Automorphic Forms and Heterotic Vacua

Nicole Righi

based on work with J. M. Leedom, A. Westphal and A. Kidambi

Strings Breaking SUSY, November 22, 2023

Cosmology & String Vacua

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Can we explain this from string theory?

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→ No-go results for dS in heterotic string theory

[Maldacena, Nunez '00]	[Green, Martinec, Quigley, Sethi '11]	[Gautason, Junghans, Zagermann '12]	[Kutasov, Maxfield, Melnikov, Sethi '15]	[Quigley '15]	[Gonzalo, Ibanez, Uranga '18]
Classical SUGRA?	Leading α'?	Infinite α' tower?	Nonperturbative α'?	Nonperturbative g_s, Gaugino Condensation?	Instantons,Condesates, Threshold Corrections*?
no dS	no dS	no dS	no dS	no dS	no dS (numerically)
AdS ok	AdS ok	no AdS	AdS ok	no AdS	AdS ok

Overview

- review: 4d, N=1 toroidal orbifold compactification of the heterotic string

- prove no-go theorems for dS minima

- provide a way to evade those theorems and potentially construct dS minima

[Leedom, NR, Westphal '22]

- extension to higher genera [Kidambi, Leedom, NR, Westphal *WiP*]

Two moduli: *ST* model

Kahler modulus $T = a + it$

Dilaton $S = 1/g_s^2 + i\theta$

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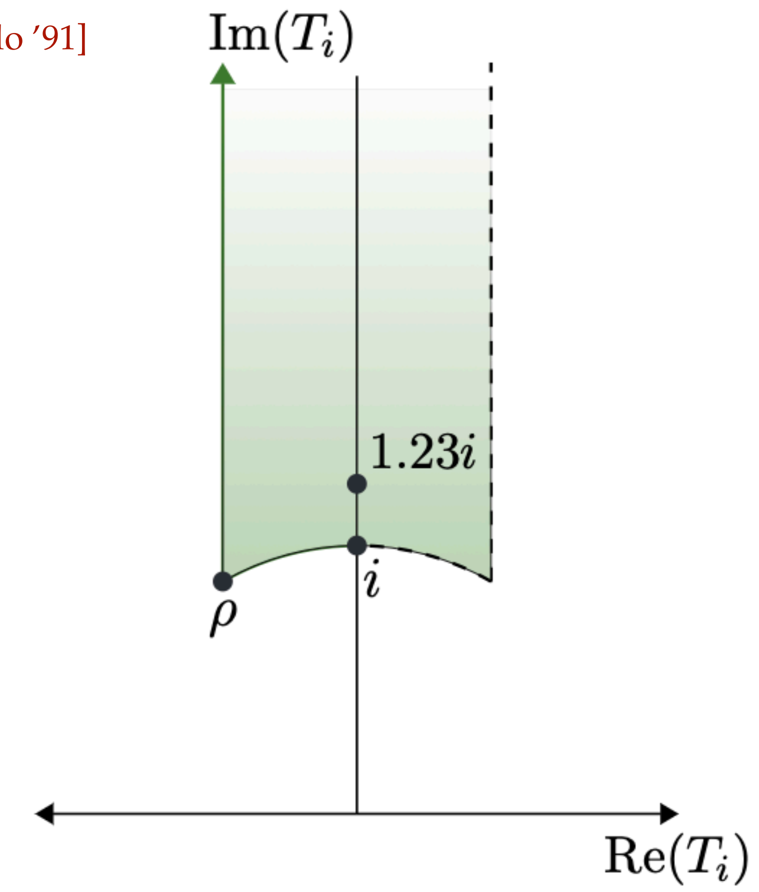

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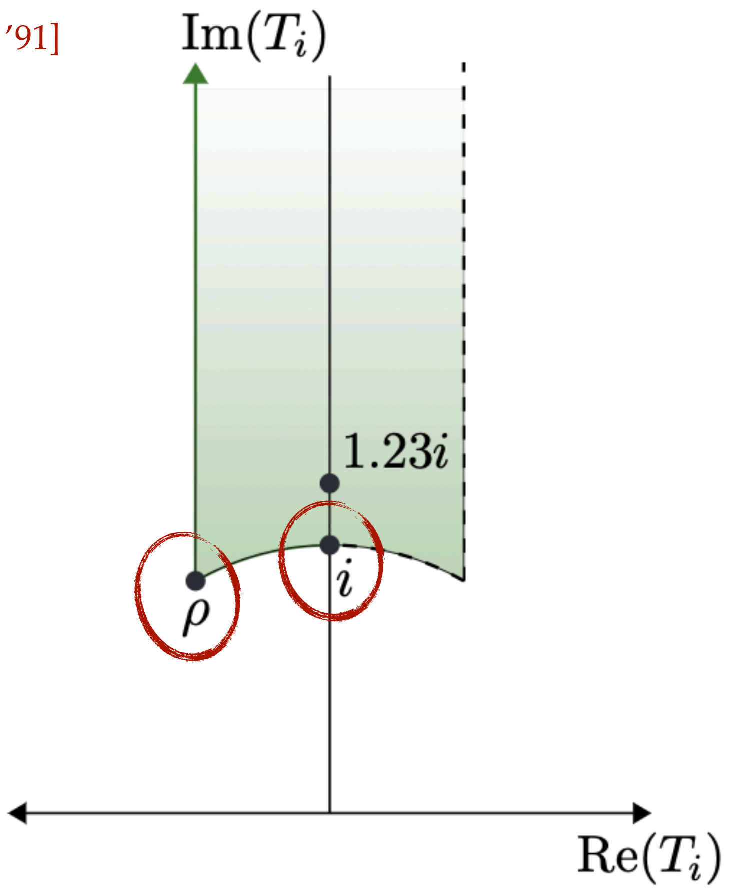

