

# CHARGED VS NEUTRAL CORRELATORS

## ISOSPIN BREAKING IN $\tau$ DATA FOR $(g - 2)_\mu$

Mattia Bruno

work in collab. with T. Izubuchi, C. Lehner, A. Meyer, X. Tuo  
for the RBC/UKQCD collaborations



Isospin-breaking effects on precision observables in Lattice QCD  
University of Mainz, Germany, July 24th

# INTRODUCTION

## Hadronic spectral densities

at low energies dominated by non-perturbative effects

extracted from experiments

extracted from lattice calculations, require solving inverse problems

## Integrated hadronic densities relevant for SM phenomenology

Hadronic-Vacuum-Polarization piece of  $(g - 2)_\mu$

directly in Euclidean space-time [Blum '02][Bernecker,Meyer '11]

neutral (EM current) spectral density

inclusive  $\tau$  decays

(mild) inverse problem

[ETMC '23]

charged (weak current) spectral density

Goal: study relation charged vs neutral densities for  $(g - 2)_\mu$

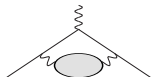
Compared with the traditional prediction, the latest  $g-2$  measurement has a **discrepancy of over 5-sigma**, which corresponds to a one in 3.5 million chance that the result is a fluke, ...

**A newer technique called a lattice calculation**, which uses supercomputers to model the universe as a four-dimensional grid of space-time points, has also emerged. There's just one problem: It generates a  $g-2$  prediction that **differs from the traditional approach**.

Rarely in physics does an experiment surpass the theory, but this is one of those times, Dr. Pitts said. "The attention is on the theoretical community," he added. "The limelight is now on them."

# DISPERSIVE APPROACH

Method



$$a_\mu = \frac{\alpha}{\pi} \int \frac{ds}{s} K(s, m_\mu) \frac{\text{Im}\Pi(s)}{\pi} \quad [\text{Brodsky, de Rafael '68}]$$

analyticity  $\hat{\Pi}(k^2) = \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi} \int_0^\infty ds \frac{\text{Im}\Pi(s)}{s(s - k^2 - i\varepsilon)}$

unitarity

$$\text{Im} \left[ \text{Feynman diagram with muon loop} \right] = \sum_X \left| \text{Feynman diagram with muon and hadrons} \right|^2$$

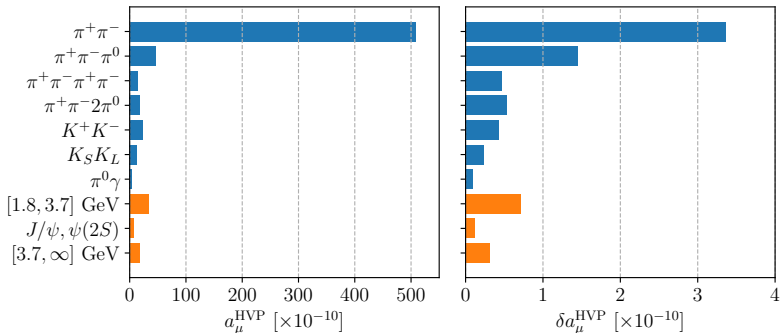
$$\frac{4\pi^2\alpha}{s} \frac{\text{Im}\Pi(s)}{\pi} = \sigma_{e^+e^- \rightarrow \gamma^* \rightarrow \text{had}}$$

$$v_0(s) = \frac{\text{Im}\Pi(s)}{\pi} = \frac{s}{4\pi\alpha^2} \sigma_{\text{had}}(s) \quad \text{spectral function}$$

# DISPERSIVE APPROACH

## Breakdown

[White Paper '20, DHMZ19]



Combination of  $m_\mu$  and  $\rho$ -meson physics  $\rightarrow \pi^+\pi^-$  dominant channel

# DISPERSIVE APPROACH

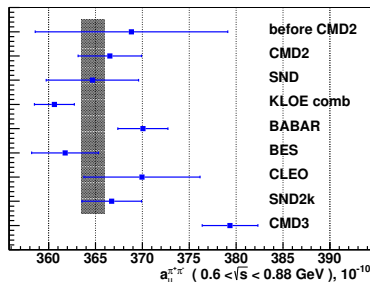
Tensions in  $\pi^+\pi^-$  channel

Large tensions among experiments: BaBar, KLOE, CMD3

[CMD3 2302.08834]

combination of different experiments?

error of  $\pi\pi$  contribution to  $a_\mu$ ?

