Technische Universität München
T31 SFB 1258 Projekt D04
„Neutrinos and Dark Matter in Astro- and Particle Physics“

Excited bound states and their role in Dark Matter production

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Based on:

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In collaboration with:

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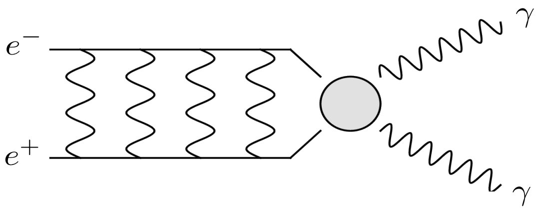
The Dark Matter Landscape: From Feeble to Strong Interactions, MITP

Positronium example

Bound-state decay and Sommerfeld enhancement:

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

Pirenne & Wheeler 1946



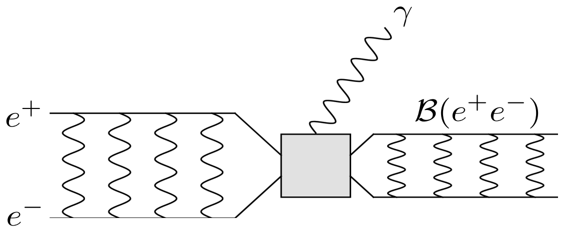
$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v) \,, \text{ for } v \lesssim \alpha.$$

Sakharov 1948 (Sommerfeld 1931)

Bound-state formation (recombination):

$$(\sigma v)_{nl} = \frac{4\alpha}{3} |\langle \psi_{nl}| \, \mathbf{r} \, |\psi_v\rangle|^2 \Delta E^3$$
$$\sim 3 \times \text{annihilation}, \text{ for } v \lesssim \alpha.$$

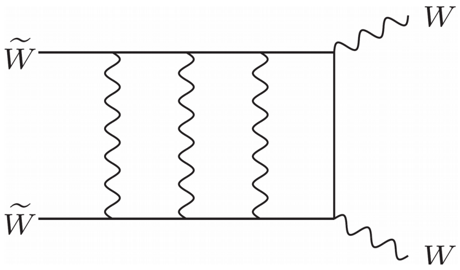
(and n=1,l=0.)

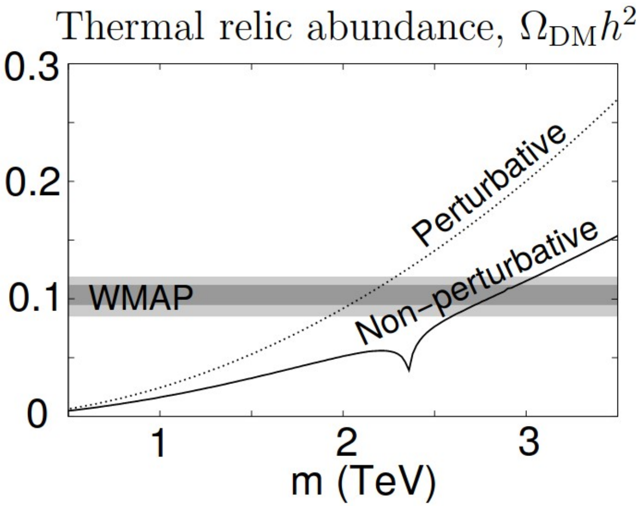


(originates from the Electric Dipole Operator „gr.E“, see e.g. Landau&Lifshitz)

Wino Dark Matter example

$$\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}^0 \\ \widetilde{W}^- \end{pmatrix}$$



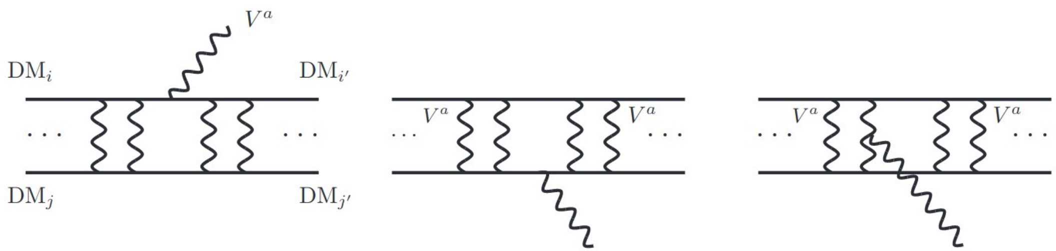


[Hisano et al. 03,05,06]

