Using gauge/gravity duality for calculations in Composite Higgs models

Werner Porod

(Uni. Würzburg)

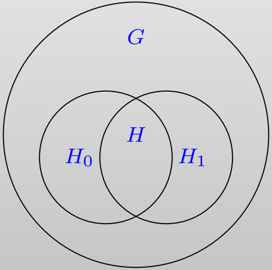
Work with Johanna Erdmenger, Nick Evans, Yang Liu, Kostas Rigatos
PRL **126** (2021), 071602; JHEP **02** (2021), 058; Universe **9** (2023), 289; JHEP **07** (2024), 169

Effective Theories for Nonperturbative Physics
7 August 2024

‘Minimal Composite Higgs framework’

K. Agashe, R. Contino and A. Pomarol, NPB **719** (2005), 165
R. Contino, TASI lectures 2009

Assumes there is an additional strong force, often called hyper-color, and new ‘quarks’



G: $SO(5) \times U(1)_X$, global symmetry of the strong sector above confinement scale

H_1: $SO(4) \times U(1)_X \sim SU(2)_L \times SU(2)_R \times U(1)_X$, global symmetry group in confined phase

H_0: $SU(2)_L \times U(1)_Y$, SM electroweak gauge group

H: $U(1)_{em}$, unbroken gauge group

- $SO(5) \to SO(4)$ breaking $\Rightarrow$ 4 Nambu-Goldstone bosons in $(2,2)$ of $SU(2)_L \times SU(2)_R$
- $Y = T^{3R} + X$, $U(1)_X$ needed to get correctly the hypercharges of the fermions

‘Minimal Composite Higgs framework’

Fermion masses and couplings: partial compositeness

Higgs transforms non-linearly under G.
$\Rightarrow$ no Yukawa interaction if fermion are elementary (transform linearly).
Possible solution: mix elementary fermions with composite resonances.

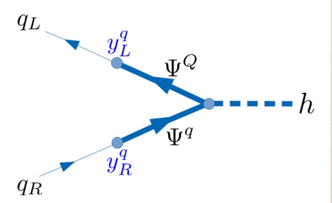
Elementary fermions (in $SO(5)$) rep.)

$$q_L = \frac{1}{\sqrt{2}}(\mathrm{i}\,d_L, d_L, \mathrm{i}\,u_L, -u_L, 0)^T$$

$$q_R = (0,0,0,0,u_R)^T$$

Composite fermions (in $SO(5)$) rep.)

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{i}B - \mathrm{i}X_{5/3} \\ B + X_{5/3} \\ \mathrm{i}T + \mathrm{i}X_{2/3} \\ -T + X_{2/3} \\ \sqrt{2}\tilde{T} \end{pmatrix}$$



Generic Composite Higgs set-up

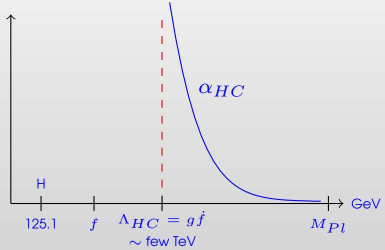
Possible solution to hierarchy problem

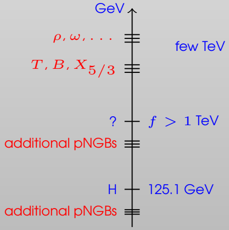
- Generate a scale $\Lambda_{HC} \ll M_{pl}$ through a new confining gauge group
- Interpret Higgs as a pseudo-Nambu-Goldstone boson (pNGB) of a spontaneously broken global symmetry of the new strong sector, G/H

(Georgi, Kaplan, PLB **136** (1984), 136)

'Price' to pay

- additional resonances at the scale Λ_{HC} (vectors, vector-like fermions, scalars)
- additional light pNGBs/ extended scalar sector
- deviations of the Higgs couplings from their SM values of O(v/f)