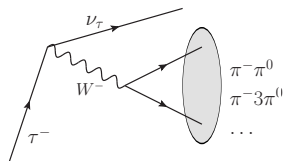
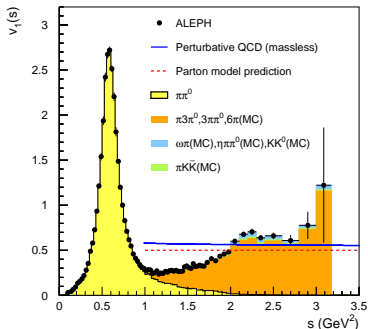


$\tau$  decays: more consistent experimental picture

CLEO, OPAL, Aleph, Belle08 [Davier et al][Golterman et al]

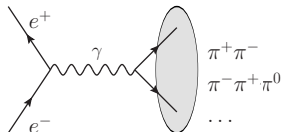
# MOTIVATIONS

## $\tau$ decays



$V - A$  current

Final states  $I = 1$  charged



EM current

Final states  $I = 0, 1$  neutral

$\tau$  data can improve  $a_\mu[\pi\pi]$

→ 72% of total Hadronic LO

→ competitive precision on  $a_\mu^W$

# ISOSPIN CORRECTIONS

Status

Restriction to  $e^+e^- \rightarrow \pi^+\pi^-$  and  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2} \sigma_{\pi^+\pi^-(\gamma)}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2} \frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds} \left(1 - \frac{s}{m_\tau^2}\right)^{-1} \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \frac{1}{S_{EW}}$$

Isospin correction  $v_0 = R_{IB}v_-$   $R_{IB} = \frac{\text{FSR}}{G_{EM}} \frac{\beta_0^3 |F_\pi^0|^2}{\beta_-^3 |F_\pi^-|^2}$  [Alemani et al. '98]

0.  $S_{EW}$  electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of  $\pi^+\pi^-$  system [Schwinger '89][Drees, Hikasa '90]

2.  $G_{EM}$  (long distance) radiative corrections in  $\tau$  decays

Chiral Resonance Theory [Cirigliano et al. '01, '02]

Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ( $\beta_{0,-}$ ) due to  $(m_{\pi^\pm} - m_{\pi^0})$



# RADIATIVE CORRECTIONS

## Long-distance effects

At low energies relevant degrees of freedom are mesons

Chiral Perturbation Theory

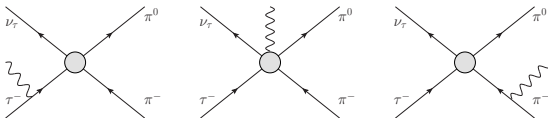
[Cirigliano et al. '01, '02]

Meson dominance model

[Flores-Talpa et al. '06, '07]

Corrections casted in one function  $v_-(s) \rightarrow v_-(s)G_{EM}(s)$

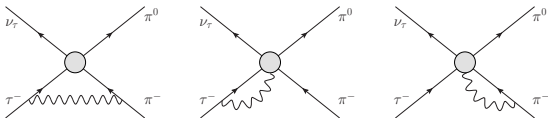
## Real photon corrections



Real + virtual

→ IR divergences cancel

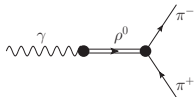
## Virtual photon corrections



# FORM FACTORS

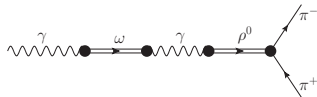
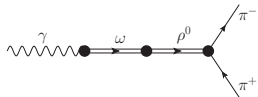
## Pheno models

$$F_{\pi}^0(s) \propto \frac{m_{\rho}^2}{D_{\rho}(s)}$$

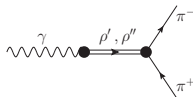


[Gounaris, Sakurai '68]  
[Kühn, Santamaria '90]

