Precision calculation of relic abundance for two-component dark matter: out-of-kinetic equilibrium effects

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based on ongoing work with Andrzej Hryczuk

MITP
SCIENTIFIC
PROGRAM

The Dark Matter Landscape:
From Feeble to Strong Interactions
August 26 – September 13, 2024
https://indico.mitp.uni-mainz.de/event/366

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August 30th, 2024

Outline

- Brief recap of the standard calculation of Dark Matter (DM) abundance
- Towards a more precise calculation when the underlying assumption of kinetic equilibrium as in the canonical case is not met
    - When does DM freeze-out outside of kinetic equilibrium
    - How is the Boltzmann equation solved without this simplifying assumption: challenges and solutions
- Non minimal dark sector: two-component
- Summary

Production of DM by freeze-out

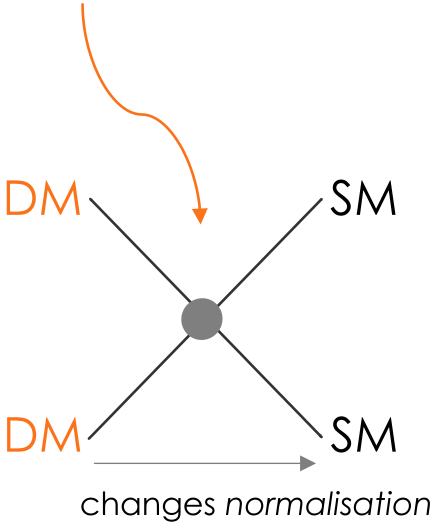
Dark matter relic density measurement from the CMB is a well-measured quantity

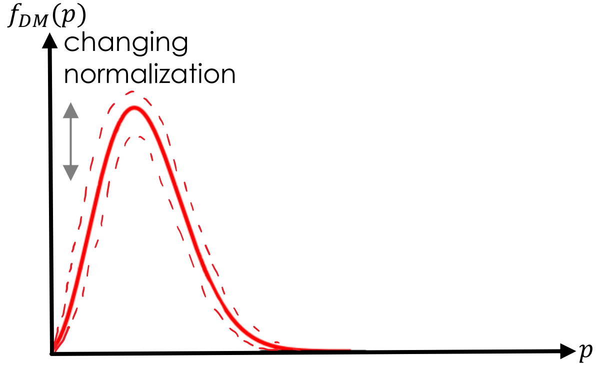
$$\Omega_c h^2 = 0.1198 \pm 0.0012 \quad \text{PLANCK 2018}$$

- Obtained from solving the Boltzmann equation

$$L[f_{DM}] = C[f_{DM}]$$

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}] + \ldots$$





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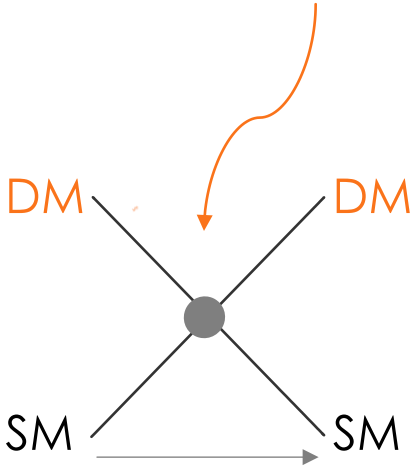
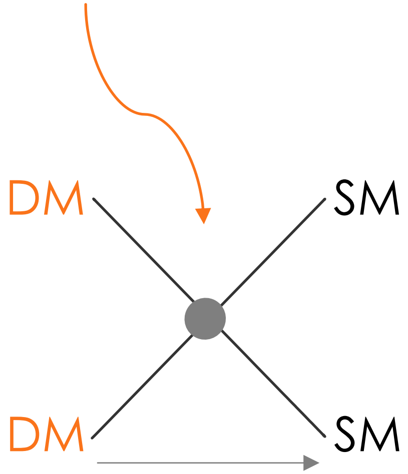
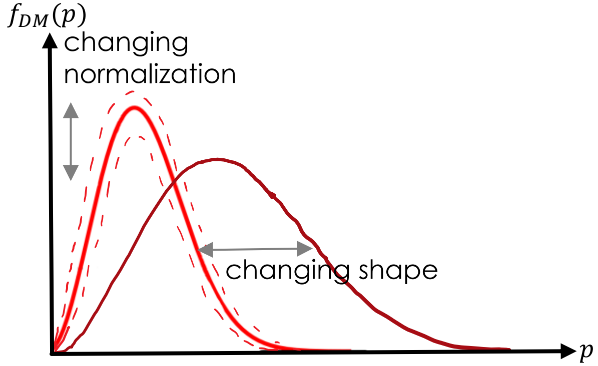


changes *shape*



changes *normalisation*



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$$\dot{n} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

Kinetic equilibrium Bernstein, Brown, Feinberg 1985

$$f_{DM}(T) \propto f_{eq}(T)$$

- Although typically a good assumption for $m_{DM} \gg m_{SM}$ …

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~~Kinetic equilibrium~~ Bernstein, Brown, Feinberg 1985

$\cancel{f_{DM}(T) \propto f_{eq}(T)}$

- Although typically a good assumption for $m_{DM} \gg m_{SM}$ …
  there exist scenarios where kinetic decoupling PRECEEDES freeze-out

When can Kinetic Decoupling precede freeze-out?

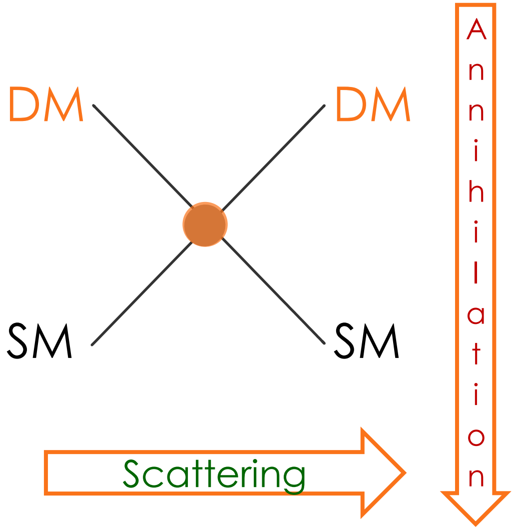
Bringmann, Hoffman 2006
Binder, Bringmann, Gustafsson, Hryczuk 2017
Ala-Mattinen, Kaunilainen 2019
Gondolo, Hisano, Kadota 2012
Abe 2004

When can Kinetic Decoupling precede Freeze-out?

Freeze-out (FO) occurs in Kinetic Equilibrium in typical WIMP models when:

$$n_{SM}^{eq}\langle\sigma v\rangle_s \gg n_{DM}^{eq}\langle\sigma v\rangle_a \simeq H \qquad \text{with} \qquad \langle\sigma v\rangle_a \simeq \langle\sigma v\rangle_s\,, n_{SM}^{eq} \gg n_{DM}^{eq}$$

1. Same coupling fully controls annihilation and elastic scattering
2. # scattering partners (n_{SM}) >> # annihilating partners (n_{DM}) at FO



When can Kinetic Decoupling precede Freeze-out?

(I) Resonant annihilation:

I. Resonant annihilation
II. Sommerfeld enhanced annihilation
III. Heavy scattering partner
IV. DM stabilized by Z_3
V. Multicomponent dark sector …

✗ Same coupling fully controls annihilation and elastic scattering

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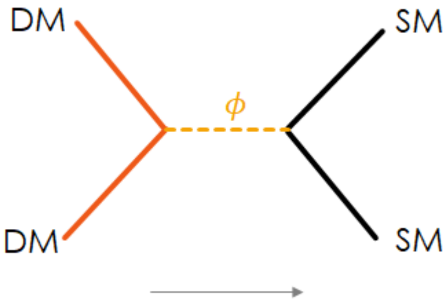
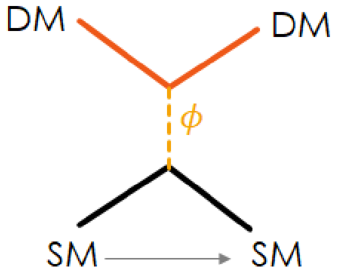
Annihilation



Resonance for $m_\phi \simeq 2m_{DM}$

>>

Scattering



no resonance for $m_\phi \simeq 2m_{DM}$

$$n_{SM}^{eq}\langle\sigma v\rangle_s \cancel{\gg} n_{DM}^{eq}\langle\sigma v\rangle_a \simeq H \qquad \text{with} \qquad \langle\sigma v\rangle_a \cancel{\simeq} \langle\sigma v\rangle_s \,, n_{SM}^{eq} \gg n_{DM}^{eq}$$

(Binder, Bringmann, Gustafsson, Hryczuk '17, '21)

When can Kinetic Decoupling precede Freeze-out?

(II) Sommerfeld enhanced annihilation:

✗ Same coupling fully controls annihilation and elastic scattering

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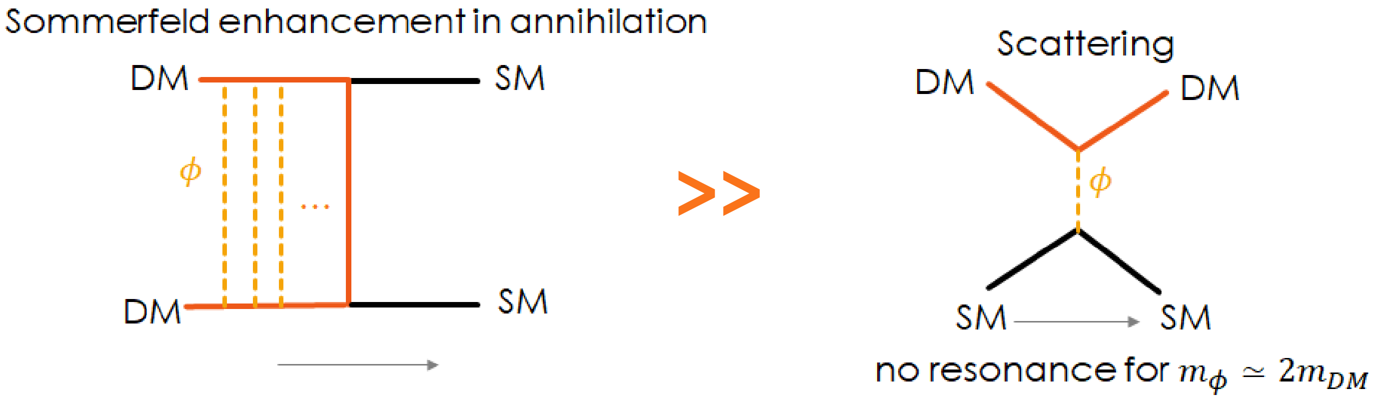
Sommerfeld enhancement in annihilation

Scattering

no resonance for $m_\phi \simeq 2m_{DM}$

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(Binder, Bringmann, Gustafsson, Hryczuk ‘17, ‘21)

When can Kinetic Decoupling precede Freeze-out?

(III) Scattering partner is heavy and also Boltzmann suppressed at FO

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- II. Sommerfeld enhanced annihilation
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✓ Same coupling fully controls annihilation and elastic scattering

✗ # scattering partners (n_{SM}) >> # annihilating partners (n_{DM}) at FO

$$m_{DM} \sim m_{SM} \Longrightarrow n_{DM}^{eq} \simeq n_{SM}^{eq}$$

$$n_{SM}^{eq} \langle \sigma v \rangle_s \cancel{\gg} n_{DM}^{eq} \langle \sigma v \rangle_a \simeq H \qquad \text{with} \qquad \langle \sigma v \rangle_a \simeq \langle \sigma v \rangle_s \, , n_{SM}^{eq} \cancel{\gg} n_{DM}^{eq}$$

(Binder, Bringmann, Gustafsson, Hryczuk ‘17, ‘21)
Gondolo, Hisano, Kadota 2012

When can Kinetic Decoupling precede Freeze-out?

(IV) Non-minimal dark sector – DM stabilised by Z_3 or larger group

I. Resonant annihilation
II. Sommerfeld enhanced annihilation
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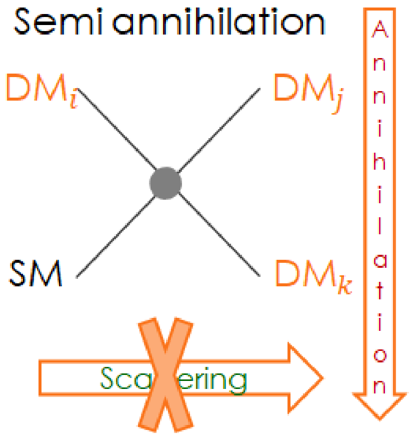

$$n_{SM}^{eq}\langle\sigma v\rangle_s \not\gg n_{DM}^{eq}\langle\sigma v\rangle_a \simeq H \qquad \text{with} \qquad \langle\sigma v\rangle_a \not\simeq \langle\sigma v\rangle_s \,, n_{SM}^{eq} \gg n_{DM}^{eq}$$

(Eramo, Thaler 2010; Cai, Spray 2016)

When can Kinetic Decoupling precede Freeze-out?

