Nonperturbative effects from gauging higher-form symmetries

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Based on arXiv:2110.02981 with Erich Poppitz and arXiv:2407.03416 with Samson Chan

Outline

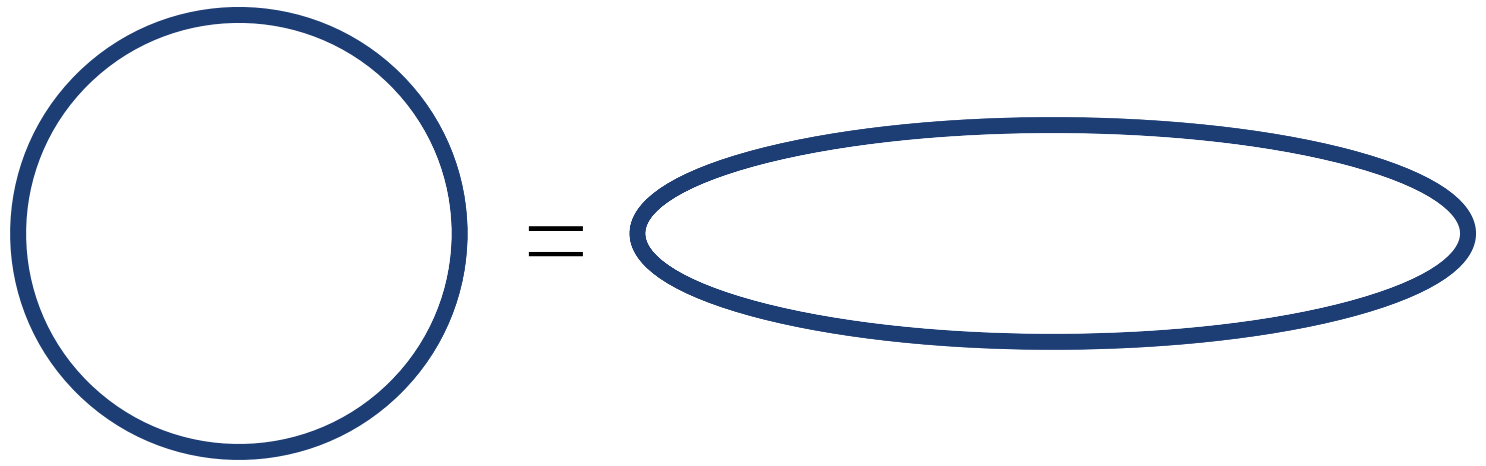
- Review of Higher-form symmetries
- 1-form global symmetry in SM: gauging and new B&L violation processes
- 2-form global symmetry in axion-YM: gauging, anomalies, and hadronic structure

Symmetries: the new perspective

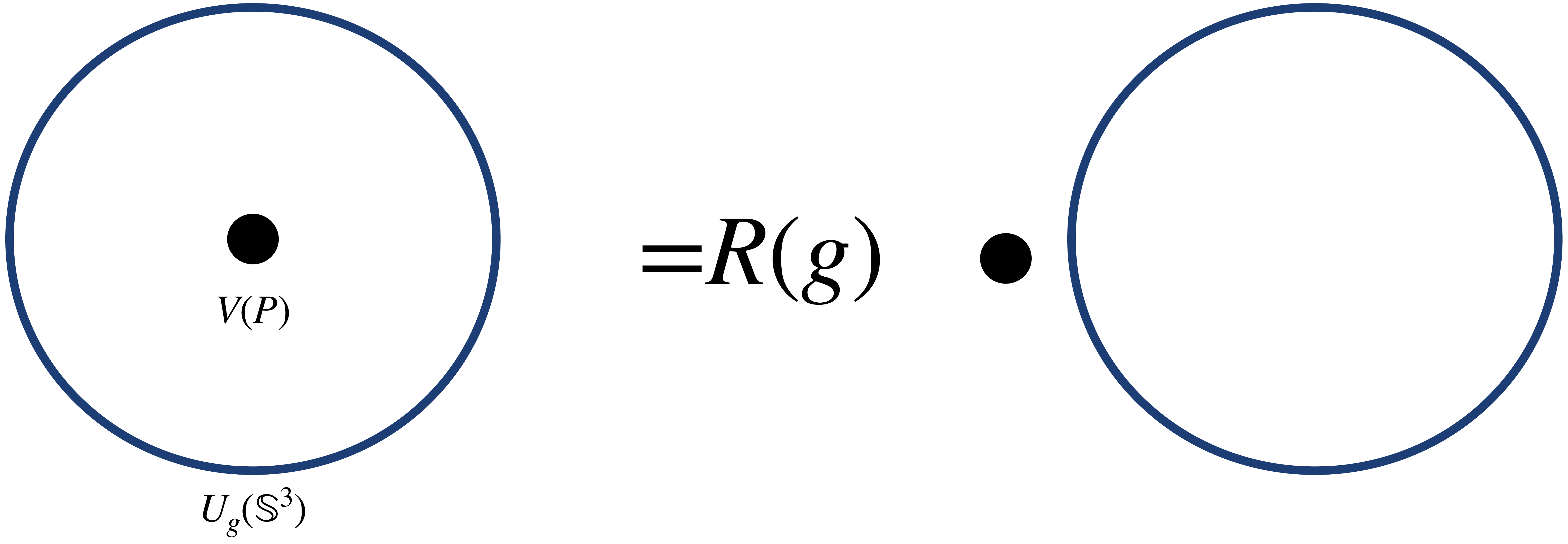
- 0-form **global** symmetries act on local fields: $\phi(P) \to R(G)\phi(P)$
- E.g. $G = U(1), SU(N_f)$ or $\mathbb{Z}_N$
- G continuous ➡ Conserved Noether's current $d \star j^{(1)} = 0 \leftrightarrow \partial_\mu j_\mu = 0$
- Modern way: associate the action of G to a 3-manifold (4D theory)
- $$Q(\mathbb{M}_3) = \underbrace{\oint_{\mathbb{M}^3} \star j^{(1)}}_{\text{topological}} = \oint_{\mathbb{M}^3} ds\, n_\mu j_\mu, \quad \underbrace{U_g(\mathbb{M}^3) = e^{i\alpha Q(\mathbb{M}_3)}}_{\text{symmetry defect}}$$

