The exploration into $U(1)$ hidden sectors

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This talk is based on

Aboubrahim, WZF, Nath, "A long-lived stop with freeze-in and freeze-out dark matter in the hidden sector," 1910.14092.
Aboubrahim, WZF, Nath, "Expanding the parameter space of natural supersymmetry," 2003.02267.

Aboubrahim, WZF, Nath, Wang, "Self-interacting hidden sector dark matter, small scale galaxy structure anomalies," 2008.00529.
Aboubrahim, WZF, Nath, Wang, "A multi-temperature universe can allow a sub-MeV dark photon dark matter," 2103.15769.

WZF, Zhang, Zhang, "Sub-GeV millicharge dark matter from the U(1) hidden sector," 2312.03837.
WZF, Nath, Li, "Cosmologically consistent analysis of gravitational waves from hidden sectors," 2403.09558.
WZF, Zhang, "Freeze-in Dark Matter Explanation of the Galactic 511 keV Signal," 2405.19431.

Perspective as a dark matter model builder

$\Omega h^2 = 0.12$ now becomes a constraint to the dark matter models rather than an ultimate goal to achieve.

Overview

Matters from $U(1)$ hidden sectors

- Minimal setup: Z' (or dark photon γ' for small mass), one or more dark fermion(s), with or without a dark Higgs.
- The dark fermion carrying the extra $U(1)$ charge is a natural dark matter candidate. Light dark photon can also be a dark matter candidate.
- $U(1)$ extension of the Standard Model: $U(1)_v$, $U(1)_h$ (kinetic and/or mass mixing).
- Freeze-out scenarios (dark matter is initially in thermal equilibrium with SM particles) – various problems: direct/indirect detection constraints, collider constraints... It is now *very difficult* to satisfy the relic density together with various experimental constraints.
- Turn to freeze-in (dark matter never achieves thermal equilibrium with SM particles) – one major problem: why there exists such feeble coupling? *Mixings* can just provide such smallness of the coupling constants.

A brief review of $U(1)$ extensions of the SM

$U(1)_v$ extensions – All of or some of the SM particles are charged under this $U(1)$.

1. Anomaly free $U(1)$'s: for example $U(1)_{\alpha Y+\beta(B-L)}$, ...
2. Anomalous $U(1)$'s: for example $U(1)_B$, $U(1)_L$, $U(1)_{PQ}$, ...
   - Adding chiral exotics: making the theory look ugly, also there is no good explanation why these chiral fields are much heavier compared with SM particles, since they haven't been discovered.
   - Green-Schwarz mechanism. The coupling of the $U(1)$ gauge field and the axion is responsible to cancel the anomalies, and at the same time this coupling also give the $U(1)$ gauge boson a Stueckelberg mass.

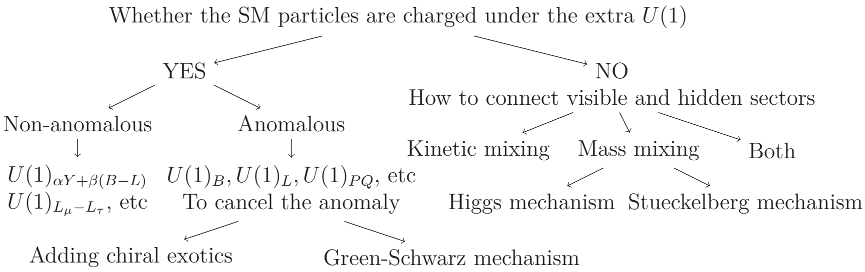
Family-dependent $U(1)_v$ extensions: for example $U(1)_{L_\mu-L_\tau}$ [WZF, Nath, Peim, 1204.5752], $U(1)_{B_1+B_2-2B_3}$ [Celis, WZF, Vollmann, 1608.03894], ...

$U(1)_h$ extensions – All of the SM particles are neutral under this $U(1)$. Then one needs to find a mechanism to connect the visible sector and the hidden sector:

1. Kinetic mixing: A term $\sim \frac{\delta}{2} F^Y_{\mu\nu} F^{\mu\nu}_h$ can be induced by loop effects.
2. Mass mixing: A term $\sim m^2 A_Y A_h$ can be induced by either Higgs mechanism or Stueckelberg mechanism.
3. Both kinetic mixing and mass mixing. A simultaneous diagonalization is required.

After the diagonalization (either the kinetic terms or the mass terms or both), in the final (physical) eigenbasis $U(1)$'s couple to both hidden sector fields and visible sector fields.

Summary

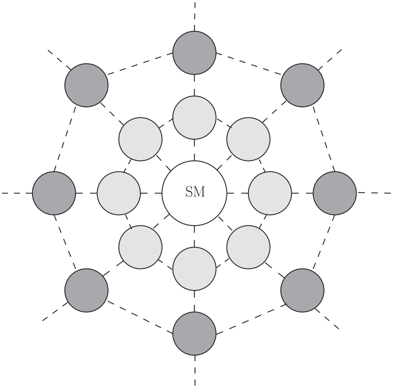


$U(1)$ extensions beyond the SM.

Explorations into hidden sectors

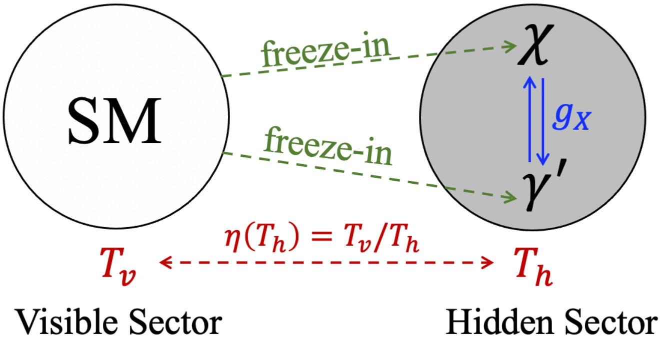
- Hidden sectors generally exist in string theory and various GUT models.
- SM extensions with one or more hidden sectors is one possibility of our true world.
- Evolutions of hidden sector particles and hidden sector temperature are important in determining new physics signals, in concordance with various experimental constraints and dark matter relic density constraint.
- YES WE CAN *now* calculate the full evolutions of hidden sector particles as well as hidden sector temperatures.

Multiple hidden sectors beyond the SM



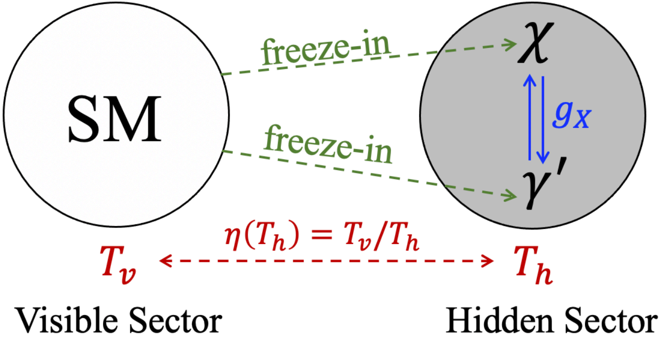
Our SM is in general directly or indirectly connected with multiple hidden sectors, with either weak scale coupling or ultraweak coupling. A hidden sector feebly interacted with the SM will evolve almost independently wrt. the universe, and it possesses its own temperature.

A graphic illustration of the simplest $U(1)_X$ model



A graphic illustration of the freeze-in generation of the simplest $U(1)_X$ model.

A graphic illustration of the simplest $U(1)_X$ model



Because of the self-interaction inside the hidden sector, this simplest setup *was* difficult to calculate.

Why difficult?

- Indeed, *previously*, the evolution of *any* models with a freeze-in produced hidden sector involving hidden sector self-interactions, cannot be computed accurately. RH neutrino portal dark matter is also one of such examples.
- The thermal averaged cross-section of hidden sector interactions depend on the hidden sector temperature. We have no clue what the hidden sector temperature is.
- How to setup a connection between the hidden sector temperature and the visible sector (SM) temperature.
- Earlier works mostly used simplified calculation, with strong assumptions. The limitation of the calculation forces people to focus on very small regions in parameter space. Later I will show in the slides: even under extreme assumptions early calculations in the literature were not rigorous nor reliable.