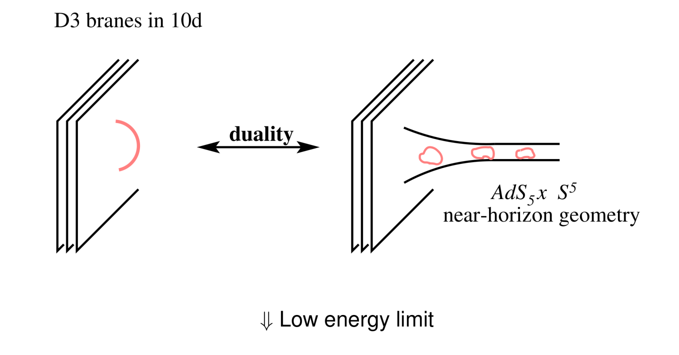


List of "minimal" CHM UV embeddings

$G_{\rm HC}$	ψ	χ	Restrictions	$-q_\chi/q_\psi$	Y_χ	Non Conformal	Model Name
	Real	Real	SU(5)/SO(5) × SU(6)/SO(6)				
$SO(N_{\rm HC})$	$5\times\mathbf{S_2}$	$6\times\mathbf{F}$	$N_{\rm HC}\geq 55$	$\frac{5(N_{\rm HC}+2)}{6}$	1/3	/	
$SO(N_{\rm HC})$	$5\times\mathbf{Ad}$	$6\times\mathbf{F}$	$N_{\rm HC}\geq 15$	$\frac{5(N_{\rm HC}-2)}{6}$	1/3	/	
$SO(N_{\rm HC})$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$N_{\rm HC}=7,9$	$\frac{5}{6},\frac{5}{12}$	1/3	$N_{\rm HC}=7,9$	M1, M2
$SO(N_{\rm HC})$	$5\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\rm HC}=7,9$	$\frac{5}{6},\frac{5}{3}$	2/3	$N_{\rm HC}=7,9$	M3, M4
	Real	Pseudo-Real	SU(5)/SO(5) × SU(6)/Sp(6)				
$Sp(2N_{\rm HC})$	$5\times\mathbf{Ad}$	$6\times\mathbf{F}$	$2N_{\rm HC}\geq 12$	$\frac{5(N_{\rm HC}+1)}{3}$	1/3	/	
$Sp(2N_{\rm HC})$	$5\times\mathbf{A_2}$	$6\times\mathbf{F}$	$2N_{\rm HC}\geq 4$	$\frac{5(N_{\rm HC}-1)}{3}$	1/3	$2N_{\rm HC}=4$	M5
$SO(N_{\rm HC})$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$N_{\rm HC}=11,13$	$\frac{5}{24},\frac{5}{48}$	1/3	/	
	Real	Complex	SU(5)/SO(5) × SU(3)^2/SU(3)				
$SU(N_{\rm HC})$	$5\times\mathbf{A_2}$	$3\times(\mathbf{F},\overline{\mathbf{F}})$	$N_{\rm HC}=4$	$\frac{5}{3}$	1/3	$N_{\rm HC}=4$	M6
$SO(N_{\rm HC})$	$5\times\mathbf{F}$	$3\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$N_{\rm HC}=10,14$	$\frac{5}{12},\frac{5}{48}$	1/3	$N_{\rm HC}=10$	M7
	Pseudo-Real	Real	SU(4)/Sp(4) × SU(6)/SO(6)				
$Sp(2N_{\rm HC})$	$4\times\mathbf{F}$	$6\times\mathbf{A_2}$	$2N_{\rm HC}\leq 36$	$\frac{1}{3(N_{\rm HC}-1)}$	2/3	$2N_{\rm HC}=4$	M8
$SO(N_{\rm HC})$	$4\times\mathbf{Spin}$	$6\times\mathbf{F}$	$N_{\rm HC}=11,13$	$\frac{8}{3},\frac{16}{3}$	2/3	$N_{\rm HC}=11$	M9
	Complex	Real	SU(4)^2/SU(4) × SU(6)/SO(6)				
$SO(N_{\rm HC})$	$4\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$6\times\mathbf{F}$	$N_{\rm HC}=10$	$\frac{8}{3}$	2/3	$N_{\rm HC}=10$	M10
$SU(N_{\rm HC})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$6\times\mathbf{A_2}$	$N_{\rm HC}=4$	$\frac{2}{3}$	2/3	$N_{\rm HC}=4$	M11
	Complex	Complex	SU(4)^2/SU(4) × SU(3)^2/SU(3)				
$SU(N_{\rm HC})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{A_2},\overline{\mathbf{A}}_2)$	$N_{\rm HC}\geq 5$	$\frac{4}{3(N_{\rm HC}-2)}$	2/3	$N_{\rm HC}=5$	M12
$SU(N_{\rm HC})$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{S_2},\overline{\mathbf{S}}_2)$	$N_{\rm HC}\geq 5$	$\frac{4}{3(N_{\rm HC}+2)}$	2/3	/	

G. Ferretti, JHEP **06** (2016), 107; A. Belyaev et al. JHEP **01** (2017), 094

String theory origin of the gauge/gravity duality

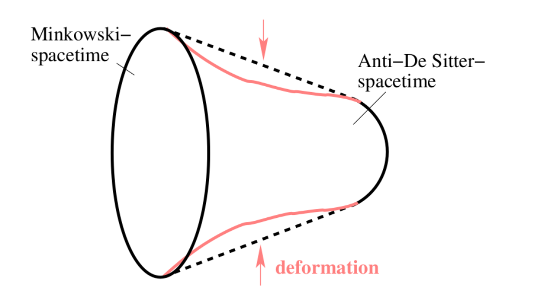


$\mathcal{N} = 4$ Supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$ limit) $\Leftrightarrow$ Supergravity on the space $\mathrm{AdS}_5 \times S^5$

J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998), 231

Breaking of conformal symmetry and supersymmetry

Deformation of AdS_5



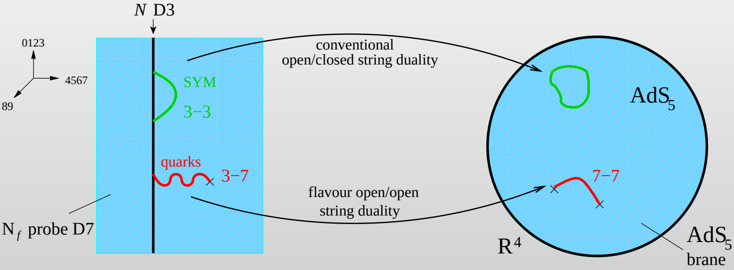
5^{th} dimension $\Leftrightarrow$ energy scale

