Jožef Stefan Institute

Constraining Effective HNL Interactions at Future Lepton Colliders

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YOUNGST@RS - Interacting dark sectors in astrophysics, cosmology, and the lab
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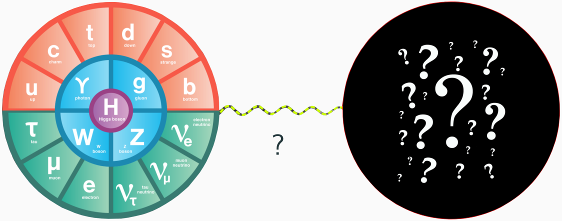
Dark Sectors

Standard Model (SM) Dark Sector



$\nparallel$ $\parallel$?

- Dark matter
- Neutrino masses $\Leftrightarrow$ SM-singlet fermions N_R?
- Baryon asymmetry

Right-Handed Neutrinos

As N_R is a SM-singlet, additional renormalisable terms ($d = 4$):

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} + i\bar{N}_R \partial\!\!\!/ N_R - \left[\frac{1}{2}\bar{N}_R^c M_R N_R + \bar{L} Y_\nu N_R \tilde{H} + \text{h.c.}\right]$$

Introduce $\mathcal{N}$ states N_R:

$$-\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{N}_R^c\end{pmatrix}\mathcal{M}_\nu \begin{pmatrix}\nu_L^c \\ N_R\end{pmatrix} + \text{h.c.}\,; \quad \mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}\,, \quad M_D = \frac{v}{\sqrt{2}} Y_\nu$$

Λ $\uparrow$ $m_N \approx M_R$ (eV to 10^{19} GeV)

$M_D \ll M_R$

$\mathcal{U}^\dagger \mathcal{M}_\nu \mathcal{U}^*$

$U_{\nu N} \approx M_D M_R^{-1}$

$m_\nu \approx -M_D M_R^{-1} M_D^T$ ($<$ eV)

$N_M = N_R + N_R^c$

Majorana vs Dirac Heavy Neutrinos

Other scenario: $2\mathcal{N}$ SM-singlet states

$$-\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{N}_R^c & N_R'\end{pmatrix}\mathcal{M}_\nu\begin{pmatrix}\nu_L^c \\ N_R \\ N_R'^c\end{pmatrix} + \text{h.c.}\,; \quad \mathcal{M}_\nu = \begin{pmatrix}0 & M_D \\ M_D^T & M_R\end{pmatrix}, \quad M_R = \begin{pmatrix}0 & M_S^T \\ M_S & \mu\end{pmatrix}$$

In $\mu \ll 1$ limit (approximate $\Delta L = 0$), obtain Dirac fields

$$N_D = N_R + N_R'^c$$

$\Rightarrow N_D \equiv 2 \times N_M$ with degenerate masses and opposite CP phases

Two low-scale seesaw scenarios:

$$1)\ Y_\nu \ll 1 \ \Rightarrow\ m_\nu \approx -M_D M_R^{-1} M_D^T, \quad U_{\nu N} \sim \sqrt{m_\nu / m_N}$$
$$2)\ \Delta L \approx 0 \ \Rightarrow\ m_\nu \approx -M_D M_R^{-1} M_D^T, \quad U_{\nu N} \gtrsim \sqrt{m_\nu / m_N}$$

Active-Sterile Mixing Phenomenology

The active-sterile mixing induces

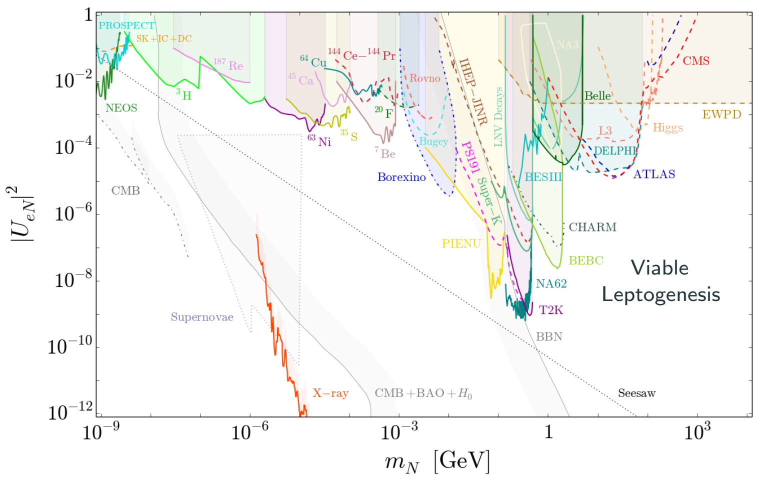
$$\mathcal{L} \supset \left[-\frac{g}{\sqrt{2}} U_{\alpha N_i} \bar{\ell}_\alpha \not{W} P_L N_i + \text{h.c.} \right] - \frac{g}{2c_W} U_{\alpha N_i} \bar{\nu}_\alpha \not{Z} P_L N_i - \frac{g}{2c_W} U^*_{\alpha N_i} U_{\alpha N_j} \bar{N}_i \not{Z} P_L N_j$$

Any SM CC or NC process can produce single N (suppressed by U^2) or N pair (U^4)

Constraints on U^2 for $m_N \in [\text{eV}, \text{TeV}]$:

- β decays (β, $2\nu\beta\beta$, $0\nu\beta\beta$)
- Charged meson decays ($\pi^+ \to e^+ N$, $K^+ \to \mu^+ N$)
- Collider (Same-sign leptons, electroweak precision)
- Astrophysics + cosmology (supernovae, BBN, ΔN_{eff})