Solution

In [Aboubrahim, WZF, Nath, Wang, 2008.00529], a general formalism was established to compute the *complete* evolution of the hidden sector (produced through freeze-in mechanism) particle number densities as well as the hidden sector temperature.

For a general hidden sector feebly coupled to the visible sector, its temperature T_h is linked to the visible sector temperature (the temperature of the observed Universe) T by a function $\eta(T_h) = T/T_h$.

The continuity equation derived from Friedmann equations is now modified to be

$$\frac{\mathrm{d}\rho_h}{\mathrm{d}t} + 3H\left(\rho_h + p_h\right) = j_h\,,$$
$$\frac{\mathrm{d}\rho_v}{\mathrm{d}t} + 3H\left(\rho_v + p_v\right) = -j_h\,,$$

where j_h is the source term arising from the freeze-in.

Temperature dependence

One can further deduce

$$\rho\frac{\mathrm{d}\rho_h}{\mathrm{d}T_h} = \left(\frac{\zeta_h}{\zeta}\rho_h - \frac{j_h}{4H\zeta}\right)\frac{\mathrm{d}\rho}{\mathrm{d}T_h}\,,$$

$\zeta_h = \frac{3}{4}(1 + p_h/\rho_h)$ and $\zeta_h = 1$ for radiation dominated hidden sector.

Using the fact $\rho = \rho_v + \rho_h$, and the total entropy of Universe is conserved and the Hubble parameter is given by

$$H^2 = \frac{8\pi G_N}{3}\left[\rho_v(T) + \rho_h(T_h)\right],$$

one can finally obtain

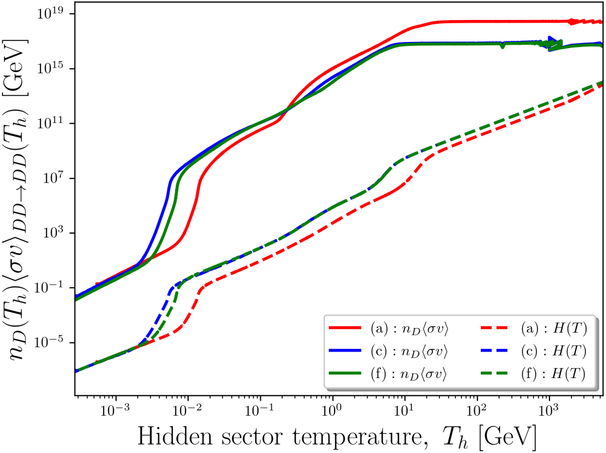
$$\frac{\mathrm{d}\eta}{\mathrm{d}T_h} = -\frac{A_v}{B_v} + \frac{\zeta\rho_v + \rho_h(\zeta - \zeta_h) + j_h/(4H)}{B_v[\zeta_h\rho_h - j_h/(4H)]}\frac{\mathrm{d}\rho_h}{\mathrm{d}T_h}\,,$$

with A_v, B_v functions of T_h and g_{eff}.

The complete coupled Boltzmann equations for $U(1)_X$

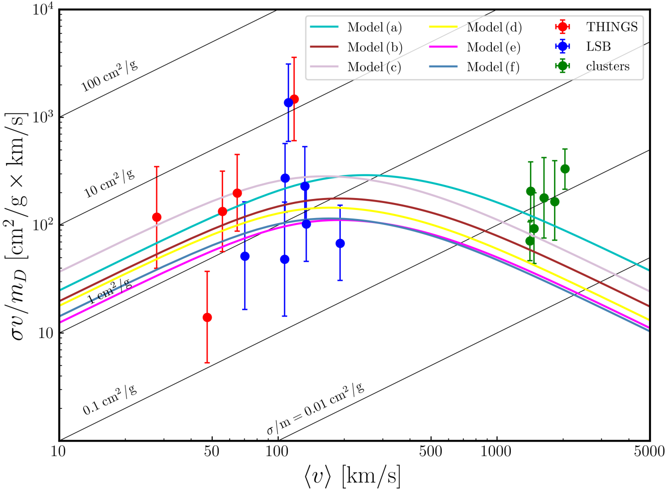
$$\begin{aligned}
\frac{\mathrm{d}Y_\chi}{\mathrm{d}T_h} = &- \frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{(Y_\chi^{\mathrm{eq}})^2\langle\sigma v\rangle^{T_h\eta}_{\chi\bar\chi\to i\bar i} + \frac{1}{s}Y_{\gamma^*}\langle\Gamma\rangle^{T_h\eta}_{\gamma^*\to\chi\bar\chi} \\
&+ \theta(M_{\gamma'} - 2m_\chi)\big[-Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} + \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
&- Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} + Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi}\Big\}, \\
\frac{\mathrm{d}Y_{\gamma'}}{\mathrm{d}T_h} = &- \frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} - Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi} \\
&+ \theta(M_{\gamma'} - 2m_\chi)\big[Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
&+ \theta(M_{\gamma'} - 2m_i)\big[Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to i\bar i}\big] \\
&+ Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'\gamma} + 2Y_iY_{\gamma'}^{\mathrm{eq}}\langle\sigma v\rangle^{T_h\eta}_{i\gamma'\to i\gamma}\Big\}.
\end{aligned}$$

Self-interacting DM explanation of small scale structures



[Aboubrahimb, WZF, Nath, Wang, 2008.00529]

Velocity-dependent DM self-interaction cross-sections



[Aboubrahimb, WZF, Nath, Wang, 2008.00529]

Summary

- We focus on a $U(1)_X$ beyond the SM, where all SM particles are not charged under the $U(1)_X$. The connection between the hidden sector and the SM can be kinetic mixing and/or mass mixing.
- The strong interaction among dark matter, which might be the answer to various small scale structure issues, can be addressed in a $U(1)_X$ hidden sector produced via freeze-in beyond the SM.
- We have developed a formalism to compute the evolution of hidden sector temperature as well as the hidden sector particles produced through freeze-in mechanism.
- This formalism can be applied to any models with a freeze-in produced hidden sector involving hidden sector self-interactions, and compute the complete evolution of the hidden sector particles.

History of kinetic mixing and millicharge dark matter

- Holdom 1986: The kinetic mixing between two massless $U(1)$'s can generate a millicharge.
- Goldberg and Hall 1986: millicharge dark matter.
- Feldman, Liu and Nath 2007: a kinetic mixing cannot generate a millicharge considering the extra $U(1)$ mixed with the full electroweak theory. Mass mixing can be the only source.
- [WZF, Zhang, Zhang, 2312.03837] provides a comprehensive review on this subject.

A deeper look at two $U(1)$ mixing

Kinetic mixing between two massless $U(1)$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu}\,,$$
$$\mathcal{L}_{\rm int} = eA_\mu J^\mu_{\rm em} + g_X C_\mu J^\mu_{\rm d}\,.$$

The kinetic terms can be diagonalized by a non-unitary transformation (to keep the gauge kinetic term in the canonical form)

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\boldsymbol{\delta}^2}} & 0 \\ \frac{-\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}} & 1 \end{pmatrix} \begin{pmatrix} A_\gamma \\ A_X \end{pmatrix}.$$

In the physical eigenbasis, the interactions can be rewritten as

$$\mathcal{L}_{\rm int} = \frac{e}{\sqrt{1-\boldsymbol{\delta}^2}}A_\gamma J_{\rm em} + g_X\left(\frac{\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}}A_\gamma + A_X\right)J_{\rm d}\,.$$

A deeper look at two $U(1)$ mixing

For the case of kinetic mixing between a massless $U(1)$ and a massive $U(1)$, the Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\rm eff} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu} - \frac{1}{2}M^2C^2\,,\\ \mathcal{L}_{\rm int} &= eA_\mu J^\mu_{\rm em} + g_X C_\mu J^\mu_{\rm d}\,.\end{aligned}$$

Now the only possible way of eliminating the kinetic mixing term is given by the following transformation

$$\begin{pmatrix} C \\ A \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\boldsymbol{\delta}^2}} & 0 \\ \frac{-\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}} & 1 \end{pmatrix} \begin{pmatrix} A_X \\ A_\gamma \end{pmatrix},$$

which gives rise to the interaction in the physical eigenbasis as

$$\mathcal{L}_{\rm int} = \frac{g_X}{\sqrt{1-\boldsymbol{\delta}^2}}A_X J_{\rm d} + e\left(\frac{\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}}A_X + A_\gamma\right)J_{\rm em}\,.$$

In this case, the dark particle carries exactly zero electric charge.

