Cartography of gauge-chiral dynamics

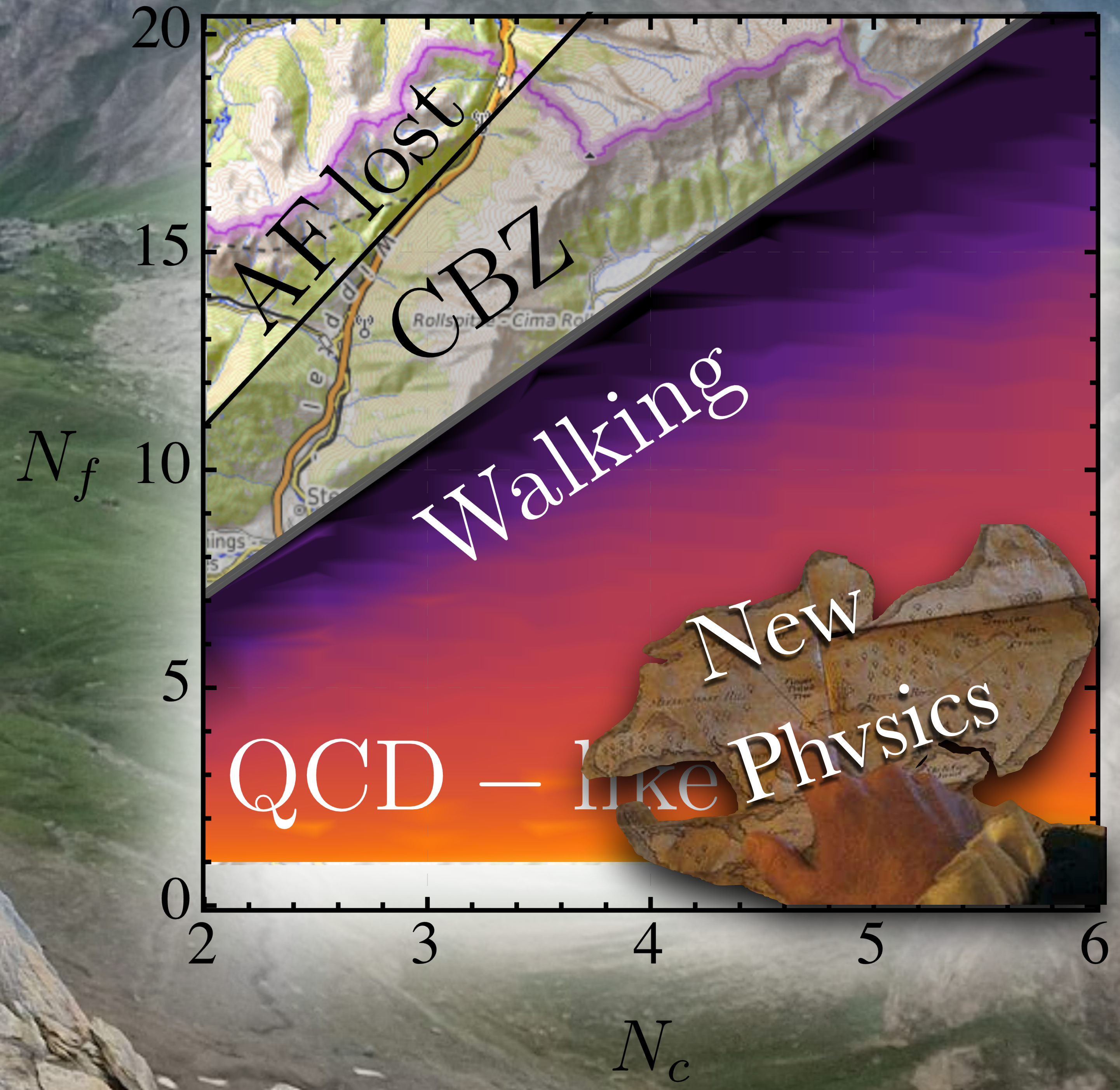
MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK
HEIDELBERG

1. confinement

2. dχSB

3. conformality

Work in collaboration with
Florian Goertz and Jan M. Pawlowski



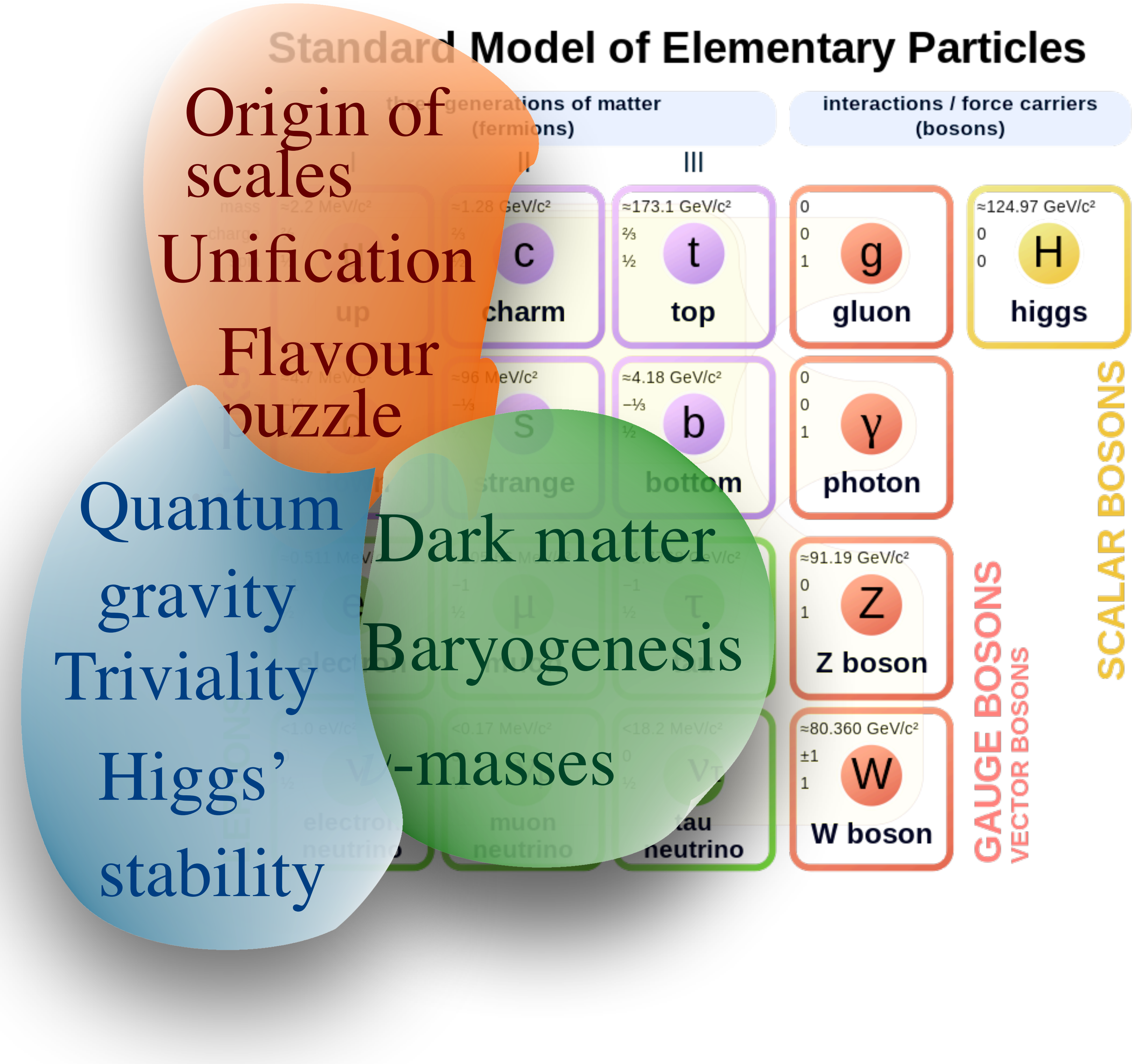
Álvaro Pastor Gutiérrez

MITP 5.08.2024

Gauge-chiral theories

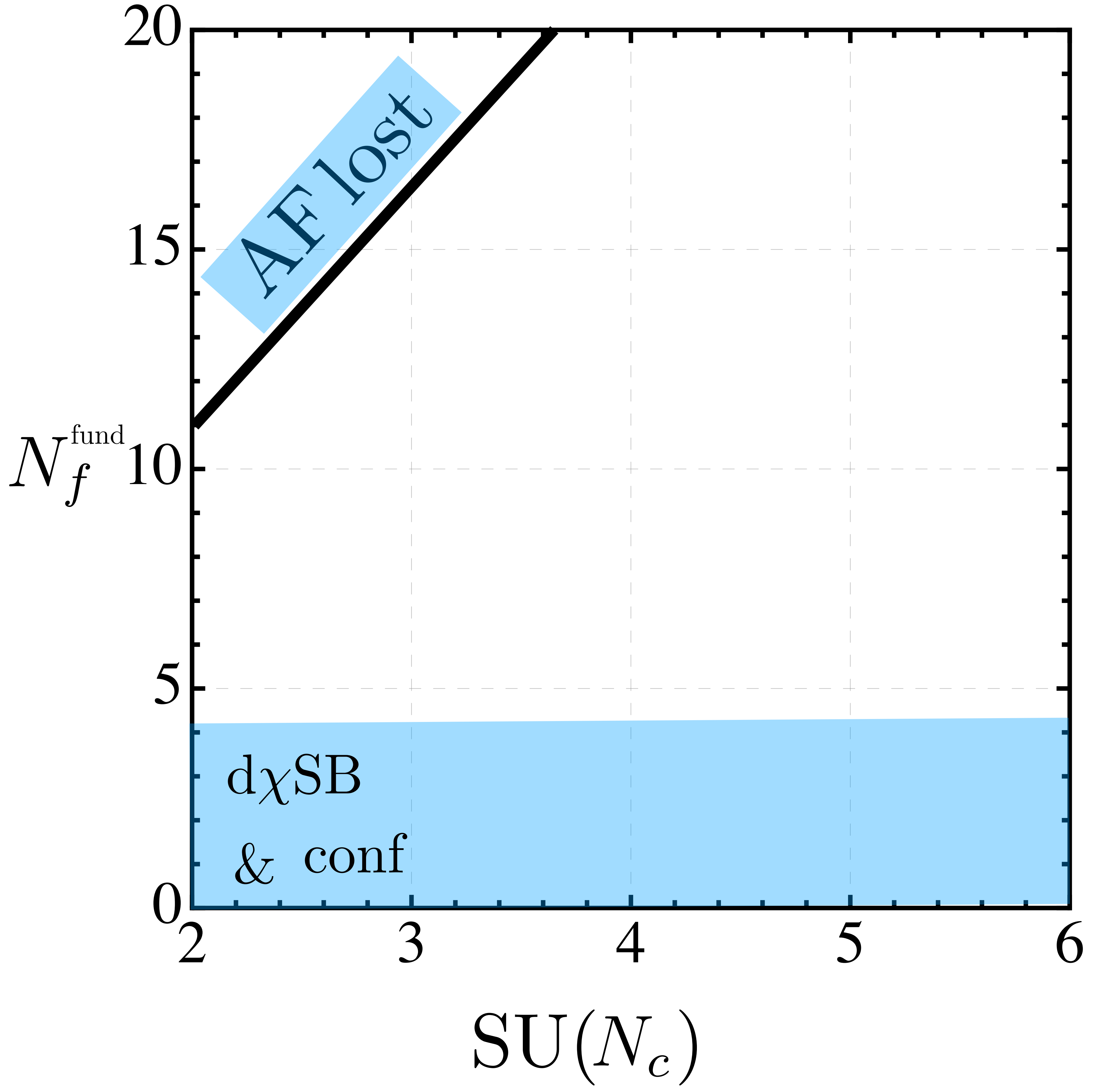
$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}} + \bar{\psi} \not{D} \psi$$

- Importance in the **natural world**
- Non-trivial **dynamics** and **phenomena**:
  - Confinement
  - Dynamical chiral symmetry breaking (dχSB)
  - Near-conformality (walking)
- Can be the **solution** to our **problems** and **puzzles**
  - Dark strong sectors
  - Composite Higgs and Technicolour
  - Cosmological phase transitions and gravitational wave signatures



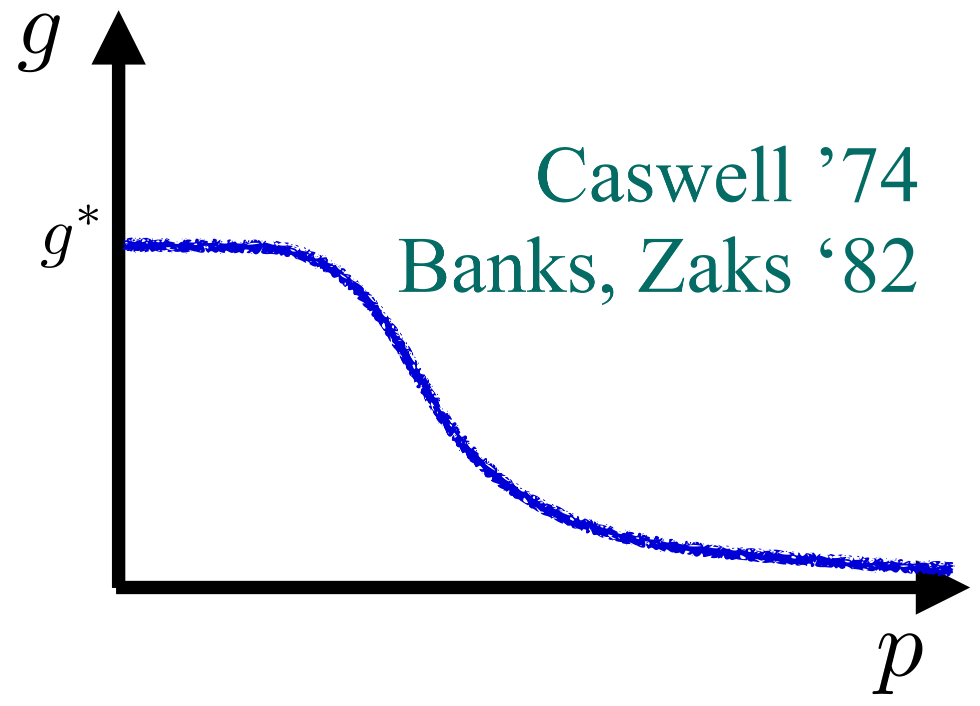
What we know so far from first-principles?

- Where **asymptotic freedom** is lost
- **Few flavours**: QCD-like theories

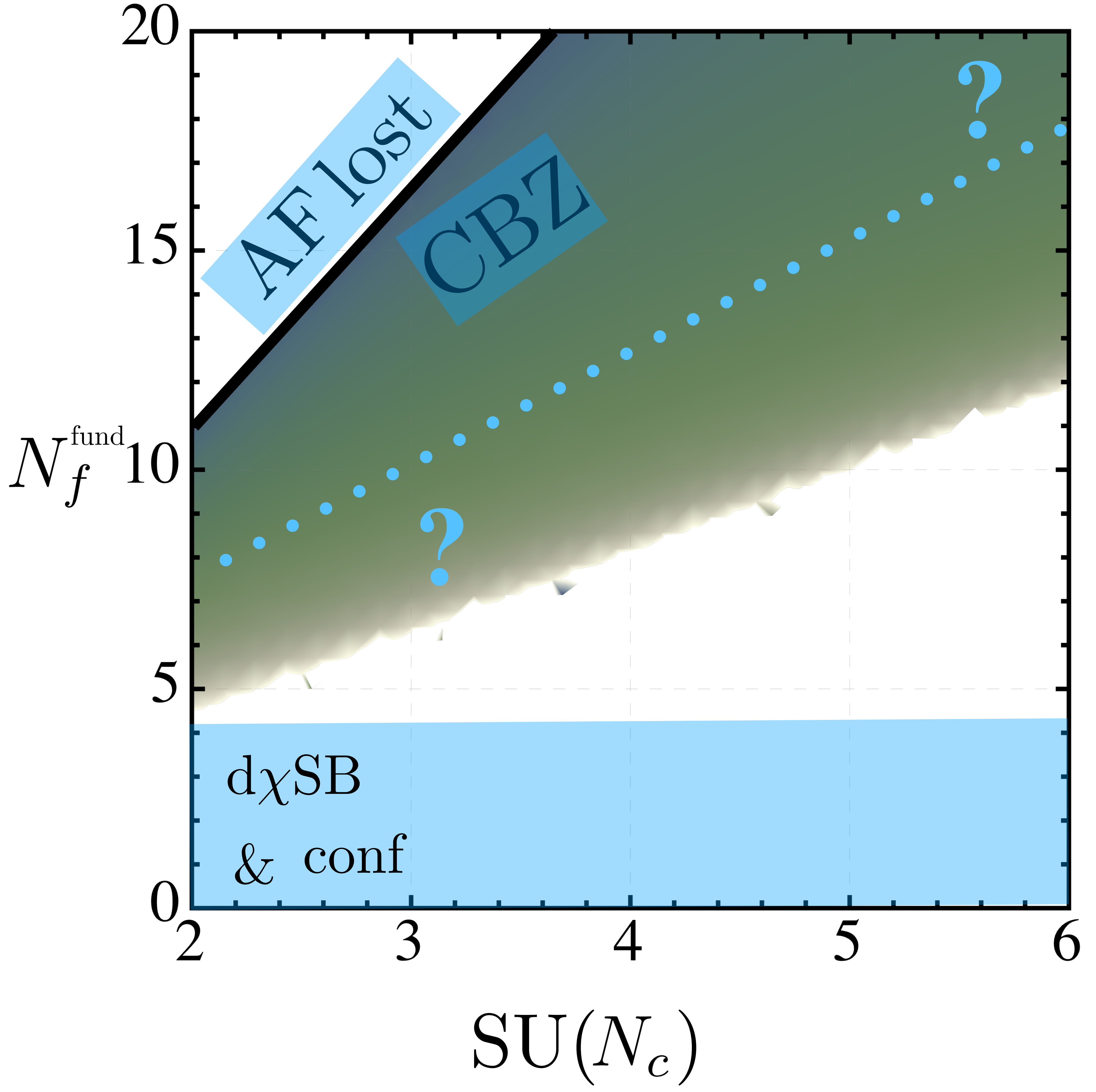


What we know so far from first-principles?

- ✦ Where **asymptotic freedom** is lost
- ✦ **Few flavours**: QCD-like theories
- ✦ **CBZ IR fixed-point** (perturbative)



$$\beta_g = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_{\mathrm{A}} - \frac{4}{3}T_{\mathrm{F}}N_f\right) - \frac{g^5}{(4\pi)^4}\left(\frac{34}{3}C_{\mathrm{A}} - 4C_{\mathrm{F}}T_{\mathrm{F}}N_f - \frac{20}{3}C_{\mathrm{A}}T_{\mathrm{F}}N_f\right) + \cdots$$



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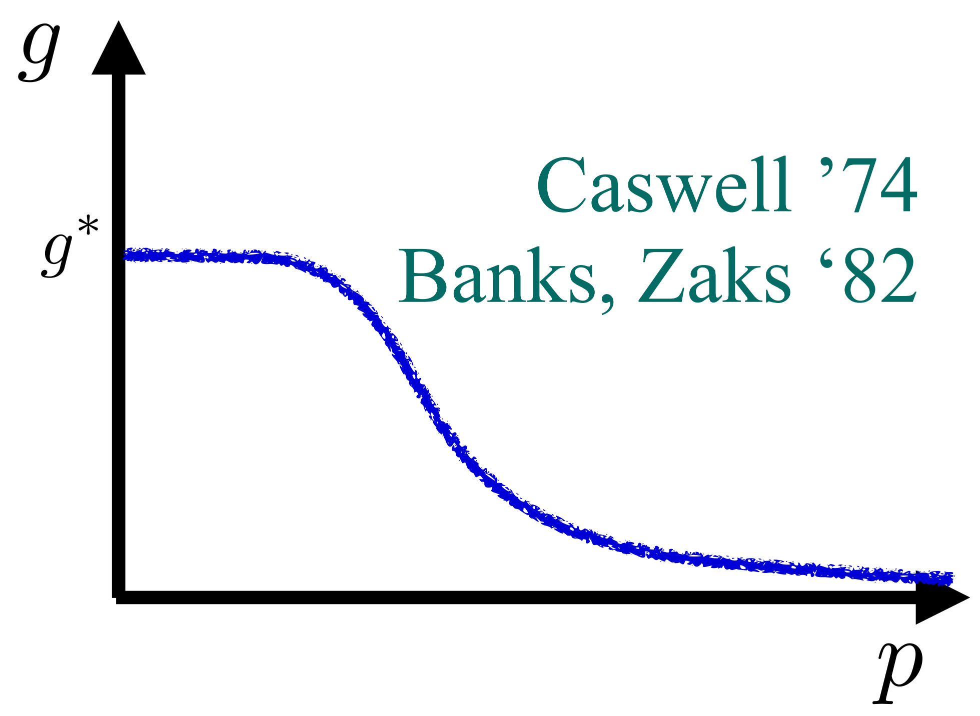

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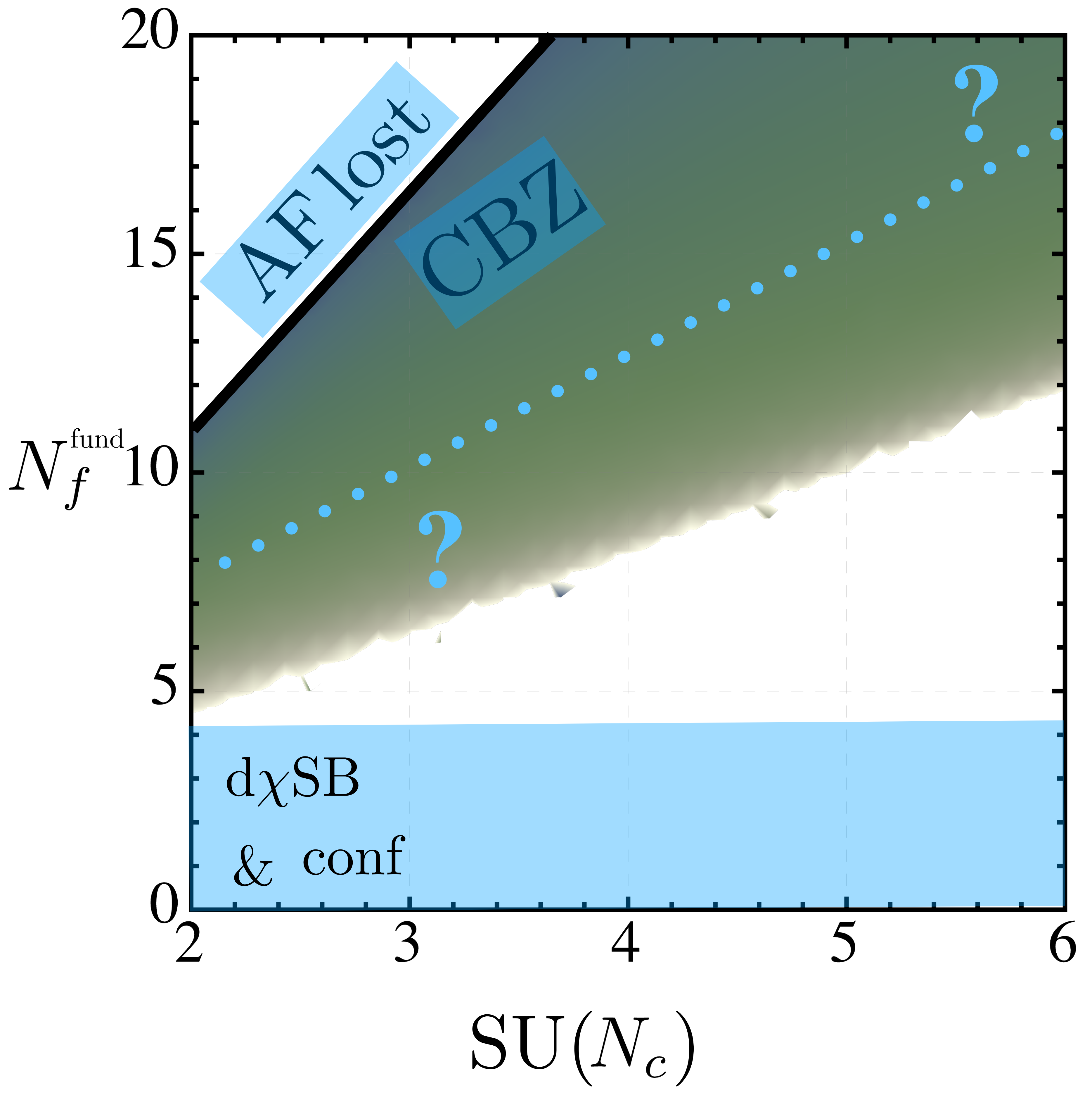
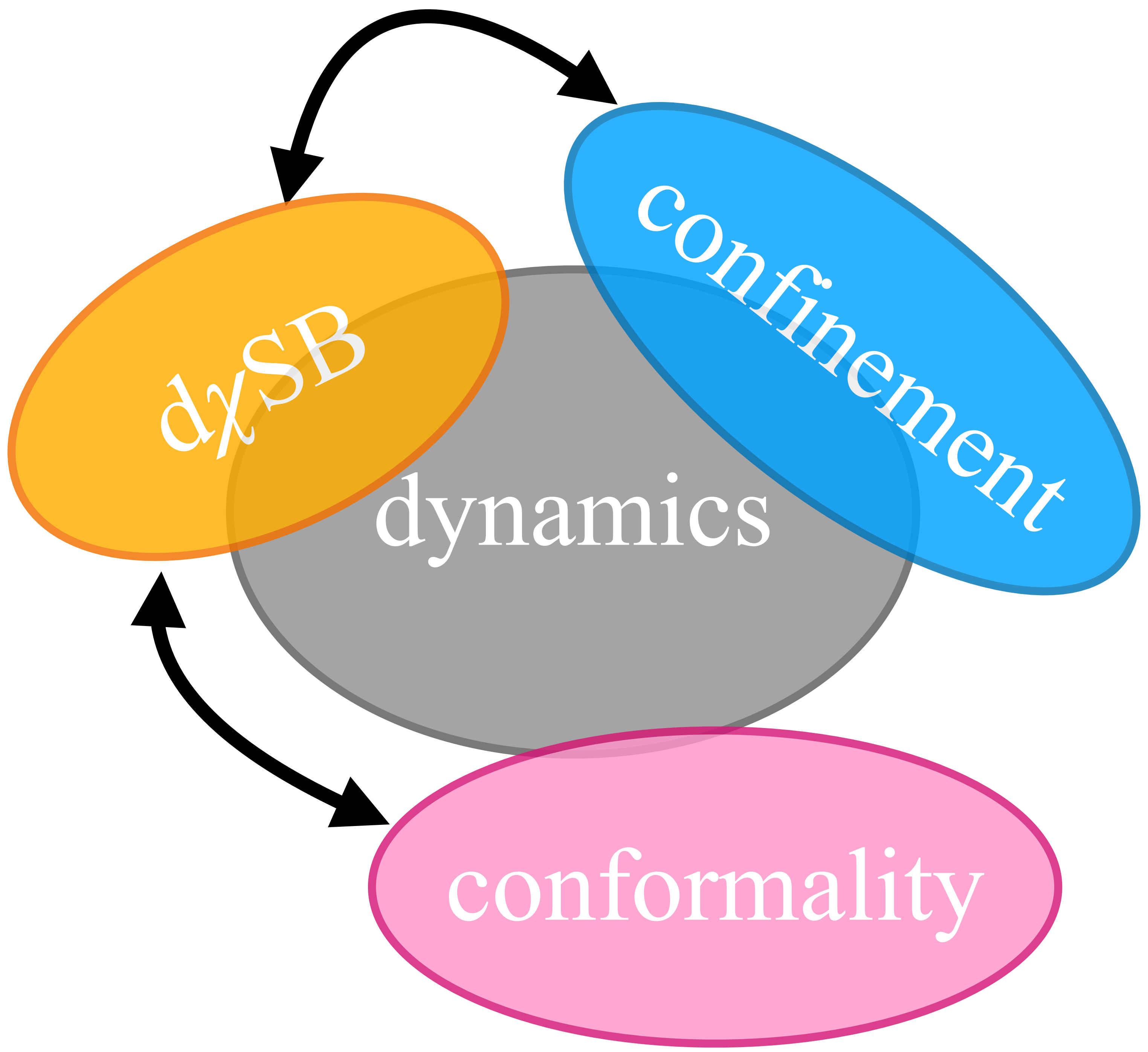


- ✦ Lattice deficiencies:
  - Individual theory investigations
  - The chiral limit is difficult: singular fermion determinant

Towards the uncharted regime



1. Chart the **landscape** of gauge-chiral theories
2. Understand **dynamics** in the many-flavour limit from **first principles**
3. Obtain **fundamental parameters** and the relation between scales
4. Determine the **boundary** between phases: conformal & non-conformal
5. Provide **UV input** to low-energy effective theories

✦ Functional methods in the continuum limit: the **functional Renormalisation Group**

- No difficulties in the exact chiral limit

$$\Gamma_k[\phi]$$

Functional Renormalisation Group

✦ Effective **average** action: $\Gamma_k[\phi]$

- Average action of fields over a k^{-d} volume
- Kadanoff's block-spinning idea in the continuum limit

✦ Pedagogical introduction to fRG: Gies [0611146]

✦ Review of achievements: Dupuis,Canet,Eichhorn, Metzner,Pawlowski,Tissier,Wschebor [2006.04853]

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$$\Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$

Wetterich '89

IR cutoff scale: $k \sim T \sim \langle p \rangle$

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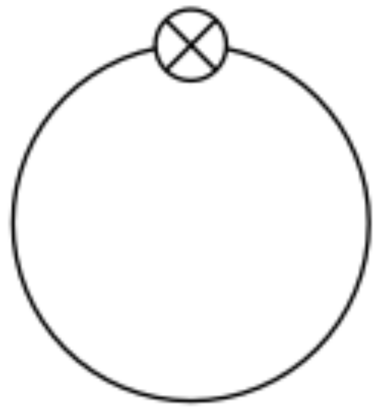
Flow equation:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k\right] = \frac{1}{2}$$



$\partial_t \equiv k\,\partial_k$ Wetterich '93

✦ One loop exact

✦ **Non-perturbative**

✦ **Mass-dependent**

✦ **Analytic regulators**

✦ **Versatile**

✦ Systematic expansion schemes

✦ UV-IR finite

✦ Diagrammatic

✦ Real-time formulation

✦ Pedagogical introduction to fRG: Gies [0611146]

✦ Review of achievements: Dupuis, Canet, Eichhorn, Metzner, Pawlowski, Tissier, Wschebor [2006.04853]

Deriving correlation functions

$$\Gamma_k[\phi] = \sum_n^{N_{\rm tr}} \int_p \Gamma^{(n)}_{k,\phi_{i_1}\cdots\phi_{i_n}}(p_1,...,p_n)\,\phi_{i_n}(p_n)\cdots\phi_{i_1}(p_1)$$

$$\Gamma^{(n)}_{k,\phi_{i_1}\cdots\phi_{i_n}}(p_1,...,p_n) = \frac{\delta}{\delta\phi_{i_1}(p_1)}\cdots\frac{\delta}{\delta\phi_{i_n}(p_n)}\Gamma_k[\phi]$$

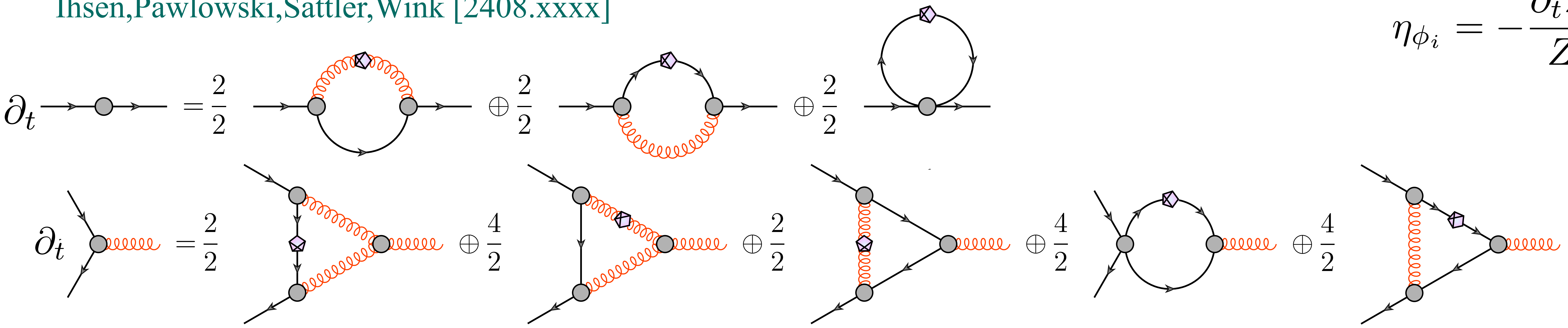
Example: quark-gluon vertex and gauge coupling

$$\partial_t \Gamma_k^{(\bar\psi\psi A)} = \partial_t\left(Z_A^{1/2} Z_\psi\ g_{\bar\psi\psi A}\cdot\mathcal{T}_\mu\right) \longrightarrow \partial_t g_{\bar\psi\psi A} = \frac{\mathrm{Tr}\left[\mathcal{T}_\mu\,\partial_t\Gamma_k^{(\bar\psi\psi A)}\right]}{\mathrm{Tr}\left[\mathcal{T}_\mu^2\right] Z_A^{1/2} Z_\psi} + \left(\frac{1}{2}\eta_A + \eta_\psi\right) g_{\bar\psi\psi A}$$

with anomalous dimensions:

$$\eta_{\phi_i} = -\frac{\partial_t Z_{\phi_i}}{Z_{\phi_i}} = -\frac{\partial_{p^2}\,\partial_t\Gamma_k^{(\phi_i\phi_i)}}{Z_{\phi_i}}$$

