Misaligned SUSY in String Vacua

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C. Angelantonj, I. Florakis, G. L., arXiv:2302.13702

G.L., arXiv: 2308.09757

Strings Breaking SUSY, November 2023

SUSY breaking at the string scale induces instabilities

- Best case scenario ⟶ tachyon free vacua with non vanishing contributions to dilaton potential from tadpoles or perturbative corrections

- Worst case scenario ⟶ tachyons in the tree level spectrum

⟶ can we distinguish the two?

Misaligned SUSY for Closed Strings

Interplay between IR and UV properties

[Dienes, 1994]

$$\mathcal{T} = \sum_{i,\bar{\imath}} N_{i\bar{\imath}}\, \chi_i(\tau)\, \bar{\chi}_{\bar{\imath}}(\bar{\tau})$$

$$\sum_{n=0}^{\infty} d_i(n)\, q^{n+H_i}$$

$$H_i = h_i - \frac{c}{24}$$

$$\langle d(n)\rangle = \sum_{i\bar{\imath}} N_{i\bar{\imath}}\, d_i(n) \bar{d}_{\bar{\imath}}(n)$$

Sector averaged sum ➡ counts dof in the large mass regime

Rademacher expansion

The degeneracies $d_i(n)$ from Cauchy integral

$$d_i(n) = \oint \frac{dq}{2\pi i} \frac{\chi_i(q)}{q^{n+H_i+1}}$$

Integral hard to evaluate ➡ *Circle Method*

[Hardy Ramanujan, 1917]
[Rademacher, 1938]

sum over essential singularities at $\tau = \frac{p}{l}$

contour from arcs of Ford Circles of each irreducible fraction

Rademacher *exact* formula

Generalised Kloosterman sum → Moebius $\hat{Q}^{(l,n)} = (-1)^n Q^{(l,n)}$

$$d_i(n) = \sum_{l=1}^{\infty} \sum_{H_j<0} Q_{ij}^{(l,n)} \left(\frac{|H_j|}{n+H_i}\right)^{d/2} I_{\frac{d}{2}}\left(\frac{4\pi}{l}\sqrt{|H_j|(n+H_i)}\right)$$

$$\sum_{\substack{p=0 \\ \gcd(l,p)=1}}^{l-1} \left(\gamma_{l,p}^{-1}\right)_{ij} e^{-\frac{2\pi i}{l}(p(n+H_i)-p'H_j)}$$

$f_j(l,n)$

sum over tachyonic characters

modular transformation mapping rational points on the unit circle $|q| = 1$ to $q = 0$

[Sussman, 2017]
[Manschot et al., 2010]
[Dijkgraaf et al., 2007]

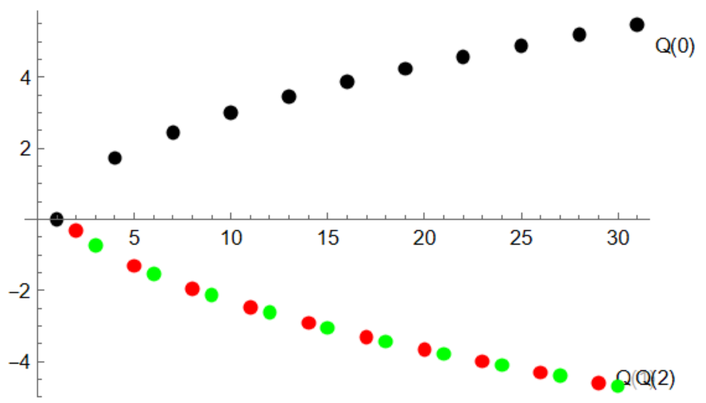
Dof's rearranged into a finite number of subclasses

[Cribiori et al., 2021]

$$n = w + l\ m$$

$$Q^{(l,w+l\ m)} = Q^{(l,w)}$$

Example with period 3



Refined degeneracies

$$d_i(n) \longrightarrow \sum_{l\geq 1} \sum_{w=0}^{l-1} \sum_{H_j<0} Q_{ij}^{(l,w)} f_j(n,l)$$