A deeper look at two $U(1)$ mixing

For the case of kinetic mixing between two massive $U(1)$'s (may produce a massless in the final mass eigenbasis), the Lagrangian reads

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu}\,, \\
\mathcal{L}_{\text{mass}} &= -\frac{1}{2}M_2^2A^2 - \frac{1}{2}M_1^2C^2 - M_1M_2AC\,, \\
\mathcal{L}_{\text{int}} &= eA_\mu J^\mu_{\text{em}} + g_XC_\mu J^\mu_{\text{d}}\,.
\end{aligned}$$

To obtain the physical eigenbasis, one needs to diagonalize the kinetic mixing matrix and mass mixing matrix simultaneously for the gauge eigenbasis $V^T = (C, A)$,

$$\mathcal{K} = \begin{pmatrix} 1 & \boldsymbol{\delta} \\ \boldsymbol{\delta} & 1 \end{pmatrix}, \qquad M^2_{\text{St}} = \begin{pmatrix} M_1^2 & M_1M_2 \\ M_1M_2 & M_2^2 \end{pmatrix} = M_1^2\begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix},$$

where we define the mass mixing parameter $\epsilon = M_2/M_1$.

Kinetic and mass mixing between two massive $U(1)$'s

The interaction terms are

$$\mathcal{L}_{\rm int} = \frac{1}{\sqrt{1-2\epsilon\boldsymbol{\delta}+\epsilon^2}}\frac{1}{\sqrt{1-\boldsymbol{\delta}^2}}A_X\left[(1-\epsilon\boldsymbol{\delta})g_X J_{\rm d} + eJ_{\rm em}(\epsilon-\boldsymbol{\delta})\right]$$
$$+\frac{1}{\sqrt{1-2\epsilon\boldsymbol{\delta}+\epsilon^2}}A_\gamma(eJ_{\rm em}-\epsilon g_X J_{\rm d})\,.$$

String origin of the millicharge: small fractional charge and the fraction is proportional to the D-brane wrapping numbers on the 3-cycles of the 6D internal manifold [WZF, Shiu, Soler, Ye, 1401.5880, 1401.5890].

The extra $U(1)$ mixed with the full electroweak theory

Indeed, one should consider $F_{\rm em}^{\mu\nu} \to F_Y^{\mu\nu}$:

The Lagrangian are written as: (1) Kinetic mixing

$$\mathcal{L}_{\rm mix}^{\rm kin} = -\frac{\delta}{2} F_{Y\,\mu\nu} F_X^{\mu\nu}\,,$$

(2) Mass mixing: Stueckelberg mass mixing [Cheng and Yuan 2007, Feldman, Liu and Nath 2007]

$$\mathcal{L}_{\rm mix}^{\rm st} = -\frac{1}{2}(M_1 C_\mu + M_2 B_\mu + \partial_\mu \sigma)^2\,,$$

or extra Higgs mixing [Zhang, WZF, 2204.08067]

$$D_\mu H = \left(\partial_\mu - i g_2 T^a A_\mu^a - \frac{i}{2} g_Y Y B_\mu - \frac{i}{2} g_Y y C_\mu\right) H\,,$$

$$D_\mu \phi = (\partial_\mu - i g_X C_\mu)\,\phi\,,$$

Mass mixing: Stueckelberg case

In the gauge eigenbasis of the $U(1)_X$, hypercharge and the neutral $SU(2)$ gauge field $V^T = (C, B, A_3)$, the mixing matrices can be written as [Feldman, Liu, Nath, 2007]

$$\mathcal{K} = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M^2_{\rm St} = \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4} v^2 g_Y^2 & -\frac{1}{4} v^2 g_2 g_Y \\ 0 & -\frac{1}{4} v^2 g_2 g_Y & \frac{1}{4} v^2 g_2^2 \end{pmatrix}.$$

$$\begin{pmatrix} C \\ B \\ A_3 \end{pmatrix} = c \begin{pmatrix} \times & -\frac{g_2 \epsilon}{\sqrt{g_2^2 + g_Y^2}} & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} A' \\ A^\gamma \\ Z \end{pmatrix}$$

$$\mathcal{L} \sim \epsilon g_X Q_\chi \cos\theta_W \bar{\chi} \gamma^\mu \chi A^\gamma_\mu \equiv Q_\epsilon \bar{\chi} \gamma^\mu \chi A^\gamma_\mu ,$$

where $\epsilon = M_2/M_1$ is the mass mixing parameter.

Mass mixing: Extra Higgs case

In the gauge eigenbasis of the $U(1)_{y+Q}$, hypercharge and the neutral $SU(2)$ gauge field $V^T = (C, B, A_3)$, the mixing matrices can be written as [Zhang, WZF, 2204.08067]

$$\mathcal{K} = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; M^2 = \begin{pmatrix} g_X^2 u^2 + \frac{1}{4} y^2 g_Y^2 v^2 & \frac{1}{4} y g_Y^2 v^2 & -\frac{1}{4} y g_2 g_Y v^2 \\ \frac{1}{4} y g_Y^2 v^2 & \frac{1}{4} g_Y^2 v^2 & -\frac{1}{4} g_2 g_Y v^2 \\ -\frac{1}{4} y g_2 g_Y v^2 & -\frac{1}{4} g_2 g_Y v^2 & \frac{1}{4} g_2^2 v^2 \end{pmatrix}$$

$$\begin{pmatrix} C \\ B \\ A_3 \end{pmatrix} = \begin{pmatrix} \times & 0 & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} A' \\ A^\gamma \\ Z \end{pmatrix}$$

The photon may couple to a dark fermion originally carry a small amount of hypercharge y, although this coupling is not generated by the mixing effect.

Summary of the couplings from the mixing

A summary of the order of the couplings induced by the kinetic mixing and the mass mixing:

$$
\begin{aligned}
A^\gamma i\bar{i} &\sim eQ_i\,, &\qquad A^\gamma \chi\bar{\chi} &\sim \epsilon g_X\,,\\
Z i\bar{i} &\sim g_2\,, &\qquad Z\chi\bar{\chi} &\sim \delta g_X\,,\\
A' i\bar{i} &\sim |\delta-\epsilon| g_Y\,, &\qquad A'\chi\bar{\chi} &\sim g_X\,.
\end{aligned}
$$

$U(1)$ mixings and the millicharge

1. Without inducing other modifications to the SM, the effective term $\mathcal{L} \sim -\frac{\delta}{2} F^{\rm em}_{\mu\nu} F^{\mu\nu}_X$ is difficult to generated from a renormalized model in UV. Thus considering $\mathcal{L} \sim -\frac{\delta}{2} F^{\rm em}_{\mu\nu} F^{\mu\nu}_X$ is not appropriate, especially from the theoretical perspective.
2. Even one consider such mixing term, the millicharge cannot be generated if the extra $U(1)_X$ is massive.
3. The millicharge can be only generated in three ways:
    1. The dark particle carries a tiny amount of hypercharge as a prior.
    2. A kinetic mixing between a *massless* $U(1)$ and the hypercharge gauge field, and the generated millicharge is proportional to the kinetic mixing parameter.
    3. The mass mixing between a *massive* $U(1)$ and the hypercharge gauge field, and the generated millicharge is proportional to the mass mixing parameter [Cheng and Yuan 2007, Feldman, Liu and Nath 2007]. In this case the kinetic mixing does not play any role in generating the millicharge, and the millicharge generated is proportional to the mass mixing parameter.

The destination of the dark photon

Decay channels of the dark photon for $M_{\gamma'} < 2m_e$: to neutrinos, and to three photons

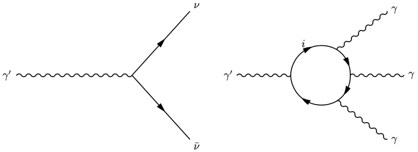


Figure: Dark photon decay channels for $M_{\gamma'} < 2m_e$, including the decay to pairs of neutrino, and to three photons. Considering various constraints, the decay of the dark photon to neutrinos due to the mixing effect is always suppressed compared to the three-photon decay channel.