  Gaiotto, Kapustin, Seiberg, Willett, 2014
- Fusion rule: $U_{g_1}(\mathbb{M}^3)U_{g_2}(\mathbb{M}^3) = U_{g_1 g_2}(\mathbb{M}^3) \quad g_1, g_2 \in G$

Symmetries: the new perspective

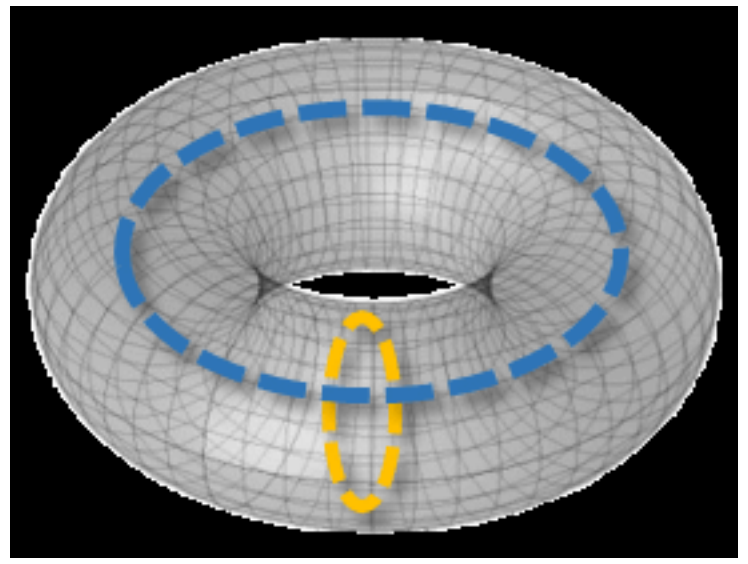
- $\mathbb{M}^3$ is a topological surface: 

- $V(P) \equiv e^{i\phi(P)}$
- $U_g(\mathbb{M}^3)V(P) = R(g)V(P)U_g(\mathbb{M}^3)$



1-form global symmetry

- 1-form **global** symmetry: charged objects are 1-dimensional **Wilson lines**
- E.g. $W(\mathbb{S}^1) = e^{i\oint_{\mathbb{S}^1} A^{(1)}}$, **probe** $n = 1$ for $U(1)$ EM
- We have $U(1)^{(1)}$-form symmetry

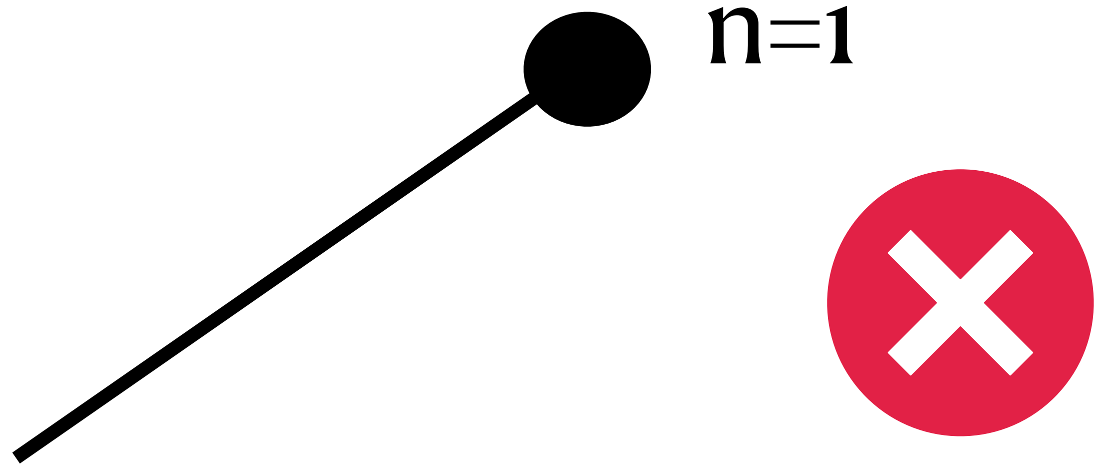


$\mathbb{M}^4 = \mathbb{T}^4$

- **Free Maxwell's** eq. $d \star F^{(2)} = \partial_\mu F_{\mu\nu} = 0$ ➡ $d \star j^{(2)} = d \star F^{(2)} = 0$
- We define $U_{g=e^{i\alpha}}(\mathbb{M}^2) = e^{i\alpha \oint_{\mathbb{M}^2} \star j^{(2)}} = e^{i\alpha \oint_{\mathbb{M}^2} \star F^{(2)}}$: measures the electric flux
- $U_{g_1}(\mathbb{M}^2) U_{g_2}(\mathbb{M}^2) = U_{g_1 g_2}(\mathbb{M}^2)\,, \quad U_g(\mathbb{M}^2) W(\mathbb{S}^1) = e^{i\alpha \mathrm{Link}(\mathbb{S}^1, \mathbb{M}^2)} W(\mathbb{S}^1) U_g(\mathbb{M}^2)$