Ihsen,Pawlowski,Sattler,Wink [2408.xxxx]



Colour confinement: gluon mass gap

- ✦ **Absence** of **colour asymptotic states**
- ✦ **Massive spectrum** of bound states (glueballs)

- ✦ Existence of a gluon **mass gap:**
  - Wilson area law
  - Kugo-Ojima conditions
  - Confinement-deconfinement phase transition

$$Z_c(p) \propto \left(p^2\right)^{\kappa_{\text{ghost}}}$$

$$Z_A(p) \propto \left(p^2\right)^{-2\kappa_{\text{gluon}}}$$

$$\kappa_{\text{ghost}} = 0.579 \pm 0.005$$
$$\kappa_{\text{gluon}} = 0.573 \pm 0.002$$

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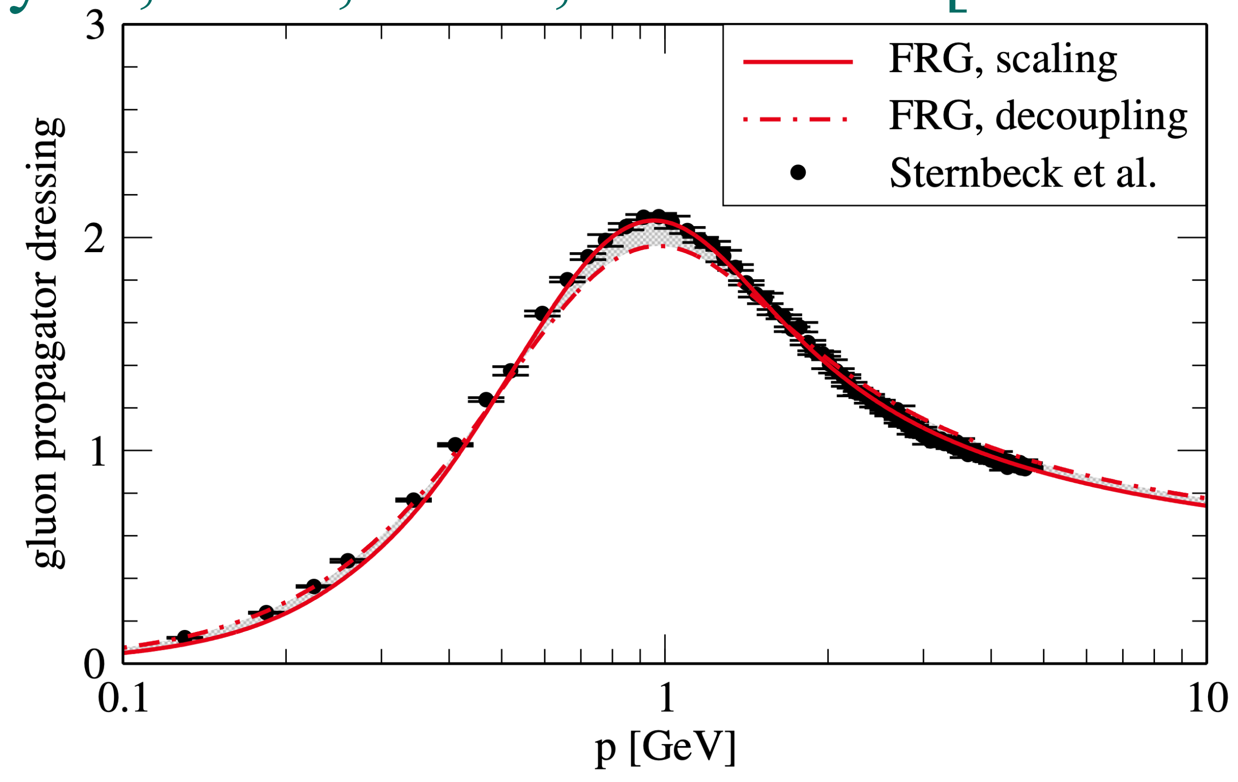
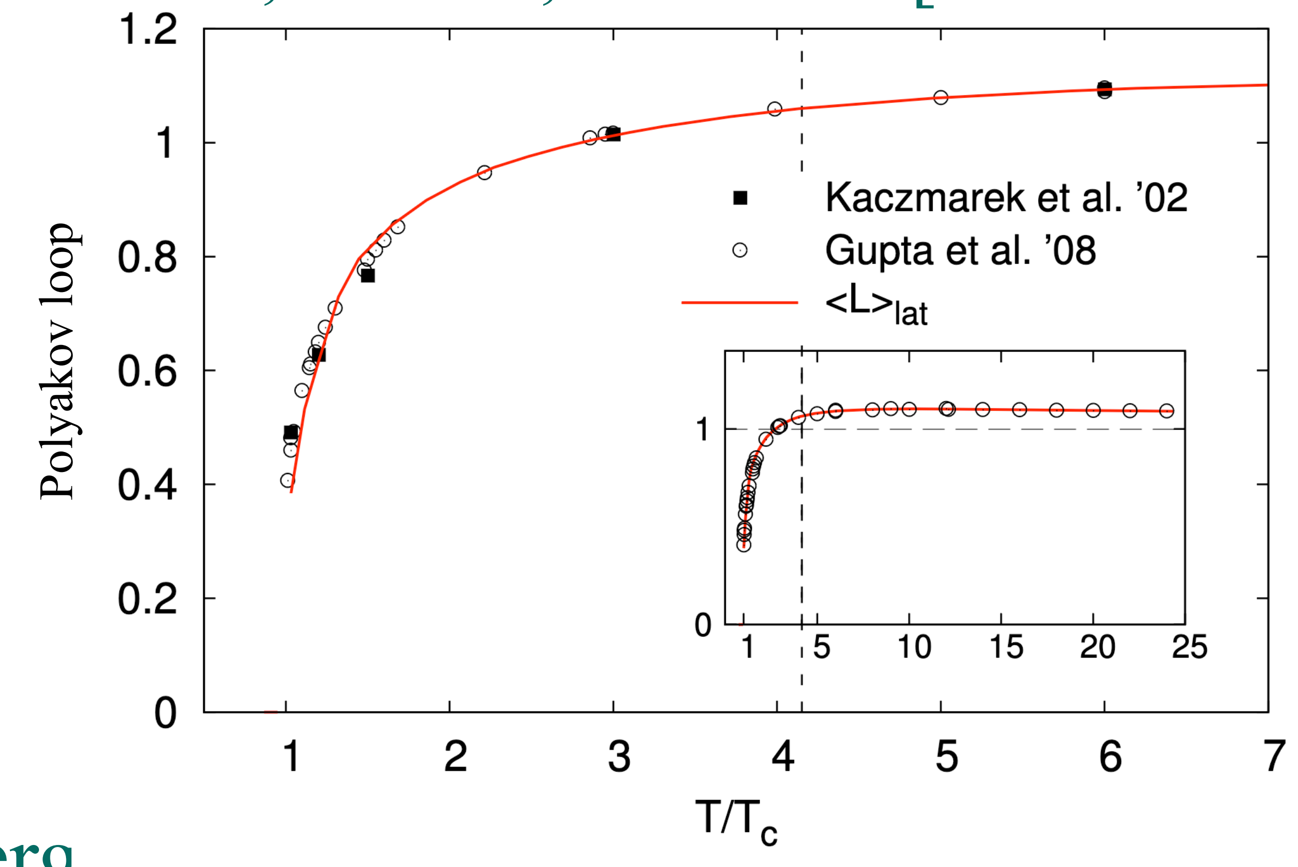
Cyrol,Fister,Mitter,Pawlowski [1605.01856]



Herbst,Luecker,Pawlowski[1510.03830]



✦ **fRG bootstrap approach** to confinement:

- **Mass gap** generated by **quantum fluctuations**

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p)\left(p^2 + m_{A,k}^2\right) = \hat{Z}_{A,k}(p)\, p^2$$

- **Uniquely defined** confinement condition
- Confinement from correlation function

✦ New: "*easy*" confinement

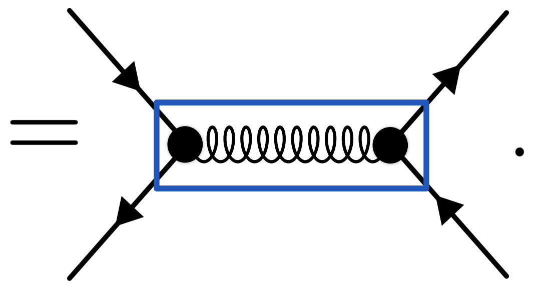
- **Cutoff dependences suffice**
- Semi-analytical
- Facilitate study beyond QCD-limit

Confinement in correlation functions

✦ Flows computed: $\{g_{A\bar\psi\psi}, g_{A\bar c c}, g_{A^3}, g_{A^4}, \bar m_A, Z_A, Z_c\}$

✦ **Exchange couplings**:

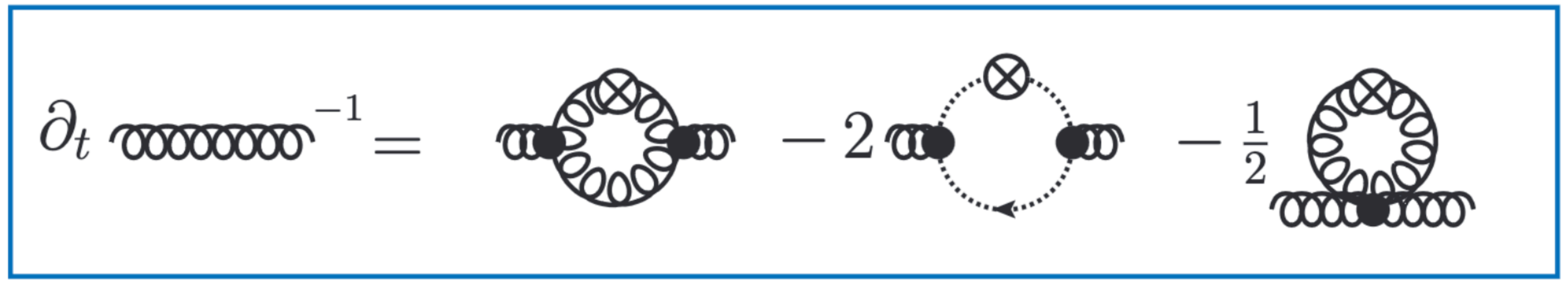
$$\alpha_{\psi\bar\psi A} = \frac{\left[\Gamma_k^{(\psi\bar\psi A)}\right]^2_{\mathcal{T}=1}}{4\pi\, Z_A\, Z_\psi^2} = \frac{g^2_{\psi\bar\psi A}}{4\pi\,(1+\bar m_A^2)}$$

$\alpha_{\psi\bar\psi A} =$  $\cdot\, p^2$

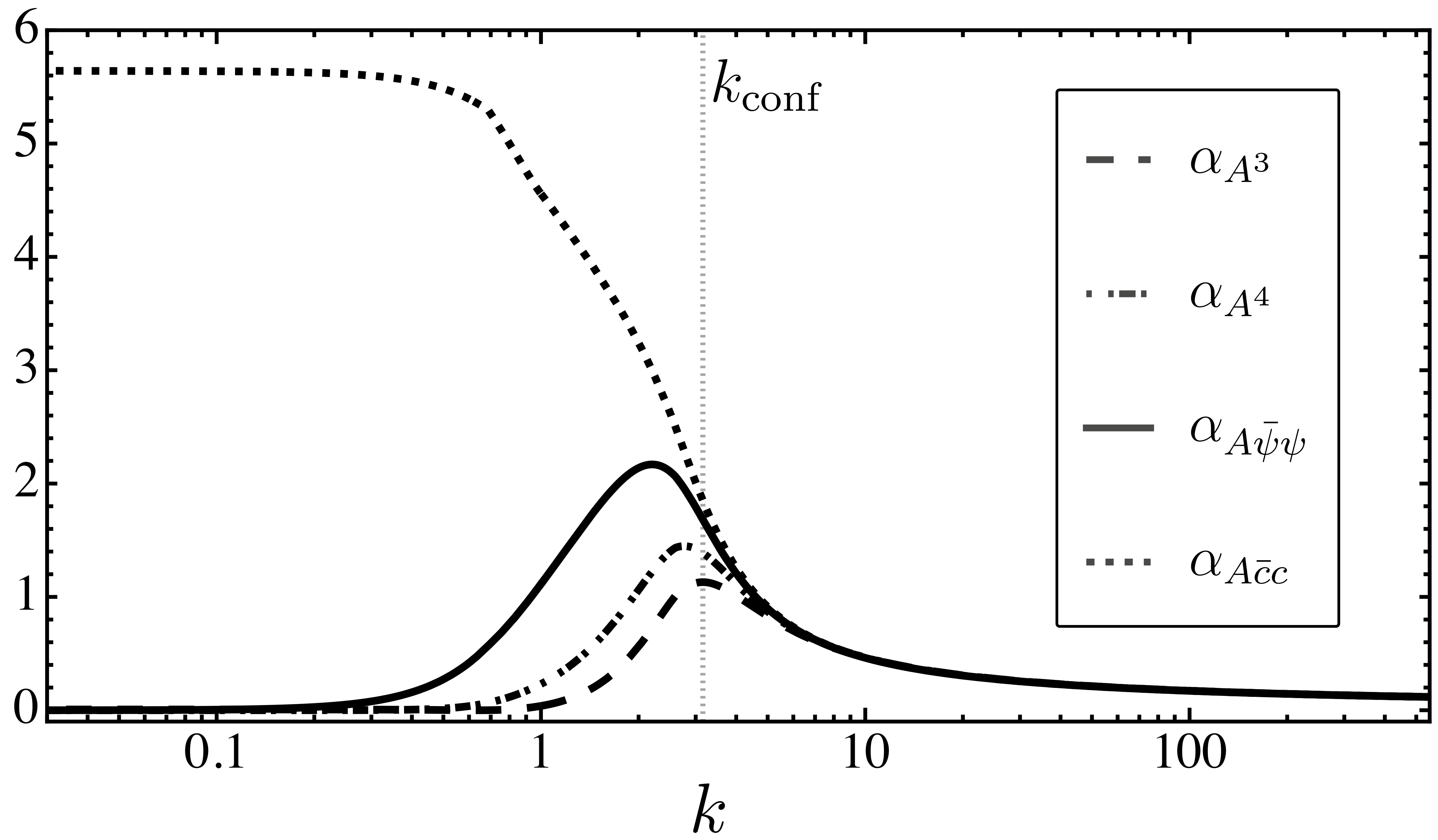
✦ **Decay** of correlation functions below the **mass gap scale**

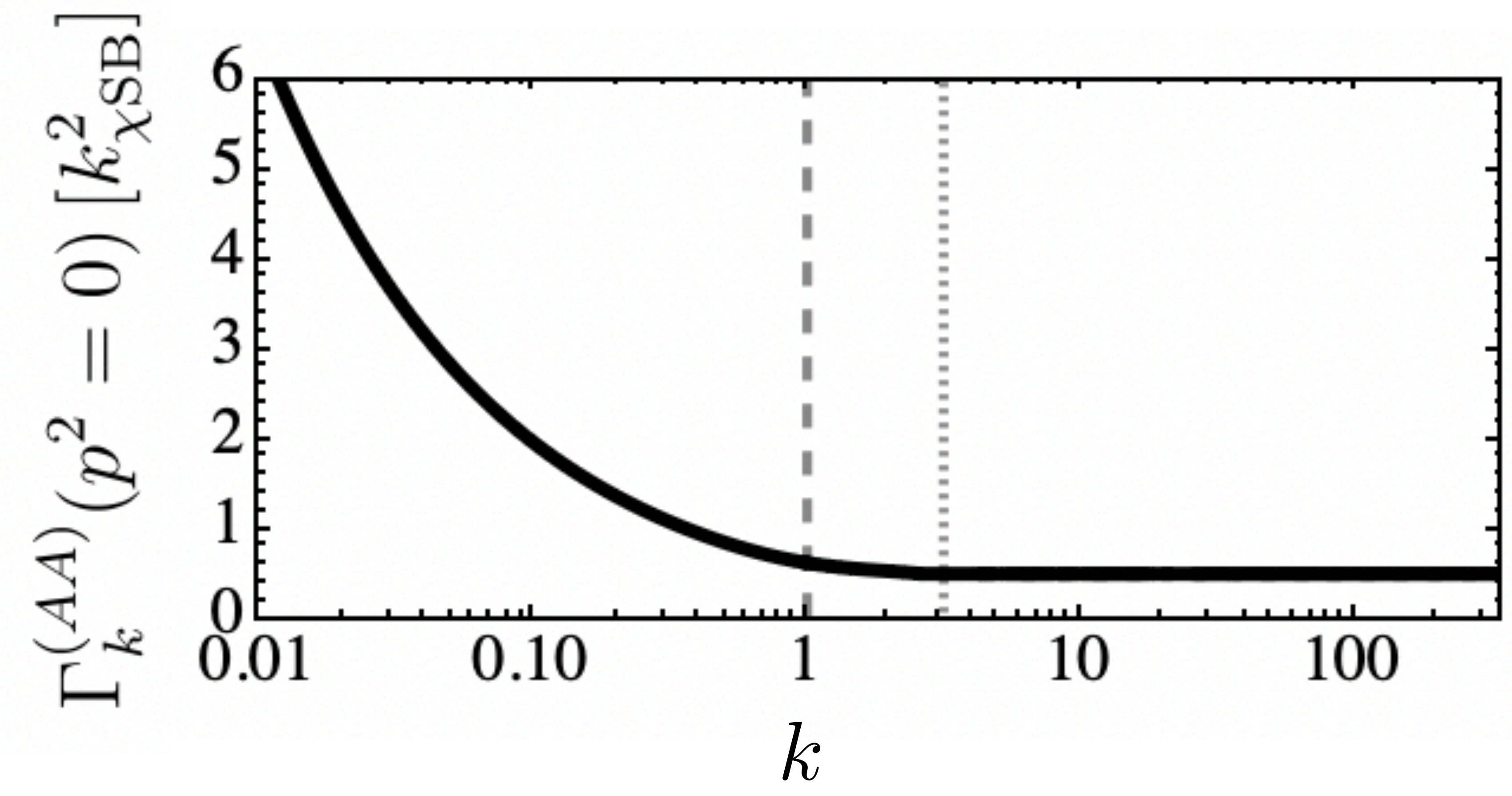
$$k_{\rm conf} \sim m_{A,\,\rm gap} \sim T_{\rm conf} \sim \Lambda_{\rm QCD}$$

✦ The **interplay** of **gapped gauge** and **ghost** contributions



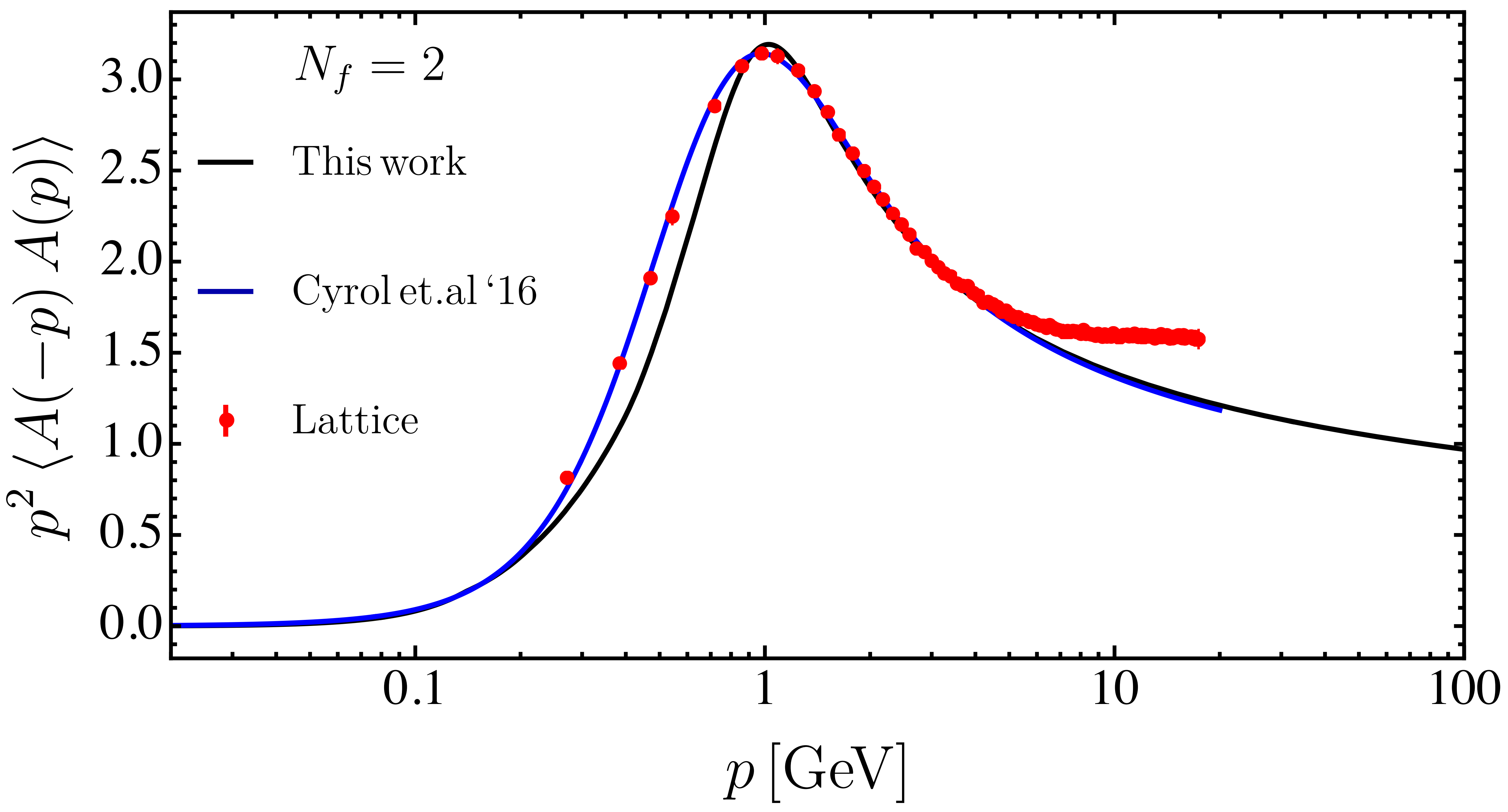
Cyrol,Fister,Mitter,Pawlowski [1605.01856]

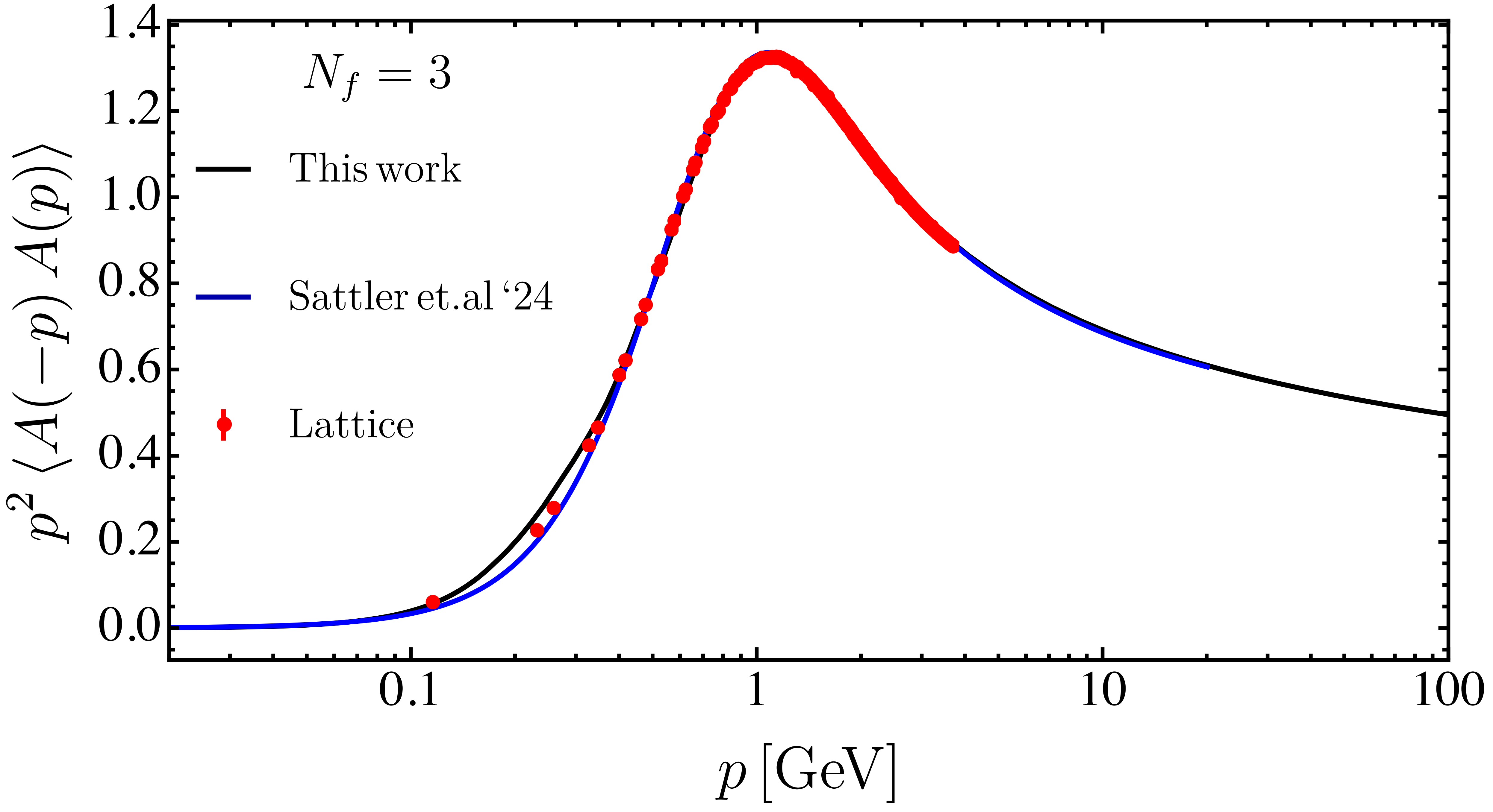




"Easy" confinement

Gluon propagator dressing: $$p^2 \langle A(-p) A(p) \rangle = p^2 \left[\Gamma_k^{(AA)}(p^2 = 0) \right]^{-1}_{k = c\,p}$$





Dynamical χSB

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi}\gamma_\mu D_\mu \, \psi$$

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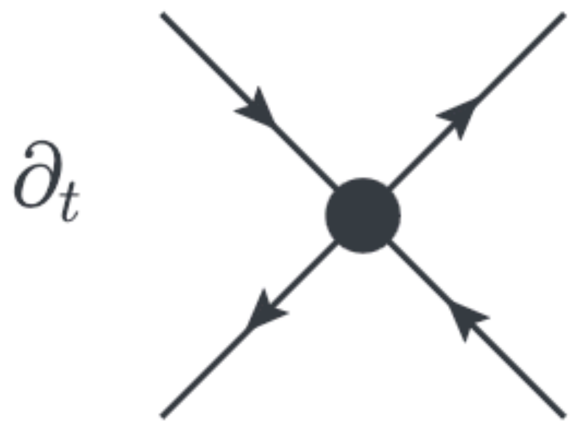

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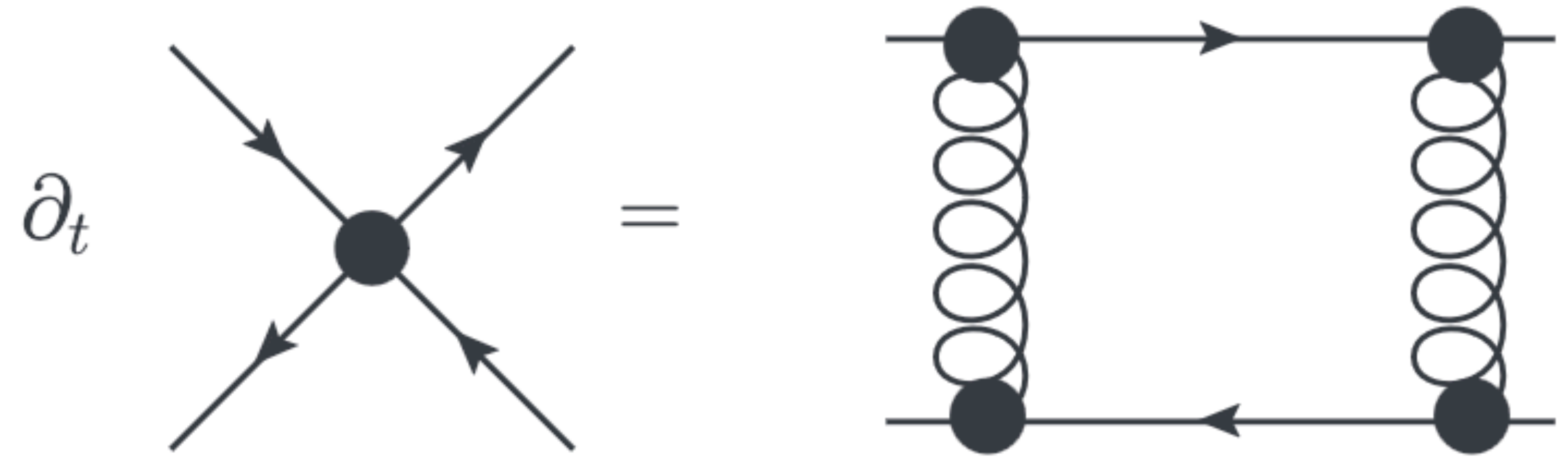

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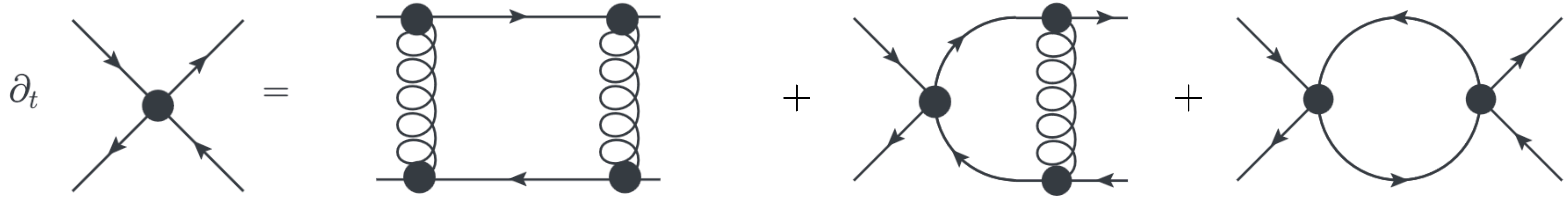

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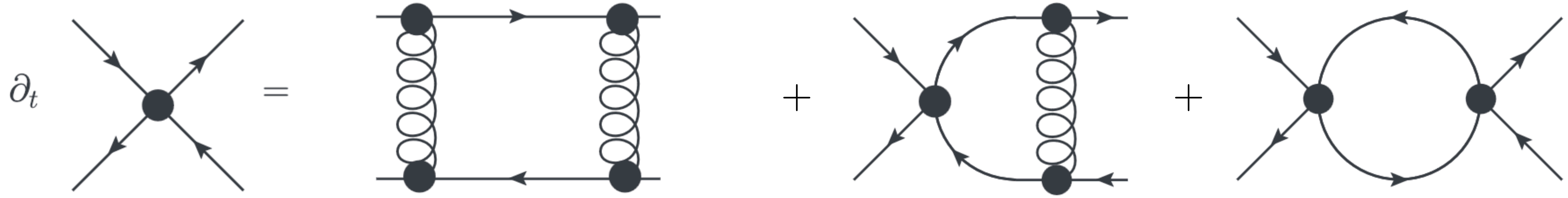

