



*GRAVITATIONAL  
WAVES FROM DARK  
CONFINEMENT  
WITH HOLOGRAPHY*

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2210.11821, 240X.XXXXX, (2XXX.XXXXX)

# MOTIVATION & MAIN POINTS



- Gravitational Waves from FOPT in Strongly coupled QFTs (“resembling known theories”) is likely not visible in the future due to lack of supercooling. (In contrast to previous attempts in RS using AdS/CFT)
- The AdS/CFT Correspondence for FOPTs in strongly coupled QFTs described by HP PT provide an bound on the amount of supercooling.

# OUTLINE

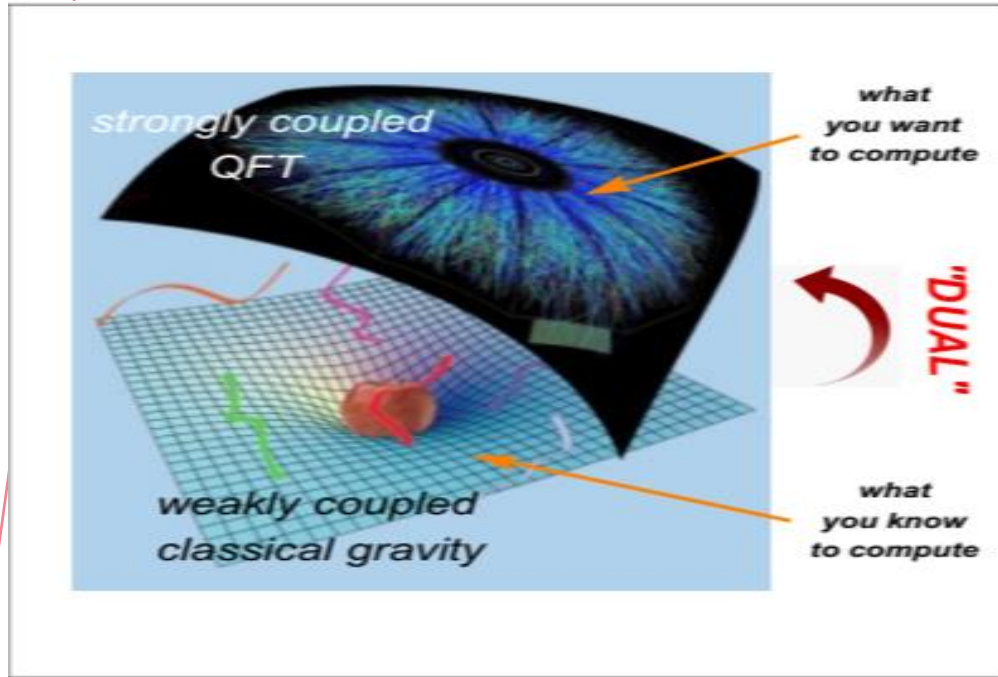


Figure from Baggioli 1908.02667

Interlude to AdS/CFT

Our Holographic model of interest for  $SU(N)$  pYM.

Gravitational waves from an FOPT

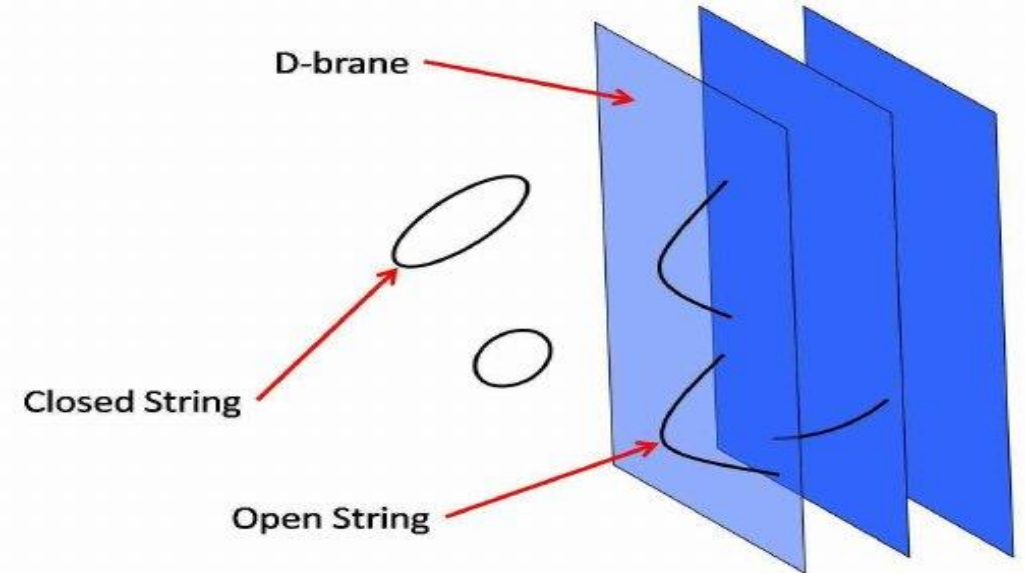
Outlook & Future interests

# ADS/CFT CONJECTURE! (9711200)

In this work we wish to study  $SU(N)$  pYM

$\mathcal{N} = 4, \text{SUSY } SU(N_c) \text{ YM} = \text{IIB String Theory on } AdS_5 \times S_5$

C(Q)FT in d-Dimensions	String/Gravity in d+1-Dimensions
$\mathcal{Z}_{QFT}$	$\mathcal{Z}_{String/Gravity}$
$\mathcal{W}_{QFT}$	$-\mathcal{S}_{0\delta}$
Temperature T	Hawking Temperature BH
Entropy S	Hawking Entropy
Large N Limit	Classical GR in $AdS_5 \times S_5$



Two immediate questions emerge:

- $SU(N)$  pYM Theory is not a CFT?
- Furthermore it is also not Supersymmetric.

# *NON CONFORMALITY/ NO SUSY?*

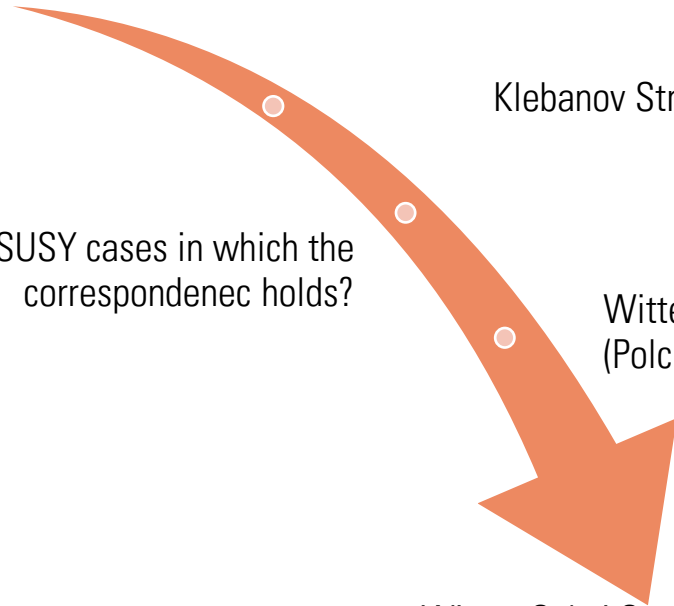
Are there gravity duals to non-conformal  
Strongly coupled QFT's?