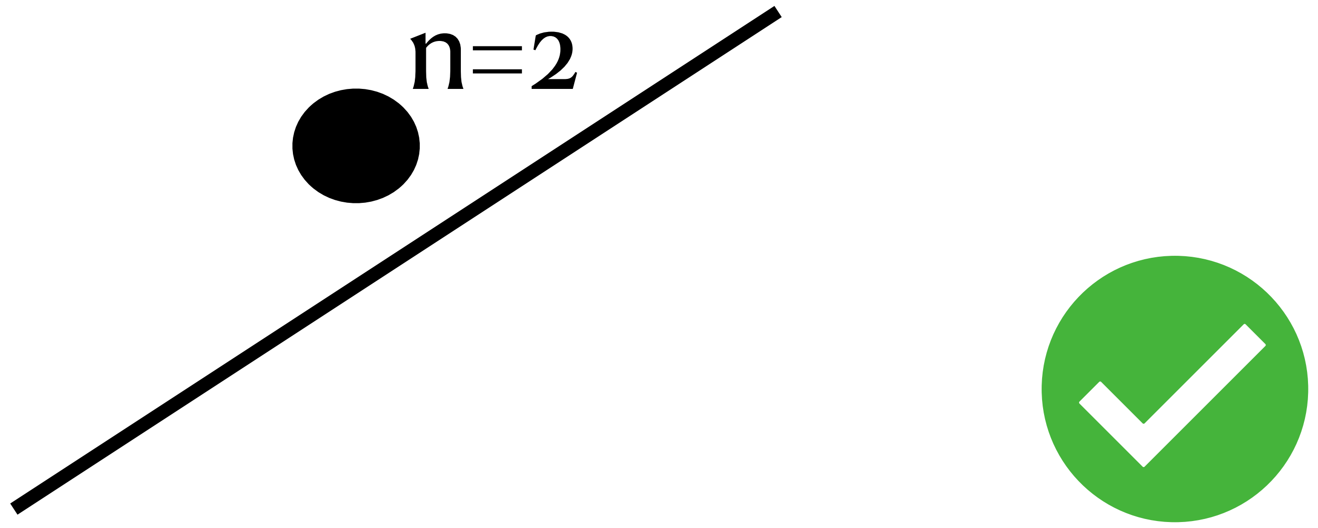
1-form global symmetry

- Assume $n = 1$ dynamical charges are present



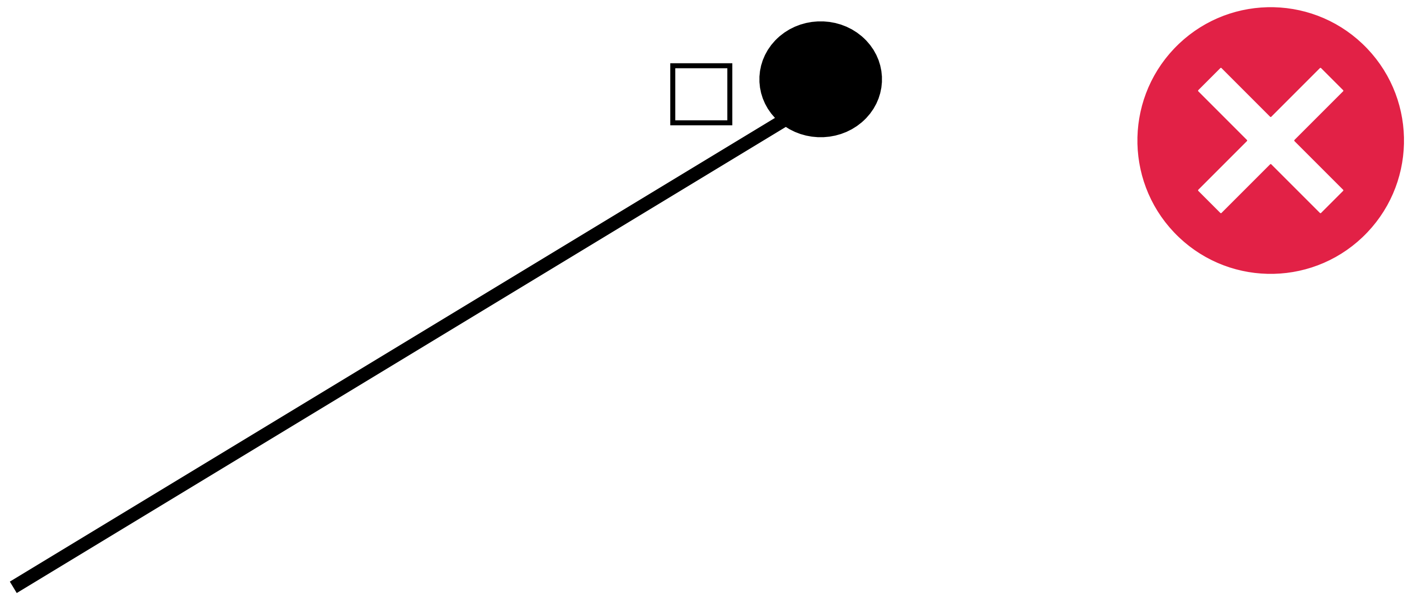
- If only $n > 1$ charges are present, fundamental Wilson lines are topological

$\mathbb{Z}_2^{(1)}$



1-form global symmetry in YM

- SU(N) pure Yang-Mills has a $\mathbb{Z}_N^{(1)}$ $1-$form global symmetry:
- $W(\mathbb{S}^1) = \mathrm{Tr}_{\square} e^{i\oint_{\mathbb{S}^1} A^{(1)}}$
- $U_{g=\mathbb{Z}_N}(\mathbb{M}^2) W(\mathbb{S}^1) = e^{i\frac{2\pi k}{N}\mathrm{Link}(\mathbb{S}^1,\mathbb{M}^2)} U(\mathbb{M}^2) W(\mathbb{S}^1) U_{g=\mathbb{Z}_N}$
- Or $\mathbb{Z}_N^{(1)}: W \to e^{i\frac{2\pi}{N}} W$
- No topological Wilson lines with dynamical matter in $\square$:



1-form global symmetry in SM

- Matter contents:

field	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_B$	$U(1)_L$
q_L	$\square$	$\square$	1	$\frac{1}{3}$	0
l_L	$\mathbf{1}$	$\square$	-3	0	1
$\tilde{e}_R$	$\mathbf{1}$	$\mathbf{1}$	6	0	-1
$\tilde{u}_R$	$\overline{\square}$	$\mathbf{1}$	-4	$-\frac{1}{3}$	0
$\tilde{d}_R$	$\overline{\square}$	$\mathbf{1}$	2	$-\frac{1}{3}$	0
h	$\mathbf{1}$	$\square$	3	0	0

