Domain wall networks
and their cosmological signatures

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MITP Program
Effective Theories for non-perturbative Physics
5-17 August 2024
14-8-24

Preliminary evidence : Römische Theater

Overview

→ Left-right symmetric model

  – A minimal esthetic extension of SM

→ Generic domain walls in left-right symmetric models

  – Leptogenesis

  – Gravitational waves

→ Supersymmetric version and SO(10) embedding

  – strategies for DW removal

  – Relating CP violation to EDM

→ Possibility of a two peak Gravitational Wave signal

1 Genesis of baryogenesis

Just a little asymmetry

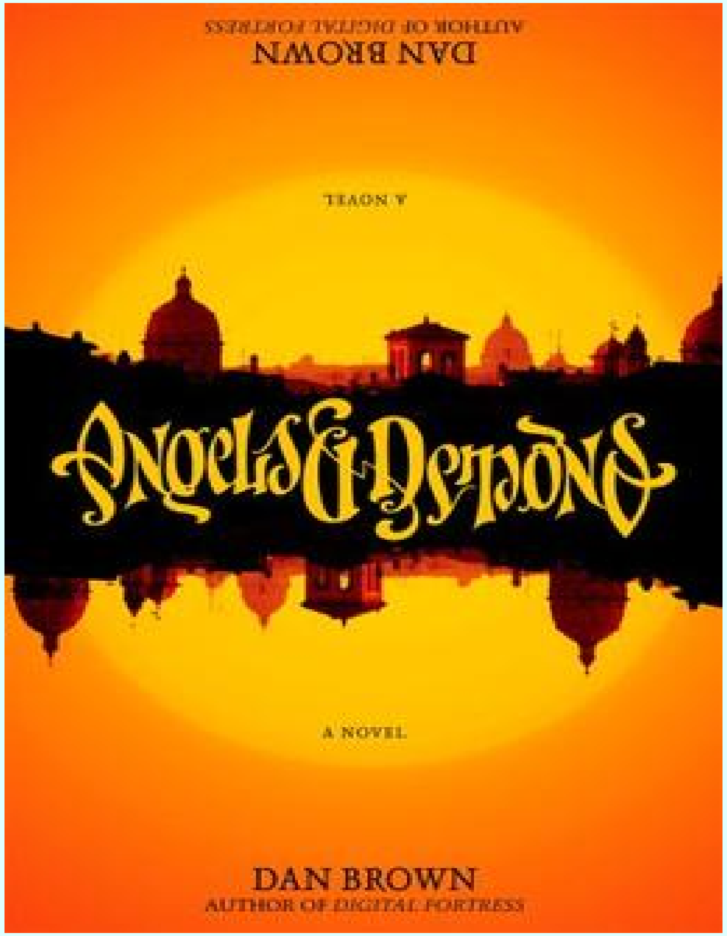
Annus Mirablis of Cosmology
Two discoveries of 1964

→ CP violation in $K^0 - \overline{K^0}$

→ CMB !!!

Weinberg comment in Brandeis Lectures 1964.

Sakharov model elucidating the criteria 1967



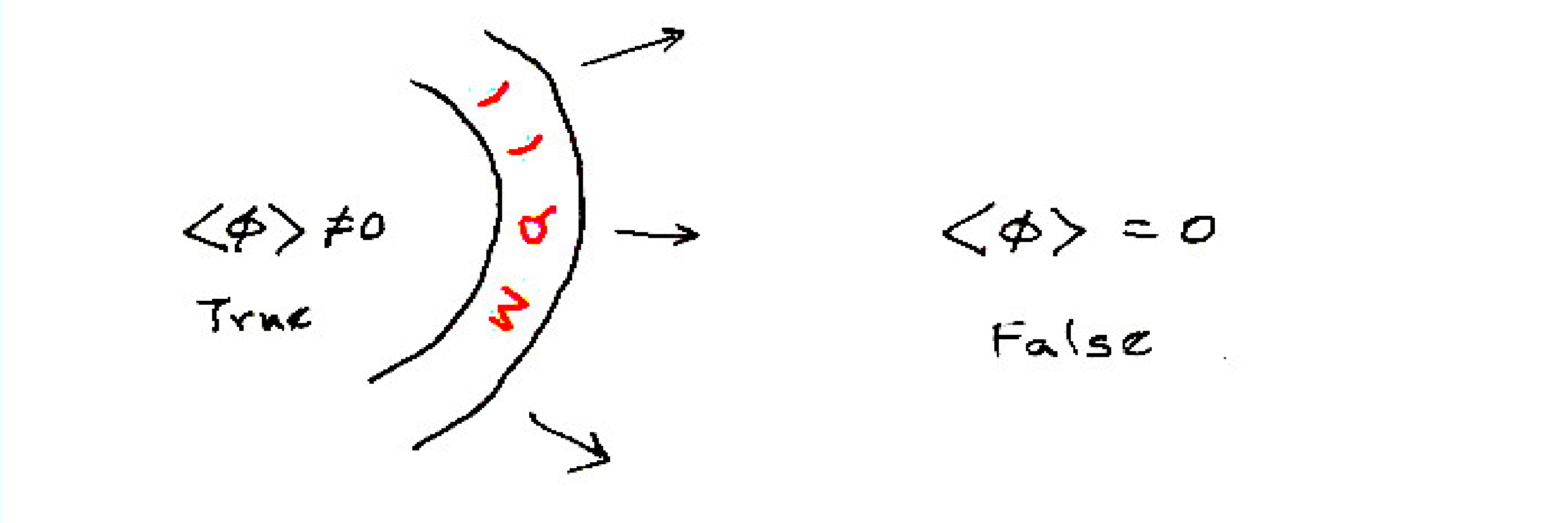
1.1 GUT baryogenesis

- Out of equlibrium decays of GUT scale leptoquarks
- The Particle Physics rates and expansion rate of the Universe compete : out-of-equilibrium decays

$$\Gamma_X \cong \alpha_X m_X^2 / T \; ; \qquad H \cong g_*^{1/2} T^2 / M_{Pl}$$

1.2 Electroweak baryogenesis (Low scale)

- Expansion rate H too slow at electroweak scale
  - need another source of out of equilibrium conditions
  - moving phase boundaries of a First Order Phase Transition (FOPT)



- But First order phase transition (FOPT) in SM requires Higgs mass to be $\lesssim$90GeV

2 Baryogenesis from leptogenesis

Replay the Baryon asymmetry recipes for Leptons

- Thermal solution : Out of equilibrium decays of Majorana neutrinos – high scale
- Phase transition : Moving phase boundaries of Left-Right symmetric model – can be low scale

2.1 Difficulties of High scale leptogenesis

out-of-equilibrium decays of heavy Majorana neutrinos
[see eg. Buchmüller, Di Bari and Plümacher (2004)]

- Getting Majorana neutrinos to be in equilibrium

$$M_N \gtrsim O(10^9)\mathrm{GeV}\left(\frac{2.5\times10^{-3}}{Y_N}\right)\left(\frac{0.05\mathrm{eV}}{m_\nu}\right)$$

- Have sufficiently large CP violation – assuming see-saw mechanism and 3 generations

$$|\varepsilon_{CP}| \lesssim 10^{-7}\left(\frac{M_1}{10^9\mathrm{GeV}}\right)\left(\frac{m_3}{0.05\mathrm{eV}}\right)$$

- Preventing washout of the produced asymmetry by the same Majorana neutrino mediated processes

3 Left-right symmetric model

(Mohapatra and Senjanovic 1970's; predecessor Pati and Salam 1974)