Although the dark photon's lifetime is extended beyond the age of the Universe, it can still undergo decay, even in minuscule amounts. This decay contributes to the isotropic diffuse photon background (IDPB), and thus the model suffer even more stringent constraints.

IDPB constraint

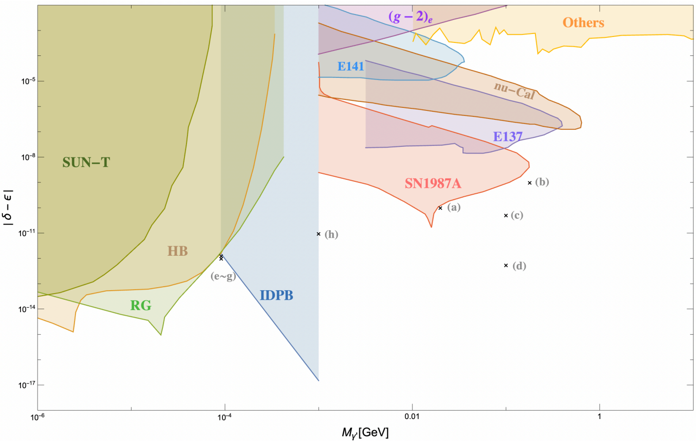


Figure: A display of current constraints (colored regions) on the absolute value of the kinetic mixing parameter minus the mass mixing parameter.

Sub-GeV millicharge dark matter

[WZF, Zhang, Zhang, 2312.03837] focus on sub-GeV mass region of dark matter. We consider six different cases, which have very distinct evolution details:

Case		Model	$M_{\gamma'}$	m_χ	g_X	δ	ϵ	ε	$\Omega_\chi h^2$	$\Omega_{\gamma'} h^2$	$\tau_{\gamma'}$
1	$m_\chi > M_{\gamma'} > 2m_e$	a	20	100	0.0054	1×10^{-13}	1×10^{-10}	1.57×10^{-12}	0.120	0	0.616
		b	180	250	0.015	1×10^{-12}	1×10^{-9}	4.36×10^{-11}	0.120	0	6.82×10^{-4}
2	$2m_\chi > M_{\gamma'} > m_\chi > 2m_e$	c	100	60	1.59×10^{-5}	1×10^{-14}	5×10^{-11}	2.29×10^{-15}	0.120	0	0.491
3	$M_{\gamma'} > 2m_\chi > 2m_e$	d	100	10	0.01	1×10^{-14}	5.6×10^{-13}	1.62×10^{-14}	0.120	0	2.48×10^{-19}
4	$2m_e > m_\chi > M_{\gamma'}$	e	0.09	1	0.20	1×10^{-14}	1.27×10^{-12}	7.34×10^{-13}	7.42×10^{-12}	4.43×10^{-3}	2.67×10^{30}
5	$2m_e > 2m_\chi > M_{\gamma'} > m_\chi$	f	0.09	0.06	0.01	1×10^{-14}	1.27×10^{-12}	3.67×10^{-14}	5.94×10^{-3}	2.58×10^{-9}	2.67×10^{30}
		g	0.09	0.06	1.5×10^{-4}	1×10^{-14}	1.27×10^{-12}	5.50×10^{-16}	1.85×10^{-3}	3.03×10^{-3}	2.67×10^{30}
6	$2m_e > M_{\gamma'} > 2m_\chi$	h	1	0.05	0.001	4×10^{-13}	1×10^{-11}	2.9×10^{-14}	0.120	0	6.20×10^{-18}

The benchmark models we consider for six different types of models. The lifetimes (in the unit of seconds) of the dark photon for each model are listed in the last column. $M_{\gamma'}$ and m_χ are in MeVs.

For a single $U(1)$ extension, a dark photon dark matter can occupy at most $\sim 5\%$ of the total dark matter relic density. $\Longrightarrow$ Extension with more $U(1)$'s.

Case 1, model a: $M_{\gamma'} = 20\text{MeV}, M_\chi = 100\text{MeV}$

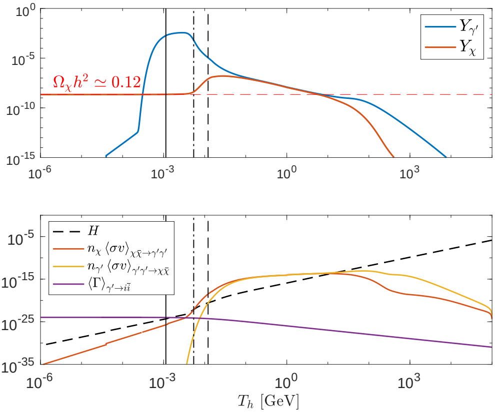


Figure: One can see apparent dark freeze-out ($\chi\bar{\chi} \to \gamma'\gamma'$) and the hidden sector interactions reach equilibrium (inside the hidden sector).

Case 2, model c: $M_{\gamma'} = 100\text{MeV}, M_\chi = 60\text{MeV}$

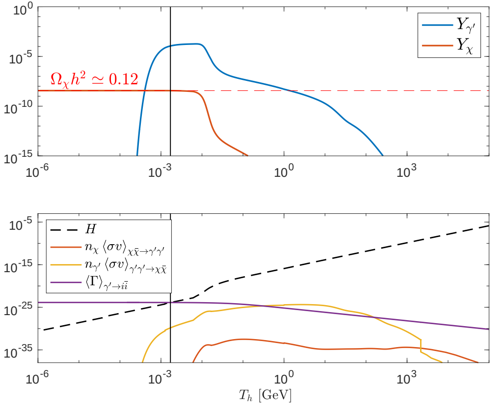


Figure: In this case we choose a rather small $g_X \sim 10^{-5}$, thus hidden sector interactions never reach equilibrium inside the hidden sector. However, these ultraweak interactions still play significant roles.

Case 2, model c: $M_{\gamma'} = 100\text{MeV}, M_\chi = 60\text{MeV}$

Comparison of different calculations	$\Omega_\chi h^2$
All freeze-in processes included	0.1195
Plasmon decay process excluded	0.1195
Four-point γ' freeze-in processes excluded	0.0643
Pure freeze-in for χ	10^{-9}

Table: A comparison of different calculations of the dark matter relic density for the benchmark model c. There is no such limit called “pure freeze-in”.

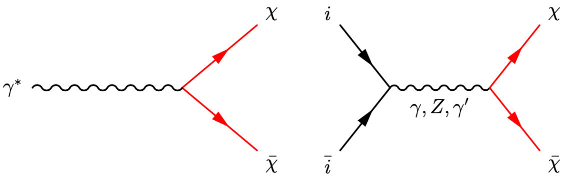


Figure: Freeze-in processes for the dark fermion χ.

Dark photon freeze-in

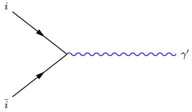


Figure: Three-point freeze-in processes for the dark photon γ'.

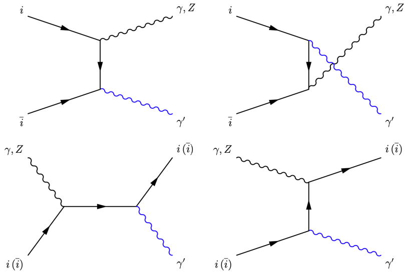


Figure: Four-point freeze-in processes for the dark photon γ'.

General discoveries from $U(1)$ sector freeze-in

In [Aboubrahim, WZF, Nath, Wang, 2008.00529], a general formalism was established to compute the *complete* evolution of the hidden sector (produced through freeze-in) particle number densities as well as the hidden sector temperature.

In [WZF, Zhang, Zhang, 2312.03837] we find some general results which may apply to general freeze-in scenarios:

1. The hidden sector interactions never reach equilibrium, does not indicate such interactions don't occur. On the contrary, these interactions inside the hidden sector play significant role in determining the dark particle number densities.
2. The hidden sector interactions (even ultraweak) must be taken into account at all times. There is no such limit called "pure freeze-in".
3. Four-point freeze-in processes must be kept at all times, even the three-point freeze-in production channels are present for the same freeze-in particle.

The 511 keV photon signal

Longstanding discovery and seen from many collaborations:

The galactic 511 keV photon line emission has been firstly observed for more than 50 years [Johnson, Harnden, Haymes, 1972], and confirmed by recent measurements including the SPI spectrometer on the INTEGRAL observatory [astro-ph/0309442, ...] and COSI balloon telescope [arXiv:1912.00110], see [arXiv:1009.4620] for an early review.