Deformation of S^5: breaking of supersymmetry

$\Rightarrow$ Generalized AdS/CFT Correspondence: Gauge/Gravity Duality

see e.g. J. Erdmenger et al, Eur. Phys. J. A **35** (2008), 81 for a review

Quarks in the gauge/gravity duality



(from J. Erdmenger et al, Eur. Phys. J. A **35** (2008), 81)

$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

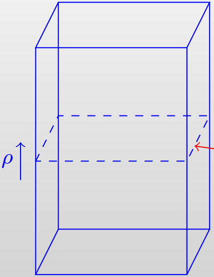
duality acts twice

$\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet $\Leftrightarrow$ IIB supergravity on AdS$_5 \times S^5$ + Probe brane DBI on AdS$_5 \times S^3$

A. Karch and E. Katz, JHEP **06** (2002), 043

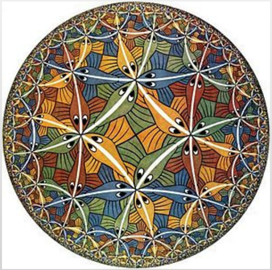
(DBI: Dirac-Born-Infeld)

How does gauge/gravity duality work?



$$d^2 s = \frac{d^2\rho}{\rho^2} + \rho^2 d^2 x_{3+1}$$

3+1d slice parallel to D3 brane
on which field theory lives



dilation $\Leftrightarrow$ running of gauge couplings

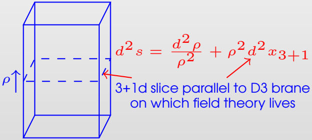
$$\int d^4x \partial_\mu \phi \partial^\mu \phi \,, \quad x \to \mathrm{e}^{-\alpha} x \quad \phi \to \mathrm{e}^{\alpha} \phi$$

dilaton become spacetime symmetry of AdS

$$\rho \to \mathrm{e}^{\alpha} \rho$$

ρ is a continuos mass dimension $\to$ RG scale

How does gauge/gravity duality work?



$$\sqrt{-\det g}$$

$$= \left| \begin{pmatrix} -\rho^2 & 0 & 0 & 0 & 0 \\ 0 & \rho^2 & 0 & 0 & 0 \\ 0 & 0 & \rho^2 & 0 & 0 \\ 0 & 0 & 0 & \rho^2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\rho^2} \end{pmatrix} \right|^{1/2}$$

$$= \rho^3$$

Field theory side

Operators and sources appear as fields in the bulk, e.g.

$$\int d^4x \, m \, \bar{\psi}\psi$$

m is the quark mass and c the condensate

$$c = \langle \bar{\psi}\psi \rangle$$

AdS side

A field for the mass/condensate

$$\int d^4x \int d\rho \frac{1}{2} \rho^3 (\partial_\rho L)^2$$

$$\Rightarrow \quad \partial_\rho \left(\rho^3 \partial_\rho L \right) = 0$$

$$\Rightarrow L = m + \frac{c}{\rho^2}$$

Running Dimensions in Holography

Holographically we can change the dimension of our operator by adding a mass term

$$\partial_\rho \left(\rho^3 \partial_\rho L\right) - \rho\, \Delta m^2 L = 0 \quad , \quad \gamma(\gamma - 2) = \Delta m^2$$

$$\Rightarrow L = \frac{m}{\rho^\gamma} + \frac{c}{\rho^{2-\gamma}}$$

$\Delta m^2 = -1$ corresponds to $\gamma = 1$ and is special – the Breitenlohner Freedman bound instability . . .

So we can include a running coupling by a running mass squared for the scalar.

Top down derivation:

- several string constructions e.g. probe D7 branes in D3 backgrounds
- neglect back reaction of probe branes on metric '⇔' quenched approximation on lattice

R. Alvares, N. Evans, K.-Young arXiv:1204.2474 (hep-ph); M. Jarvinen, E. Kiritsis arXiv:1112.1261 (hep-ph)

Dynamic AdS/YM

$$S = \int d^4x \int d\rho \, \mathrm{Tr} \left[\frac{1}{\rho^2 + |X|^2} |DX| + \frac{\Delta m^2}{\rho^2} |X|^2 \right]$$

$$X = L(\rho) \mathrm{e}^{2\mathrm{i}\pi \xi^a(x) T^a} , \qquad d^2 s = \frac{d^2 \rho}{\rho^2 + |X|^2} + (\rho^2 + |X|^2) d^2 x$$

$L = |X|$ is now the dynamical field whose solution will determine the condensate as a function of m - the phase is the pion.

We use the top-down IR boundary condition on mass-shell: $L'(\rho = L) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate - no hard wall.
gauge/gravity duality: $X \Leftrightarrow \bar{q}q$

The gauge dynamics is input through a guess for $\Delta m = \gamma(\gamma - 2)$ with

$$\gamma = \frac{3(N_c^2 - 1)}{4\pi N_c} \alpha$$

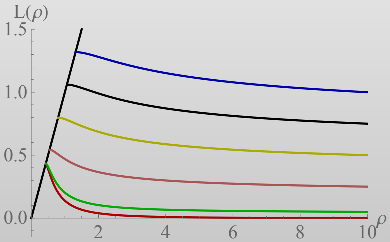
in case of $SU(N_c)$. The only free parameters are $N_c, N_f, m, \Lambda_{UV}$

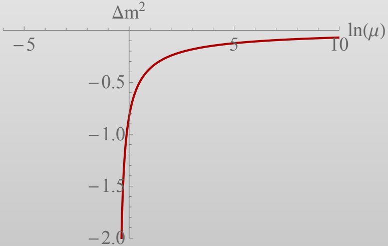
T. Alho, N. Evans, K. Tuominen arXiv:1307.4896 (hep-ph)