But left and right sectors imply the periodicity be

$$\mathrm{lcm}(l, \bar{l})$$

Modular invariance, level rearrangement and Rademacher expansion imply

$$\langle d(n) \rangle (\mathcal{T}) = \sum_{\substack{i,\bar{\imath} \\ H_i = \bar{H}_{\bar{\imath}} < 0}} N_{i\bar{\imath}}\, d_i(0)\, \bar{d}_{\bar{\imath}}(0) \sum_{\ell=1}^{\infty} \varphi(\ell)\, e^{\frac{8\pi}{\ell}\sqrt{|H_i|\, n}}$$

⟶ contributions only from level matched tachyons

⟶ Orientifold vacua?

Elephant in the room:

absence of modular invariance

→ connects $\langle d(n) \rangle$ with tachyons

→ how is it translated in orientifolds?

Orientifold vacua

Modular action is trivialised

$$\mathcal{Z} = \mathcal{K}, \mathcal{A}, \mathcal{M}$$

$$\mathcal{Z} = \sum_i \zeta_i \, \chi_i(\tau) \quad \longrightarrow \quad \langle d(n) \rangle \, (\mathcal{Z}) = \sum_i \zeta_i \, d_i(n)$$

$\longrightarrow$ Modular group acts covariantly

$\longrightarrow$ S and P map different descriptions of the same amplitudes

$$\tilde{\mathcal{Z}} = \sum_i \tilde{\zeta}_i \, \chi_i(\tau) \quad \longrightarrow \quad \langle d(n) \rangle (\tilde{\mathcal{Z}}) = \sum_i \tilde{\zeta}_i \, d_i(n)$$

For orientifolds only holomorphic sector

Refined degeneracies

$$d_i(n) \longrightarrow \sum_{l\geq 1}\sum_{w=0}^{l-1}\sum_{H_j<0} Q_{ij}^{(l,w)}\, f_j(n,l)$$

But

$$\sum_{w=0}^{l-1} Q^{(l,w)} = 0\,, \qquad l>1$$

➡ only S survives

$$\sum_{w=0}^{\mathrm{lcm}(l,2)-1} \hat{Q}^{(l,w)} = 0\,, \qquad l\neq 2$$

➡ only P survives

Direct channel

$$\langle d(n)\rangle(\mathcal{Z}) = \sqrt{\frac{\ell_{\mathcal{Z}}}{2}} \sum_{i\,|\,H_i<0} \tilde{\zeta}_i\, d_i(0)\ \ e^{\frac{4\pi}{\ell_{\mathcal{Z}}}\sqrt{|H_i|\,n}}$$

➡ vanishes only if tachyons are absent in the transverse channel

Transverse channel

$$\langle d(n)\rangle(\tilde{\mathcal{Z}}) = \sqrt{\frac{\ell_{\mathcal{Z}}}{2}} \sum_{i\,|\,H_i<0} \zeta_i\, d_i(0)\ \ e^{\frac{4\pi}{\ell_{\mathcal{Z}}}\sqrt{|H_i|\,n}}$$

➡ vanishes only if tachyons are absent in the direct channel

Misaligned SUSY in Orientifolds

No closed tachyons $\longrightarrow$ $\langle d(n)\rangle(\mathcal{Z}) = 0$

↳ Necessary condition

$$\langle d(n)\rangle(\tilde{\mathcal{Z}}) = 0$$

$$\langle d(n)\rangle(\mathcal{T}) = 0$$

$\longrightarrow$ No closed and open tachyons

↳ Sufficient condition

$\longrightarrow$ Necessary and sufficient condition?