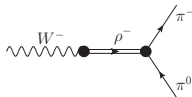
$$\times \left[ 1 + \delta_{\rho\omega} \frac{s}{D_{\omega}(s)} \right]$$



$$+ \frac{m_X^2}{D_X(s)} \quad X = \rho', \rho''$$



$$F_{\pi}^{-}(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho', \rho'')$$



Sources of IB breaking in phenomenological models

$$m_{\rho^0} \neq m_{\rho^{\pm}}, \quad \Gamma_{\rho^0} \neq \Gamma_{\rho^{\pm}}, \quad m_{\pi^0} \neq m_{\pi^{\pm}}$$

$$\rho - \omega \text{ mixing } \delta_{\rho\omega} \simeq O(m_u - m_d) + O(e^2)$$

# STATUS

From the  $(g - 2)$  White Paper

“ ... it appears that, at the required precision to match the  $e^+e^-$  data, the present understanding of the IB corrections to  $\tau$  data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. ”

“The ratio  $|F_0(s)/F_-(s)|^2$  is the most difficult to estimate reliably, since a number of different IB effects may contribute.”

# ROADMAP

1. EM corrections to hadronic  $\tau$  decays
2. connect differential rates w/ Euclidean correlators
3. charged vs neutral correlators in Lattice QCD+QED

# DEFINITIONS

Hadronic currents

$$\begin{aligned}\mathcal{J}_\mu^\gamma &= Q_u \bar{u} \gamma_\mu u + Q_d \bar{d} \gamma_\mu d \\ \mathcal{J}_\mu^- &= \bar{u} \gamma_\mu d, \quad \mathcal{J}_\mu^1 = \frac{Q_u - Q_d}{\sqrt{2}} \bar{u} \gamma_\mu d\end{aligned}$$

Hadronic phase-space factor,  $i$  labels hadrons

$$d\Phi_f(p) \equiv (2\pi)^4 \delta^4(p - \sum_i p_i) S_f \prod_i \frac{d^3 p_i}{(2\pi)^3 2\omega_i}$$

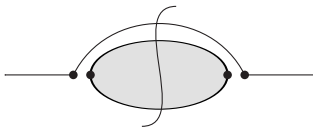
Charged spectral densities

$$\begin{aligned}\rho_{\mu\nu}^w(p) &= \frac{1}{2\pi} \int d^4 x e^{ipx} \langle 0 | \mathcal{J}_\mu^+(x) \mathcal{J}_\nu^-(0) | 0 \rangle \\ &= \frac{1}{2\pi} \sum_f \int d\Phi_f \langle 0 | \mathcal{J}_\mu^+(0) | p_1 \dots, \text{out} \rangle \langle p_1 \dots, \text{out} | \mathcal{J}_\nu^-(0) | 0 \rangle \\ &= (g_{\mu\nu} - p_\mu p_\nu) \rho^w(s) \quad [s = p^2]\end{aligned}$$

# HADRONIC $\tau$ DECAYS

Fermi theory

$$\mathcal{M}_f(P, q, p_1 \cdots p_{n_f}) = \frac{G_F V_{ud}}{\sqrt{2}} \bar{u}_\nu(-q) \gamma_\mu^L u_\tau(P) \langle \text{out}, p_1 \cdots p_{n_f} | \mathcal{J}_\mu^-(0) | 0 \rangle$$



$$\begin{aligned} d\Gamma &= \frac{1}{4m} d\Phi_q \sum_f d\Phi_f \sum_{\text{spin}} |\mathcal{M}_f|^2 \\ &= \frac{1}{4m} d\Phi_q \frac{G_F^2 |V_{ud}|^2}{2} \mathcal{L}_{\mu\nu}(P, q) \rho_{\mu\nu}^w(p) \end{aligned}$$

Charged spectral density isospin limit =  $\rho^{w,0}$   $\left[ d\Phi_q = \frac{d^3q}{(2\pi)^3 2\omega_q} \right]$

$$\begin{aligned} \frac{d\Gamma(s)}{ds} &= G_F^2 |V_{ud}|^2 \frac{m^3}{16\pi^2} \left(1 + \frac{2s}{m^2}\right) \left(1 - \frac{s}{m^2}\right)^2 \rho^{w,0}(s) \\ &= G_F^2 |V_{ud}|^2 \frac{m^3}{16\pi^2} \kappa(s) \rho^{w,0}(s) \end{aligned}$$