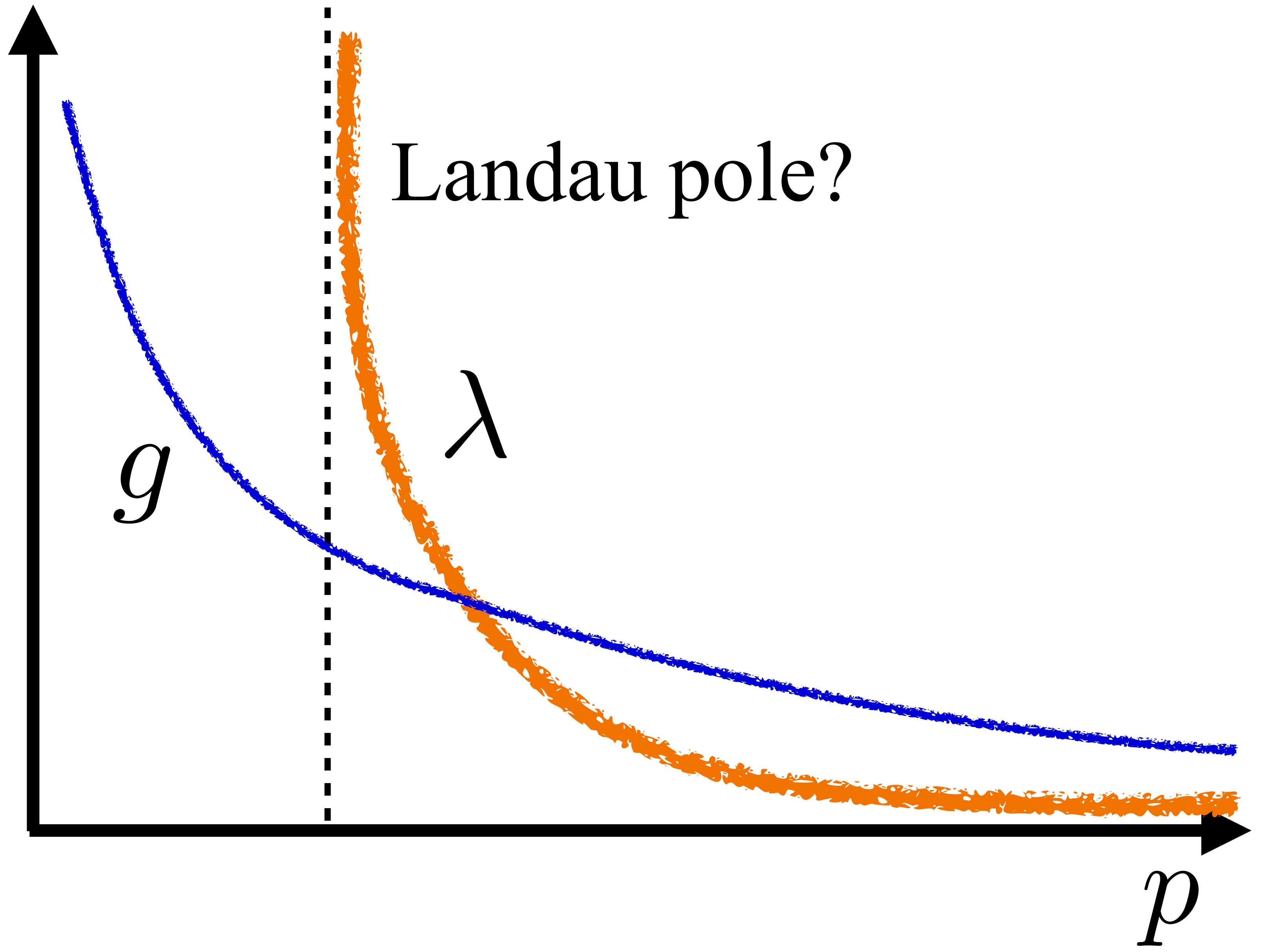

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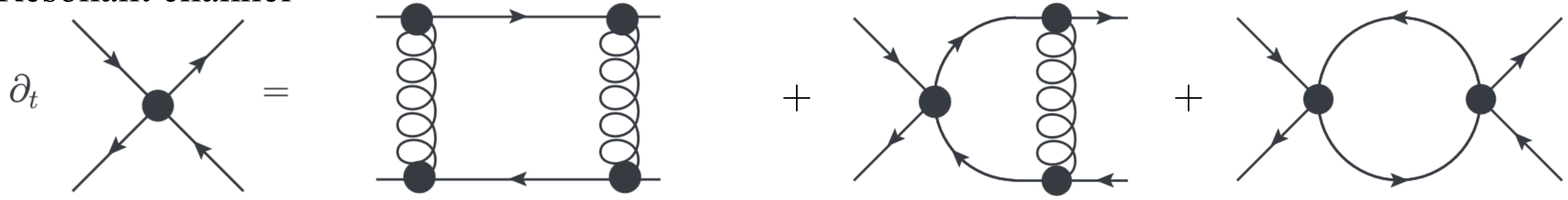
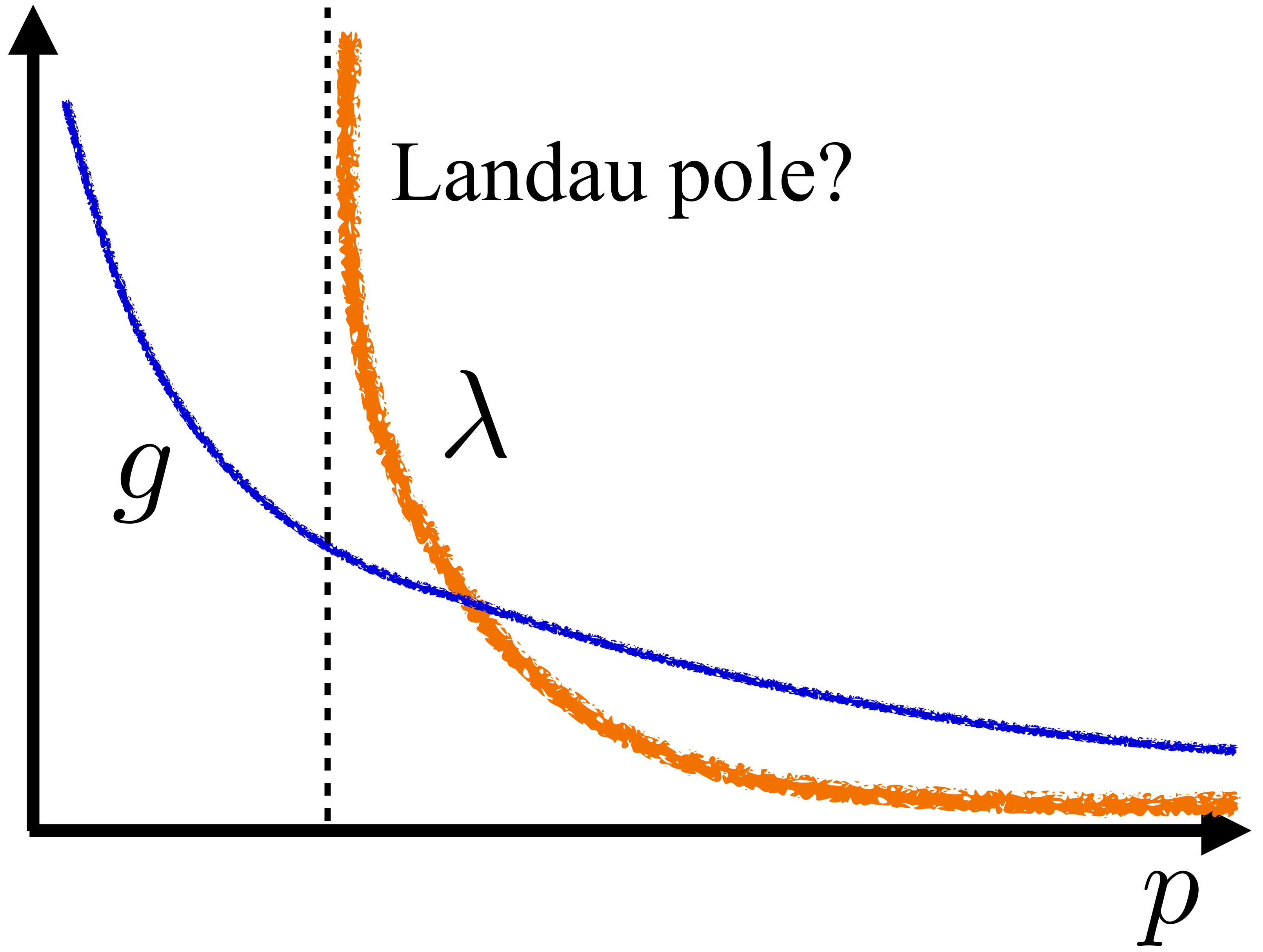
Resonant channel





$$\langle(\bar{\psi}\psi)^2\rangle \sim \langle\phi\phi\rangle^{-1} \longrightarrow \lambda \propto \frac{1}{p^2 + m_\phi^2}$$

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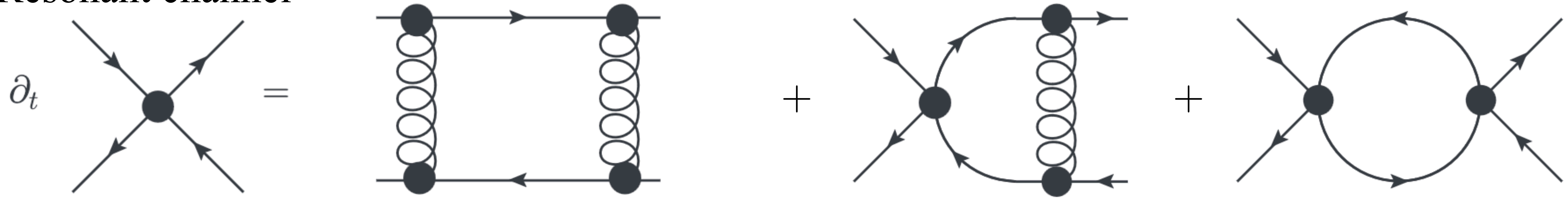

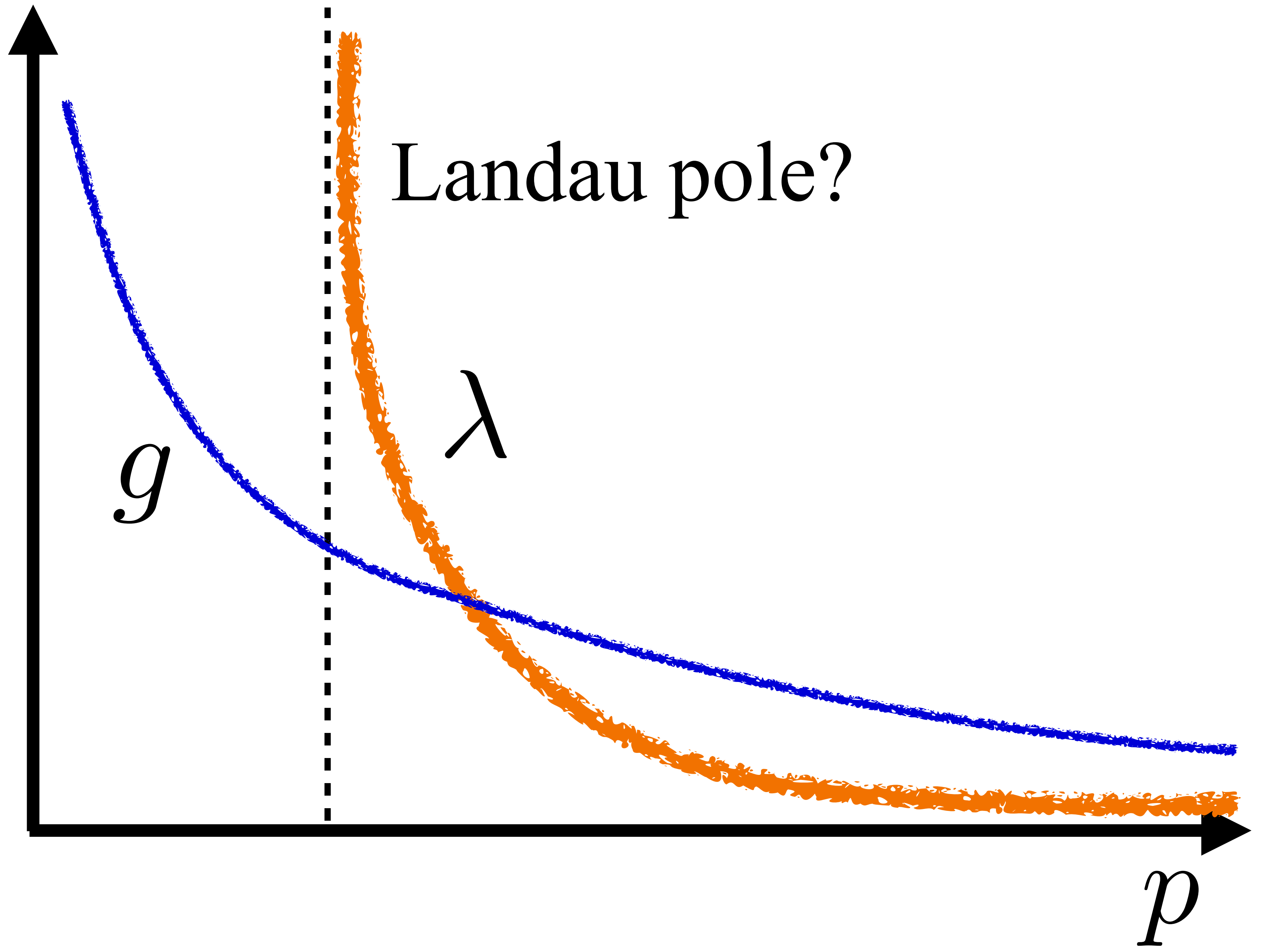

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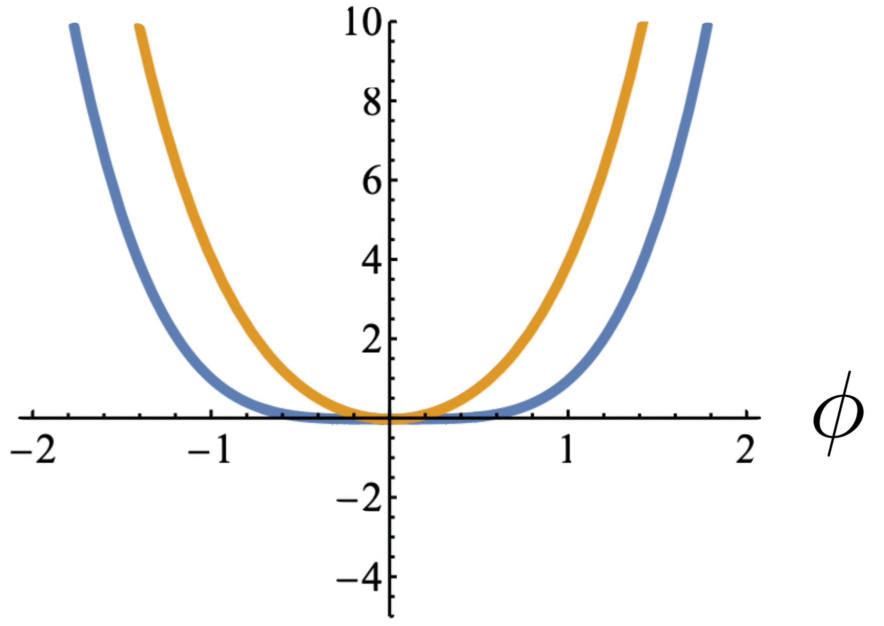
$$\chi\text{SB} \longrightarrow (\lambda = \infty) \equiv (m_\phi = 0)$$

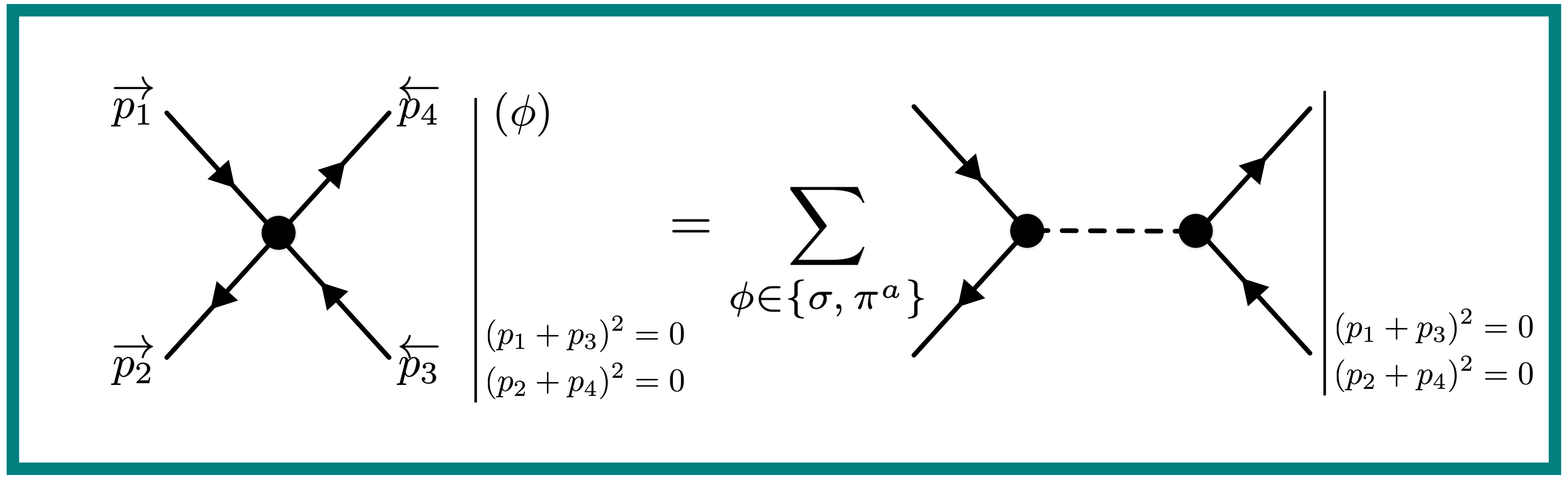


Higher dimensional operators carry information of **bound states formation** and **dχSB**

Emergent composites

$$\Gamma = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 + \bar{\psi} \left[(\gamma_\mu D_\mu) \right] \psi - \lambda \left[\left(\bar{\psi}\, T^0_f \, \psi \right)^2 + (\bar{\psi}\, \mathrm{i}\gamma_5 \, T^a_f \, \psi)^2 \right]$$

Stratonovich'57 Hubbard'59 Gies, Wetterich '01



Pawlowski [hep-th/0512261] Fukushima,Pawlowski,Strodthoff [2103.01129]

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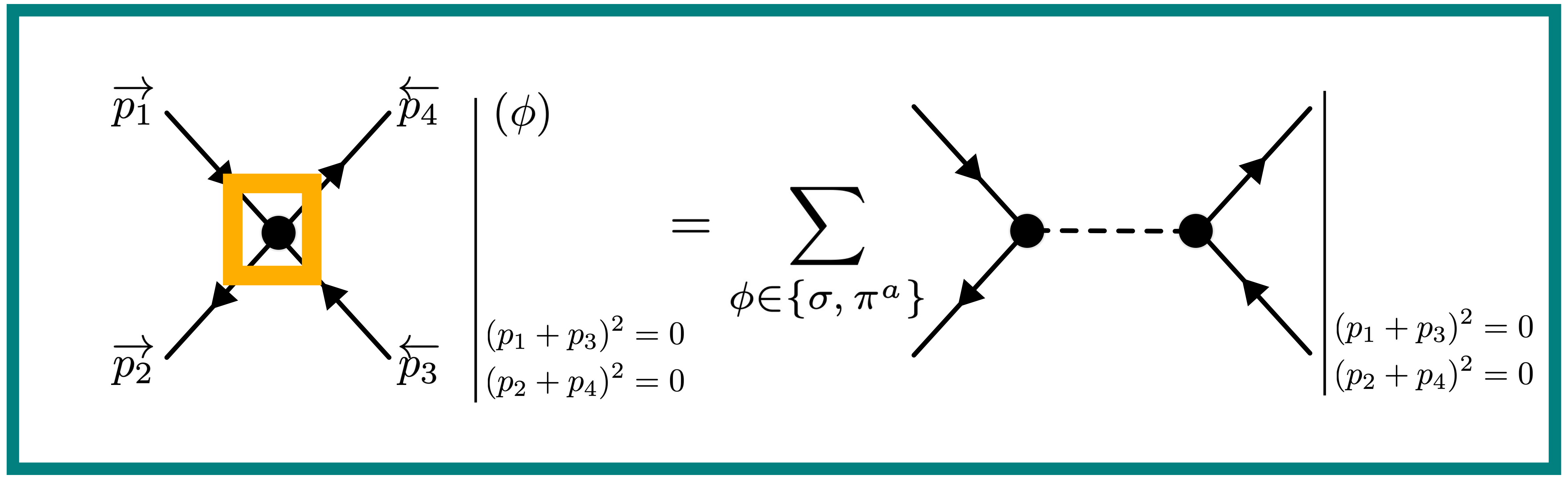

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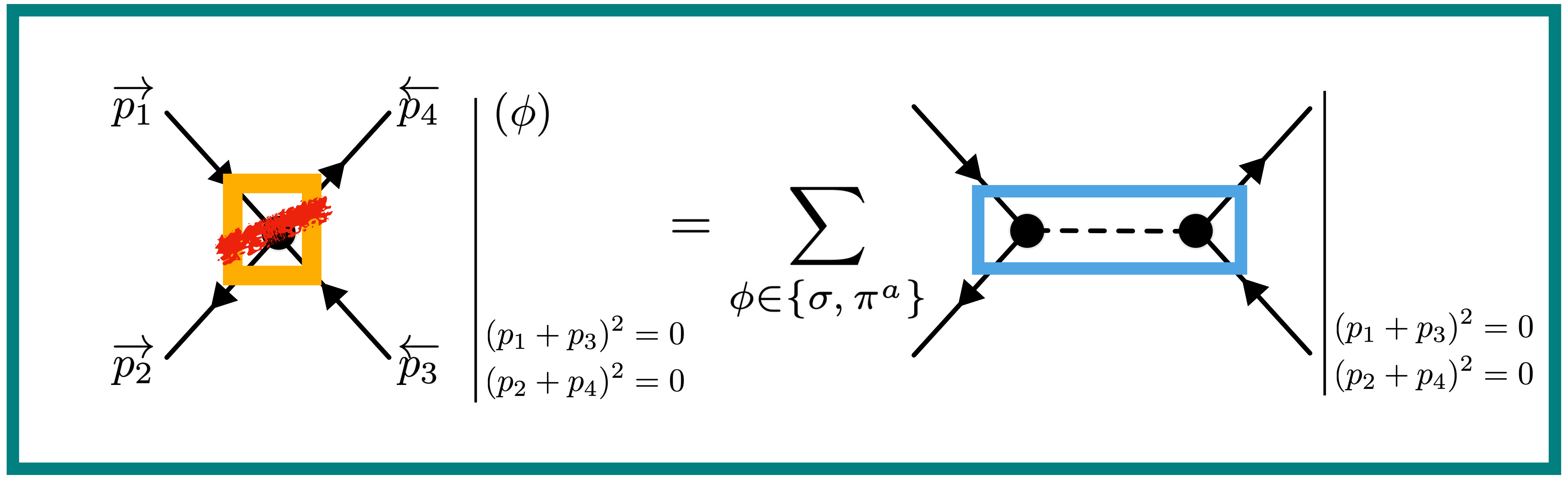

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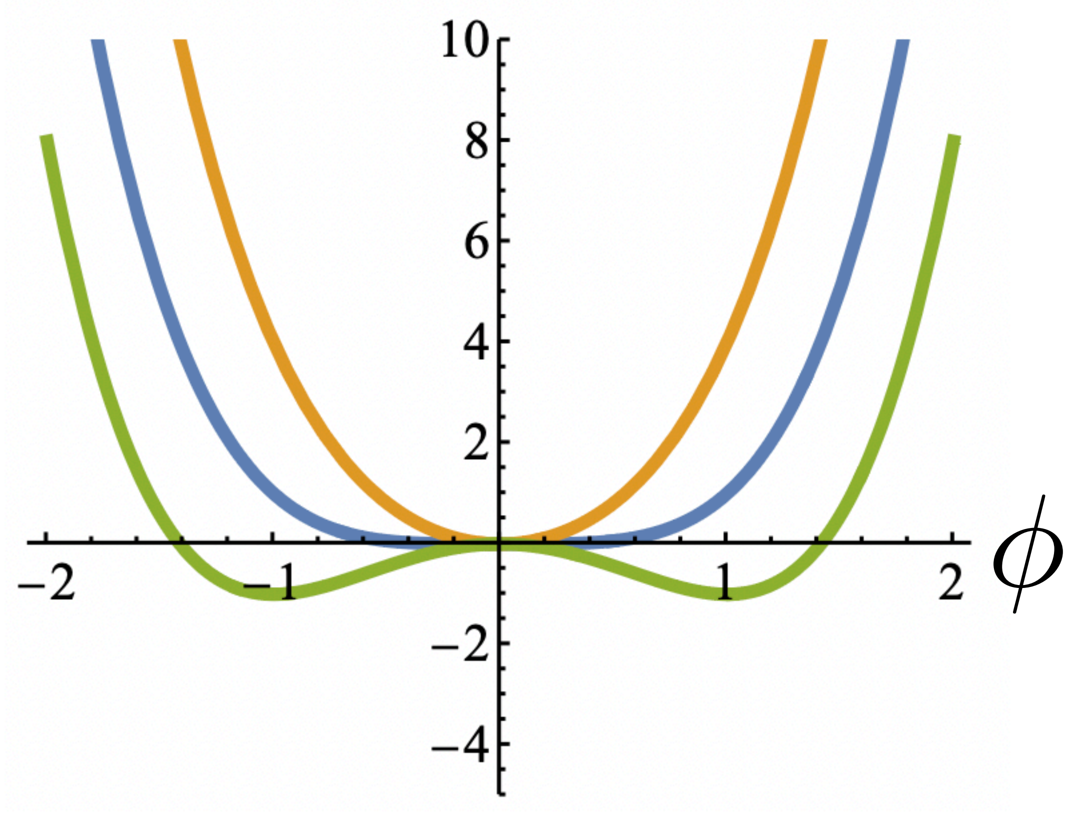

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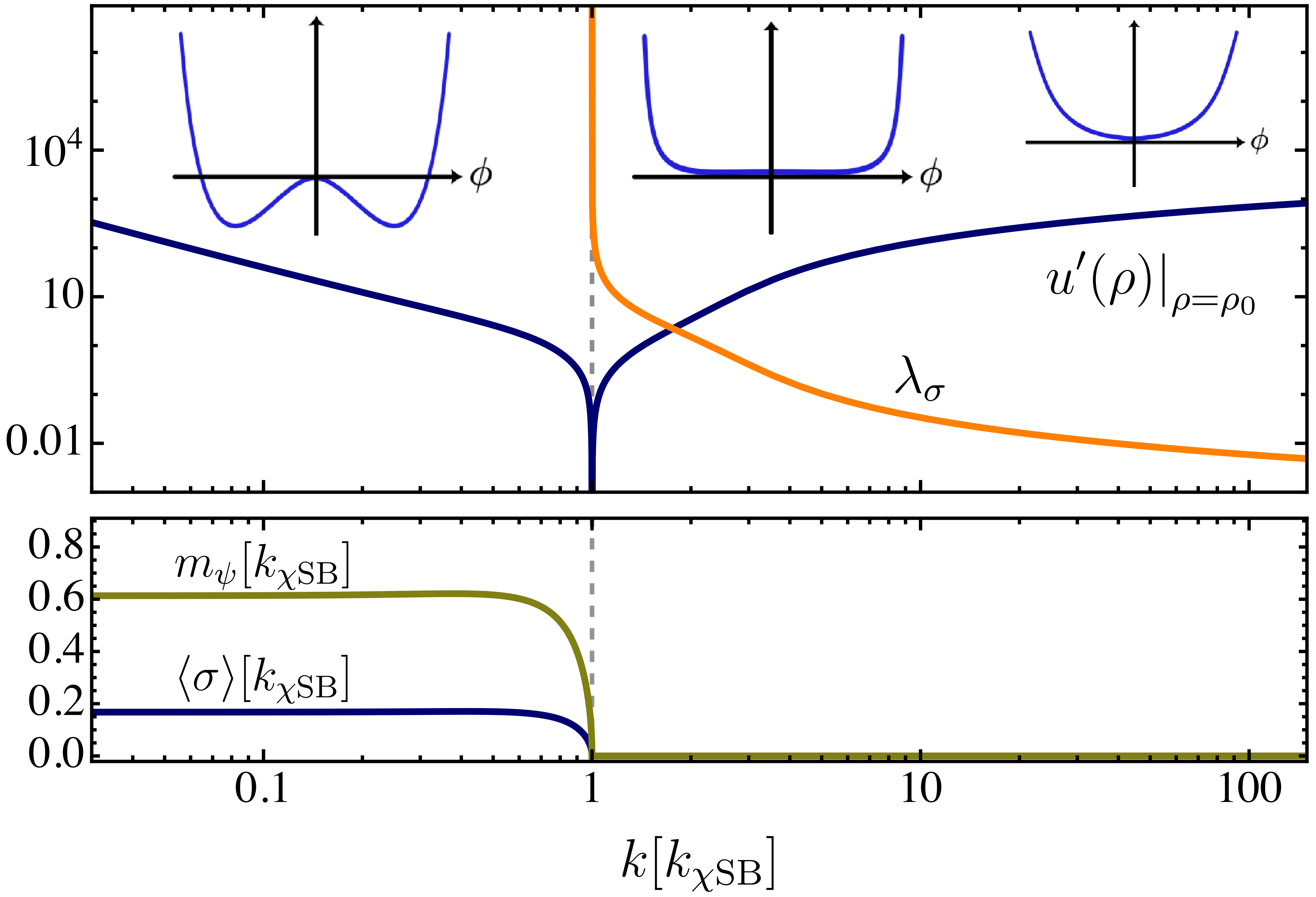
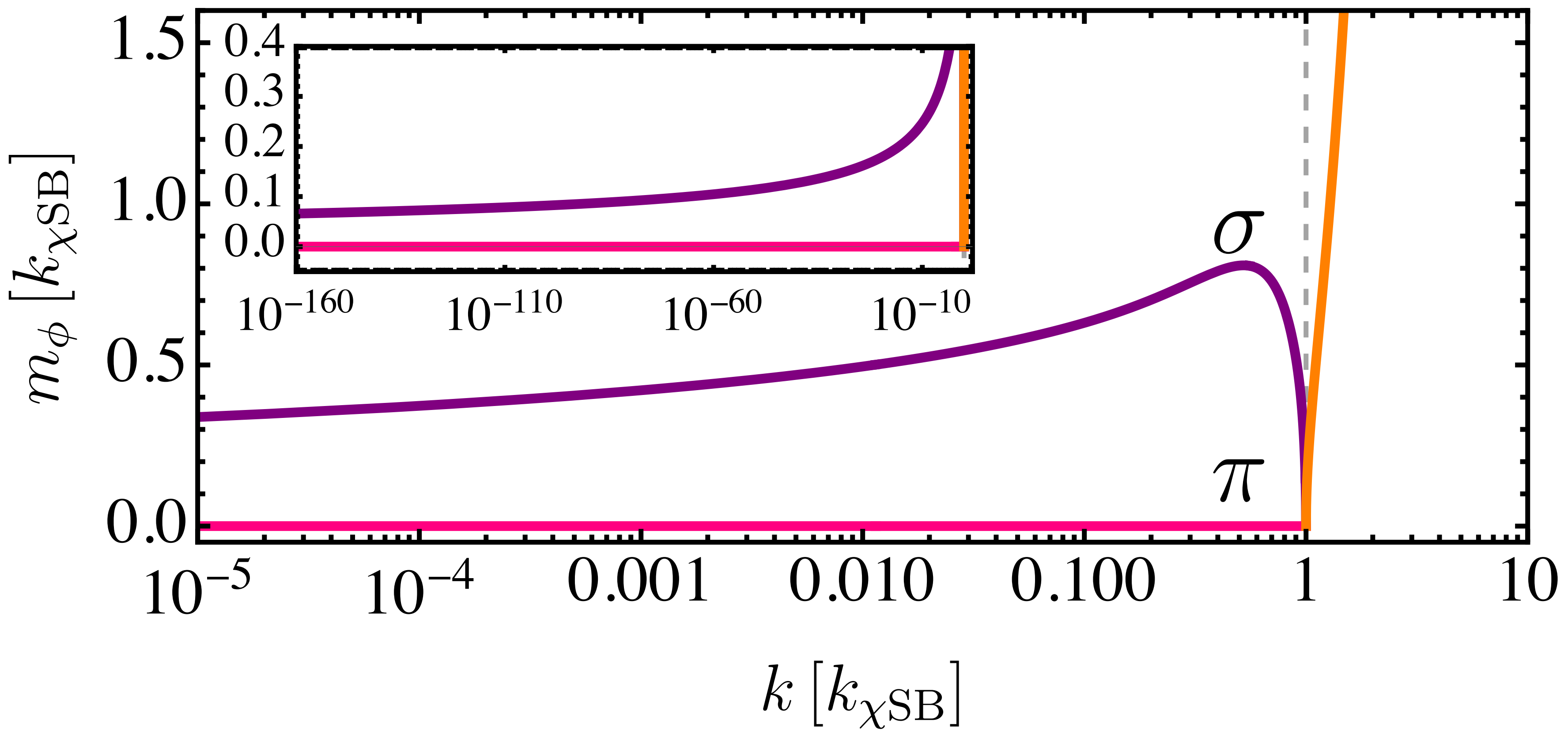
$$\phi = (\sigma, \mathrm{i}\gamma_5 \pi^a)$$

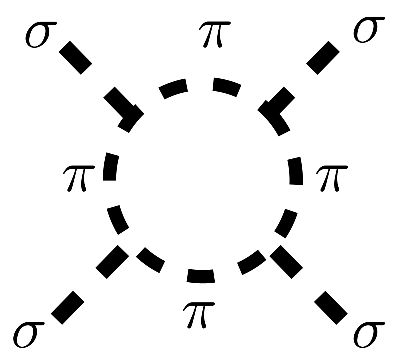
$$+ h\,\bar{\psi}\,\left(T^0_f \sigma + \mathrm{i}\gamma_5\, T^a_f \pi^a\right)\,\psi + \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi^2)$$

$$V(\phi^2) = \sum_{n=1}^{N_{\max}} \frac{\lambda_n}{n!}\left(\frac{\phi^2}{2}\right)^n$$