	T_L^3	T_R^3	$\frac{1}{2}X$	Q
ν_L	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0
e_L^-	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1
ν_R	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0
e_R^-	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1

	T_L^3	T_R^3	$\frac{1}{2}X$	Q
u_L	$+\frac{1}{2}$	0	$+\frac{1}{6}$	$+\frac{2}{3}$
d_L	$-\frac{1}{2}$	0	$+\frac{1}{6}$	$-\frac{1}{3}$
u_R	0	$+\frac{1}{2}$	$+\frac{1}{6}$	$+\frac{2}{3}$
d_R	0	$-\frac{1}{2}$	$+\frac{1}{6}$	$-\frac{1}{3}$

3.0.1 Higgs sector - suitable for neutrino see-saw

$$\Phi = (1, 2, 2, 0)$$

$$\Delta_L = (1, 3, 1, 2), \quad \Delta_R = (1, 1, 3, 2)$$

In the notation

$$\Delta_L \equiv \Delta_L^i T_L^i = \begin{pmatrix} \Delta^+ & \Delta^{++} \\ \Delta^0 & \Delta^- \end{pmatrix}$$

Choice of vev

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ l & 0 \end{pmatrix}, \qquad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ r & 0 \end{pmatrix},$$

$$\Phi = \begin{pmatrix} \kappa & 0 \\ 0 & k' \end{pmatrix}$$

- Introduced new species ν_R --> as a partner to e^-_R
- New gauge symmetry $SU(2)_R$
- Need a new hypercharge X --> turns out to be exactly $B - L$
- In praise of $B - L$ ...
  - the only conserved charge of SM which is not gauged!
  - Hereby it gains the status of being gauged
- Emerges naturally in $SO(10)$ unification

4 L-genesis in left-right symmetric world

- Assume flip symmetry $SU(2)_L \leftrightarrow SU(2)_R$ (possible in SO(10))
- $B - L$ automatically a local symmetry – ensures we start with a clean slate $B - L = 0$ at the Big Bang
- Two kinds of vacua - $SU(2)_L$ breaking or $SU(2)_R$ breaking
  - one desirable, the other accidental
- Big Bang universe has horizons
  - patchwork of both kinds of domains
- So we have

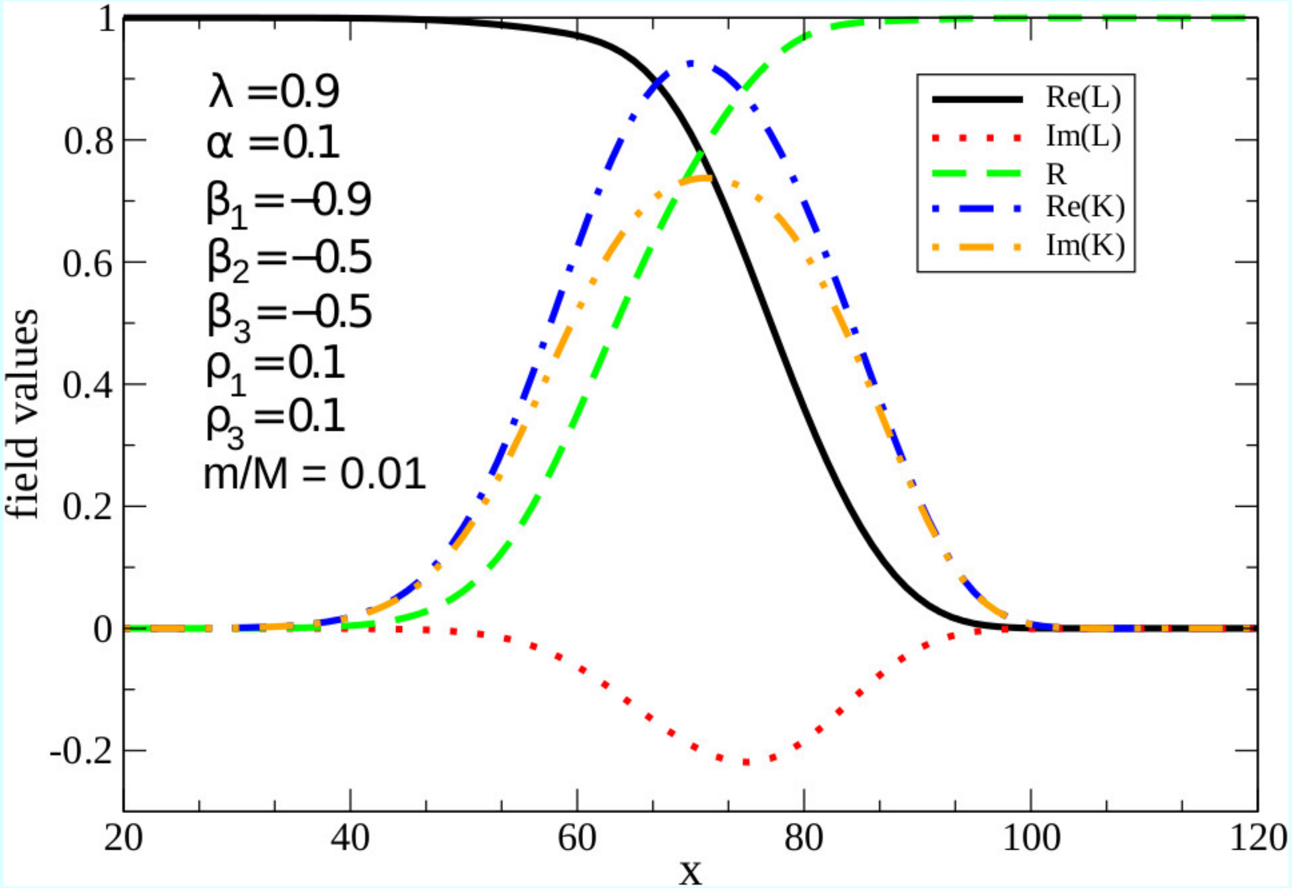
  → L number violation for Majorana neutrinos

  → Out of equilibrium wall motion (bring us to SM)

  → CP violation – transient values in the core of the DW

Moving phase boundaries at $SU(2)_R$ breaking

[Sarkar, Abhishek and UAY]



How can we verify this? The parable of the cow and the grass.
Electroweak baryogenesis models rely on

- a cosmological phase transition
- Movement of bubble walls
- CP violation within the width of the wall

The CP phase is transient : both time and space dependent.

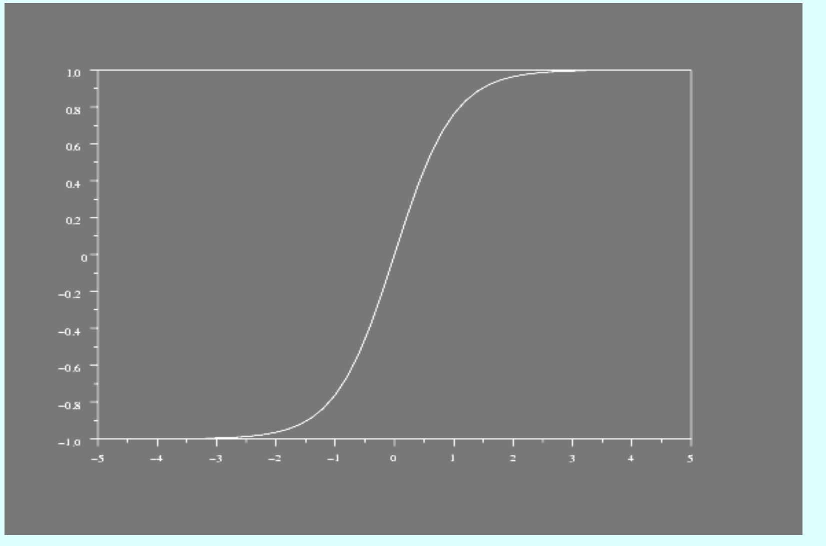
Further,

- Thermal letpgenesis with high scale has difficulties
- The EW Bgenesis scenarios can be adapted to BSM for leptogenesis

How to relate transient CP violation to low energy physics.