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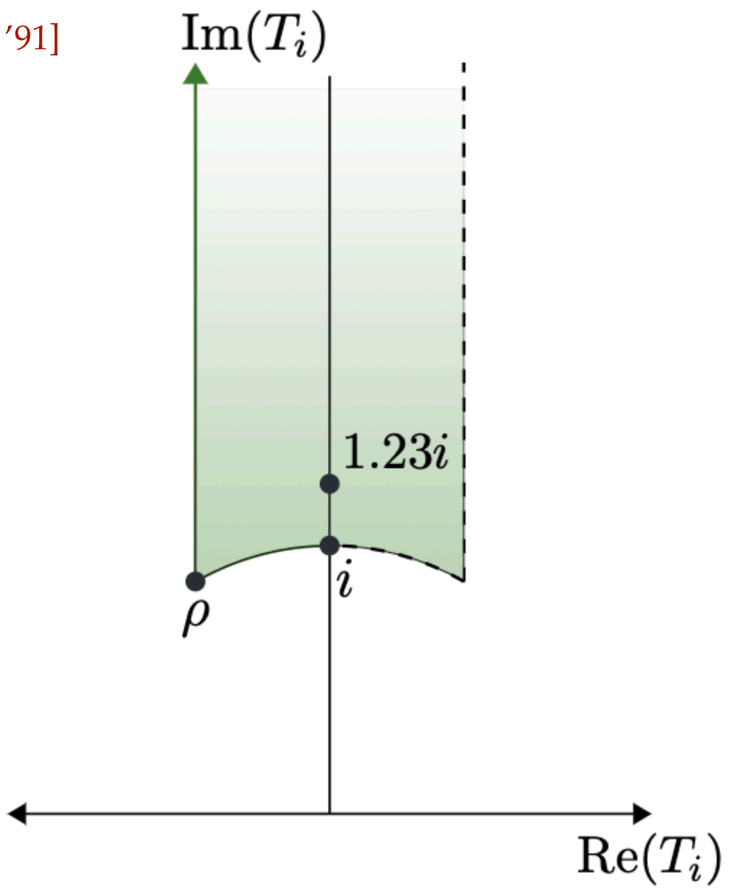

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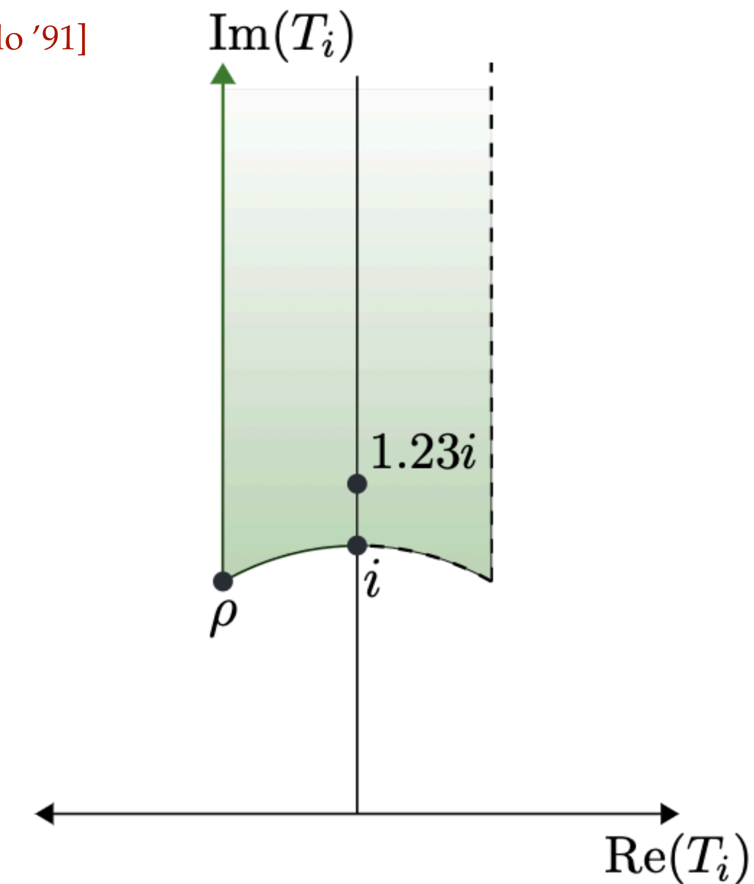
- Superpotential $W = \dfrac{\Omega(S)H(T)}{\eta^6(T)}$ $\quad \Omega(S) \sim e^{-S}$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$$

[Rademacher, Zuckerman '38]