Dynamical chiral symmetry breaking







- ✦ Flows computed: $\{h, V(\phi), Z_\psi, Z_\phi, \lambda_i\}$
- ✦ **Continuous** interpolation between chirally **symmetric** and **broken** regimes
- ✦ A **clear** and **precise** way to **diagnose** χ**SB**

- ✦ Obtaining **fundamental parameters**
  - Constituent fermion masses: m_ψ
  - Chiral condensate: $\langle\sigma\rangle \sim f_\pi$
  - Composite masses of bosonised channels: m_σ, m_π
- ✦ Account for **higher dimensional fermionic** operators via higher-order scalar potential

Simplified truncation

✦ Purpose:

- **Practical** simplification of the pure gauge sector
- **Connection** with high-order **perturbative computations** towards the **CBZ** limit
- Implement **IR fixed-point** solutions from perturbative approaches

✦ Implementation:

- Employ a **single adapted** gauge coupling **avatar**
- **Bosonised sector** in **full glory**

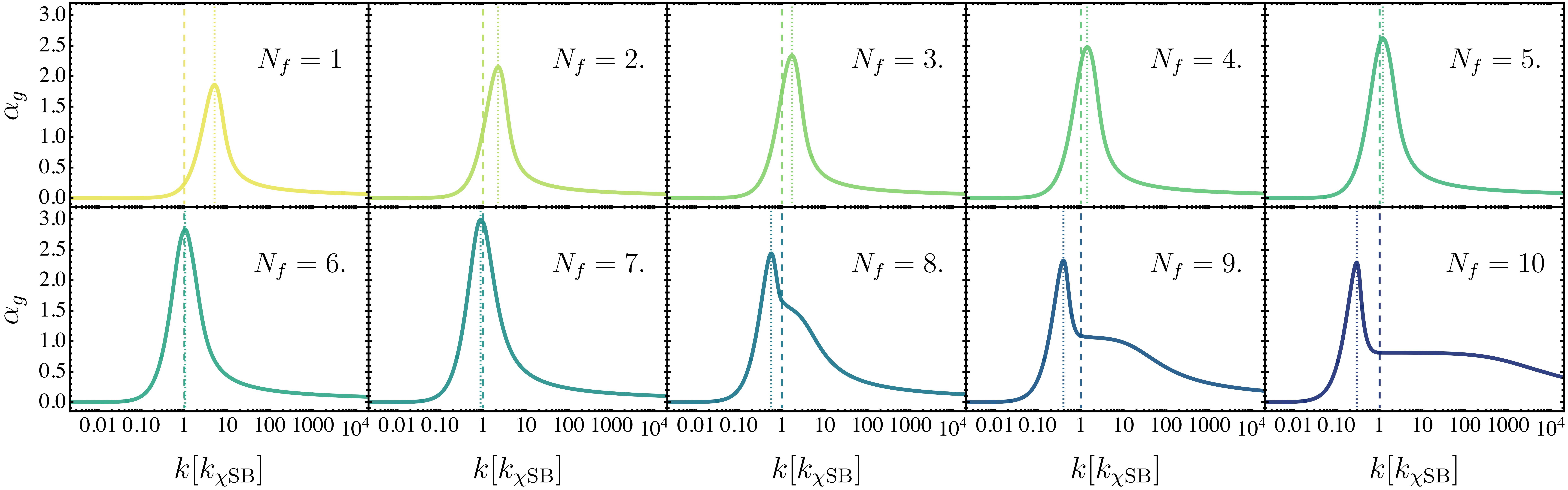
Smooth decoupling of dof

$$\partial_t g = \boxed{\partial_t g^{(1)}_{A\bar\psi\psi}} + \boxed{\frac{\partial_t g^{(>1)}_{A\bar\psi\psi}}{\partial_t g^{(>1)}_{A\bar\psi\psi}\Big|_{\{\bar m_A,\bar m_\psi\}=0}}} \sum_{n=2}^{N_{\max}} \left\{ \sum_{i=1}^{i_{\max}} \boxed{\frac{1}{(1+i_{\max})}\frac{1}{(1+\bar m_A^2)^{3n+i}}} \left(\boxed{\beta_A^{(n)}} + \left[\frac{(2+\bar m_\psi^2)}{2\,(1+\bar m_\psi^2)^3}\right]^{N_f \text{ in loop}} \boxed{\beta_\psi^{(n)}} \right)\right.$$

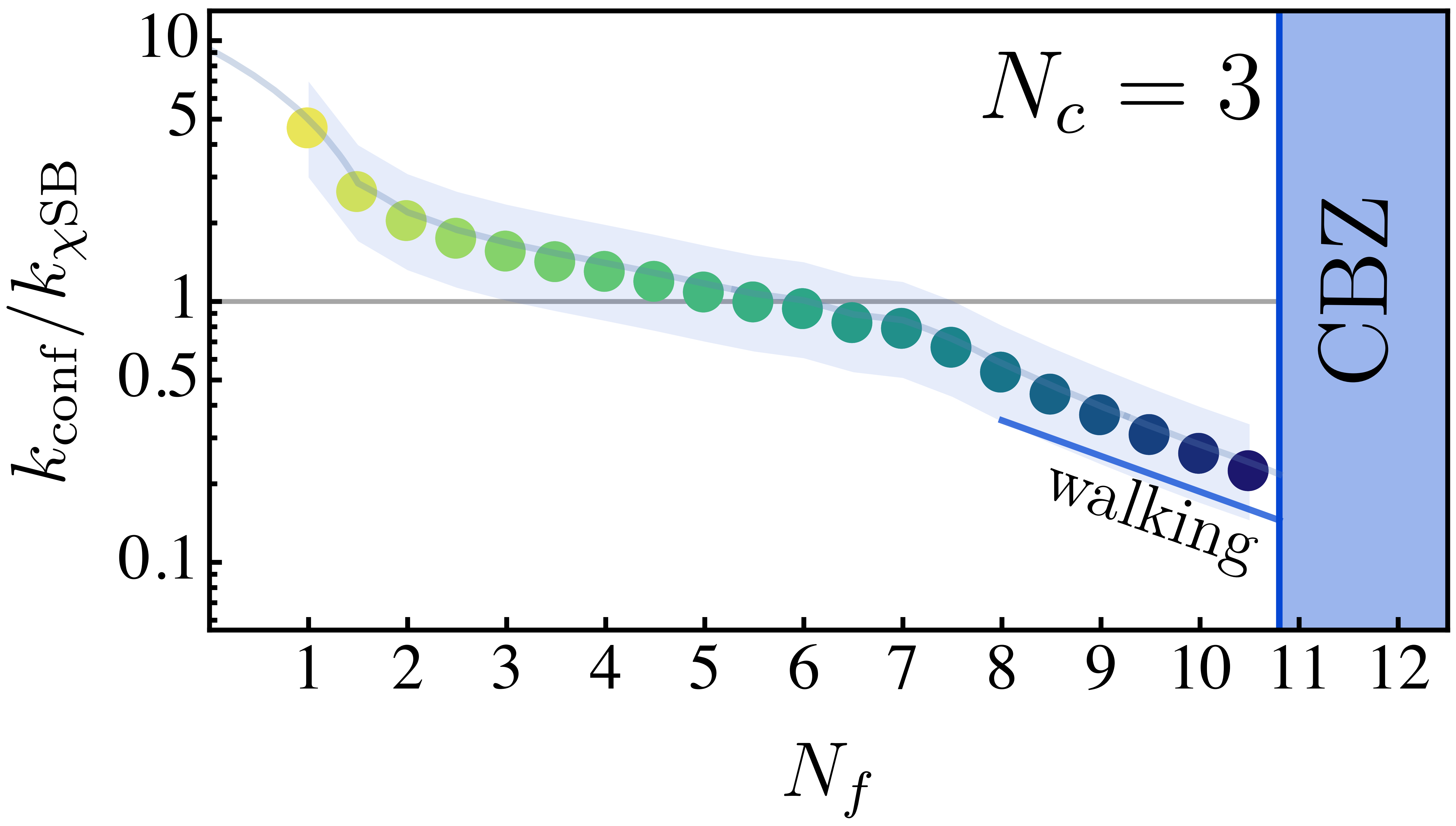
Non-perturbative **Perturbative**

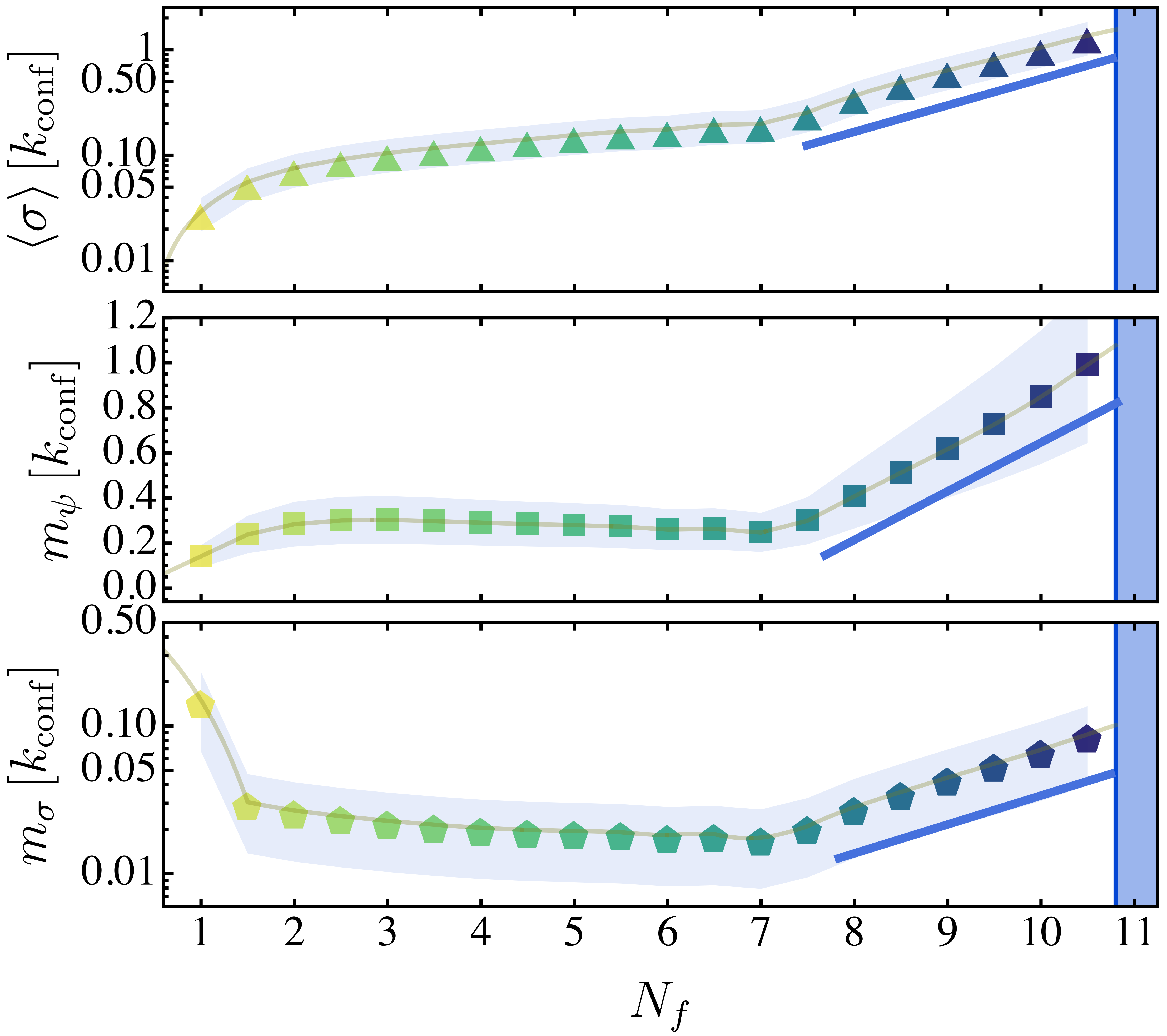
* Fundamental parameters in good agreement with best truncations

Many flavour gauge dynamics

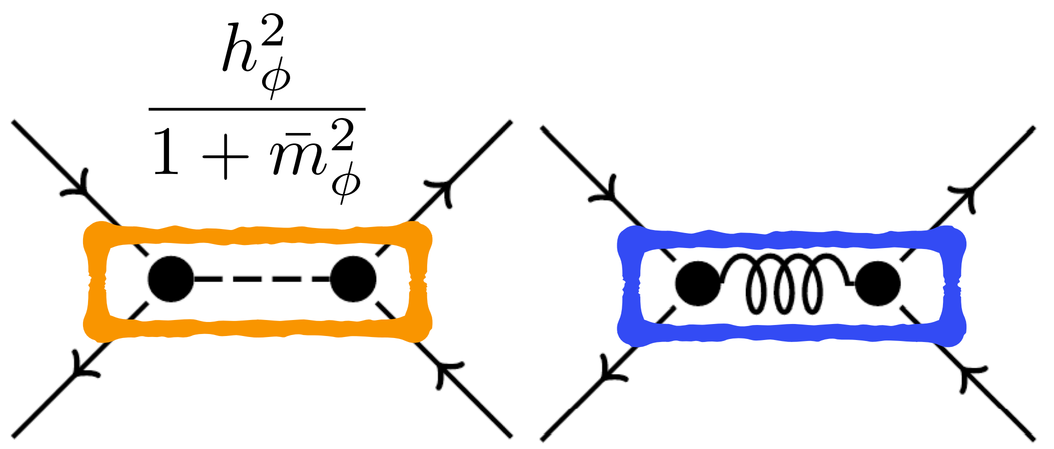


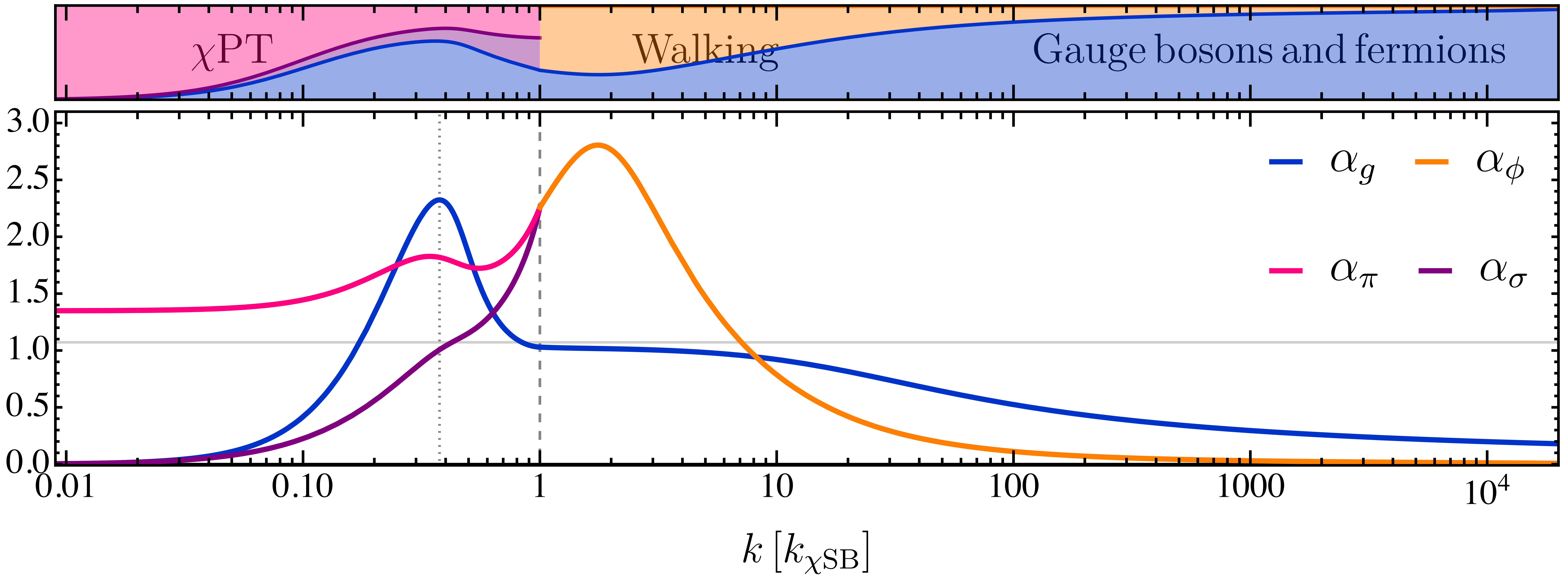
Interplay of scales and fundamental parameters