Basic interpretation

Low-energy positrons can annihilate with electrons and produce two 511 keV photons directly in a small fraction (faction $(1 - f_p)$), or form a bound state known as positronium (fraction f_p) with two possible states.

The singlet state (para-positronium/p-Ps) with a zero total spin angular momentum $s = 0$, which occupies $1/4$ of the fraction of the total positronium can annihilate into two photons with energies equal to 511 keV.

Thus the total production rate of 511 keV photons is given by

$$\dot{n}_\gamma = 2\left[(1 - f_p) + \frac{1}{4} f_p\right]\dot{n}_{e^+} = 2\left(1 - \frac{3}{4} f_p\right)\dot{n}_{e^+} \,.$$

The dark matter interpretation

The positron production rates in the case of annihilation and decays are given by ($\rho(r)$ is the dark matter density)

$$\dot{n}_{e^+}^{\rm ann} = f_X \frac{\rho^2(r)}{4m_X^2} \langle \sigma v \rangle_{X\bar{X}\to e^+e^-} \ ,$$
$$\dot{n}_{e^+}^{\rm dec} = f_X \frac{\rho(r)}{m_X} \Gamma_{X\to e^+e^-} {\rm Br}(X \to e^+e^-) \, ,$$

Two types of dark matter density profiles widely adopted:

1. The Navarro-Frenk-White (NFW) profile [Navarro, Frenk, White 1996]
$$\rho_{\rm NFW}(r) = \rho_s \left(\frac{r}{r_s}\right)^{-\gamma} \left(1 + \frac{r}{r_s}\right)^{\gamma-3} .$$
2. The Einasto profile [Einasto, arXiv:0901.0632]
$$\rho_{\rm Einasto}(r) = \rho_s \exp\left\{ -\left[\frac{2}{\alpha} \left(\frac{r}{r_s} \right)^{\alpha} - 1 \right] \right\} .$$

Constraints on dark matter responsible for 511

- Internal bremsstrahlung $\chi\bar{\chi} \to e^+e^-\gamma$, constrains the dark matter mass $\lesssim 20$ MeV [Beacom, Bell, Bertone, 2004].
- Positron in-flight annihilation [Beacom, Yuksel, 2005]. Small fraction of energetic positrons will annihilate with electrons during their energy-loss, which sets constraint on annihilation dark matter mass $\lesssim 3$ MeV, and decaying dark matter mass $\lesssim 6$ MeV.
- Additional constraints on feebly interacting particles from supernova [Calore, Carenza, Giannotti, Jaeckel, Lucente, Mastrototaro, Mirizzi, 2021].

Dark matter annihilation

The light WIMP achieved their final relic abundance through the freeze-out mechanism with the (total) annihilation cross-section

$$\langle \sigma v \rangle_{\rm ann} \simeq 3 \times 10^{-26} \left(\frac{m_{\rm DM}}{\rm MeV} \right)^2 \ {\rm cm}^3/{\rm s}$$

at the freeze-out.

To explain the 511 keV signal the annihilation of dark matter at late times needs to be

$$\langle \sigma v \rangle_{e^+e^-}^{511} \simeq 5 \times 10^{-31} \left(\frac{m_{\rm DM}}{\rm MeV} \right)^2 \ {\rm cm}^3/{\rm s}\,.$$

[Boehm, Hooper, Silk, Casse, Paul, 2003][Ascasibar, Jean, Boehm, Knoedlseder, 2005][Gunion, Hooper McElrath, 2005][Huh, Kim, Park, Park, 2007][Vincent, Martin Cline, 2012][Wilkinson, Vincent, Boehm, McCabe, 2016][Ema, Sala, Sato, 2020][Boehm, Chu, Kuo, Pradler, 2020][Drees, Zhao, 2021][De la Torre Luque, Balaji, Silk, 2021]

Annihilation DM from freeze-in is highly implausible

To explain the 511 keV signal, the effective coupling between $\chi\bar{\chi}$ and e^+e^- is too large for the freeze-in production (overproduction of the dark matter from freeze-in).

One can introduce mediators (which can decay or annihilate into SM completely)

$$\chi\bar{\chi} \xrightarrow{\text{scalar } S, \text{dark photon } \gamma' \cdots} e^+e^-$$

such that one can arrange the interactions among the hidden sector

$$\chi\bar{\chi} \to SS,\ \gamma'\gamma' \to e^+e^- \text{ before BBN}$$

such that the overproduction of the dark matter χ can be depleted.

But now the mediators will receive stringent constraints from experiments and thus are already ruled out [Calore, Carenza, Giannotti, Jaeckel, Lucente, Mastrototaro, Mirizzi, 2021].

Dark matter decay

To explain the 511 keV signal, based on the dark matter profiles widely adopted, one needs

$$\frac{\tau_{X\to e^+e^-}(\mathrm{sec}) \times M_X(\mathrm{MeV})}{f_X \times \mathrm{Br}(X\to e^+e^-)} \sim 10^{26} - 10^{29}\,,$$

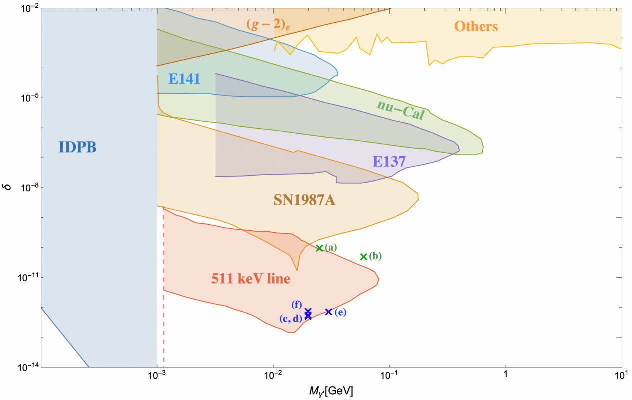
which can be also written as

$$g^2 \times f_X \times \mathrm{Br}(X\to e^+e^-) \sim 10^{-50} - 10^{-47}\,, \qquad \Longrightarrow g \lesssim 10^{-16}$$

where g is the coupling of X with e^+e^-, f_X is the fraction of X in the total dark matter relic density, and $\mathrm{Br}(X\to e^+e^-)$ is the branching fraction of X decay to e^+e^-. The coupling g is toooooo small even for the freeze-in production.

[Picciotto, Pospelov, 2004][Hooper, Wang, 2004][Takahashi, Yanagida, 2005][Finkbeiner, Weiner, 2007][Pospelov, Ritz, 2007][Cembranos, Strigari, 2008][Vincent, Martin Cline, 2012][Cai, Ding, Yang, Zhou, 2020][Lin, Yanagida, 2022][Cappiello, Jafs, Vincent, 2023][Cheng, Lin, Sheng, Yanagida, 2023]

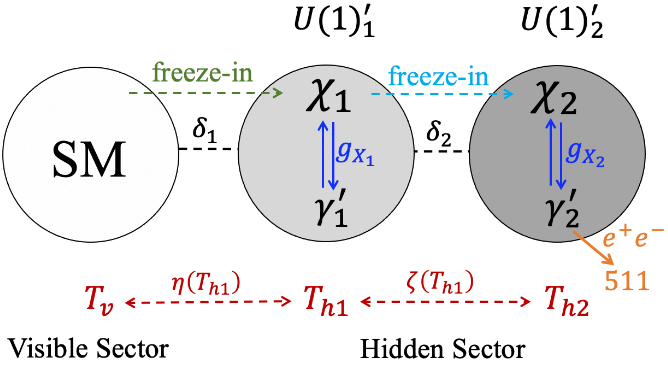
Two types of benchmark models



[WZF, Zhang, 2405.19431]

511 constraints for feebly interacting particles: [Calore, Carenza, Giannotti, Jaeckel, Lucente, Mastrototaro, Mirizzi, 2021]