Active-Sterile Mixing Constraints



[PDB, Deppisch, Dev, 2019]

SMEFT + Right-Handed Neutrinos

SM and N_R may couple to new degrees of freedom at Λ

For $E \ll \Lambda$, integrate out new physics $\Rightarrow \mathcal{O}^{(d)}$, $d > 5$

- Building blocks: $Q, u_R, d_R, L, e_R, N_R, H$

Λ — e.g. Z', leptoquarks, $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$\mu = m_Z$ — $\bar{L} N_R \tilde{H},\ \bar{N}_R^c N_R,\ \sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$ (SMEFT)

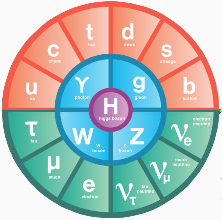
$\mathrm{SU}(3)_c \times \mathrm{U}(1)_Q$

$\mu \ll m_Z$ — $m_\nu,\ m_N,\ U_{\nu N},\ \sum_k C_k^{(\bar{d})} \mathcal{O}_k^{(\bar{d})}$ (LEFT)

- Complete basis of operators written up to $d = 9$

[Bhattacharya, Wudka, 2015]
[Li, Ren, Xiao, Yu, Zheng, 2021]

Standard Model (SM)



Dark Sector

$\bar{L} Y_\nu N \tilde{H}$

$\sum_i C_i^{(d)} \mathcal{O}_i^{(d)}$

$\nparallel$ $\qquad\qquad$ $\parallel ?$

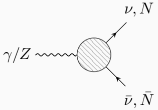
- Dark matter
- Neutrino masses
- Baryon asymmetry

$\Leftrightarrow$

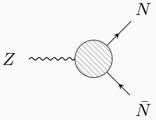
$$N_R \begin{cases} N_M = N_R + N_R^c \\ N_D = N_R + N_L \end{cases}$$

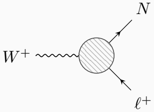
Phenomenology - Leading Effects

- (Dressed) Active-sterile mixing: $\bar{L}N\tilde{H}$, $\bar{L}N\tilde{H}(H^\dagger H)$
- Dipole moments: $(\bar{N}\sigma_{\mu\nu}N)B^{\mu\nu}$, $(\bar{L}\sigma_{\mu\nu}N)\tilde{H}B^{\mu\nu}$, $(\bar{L}\sigma_{\mu\nu}N)\tau^I\tilde{H}W^{I\mu\nu}$

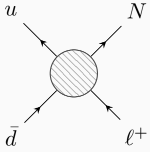


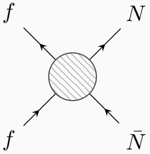
- Bosonic currents: $(H^\dagger iD_\mu H)(\bar{N}\gamma^\mu N)$, $(\tilde{H}^\dagger iD_\mu H)(\bar{N}\gamma^\mu e_R)$





- 4-Fermion CC: $(\bar{L}e_R)\epsilon(\bar{L}N)$, $(\bar{Q}d_R)\epsilon(\bar{L}N)$, $(\bar{Q}u_R)\epsilon(\bar{N}L)$, $(\bar{d}_R\gamma_\mu u_R)(\bar{N}\gamma^\mu e_R)$
- 4-Fermion NC: $(\bar{f}_R\gamma_\mu f_R)(\bar{N}\gamma^\mu N)$, $(\bar{L}\gamma_\mu L)(\bar{N}\gamma^\mu N)$, $(\bar{Q}\gamma_\mu Q)(\bar{N}\gamma^\mu N)$





Constraints on SMEFT + N_R

- (Dressed) Active-sterile mixing – β decays, charged meson decays, collider, ...

- Dipole moments – Mono-γ, mono-j, decay signatures (LEP, LHC)
  – Fixed target experiments (CHARM, NuCal, NA64)
  – Neutrino (up)scattering (COHERENT, NUCLEUS)
  – Inv. vector meson decays $\{\pi^0, \eta, \eta', \phi, J/\psi\} \to$ inv.
  – Supernova cooling (SN1987A)

- Bosonic currents – Invisible Z decays
  – Peak searches in meson and τ decays (PIENU, NA62, T2K)

- 4-Fermion CC & NC – Mono-γ, mono-j
  – Inv. vector meson decays $\{\pi^0, \eta, \eta', \phi, J/\psi\} \to$ inv.
  – τ decays (Belle II)
  – Supernova cooling

[Li, Ma, Schmidt, 2020]

[PDB, Deppisch, Fridell, Hati, Harz, Kulkarni, 2021]

[Zhou, Gunther, Wang, de Vries, Dreiner, 2022]

[Barducci, Bertuzzo, Taoso, Toni, 2023]

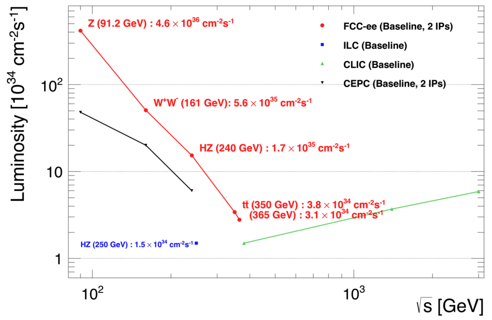
[Fernandez-Martinez, Gonzalez-Lopez, Hernandez-Garcia, Hostert, Lopez-Pavon, 2023]

Future Lepton Collider Constraints

Case Study: FCC-ee

Proposed Future Circular Collider (FCC):