Klebanov Strassler

Non SUSY cases in which the  
correspondence holds?

Witten Yang Mills Model,  
(Polchinski Strassler)

Witten Sakai Sugimoto Model is  
a Top down example



# *PROS/CONS ??*

## TOP DOWN HOLOGRAPHY

- Dictionary known precisely, one knows exactly what one talks about.
- Dual theories stemming from string theory somewhat confident in no pathologies
- Limited set of theories with known explicit duals (SUSY, CFT)
- Enormous large amounts of fields and content in each theory to account for

## BOTTOM UP HOLOGRAPHY

- Large amount of flexibility as one imposes the sought dynamics one wish to study
- In certain cases the models can be relatively simple yet possess rich dynamics
  - This approach has been useful for understanding various dynamics and features in Hydrodynamics, QCD and Condensed Matter

In principle there could be unwanted/hidden pathologies in the constructed theory

# *IMPROVED HOLOGRAPHIC*

# *QCD*

0707.1324, 0707.1349, 0812.0792,  
0903.2859, 0906.1980

This model is not provided from a string theory. Hence we possess a large  $N$  gauge theory at strong coupling with certain features.

# *IHQCD VACUUM THEORY*

$$ds^2 = b^2(r)(dr^2 - dt^2 + dx^m dx_m), \quad \Phi(r),$$

$$\mathcal{S}_5 = -M_p^3 N_c^2 \int d^5 x \sqrt{g} \left( R - \frac{4}{3} (\partial\Phi)^2 + V(\Phi) \right) + 2M_p^3 \int_{\partial\mathcal{M}} d^4 x \sqrt{h} \mathcal{K} + \text{CT}$$

- AdS<sub>5</sub> Einstein-dilaton gravity
- Radial 5-D coordinate  $r$
- Scalar field  $\lambda = e^\Phi$
- 4D Strongly coupled QFT
- RG scale
- T'hooft coupling  $\lambda_t = N_c g_{YM}^2$



# IHQCD VACUUM IN THE UV REGIME

- Vacuum equations of motion

$$6 \frac{\dot{b}^2}{b^2} + 3 \frac{\ddot{b}}{b^2} = b^2 V, \quad 6 \frac{\dot{b}^2}{b^2} - 3 \frac{\ddot{b}}{b^2} = \frac{4}{3} \dot{\Phi}^2,$$

- 4-D energy scale given by  $E = E_0 b(r)$ .

- Furthermore imposing the identification  $\lambda = \kappa \lambda_t$

- Defining the holographic beta function as

$$\beta(\lambda) = \frac{d\lambda}{d \log E} = \lambda \frac{\dot{\Phi}}{\dot{A}}, \quad A(r) = \log b(r),$$

- Introducing scalar variables  $X(\lambda) = \frac{\beta(\lambda)}{3\lambda}$ . ("Gluon Condensate")

$$\lambda \frac{dX}{d\lambda} = -\frac{4}{3} (1 - X^2) \left( 1 + \frac{3}{8X} \lambda \frac{d \log V}{d\lambda} \right),$$

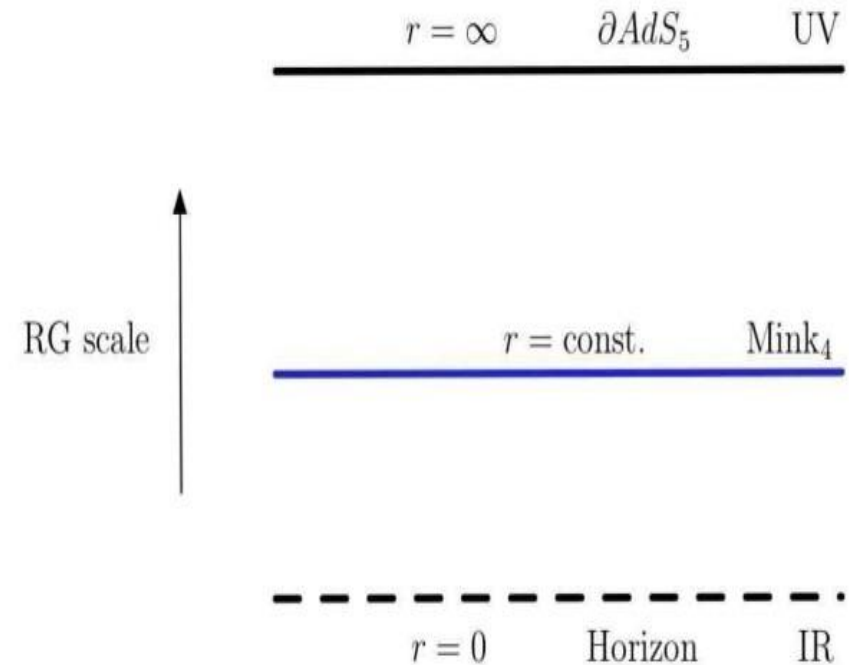
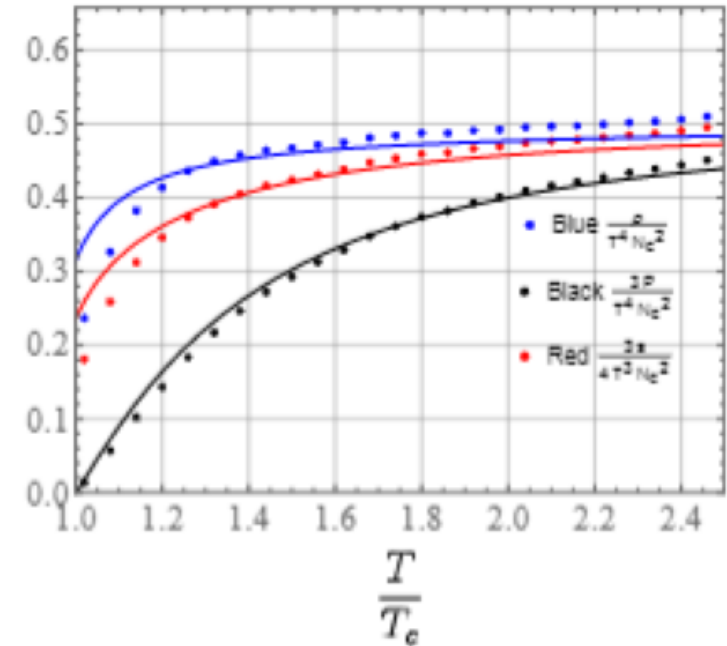


Figure from Mateos 0709.1523

# MAIN BOTTOM UP ASPECT!

$$V = \frac{12}{l^2} \left( 1 + V_0 \lambda + V_1 \lambda^{\frac{4}{3}} \left( \log \left[ 1 + V_2 \lambda^{\frac{4}{3}} + V_3 \lambda^2 \right]^{\frac{1}{2}} \right) \right)$$

- Fix available parameters  $V_0, V_1, V_2, V_3$
- $V_0, V_2$  fixed to reproduce YM  $\beta$  function at 2 loops.
- $V_1, V_3$  fitted against SU(3) lattice data in the IR.



