# The Heterotic String Potential Energy

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# Outline.

Here is an outline of my talk. The first part of the talk is an overview of recent results. The second part is a brief sketch of work in progress.

1. Part 1: The $O(16) \times O(16)$ String on $S^1$
2. Part 2: Heterotic Potential Energy

# Part 1: The $O(16) \times O(16)$ String on $S^1$

# Acknowledgements

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This section is based on arXiv:2307.13745 .

# Quantum Gravity without SUSY?

Supersymmetry has been critical to our exploration of quantum gravity via string theory. We might formulate a series of questions about the importance of supersymmetry:

- Is SUSY required at the Planck scale?
- Is SUSY required for Minkowski spacetimes?
- Is SUSY required for *AdS* solutions, or equivalently for holographic CFTs?

Let's exclude very low-dimensional spacetimes and consider $D > 2$.

It is easy to find supersymmetric Minkowski and *AdS* solutions. It is hard to find non-supersymmetric solutions!

# Non-supersymmetric strings

Given that the universe appears to be very non-supersymmetric at observed energy scales, we should explore quantum gravity without supersymmetry.

String theory without supersymmetry at the string scale provides a perturbative arena for such a study.

There are three known non-supersymmetric strings in $D = 10$ without tachyons:

- $O(16) \times O(16)$ heterotic string (Dixon, Harvey; Alvarez-Gaume, Ginsparg, Moore, Vafa)
- type 0'B (Sagnotti)
- $USp(32)$ open string theory (Sugimoto)

Because supersymmetry is broken, there will generically be a 1-loop potential for the dilaton:

$$V_{1-\mathrm{loop}} = \int_{\mathcal{F}} Z_{1-\mathrm{loop}}.$$

Each of the three non-SUSY strings in $D = 10$ has a positive 1-loop dilaton potential,

$$V \sim e^{\alpha\phi}$$

with $\alpha > 0$.

A conceptual picture of strings breaking supersymmetry, generated by the AI DALL-E.

# Connections to the swampland program

There are very interesting swampland conjectures roughly suggesting there are no stable $AdS$ spacetimes and no $AdS/CFT$ holography without supersymmetry.

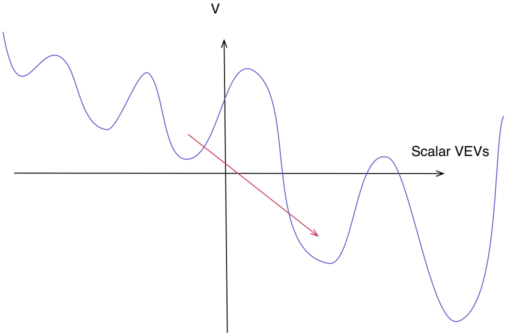
(Ooguri & Vafa, Freivogel & Kleban)

On the other hand, there are explicit field theory arguments suggesting that suitable non-supersymmetric CFTs might exist.

(Giombi & Perlmutter)

These constructions involve perturbing UV SUSY fixed points by double trace operators and flowing. Whether the end point of the flow is a CFT at finite $N$ is still unsettled.

Today we will revisit one of the more basic ingredients in the landscape versus swampland debates. We will want to understand to what extent is the picture of an effective potential correct?




Understanding features of the potential energy for the $O(16) \times O(16)$ heterotic case, and even when the potential makes sense, is the goal of today's talk.

# Heterotic string on $S^1$

The $O(16) \times O(16)$ heterotic string can be constructed from either heterotic superstring as a $\mathbb{Z}_2$ orbifold.

For the $O(16) \times O(16)$ heterotic string, the 1-loop potential in $D = 10$ takes the form

$$V_{1-\mathrm{loop}} \sim e^{\frac{5}{2}\phi}.$$

Toroidal compactifications on $T^d$ with flat gauge bundles (deformable to the trivial connection) provide tree-level solutions with a Narain moduli space:

$$\frac{O(d+16, d; \mathbb{R})}{O(d+16, \mathbb{R}) \times O(d, \mathbb{R}) \times O(d+16, d; \mathbb{Z})}.$$

We will focus on this string compactified on $S^1$. This case was first studied by Ginsparg and Vafa more than 37 years ago .

