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Isospin-Breaking Effects on Precision Observables in Lattice QCD
MITP: 22-26 July, 2024

André Walker-Loud
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Title: Three topics related to isospin breaking

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Three topics related to isospin breaking

- Scheme that separates γ and $m_d - m_u$ corrections at leading order in isospin breaking
- QED corrections to g_A: estimates from χPT
- Non-monotonic FV corrections to g_A

Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- GOAL
  - Well defined scheme that separates QED and $m_d - m_u$ isospin breaking effects at leading order
- WHY?
  - This is implicitly what is done with Cottingham estimate of QED corrections to the spectrum, as well as other hadronic processes
  - Easy to incorporate both corrections in perturbation theory
- What are complications?
  - QED renormalizes the quark masses
  - The quark mass operators serve as counter-terms for UV divergences from QED
  - The intertwining of these effects is necessarily renormalization scheme and scale dependent

Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- Find a quantity that is not sensitive to isospin breaking to define isospin symmetric world
- Find a quantity that is not sensitive to QCD isospin breaking to define a line of constant physics with only QED isospin breaking
- Find a quantity that is not sensitive to QED isospin breaking to define a line of constant physics with only QCD isospin breaking

Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- Find a quantity that is not sensitive to isospin breaking to define isospin symmetric world
- m_{π^0}
  - strong isospin breaking shifts the π^0 mass but at 2nd order

    $$m^2_{\pi^0} = m^2_{\pi^\pm} + \frac{2B\delta)^2}{(4\pi F_\pi)^2}(4\pi)^2 l_7 \qquad \delta \equiv \frac{1}{2}(m_d - m_u)$$

  - The QED corrections to the neutron pion are suppressed in the chiral expansion and ``tiny'' [Bijnens & Prades hep-ph/9610360] $\Delta_\gamma m^2_{\pi^0} \propto \frac{e^2}{4\pi} \times \frac{m^2_\pi}{(4\pi F_\pi)^2}$
    - While this is formally leading order in isospin breaking
      $\delta \sim m_d \sim m_u$
      it is numerically 2nd order in isospin breaking
      Collins Nucl.Phys.B149 (1979)
- Define: $$m_{\pi^\pm}\Big|_{\text{iso}-\text{symmetric}} \equiv m^{\text{PDG}}_{\pi^0}$$

Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- Find a quantity that is not sensitive to QCD isospin breaking to define a line of constant physics with only QED isospin breaking
- $m_{\pi^\pm} - m_{\pi^0}$
  - Tune $m_l = m_u = m_d$ until $m_{\pi^\pm} = m_{\pi^0}^{\rm PDG}$ with iso-symmetric LQCD
  - Turn on QED with physical $\alpha_{f.s.}$
  - adjust m_l until $m_{\pi^\pm} = m_{\pi^\pm}^{\rm PDG}$
    - if a regulator that respects ``chiral symmetry'' is used, this should be a small change $\delta m_l \propto \alpha_{f.s.} \times m_l$
      and if the scheme is working, we should find that m_{π^0} with the adjusted quark mass with and without QED is still equal to the PDG mass up to 2nd order corrections

Isospin breaking scheme

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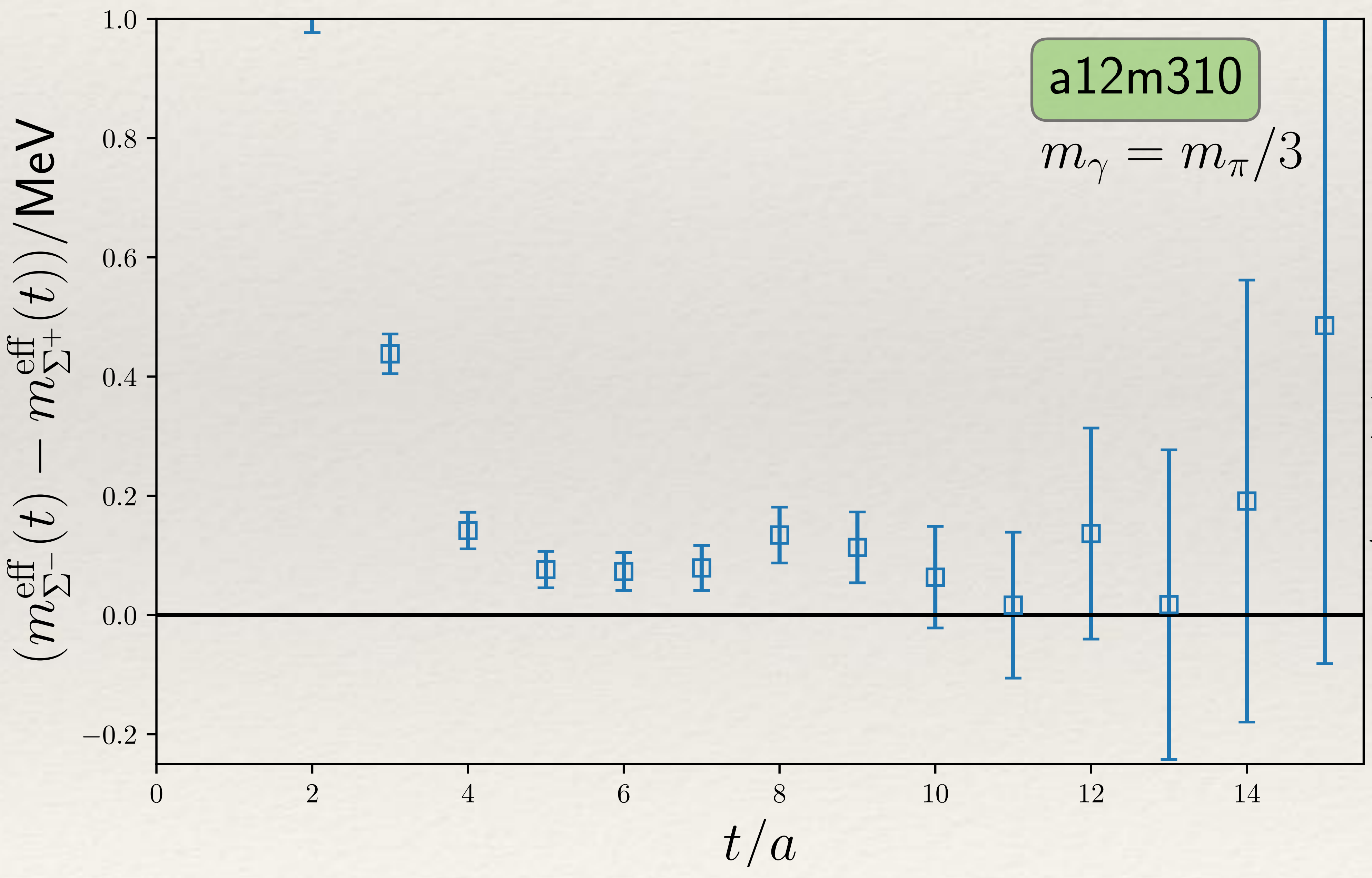
- Find a quantity that is not sensitive to QED isospin breaking to define a line of constant physics with only QCD isospin breaking
- $m_{\Sigma^-} - m_{\Sigma^+}$
  - up to inelastic structure corrections, this mass splitting should be insensitive to QED They have been estimated to be O(0.1 MeV) [Erben, Shanahan, Thomas, Young, 1408.6628]



QED$_M$ — in preparation
Della Morte, Hall, Hörz, Monge-Camacho, Nicholson, Shindler, Tsang, Walker-Loud

  - Tune $2\delta = m_d - m_u$ until $m_{\Sigma^-} - m_{\Sigma^+} = 8.08(8)$ MeV [= 1197.45(4) – 1189.37(7)] (tuning just the valence quark mass is sufficient for the tuning)