Here we display the direct comparison using the set of potential parameters provided by original authors. Data from

# SOLUTIONS AT $T \neq 0$

AdS Schwarzschild BH solution Deconfined Phase

$$ds^2 = b^2(r) \left( \frac{dr^2}{f(r)} - f(r)dt^2 + dx^m dx_m \right)$$

$$\Phi = \Phi(r), \quad r \in (0, r_h), \quad f(r_h) = 0$$

Thermal Graviton Gas Solution  
Confined Phase

$$ds^2 = b_0^2(r)(dr^2 - dt^2 + dx^m dx_m)$$

$$\Phi = \Phi_0(r), \quad r \in (0, \infty)$$

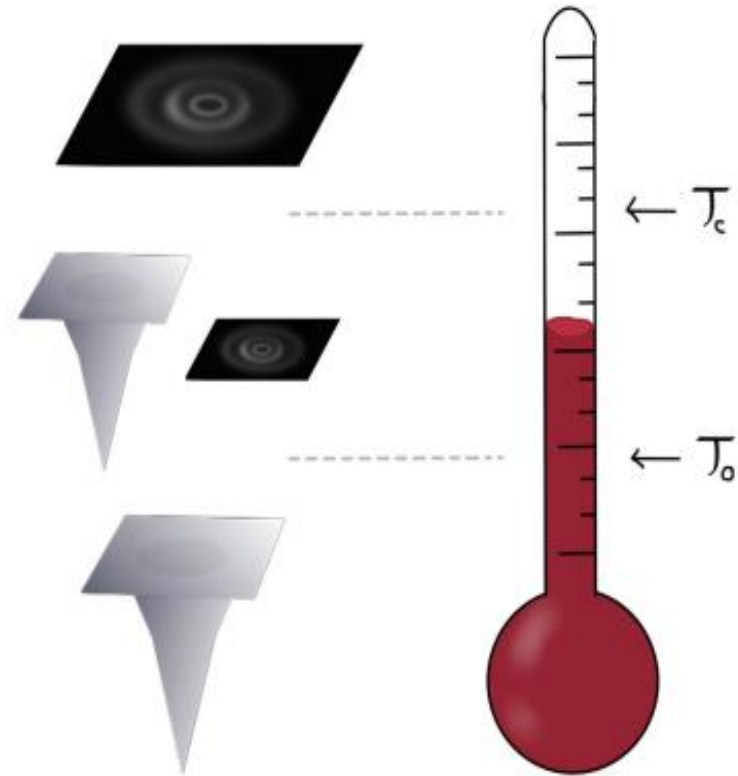
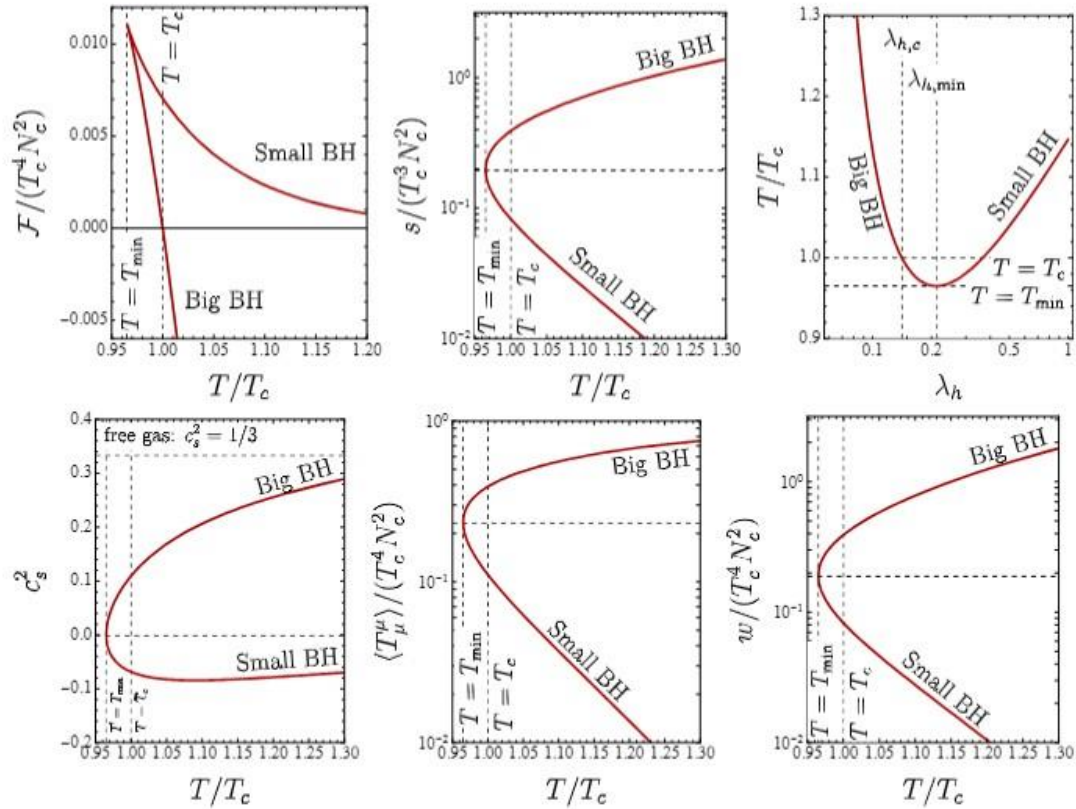


Figure from Dewolfe 1304.7794

# THERMODYNAMICS



- Temperature, Time periodicity  $\tau \rightarrow \tau + \frac{1}{T}$ .
- $S = \frac{\text{Area}}{4G_5} =$
- $4\pi M_p^3 N_c^2 V_3 b(r_h)^3$
- $\mathcal{F} = \frac{\beta}{V_3} (S_{dc} - S_{cn.})$
- $T_h \equiv \frac{|\dot{f}(r_h)|}{4\pi} = T$

# CENTRAL IDEA/WHY USING ADS/CFT!!

$$\mathcal{Z}_\phi[\Phi_0]_{\text{Gravity}} = \mathcal{Z}_O[\Phi_0]_{\text{CFT}}$$