Formation of the Chiral Condensate

We solve for the vacuum configuration of $L = |X|$

$$\partial_\rho \left(\rho^3 \partial_\rho L(\rho)\right) - \rho\, \Delta m^2(\rho) L(\rho) = 0\,, \quad L'(\rho = L) = 0 \quad \text{and} \quad L(\rho) = \rho$$





$N_c = 3, N_f = 2, \mu = \sqrt{\rho^2 + L^2}$
$m = 0$ the $L(\rho)$: $L_{IR} = 0.43$
quark masses from bottom to top: 0, 0.05, 0.25, 0.5, 0.75, 1

Meson Fluctuations

$$S = \int d^4x \int d\rho \, \mathrm{Tr} \left[\frac{1}{\rho^2 + |X|^2} |DX| + \frac{\Delta m^2}{\rho^2} |X|^2 \right] + \frac{1}{2\kappa^2} (F_V^2 + F_A^2)$$

$$L = L_0 + \delta(x) e^{ikx} \qquad k^2 = -M^2$$

$$\partial_\rho(\rho^3 \partial_\rho \delta) - \Delta m^2 \, \rho \, \delta - \rho \, L_0 \, \delta \frac{\partial \Delta m^2}{\partial L} \Big|_{L_0} + M^2 \frac{\rho^3}{(L_0^2 + \rho^2)^2} \delta = 0$$

The source free solutions pick out particular mass states, the σ (or f_0) and its radial excited states

The gauge fields let us also study the operators and states

$\bar{u}\gamma_\mu u \to \rho$ meson , $\bar{u}\gamma_\mu \gamma_5 u \to a$ meson

QCD Dynamics – $N_c = 3, N_f = 2, m_q = 0$

$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2 \qquad b_0 = \frac{1}{6\pi}\left(11 N_c - (N_f + \bar{N}_f)\right), \qquad \gamma = \frac{3({N_c}^2 - 1)}{4 N_c \pi}\alpha\,.$$

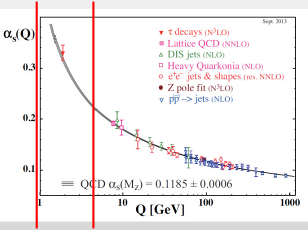
Two-loop contribution included as well

Observables (MeV)	QCD	AdS/SU(3) 2 F 2 $\bar{F}$	Deviation
M_ρ	775	775*	fitted
M_A	1230	1183	- 4%
M_S	500/990	973	+64%/-2%
M_B	938	1451	+43%
f_π	93	55.6	-50%
f_ρ	345	321	- 7%
f_A	433	368	-16%
$M_{\rho,n=1}$	1465	1678	+14%
$M_{A,n=1}$	1655	1922	+19%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%
$M_{B,n=1}$	1440	2406	+50%

The predictions for masses and decay constants (in MeV) for $N_f = 2$ massless QCD. The ρ-meson mass has been used to set the scale (indicated by the *).

- scale fixed by V-meson
- f_π needs a mass term
- baryon mass high
- radial exitations wrong – no string physics included

Perfecting with HDOs



The weakly coupled gravity dual should only live between the red lines
probably we need HDOs at the UV scale to include matching effect and stringy effects in the gravity model

$$\frac{g_S^2}{\Lambda_{UV}^2}|\bar{q}q|^2 \quad \frac{g_V^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu q|^2 \quad \frac{g_A^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu\gamma_5 q|^2$$

$$\frac{g_B^2}{\Lambda_{UV}^5}|qqq|^2$$

Observables (MeV)	QCD	Dynamic AdS/QCD	HDO coupling
M_V	775	775	sets scale
M_A	1230	1230	fitted by $g_A^2 = 5.76149$
M_S	500/990	597	prediction $+20\%/-40\%$
M_B	938	938	fitted by $g_B^2 = 25.1558$
f_π	93	93	fitted by $g_S^2 = 4.58981$
f_V	345	345	fitted by $g_V^2 = 4.64807$
f_A	433	444	prediction $+2.5\%$
$M_{V,n=1}$	1465	1532	prediction $+4.5\%$
$M_{A,n=1}$	1655	1789	prediction $+8\%$
$M_{S,n=1}$	990/1200-1500	1449	prediction $+46\%/0\%$
$M_{B,n=1}$	1440	1529	prediction $+6\%$

The spectrum and the decay constants for two-flavour QCD with HDOs used to improve the spectrum.