Isospin breaking scheme

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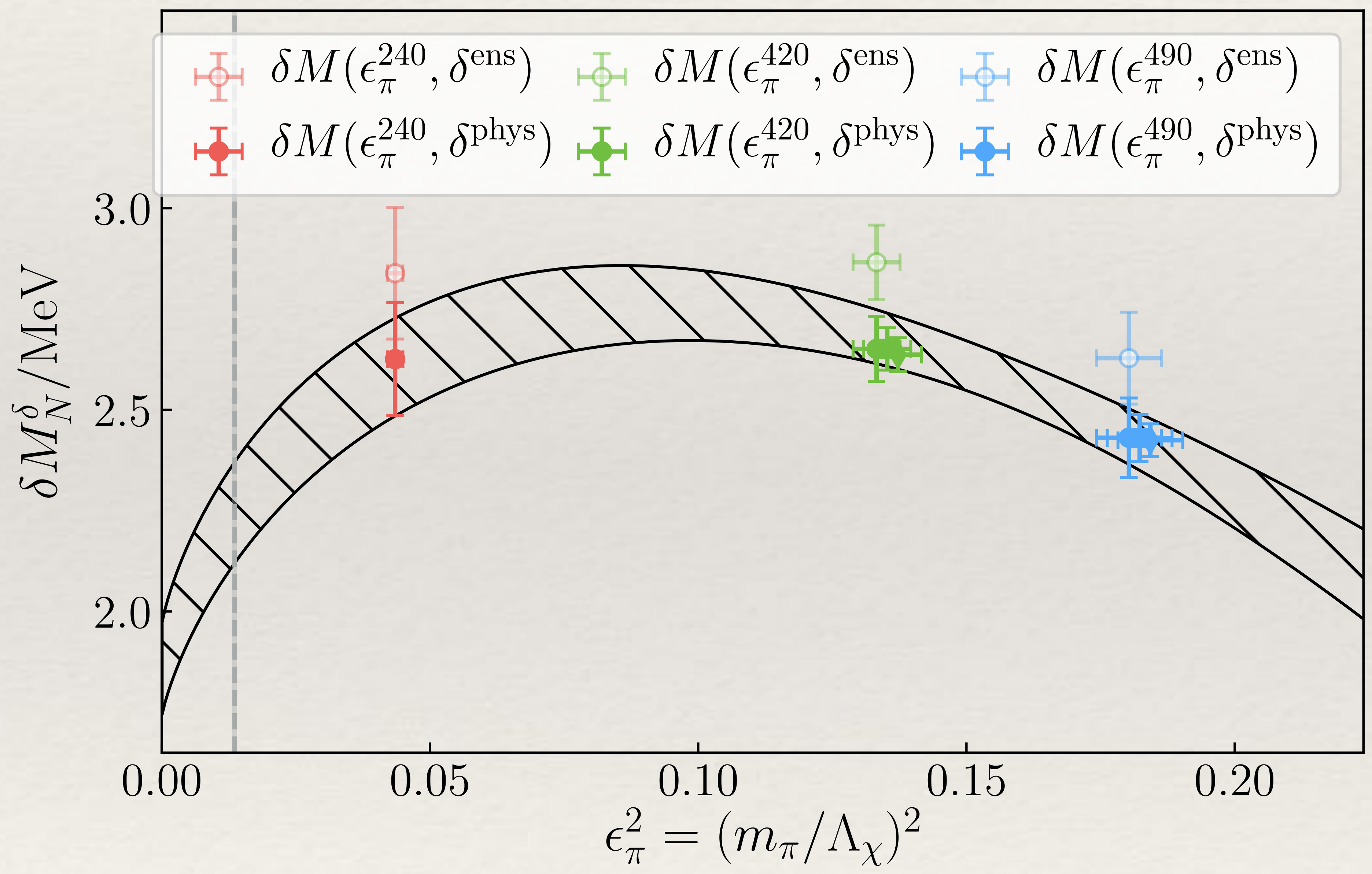
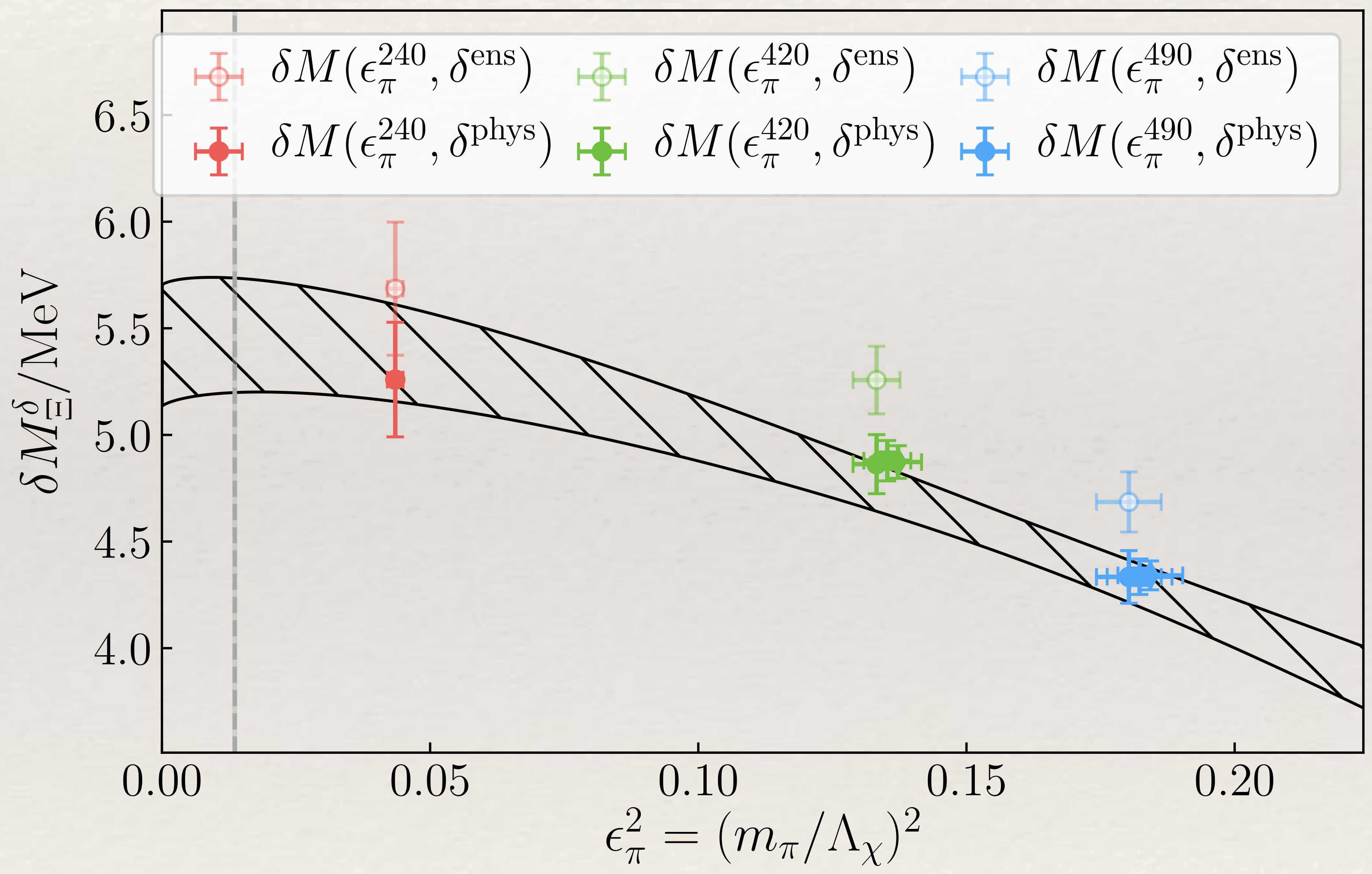
- Find a quantity that is not sensitive to QED isospin breaking to define a line of constant physics with only QCD isospin breaking
- $m_{\Sigma^-} - m_{\Sigma^+}$
  - Caution: there are potentially significant m_π corrections to $m_{\Sigma^-} - m_{\Sigma^+}$





Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- What about the strange quark mass? I don't (yet) have a good strategy without combined QCD + QED calculations (if m_Ω is used for scale setting)
- QED corrections to m_{K^0} are also suppressed in the chiral expansion, but as

$$\Delta_\gamma m_{K^0} \propto \frac{e^2}{4\pi} \frac{m_K^2}{(4\pi F)^2}$$

  - We can parameterize the isospin breaking corrections to the kaon masses as

$$m_{K^0} = m_K^{\text{iso}} + \frac{1}{2}\Delta_\delta m_K + \epsilon_\gamma \Delta_\gamma m_K \qquad m_{K^\pm} = m_K^{\text{iso}} - \frac{1}{2}\Delta_\delta m_K + (1+\epsilon_\gamma)\Delta_\gamma m_K$$

$$m_{K^0} - m_{K^\pm} = \Delta_\delta m_K - \Delta_\gamma m_K$$

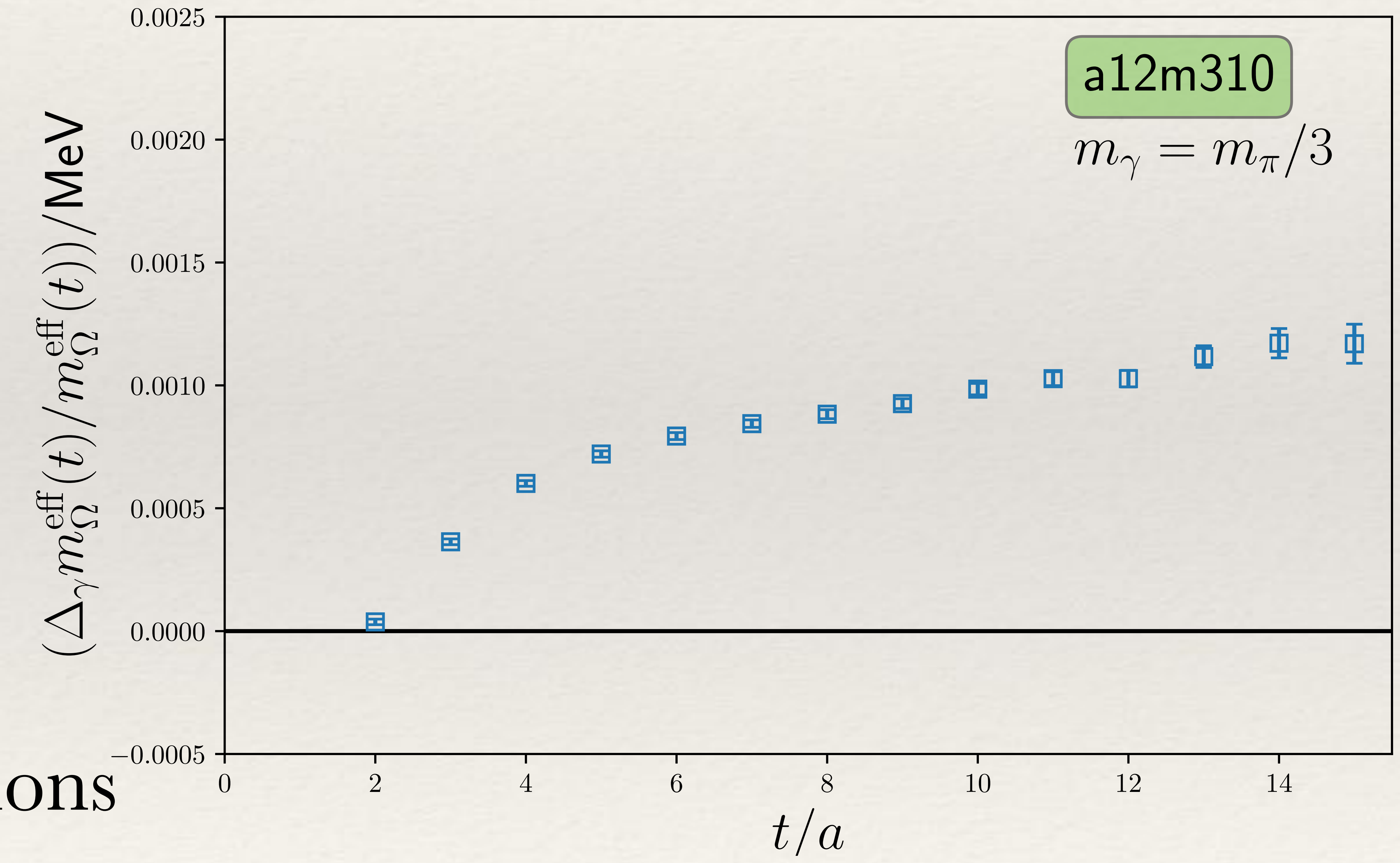
    where we anticipate $\epsilon_\gamma \sim 1/4 \sim m_K^2/(4\pi F)^2$

  - $\Delta_\delta m_K$ is determined by setting $2\delta = m_d - m_u$ from the $m_{\Sigma^-} - m_{\Sigma^+}$ determination
  - We need to turn on QED to determine both $\Delta_\gamma m_K$ and ϵ_γ
    Starting near m_s^{phys} (from iso-symmetric LQCD) a small change in m_s will allow for these determinations and m_K^{iso}

Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- What about scale setting?
- m_Ω
  - The omega mass has very mild light quark mass dependence
  - The omega is heavy with little relative sensitivity to QED corrections
  - electro-quenched correction to m_Ω O(0.1%)
  - sea-quark corrections will be $\lesssim$ valence corrections



QED$_M$ — in preparation

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Isospin breaking scheme

Bussone, Della Morte, Janowski, Walker-Loud — Lattice2018 — arXiv:1810.11647

- A scheme that separates QED and QCD isospin breaking at (numerical) leading order in isospin breaking
  - m_{π^0}: determines $m_l = \frac{1}{2}(m_u + m_d)$
  - $m_{\Sigma^-} - m_{\Sigma^+}$: determines $\delta = \frac{1}{2}(m_d - m_u)$
  - $m_{K^0}, m_{K^\pm}$: a small iterative procedure with QCD and QCD+QED determines m_s
  - m_Ω: determines the scale

- This scheme is implicitly used in much phenomenology — Cottingham
  It is theoretically nice as it allows for an exploration of QED and QCD isospin breaking independently (until 2nd order corrections are needed)