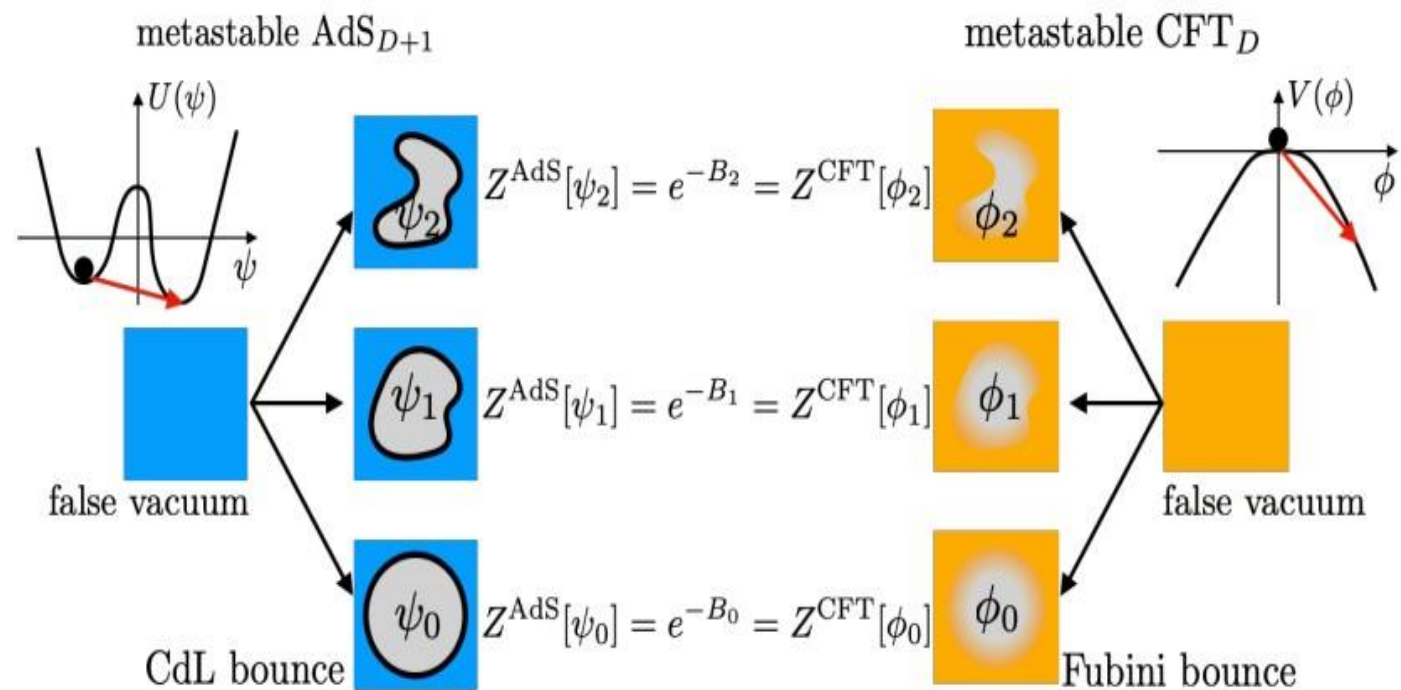
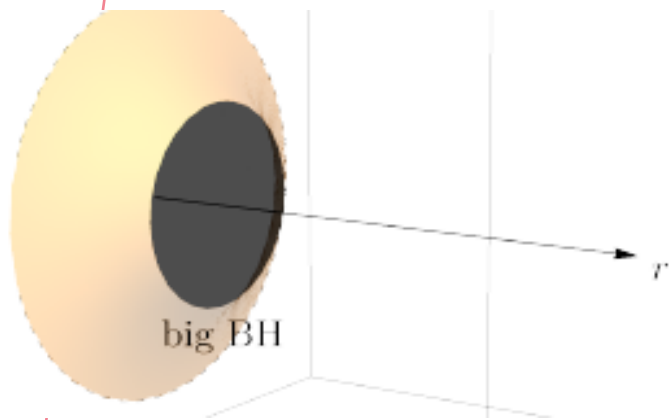


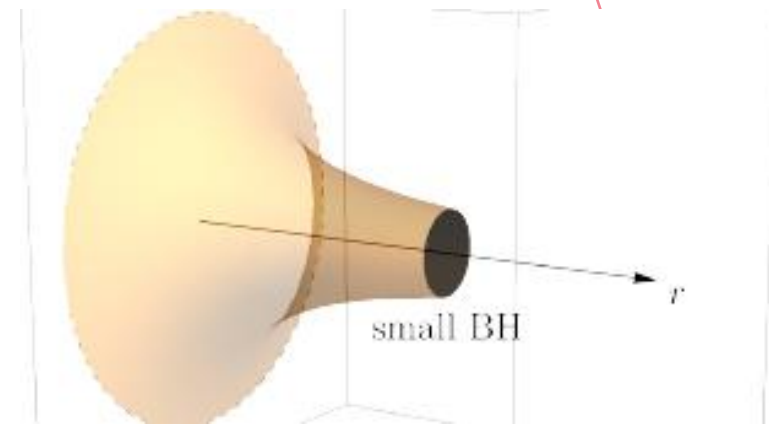
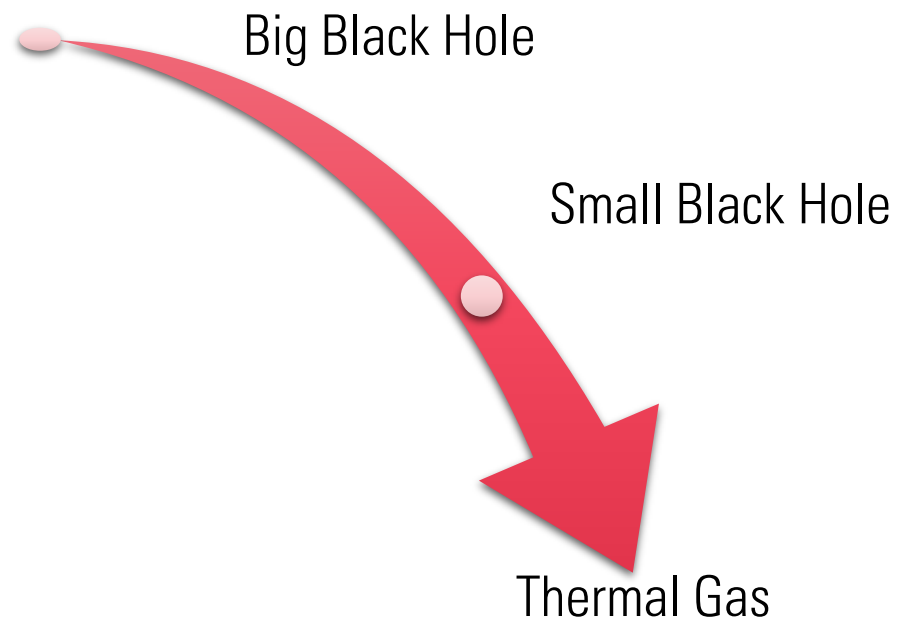
Figure from 2308.02159.