1-form global symmetry in SM

- SM accommodates a $\mathbb{Z}_6^{(1)}$ 1-form global symmetry

Tong, 2017
MA, Poppitz, 2021

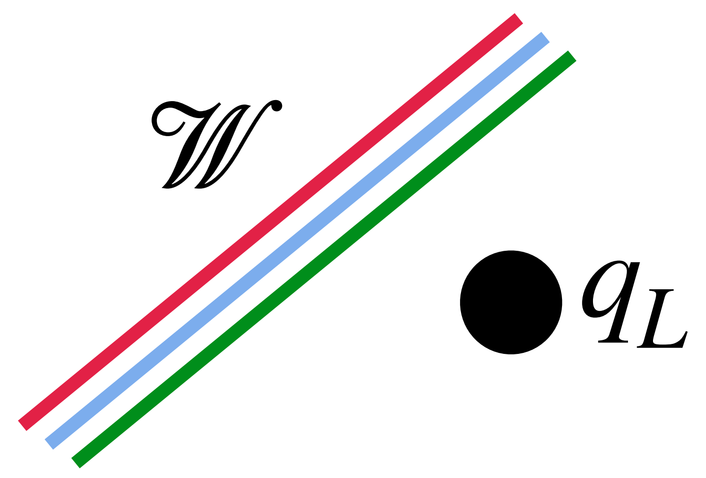
- Three distinct Wilson lines:
- $W_3 = \mathrm{Tr}_{\square} e^{i\oint_{\mathbb{S}^1} A_3^{(1)}}, \mathbb{Z}_3^{(1)} : W_3 \to e^{i\frac{2\pi}{3}} W_3$
- $W_2 = \mathrm{Tr}_{\square} e^{i\oint_{\mathbb{S}^1} A_2^{(1)}}, \mathbb{Z}_2^{(1)} : W_2 \to e^{i\frac{2\pi}{2}} W_2$
- $W_1 = e^{i\oint_{\mathbb{S}^1} A_1^{(1)}}, U(1)^{(1)}_{g=e^{i(\alpha=\frac{2\pi}{6})}} : W_1 \to e^{i\frac{2\pi}{6}} W_1$

$\mathrm{LCM}(2,3) = 6$

$\mathscr{W} = W_1 W_2 W_3$

$\mathbb{Z}_6^{(1)} : \mathscr{W} \to e^{i\frac{2\pi}{6}} \mathscr{W}$

- $\mathscr{W}$ is not screened by SM particle
- E.g. $q_L : \underbrace{e^{i\frac{2\pi}{3}}}_{\square \text{ in } SU(3)} \underbrace{e^{i\frac{2\pi}{2}}}_{\square \text{ in } SU(2)} \underbrace{e^{i\frac{2\pi}{6}}}_{q=1 \text{ under } U(1)} = 1$



Gauging a global symmetry

- **Gauging 0-form symmetry, 2 steps:**
  $d\star j^{(1)}=0 \leftrightarrow \oint_{\mathbb{M}^4} A^{(1)}\star j^{(1)}\,,\quad G^{(0)}: A^{(1)}\to A^{(1)}+d\Lambda^{(0)}$
- $\mathscr{Z}=\int [DA^{(1)}]e^{-S_{YM}-\theta\underbrace{\frac{1}{8\pi^2}\int_{\mathbb{M}^4}F^{(2)}\wedge F^{(2)}}_{Q}}\,,\quad F^{(2)}=dA^{(1)}+[A^{(1)},A^{(1)}]\,,\quad Q\in\mathbb{Z}$
- BPST instantons have $Q\in\mathbb{Z}$, e.g., $G=SU(2)$ instantons in the weak sector
- Well understood explicit solutions on $\mathbb{M}^4=\mathbb{R}^4$
- 't Hooft vertex: $e^{-\frac{8\pi^2}{g_2^2}}q_Lq_Lq_Ll_L\to|\Delta B|=1$, applications to baryogengesis

Gauging the $\mathbb{Z}_6^{(1)}$ symmetry

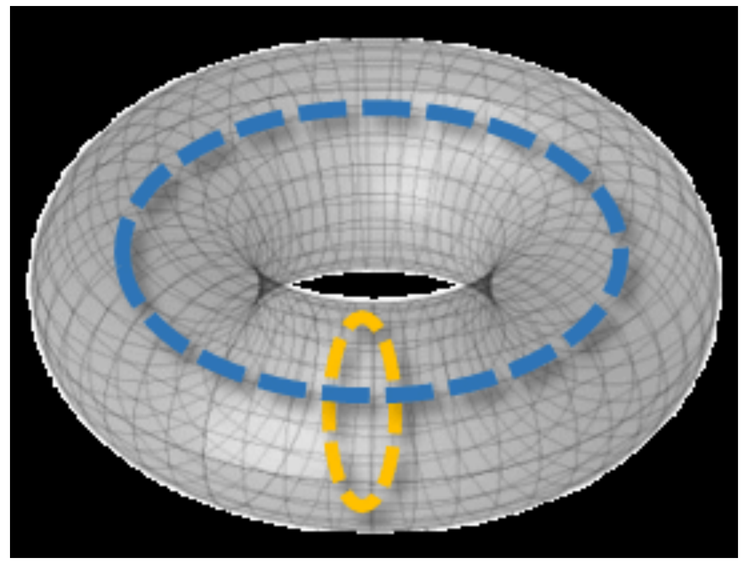
- **Gauging $\mathbb{Z}_n^{(1)}$ 1-form symmetry**:
- $d \star j^{(2)} = 0 \leftrightarrow \oint_{\mathbb{M}^4} B^{(2)} \star j^{(2)}, \quad G^{(1)} : B^{(2)} \to B^{(2)} + d\Lambda^{(1)}, \quad dB^{(2)} = 0$
- $\mathcal{Z} = \int [DB^{(2)}] e^{-S_{YM} - \underbrace{\frac{n}{8\pi^2} \int_{\mathbb{M}^4} B^{(2)} \wedge B^{(2)}}_{Q}}, \quad Q \in \frac{\mathbb{Z}}{n}$
- We may gauge $\mathbb{Z}_n^{(1)} \subseteq \mathbb{Z}_6^{(1)}, \quad n = 1,2,3,6$
- Four distinct SM(s): $G_n = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_n}, \quad n = 1,2,3,6$

