Instituto de Física Teórica
UAM-CSIC

Dynamical Cobordism without Supersymmetry

Based on work with R. Blumenhagen, N. Cribiori, C. Kneissl

Andriana Makridou

Strings Breaking SUSY @ MITP

November 22, 2023

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Based on work with R. Blumenhagen, N. Cribiori, C. Kneissl

See also talk by Roberta Angius!

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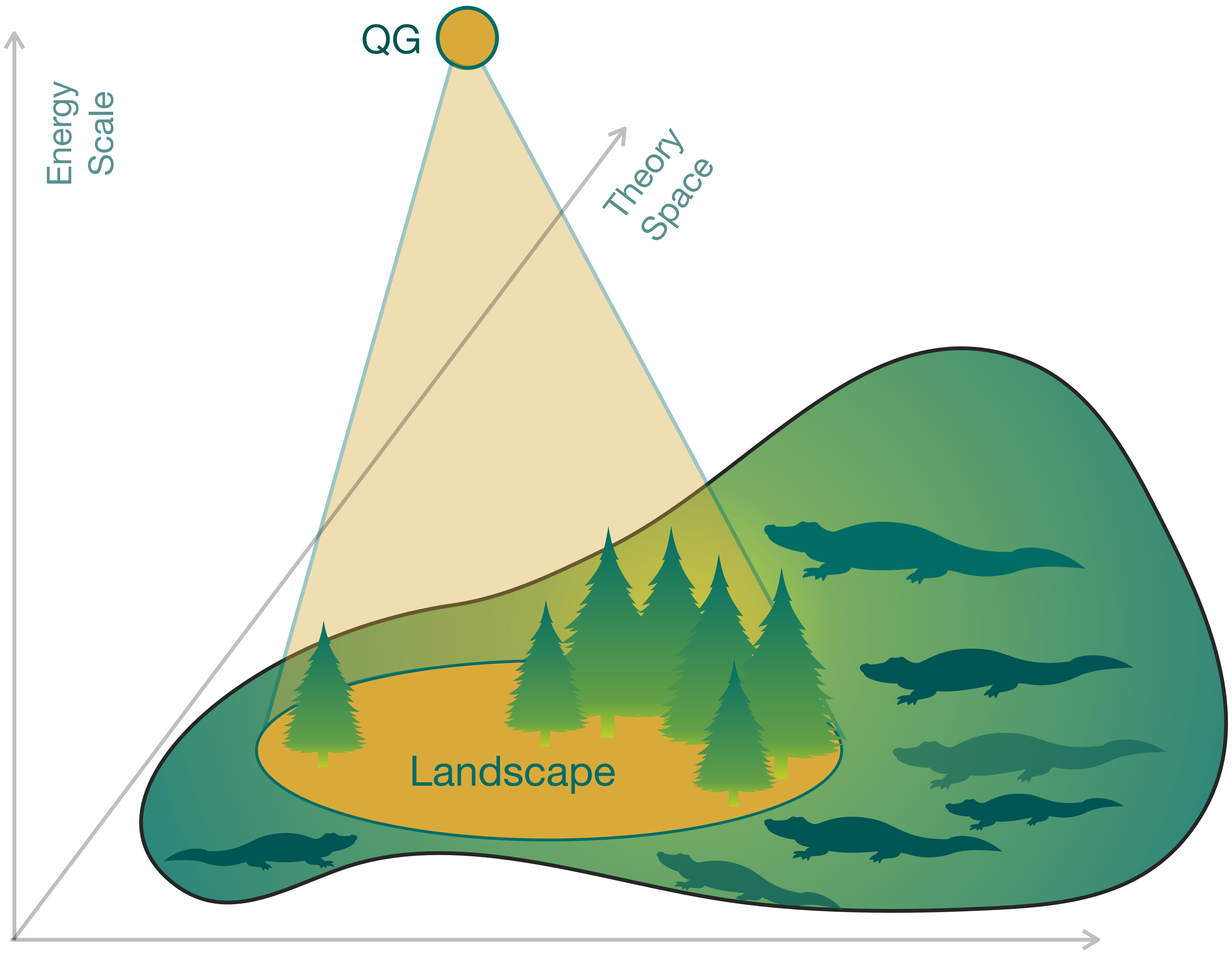
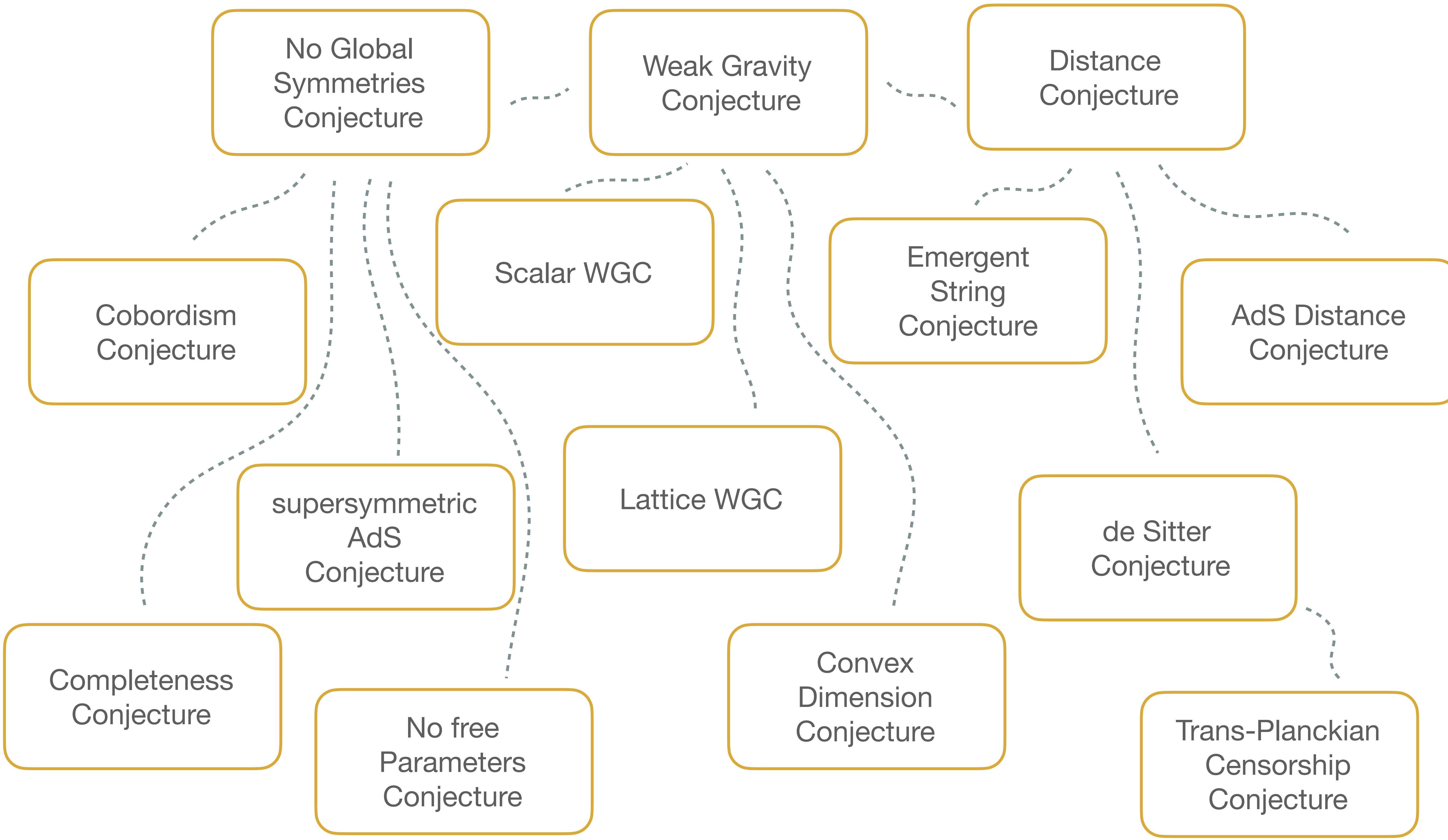
The Swampland Program