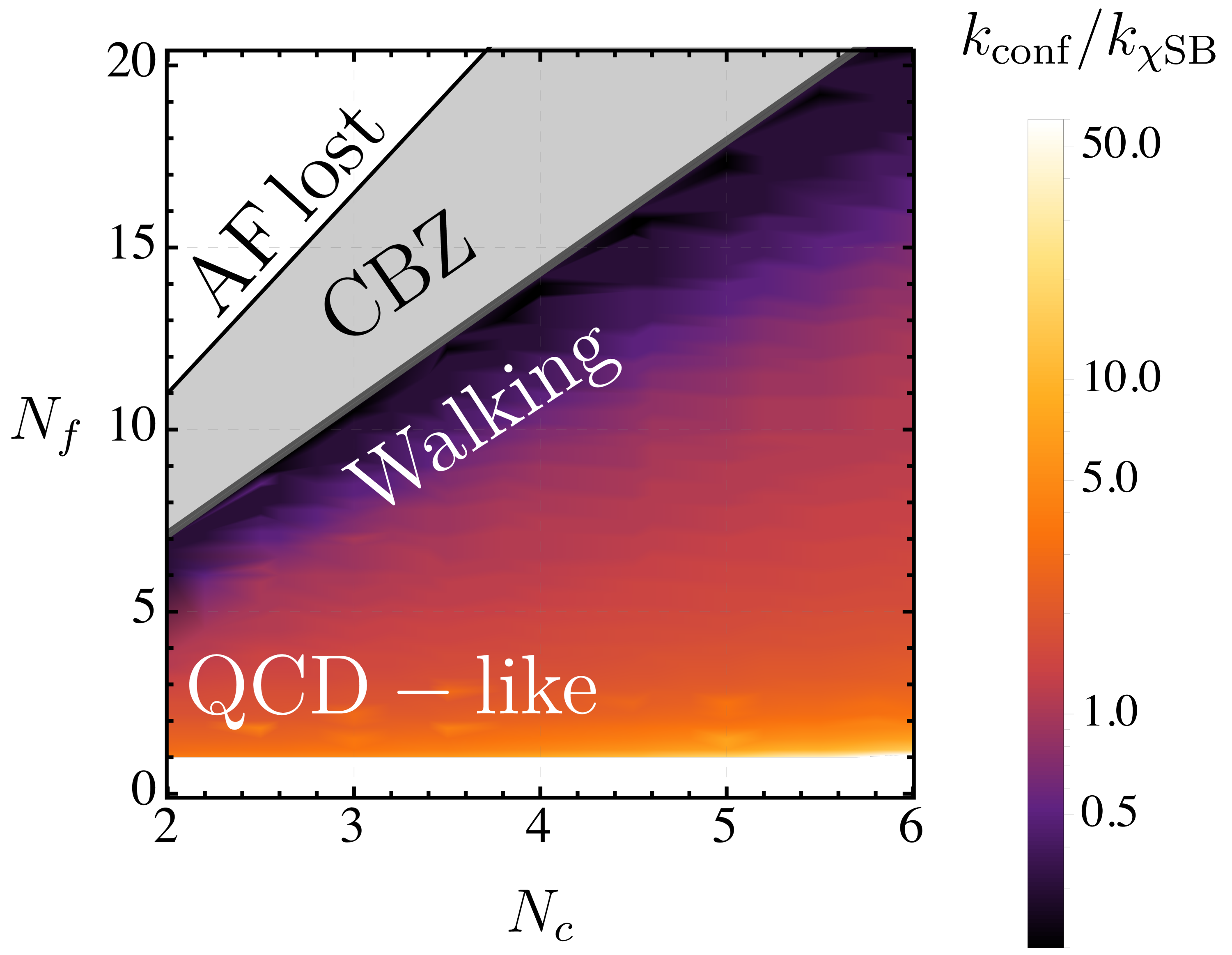


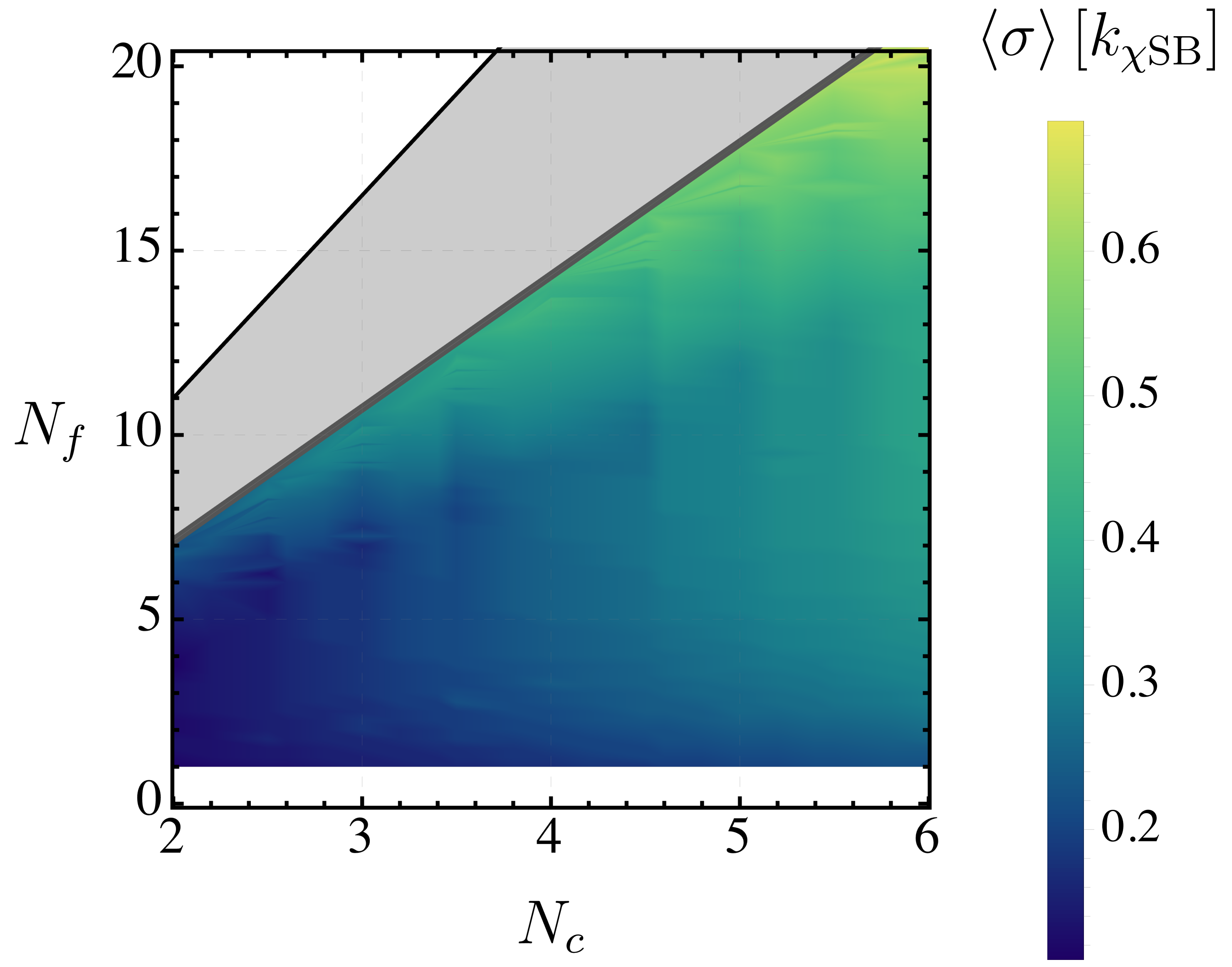
Walking dynamics and flow over EFTs

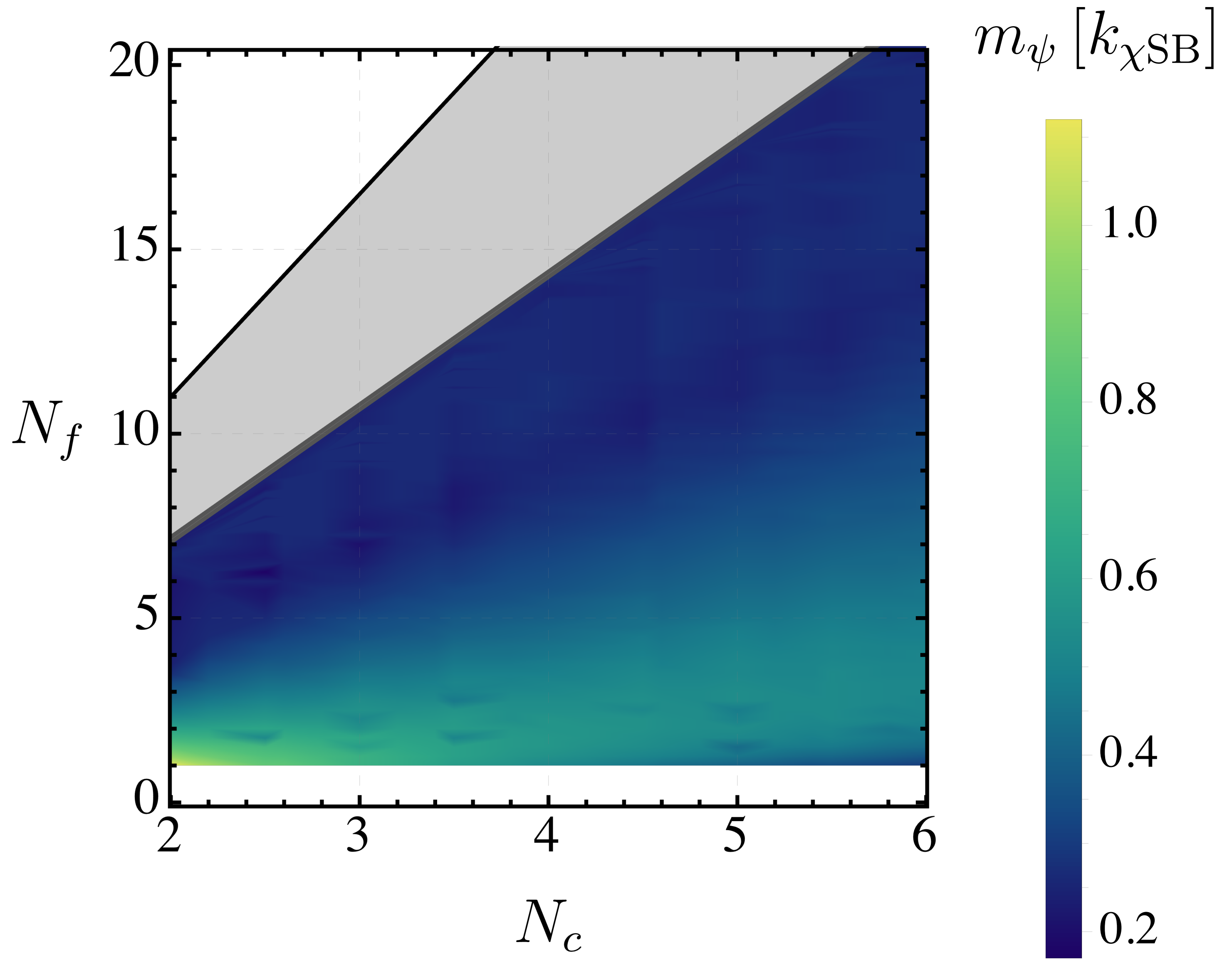


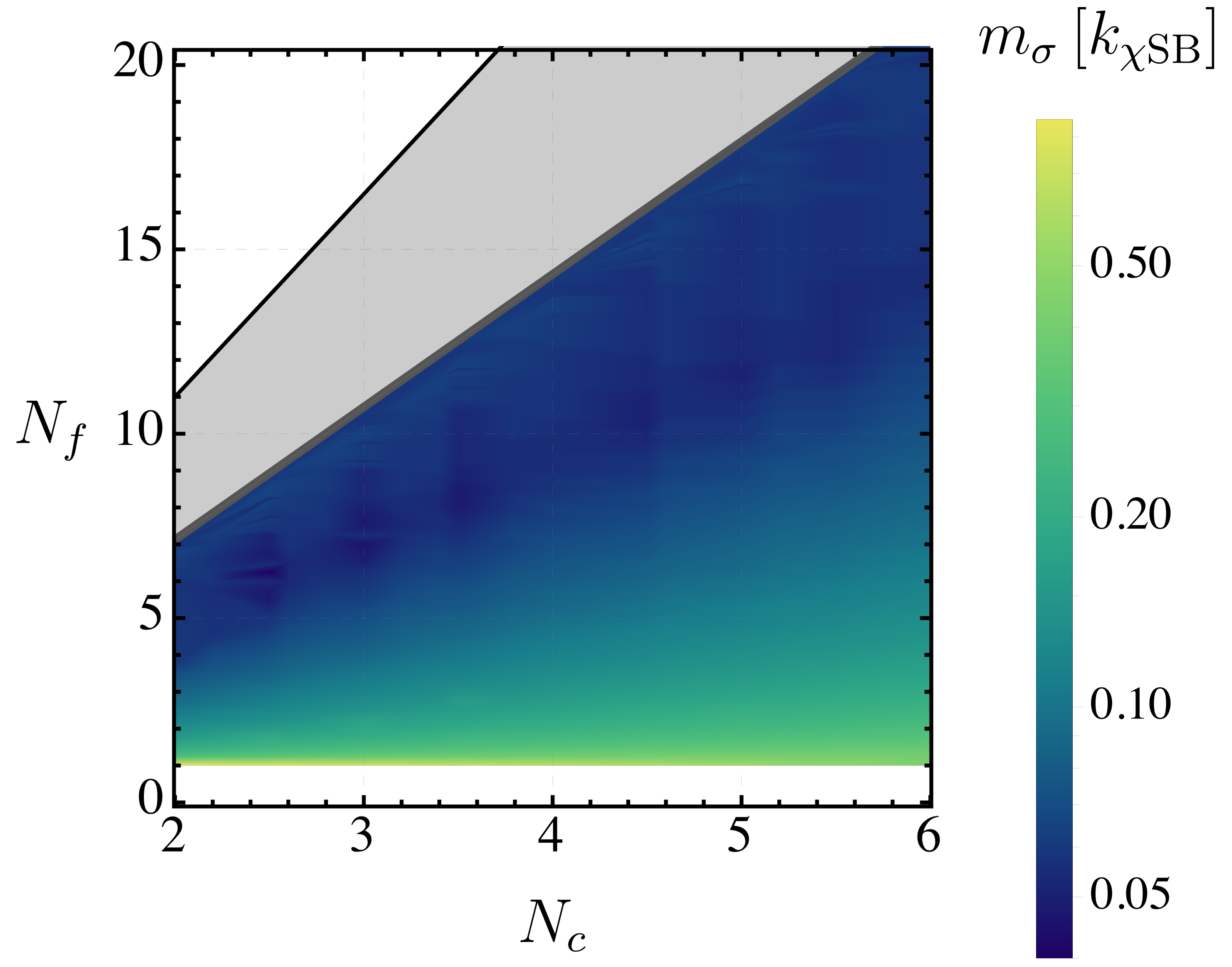


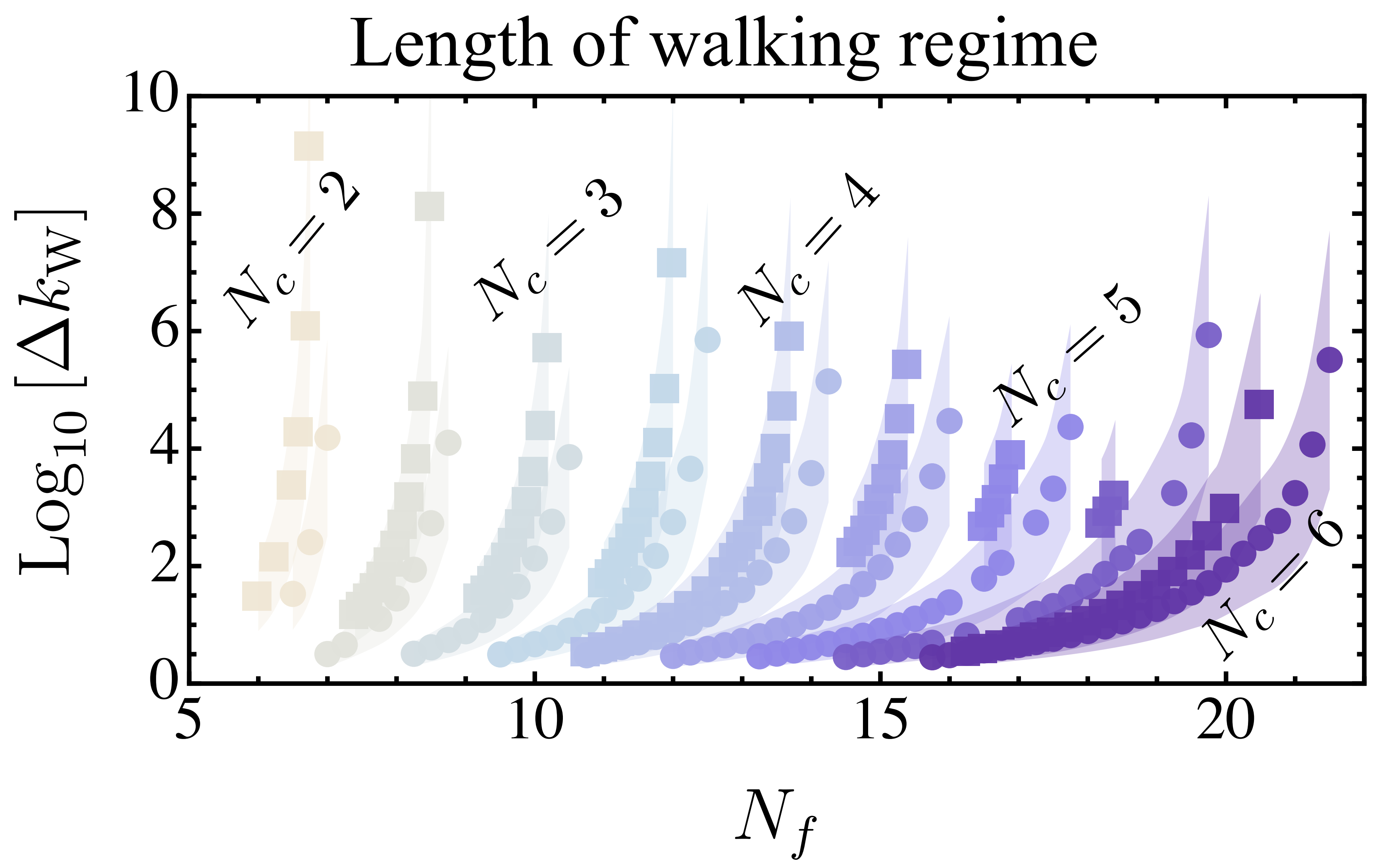
Cartography





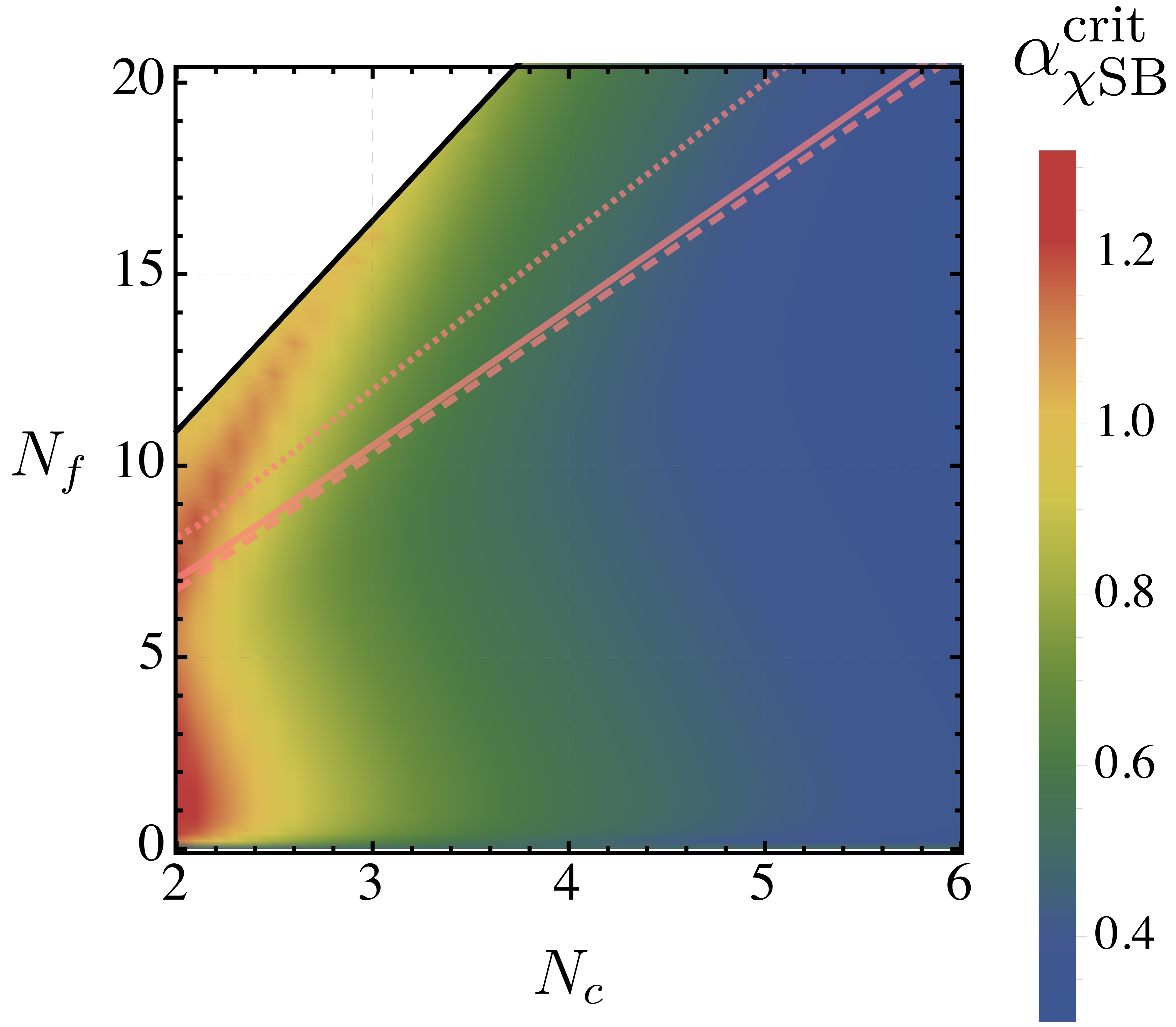




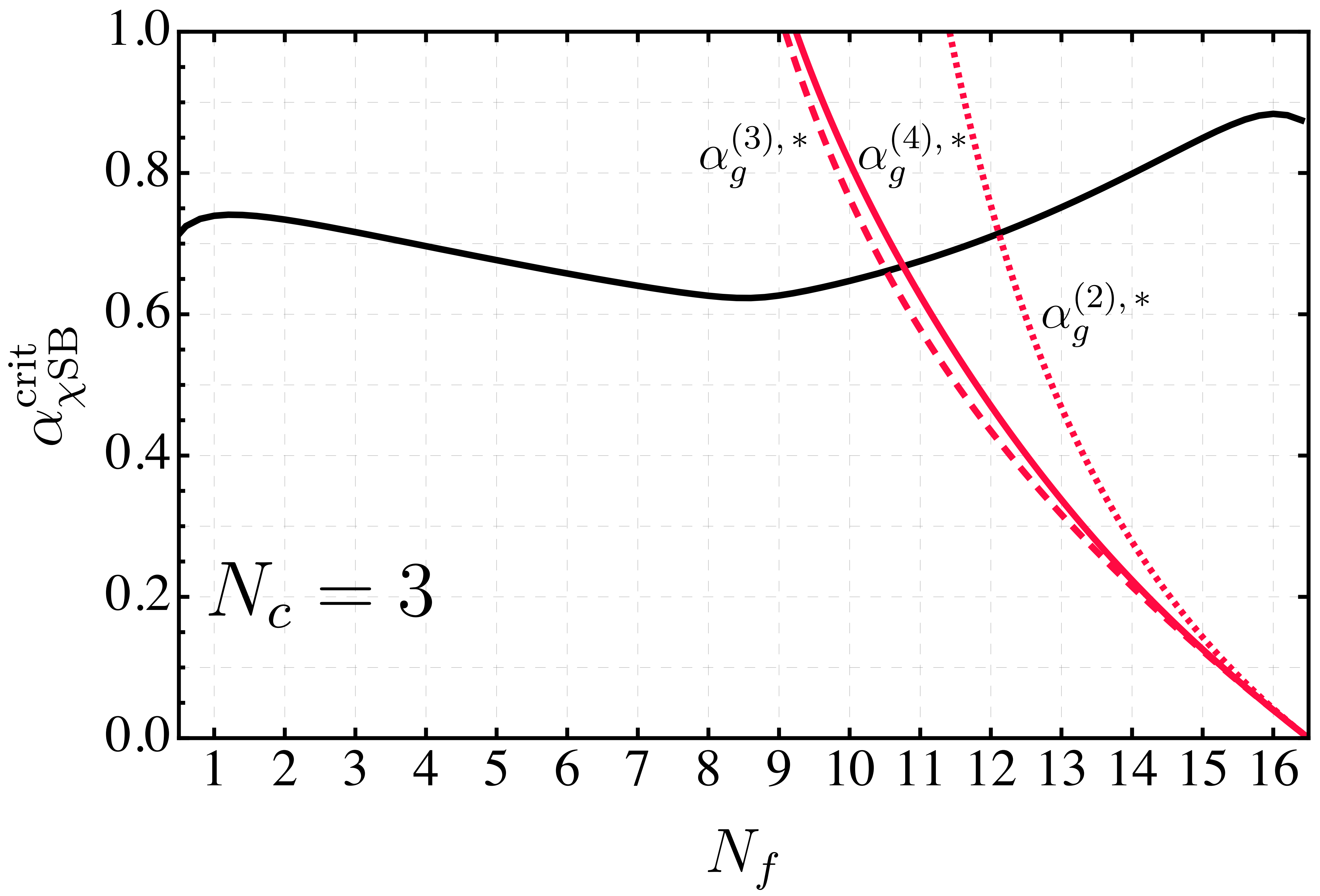


Boundary of the conformal window

$\alpha_{\chi\mathrm{SB}}^{\mathrm{crit}}$: Critical strength of **gauge dynamics** necessary to trigger dynamically χSB



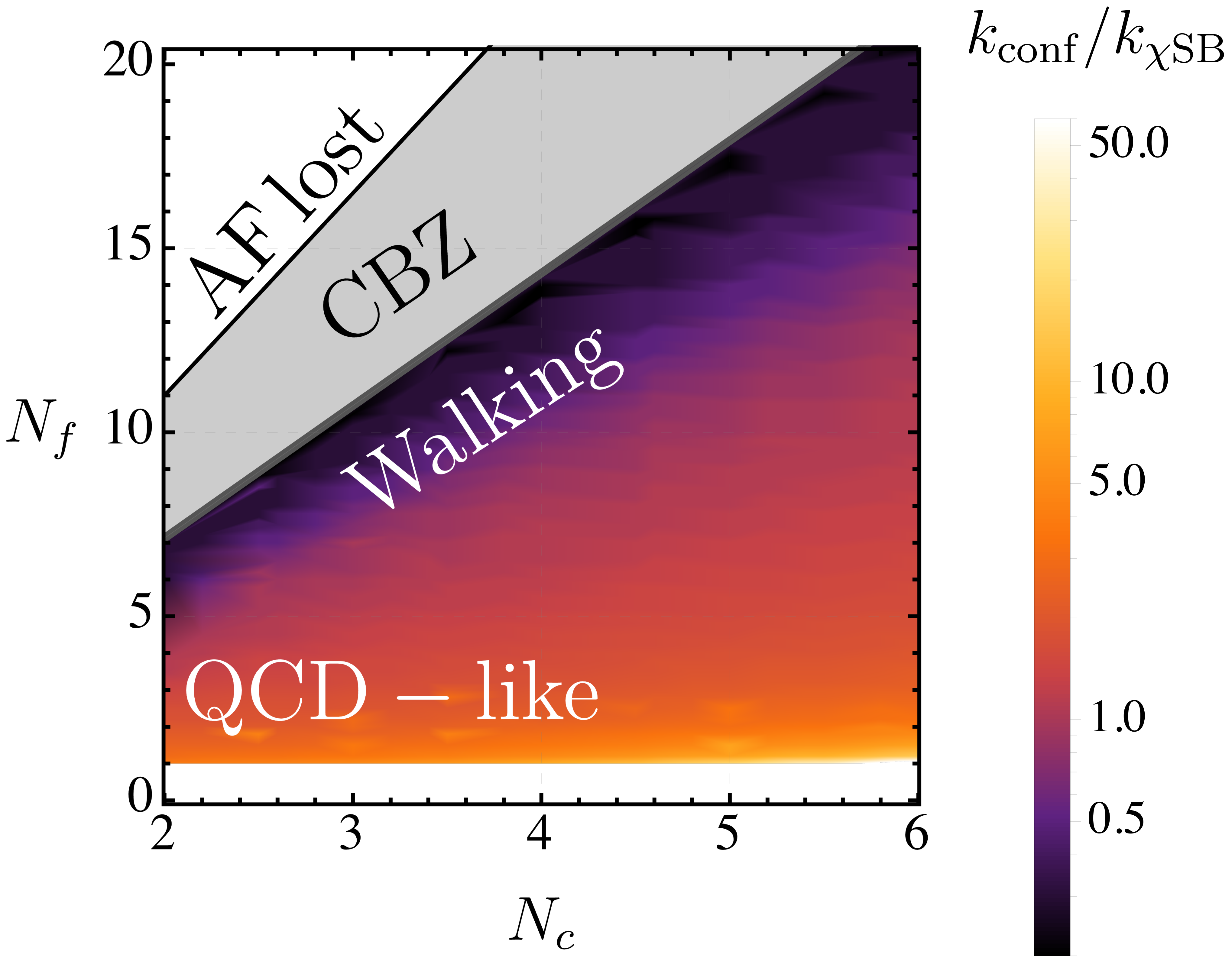
$N_f^{\mathrm{crit}} = 10.8$



Summary and conclusions

✦ The fRG is a **powerful approach** to strong dynamics:

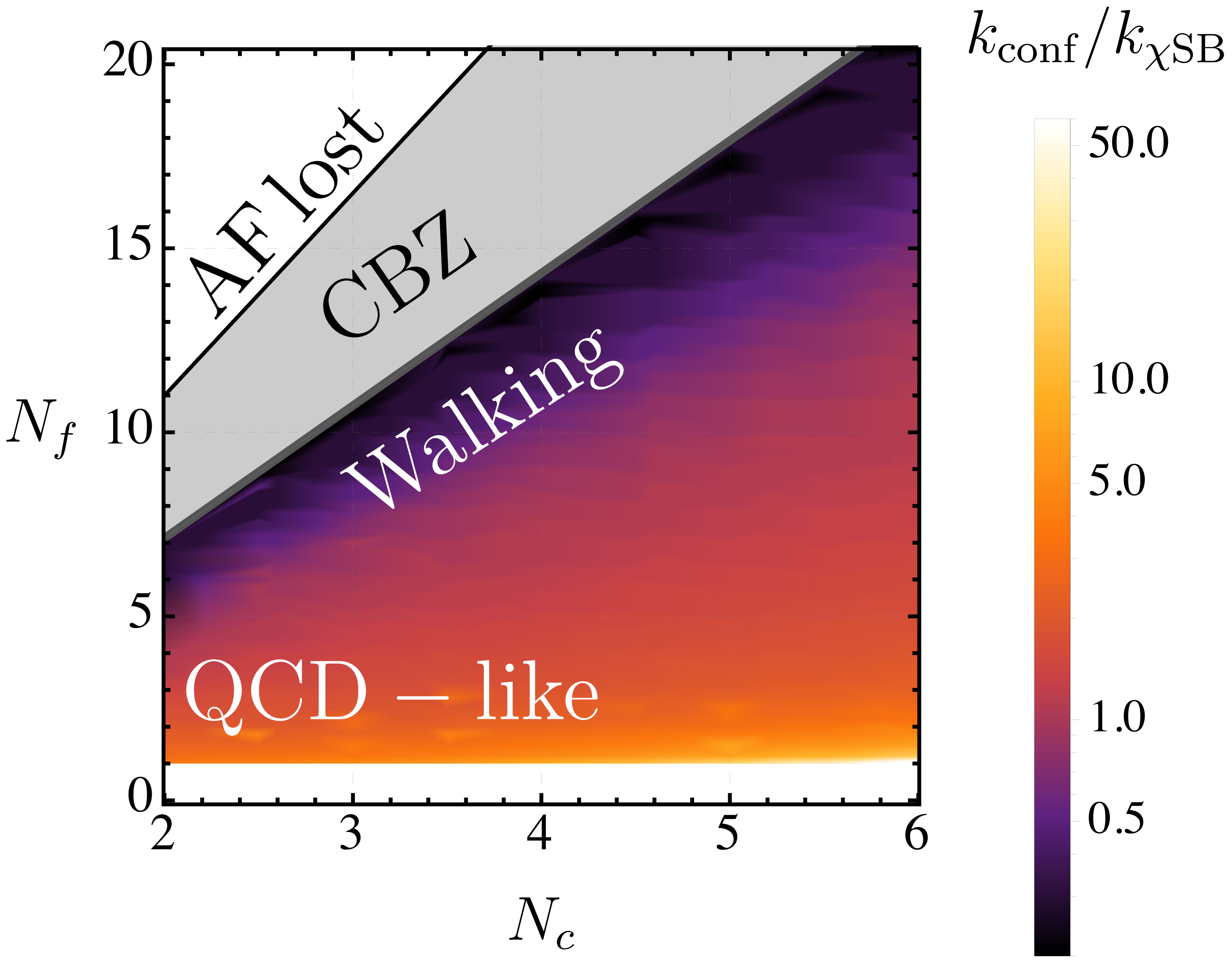
- Qualitatively new **treatment** of **confinement**
- Charted the **many-flavour landscape** from **first principles**
- Determination of **fundamental parameters**: m_ψ, $\langle\sigma\rangle$, m_σ, size of walking regime, …
- Provided a new **prediction** for the **lower boundary** of the CBZ window



Outlook

✦ A large number of **applications**:

- **Ph: Solving the Dark Matter - Baryon Coincidence:** Exploring Theories of Dark-Color Unification (with Yi Chung, Florian Goertz and Aika Tada)
- **Th:** Dynamical symmetry breaking in the **Bars-Yankielowicz** model (with Shahram Vatani and Haolin Li)
- **Th-Ph:** Phase diagram of **scalar-QCD** theories (with Maciej Kierkla, Masatoshi Yamada and Jisuke Kubo)
- …



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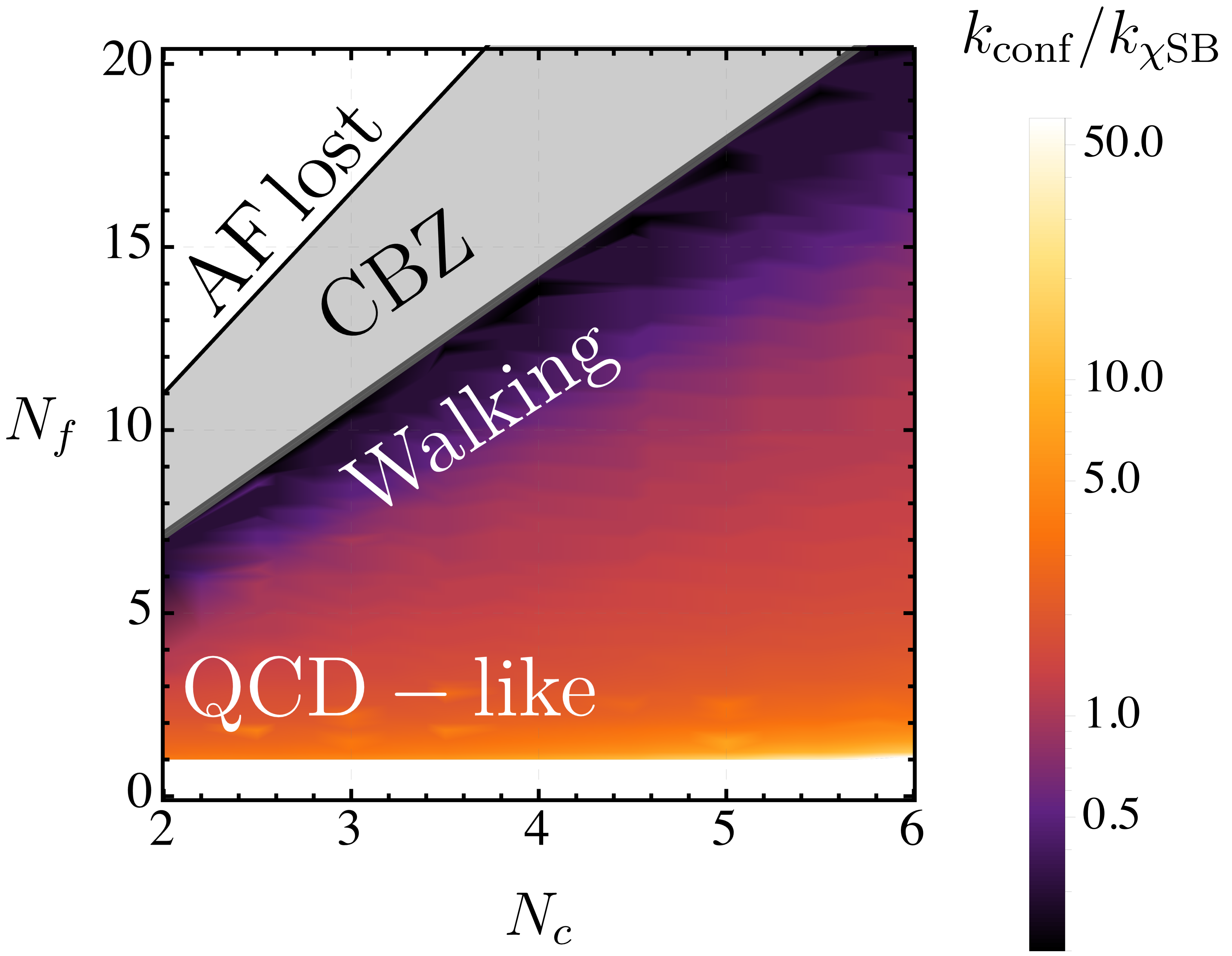
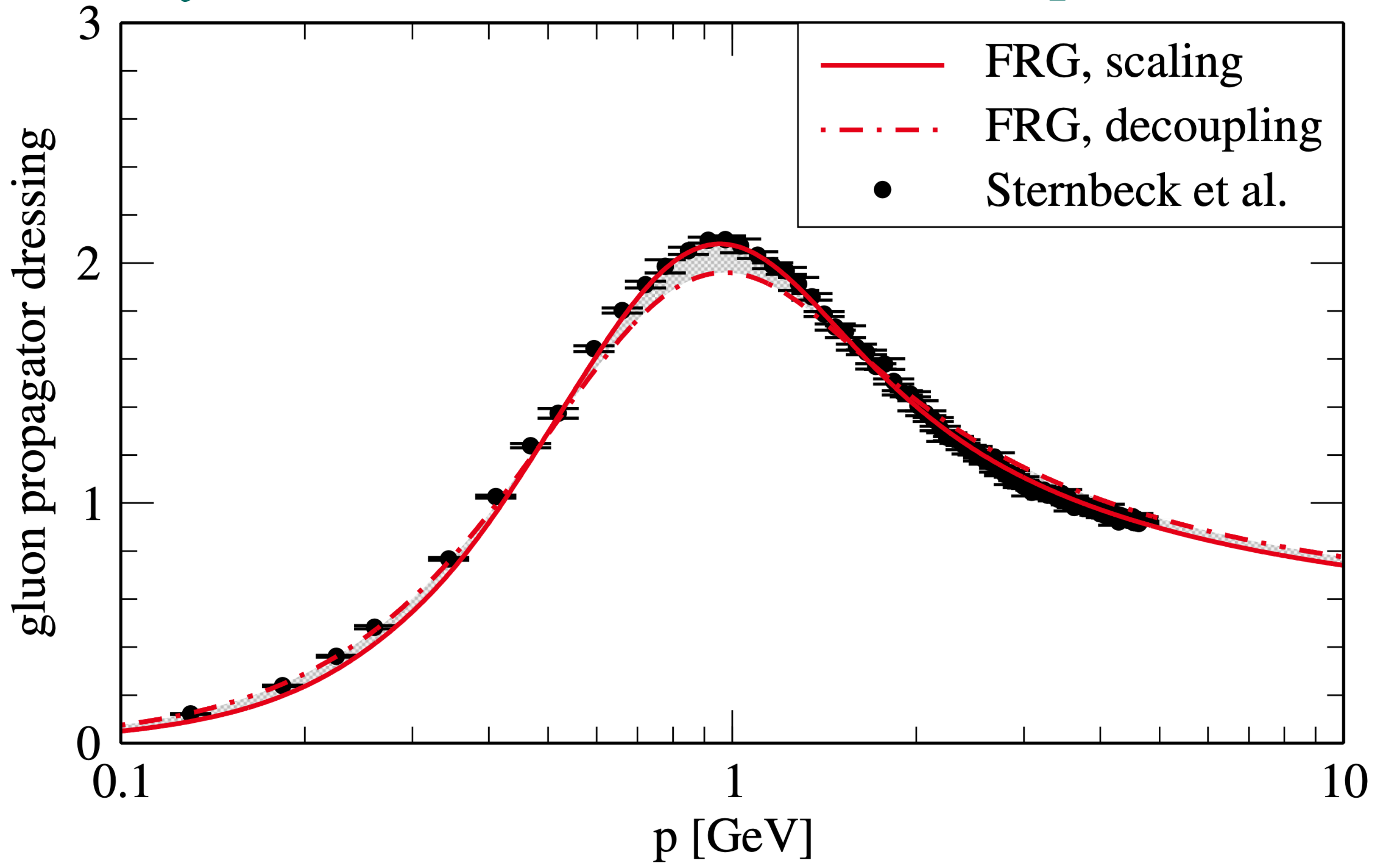


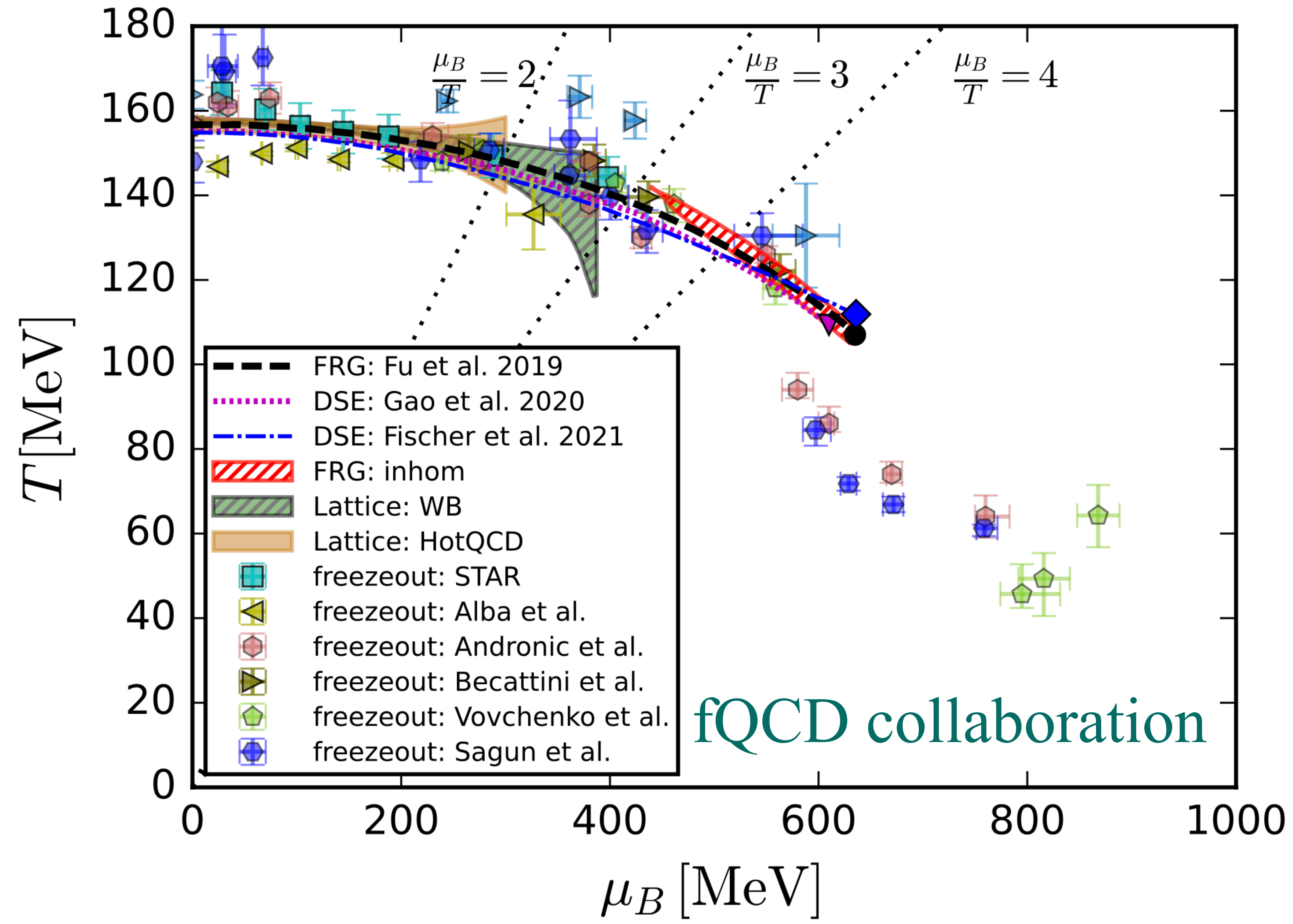
Thank you for your attention!