Key issues :

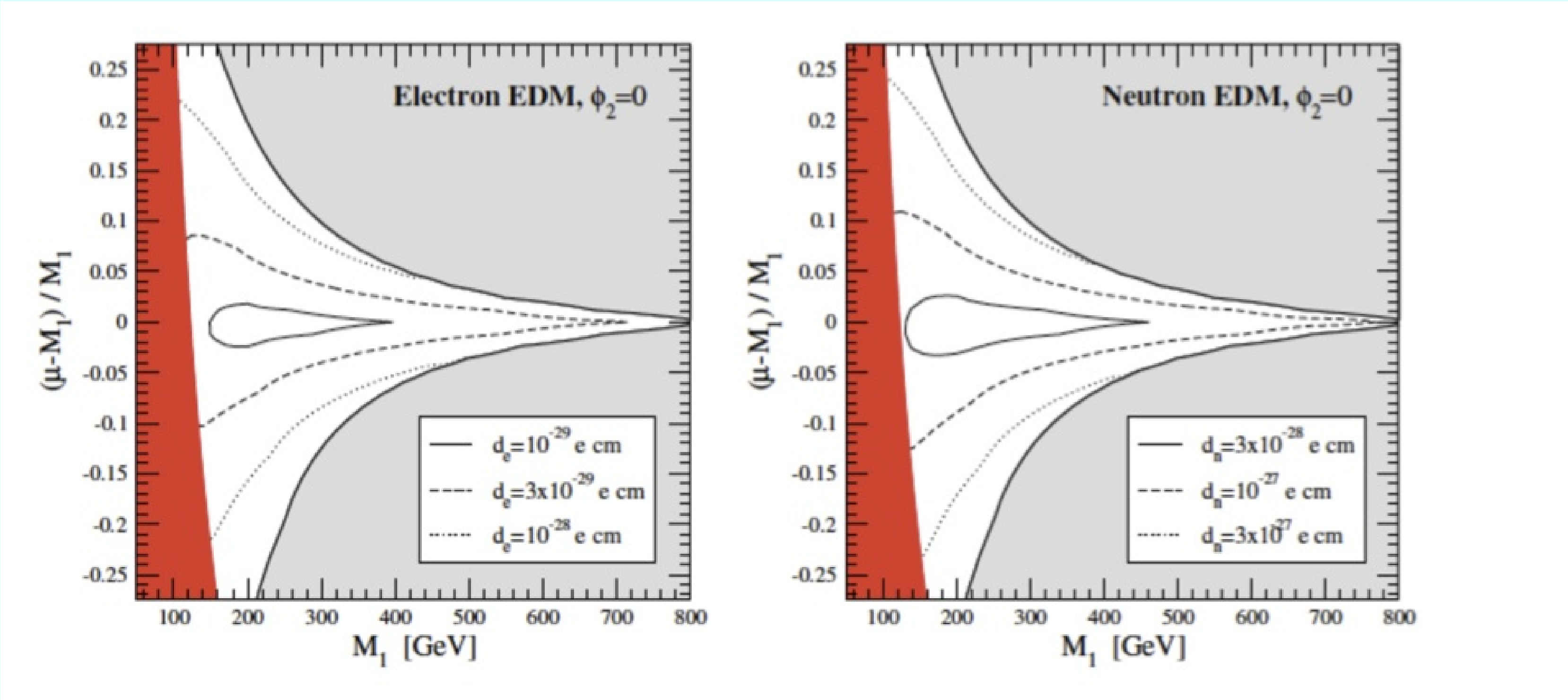
- Bubble walls are solitonic solutions
  - Space dependent *CP* phase is also solitonic
  - Machine errors in end values can produce a completely different curve.
    Difficulty relating values at a finite boundary to interior values.



- Bubble walls occur at finite temprature. Need to relate finite temprature parameter values to observable zero temperature values.

4.1 Relating *CP* violation to EDM

In MSSM++ models,



Morrissey and Ramsey-Musolf (2012)

$$d_f \cong \sin\delta_{CP}\left(\frac{m_f}{\mathrm{MeV}}\right)\left(\frac{1\mathrm{TeV}}{M}\right)^2 \times 10^{-26}\, e\,\mathrm{cm}$$