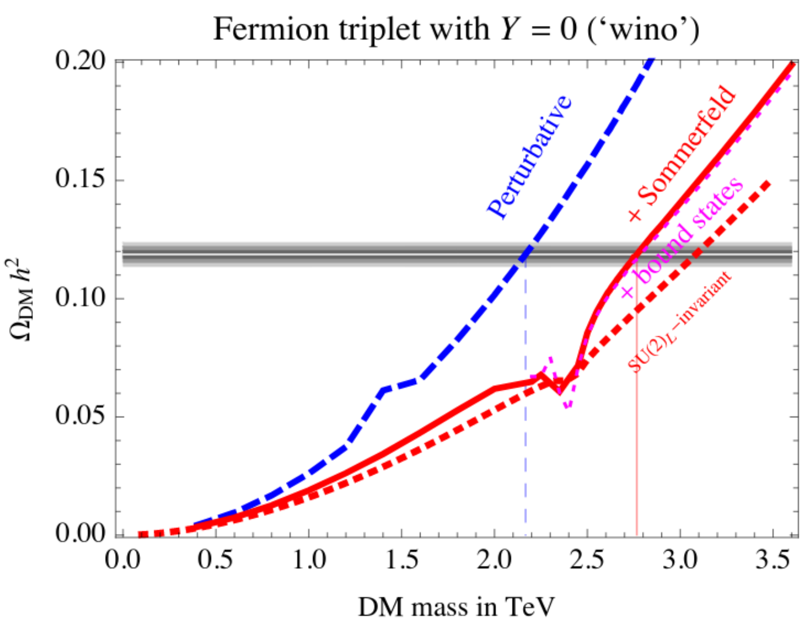
- Majorana Fermion, SU(2) Triplet, zero Hypercharge („most minimal WIMP“)
- Sommerfeld-enhanced annihilation allows for heavier Wino masses
- ID signal mass sensitive, see e.g.

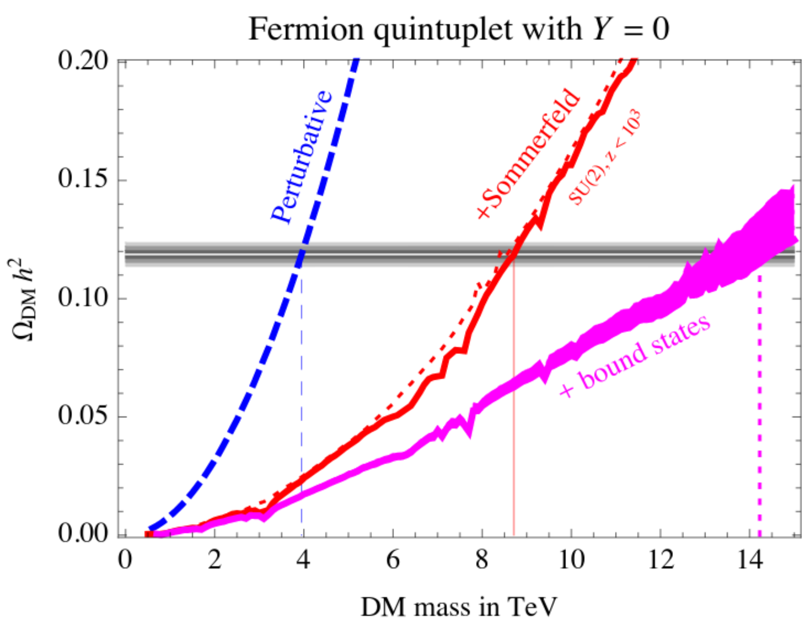
[Rinchiuso, Slatyer et al. 20]

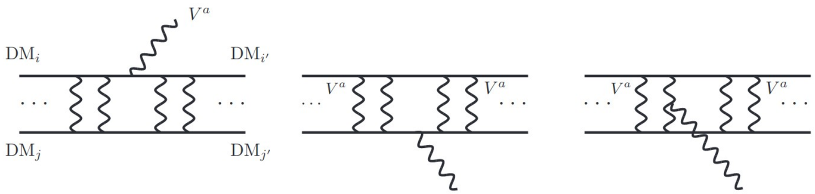
Minimal Dark Matter examples



[Mitridate *et al.* 17]







Colored co-annihilation examples

- **I.e., co-annihilating partner charged under SM SU(3)**
- **Longe-range effects impact** $(\Delta m_\chi, m_\chi)$**- plane**
  - Squark (scalar triplet)
  - Gluino (fermion octet)

[Ellis *et al.* 15, Liew & Luo 16, Mitridate *et al.* 17]

[Harz & Petraki 18,19]

- **+ Higgs**
  - Additional attractive contribution
  - (squark) octet can be bounded

[Gross *et al.* 18, Fukuda & Luo & Shirai 18]

- **Non-perturbative regime**
  (for mass splitting below confinging scale)

Classification of bound-state formation

Leading multipole: $\langle \psi_{nl} | \, r^X \, | \psi_{\mathbf{p}} \rangle$

- **Monopole (X=0):** via *charged scalar* emission [Oncala & Petraki 19,21]
  - Matrix elements fully known
  - Partial-wave unitarity can be problematic already for ground state capture
- **Dipole (X=1):** via *vector gauge field*
  - SM charged: Wino, Minimal DM, Colored co-annihilation [Ellis et al. 15, Mitridate et al. 17, Harz et al. 18, ...]
  - Dark U(1) [Harling et al. 14, ...]
  - Dark SU(N) [..., Asadi 21, Biondini et al 23]
- **Quadrupole (X=2):** via *neutral scalar* emission [Wise et al. 14,16, Petraki et al. 15]
  - „Dark Yukawa“ (pNRY [Biondini 21,22])

This talk: highly excited bound states in perturbative, unbroken gauge theories (dipole).

