Strings Breaking SUSY
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https://indico.mitp.uni-mainz.de/event/363

The potential and cosmological constant in non-SUSY strings and comments on the distance conjecture

Steve Abel *(IPPP)*

Mainly based on …

- w/ Dienes and Nutricati — arXiv:2303.08534; arXiv:2311.NNNNN; arXiv:23MM.NNNNN
- w/ Keith Dienes arXiv:2106.04662
- w/ Dienes+Mavroudi Phys.Rev.D 97 (2018) 12, 126017 arXiv: 1712.06894
- w/ Stewart, Phys.Rev.D 96 (2017) 10, 106013 arXiv:1701.06629
- Aaronson, SAA, Mavroudi, Phys. Rev. D 95, (2016) 106001, arXiv:1612.05742
- SAA JHEP 1611 (2016) 085, arXiv:1609.01311
- w/ Dienes+Mavroudi Phys.Rev. D 91, (2015) 126014, arXiv:1502.03087

It is difficult to make rigorous statements about the potential in string theory without going to a particular model.

Why is this so much harder than in EFT where we work with the radiative potential for any theory which is just a function of the mass spectrum (Coleman-Weinberg)?

- In EFT we subtract all the UV mystery with counter terms and use renormalization, but string theory is UV-complete and UV/IR mixed. If we are really doing string theory we are not allowed to ignore anything.

- In string theory there are both physical (level matched / on-shell) and non-physical states contributing. It is not obvious what "mass spectrum" means.

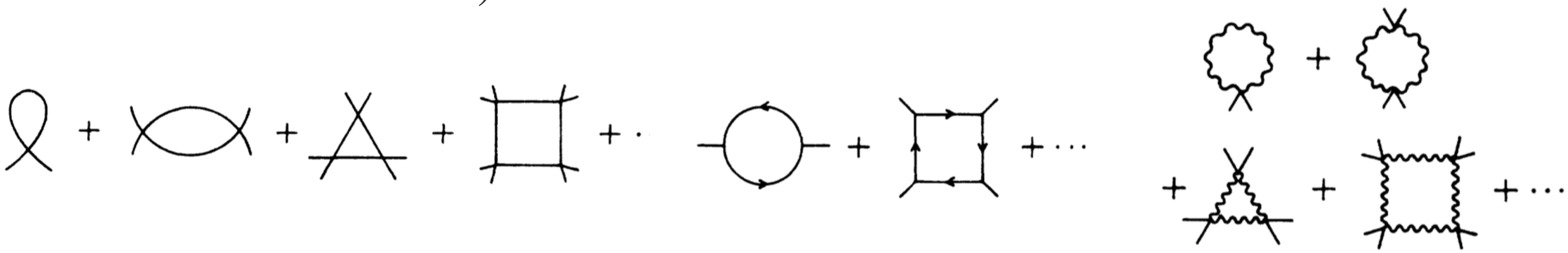
Is it possible to deal with the potential in string theory in the same model-agnostic way that we understand radiative potentials in field theory, but in a UV-complete way?

Outline

- Vacuum energy in field theory
- Vacuum energy in string theory
- But what is misaligned SUSY?
- Decompactification limits
- Comments on the distance conjecture
- Emergence of the effective theory

Vacuum energy in field theory

Let's start our story by examining the one-loop CW effective potential in field theory (and similar amplitudes where we don't care about the external momenta):



giving …

$$\overset{(4)}{\Lambda}(\phi) = \sum_n \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_n^2)$$

$$= \frac{1}{2} M_{UV}^4 + \frac{M_{UV}^2}{32\pi^2} \underset{\text{EFT}}{\mathrm{Str}} M^2 - \frac{1}{64\pi^2} \underset{\text{EFT}}{\mathrm{Str}} M^4 \log\left(c \frac{M^2}{M_{UV}^2} \right)$$

where masses can be functions of the Higgs ϕ and we are forced to put in a cut-off and

$$\underset{\text{EFT}}{\mathrm{Str}}\, X = \sum_{\text{states}} (-1)^F X_{\text{state}}$$

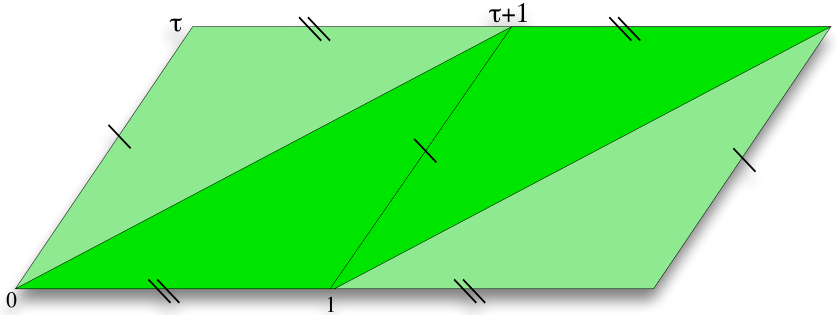
Note from this can infer a catastrophic Higgs mass-squared from the double derivative:

$$m_\phi^2 = \frac{M_{\rm UV}^2}{32\pi^2}\,\underset{\rm EFT}{\rm Str}\,\partial_\phi^2 M^2 - \underset{\rm EFT}{\rm Str}\,\partial_\phi^2\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{M_{\rm UV}^2}\right)\right]$$

This is the origin of the unfortunate naturalness problem associated with the Higgs mass. It is associated with the quadratic UV divergence in the EFT.