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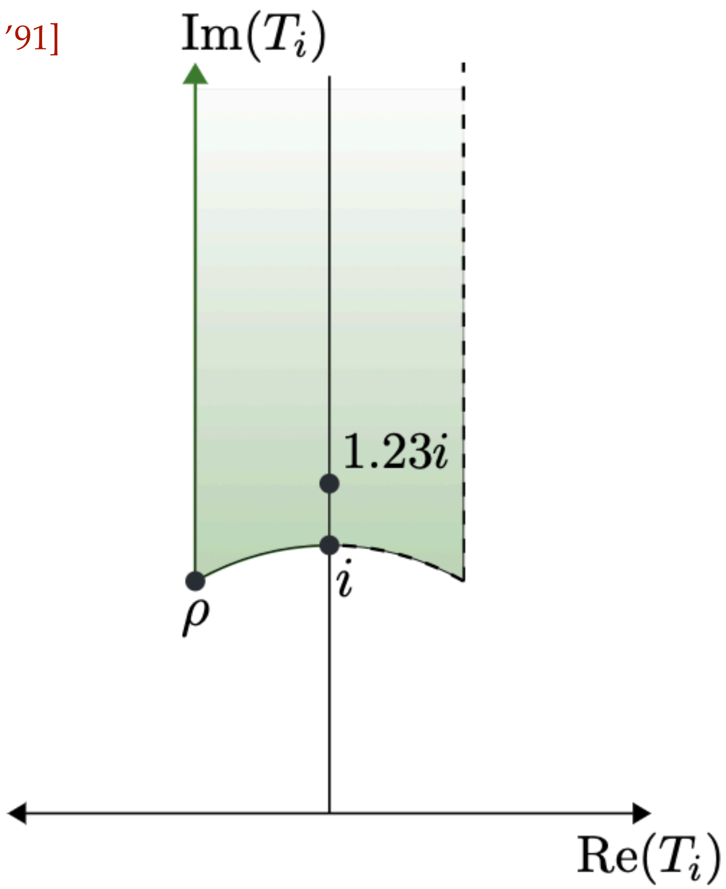

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$$V(T,S) = e^{k(S,\bar{S})} Z(T,\bar{T})\, |\Omega(S)|^2 \left[\left(A(S,\bar{S}) - 3\right)|H(T)|^2 + \widehat{V}(T,\bar{T})\right]$$

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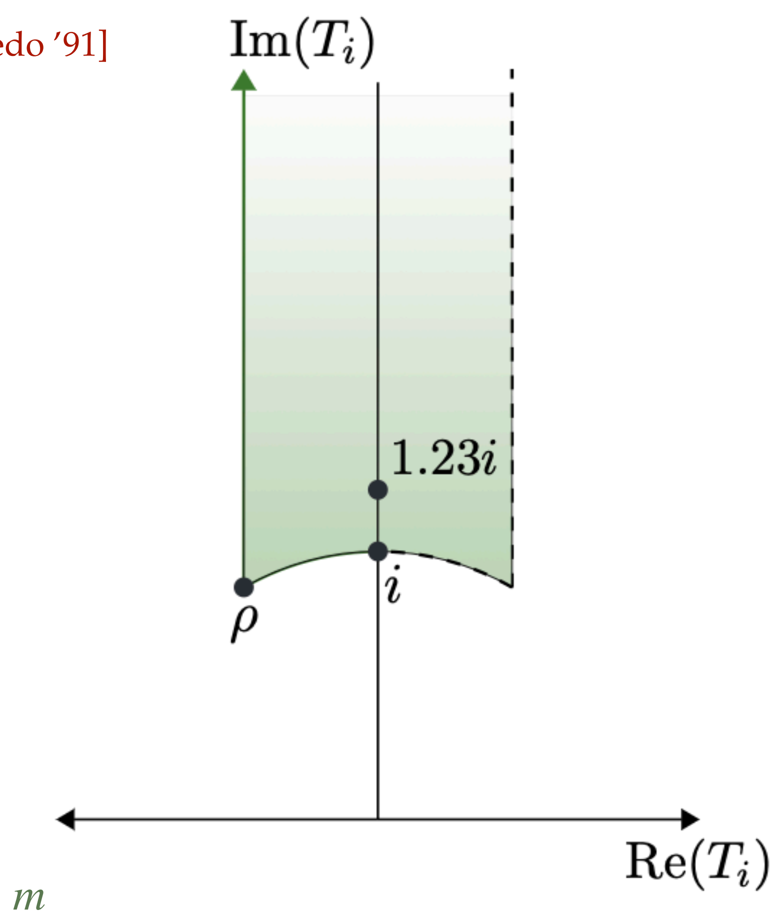

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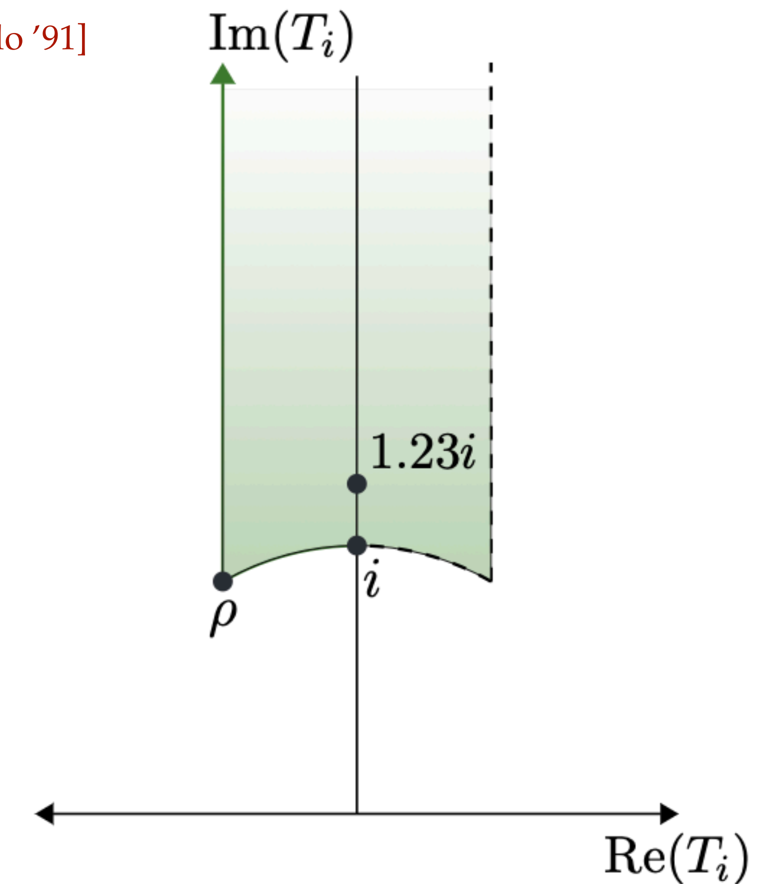