Overview

- Dipole matrix elements: the general case
- Freeze-out or „eternal depletion“?
- Example: t-channel simplified model in the superWIMP regime
- Partial wave unitarity

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

$$\mathcal{L} \xrightarrow{m} \mathcal{L}^{\rm NR} \xrightarrow{\alpha m} \mathcal{L}^{\rm pNR}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) QED:

$$\mathcal{L}^{\rm pNRQED} \supset \int \mathrm{d}^3 r\, S^\dagger(\mathbf{x},\mathbf{r},t)\left[i\partial_t + \frac{\nabla_{\mathbf{x}}^2}{4m} + \frac{\nabla_{\mathbf{r}}^2}{m} - V(r) + i2\frac{\pi\alpha^2}{m^2}\delta^3(\mathbf{r}) + \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)\right] S(\mathbf{x},\mathbf{r},t)$$

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$

$$(\sigma v)_{nl} = \frac{4\alpha}{3} |\langle \psi_{nl}| \,\mathbf{r}\, |\psi_v\rangle|^2 \Delta E^3$$

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

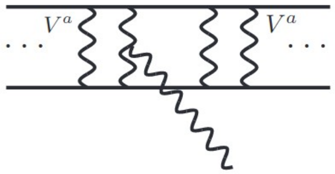
$$\mathcal{L} \xrightarrow{m} \mathcal{L}^{\mathrm{NR}} \xrightarrow{\alpha m} \mathcal{L}^{\mathrm{pNR}}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) SU(N) in the weakly coupled regime:

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$$

$$\mathcal{L}_{\mathrm{pNREFT}} \supset \int \mathrm{d}^3 r \ \mathrm{Tr}\Big[\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger(iD_0 - H_{\mathrm{adj}})\mathrm{Adj} - V_A(\mathrm{Adj}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) - \frac{V_B}{2}\mathrm{Adj}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, \mathrm{Adj}\} + \cdots\Big].$$



Included, e.g. [Binder et al. 2021]

e.g. quarkonium, squark:

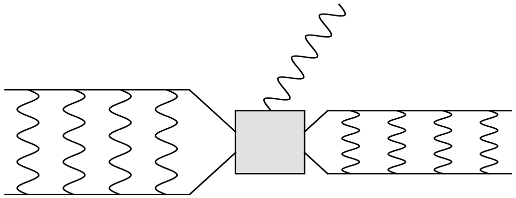
$3 \otimes \bar{3} = 1 \oplus 8$ $\qquad$ $\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$ $\qquad$ $(\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 |\langle \psi^1_{nl}| \mathbf{r} |\psi^8_{\mathbf{p}}\rangle|^2$

General dipole transition matrix elements

- „gr.E“ leads to matrix elements of the form:

$$\langle\psi_f|\,\mathbf{r}\,|\psi_i\rangle = \int \mathrm{d}^3r\ \psi_f^\star(\mathbf{r})\ \mathbf{r}\ \psi_i(\mathbf{r}).$$

$$\mathrm{V}_{i/f} = -\frac{\alpha_{i/f}^{\mathrm{eff}}}{r}$$



- E.g.: (chromo-) electric dipole transitions of pairs in unbroken U(1) and SU(N) gauge theories

- Analytic result in terms of recurrence relations* allows for efficient and numerically stable evaluation.

- Tested against know results for low excitations

*) in QED limit, reduces to 2 Hypergeometric functions, consistent with
[W. Gordon, Zur Berechnung der Matrizen beim Wasserstoffatom, Annalen der Physik 394 (1929)]

Bound-state formation in U(1) gauge theory

$\mathcal{S}(\chi\bar{\chi}) \to \mathcal{B}(\chi\bar{\chi})_{nl} + \gamma$

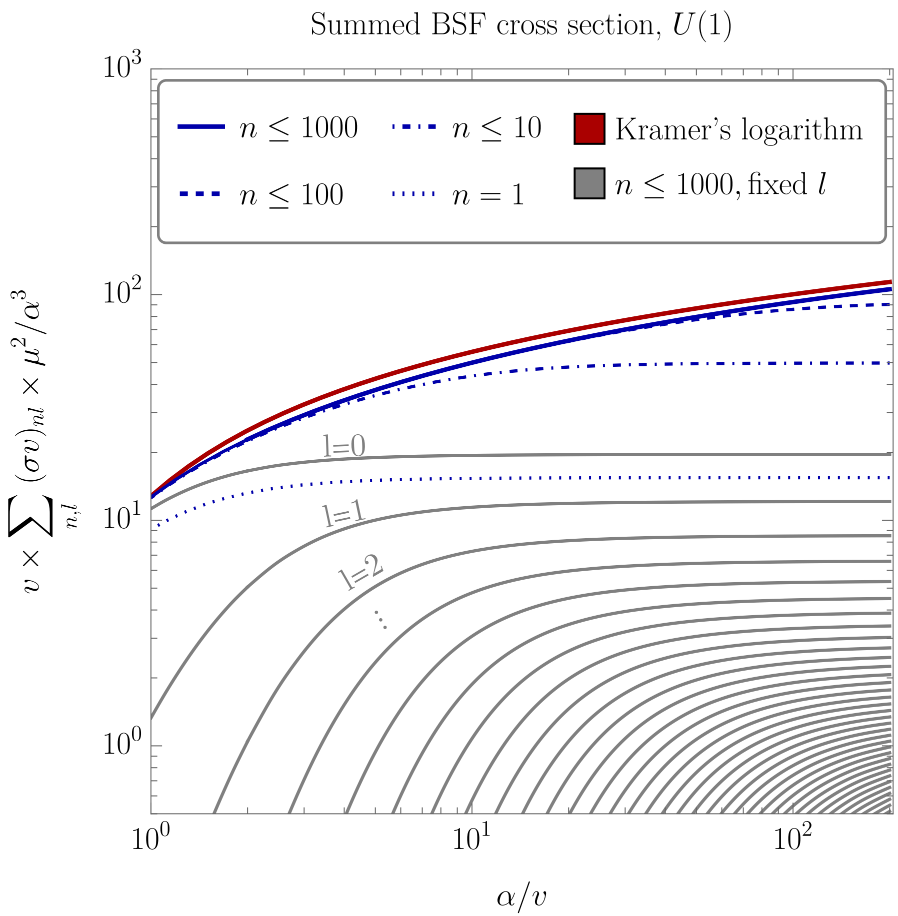
$(\sigma v)_{nl} = \frac{4\alpha}{3}\Delta E^3 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{\mathbf{p}}\rangle|^2$