Idea: Not all low-energy EFTs can be UV-completed to Quantum Gravity

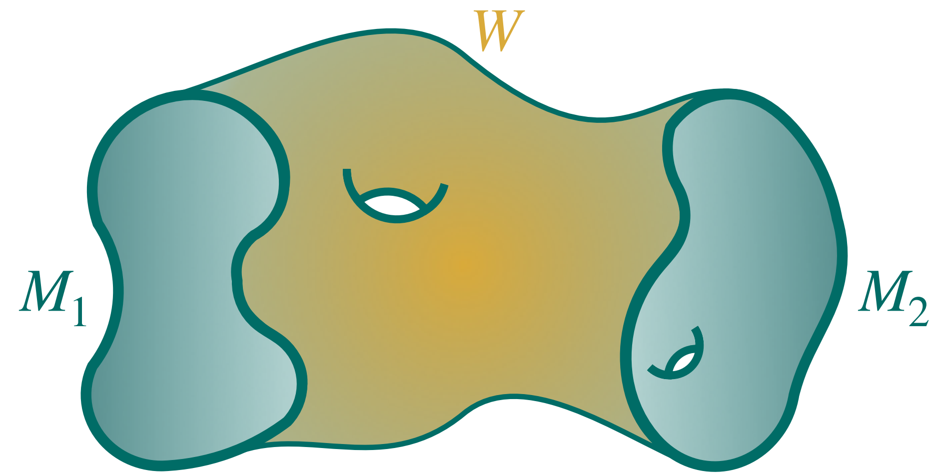


[Recent reviews: Palti ’19, Valenzuela et al ’21, Vafa et al ’22]

A web of swampland conjectures



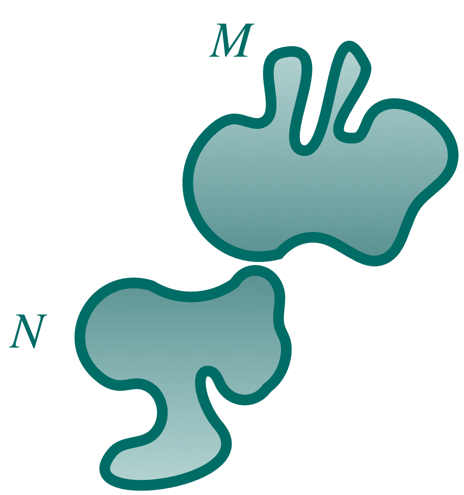
Cobordism

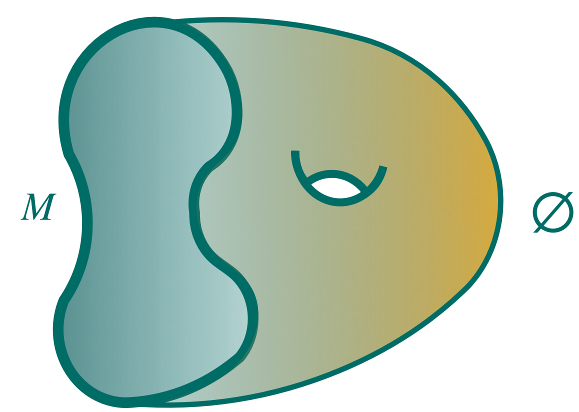


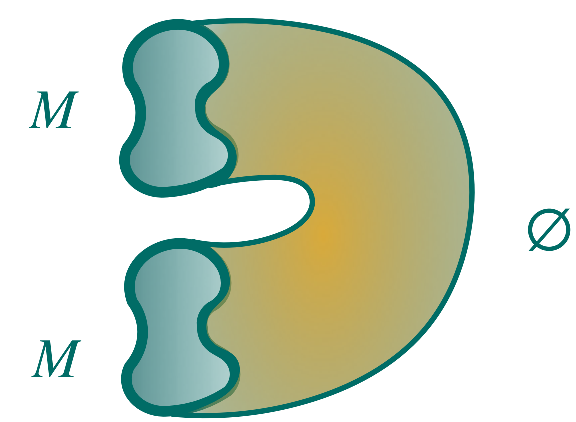
Allowed topology changes
(Encoded in ξ)

$$M_1 \sim M_2 \Leftrightarrow \exists W \text{ s.t. } \partial W = M_1 \sqcup M_2$$

$$\Omega_k^{\xi} = \{\text{compact, closed, k} - \text{dimensional manifolds}\}/\sim$$





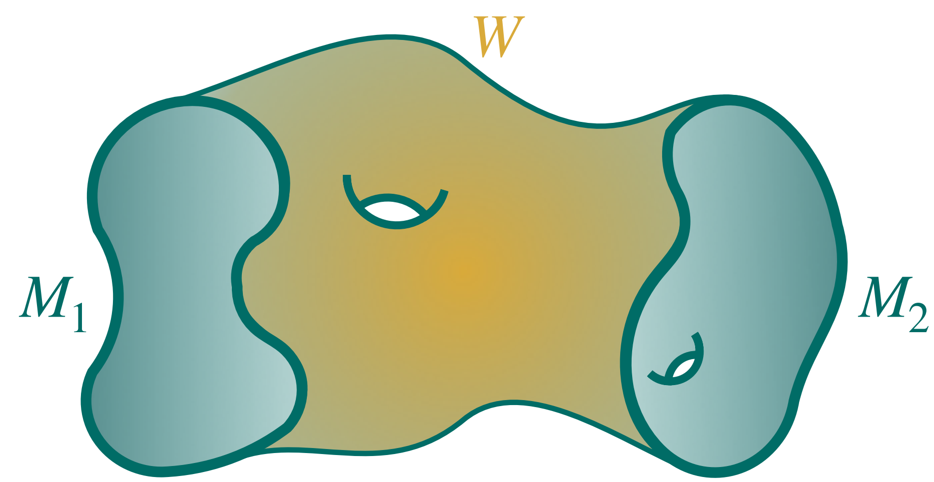


$$[M \sqcup N] = [M] + [N]$$

$$[\varnothing] = 0$$

$$[M] + [M] = [M \sqcup M] = 0$$

Cobordism



Allowed topology changes
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$$M_1 \sim M_2 \Leftrightarrow \exists W \text{ s.t. } \partial \mathrm{W} = \mathrm{M}_1 \sqcup \mathrm{M}_2$$

$$\Omega_k^{\xi} = \{\text{compact, closed, k} - \text{dimensional manifolds}\} / \sim$$

