# Self-consistent bounce from the 2PI effective action formalism

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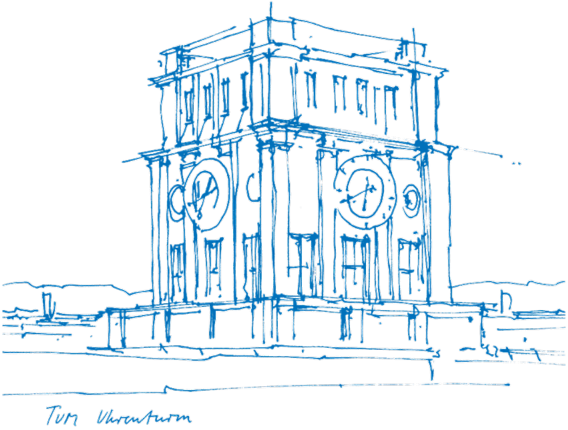
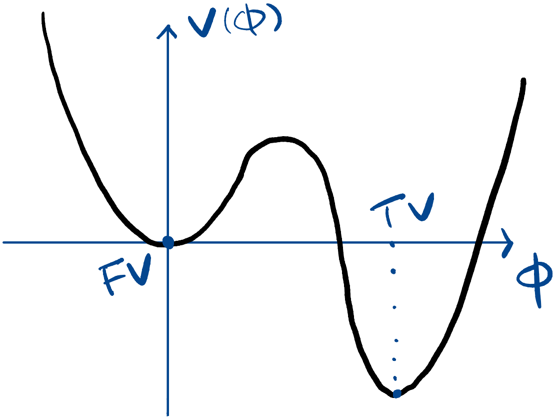


# Outline


1. Introduction and motivation
   - False vacuum decay and phase transitions
   - Computing the decay rate
2. FV decay within the 2PI effective action formalism
3. Obtaining the self-consistent bounce
4. Conclusions and outlook

# False vacuum decay and phase transitions



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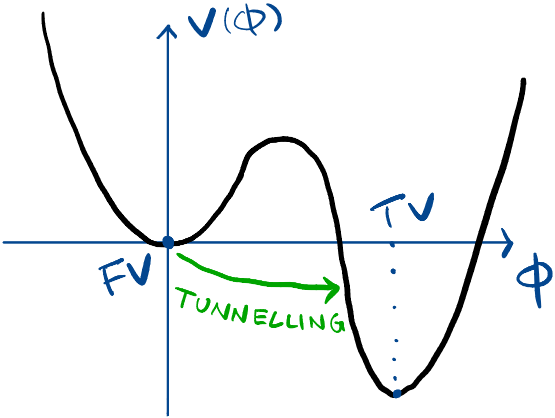

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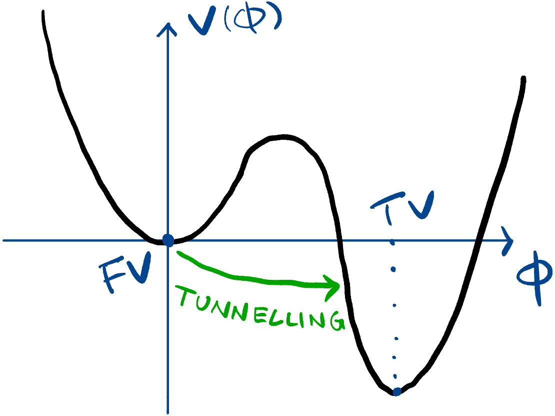
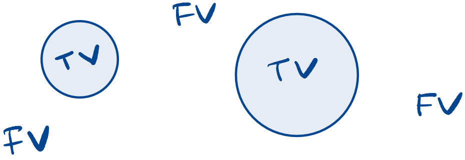



⟹ Bubble nucleation



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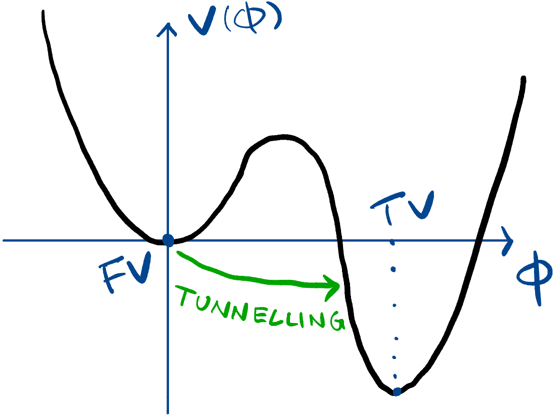


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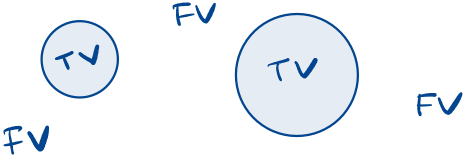


→ EW Baryogenesis

→ Formation of top. def.