Additional slides

Functional Renormalisation Group in QCD

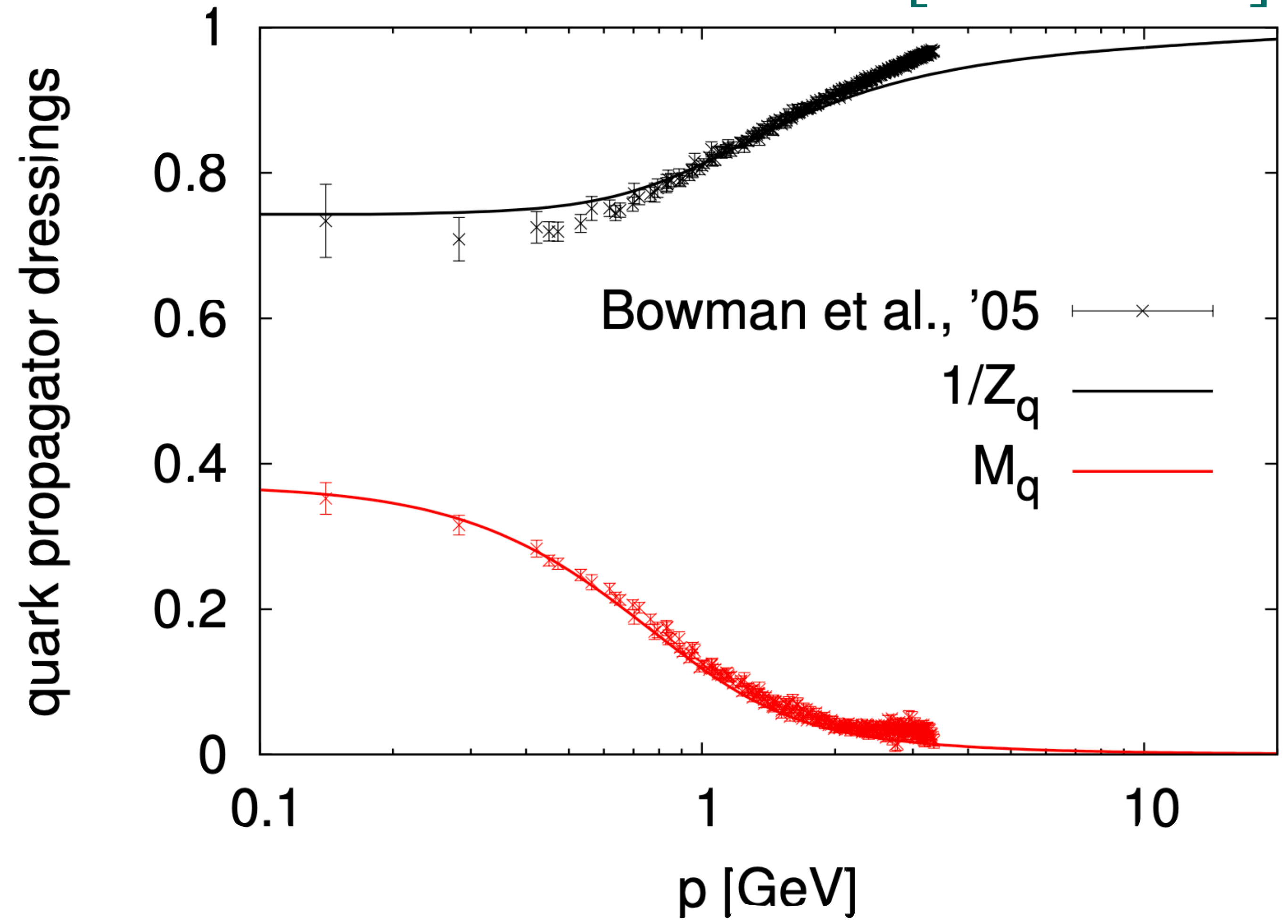
Cyrol,Fister,Mitter,Pawlowski [1605.01856]



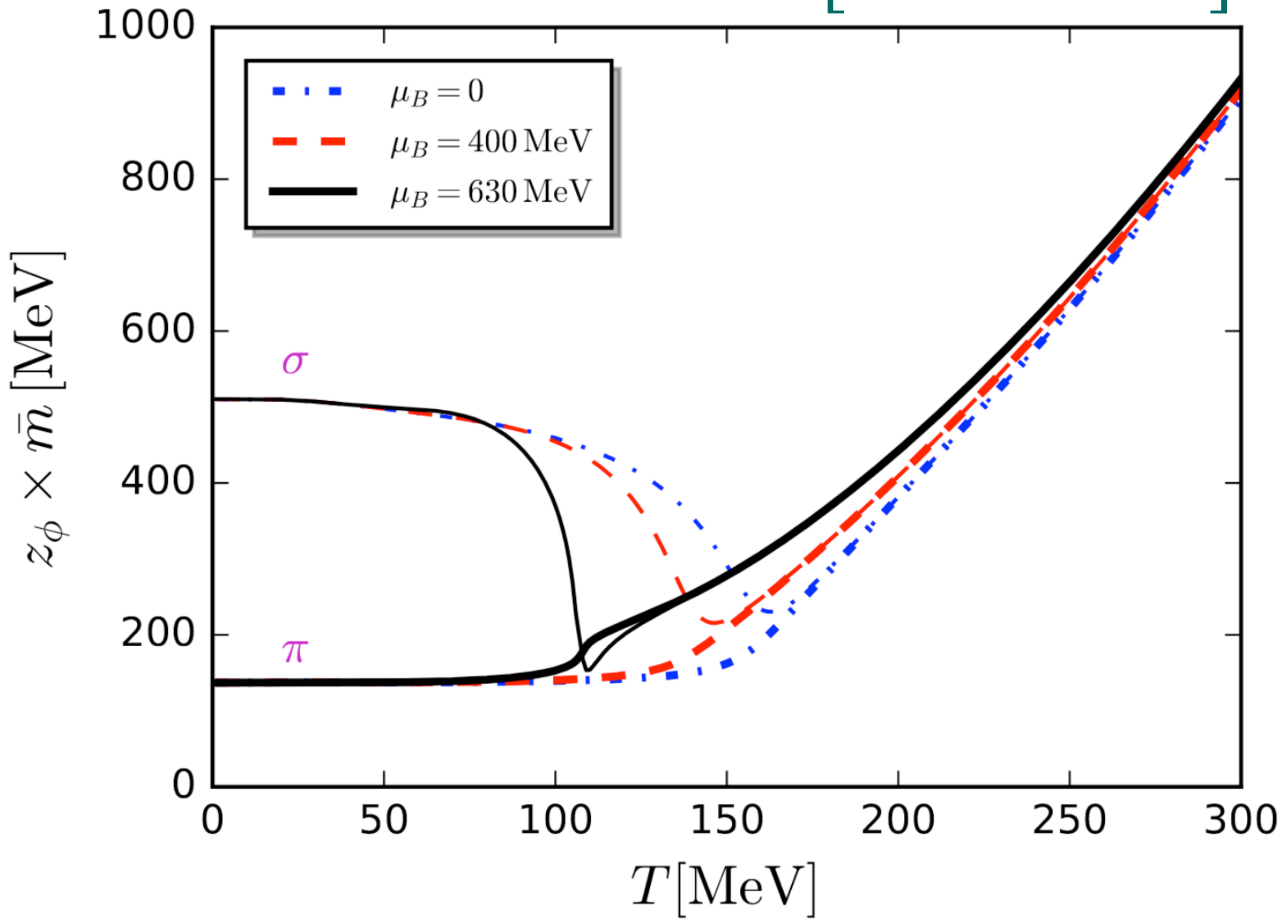


fQCD collaboration

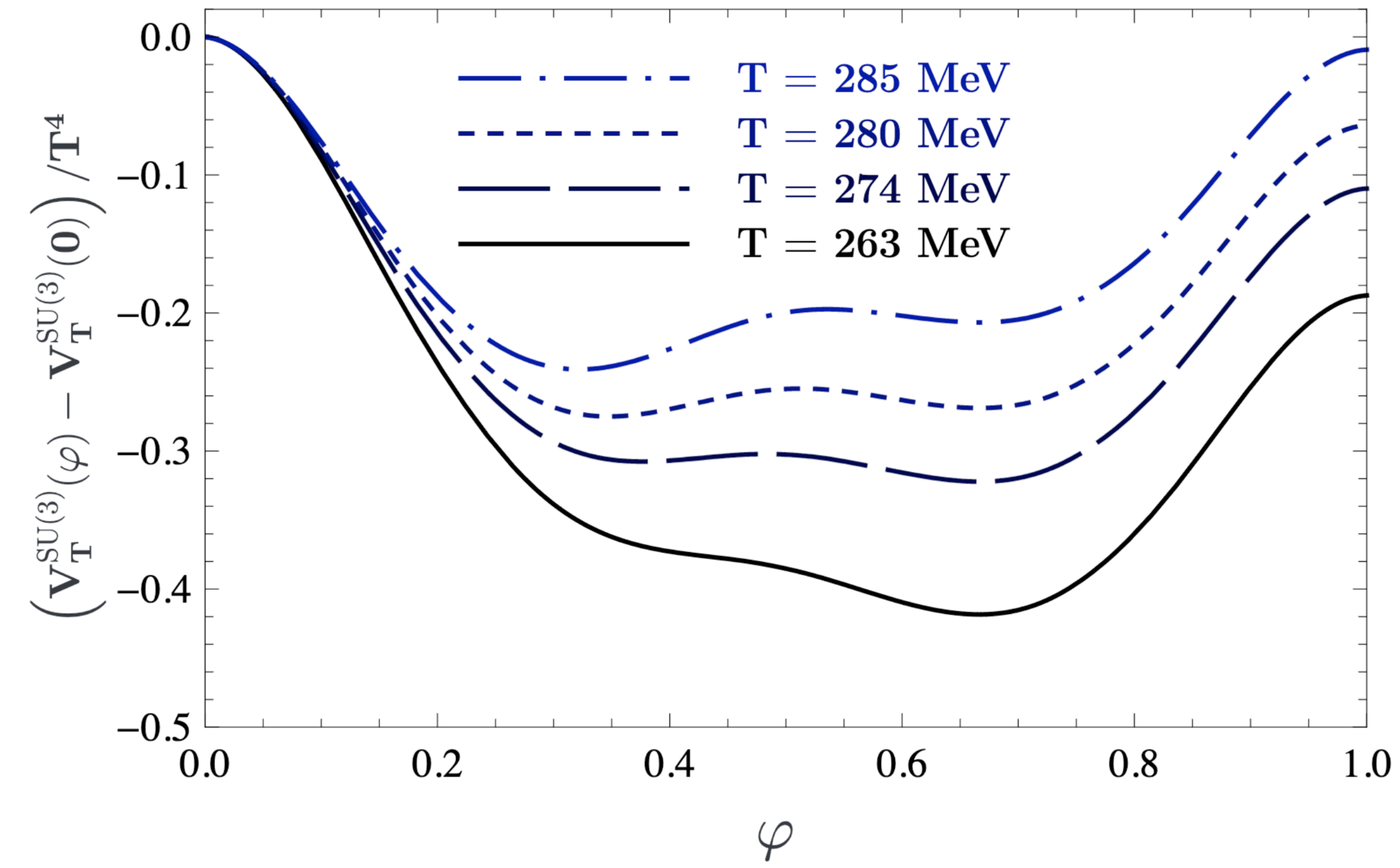
Mitter,Pawlowski, Strodthoff[1411.7978]



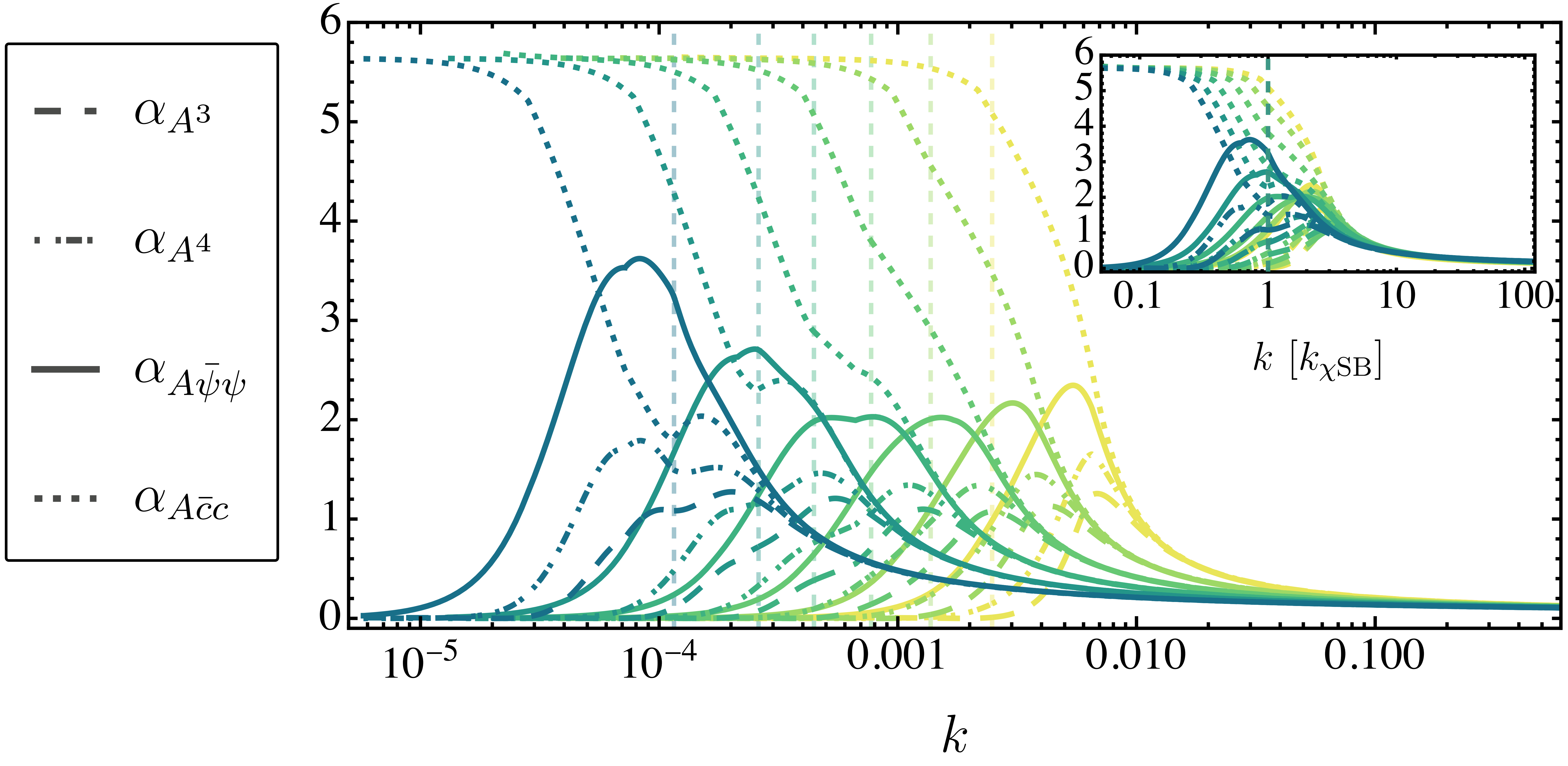
Fu,Pawlowski, Rennecke [1909.02991]



Fister,Pawlowski [1301.4163]



Many flavour full truncation



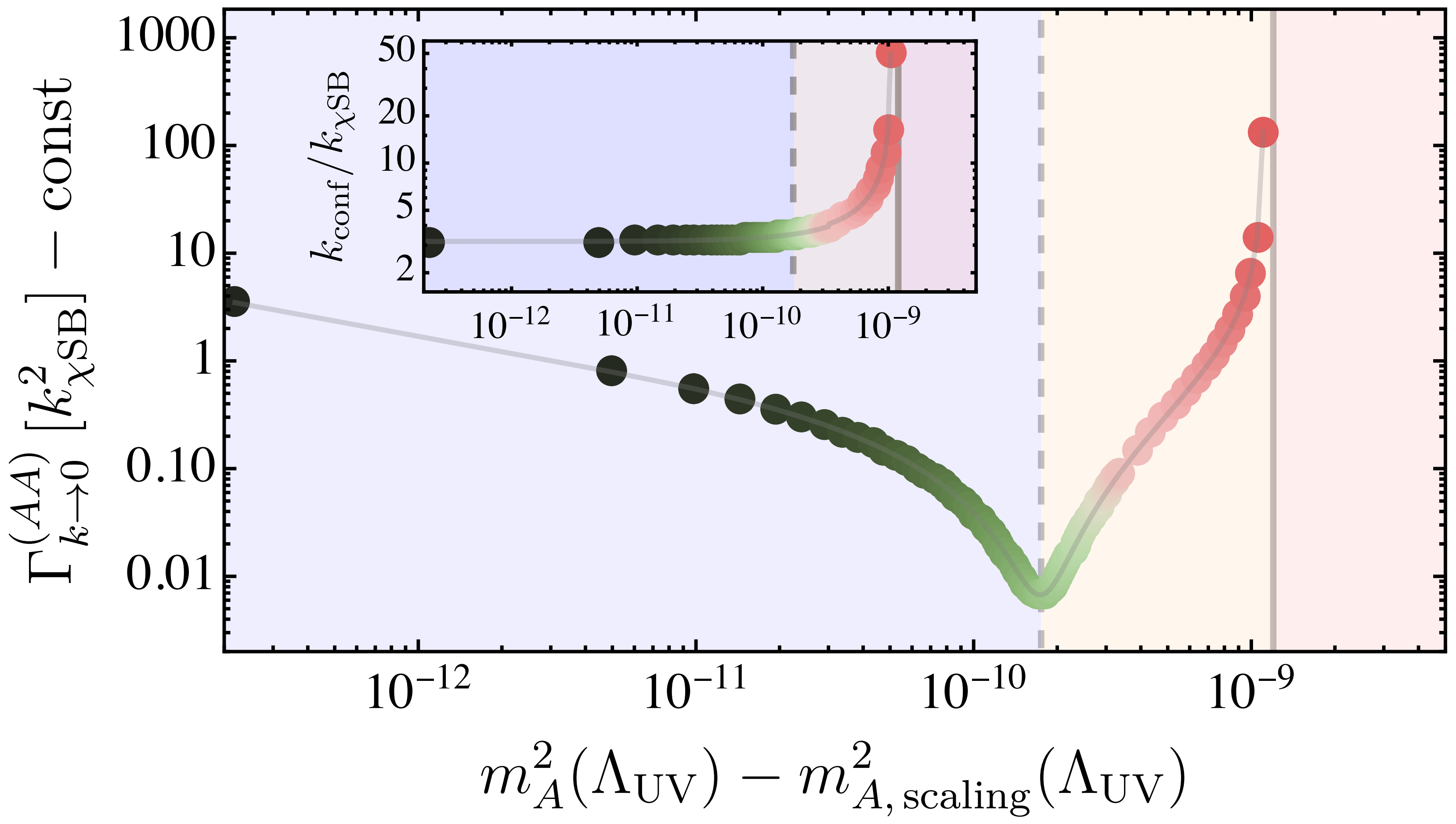
✦ Observations:

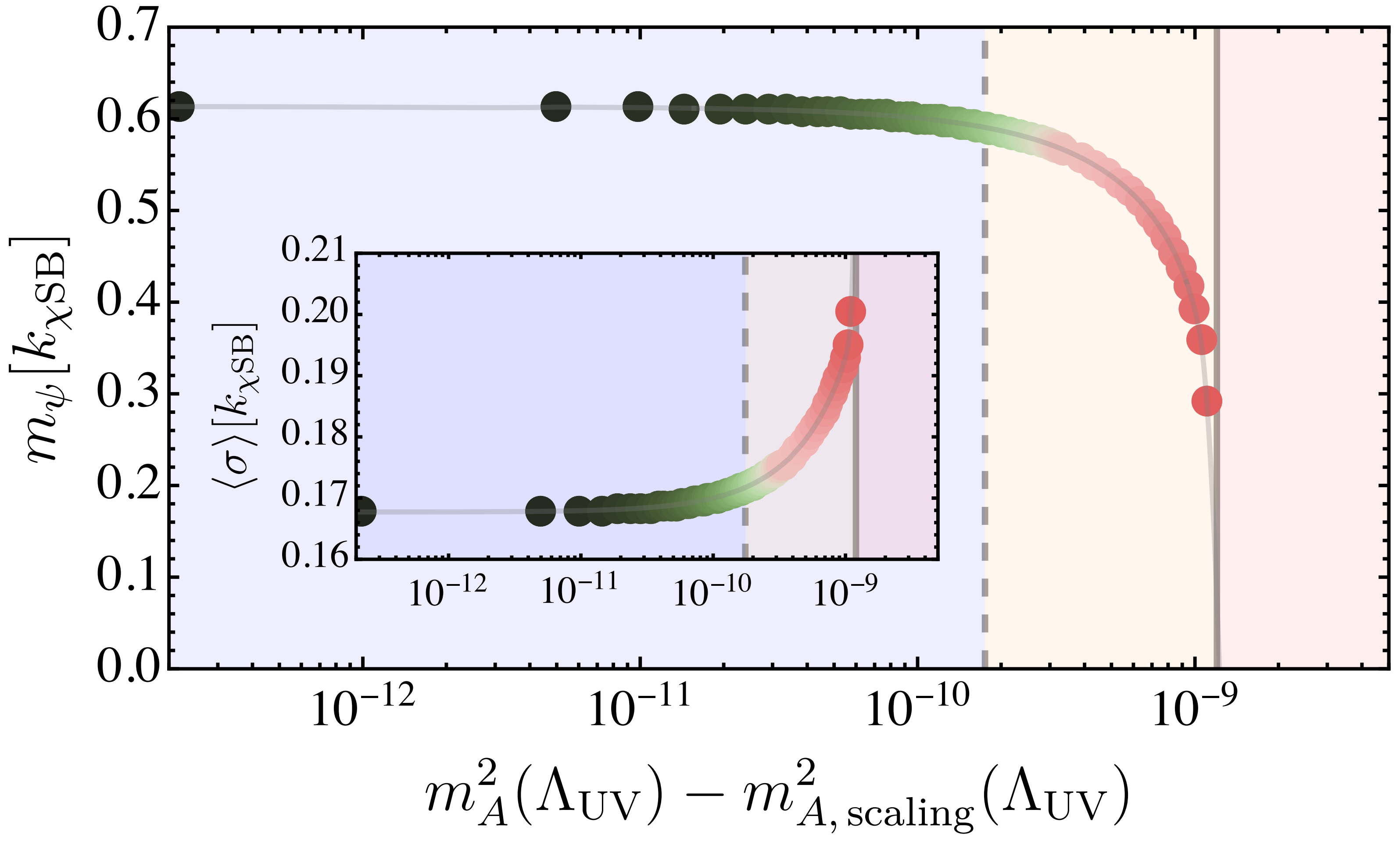
- The peak of the pure-glue vertex is rather constant and independent of the number of flavours
- The ghost-gauge coupling in the deep IR always reaches the same constant coupling given it is only colour-group dependent
- Increment of gauge-chiral coupling strength. The stronger value of this avatar causes a flattening of the chiral potential and consequently an earlier χSB.

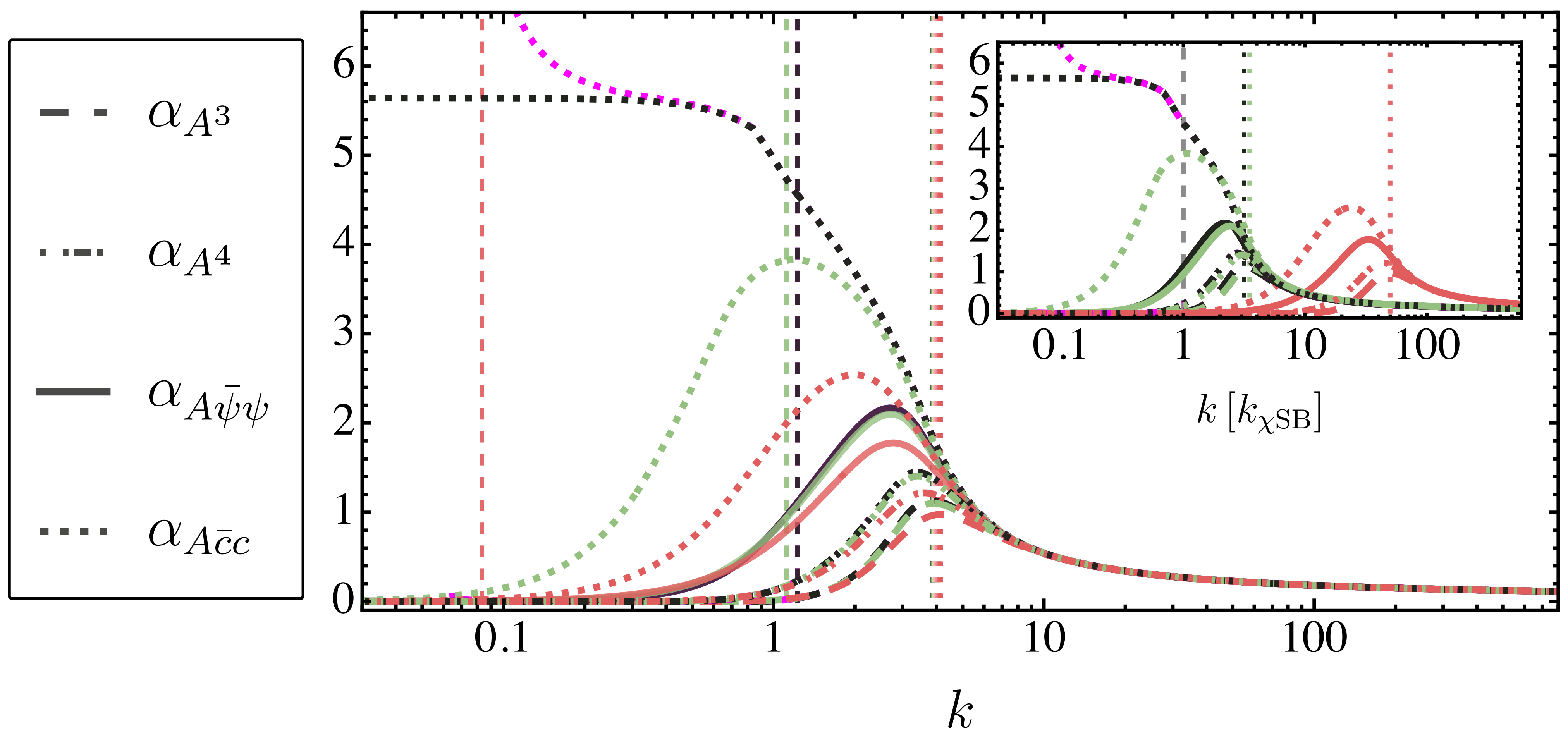
Yang-Mills phases and confinement

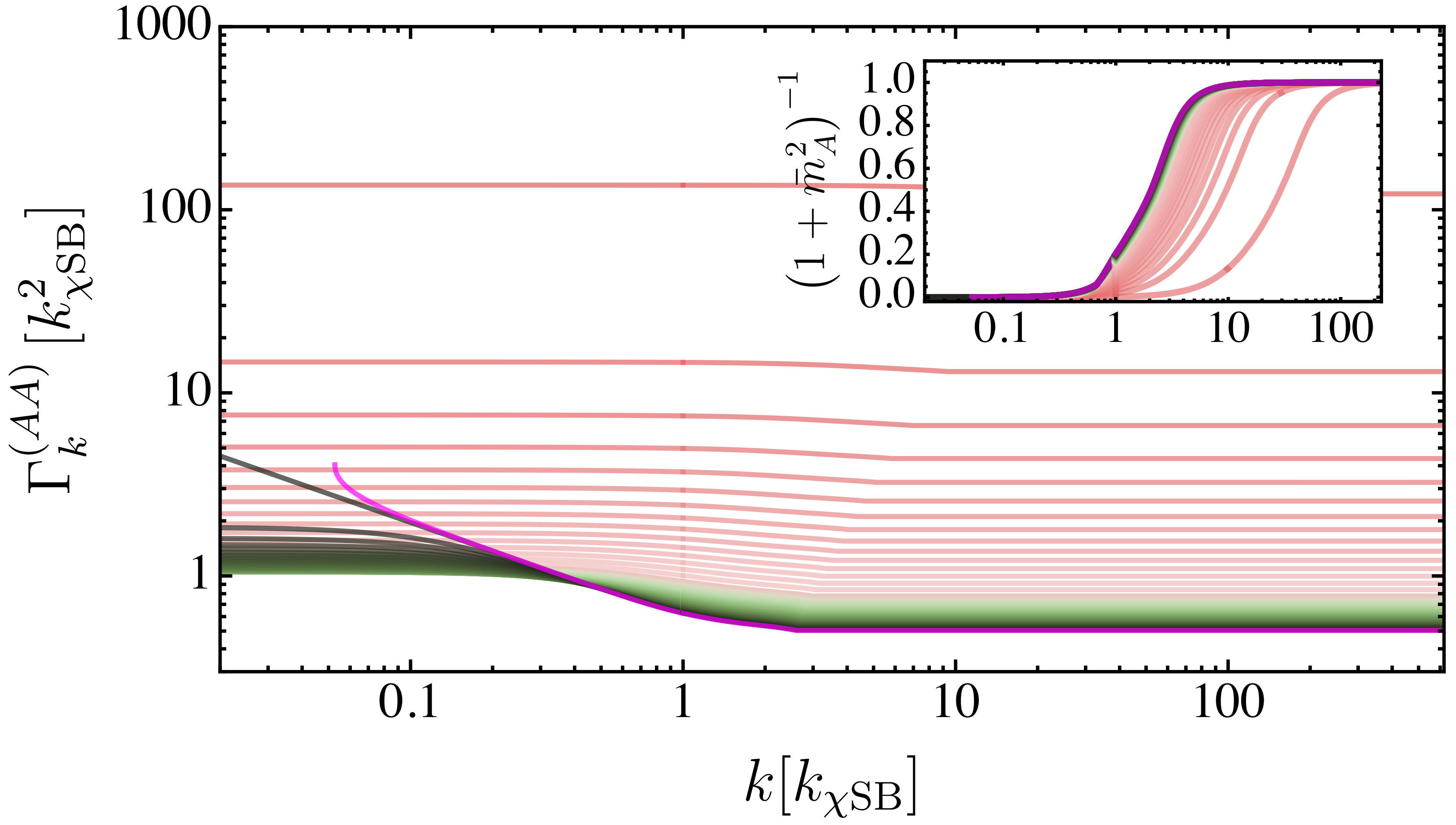
$$\Gamma_k^{(AA)}(p^2) = Z_{A,\,k}(p)\left(p^2 + m^2_{A,\,k}\right)$$

- ✦ Mass gap as a dialing parameter
- ✦ Confining solutions:
  - Scaling
  - Decoupling
- ✦ Non-confining solutions:
  - Coulomb phase
  - Massive Yang-Mills

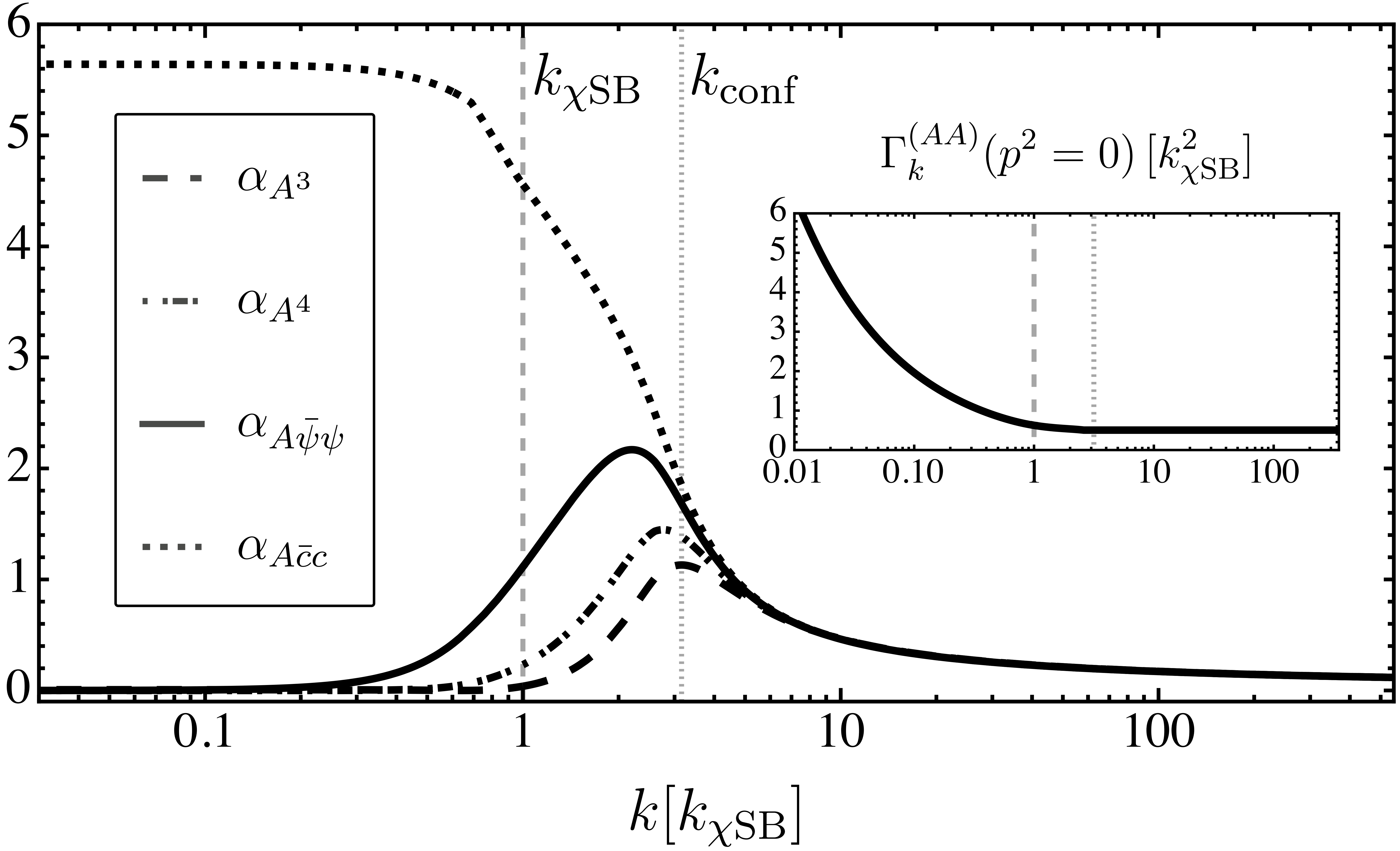








Criticality of low Nf



Conformality and scale invariance

$$S = \int_x \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm gh} + \bar{\psi} \not{D} \psi$$

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$\mathcal{G}_{\text{TC}}$

$\text{SU}(N_f)_L \times \text{SU}(N_f)_R$

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$\rightarrow$ $\beta_g = -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3} C_{\text{A}} - \frac{4}{3} T_{\text{F}} N_f \right)$

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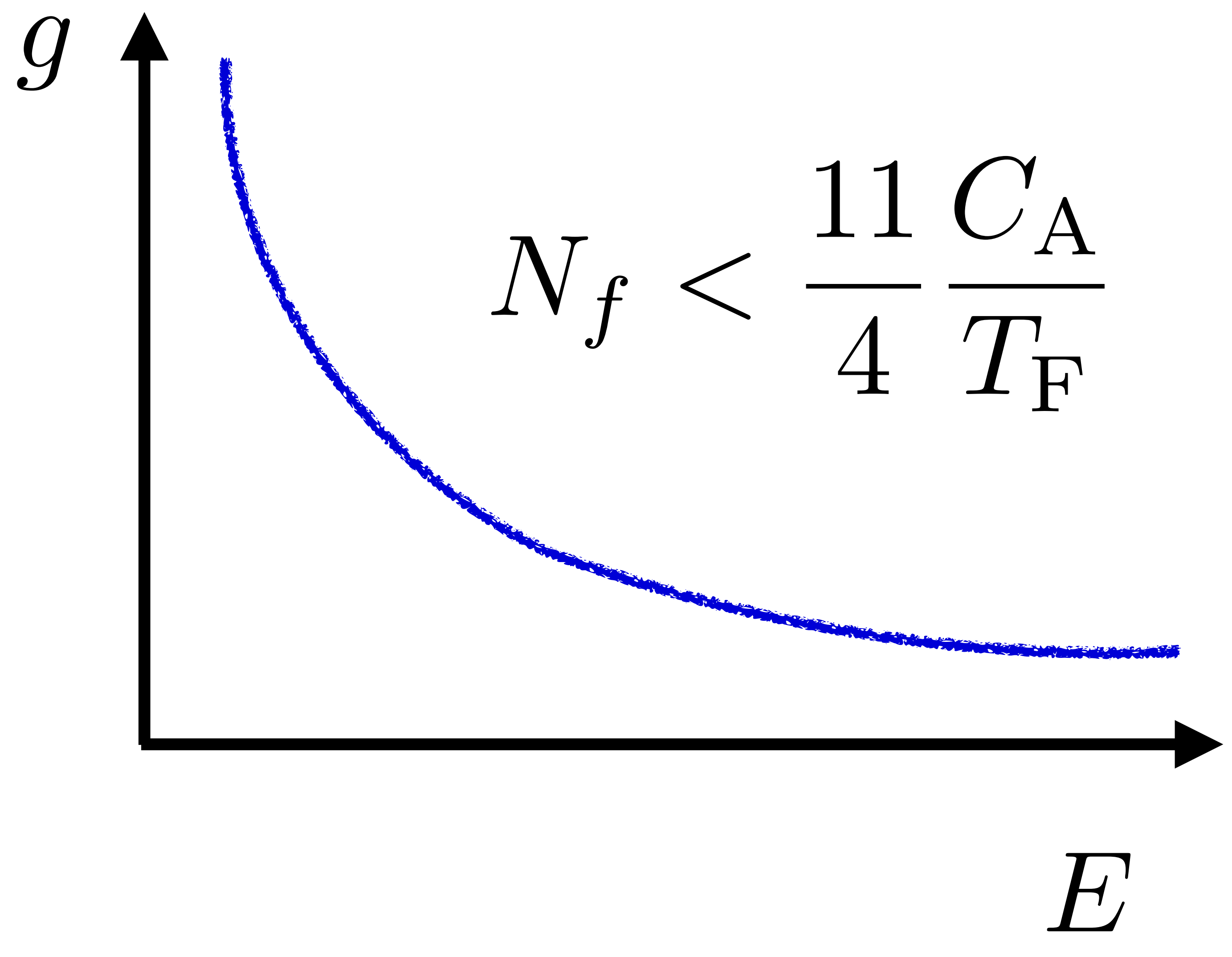

