Intersecting End of The World branes: an effective approach beyond SUSY

Roberta Angius

Instituto de Física Teórica
UAM-CSIC

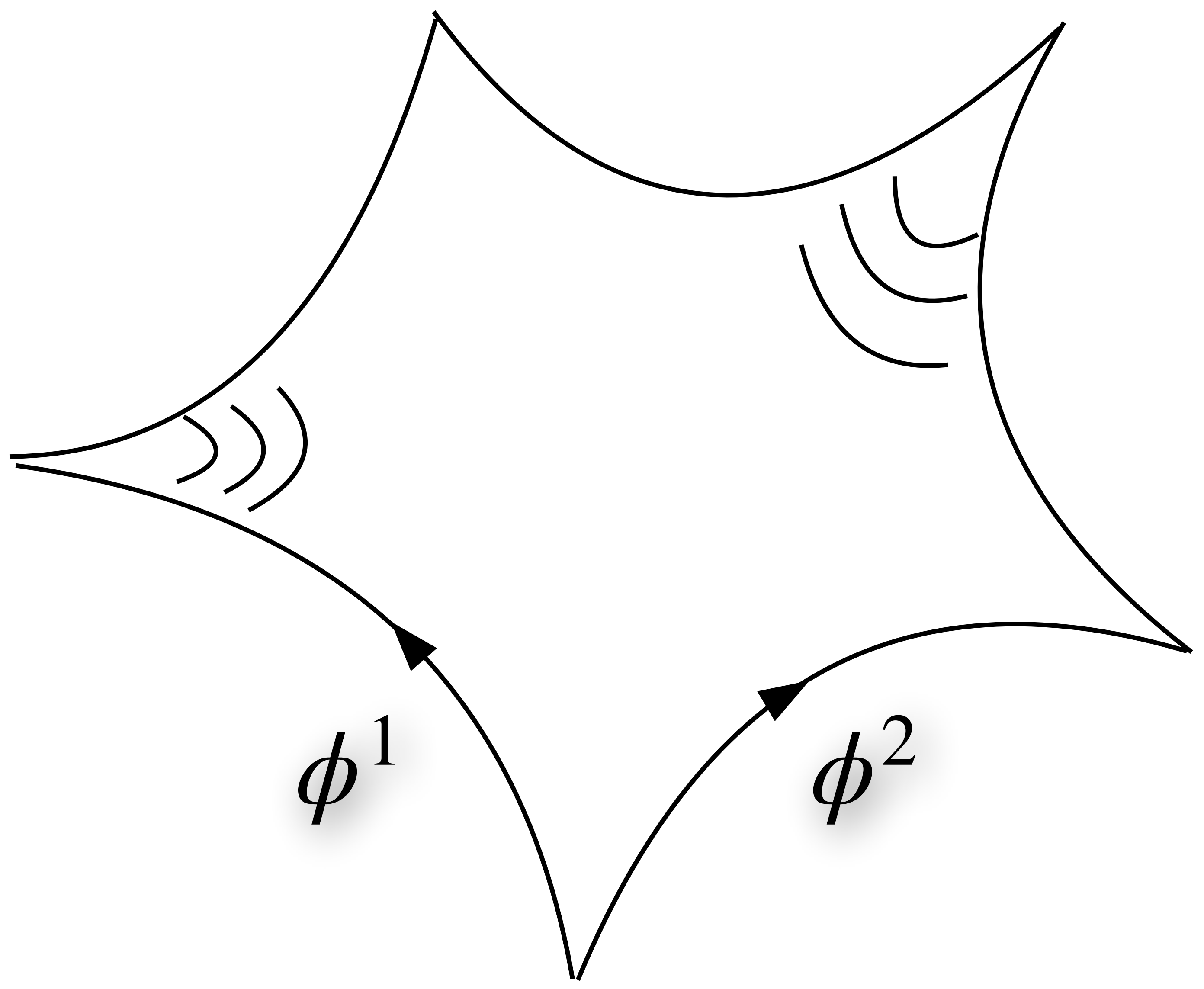
Based on: **2312.xxxx** with **A.Uranga** and **A. Makridou**

Strings Breaking SUSY, November 22, 2023

➔ To study spacetime dependent solutions of:

$$S = \int d^d x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} G_{ij} \partial\phi^i \partial\phi^j - V(\phi^1, \phi^2, \dots) \right\}$$

Motivations



- We explore infinite distance limits in the field space:
  - where the physics is under control
  - they are test for Swampland conjectures and dualities

- The method apply to any String Theory setup, including those not protected by SUSY

[R.A., M.DELGADO, A.URANGA, JHEP08 (2022) 285 , ArXiv: 2207.13108]

Computing the equations of motion associated to the action:

$$S = \int d^d x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} \partial\phi\partial\phi - V(\phi) \right\}$$

we look for solutions such that:

- are spacetime dependent,
- show a Ricci singularity in the metric located at finite distance in spacetime,
- the scalar diverges when we approach to such singularity.

NOTHING

x

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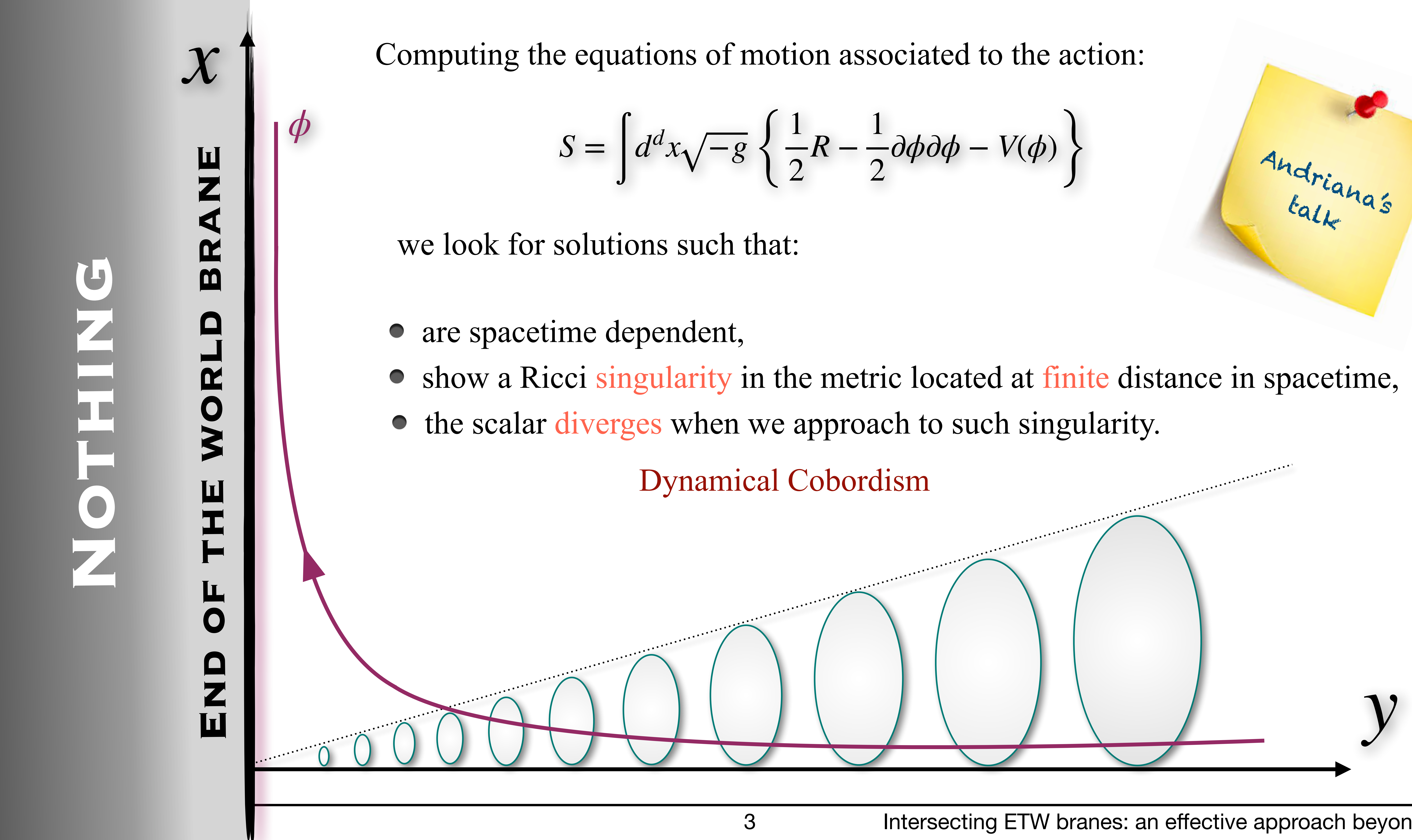
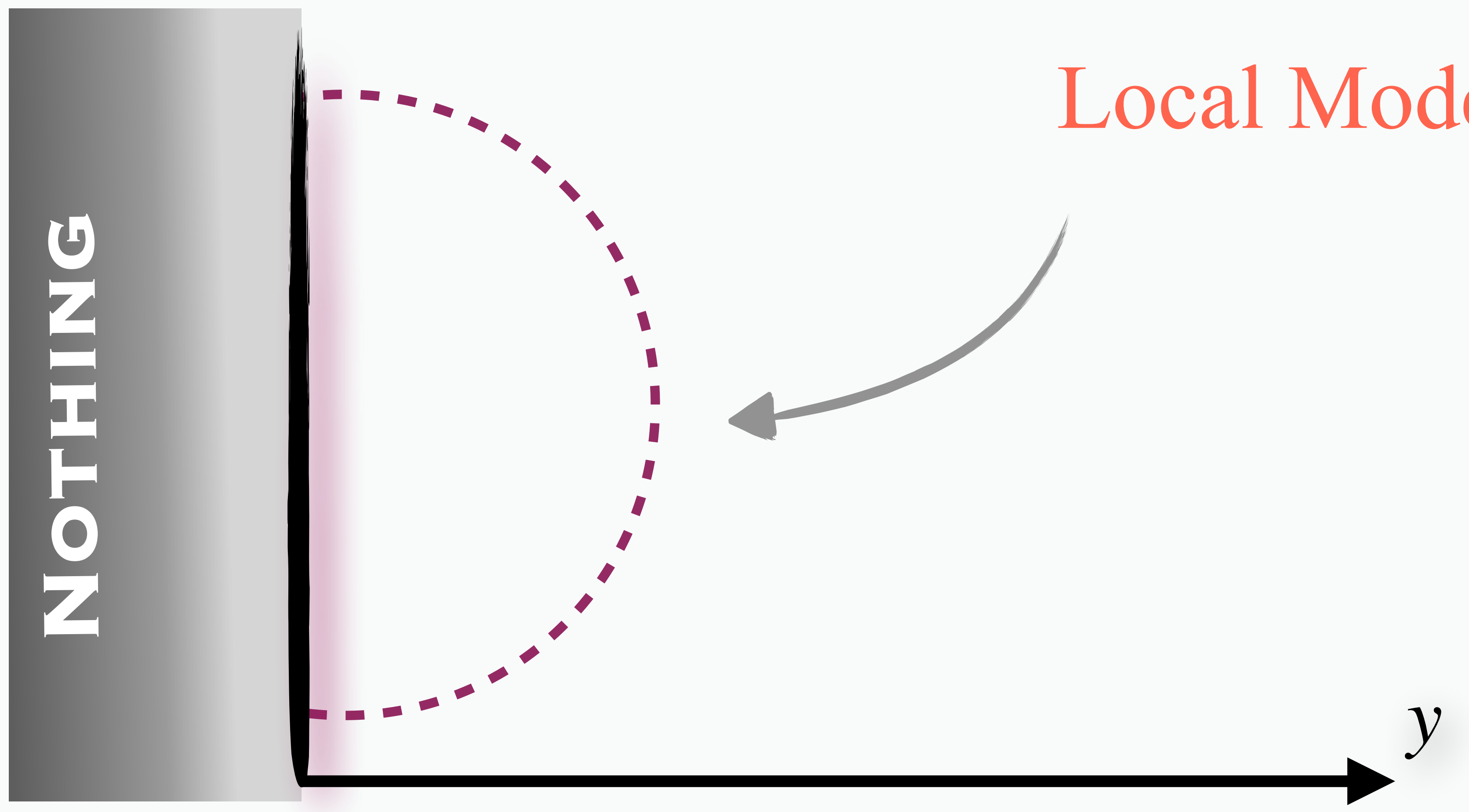
Dynamical Cobordism