Pretty good
but we've lost some predictivity

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

- The sextet quarks A_2 are expected to condense first and break $SU(6) \to SO(6)$. Their main job: form FA_2F baryon top partners.
- Then the fundamentals F break $SU(4) \to Sp(4)$ – this is where the Higgs is generated. (It's the same condensation as in QCD)

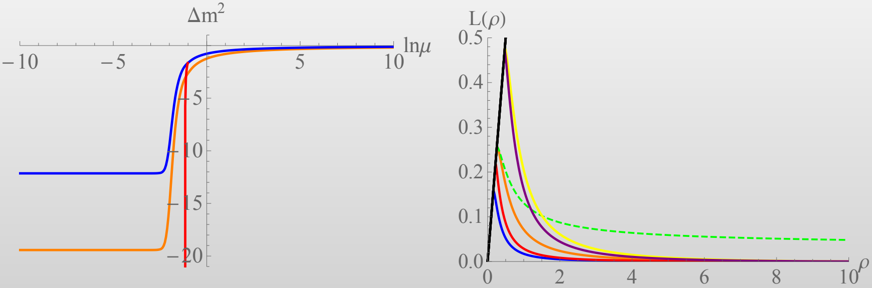
$$b_0 = \frac{1}{6\pi}\left(11(N+1) - N_{f_1} - 2(N-1)N_{f_2}\right)$$
$$\gamma_{A_2} = \frac{3}{2\pi}N\alpha\,,$$
$$\gamma_F = \frac{3}{2\pi}\frac{2N+1}{4}\alpha\,,$$

with $N=4$, $n_{f_2}=4$ and $N_{f_2}=6$ (two-loop contributions included as well)

These fix Δm^2 and hence the model

Quenching: set $N_{f_1}=N_{f_2}=0$ in b_i

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$



- blue line: F
- orange line: A_2
- red line: F but A_2 integrated out when it condensates
- dashed green: F + additional HDO-terms such that it matches in the IR the A_2 representation.
- yellow line: quenched models for the A_2
- purple line: quenched models for the F

How you decouple the quarks is important and unknown

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

	AdS/$Sp(4)$ no decouple	AdS/$Sp(4)$ A2 decouple	AdS/$Sp(4)$ quench	lattice[a] quench	lattice[b] unquench
$f_{\pi A_2}$	0.120	0.120	0.103	0.1453(12)	
$f_{\pi F}$	0.0569	0.0701	0.0756	0.1079(52)	0.1018(83)
$M_{V A_2}$	1*	1*	1*	1.000(32)	
$f_{V A_2}$	0.517	0.517	0.518	0.508(18)	
$M_{V F}$	0.61	0.814	0.962	0.83(19)	0.83(27)
$f_{V F}$	0.271	0.364	0.428	0.411(58)	0.430(86)
$M_{A A_2}$	1.35	1.35	1.28	1.75 (13)	
$f_{A A_2}$	0.520	0.520	0.524	0.794(70)	
$M_{A F}$	0.938	1.19	1.36	1.32(18)	1.34(14)
$f_{A F}$	0.303	0.399	0.462	0.54(11)	0.559(76)
$M_{S A_2}$	0.375	0.375	1.14	1.65(15)	
$M_{S F}$	0.325	0.902	1.25	1.52 (11)	1.40(19)
$M_{B A_2}$	1.85	1.85	1.86		
$M_{B F}$	1.13	1.53	1.79		

[a] Ed Bennett et al., PRD **101** (2020), 074516; [b] Ed Bennett et al., JHEP **12** (2019), 053

- We set the scale in the A_2 sector
  the pattern of mass scales is right
  A, S meson sectors are a little light
- The F sector is lighter than the A_2s and then also in the right pattern

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

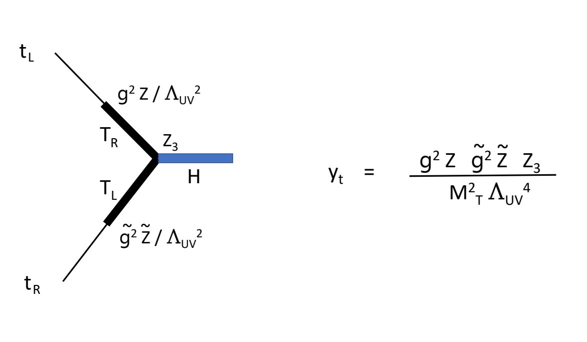
	AdS/$Sp(4)$ no decouple	AdS/$Sp(4)$ A2 decouple	AdS/$Sp(4)$ quench	lattice[a] quench	lattice[b] unquench
$f_{\pi A_2}$	0.120	0.120	0.103	0.1453(12)	
$f_{\pi F}$	0.0569	0.0701	0.0756	0.1079(52)	0.1018(83)
$M_{V A_2}$	1*	1*	1*	1.000(32)	
$f_{V A_2}$	0.517	0.517	0.518	0.508(18)	
$M_{V F}$	0.61	0.814	0.962	0.83(19)	0.83(27)
$f_{V F}$	0.271	0.364	0.428	0.411(58)	0.430(86)
$M_{A A_2}$	1.35	1.35	1.28	1.75 (13)	
$f_{A A_2}$	0.520	0.520	0.524	0.794(70)	
$M_{A F}$	0.938	1.19	1.36	1.32(18)	1.34(14)
$f_{A F}$	0.303	0.399	0.462	0.54(11)	0.559(76)
$M_{S A_2}$	0.375	0.375	1.14	1.65(15)	
$M_{S F}$	0.325	0.902	1.25	1.52 (11)	1.40(19)
$M_{B A_2}$	1.85	1.85	1.86		
$M_{B F}$	1.13	1.53	1.79		

[a] Ed Bennett et al., PRD **101** (2020), 074516; [b] Ed Bennett et al., JHEP **12** (2019), 053

- holographic modeling useful to study effect of changes in the running
- the gap between the F and A_2 sector grows (in particular if A_2 not decoupled)
- the slower the running the lighter the scalar mass becomes – is the biggest change
- prediction for baryons: between FFF and $A_2A_2A_2$ values