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(V) Non-minimal dark sector

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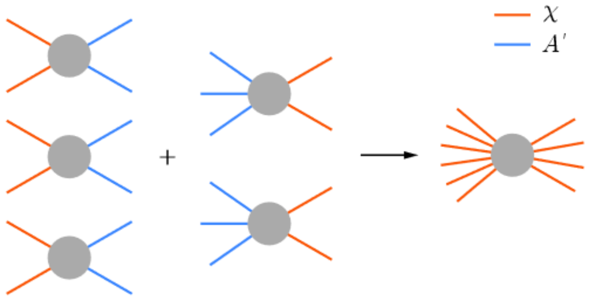

FIG. 1. Schematic illustration of the catalyzed annihilation of DM χ (red line) with a catalyst A' (blue line). Three $2\chi \to 2A'$ processes plus two $3A' \to 2\chi$ effectively deplete the number of DM particles by two.

fig. from Xing, Zhu '21; hep-ph: 2102.02447

$$n_{SM}^{eq}\langle\sigma v\rangle_s \cancel{\gg} n_{DM}^{eq}\langle\sigma v\rangle_a \simeq H \qquad \text{with} \qquad \langle\sigma v\rangle_a \cancel{\simeq} \langle\sigma v\rangle_s \,, n_{SM}^{eq} \gg n_{DM}^{eq}$$

Dark Matter Freeze-out production out of Kinetic Equilibrium

solve for *one* variable n → *two* variables n and T

Review of current literature in solving for abundance of DM out of Kinetic equilibrium:

1. Solve for DM temperature along with abundance (coupled BE)
   assume: DM distribution still has an equilibrium shape, only at a temperature $T_{DM} \neq T_{SM}$
   (Binder, Bringmann, Gustafsson, Hryczuk 2017, 2021; Hryczuk, Laletin 2021; Benincasa, Hryczuk, Kannike, Laletin 2023)

Dark Matter Freeze-out production out of Kinetic Equilibrium

solve for *one* variable n → *two* variables n and T→ *full phase space* with approximations (maintaining detailed balance)

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2. A generalized relaxation approximation agrees with fBE in specific cases; but difficult not justified In full generality
   assume: $f_{DM}(p,t) = g(t)f_{eq}(p,t) + \delta f(p,t)$ and that the integrated difference between the exact collision term and this momentum dependent approximation is small
   (Ala-Mattinen, Kainulainen 2019; Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen 2022)
3. Langevin simulations confirms the predictions from cBE in studied case
   Stochastic differential equation for studying the efficiency of kinetic equilibration in the non-relativistic regime (Kim, Laine 2023)

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4. Solving the DM distribution function at the full phase space level: numerically very challenging

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}]$$

(Du, Huang, Li, Li, Yu '21; Hryczuk, Laletin '22; Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen '22; Aboubrahim, Klasen, Wiggering '23; Brahma, Heeba, Schutz '23)

Boltzmann equation at the phase space level

Solving the DM distribution function at the full phase space level:

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}] \qquad \text{where, } f_{DM} \equiv f_{DM}(p,T).$$

CAN proceed fully numerically but it is time and CPU costly, due to the multidimensional integrations in the collision operators:

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM\rightarrow DM,SM}\,\big(\underbrace{f_{DM}(p_1)f_{eq}(p_3)}_{\text{easier}} - \underbrace{\boldsymbol{f_{DM}(p_2)}f_{eq}(p_4)}_{\text{harder}}\big)$$

$$C_{ann}[f_{DM}] = \int d\Pi\, |M|^2_{DM,DM\rightarrow SM,SM}\big(\underbrace{f_{DM}(p_1)\boldsymbol{f_{DM}(p_2)}}_{\text{harder}} - \underbrace{f_{eq}(p_3)f_{eq}(p_4)}_{\text{easier}}\big)$$

Typically the average momentum transferred during the scattering events is small

$$\delta^{(3)}(\vec{p_3}+\vec{p_4}-\vec{p_1}-\vec{p_2}) \approx \sum_n \left(\frac{1}{n!}\left(\vec{q}.\vec{\nabla}_{p_3}\right)^n \delta^{(3)}(\vec{p_3}-\vec{p_1})\right)$$

$$C_{el}[f_{DM}] = C_2 + C_4 + C_6 + C_8 + \cdots$$

$$C_{el}[f_{DM}] \simeq C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

no integration on f_{DM}

$$\frac{\Delta\vec{p}}{\vec{p}} < 1, \frac{p_1}{E_1} < 1$$

Fokker Planck approximation

----DRAKE: publicly available code for solving at this full phase space level

Bringmann, Hoffman '06
Gondolo, Hisano, Kadota '12
Binder, Bringmann, Gustafsson, Hryczuk '17, '21

$$d\Pi \equiv d\pi_{p_2} d\pi_{p_3} d\pi_{p_4}\, \delta^{(4)}(p_1 + p_3 - p_2 - p_4)$$

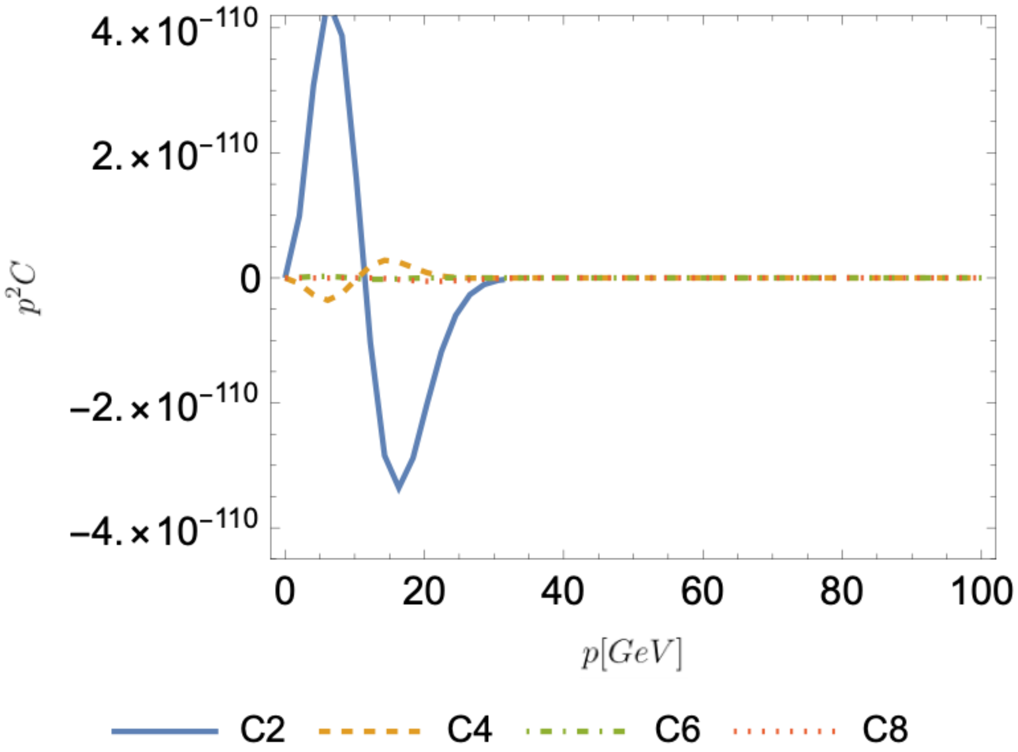
The Fokker Planck approximation

$$C_{el}[f_{DM}] = \underbrace{C_2 + C_4}_{C_{FP} = \frac{1}{2E_1}\gamma\left(f_{eq}\right)\,\widehat{FP}(p_1).\,\boldsymbol{f_{DM}(p_1)}} + C_6 + C_8 + \cdots$$

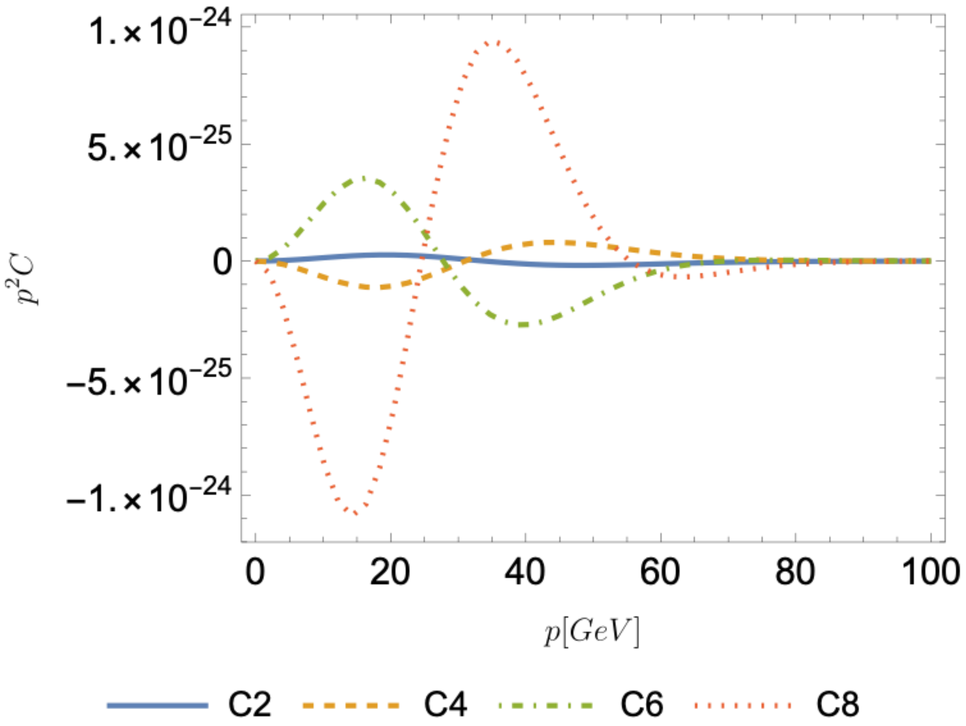
Has all the nice features:
- ✓ no integration on f_{DM}
- ✓ number conserving
- ✓ 0 on equilibrium distribution

$$x \equiv \frac{m_{DM}}{T} = \frac{m_\chi}{T}$$

$$\xcancel{\frac{\Delta\vec{p}}{p} < \,,\ \frac{p_1}{E_1} < 1}$$



$m_\chi = 100$ GeV, $m = 1$ GeV, $x = 250$



$m_\chi = 100$ GeV, $m = 100$ GeV, $x = 25$

When does the Fokker Planck approx. work?

- Arrived at by *dropping higher order terms* in $\Delta\vec{p}/\vec{p}$ and p_1/E_1.
- Very good "approximation" (O(1%)) while the conditions of the expansion hold true.

Q: How to know when the FP approximation works?

$$|M|^2 \longrightarrow \underbrace{t^{n_1}}_{\propto \text{ transfer momentum}}\underbrace{(s-(m_{DM}+m_{SM})^2)^{n_2}}_{\propto \text{ relative velocity}}\underbrace{(u-(m_{DM}-m_{SM})^2)^{n_3}}_{\propto \text{ velocities}}$$

With an efficiently implemented fully numerical[1] solver for the Boltzmann equation into DRAKE, we find that The Fokker Planck approximation works well for:

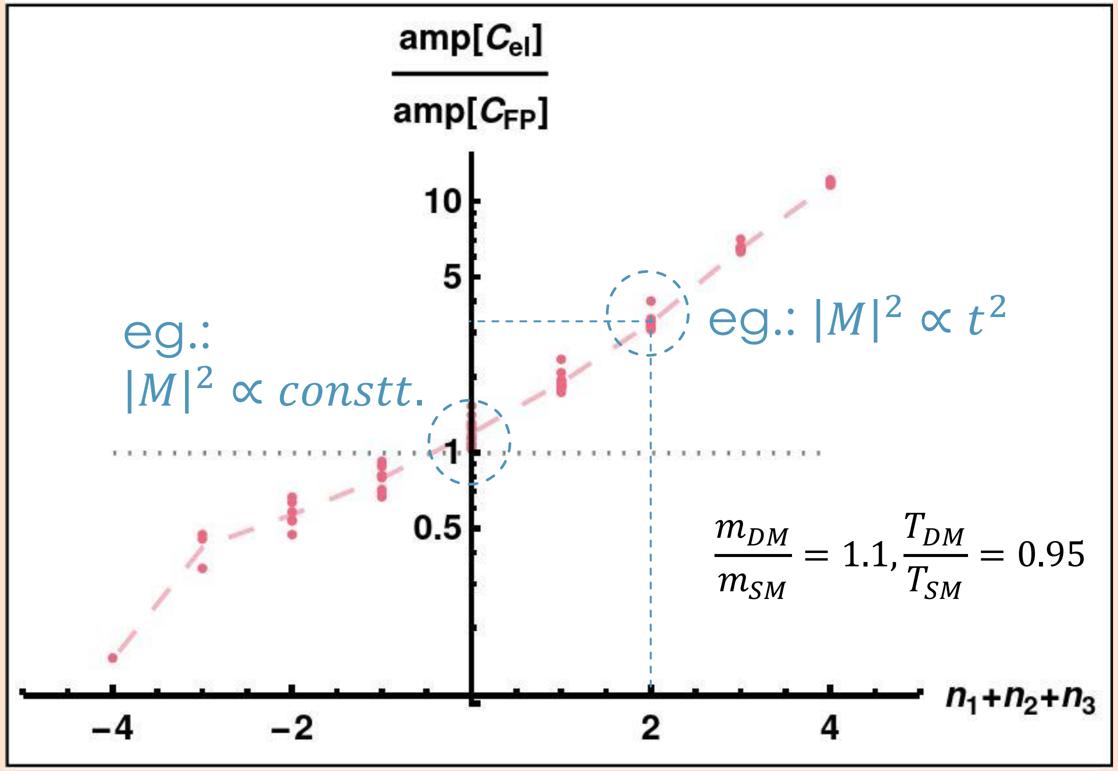
1. Scattering particle with masses significantly smaller than DM mass (small reduced mass ⇒ small momentum transfer

   &

2. DM temperatures close to the SM temperature (eg.: near kinetic decoupling)

   &

3. Scattering amplitudes that aren't strongly dependent on momentum transfer (the dropped higher order terms are more relevant for an amplitude sensitive to said dropped quantity)



eg.: $|M|^2 \propto constt.$

eg.: $|M|^2 \propto t^2$

$\frac{m_{DM}}{m_{SM}} = 1.1, \frac{T_{DM}}{T_{SM}} = 0.95$

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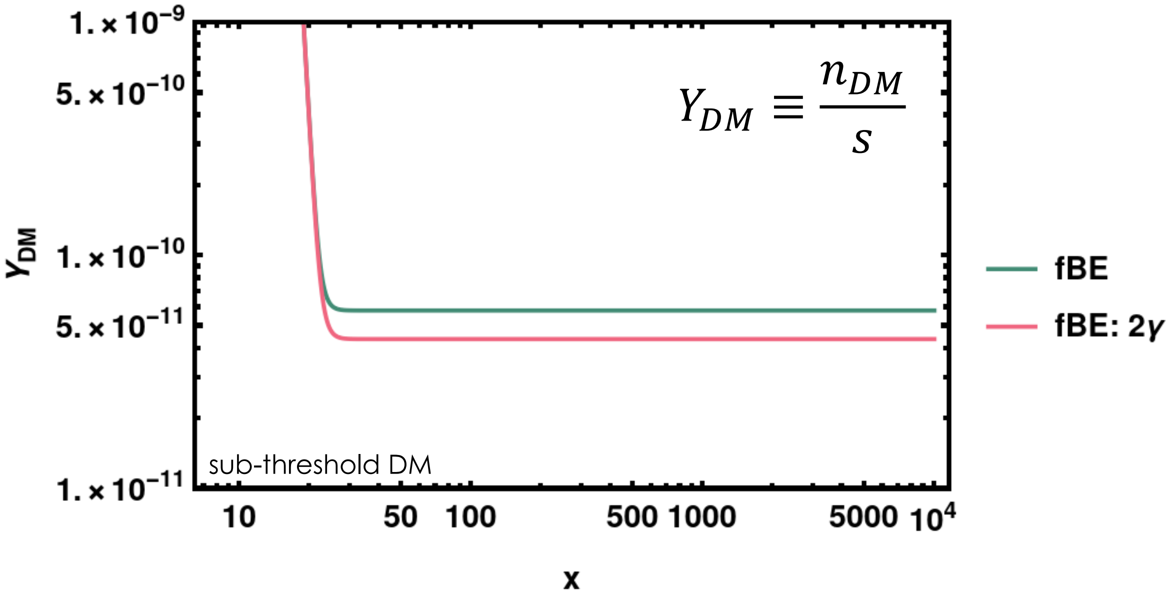
[1] Ala-Mattinen, Kainulainen '19
Hryczuk, Laletin '20
Aboubrahim, Klasen, Wiggering '23
Beauchesne, Chiang '24;