Problem:

Tachyon can be removed by orientifold projection

→ non vanishing sector averaged sums even without tachyons

→ mutual cancellation required

↓

impossible to achieve

Examples in 10d

Type 0B superstring

$$\mathcal{T} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2$$

$\longrightarrow$ tachyon level matched

Sector averaged sum

$$\langle d(n) \rangle (\mathcal{T}) = \sum_{\ell=1}^{\infty} \varphi(\ell) \frac{e^{\frac{8\pi}{\ell}\sqrt{n/2}}}{(2n)^{11/2}}$$

Type 0B2

[Sagnotti,1995]

Closed sector ➡ tachyon survives

$$\langle d(n)\rangle\,(\tilde{\mathcal{K}}) = \frac{e^{4\pi\sqrt{n/2}}}{(2n)^{11/4}}$$

$$\mathcal{K} = O_8 + V_8 + S_8 + C_8 \qquad \tilde{\mathcal{K}} = O_8$$

$$\langle d(n)\rangle\,(\mathcal{K}) = 2\frac{e^{4\pi\sqrt{n/2}}}{(2n)^{11/4}}$$

Open sector ➡ tachyon in antisymmetric of $U(1)$

$$\langle d(n)\rangle\,(\tilde{\mathcal{A}}) = \frac{e^{4\pi\sqrt{n/2}}}{(2n)^{11/4}}\,2$$

$$\langle d(n)\rangle\,(\mathcal{A}) = \frac{1}{2}\frac{e^{4\pi\sqrt{n/2}}}{(2n)^{11/4}}4 \qquad \mathcal{A} = O_8 + V_8 \qquad \tilde{\mathcal{A}} = O_8 + V_8$$

$$\mathcal{M} = -\hat{O}_8 \qquad \tilde{\mathcal{M}} = \hat{O}_8$$

$$\langle d(n)\rangle\,(\mathcal{M}) = \frac{e^{2\pi\sqrt{n/2}}}{2^{9/4}\;n^{11/4}}\,2$$

$$\langle d(n)\rangle\,(\tilde{\mathcal{M}}) = -\frac{e^{2\pi\sqrt{n/2}}}{2^{9/4}\;n^{11/4}}\,2$$

Type 0B3

[Sagnotti,1995]

Closed sector ➡ tachyon projected away

$$\langle d(n)\rangle\,(\mathcal{K}) = 0$$

$$\mathcal{K} = -O_8 \; + \; V_8 \; + \; S_8 \; - \; C_8 \qquad \tilde{\mathcal{K}} = -C_8$$

$$\langle d(n)\rangle\,(\tilde{\mathcal{K}}) = -\frac{e^{4\pi\sqrt{n/2}}}{(2n)^{11/4}}$$

Open sector ➡ tachyon in bifundamental of $U(32+n)\times U(n)$

tachyon not projected

Unable to cancel

$$\langle d(n)\rangle\,(\mathcal{T}) = \sum_{\ell=1}^{\infty}\varphi(\ell)\frac{e^{\frac{8\pi}{\ell}\sqrt{n/2}}}{(2n)^{11/2}}$$

Conclusions

Oriented Closed Strings

No physical tachyons $\Longleftrightarrow$ $\langle d(n) \rangle = 0$

Orientifolds

Direct (transverse) $\langle d(n) \rangle = 0$ $\Longleftrightarrow$ No transverse (direct) tachyons

Outlook

Connection with Rankin-Selberg-Zagier? [Kutasov, Seiberg, 1990] [Angelantonj et al., 2011]

New technology for orientifolds?