Compactify d-dimensional theory on M^k down to D=d-k dimensions:

EFT_1 $(d-k)-\text{dimensional}$

Domain Wall $(d-k-1)-\text{dimensional}$

EFT_2 $(d-k)-\text{dimensional}$

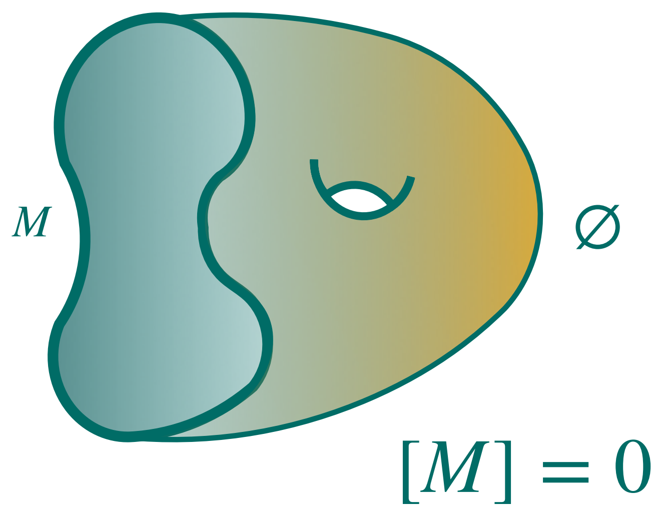
→ *Finite-energy transition between EFTs*

→ *Domain walls detected by cobordism can be well-known objects, e.g. D-branes*

Cobordism Conjecture

No global symmetries in quantum gravity → Cobordism Conjecture

[e.g., Banks, Seiberg ’11]



[McNamara, Vafa ’19]

Cobordism Conjecture:

All Cobordism Classes should be trivial

$$\Omega_k^{QG} = 0$$

Cobordism Group Ω_k^{ξ} ↔ Cobordism Invariant μ_k

For trivial cobordism class: $\mu_k[\varnothing] = 0$

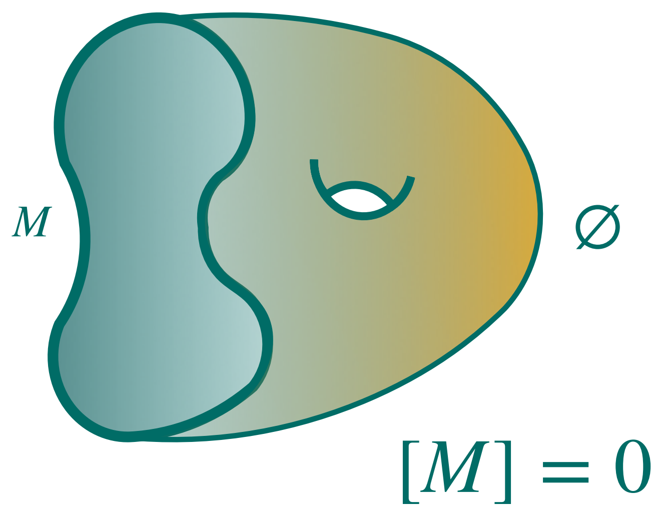
If cobordism class $[M] \neq 0$ ↔ obstruction to decay into “nothing”

$\Omega_k^{\xi} \neq 0 \Leftrightarrow (d-k-1)$-dim. global symmetry

with charges labelled by classes $[M] \in \Omega_k^{\xi}$

Cobordism Conjecture

No global symmetries in quantum gravity → Cobordism Conjecture
[e.g., Banks, Seiberg '11]



[McNamara, Vafa '19]

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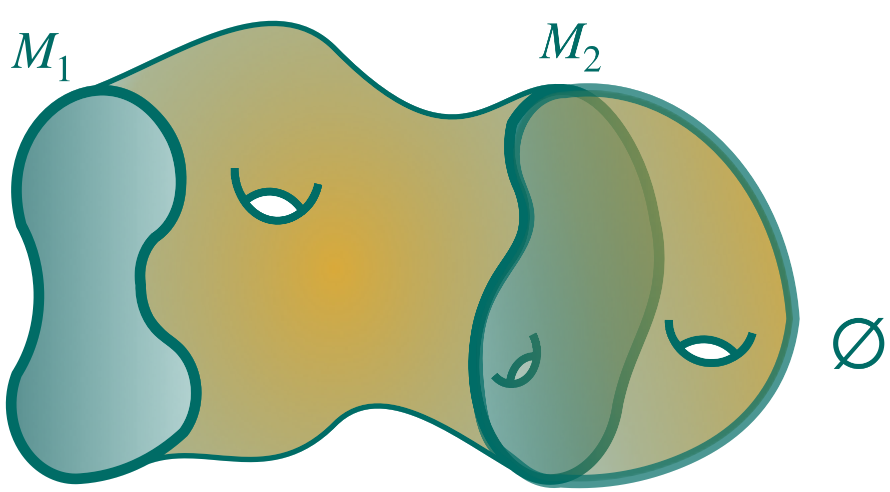


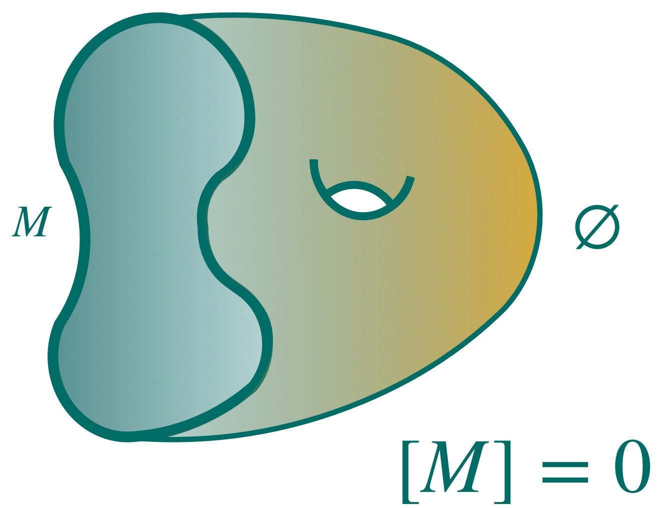
$\longleftrightarrow$

In D=d-k dimensions:

Major Implication: there is a "single" theory of quantum gravity

Cobordism Conjecture

No global symmetries in quantum gravity → Cobordism Conjecture
[e.g., Banks, Seiberg '11]



[McNamara, Vafa '19]

Cobordism Conjecture:

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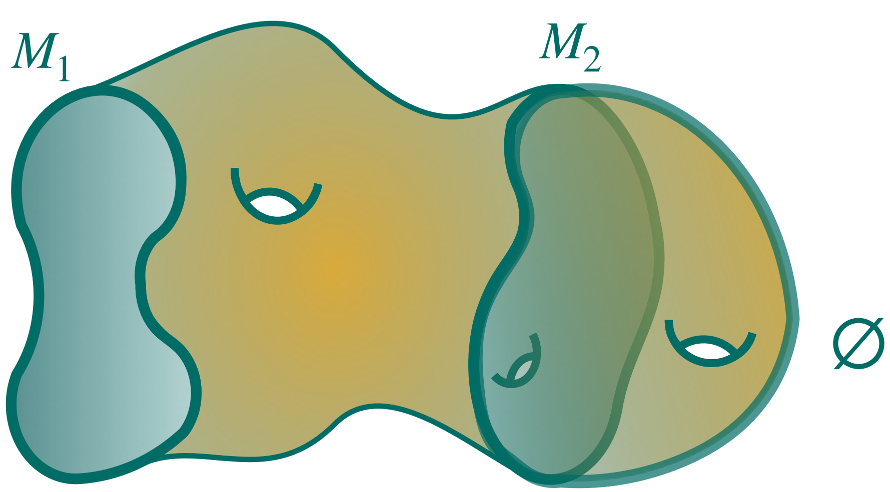
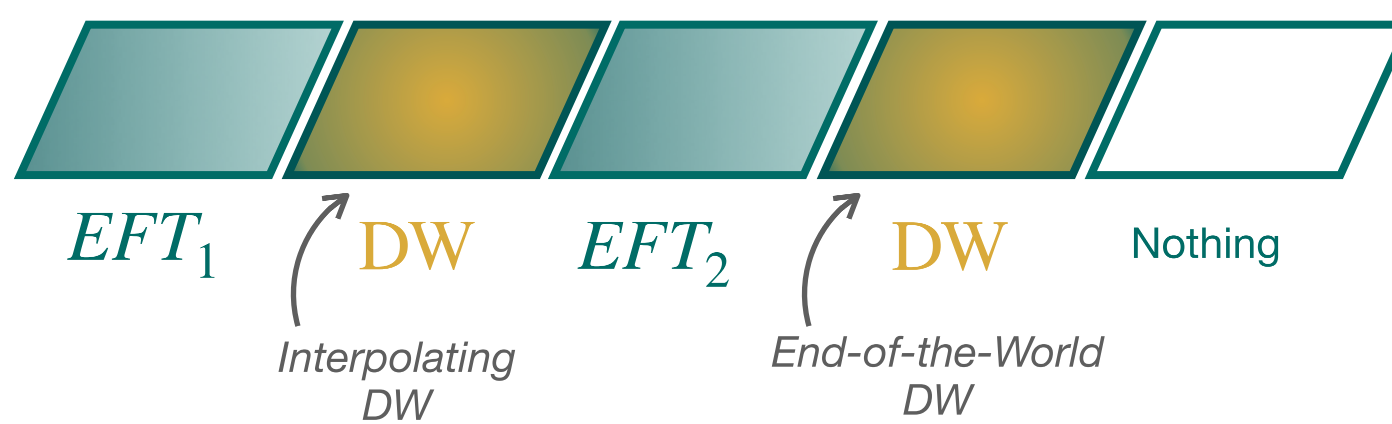
$$\Omega_k^{QG} = 0$$



$\longleftrightarrow$

In D=d-k dimensions:



Major Implication: there is a "single" theory of quantum gravity

Cobordism Conjecture in practice

$$\Omega_k^{\widetilde{QG}} \neq 0$$

Breaking

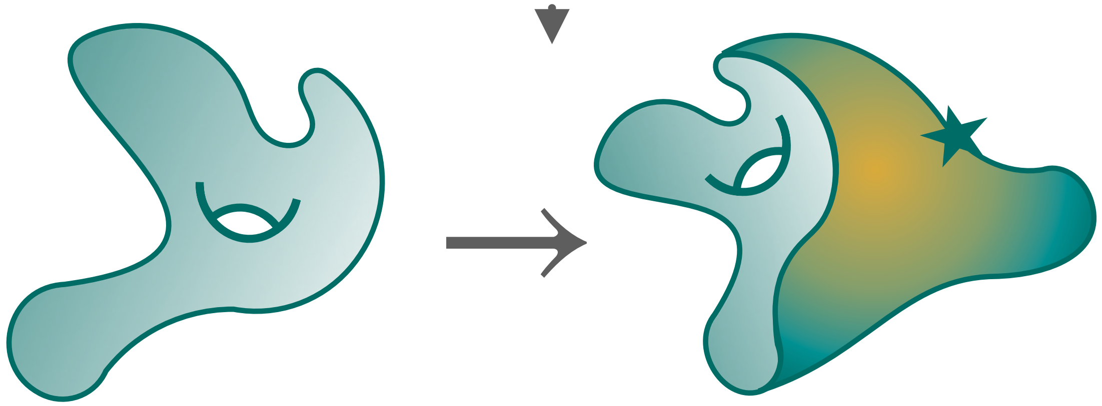
$$\Omega_k^{\widetilde{QG}} \to \Omega_k^{\widetilde{QG}\text{+defects}} = 0$$

Gauging

$$0 = [M] \in \Omega^{\widetilde{QG}} \neq 0$$

$$\Omega^{\widetilde{\mathrm{QG}}\text{+g.fields}} \to \Omega^{\widetilde{\mathrm{QG}}}$$

[Blumenhagen, Cribiori '21],
[Blumenhagen, Cribiori, Kneissl, AM '22/2]



$0 \neq [M] \in \Omega_k^{\widetilde{QG}}$

$[M] = 0 = \Omega_k^{\widetilde{QG}\text{+defects}}$

New defects possibly detected!