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F-terms $F_S = \dfrac{H(T)}{\eta^6(T)}\left(\Omega_S + k_s \Omega(S)\right)$

New dS no-go theorem

$$V(T,S) = e^{k(S,\bar{S})} Z(T,\bar{T})\, |\Omega(S)|^2 \left[\left(A(S,\bar{S}) - 3 \right) |H(T)|^2 + \widehat{V}(T,\bar{T}) \right]$$

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- $F_S = 0$
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Two classes of minima:

- $F_S = 0$ $\longrightarrow$ **Theorem 1.** no dS minima in SUSY preserving setups
- $F_S \neq 0$ $\longrightarrow$ *1.* how to break SUSY?

  2. when are $T = i, \rho$ **dS** minima?

Modular landscape of heterotic vacua

- Self dual points: $W = \dfrac{\Omega(S)H(T)}{\eta^6(T)}$ where $H(T) = \left(\dfrac{G_4(T)}{\eta^8(T)}\right)^n \left(\dfrac{G_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$

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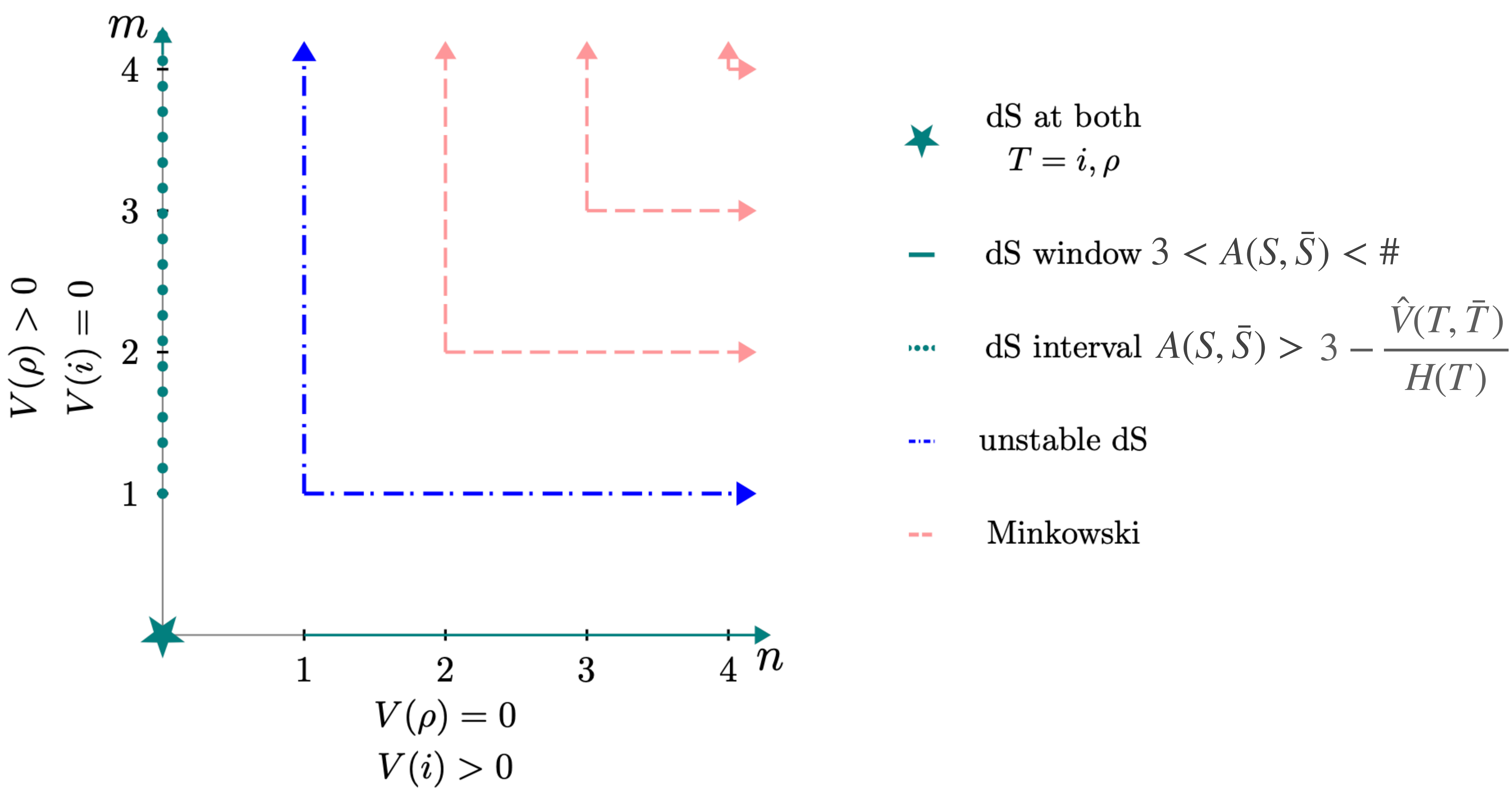
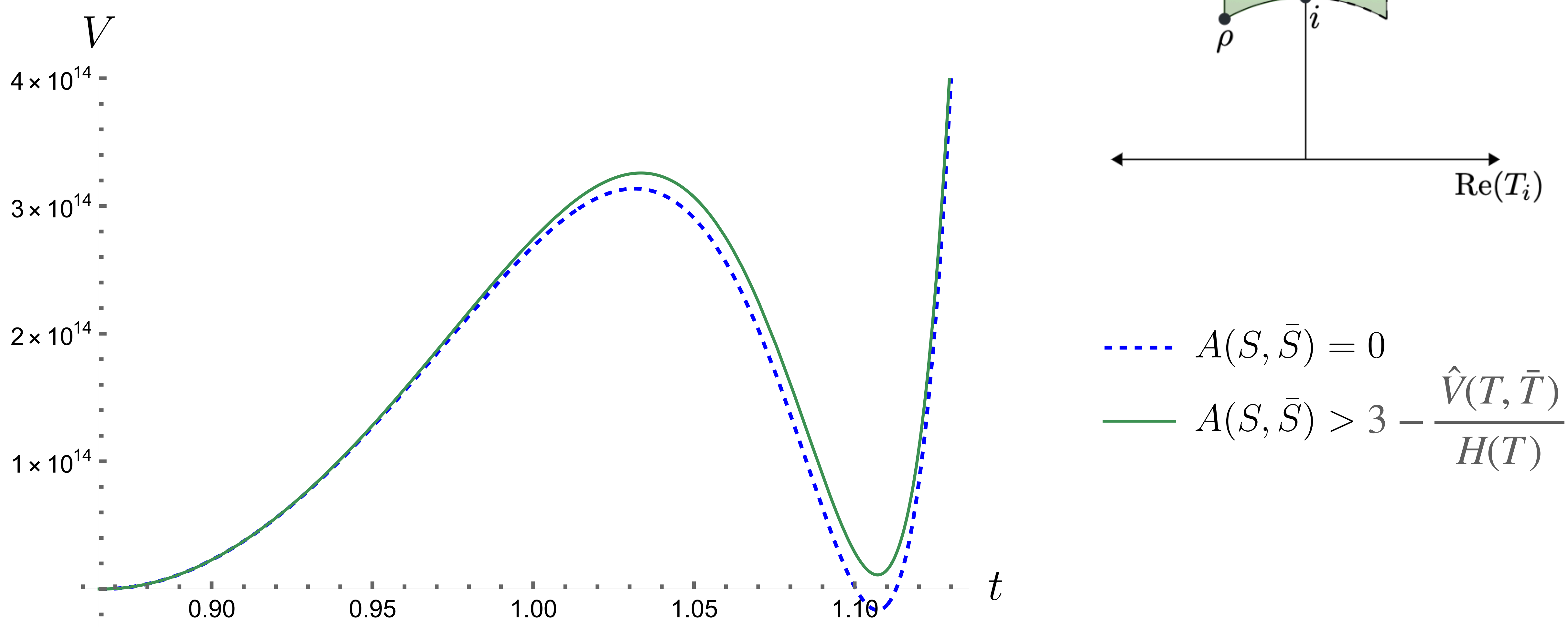
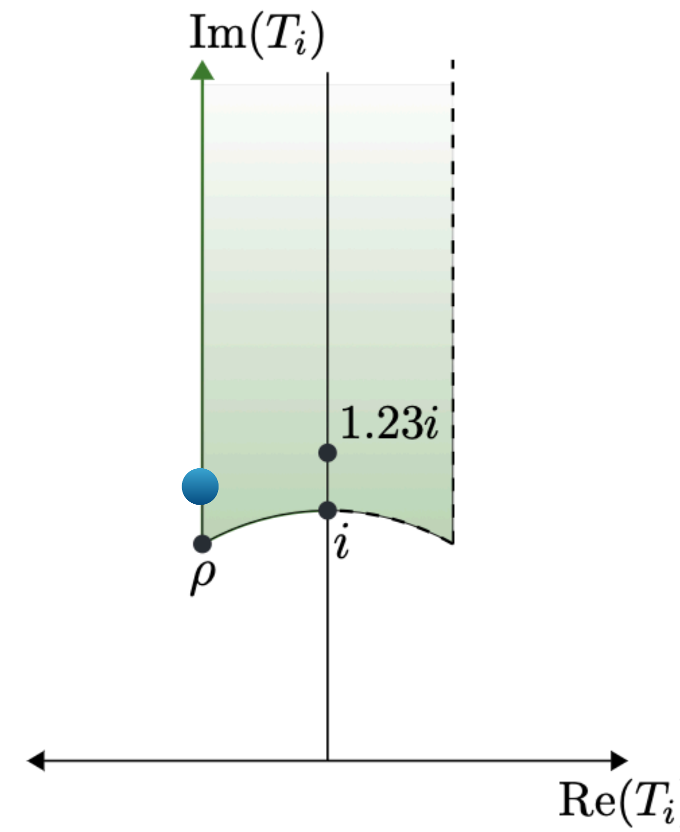


Modular landscape of heterotic vacua

- Into the fundamental domain: an example





Stringy instantons and de Sitter

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Theorem 2. no dS minima for $F_S \neq 0,\, F_T = 0$ and $K(S,\bar{S}) = -\ln(S+\bar{S})$

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[Shenker '90]

$\longrightarrow$ Quite odd in heterotic — no D-branes!