- Post-LHC particle accelerator at CERN, labeled a *high-priority future initiative* by the 2020 Update of the European Strategy for Particle Physics
- Frontier factory for Higgs, top, flavour and electroweak physics
- 100 km circular tunnel, FCC-ee (e^+e^-) followed by $\sqrt{s} \gtrsim 100$ TeV FCC-hh (pp)
- Opportunities: High precision SM measurements + searches for **new physics**



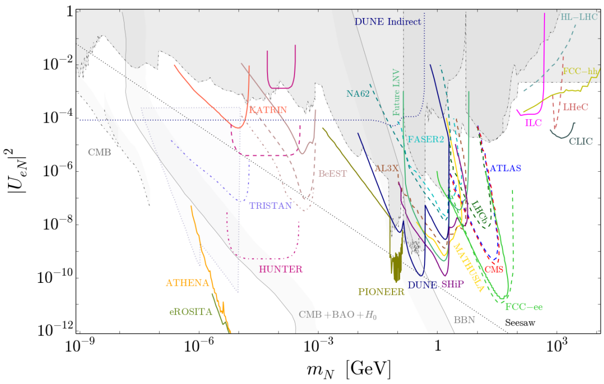
[FCC Collaboration, 2019]

FCC-ee LLP Search

Active-sterile mixing $\Rightarrow$ $e^+e^- \to Z \to \nu N$ $\Rightarrow$ Long-lived N + displaced vertex

$$\ell_N \sim \frac{3\text{ cm}}{U^2}\left(\frac{1\text{ GeV}}{m_N}\right)^6\left(1-\frac{m_N^2}{m_Z^2}\right)$$

$\Rightarrow$ Negligible background for decay at $\ell_N \sim 1$ m
$\Rightarrow$ With 5×10^{12} Z bosons, expect a few clear signal events for $U^2 \sim 10^{-12}$



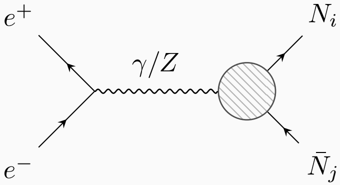
[Alimena *et al.*, 2022]

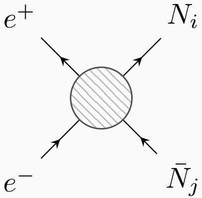
Relevant SMEFT + N_R operators

Interesting operators in the context of FCC-ee are:

$$\mathcal{L} \supset (\bar{N}\sigma_{\mu\nu}N)B^{\mu\nu} + (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{N}\gamma^\mu C_{HN}N)$$
$$+ (\bar{L}\gamma_\mu L)(\bar{N}\gamma^\mu C_{LN}N) + (e_R\gamma_\mu e_R)(\bar{N}\gamma^\mu C_{eN}N)$$
$$+ \left[H^\dagger(\bar{e}_R L)(\bar{N}C_{eLNH}N) + (\bar{L}N)(\bar{N}C_{LNeH}e_R)H + \text{h.c.}\right]$$

$\Rightarrow$ Another way to induce $e^+e^- \to N_iN_j$ at colliders, not suppressed by U^4

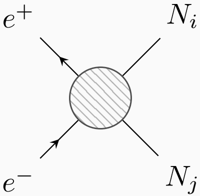




LEFT Operators (Majorana Case)

Below the EW scale, the NC-like operators induce for Majorana neutrinos (N_M):

$$\mathcal{L}^{(6)} \supset (\bar{e}_L\gamma_\mu e_L)(\bar{N}_R\gamma^\mu C_{V,LR} N_R) + (\bar{e}_R\gamma_\mu e_R)(\bar{N}_R\gamma^\mu C_{V,RR} N_R)$$
$$+ \Big[(\bar{e}_R e_L)(\bar{N}_R^c C_{S,LR} N_R) + (\bar{e}_L e_R)(\bar{N}_R^c C_{S,RR} N_R)$$
$$+ (\bar{e}_L\sigma_{\mu\nu} e_R)(\bar{N}_R^c \sigma^{\mu\nu} C_{T,RR} N_R) + \text{h.c.}\Big]$$

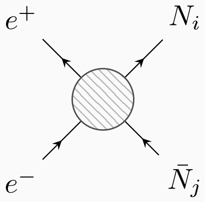


$$\begin{matrix} C_{V,XY} = C_{V,XY}^\dagger \\ C_{S,XY} = C_{S,XY}^T,\ C_{T,XX} = -C_{T,XX}^T \end{matrix} \Rightarrow \begin{cases} \#(\text{free parameters}) = \mathcal{N}(5\mathcal{N}+1) = 22 \\ \#(\text{CP-invariant}) = \frac{1}{2}\mathcal{N}(5\mathcal{N}+3) = \mathbf{13} \end{cases}$$

LEFT Operators (Dirac Case)

