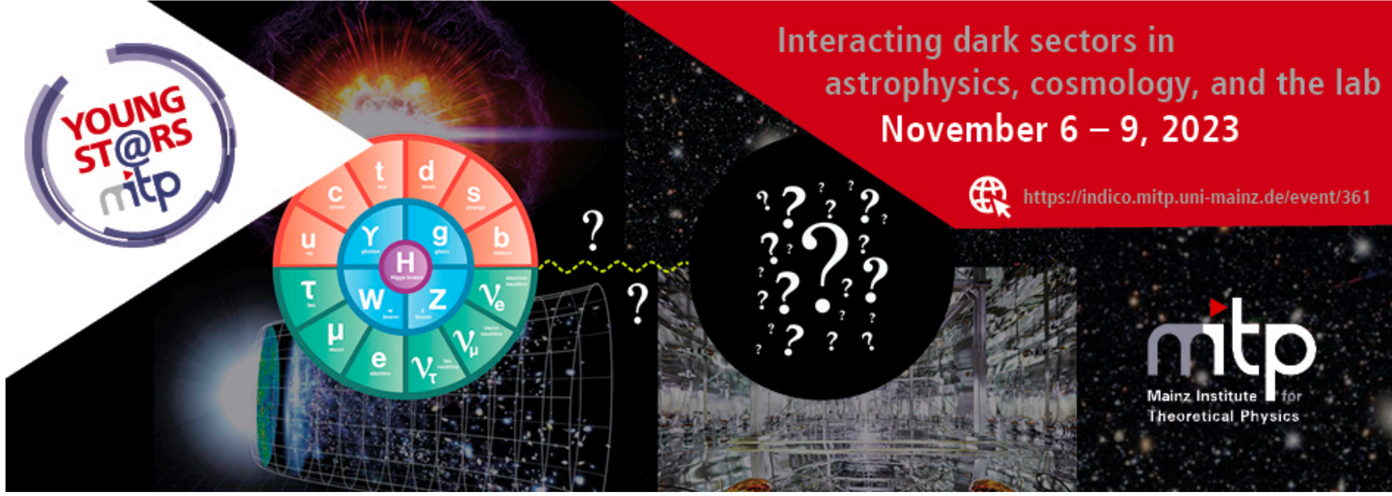
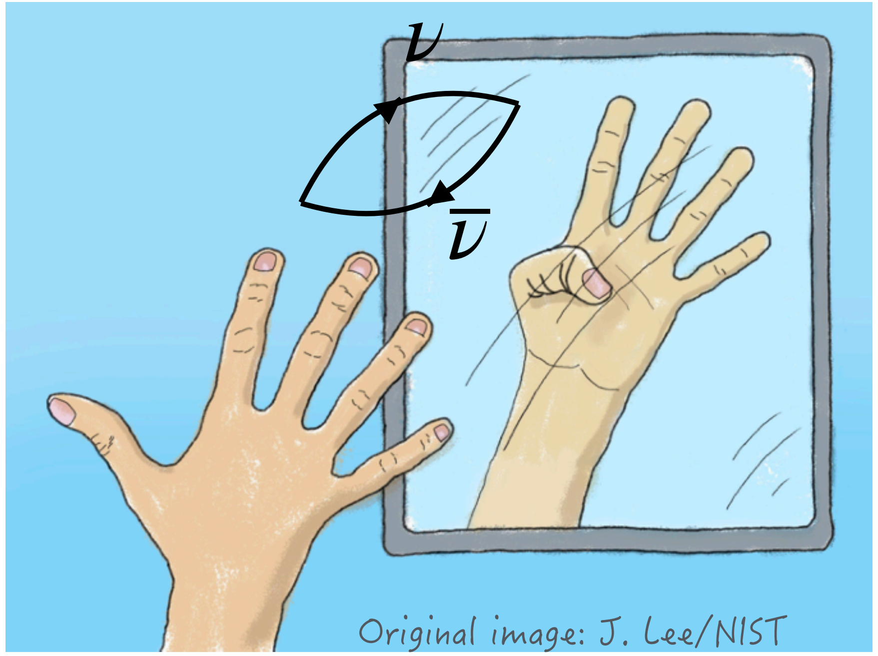


Neutrino forces and where to find them

Walter Tangarife



Collaborators:
Mitrajyoti Ghosh, Yuval Grossman, Xunjie Xu, and Bingrong Yu

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JHEP 02 (2023) 092

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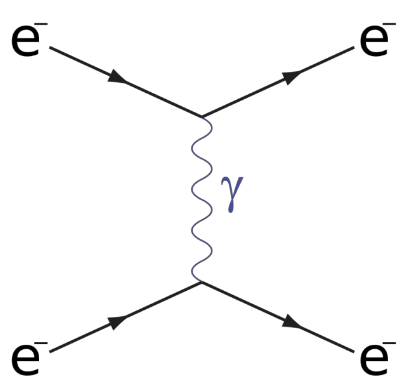
PHY-2310224

Motivation

We learned to compute the classical Coulomb potential with Peskin



Non-relativistic limit

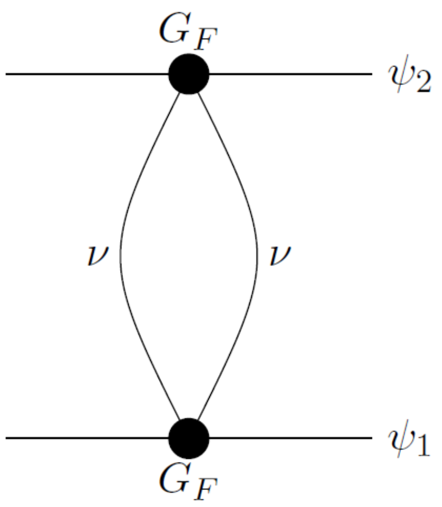
$$\mathcal{M} \sim \frac{1}{\mathbf{q}^2} \longrightarrow V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \;\; \frac{1}{\mathbf{q}^2} \, e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{1}{r}$$

For a massive mediator: $$V(\mathbf{r}) \sim \int \mathrm{d}^3\mathbf{q} \;\; \left(\frac{1}{\mathbf{q}^2 + m^2}\right) \, e^{-i\mathbf{q}\cdot\mathbf{r}} \sim \frac{e^{-mr}}{r}$$

$V(r)$ is computed by taking the Fourier transform of the amplitude. The range of the force is given by the location of the branch cut in the matrix element in the t-plane.

Motivation

A pair of massless neutrinos mediate a long-range force via one-loop diagrams



At leading order $$V(r) = \frac{G_F^2}{4\pi^3 r^5}$$

Feinberg & Sucher (1968)
Feinberg, Sucher & Au (1989)
Hsu & Sikivie (1994)
…

At distances larger than 1 nm, this force is weaker than the gravitational force between two protons

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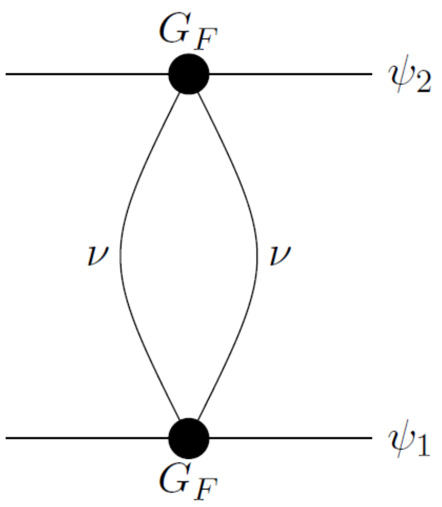

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At distances larger than 1 nm, this force is weaker than the gravitational force between two protons

Is there any way to probe this force that has not been explored yet?