# ELECTRONIC RATE

Eliminating  $G_F$

$$\Gamma_e = \Gamma(\tau \rightarrow e\bar{\nu}\nu) = \frac{\mathcal{B}_e \Gamma}{\mathcal{B}} = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

$$\text{conventionally } \rho^{w,0}(s) = \frac{m_\tau^2}{12\pi^2 |V_{ud}|^2 \kappa(s)} \frac{\mathcal{B}}{\mathcal{B}_e} \frac{1}{\Gamma} \frac{d\Gamma}{ds}$$

$O(\alpha)$  correction finite in Fermi theory

[Kinoshita, Sirlin '59]

$$\Gamma_e = \frac{G_F^2 m_\tau^5}{192\pi^3} \left[ 1 + \frac{\alpha}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right] \left[ 1 + O(m_W^2/m_\tau^2) + O(m_e^2/m_\tau^2) \right]$$

→ 0.4% correction

# W REGULARIZATION

## Short-distance effects

[Sirlin '82][Marciano, Sirlin '88][Braaten, Li '90]

Effective Hamiltonian  $H_W \propto G_F O_{\mu\nu}$

$G_F$  low-energy constant; 4-fermion operator  $O_{\mu\nu}$

At  $O(\alpha)$  new divergences in EFT  $\rightarrow$  need regulator,  $Z$  factors



$$\frac{1}{k^2} = \frac{1}{k^2 - m_W^2} - \frac{m_W^2}{k^2(k^2 - m_W^2)}$$

[Sirlin '78]

1. universal UV divergences re-absorbed in  $G_F$
2. process-specific corrections in  $S_{EW}$ , like a  $Z$  factor

Effective Hamiltonian at  $O(\alpha)$ :  $H_W \propto G_F S_{EW}^{1/2} O_{\mu\nu}$

matching required as noted by [Carrasco et al '15][Di Carlo et al '19]



# IR DIVERGENCES

Book-keeping tool

Collect Feynman graphs in 3 classes which are individually IR safe:

Factorizable leptonic corrections (initial state)

Factorizable QCD corrections (final state)

Non-factorizable corrections (initial-final state)

# ISOSPIN BREAKING

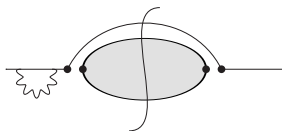
Initial state

Wave-function renormalization

$$Z_\tau = 1 + \frac{\alpha}{2\pi} \left[ \log \frac{m_\tau}{\mu} + 2 \log \frac{m_\gamma}{m_\tau} + \dots \right]$$

$$\frac{d\Gamma}{ds} \simeq 2 \times \frac{1}{2} [Z_\tau - 1] |\mathcal{M}|^2$$

$$\delta Z_\tau \equiv \frac{\alpha}{2\pi} \log(m_W/m_\tau) \quad [\text{Sirlin '82}]$$



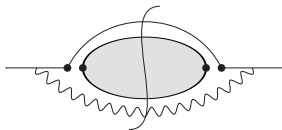
$\tau$  Bremsstrahlung

[Cirigliano et al '00, '01][MB et al, in prep]

$$\frac{d\Gamma}{ds} \frac{\alpha}{\pi} [G_{\log}(s, m_\gamma) + G_1(s) + G_2(s)]$$

$$G_{\log}(s, m_\gamma) = \log \frac{m_\gamma}{m_\tau} + \dots$$

$$\delta\kappa(s) \equiv G_{\log}(s, m_\tau) + G_1(s) + G_2(s)$$



$$\frac{d\Gamma}{ds} \simeq G_F^2 |V_{ud}|^2 \frac{m^3}{16\pi^2} \kappa(s) \rho^{w,0}(s) [\delta Z_\tau + \delta\kappa(s)]$$

# ISOSPIN BREAKING

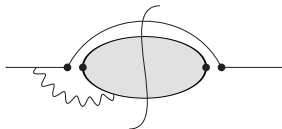
Initial-final state

Virtual photon loop

$$\delta Z_{\kappa\rho} = \frac{2\alpha}{\pi} \left(1 + \frac{3}{2}\bar{Q}\right) \log(m_W/m_\tau) \quad [\text{Sirlin '82}]$$