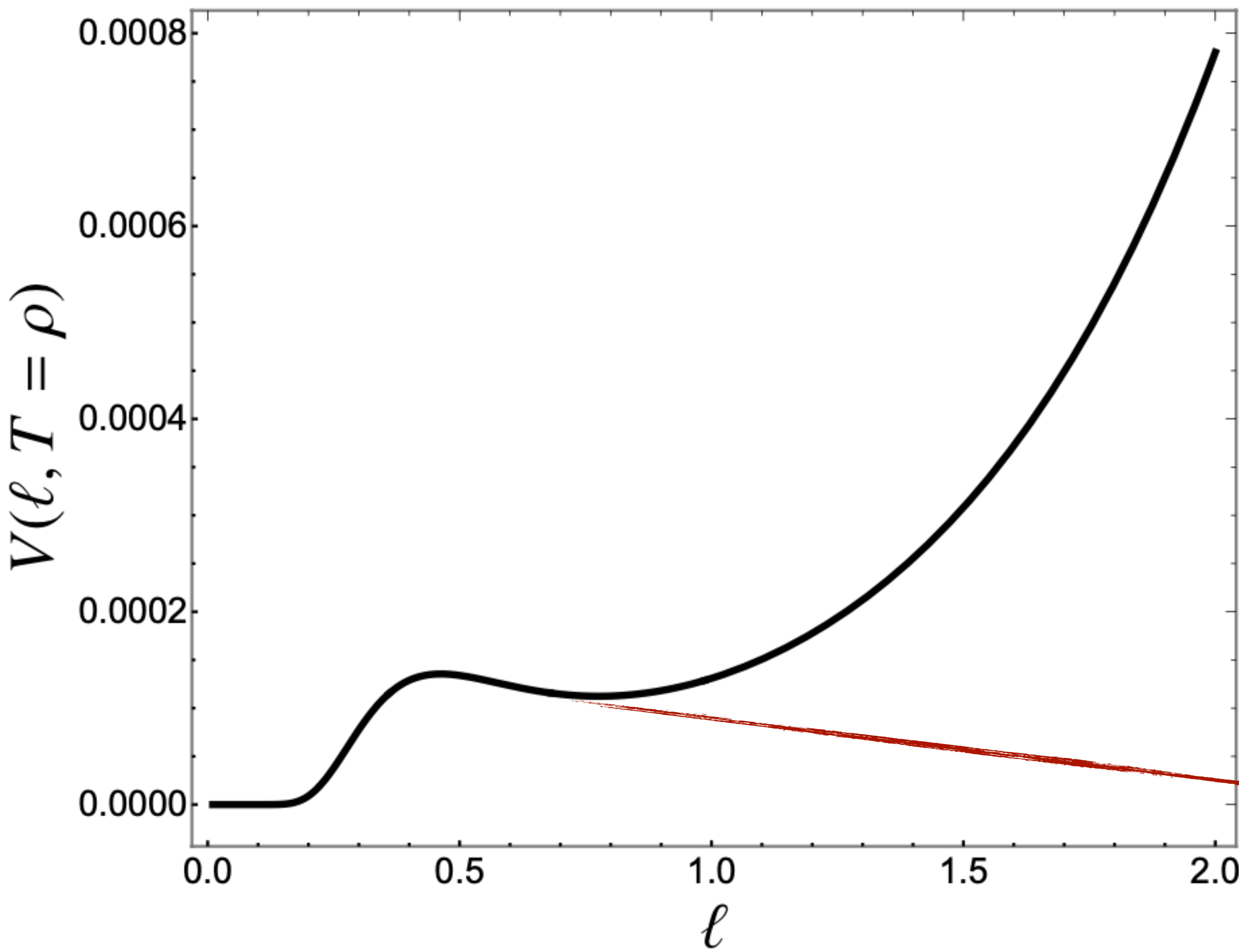
Dig in the literature: very few studies

A working example

Scalar potential in the linear multiplet formalism: $\frac{g_s^2}{2} = \left\langle \frac{\ell}{1+f(\ell)} \right\rangle$

Parametrise Shenker term: $f(\ell) = \sum_{n=0} A_n \ell^{-q_n} e^{-B/\sqrt{\ell}}$

[Gaillard, Nelson '07]



metastable dS vacuum

$g_s \simeq 0.7$

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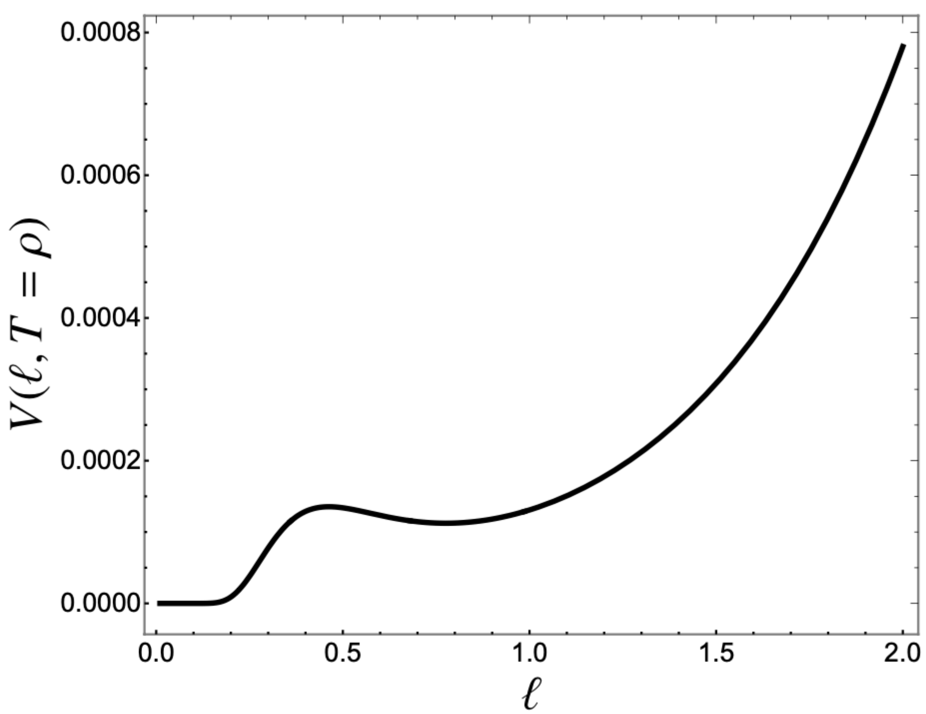

metastable dS vacuum

↓

? what generates these corrections ?