Possible answers

To observe a small effect, look for symmetries that this force violates:

The two-neutrino force is the largest long-range parity-violating interaction in the Standard Model

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Spoiler:

We find that the effect is tiny

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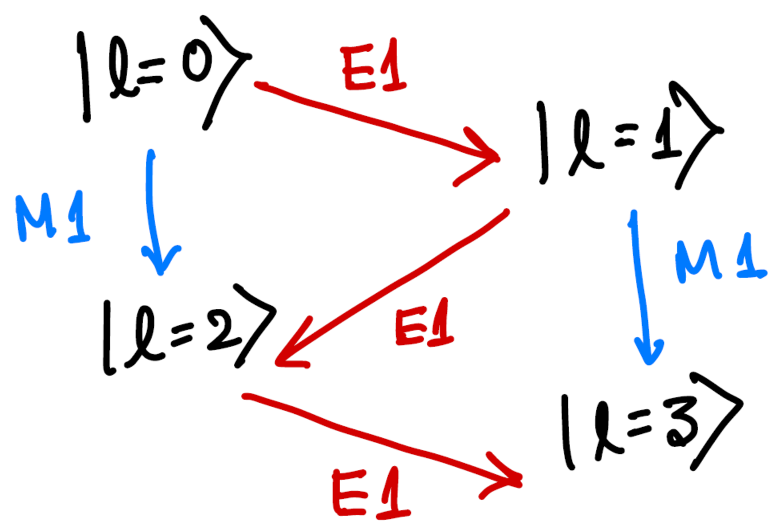
Observing atomic parity violation in atoms

Consider stimulated emission in an atom:

- Electric dipole transitions $E1$: between states of opposite parity
- Magnetic dipole transitions $M1$: between states of same parity



Parity is conserved

Walter Tangarife (Loyola Chicago)

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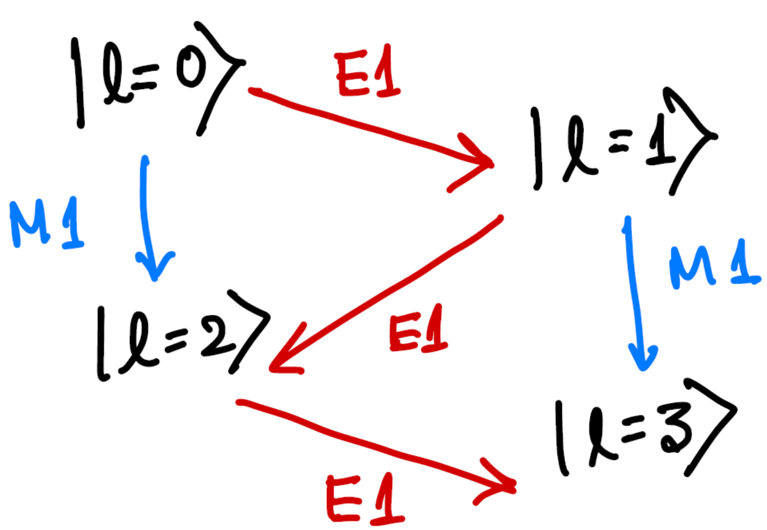
If the Hamiltonian contains a perturbation that violates parity, its eigenstates will contain a small mixture of opposite-parity corrections



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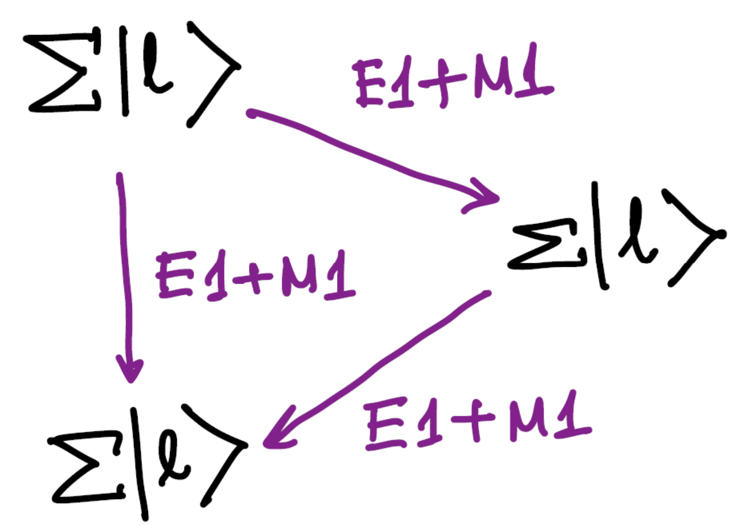

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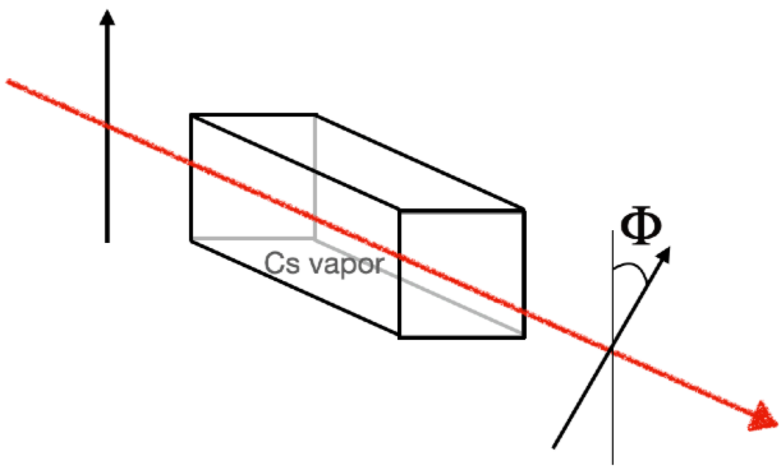
↓

Optical rotation: Left-polarized and right-polarized light will refract with different index of refraction in a sample of atomic vapors



$$\Phi = \frac{\pi L}{\lambda}\mathrm{Re}\left(n_R(\lambda) + n_L(\lambda)\right) \approx \frac{2\pi L}{\lambda}\mathrm{Re}\left(n_R(\lambda) + n_L(\lambda) - 2\right) R$$

near resonance

$$R \equiv \mathrm{Im}\left(\frac{E1_{PV}}{M1}\right)$$

Reviews: Khriplovich (1991), Bouchiat & Bouchiat (1997),…

Parity violating forces in the hydrogen atom

Assuming a) a static nucleus and b) that the electron velocity is a small parameter, the most general PV-potential is

$$V_{PNC}(r) = H_1 F(r)\vec{\sigma}_e \cdot \vec{v}_e + H_2 F(r)\vec{\sigma}_N \cdot \vec{v}_e + C(\vec{\sigma}_e \times \vec{\sigma}_N) \cdot \vec{\nabla}\left[F(r)\right]$$

Dobrescu & Mocioiu (2006)