A two-$U(1)$ model



Full Boltzmann equations

$$\frac{dY_{\chi_1}}{dT_{h_1}} = -s\frac{d\rho/dT_{h_1}}{4\rho H}\sum_{i\in\mathrm{SM}}\Big\{(Y^{\rm eq}_{\chi_1})^2\langle\sigma v\rangle^{T_{h_1}\eta}_{\chi_1\bar\chi_1\to i\bar i} - Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\chi_2\bar\chi_2}$$
$$- Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1\gamma'_1} + Y^2_{\gamma'_1}\langle\sigma v\rangle^{T_{h_1}}_{\gamma'_1\gamma'_1\to\chi_1\bar\chi_1} - Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1\gamma'_2}$$
$$+\theta(M_{\gamma'_1}-2m_{\chi_1})\Big[-Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1} + \frac{1}{s}Y_{\gamma'_1}\langle\Gamma\rangle^{T_{h_1}}_{\gamma'_1\to\chi_1\bar\chi_1}\Big]\Big\},$$

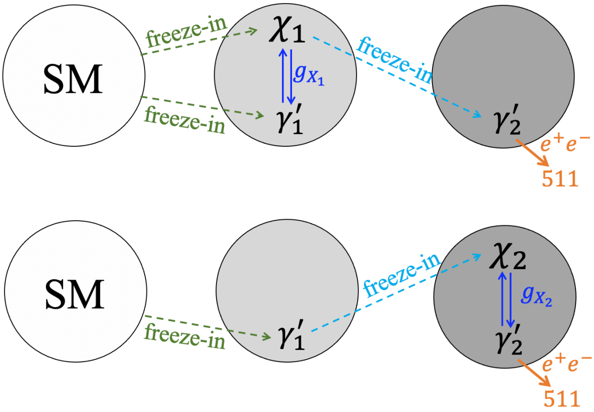
$$\frac{dY_{\gamma'_1}}{dT_{h_1}} = -s\frac{d\rho/dT_{h_1}}{4\rho H}\sum_{i\in\mathrm{SM}}\Big\{Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1\gamma'_1} - Y^2_{\gamma'_1}\langle\sigma v\rangle^{T_{h_1}}_{\gamma'_1\gamma'_1\to\chi_1\bar\chi_1} + Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1\gamma'_2}$$
$$- Y^2_{\gamma'_1}\langle\sigma v\rangle^{T_{h_1}}_{\gamma'_1\gamma'_1\to\chi_2\bar\chi_2} + \theta(M_{\gamma'_1}-2m_i)\Big[Y^2_i\langle\sigma v\rangle^{T_{h_1}\eta}_{i\bar i\to\gamma'_1} - \frac{1}{s}Y_{\gamma'_1}\langle\Gamma\rangle^{T_{h_1}}_{\gamma'_1\to i\bar i}\Big]$$
$$+\theta(M_{\gamma'_1}-2m_{\chi_1})\Big[Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1} - \frac{1}{s}Y_{\gamma'_1}\langle\Gamma\rangle^{T_{h_1}}_{\gamma'_1\to\chi_1\bar\chi_1}\Big]$$
$$+\theta(M_{\gamma'_1}-2m_{\chi_2})\Big[Y^2_{\chi_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\chi_2\bar\chi_2\to\gamma'_1} - \frac{1}{s}Y_{\gamma'_1}\langle\Gamma\rangle^{T_{h_1}}_{\gamma'_1\to\chi_2\bar\chi_2}\Big]\Big\},$$

$$\frac{dY_{\chi_2}}{dT_{h_1}} = -s\frac{d\rho/dT_{h_1}}{4\rho H}\sum_{i\in\mathrm{SM}}\Big\{Y^2_{\gamma'_1}\langle\sigma v\rangle^{T_{h_1}}_{\gamma'_1\gamma'_1\to\chi_2\bar\chi_2} + Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\chi_2\bar\chi_2} - Y^2_{\chi_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\chi_2\bar\chi_2\to\gamma'_2\gamma'_2}$$
$$+ Y^2_{\gamma'_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\gamma'_2\gamma'_2\to\chi_2\bar\chi_2} + \theta(M_{\gamma'_1}-2m_{\chi_2})\Big[-Y^2_{\chi_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\chi_2\bar\chi_2\to\gamma'_1} + \frac{1}{s}Y_{\gamma'_1}\langle\Gamma\rangle^{T_{h_1}}_{\gamma'_1\to\chi_2\bar\chi_2}\Big]\Big\},$$

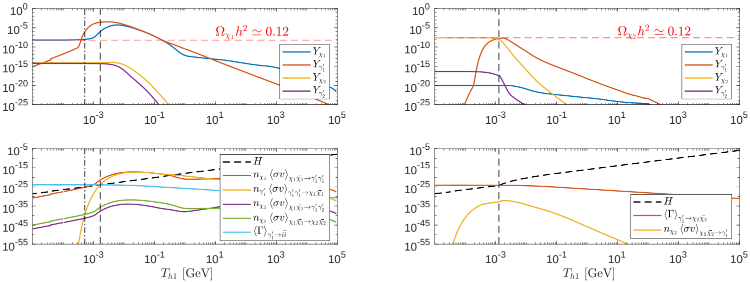
$$\frac{dY_{\gamma'_2}}{dT_{h_1}} = -s\frac{d\rho/dT_{h_1}}{4\rho H}\sum_{i\in\mathrm{SM}}\Big[Y^2_{\chi_1}\langle\sigma v\rangle^{T_{h_1}}_{\chi_1\bar\chi_1\to\gamma'_1\gamma'_2} + Y^2_{\chi_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\chi_2\bar\chi_2\to\gamma'_2\gamma'_2} - Y^2_{\gamma'_2}\langle\sigma v\rangle^{T_{h_1}\zeta}_{\gamma'_2\gamma'_2\to\chi_2\bar\chi_2}\Big],$$

$$\frac{d\eta}{dT_{h_1}} = -\frac{\eta}{T_{h_1}} + \frac{1}{T_{h_1}}\left(\frac{4H\rho_v + j_{h_1} + j_{h_2}}{4H\rho_{h_1} - j_{h_1}}\right)\frac{d\rho_{h_1}/dT_{h_1}}{d\rho_v/dT_v},\quad \frac{d\zeta}{dT_{h_1}} = -\frac{\zeta}{T_{h_1}} + \frac{1}{T_{h_1}}\left(\frac{4H\rho_{h_2} - j_{h_2}}{4H\rho_{h_1} - j_{h_1}}\right)\frac{d\rho_{h_1}/dT_{h_1}}{d\rho_{h_2}/dT_{h_2}},$$

Two types of benchmark models



Evolution of the two models



Dark photon dark matter explanation of the 511 signal

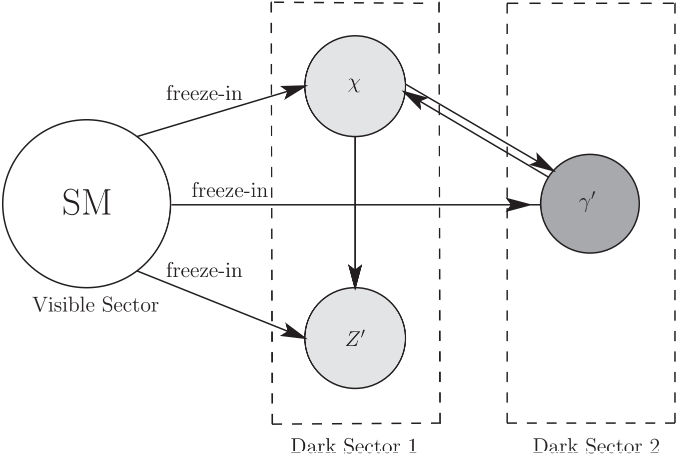
The 511 keV photon flux generated by the decay of the dark photon γ_2' is computed to be

$$\Phi_{511} = \frac{f_{\gamma_2'}}{8\pi} \int \mathrm{d}\Omega \int_{\mathrm{l.o.s}} \frac{\rho(r)}{M_{\gamma_2'}} \Gamma_{\gamma_2' \to e^+ e^-} \mathrm{d}s \,.$$

The bulge flux	$\Phi_{511}^{\mathrm{NFW}}$	$\Phi_{511}^{\mathrm{Einasto}}$
$\lvert l\rvert \lesssim 30^\circ$, $\lvert b\rvert \lesssim 15^\circ$	8.3	9.7
FWHM $\simeq 20.55^\circ$	4.8	6.0

The results of bulge flux for benchmark model a using different integral range for the two dark matter density profile we consider. All fluxes are in units of 10^{-4} ph cm^{-2} s^{-1}, which are consistent with the measured bulge flux ~ 9.6 [Siegert, Diehl, Khachatryan, Krause, Guglielmetti, Greiner, Strong, Zhang, 2016].