Below the EW scale, the NC-like operators induce for Dirac neutrinos (N_D):

$$\begin{aligned}\mathcal{L}^{(6)} \supset\ & (\bar{e}_L\gamma_\mu e_L)(\bar{N}_L\gamma^\mu C_{V,LL}N_L) + (\bar{e}_L\gamma_\mu e_L)(\bar{N}_R\gamma^\mu C_{V,LR}N_R) \\ & + (\bar{e}_R\gamma_\mu e_R)(\bar{N}_L\gamma^\mu C_{V,RL}N_L) + (\bar{e}_R\gamma_\mu e_R)(\bar{N}_R\gamma^\mu C_{V,RR}N_R) \\ & + (\bar{e}_R e_L)(\bar{N}_R C_{S,LL}N_L) + (\bar{e}_R e_L)(\bar{N}_L C_{S,LR}N_R) \\ & + (\bar{e}_L e_R)(\bar{N}_R C_{S,RL}N_L) + (\bar{e}_L e_R)(\bar{N}_L C_{S,RR}N_R) \\ & + (\bar{e}_R\sigma_{\mu\nu} e_L)(\bar{N}_R\sigma^{\mu\nu} C_{T,LL}N_L) + (\bar{e}_L\sigma_{\mu\nu} e_R)(\bar{N}_L\sigma^{\mu\nu} C_{T,RR}N_R)\end{aligned}$$



$$\begin{gathered} C_{V,XY} = C^\dagger_{V,XY} \\ C_{S,XY} = C^\dagger_{S,YX},\ C_{T,XX} = C^\dagger_{T,YY}\end{gathered} \Rightarrow \begin{cases} \#(\text{free parameters}) = 10\mathcal{N}^2 = 40 \\ \#(\text{CP-invariant}) = \mathcal{N}(5\mathcal{N}+2) = \mathbf{24}\end{cases}$$

Matching

Matching of the NC-like SMEFT operators on to the LEFT (around $\mu \sim m_Z$)

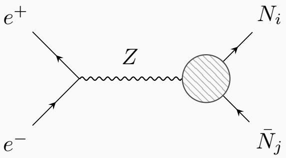
$$C_{\underset{\alpha\beta ij}{V,LR}} = C_{\underset{\alpha\beta ij}{LN}}\,, \quad C_{\underset{\alpha\beta ij}{V,RR}} = C_{\underset{\alpha\beta ij}{eN}}\,,$$

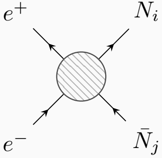
$$C_{\underset{\alpha\beta ij}{S,LR}} = \frac{v}{\sqrt{2}}\,C_{\underset{\alpha\beta ij}{eLNH}}\,, \quad C_{\underset{\alpha\beta ij}{S,RR}} = -\frac{v}{4\sqrt{2}}\Big(C_{\underset{\alpha ij\beta}{LNeH}} + C_{\underset{\alpha ji\beta}{LNeH}}\Big)\,,$$

$$C_{\underset{\alpha\beta ij}{T,RR}} = \frac{v}{16\sqrt{2}}\Big(C_{\underset{\alpha ij\beta}{LNeH}} - C_{\underset{\alpha ji\beta}{LNeH}}\Big)$$

and

$$-\frac{g}{2c_W}\bar{N}_i \not{Z}\Big(C_{ij} + v^2 C^*_{\underset{ij}{HN}}\Big)P_L N_j\,; \quad C_{ij} = \sum_\alpha U^*_{\alpha N_i} U_{\alpha N_j}$$





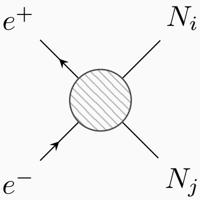
Production of HNL Pair

From effective Lagrangian, the cross section for $e^+e^- \to N_i N_j$ is

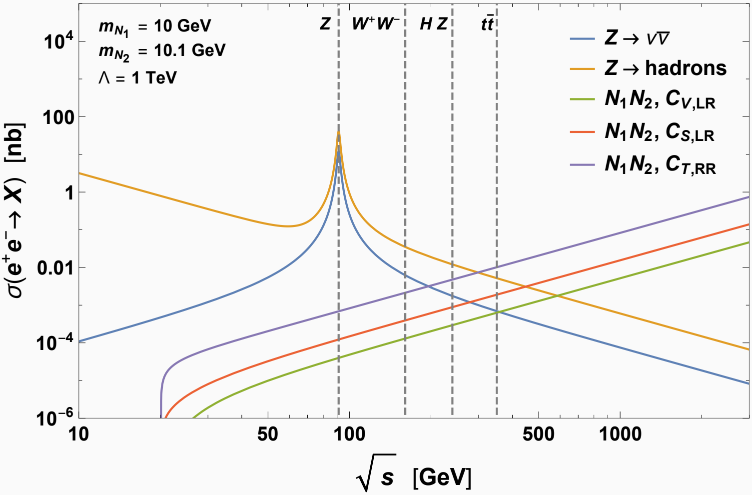
$$\sigma(e^+e^- \to N_i N_j)\big|_{\text{Maj}} = \frac{s}{64\pi}\sqrt{(1-\Sigma_{ij})(1-\Delta_{ij})}\,\mathcal{F}(\Sigma_{ij}, \Delta_{ij}, \boldsymbol{C}|_{\text{Maj}})$$

$$\sigma(e^+e^- \to N_i N_j)\big|_{\text{Dirac}} = \frac{s}{192\pi}\sqrt{(1-\Sigma_{ij})(1-\Delta_{ij})}\,\mathcal{G}(\Sigma_{ij}, \Delta_{ij}, \boldsymbol{C}|_{\text{Dirac}})$$

with $\Sigma_{ij} \equiv (m_{N_i} + m_{N_j})^2/s$ and $\Delta_{ij} \equiv (m_{N_i} - m_{N_j})^2/s$



