Non-Supersymmetric Heterotic Branes

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Strings Breaking SUSY
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[2303.17623] **JK**, Ohmori, Tachikawa, Yonekura

[2010.10521] **JK**

Non-Supersymmetric Heterotic Strings

- **We all are familiar with the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ and $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic string theories.**

- **But there are known to be 7 other heterotic strings:**

$$\mathfrak{e}_8\ , \quad \mathfrak{u}(16)\ , \quad (\mathfrak{e}_7 \times \mathfrak{su}(2))^2\ , \quad \mathfrak{o}(8) \times \mathfrak{o}(24)\ ,$$
$$\mathfrak{o}(16) \times \mathfrak{e}_8\ , \quad \mathfrak{o}(32)\ , \quad \mathfrak{o}(16) \times \mathfrak{o}(16)$$

- **These were originally found in [Kawai, Lewellen, Tye '86; Dixon, Harvey '86].**
  - Until very recently, it was unknown if these were all of the possibilities, or if there could be others. The completeness of this list was proven in [Boyle Smith, Lin, Tachikawa, Zheng '23] (see also [Rayhaun '23; Höhn, Möller '23]), where the set of all $c = 16$ spin-CFTs was classified.

- **All of these theories are non-supersymmetric. All but the last one has a closed string tachyon.**

Non-Supersymmetric Heterotic Strings

- **The closed string tachyon is worrying, but not fatal. It simply indicates that we are expanding around the wrong vacuum.**

- **We will see that we can** $condense$ **the tachyon to obtain theories in** $d = 9, 8, 6, 2$**, which have no tachyon, but have a linear dilaton** [**JK** '20]**.**

  - Stable, but the linear dilaton is a bit weird.

- **What do these stable vacua describe? Why do they exist??**

- **One place where we are familiar with a linear dilaton is in the near-horizon region of an** $\mathrm{NS5}$**-brane.**

- **So maybe these non-supersymmetric strings exist in order to describe the near-horizon regions of some branes?**

  - The $d = 9, 8, 6, 2$ vacua would describe near-horizons of 7-,6-,4-,0-branes.

- **The branes would be in the supersymmetric** $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ **or** $\mathrm{Spin}(32)/\mathbb{Z}_2$ **heterotic strings, and would break all supersymmetries.**

Non-Supersymmetric Heterotic Branes

- **No such branes are known in heterotic string theory. . .**

- **A related question: the old quantum gravity lore states that (see** [McNamara, Vafa '19] **for a modern version):**

  Any consistent theory of quantum gravity must contain objects carrying all possible charges.

- **Basically any topological invariant should count as a charge. Note that:**

$$\pi_0\left((E_8\times E_8)\rtimes\mathbb{Z}_2\right)\simeq\mathbb{Z}_2 \qquad \pi_1(\text{Spin}(32)/\mathbb{Z}_2)\simeq\mathbb{Z}_2$$
$$\pi_3\left((E_8\times E_8)\rtimes\mathbb{Z}_2\right)\simeq\mathbb{Z}\times\mathbb{Z} \qquad \pi_7(\text{Spin}(32)/\mathbb{Z}_2)\simeq\mathbb{Z}$$

- **What carries the charges?**

- **They would capture non-trivial configurations on S^1, S^2, S^4, S^8, which is just what is needed to surround a 7-,6-,4-,0-brane!**

Outline

- **In the rest of this talk we will try to:**

  Part I: Review the tachyonic heterotic strings, and show how to condense the tachyons to get stable, lower-dimensional vacua. [**JK** '20]

  Part II: Explicitly construct the non-supersymmetric branes whose existence we hinted at earlier. [**JK**, Ohmori, Tachikawa, Yonekura '23]