(e.g. hydrogen, (dark) positronium,
complex scalars)

- Up to half a million bound states:
  all $\mathrm{n} \leq 1000, l \leq n-1.$

- Confirm Kramer's logarithm within expected error as a check:

$$\sum_{n,\ell}(\sigma v)_{n\ell} \simeq \frac{32\pi}{3\sqrt{3}}\frac{\alpha^2}{\mu^2}\frac{\alpha}{v}[\log(\alpha/v)+\gamma_E]\,,\ \text{for}\ v \ll \alpha.$$



Bound-state formation in SU(3) gauge theory

$3 \otimes \bar{3} = 1 \oplus 8$

$\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$

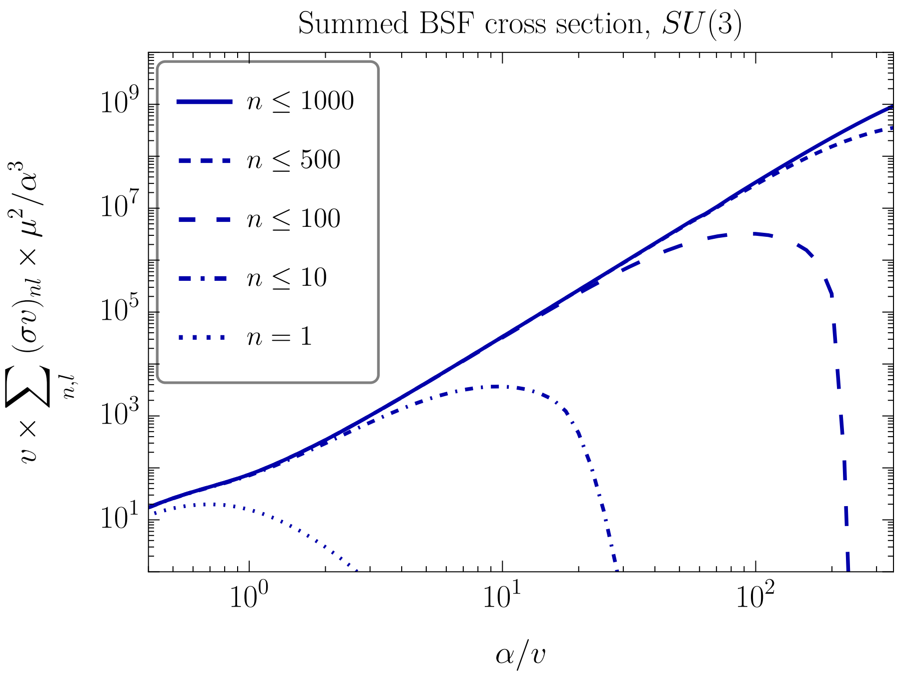
$(\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 |\langle\psi^1_{nl}|\,\mathbf{r}\,|\psi^8_{\mathbf{p}}\rangle|^2$

(e.g. quarkonium, squark)

- Assume constant coupling
- Low velocity scaling much stronger:

  $\sum_{nl}(\sigma v)_{nl} \propto v^{-4}$ for $v \ll \alpha$

- Raises concerns about partial wave-unitarity violation



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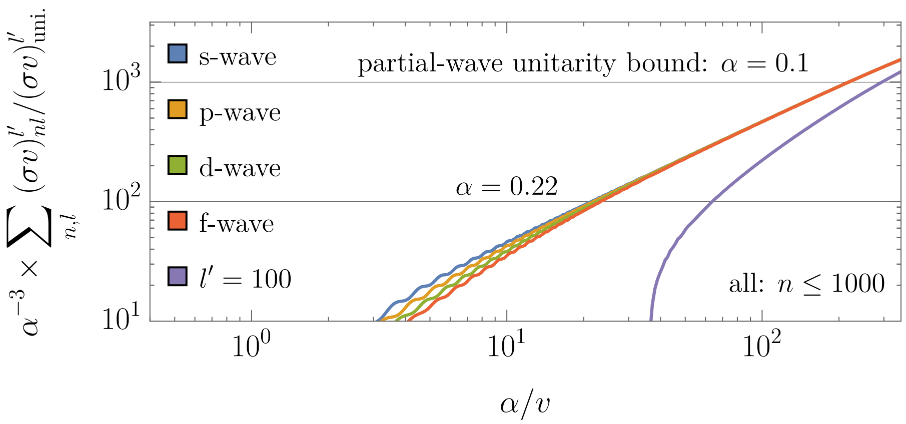
(e.g. quarkonium, squark)