End of The World branes



Local Model:

- $$\varphi(y) \simeq -\frac{2}{\delta}\log y$$
- $$ds^2_{n+1} = e^{-2\sigma(y)}ds^2_n + dy^2$$

with $$\sigma(y) \simeq \pm \frac{4}{(n-1)\delta^2}\log y$$

- At fixed spacetime dimension, the parameter δ specifies our ETW-brane in this effective perspective
- It doesn't provide a microscopic description of the defect (Dp-branes, bubbles of nothing, unknown 7-branes ecc…)

[R.A., J.CALDERON-INFANTE, M.DELGADO, J.HUERTAS, A.URANGA, '22]
[R.BLUMENHAGEN,N.CRIBIORI, C.KNEISSL, A.MAKRIDOU, '22]

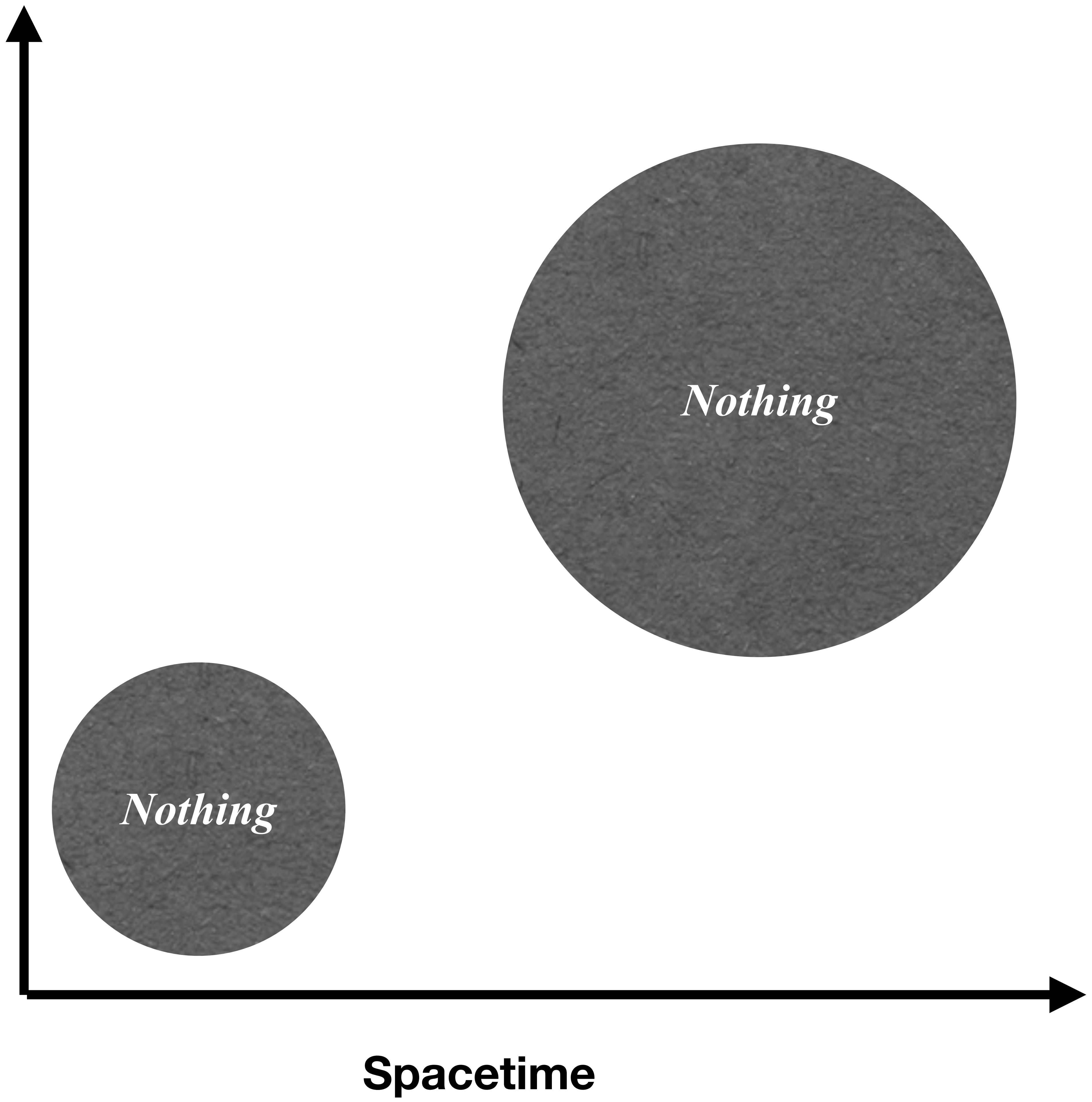
Critical exponent: $$\delta = 2\sqrt{\frac{n}{n-1}(1-a)}$$

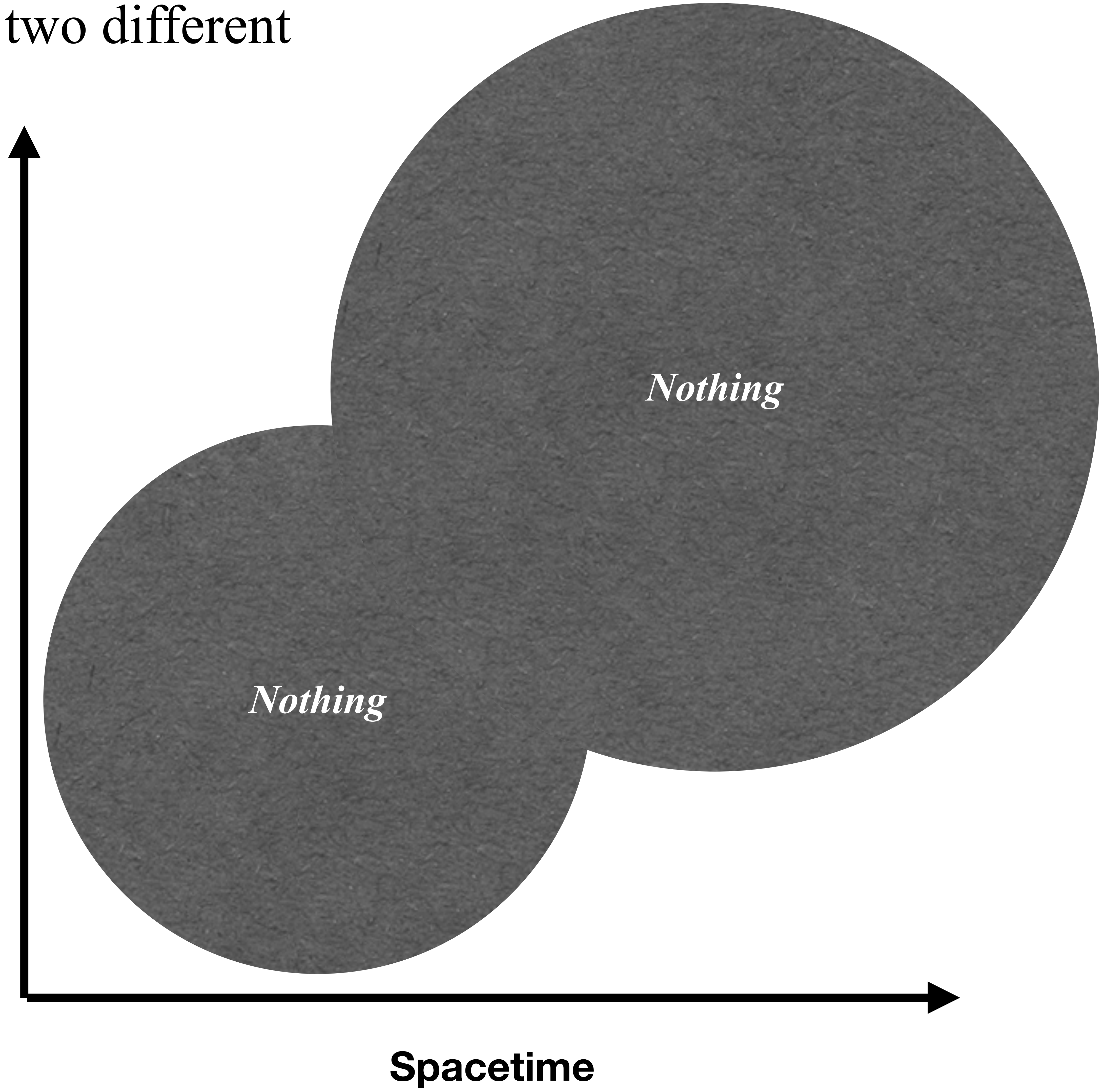
Potential class: $$V(\phi) \simeq -cae^{\delta\phi}$$

Microscopic defect	$n+1$	δ	a
Bubble of Nothing	4	$\sqrt{6}$	0
D2 brane	4	$\sqrt{14/7}$	20/21
D2/D6 on $T^4 \times S^2$	4	$\sqrt{2}$	2/3

Cosmic Bubbles Collision

The first motivation came from cosmology, in particular from the interest to study the physics in the region around the intersection between two different Bubbles of Nothing





Moduli space of CY compactifications

Let $\mathscr{M}_{cs}$ be the complex structure moduli space of CY_D manifolds:

- It has complex dimension $h^{D-1,1}$
- It is not smooth nor compact
- It is a *quasi-projective* manifold