If we only consider the radius $R$ of the circle, there is a special point under T-duality

$$R \to \frac{\alpha'}{R},$$

which is the self-dual radius. At this point the gauge symmetry is enhanced to

$$SO(16) \times SO(16) \times SU(2).$$

The moduli space, however, is 17-dimensional including the Wilson lines and there are other special points where the gauge symmetry is enhanced.

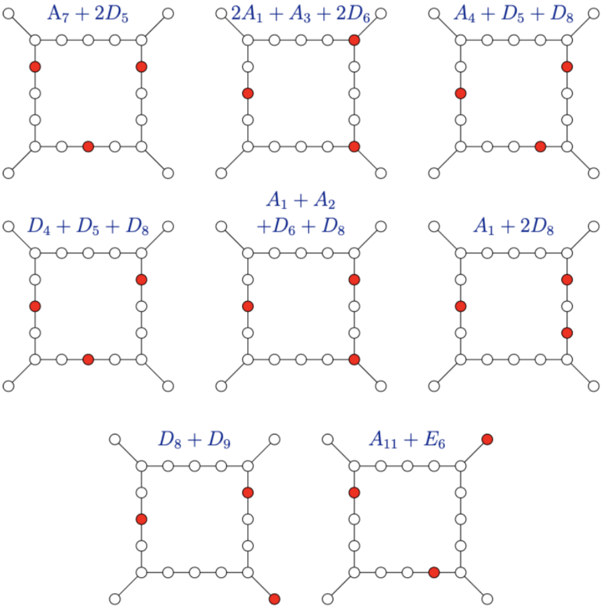
# Critical points

Each point of enhanced symmetry is an extremum of the 1-loop potential.

We've found a list of all the points of maximal enhancement (rank 17 without abelian factors other than the graviphoton). There are 107 such points though most have tachyons.

There are also critical points which are *not* maximal enhancements. We found 4 such points without tachyons.

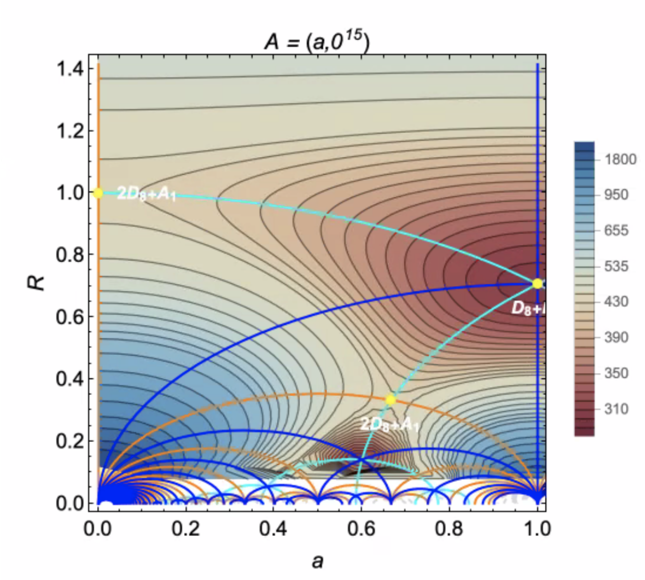
There are 8 maximal enhancements without tachyons.



Some of these critical points are very strange and highlight the extremely rich and peculiar nature of the potential energy.

Ginsparg and Vafa called them ‘knife edges’ in analogy with the knife-edge of Capitol Peak near Aspen:

Here is another look at the potential landscape:



Every extremum has positive cosmological constant. Even the last one listed below with *no* massless fermions.

*Saddle points*

$$SO(16) \times SO(16) \times SU(2)$$
$$SO(16) \times SO(10) \times SU(5)$$
$$SO(16) \times SO(12) \times SU(3) \times SU(2)$$

*Local maximum*

$$SO(10) \times SO(10) \times SU(8)$$

*Edge of tachyonic regions*

$$SO(16) \times SO(18)$$
$$SO(16) \times SO(10) \times SO(8)$$
$$(SO(12) \times SU(2))^2 \times SU(4)$$
$$E_6 \times SU(12)$$

# Cosmological constant

The cosmological constant for the 8 extremal maximum enhancement points.