- **If time permits, we will also discuss one of the branes in M-theory.**

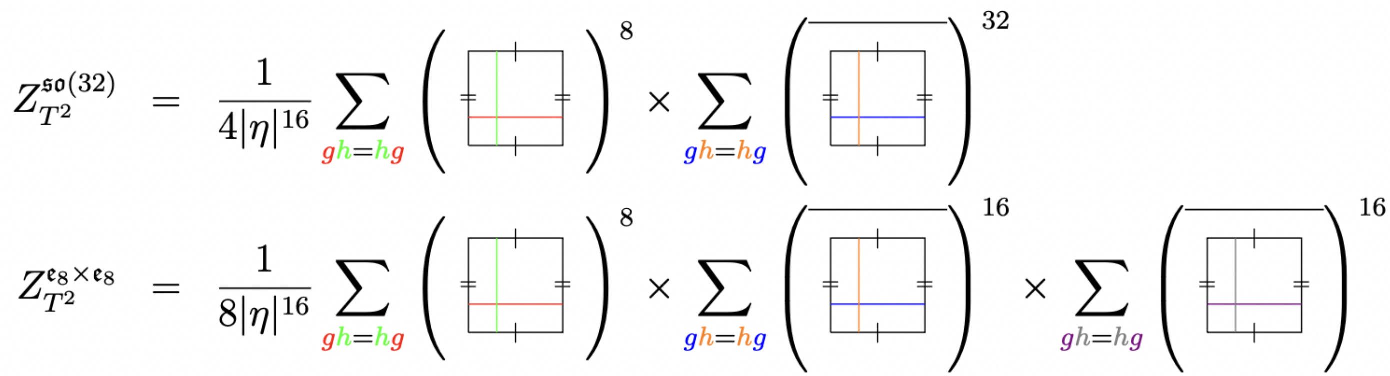
Part I

Part I: Stable vacua for heterotic strings

Tachyonic Heterotic Strings

- **The standard heterotic strings have the following worldsheet content:**
  - Bosons: $X^i_{L,R}$ with $i = 1, \ldots, 8$
  - Right-moving fermions: ψ^i_R with $i = 1, \ldots, 8$
  - Left-moving fermions: λ^a_L with $a = 1, \ldots, 32$

- **When we perform the GSO projection, we have the freedom of assigning different spin structures for the left- and right-movers. Standard options:**

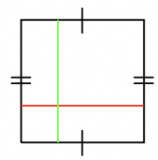


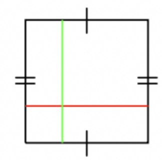
- **This gives the two SUSY heterotic strings.**

Tachyonic Heterotic Strings

- **If we identify the left- and right-moving spin structures, we get a non-SUSY string** [Kawai, Lewellen, Tye ‘86; Dixon, Harvey ‘86]

$$Z^{?}_{T^2} = \frac{1}{2|\eta|^{16}} \sum_{gh=hg} \left(\quad \right)^{8} \times \left(\overline{\quad} \right)^{32}$$





- **To understand the properties of this string, we consider the level-matched partition function:**

$$\int_0^1 d\tau_1 \; Z^{?}_{T^2} = 32(q\bar{q})^{-\frac{1}{2}} + 4032 + 188928(q\bar{q})^{\frac{1}{2}} + O(q\bar{q})$$

- **The result is non-zero, so no spacetime SUSY. The spectrum contains:**
  - 32 tachyons
  - 4032 massless bosons: can split into graviton (35), B-field (28), dilaton (1), and 496 gauge bosons of $\mathfrak{so}(32)$.

- **So this is a non-SUSY, tachyonic $\mathfrak{so}(32)$ string.**

Tachyonic Heterotic Strings

- **Can construct other heterotic strings by further splitting the spin structures. Alternatively, note that the worldsheet has a $(\mathbb{Z}_2)^5$ global symmetry generated by:**

$$
\begin{aligned}
g_1 &= \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \ , & g_2 &= \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \ , \\
g_3 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \ , & g_4 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \otimes \mathbb{1}_2 \ , \\
g_5 &= \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \mathbb{1}_2 \otimes \sigma_3 \ .
\end{aligned}
$$