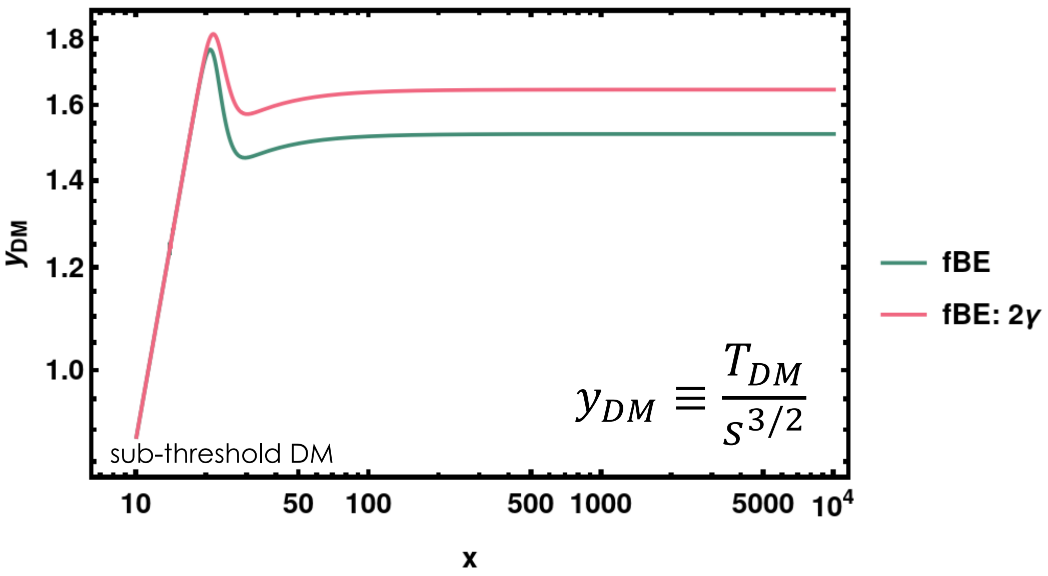
Improvement on Fokker Planck: Relic density

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}]$$

$$C_{el}[f_{DM}] \simeq C_{FP} = \frac{1}{2E_1}\gamma\left(f_{eq}\right)\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

An overall factor 2 at the level of collision operator ⟹ 25% change in DM relic density

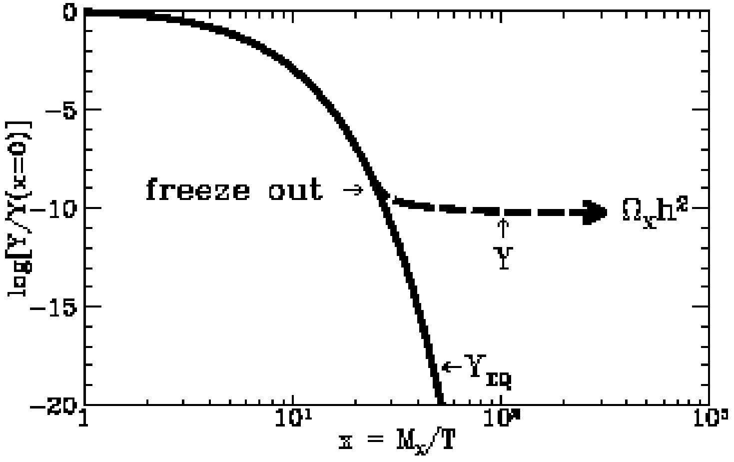


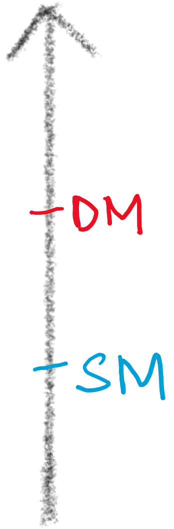


Non-minimal Dark Sector

Dark Matter production:

- In the simplest freeze-out production of WIMP (weakly interacting massive particle) DM, there is one DM particle, initially in kinetic and chemical equilibrium with the SM plasma.





Dark Matter production: why multiparticle?

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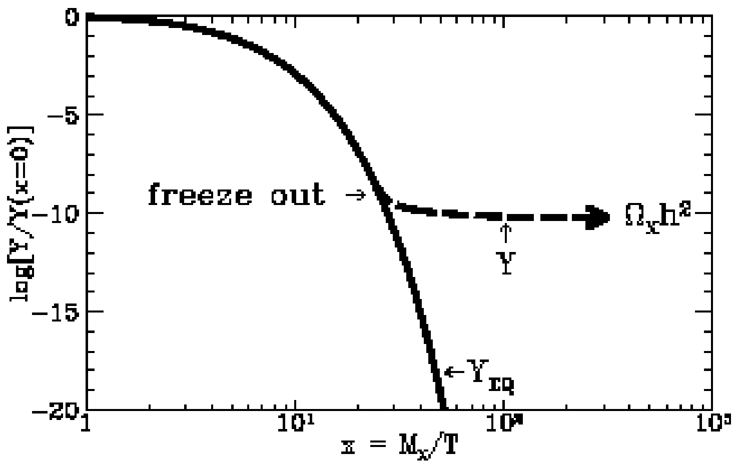

- DM could be part of a sector of particles charged under the parity that stabilizes the DM particles.

- If the DM is well separated from the rest --- the one-particle freeze-out picture holds

Dark Matter production: why multiparticle?

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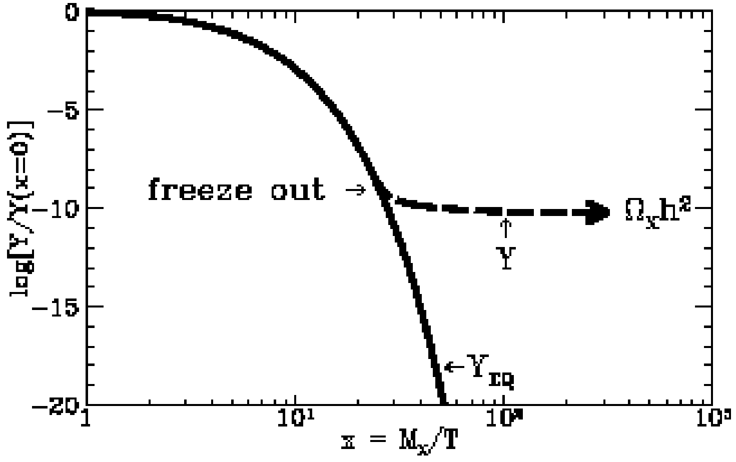


extended “dark” sector

DM

SM

- DM could be part of a sector of particles charged under the parity that stabilizes the DM particles.
- If the DM is well separated from the rest --- the one-particle freeze-out picture holds
- $m_{NLSP} \simeq m_{DM}$ $\Longrightarrow$ “multiparticle” freeze-out

What if dark sector had more than one particle

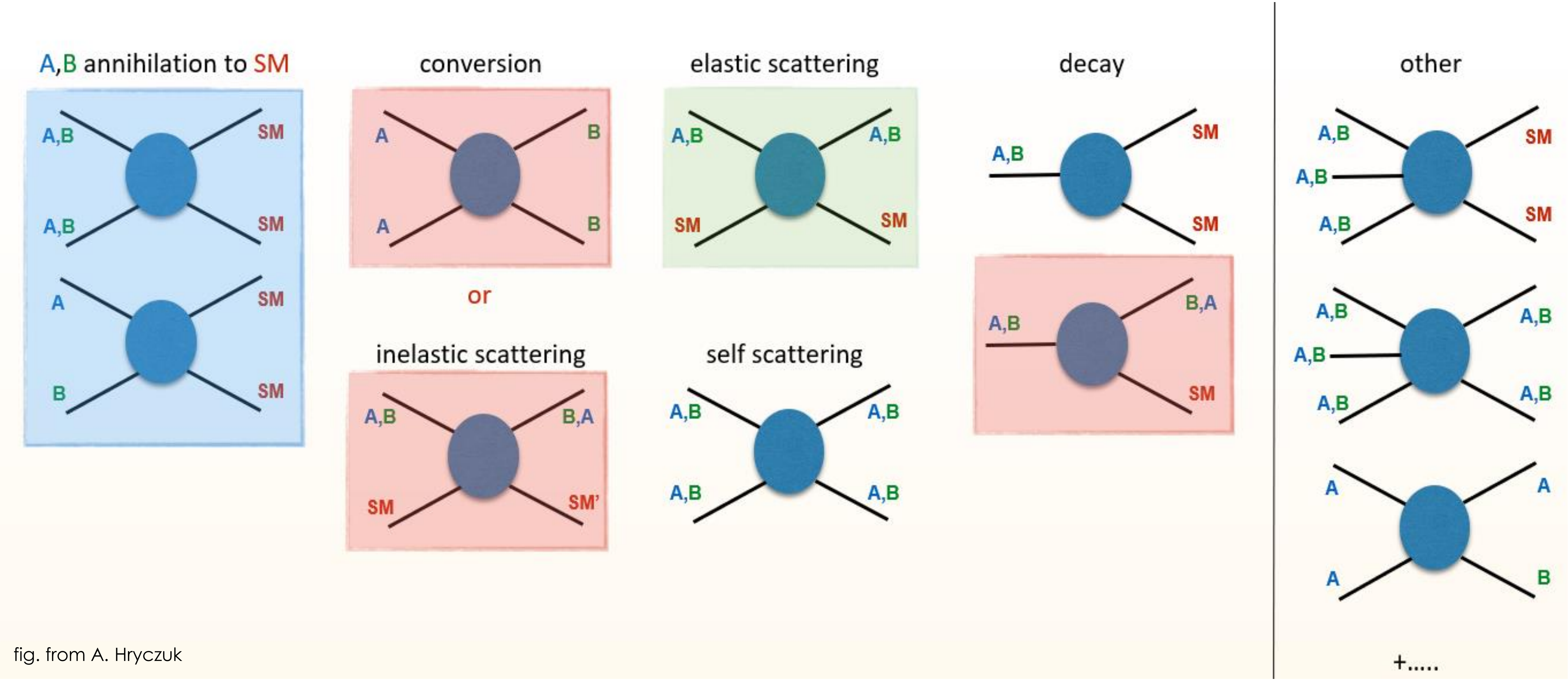

fig. from A. Hryczuk

computationally more challenging…

2-particle freeze-out:

$\mathrm{A} = \chi_1;\ \mathrm{B} = \chi_2;\ m_{\chi_2} > m_{\chi_1}$

Coupled Boltzmann equation:

$$\frac{dY_1}{dT} = \frac{1}{3H}\frac{ds}{dx}\left[\langle\sigma_{11}v\rangle\left(Y_1^2 - Y_{1,eq}^2\right) + \langle\sigma_{12}v\rangle\left(Y_1 Y_2 - Y_{1,eq}Y_{2,eq}\right) + \frac{\Gamma_{1\to 2}}{s}\left(Y_1 - Y_2\frac{Y_{1,eq}}{Y_{2,eq}}\right) - \frac{\Gamma_2}{s}\left(Y_2 - Y_1\frac{Y_{2,eq}}{Y_{1,eq}}\right) + \langle\sigma_{11\to 22}v\rangle\left(Y_1^2 - Y_2^2\frac{Y_{1,eq}^2}{Y_{2,eq}^2}\right)\right]$$

$$\frac{dY_2}{dT} = \frac{1}{3H}\frac{ds}{dx}\left[\langle\sigma_{22}v\rangle\left(Y_2^2 - Y_{2,eq}^2\right) + \langle\sigma_{12}v\rangle\left(Y_1 Y_2 - Y_{1,eq}Y_{2,eq}\right) - \frac{\Gamma_{1\to 2}}{s}\left(Y_1 - Y_2\frac{Y_{1,eq}}{Y_{2,eq}}\right) + \frac{\Gamma_2}{s}\left(Y_2 - Y_1\frac{Y_{2,eq}}{Y_{1,eq}}\right) - \langle\sigma_{11\to 22}v\rangle\left(Y_1^2 - Y_2^2\frac{Y_{1,eq}^2}{Y_{2,eq}^2}\right)\right]$$

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Coupled Boltzmann equation:

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$$\frac{dY_2}{dT} = \frac{1}{3H}\frac{ds}{dx}\left[\langle\sigma_{22}v\rangle\left(Y_2^2 - Y_{2,eq}^2\right) + \langle\sigma_{12}v\rangle\left(Y_1Y_2 - Y_{1,eq}Y_{2,eq}\right) - \frac{\Gamma_{1\to2}}{s}\left(Y_1 - Y_2\frac{Y_{1,eq}}{Y_{2,eq}}\right) + \frac{\Gamma_2}{s}\left(Y_2 - Y_1\frac{Y_{2,eq}}{Y_{1,eq}}\right) - \langle\sigma_{11\to22}v\rangle\left(Y_1^2 - Y_2^2\frac{Y_{1,eq}^2}{Y_{2,eq}^2}\right)\right]$$

$$\frac{n_i}{n} \simeq \frac{n_{i,eq}}{n_{eq}}$$

$$\frac{dY}{dx} \propto \langle\sigma_{eff}v\rangle\left(Y^2 - Y_{eq}^2\right)$$

$\Gamma_{\chi_{1,SM}\leftrightarrow\chi_{2,SM}} \gg H$ Coannihilation

Edsjo, Gondolo '97

2-particle freeze-out:

$\mathrm{A} = \chi_1; \;\; \mathrm{B} = \chi_2; \;\; m_{\chi_2} > m_{\chi_1}$

Coupled Boltzmann equation:

$$\frac{dY_1}{dT} = \frac{1}{3H}\frac{ds}{dx}\left[\langle\sigma_{11}v\rangle\left(Y_1^2 - Y_{1,eq}^2\right) + \langle\sigma_{12}v\rangle\left(Y_1 Y_2 - Y_{1,eq}Y_{2,eq}\right) + \frac{\Gamma_{1\to2}}{s}\left(Y_1 - Y_2\frac{Y_{1,eq}}{Y_{2,eq}}\right) - \frac{\Gamma_2}{s}\left(Y_2 - Y_1\frac{Y_{2,eq}}{Y_{1,eq}}\right) + \langle\sigma_{11\to22}v\rangle\left(Y_1^2 - Y_2^2\frac{Y_{1,eq}^2}{Y_{2,eq}^2}\right)\right] \dots$$

$$\frac{dY_2}{dT} = \frac{1}{3H}\frac{ds}{dx}\left[\langle\sigma_{22}v\rangle\left(Y_2^2 - Y_{2,eq}^2\right) + \langle\sigma_{12}v\rangle\left(Y_1 Y_2 - Y_{1,eq}Y_{2,eq}\right) - \frac{\Gamma_{1\to2}}{s}\left(Y_1 - Y_2\frac{Y_{1,eq}}{Y_{2,eq}}\right) + \frac{\Gamma_2}{s}\left(Y_2 - Y_1\frac{Y_{2,eq}}{Y_{1,eq}}\right) - \langle\sigma_{11\to22}v\rangle\left(Y_1^2 - Y_2^2\frac{Y_{1,eq}^2}{Y_{2,eq}^2}\right)\right] \dots$$

$$\dots$$

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$\Gamma_{\chi_{1,SM}\leftrightarrow\chi_{2,SM}} \simeq H$: Conversion-driven