| | $N_v$ | $N_s$ | $N_c$ | $N_0$ | $N_0'$ | $\Lambda_{m=0}$ | $\Lambda$ |
|---|---|---|---|---|---|---|---|
| $\mathrm{A}_1 + 2\mathrm{D}_8$ | 226 | 256 | 256 | 0 | 0 | 341.6 | 431.4 |
| $\mathrm{A}_1 + \mathrm{A}_2 + \mathrm{D}_6 + \mathrm{D}_8$ | 180 | 192 | 192 | 0 | 0 | 251.8 | 383.5 |
| $\mathrm{A}_4 + \mathrm{D}_5 + \mathrm{D}_8$ | 172 | 128 | 160 | 0 | 0 | 155.4 | 359.2 |
| $\mathrm{A}_7 + 2\mathrm{D}_5$ | 136 | 0 | 170 | 0 | 0 | 65.6 | 303.8 |
| $\mathrm{D}_8 + \mathrm{D}_9$ | 256 | 128 | 288 | 256 | 0 | 168.7 | 305.0 |
| $\mathrm{D}_4 + \mathrm{D}_5 + \mathrm{D}_8$ | 176 | 208 | 128 | 256 | 0 | 168.7 | 305.0 |
| $2\mathrm{A}_1 + \mathrm{A}_3 + 2\mathrm{D}_6$ | 136 | 128 | 168 | 256 | 0 | 168.7 | 305.0 |
| $\mathrm{A}_{11} + \mathrm{E}_6$ | 204 | 0 | 0 | 408 | 2 | $-243.7$ | 180.4 |

Now this talk is about saddles because we found a surprise.

Every critical point is a saddle or a knife-edge!

# Hessian

The Hessian for the 8 extremal maximum enhancement points. For the knife-edge points, this will actually diverge.

| Group | $R^2$ | Wilson line | $\Lambda$ | $\nabla_i \nabla_j \Lambda$ |
|---|---|---|---|---|
| $\left[Spin(16)^2\right]/\mathbb{Z}_2 \times SU(2)$ | 1 | $0^{16}$ | 431.4 | $-306^{16}, 831$ |
| $\left[Spin(16) \times Spin(12) \times SU(2)\right]/\mathbb{Z}_2 \times SU(3)$ | $\frac{3}{4}$ | $0^{14}, \frac{1}{2}^2$ | 383.5 | $-307^{15}, 544^2$ |
| $Spin(16) \times Spin(10) \times SU(5)$ | $\frac{5}{8}$ | $0^{13}, \frac{1}{2}^3$ | 359.2 | $-569^5, -256^8, 355^4$ |
| $\left[Spin(10)^2 \times SU(8)\right]/\mathbb{Z}_4$ | $\frac{1}{4}$ | $0^4, \frac{1}{2}^4, \frac{1}{4}^8$ | 303.8 | $-195^{17}$ |
| $Spin(18) \times Spin(16)$ | $\frac{1}{2}$ | $0^{15}, 1$ | 305.0 | $-1283^8, 588^9$ |
| $\left[Spin(16) \times Spin(10) \times Spin(8)\right]/\mathbb{Z}_2$ | $\frac{1}{2}$ | $0^{12}, \frac{1}{2}^4$ | 305.0 | $-1283^4, -347^8, 588^5$ |
| $\left[Spin(12)^2 \times SU(4) \times SU(2)^2\right]/\mathbb{Z}_2^2$ | $\frac{1}{2}$ | $0^6, \frac{1}{2}^2, 0^6, \frac{1}{2}^2$ | 305.0 | $-1283^2, -347^{12}, 588^3$ |
| $\left[E_6 \times SU(12)\right]/\mathbb{Z}_3$ | $\frac{1}{8}$ | $0^3, \frac{1}{2}^5, -\frac{1}{4}, \frac{1}{4}^7$ | 180.4 | $-72^{17}$ |

# Refined de Sitter conjecture

For the 4 non-knife-edge extremal maximum enhancement points the refined de Sitter conjecture,

$$-M_P^2 \frac{V''}{V} \equiv c^2 \gtrsim O(1),$$

(Obied, Ooguri, Spodyneiko, Vafa)

is satisfied with $c = 0.64$.

| Group | $\Lambda$ | $\min(\nabla_i \nabla_j V)/V$ |
|---|---|---|
| $\left[Spin(16)^2\right]/\mathbb{Z}_2 \times SU(2)$ | 431.4 | $-0.71$ |
| $\left[Spin(16) \times Spin(12) \times SU(2)\right]/\mathbb{Z}_2 \times SU(3)$ | 383.5 | $-0.80$ |
| $Spin(16) \times Spin(10) \times SU(5)$ | 359.2 | $-1.58$ |
| $\left[Spin(10)^2 \times SU(8)\right]/\mathbb{Z}_4$ | 303.8 | $-0.64$ |

This is in a highly stringy regime!