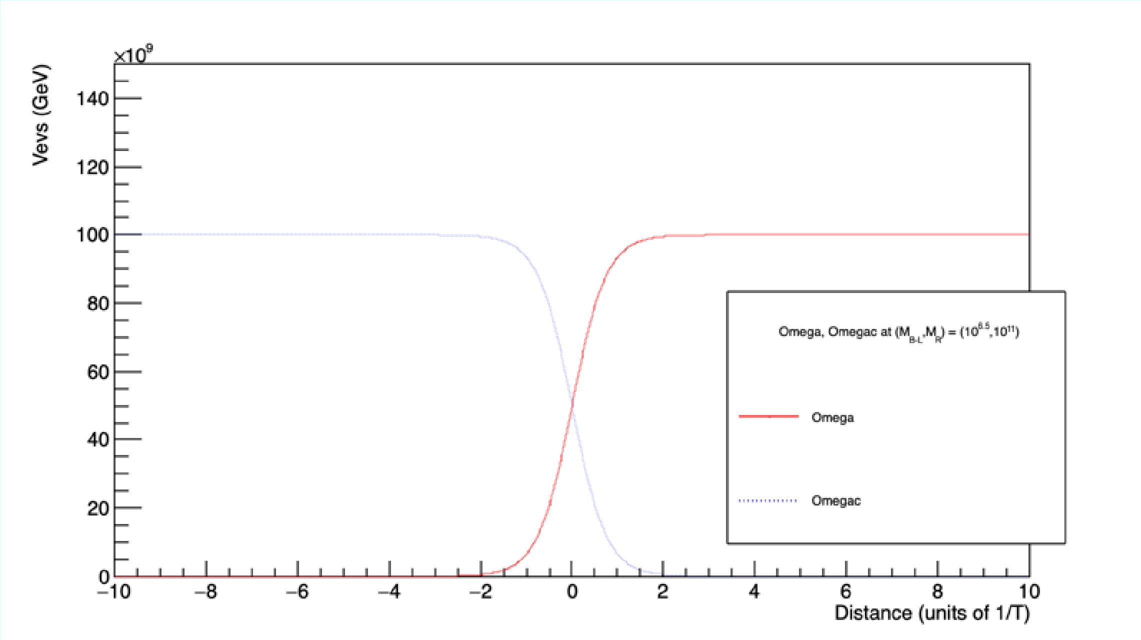
5 A renormalisable SUSY LR model

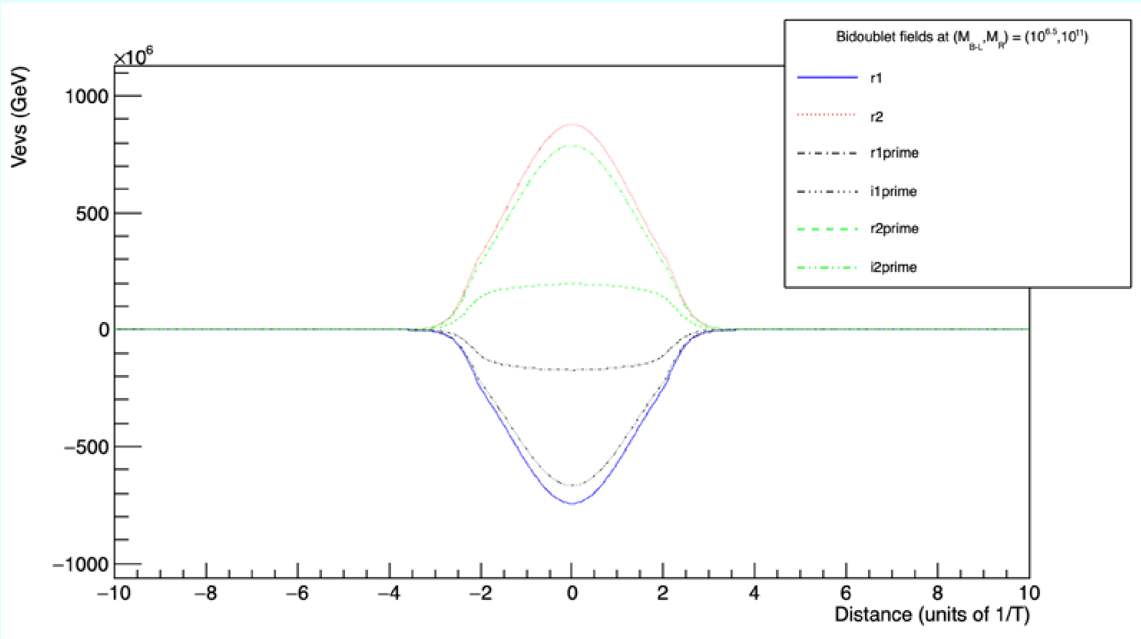
[Benakli, Aulakh, Senjanovic (1997)]

- Higgs content : superfields Bidoublet Φ, Triplets Δ_L, Δ_R, Δ_L^c, Δ_R^c with $B-L=\pm 2$, and new Triplets Ω_L, Ω_R with $B-L=0$.
  - Renormalisable model
- Two stage gauge symm breaking : $M_R \sim SU(2)_R \to U(1)_R$ and $M_{B-L} \sim U(1)_R \otimes U(1)_{B-L} \to U(1)_Y$

– Avoid new mass scale by imposing an R symmetry $\to M_{B-L}^2 \approx M_{EW} M_R$

$$
\begin{aligned}
W \;=\; & m_\Delta (\mathrm{Tr}\Delta\,\bar{\Delta} + \mathrm{Tr}\Delta_c\,\bar{\Delta}_c) + \frac{m_\Omega}{2}(\mathrm{Tr}\Omega^2 + \mathrm{Tr}\Omega_c^2) + \mu_{ij}\mathrm{Tr}\tau_2\,\Phi_i^T\,\tau_2\,\Phi_j \\
& + a\,(\mathrm{Tr}\Delta\,\Omega\,\bar{\Delta} + \mathrm{Tr}\Delta_c\,\Omega_c\,\bar{\Delta}_c) \\
& + \alpha_{ij}\,(\mathrm{Tr}\Omega\,\Phi_i\,\tau_2\,\Phi_j^T\,\tau_2 + \mathrm{Tr}\Omega_c\,\Phi_i^T\,\tau_2\,\Phi_j\,\tau_2)
\end{aligned}
$$





Piyali Banerjee
and UAY JHEP
(2021)

5.1 Corroborating with EDM

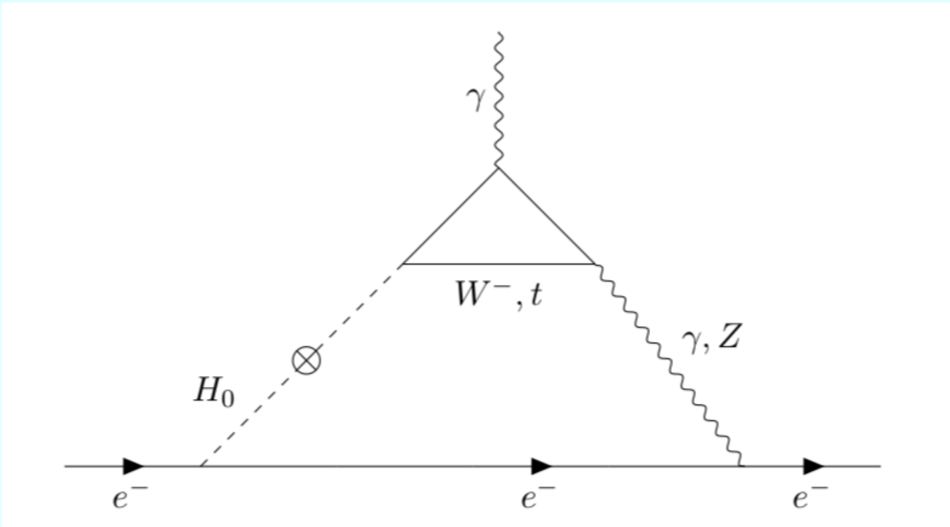
- Assume all scalar vevs entering the DW are taken to be corrected by temperature correction $O(g^2T^2)$

- In a simple bidouble Higgs model, a 1-loop formula for EDM is

$$\frac{d_e}{e} \sim \frac{\alpha\, m_e}{4\pi M_h^2} \sin\delta$$

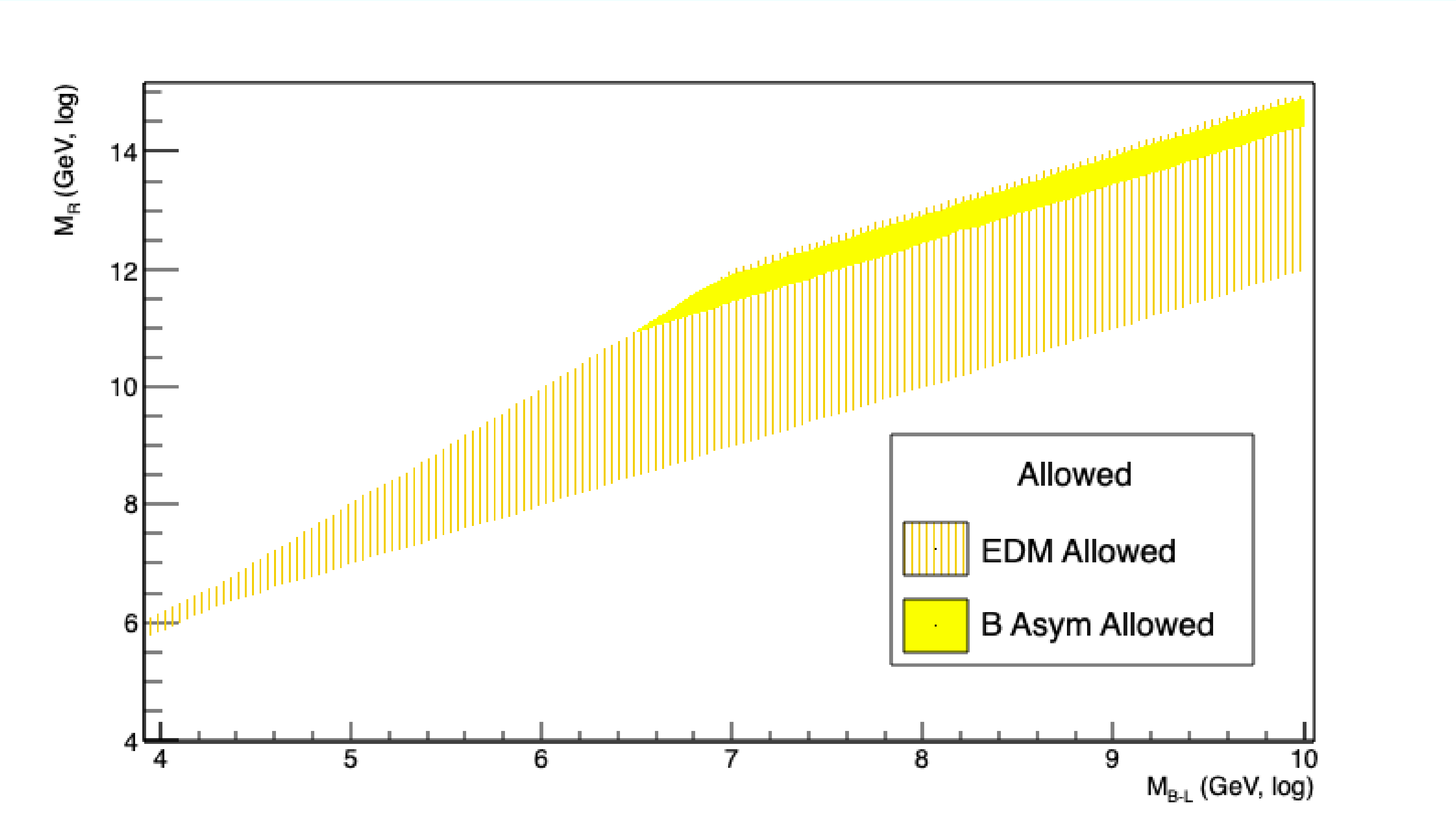
- For large values of M_{B-L} and M_R two loop effects arising from the neutral scalars dominate the one loop