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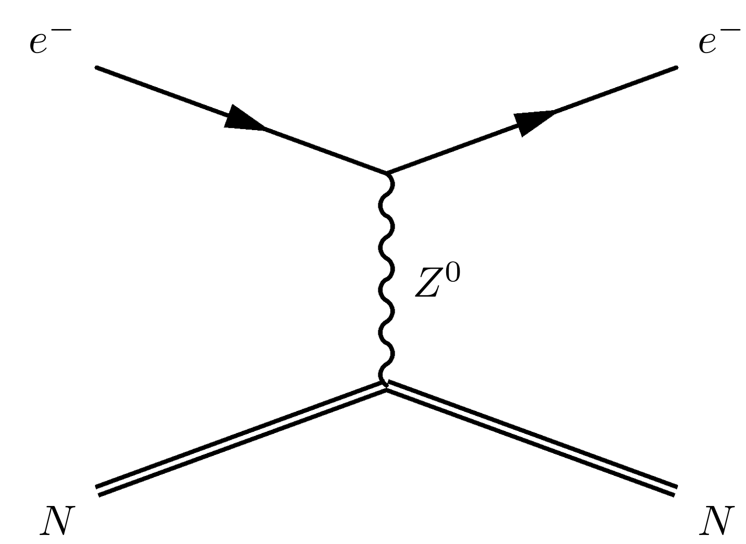
Tree-level



$$\mathcal{L}_{Z\bar{\psi}\psi} = \frac{1}{2}\frac{g}{\cos\theta_W}\bar{\psi}\left[(g_V^\psi - g_A^\psi\gamma^5)\not{Z}\psi\right]$$

$$H_1 = H_1^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W} g_A^e g_V^p,$$

$$H_2 = H_2^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W} g_V^e g_A^p,$$

$$C = C^{\text{tree}} = \frac{g^2}{2\cos^2\theta_W}\frac{g_V^e g_A^p}{2m_e},$$

$$F(r) = F^{\text{tree}}(r) = \frac{e^{-m_Z r}}{4\pi r}.$$

$$V_{PNC}^{\text{tree}} \sim \frac{g^2}{m_e}\left[\frac{e^{-m_Z r}}{r}\vec{\sigma}_e \cdot \vec{p} + \frac{e^{-m_Z r}}{r}\vec{\sigma}_p \cdot \vec{p} + (\vec{\sigma}_e \times \vec{\sigma}_p) \cdot \vec{\nabla}\left(\frac{e^{-m_Z r}}{r}\right)\right]$$

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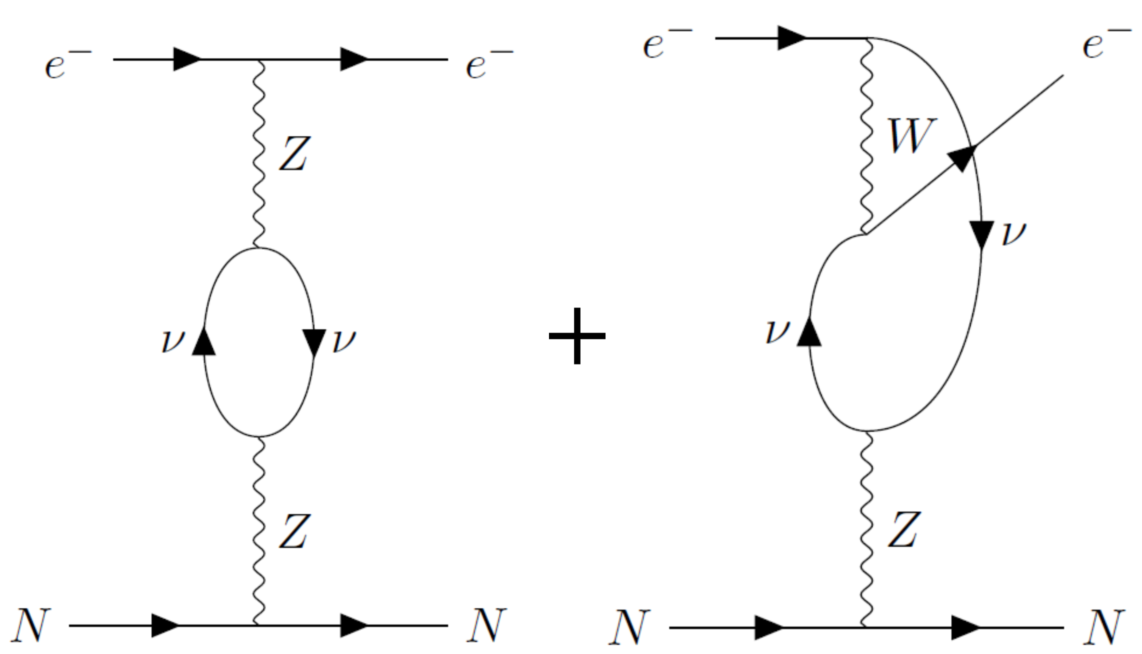
Loop-level: Enter the neutrino force

$$(\mathcal{O}_Z)_{ij} = -\frac{g^2}{8m_Z^2 c_W^2}[\bar{\psi}\gamma^\mu(g_V^\psi - g_A^\psi\gamma^5)\psi]\delta_{ij}[\bar{\nu}_j\gamma_\mu(1-\gamma^5)\nu_i] + (\mathcal{O}_W)_{ij} = -\frac{g^2}{8m_W^2}U_{\alpha j}U^*_{\alpha i}[\bar{\psi}\gamma^\mu(1-\gamma^5)\psi][\bar{\nu}_j\gamma_\mu(1-\gamma^5)\nu_i]$$



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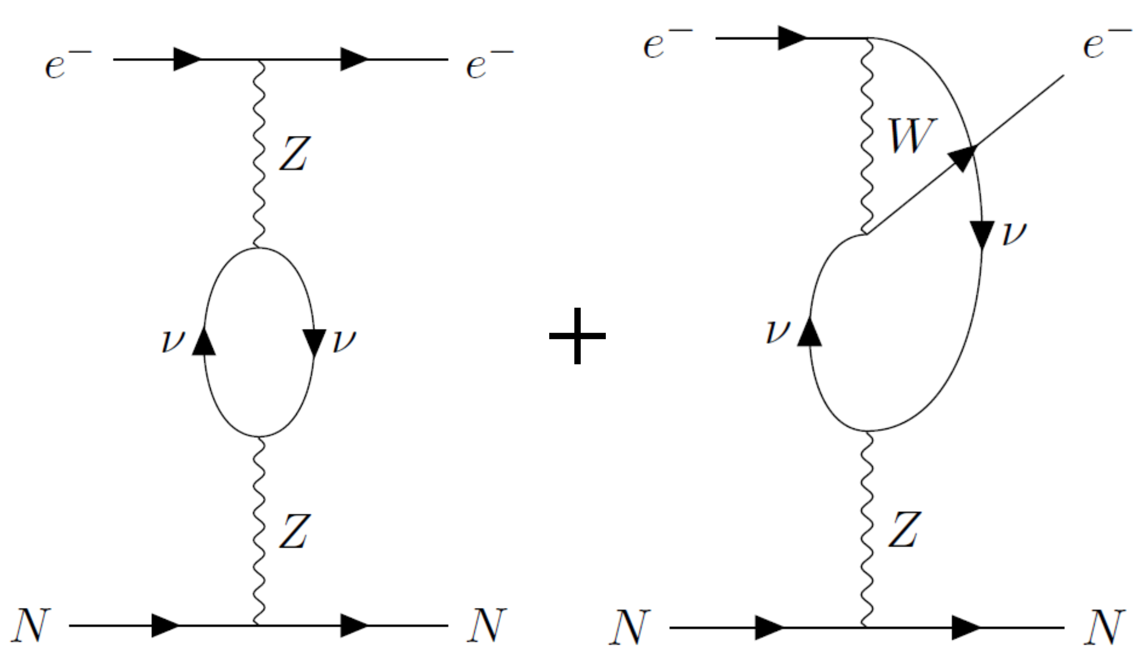
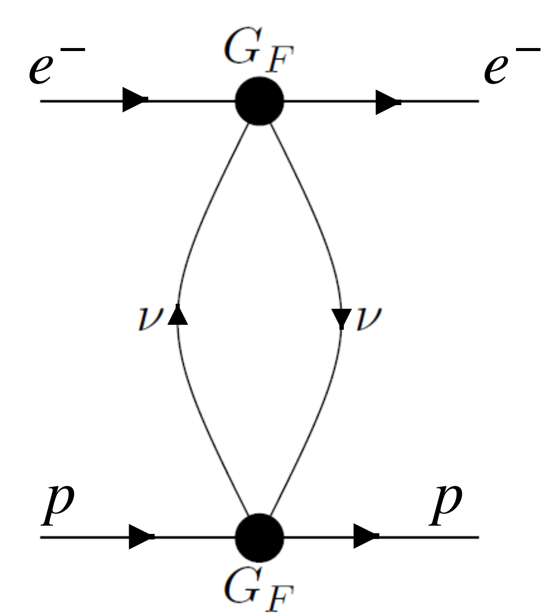
$$\mathcal{O}_4 = -\frac{G_F}{\sqrt{2}}[\bar{\psi}\gamma^\mu(a^\psi - b^\psi\gamma^5)\psi][\bar{\nu}\gamma_\mu(1-\gamma^5)\nu]$$