Dark photon dark matter from $U(1)$ mixings



[Aboubrahim, WZF, Nath, Wang, 2103.15769]: dark photon dark matter from two $U(1)$'s can occupy almost 100% of the relic density.

SUSY signatures

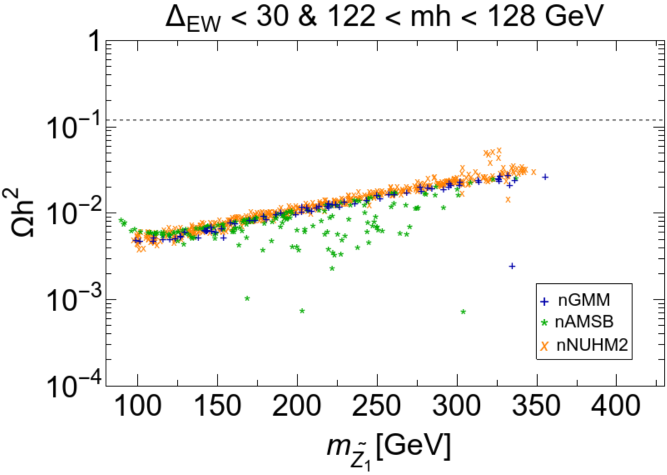
- SUSY is the natural solution to the hierarchy problem and also offers a natural dark matter candidate – the LSP.
- Sadly there is no signal for supersymmetry (yet). LHC has analyzed up to 139 fb^{-1} of data for each of ATLAS and CMS and the results are consistent with the SM.
- Traditional SUSY signals, such as final states with large missing energy due to a neutralino as the LSP, hard jets arising from the decay of squarks and gluinos, or high momentum leptons coming from the decay of electroweak gauginos are now significantly more constrained.
- From model building point of view, constraints are less severe for more rare processes because of their small production cross-sections. Another search which is still not highly constrained is long-lived particles.

Issues of neutralino dark matter

- The measured value of the Higgs boson mass at 126 GeV indicates the size of weak scale supersymmetry lies in the TeV region.
- Direct detection of stop and gluino at the LHC also point to a SUSY breaking scale in the multi-TeV regime.
- Meanwhile, direct searches for relic WIMP dark matter by LUX PandaX failed to detect the SUSY WIMP.
- Indirect WIMP searches from Fermi-LAT (expecting to detect WIMP annihilation to gamma rays) also place strong limits on SUSY WIMPs.

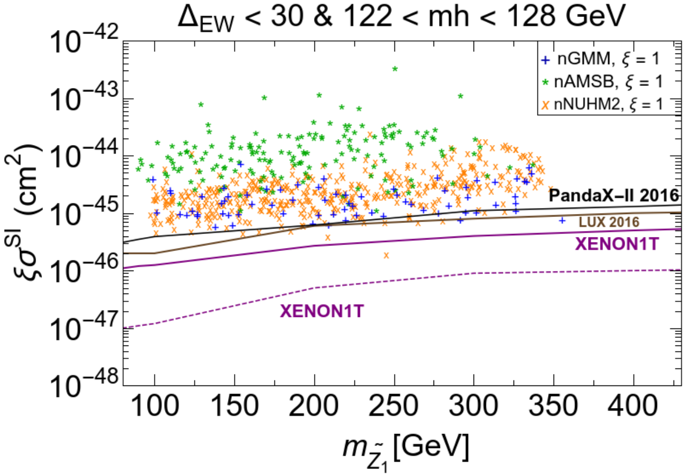
The requirement of naturalness in SUSY models necessitates light higgsinos not too far from the weak scale. The LSP is expected to be a mainly higgsino-like neutralino with non-negligible gaugino components. The computed thermal WIMP abundance in natural SUSY models is then found to be typically a factor 5-20 below the observed relic density [Baer, Barger, Sengupta, Tata, 2018].

Natural SUSY WIMP relic density



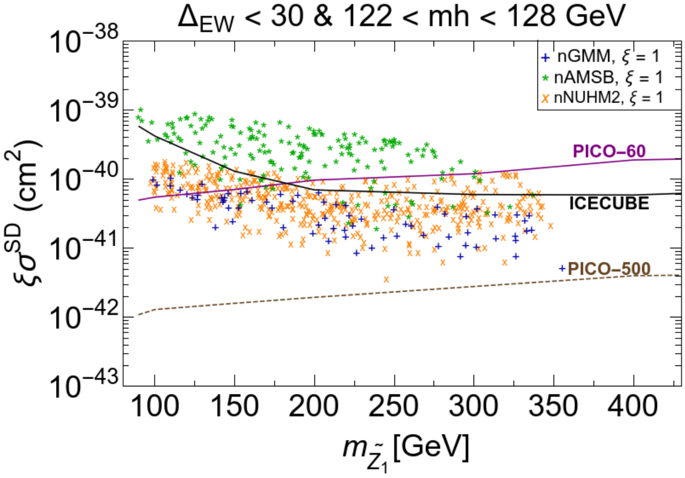
[Baer, Barger, Sengupta, Tata, 2018].

Direct detection bounds



[Baer, Barger, Sengupta, Tata, 2018].

Indirect detection bounds



[Baer, Barger, Sengupta, Tata, 2018].

Solution

- Models where the WIMP relic density (taken to be its thermal value) forms just $\sim 5-20\%$ of the measured CDM density survive the combined constraints from LHC as well as from direct and indirect searches.
- To gain concordance with observations, either additional DM particles must be present or additional non-thermal mechanisms must augment the neutralino abundance.
- Freeze-in hidden sectors allow us to reconstruct the MSSM spectrum, and gives us more choices for the SUSY parameter space.
- Thus in general, freeze-in mechanism provides us more possibilities in constructing particle physics models to explain various unknown problems.

$U(1)$ extension of the SUSY model

- Consider an MSSM/SUGRA model extended by an $U(1)$ sector feebly interacted with the MSSM.
- The model contains additional chiral scalar superfields S and $\bar{S}$ and a vector superfield C. The fermionic component of S and $\bar{S}$ and the gaugino components of C mix with the MSSM neutralino fields producing a 6×6 neutralino mass matrix.
- Two different neutralino mass hierarchies:
  1. the real LSP is the dark neutralino,
  2. the real LSP is the MSSM neutralino,

  which will be the dark matter candidate of the model.

Case 1: The real LSP is the dark neutralino

In the neutralino sector, we label the mass eigenstates as

$$\tilde{\xi}_1^0,\ \text{MSSM LSP},\ \tilde{\xi}_2^0,\ \tilde{\chi}_1^0,\ \tilde{\chi}_2^0,\ \tilde{\chi}_3^0, \tilde{\chi}_4^0.$$

Since the mixing parameter δ is small, $\tilde{\xi}_1^0$ and $\tilde{\xi}_2^0$ are mostly the dark neutralinos while the remaining four $\tilde{\chi}_i^0$ $(i = 1 \cdots 4)$ are mostly MSSM neutralinos.

In this case, the real $\tilde{\xi}_1^0$ is the DM candidate and the MSSM LSP can be long-lived squarks or sleptons.

Long-lived particles

- If the particle is charged and stable over detector length it can be identified by the track it leaves in the inner tracker and in the muon spectrometer. Other signatures are possible such as a disappearing track where a charged particle can decay into very soft final states which escape the trigger threshold.
- ATLAS and CMS were not designed to look for long-lived particles and part of the upcoming upgrade is to further their capabilities to become more sensitive to such searches.
- Most long-lived particle searches at the LHC consider an NLSP very close in mass to the LSP ($\Delta m \sim$ few GeV down to MeV) resulting in a highly suppressed phase space. This leads to a small decay width for the NLSP and thus a long-lived particle.
- Long-lived particles can also arise in SUSY models with a hidden sector if the hidden sector has ultraweak interactions with the visible sector and the LSP of the visible sector decays into the hidden sector.