- **We can now gauge $(\mathbb{Z}_2)^n$, which breaks $\mathfrak{so}(32) \not\to \mathfrak{so}(2^{5-n}) \times \mathfrak{so}(32-2^{5-n})$**

n	tachyons	massless fermions	gauge bosons	gauge algebra
0	32	0	496	$\mathfrak{so}(32)$
1	16	256	368	$\mathfrak{o}(16) \times \mathfrak{e}_8$
2	8	384	304	$\mathfrak{o}(8) \times \mathfrak{o}(24)$
3	4	448	272	$(\mathfrak{e}_7 \times \mathfrak{su}(2))^2$
4	2	480	256	$\mathfrak{u}(16)$
5	1	496	248	$\mathfrak{e}_8$

Tachyon Condensation

- **All of the heterotic strings above have tachyons. We now try to condense them** [Hellerman, Swanson '06; '07]

- **Say $\tilde{\lambda}^a$ are the subset of λ_L^a covariant under $\mathfrak{so}(2^{5-n})$. Condensation produces a superpotential**

$$W = \sum_{a=1}^{2^{5-n}} \tilde{\lambda}^a \mathcal{T}^a(X)$$

- **Equation of motion for $\mathcal{T}^a(X)$ is**

$$\partial^\mu \partial_\mu \mathcal{T}^a - 2\partial^\mu \phi \, \partial_\mu \mathcal{T}^a + \frac{2}{\alpha'}\mathcal{T}^a = 0$$

which admits the following solution:

$$\phi = -\frac{2^{\frac{3-n}{2}}}{\sqrt{\alpha'}} X^- \,, \qquad \mathcal{T}^a = m\sqrt{\frac{2}{\alpha'}} e^{\beta X^+} X^{a+1}$$

Condensation to $d > 2$

- **Plugging back in and computing the scalar potential gives**

$$V = A\, e^{2\beta X^+} \sum_{a=1}^{2^{5-n}} (X^{a+1})^2 - B\, e^{\beta X^+} \sum_{a=1}^{2^{5-n}} \tilde{\lambda}^a \psi^{a+1} + \dots$$

- **As $X^+ \to \infty$, fluctuations along $X^2, \dots, X^{2^{5-n}+1}$ are suppressed, and we get a theory in $d = 10 - 2^{5-n}$ localized at $X^2 = \dots = X^{2^{5-n}+1} = 0$.**

n	d	massless fermions	gauge bosons	gauge algebra
3	6	112	266	$\mathfrak{e}_7 \times \mathfrak{e}_7$
4	8	240	255	$\mathfrak{su}(16)$
5	9	248	248	$\mathfrak{e}_8$

- **Low-energy gravity+gauge theories can be checked to be anomaly-free!**

Condensation to $d = 2$

- **For $n < 2$, then $d = 10 - 2^{5-n} < 0$ so it's a bit different.**
- **In these cases we simply condense to $d = 2$, where dilaton background lifts remaining tachyons. This gives 2d strings found in** [Davis, Larsen, Seiberg '05].
- **To summarize, we have found that all non-SUSY heterotic strings admit lower-dimensional stable vacua, given by:**

d	gauge algebra
9	$\mathfrak{e}_8$
8	$\mathfrak{su}(16)$
6	$\mathfrak{e}_7 \times \mathfrak{e}_7$
2	$\mathfrak{so}(24),\ \mathfrak{o}(8) \times \mathfrak{e}_8,\ \times \mathfrak{so}(24)$

- **All of these vacua are anomaly-free and perturbatively stable. They all have a linear dilaton though.**

Part II

Part II: Non-supersymmetric heterotic branes

The NS5-brane

- **Recall another context in which a linear dilaton arises: the NS5-brane.**
- **In supergravity, we have the following extremal solution:**

$$\begin{aligned} ds^2_{\text{NS5}} &= dx_{\parallel}^2 + e^{2\phi} dx_{\perp}^2 \\ e^{2\phi} &= e^{2\phi(\infty)} + \frac{r_0^2}{r^2} \end{aligned} \qquad H_{mnp} = -\epsilon_{mnp}{}^{q}\,\partial_q \phi \qquad A_m = -2\rho^2\,\overline{\Sigma_{nm}}\frac{x^n}{r^2(r^2+\rho^2)}$$

where r is the transverse radial direction. [Callan, Harvey, Strominger '91]