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

Top Yukawa coupling:



Plausible forms for the Z factors up to O(1) couplings
(this is beyond quadratic order in the holographic model)

$$Z_3 \simeq \int d\rho\, \rho^3 \frac{\partial_\rho \pi(\rho)\, \psi_B(\rho)^2}{(\rho^2 + L^2)^2}\,, \qquad Z = \tilde{Z} \simeq \int d\rho\, \rho^3 \partial_\rho \psi_B(\rho)$$

$\Rightarrow Y_t \simeq 0.01 - 0.1$ naturally if $\Lambda_{UV} \simeq$ few TeV

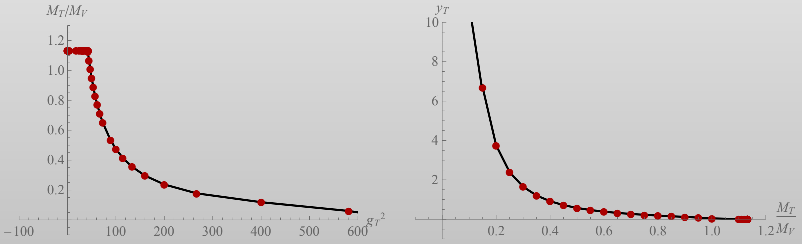
Confirmed on the lattice

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

Important: We can lower the top partner mass using a HDO

$$\mathcal{L}_{HDO} = \frac{g_T^2}{\Lambda_{UV}^5} |F A_2 F|^2 \, .$$

which raises Y_t:



This is a new mechanism to generate the large top mass in these models – drives the top partner baryon mass to half the vector meson mass

Non-abelian set-up

Basic idea: mass differences correspond to D7 brane separation
⇒ starting point non-abelian DBI action

$$S_{N_f} = -\tau_p \int d^{p+1}\xi e^{-\phi}\, \mathrm{STr}\left(\sqrt{-\det(P[G_{rs} + G_{ra}(Q^{-1}-\delta)^{ab}G_{sb}] + T^{-1}F_{rs})}\sqrt{\det Q^a{}_b}\right)$$

$$Q^a{}_b = \delta^a{}_b + \mathrm{i}\,T\,[X^a, X^c]G_{cb}$$

$$\mathrm{STr}(A_1 \ldots A_n) \equiv \frac{1}{n!}\sum \mathrm{Tr}\,(A_1 \ldots A_n + \text{ all permutations}),$$

$G_{ab}(r^2)$ have matrix structure, e.g., for the case of diagonal real masses in SU(2)

$$G_{\rho\rho} = \frac{1}{r^2} \quad \text{with} \quad r^2 = \begin{pmatrix} r_u^2 & 0 \\ 0 & r_d^2 \end{pmatrix}.$$

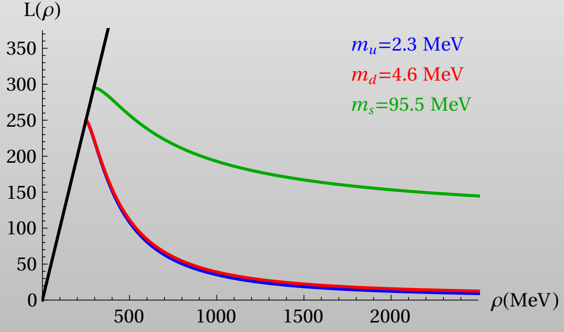
⇒ coupled Sturm-Liouville problem

† J. Erdmenger, N. Evans, Y. Liu and W.P., Universe **9** (2023) no.6, 289; JHEP **07** (2024), 169

QCD, $N_f = 3$, Embeddings

$$G_{\rho\rho} = \frac{1}{r^2} \quad \text{with} \quad r^2 = \begin{pmatrix} r_u^2 & 0 & 0 \\ 0 & r_d^2 & 0 \\ 0 & 0 & r_s^2 \end{pmatrix} \quad \text{and} \quad r_i^2 = \rho^2 + L_i^2$$

⇒ eqs. for L_i ($i = u, d, s$) decouple:



QCD, $N_f = 3$, meson masses

Observables	QCD (MeV)	$N_f = 3$ –Split Masses (MeV)	Deviation
$M_{\rho(770)}, \omega(782)$	775.26 ± 0.23	775*	fitted
$M_{K^*(892)}$	891.67 ± 0.26	1009	12%
$M_{\phi(1020)}$	1019.46 ± 0.02	1048	3%
$M_{a_1(1260)}, M_{f_1(1285)}$	1230 ± 40	1104	11%
$M_{K_1(1400)}$	1403 ± 7	1377	2%
$M_{f_1(1420)}$	1426.3 ± 0.9	1713	18%
$M_{a_0(980)}, M_{f_0(980)}$	980 ± 20	929	5%
$M_{K_0^*(700)}$	845 ± 17	876	4%
$M_{f_0(1370)}$	1350±150	970	34%
M_π	$139.57 \pm O(10^{-4})$	139	<1%
M_K	497.61 ± 0.01	584	16%
$M_{\eta'}$	957.78 ± 0.06	791	19%

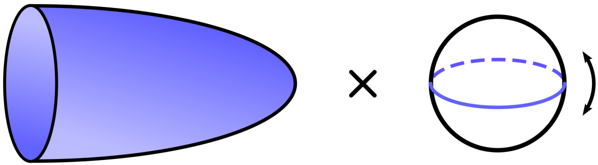
quark masses: $m_u = m_d = 3.1$ MeV and $m_s = 95.7$ MeV