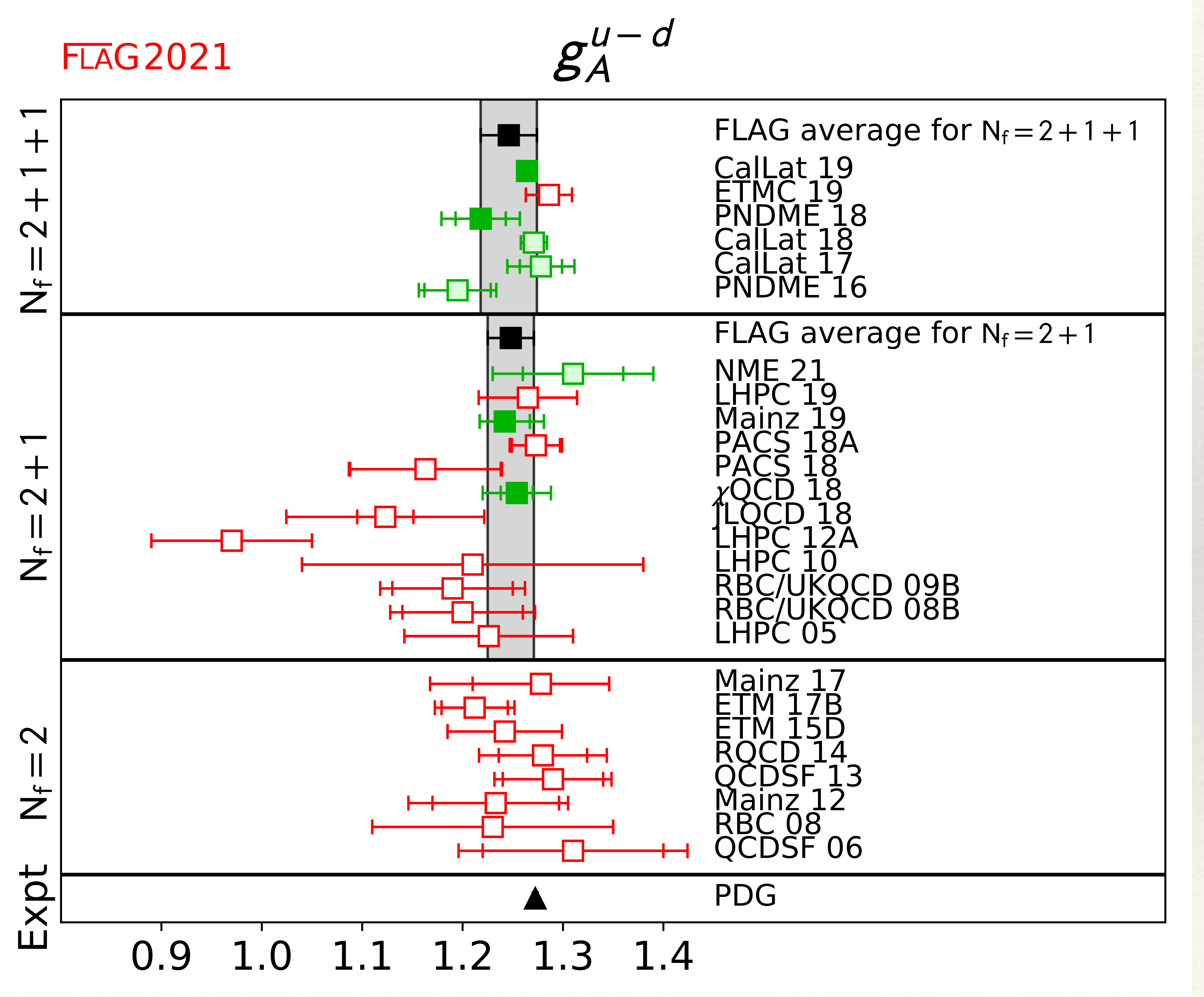
QED corrections to g_A

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- We compare our LQCD calculations of $g_A^{\rm iso}$ to $g_A^{\rm PDG}$

- $g_A^{\rm PDG}$ is determined from an experimental measurement of $\lambda = g_A/g_V$ after some analytic long-distance QED effects are subtracted — see Hayen & Young, 2009.11364 for discussion

- But it turns out - potentially significant low-energy nucleon structure corrections may spoil this comparison



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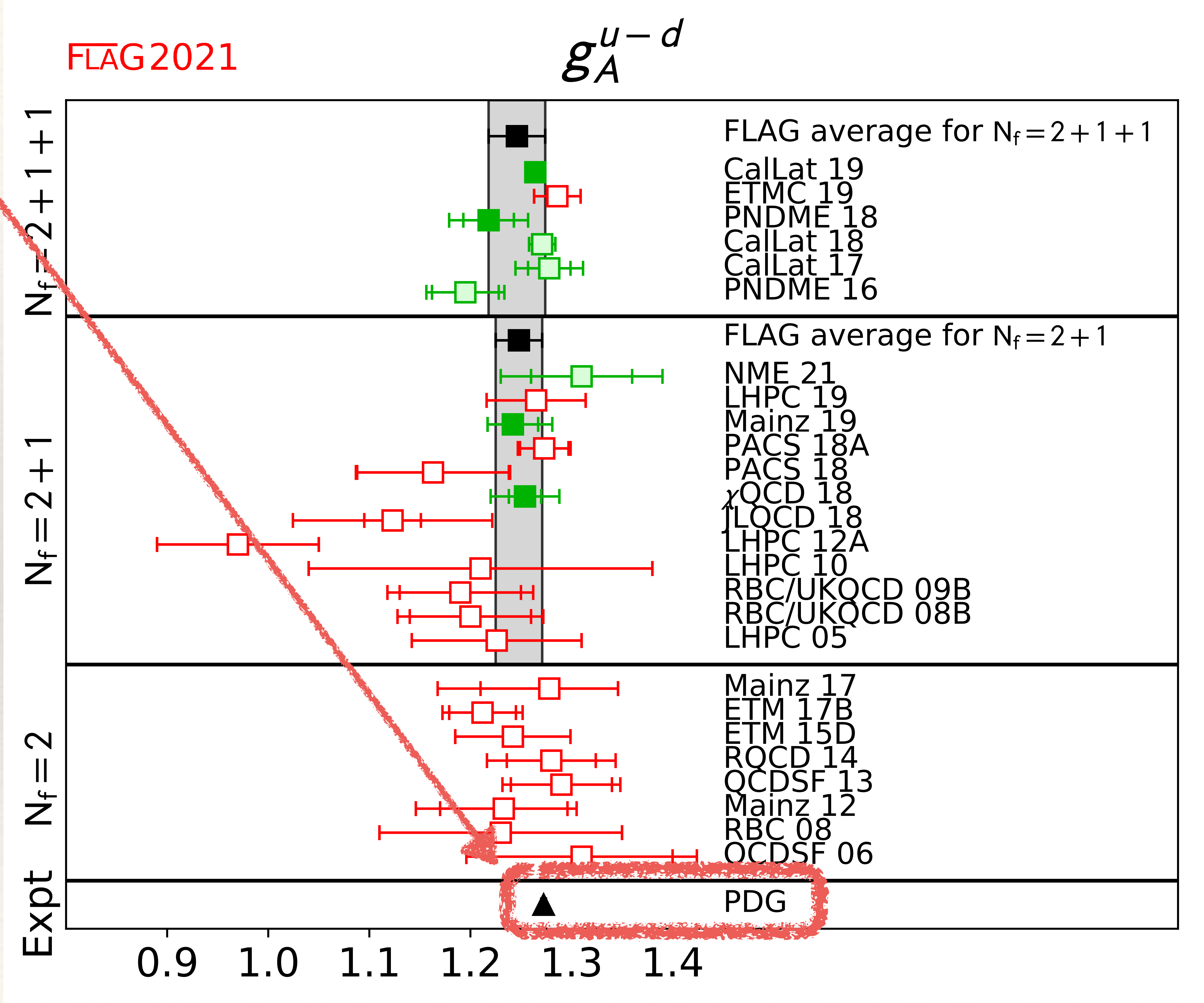


Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Systematic, EFT treatment of neutron β-decay

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3\lambda^2)w(E_e) \times \left[1 + \bar{a}(\lambda)\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \bar{A}(\lambda)\frac{\vec{\sigma}_n\cdot\vec{p}_e}{E_e} + \ldots\right]$$

The parameters can be measured

If we want to connect them to Standard Model (SM) parameters
we need to start from a Lagrangian with parameters related to SM parameters

pion-less low-energy EFT

$$\lambda = \frac{g_A}{g_V}$$

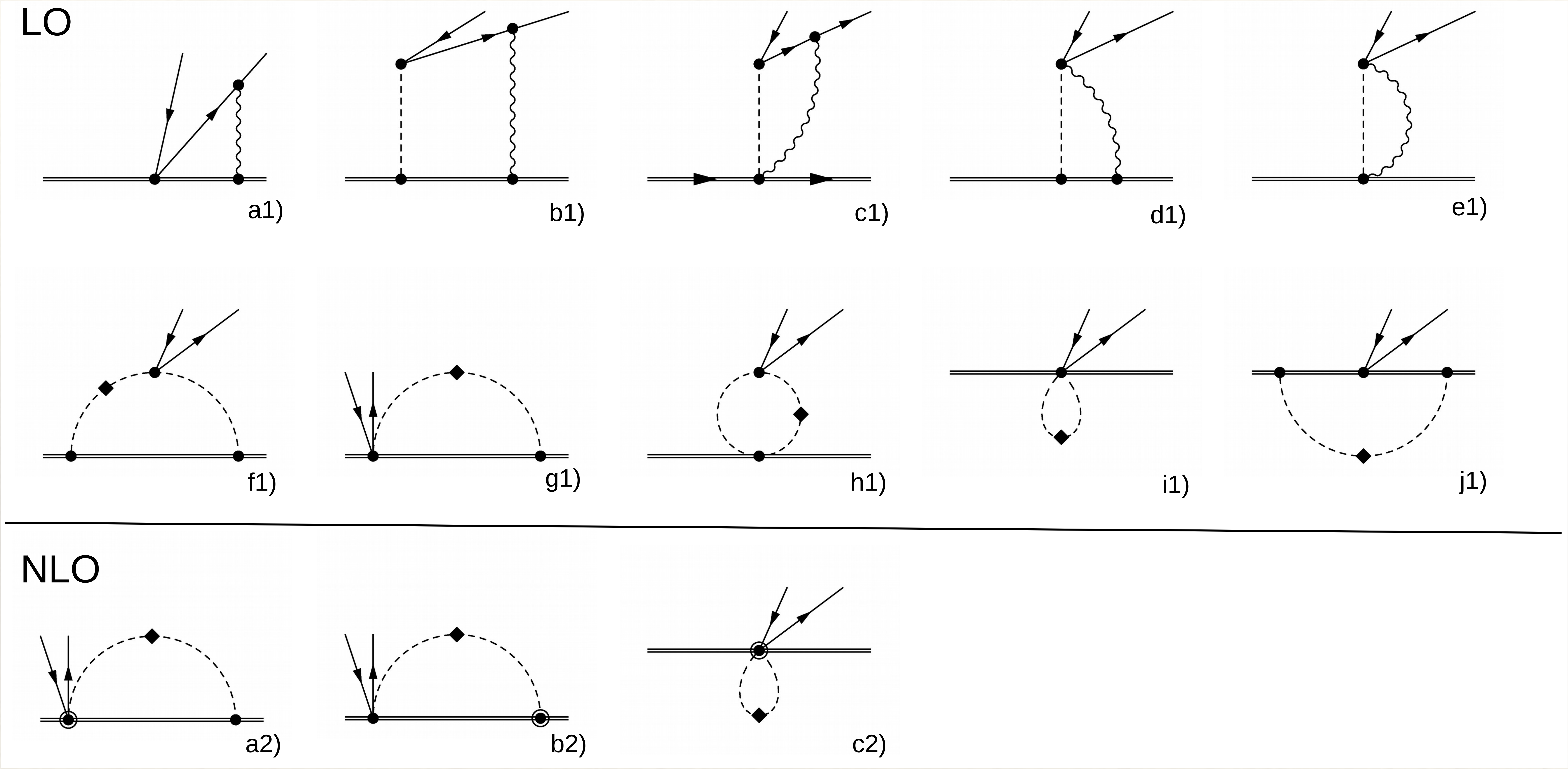
$$\begin{aligned}\mathcal{L}_{\not\pi} = -\sqrt{2}G_F V_{ud}\Big[\bar{e}\gamma_\mu P_L \nu_e \Big(\bar{N}\left(g_V v_\mu - 2g_A S_\mu\right)\tau^+ N \\ + \frac{i}{2m_N}\bar{N}(v^\mu v^\nu - g^{\mu\nu} - 2g_A v^\mu S^\nu)(\overleftarrow{\partial} - \overrightarrow{\partial})_\nu \tau^+ N\Big) \\ + \frac{i c_T m_e}{m_N}\bar{N}\left(S^\mu v^\nu - S^\nu v^\mu\right)\tau^+ N\ (\bar{e}\sigma_{\mu\nu}P_L\nu) \\ + \frac{i\mu_{\text{weak}}}{m_N}\bar{N}[S^\mu, S^\nu]\tau^+ N\,\partial_\nu\left(\bar{e}\gamma_\mu P_L \nu\right)\Big] + \ldots \qquad (2)\end{aligned}$$