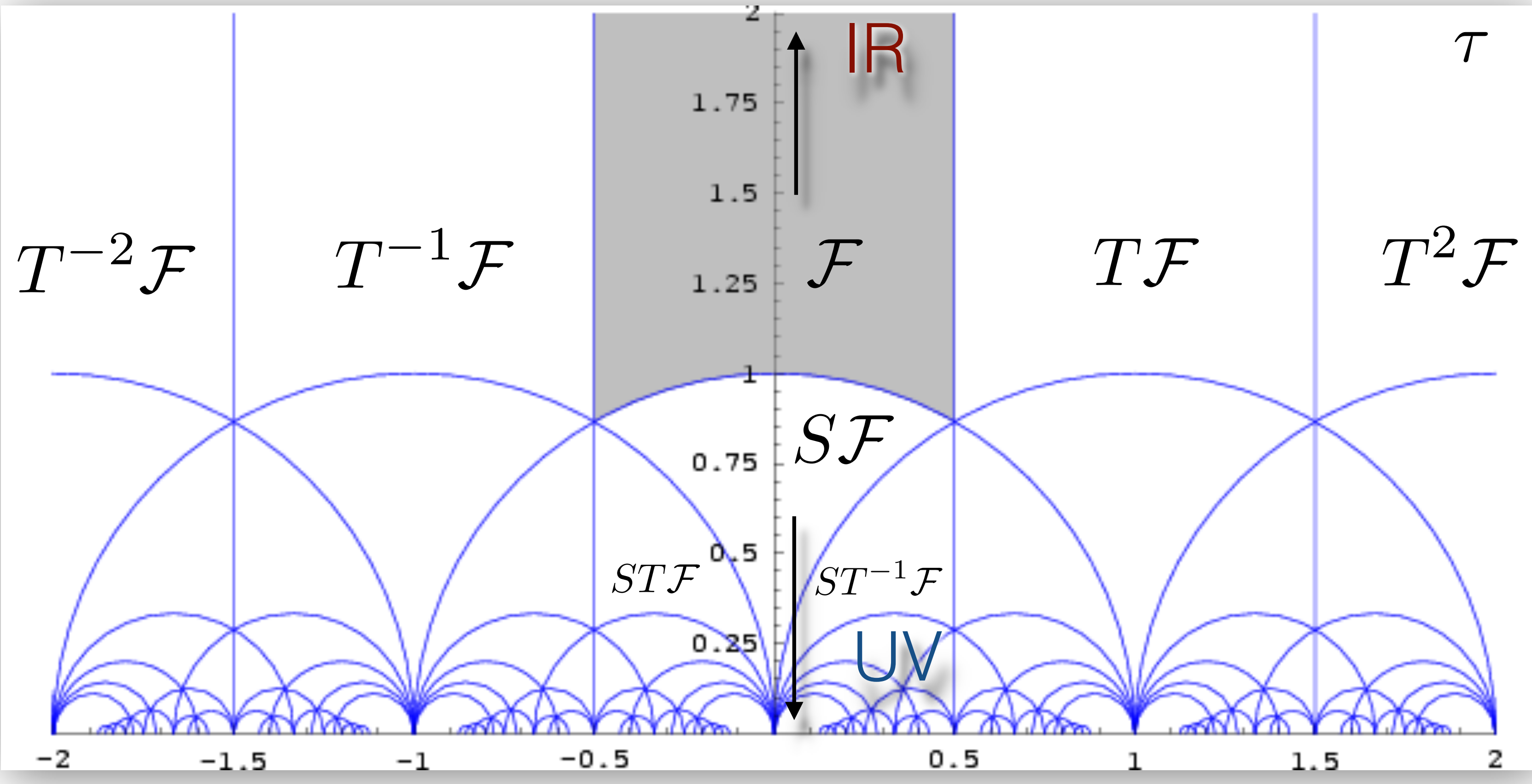
Vacuum energy in string theory

Closed strings: Everything governed by Modular Invariance …



$T:\ \boldsymbol{\tau}\to\boldsymbol{\tau}+1$ redefines torus :

$S:\ \boldsymbol{\tau}\to -1/\boldsymbol{\tau}$ swops σ_1 and σ_2 and just reorients torus



We simply have to integrate over all inequivalent tori, i.e. over the complex τ, with the string partition function $Z(\tau)$ in place of the particle partition function $Z(t)$

A bit of notation: $\tau = \tau_1 + i\tau_2$ $\left(\mathcal{M}^2 = \frac{1}{4\pi^2\alpha'} = \frac{M_s^2}{4\pi^2}\right)$

$$\Lambda^{(D)} \equiv -\tfrac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^2} Z(\tau) \qquad q = e^{2\pi i\tau}$$

$$= -\frac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^{\frac{D}{2}+1}} \sum_{m,n} a_{mn}\overline{q}^m q^n$$

Thus in principle $Z(\tau)$ holds all the information: it is non-zero in a non-SUSY model but this is hard to evaluate — looks like we'll need to know the specific model ???

Can this be made to look more like CW?

The integral we need to do in 4D is:

$$\Lambda^{(4)} = -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\mathcal{Z}(\tau)$$

Let us guess that we can write this in terms of only physical (level-matched) states whose nett spectral density can be always be found by doing the τ_1 integral:

$$\begin{aligned} g(\tau_2) &= -\frac{\mathcal{M}^4}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau) \\ &= -\frac{\mathcal{M}^4}{2}\tau_2^{-1}\sum_{\text{states}}(-1)^F e^{-\pi\tau_2\alpha' M^2_{\text{state}}} \end{aligned}$$

The tricky part is that if we want the answer to be given by this object, it implies an integral over the critical strip not the fundamental domain: we need to unfold $\mathcal{F}$ to S

Rankin-Selberg: unfold integral to the "critical strip" by convoluting it with an Eisenstein

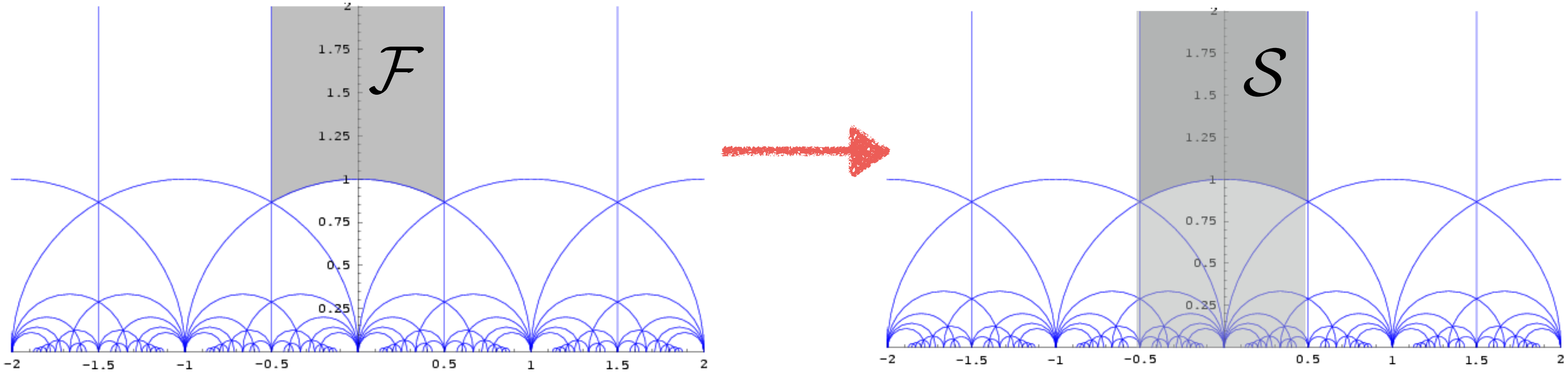
- Rankin, Selberg (1939,40)
- In string theory: McClain, Roth, O'Brien, Tan; Angelantonj, Florakis, Pioline, Rabinovici