- We have holographic models that describe chiral symmetry breaking due to the running of γ and HDO interactions
- compare to lattice results and look for changes as we unquench, and extra flavours beyond the lattice
- We have proposed a new HDO method to raise the top Yukawa coupling in these models
- non-Abelian DBI to get mass splitting between different representations for mesons
- What next
  - non-Abelian DBI to get mass splitting between different representations for baryons
  - inclusion of tow-index representations in brane picture

Quarks in the gauge/gravity duality

Add D7-Branes (eight-dimensional surfaces) to ten-dimensional space

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		



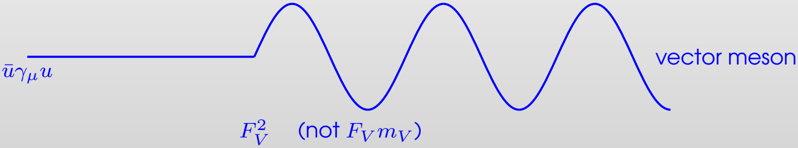
(thanks to J. Erdmenger)

Quarks: Low energy limit of open strings between D3- and D7-Branes

Decay Constants

a la AdS/QCd, see J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, PRL **95** (2005), 261602

Decay constants are determined by allowing a source to couple to a physical state



Now we need to fix the normalizations of the holographic linear perturbations . . .

For the physical states we canonically normalize the kinetic terms. . .

For the source solutions we fix κ and the norms so that we match perturbative results for e.g. Π_{VV} in the UV

$$N_V^2 = N_A^2 = \frac{g_5^2 d(R)\, N_f(R)}{48\pi^2}$$

Baryons

In D3/D7 system some quark-gaugino-quark tri-fermion states are described by world volume fermions on the D7 – it does not seem unreasonable to include three quark states in this way therefore

$$S_{1/2} = \int d^4x \int \rho\, \rho^3 \bar{\Psi}(\not{D}_{AdS} - m)\Psi$$

The four component fermion satisfies the second order equation

$$\left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{M_B^2}{r^4} + \mathcal{P}_2 \frac{1}{r^4} - \frac{m^2}{r^2} - \mathcal{P}_3 \frac{m}{r^3}\, \gamma^\rho\right) \psi = 0\,,$$

where M_B is the baryon mass and

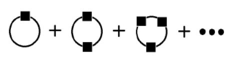
$$\mathcal{P}_1 = \frac{6}{r^2}\,(\rho + L_0\, \partial_\rho L_0)\ ,$$
$$\mathcal{P}_2 = 2\left((\rho^2 + L_0^2) L \partial_\rho^2 L_0 + (\rho^2 + 3L_0^2)(\partial_\rho L_0)^2 + 4\rho L_0 \partial_\rho L_0 + 3\rho^2 + L_0^2\right)\ ,$$
$$\mathcal{P}_3 = (\rho + L_0\, \partial_\rho L_0)\ .$$

G. F. de Teramond and S. J. Brodsky, PRL **94** (2005), 201601; R. Abt, J. Erdmenger, N. Evans and K. S. Rigatos, JHEP **11** (2019), 160

Higher Dimension/Nambu Jona-Lasinio Operators

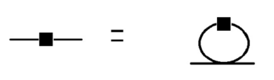
$$\mathcal{L} = \bar{\psi}_L \not{\partial} \psi_L + \bar{\psi}_R \not{\partial} \psi_R + \frac{g^2}{\Lambda_{UV}} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L$$

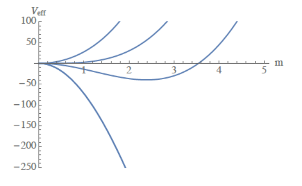
Calculate effective potential

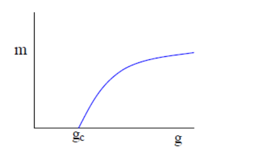


$$\Delta V_{eff} = - \int_0^{\{} \Lambda_{UV} \frac{d^4 k}{(2\pi)^4} \mathrm{Tr} \log(k^2 + m^2)$$

$$\frac{g^2}{\Lambda_{UV}} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \rightarrow \frac{g^2}{\Lambda_{UV}} \langle \bar{\psi}_L \psi_R \rangle \bar{\psi}_R \psi_L \rightarrow \frac{m^2 \Lambda_{UV}}{g^2}$$







Witten´s Multi-Trace Operator Prescription

E. Witten hep-th/0112258; N. Evans + K. Kim arXiv:1601.02824 (hep-th)

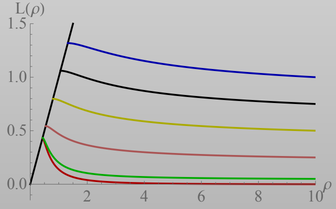
$$\frac{g^2}{\Lambda_{UV}}\,\bar\psi_L\psi_R\,\bar\psi_R\psi_L \to \frac{g^2}{\Lambda_{UV}}\,\langle\bar\psi_L\psi_R\rangle\,\bar\psi_R\psi_L \to \frac{m^2\Lambda_{UV}}{g^2}$$ so add $$S=\int\mathcal{L}+\frac{L^2\rho^2}{g^2}\Big|_{\Lambda_{UV}}$$