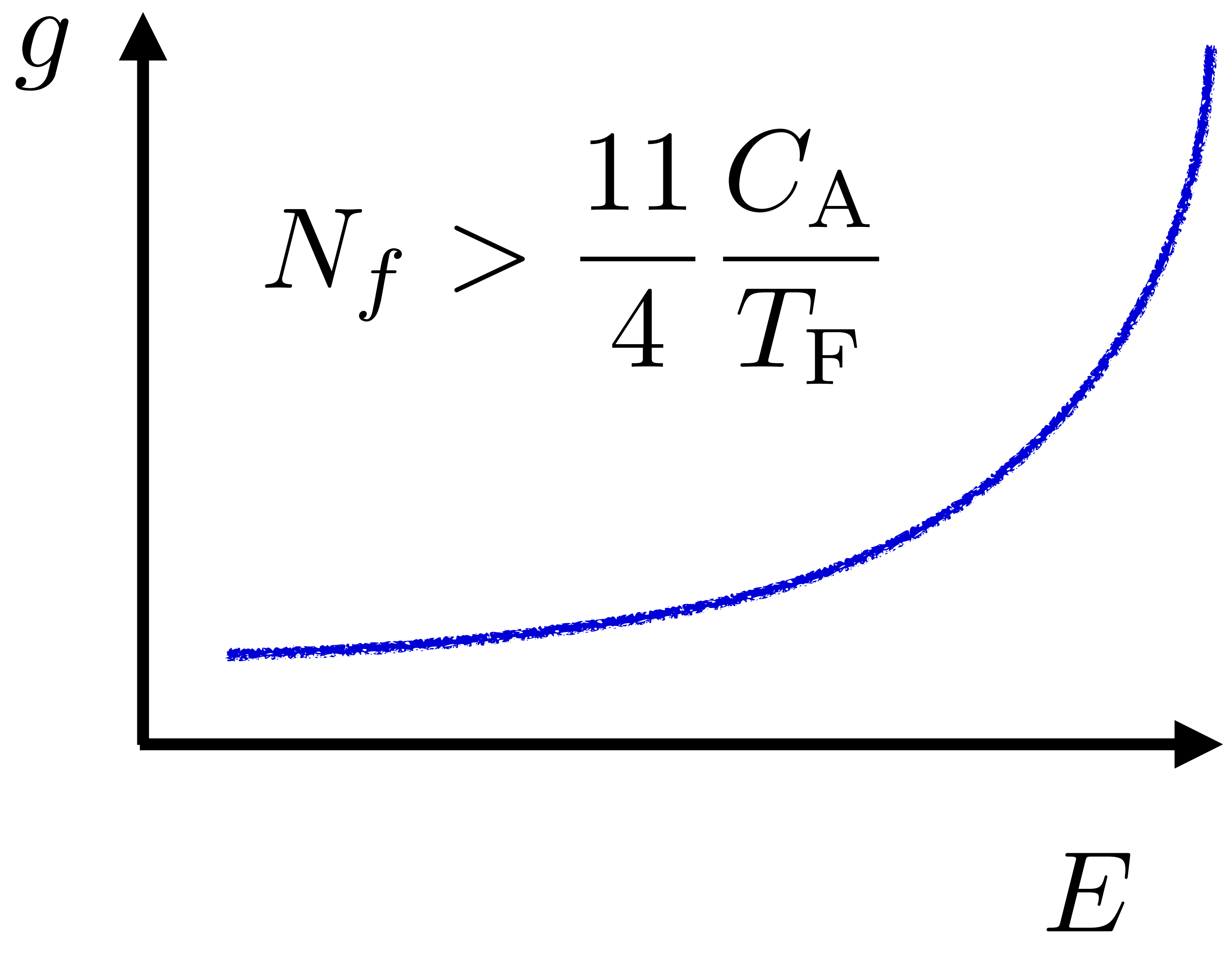

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$\mathcal{G}_{\rm TC}$ $\quad$ $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$

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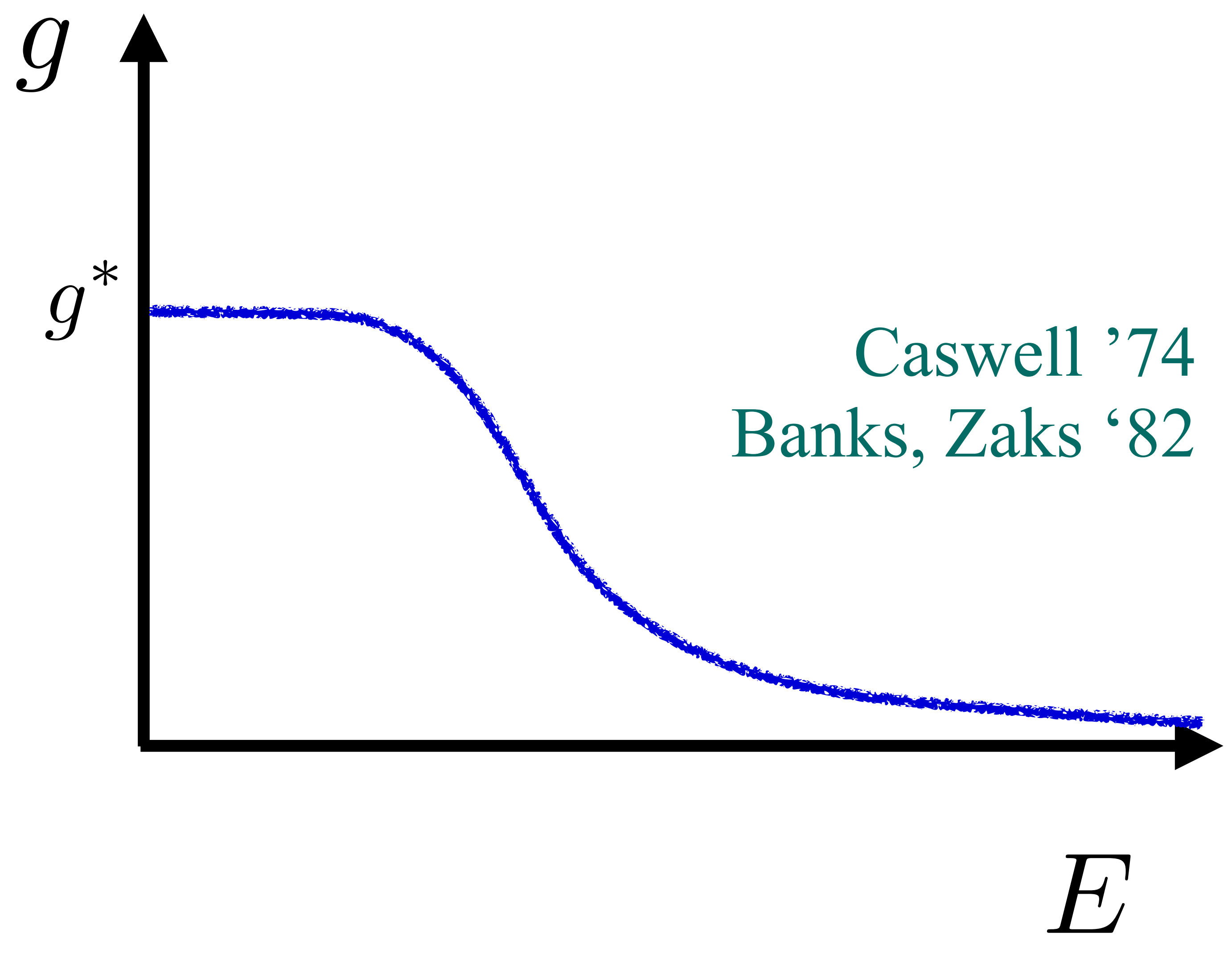
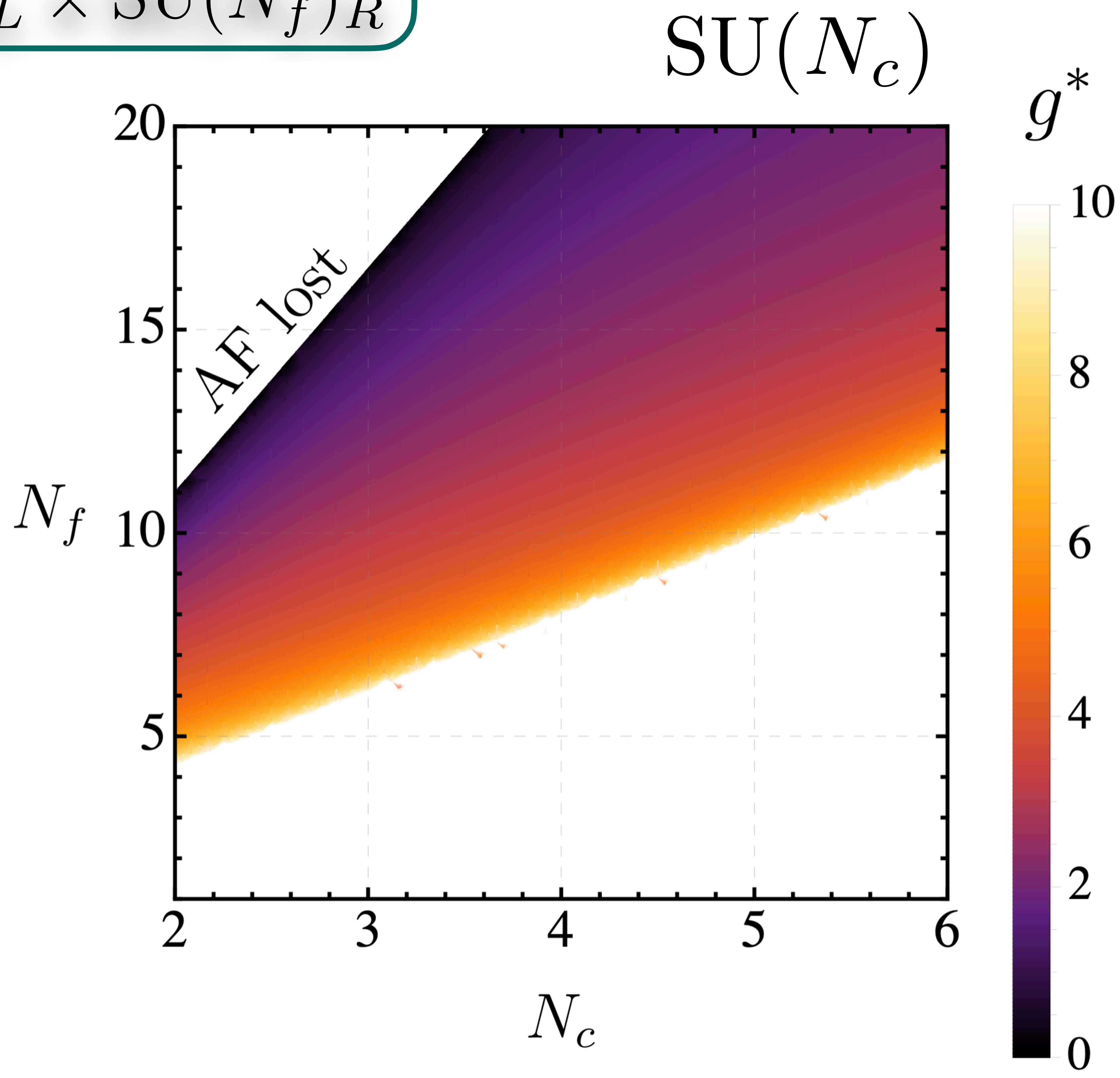
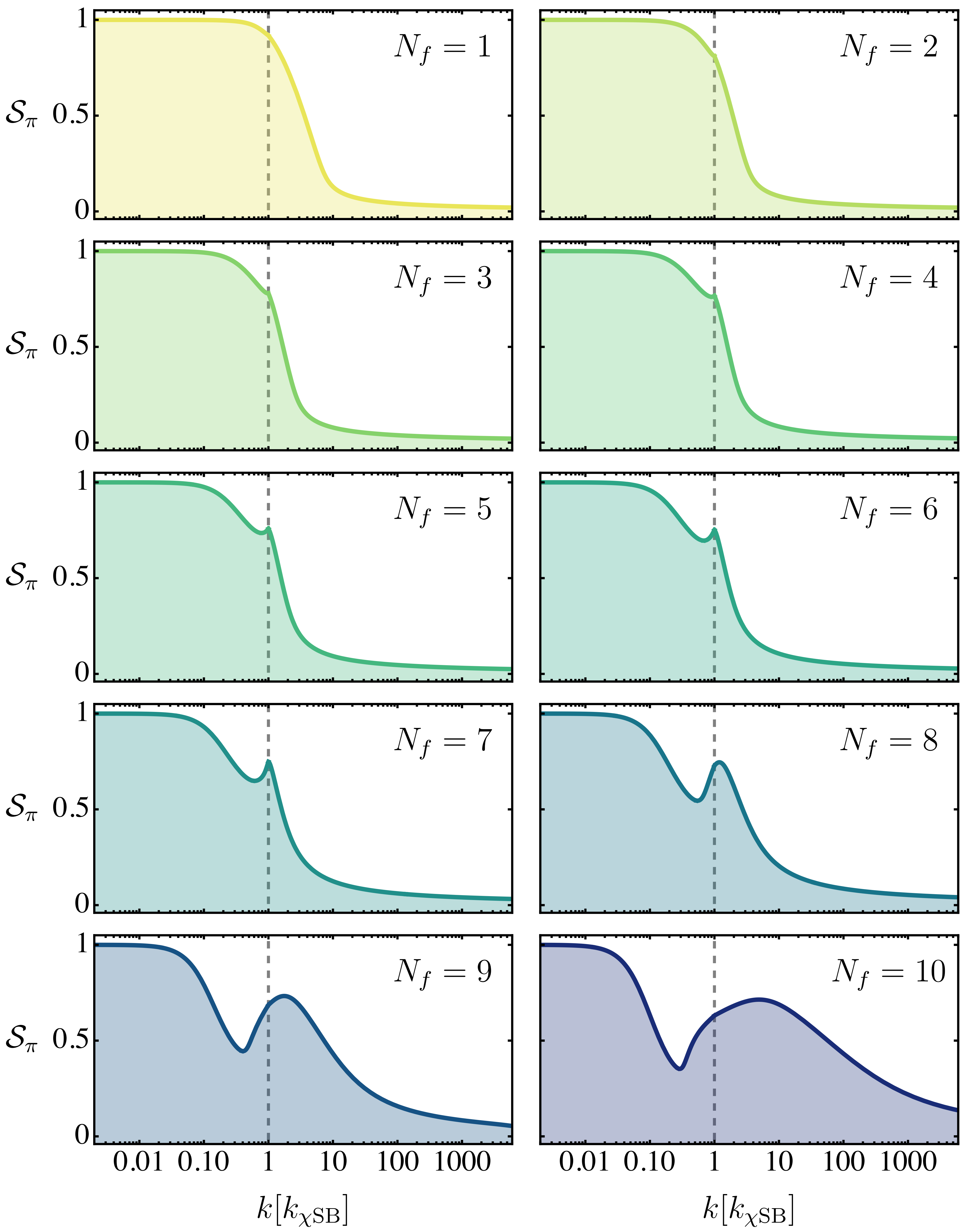


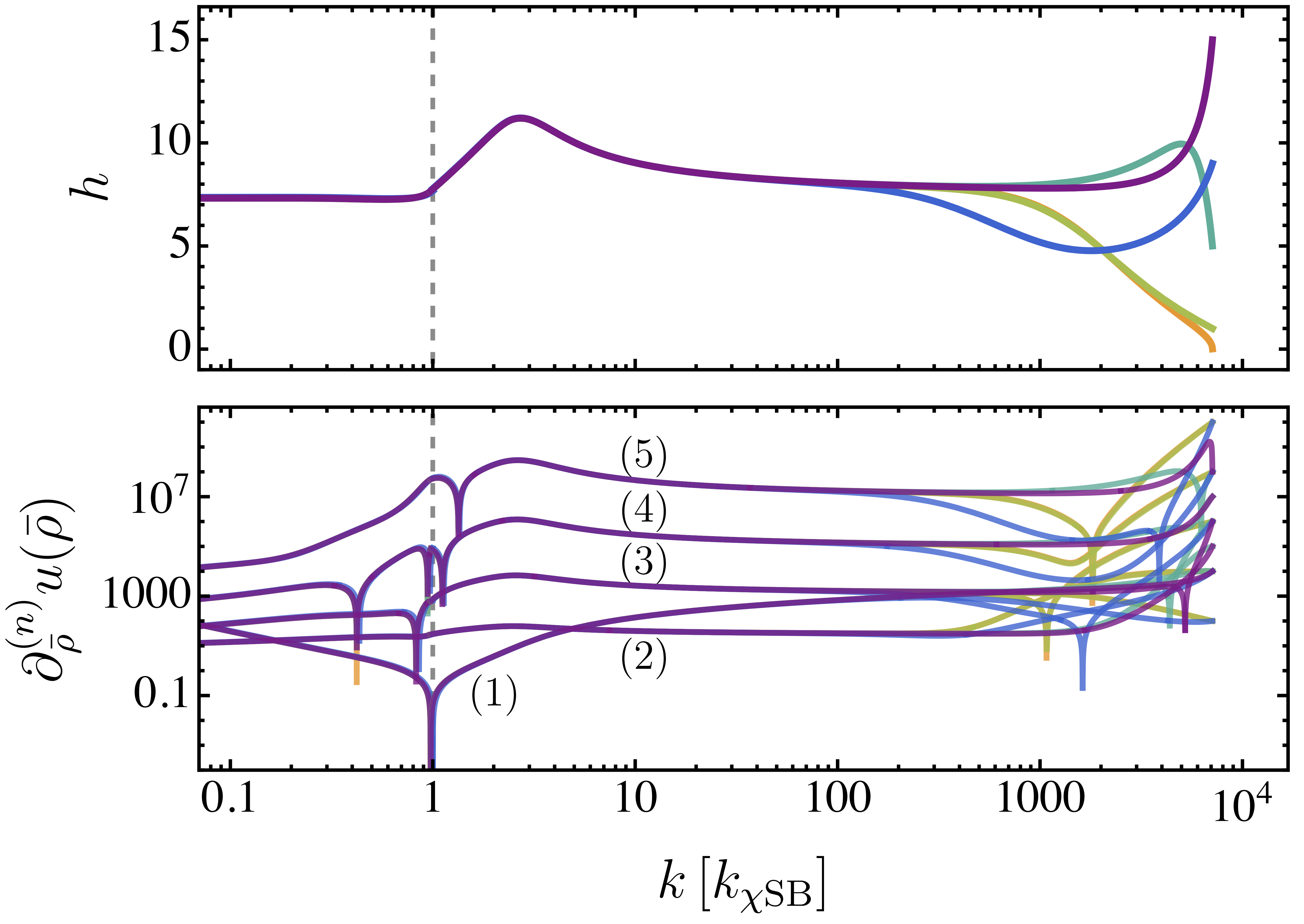
Caswell '74
Banks, Zaks '82



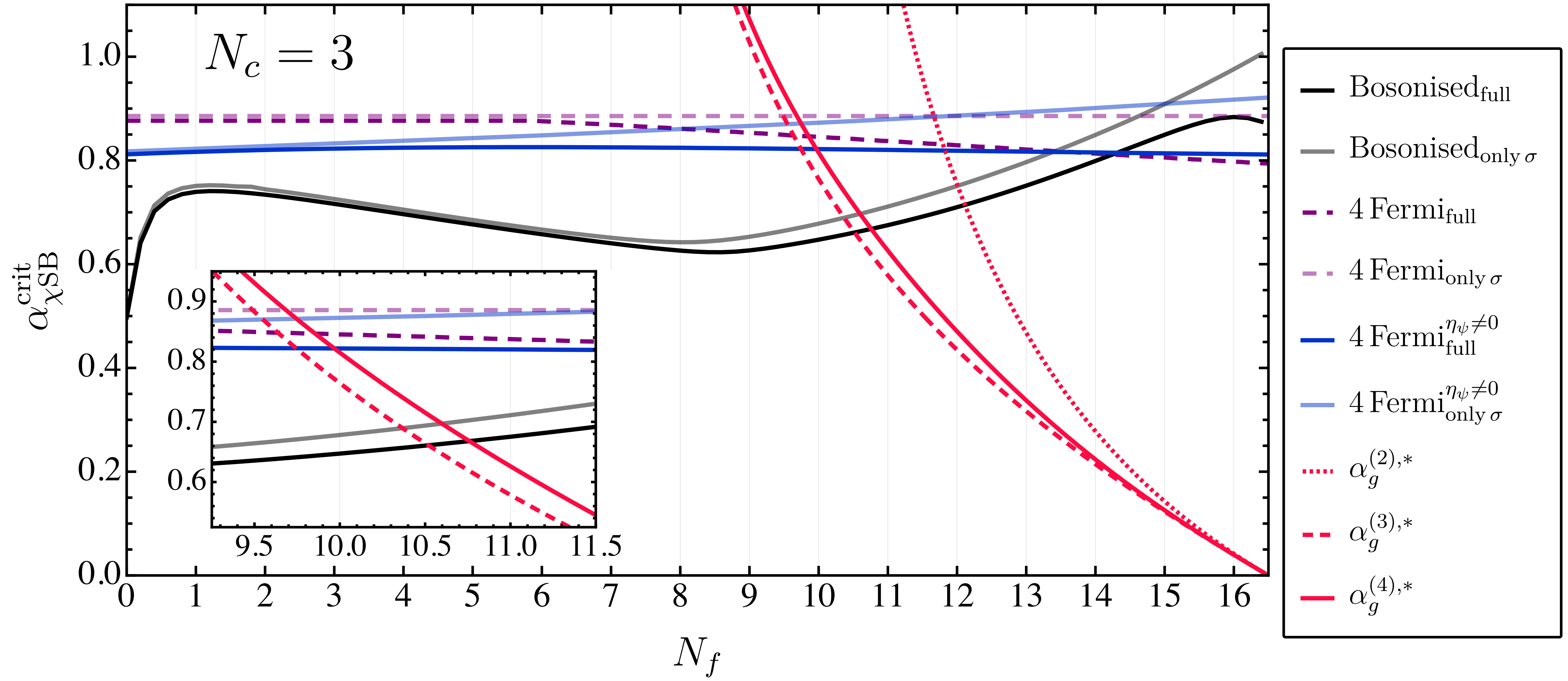
Validity of LEFTS

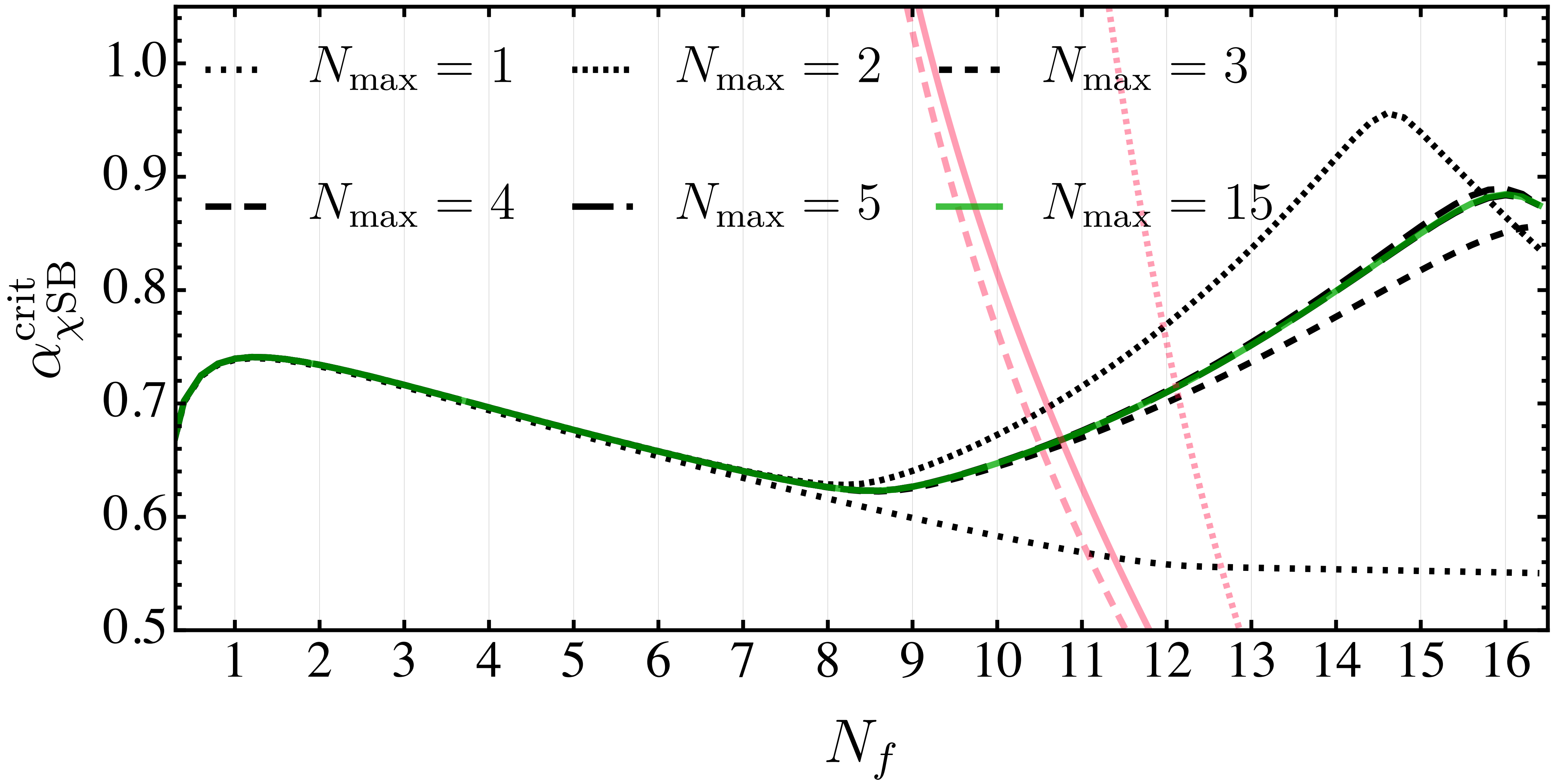


Dynamical bosonisation



Boundary of the conformal window: systematics

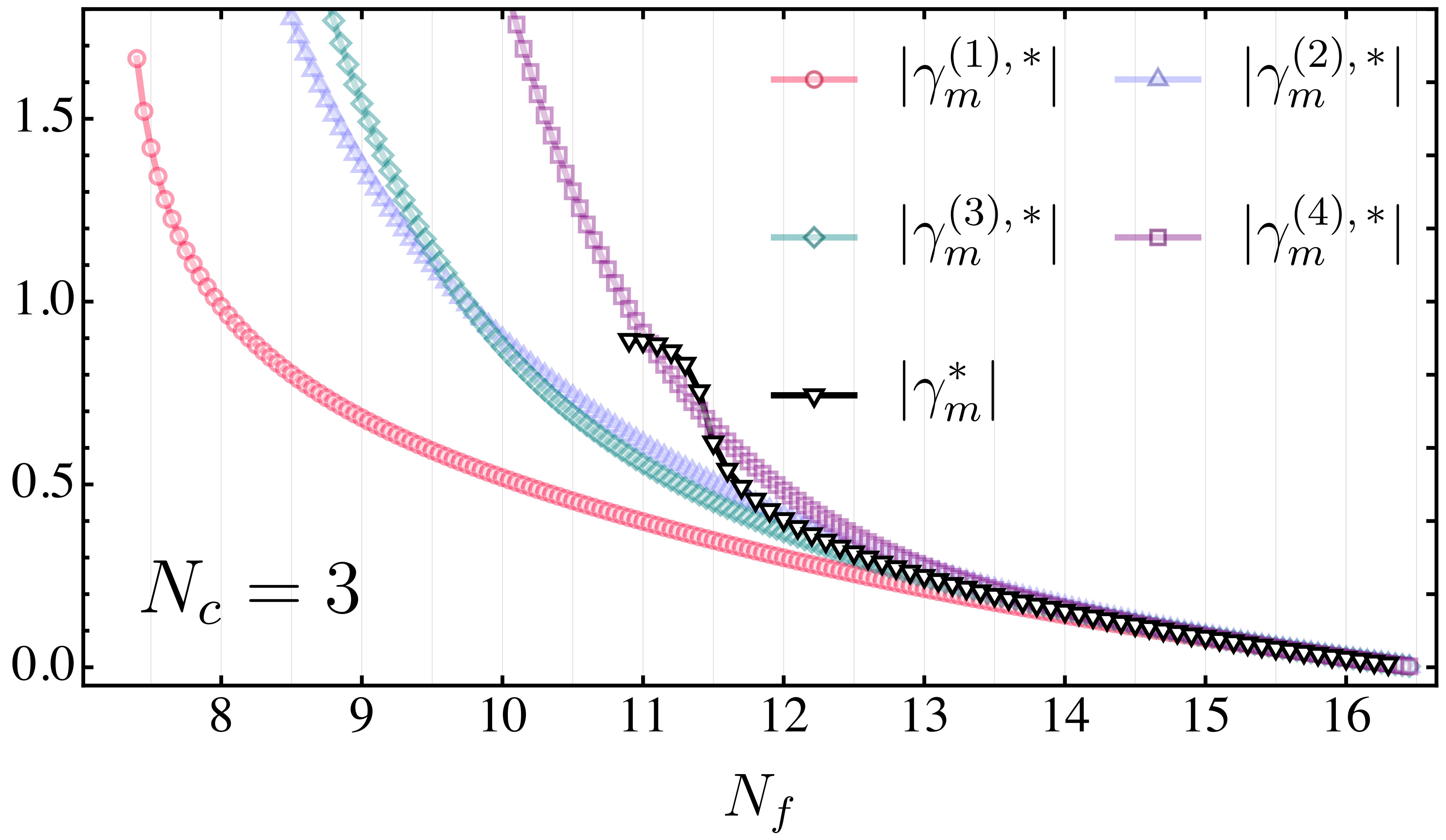


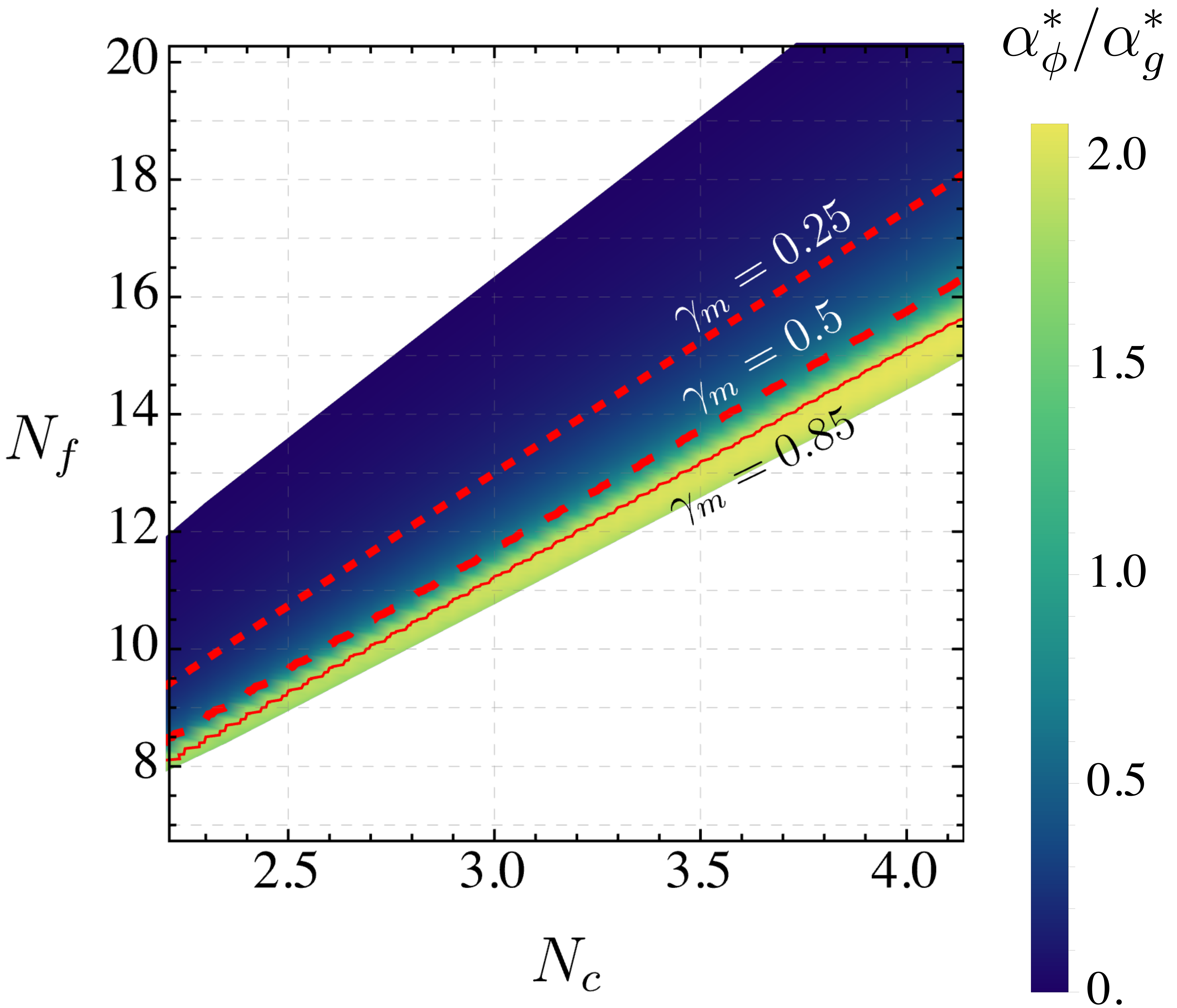


Boundary of the conformal window: γ_m

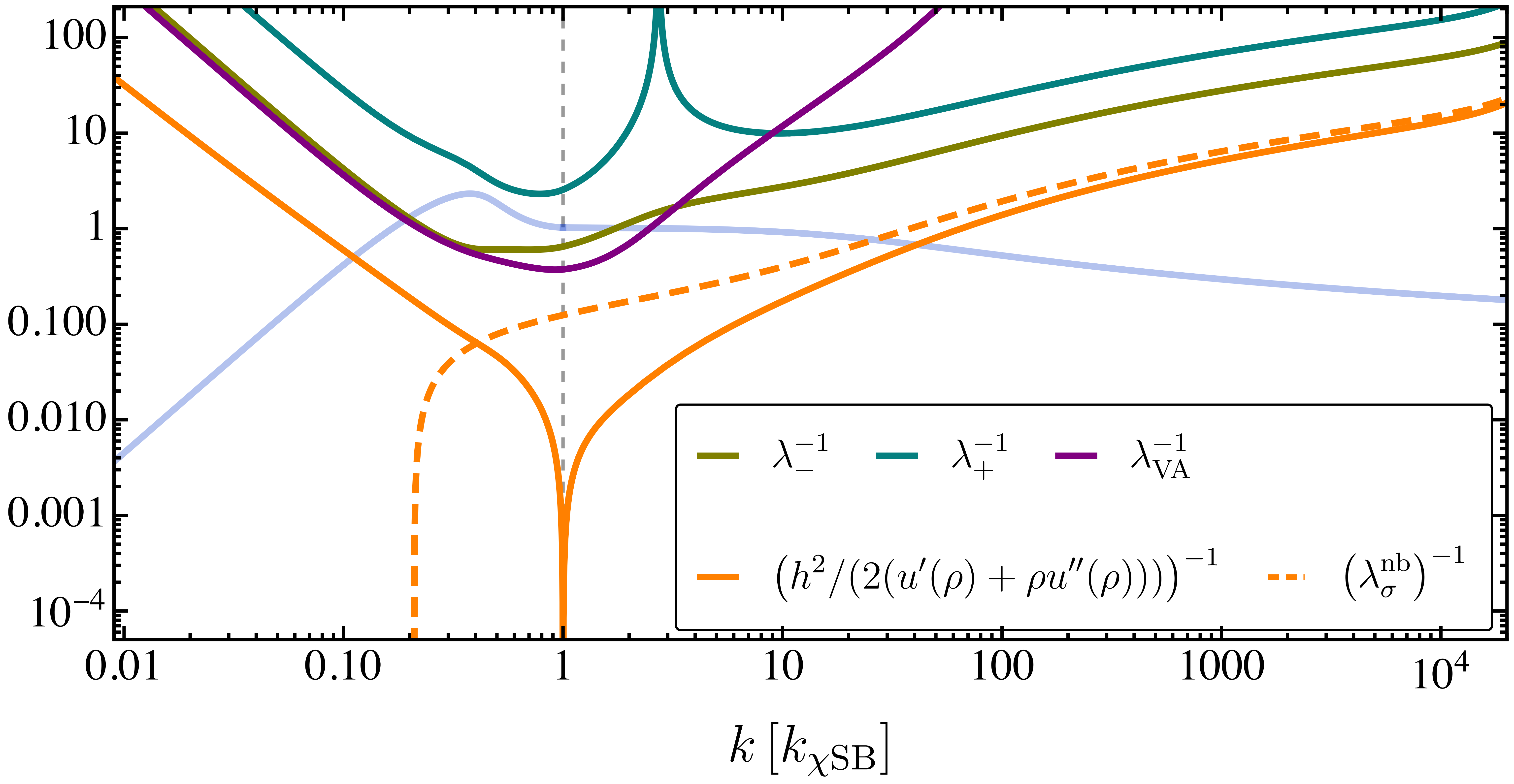
$$\gamma_m = \frac{\partial_t \bar{m}_\psi}{\bar{m}_\psi} = \left[\frac{\partial_t \left(T_f^0 \, \Gamma_k^{(\bar\psi\psi)}(p^2=0) \right)}{Z_\psi m_\psi} + \eta_\psi \right]$$