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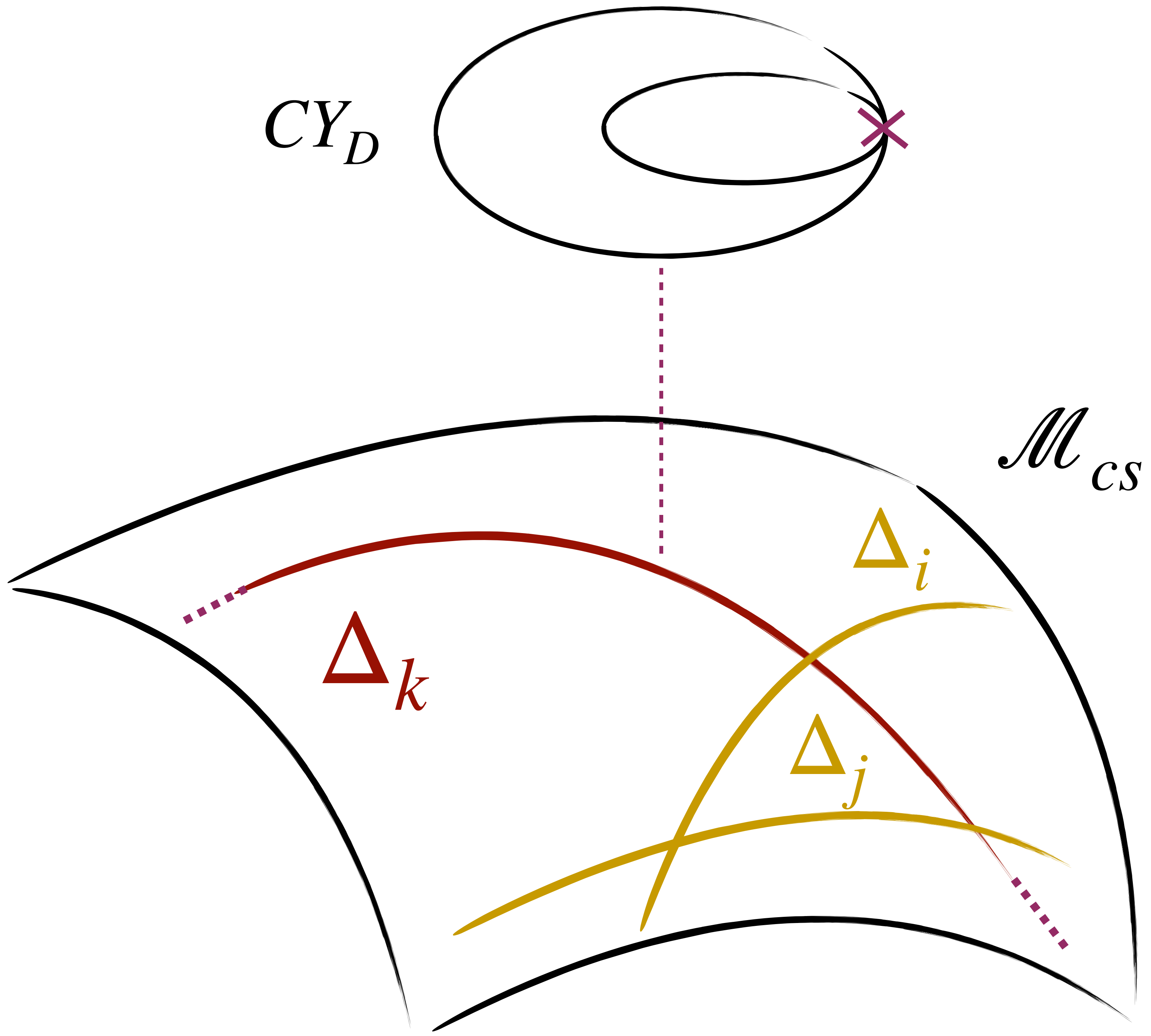
Discriminant locus

$$\Delta = \bigcup_k \Delta_k$$



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Discriminant locus

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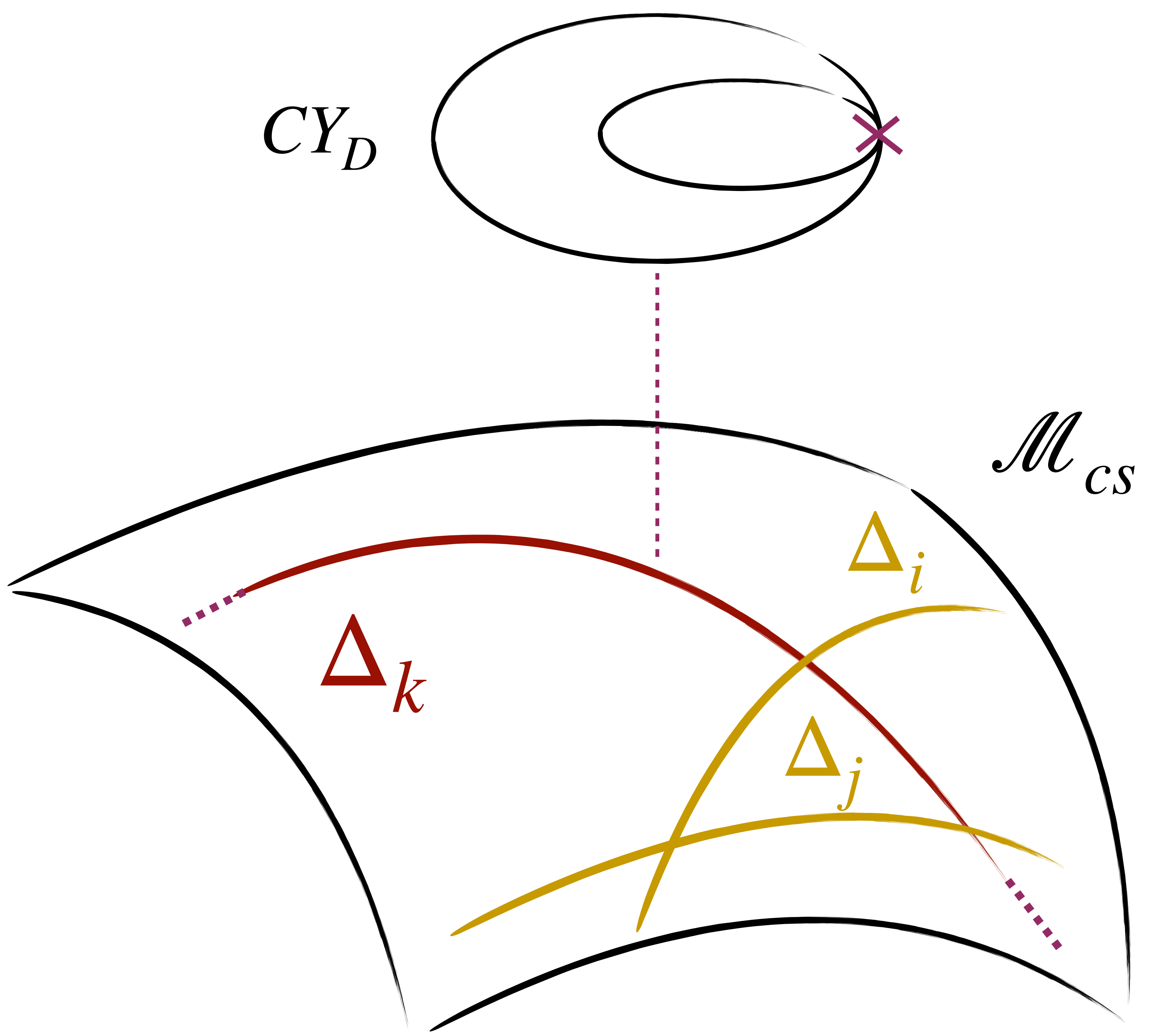
- Local set of coordinates $\{\xi^i, t^k\}$ near the divisor Δ_k s.t.:

$$t^k = a^k + is^k \mapsto const + i\infty$$

The types of singularities located at infinite distance are

- II, III, IV for CY_3 [T.GRIMM, E.PALTI, I.VALENZUELA, '18] [T.GRIMM, C.LI, E.PALTI, '19]
- II, III, IV, V for CY_4 [T.GRIMM, C.LI, I. VALENZUELA, '20]

$$S = \int d^3x\sqrt{-g}\left\{\frac{1}{2}R - G_{kk}\partial s^k \partial s^k + \dots\right\}$$



- Putting a G_4-flux in our CY_4 compactification we generate a 3d potetial:

$$V = \frac{1}{\mathscr{V}_4^3}\left(\int_{Y_4} G_4 \wedge \star \bar{G}_4 - \int_{Y_4} G_4 \wedge G_4\right)$$

- Using the *Growth Theorem* for the Hodge norm we get the leading potential near the boundary Δ_k:

$$V_M \sim \frac{||\rho||_\infty}{\mathscr{V}_4^3}(s^1)^{l_1-4}$$

Redefining the field $s^k \mapsto \phi^k$ in a suitable way, the action becomes:

$$S = \int d^3x\sqrt{-g}\left\{\frac{1}{2}R - G_{kk}\partial\phi^k\phi^k + V(\phi^k)\right\}$$

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Type	**d**	l_1	**Sector**	δ
$II_{0,0}$	1	3 5	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$
$II_{0,1}$	1	3 5	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$
$II_{1,1}$	1	3 5	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$
$II_{1,1}$	1	2 6	*light* *heavy*	$-2\sqrt{2}$ $+2\sqrt{2}$
$III_{0,0}$	2	2 6	*light* *heavy*	-2 $+2$
$III_{0,1}$	2	2 6	*light* *heavy*	-2 $+2$
$III_{0,1}$	2	3 5	*light* *heavy*	-1 $+1$
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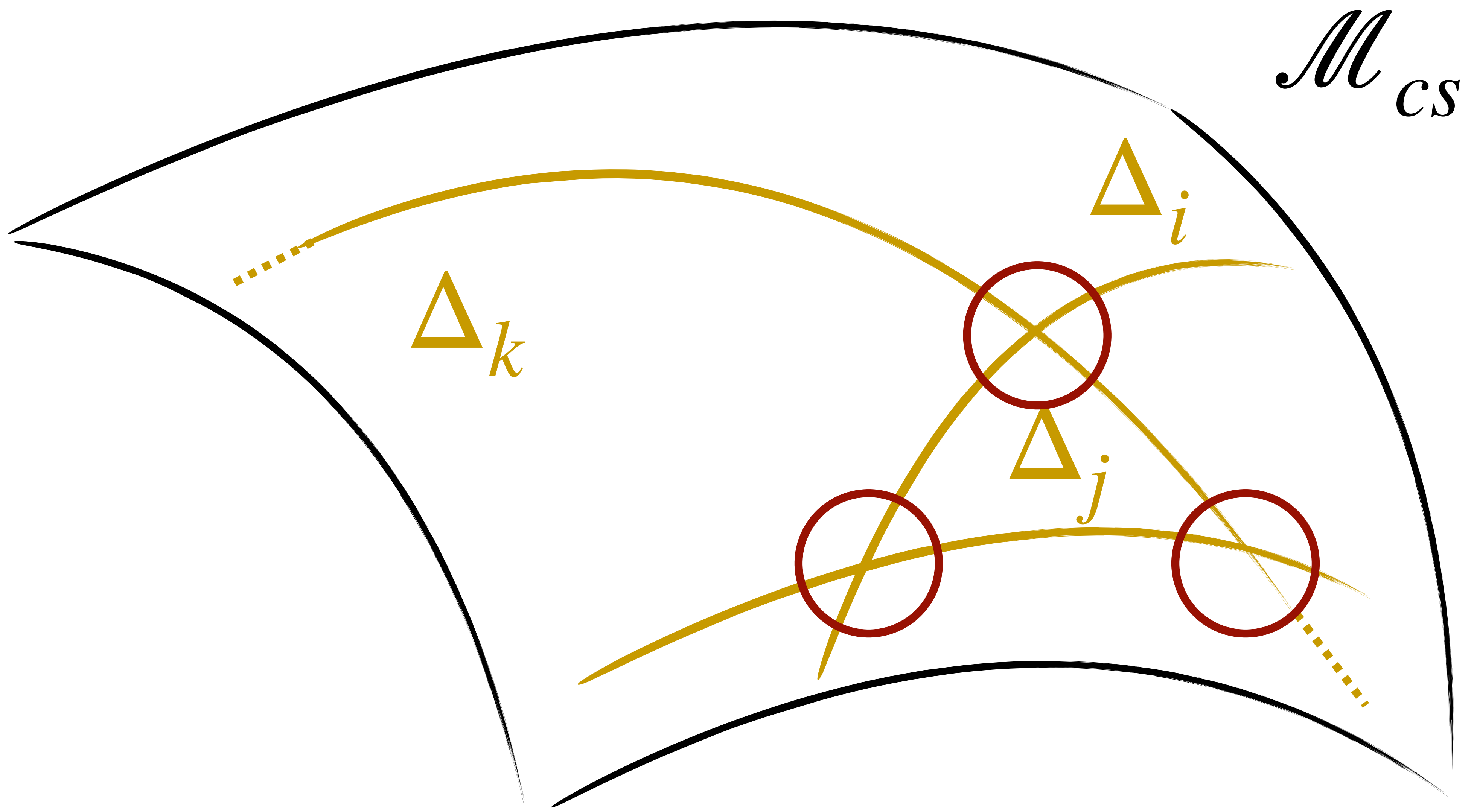
$IV_{0,1}$	3	1 7	*light* *heavy*	$-\sqrt{6}$ $+\sqrt{6}$
$IV_{0,1}$	3	3 5	*light* *heavy*	$-\frac{\sqrt{6}}{3}$ $+\frac{\sqrt{6}}{3}$
$V_{1,1}$	4	0 8	*light* *heavy*	$-2\sqrt{2}$ $+2\sqrt{2}$
$V_{1,1}$	4	2 6	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$
$V_{1,2}$	4	0 8	*light* *heavy*	$-2\sqrt{2}$ $+2\sqrt{2}$
$V_{1,2}$	4	2 6	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$
$V_{1,2}$	4	3 5	*light* *heavy*	$-\frac{1}{\sqrt{2}}$ $+\frac{1}{\sqrt{2}}$
$V_{2,2}$	4	0 8	*light* *heavy*	$-2\sqrt{2}$ $+2\sqrt{2}$
$V_{2,2}$	4	2 6	*light* *heavy*	$-\sqrt{2}$ $+\sqrt{2}$