→ GWs

→ SM stability

# Computing the decay rate

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prefactor: encodes the contribution of quantum fluctuations

Euclidean action of a solution interpolating between the minima: the bounce

# The bounce action

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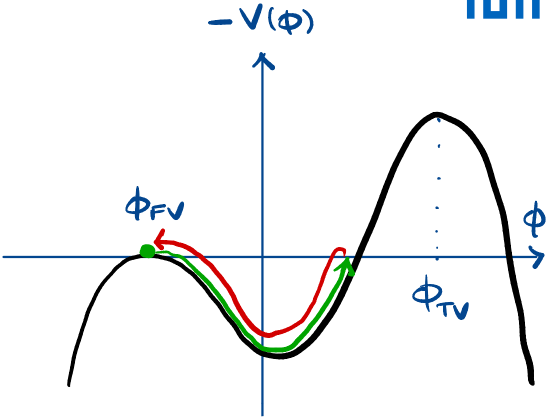
- using $O(d)$ invariance: $r^2 = x_E^i x_E^i$

$$-\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} - \frac{d-1}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} + V'(\phi) = 0 \qquad (1.4)$$




$$\begin{cases} \lim_{r\to\infty} \phi(r) = \phi_{FV} \\ \dot{\phi}(0) = 0 \end{cases}$$

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  - higher precision
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- To solve these issues we employ the **2PI effective action** [Bergner&Bettencourt 2003, although only Hartree approximation]

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1 Introduction and motivation

2 FV decay within the 2PI effective action formalism
- The 2PI effective action
- The bounce equation of motion
- The 2pt function equation of motion
- Expanding the self-energy

3 Obtaining the self-consistent bounce

4 Conclusions and outlook

# The 2PI effective action

- Define a generating functional for the connected 1- and 2-point functions [see e.g. Berges 2004, Introduction to Nonequilibrium QFT]

$$e^{W[J,R]} = \mathcal{N} \int [\mathcal{D}\phi] \; e^{-S_E[\phi] - \int_x J_x \phi_y - \frac{1}{2} \int_{x,y} \phi_x R_{xy} \phi_y} \tag{2.1}$$

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- Perform a Legendre transform to obtain the 2PI effective action

$$\Gamma_{2PI}[\varphi, G] = W[J,R] - \int_x J_x \varphi_x - \frac{1}{2} \int_{x,y} G_{xy} R_{xy} \tag{2.2}$$

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- Do a perturbative expansion in loops

$$\Gamma_{2PI}[\varphi, G] = S_E[\varphi] + \frac{1}{2} \operatorname{Tr} \Delta^{-1} G - \frac{1}{2} \operatorname{Tr} \log G^{-1} + \Gamma_2[\varphi, G] \tag{2.3}$$

# The bounce equation of motion

- The EoM for the 1pt function is easily obtained

$$\frac{\delta\Gamma_{2PI}}{\delta\varphi(x)} = 0$$

$$\implies \frac{\delta S_E[\varphi]}{\delta\varphi(x)} + \frac{1}{2}V'''(\varphi(x))G(x,x) + \frac{\delta\Gamma_2}{\delta\varphi} = 0 \tag{2.4}$$