- Unitarity condition:

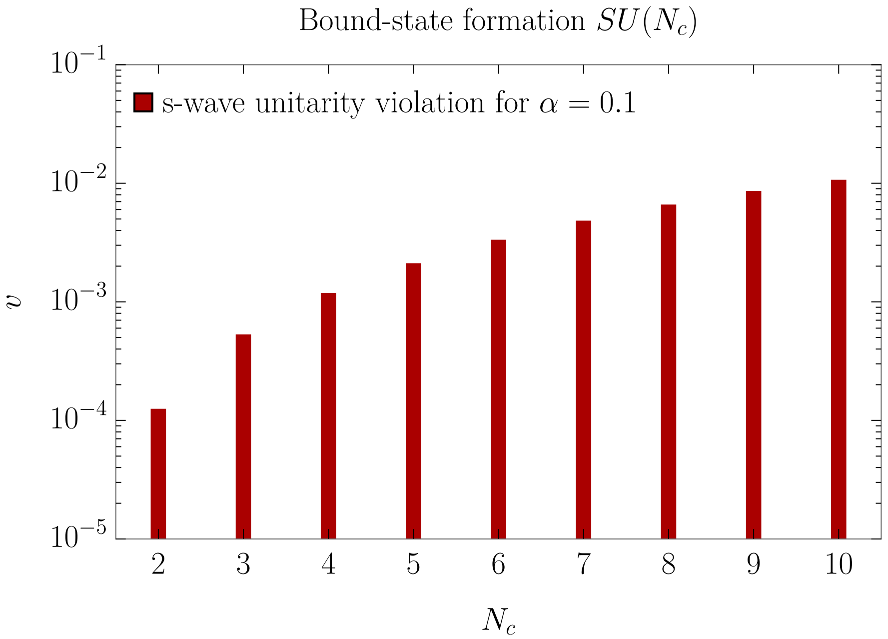
$$\sum_{nl}(\sigma v)^{l'}_{nl} \leq (\sigma v)^{l'}_{\text{uni.}} = \frac{\pi(2l'+1)}{\mu^2 v}$$

- Observe partial-wave unitarity violation in the perturbative regime



Partial wave unitarity violation in SU(N)

- We observe partial wave unitarity violation in SU(N) gauge theories for perturbatively small couplings
- More generally: if the initial state is less attractive than the final state partial wave unitarity will be violated at a finite velocity
- Mechanism behind unitarization unknown



In the following, focussing on the regime consistent with perturbativity and unitarity

Overview

- Dipole matrix elements: the general case
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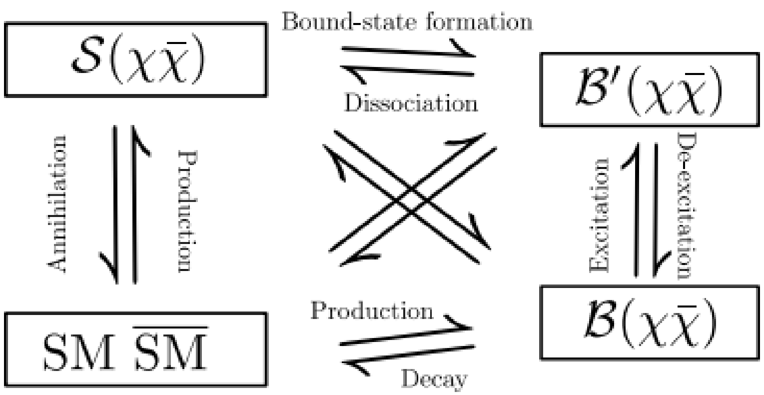
- Example: t-channel simplified model in the superWIMP regime
- Partial wave unitarity