Perform the calculation with SU(2) heavy-baryon χPT and match the results to this pion-less EFT whose parameters can be matched to experimentally measured quantities

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Sub-set of O(50) diagrams



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$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$

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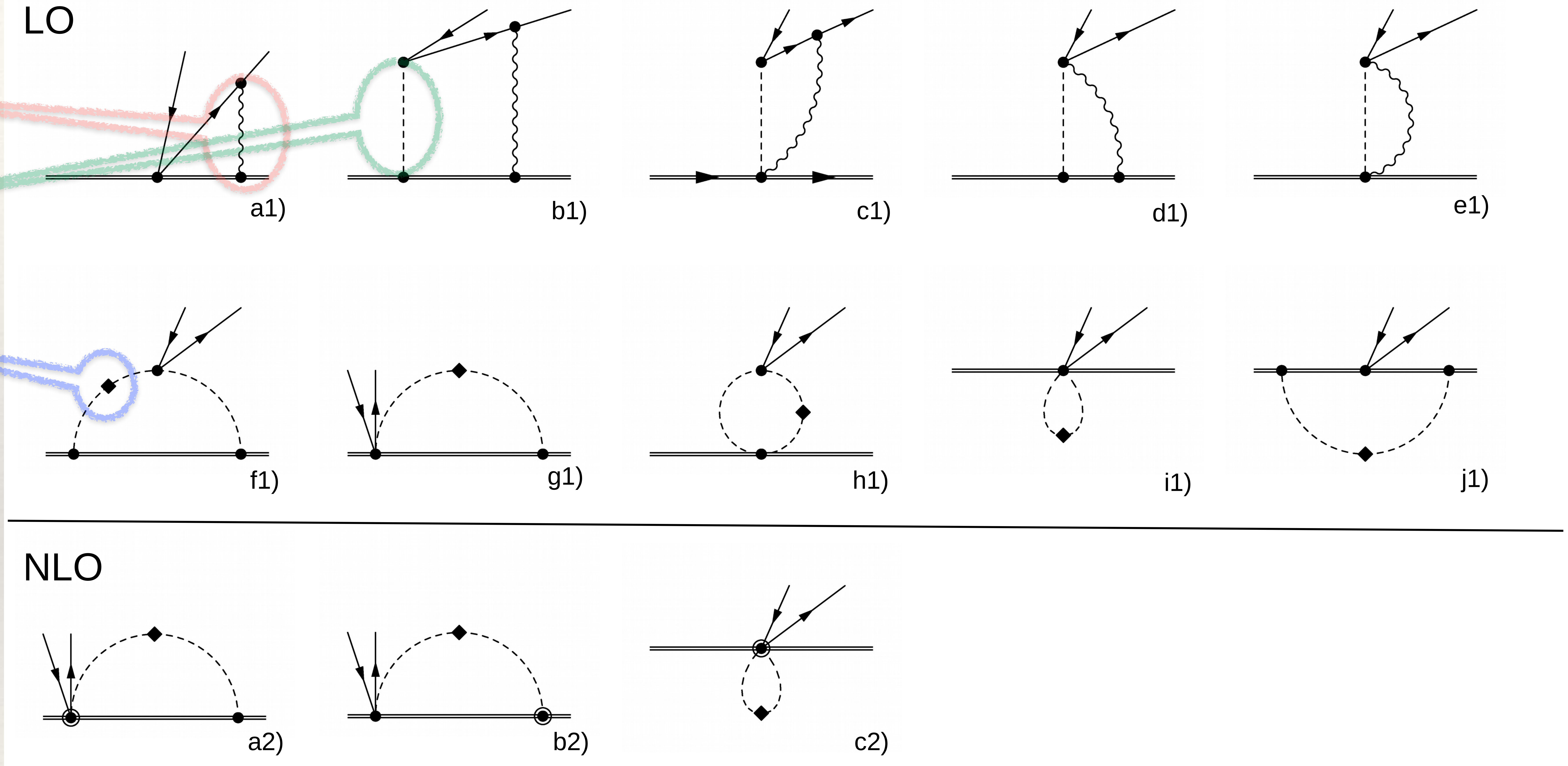
- Sub-set of O(50) diagrams

photons

pions

pion electromagnetic mass splitting

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$



NOTE: at this order, we also include QED, m_d-m_u corrections to M_n-M_p

- iso-vector contributions to M_n-M_p vanish from symmetry constraints for τ^+ current
- iso-scalar contributions do not vanish - but the sum of all of them does vanish through NLO

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Matching $\lambda = g_A^{\rm QCD}\left(1+\delta_{\rm RC}^{(\lambda)} - 2{\rm Re}(\epsilon_R)\right)$ $\qquad \delta_{\rm RC}^{(\lambda)} = \frac{\alpha}{2\pi}\left(\Delta^{(0)}_{A,{\rm em}} + \Delta^{(1)}_{A,{\rm em}} - \Delta^{(0)}_{V{\rm em}}\right)$

$$g_{V/A} = g^{(0)}_{V/A}\left[1+\sum_{n=2}^{\infty}\Delta^{(n)}_{V/A,\chi} + \frac{\alpha}{2\pi}\sum_{n=0}^{\infty}\Delta^{(n)}_{V/A,{\rm em}} + \left(\frac{m_u-m_d}{\Lambda_\chi}\right)^{n_{V/A}}\sum_{n=0}^{\infty}\Delta^{(n)}_{V/A,\delta m}\right]$$

$g_V^{(0)} = 1 \qquad \Delta^{(n)}_{\chi,{\rm em},\delta m} \sim O(\epsilon_\chi^n)$

$n_V = 2$ CVC $\qquad n_A = 1$ explicit calculation:

$$\Delta^{(0),(1)}_{A,\delta m} = 0$$
$$\Delta^{(0)}_{V,\delta m} = 0$$

$$\Delta^{(0)}_{A,{\rm em}} = Z_\pi\left[\frac{1+3g_A^{(0)2}}{2}\left(\log\frac{\mu^2}{m_\pi^2}-1\right) - g_A^{(0)2}\right] + \hat{C}_A(\mu)$$

Low-Energy-Constants (LECs)

$$\Delta^{(1)}_{A,{\rm em}} = Z_\pi\, 4\pi m_\pi\left[c_4 - c_3 + \frac{3}{8m_N} + \frac{9}{16m_N}g_A^{(0)2}\right]$$

$C_A(\mu)$ - completely unknown

c_3 & c_4 are estimated from literature

Using Naive Dimensional Analysis (NDA) to estimate $C_A(\mu)$ and $c_{3,4}$ from the literature

$\delta_{\rm RC}^{(\lambda)} \in \{1.4,\ 2.6\}\cdot 10^{-2}$ an order of magnitude larger than previous estimates

Pion-induced radiative corrections to neutron beta-decay

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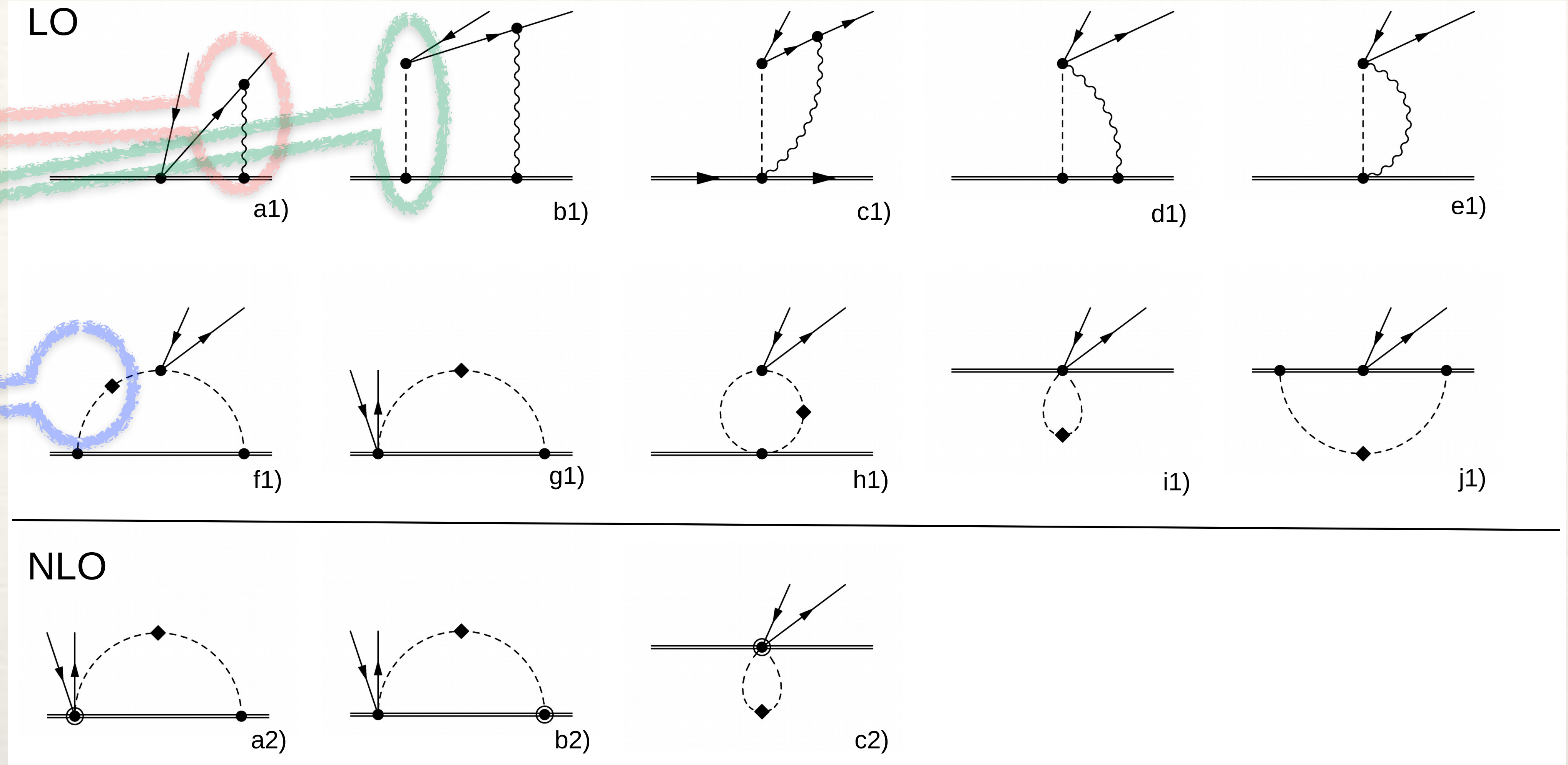