# Hawking Page Phase Transition "dual" to YM confinement PT

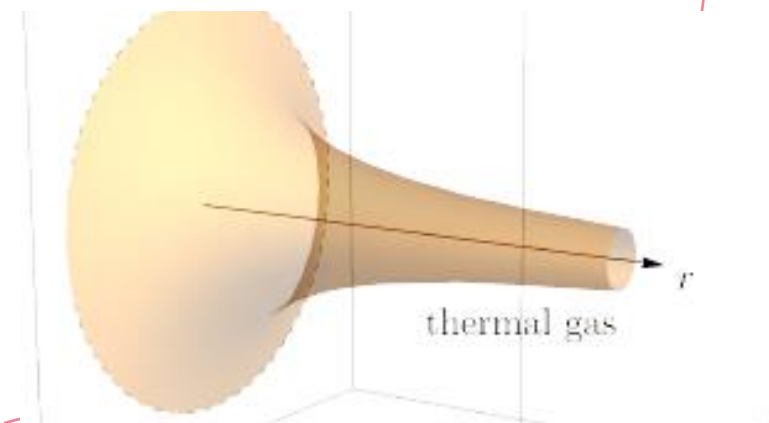


Calculation of the evaporation of a BBH/ Nucleation of Thermal Gas

$$\Gamma = Ae^{-(S_{BH} - S_{TG})}$$

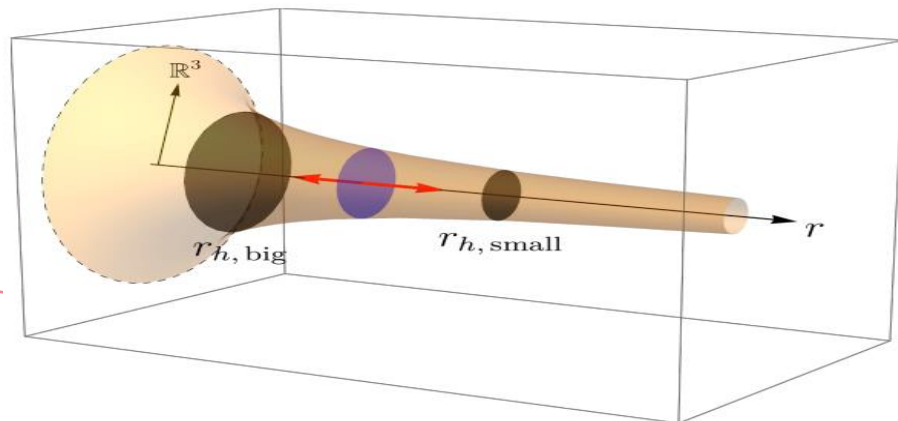


SBH fits the description of an instanton! (Negative Eigenvalue)

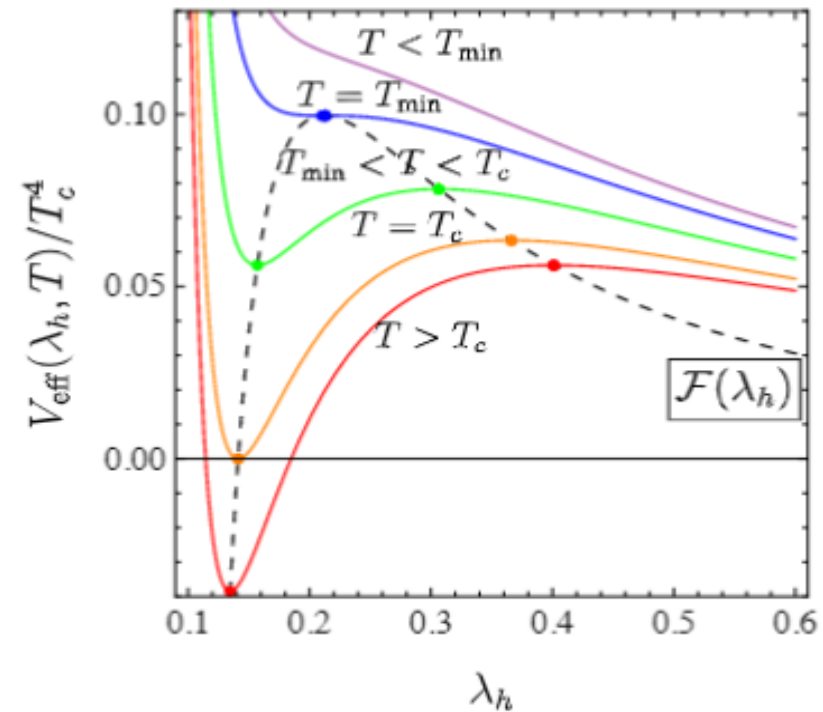


# EFFECTIVE ACTION FOR TUNNELING I

- Interpolate between BBH and SBH
- Violate the condition  $T_h \neq T$ 
  1. BH not in thermal eq.
  2. Conical singularity
- Regularize with spherical cap

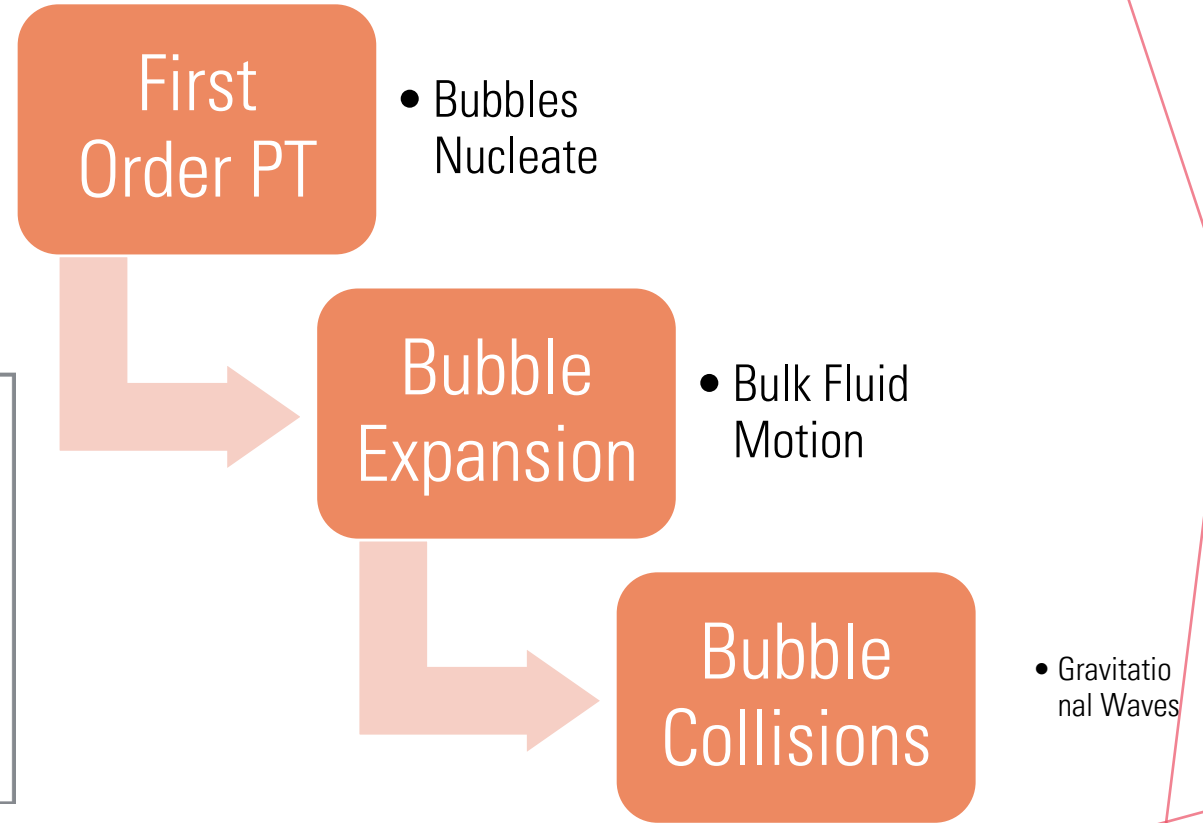
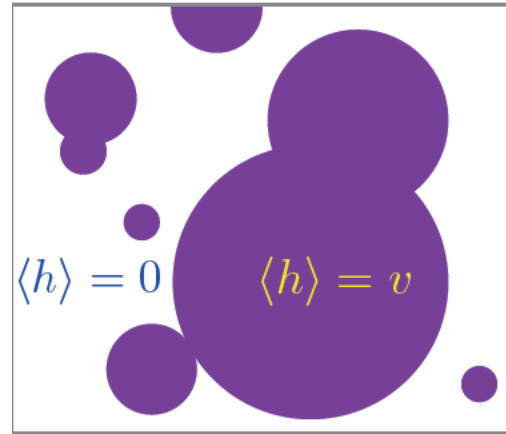
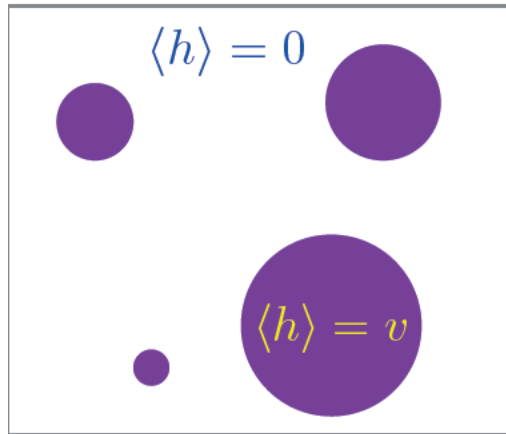