$$\Lambda^{(4)} = 2\,\mathrm{Res}_{s=1}(\mathcal{R}^\star(Z,s))$$

where $\mathcal{R}^*$ is the Rankin-Selberg transform:

$$\mathcal{R}^\star(Z,s) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} E(s,\tau)\, Z(\tau) = \int_0^\infty \frac{d\tau_2}{\tau_2^2} \tau_2^s \pi^{-s} \Gamma(s) \zeta(2s) g(\tau_2)$$

The whole integral including the projection to physical states now looks like:



This way of doing the integral can give us amazing insights …

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau,\overline{\tau}) = \frac{\pi}{3} \operatorname*{Res}_{s=1} \int_0^\infty d\tau_2 \tau_2^{s-2} g(\tau_2)$$

Inverse
Mellin
transform

$$= \frac{\pi}{3} \lim_{\tau_2 \to 0} g(\tau_2)$$

- Rankin, Selberg
- Zagier (1981)
- Kutasov, Seiberg, 1991

But in this case $g(\tau_2) = -\frac{\mathcal{M}^4}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau)$

$$= -\frac{\mathcal{M}^4}{2} \tau_2^{-1} \sum_{\text{states}} (-1)^F e^{-\pi\tau_2 \alpha' M_{\text{state}}^2}$$

where "states" means physical level-matched states only: this looks superficially like it should diverge in the $\tau_2 \to 0$ limit!

So the incredible fact that this infinite sum is finite can be put down to the fact that the spectral density functions behave as follows as $\tau_2 \to 0$:

$$g(\tau_2) \sim \tau_2^{-1} \sum_{\text{states}} (-1)^F e^{-\pi\tau_2\alpha' M^2_{\text{state}}} \quad \longrightarrow \quad c_0$$

If we define the following natural definition of a regulated stringy supertrace over the infinite towers of physical states:

$$\mathrm{Str}\, X \equiv \lim_{y\to 0} \sum_{\substack{\text{physical}\\ \text{states}}} (-1)^F X e^{-yM^2/M_s^2}$$

Then we see **Str(1) = 0** ***even when no SUSY!***

- Dienes, Misaligned SUSY, 1994

So what is left? Roughly speaking we expand the exponential and take the next term:

$$\Lambda^{(4)} = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

- Dienes, Misaligned SUSY, 1994
- Dienes, Moshe, Myers 1995

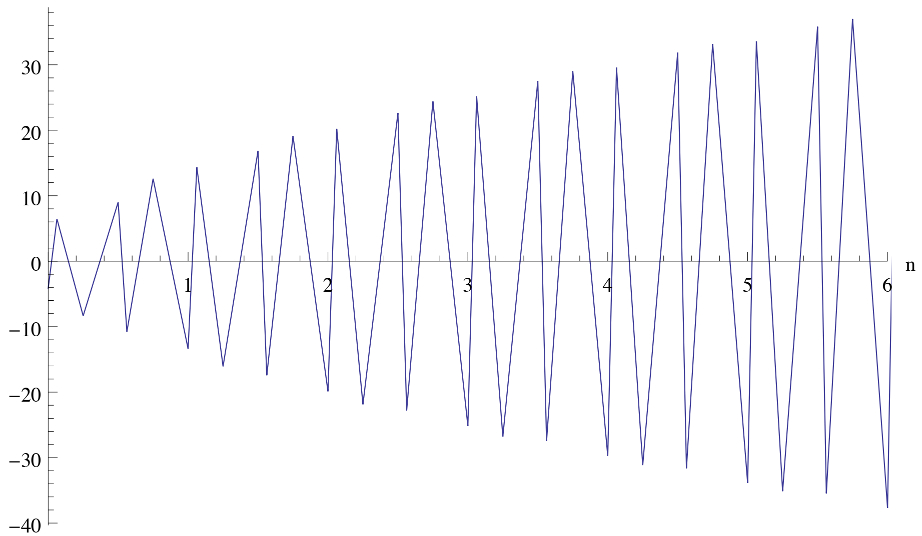
Looks like the CW potential (if we for the moment ignore the logarithmic part) with misaligned SUSY explaining the lack of quartic term!

But what is misaligned SUSY?

$$\Lambda^{(4)} = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

Looks like the quadratic piece in the CW potential but this definitely is *not* a normal field theory object — this supertrace is over the *infinite* string tower of physical states!! e.g. in non-supersymmetric models …

$\pm \log |a_{nn}|$



- This spectrum has finite $\Lambda^{(4)}$!!
- Note: modular invariance is so constraining that it has rendered the contribution from *everything* in terms of just level-matched physical states.
- *Absolutely no SUSY pairing at all!*

Misaligned SUSY: How on earth can a such a non-paired spectrum growing with a Hagedorn like $g(n) \sim e^{\sqrt{n}}$ ever cancel? …

You might (wrongly) think there is secretly just some kind of split SUSY like this …

$$
\begin{aligned}
f_{\mathrm{B}}(y) &= \sum_{n=0}^{\infty} g(n)\, e^{-yn} \\
f_{\mathrm{F}}(y) &= \sum_{n=0}^{\infty} g(n)\, e^{-y(n+\delta n)}
\end{aligned}
$$

so that the regulated supertrace requires the $y \to 0$ limit of

$$
\begin{aligned}
f_{\text{field}}(y) &= f_B(y) - f_F(y) \\
&= \sum_{n=0}^{\infty} \left\{ g(n) \left[e^{-yn} - e^{-y(n+\delta n)} \right] \right\}
\end{aligned}
$$

but this would not produce a strong enough cancellation to offset the Hagedorn behaviour and yield a finite limit … instead …