Walter Tangarife (Loyola Chicago)

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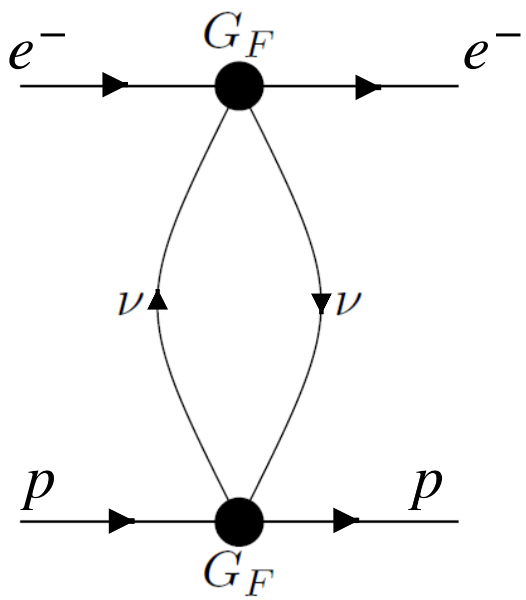
$$i\mathcal{M} = -\frac{(-iG_F)^2}{2}\bar{e}\bar{N}\left[\Gamma^e_\mu\Gamma^N_\nu\right]\int\frac{\mathrm{d}^4k\mathrm{d}^4k'}{(2\pi)^4}\delta^4(q-k-k')\mathrm{Tr}\left[i\Gamma^\mu\frac{i(-\not{k}'+m)}{k'^2-m^2}i\Gamma^\nu\frac{i(\not{k}+m)}{k^2-m^2}\right]eN.$$



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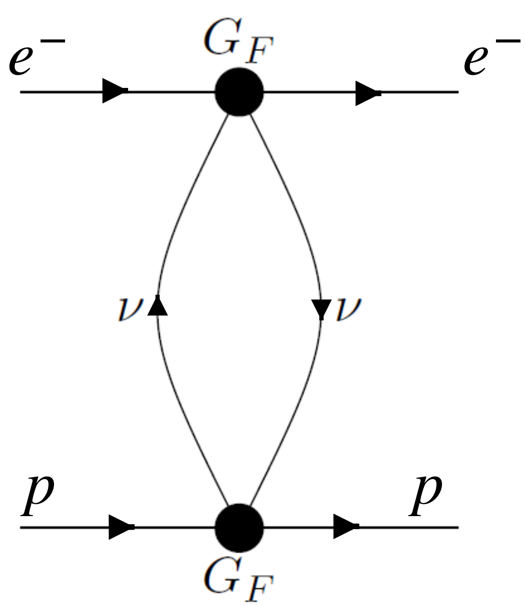

$$V^{\text{loop}}_{PNC} \approx \sum_i \frac{G_A}{m_e}\left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)\left[(2\vec{\sigma}_p \cdot \vec{p}_e)V_{\nu_i\nu_i}(r) + (\vec{\sigma}_e \times \vec{\sigma}_p)\cdot \vec{\nabla}V_{\nu_i\nu_i}(r)\right]$$

$V_{\nu\nu}$ is computed by taking the Fourier transform of the parity-conserving part of the amplitude (using the Cutkosky cutting rules)

$$V^{\text{Dirac}}_{\nu\nu}(r) = \frac{G_F^2 m_\nu^3}{4\pi^3}\frac{K_3(2m_\nu r)}{r^2} \qquad V^{\text{Majorana}}_{\nu\nu}(r) = \frac{G_F^2 m_\nu^2}{2\pi^3}\frac{K_2(2m_\nu r)}{r^3}$$

Effect of the neutrino force on the hydrogen atom

Ghosh, Grossman & Tangarife PRD (2020)

Unperturbed eigenstates $|n, f, m_f, j, \ell, s_p, s_e\rangle$

$\hat{H}_0$ $\vec{F} \equiv \vec{L}_e + \vec{S}_e + \vec{S}_p$ $\vec{J} \equiv \vec{L}_e + \vec{S}_e$

The energy of a state with $f, j, \ell, s_e = s_p = \frac{1}{2}$

$$E_{nfj\ell} = (E_0)_n + (E_{\rm fine})_{nj} + (E_{\rm hyperfine})_{nfj\ell}$$

$$(E_0)_n = -\frac{\alpha^2 m_e}{2n^2} \qquad (E_{\rm fine})_{nj} = -\frac{\alpha^4 m_e}{2n^4}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)$$

$$(E_{\rm hyperfine})_{nfj\ell} = \frac{\alpha^4 g_p}{m_p} a_0^3 \frac{\ell(\ell+1) m_e^2 \left(f(f+1) - j(j+1) - \frac{3}{4}\right)}{4j(j+1)} \left\langle \frac{1}{r^3} \right\rangle_{n\ell}$$

The only degeneracy remains in m_f

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Ghosh, Grossman & Tangarife PRD (2020)

We treat $V_{\nu\nu}$ as a perturbation $$|\psi_q^1\rangle = |\psi_q^0\rangle + \sum_{p\neq q} \frac{\langle\psi_p^0|V|\psi_q^0\rangle}{E_q^0 - E_p^0}|\psi_p^0\rangle$$

$\eta \equiv r/a_0$

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$$\langle n\ell m|V_{PNC}^{\rm tree}|n'\ell' m'\rangle \sim \int_0^\infty {\rm d}\eta\; \eta^2\; \eta^{\ell'} V_{PNC}^{\rm tree}(\eta)\eta^{\ell} \;\sim \frac{\alpha^{2\ell+5} m_e^{2\ell+3}}{m_Z^{2\ell+2}} = m_e \alpha^{2\ell+5}\left(\frac{m_e}{m_Z}\right)^{2\ell+2}$$