Production of HNL Pair



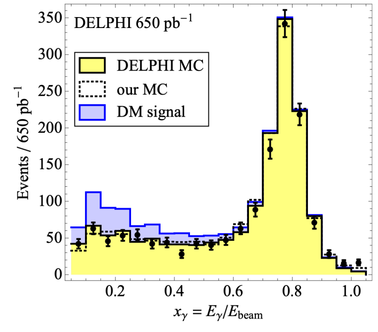
$$\sigma \propto \frac{s}{\Lambda^4}$$

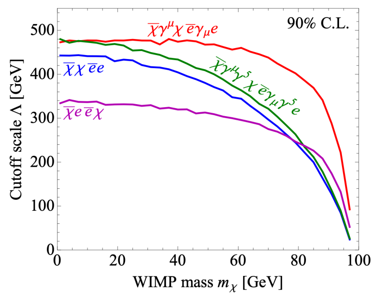
$$\sigma \propto \frac{G_F^2 m_Z^4}{s}$$

Dark Matter Searches at Colliders

DM searches at colliders:

- Monophoton ($e^+e^- \to \chi\chi\gamma$) and monojet signals ($pp \to \chi\chi g$)
- Signal: Hard photon or jet and missing E_T
- Complementary to direct DM searches from scattering ($p\chi \to p\chi$)





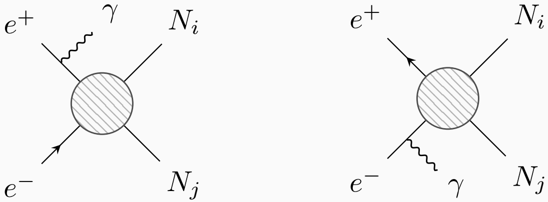
[Fox, Harnik, Kopp, Tsai, 2011]

The cross section for $e^+e^- \to N_i N_j \gamma$ can be calculated analytically

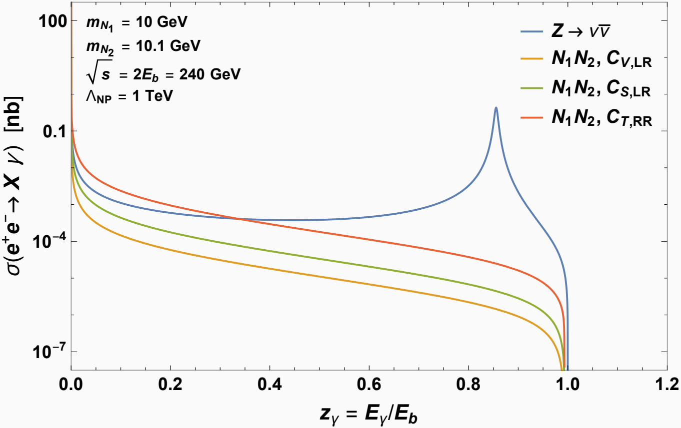
- To leading order:

$$\frac{d\sigma}{dz_\gamma \, d\cos\theta_\gamma} \simeq \sigma\left(s(1-z_\gamma)\right) \frac{\alpha}{2\pi} \frac{1+(1-z_\gamma)^2}{z_\gamma} \frac{1}{1-\beta_e^2\cos^2\theta_\gamma} \, ; \quad \beta_e = \sqrt{1-\frac{4m_e^2}{s}}$$

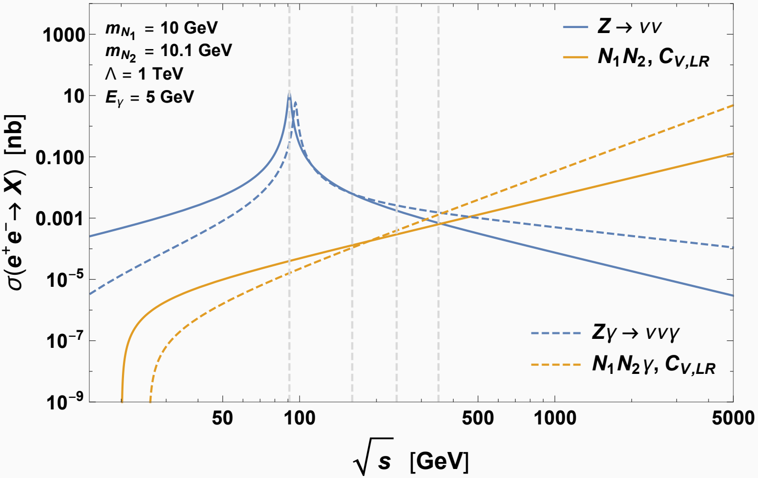
where $z_\gamma = 2E_\gamma/\sqrt{s} = E_\gamma/E_b$



$$\frac{d\sigma}{dz_\gamma} \simeq \sigma\left(s(1-z_\gamma)\right)\frac{\alpha}{2\pi}\frac{1+(1-z_\gamma)^2}{z_\gamma}\ln\frac{s}{m_e^2}$$



FCC-ee Monophoton Signatures: Analytical



$$\sigma \propto \frac{s}{\Lambda^4} \ln \frac{s}{m_e^2}$$

$$\sigma \propto \frac{s}{\Lambda^4}$$

Validity of EFT Approach

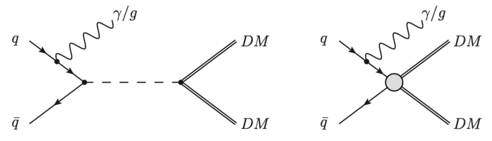
Note (in the context of DM searches) that the EFT approach is not always valid:

- Integrating out a heavy mediator of mass M:

$$\frac{g_e g_N}{s - M^2} = -\frac{g_e g_N}{M^2}\left(1 + \frac{s}{M^2} + \mathcal{O}\left(\frac{s^2}{M^4}\right)\right)$$

$$\frac{1}{\Lambda^2} = \frac{g_e g_N}{M^2} \quad \Rightarrow \quad \begin{array}{c} M > 2m_N \\ \sqrt{s} < M < \sqrt{g_e g_N}\,\Lambda \lesssim 4\pi\Lambda \end{array}$$