What are we to make of such a bizarre and rich potential landscape?

- The potential is discontinuous when computed around this unstabilized configuration.
- Are the peculiar features like the knife-edge smoothed when we include backreaction? i.e. from replacing $\mathbb{R}^9$ by a solution that stabilizes the dilaton? More on this in a moment.
- Can we build domain walls where the moduli interpolate from extremum to extremum along geodesics in field space?
- Are there any actual minima on higher tori?
- Is the cosmological constant always positive? This is not expected.

There are many more fascinating questions about off-shell quantum gravity along these lines.

# Embedding in $AdS$

Let's examine a perturbatively stable solution. Consider

$$AdS_3 \times S^3 \times \hat{S}^3 \times S^1.$$

This only has NS fields turned on. There are 3 integers parametrizing the background:

$$(n_1, n_5, \hat{n}_5).$$

It is well studied in the supersymmetric context. Let's embed it in the $O(16) \times O(16)$ heterotic string (Baykara, Robbins, S.S.).

You might worry right away that we have a circle, a dilaton and Wilson line moduli that might destabilize the background. We will cure these problems.

Minimizing the tree-level potential potential gives,

$$L = \sqrt{\alpha' |n_5|}, \qquad \hat{L} = \sqrt{\alpha' |\hat{n}_5|}, \qquad \frac{g_s^4}{r^2} = \frac{n_5^2 \hat{n}_5^2 (|n_5| + |\hat{n}_5|)}{16\pi^2 \alpha' n_1^2},$$

with cosmological constant:

$$\Lambda = -\left(\frac{1}{L^2} + \frac{1}{\hat{L}^2}\right) = -\frac{1}{\alpha'}\left(\frac{1}{|n_5|} + \frac{1}{|\hat{n}_5|}\right).$$

Note that the string coupling can be made arbitrarily small independently of the value of $\Lambda$,

$$g_s \to 0, \qquad n_1 \to \infty.$$

This is identical to the SUSY analysis so there should be no BF-violating tachyons in the story at all at tree-level.

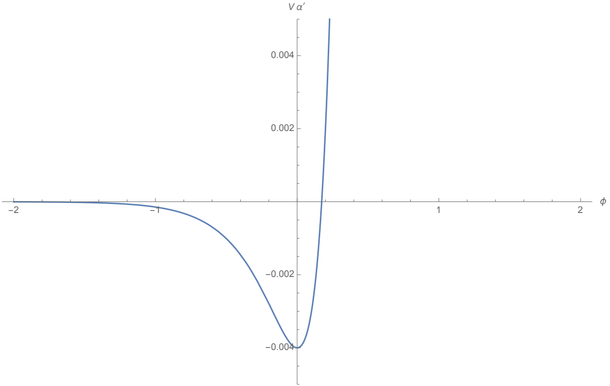



Figure: The potential $V(\phi)$ with $n_5 = \hat{n}_5 = 10^3$.

Now we can use the string 1-loop potential on $\mathbb{R}^9 \times S^1$ discussed earlier, at least when the $AdS$ and sphere length scales are large.

For example, at the $SO(16) \times SO(16) \times SU(2)$ point, the radius of the circle is frozen at the self-dual value,

$$r = \sqrt{\alpha'}.$$

The $\sigma$ field is now massed up. Fortunately all the Wilson line moduli also seemed to be massive at this point so we should have been able to ignore them.

If this were correct, there would be no massless moduli left!

This would have been the case if this point were a minimum but now we know it's a saddle. So there are tachyons.

Here is a comparison of tree-level versus loop-corrected coupling:

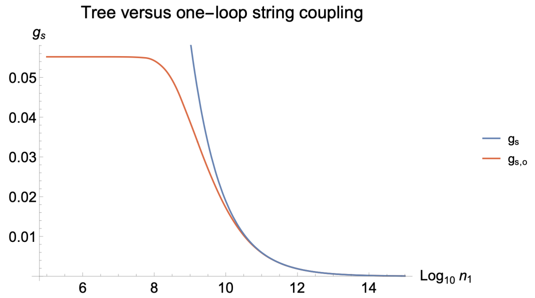



Figure 3: Comparison of tree level $g_s$ and one-loop corrected $g_{s,\circ}$ for $n_5 = \hat{n}_5 = 10^3$. Tree level $g_s$ diverges in the small $n_1$ limit, but $g_{s,\circ}$ is bounded.