Garny, Heisig, Lulf, Vogl '17;
D'Agnnolo et al '17;
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…

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$$\cancel{\frac{dY}{dx} \propto \langle\sigma_{eff}v\rangle\left(Y^2 - Y_{eq}^2\right)}$$

$\frac{\Gamma_{\chi_1,SM\to\chi_1,SM}}{H} \gg 1$ & $\Gamma_{\chi_1,SM\leftrightarrow\chi_2,SM} \simeq H$: Conversion-driven

Assumes efficient processes to restore equilibrium distribution

Garny, Heisig, Lulf, Vogl '17;
D'Agnnolo et al '17;
Garny, Heisig, Hufnagel, Lulf, Vogl '19;

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$$\dots$$

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$$\frac{\Gamma_{\chi_1,SM\to\chi_1,SM}}{H} \cancel{\gg} 1$$

- Process to restore equilibrium distribution is inefficient

Garny, Heisig, Lulf, Vogl '17;
D'Agnnolo et al '17;
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full Boltzmann equation (fBE) must be solved when:

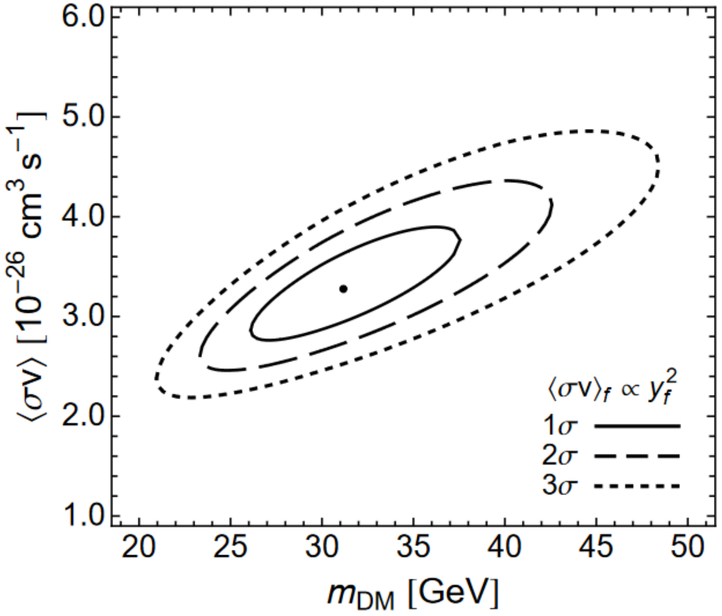
- Process to restore equilibrium distribution is inefficient
- Strongly momentum dependent/ selective processes

Brummer '19;
Garny, Heisig, Lulf, Vogl '17;
D'Agnnolo et al '17;
Garny, Heisig, Hufnagel, Lulf, Vogl '19;

Coy Dark Matter:

1. A DM interpretation of the extended Galactic gamma-ray excess from Fermi-LAT
2. Dirac DM (χ) with interaction mediated by a light pseudoscalar, with couplings to SM particles proportional to Yukawa couplings per Minimal Flavour Violation (MFV)
3. Direct detection rates suppressed by square of the nuclear recoil energy

$$\mathcal{L} \supset -i\frac{g_{DM}}{\sqrt{2}}\, a\,\bar{\chi}\gamma^5\chi - i\sum_{f\in SM}\frac{g_f}{\sqrt{2}} a\,\bar{f}\gamma^5 f$$



Boehm et al 2014

hep-ph: 1612.05687

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- “crossing symmetry” between annihilation and scattering is broken ⟹ DM distribution can veer away from equilibrium shape

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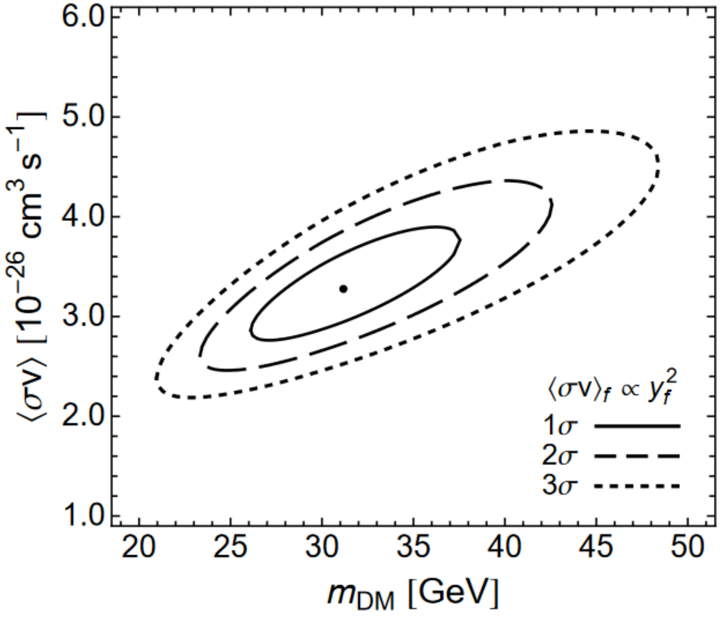


Boehm et al 2014

hep-ph: 1612.05687

Coy Dark Matter: 2-component

1. A DM interpretation of the extended Galactic gamma-ray excess from Fermi-LAT
2. Dirac fermions (χ_1, χ_2) with interaction mediated by a light pseudoscalar (a), with couplings to SM particles proportional to Yukawa couplings per Minimal Flavour Violation (MFV)
3. Direct detection rates suppressed by square of the nuclear recoil energy

- momentum-dependent scattering rates
- "crossing symmetry" between annihilation and scattering is broken ⟹ DM distribution can veer away from equilibrium shape

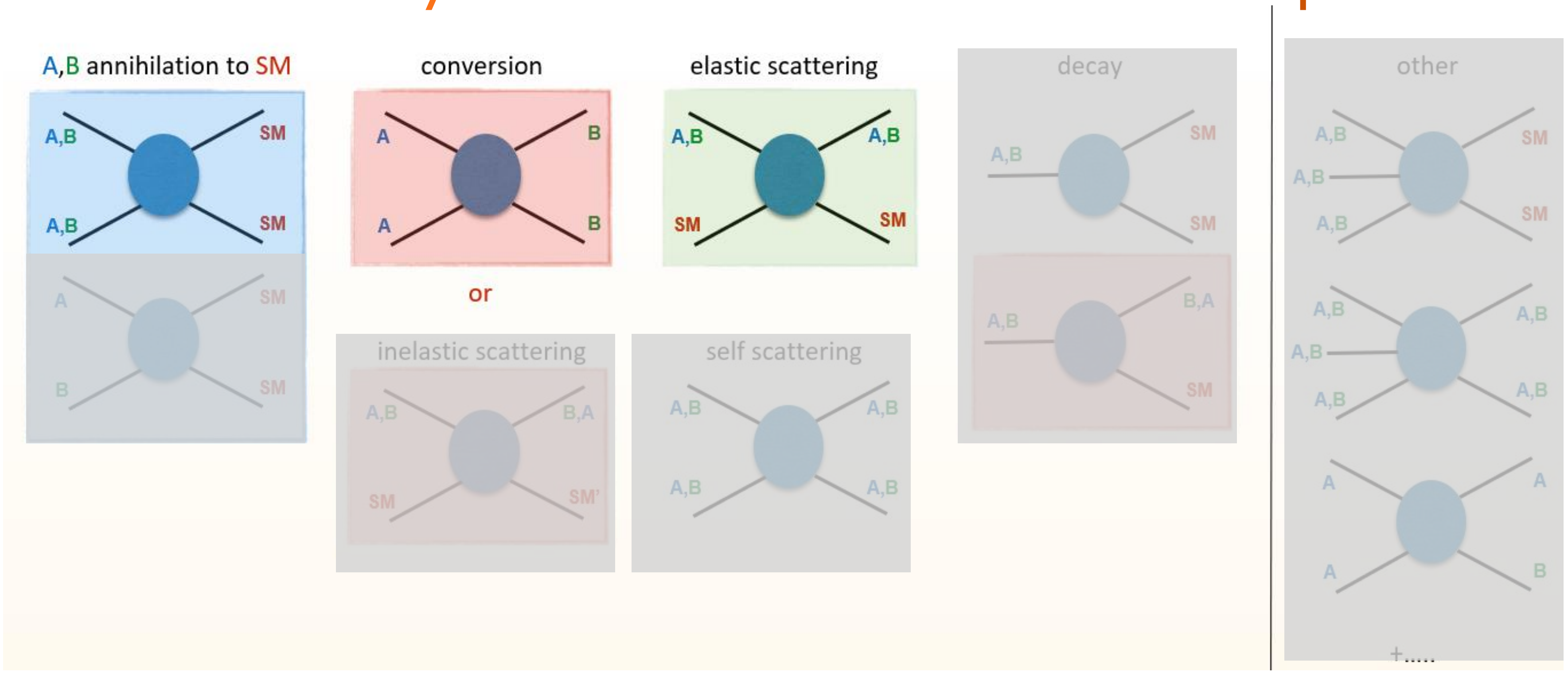
Can potentially depend on particle momentum distributions

⇓

Solve full coupled Boltzmann equation to investigate all the effects of conversions, annihilations and scatterings.

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1\gamma^5\chi_1 - i\lambda_2 a\, \bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f\, a\, \bar{f}\gamma^5 f$$

Coy Dark Matter: 2-component

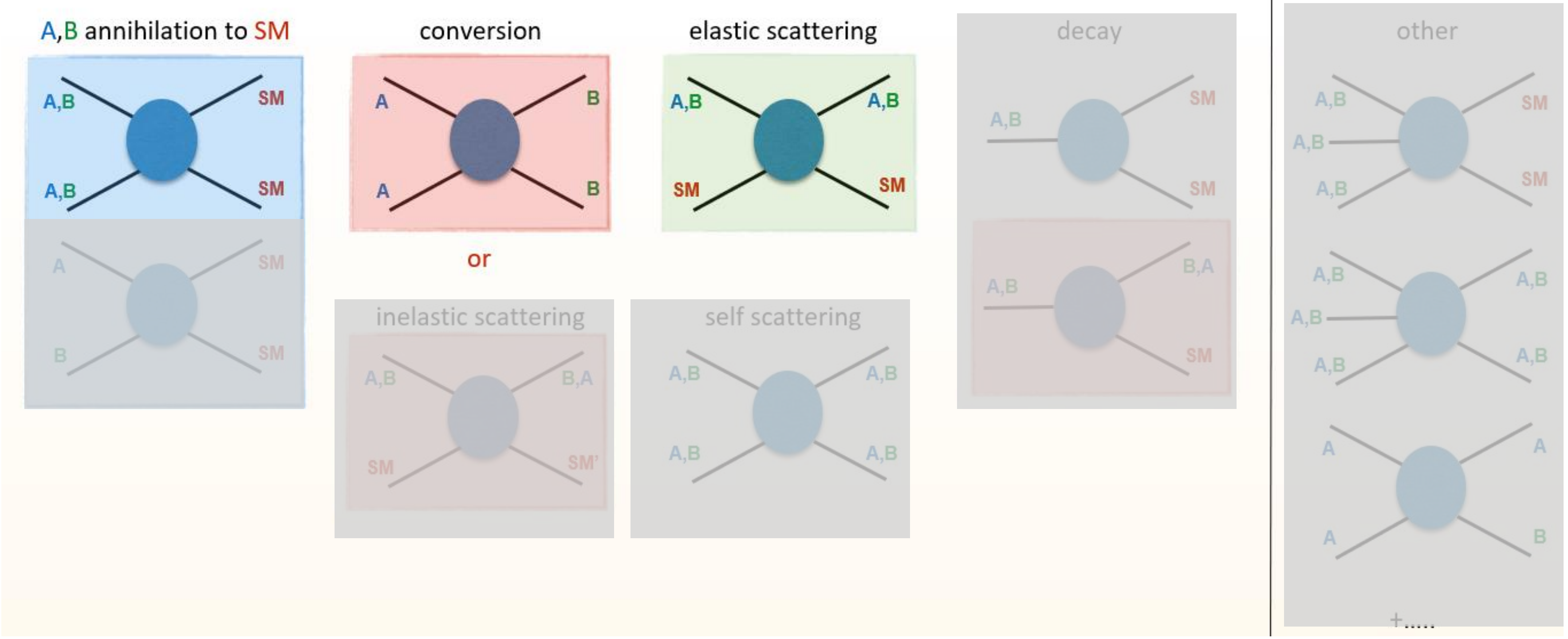


- Code to solve at Yield level: micrOMEGAs 6.0: N-component DM
- We develop a code to solve for this multicomponent DM at phase space level: extending the publicly available code DRAKE

$$(\partial_t - p_i H \partial_{p_i}) f_i(p_i,t) = \underbrace{\hat{C}_{\chi_i,SM\to\chi_i,SM}(p_i,t)}_{Elastic\ scattering} + \underbrace{\hat{C}_{\chi_i,\chi_i\to SM,SM}(p_i,t)}_{Annihilations} + \underbrace{\sum_{i\neq j}\hat{C}_{\chi_i,\chi_i\to\chi_j,\chi_j}(p_i,t)}_{Conversions}$$

$$\boxed{\begin{gathered}\mathcal{L} \supset -i\lambda_1 a\,\bar{\chi}_1\gamma^5\chi_1 - i\lambda_2 a\,\bar{\chi}_2\gamma^5\chi_2 \\ -i\lambda_y \sum_{f\in SM} y_f\, a\,\bar{f}\gamma^5 f\end{gathered}}$$

Coy Dark Matter: 2-component



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Collision operators:

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM\to DM,SM}\left(f_{DM;A,B}(p_1) f_{eq}(p_3) - \boldsymbol{f_{DM;A,B}(p_2)} f_{eq}(p_4)\right)$$