… string theory solves this conundrum in a remarkably simple way: states in the spectrum get moved around dramatically to adjust the spectral density such that:

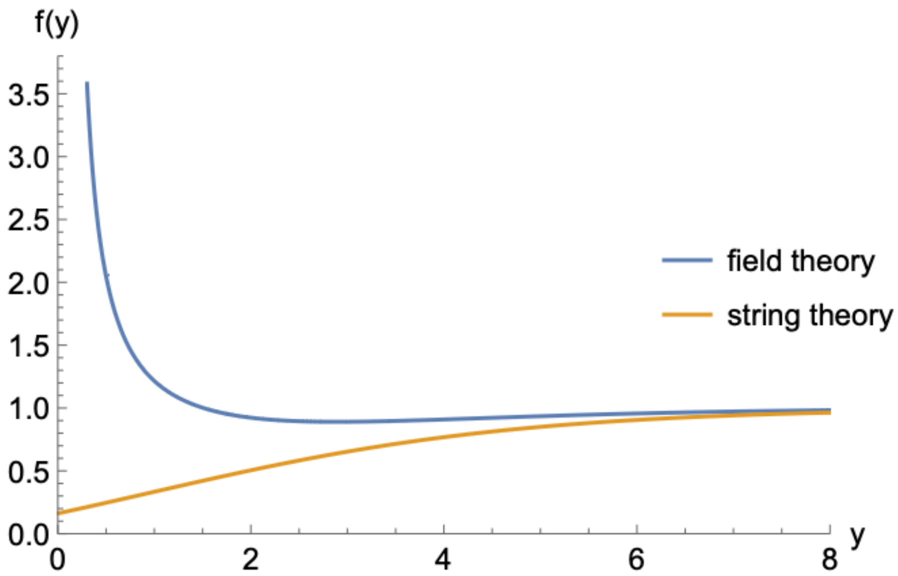
$$g(n) \longrightarrow g(n+\delta n)$$

Thus

$$f_{\text{string}}(y) \;=\; \sum_{n=0}^{\infty}\left[g(n)e^{-yn} \;-g(n+\delta n)e^{-y(n+\delta n)}\right]$$

Toy example with $g(n) = e^{\sqrt{n}}$ and $\delta n = 1/2$ which generally gives

$$f_{\text{string}}(y) \;=\; \sum_{n\in\mathbb{Z}/2} (-1)^{2n}\, g(n)\, e^{-yn}$$



The supertrace relation Str*(1)=0* is just one example of this ***magic cancellation*** that runs across the entire string spectrum, supressing divergences and/or ensuring the finiteness of string amplitudes relative to naive QFT expectations.

It turns out the Str*(1)=0* relation is just the tip of the iceberg!
One already known example that will be relevant for us where there are more cancellations is in higher dimensional theories. Indeed in D dimensions we have

$$\Lambda^{(D)} = (-1)^{D/2} \left(\frac{\pi}{6}\right) \frac{1}{(4\pi)^{D/2-1}\Gamma(D/2)} \mathcal{M}^2 \mathrm{Str} M^{D-2}$$

This requires a whole load of vanishing supertraces! (Note that D=2 is the only case where no exact cancellation of supertraces is required):

$$\mathrm{Str}\, M^{2k} \;=\; 0 \quad \text{for all} \quad 0 \leq k \leq D/2-2$$

- Dienes, 1994
- Dienes, Moshe, Myers

Decompactification limits

What does this tell us about what happens when a theory decompactifies?

To be able to reach a higher dimensional theory the partition function looks as follows …

$$Z^{(4)} = \sum_{i=1}^{N} Z'_i \Theta_i$$

The i indicates a sum over different sectors … each with a P.F. contribution Z'_i multiplying a radius dependent factor Θ_i which turns into a volume at large radius.

When δ dimensions become large some the Θ_i factors contribute to a modular invariant combination Θ yielding what we call the T-volume with the remaining contributions going exponentially fast to zero:

- SAA, Dienes, Nutricati to appear

$$\mathcal{M}^\delta V_T \equiv \frac{3}{\pi} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \Theta \quad \sim \mathcal{M}^\delta V_\delta$$

Thus we have the following for ANY modular invariant theory …

$$\Lambda^{(4)} \;\approx\; V_T\,\Lambda^{(4+\delta)} \qquad \text{for} \quad \mathcal{M}^\delta V_T \gg 1$$

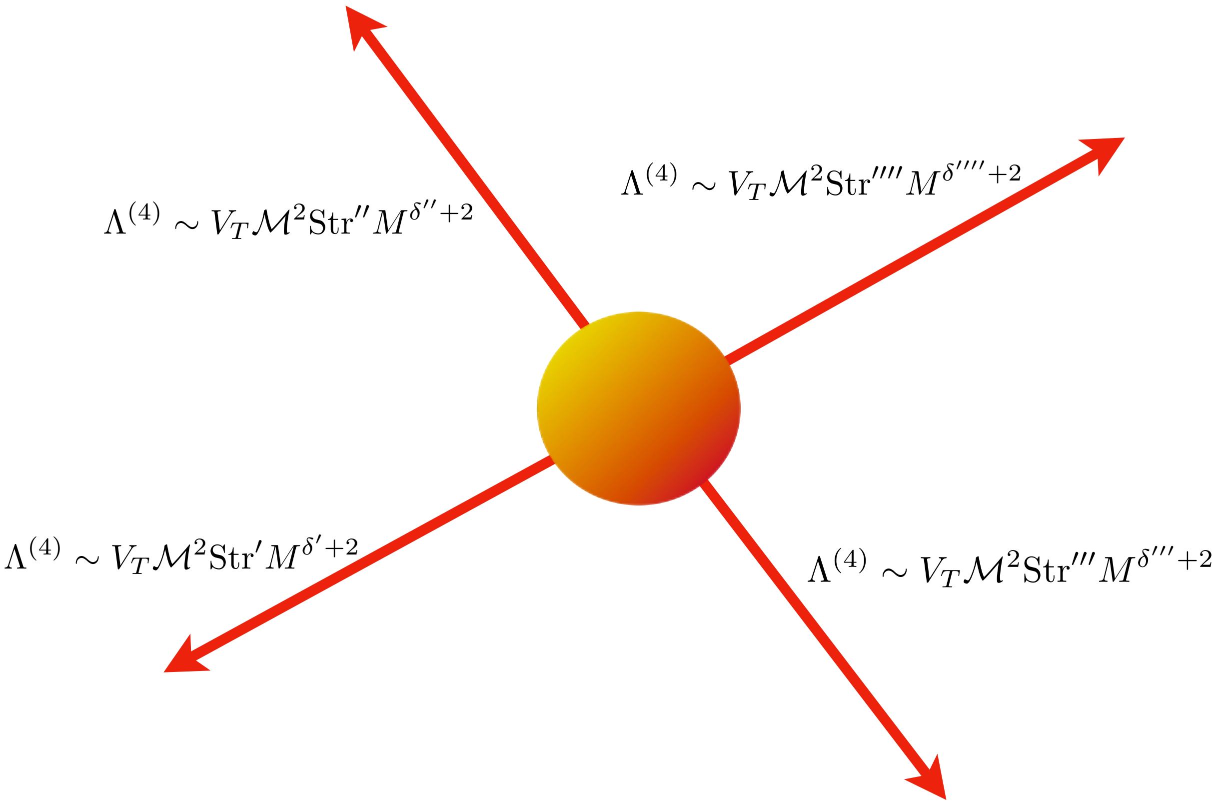
Hence the supertrace constraints of all the theories at the endpoints of a decompactification must also be satisfied!