Intersecting Loci

- We can associate to each singular divisor Δ_k a specific scalar field and a specific ETW-brane in the spacetime picture

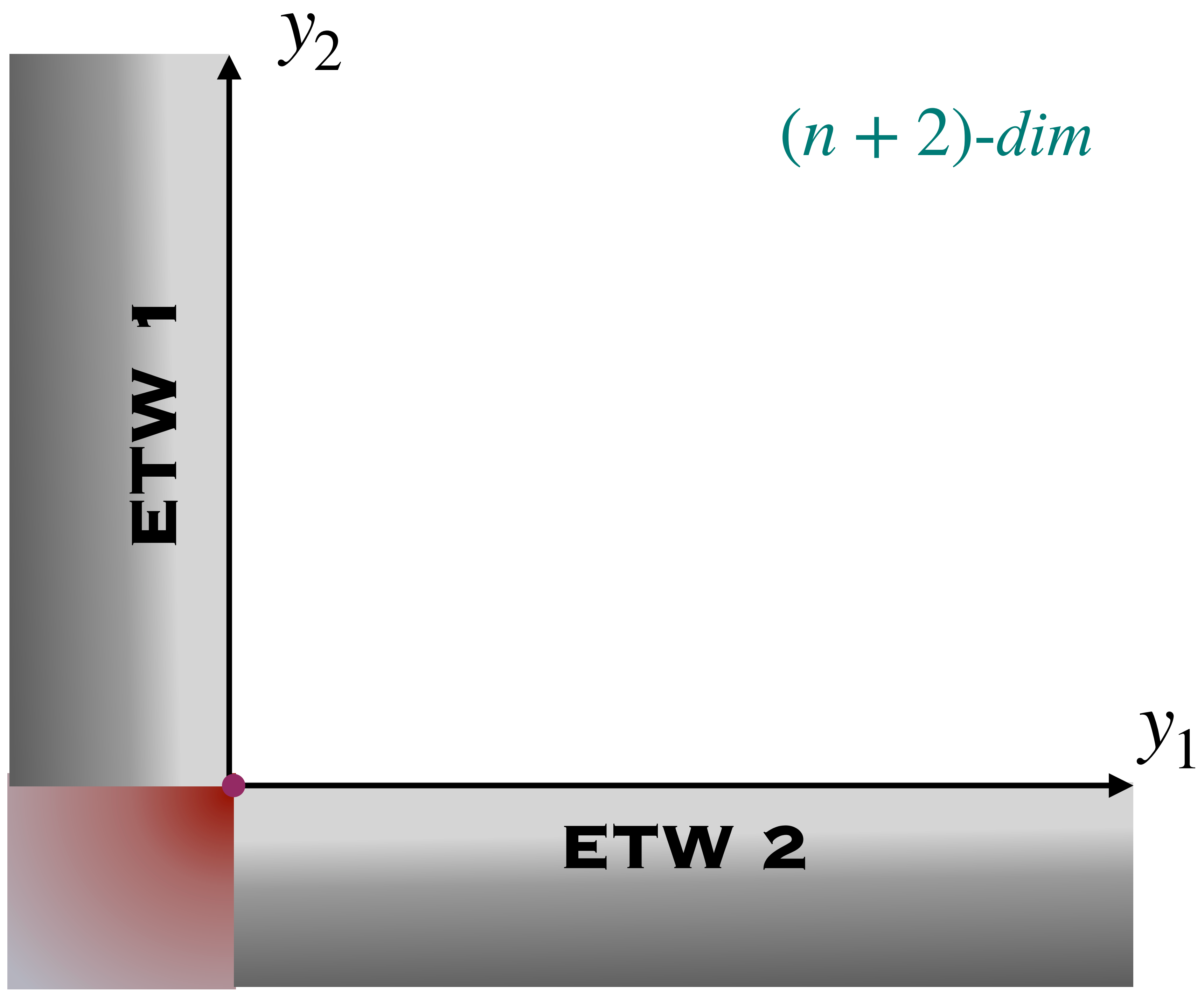


What happens at the intersection points among divisors?

- The spacetime picture is a regime with two scalars where the corresponding ETW-branes intersect.

The classification of the intersection loci in the CY moduli space is already known.

? We need a physical description of the spacetime at the intersection among different ETW-branes **?**



Generalized setup

- Two independent real scalars
- They are associated to two individual ETW-brane
- These two ETW brane are cod-1 object in the $(n+2)$-*dim* spacetime
- They intersect in a cod-2 locus

The idea is to extract a Universal behavior for the spacetime solution around the intersection determined just by the nature of the two individual ETW branes.

$$S = \int d^{n+2}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\phi_1\right)^2 - \frac{1}{2}\left(\partial\phi_2\right)^2 - \frac{\alpha}{2}\partial_\rho\phi_1\partial^\rho\phi_2 - V(\phi_1,\phi_2)\right\}$$

Spacetime solutions

- We compute the Equations of Motion
- We look for solutions satisfying the ansatz:

$$ds^2_{n+2} = e^{2(A_1(y_1)+A_2(y_2))}ds^2_n + e^{2B(y_2)}dy_1^2 + e^{2C(y_1)}dy_1^2$$
$$\phi_1 = \phi_1(y_1)$$
$$\phi_2 = \phi_2(y_2)$$

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→ We obtain the following class of solutions:

$$ds^2_{n+2} = e^{-2\sigma_1(y_1)-2\sigma_2(y_2)}ds^2_n + e^{-2\sigma_2(y_2)}dy_1^2 + e^{-2\sigma_1(y_1)}dy_2^2$$
$$\phi_1 = -\frac{2}{\delta_1}\log y_1$$
$$\phi_2 = -\frac{2}{\delta_2}\log y_2$$

where

$$\sigma_1(y_1) = \pm\frac{4}{n\delta_1^2}\log y_1$$
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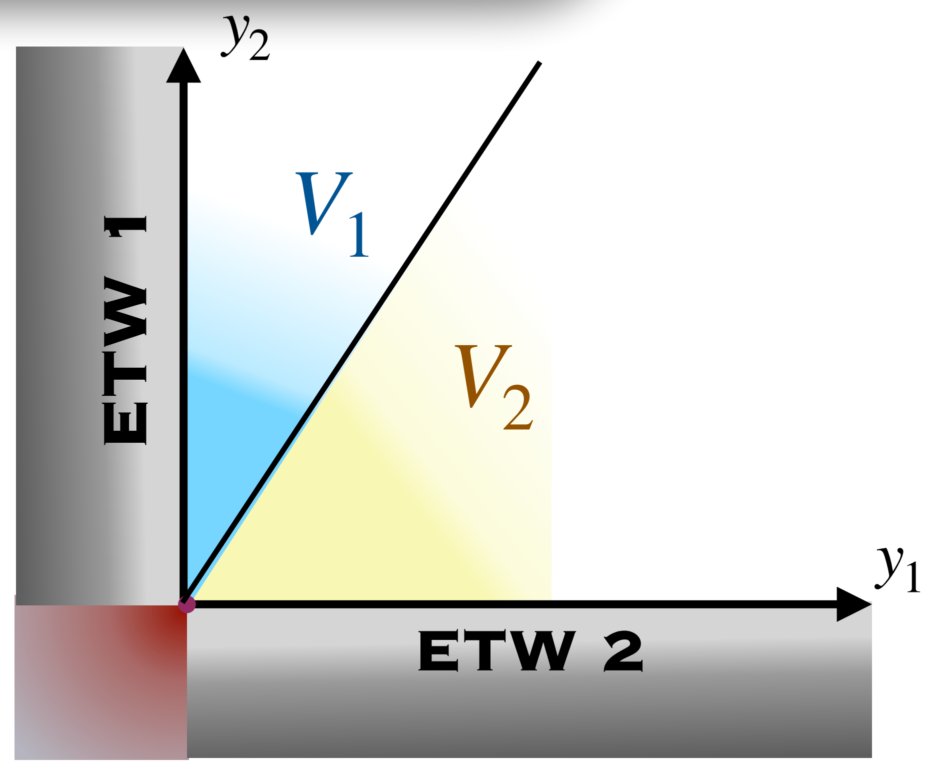
$$\delta_1^2 = \frac{8(n+1)}{n \pm \sqrt{n\left[n+8v_1(n+1)\right]}}$$
$$\delta_2^2 = \frac{8(n+1)}{n \pm \sqrt{n\left[n+8v_2(n+1)\right]}}$$

$$V \simeq V_1 + V_2$$
$$V_1 = -k_1 v_1 c_1(\phi_2)e^{\delta_1\phi_1}$$
$$V_2 = -k_2 v_2 c_2(\phi_1)e^{\delta_2\phi_2}$$



Scaling Relations

$$|R| \sim e^{\delta \mathscr{D}} \qquad \Delta \sim e^{-\frac{\delta}{2}\mathscr{D}}$$

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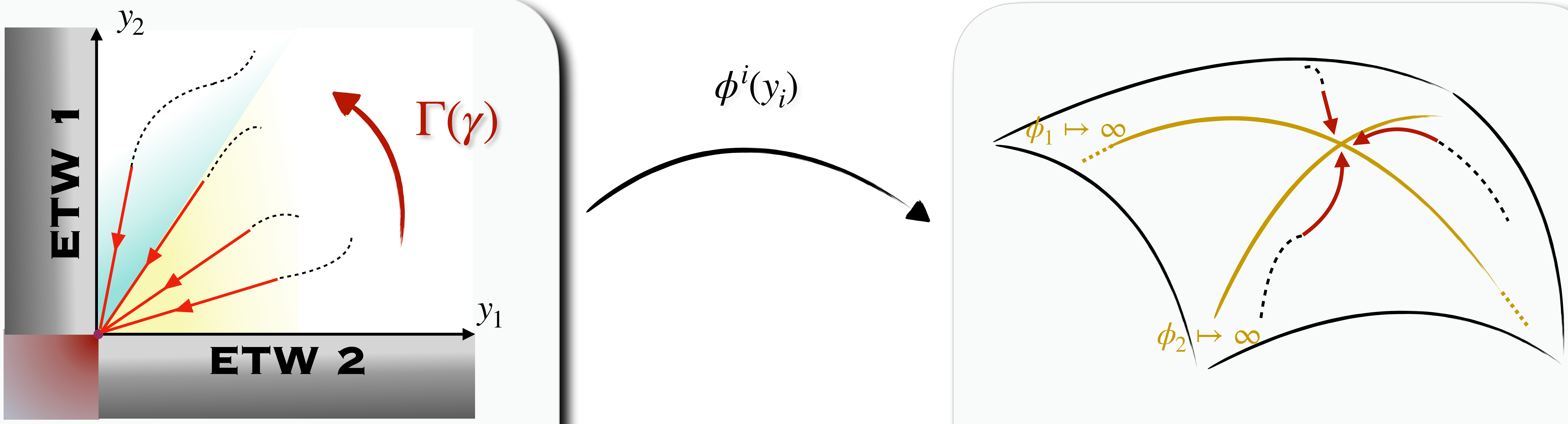
Spacetime