[Cirigliano et al '01]

Finite parts EFT and  $2\pi$



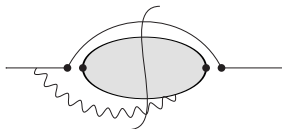
Lepton-Hadrons bremsstrahlung interference

From EFT and  $2\pi$  [Cirigliano et al' 00, '01]

Structure-independent captured by EFT

Structure-dependent meson dominance

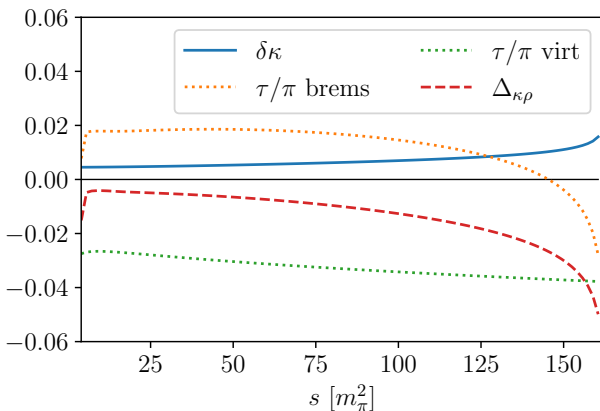
[Flores-Talpa et al. '06, '07]



$$\frac{d\Gamma}{ds} += G_F^2 |V_{ud}|^2 \frac{m^3}{16\pi^2} \kappa(s) \rho^{w,0}(s) [\delta Z_{\kappa\rho} + \Delta_{\kappa\rho}(s)]$$

# LONG-DISTANCE CORRECTIONS

Let's take a look



$\delta\kappa$  is channel and  $m_\gamma$  independent [MB et al, in prep]

$\Delta_{\kappa\rho} \rightarrow 2\pi$ , point-like,  $m_\gamma$  independent [Cirigliano et al '01, '02]

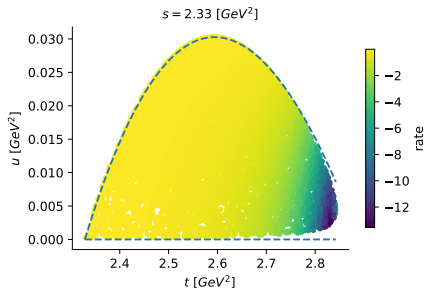
A numerical  $n$ -particle phase-space integrator

Grid/GPT backend, support for several parallelization schemes

partial support for 1-loop Passarino-Veltman functions

no support for MCMC yet (needed for  $\geq 6$  particles)

currently private, soon public [github.com/mbruno46](https://github.com/mbruno46)



Used to cross-check analytic formulae

Example: Dalitz plot  $\tau$  Bremsstrahlung  
 $\rightarrow$  wrong boundary: finite  $m_\gamma$  effects

# $a_\mu$ ON THE LATTICE

Window fever

Hadronic Vacuum Polarization (HVP) contribution to  $a_\mu$

Time-momentum representation

[Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^\gamma(x) j_k^\gamma(0) \rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Windows in Euclidean time

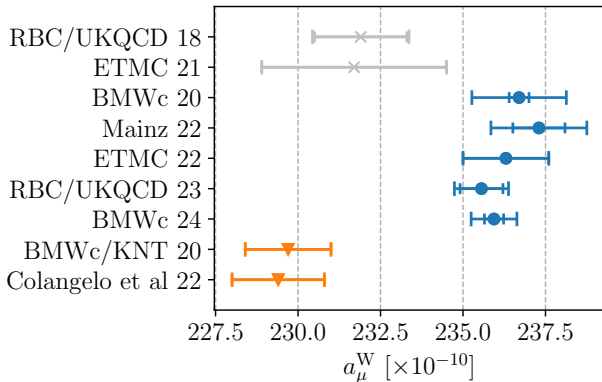
[RBC/UKQCD '18]

$$a_\mu^W = 4\alpha^2 \sum_t w_t G^\gamma(t) [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$$

$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm} \quad \Delta = 0.15 \text{ fm}$