$$V_{\text{eff}}(\lambda_h, T) = \mathcal{F}(\lambda_h) - 4\pi M_p^3 N_c^2 b(\lambda_h)^3 \left(1 - \frac{T_h}{T}\right)$$



# GRAVITATIONAL WAVES FROM FOPT

- Release of vacuum energy  $\Delta v$ ,
- Drives bubble expansion and fluid motion which sources Gravitational waves





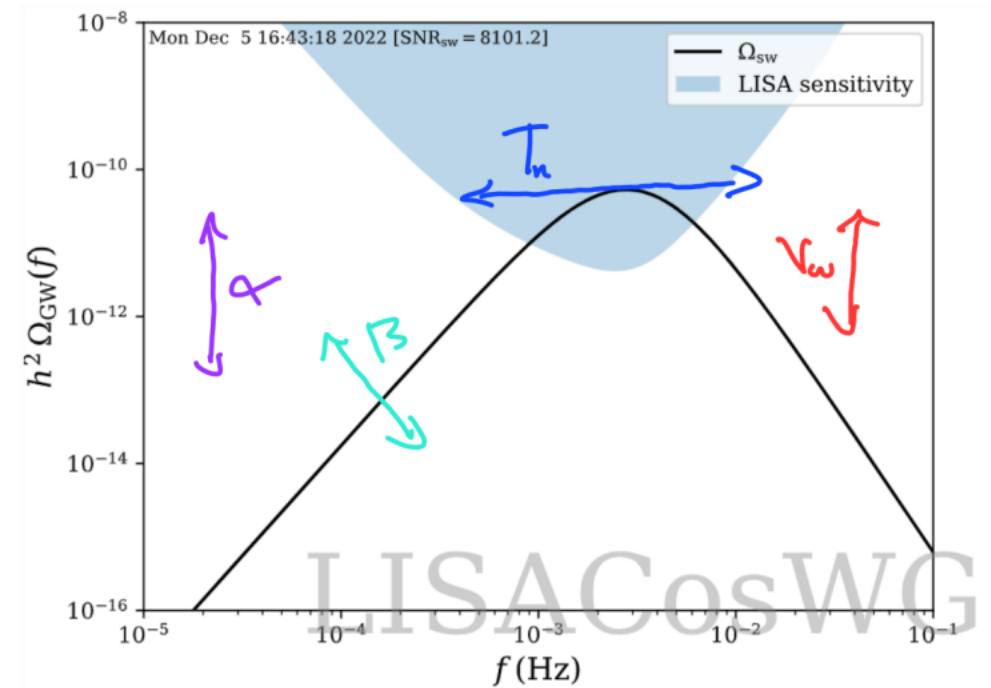
# GRAVITATIONAL WAVES FROM FOPT II

## How to compute GW signal?

- Hydrodynamical lattice simulations of a scalar field coupled to the plasma.
- Imposing Local Thermal Equilibrium simplifies things

## PT is characterized by few parameters:

- PT strength (Latent Heat)  $\alpha \sim \frac{\Omega_{\text{vac}}}{\Omega_{\text{rad}}}$
- Bubble wall velocity  $v_w$
- Bubble nucleation rate  $\beta_*$
- PT temperature  $T_*$



# GRAVITATIONAL WAVES FROM FOPT III

$$F_{GW,0} = (3.57 \pm 0.05) \cdot 10^{-5} \left( \frac{100}{g_*} \right)^{\frac{1}{3}}$$

Numerical  
Prefactor  $\sim 10^{-2}$

$$\begin{aligned} f &= f^3, & f < f_p \\ f &= f^{-4}, & f > f_p \end{aligned}$$

- $$\frac{d\Omega_{GW,0}}{d \ln f} = 0.687 F_{GW,0} K^{\frac{3}{2}} (H(T_*) R(T_*))^2 \overline{\Omega_{GW} C} \left( \frac{f}{f_{p,0}} \right)$$

$$K = \frac{\kappa(\alpha)\alpha}{1 + \alpha}$$

$$\kappa(\alpha) = \frac{\alpha}{0.73 + 0.083\sqrt{\alpha} + \alpha}$$

$$f_{p,0} \simeq 26 \left( \frac{1}{R_* H(T_*)} \right) \left( \frac{z_p}{10} \right) \left( \frac{T_*}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{\frac{1}{6}} \mu\text{Hz}$$