Field space



$$\Delta = \int_{\Gamma(\gamma)} \left[e^{-2\sigma_2} dy_1^2 + e^{-2\sigma_1} dy_2^2\right]^{1/2}$$

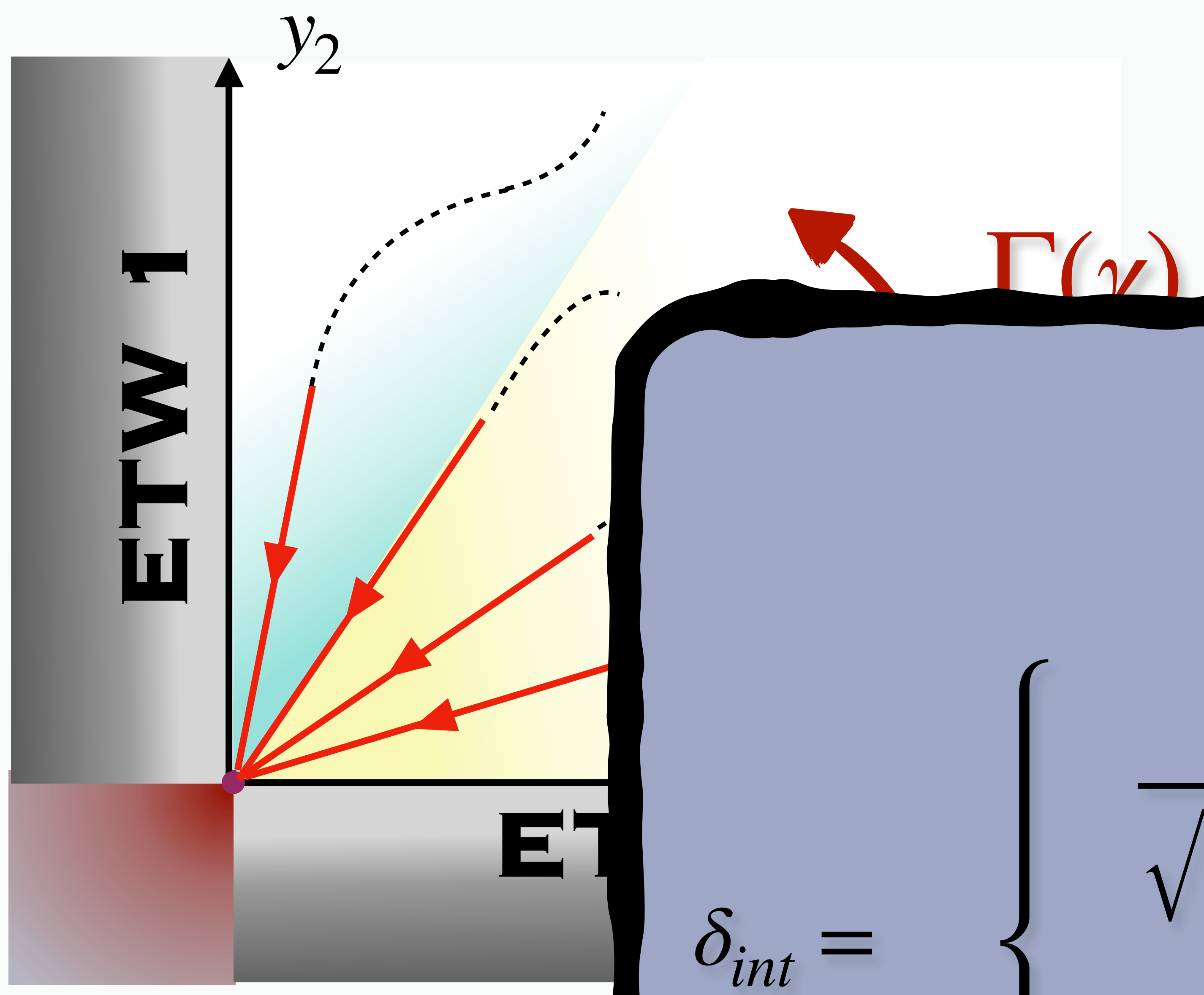
$$\mathscr{D} = \int_{\Gamma(\gamma)} \left[d\phi_1^2 + d\phi_2^2\right]^{1/2}$$

Scaling Relations

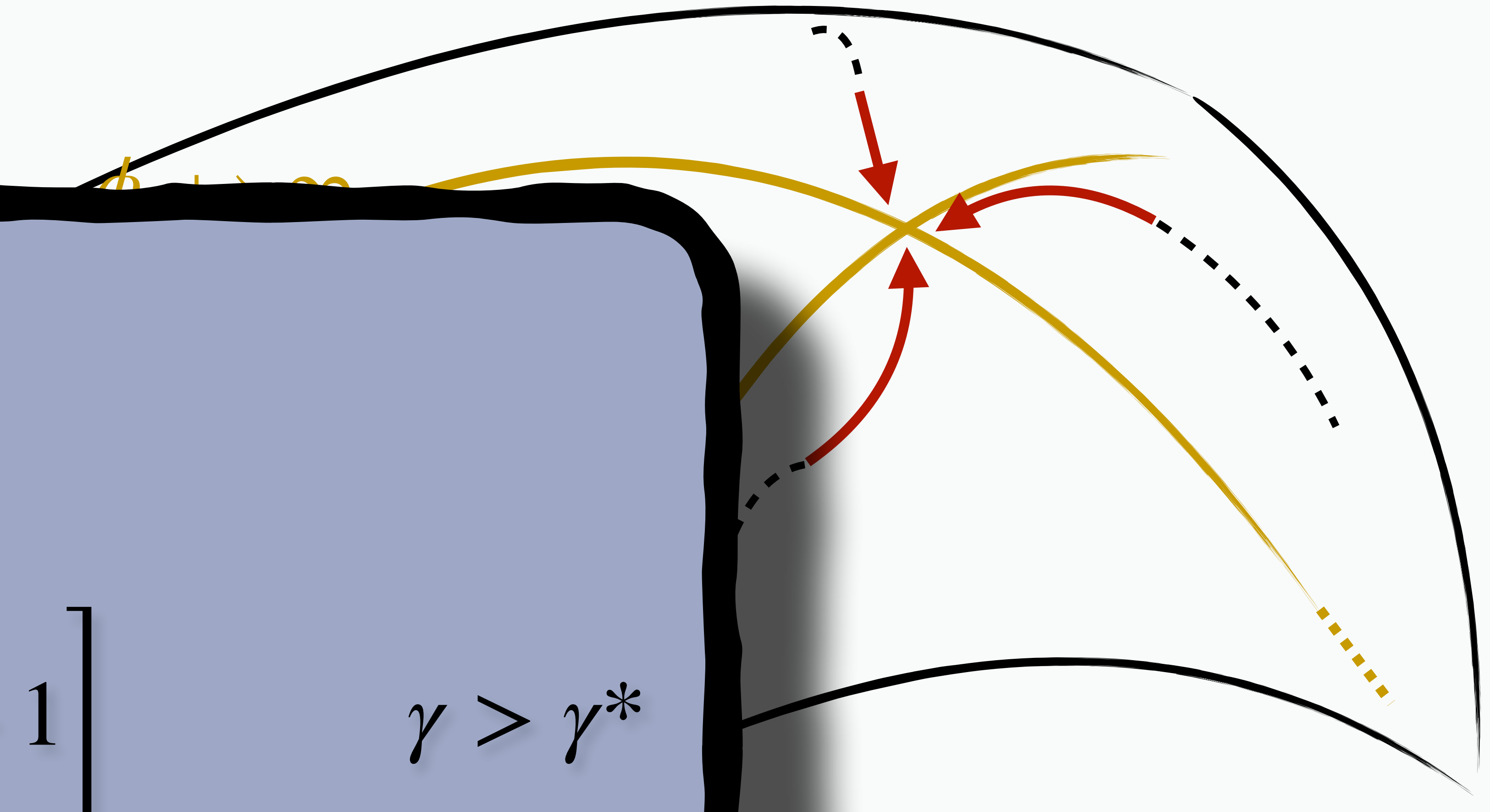
$$|R| \sim e^{\delta_{int}\mathcal{D}} \qquad \Delta \sim e^{-\frac{\delta_{int}}{2}\mathcal{D}}$$

Spacetime

Field space



$\phi^i(y_i)$

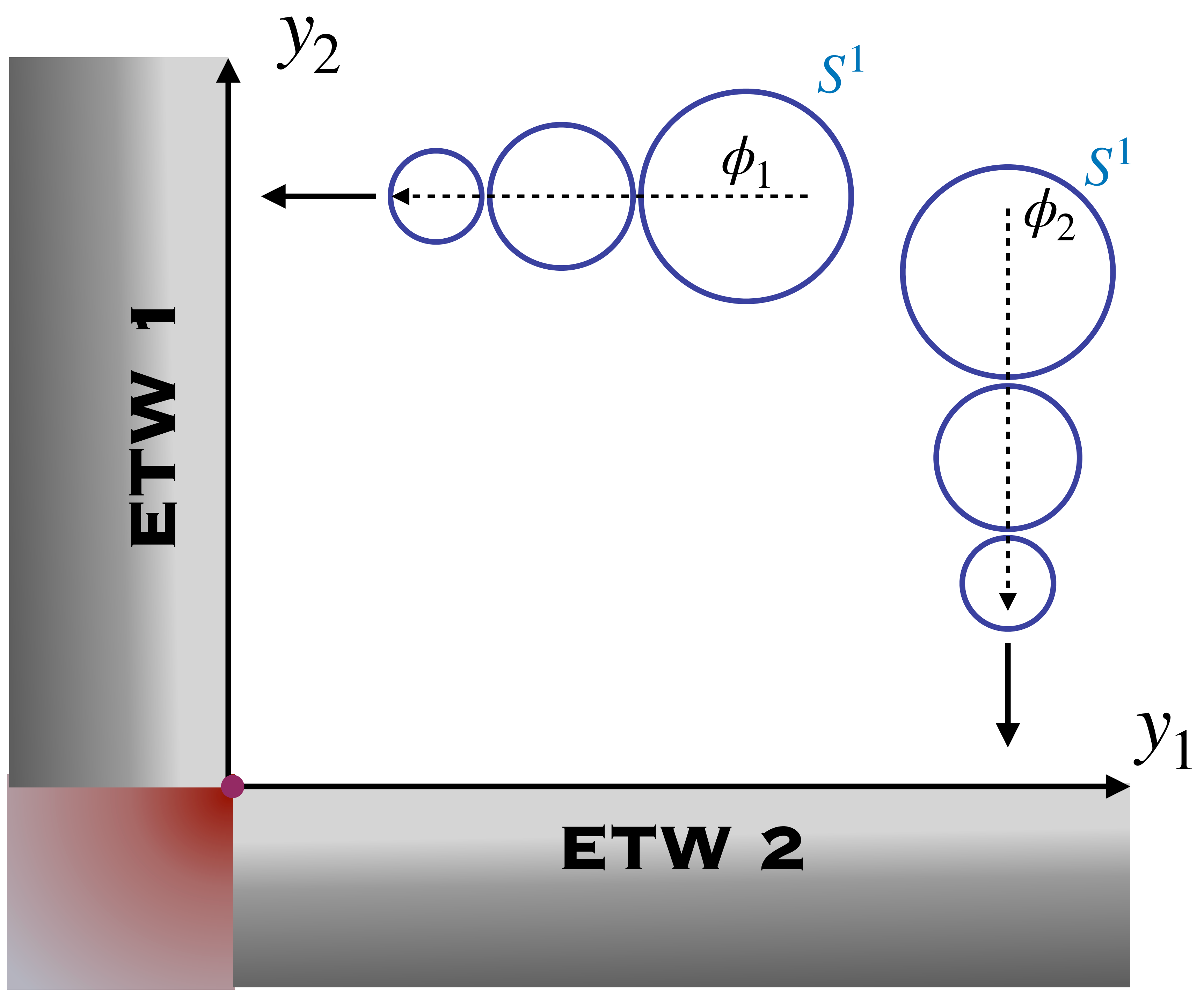


$$\Delta = \int_{\Gamma(\gamma)} \left[e^{-2\sigma_2}dy_1^2 \right.$$

$$\left. \phi_1^2 + d\phi_2^2\right]^{1/2}$$

$$\delta_{int} = \begin{cases} \dfrac{\delta_1\delta_2}{\sqrt{\gamma^2\delta_2^2 + (1-\gamma)^2\delta_1^2}}\left[\dfrac{(1-\gamma)}{2}\left(\dfrac{8}{n\delta_2^2} - 2\right) + 1\right] & \gamma > \gamma^* \\ \dfrac{\delta_1\delta_2}{\sqrt{\gamma^2\delta_2^2 + (1-\gamma)^2\delta_1^2}}\left[\dfrac{(\gamma)}{2}\left(\dfrac{8}{n\delta_1^2} - 2\right) + 1\right] & \gamma > \gamma^* \end{cases}$$