# The 2pt function equation of motion

- The EoM for the connected 2pt function is obtained analogously

$$\frac{\delta \Gamma_{2PI}}{\delta G(x,y)} = 0$$

$$\implies \Delta^{-1}(x,y) - G^{-1}(x,y) + \Sigma(x,y) = 0 \tag{2.5}$$

- having defined

$$\Sigma(x,y) = 2\frac{\delta \Gamma_2}{\delta G(x,y)} \tag{2.6}$$

# The 2pt function equation of motion

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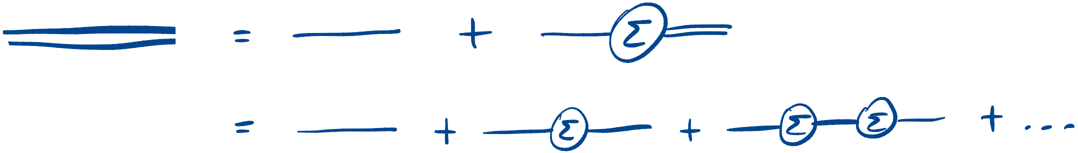
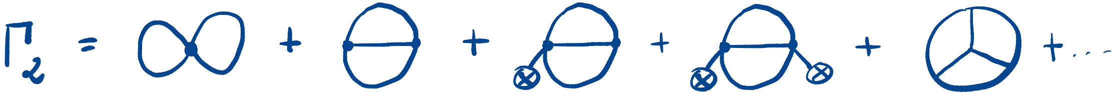
- Representing it diagrammatically makes it clear that $G$ satisfying eq. (2.7) is the resummed propagator

# Expanding the self-energy

- The term $\Gamma_2$ contains all 2PI vacuum diagrams



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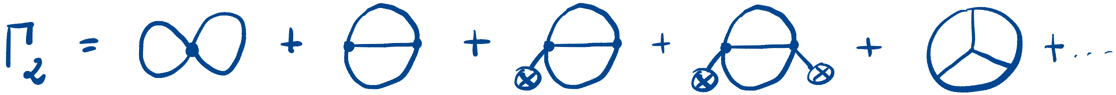
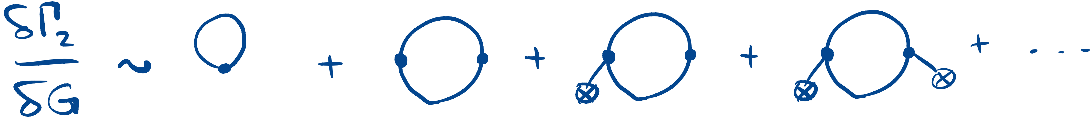


- The self-energy $\Sigma$ is obtained by differentiation, i.e. cutting one leg



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3 Obtaining the self-consistent bounce
- A system of coupled equations
- The self-consistent procedure
- Renormalising the tadpole
- Renormalising the bubble

4 Conclusions and outlook

# A system of coupled equations

- At 1-loop order we obtain a system of coupled *non-linear integrodifferential* equations

$$-\Delta_x \varphi_x + V'(\varphi_x) + \frac{1}{2}V'''(\varphi_x)G_{xx} = 0 \tag{3.1}$$

$$\left(-\Delta_x + V''(\varphi_x)\right) G_{xy} + \int_z \Sigma_{xz} G_{zy} = \delta^{(d)}_{xy} \tag{3.2}$$

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- Let's make use of the central symmetry of the problem to simplify the equations

# Angular momentum decomposition

- Introduce an angular momentum decomposition

$$G(x,y) = \frac{1}{(r_x r_y)^\kappa} \sum_{j,\{\ell\}} Y_{j,\{\ell\}}(\Omega_x)\, Y_{j,\{\ell\}}(\Omega_y)\, G_j(r_x, r_y) \tag{3.3}$$

- Split the self-energy into local and non-local contributions

$$\Sigma(x,y) = \delta^{(d)}(x-y)\Pi(x) + \Sigma_{\text{n.l.}}(x,y) \tag{3.4}$$

- Make a similar ansatz for the non-local term

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- We get a system of ordinary integro-differential equations, though now we have infinitely many of them, all coupled!

$$-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}\varphi(r) + V'(\varphi(r)) + \frac{1}{2}V'''(\varphi(r))G_{xx} = 0 \tag{3.6}$$

$$\left(-\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}}{\mathrm{d}r} + \frac{(j+\kappa)^2}{r^2} + V''(\varphi(r)) + \Pi(r)\right)G_j(r,r') + \int_0^\infty \mathrm{d}r''\, r''\, \Sigma_j(r,r'')\, G_j(r'',r') = \frac{1}{r}\delta(r-r') \tag{3.7}$$

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- We can only solve this **self-consistently**