[McNamara, Vafa '19], [Montero, Vafa '20], [Dierigl, Heckman '21]
[Debray, Dierigl, Heckman, Montero '21]
[Dierigl, Heckman, Montero, Torres '22/23]
[Blumenhagen, Cribiori, Kneissl, AM '22]
[Blumenhagen, Kneissl, Wang '23]
[Kaidi, Ohmori, Tachikawa, Yonekura '23]
[Friedrich, Hebecker, Walcher '23]

Cobordism and string theory

What are the relevant structures for String Theory?

e.g. Type I $\leftrightarrow \xi = \mathrm{Spin}$, Type II $\leftrightarrow \xi = \mathrm{Spin}^c$

[see also e.g. Andriot, Carqueville, Cribiori '22, Debray '23]

n	0	1	2	3	4	5	6	7	8	9	10
Ω_n^{Spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$2\mathbb{Z}_2$	$3\mathbb{Z}_2$
$\Omega_n^{\mathrm{Spin}^c}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2\mathbb{Z}$	0	$4\mathbb{Z}$	0	$4\mathbb{Z} \oplus \mathbb{Z}_2$

Example: $\Omega_{p+1}^{\mathrm{Spin},\mathrm{U}(1)_\mathrm{p}} \supset \mathbb{Z}$

[McNamara, Vafa '19, see also Dierigl @ASC '23]

$$dF_{p+1} = 0 \Leftrightarrow \int_M F_{p+1} \in \mathbb{Z}$$

Can kill cobordism charges by adding source in Bianchi identity: $dF_{p+1} \neq 0$

Physical object: magnetically charged (p-3)-manifold$\rightarrow$ Dp-branes

Predicted objects can be totally new! e.g. R7-branes, heterotic branes

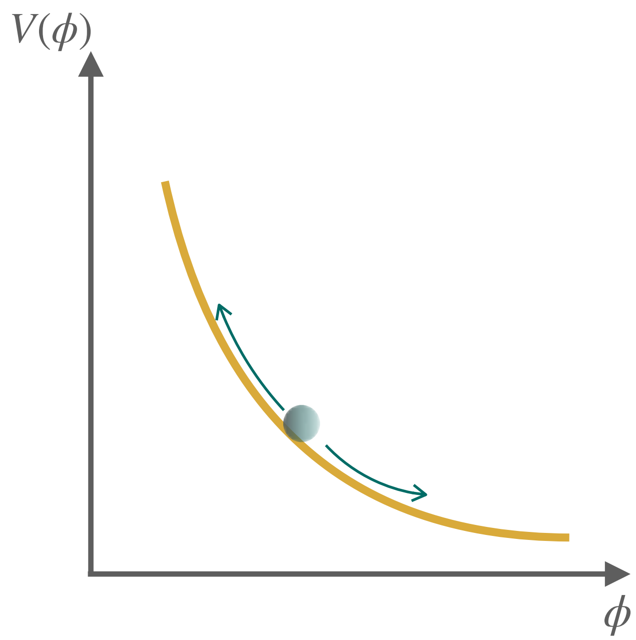
[Dierigl, Heckman, Montero, Torres '22/23] [Kaidi, Ohmori, Tachikawa, Yonekura '23]

See talk by Justin Kaidi this afternoon!

Side remark: Cobordism also relevant in study of anomalies, with recent application to non-SUSY strings!

[Basile, Debray, Delgado, Montero '23]

Dynamical Cobordism : Motivation



[Sugimoto '99]
[Antoniadis, Dudas, Sagnotti '99]
[Angelantonj '99]
...
[Raucci '22]
[Basile, Raucci, Thomée '22]
...
[Mourad, Sagnotti '23]

Dynamical tadpoles (vs RR tadpoles)

Naturally occurring in supersymmetry-breaking potentials

Indicate lack of maximally-symmetric vacuum

Example: Sugimoto Model (USp(N) Type I with N $\bar{D}8$ and N $O9^+$) [Sugimoto '99]

Action:

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(\mathscr{R} - \frac{1}{2}(\partial\Phi)^2\right) - T_9\int d^{10}x\left((N+32)\sqrt{-G}e^{\frac{3}{2}\Phi} - (N-32)A_{10}\right) + \ldots .$$

Dudas-Mourad solution - preserving 9d Poincaré invariance: *See talk by Salvatore Raucci tomorrow!*

[Dudas, Mourad '00]

$$ds_E^2 = \sqrt{\alpha_E}^{\;1/9} e^{-\frac{\alpha_E}{8}y^2}\eta_{\mu\nu}dx^\mu dx^\nu + \sqrt{\alpha_E}y^{\;-1} e^{-\frac{3\phi_0}{2}}e^{-\frac{9\alpha_E}{8}y^2}dy^2, \quad \alpha_E = 64k^2T_9$$

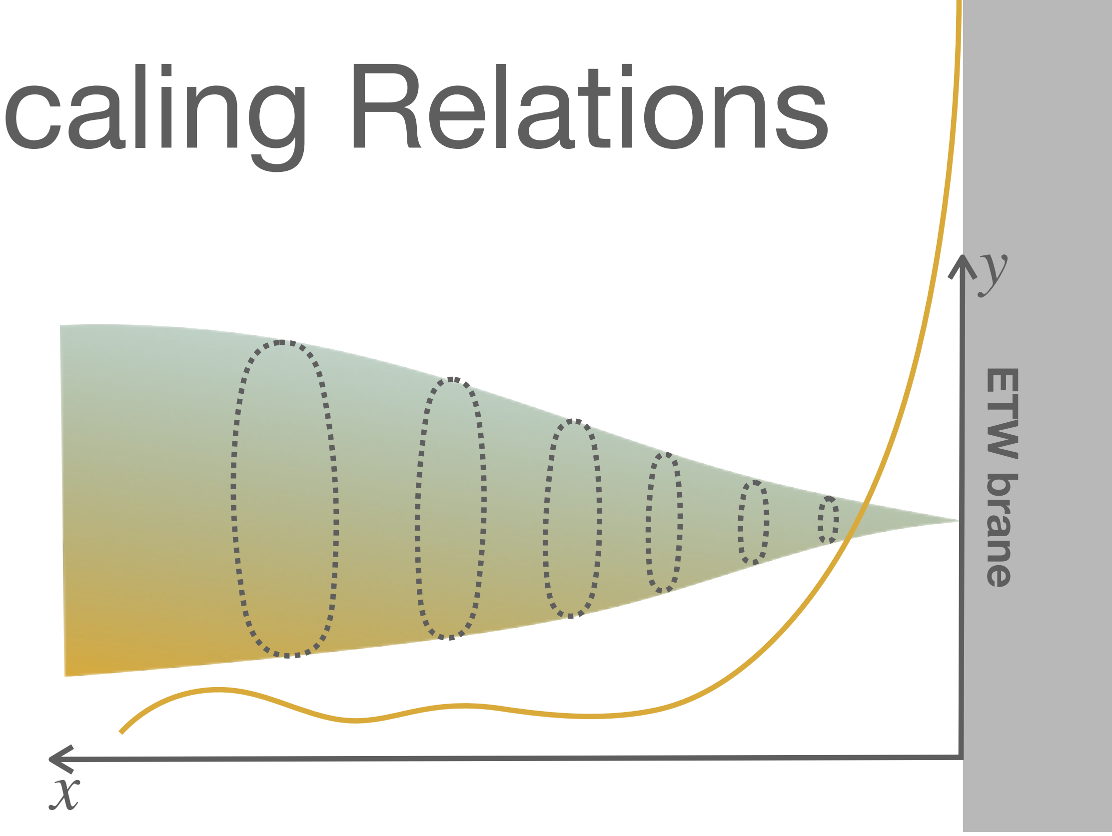
→ singularities at finite spacetime distance, spontaneous compactification to 9d