- **In the near-horizon limit $r \to 0$:**

$$ds^2_{\text{NS5}} = dx_{\parallel}^2 + \frac{r_0^2}{r^2} dr^2 + r_0^2 d\Omega_3^2 \qquad e^{2\phi} = \frac{r_0^2}{r^2}$$

- **Or defining $y := \log \frac{r}{r_0}$,**

$$ds^2_{\text{NS5}} = dx_{\parallel}^2 + dy^2 + r_0^2 d\Omega_3^2 \qquad \phi = -y$$

The NS5-brane

- **So the extremal, near-horizon solution is:**

$$ds^2_{\text{NS5}} = dx_{\|}^2 + dy^2 + r_0^2\, d\Omega_3^2 \qquad\qquad \phi = -y$$

with one unit of H flux through S^3 (in general r_0 is small though!)

- **There is an exact worldsheet description for this near-horizon solution** [Callan, Harvey, Strominger '91]

$$\mathbb{R}^{1,5} \times \mathbb{R}_{\text{linear dilaton}} \times \mathfrak{su}(2)_\bullet \times \mathfrak{g}$$

- **Intuitively: think of this as a 7d vacuum with a linear dilaton and $\mathfrak{su}(2) \times \mathfrak{g}$ gauge group.**

- **This is expected to be the holographic dual to the 6d LST living on the NS5 brane.** [Aharony, Berkooz, Kutasov, Seiberg '98]

The 6-brane

- **We now try an analogous thing for the 6-brane. Our considerations before suggest that the near-horizon limit is described by**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\text{linear dilaton}} \times \mathfrak{su}(16)_{\bullet}$$

- **To reproduce this, we note that there exists a black 6-brane solution for the $\mathfrak{so}(32)$ heterotic string** [Horowitz, Strominger '91]

$$ds^2 = -\frac{(1-\frac{r_+}{r})}{(1-\frac{r_-}{r})}dt^2 + d\vec{x}^2 + \frac{dr^2}{(1-\frac{r_+}{r})(1-\frac{r_-}{r})} + r^2 d\Omega_2^2$$

$$e^{-2\phi} = e^{-2\phi(\infty)}\left(1-\frac{r_-}{r}\right) \qquad 8r_+r_- = \alpha' \sum_{i=1}^{16} q_i^2$$

$$\frac{F_{\mathfrak{so}(32)}}{2\pi} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \frac{\text{vol}(S^2)}{4\pi}$$

The 6-brane

- Taking the extremal, near-horizon limit gives:

$$ds^2 = dx_{\parallel}^2 + dy^2 + r_0^2\, d\Omega_2^2 \qquad \phi = -y$$

$$\frac{F_{\mathfrak{so}(32)}}{2\pi} = \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & q_i \\ -q_i & 0 \end{pmatrix} \frac{\mathrm{vol}(S^2)}{4\pi}$$

so we have an infinite throat with S^2 of constant size $r_0 := \sqrt{\frac{\alpha'}{8} \sum_{i=1}^{16} {q_i}^2}$.

- An important fact about the $\mathfrak{so}(32)$ heterotic string is that the global form of the gauge group is $\mathrm{Spin}(32)/\mathbb{Z}_2$. This makes it possible to choose $q_i = 1/2$.

- If we choose $q_i = 1/2$ for all i, then we preserve $\mathfrak{u}(16) \subset \mathfrak{so}(32)$.

- We expect this to give the six-brane, but note that $r_0 = (\alpha'/2)^{1/2}$, so that supergravity is not reliable.