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4. Solve for the bounce with 1-loop self-energies
5. Solve for the Green's function with 1-loop self-energies
6. Go back to step 3 and repeat until convergence

# Renormalising the tadpole

- Take the potential

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4 \tag{3.8}$$

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- We must renormalise the coincident Green's function

$$\begin{aligned} G(x, x) &= \frac{1}{r^{2\kappa}} \sum_{j,\{\ell\}} Y_{j,\{\ell\}}(\Omega) \, Y_{j,\{\ell\}}(\Omega) \, G_j(r, r) \\ &= \frac{2}{(4\pi)^{\kappa+\frac{1}{2}}} \frac{1}{\Gamma\left(\kappa + \frac{1}{2}\right)} \frac{1}{r_x^{2\kappa}} \sum_{j=0}^{\infty} (j + \kappa) \frac{\Gamma(j + 2\kappa)}{\Gamma(j + 1)} G_j(r, r) \end{aligned} \tag{3.10}$$

# Renormalising the tadpole

- The UV divergence is due to the large angular momentum modes: use WKB to obtain an expression for these

$$G_j^{\mathrm{WKB}}(r,r) = \frac{1}{2(j+\kappa)}\left(1 - \frac{m_\phi^2(r)r^2}{2(j+\kappa)^2} + \mathcal{O}\left((j+\kappa)^{-4}\right)\right) \tag{3.11}$$

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$m_\phi^2(r) = V''(\varphi(r)) + \Pi(r)$

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- We can use dim. reg. to regularise the sum

$$\begin{aligned}[G(x,x)]_{\kappa=1-\epsilon} &= \frac{1}{2\pi^2 r^2}\sum_{j=0}^{\infty}(j+1)^2\left(G_j(r,r) - G_j^{\mathrm{WKB}}(r,r)\right) + \left[G^{\mathrm{WKB}}(x,x)\right]_{\kappa=1-\epsilon} \\ &= \frac{1}{2\pi^2 r^2}\sum_{j=0}^{\infty}(j+1)^2\left(G_j(r,r) - G_j^{\mathrm{WKB}}(r,r)\right) \\ &\quad - \frac{1}{32\pi^2 r^2}\left[\frac{m_\phi^2(r)r^2}{\epsilon} + \frac{1}{3} + m_\phi^2(r)r^2 \log\frac{1}{4}e^2 r^2\mu^2\right] \end{aligned} \quad (3.12)$$

# Renormalising the tadpole

- We can then define the $\overline{\mathrm{MS}}$-renormalised coincident Green's function

$$[G(x,x)]^{\overline{\mathrm{MS}}} = \frac{1}{2\pi^2 r^2}\sum_{j=0}^{\infty}(j+1)^2\left(G_j(r,r) - G_j^{\mathrm{WKB}}(r,r)\right) - \frac{1}{32\pi^2 r^2}\left[\frac{1}{3} + m_\phi^2(r) r^2 \log\frac{1}{4}e^2 r^2 \mu^2\right] \tag{3.13}$$

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- We can then obtain the on-shell one by subtracting the $\phi \equiv \phi_{\text{FV}}$ result

$$\begin{aligned}[G(x,x)]^{\text{OS}} &= [G(x,x)]^{\overline{\text{MS}}} - [G(x,x)]^{\overline{\text{MS}}}_{\phi\equiv\phi_{\text{FV}}} \\ &= \frac{1}{2\pi^2 r^2}\sum_{j=0}^{\infty}(j+1)^2\left[G_j(r,r) - G_{0,j}(r,r) + \frac{(m_\phi^2(r) - m^2)r^2}{4(j+1)^3}\right] \\ &\quad - \frac{1}{32\pi^2}(m_\phi^2(r) - m^2)\log\frac{1}{4}e^2 r^2 \mu^2 \end{aligned} \quad (3.14)$$

# Renormalising the bubble

- Renormalising the bubble is much harder. It requires finding the divergent structure of $\Sigma_j$ defined by

$$G(x,y)^2 = \left( \frac{2}{(4\pi)^{\kappa+\frac{1}{2}}} \frac{\Gamma(2\kappa)}{\Gamma\left(\kappa+\frac{1}{2}\right)} \frac{1}{(r_x r_y)^\kappa} \sum_{j=0}^{\infty} (j+\kappa) C_j^\kappa(\cos\theta) G_j(r_x, r_y) \right)^2$$
$$= \frac{2}{(4\pi)^{\kappa+\frac{1}{2}}} \frac{\Gamma(2\kappa)}{\Gamma\left(\kappa+\frac{1}{2}\right)} \frac{1}{(r_x r_y)^\kappa} \sum_{j=0}^{\infty} (j+\kappa) C_j^\kappa(\cos\theta) \Sigma_j(r_x, r_y) \tag{3.15}$$