Dynamical Cobordism : Scaling Relations

Common features:

- Solution extends over finite spacetime distance Δ
- Ricci curvature singularity
- Scalar diverges close to the singularity

Field distance $D \to \infty$



Interpretation: [Buratti, Delgado, Uranga '21]

Physical mechanism cutting off spacetime = cobordism defect of the initial theory

$\rightarrow$ *End-of-the-World (ETW) brane*

[Buratti, Calderon-Infante, Delgado, Uranga '21]

Cobordism Distance Conjecture:

- An infinite field distance limit is realized as running into a cobordism wall of nothing.

- In this limit one has the scaling relations

$$\Delta \sim e^{-\frac{1}{2}\delta D}, \qquad \mathcal{R} \sim e^{\delta D}, \delta > 0.$$

Universal local description possible in terms of critical exponent δ

[Angius, Calderon-Infante, Delgado, Huertas, Uranga '21] ***See talk by Roberta Angius later!***

Application to a non-supersymmetric setup

[Blumenhagen, Cribiori, Kneissl, AM'22]

Setup: Gauge neutral, non-supersymmetric 9d object w/ brane-like dilaton coupling

Physical realisation: non-BPS $\widehat{D8}$ -brane, non-SUSY stack of $16 \times \bar{D}8 + O8^{++}$

[Blumenhagen, Font '00]

Action: $$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(\mathscr{R} - \frac{1}{2}(\partial\Phi)^2\right) - T\int d^{10}\sqrt{-g}\,e^{\frac{5}{4}\Phi}\delta(r)\,.$$

transverse direction

Solution Ansatz: $$ds^2 = e^{2\mathscr{A}(r,y)}ds_8^2 + e^{2\mathscr{B}(r,y)}(dr^2 + dy^2)\,.$$

$$\mathscr{A} = A(r) + U(y) \qquad \mathscr{B} = B(r) + V(y) \qquad \Phi = \chi(r) + \psi(y)$$

Solution I

[Blumenhagen, Font '00]

$$A(r) = \frac{1}{8}\log\left|\sin\left(8K(\ r\ -\frac{R}{2})\right)\right| \qquad U(y) = -\,Ky$$

$$B(r) = \frac{1}{8}\log\left|\sin\left(8K(\ r\ -\frac{R}{2})\right)\right| \qquad V(y) = 0$$

$$\chi(r) = -\frac{3}{2}\log\left|\tan\left(4K(\ r\ -\frac{R}{2})\right)\right| + \phi_0 \qquad \psi(y) = 0$$

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Solutions $II^{\pm}$

[Blumenhagen, Font '00]

$$A(r) = \frac{1}{8} \log \left| \sin\left(8K\left(r - \frac{R}{2}\right)\right) \right|$$

$$B(r) = \frac{\alpha^{\pm}}{8} \log \left| \sin\left(8K\left(r - \frac{R}{2}\right)\right) \right| \mp 2 \log \left| \tan\left(8K\left(r - \frac{R}{2}\right)\right) \right| + \phi_0$$

$$\chi(r) = \frac{\mu}{8} \log \left| \sin\left(8K\left(r - \frac{R}{2}\right)\right) \right| \mp \frac{\alpha^{\pm}}{8} \log \left| \tan\left(8K\left(r - \frac{R}{2}\right)\right) \right|$$

$$U(y) = \frac{1}{8} \log\left(\cosh(8Ky)\right)$$

$$V(y) = \alpha^{\pm} U(y)$$

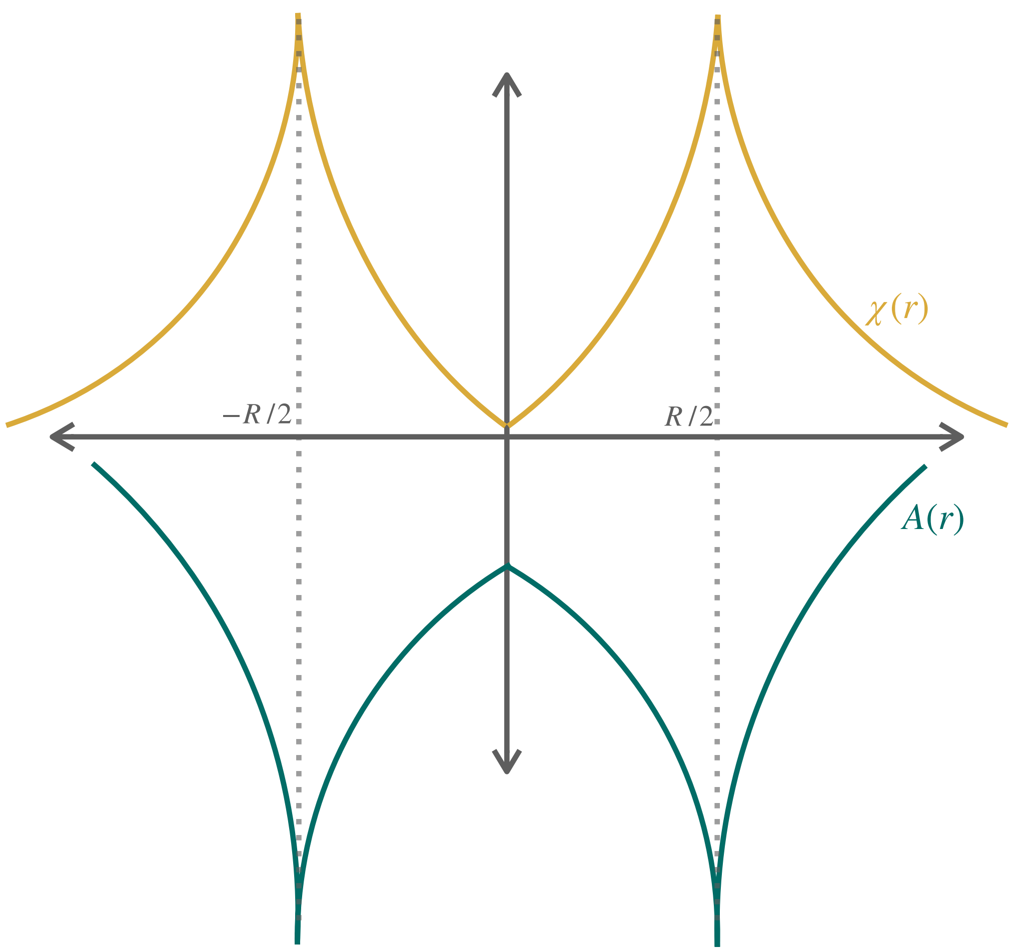
$$\psi(y) = -\frac{5\alpha^{\pm}}{4} U(y)$$