Effective cross section & critical scaling



Quasi steady-state

$$\dot{n} + 3Hn = -\langle\sigma v\rangle_{\text{eff}}(n^2 - n_{\text{eq}}^2)$$

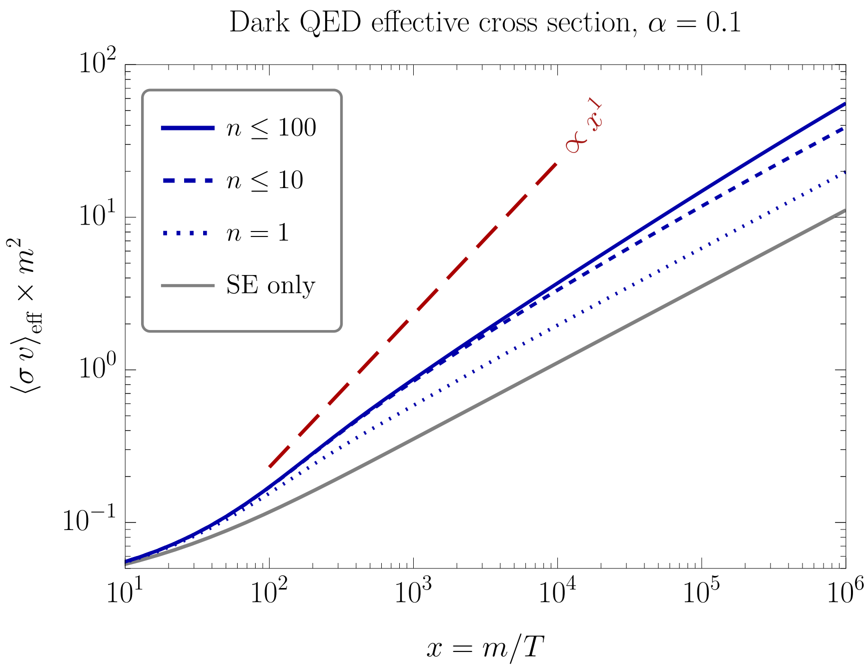
$$\langle\sigma v\rangle_{\text{eff}} = \langle\sigma v\rangle_{\text{ann}} + \sum_{\mathcal{B}} \langle\sigma v\rangle_{\mathcal{B}} R_{\mathcal{B}}$$

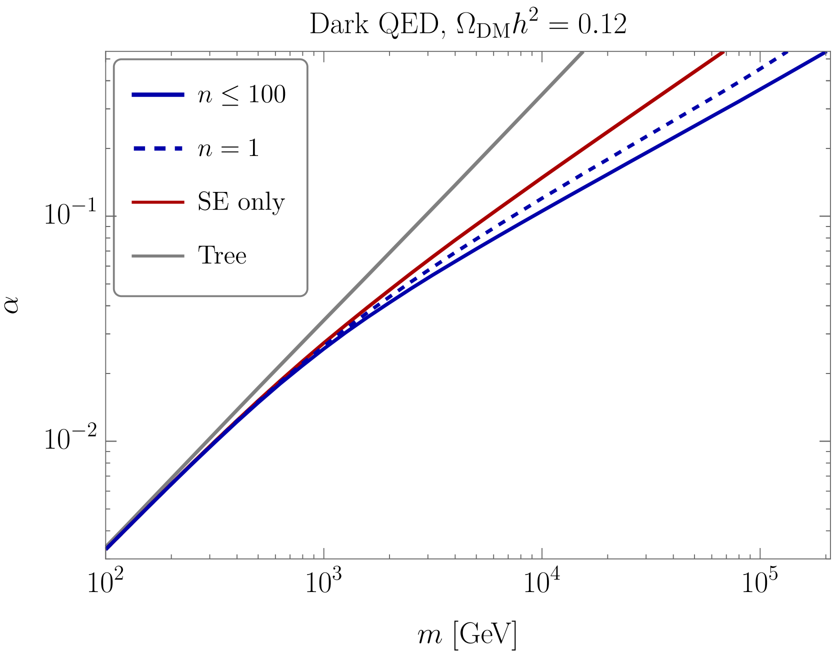
[Ellis et al. 15, Mitridate et al. 17,
Binder et al. 21, Garny & Heisig 21]

- Effective cross section encodes complex interplay between annihilation, scattering-to-bound, bound-to-bound and bound-state decay processes.
- *Super critical scaling:* $\langle\sigma v\rangle_{\text{eff}} \propto (m/T)^{\gamma}$, where $\gamma \geq 1.$

  i.e. particles do not freeze-out but continue depletion.