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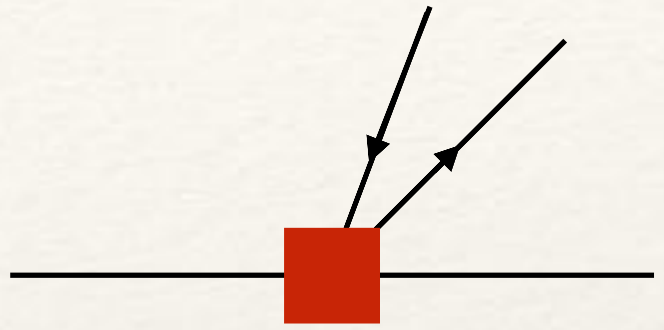


$\hat{C}_A(\mu)$

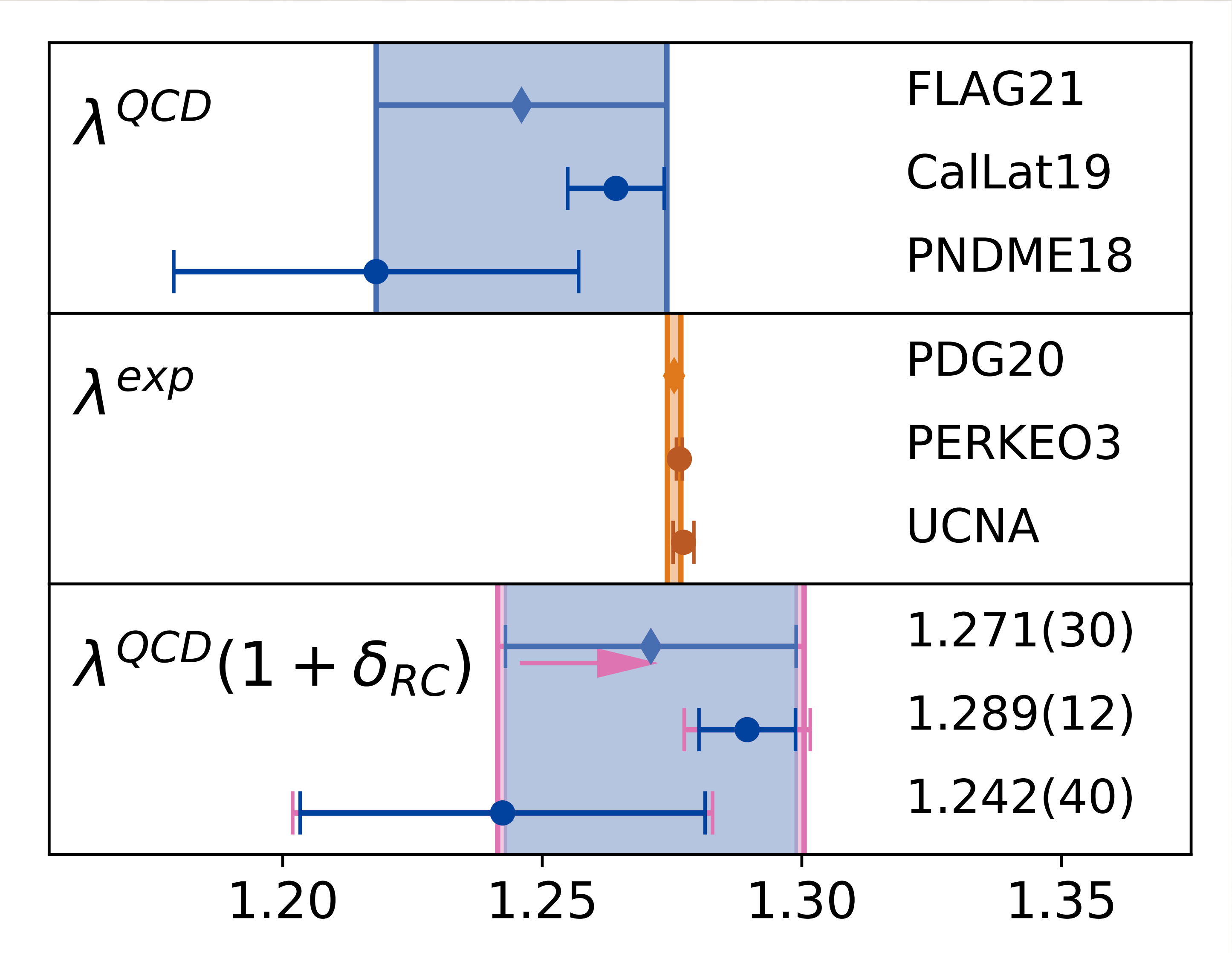
Low-Energy-Constants (LECs)

pion electromagnetic mass splitting

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$

$$\delta_{\text{RC}}^{(\lambda)} \in \{1.4,\ 2.6\} \cdot 10^{-2}$$

$$g_A^{\text{SM}} = g_A^{\text{QCD}} + \delta_{\text{RC}}^{(\lambda)}(\alpha_{fs}, \hat{C}_A(\mu), \dots)$$



$\hat{C}_A(\mu)$ - completely unknown	estimate by varying μ (NDA)
other LECs (c_3, c_4)	estimate from literature

- seems to move g_A^{QCD} towards g_A^{exp}
- need LQCD+QED calculation to determine $\delta_{\text{RC}}^{(\lambda)}$

QED corrections to g_A

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- An O(2%) QED correction to g_A was estimated with χPT
  - Assume χPT is at least qualitatively correct (if not accurate) (no significant cancellation between analytic terms and LECs)
- In order to compare LQCD results of g_A to experiment, this QED correction MUST be determined — LQCD + QED is the only way
  - It is a scheme (and possibly QED-gauge) dependent quantity
- This correction does NOT impact extraction of V_{ud} — it is a "right handed" correction
  - The λ in Γ is the same as in beta-assymetry (A)
- It does prevent us from using LQCD to constrain BSM right-handed currents better than a few percent

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3\lambda^2)w(E_e) \times \left[1 + \bar{a}(\lambda)\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + \bar{A}(\lambda)\frac{\vec{\sigma}_n\cdot\vec{p}_e}{E_e} + \ldots\right]$$

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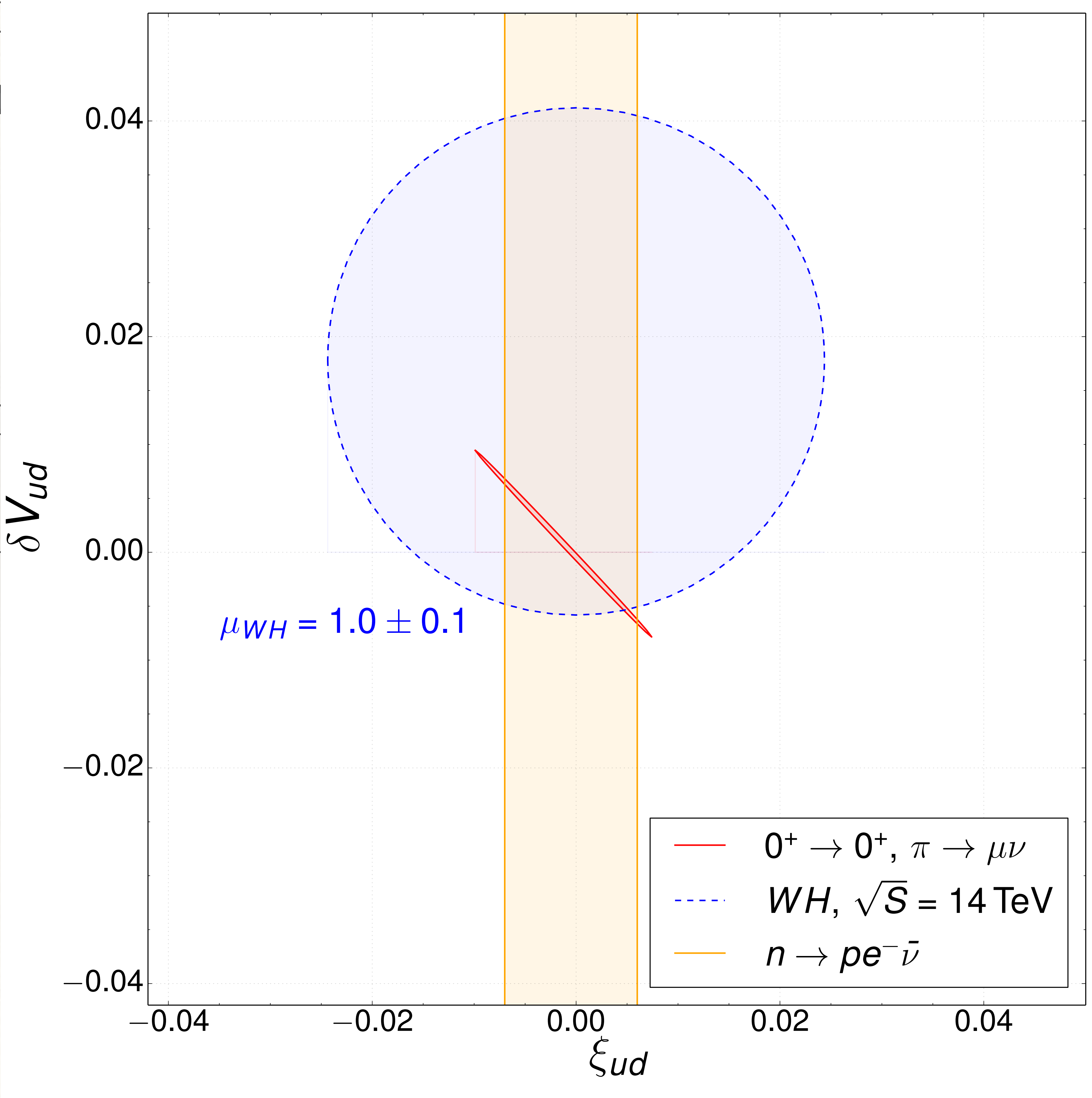