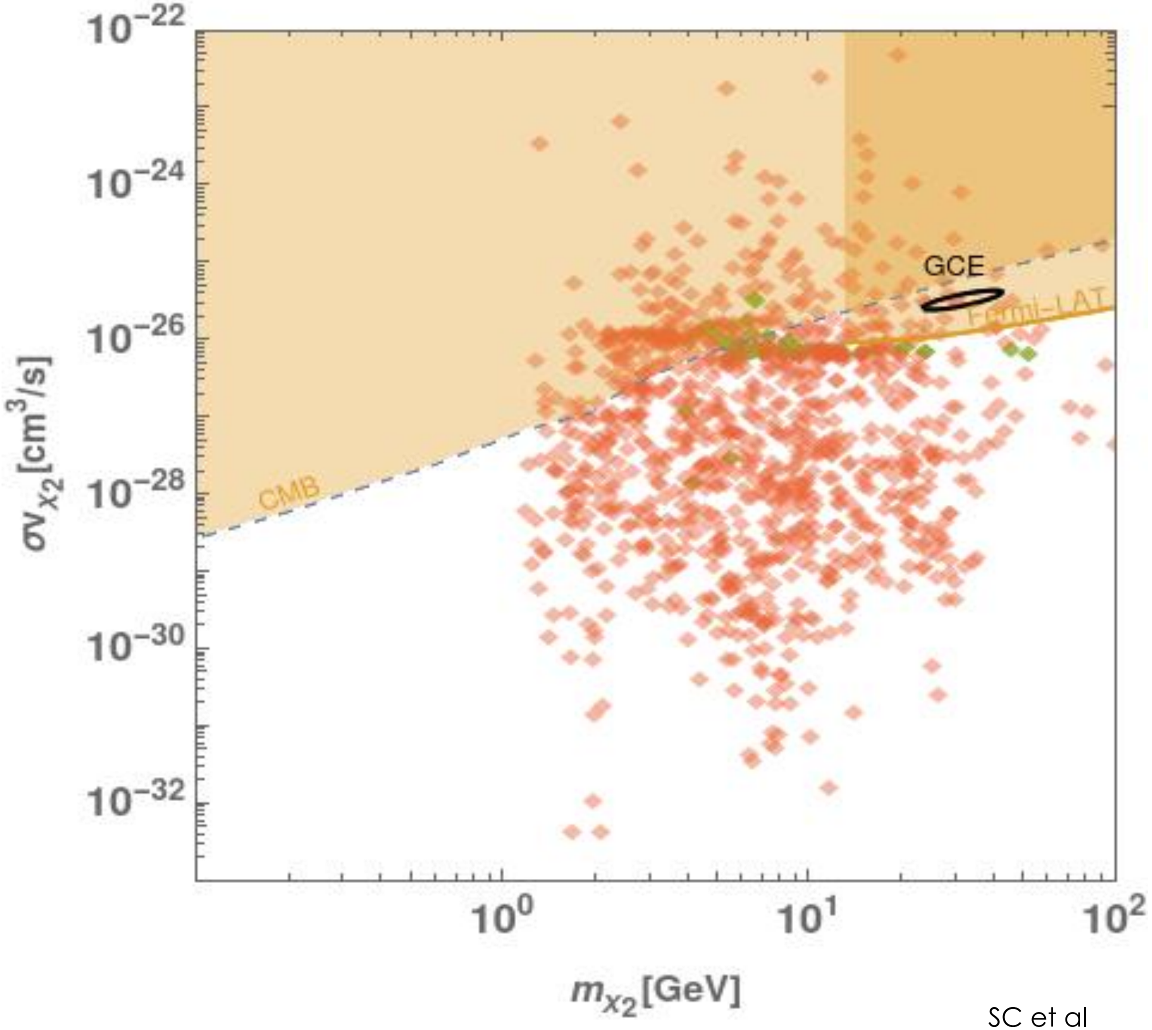
$$C_{ann}[f_{DM}] = \int d\Pi\, |M|^2_{DM,DM\to SM,SM}\left(f_{DM;A,B}(p_1) \boldsymbol{f_{DM;A,B}(p_2)} - f_{eq}(p_3) f_{eq}(p_4)\right)$$

$$C_{conv}[f_{DM}] = \int d\Pi\, |M|^2_{A,A\to B,B}\left(f_{DM,A}(p_1) \boldsymbol{f_{DM,A}(p_2)} - \boldsymbol{f_{DM,B}(p_3) f_{DM,B}(p_4)}\right)$$

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1 \gamma^5 \chi_1 - i\lambda_2 a\, \bar{\chi}_2 \gamma^5 \chi_2 - i\lambda_y \sum_{f\in SM} y_f\, a\, \bar{f}\gamma^5 f$$

Coy Dark Matter: 2-component

Indirect Detection:



SC et al

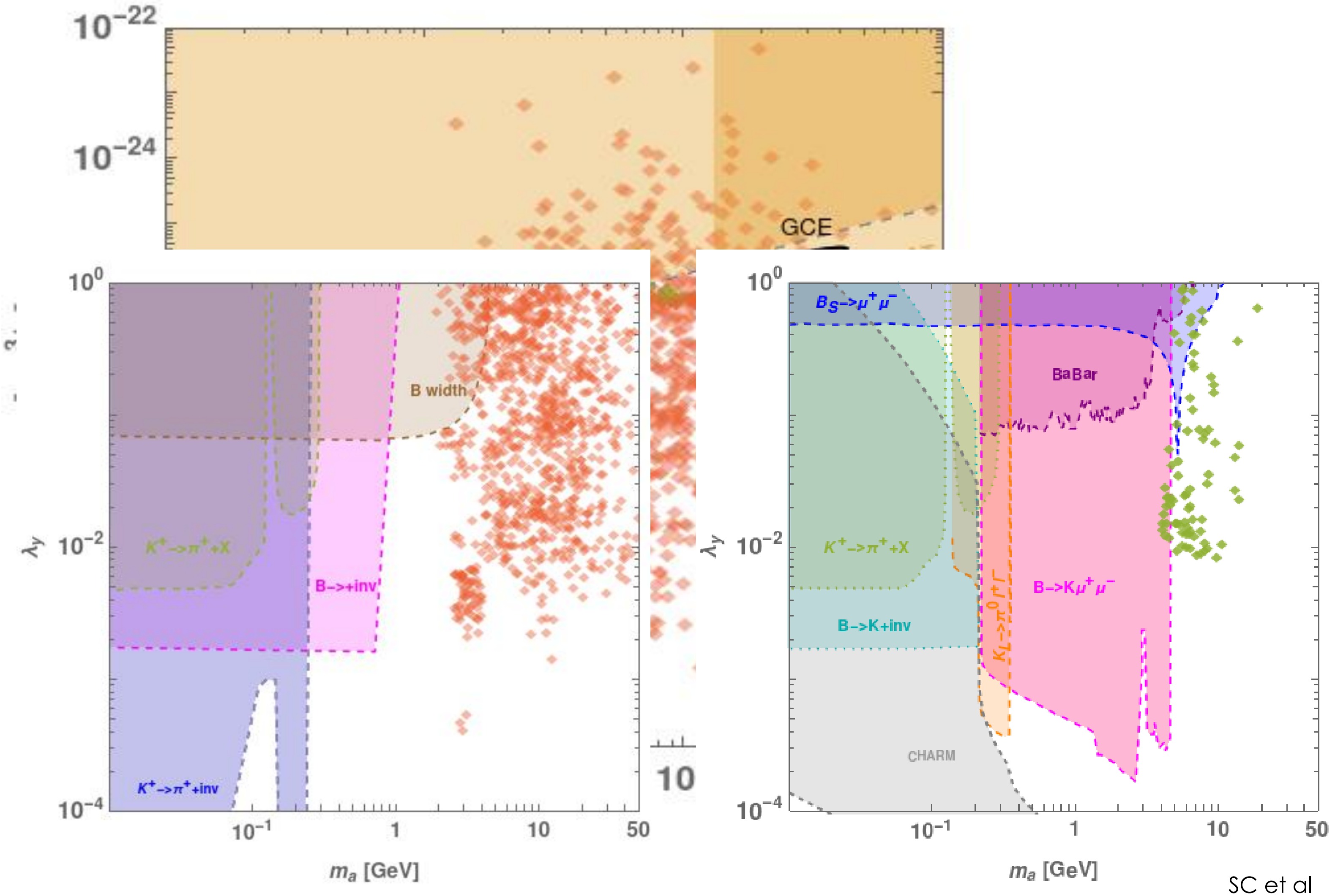
Scan results: $m_{\chi_2} \leq m_{\chi_1}, m_a \geq 1GeV$

- Sum of χ_1, χ_2 relic densities reproduces observed $\Omega h^2 = 0.12 \pm 0.012$
- Indirect detection constraint on χ_2 which is the dominant relic
- Red-- $m_{\chi_2} < \frac{m_a}{2}$ a decays dominantly to SM
- Green-- $m_{\chi_2} > \frac{m_a}{2}$ a decays dominantly to DM
- Shown is the 2σ preferred region to explain the Galactic Centre excess (Boehm et al 2014)

MFV pseudoscalar cnstraints from Dolan et al 1412.5174

Coy Dark Matter: 2-component

Indirect Detection:



SC et al

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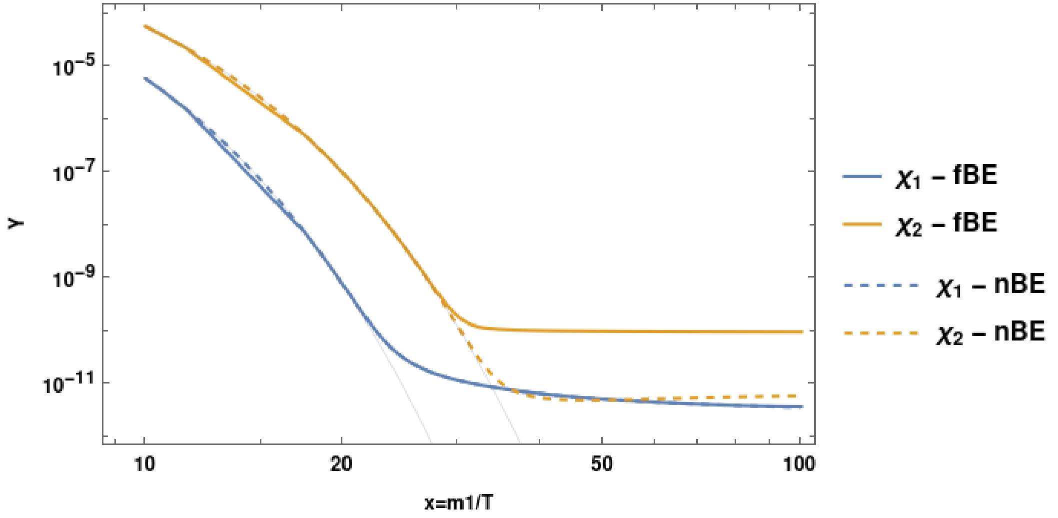
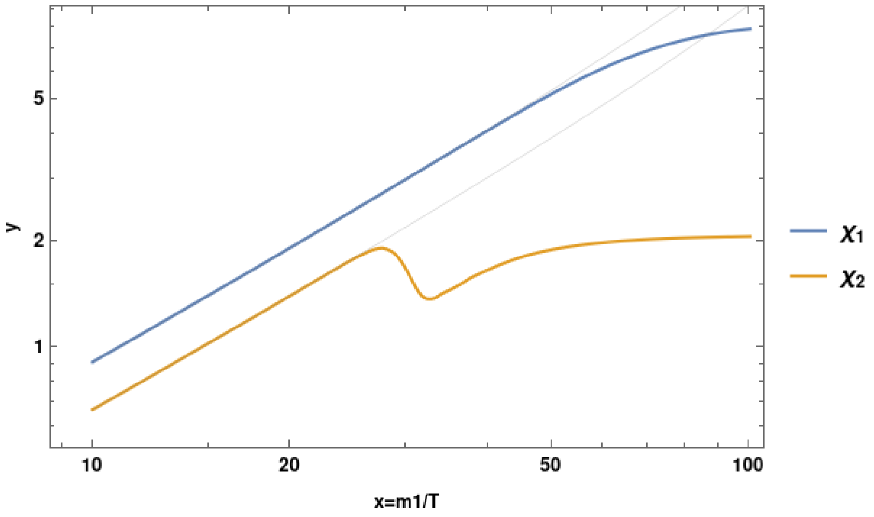
- Shown is the 2σ preferred region to explain the Galactic Centre excess (Boehm et al 2014)

- Bounds on pseudoscalar a from flavor factories and fixed-target experiments (MFV interaction with SM) (Dolan et al 1412.5174)

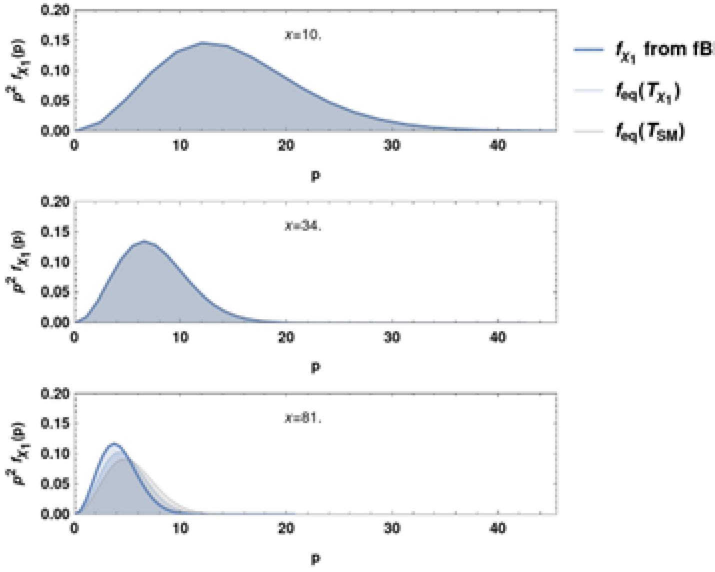
2-component Coy Dark Matter: Resonant case

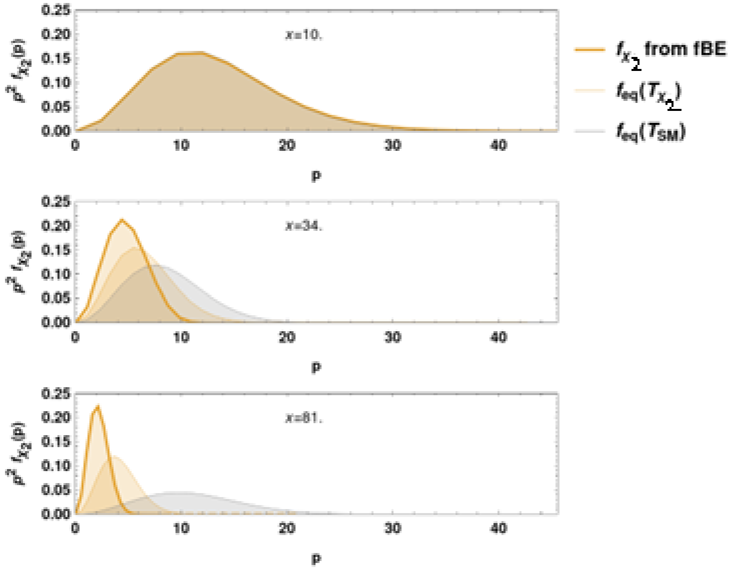




$$Y_i \equiv \frac{n_i}{s}, y_i \equiv \frac{m_1 T_i}{s^{2/3}}$$

$m_{\chi_1} = 26.6GeV$
$m_{\chi_2} = 19.54GeV$
$m_a = 43.34GeV$
$\lambda_1 = 0.4, \lambda_2 = 0.28, \lambda_y = 0.16$