The 6-brane

- **At this point we transition to a worldsheet analysis. Worldsheet version of near-horizon limit would be (ignoring flux):**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\textsf{linear dilaton}} \times (\mathcal{N}=(1,0)\;\; S^2) \times \mathfrak{so}(32)_1$$

- **It turns out that**

$$\mathfrak{so}(32)_1 = [\mathfrak{su}(16)_1 \times \mathfrak{so}(2)_1]/(-1)^{\mathsf{F_L}}$$

- **With the flux, we can reorganize the worldsheet theory as**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\textsf{linear dilaton}} \times [(\mathcal{N}=(1,1)\;\; S^2) \times \mathfrak{su}(16)_1]/(-1)^{\mathsf{F_L}}$$

(because vector bundle of $\mathfrak{so}(2)_1$ = tangent bundle of S^2)

- **This is not a solution to the string equations of motion since it is not conformal. But it flows in the IR to the following theory:**

$$\mathbb{R}^{1,6} \times \mathbb{R}_{\textsf{linear dilaton}} \times \mathfrak{su}(16)_1$$

The 6-brane

- **The interpretation of this RG flow is that the S^2 dynamically shrinks away, leaving an 8d spacetime with linear dilaton and $\mathfrak{su}(16)$ gauge group. This is precisely the 8d vacuum from before!**

- **So indeed, the near-horizon limit of the 6-brane is described by this vacuum.**

- **What charge does the 6-brane carry?**
  - Recall that it sources the following flux: $\int_{S^2} \frac{F_{\mathfrak{so}(32)}}{2\pi} = \frac{1}{2} \bigoplus_{i=1}^{16} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
  - This is incompatible with the vector representation, but is compatible with the adjoint and one of the spinor representations.
  - Given a $\mathrm{Spin}(32)/\mathbb{Z}_2$ bundle, the obstruction to it being a $SO(32)$ bundle is captured by a class $\widetilde{w}_2$ (c.f. $\pi_1(\mathrm{Spin}(32)/\mathbb{Z}_2) = \mathbb{Z}_2$).
  - This is the charge carried by the brane.

Other branes

- **The key tool in the above analysis was the identity**

$$\mathfrak{so}(32)_1 = [\mathfrak{su}(16)_1 \times \mathfrak{so}(2)_1]/(-1)^{\mathsf{F_L}}$$

- **There are three (actually 5) similar identities:**

$$\begin{aligned} \mathfrak{so}(32)_1 &= [\mathfrak{so}(24)_1 \times \mathfrak{so}(8)_1]/(-1)^{\mathsf{F_L}} \\ (\mathfrak{e}_8 \times \mathfrak{e}_8)_1 &= [(\mathfrak{e}_7 \times \mathfrak{e}_7)_1 \times \mathfrak{so}(4)_1]/(-1)^{\mathsf{F_L}} \\ (\mathfrak{e}_8 \times \mathfrak{e}_8)_1 &= [(\mathfrak{e}_8)_2 \times \mathfrak{so}(1)_1]/(-1)^{\mathsf{F_L}} \end{aligned}$$

- **These can be used to give exact worldsheet descriptions for 0-, 4-, and 7-branes, respectively:**
  - The 0-brane is an endpoint for the $\mathfrak{so}(32)$ heterotic string [Polchinski '05].
  - The 4-brane can be interpreted as an $M5$ stretched between two $M9$s [Bergshoeff, Gibbons, Townsend '06] .
  - Going around the 7-brane flips the two $\mathfrak{e}_8$ factors.

Summary

- **We have found 0- and 6-branes in the $Spin(32)/\mathbb{Z}_2$ heterotic string, as well as 4- and 7-branes in the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic string.**

- **The near-horizon limits of these branes were shown to be described by the stable, lower-dimensional vacua of the non-SUSY heterotic strings.**

- **By the holographic dictionary, the latter should provide holographic descriptions of the worldvolume theories of the branes.**

- **Phrased provocatively: the reason that the non-SUSY heterotic strings exist is to describe the worldvolume theories of the non-SUSY branes!**

- **There exist even more non-SUSY branes, which we are in the process of uncovering!** [Dierigl, Heckman, Montero, Torres '22; . . .]

The End (for now)

Thank you!