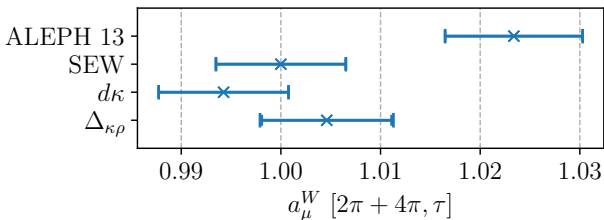
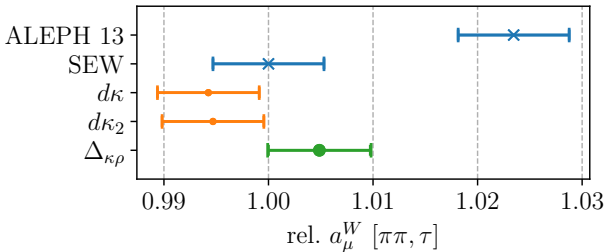
allow for in-depth cross-checks

# TENSIONS



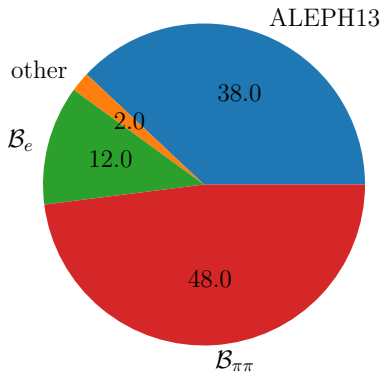
# TOWARDS $a_\mu^W$

$2\pi$  error breakdown



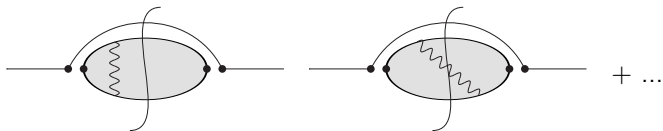


TOWARDS  $a_\mu^W$   
 $2\pi$  error breakdown



# ISOSPIN BREAKING

Final state



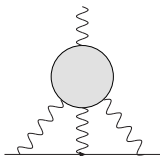
# ISOSPIN BREAKING

## Strategy

1. take experimental  $d\Gamma/ds$  (e.g. Aleph13, Belle08)
2.  $\delta\kappa$  initial state corrections: analytic, under control
3.  $\Delta_{\kappa\rho}$  initial-finite mixed rad. corr:
  - analytically known for intermediate two-pion channel
  - effective field theory [Cirigliano et al '01, '02]
  - meson dominance models [Flores-Talpa et al. '06, '07]
  - new results from phenomenological models [Roig et al '23]
4. define  $\delta\Gamma_{EM} \equiv \delta\kappa(s) + \Delta_{\kappa\rho}(s)$  and calculate:

$$\frac{m_\tau^2}{12\pi^2 G_F^2 |V_{ud}|^2 \kappa(s)} \frac{1}{S_{EW}} \frac{1}{1 + \frac{\alpha}{\pi} \delta\Gamma_{EM}(s)} \left[ \frac{\mathcal{B}_e}{\mathcal{B}} \frac{1}{\Gamma} \frac{d\Gamma}{ds} \right]_{\text{exp}} = \rho^{w,0}(s) + \delta\rho(s)$$

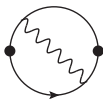
5. Laplace transform to Euclidean time
6. add difference  $e\ell - \tau$  evaluated from LQCD+QED



from QCD we need a **4-point function**  $f(x, y, z, t)$ :  
**known kernel** with details of photons and muon line  
 1 pair of point sources  $(x, y)$ , sum over  $z, t$  exact at sink  
 stochastic sampling over  $(x, y)$  (based on  $|x - y|$ )

**Successful strategy:** x10 error reduction

[RBC '16]



from QCD we need a **4-point function**  $f(x, y, z, t)$ :  
 $(g - 2)_\mu$  kernel + photon propagator

**Similar problem** → re-use HLbL point sources!

## The RBC & UKQCD collaborations

### [University of Bern & Lund](#)

Dan Hoying

### [BNL and BNL/RBRC](#)

Peter Boyle (Edinburgh)

Taku Izubuchi

Yong-Chull Jang

Chulwoo Jung

Christopher Kelly

Meifeng Lin

Nobuyuki Matsumoto

Shigemi Ohta (KEK)

