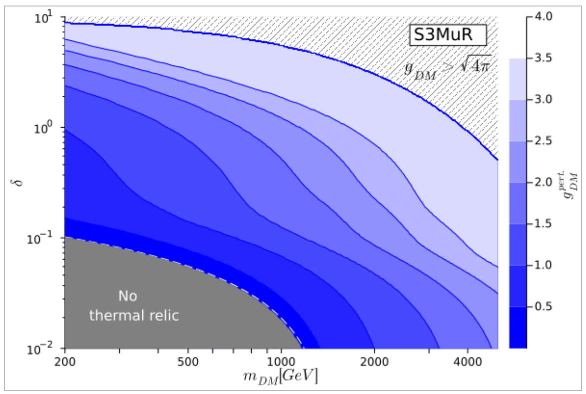


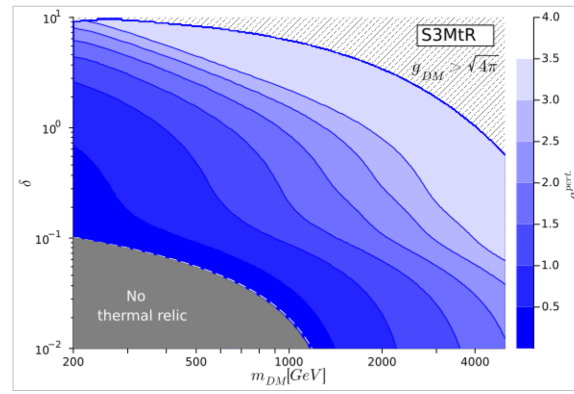
Preliminary!

Results without long-range effects

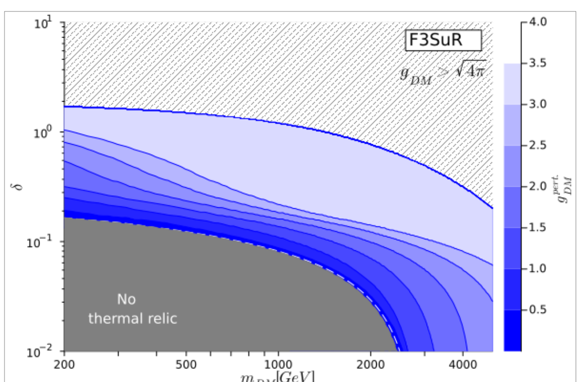
up-quark | **top-quark**

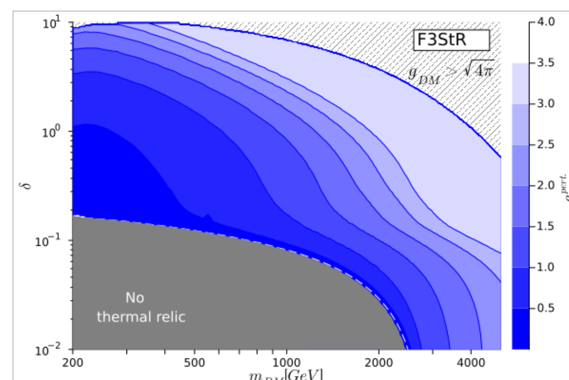
Scalar mediators





Fermionic mediators



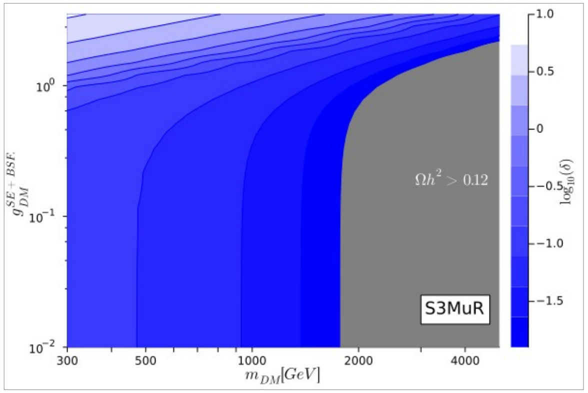


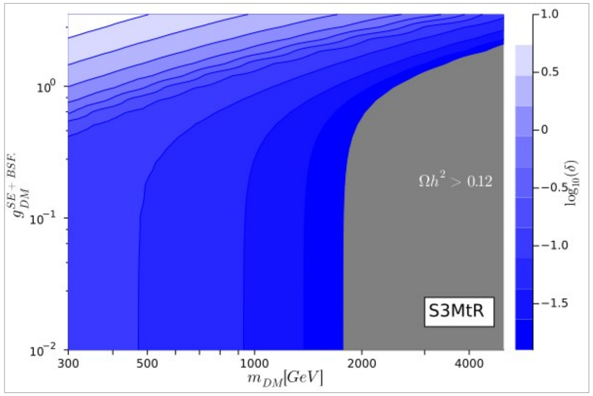
Preliminary!