$$(d_e/e)|_{twoloop} = \frac{G_F m_e \alpha \sin\delta}{\pi^3\sqrt{2}}(f_{W,H\gamma\gamma}(M_W^2/M_h^2) + f_{W,HZ\gamma}(M_W^2/M_h^2)$$
$$+f_{t,H\gamma\gamma}(M_t^2/M_h^2) + f_{t,HZ\gamma}(M_t^2/M_h^2)).$$



and four other such diagrams

Thus we obtain constraints on the mass scales M_R and M_{B-L}.



→ Interesting lesson : the R-symmetry compatible formula of Benaqli, Aulakh and Senjanovic is in tension.

→ Can be repaired by including the $m_\Omega \Omega\Omega$ term in superpotential

Left – Right models
a tale of two
phase transitions

6 Phase transition gravitational waves

mostly Caprini, Durrer, Servant, Binétruy, Hindmarsh ... 2009 - 2019;

Kosowsky and Turner(1993); Huber and Konstandin(2008); Weir (2016)

1. Bubble collisions

   $$h^2\,\Omega_{\mathrm{env}}(f) = 1.67 \times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_c\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11\,v_w^3}{0.42+v_w^2}\right)\frac{3.8\,(f\,/\,f_{\mathrm{env}})^{2.8}}{1+2.8\,(f\,/\,f_{\mathrm{env}})^{3.8}}$$

   where $H_* = H(T_n)$

2. Sound waves

   Pressure waves created in the plasma by movement of DW

   $$h^2\,\Omega_{\mathrm{sw}}(f) = 2.65 \times 10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w\left(\frac{f}{f_{\mathrm{sw}}}\right)^3\left(\frac{7}{4+3\,(f\,/\,f_{\mathrm{sw}})^2}\right)^{7/2}$$

   with peak frequency

   $$f_{\mathrm{sw}} = \frac{1.9\times 10^{-5}}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\mathrm{Hz}$$

3. MHD turbulence

$$h^2\Omega_{turb}(f) = \left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{turb}\ \alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w \frac{3.35\times 10^{-4}(f/f_{turb})^3}{[1+(f/f_{turb})]^{\frac{11}{3}}(1+8\pi f/h_*)}$$

6.1 The two phase transitions

Zafri A. Borboruah and UAY ArXiv 2022; PRD2024

Tree level Higgs potential for Δ_R vev r

$$V_0(r) \approx \frac{\rho_1}{4}(r^2 - \eta^2)^2$$

and similar for l.

The effective potential to be used

$$V_{\rm eff}(r, T) = V_0(r) + V_{\rm CW}(r) + V_{\rm FT}(r, T) + V_{\rm D}(r, T)$$

including Coleman-Weinberg, Finite temperature and daisy digrams.

Finally, with both L and R contributions :

$$V_{eff}^{total}(r, l, T) = V_{eff}(r, T) + V_{eff}(l, T)$$

Two types of phase transitions

I. **Kibble mechanism – "Causal horizon limited SOPT"**

Characterised by Ginzberg temperature

$$\xi_G^3 \, \Delta_c \, V_{eff} = T_G$$

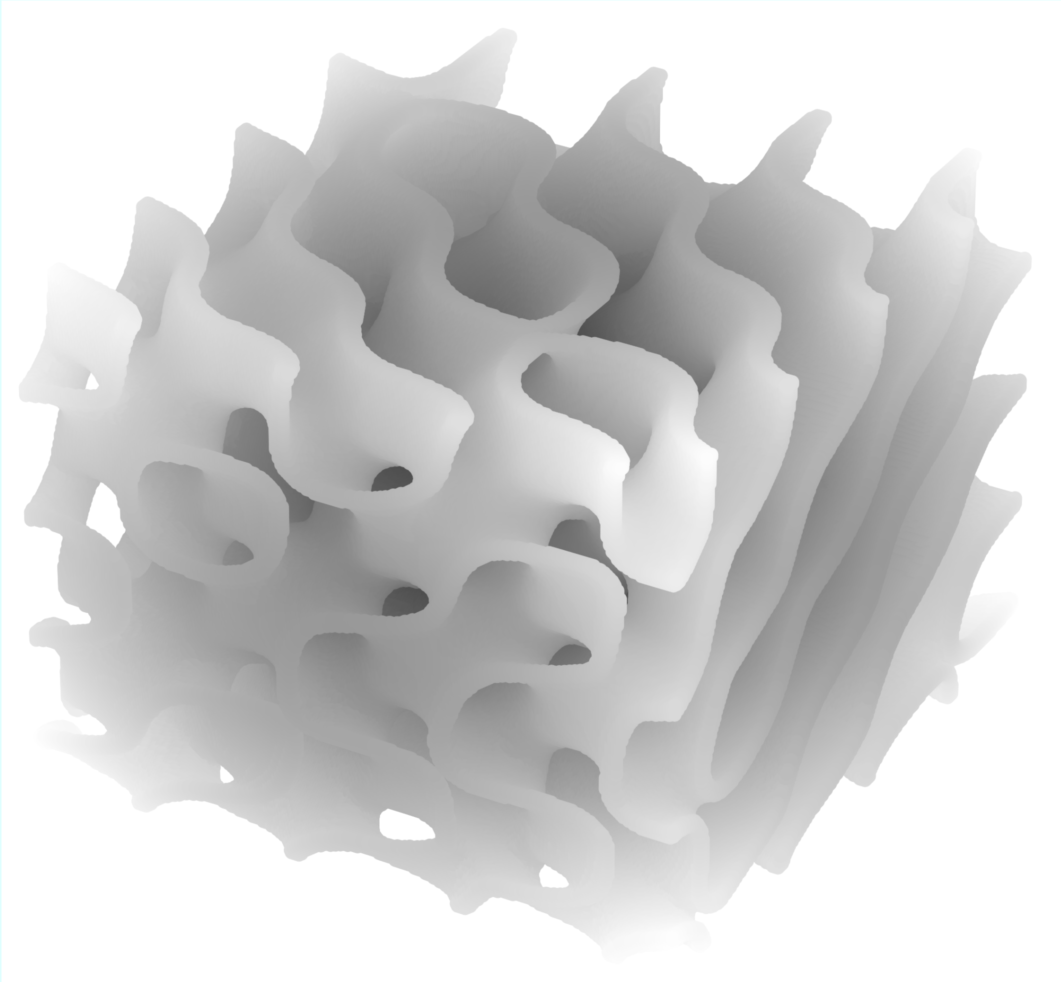
and its length scale ρ_1 a quartic coupling, η a vev