Amarjit Soni

Raza Sufian

Tianle Wang

### [CERN](#)

Andreas Jüttner (Southampton)

Tobias Tsang

### [Columbia University](#)

Norman Christ

Sarah Fields

Ceran Hu

Yikai Huo

Joseph Karpie (JLab)

Erik Lundstrum

Bob Mawhinney

Bigeng Wang (Kentucky)

### [University of Connecticut](#)

Tom Blum

Luchang Jin (RBRC)

Douglas Stewart

Joshua Swaim

Masaaki Tomii

### [Edinburgh University](#)

Matteo Di Carlo

Luigi Del Debbio

Felix Erben

Vera Gülpers

Maxwell T. Hansen

Tim Harris

Ryan Hill

Raoul Hodgson

Nelson Lachini

Zi Yan Li

Michael Marshall

Fionn Ó hÓgáin

Antonin Portelli

James Richings

Azusa Yamaguchi

Andrew Z.N. Yong

### [Liverpool Hope/Uni. of Liverpool](#)

Nicolas Garron

### [LLNL](#)

Aaron Meyer

### [University of Milano Bicocca](#)

Mattia Bruno

### [Nara Women's University](#)

Hiroshi Ohki

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Xu Feng

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### [RIKEN CCS](#)

Yasumichi Aoki

### [University of Siegen](#)

Matthew Black

Anastasia Boushmelev

Oliver Witzel

### [University of Southampton](#)

Alessandro Barone

Bipasha Chakraborty

Ahmed Elgaziari

Jonathan Flynn

Nikolai Husung

Joe McKeon

Rajnandini Mukherjee

Callum Radley-Scott

Chris Sachrajda

### [Stony Brook University](#)

Fangcheng He

Sergey Syritsyn (RBRC)

# CONTRIBUTION TO $a_\mu$

Time-momentum representation

[Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^\gamma(x) j_k^\gamma(0) \rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of  $u, d$  current

$$j_\mu^\gamma = \frac{i}{6} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \frac{i}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) = j_\mu^{(0)} + j_\mu^{(1)}$$

---

$$\frac{i}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \left[ \begin{array}{c} I = 1 \\ I_3 = 0 \end{array} \right] \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}} (\bar{u}\gamma_\mu d), \left[ \begin{array}{c} I = 1 \\ I_3 = -1 \end{array} \right]$$

Isospin 1 charged correlator  $G_{11}^W = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0) \rangle$

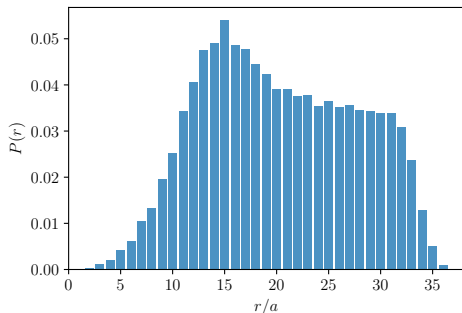
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$$G_{II'}^\gamma \equiv \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^{(I)}(x) j_k^{(I')}(0) \rangle, \quad \delta G^{11} \equiv G_{11}^\gamma - G_{11}^W$$

# SAMPLING STRATEGY

$$\tilde{V}_\Gamma(x_0, z_0, r) = \sum_{\vec{x}, \vec{z}} \text{tr} \left[ \Gamma D^{-1}(x, 0) \gamma_\nu D^{-1}(0, z) \Gamma D^{-1}(z, r) \gamma^\nu D^{-1}(r, x) \right]$$
$$V_\Gamma(|x_0 - z_0|) = \sum_r \Delta(r) \tilde{V}_\Gamma(x_0, z_0, r)$$