Contours of constant δ

up-quark | **top-quark**

Scalar mediators





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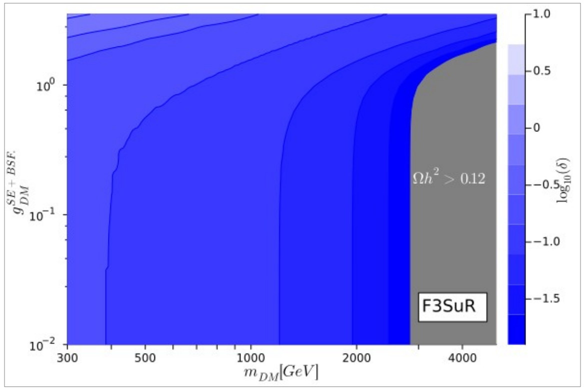
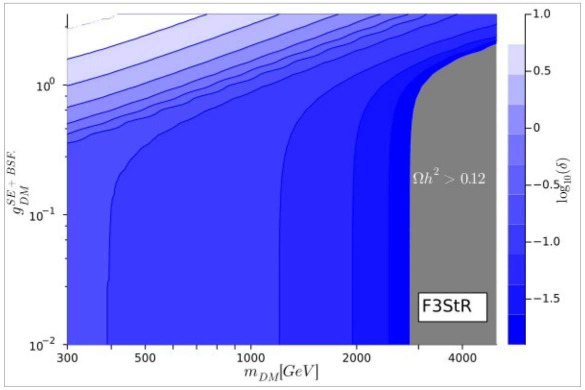


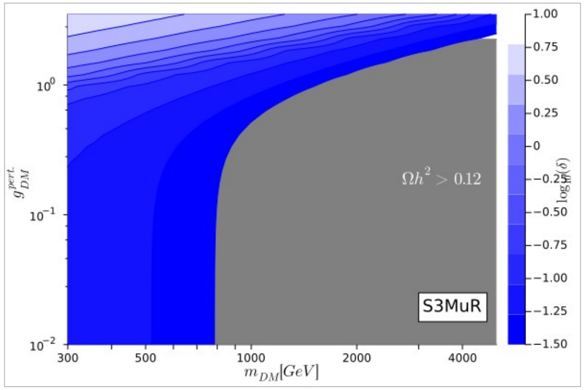


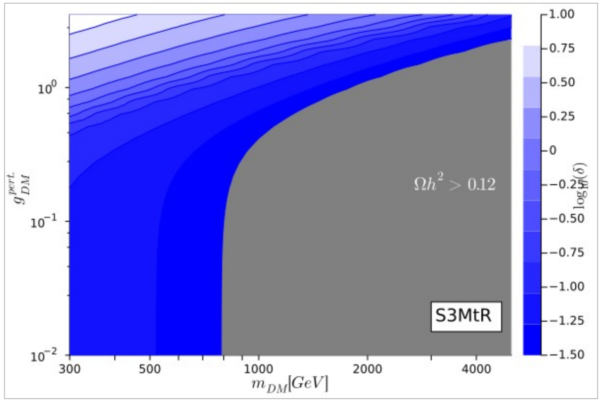
Preliminary!

Contours of constant δ (without long-range effects)

up-quark | **top-quark**

Scalar mediators





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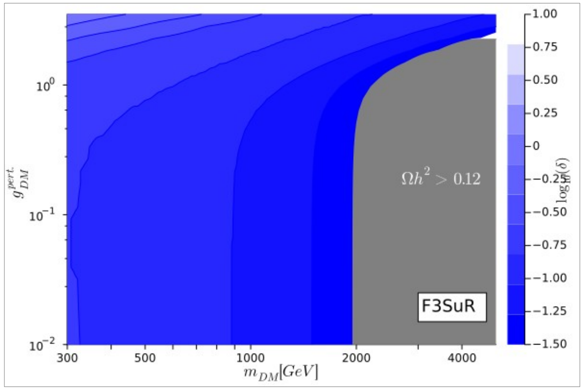

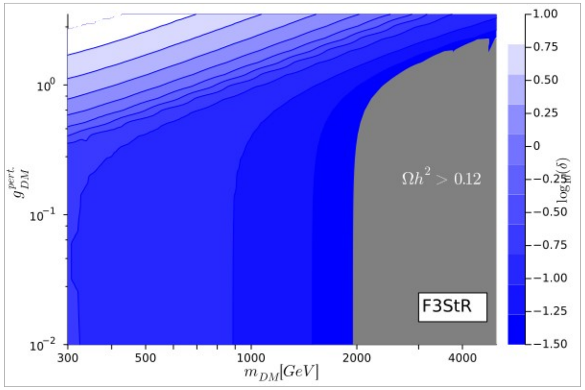

Preliminary!