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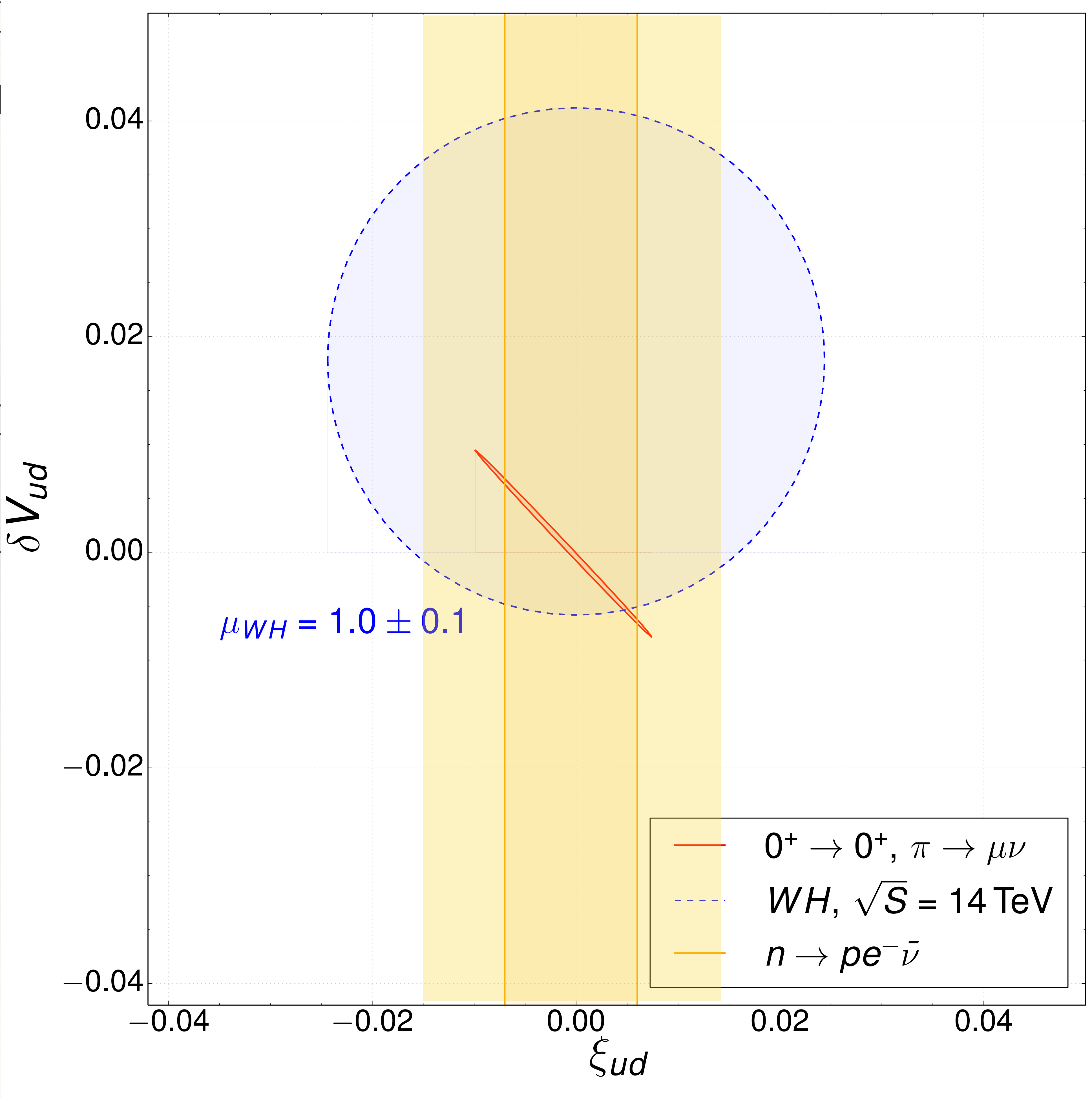

Non-monotonic FV corrections to g_A

Z. Hall, D. Pefkou, A.S. Meyer, R. Briceño, M.A. Clark, M. Hoferichter, E. Mereghetti,
H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vranas, A. Walker-Loud — In preparation

- What is the issue?
- We (the LQCD community) think of FV corrections in the asymptotic scaling regime
- We have numerical evidence that the sign of the FV correction depends upon m_π 😱
- We have qualitative evidence that the sign of FV corrections at $m_\pi \approx 300$ MeV is not the same as at m_π^{phys}
- We have qualitative evidence that the sign of the FV corrections can change
  - at fixed $m_\pi L$ as one varies m_π
  - at fixed m_π as one varies $m_\pi L$
- We should not find this surprising, after all, for nucleon quantities

Non-monotonic FV corrections to g_A

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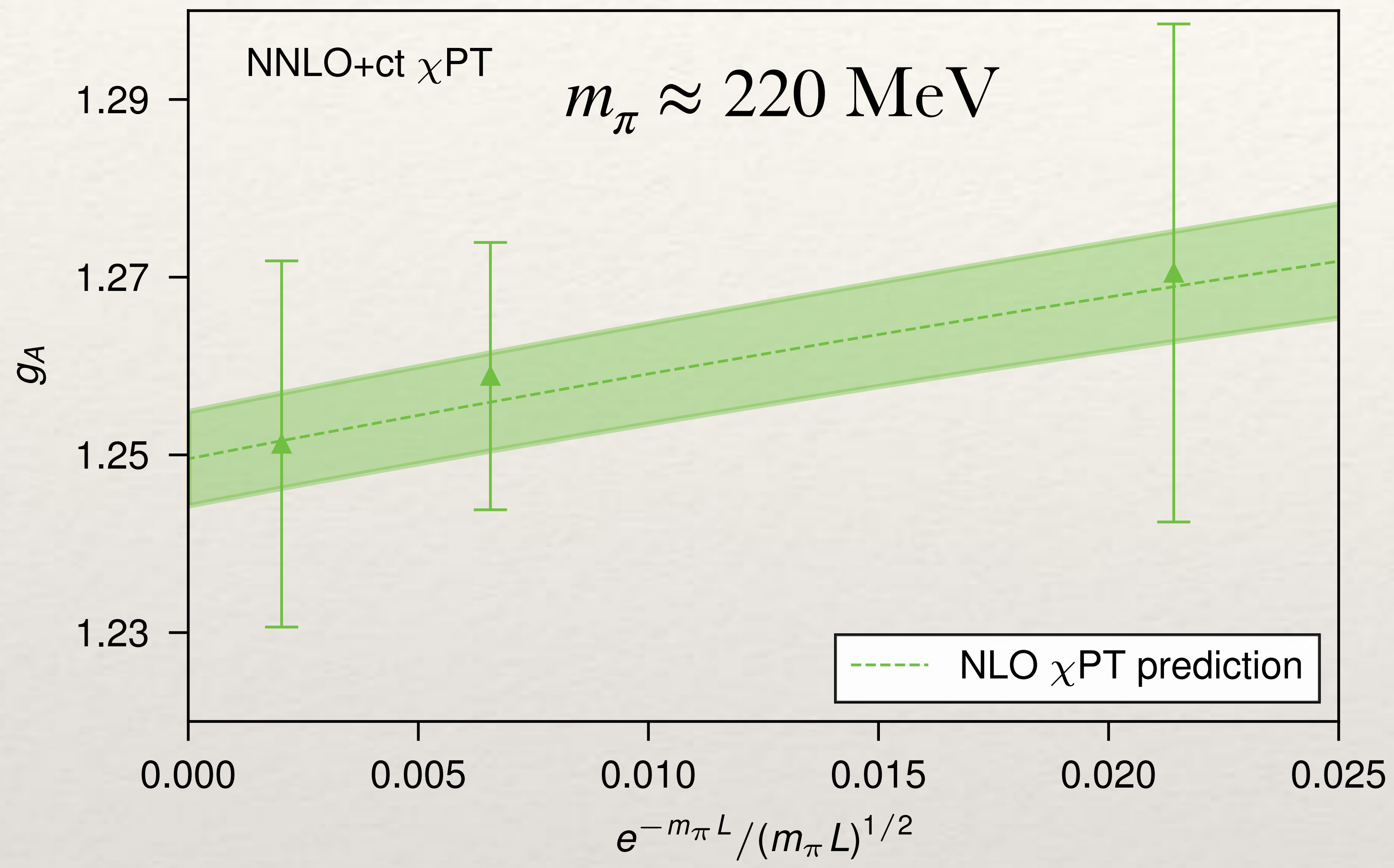
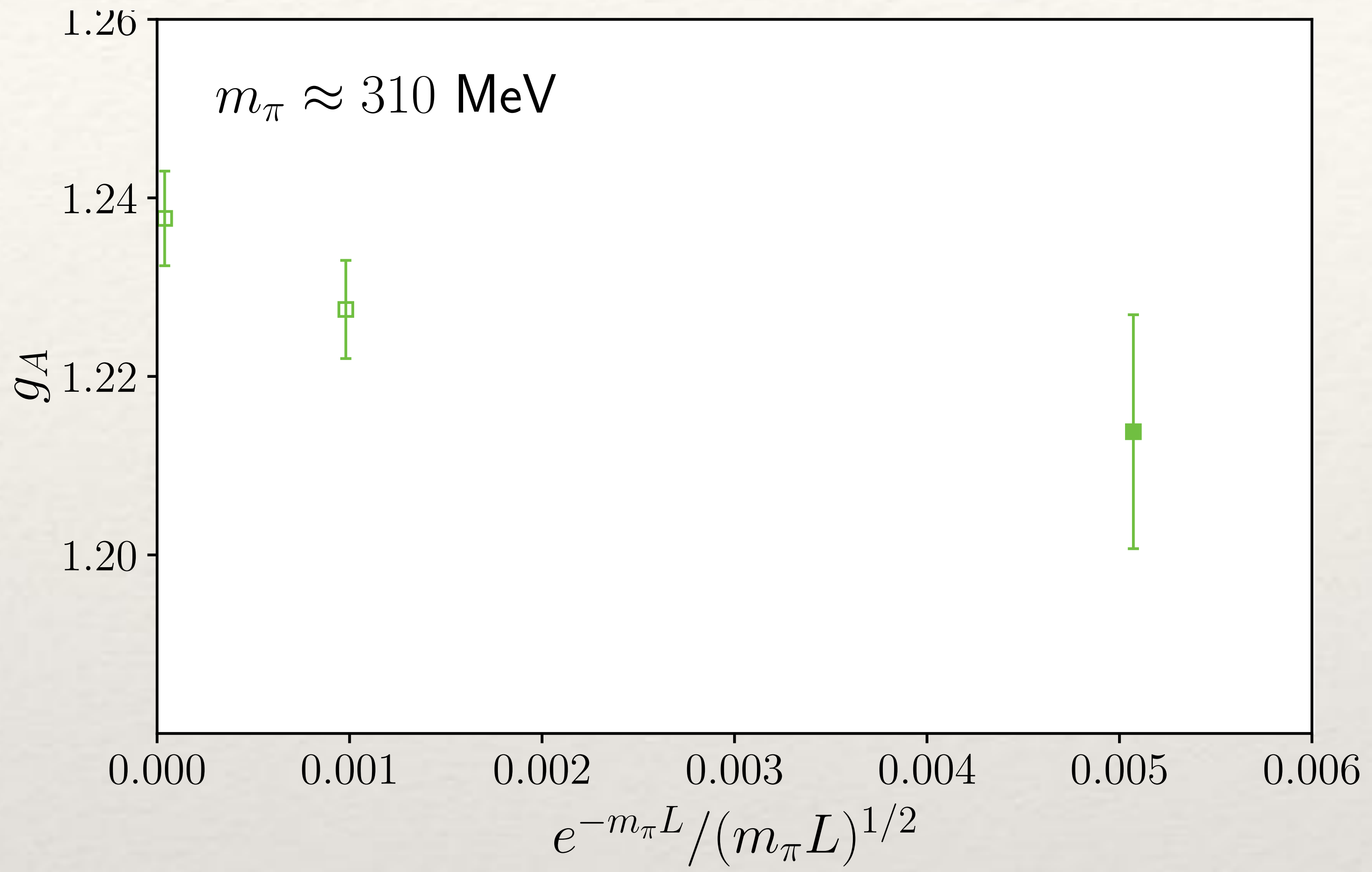
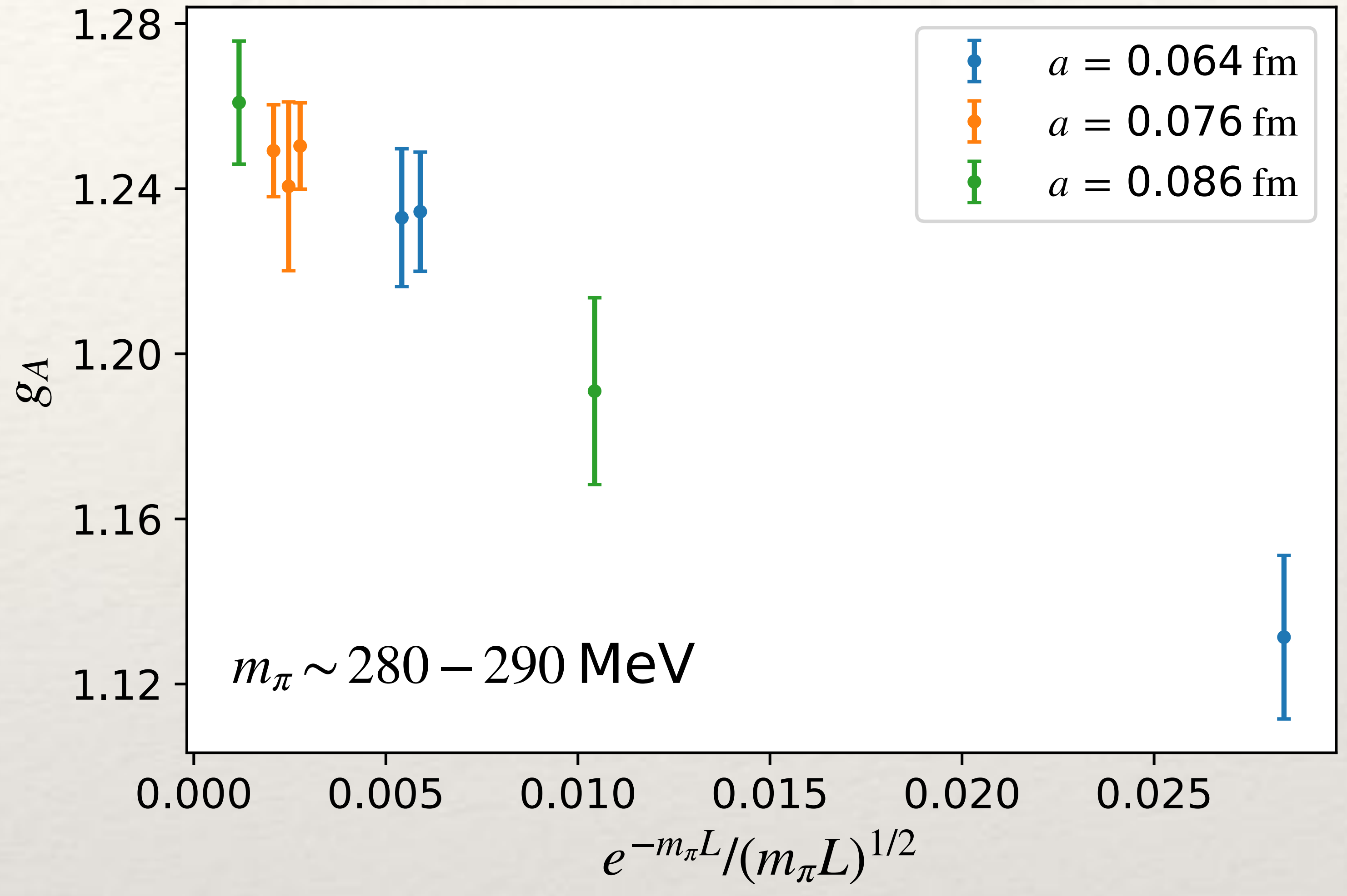
- Numerical Evidence:



CalLat [1805.12130]



CalLat - unpublished



RQCD - 2305.04717

- At $m_\pi \approx 220$ MeV, results are consistent with leading prediction from χPT (and also consistent with no correction or opposite sign)
- At $m_\pi \approx 300$ MeV, results constrain the sign of the volume correction opposite of χPT prediction

Non-monotonic FV corrections to g_A

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- Expectations from χPT
  - The chiral expansion for nucleons is a series in $\epsilon_\pi = \dfrac{m_\pi}{4\pi F_\pi}$, while for pions, it is in ϵ_π^2
    - therefore, higher order corrections are relatively more important
  - The nucleon has a much richer spectrum of virtual excited states $(N\pi, \Delta\pi, \ldots)$
  - In the large N_c limit, there is an exact cancellation of most NLO corrections to g_A
    - The finite volume corrections also respect this cancellation and lead to a sign change at fixed m_π vs $m_\pi L$
  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

Non-monotonic FV corrections to g_A

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- Expectations from χPT
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$$g_A = g_0 + \Delta^{(2)} + \delta^{(2)}_{\rm FV} + \Delta^{(3)} + \delta^{(3)}_{\rm FV}$$

$$\Delta^{(2)} = \epsilon_\pi^2 \left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3 \right]$$

$$\Delta^{(3)} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3) \right]$$

$$\delta^{(2)}_{\rm FV} = \frac{8}{3}\epsilon_\pi^2 \left[g_0^3 F_1^{(2)}(m_\pi L) + g_0 F_3^{(2)}(m_\pi L) \right]$$

$$F_1^{(2)}(x) = \sum_{\vec{n}\neq 0}\left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|} \right]$$

$$F_3^{(2)}(x) = -\frac{3}{2}\sum_{\vec{n}\neq 0}\frac{K_1(x|\vec{n}|)}{x|\vec{n}|}\,,$$

$$\delta^{(3)}_{\rm FV} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left\{ g_0^2 \frac{4\pi F}{M_0} F_1^{(3)}(m_\pi L) - \left[\frac{4\pi F}{M_0}(3+2g_0^2) + 4(2\tilde{c}_4 - \tilde{c}_3) \right] F_3^{(3)}(m_\pi L) \right\}$$

$$F_1^{(3)}(x) = \sum_{\vec{n}\neq\vec{0}} \frac{K_{\frac{1}{2}}(x|\vec{n}|)}{\sqrt{\frac{\pi}{2}x|\vec{n}|}}\, x|\vec{n}| \quad = \sum_{\vec{n}\neq\vec{0}} e^{-x|\vec{n}|}$$

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Non-monotonic FV corrections to g_A

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H. Monge-Camacho, C. Morningstar, A. Nicholson, P. Vranas, A. Walker-Loud — In preparation

- Expectations from χPT
  - SU(2) HBχPT($\not\Delta$) at NNLO also predicts change in sign of FV corrections

$$g_A = g_0 + \Delta^{(2)} + \delta_{\rm FV}^{(2)} + \Delta^{(3)} + \delta_{\rm FV}^{(3)}$$

$$\delta_{\rm FV}^{(2)} = \frac{8}{3}\epsilon_\pi^2 \left[g_0^3 F_1^{(2)}(m_\pi L) + g_0 F_3^{(2)}(m_\pi L)\right]$$

$$\Delta^{(2)} = \epsilon_\pi^2 \left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}_{16}^r - g_0^3\right]$$

$$\Delta^{(3)} = \epsilon_\pi^3 g_0 \frac{2\pi}{3}\left[3(1+g_0^2)\frac{4\pi F}{M_0} + 4(2\tilde{c}_4 - \tilde{c}_3)\right]$$

NOTE: the leading FV correction is a prediction
g_0 is determined in the chiral extrapolation

$$F_1^{(2)}(x) = \sum_{\vec{n}\neq 0}\left[K_0(x|\vec{n}|) - \frac{K_1(x|\vec{n}|)}{x|\vec{n}|}\right]$$

$$F_3^{(2)}(x) = -\frac{3}{2}\sum_{\vec{n}\neq 0}\frac{K_1(x|\vec{n}|)}{x|\vec{n}|},$$

for $g_0 \sim 1.2$, $\delta_{\rm FV}^{(2)} > 0$

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$$\tilde{c}_i = (4\pi F)\, c_i$$

$$\Delta^{(2)} = \epsilon_\pi^2\left[-g_0(1+2g_0^2)\ln\epsilon_\pi^2 + 4\tilde{d}^r_{16} - g_0^3\right]$$

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in SU(2) HBχPT($\not\Delta$), with N³LO $N\pi$ phase shift analysis
Siemens et al, 1610.08978
$c_3 = -5.60(6)$ GeV^{-1}
$c_4 = \;\; 4.26(4)$ GeV^{-1}

This leads to LARGE, negative FV correction
Fitting $2c_4 - c_3$ to our LQCD results yields a value $\sim 10\times$ smaller — leads to change in sign of $\delta_{\rm FV}$ as function of m_π