THANK YOU FOR THE ATTENTION

Examples in 9d

M-theory breaking compactification

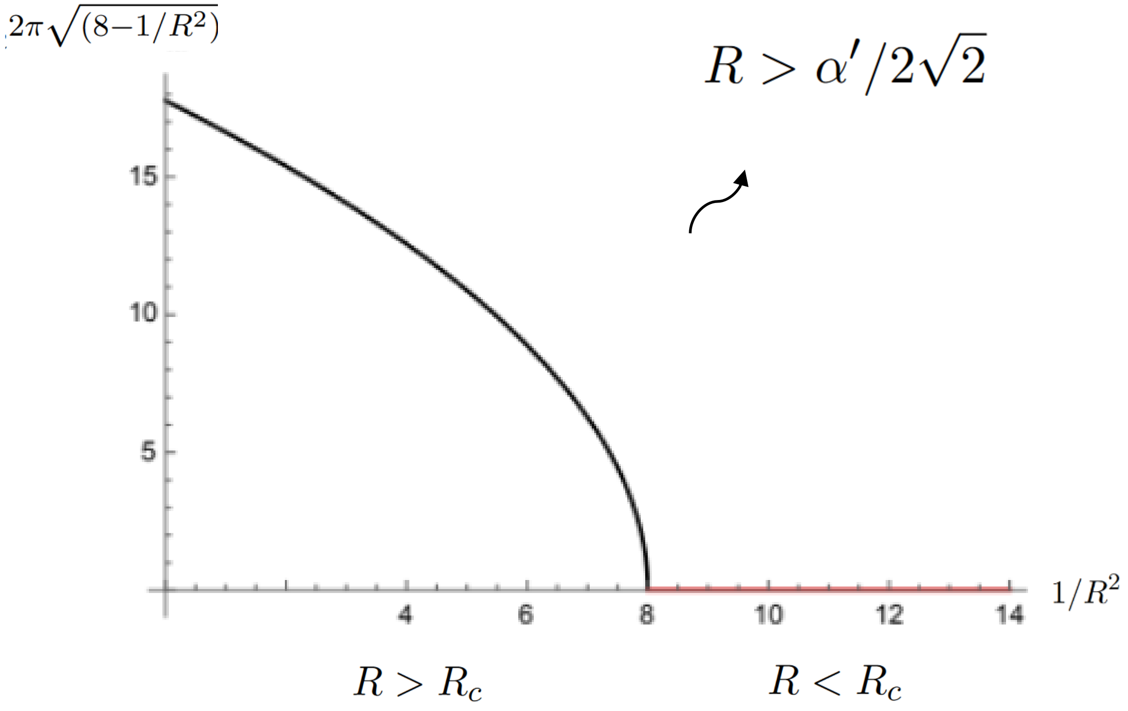
$$\langle d(n) \rangle (\mathcal{T}) = \begin{cases} 0 & R \leq 1/8 \\ e^{2\pi\sqrt{(8-1/R^2)n}} & R > 1/8 \end{cases}$$

$R \to \infty$ ➡ 0B (purely bosonic)

$R \to 0$ ➡ IIB in 10d (SUSY)

tachyons if

$R > \alpha'/2\sqrt{2}$



M-theory breaking orientifolds

[Antoniadis,1998]

World-sheet parity allows

$$\mathcal{K} = (V_8 - S_8)\, P_m + (O_8 - C_8)\, P_{m+1/2}$$

Tachyon survives

$$\langle d(n)\rangle\,(\tilde{\mathcal{K}}) \sim e^{\pi\sqrt{(8-\alpha'/R^2)\,n}}$$

$$\langle d(n)\rangle\,(\tilde{\mathcal{M}}) = 0$$

Open sector ➡ tachyon in bifundamental of $SO(16) \times SO(16)$

$$\langle d(n)\rangle\,(\tilde{\mathcal{A}}) \sim 2\ 16\,16\ e^{\pi\sqrt{(8-\alpha'/R^2)\,n}}$$

No tachyonic tadpoles

$$\langle d(n)\rangle(\mathcal{Z}) = 0$$

A variation on M-theory breaking orientifolds

Oreintifold projection dressed with extra symmetries [Dudas, 2002]

$$\mathcal{K} = (V_8 - S_8)\, P_m - (O_8 - C_8)\, P_{m+1/2}$$

No open tachyons and no tachyonic tadpoles

$$\langle d(n) \rangle\, (\tilde{\mathcal{A}}) = 0 \quad \langle d(n) \rangle\, (\tilde{\mathcal{M}}) = 0 \qquad \langle d(n) \rangle (\mathcal{Z}) = 0$$

Tachyon projected away

$$\langle d(n) \rangle\, (\tilde{\mathcal{K}}) \sim -e^{\pi \sqrt{(8-\alpha'/R^2)\, n}}$$

⟶ but unable to cancel $\langle d(n) \rangle (\mathcal{T}) = \sum_{\ell=1}^{\infty} \varphi(\ell)\, e^{\frac{2\pi}{\ell} \sqrt{(8-\alpha'/R^2)\, n}}$