Effective cross section: Dark QED sector

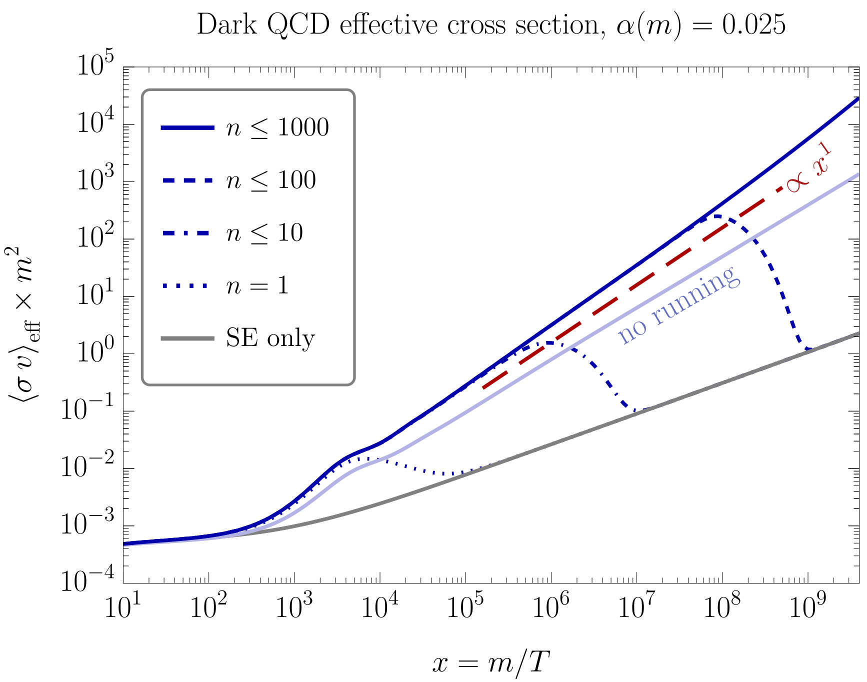




- Includes about 5000 bound states and all possible transitions (~10^6).
- Dark QED indeed freezes out.
- Upper bound on DM mass consistent with perturbative unitarity is 0.2 PeV.

Effective cross section: Dark QCD sector

- Only s-wave bound states included, which are dominant decay channel.
- Transitions among color singlet bound states not possible via chromo-electric dipole operator.
- Running coupling effects lead to supercritical behaviour, i.e. no freeze-out in the perturbative regime.
- SU(N) with N>3: all supercritical
- Simultaneous SU(N) and U(1) charge allows for transitions among singlets → enhanced scaling



Overview

- Dipole matrix elements: the general case
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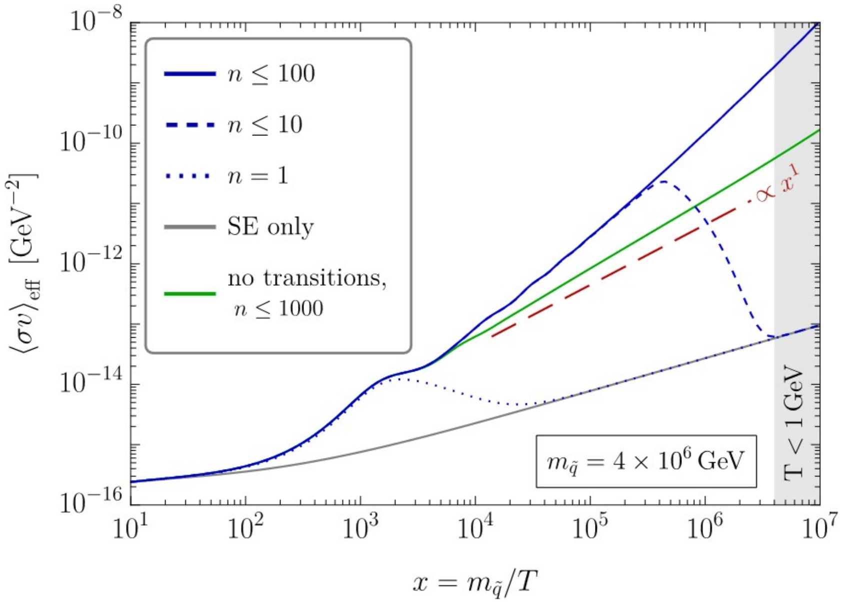
SM SU(3) and U(1) charged mediator model

„t-channel“ simplified model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q} \bar{q}_R \chi + h.c.$$

- $\tilde{q}$: scalar mediator, carries SM electric and color charge
- q_R: right handed SM quark
- χ : Majorana Fermion Dark Matter



DM production scenarios

„t-channel" simplified model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q}\bar{q}_R\chi + h.c.$$

DM production can be classified into:

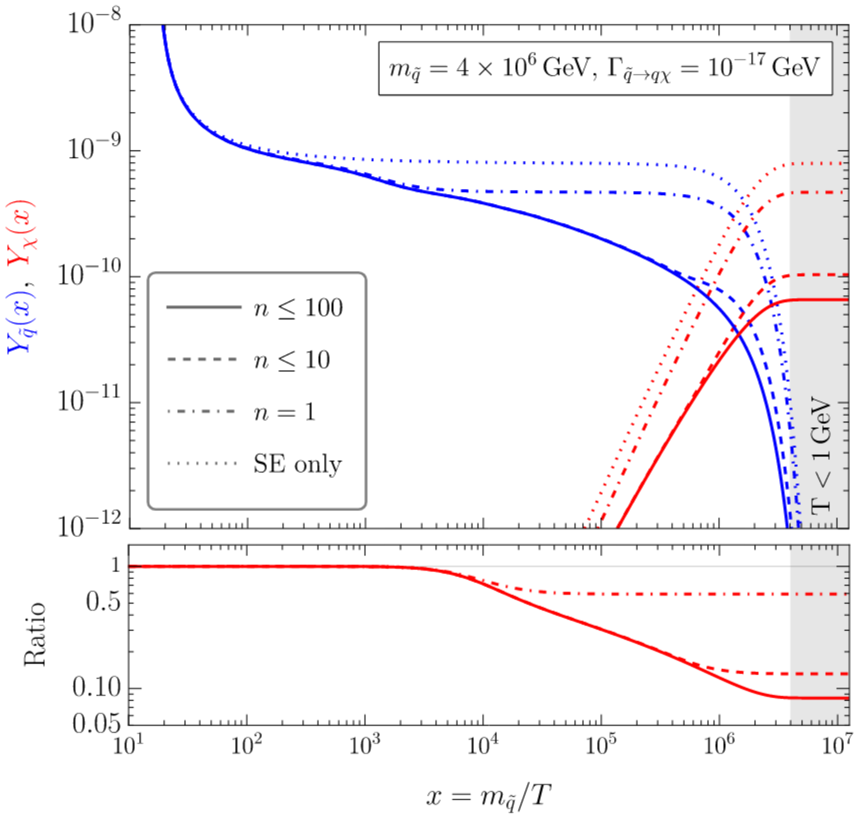
$$\begin{aligned} \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\gg H(m_{\tilde{q}}) && \text{coannihilation}\,, \\ \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\sim H(m_{\tilde{q}}) && \text{conversion-driven}\,, \\ \Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\ll H(m_{\tilde{q}}) && \text{superWIMP/freeze-in}\,. \end{aligned}$$