Gauging the $\mathbb{Z}_6^{(1)}$ symmetry

- E.g. gauging the full $\mathbb{Z}_6^{(1)}$, i.e., $G_6 = \dfrac{SU(3)\times SU(2)\times U(1)}{\mathbb{Z}_6}$
- Recalling $\mathscr{W} = \underbrace{W_1}_{e^{i\frac{2\pi}{6}}}\ \underbrace{W_2}_{e^{i\frac{2\pi}{2}}}\ \underbrace{W_3}_{e^{i\frac{2\pi}{3}}}$, three backgrounds are needed
- $\oint_{\mathbb{M}^2} B_1^{(2)} \in \frac{2\pi}{6}\mathbb{Z}\,,\quad \oint_{\mathbb{M}^2} B_2^{(2)} \in \frac{2\pi}{2}\mathbb{Z}\,,\quad \oint_{\mathbb{M}^2} B_3^{(2)} \in \frac{2\pi}{3}\mathbb{Z},$
- $\underbrace{\oint_{\mathbb{M}^2} B_1^{(2)} = \oint_{\mathbb{M}^2} B_2^{(2)} + B_3^{(2)}}_{\text{constraint}}$ (SM particles are blind to the combined fluxes)

2-cycles are needed

e.g. $\mathbb{M}^2 = \mathbb{T}^2 \subset \mathbb{M}^4 = \mathbb{T}^4$



Gauging the $\mathbb{Z}_6^{(1)}$ symmetry

- Sum over backgrounds of $B_1^{(2)}, B_2^{(2)}, B_2^{(3)}$ (omitting the details)
- $\mathcal{Z}_{G_6} = \int [DB_1^{(2)} DB_2^{(2)} DB_3^{(2)}] e^{-S_{SM}} = \sum_{Q_1, Q_2, Q_3} e^{-S_{SM}} \underbrace{\ldots.}_{\text{extra stuff}}$
- $Q_2 \in \frac{1}{2} + \mathbb{Z}\,, \quad Q_3 \in \frac{1}{3} + \mathbb{Z}\,, \quad Q_1 = \underbrace{\left(n_1 - \frac{1}{2} - \frac{1}{3}\right)\left(n_2 - \frac{1}{2} - \frac{1}{3}\right)}_{\oint_{\mathbb{M}^2} B_1^{(2)} = \oint_{\mathbb{M}^2} B_2^{(2)} + B_3^{(2)}}$
- Are these objects physical?

Fractional instantons

- Exact (Anti)self-dual instnaton solutions on $\mathbb{T}^4$
- E.g. $SU(2)/\mathbb{Z}_2$ bundle: $S_2 = \frac{1}{4g_2^2}\int_{\mathbb{T}^4} F_{\mu\nu}F_{\mu\nu} = \pm\frac{8\pi^2}{g_2^2}Q_2 + \int_{\mathbb{T}^4}\left(F_{\mu\nu} \mp \tilde{F}_{\mu\nu}\right)^2$
- (Anti)self-dual solutions: $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}$ ➡ $S_2 = \frac{8\pi^2}{g_2^2}|Q_2|, \quad |Q_2| = \frac{1}{2}$
- Explicit solution: symmetric $\mathbb{T}^4$ with 0 Higgs vev
- $$\underbrace{A_1 = \frac{2\pi x^2}{L^2}\frac{\tau^3}{2}, \quad A_3 = \frac{2\pi x^4}{L^2}\frac{\tau^3}{2}, \quad A_2 = A_4 = 0}_{\text{constant abelian field strength}}$$