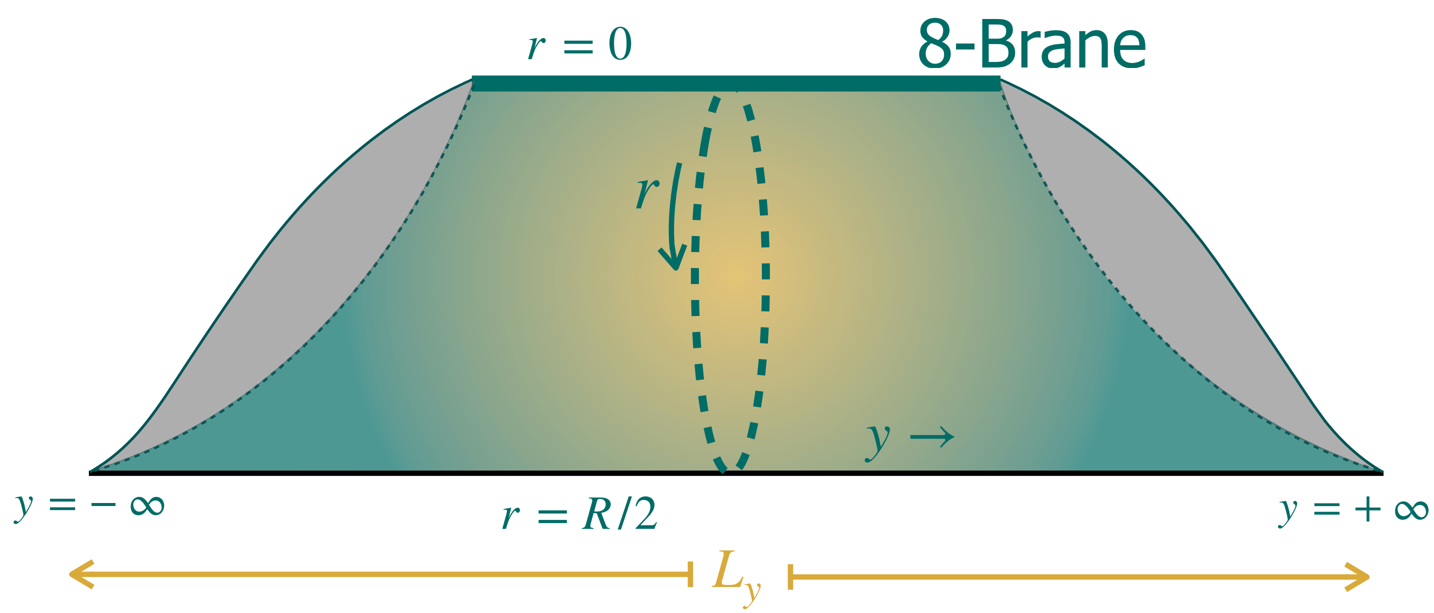
Qualitative behaviour

r-direction spontaneously compactified on S^1 with radius R: $e^{\frac{5}{4}\phi_0} \sim \frac{1}{\lambda R}$ $\kappa_{10}^2 T$

Logarithmic singularities at $r = \pm \frac{R}{2}$, string coupling diverges

y-direction: infinite length in sols I, II^-, becomes finite interval in sol II^+





At $(r, y) = (R/2, \pm\infty)$: $\left\{ \begin{matrix} \Delta \sim L_y \sim e^{-\frac{5}{4}\sqrt{2}D} \\ \mathscr{R} \sim e^{\frac{5}{2}\sqrt{2}D} \end{matrix} \right\}$ $\delta = \frac{5\sqrt{2}}{2}$

ETW Defects

Input: 8-dimensional defect : log-singularity, S^1 direction capped off

Poincaré symmetry along the brane preserved

2d transversal rotational symmetry broken

Non-Isotropic Solution Ansatz: $$ds^2 = e^{2\hat{\mathscr{A}}(\rho,\varphi)} ds_8^2 + e^{2\hat{\mathscr{B}}(\rho,\varphi)} (d\rho^2 + \rho^2 d\varphi^2)\,.$$

Separation of variables

Solutions: $$\hat{A}(\rho) = \frac{1}{8}\log\Big|\cosh\Big(8\hat{K}\log(\frac{\rho}{\rho_0})\Big)\Big|\,,\quad \hat{B}(\rho) = -\log\Big(\frac{\rho}{\rho_0}\Big) + \Big(\frac{\hat{\alpha}^2}{32} - \frac{7}{2}\Big)\hat{A}(\rho)$$

$$\hat{\chi}(\rho) = \hat{\alpha}\hat{A}(\rho)$$

$$\hat{U}(\varphi) = \frac{1}{8}\log\Big|\cos\Big(8\hat{K}\varphi\Big)\Big|\,,\qquad \hat{V}(\varphi)] = -\Big(\frac{\hat{\alpha}^2}{32} - \frac{9}{2}\Big)\hat{U}(\varphi) + \frac{\hat{\alpha}}{16}\hat{\psi}(\varphi)$$

$$\hat{\psi}(\phi) = \frac{\hat{\alpha}}{8}\log\Big|\cos(8\hat{K}\phi)\Big| \pm 2\log\Big|\tan(4\hat{K}\phi + \frac{\pi}{4})\Big|$$