Example: $S^1 \times S^1$



- We start from a $(n+4)$-*dim* Einstein gravity
- We compactify two directions on $S^1 \times S^1$

The reduced $(n+2)$-*dim* action is:

$$S = \int d^{n+2}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\phi_1\right)^2 - \frac{1}{2}\left(\partial\phi_2\right)^2 - \frac{\alpha}{2}\partial_\rho\phi_1\partial^\rho\phi_2\right\}$$

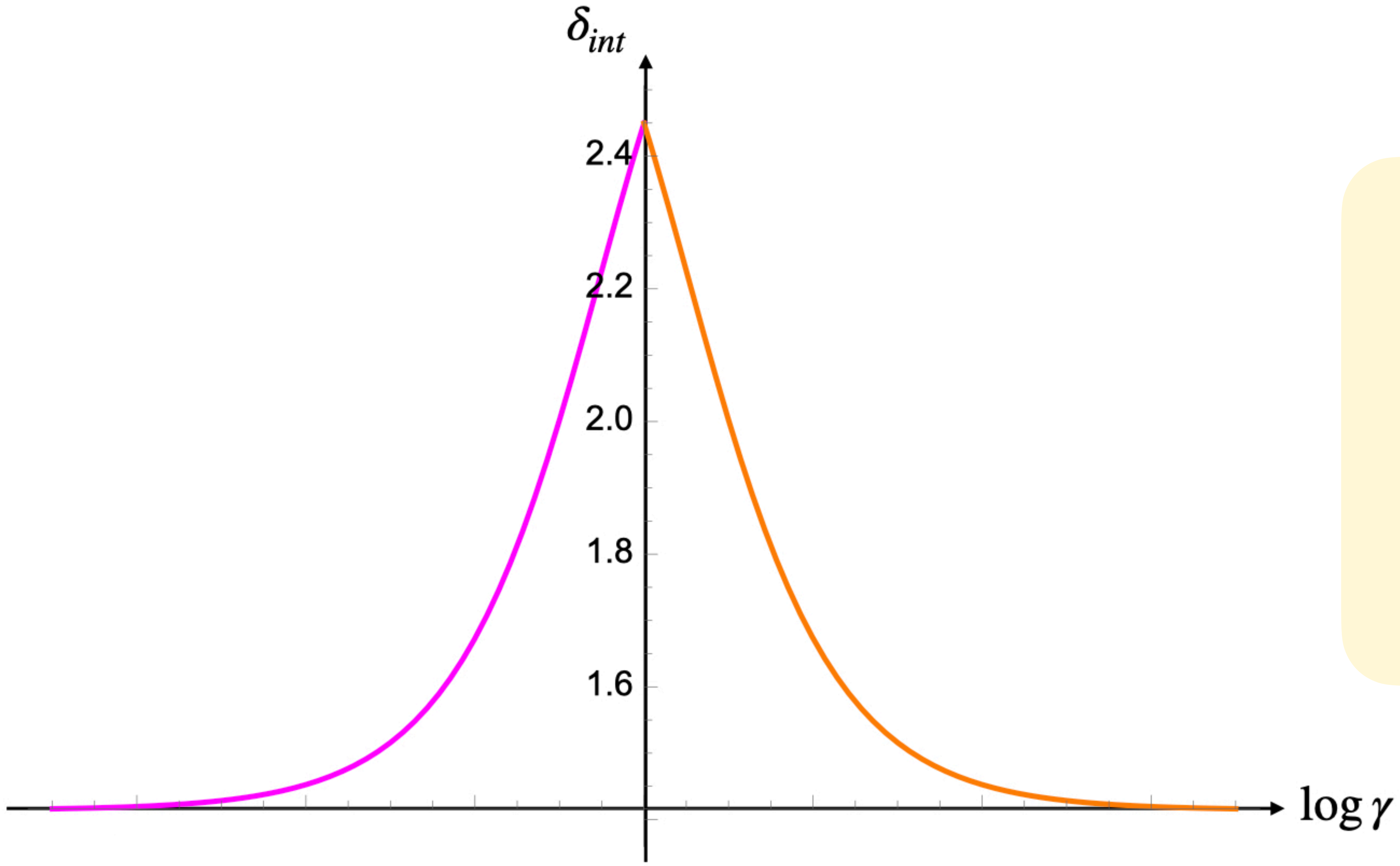
The spacetime solutions are:

- $ds^2_{n+2} = y_1^{\frac{2}{1+n}} y_2^{\frac{2}{1+n}} ds_n^2 + y_2^{\frac{2}{1+n}} dy_1^2 + y_1^{\frac{2}{1+n}} dy_2^2$
- $\phi_1 = -\sqrt{\frac{n}{n+1}}\log y_1$
- $\phi_2 = -\sqrt{\frac{n}{n+1}}\log y_2$

Critical Exponents

$$\delta_{1/2} = 2\sqrt{\frac{n+1}{n}}$$

- The path-dependent critical exponent controlling the scaling relations is now a symmetric function:

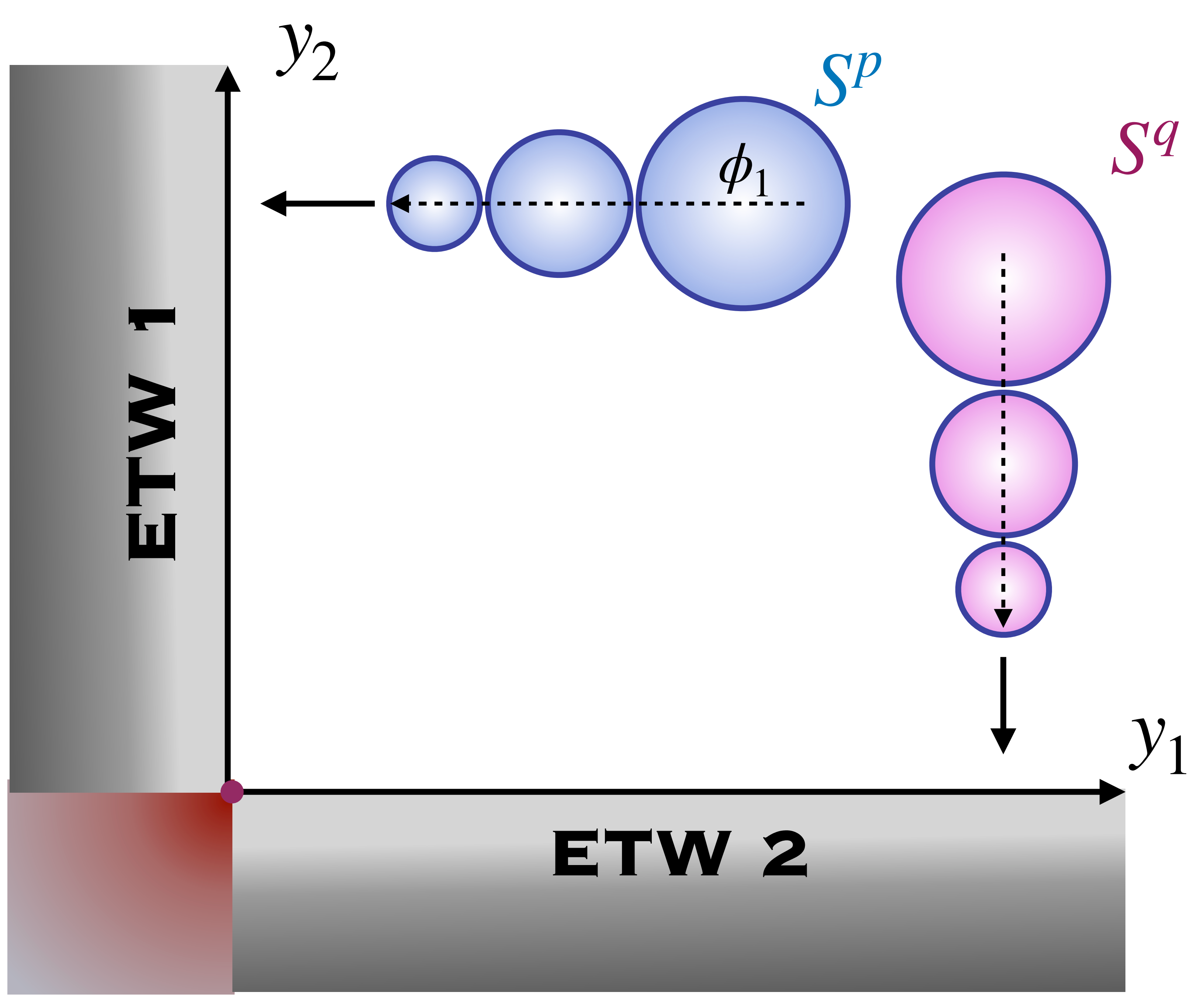


$$\delta_{int} = \begin{cases} 2\sqrt{\frac{n+1}{n}}\left[\frac{n(1-\gamma)}{n+1}+1\right] & \gamma > \frac{1}{2} \\ 2\sqrt{\frac{n+1}{n}}\left[\frac{n\gamma}{n+1}+1\right] & \gamma < \frac{1}{2} \end{cases}$$

- The maximum is obtained following the path reaching the intersection along the diagonal line between the two ETW-branes

- The asymptotic values are the critical exponents δ_1 and δ_2 associated to the two individual ETW-branes

Example: $S^p \times S^q$



- We start from a $(n+2+p+q)$-*dim* Einstein gravity
- We compactify two directions on $S^p \times S^q$

The reduced $(n+2)$-*dim* action is:

$$S = \int d^{n+2}x\sqrt{-g}\left\{\frac{1}{2}R - \frac{1}{2}\left(\partial\phi_1\right)^2 - \frac{1}{2}\left(\partial\phi_2\right)^2 - \frac{\alpha}{2}\partial_\rho\phi_1\partial^\rho\phi_2 + \right.$$
$$\left. -\frac{1}{2}\left(\frac{n}{n+p}\right)^2 p(p-1)e^{\pm 2\sqrt{\frac{q}{n(n+q)}}\phi_2}e^{\pm 2\sqrt{\frac{n+p}{np}}\phi_1} - \frac{1}{2}\left(\frac{n}{n+q}\right)^2 q(q-1)e^{\pm 2\sqrt{\frac{p}{n(n+p)}}\phi_1}e^{\pm 2\sqrt{\frac{n+q}{nq}}\phi_2}\right\}$$