$$\text{Str}'\,M^{2k} \;=\; 0 \qquad \text{for all} \quad 0 \leq k \leq \frac{\delta}{2}$$

Meanwhile as we saw the 4D vacuum energy is governed by the *next* supertrace in the series …

$$\Lambda^{(4)} \sim V_T \mathcal{M}^2 \text{Str}'\,M^{\delta+2}$$

So the picture looks like this …



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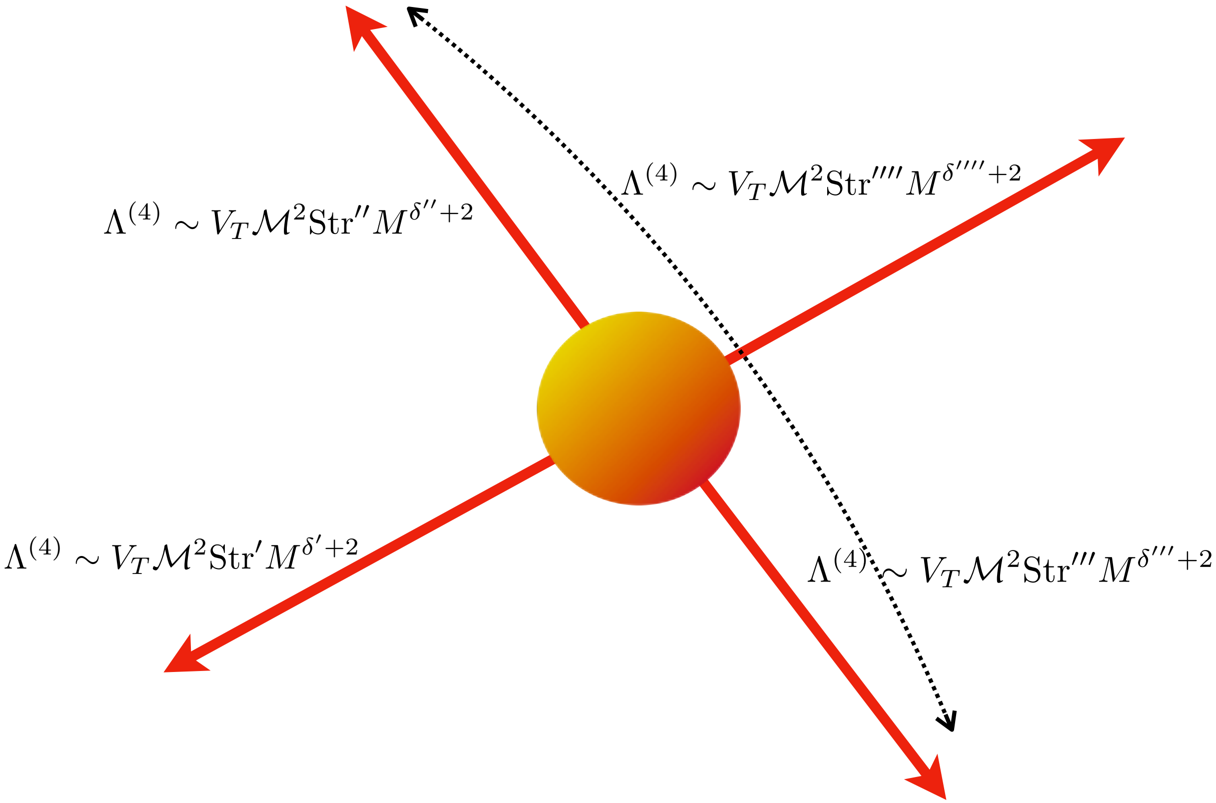

Some of these endpoint theories related by T-duality transformations - in general some endpoints are supersymmetric while others are not

Comments on the distance conjecture

- SAA, Dienes, Nutricati - imminent!!

- Luest, Palti, Vafa
- Grana, Herraez
- Ooguri, Vafa
- Ibanez, Martin-Lozano, Valenzuela
- Gonzalo, Herraez, Ibanez
- Rudelius …

According to the de-Sitter distance conjecture:

$$M_{\mathrm{KK}} \sim \left| \frac{\Lambda^{(4)}}{M_{\mathrm{Pl}}^4} \right|^{\alpha} M_{\mathrm{Pl}}$$

$$1/4 \leq \alpha \leq 1/2,$$

where M_{KK} represents the inverse distance scale.

Various arguments for this: lower bound is the "Casimir energy" behaviour and the "Dark dimension" scenario. Upper bound is the Higuchi bound

- Montero, Vafa, Valenzuela
- Anchordoqui, Antoniadis, Lust
- Anchordoqui, Antoniadis, Cribiori, Lust, Scalisi
- Burgess, Quevedo
- Anchordoqui, Antoniadis, D.Lust, S.Lust
- Higuchi
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- Luben, Lust

However our supertrace formula for the C.C can be turned into …

$$\sqrt{{\rm Str}\, M^2} \;=\; 2\sqrt{6}\,\left|\frac{\Lambda^{(4)}}{\mathcal{M}^4}\right|^{1/2} \mathcal{M}$$

- Luest, Palti, Vafa
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!!!