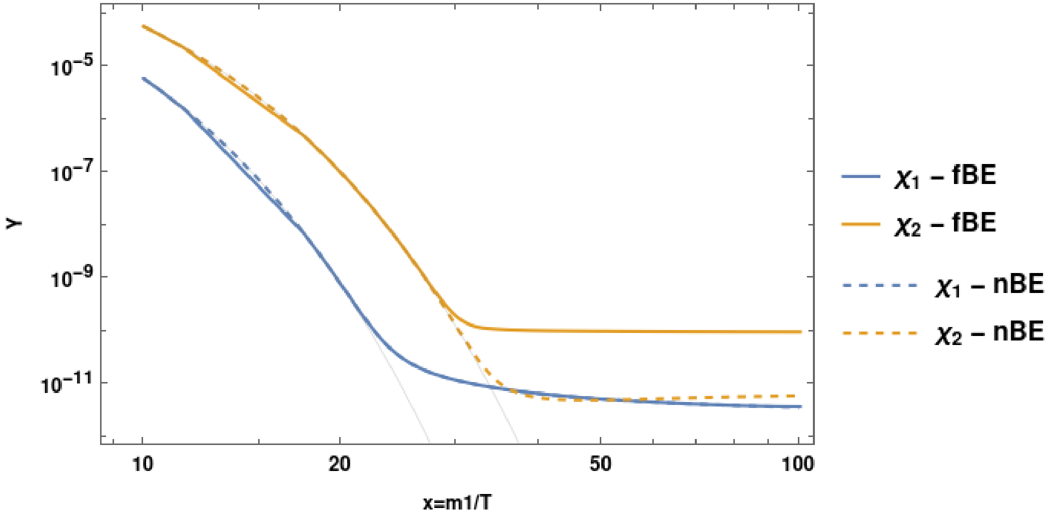
Resonant annihilation of χ_2

$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 0.975, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.058$$

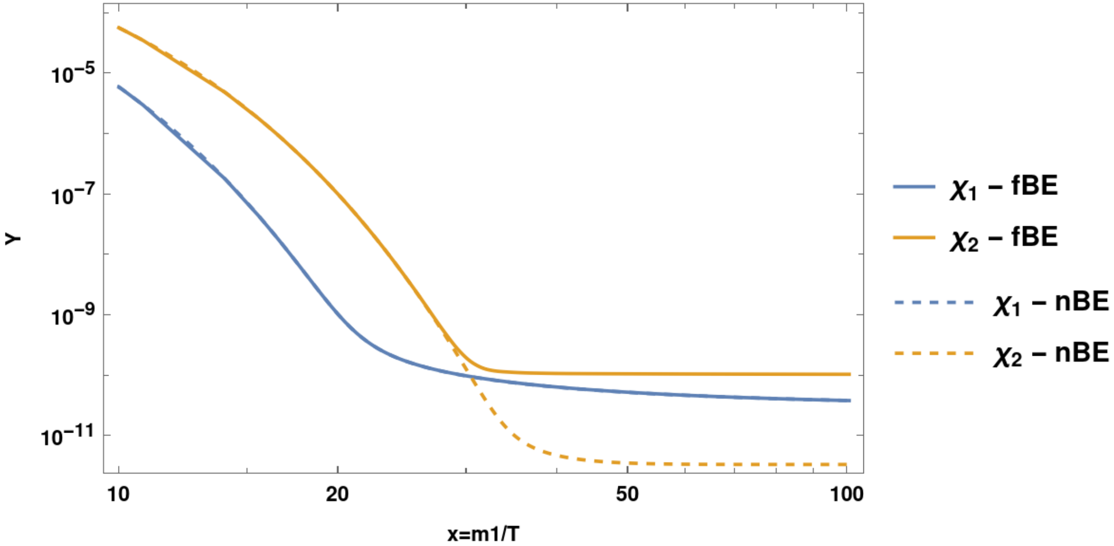
nBE: $(\Omega h^2)_1 = 0.05, (\Omega h^2)_2 = 0.06$

2-component Coy Dark Matter: Resonant case

With conversions:



Without conversions:



$$Y_i \equiv \frac{n_i}{s},$$
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Resonant annihilation of χ_2

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nBE: $(\Omega h^2)_1 = 0.054, (\Omega h^2)_2 = 0.067$

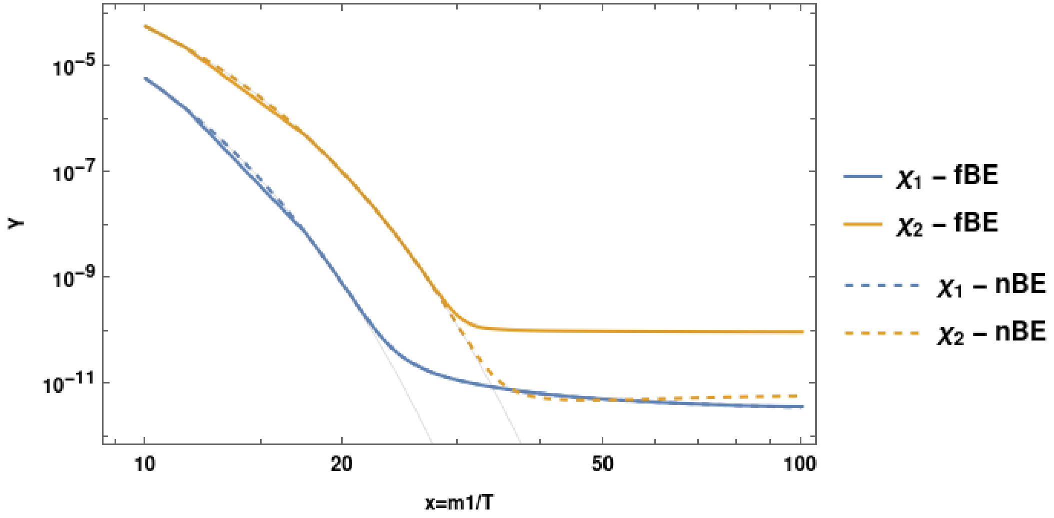
$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 1.00, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.03$$

nBE: $(\Omega h^2)_1 = 0.57, (\Omega h^2)_2 = 0.036$

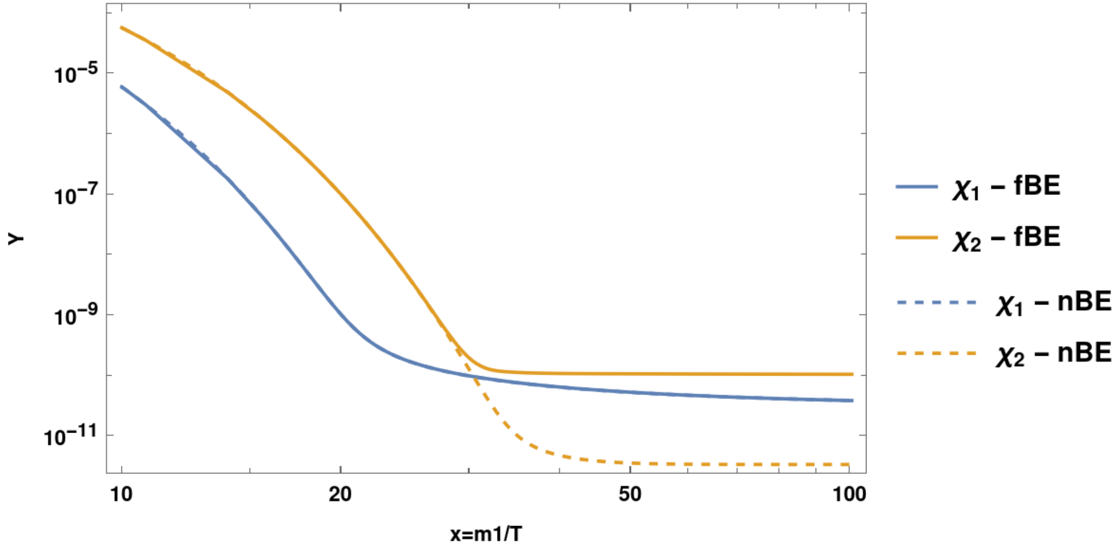
2-component Coy Dark Matter: Resonant case

$$Y_i \equiv \frac{n_i}{s},\quad y_i \equiv \frac{m_1 T_i}{s^{2/3}}$$

With conversions:



Without conversions:



$m_{\chi_1} = 26.6 GeV$

$m_{\chi_2} = 19.54 GeV$

$m_a = 43.34 GeV$

$\lambda_1 = 0.4, \lambda_2 = 0.28, \lambda_y = 0.16$

Resonant annihilation of χ_2

$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 0.975, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.058$$

Conversions +
Resonant annihilation

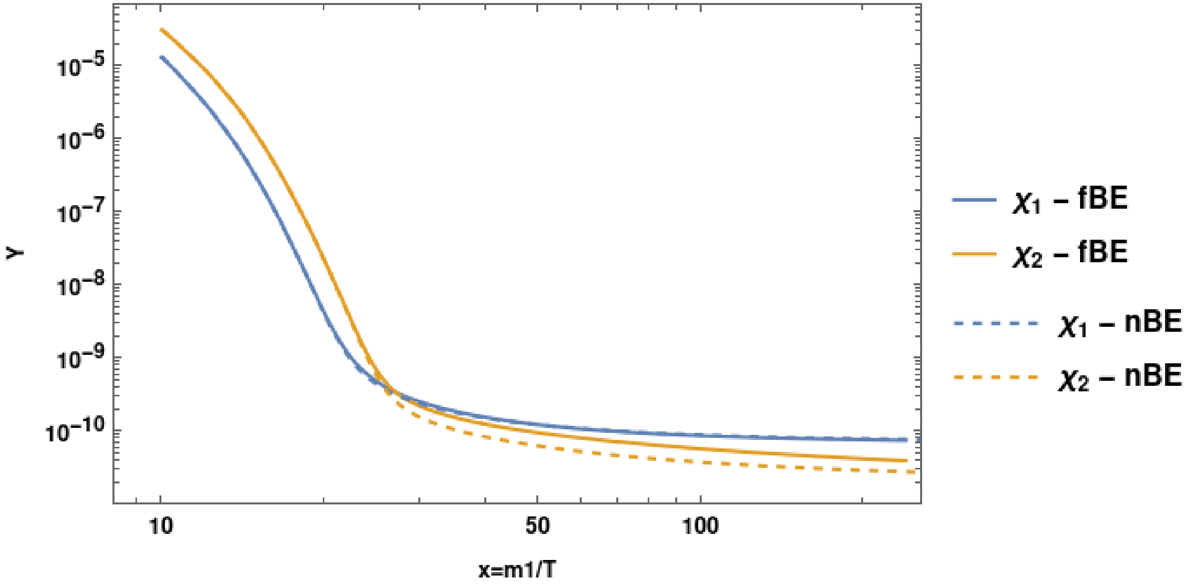
nBE: $(\Omega h^2)_1 = 0.05, (\Omega h^2)_2 = 0.06$

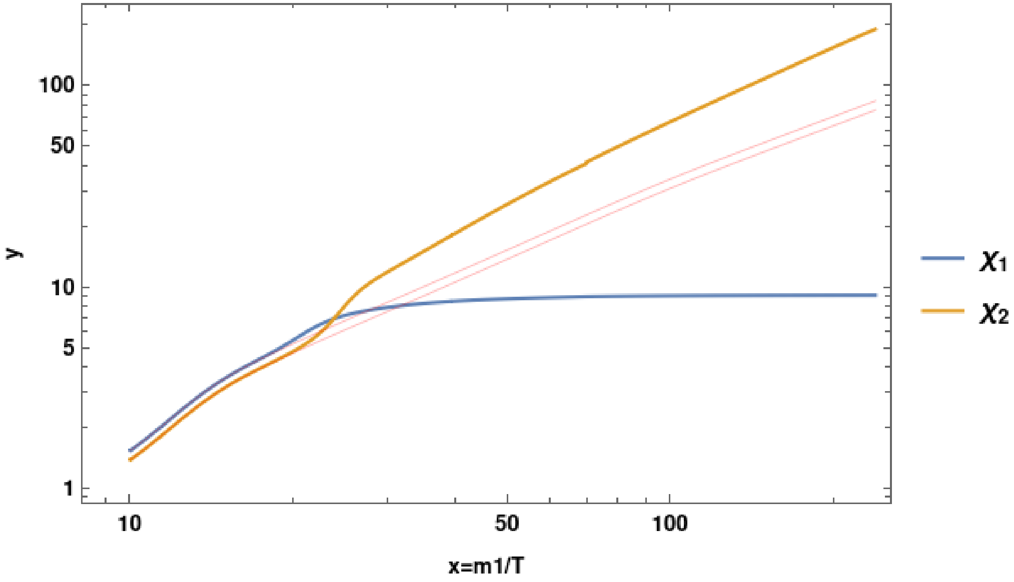
$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 1.00, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.03$$

nBE: $(\Omega h^2)_1 = 0.57, (\Omega h^2)_2 = 0.036$

2-component Coy Dark Matter: Near-resonant

$$Y_i \equiv \frac{n_i}{s},$$
$$y_i \equiv \frac{m_1 T_i}{s^{2/3}}$$



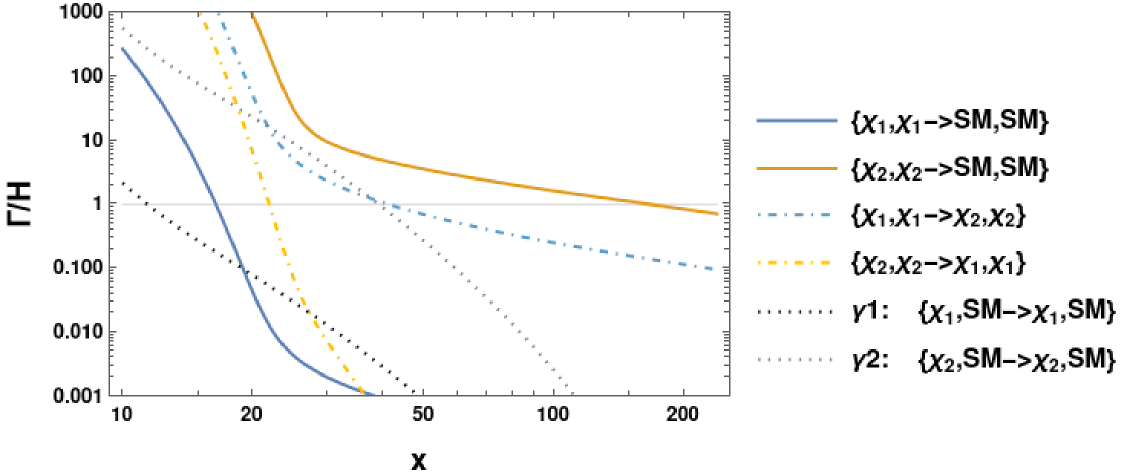


$$m_{\chi_1} = 1.86\ GeV$$
$$m_{\chi_2} = 1.67\ GeV$$
$$m_a = 3.31\ GeV$$
$$\lambda_1 = 0.0067, \lambda_2 = 0.11, \lambda_y = 0.17$$