$$= \gamma_m^{(1)} + \gamma_m^{(>1)} = \frac{3\,\alpha_g\, C_\mathrm{F}}{2\pi} + \ldots$$





Different tensor structures



Dynamical hadronisation and flow over EFTs

✓ Well defined implementation with the fRG

$$\left(\partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c}\right) \Gamma_k[\phi]$$

$$= \frac{1}{2} \mathrm{Tr} \left[\left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left(\partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{mi} \right]$$

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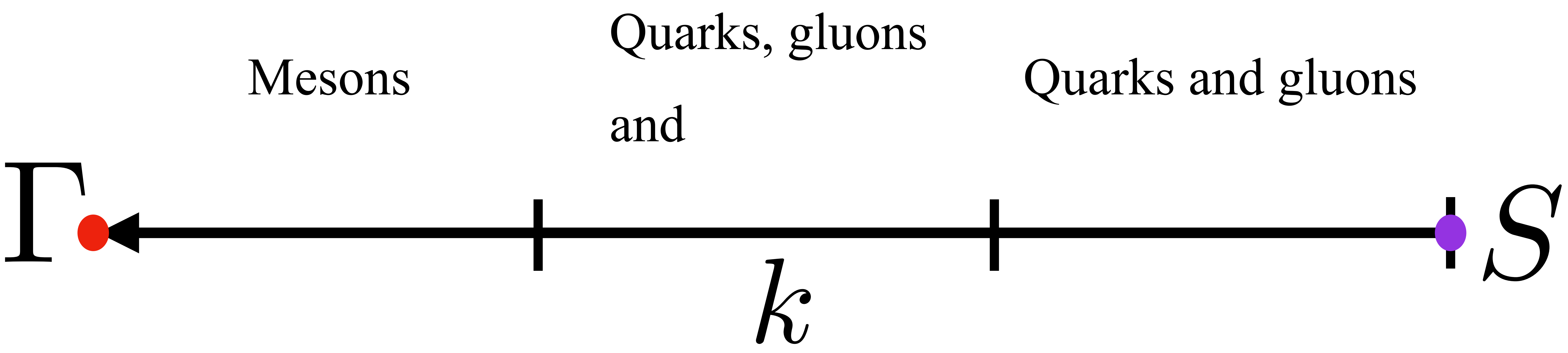

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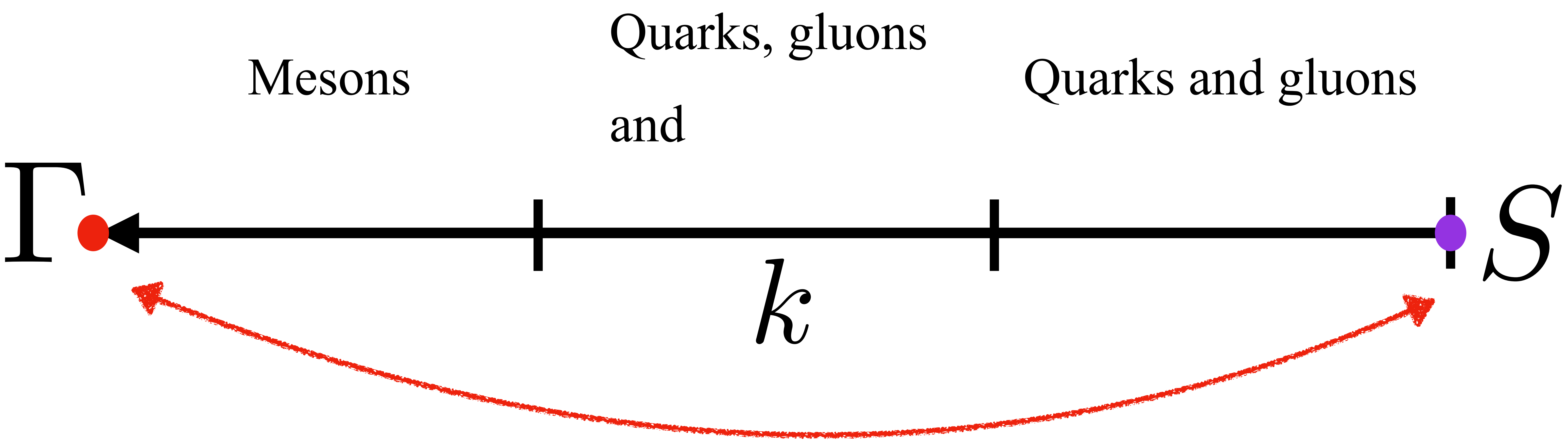

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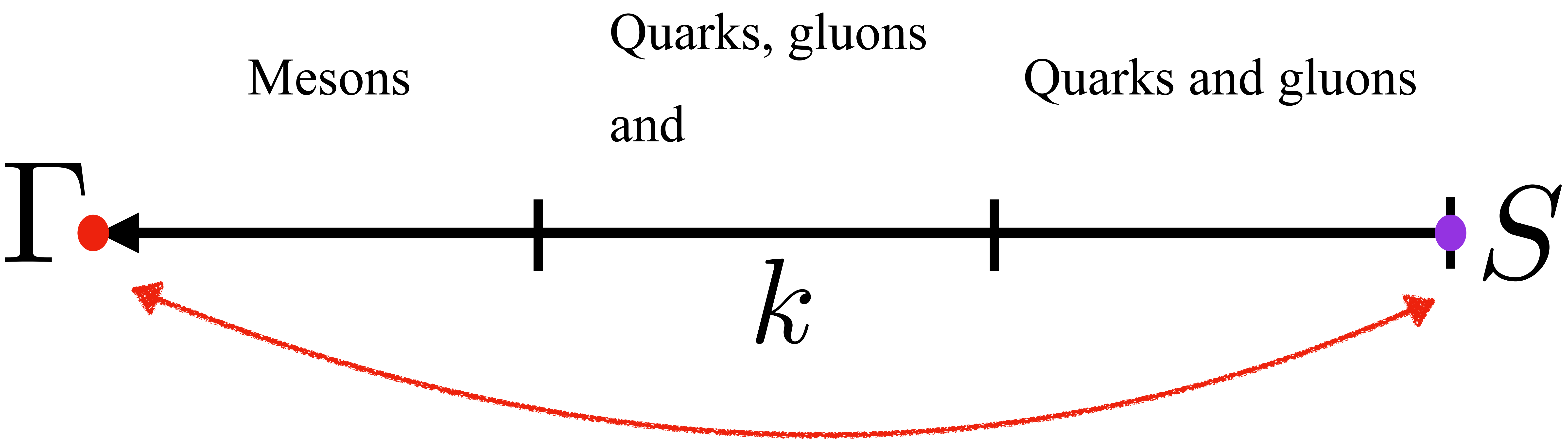


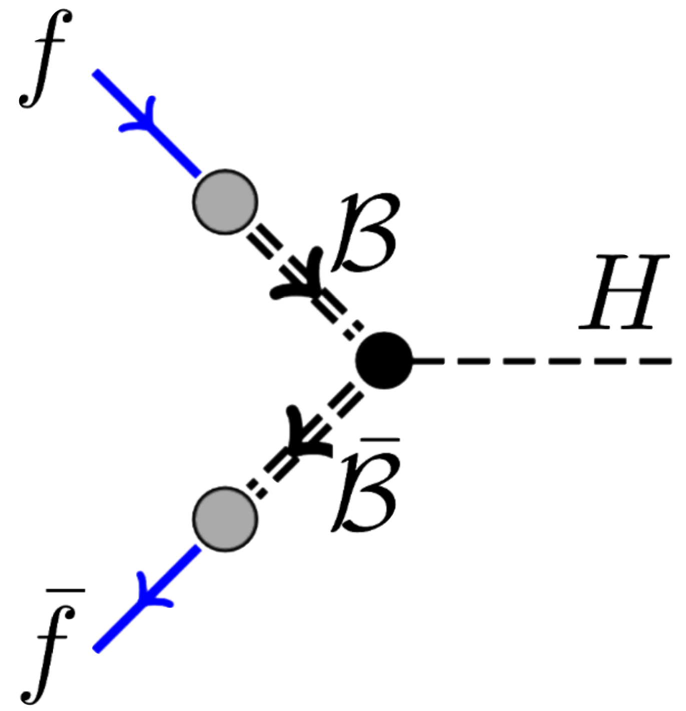
- ✳ Composites at all scales
- ✳ Address theories in a full and complete manner
- ✳ Input parameters in the fundamental theory predict the composite sector

Flavour hierachies from Fundamental Partial Compositeness

Sannino,Strumia,Tesi,Vigiani [1607.01659]
Cacciapaglia,Gertov,Sannino,Thomsen [1704.07845]

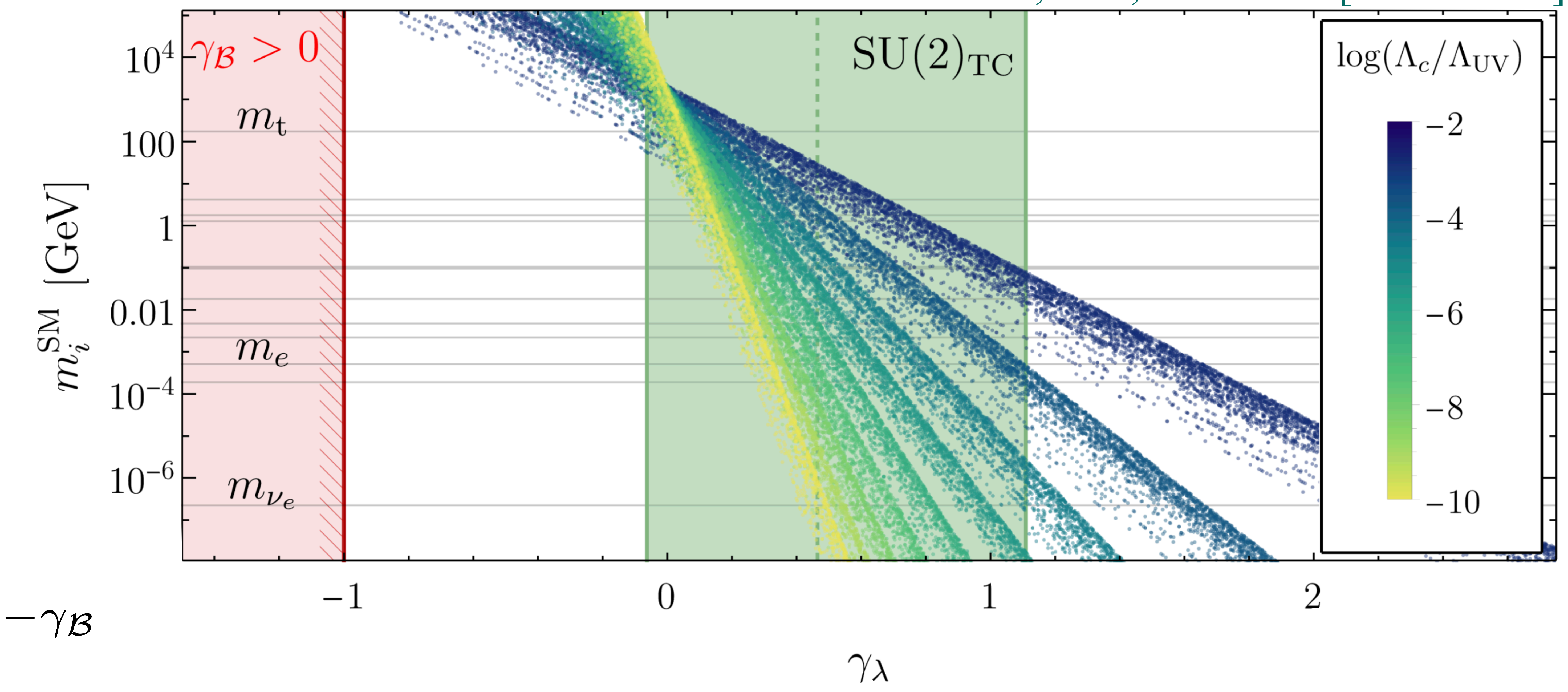
$$\bar{\Gamma}_{\text{mix}} = \int_x \left\{ \lambda_t^L \; \bar{q}_L \mathcal{B}_R^q + \lambda_t^R \; \bar{t}_R \mathcal{B}_L^t + \text{ h.c.} \right\}$$

$$\mathcal{B} \sim \mathcal{F}\mathcal{S} \qquad \underline{\text{MFPC}} \begin{cases} \text{Sp}(24)_{\mathcal{S}} \\ \text{SU}(4)_{\mathcal{F}} \end{cases}$$



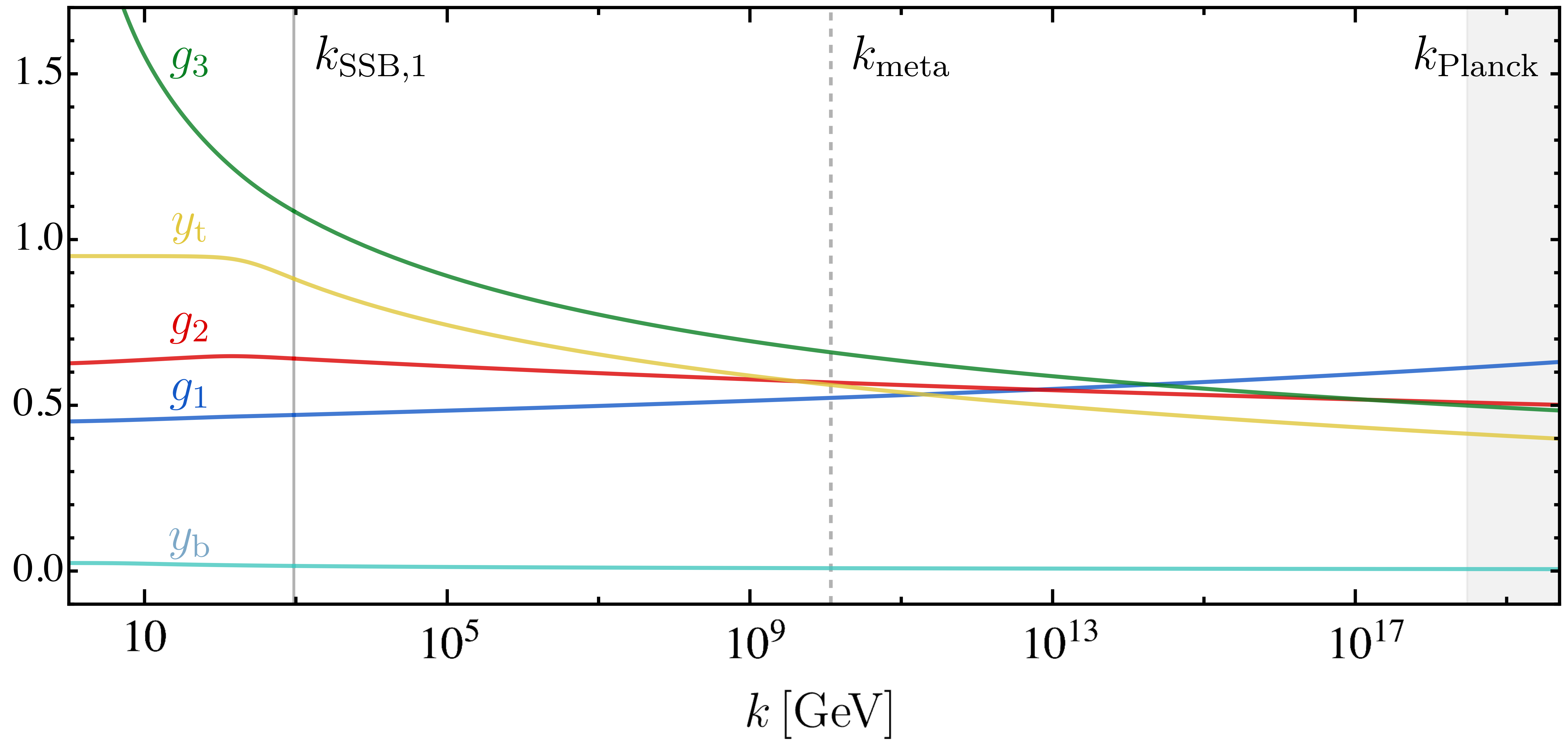
$$m_f \sim \frac{\mathcal{Y}_f \, v}{\sqrt{2}} \left(\frac{\Lambda_c}{\Lambda_{\text{UV}}} \right)^{-2-\gamma_{\mathcal{B}}}$$

Goertz,ÁPG,Pawlowski [2307.11148]



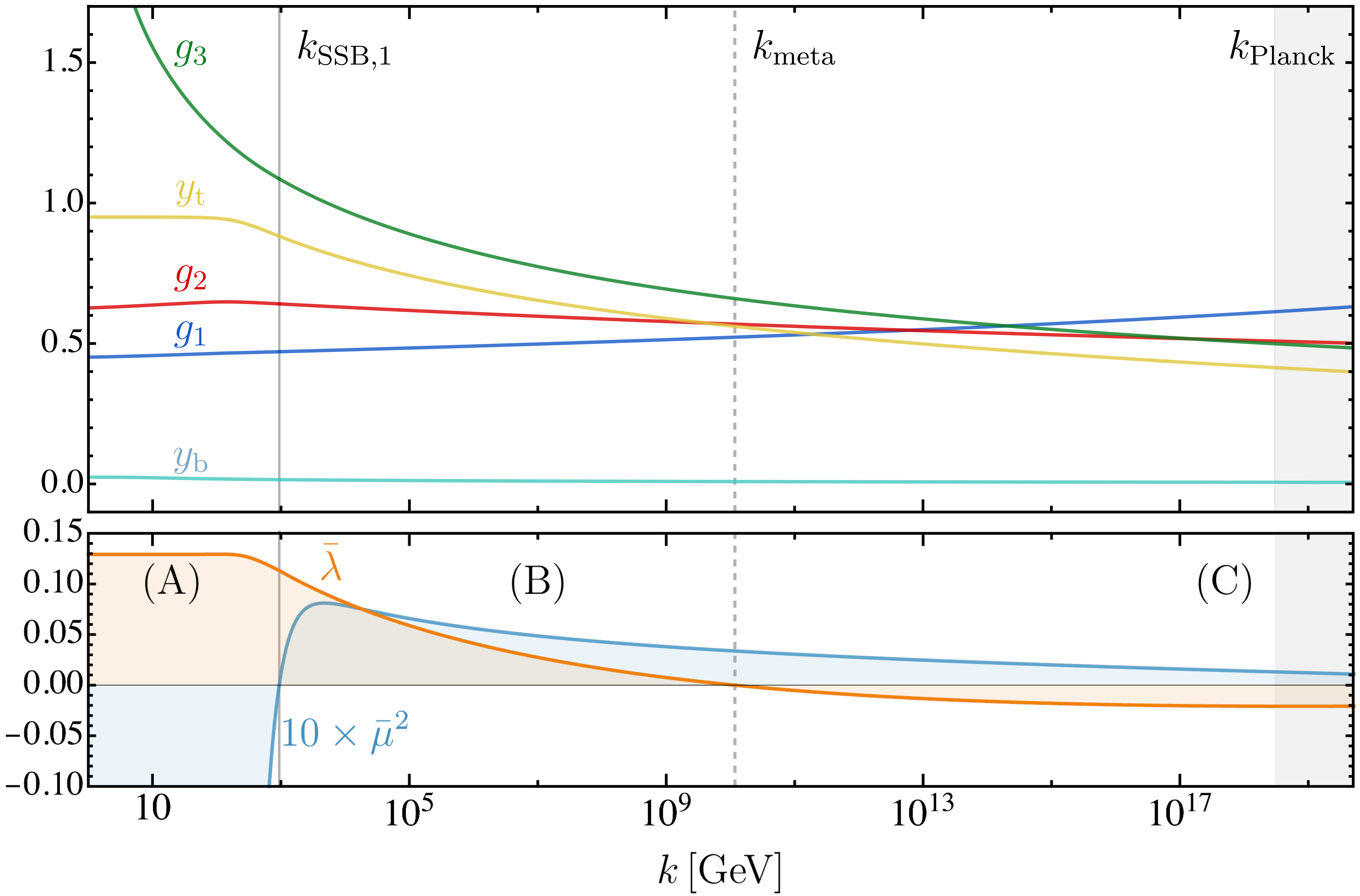
The SM from the fRG

ÁPG,Pawlowski,Reichert [2207.09817] Goertz,ÁPG [2308.13594]



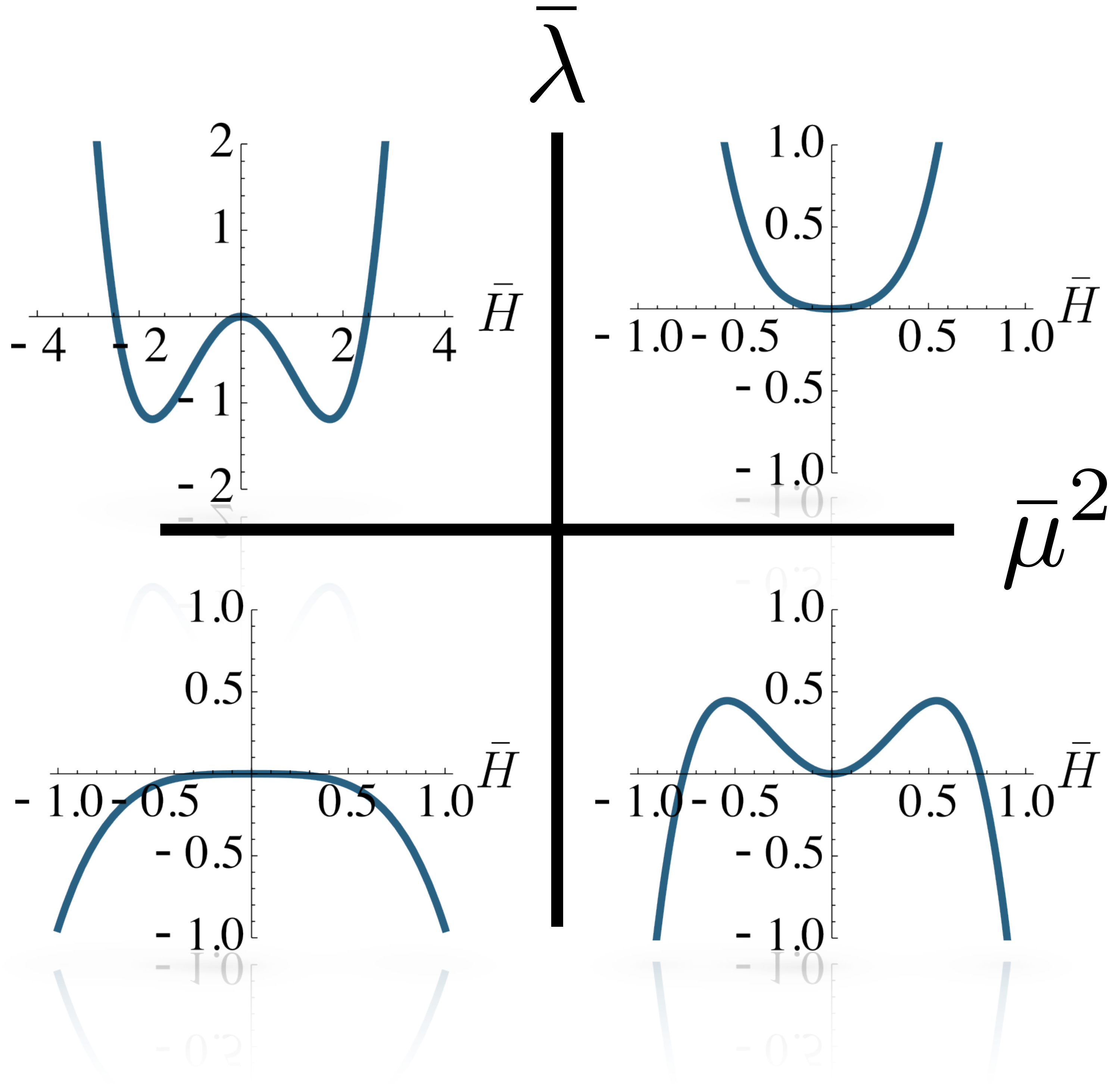
The SM from the fRG

ÁPG,Pawlowski,Reichert [2207.09817]   Goertz,ÁPG [2308.13594]



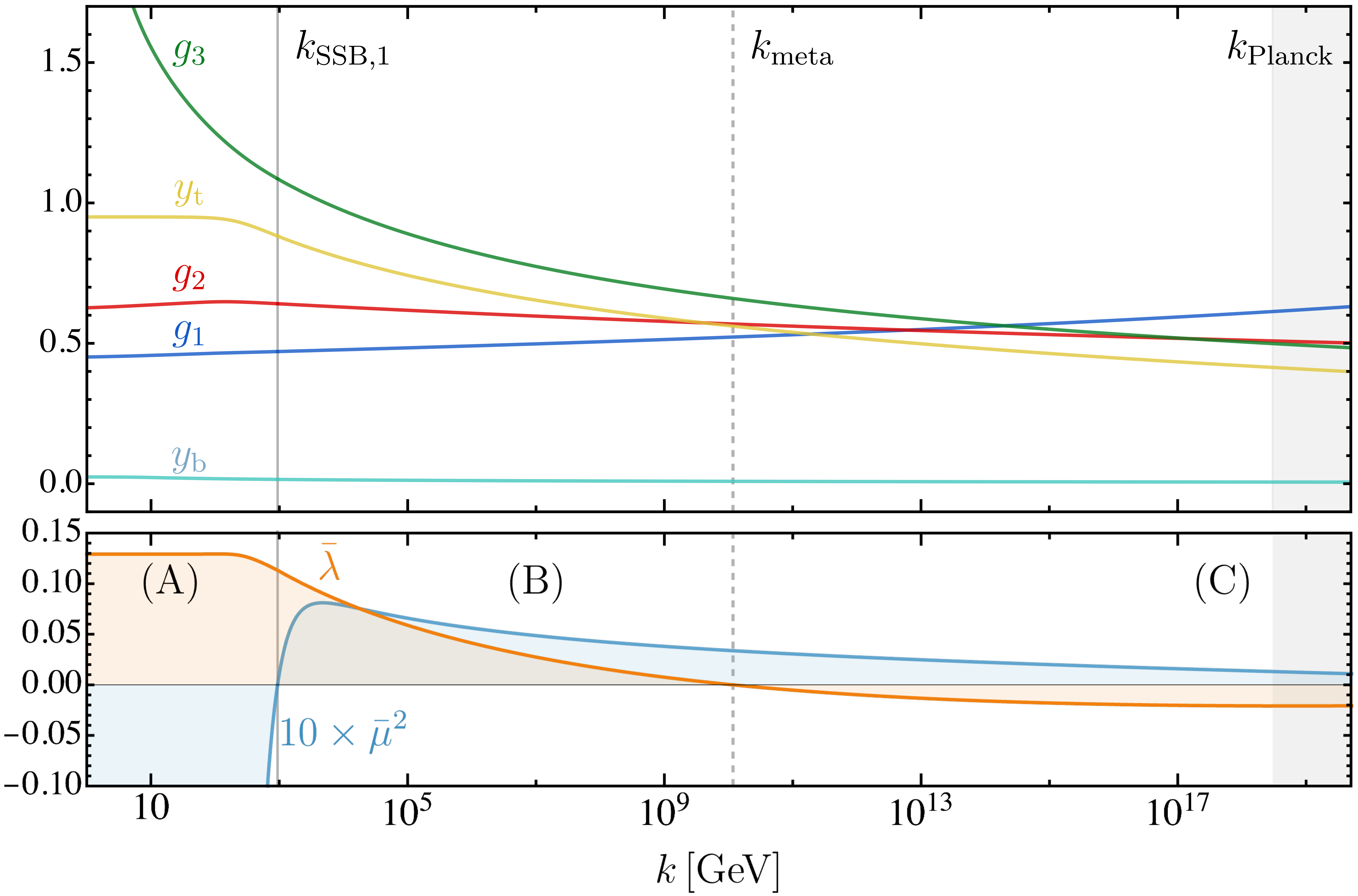
$$u(\bar{\rho}) = V_{\mathrm{eff}}(\rho) \,/\, k^4 = \bar{\mu}^2\bar{\rho} + \bar{\lambda}\bar{\rho}^2$$

$$\bar{\rho} = Z_\Phi \frac{\mathrm{tr}\,\Phi^\dagger\Phi}{k^2} \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ H + i\mathcal{G}_3 \end{pmatrix}$$



The SM from the fRG

ÁPG,Pawlowski,Reichert [2207.09817] Goertz,ÁPG [2308.13594]



ÁPG,Pawlowski,Reichert [2207.09817]