Note that we never appealed to any model or background to get this formula. Thus the distance conjecture in any modular invariant theory is more appropriately expressed as supertrace behaviour:

$$\sqrt{\mathrm{Str}\, M^2} \;=\; 2\sqrt{6}\,\left|\frac{\Lambda^{(4)}}{\mathcal{M}^4}\right|^{1/2}\mathcal{M}$$

Indeed converting the string scale to the Planck scale, $V_\delta \mathcal{M}^\delta \to M_{\mathrm{Pl}}^2/\mathcal{M}^2$, gives

$$\boxed{\sqrt{\mathrm{Str}\, M^2} \;=\; \frac{16\sqrt{3\pi^3}}{g_s}\left|\frac{\Lambda^{(4)}}{M_{\mathrm{Pl}}^4}\right|^{1/2} M_{\mathrm{Pl}}}$$

But we know that generically $\Lambda^{(4+\delta)} \sim \mathcal{M}^2 \mathrm{Str}' M^{\delta+2} \sim \mathcal{M}^{4+\delta}$, so that using …

$$\Lambda^{(4)} \approx V_T \, \Lambda^{(4+\delta)} \quad \text{for} \quad \mathcal{M}^\delta V_T \gg 1$$

gives us

$$M_{\mathrm{KK}} \sim \left| \frac{\Lambda^{(4)}}{M_{\mathrm{Pl}}^4} \right|^{\frac{2+\delta}{2\delta}} M_{\mathrm{Pl}}$$

Interestingly saturates at the Higuchi bound:

$$\alpha = \frac{2+\delta}{2\delta} > \frac{1}{2}$$

Note that the SO(16)xSO(16) string has a positive C.C. so (if we ignore the stability issue) compactifications of it seem to violate the distance conjecture

Two other options:

Option A: $$\Lambda^{(4+\delta)} = 0$$

Note this does not require SUSY - could just be accidentally vanishing supertrace: then what generically dominates at large distance is the "Casimir energy" contribution

$$\Lambda^{(4)} \approx \frac{(4\pi)^{\delta/2}\,\Gamma(2+\delta/2)}{16\pi^6}\frac{(n_F^0 - n_B^0)}{R^4}$$

where $(n_F^0 - n_B^0)$ is the nett exactly-massless non-degeneracy, which is the dark dimension assumption …

$$M_{\rm KK} \sim \left|\frac{\Lambda^{(4)}}{M_{\rm Pl}^4}\right|^{1/4} M_{\rm Pl}$$

Option B: $$\Lambda^{(4+\delta)} = 0 \; ; \quad n_F^0 = n_B^0$$

Non-SUSY models that satisfy this latter condition can be constructed using the Scherk-Schwarz mechanism and have been of some interest …

- SAA, Dienes, Mavroudi 2015
- Kounnas, Partouche

$$\mathrm{Str} M^2 \approx \sum_{n>0} \frac{96(n_B^n - n_F^n)}{\mathcal{M}^{1+\delta}} \times (M_n M_{\mathrm{KK}})^{\frac{3+\delta}{2}} \; e^{-2\pi \frac{M_n}{M_{\mathrm{KK}}}}$$

Summary:

- SAA, Dienes, Nutricati

Option A:	$\Lambda^{(4+\delta)} = 0\ ;\quad n_F^0 \neq n_B^0$	$M_{\rm KK} \sim \left\vert \frac{\Lambda^{(4)}}{M_{\rm Pl}^4} \right\vert^{1/4} M_{\rm Pl}$
Option B:	$\Lambda^{(4+\delta)} = 0\ ;\quad n_F^0 = n_B^0$	$\Lambda^{(4)} \sim \mathcal{M}^4 \left(\frac{M_{\rm KK}}{\mathcal{M}} \right)^{\frac{3+\delta}{2}} e^{-2\pi \mathcal{M}/M_{\rm KK}}$
Option C:	$\Lambda^{(4+\delta)} \neq 0$	$M_{\rm KK} \sim \left\vert \frac{\Lambda^{(4)}}{M_{\rm Pl}^4} \right\vert^{\frac{2+\delta}{2\delta}} M_{\rm Pl}$

Only one of these obviously satisfies the conjecture. Interesting that Option C violates the Higuchi bound so may be excluded for physical reasons. Option B doesn't look much like the distance conjecture. Two-loops to the rescue? Debatable.

Emergence of the effective theory

So … whatever happened to the logarithm of Coleman-Weinberg?