$ETW\ 7^{\pm}$ solutions

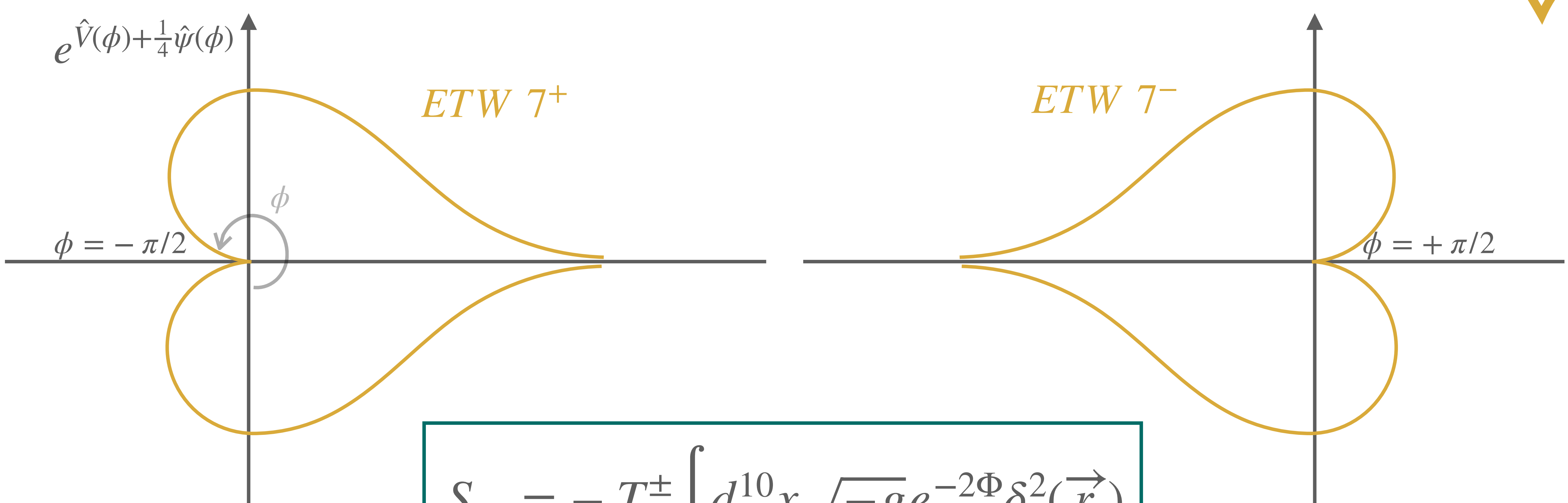
Qualitative behavior

Logarithmic singularities at $\rho = 0$, string coupling diverges

For appropriate constant ($\hat{\alpha} = \alpha^+$) same scaling as 9d defect

Dynamical Cobordism scaling satisfied: $\Delta \sim e^{-\frac{5}{4}\sqrt{2}D}$, $\mathscr{R} \sim e^{\frac{5}{2}\sqrt{2}D}$

$\delta = \dfrac{5\sqrt{2}}{2}$



$$S_{str} = -T_7^{\pm} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \delta^2(\vec{r})$$

with $\kappa_{10}^2 T_7 = 2\pi$

Cobordism Interpretation

Backreaction of 9d Domain Wall

Spontaneous compactification on S^1

Cobordism Group: $\Omega_1^{\xi} \neq 0$

Solution II^+

Finite L_y, breaking possible

Solutions $ETW\ 7^{\pm}$

$\Delta \sim e^{-\sqrt{2}D}$

$\mathscr{R} \sim e^{2\sqrt{2}D}$

Spacetime capped off

$\Omega^{\xi + ETW7^{\pm}} = 0$

Solutions I, II^-

Infinite L_y, defects not in EFT, gauging

non-BPS $\widehat{D8}$ brane

$\Omega_1^{\mathrm{Spin}} \to KO^{-1}(pt)$

$\mathbb{Z}_2$ charge cancellation

Single $\widehat{D8}$ inconsistent

Generalisation and more evidence

[Blumenhagen, Kneissl, Wang '23]

Setup: Gauge neutral, non-supersymmetric codimension-1 object w/ arbitrary dilaton coupling in arbitrary dimensions D

Physical realisation: Generalised BF model

Action: $$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-G}\left(\mathscr{R} - \frac{1}{2}(\partial\Phi)^2\right) - T\int d^D x\sqrt{-G}e^{a\Phi}\delta(r)\,.$$

ETWs: neutral (d-2)-dimensional branes

Generalisation and more evidence

[Blumenhagen, Kneissl, Wang ’23]

Setup: Gauge neutral, non-supersymmetric codimension-0 object w/ arbitrary dilaton coupling in arbitrary dimensions D

Physical realisation: Generalised DM model, $SO(16) \times SO(16)$, IIA with Romans mass

Action: $$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} \left(\mathscr{R} - \frac{1}{2} (\partial \Phi)^2 \right) - T \int d^D x \sqrt{-G} e^{b\Phi} \, .$$

Important difference: depending on value of b, charged ETWs also required!!

$$b_{cr} = \sqrt{\frac{2(D-1)}{D-2}}$$

$b \leq b_{cr}$

2 neutral ETW branes

$b > b_{cr}$

1 neutral ETW brane
+
1 charged ETW brane

$b_{DM} = \sqrt{\frac{2(3D-5)}{(D-2)}}$: charged ETW = O(D-2)-planes!

Summary and Outlook

- ETW-branes can be detected within the Dynamical Cobordism framework.

- Previously puzzling singularities in non-SUSY setups explained
  $\rightarrow$ dynamical cobordism scalings verified.

- Explicit descriptions of cobordism-predicted ETW branes possible.
  $\rightarrow$ possibly new objects?

- Independent verification of new ETW branes? Stability?

- Relation with other cobordism detected ETW-objects?

- Physical meaning of δ, relation with Distance Conjecture?

- Universal Local Dynamical Cobordism description for higher codimension objects?

Thank you for the attention!

Gauging Cobordism Charges

[Blumenhagen, Cribiori ’21]

K-theory charges are gauged
[Freed ’00]

Gauged symmetries accompanied by tadpole conditions: $0 = \int_M dF_{n-1} = \int_M J_n$

Idea: Cobordism and K-theory can be combined in single tadpole

In practice: add K-theory + cobordism contributions

$$0 = \int_M dF_{n-1} = \sum_{j \in \text{def}} \int_M \underbrace{Q_j \delta^{(n)}(\Delta_{10-n,j})}_{\text{D-brane contributions}} + \sum_{\text{i} \in \text{inv}} \underbrace{a_i \mu_{n,i}}_{\text{Geometric contributions}}$$

Bottom-up way to construct tadpoles without string theoretical input!
(up to constants!)

An example:

[Blumenhagen, Cribiori '21]

Focus on interplay of $K^{-6}(pt) = \mathbb{Z}$, $\Omega_6(pt) = 2\mathbb{Z}$:

Quantities entering tadpole:

D3-branes

Cobordism invariants

$$0 = \sum_i \int_M N_i \delta^{(6)}(\Delta_{4,i}) + a_1^{(6)} \frac{c_2(M) c_1(M)}{24} + a_2^{(6)} \frac{c_1^3(M)}{2} \, .$$

D3-tadpole for F-theory compactified on CY 4-fold Y over base B:

[Sethi, Vafa, Witten '96]

$$\int_B \sum_i \delta^{(6)}(\Delta_{4,i}) + \ldots = \frac{\chi}{24} = \int_B \Big(\frac{1}{2} c_2(B) c_1(B) + 15 c_1^3(B) \Big)$$

Perfectly reproduced for $a_1^{(6)} = -12, \;\; a_6^{(2)} = -30$