- For the monophoton signal, $s \to s_\gamma = s(1 - z_\gamma)$
- For hadron colliders, $s_{\gamma,g} = x_1 x_2 s - \sqrt{s} p_T (x_1 e^{-\eta} + x_2 e^{\eta})$, but higher $\sqrt{s}$
  $\Rightarrow$ Simplified models favoured



FCC-ee Monophoton Signatures: Simulation

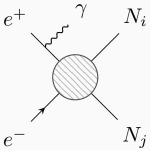
Simulation:

- Model file for effective Lagrangian in Dirac and Majorana cases
- SM backgrounds: $e^+e^- \to e^+e^-\gamma$ (reducible) and $e^+e^- \to \nu\nu\gamma$ (irreducible)

MadGraph: Generate events for the following scenarios:

1) $e^+e^- \to N_1 N_1$ (Majorana)
2) $e^+e^- \to N_1 N_2$ (Majorana)
3) $e^+e^- \to N_1 N_1$ (Dirac)
4) $e^+e^- \to N_1 N_2$ (Dirac)

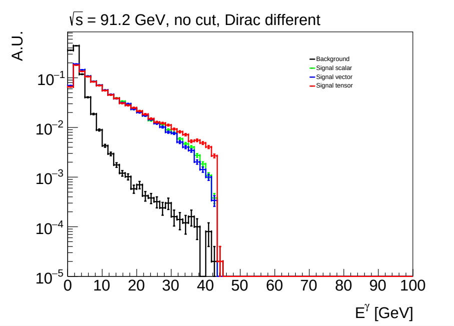
$\sqrt{s} = 91.2$ GeV (100 ab^{-1}), 240 GeV (5 ab^{-1})
$C_{V,XY},\ C_{S,XY},\ C_{T,XX}$

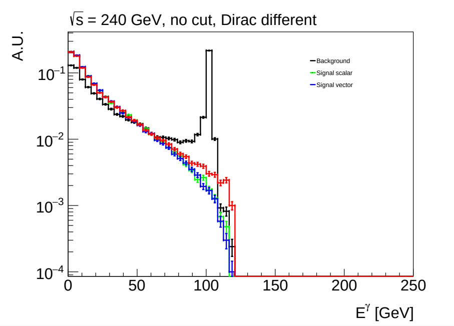


FCC-ee Monophoton Signatures: Simulation

- Event generation cuts: $p_T^\gamma > 1$ GeV, $E_\gamma > 1$ GeV (avoid soft singularity)
- Veto prompt electron to remove Bhabha scattering background
- Additional cuts on $(E_\gamma, \eta, \cos\theta_\gamma)$ to maximise signal-to-background ratio (S/B)

$\Rightarrow$ For each coefficient $C = 1/\Lambda^2$, set lower limits on Λ

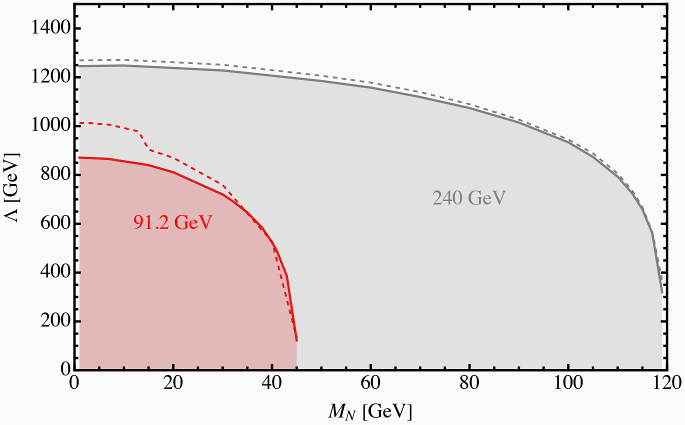




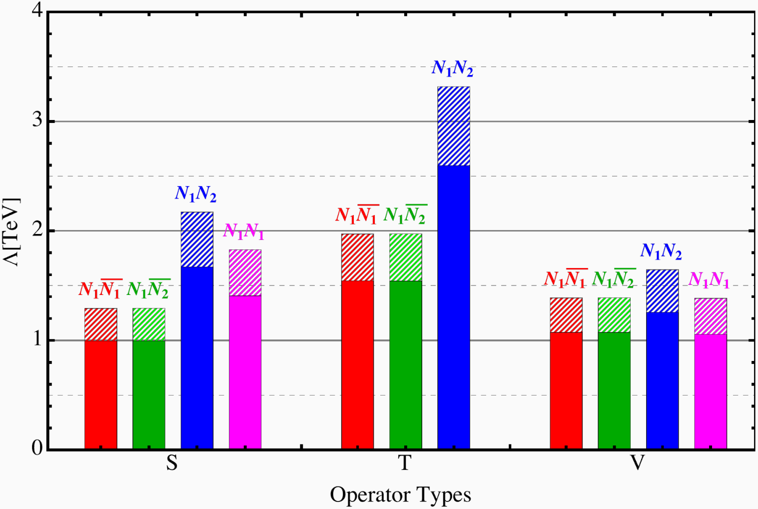
Constraints

Example:

- Constraints on $C_{S,RR} = 1/\Lambda^2$ (90% CL) with $m_{N_1} = m_{N_2} = m_N$
- Dashed lines: For each m_N, cuts on $(E_\gamma, \eta, \cos\theta_\gamma)$ maximise S/B