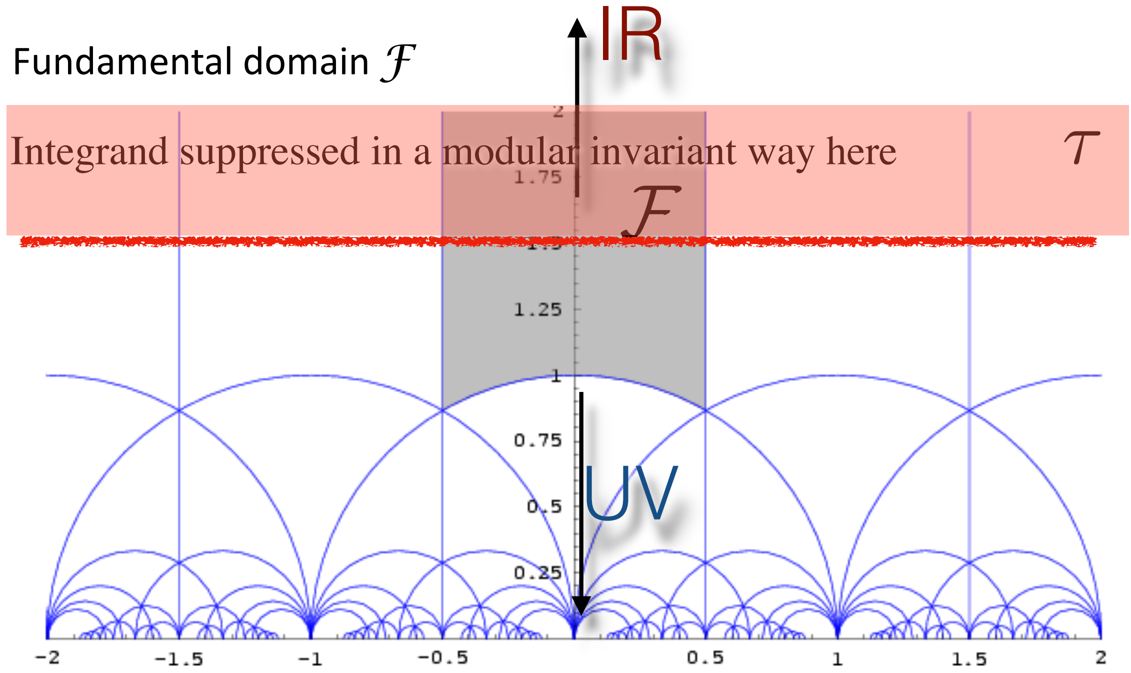
- SAA, Dienes, 2021

What we found for the vacuum energy looks like just the quadratically sensitive part of the CW potential with vanishing quartic term.

This is correct because from a field theory perspective it is the *deep-IR effective action*. Coleman-Weinberg in their analysis could not calculate this: they need renormalisation conditions which requires an energy scale. *(Note the UV-IR mixing going on here: it is the UV finiteness of string theory that allowed us to write down the deep IR object)*.

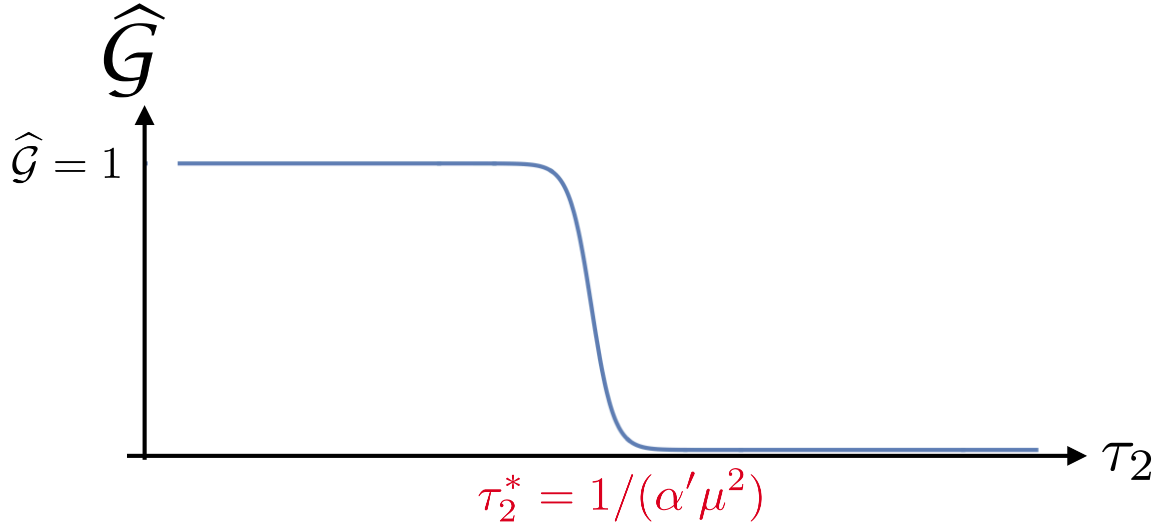
If we want to see how an EFT *emerges* from string theory we must insert an energy scale as well which is defined with a modular invariant "Wilsonian" cut-off instead:

$$I = \int \frac{d^2\tau}{\tau_2^2} F(\tau,\overline{\tau}) \;\longrightarrow\; \widehat{I}(\mu) = \int \frac{d^2\tau}{\tau_2^2} \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) F(\tau,\overline{\tau})$$



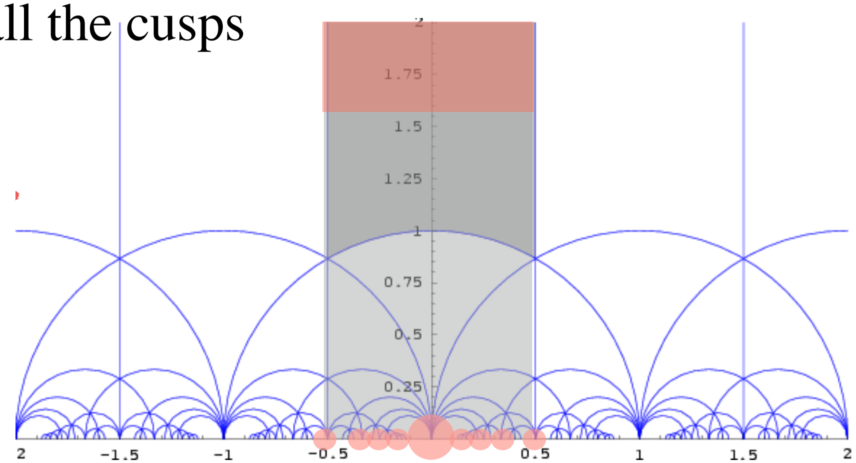
Required properties of Wilsonian regulator, $\widehat{\mathcal{G}}$: $\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$

- a) Is itself a modular function
- b) Should look like this ….



- c) Remember, our goal is to write everything as a supertrace which ultimately means an integral over the critical strip … all the cusps are quenched equally. In other words: all the cusps are equivalent IR cusps, implying…

$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu,\tau,\overline{\tau})$$



The result is a smooth modular invariant stringy Coleman-Weinberg potential

Complicated infinite sum of Bessel functions, which by magic gives …

$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2 - c'\underset{M\gtrsim\mu}{\mathrm{Str}}\,M^2\mu^2 \quad - \underset{0\le M\lesssim\mu}{\mathrm{Str}}\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right) + c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$.

Fully UV complete one-loop effective potential for any modular invariant theory

Below the mass of a state it no longer contributes to running

Parameter c depends on the choice of regulator ~ the RG scheme

At some intermediate energy scale the result is a sum over all states *as **if they had all logarithmically run up from their mass**.*

It is by construction symmetric around the string scale: $\widehat{\Lambda}(\mu) = \widehat{\Lambda}(M_s^2/\mu)$

Note that $\underset{0\le M\lesssim\mu}{\mathrm{Str}} \equiv \underset{\mathrm{EFT}}{\mathrm{Str}}$ so the supertrace of the EFT drops out of the string one!