’t Hooft, 1981
Van Paal, 1984
MA, Poppitz, 2021

Fractional instantons

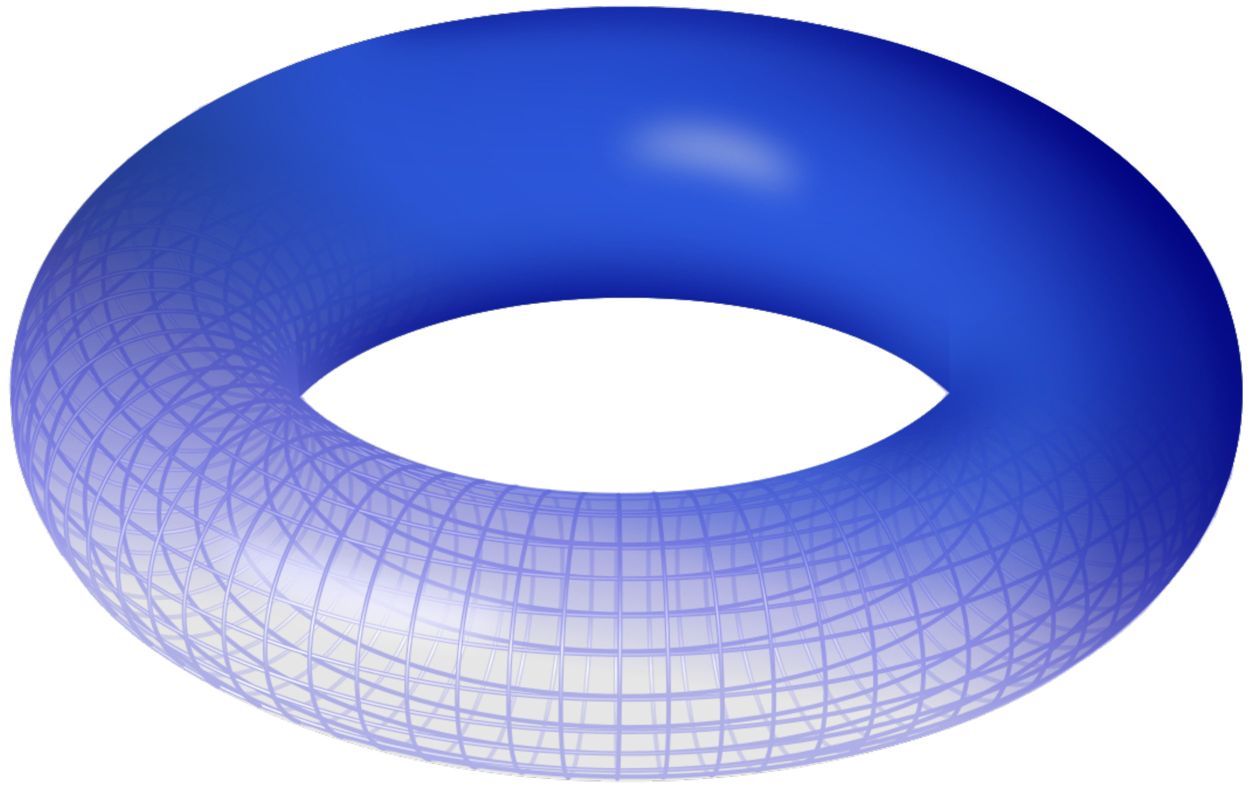
- Similar $SU(3)$ and $U(1)$ solutions exists: $Q_3 = \frac{2}{3}, \quad Q_1 = \left(n_1 - \frac{1}{2} - \frac{1}{3}\right)\left(n_2 - \frac{1}{2} - \frac{1}{3}\right)$
- Self-duality ➡ **stability** of $SU(2)$ and $SU(3)$ solutions
- We can relax the symmetric $\mathbb{T}^4$:
- Exact self-dual solutions exist if $L_1 L_2 = L_3 L_4$ 't Hooft 1981, Van Ball, 1984
- Approximate self-dual solutions for $L_1 L_2 \neq L_3 L_4$ (expansion in $\Delta = \frac{L_1 L_2 - L_3 L_4}{\sqrt{L_1 L_2 L_3 L_4}}$)
- All known **classical** solutions are **not localized**! Antonio González-Arroyo, 2020, M.A., Poppitz 2023

Baryon number violation

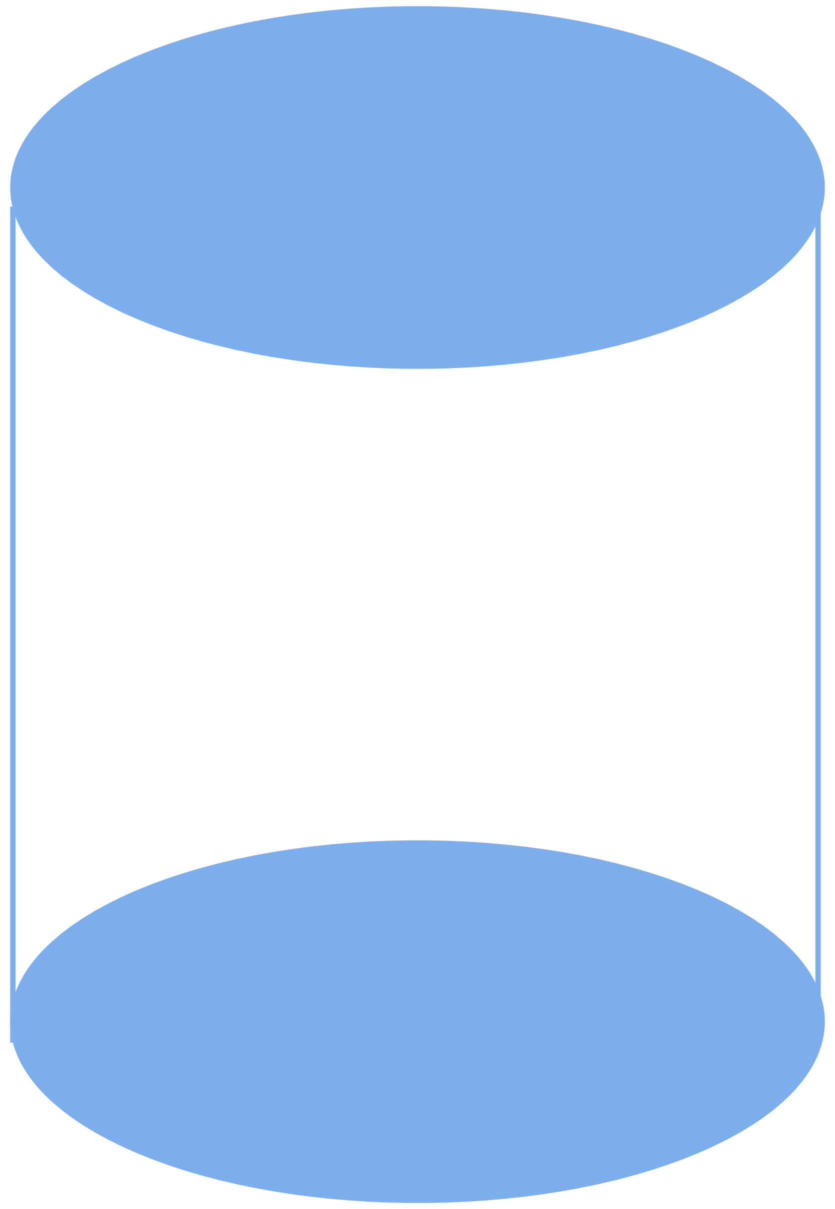
- $\mathcal{Z}_{G_6} = \sum_{\text{fractional or integer } Q} e^{-S_{SM}}$
- $U(1)_B - U(1)_L$ is a good symmetry, $U(1)_B + U(1)_L$ is not
- ΔB from fractional instantons:
- New 't Hooft vertex $\sim e^{-(S_1+S_2+S_3)}(q_L)^{I_1}(l_L)^{I_2}(\tilde{e}_R)^{I_3}(\tilde{u}_R)^{I_4}(\tilde{d}_R)^{I_5}$ MA, Poppitz, 2021
- BPST 't Hooft vertex $\sim e^{-\frac{8\pi^2}{g_2^2}} q_L q_L q_L l_L$