$O(10^3)$  points  $\rightarrow O(10^6)$  pairs

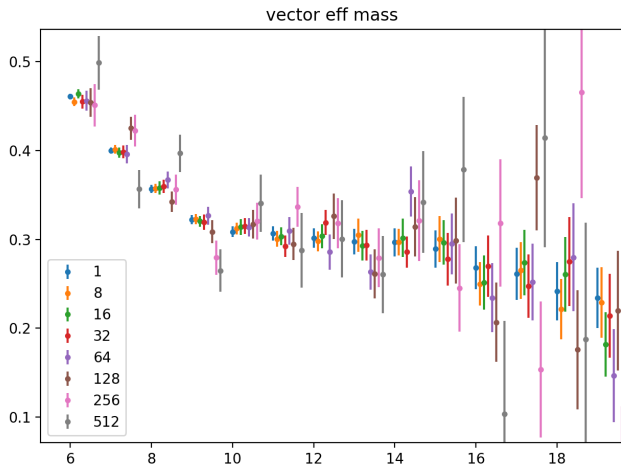


contract photon offline  
 $\rightarrow$  study  $\text{QED}_L$  vs  $\text{QED}_\infty$

# SPARSE PROPAGATORS

Vector effective mass

observable dependent, low stat. but good guidance





# EXAMPLE

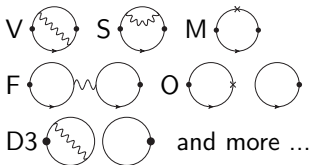
## Diagram V

[MB et al PoS'18]

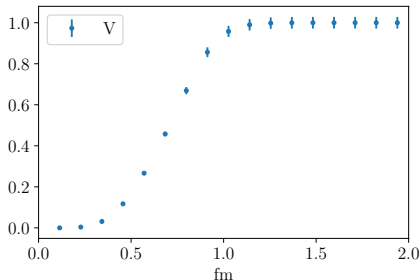
IB corrections for charged ( $\tau$ ) and neutral ( $ee$ )  
difference of  $\tau$  and  $ee$  spectral densities in Euclidean time

[preliminary]

many (quark) diagrams involved:



plot = cumsum[win  $\times$   $\mu$ -kernel  $\times$  V]  
64l phys.pion mass ensemble



first calculations of all diagrams [BMWc '20,'24]

ongoing RBC/UKQCD effort

significant stat. improvement for leading-diagrams

first results for sub-leading diagrams

# INCLUSIVITY PROBLEM

Take  $\Delta_{\kappa\rho}$  from EFT  $\rightarrow$  restrict to two-pion channel  
discard  $G_{00}^\gamma$ , keep  $G_{01}^\gamma$

Lattice calculation fully inclusive in energy (cut at  $m_\tau$ ) and channels

$G_{01}$  **mostly** dominated by  $\pi\pi$ . Is it correct?

simple estimate  $a^W[3\pi] \leq 20\%$  of  $a^W[2\pi]$  [MB Edinburgh '22]

Isospin-breaking in  $2\pi$  and  $3\pi$  from [Colangelo et al 22][Hofericther et al '23]

IB correction of  $a^W[3\pi] \approx -1 \cdot 10^{-10}$

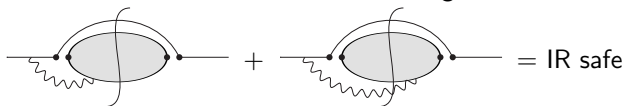
IB correction of  $a^W[2\pi] \approx +1 \cdot 10^{-10}$

**warning** if precision from Lattice  $\ll 2 \cdot 10^{-10}$

# LONG-DISTANCE

## Final dream

Infrared divergences cancel out in total rate  
cancellations occur in subset of diagrams



Intermediate two-pion channel  
effective field theory  
meson dominance models

[Cirigliano et al '01, '02]

[Flores-Talpa et al. '06, '07]

Full estimate in LQCD+QED  
integral of inf.vol. kernel w/ 3-point QCD correlators

# CONCLUSIONS

Windows very powerful quantities: **intermediate window**  $a_\mu^W$   
**hadronic  $\tau$ -decays** can shed light on tension lattice vs  $e^+e^-$

$\tau$  data **very competitive** on intermediate window  
historic tension w/  $ee$  data and in IB  $\tau$  effects  
ongoing blinded analysis of Aleph  $< 1\%$  accuracy on  $a_\mu^W$

**Work in progress** to finalize full formalism

[MB et al, in prep]

W-regularization and short-distance corrections

(re-)calculation of initial state rad.cor.

numerical calculation of final state IB corrections

relevant also for QED correction to HVP

## Thanks for your attention