For $n_5 = \hat{n}_5 = n$ and $n_1 = 0$, we find:

$$L_\circ^2 = \hat{L}_\circ^2 = \frac{\sqrt{5}}{3}\alpha'|n| \,,$$
$$g_{s,\circ}^2 = \frac{24}{5\sqrt{5}}\frac{1}{\lambda|n|} \,.$$

The upper bound of the string coupling $g_{s,\circ}$ with respect to $n_1$ for fixed $n_5 = \hat{n}_5 = n$,

$$\max_{n_1} g_{s,\circ}^2 = \frac{24}{5\sqrt{5}}\frac{1}{\lambda|n|} \approx \frac{3.04}{|n|} \,.$$

## What changes because of the saddle?

The upshot for us appears to be a phase structure in $n_1$. For sufficiently large $n_1$ and hence small string coupling, all tachyons from $S^1$ appear to be below the BF bound.

Said differently: the masses of the tachyons are all 1-loop suppressed and we can tune $g_s$ to be very small.

On the other hand, perturbative stability can potentially be lost for $n_1$ too small depending on the $AdS$ length scale. There is a problem going to the intrinsically quantum solutions except if

$$c < 0.25.$$

The $O(1)$ number in the refined de Sitter conjecture now means something precise!

# Part 2: Heterotic Potential Energy

# Current approach

There should be a deeper explanation for the features we see in the heterotic potential energy.

I'll give a brief sketch of one approach that I'm trying for the case of toroidal $O(16) \times O(16)$ heterotic compactifications.

The idea is to derive a differential equation for the potential energy.

# Type IIB

For example, in type IIB

$$S_{IIB} = \int d^{10}x\sqrt{-g}\left(R + \ldots + f(\tau,\bar{\tau})R^4 + \ldots\right). \qquad (1)$$

Here $\tau = C_0 + ie^{-\phi}$ is the complexified type IIB string coupling.

Supersymmetry requires that

$$\Delta f(\tau,\bar{\tau}) = \frac{3}{4} f(\tau,\bar{\tau}). \qquad (2)$$

This fixes $f(\tau,\bar{\tau})$ including the string 1-loop and non-perturbative corrections up to an overall constant.

The $O(16) \times O(16)$ heterotic potential takes the form (Itoyama, Koga, Nakajima)

$$\Lambda_{1-\text{loop}} = -(4\pi^2\alpha')^{-\frac{10-d}{2}} \int_{F_0} \frac{d^2\tau}{2\tau_2^2} Z(\tau) \,, \tag{3}$$

where

$$Z(\tau) = Z^{8-d} \left( \bar{V}_8 \, Z_v - \bar{S}_8 \, Z_s - \bar{C}_8 \, Z_c + \bar{O}_8 \, Z_0 \right) \,, \tag{4}$$

and

$$\begin{aligned} Z^{8-d}(\tau) &= \tau_2^{-\frac{1}{2}(8-d)} \, (\eta\bar{\eta})^{-(8-d)} \,, \\ Z_{v,s,c,0}(\tau) &= \frac{1}{\eta^{16+d} \, \bar{\eta}^d} \sum_{P \in \Gamma_{v,s,c,0}} q^{\frac{P_L^2}{2}} \, \bar{q}^{\frac{p_R^2}{2}} \,. \end{aligned} \tag{5}$$

# Moduli-dependence

Call the moduli space $\mathcal{M}$. All the target space moduli-dependence is encoded in the lattice sums. This suggests a relation

$$\Delta_{\mathcal{M}} Z(\tau) \sim \left( \Delta_\tau + \alpha \tau_2 \frac{\partial}{\partial \tau_2} + \ldots \right) Z(\tau)\,. \tag{6}$$

Applied at a critical point, this would seem to have the right ingredients to provide an explanation for the structure of the potential energy, and perhaps allow exploration beyond 1-loop.

In progress. . . .