Constraints



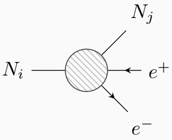
Note that for $C_{S,XY}$ and $C_{T,XX}$ (matching onto $d = 7$ operators):

$$\Lambda^2 = \frac{\sqrt{2}}{v}\Lambda^3_{\text{SMEFT}}$$
$$\Lambda_{\text{SMEFT}} < \Lambda$$

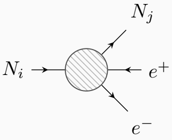
HNL Decays

HNL decays via the effective operators can also be considered

$$\Gamma(N_i \to N_j e^+ e^-)\Big|_{\text{Maj}} = \frac{m_{N_i}^5}{768\pi^3}\,\mathcal{I}(m_{N_j}, m_e, \boldsymbol{C}|_{\text{Maj}})$$



$$\Gamma(N_i \to N_j e^+ e^-)\Big|_{\text{Dirac}} = \frac{m_{N_i}^5}{1536\pi^3}\,\mathcal{J}(m_{N_j}, m_e, \boldsymbol{C}|_{\text{Dirac}})$$



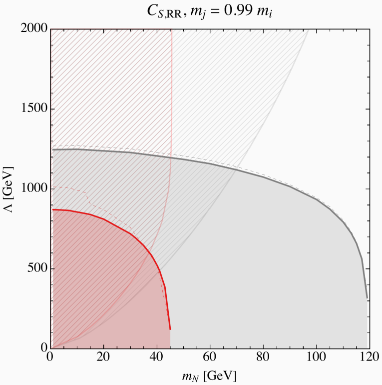
- To see monophoton signal, we require heavier state to decay outside detector

Other detection signal:

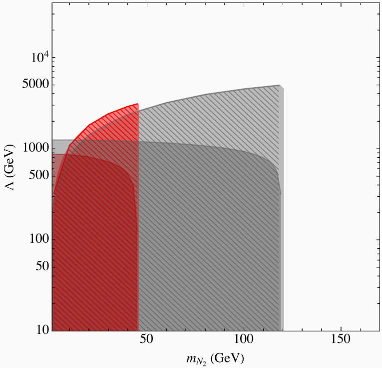
- $e^+e^- \to N_1 N_2$ followed by $N_2 \to N_1 e^+ e^-$
- Assume decays induced by active-sterile mixing negligible (e.g. $U_{\alpha N} \sim \sqrt{m_\nu/m_N}$)
- Focus of ongoing work

Constraints

$$\ell_N = \frac{\beta\gamma}{\Gamma(N_i \to N_j e^+ e^-)} > 5\,\mathrm{m}$$



$$\ell_N = \frac{\beta\gamma}{\Gamma(N_i \to N_j e^+ e^-)} < 5\,\mathrm{m}$$

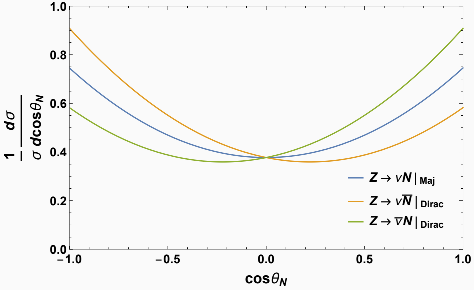


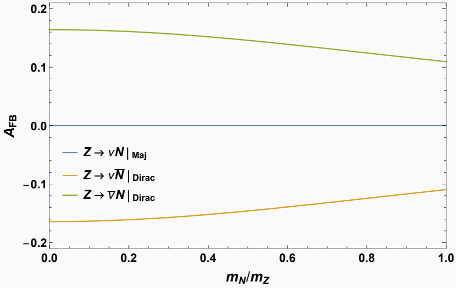
If heavy states N_i are observed via decays, possible to measure angular distributions

- Production and decay via active-sterile mixing ($y = m_N^2/m_Z^2$):

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\bigg|_{\text{Maj}} = \frac{3}{4(2+y)}\left[1+\cos^2\theta + y\sin^2\theta\right]$$

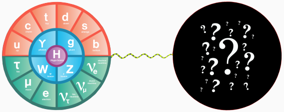
$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\bigg|_{\text{Dirac}} = \frac{3}{4(2+y)}\left[1+\cos^2\theta + y\sin^2\theta \mp 2\,\frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}\cos\theta\right]$$





[Blondel, de Gouvêa, Kayser, 2021]

Conclusions



Analysis of SMEFT + N_R at FCC-ee:

- Systematic analysis of NC-like operators inducing $e^+e^- \to N_i N_j(\gamma)$
- Simulation of monophoton signal at $\sqrt{s} = 91.2$ (100 ab^{-1}) and 240 GeV (5 ab^{-1})
- Appropriate cuts to eliminate reducible and maximise S/B for irreducible backgrounds
- $C_{V,XY}$, $C_{S,XY}$ and $C_{T,XX}$ in Majorana and Dirac scenarios
- Constraints on new physics Λ as a function of m_N

In progress:

- Simulation of $e^+e^- \to N_1 N_2$ and $N_2 \to N_1 e^+ e^-$
- HNL angular distributions and beam polarisation effects
- Other interesting phenomenology!