SUSY naturalness

- With various definitions on the SUSY naturalness, typically most natural SUSY models feature a relatively small Higgs mixing parameter μ.
- Small μ leads to a SUSY model with LSP a Higgsino-like neutralino.
- As indicated earlier, higgsino-like neutralino typically leads to a relic density that falls below the experimental value.

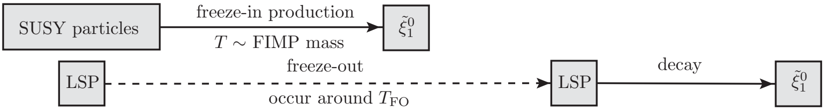
Case 2: The real LSP is the MSSM neutralino

With the freeze-in $U(1)$ hidden sector, the mass hierarchy in this setup is:

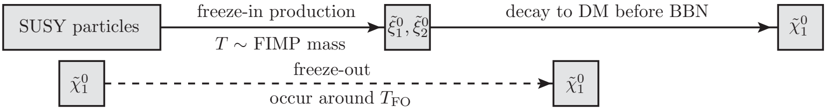
$$\tilde{\chi}_1^0,\ \tilde{\xi}_1^0,\ \tilde{\xi}_2^0,\ \tilde{\chi}_2^0,\ \tilde{\chi}_3^0,\ \tilde{\chi}_4^0.$$

In this case $\tilde{\chi}_1^0$ is the real LSP among the entire SUSY sector and is thus the dark matter candidate.

Reconstruct SUSY models with FIMPs



[Aboubrahim, WZF, Nath, 1910.14092].



[Aboubrahim, WZF, Nath, 2003.02267].

The observation of gravitational waves in black hole mergers in 2016 opened up a new avenue to explore fundamental physics.

There are various possible sources of gravitational waves:

- Compact binary inspiral gravitational waves (LIGO).
- Continuous GW (from spinning neutron stars).
- GWs from Gamma Ray Bursts and possibly from other sources of unknown origin.
- Stochastic GW.

Stochastic GW could reveal fundamental new physics:

- Inflation, Big Bang.
- Cosmic strings, primordial black hole evaporation.
- Cosmic phase transitions, specifically the First Order Phase Transition (FOPT).

FOPT: standard model vs experiment

Standard model does not produce a strong enough gravitational wave measurable by current/proposed detectors.

- $\Omega_{\rm GW} \leq 10^{-28}$ (reach of the standard model)
- $\Omega_{\rm GW} \geq 10^{-20}$ (reach of the proposed detectors)

Hidden sectors could provide much larger gravitational waves possibly in the observational range.

GW provided by the hidden sector

Consider the $U(1)$ extension of the SM

$$\Delta\mathcal{L} = -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \left|(\partial_\mu - ig_x A_\mu)\Phi\right|^2 - V^{\rm h}_{\rm eff}(\Phi) + \bar{D}(\mathrm{i}\gamma^\mu\partial_\mu - m_D)D - \frac{\delta}{2}A_{\mu\nu}B^{\mu\nu} - g_x Q_D \bar{D}\gamma^\mu D A_\mu\,,$$

with the temperature dependent potential

$$V^{\rm h}_{\rm eff}(\Phi, \mathrm{T_h}) = \mathrm{V_{0h}} + \mathrm{V^{(0)}_{1h}} + \Delta\mathrm{V^{(T_h)}_{1h}} + \mathrm{V^{daisy}_h}(\mathrm{T_h})\,.$$

For the one-loop thermal correction we have

$$\Delta V^{(T_h)}_{1h}(\chi_c, T_h) = \frac{T_h^4}{2\pi^2}\left[3J_B\left(\frac{m_A}{T_h}\right) + J_B\left(\frac{m_\chi}{T_h}\right) + J_B\left(\frac{m_{G^0_h}}{T_h}\right)\right]$$

where J_i $(i = B, F)$ is defined so that at one loop

$$J_i\left(\frac{m_i}{T_h}\right) = \int_0^\infty \mathrm{d}q\, q^2 \ln\left[1 \mp \exp\left(-\sqrt{q^2 + m_i^2/T_h^2}\right)\right], \quad i = (B, F)\,,$$

where (B, F) stand for bosonic and fermionic cases.

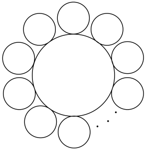
daisy resummation

The daisy loop contributions are only for the longitudinal mode of A and χ and are given for mode $i = A, \chi$ so that

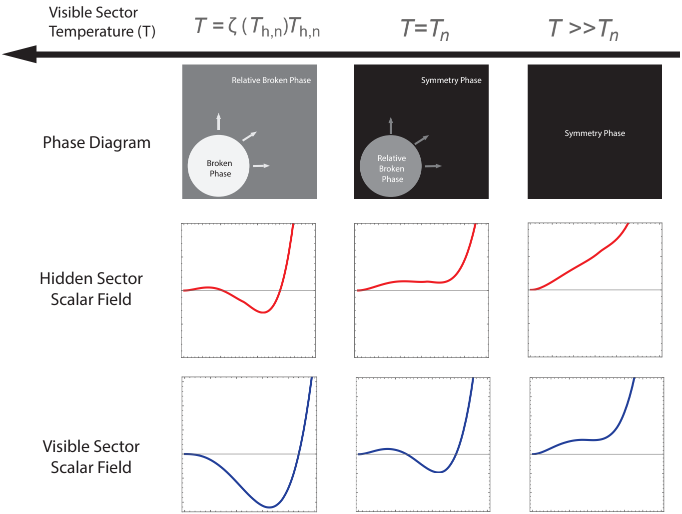
$$V^{\text{daisy}}(i, T_h) = -\frac{T_h}{12\pi}\left\{\left[m_i^2 + \Pi_i(T_h)\right]^{3/2} - m_i^3\right\},$$

where $\Pi_i(T_h)$ is thermal contribution to the zero temperature mass m_i^2. For the longitudinal mode of A and for χ they are given by

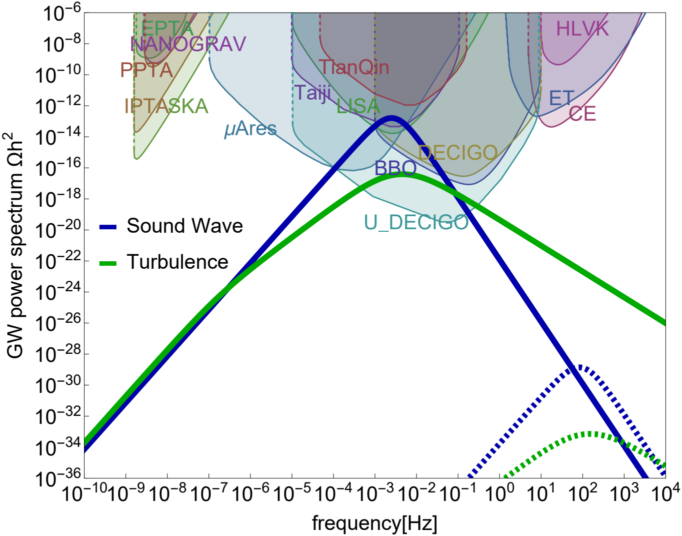
$$\Pi_A(T_h) = \frac{2}{3}g_x^2 T_h^2\,, \quad \Pi_\chi(T_h) = \frac{1}{4}g_x^2 T_h^2 + \frac{1}{3}\lambda_h T_h^2.$$



Two field tunneling: SM Higgs and hidden scalar



GW signal, SM vs Hidden



Conclusion

- A general method of computing freeze-in production of hidden sector which involves inner interactions given by [Aboubrahim, WZF, Nath, Wang, 2008.00529] is reviewed.
- A general discussion of the $U(1)$ mixings as well as the most comprehensive study of the sub-GeV freeze-in millicharge dark matter [WZF, Zhang, Zhang, 2312.03837] is presented.
- The dark matter interpretation of the 511 keV signal is discussed. In the models we study, the freeze-in mechanism generates the entire dark matter relic density, and thus any types of additional dark matter components produced from other sources are unnecessary [WZF, Zhang, 2405.19431].
- Freeze-in $U(1)$ hidden sectors reconstructing low energy SUSY spectrum is discussed. [Aboubrahim, WZF, Nath, 1910.14092, 2003.02267].
- Study of gravitational waves opens up a new avenue for the exploration of hidden sector physics [WZF, Nath, Li, 2403.09558].