Baryon number violation

- What are we comparing?
- Processes at $T = 0$



Deforming the solutions →



$$\underbrace{\mathbb{T}^4}_{T\neq 0}$$

$$\underbrace{\mathbb{T}^3 \times \mathbb{R}}_{T=0}$$

Baryon number violation

- $\mathbb{T}^3$ is symmetric: $L^{(-1)}$ = IR cutoff $\gtrsim$ TeV (0 Higgs vev)
- Find L_{critical}:
- $$\underbrace{\frac{2}{3}\frac{8\pi^2}{g_3^2(L)}}_{S_3} + \underbrace{\frac{1}{2}\frac{8\pi^2}{g_2^2(L)}}_{S_2} + \underbrace{\frac{2\pi^2}{g_1^2(L)}\left[(n_1+\frac{1}{2}+\frac{1}{3})^2+(n_2+\frac{1}{2}+\frac{1}{3})^2\right]}_{S_1} < \underbrace{\frac{8\pi^2}{g_2^2(L)}}_{\text{BPST}}$$
- RG running:
- $$\frac{8\pi^2}{g_i^2(L)} = \frac{8\pi^2}{g_i^2(M_Z)} - b_i\log(LM_Z)\,, \quad b_1 = -(80n_f + 6n_H)\,, \quad b_2 = \frac{22}{3} - \frac{4n_f}{3} - \frac{n_H}{6}\,, \quad b_3 = 11 - \frac{4}{3}n_f$$

Baryon number violation

- Results:

Gauged 1-form center	n_1	n_2	Smallest $U(1)$ action	ΔB	L^{-1}_{critical} (GeV)
$\mathbb{Z}_6^{(1)}$	-1	-1	$\frac{\pi^2}{9g_1^2}$	0	-
$\mathbb{Z}_6^{(1)}$	-1	0	$\frac{13\pi^2}{9g_1^2}$	$3n_f$	6×10^{34}
$\mathbb{Z}_6^{(1)}$	0	-1	$\frac{13\pi^2}{9g_1^2}$	$3n_f$	6×10^{34}
$\mathbb{Z}_3^{(1)}$	0	0	$\frac{4\pi^2}{9g_1^2}$	$-2n_f$	2.7×10^{33}
$\mathbb{Z}_2^{(1)}$	0	0	$\frac{\pi^2}{g_1^2}$	$-4n_f$	1.5×10^{24}

- Adding extra charged matter (under $U(1)$) can bring $L^{-1}_{critical}$ below M_P

Cosmology

- Fractional-instanton **solution is constant** over $\mathbb{T}^3$ (**not localized**). Is this interesting? Do we live on $\mathbb{T}^3$?
- Maybe: CMB analysis $L_{\mathbb{T}^3} > \mathcal{O}(\text{few})L_0$, $L_0 \sim 12\,\text{Gpc}$ Aslanyan, Manohar, Yadav, 2013
- Tracing back: $L_H \sim \dfrac{M_P}{T^2}, \quad L_{\mathbb{T}^3} \sim \dfrac{M_P}{T_l T}, \quad \dfrac{L_{\mathbb{T}^3}}{L_H} \sim \dfrac{T}{T_l}$
- Early Universe $L_{\mathbb{T}^3} > L_H$
- If fractional instantons have a cutoff scale $< L_H$, there might have played a role.

2-form symmetry in axion-YM theory

- UV: $\mathscr{L} = \text{YM}+\underbrace{\text{Dirac fermion}}_{\text{in rep. } R\to\mathbb{Z}_n^{(1)}} + |d\Phi|^2 + \lambda(|\Phi|^2 - v^2)^2 + y\Phi\tilde{\psi}\psi$
- $\Phi = \rho e^{ia}$, $a \sim a + 2\pi$, and take $v \gg \Lambda$(strong scale)
- UV symmetries:
- $U(1)^{(0)\chi} \underbrace{\to}_{\text{ABJ anomaly}} \mathbb{Z}_{2T_R}^{(0)\chi}$
- $\mathbb{Z}_n^{(1)}$ 1-form symmetry
- $U(1)_B^{(0)}$

Mixed 't Hooft anomaly (between $\mathbb{Z}_{2T_R}^{(0)\chi}$ and $\mathbb{Z}_n^{(1)}$)

2-form symmetry in axion-YM theory

- At $\Lambda \ll E \ll v$: $\mathscr{L} \supset YM + \frac{v^2}{2}|da|^2 + \underbrace{\frac{T_R\, a}{8\pi^2} F \wedge F}_{\text{ABJ anomaly}}$

- Bianchi identity: $d^2 a = 0 \leftrightarrow d \star j^{(3)} = 0$ ➡ $U(1)^{(2)}$ symmetry (couples to strings)

- $da = \star j^{(3)}, U_g = e^{i\alpha \int_{\mathbb{M}^1} da}$

Gauging the 2-form symmetry

- This is useful at $E \ll \Lambda$:
- Gauge $U(1)^{(2)}$: introduce $\mathscr{L} \supset \frac{T_R}{2\pi} \star j^{(3)} \wedge C^{(3)} = \frac{T_R}{2\pi} da \wedge C^{(3)}$
- $\mathscr{Z} = \int [dC^{(3)}][da] e^{-S_{IR}}, \quad S_{IR} = \frac{v^2}{2} |da|^2 + \frac{T_R}{2\pi} da \wedge C^{(3)} + \frac{|dC^{(3)}|^2}{\Lambda^4}$ + higher orders