On variation

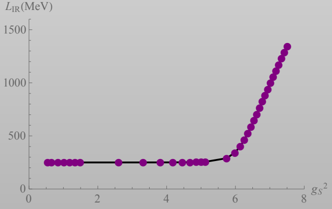
$$0=\text{E.-L. eqn}\;+\frac{\partial\mathcal{L}}{\partial L'}\delta L\Big|_{\Lambda_{IR,UV}}+\frac{2L\rho^2}{g^2}\delta L\Big|_{\Lambda_{UV}}$$

The Euler Lagrange equation solutions are left unchanged but we pick those that satisfy the UV and IR boundary conditions. Now we let the mass vary in the UV and need

$$m=\frac{g^2}{\Lambda_{UV}}c$$



Read off m, c and compute g



$SU(4), 3F, 3\bar{F}, 5A_2$; $SU(5) \times SU(3)_L \times SU(3)_R/SO(5) \times SU(3)$

(G. Ferretti, JHEP **06** (2014), 142)

Here the A_2 symmetry breaking generates the SM Higgs; FA_2F top partners

	Lattice[a] $4A_2, 2F, 2\bar{F}$ unquench	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ no decouple	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ no decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
$M_{V A_2}$	1.00(4)	1*	1*	1*	1*	1*
$f_{V A_2}$	0.68(5)	0.489	0.489	0.516	0.516	0.517
$M_{V F}$	0.93(7)	0.933	0.939	0.890	0.904	0.976
$f_{V F}$	0.49(7)	0.458	0.461	0.437	0.491	0.479
$M_{A A_2}$		1.37	1.37	1.32	1.32	1.28
$f_{A A_2}$		0.505	0.505	0.521	0.521	0.522
$M_{A F}$		1.37	1.37	1.21	1.23	1.28
$f_{A F}$		0.501	0.504	0.453	0.509	0.492
$M_{S A_2}$		0.873	0.873	0.684	0.684	1.18
$M_{S F}$		1.03	1.02	0.811	0.798	1.25
$M_{J A_2}$	3.9(3)	2.21	2.21	2.21	2.21	2.22
$M_{J F}$	2.0(2)	2.07	2.08	1.97	2.00	2.17
$M_{B A_2}$	1.4(1)	1.85	1.85	1.85	1.85	1.86
$M_{B F}$	1.4(1)	1.74	1.75	1.65	1.68	1.81

[a] V. Ayyar et al., PRD **97** (2018), 074505: (unquenched) $SU(4)\ 2F, 2\bar{F}, 4A_2$

- pattern agree quite well in particular M_{VF} and M_{JF}
- M_{JA_2} off and also M_{BF} on the lattice below our estimate

$SU(4), 3F, 3\bar{F}, 5A_2$; $SU(5) \times SU(3)_L \times SU(3)_R/SO(5) \times SU(3)$

	Lattice[a] $4A_2, 2F, 2\bar{F}$ unquench	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ no decouple	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ no decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
$M_{V A_2}$	1.00(4)	1*	1*	1*	1*	1*
$f_{V A_2}$	0.68(5)	0.489	0.489	0.516	0.516	0.517
$M_{V F}$	0.93(7)	0.933	0.939	0.890	0.904	0.976
$f_{V F}$	0.49(7)	0.458	0.461	0.437	0.491	0.479
$M_{A A_2}$		1.37	1.37	1.32	1.32	1.28
$f_{A A_2}$		0.505	0.505	0.521	0.521	0.522
$M_{A F}$		1.37	1.37	1.21	1.23	1.28
$f_{A F}$		0.501	0.504	0.453	0.509	0.492
$M_{S A_2}$		0.873	0.873	0.684	0.684	1.18
$M_{S F}$		1.03	1.02	0.811	0.798	1.25
$M_{J A_2}$	3.9(3)	2.21	2.21	2.21	2.21	2.22
$M_{J F}$	2.0(2)	2.07	2.08	1.97	2.00	2.17
$M_{B A_2}$	1.4(1)	1.85	1.85	1.85	1.85	1.86
$M_{B F}$	1.4(1)	1.74	1.75	1.65	1.68	1.81

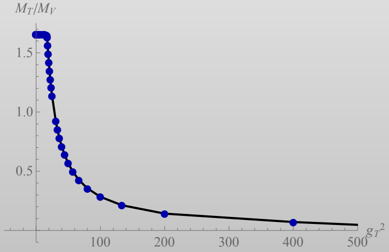
- Adding extra flavours is not a huge change
- Scalar masses get lighter by adding extra flavours

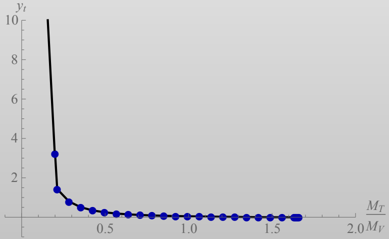
$SU(4), 3F, 3\ \bar{F}, 5A_2;\ SU(5) \times SU(3)_L \times SU(3)_R/SO(5) \times SU(3)$

Top Yukawa coupling: similar as before, need additional HDO

$$\mathcal{L}_{HDO} = \frac{g_T^2}{\Lambda_{UV}^5} |F A_2 F|^2 \, .$$

which raises Y_t:





This is a new mechanism to generate the large top mass in these models – we drive the top partner baryon mass to about $1/3$ the vector meson mass