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$$\langle n\ell m | V_{PNC}^{\text{loop}} | n'\ell' m'\rangle \sim \frac{G_F^2}{m_e a_0^6}\int \mathrm{d}\eta\ \eta^2 \eta^{\ell'} \left(\frac{1}{\eta^6}\right) \eta^\ell \exp\left[-\eta\left(\frac{1}{n}+\frac{1}{n'}\right)\right]$$

for $\ell = 0$ and $\ell = 1$, the radial integral does not converge, indicating the failure of four-Fermi theory

for $\ell \geq 2$

$$\frac{\alpha^2}{m_e m_Z^4 a_0^6}\int_0^\infty \mathrm{d}\eta\ \eta^{2\ell-3}\exp(-n_{sup}\eta) \sim m_e \alpha^8 \left(\frac{m_e}{m_Z}\right)^4 \times (f(\ell) \sim \mathcal{O}(1))$$

Effect of the neutrino force on the hydrogen atom

Ghosh, Grossman & Tangarife PRD (2020)

Let's look now at electric and magnetic transitions: Use states with $\ell = 3$ since they can mix with states with $\ell = 2$ and $\ell = 4$

Our goal: To compute $R \equiv \mathrm{Im}\left(\frac{E1_{PV}}{M1}\right)$

$|A\rangle = |4,3,3,5/2,3\rangle \equiv 4F_{5/2,F=3}$

$|B\rangle = |4,3,3,7/2,3\rangle \equiv 4F_{7/2,F=3}$

Before adding $V_{\nu\nu}$, these states have the same ℓ and there can be an $M1$ transition but not an $E1$ transition

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But now they are corrected

$$|A'\rangle = |A\rangle + \frac{\langle\Delta|V_{PNC}|A\rangle}{E_A - E_\Delta}|\Delta\rangle + \cdots \qquad |B'\rangle = |B\rangle + \frac{\langle\Delta|V_{PNC}|B\rangle}{E_B - E_\Delta}|\Delta\rangle + \cdots$$

$|\Delta\rangle = |4,3,3,5/2,2\rangle$

Effect of the neutrino force on the hydrogen atom

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So we can finally compute

$$R = \mathrm{Im}\left(\frac{E1_{PV}}{M1}\right) = \mathrm{Im}\left(\frac{\langle A'|\hat{P}|B'\rangle}{\langle A'|\hat{M}|B'\rangle}\right) \approx \left(-\frac{1}{4} + s_W^2 + \frac{1}{2}|U_{ei}|^2\right)\left[-7.7\times10^{-33} + 3.7\times10^{-32}\left(\frac{m_{\nu_i}}{\alpha\, m_e}\right)^2\right]$$

The rotation due to the neutrino force would be $\Phi \sim 10^{-32}$ rads

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This is about 23 orders of magnitude smaller than what can be measured in the lab (with Cs) Lintz, Guéna & Bouchiat (2006)

Can we probe the neutrino force?

Not yet!

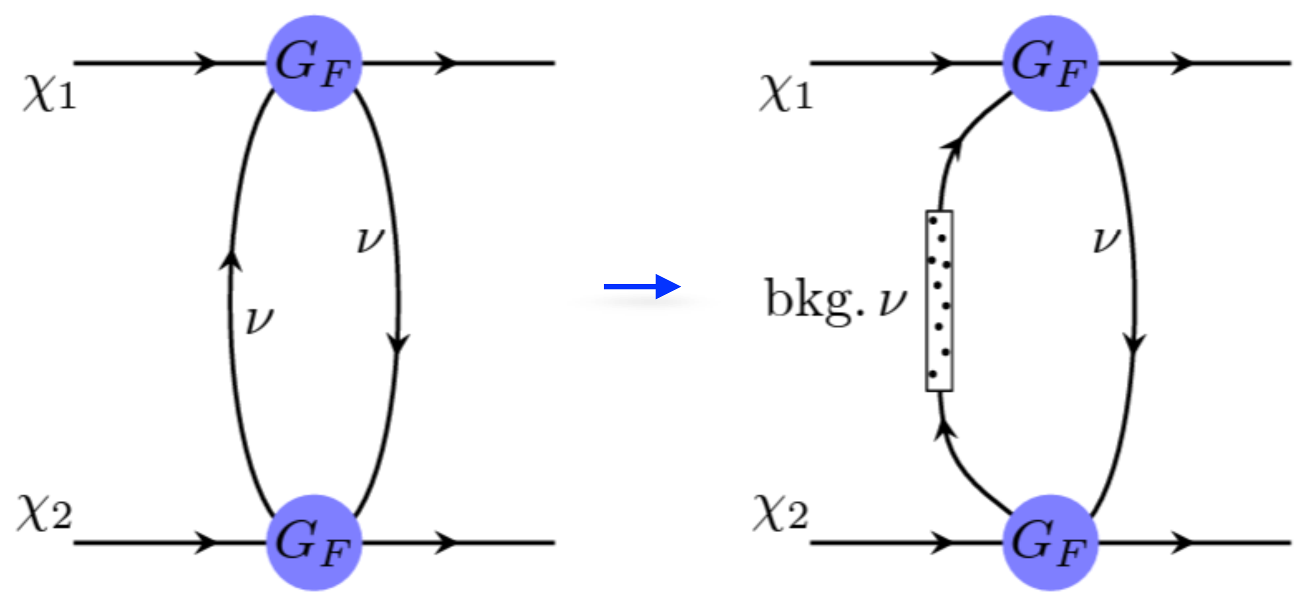
The measurement of optical rotation due to the neutrino loop is extremely challenging given the resolutions we can achieve today.

Walter Tangarife (Loyola Chicago)

Neutrino forces in a finite-density background

We move to calculate the neutrino-exchange in the presence of a background of neutrinos

Ghosh, Grossman, Tangarife, Xu, Yu JHEP (2022)



Intuition: $$V(r) \sim G_F^2 \times E_\nu \Phi \times \frac{1}{r}$$

Vacuum (G_F^2) Background ($E_\nu \Phi$)

Examples: Cosmic neutrino background, Reactor neutrino fluxes

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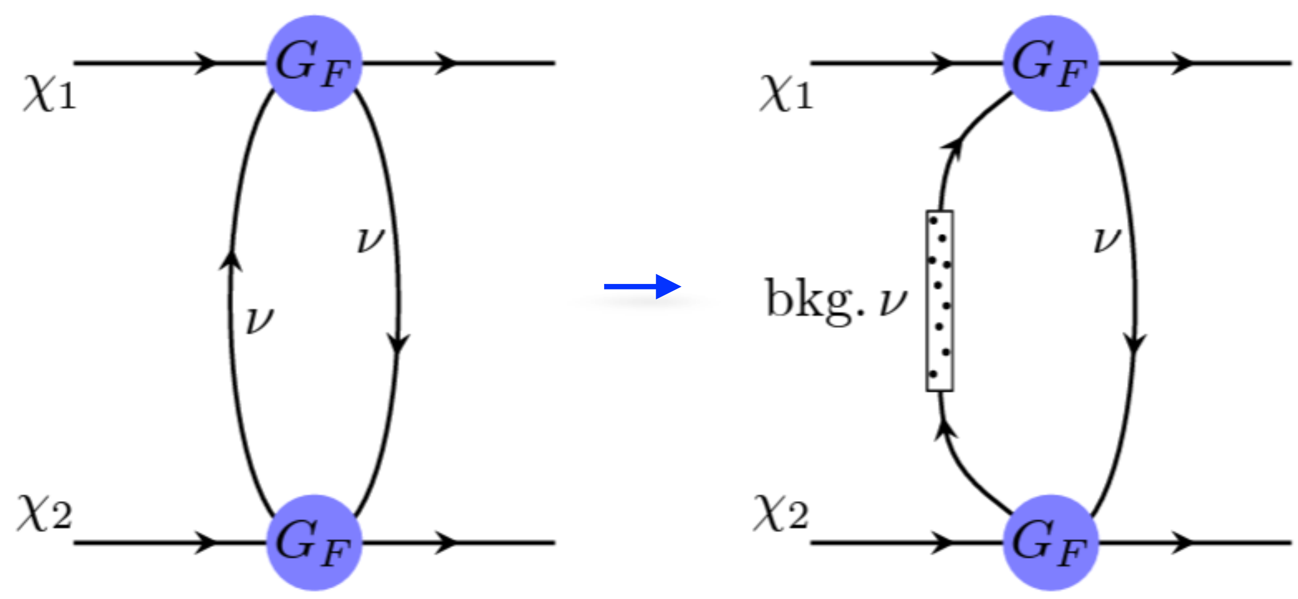