$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 1.02, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.71$$

nBE: $(\Omega h^2)_1 = 0.092, (\Omega h^2)_2 = 0.035$

Rates:



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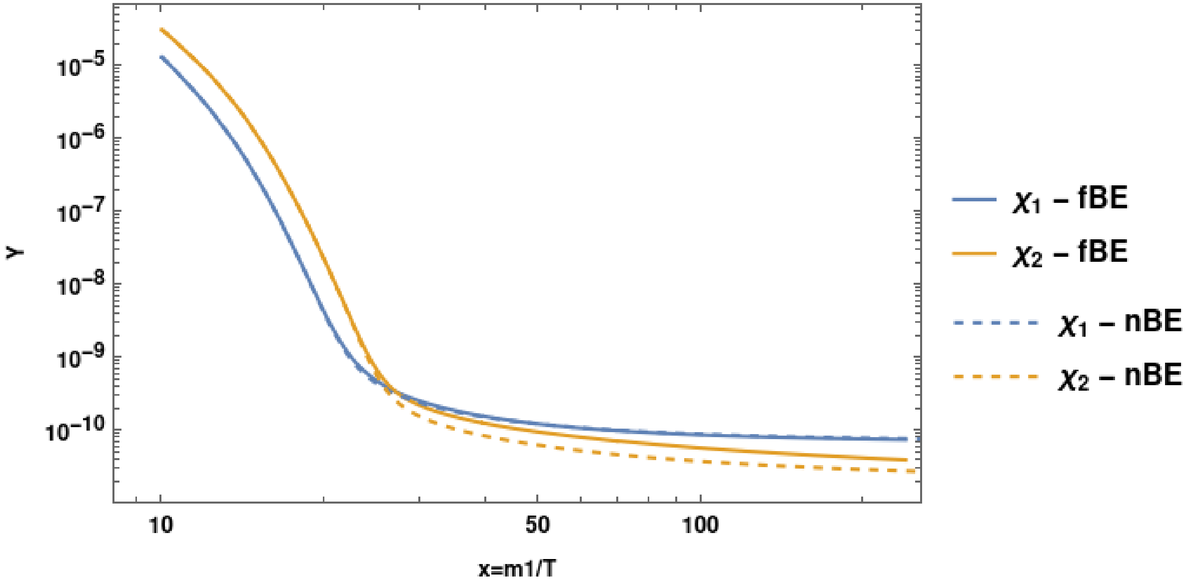

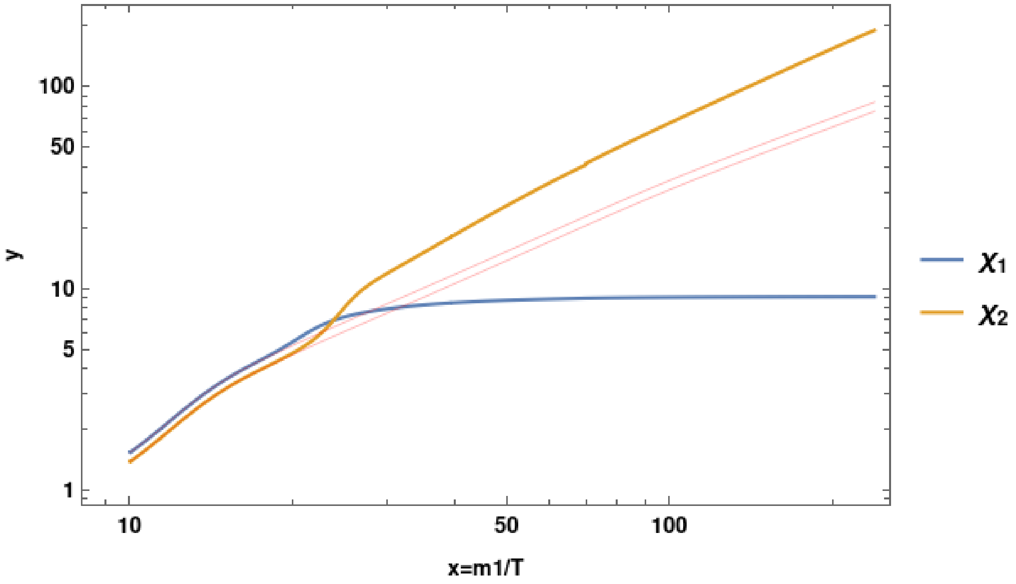

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⇒

$$\delta_1 \equiv \left(\frac{2m_{\chi_1}}{m_a}\right)^2 - 1 = 0.262$$
$$\delta_2 \equiv \left(\frac{2m_{\chi_2}}{m_a}\right)^2 - 1 = 0.019$$

$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 1.02, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 0.71$$

nBE: $(\Omega h^2)_1 = 0.092, (\Omega h^2)_2 = 0.035$

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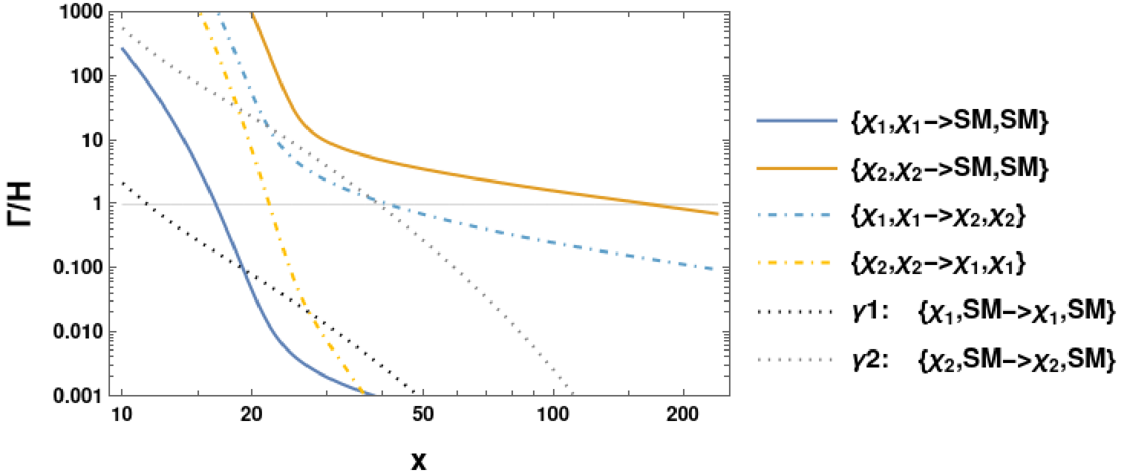

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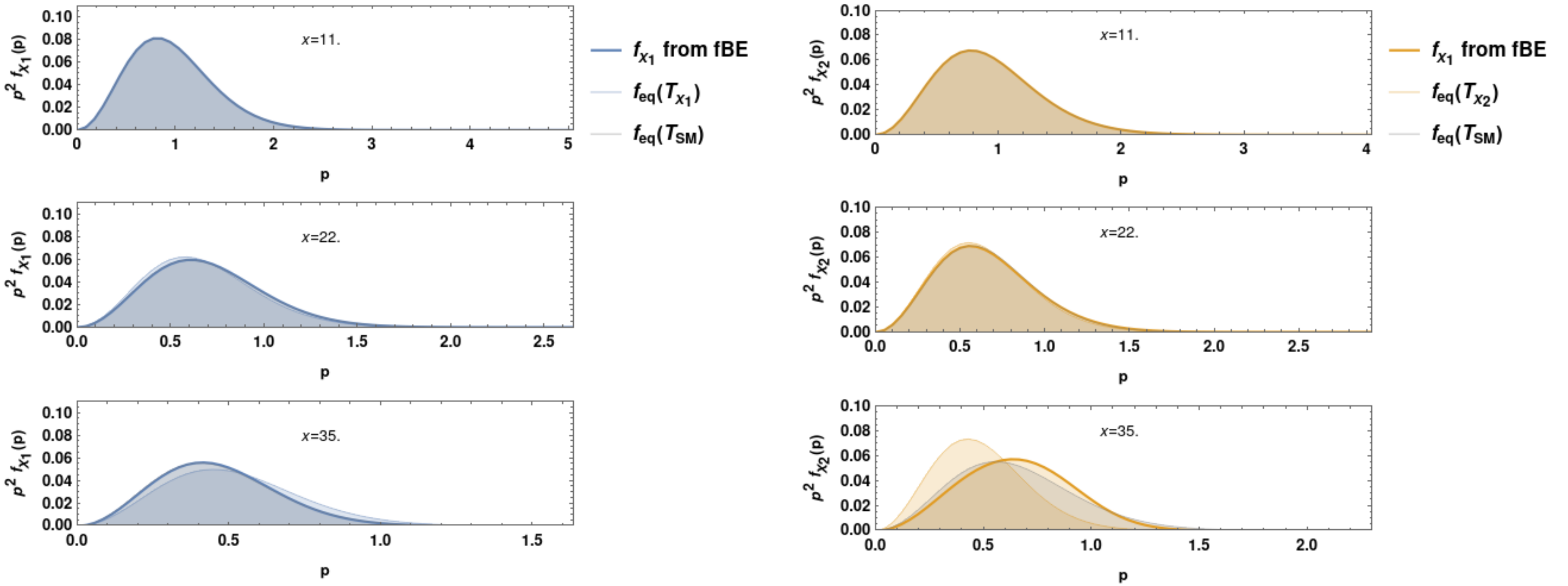

$$x = m_{DM}/T_{SM}$$

$$(m_2^2 - m_1^2)^{1/2} \simeq 0.8\ GeV$$

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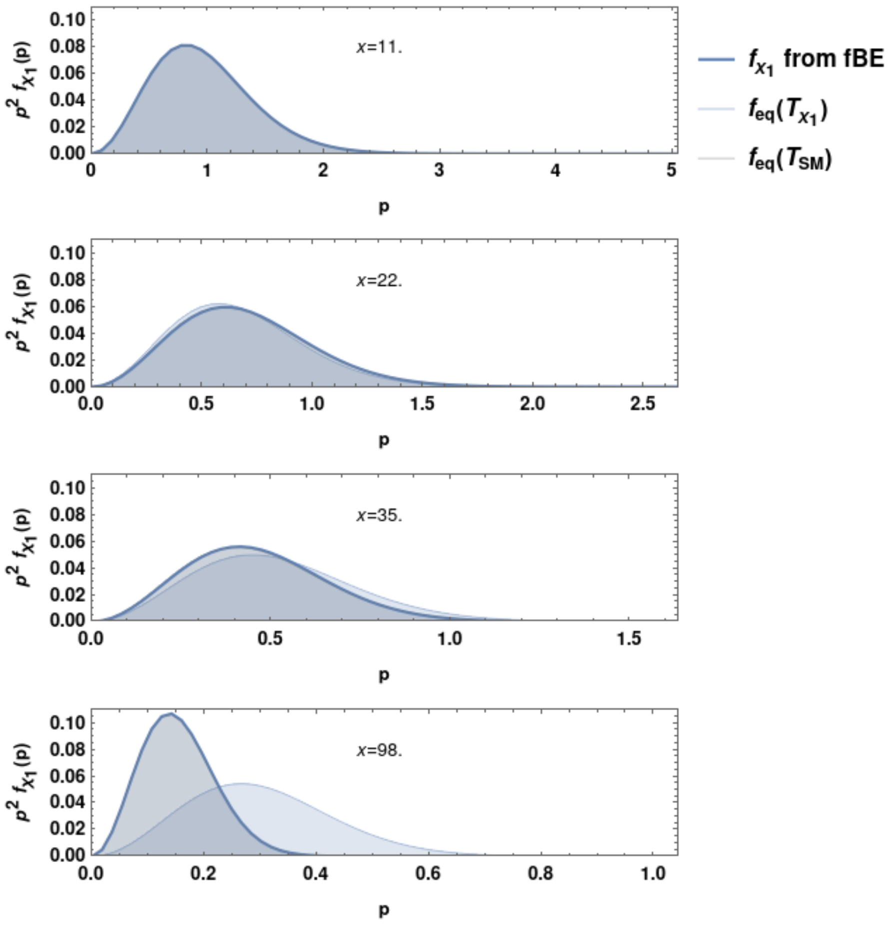