MA, Chan, 2024

- It is inevitable to IR match a mixed 1-form/0-form 't Hooft anomaly
- $dC^{(3)}$ is the long tail incarnation of $F \wedge F$ below the confinement scale Λ

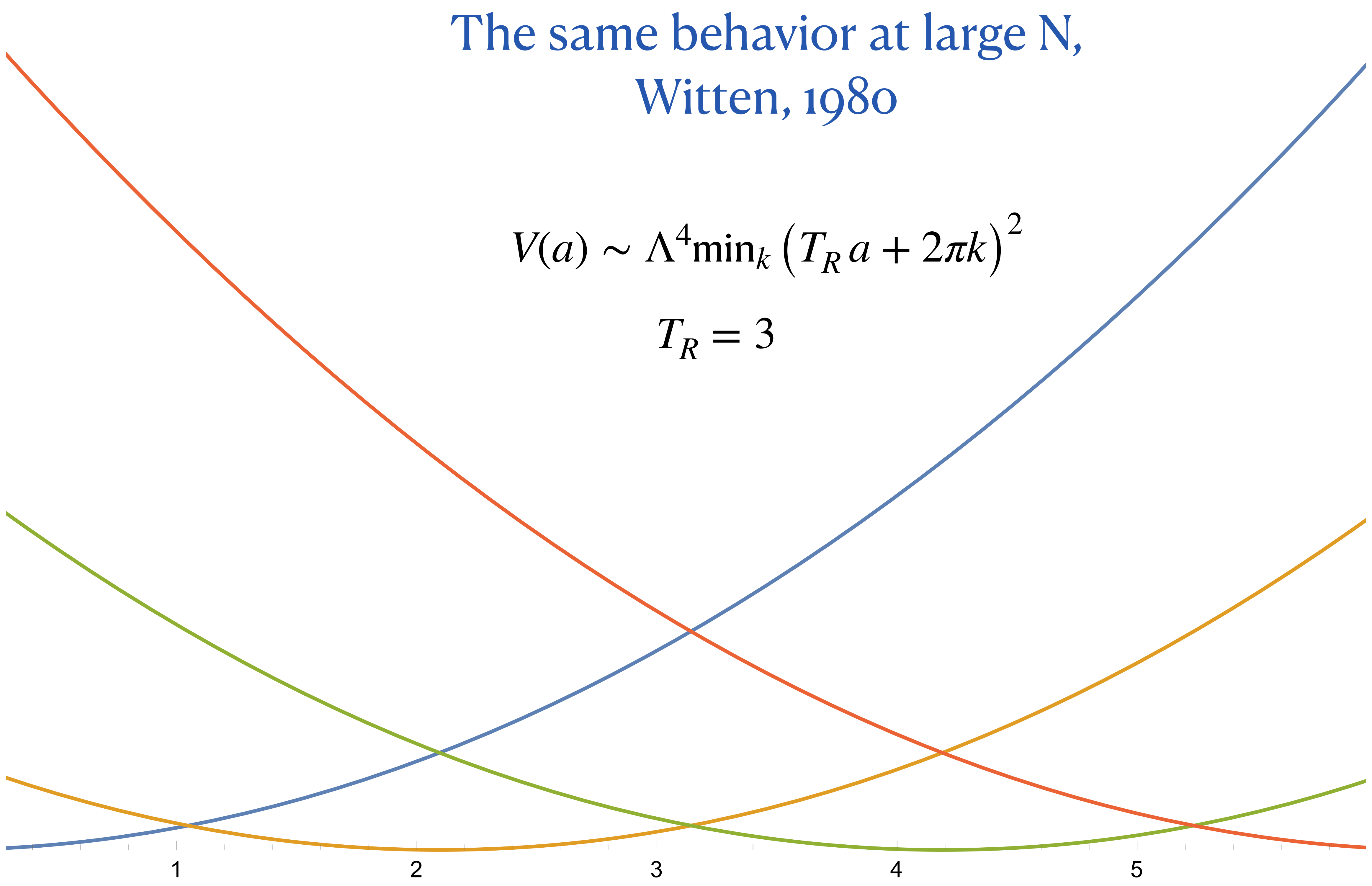
Luscher, 1978
Veneziano, 1979

Gauging the 2-form symmetry

- Quantization condition $\int_{\mathbb{M}^4} dC^{(3)} \in 2\pi m \, , m \in \mathbb{Z}$
- Performing the sum over $m \in \mathbb{Z}$:
- $$\mathcal{Z}[a] \sim \sum_{k \in \mathbb{Z}} \underbrace{\exp\left[-i\frac{Nk}{4\pi}\int_{\mathbb{M}^4} B^{(2)} \wedge B^{(2)}\right]}_{\text{mixed anomaly}: a \to a + 2\pi/T_R} \exp\left[-\int_{\mathbb{M}^4} \frac{v^2}{2}|da|^2 + \frac{\Lambda^4}{8\pi^2}\left(T_R\, a + 2\pi k\right)^2\right]$$
- $V(a) \sim \Lambda^4 \mathrm{min}_k \left(T_R\, a + 2\pi k\right)^2$
- $\text{vacua} = \dfrac{2\pi\ell}{T_R}, \quad \text{cusps} = \dfrac{\pi(2\ell + 1)}{T_R}$

Gauging the 2-form symmetry

- axion+hadronic walls: $\delta_{DW} \sim \frac{v}{\Lambda^2} \gg \Lambda^{-1} \quad \delta_H \sim \Lambda^{-1}$



Summary

- Gauging higher-form symmetries leads to new nonperturbative effects.
- SM gauge group is still an open question:
- $G_n = \dfrac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_n}, \quad n = 1,2,3,6$
- New fractional instantons on $\mathbb{M}^4$: $\Delta B = \Delta L \neq 0$
- Axion+YM encompasses 2-form symmetry, and gauging it yields a fully consistent picture