$$S_f(k) = (k\cdot\gamma + m_f)\cdot\left[\frac{i}{k^2 - m_f^2} + 2\pi\delta\left(k^2 - m_f^2\right)\Theta\left(\pm k^0\right) n_{f\pm}(\mathbf{k})\right]$$

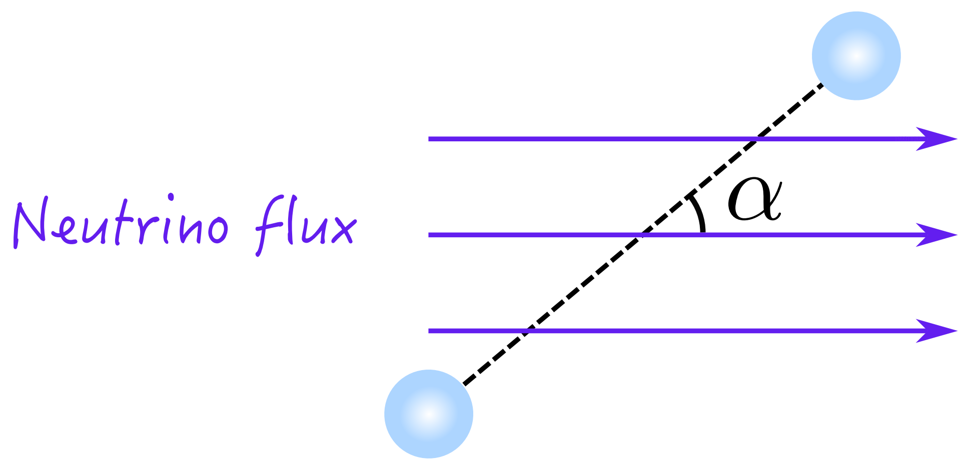
Vacuum

Background

Examples: Cosmic neutrino background, Reactor neutrino fluxes

Neutrino forces in anisotropic backgrounds

We consider backgrounds with a specific direction, e.g. solar, supernova, and reactor neutrinos



Let's assume the neutrino flux is monochromatic (big assumption)

$$n_{\pm}(\mathbf{k}) = (2\pi)^3 \delta^3 (\mathbf{k} - \mathbf{k}_0) \Phi_0$$

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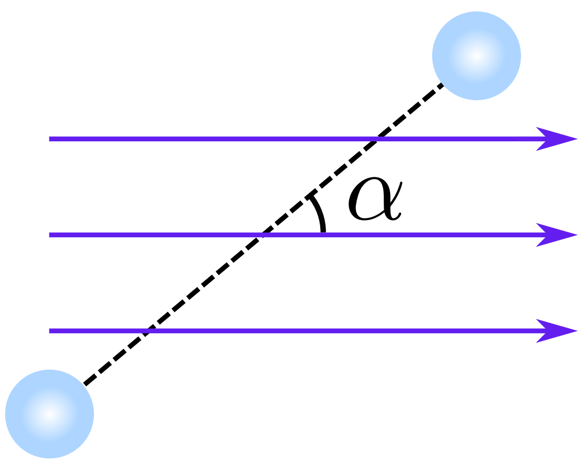


Let's assume the neutrino flux is monochromatic (big assumption)

$$n_{\pm}(\mathbf{k}) = (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}_0)\Phi_0$$

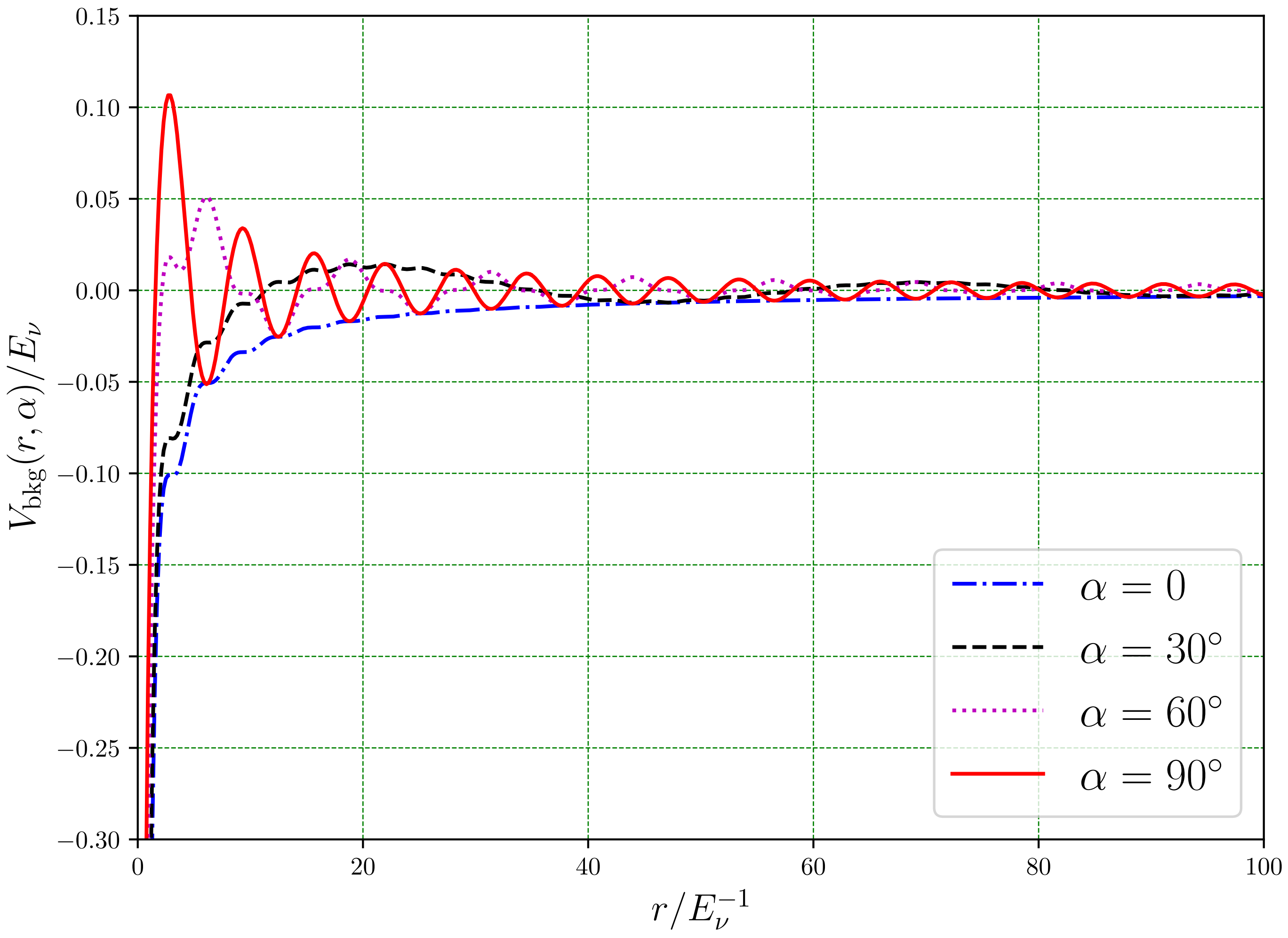
$$V(\mathbf{r}) = -\int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{A}(\mathbf{q})$$

$$i\mathcal{A}(q) = \frac{G_F^2 g_V^1 g_V^2}{2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma^0(1-\gamma_5) S_\nu(k) \gamma^0 (1-\gamma_5) S_\nu(k+q)\right]$$