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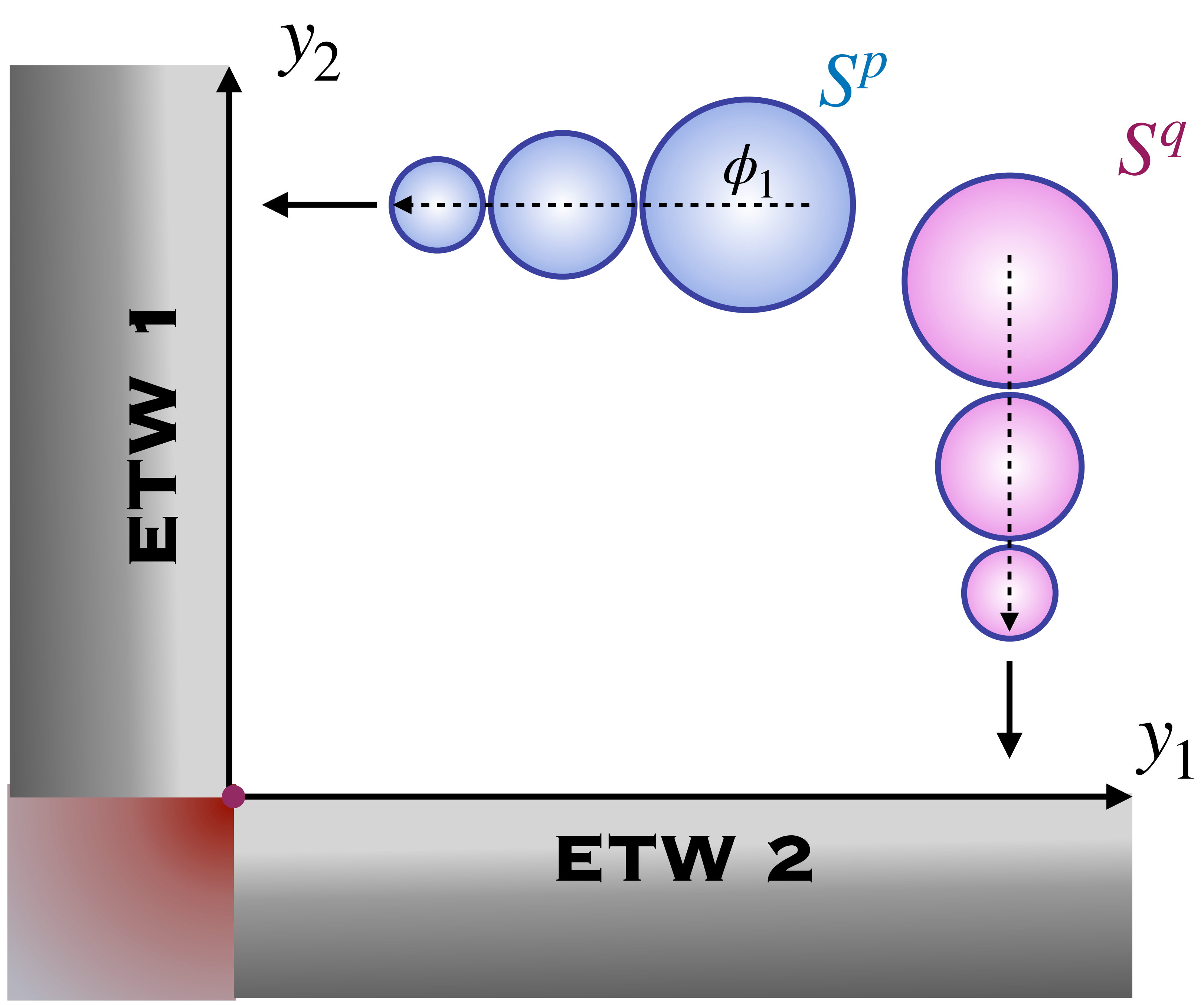

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$$\left. -\underbrace{\frac{1}{2}\left(\frac{n}{n+p}\right)^2 p(p-1)}_{v_1} e^{\pm 2\sqrt{\frac{q}{n(n+q)}}\phi_2} e^{\pm 2\sqrt{\frac{n+p}{np}}\phi_1} - \underbrace{\frac{1}{2}\left(\frac{n}{n+q}\right)^2 q(q-1)}_{v_2} e^{\pm 2\sqrt{\frac{p}{n(n+p)}}\phi_1} e^{\pm 2\sqrt{\frac{n+q}{nq}}\phi_2}\right\}$$

The spacetime solutions are:

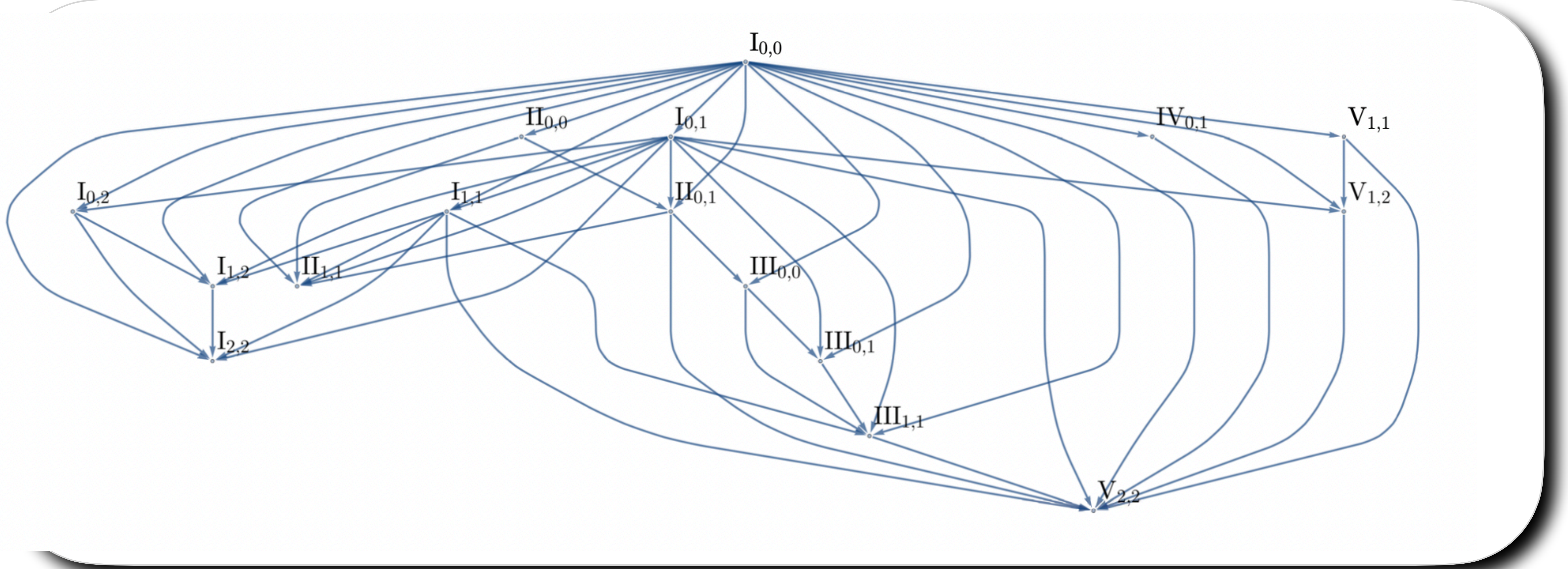
- $ds^2_{n+2} = y_1^{\frac{2p}{p+n}} y_2^{\frac{2q}{q+n}} ds_n^2 + y_2^{\frac{2q}{q+n}} dy_1^2 + y_1^{\frac{2p}{p+n}} dy_2^2$
- $\phi_1 = -\sqrt{\frac{np}{n+p}}\log y_1$
- $\phi_2 = -\sqrt{\frac{nq}{n+q}}\log y_2$

Critical Exponents

$$\delta_1 = \left[\frac{8(n+1)}{n \pm \sqrt{n\left[n + 8v_1(n+1)\right]}}\right]^{1/2} = 2\sqrt{\frac{n+p}{np}}$$

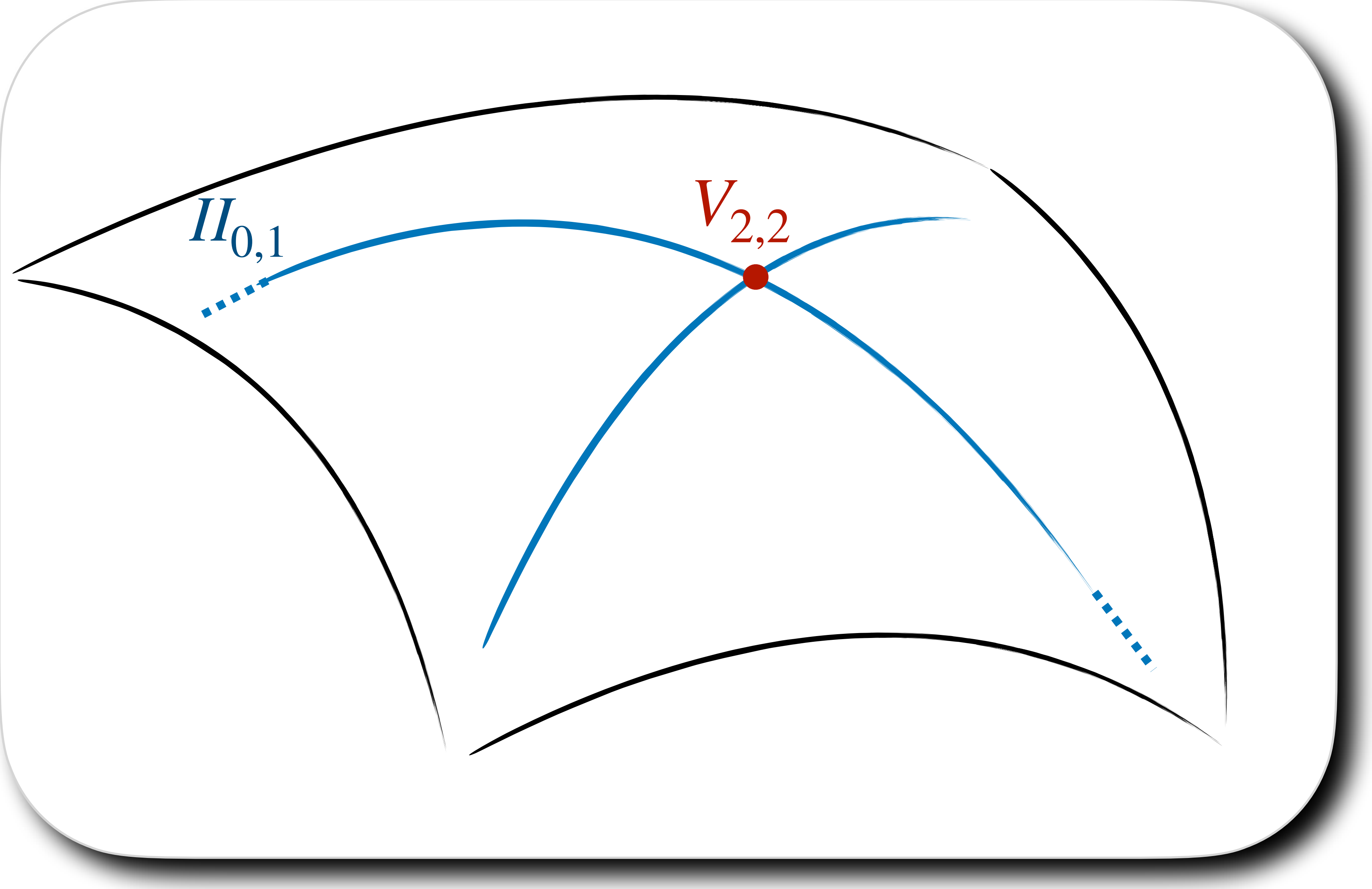
$$\delta_2 = \left[\frac{8(n+1)}{n \pm \sqrt{n\left[n + 8v_2(n+1)\right]}}\right]^{1/2} = 2\sqrt{\frac{n+q}{nq}}$$

Example: CY_4 compactification



Enhancements network in the moduli space of CY_4 with $h^{3,1} = 2$

[GRIMM, LI, VALENZUELA, '20]