We can perform the same procedure for all the couplings. e.g. the gauge couplings… e.g. in a model with 2 toroidal dimensions the threshold is the famous result of Dixon, Kaplunovsky and Louis. But note we get the entire energy dependence in Bessels.

SAA, Dienes, Nutricati

$$\begin{aligned}\widehat{\Delta}_G \;=\; \frac{-1}{1+a^2\rho}\Bigg\{ & \log(cT_2U_2|\eta(T)\eta(U)|^4) \;+\; 2\log\sqrt{\rho}a \\ & +\;\frac{8}{\rho-1}\sum_{\gamma,\gamma'\in\Gamma_\infty\backslash\Gamma}\Bigg[\tilde{\mathcal{K}}_0^{(0,1)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right) \\ & -\;\frac{1}{\rho}\tilde{\mathcal{K}}_1^{(1,2)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right)\Bigg]\Bigg\}\,,\end{aligned}$$

where

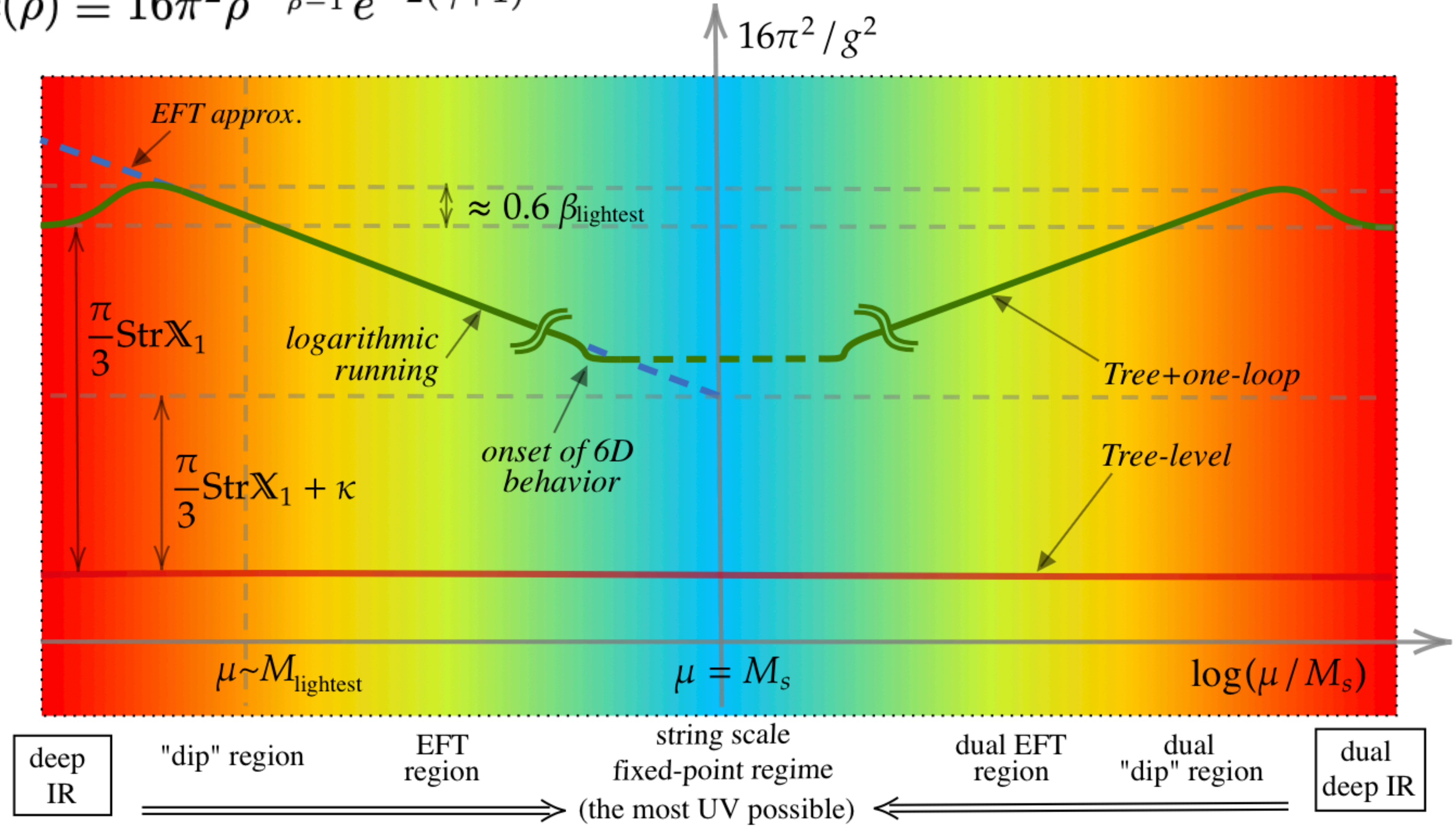
$$\tilde{\mathcal{K}}_\nu^{(n,p)}(z,\rho) \;=\; \sum_{k,r=1}^{\infty}(krz)^n\Big(K_\nu(krz/\rho)-\rho^p K_\nu(krz)\Big)$$

$$c(\rho)\equiv 16\pi^2\rho^{-\frac{\rho+1}{\rho-1}}e^{-2(\gamma+1)}$$

Analyse using asymptotics of infinite sums of Bessels:

$$\widehat{\Delta}_G \overset{a\to 0}{\approx} -\log(c\,T_2 U_2 |\eta(T)\eta(U)|^4) - 2\log\left(\frac{\mu}{M_s}\right)$$

where $c(\rho) \equiv 16\pi^2 \rho^{-\frac{\rho+1}{\rho-1}} e^{-2(\gamma+1)}$



Note only log running but from a generically large value

Conclusions

- We have developed a general supertrace formulism for understanding how we see an EFT emerge from any modular invariant theory.

- Completely model agnostic understanding of this process

- Gives us a different outlook on the distance conjecture

- A great deal hinges on the C.C. in the decompactification limits

- A modular invariant regulator provides a natural Wilsonian cut-off and definition of RG scale. Allows us to understand how an EFT Coleman-Weinberg potential emerges.

- The same techniques can be applied to all the couplings.

- Yields the symmetry $\mu \to M_s^2/\mu$ for the theory.