Neutrino forces in anisotropic backgrounds

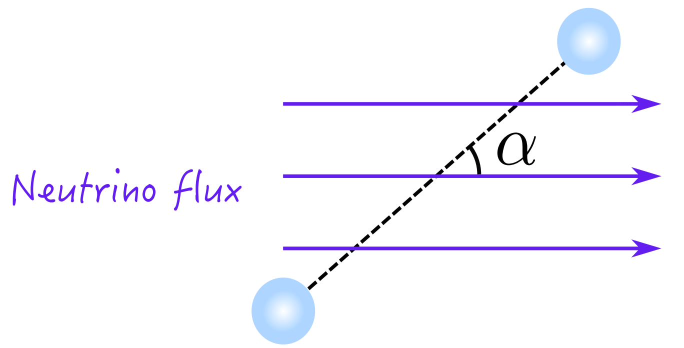
The resulting background potential at long distance is

$$V_{\rm bkg}\left(r \gg E_\nu^{-1}, \alpha\right) = -\frac{g_V^1 g_V^2}{\pi} G_F^2 \Phi_0 E_\nu \frac{1}{r}\left\{\cos^2\left(\frac{\alpha}{2}\right)\cos\left[\left(1-\cos\alpha\right)E_\nu r\right] + \sin^2\left(\frac{\alpha}{2}\right)\cos\left[\left(1+\cos\alpha\right)E_\nu r\right]\right\}$$



In the limit $r \gg E_\nu^{-1}, \alpha \ll 1$,

$$V_{\rm bkg} = -\frac{g_V^1 g_V^2}{\pi} G_F^2 \times \Phi_0 E_\nu \times \frac{1}{r} \times \cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$



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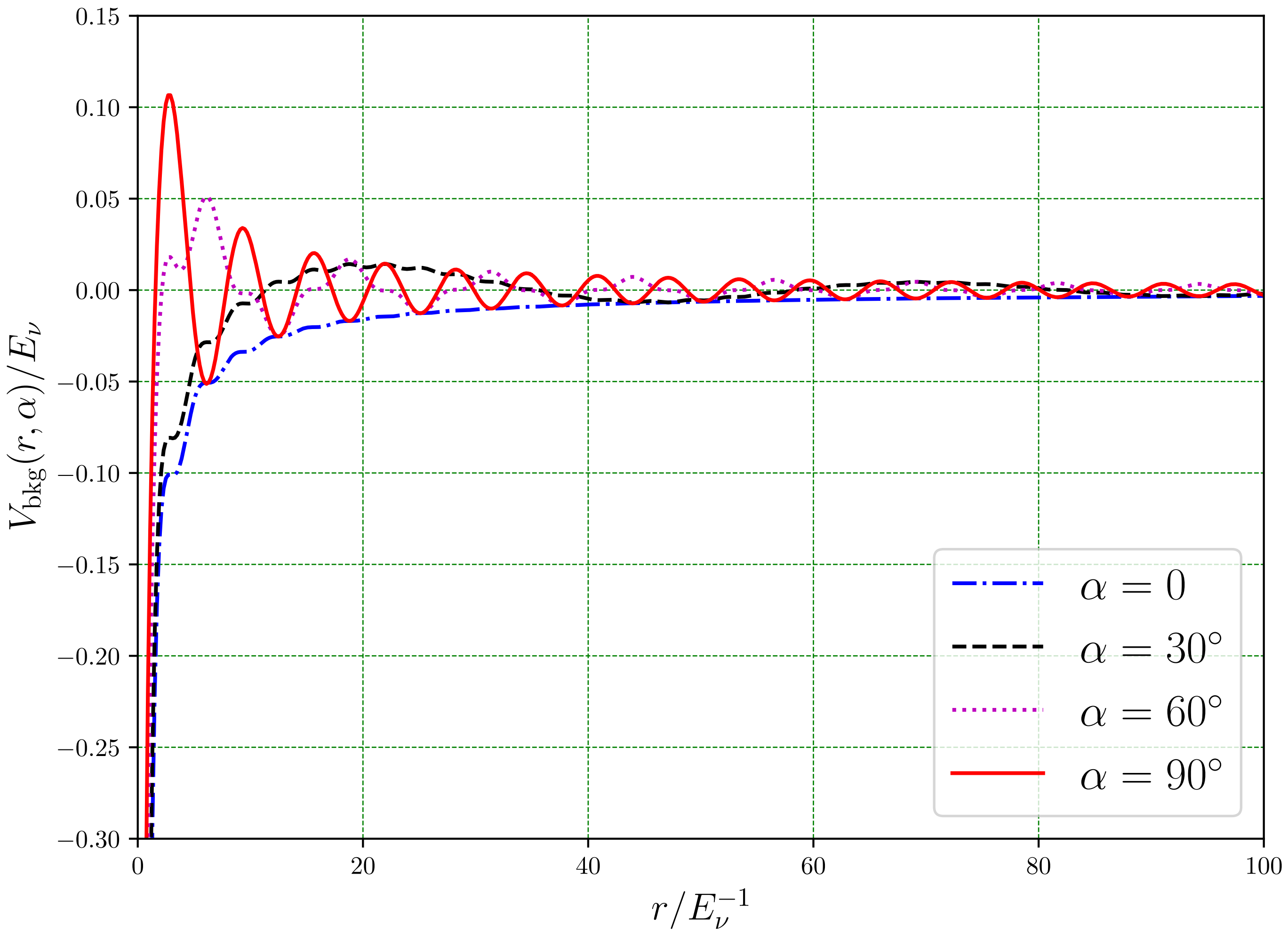

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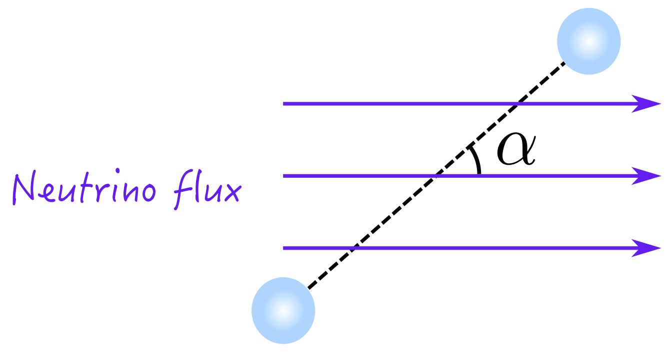
Taking into account finite size and energy spread,

$$\alpha^2 \lesssim \frac{\pi}{\Delta(E_\nu r)}$$



Walter Tangarife (Loyola Chicago)

Can we probe this force with experiments?