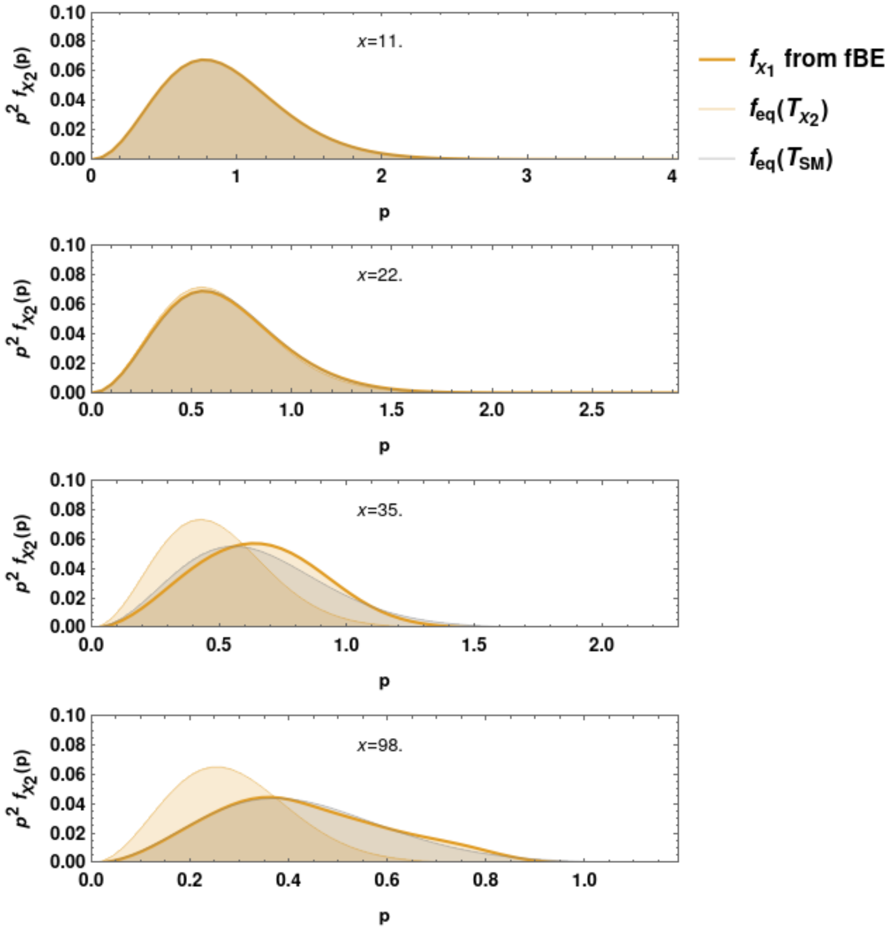

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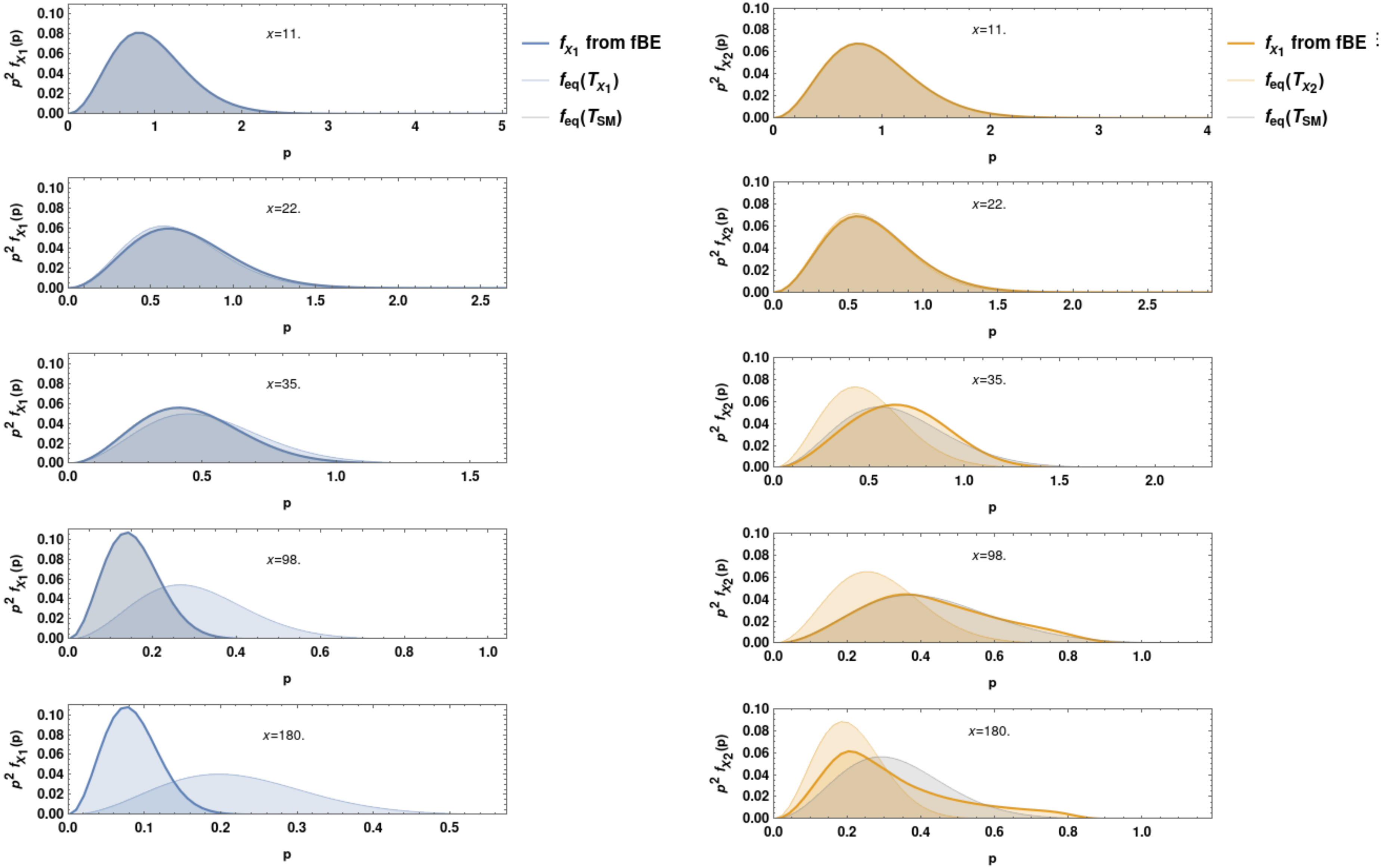

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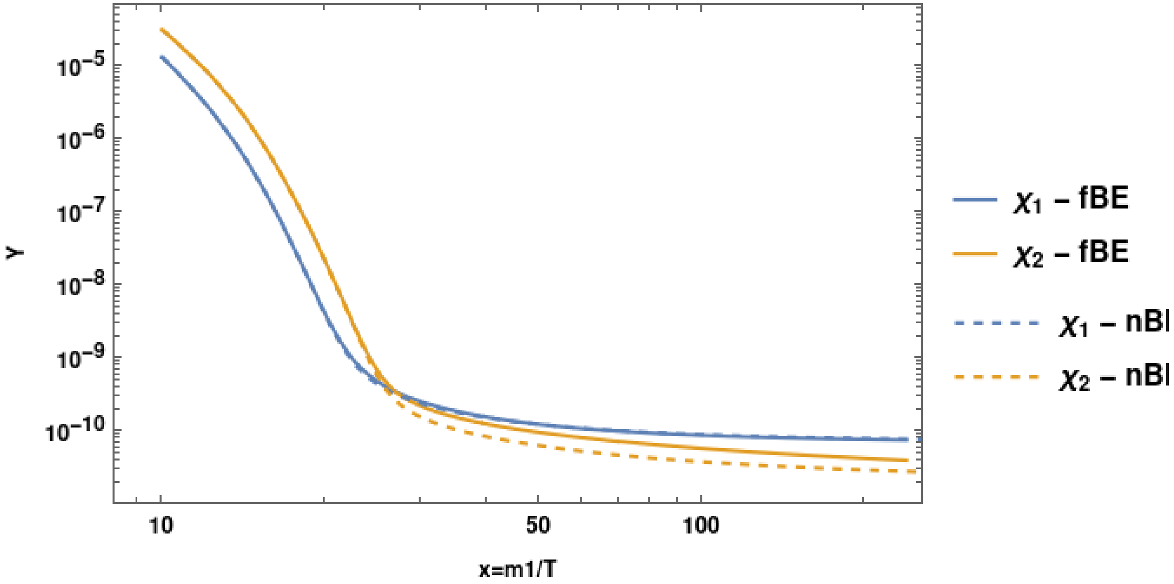
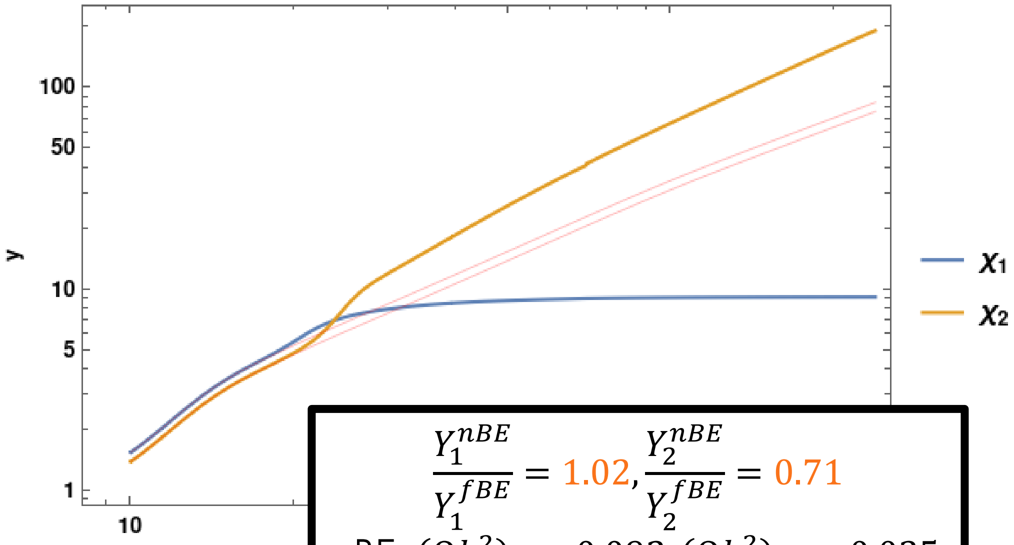
With conversions:





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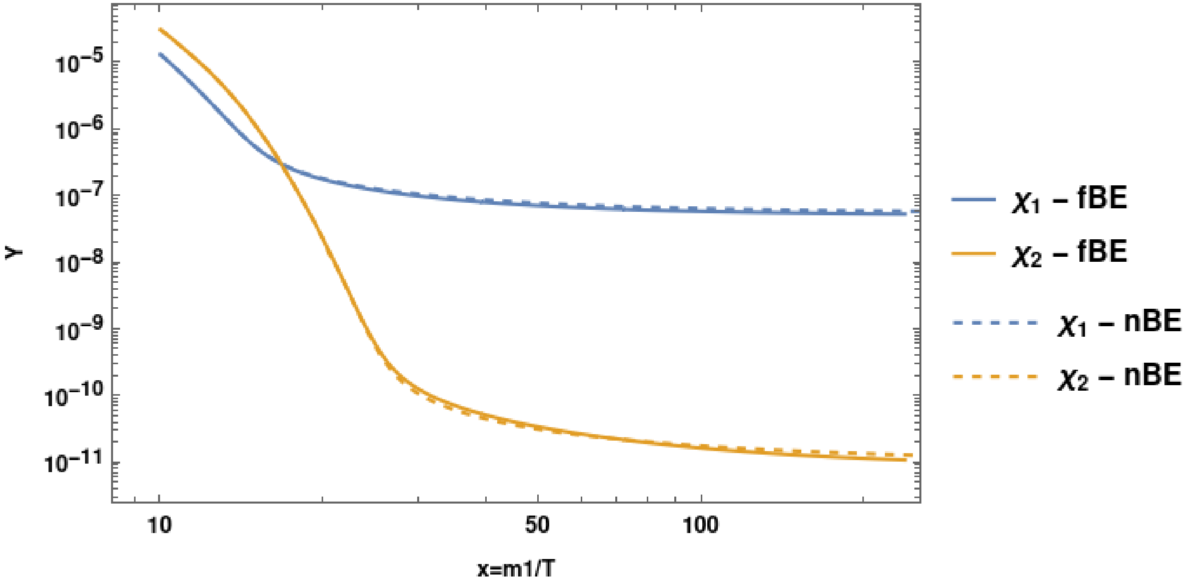

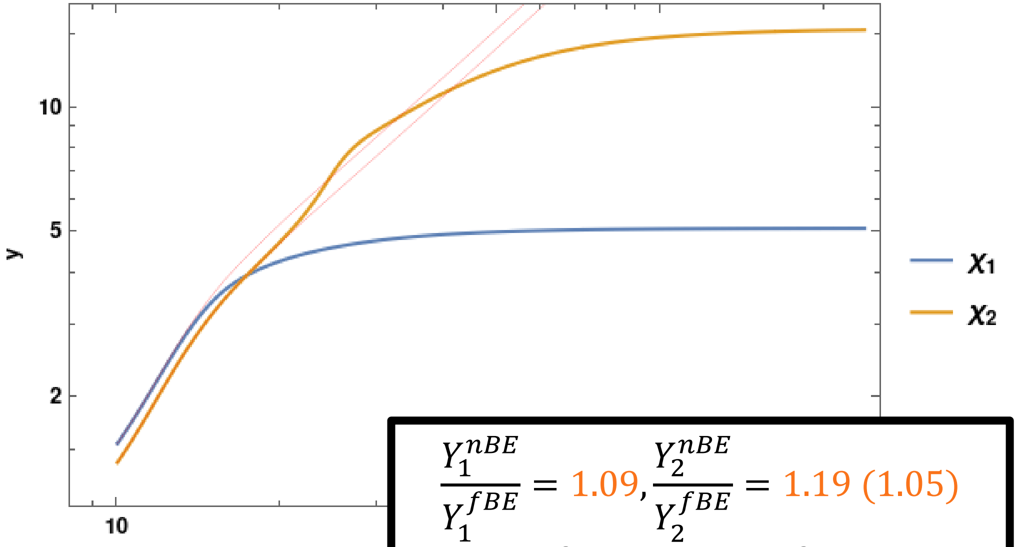

$$\frac{Y_1^{nBE}}{Y_1^{fBE}} = 1.09, \frac{Y_2^{nBE}}{Y_2^{fBE}} = 1.19\ (1.05)$$

nBE: $(\Omega h^2)_1 = 59.2\ (\Omega h^2)_2 = 0.01$

Summary

Progressing towards highly desirable state-of-the-art tool to enable a precision calculation of relic abundance of *any* frozen-out (meta-)stable particle

Challenges:

1. For m1>m2 two largely separated scales, with the two particles evolving qualitatively differently at a given time (x)

2. Precision calculation of integro-differential equation:

   - Pre-tabulate integrations and interpolate over it while solving the coupled differential equations

   - Interpolations over features in phase space distributions ---
     conversions/self-scatterings collision terms $\propto\ (q_N)^3$
     goodness of interpolation

Summary

- The sector containing DM can *in general* be richly populated with multiple particles.

- The canonical picture of a single WIMP falling out of equilibrium with the SM plasma (freeze-out) is then an approximation to the full picture: typically a good approximation, but *not always*.

- For the parameter spaces where this separation of particles cannot be made, the coupled Boltzmann equation for all particles and processes relevant to the DM freeze-out must be solved.

- Additionally, if the kinetic equilibrium of DM with SM cannot be guaranteed, a precise determination of the relic abundance requires for a solution of the full Boltzmann equation (fBE) at the phase-space level. These effects would be larger still for momentum dependent DM interactions.
- With a 2-component Coy DM model--featuring momentum dependent DM-SM scattering:
  - O(10)% deviation in relic densities of either particle is frequently observed
  - For specific points with strong resonance-effects, O(10) deviation is observed

  between the relic densities obtained from solutions of full Boltzmann equation at phase space level to the (integrated Boltzmann) equation in Yield.

- A code to solve the two-component DM Boltzmann equation at phase space level for precision calculation (to be included in a future version of the publicly available code DRAKE)

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Thank you!

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Backup

2-component Coy Dark Matter: Near-resonant

w/o conversions