[WiP]

Summary

✓ prove no-go theorems for dS minima

✓ provide a way to evade those theorems and potentially construct dS minima

[Leedom, NR, Westphal '22]

→ extension to higher genera [Kidambi, Leedom, NR, Westphal *WiP*]

Genus 2: $Sp(4,\mathbb{Z})$

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Why considering such enhancement?

$$T = B_{12} + i\sqrt{\det(G)}$$

$$U = \frac{1}{G_{11}}\left(G_{12} + i\sqrt{\det(G)}\right)$$

$$\Downarrow$$

$$SL(2,\mathbb{Z})_T \times SL(2,\mathbb{Z})_U \text{ + mirror symm.}$$

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$$T = B_{12} + i\sqrt{\det(G)} + a_1(-a_2 + Ua_1)$$

$$U = \frac{1}{G_{11}}\left(G_{12} + i\sqrt{\det(G)}\right)$$

$$Z = -a_2 + Ua_1 \qquad \text{Wilson line}$$

[Lopes Cardoso, Lüst, Mohaupt '94]

$$\Downarrow$$

$$Sp(4,\mathbb{Z})$$

$Sp(4,\mathbb{Z})$: Kahler potential

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$$M = \begin{pmatrix} T & Z \\ Z & U \end{pmatrix}$$

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- Kahler potential

$$K_{(2)} = -\ln\left[\det(M - M^\dagger)\right] = -\ln\left[(T - T^*)(U - U^*) - (Z - Z^*)^2\right]$$

[Lopes Cardoso, Lüst, Mohaupt '94]
[Ferrara, Kounnas, Lüst, Zwirner '91]

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under $Sp(4,\mathbb{Z})$:

$$K_{(2)} \to K_{(2)} + \ln\left[\det(CM^\dagger + D)\right] + \ln\left[\det(CM + D)\right]$$

$Sp(4,\mathbb{Z})$: superpotential

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- the superpotential is more involved!

$$K_{(2)} \to K_{(2)} + \ln\left[\det(CM^\dagger + D)\right] + \ln\left[\det(CM + D)\right]$$

require **invariance** for V:

defining $G \equiv K_{(2)} + \ln|W_{(2)}|^2 \;\Rightarrow\; V = e^G\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right)$

must be a modular function

$\Downarrow$

G must be invariant

$\Downarrow$

$$W_{(2)} \to \det(CM + D)^{-1} W_{(2)}$$

We have a prediction on the form!

$Sp(4,\mathbb{Z})$: threshold corrections

- $W_{(2)} \to \det(CM + D)^{-1} W_{(2)}$ is the prediction, we have to find the physics!

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$$\Rightarrow \quad W_{(2)} \sim \frac{\Omega(S)}{e^{\Delta_a(M)}}$$

threshold corrections $\quad \Delta_a \sim b_a \ln\left(|\chi_{12}(M)|^2\right)$

[Mayr, Stieberger '95]

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- what about non-perturbative effects in the moduli?

flashback to $SL(2,\mathbb{Z})$: ring of $SL(2,\mathbb{Z})$: G_4, G_6, $\Xi_{12} \equiv \eta^{24}$

$$W = \frac{\Omega(S)H(T)}{\eta^2(T)}$$

$$\Omega(S) \sim e^{-S}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$$

$\hookrightarrow$ $H(T)$ is the **most general modular function** on $SL(2,\mathbb{Z})$

[Rademacher, Zuckerman '38]

$Sp(4,\mathbb{Z})$: threshold corrections

- $W_{(2)} \to \det(CM+D)^{-1} W_{(2)}$ is the prediction, we have to find the physics!

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threshold corrections $\quad \Delta_a \sim b_a \ln\left(|\chi_{12}(M)|^2\right)$ [Mayr, Stieberger '95]

- what about non-perturbative effects in the moduli?

build the **most general modular function** using the ring of $Sp(4,\mathbb{Z})$: $\mathscr{E}_4, \mathscr{E}_6, \chi_{10}, \chi_{12}, \chi_{35}$

$$H_{(2)}(M) = \prod_{i=1}^{6} \mathscr{G}_i(M)\mathscr{P}(M)$$

$$\mathscr{G}_i(M) = \left(\frac{\mathscr{E}_4\mathscr{E}_6}{\mathscr{E}_{10}} - \left.\frac{\mathscr{E}_4\mathscr{E}_6}{\mathscr{E}_{10}}\right|_{\sigma_i}\right)^m \left(\frac{\mathscr{E}_6^2}{\mathscr{E}_{12}} - \left.\frac{\mathscr{E}_6^2}{\mathscr{E}_{12}}\right|_{\sigma_i}\right)^n \left(\frac{\mathscr{E}_4^5}{\mathscr{E}_{10}^2}\frac{\mathscr{E}_6^2}{\mathscr{E}_{12}} - \left.\frac{\mathscr{E}_4^5}{\mathscr{E}_{10}^2}\frac{\mathscr{E}_6^2}{\mathscr{E}_{12}}\right|_{\sigma_i}\right)^\ell$$

$Sp(4,\mathbb{Z})$: extrema

without computing $V(M)$, we prove:

- all 6 fixed points σ_i are extrema:

  since $V(M)$ is a Siegel modular function, $\nabla V(M)|_{M=\sigma_i} = 0$

- these extrema are always either Minkowski or AdS minima because $F_M = 0$

$$V = 0 \quad \text{if} \quad W = 0$$

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$$V = 0 \quad \text{if} \quad W = 0$$

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$\Rightarrow$ can we uplift requiring $F_S \neq 0$ as in [Leedom, NR, Westphal '22] ?