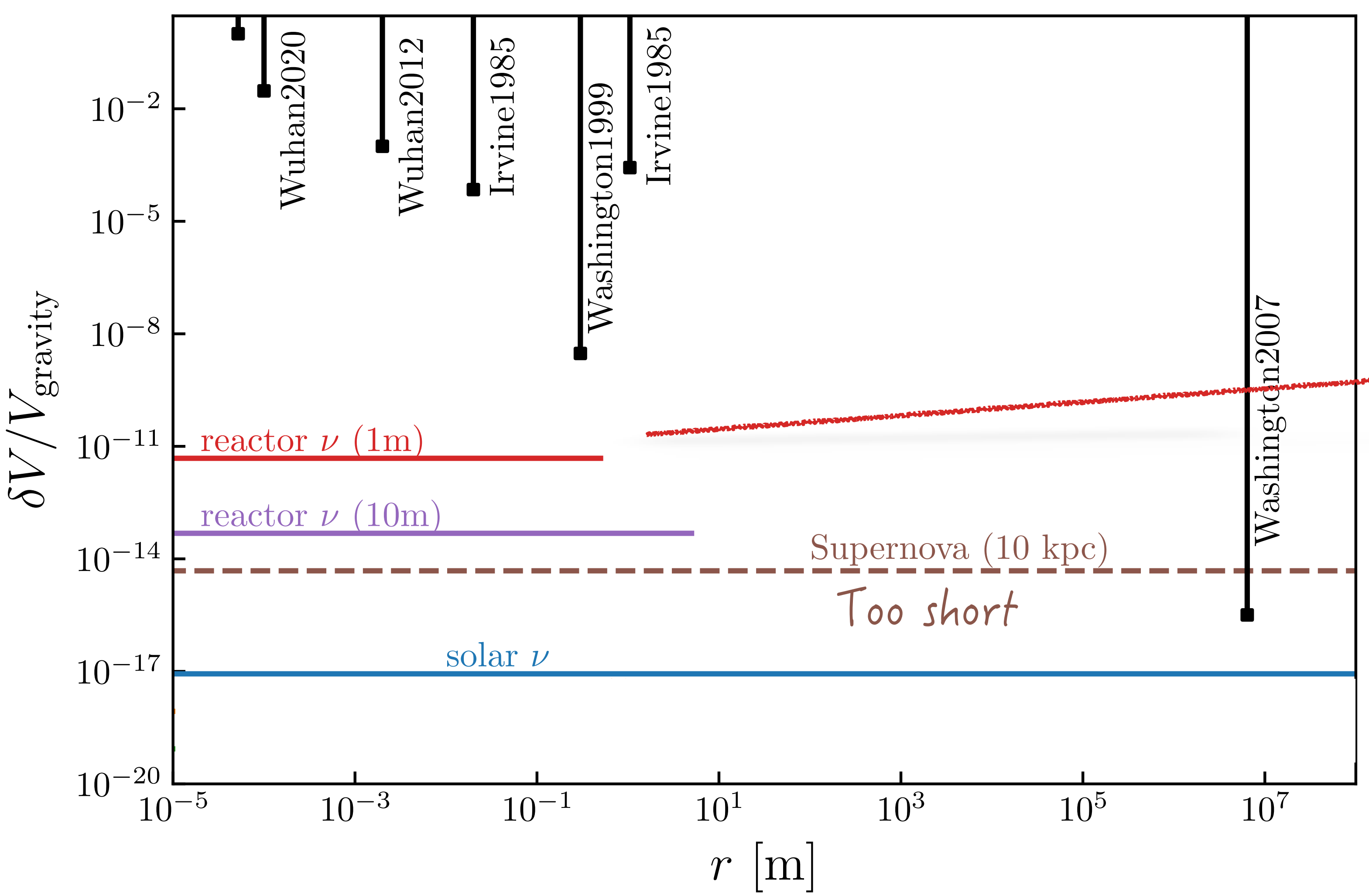
WEP: possible differences between the accelerations of different test bodies in the same gravitational field.

ISL: the variation of the gravitational attraction between two test bodies when their distance varies.

exp	$\delta V/V_{\text{gravity}}$	$\langle r \rangle$
Washington2007	3.2×10^{-16}	~ 6400 km
Washington1999	3.0×10^{-9}	~ 0.3 m
Irvine1985	0.7×10^{-4}	$2 - 5$ cm
Irvine1985	2.7×10^{-4}	$5 - 105$ cm
Wuhan2012	10^{-3}	~ 2 mm
Wuhan2020	3×10^{-2}	~ 0.1 mm
Washington2020	~ 1	52 μm
Future levitated optomechanics	$\sim 10^{4}$	1 μm

sensitivities

Can we probe this force with experiments?



Reactor neutrinos

$\Phi \sim 5 \times 10^{13}\,\mathrm{cm^{-2}s^{-1}}$

$E_\nu \sim 2\,\mathrm{MeV}$

Solar neutrinos (pp)

$\Phi \sim 6 \times 10^{10}\,\mathrm{cm^{-2}s^{-1}}$

$E_\nu \sim 0.3\,\mathrm{MeV}$

Can we probe the neutrino force?

Not yet!

The measurement of optical rotation due to the neutrino loop is extremely challenging given the resolutions we can achieve today.

Nonetheless, this calculation, performed for other systems, could lead to somewhat larger quantities and the next step would most likely be an application of this idea to many-electron atoms, beyond the simple hydrogen case. The matrix elements in these atoms are amplified by an additional Z^3 factor.

On the other hand, a strong neutrino background could significantly enhance neutrino forces. In particular, in the small-α limit, the force could be behave as $1/r$ even in the long-range regime.

The neutrino force in the solar or reactor neutrino background is much more experimentally accessible than the one in vacuum. Dedicated experimental efforts are called for to check if these enhancement factors can be exploit in order to detect the elusive neutrino force.

Thank you!

The propagator

$$S_F \propto \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\sqrt{2E_\mathbf{p}}} \frac{1}{\sqrt{2E_\mathbf{k}}} e^{-ip\cdot x + ik\cdot y} \langle 0 | a_\mathbf{p} a_\mathbf{k}^\dagger | 0 \rangle$$

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Replace $|0\rangle \to |w\rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3} w(\mathbf{p}) a^\dagger_\mathbf{p} |0\rangle \,, \; \langle w | w \rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3} |w(\mathbf{p})|^2 \equiv 1$

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$$w^*(\mathbf{k}) w(\mathbf{p}) \xrightarrow{\text{smearing}} \frac{1}{V} \int w^*(\mathbf{k}) w(\mathbf{p}) e^{i(\mathbf{p}-\mathbf{k})\cdot \Delta\mathbf{x}} d^3\Delta\mathbf{x} = \frac{(2\pi)^3 \delta^3(\mathbf{p}-\mathbf{k})}{V} |w(\mathbf{p})|^2$$

$$= (2\pi)^3 \delta^3(\mathbf{p}-\mathbf{k}) n_+(\mathbf{p})$$

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$$= -\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} (2\pi)\delta\left(p^2 - m^2\right) \Theta\left(p^0\right) n_+(\mathbf{p})$$

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$$S_F \propto \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \left\{ \frac{i}{p^2 - m^2 + i\epsilon} - (2\pi)\delta\left(p^2 - m^2\right) \Theta\left(p^0\right) n_+(\mathbf{p}) \right\}$$

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