Intersecting divisors in the moduli space

$$V_M \sim c_1(s^1)(s^2)^3 + c_2(s^1)(s^2) + c_3\frac{s^1}{s^2} + c_4\frac{s^1}{(s^2)^3} + c_5 + c_6\frac{(s^2)^3}{s^1} + c_7\frac{s^2}{s^1} + \frac{c_8}{(s^1)(s^2)} + \frac{c_9}{(s^1)(s^2)^3}$$

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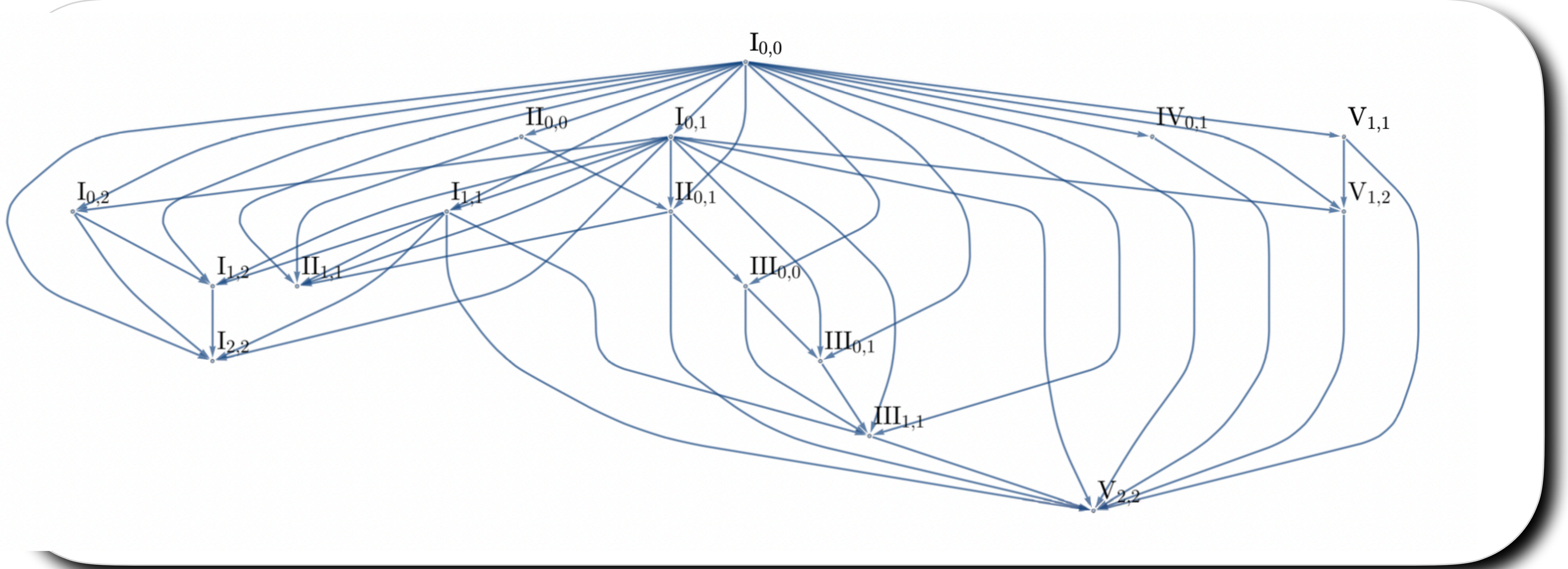

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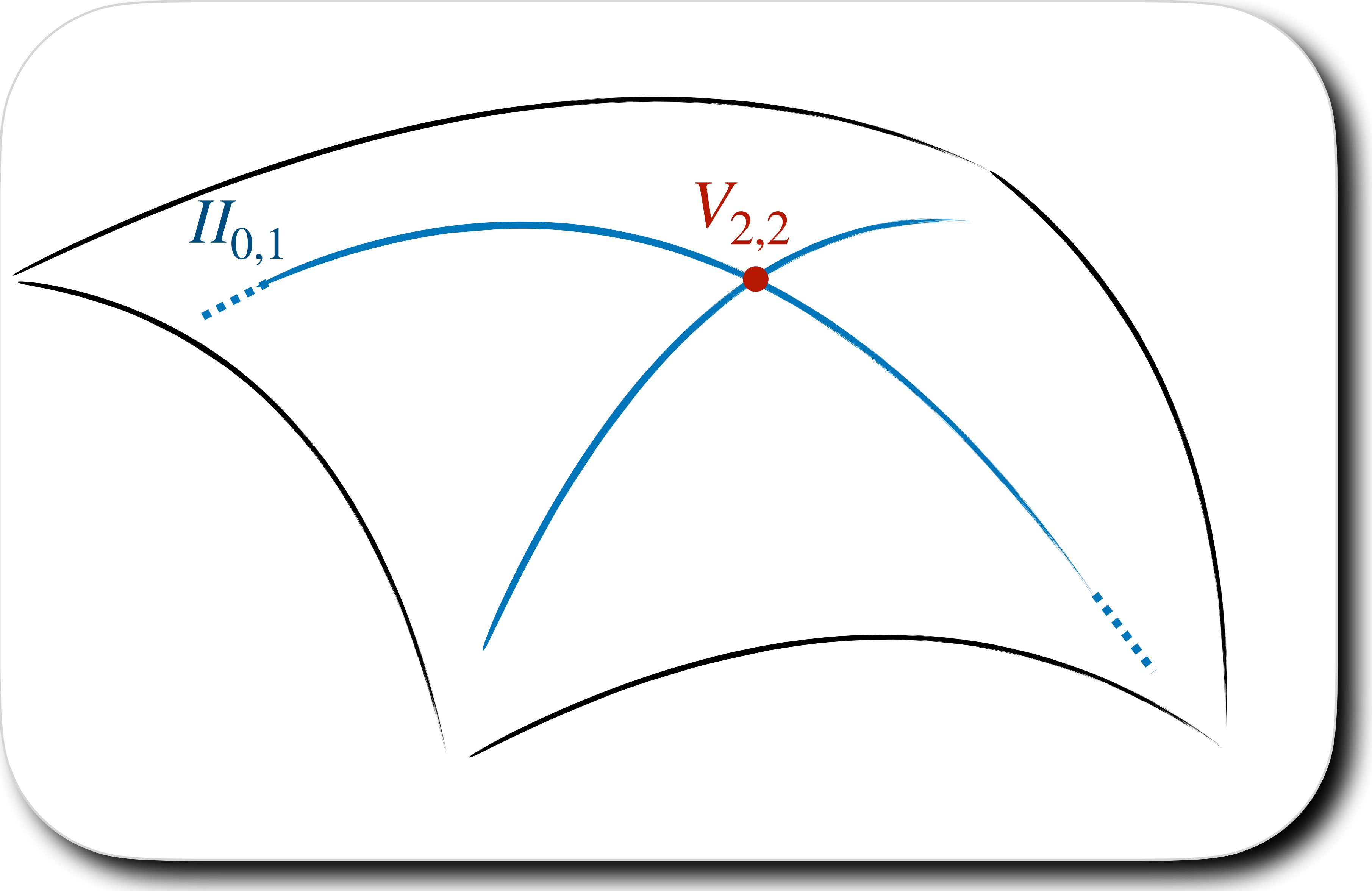
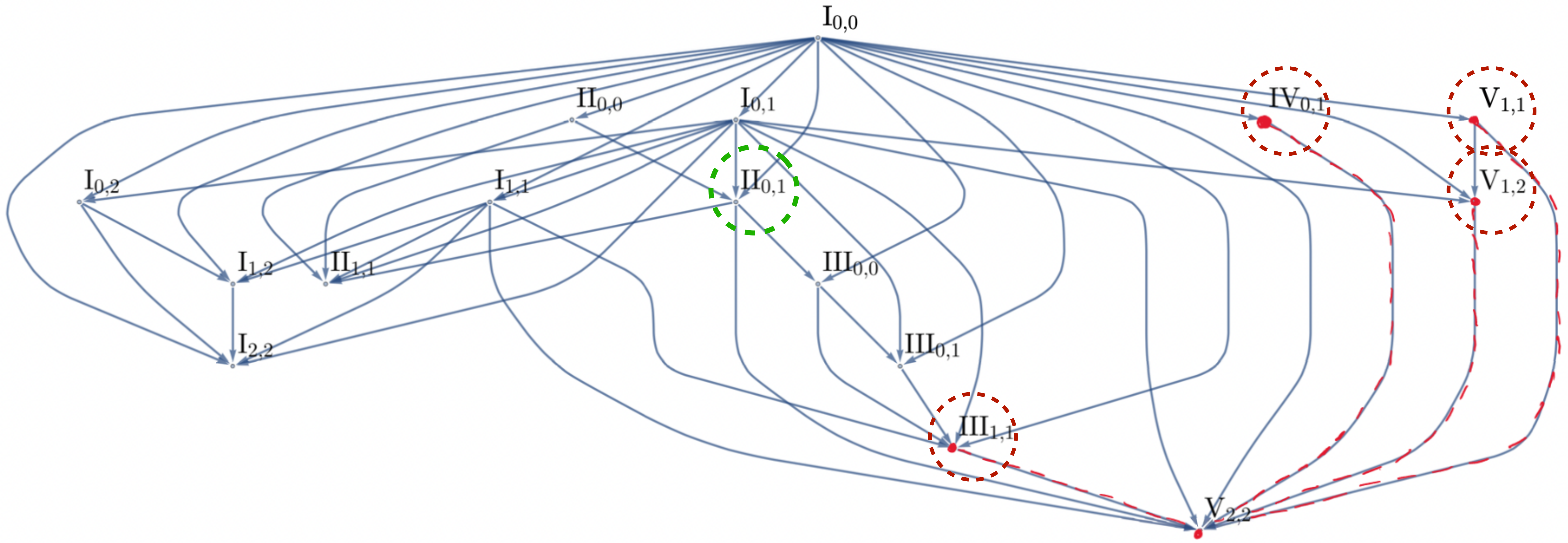


Intersecting divisors in the moduli space

$$V_M \sim c_1(s^1)(s^2)^3 + c_2(s^1)(s^2) + c_3\frac{s^1}{s^2} + c_4\frac{s^1}{(s^2)^3} + c_5 + c_6\frac{(s^2)^3}{s^1} + c_7\frac{s^2}{s^1} + \frac{c_8}{(s^1)(s^2)} + \frac{c_9}{(s^1)(s^2)^3}$$

$\rightarrow$ 'V_1' $\longrightarrow$ δ_1

where we use the redefinition: $\phi_1 = \sqrt{\frac{d_{II}}{2}}\log s^1 = \sqrt{\frac{1}{2}}\log s^1$



- The other divisor has to correspond to one of the infinite distance singularities that can enhance to $V_{2,2}$

$$II_{0,1} \quad III_{1,1} \quad IV_{0,1} \quad IV_{1,2} \quad V_{1,1}$$

- I already chose the G_4-flux: this selects a few number of possibilities in choosing the dominant term of the potential controlling the regime near to the second ETW-brane

Once selected it I can compute the corresponding δ_2

We can compute all the building blocks to construct the corresponding spacetime local model.

Conclusions

Motivated by the study of infinite distance limits in the field space, we needed to extend our Dynamical Cobordism formalism with more fields in order to include all the possible infinite distance singularities.

- We have construct a general analysis for an effective theory with two fields corrisponding to two intersecting ETW-brane.
  The model is a spacetime realisation of a cod-2 infinite distance limit in the moduli space.
- The spacetime solution is completely specified by the nature of the two intersecting brane throught their associated critical exponents.
- We showed that near a cod-2 singularity the scaling relations discovered for Dynamical Cobordism associated to cod-1 object are still valid. The only difference is that now they involve a path-dependent parameter δ_{int}.
- We discussed different examples where the method works.

Thank you!