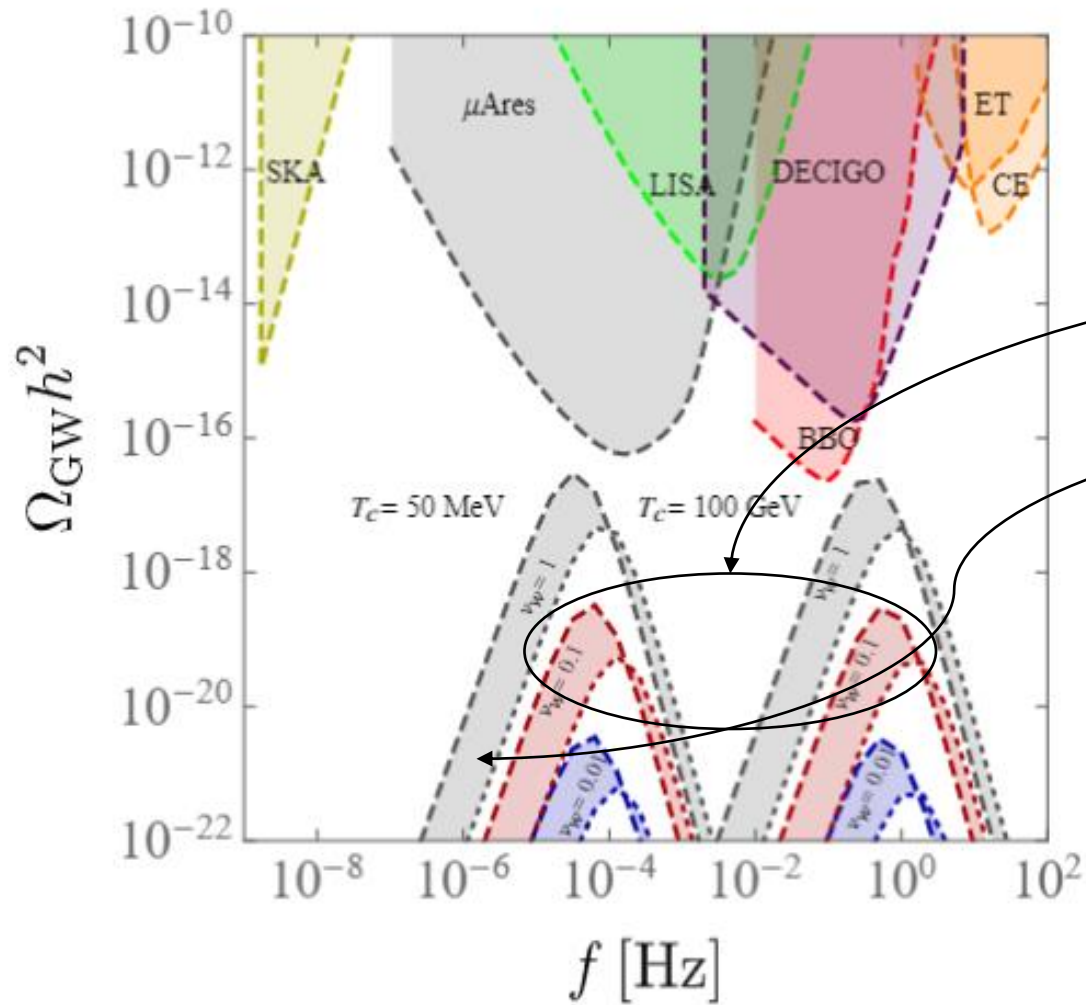
Source duration time of acoustic  
production  $\frac{\beta_*^{-2}}{H_*}$

One Major Drawback here: The simulation at which these semi-analytic templates have only been calculated at weak coupling/ weak transition strengths. Hence spectral shapes/dependencies may change at strong coupling

# *EFFECTIVE ACTION FOR TUNNELING II*

- Kinetic term normalization:  $c \frac{N_c^2}{16\pi^2} (\vec{\nabla}\lambda_h)^2$
- Effective action for  $\mathcal{O}(3)$  tunneling configurations (Thermal fluctuation!)
- $\mathcal{S}_B = \frac{4\pi}{T} \int dr r^2 \left[ c \frac{N_c^2}{16\pi^2} (\partial_r \lambda_h(r))^2 + V_{\text{eff}}(\lambda_h(r), T) \right]$
- Bubble Nucleation Rate
- $\Gamma \approx T^4 \left( \frac{\mathcal{S}_B}{2\pi} \right)^{3/2} e^{-\mathcal{S}_B}$ , Nucleation  $\Gamma \approx H^4$ .
- $T_n \approx T_p \approx 0.99T_c$
- PT strength (energy released)
- $\alpha = \frac{4}{3} \frac{\Delta\theta}{\Delta w} = \frac{1}{3} \frac{\Delta\rho - 3\Delta p}{\Delta w} \sim 0.34$
- Inverse PT rate
- $\frac{\beta}{H} = T \left( \frac{d\mathcal{S}_B}{dT} \right) \sim 10^5$

# GW SPECTRA SU(3) YANG-MILLS

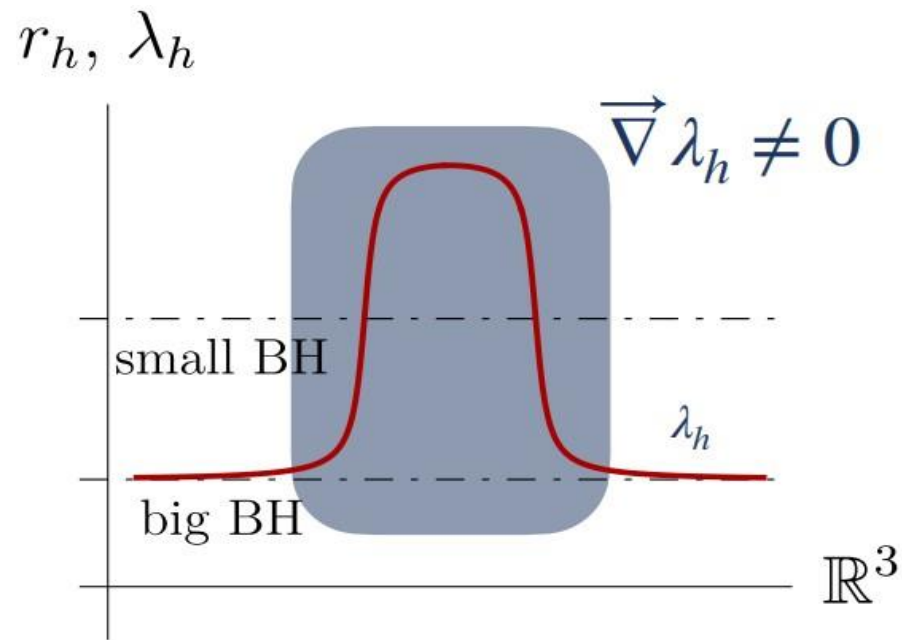


- Bubble wall velocity realistically?
- Kinetic term coefficient  $c=1$  ??
- These curves should be thought of as “naive” estimates.
- Other methods have been attempted/applied like PLM, Matrix Models, and Thin Wall estimations.
- Nothing beyond the Bounce Solution has been attempted here so far for these purposes to my knowledge

# KINETIC TERM COEFFICIENT

Preliminary

$$\mathcal{S}_B = \frac{4\pi}{T} \int dr r^2 \left[ c \frac{N_c^2}{16\pi^2} (\partial_r \lambda_h(r))^2 + V_{\text{eff}}(\lambda_h(r), T) \right]$$