$$\xi_G \simeq \frac{1}{2\,\rho_1\,\eta}$$

Instead of diverging idefinitely, the putative SOPT has a scale

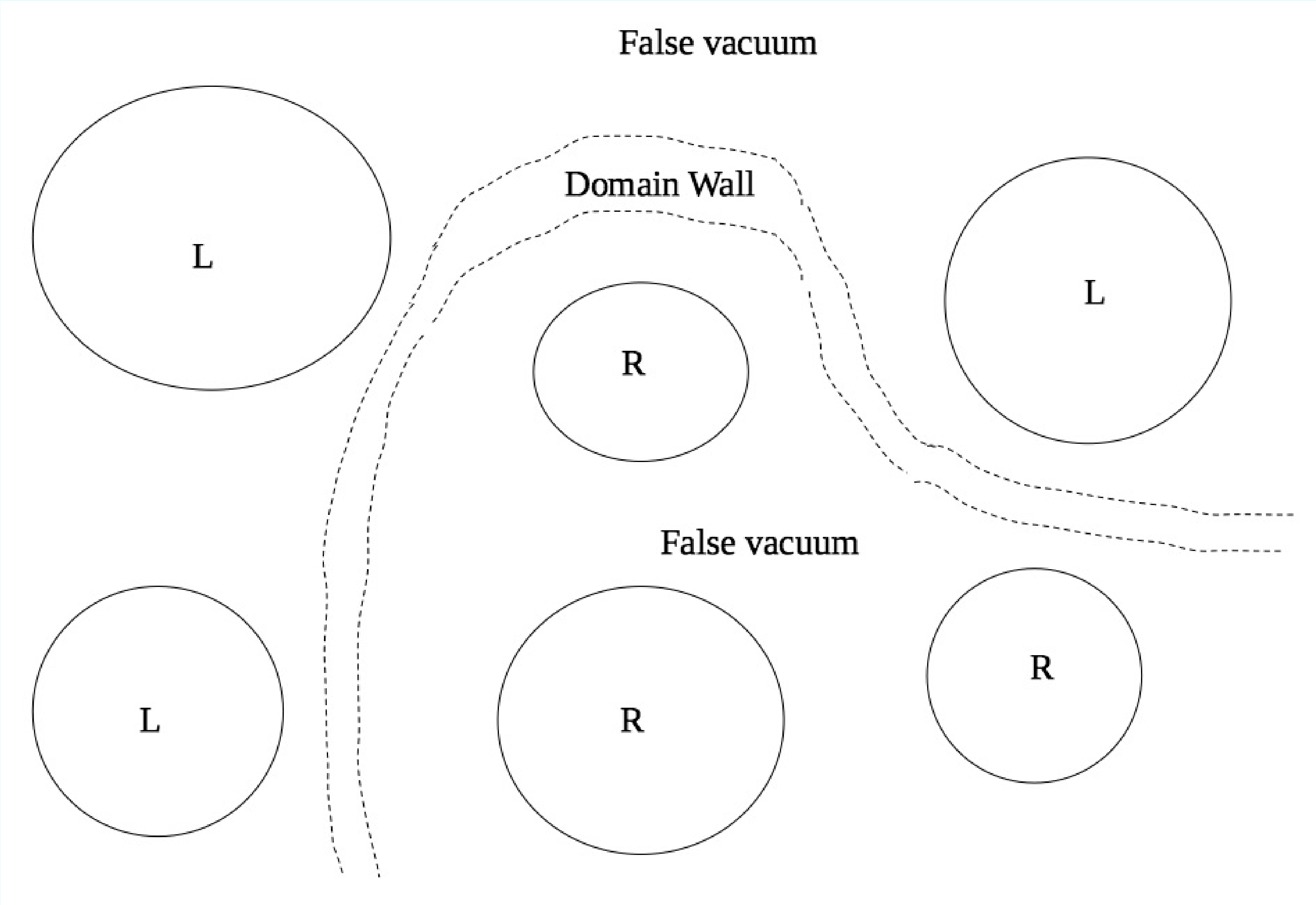
$$\xi_{causal} = \left(\frac{M_{Pl}}{\sqrt{\mathcal{N}}\; m_r^2 \, T_c^2} \right)^{1/3}$$

We treat these walls by direct simulation "crumbling walls" by introducing a bias term to break L-R