Backup

RG Running

Below the EW scale, the scalar and tensor operators run as

$$\mu \frac{d}{d\mu} C_{S,LR} = -\frac{3\alpha}{2\pi} C_{S,LR}\,, \quad \mu \frac{d}{d\mu} C_{S,RR} = -\frac{3\alpha}{2\pi} C_{S,RR}\,,$$
$$\mu \frac{d}{d\mu} C_{T,RR} = -\frac{\alpha}{2\pi} C_{T,RR}$$

Above the electroweak scale, the dimension-six operators run

$$\mu \frac{d}{d\mu} C_{\underset{\alpha\beta ij}{LN}} = \frac{\alpha_1}{6\pi}\left(C_{\underset{\rho\rho ij}{LN}} + C_{\underset{\rho\rho ij}{eN}} - C_{\underset{\rho\rho ij}{QN}} - 2C_{\underset{\rho\rho ij}{uN}} + C_{\underset{\rho\rho ij}{dN}} - \frac{1}{2} C_{\underset{ij}{HN}}\right)\delta_{\alpha\beta}\,,$$
$$\mu \frac{d}{d\mu} C_{\underset{\alpha\beta ij}{eN}} = \frac{\alpha_1}{3\pi}\left(C_{\underset{\rho\rho ij}{LN}} + C_{\underset{\rho\rho ij}{eN}} - C_{\underset{\rho\rho ij}{QN}} - 2C_{\underset{\rho\rho ij}{uN}} + C_{\underset{\rho\rho ij}{dN}} - \frac{1}{2} C_{\underset{ij}{HN}}\right)\delta_{\alpha\beta}\,,$$

with

$$\mathcal{L}^{(6)} \supset (\bar{Q}\,\gamma_\mu Q)(\bar{N}_R\,\gamma^\mu\, C_{QN}\, N_R) + (u_R\,\gamma_\mu\, u_R)(\bar{N}_R\,\gamma^\mu\, C_{uN}\, N_R)$$
$$+ (d_R\,\gamma_\mu\, d_R)(\bar{N}_R\,\gamma^\mu\, C_{dN}\, N_R)$$

From $\Lambda \sim 1$ TeV down to $\sqrt{s} = 91.2$ and 240 GeV, running is not significant

[Datta, Kumar, Liu, Marfatia, 2020]

HNL Decays: Majorana

The decay rate for a Majorana state N_i via the effective operators is:

$$\Gamma(N_i \to N_j e^+ e^-)\Big|_{\text{Maj}} = \frac{m_{N_i}^5}{768\pi^3}\Bigg[I_1(y_j, y_e, y_e)\Big(\big|C^{V,LR}_{eN_iN_j}\big|^2 + \big|C^{V,RR}_{eN_iN_j}\big|^2\Big)$$
$$+ I_2(y_j, y_e, y_e)\,\text{Re}\Big(\big(C^{V,LR}_{eN_iN_j}\big)^2 + \big(C^{V,RR}_{eN_iN_j}\big)^2\Big)$$
$$+ I_1(y_e, y_e, y_j)\Big(\big|C^{S,LR}_{eN_iN_j}\big|^2 + \big|C^{S,RR}_{eN_iN_j}\big|^2\Big)$$
$$+ 4I_2(y_j, y_e, y_e)\,\text{Re}\Big(C^{S,LR}_{eN_iN_j}C^{S,RR}_{eN_iN_j}\Big)$$
$$+ 48 I_1(y_j, y_e, y_e)\big|C^{T,RR}_{eN_iN_j}\big|^2\Bigg] + \mathcal{O}(m_e^2)$$

where

$$I_1(x, y, z) = 12\int_{(x+y)^2}^{(1-z)^2}\frac{ds}{s}(1+z^2-s)(s-x^2-y^2)\lambda(1, s, z^2)\lambda(s, x^2, y^2)$$
$$I_2(x, y, z) = 12x\int_{(y+z)^2}^{(1-x)^2}\frac{ds}{s}(s-y^2-z^2)\lambda(1, s, x^2)\lambda(s, y^2, z^2)$$

HNL Decays: Dirac

The decay rate for a Dirac state N_i via the effective operators is:

$$\begin{aligned}\Gamma(N_i \to N_j e^+ e^-)\Big|_{\text{Dirac}} = \frac{m_{N_i}^5}{1536\pi^3}\Bigg[& I_1(y_j, y_e, y_e)\Big(\big|C_{eN_iN_j}^{V,LL}\big|^2 + \big|C_{eN_iN_j}^{V,LR}\big|^2 \\ & \qquad + \big|C_{eN_iN_j}^{V,RL}\big|^2 + \big|C_{eN_iN_j}^{V,RR}\big|^2\Big) \\ & - 2I_2(y_j, y_e, y_e)\,\mathrm{Re}\Big(C_{eN_iN_j}^{V,LL} C_{eN_iN_j}^{V,LR*} + C_{eN_iN_j}^{V,RL} C_{eN_iN_j}^{V,RR*}\Big) \\ & + \frac{1}{4} I_1(y_e, y_e, y_j)\Big(\big|C_{eN_iN_j}^{S,LL}\big|^2 + \big|C_{eN_iN_j}^{S,LR}\big|^2 \\ & \qquad + \big|C_{eN_iN_j}^{S,RL}\big|^2 + \big|C_{eN_iN_j}^{S,RR}\big|^2\Big) \\ & + I_2(y_j, y_e, y_e)\,\mathrm{Re}\Big(C_{eN_iN_j}^{S,LL} C_{eN_iN_j}^{S,LR*} + C_{eN_iN_j}^{S,RL} C_{eN_iN_j}^{S,RR*}\Big) \\ & + 12 I_1(y_j, y_e, y_e)\Big(\big|C_{eN_iN_j}^{T,LL}\big|^2 + \big|C_{eN_iN_j}^{T,RR}\big|^2\Big)\Bigg] + \mathcal{O}(m_e^2)\end{aligned}$$

where $I_1(x, y, z)$ and $I_2(x, y, z)$ are given on the previous slide.