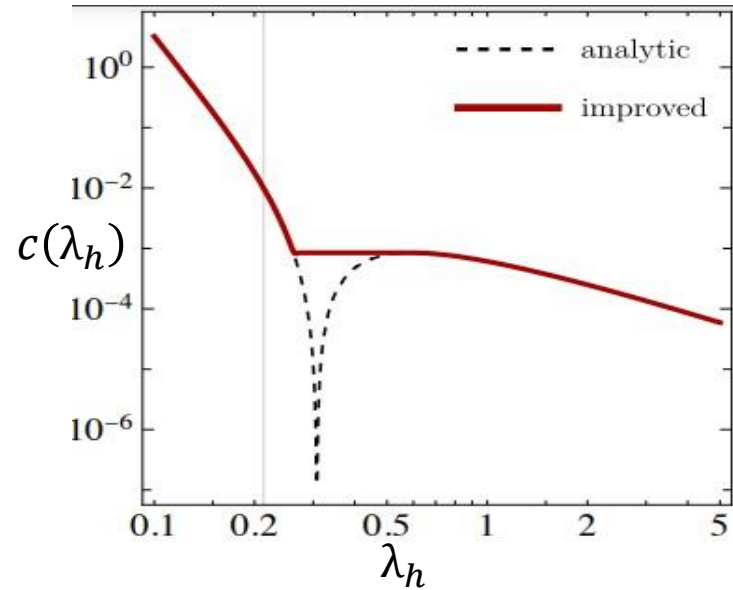
We want to evaluate the action on a bubble configuration.

$$\mathcal{S} \supset \int d^5x \dots (\vec{\nabla} \lambda_h)^2 + \dots (\vec{\nabla} \lambda_h)^4 + \dots$$

Is  $c \sim 1$  a reasonable approximation??

# KINETIC TERM COEFFICIENT II

Preliminary



Kinetic term pole at 0.3 seems to be a numerical issue. Matching at a lower cut off  $\lambda_0$  in value removes this feature.

$$\mathcal{S}_B = \frac{4\pi}{T} \int dr r^2 \left[ c(\lambda_h) \frac{N_c^2}{16\pi^2} (\partial_r \lambda_h(r))^2 + V_{\text{eff}}(\lambda_h(r), T) \right]$$

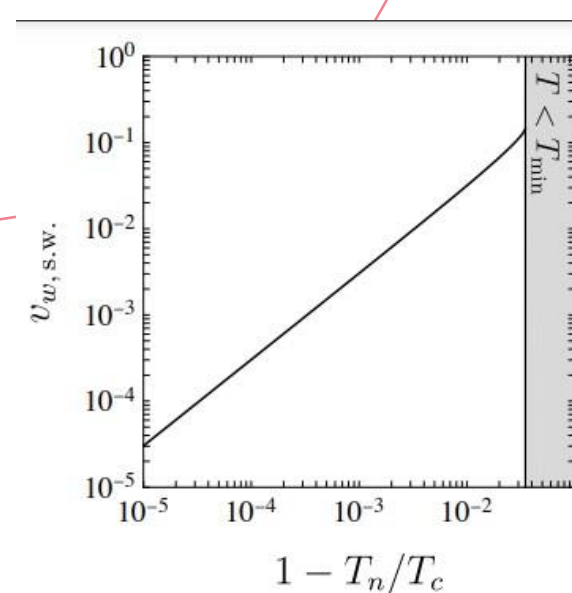
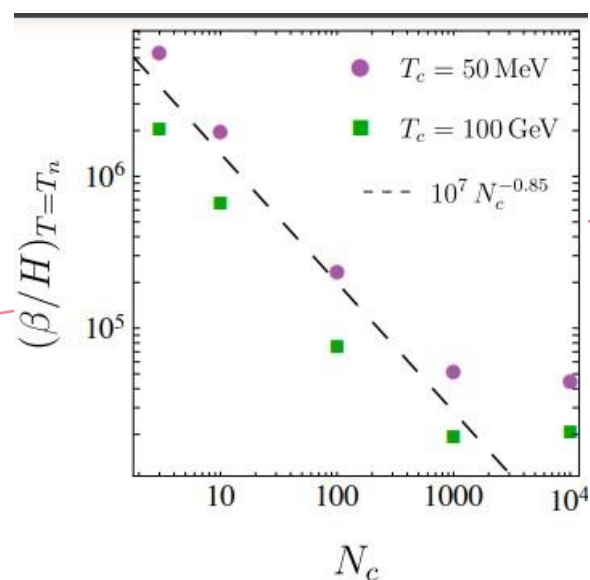
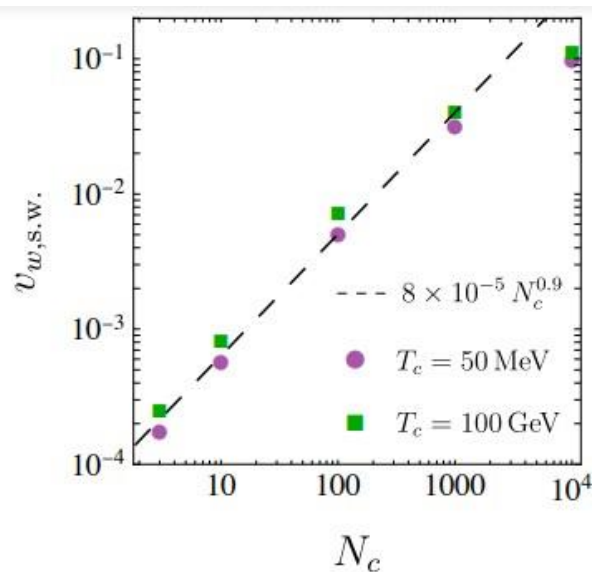
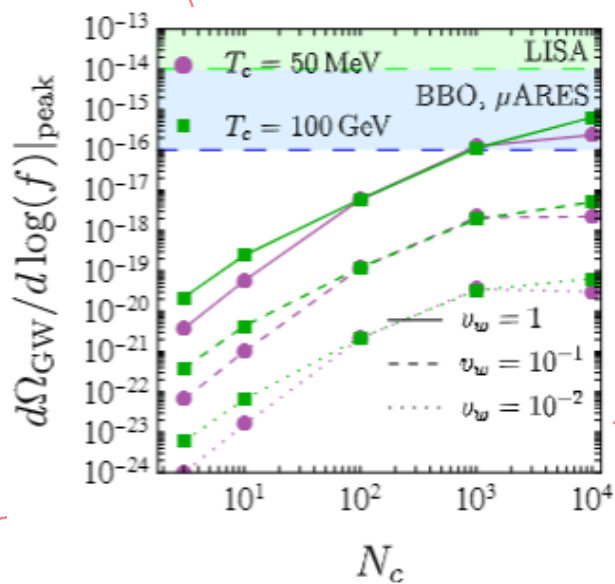
The kinetic term is small. We shall expect 2 features.

- The PT is faster hence further GW suppression
- A Thin wall approximation might be valid for these transitions

Dual Quantity: Wave function renormalization factor and its evolution w.r.t the t'hooft coupling.

# GRAVITATIONAL WAVES FROM $SU(N)$ PYM

Preliminary



# *CONCLUSIONS & OUTLOOK*

- AdS/CFT provides a upper bound on supercooling from its geometrical construction of the theories of concern.
- We should not expect to observe GWs from strongly coupled FOPT in theories with limited supercooling.
- Still much to understand in regards to frameworks methods and their various applicability.
- Exploring options for calculating bubble wall velocities
- Inclusion of flavor/axion contributions.
- Phase separated phenomena dynamical instabilities.
- Hydrodynamical analysis of plasma and its properties.
- Black Hole perturbation theory for fluctuating determinant
- What about considering Top Down examples?