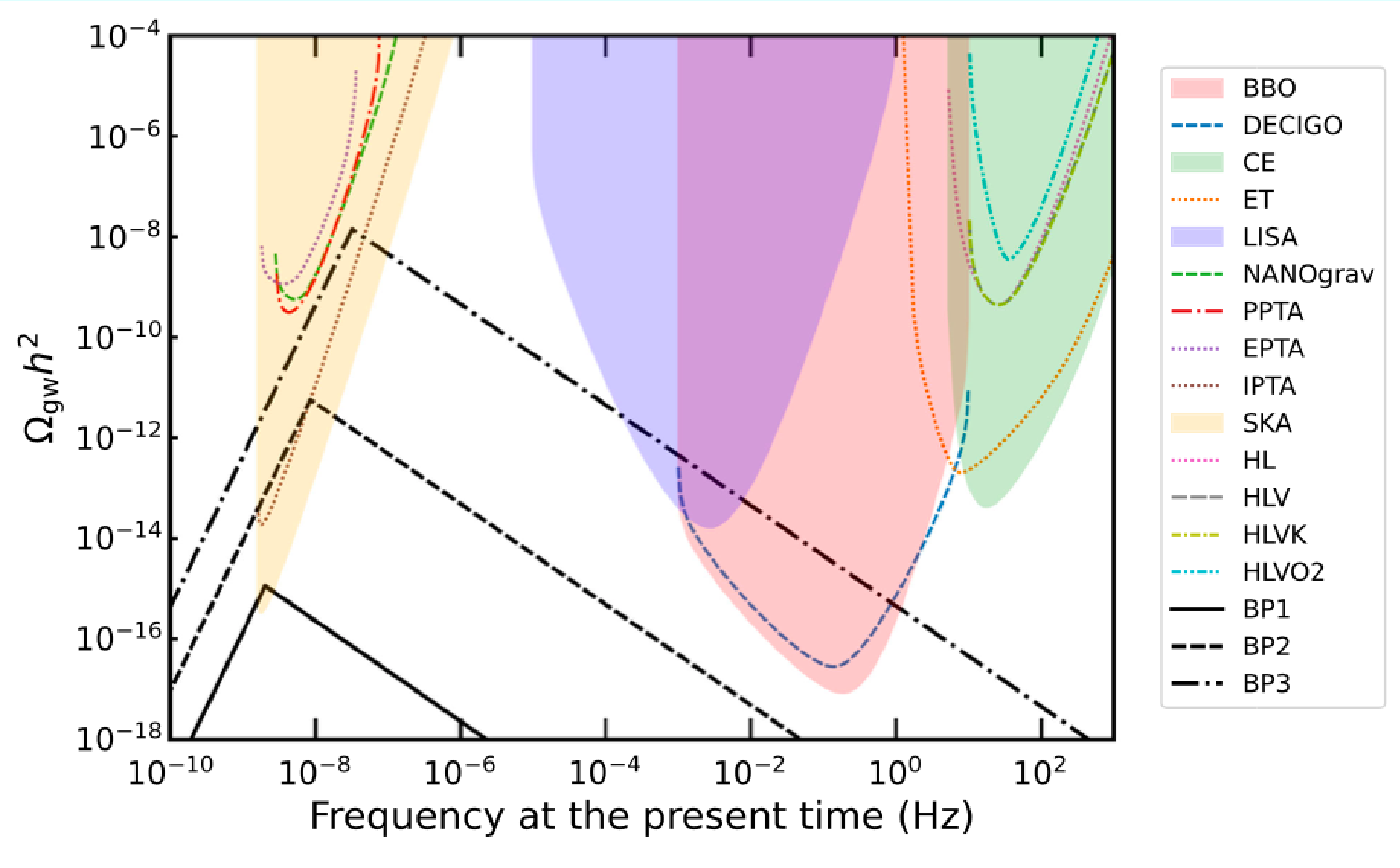
II. **Degenerate field FOPT**

- → Z_2 of D-parity for each L and R sectors effectively gives a Z_4 with two distinct fields Δ_L and Δ_R
- → The left-like and the right-like phases percolate individually
- → Where the two percolated regions meet, there is a domain wall, until the whole Universe is filled with a frustrated network of domain walls
- → This needs to be treated as
  - standard FOPT for bubbles
  - followed by the result of crumbling walls separating percolated regions