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$$\cos\theta = \frac{x \cdot y}{|x||y|}$$

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- After a long computation, we find

$$\Sigma_j(r_x,r_y) \approx \sum_q q^2 G_q(r_x,r_y)G_{q+j}(r_x,r_y) \approx \sum_q \left(\frac{r_<}{r_>}\right)^{2q}\left(1+\mathcal{O}(q^{-2})\right) \qquad (3.16)$$

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$$= \frac{2}{(4\pi)^{\kappa+\frac{1}{2}}} \frac{\Gamma(2\kappa)}{\Gamma\left(\kappa+\frac{1}{2}\right)} \frac{1}{(r_x r_y)^\kappa} \sum_{j=0}^{\infty} (j+\kappa) C_j^\kappa(\cos\theta) \Sigma_j(r_x, r_y) \tag{3.15}$$

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$$\Sigma_j(r_x, r_y) \approx \sum_q q^2 G_q(r_x, r_y) G_{q+j}(r_x, r_y) \approx \sum_q \left(\frac{r_<}{r_>}\right)^{2q} \left(1 + \mathcal{O}(q^{-2})\right) \tag{3.16}$$

$r_> = \max(r_x, r_y)$

$r_< = \min(r_x, r_y)$

# Renormalising the bubble

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- The divergence can be renormalised via local counter-terms

# Outline

1 Introduction and motivation

2 FV decay within the 2PI effective action formalism

3 Obtaining the self-consistent bounce

4 Conclusions and outlook

# Conclusions and outlook

Lessons learned

- We can use the 2PI effective action to systematically include quantum corrections around a non-trivial field configuration
- Renormalisation in position space, even of simple diagrams, is hard!
- Divergences in position space can be very subtle and must be treated with great care

Our next steps

- Solving the system of equations numerically in $d = 2$, which is expected to converge fast
- Analysing the effect of the non-local term specifically
- Extend a similar analysis to similar systems, e.g. real-time bubble wall dynamics in a first-order phase transition

# The translational zero-mode

- The fluctuation operator actually has a zero-mode, related to translational invariance. Starting from the tree-level EoM

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r}\left(-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}\varphi(r)+V'(\varphi(r))\right) &= 0\\ \Longrightarrow\left(-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}+\frac{d-1}{r^2}+V''(\varphi(r))\right)\dot{\varphi}(r) &= 0\end{aligned} \tag{5.1}$$

- There is $d$-many zero-modes in the $j=1$ sector
- Quantum corrections do not break translational symmetry, thus we must find

$$\begin{aligned}&\left(-\frac{1}{r^{d-1}}\frac{\mathrm{d}}{\mathrm{d}r}r^{d-1}\frac{\mathrm{d}}{\mathrm{d}r}+\frac{d-1}{r^2}+V''(\varphi(r))+\Pi(r)\right)\phi_{\mathrm{tr}}(r)\\ &\qquad+\int_0^\infty \mathrm{d}r''\, {r''}^{d-1}\,\frac{1}{{r''}^{\frac{d}{2}-1}}\Sigma_j(r,r'')\,\phi_{\mathrm{tr}}(r'') = 0\end{aligned} \tag{5.2}$$

# Subtracting the zero-mode

- The zero-modes are not propagating degrees of freedom and must thus be subtracted

$$\mathcal{O}G^{\perp} = \mathbb{1}^{\perp} \tag{5.3}$$

- The operator $\mathbb{1}^{\perp}$ is the identity on the orthogonal subspace to the one spanned by the zero-modes

$$\mathbb{1}^{\perp} = \mathbb{1} - \sum_i \phi_i \phi_i^* \tag{5.4}$$

- This defines the subtracted Green's function $G^{\perp}$
- Only using the subtracted Green's function we can make sure that translational modes are exact zero-modes also of the quantum theory