$$\frac{\mathrm{d}Y_{\tilde{q}}}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\frac{1}{2}\langle\sigma_{\tilde{q}\tilde{q}^\dagger}v\rangle_{\text{eff}}\left(Y_{\tilde{q}}^2 - Y_{\tilde{q}}^{\text{eq}\,2}\right)\right. \\ \left. + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) + \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\right], \tag{19}$$

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\langle\sigma_{\chi\chi}v\rangle\left(Y_\chi^2 - Y_\chi^{\text{eq}\,2}\right)\right. \\ \left. + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) - \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\right], \tag{20}$$

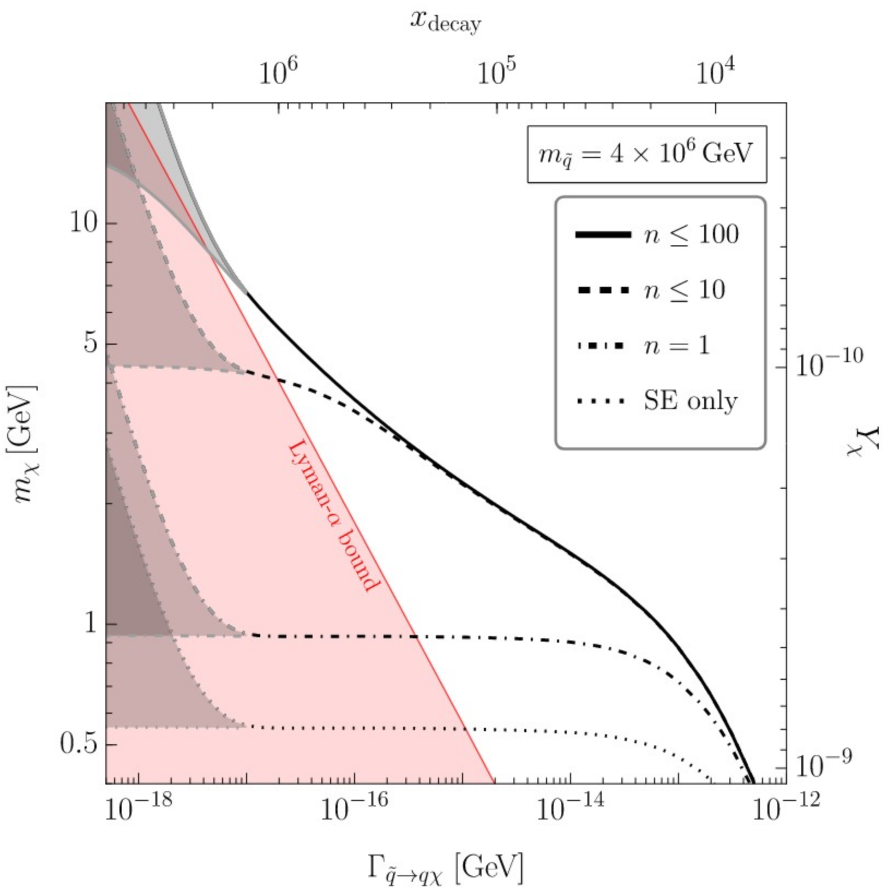
SuperWIMP regime

- Typical superWIMP mechanism: late decay of mediator into DM, final DM yield independent of actual size of the conversion rate.
- Continous depletion of mediator yield from bound state effects.
- → introduces a dependence of the DM yield on the conversion rate as a novel feature.



Constraints

- DM produced relativistically from heavy mediator decay
- DM can be „too hot“, i.e., substructure can be erased by free-streaming effect
- Substructure probed by Ly-alpha observations
- Bound state effects open up parameter space
- Corrections to the DM mass up to an order of magnitude



Summary & Conclusion

- Highly excited bound states can play an important role for predicting the DM relic abundance precisely.

- Can lead to „eternal freeze-out“ in unbroken non-abelian gauge theories

- SuperWIMP regime:

  - bound state effects can introduce a dependence of the DM yield on the mediator lifetime as a novel feature

  - corrections by up to an order of magnitude in the DM mass

- unitarization of bound state formation in unbroken non-Abelian gauge theories within the regime of perturbatively small couplings (?)