False vacuum
Domain Wall
L
R
L
False vacuum
L
R
R

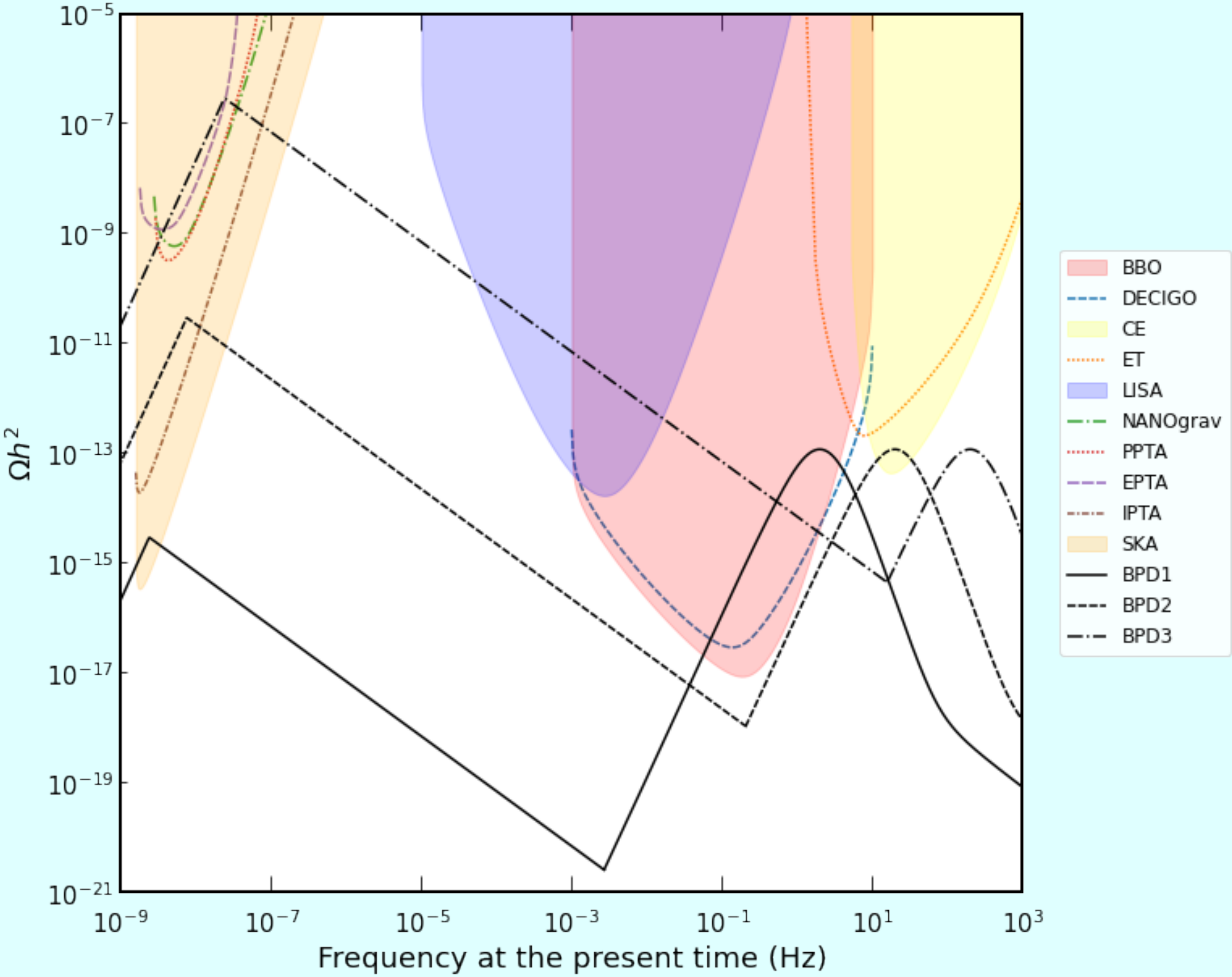
6.2 L-R two phase transition results

The Kibble or "lightcone limited SOPT" case

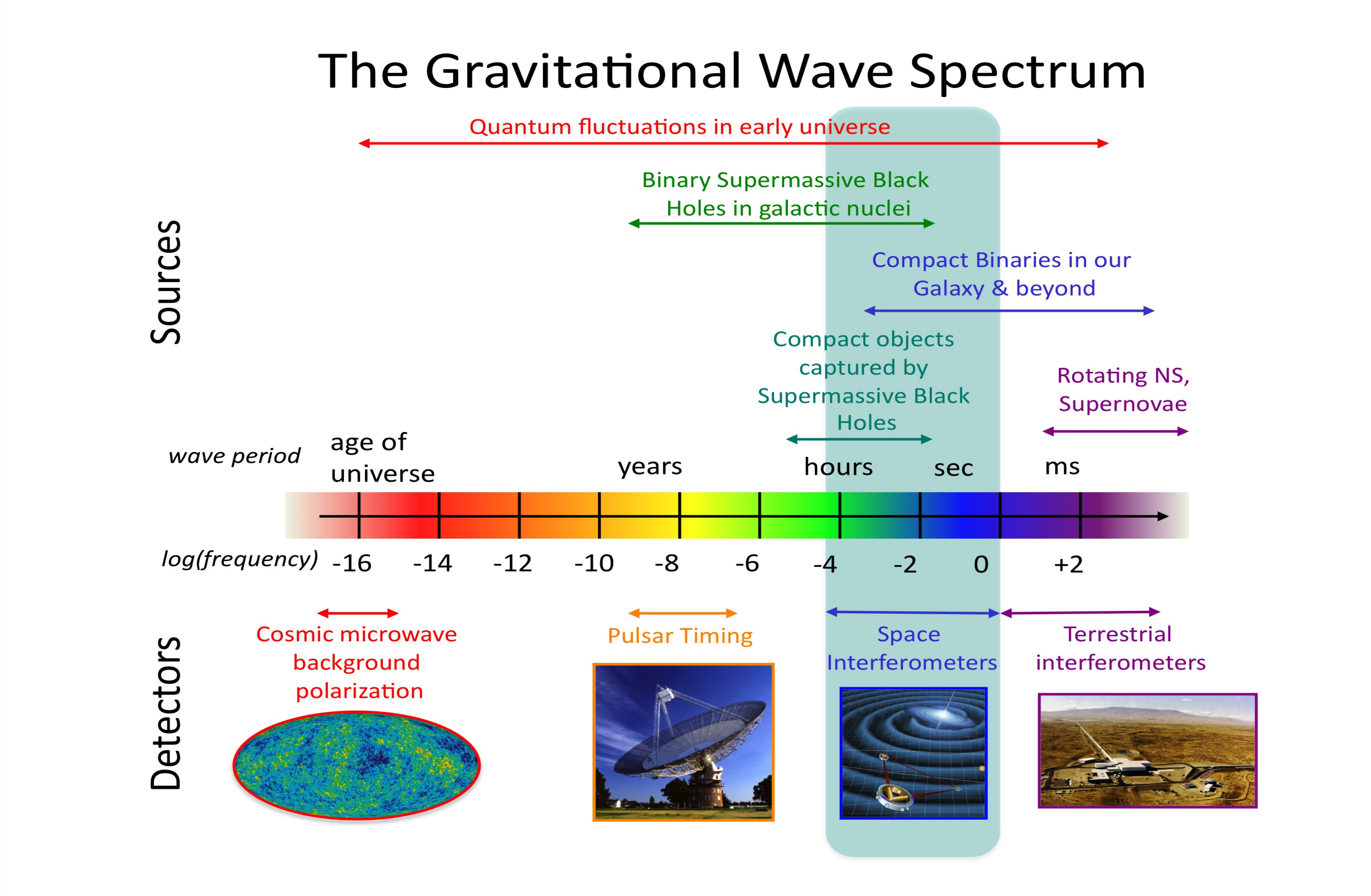


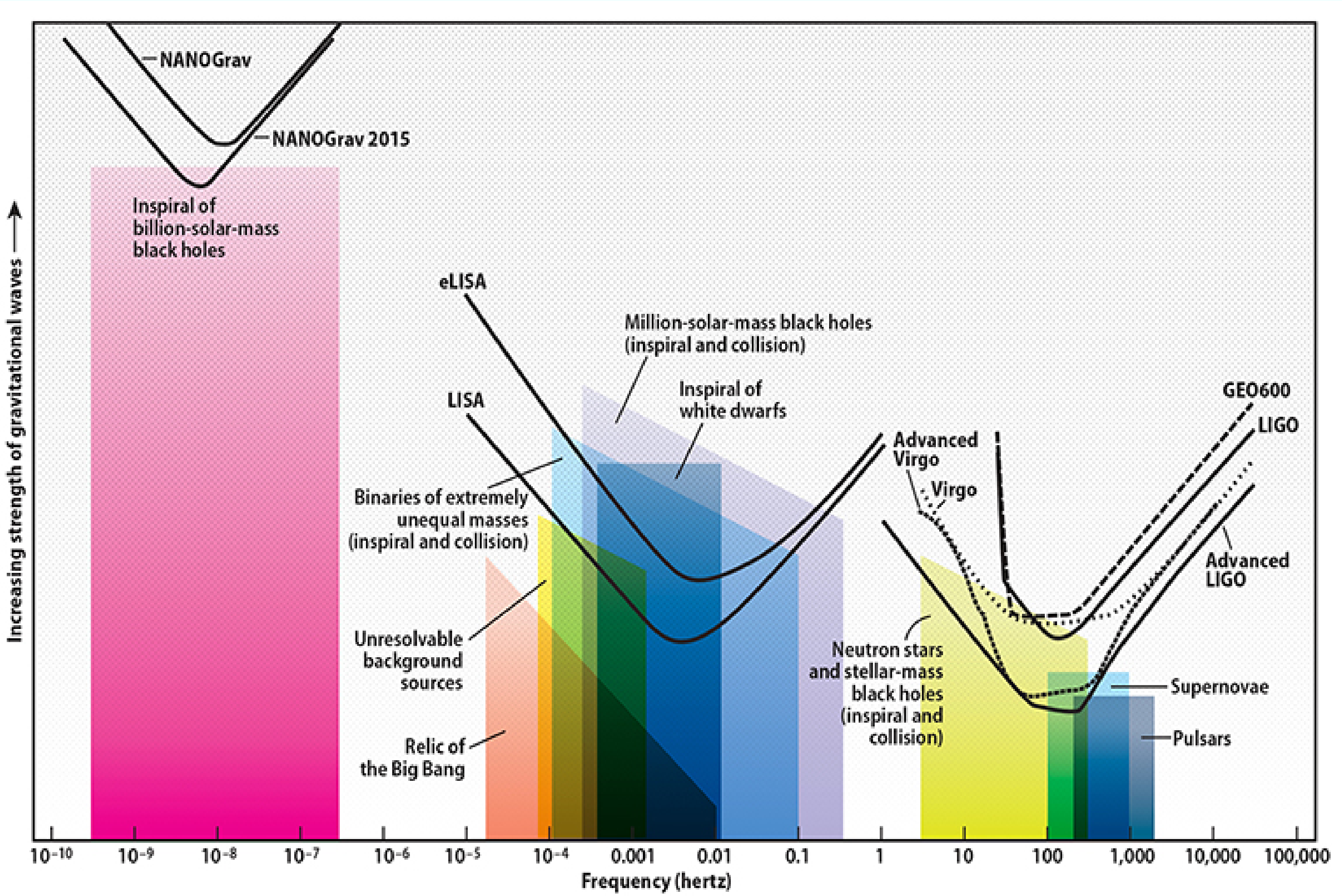
Benchmark points 1,2,3 $M_R = 10^4, 10^5, 10^6$ GeV

The Degenerate field FOPT case - two peaked spectrum



6.3 GW the observational propsects




NANOGrav
NANOGrav 2015
Inspiral of
billion-solar-mass
black holes
Increasing strength of gravitational waves
eLISA
LISA
Million-solar-mass black holes
(inspiral and collision)
Inspiral of
white dwarfs
Binaries of extremely
unequal masses
(inspiral and collision)
Unresolvable
background
sources
Relic of
the Big Bang
Advanced
Virgo
Virgo
GEO600
LIGO
Advanced
LIGO
Neutron stars
and stellar-mass
black holes
(inspiral and
collision)
Supernovae
Pulsars
10-10
10-9
10-8
10-7
10-6
10-5
10-4
0.001
0.01
0.1
1
10
100
1,000
10,000
100,000
Frequency (hertz)

6.4 Key experiments beyond LISA range

Already functioning

- Pulsar timing array - radio telescopes
  - PPTA, IPTA, EPTA -> NANOGrav

  uGMRT (2017) correlated data of 300-500 MHz and 1260-1460 MHz

- GAIA – orbiting sky scanning optical telescope till 2025

  Astrometry – μarc-sec accuracy for about 2 billion objects

  GWs from individually resolvable supermassive black hole binaries

7 Conclusion

→ GUT Bgenesis and EW Bgenesis are unrealistic possibilities

→ Thermal leptogenesis requires fine tuning

→ Low (TeV to PeV) scale leptogenesis viable through phase transition Domain Walls

→ Presented the case of L-R models transient CP phase relation to its zero temperature value Mü

  - to be verified through Electron EDM

→ Two possibilities for the L-R case; 2-peak signal for FOPT.

Danke schön!

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