What have we found

- prove new no-go theorems for dS minima

- provide a way to evade those theorems and potentially construct dS minima

[Leedom, NR, Westphal '22]

- made considerable progress in extending to genus 2 [Kidambi, Leedom, NR, Westphal *WiP*]

What have we found + What has to be done

- prove new no-go theorems for dS minima
- provide a way to evade those theorems and potentially construct dS minima

  [Leedom, NR, Westphal '22]

  - understand the origin of e^{-1/g_s} terms
  - relate our findings with the Swampland program

- made considerable progress in extending to genus 2 [Kidambi, Leedom, NR, Westphal *WiP*]
  - extend the extrema analysis made for $SL(2,\mathbb{Z})$ to $Sp(4,\mathbb{Z})$

    → new landscape of vacua

    → extension of the no-go theorems: new dS vacua?

What have we found + What has to be done

- prove new no-go theorems for dS minima
- provide a way to evade those theorems and potentially construct dS minima

[Leedom, NR, Westphal '22]

  - understand the origin of e^{-1/g_s} terms
  - relate our findings with the Swampland program

- made considerable progress in extending to genus 2 [Kidambi, Leedom, NR, Westphal *WiP*]
  - extend the extrema analysis made for $SL(2,\mathbb{Z})$ to $Sp(4,\mathbb{Z})$

    $\rightarrow$ new landscape of vacua

    $\rightarrow$ extension of the no-go theorems: new dS vacua?

Thank you

Backup: $SL(2,\mathbb{Z})$ theorems

Theorem 1. At a point (T_0, S_0), the scalar potential $V(T,S)$ can not simultaneously satisfy:

(i). $V(T_0, S_0) > 0$

(ii). $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$

(iii). $(\Omega_S + k_S \Omega)|_{S=S_0} = 0$

(iv). Eigenvalues of the Hessian of $V(T,S)$ at (T_0, S_0) are all ≥ 0

Theorem 2. At a point (T_0, S_0), the scalar potential with $k(S, \bar{S}) = -\ln(S + \bar{S})$ can not simultaneously satisfy:

(i). $V(T_0, S_0) > 0$

(ii). $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$

(iii). $F_T(T_0) = 0$

(iv). Eigenvalues of the Hessian of $V(T,S)$ at (T_0, S_0) are all ≥ 0.

Backup: $SL(2,\mathbb{Z})$ superpotential

- $W_{(2)} \to \det(CM+D)^{-1} W_{(2)}$ is the prediction, we have to find the physics!

  flashback to $SL(2,\mathbb{Z})$:

$$W = \frac{\Omega(S)H(T)}{\eta^2(T)} \qquad \Omega(S) \sim e^{-S}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathcal{P}(j(T))$$

W inherits its automorphic properties from moduli-dependent **threshold corrections**:

for gaugino condensation $W \sim \Lambda^3 \sim \mu_0^3 e^{-\frac{f_a}{b_a}}$ gauge kinetic function

$$f_a = k_a S + \left(b'_a - k_a \delta_{GS}\right) \ln \eta^2(T) + \cdots$$

tree level

threshold corrections Δ_a

[Dixon, Kaplunovsky, Louis '91]

[Kaplunovsky, Louis '95]

$$\Rightarrow \; W \sim \frac{e^{-k_a S/b_a}}{\left(\eta(T)\right)^{2-\frac{2k_a\delta_{GS}}{b_a}}} \xrightarrow{S \equiv S + \delta_{GS} \ln \eta^2(T)} W \sim \frac{e^{-k_a S/b_a}}{\eta(T)^2}$$

[Derendinger, Ferrara, Kounnas, Zwirner '95]

Backup: $Sp(4,\mathbb{Z})$ extrema

gradient: $\nabla V \to (CM+D)^T \nabla V (CM+D)$

6 fixed points σ_i, $i = 1,\ldots,6$, each with a stabiliser group Σ_{σ_i}
s.t. $\gamma_\alpha \cdot \sigma_i = \sigma_i$ for all matrices $\gamma_\alpha \in \Sigma_{\sigma_i} \subset Sp(4,\mathbb{Z})$

$\Rightarrow$ at the fixed points: $(C\sigma_i + D)^T \nabla V|_{M=\sigma_i} (C\sigma_i + D) = \nabla V(\gamma_\alpha \cdot \sigma_i) = \nabla V(\sigma_i)$

$\rightarrow$ vectorisation: given a $m \times n$ matrix A,
construct a $mn \times \mathbb{1}$ column vector vec(A) as
$\text{vec}(A) = (a_{11}, \ldots, a_{m1}, a_{12}, \ldots, a_{m2}, \ldots, a_{1n}, \ldots, a_{mn})^T$

$P^T A P = A \quad \longrightarrow \quad (P^T \otimes P^T)\text{vec}(A) = \text{vec}(A)$

$(\mathbb{1} - P^T \otimes P^T)\text{vec}(A) = 0 \quad \Rightarrow$ eigenvalue problem

but, we have α matrices $\forall\ \sigma_i$ $\Rightarrow$ diagonalisation problem