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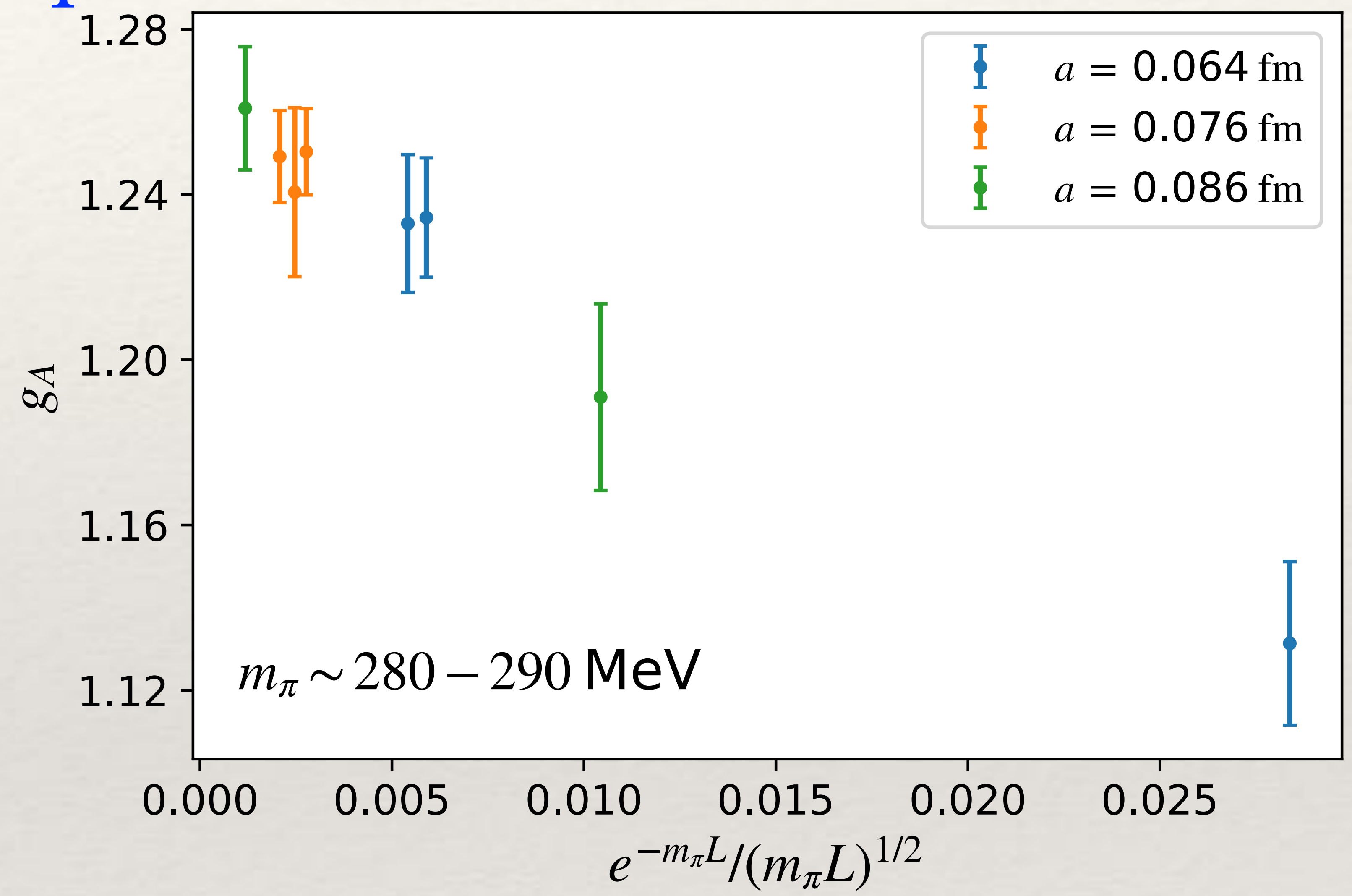
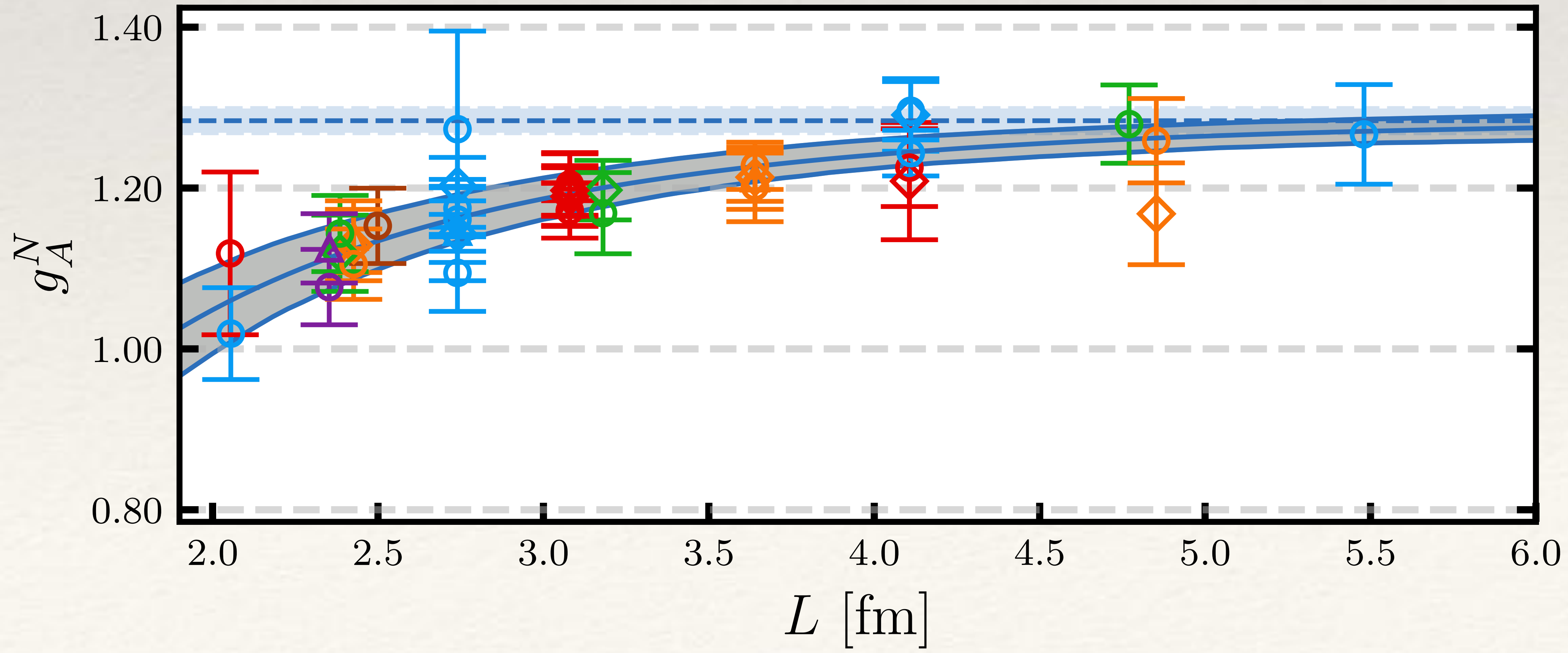
- Current strategy (of most groups)
  - take asymptotic form of Bessel functions and leading "wrap around the world" mode and only leading volume correction

$$\mathrm{L}\quad \boxed{g_A(L)} = g_A + c_2 \frac{m_\pi^2}{(4\pi F_\pi)^2} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}}$$



  - Fit c_2 essentially to heavy m_π results
  - Use this m_π-independent value of c_2 to extrapolate to infinite volume at all m_π
  - If the volume corrections do change sign (to agree with χPT prediction close to $m_\pi^{\rm phys}$) the current strategy will lead to an error
  - At what precision will this occur?



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- What should we do?
  - One needs to perform a volume study at multiple pion masses with sufficient precision to constrain the sign of the volume correction as a function of m_π

$$\boxed{g_A(L)} = g_A + c_2 \frac{m_\pi^2}{(4\pi F_\pi)^2} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}} + c_3 \frac{m_\pi^3}{(4\pi F_\pi)^3} \frac{e^{-m_\pi L}}{m_\pi L} + \cdots$$

  - Or - we need to rely only upon $m_\pi \approx m_\pi^{\rm phys}$ with sufficient precision to control the final uncertainty of g_A as well as the volume correction
  - Or - determine quantitatively that some variant of HBχPT provides an accurate description of both the m_π dependence as well as $m_\pi L$ dependence

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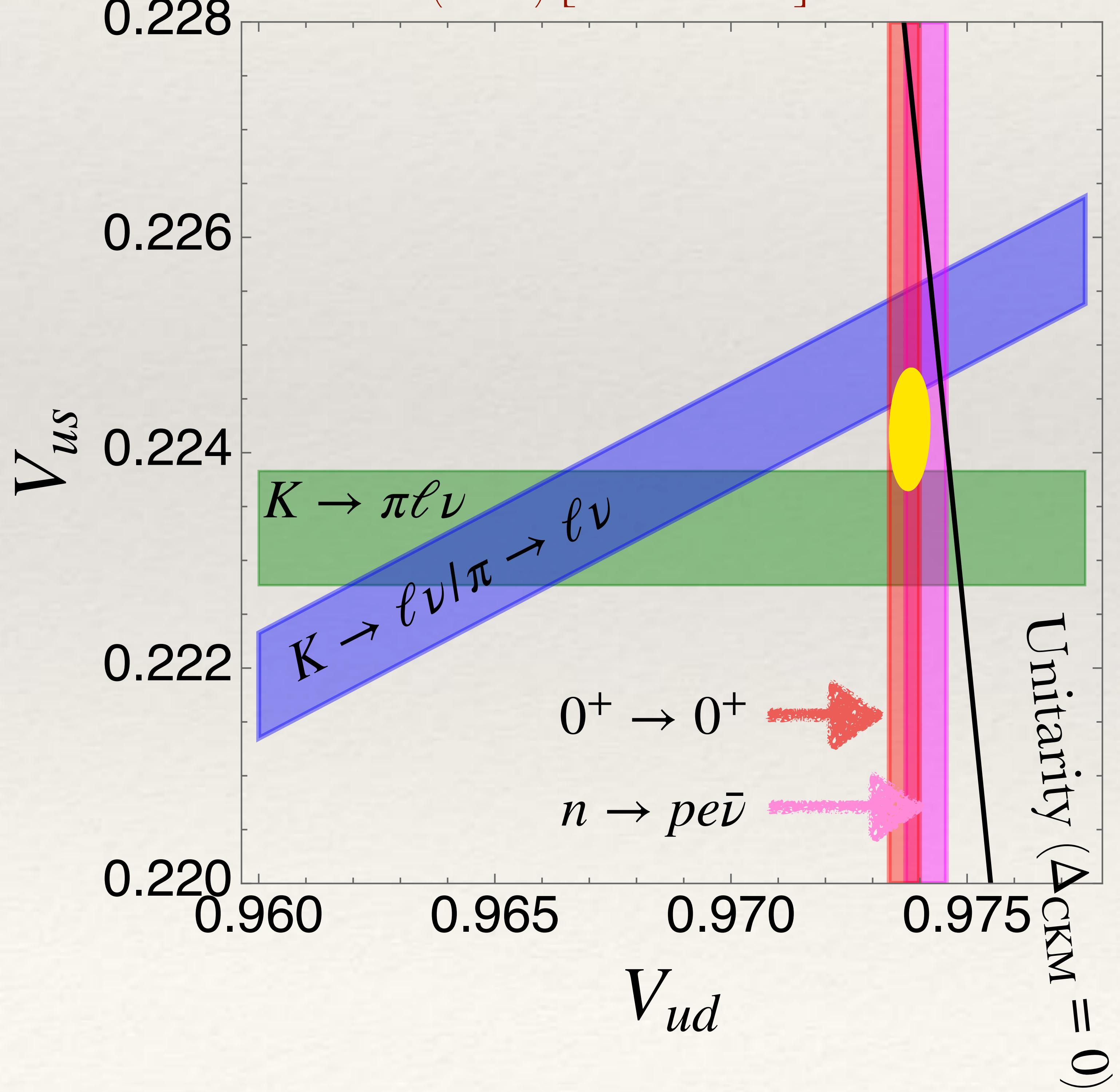
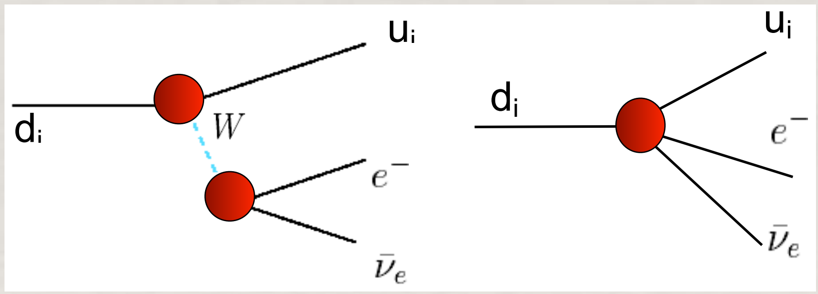
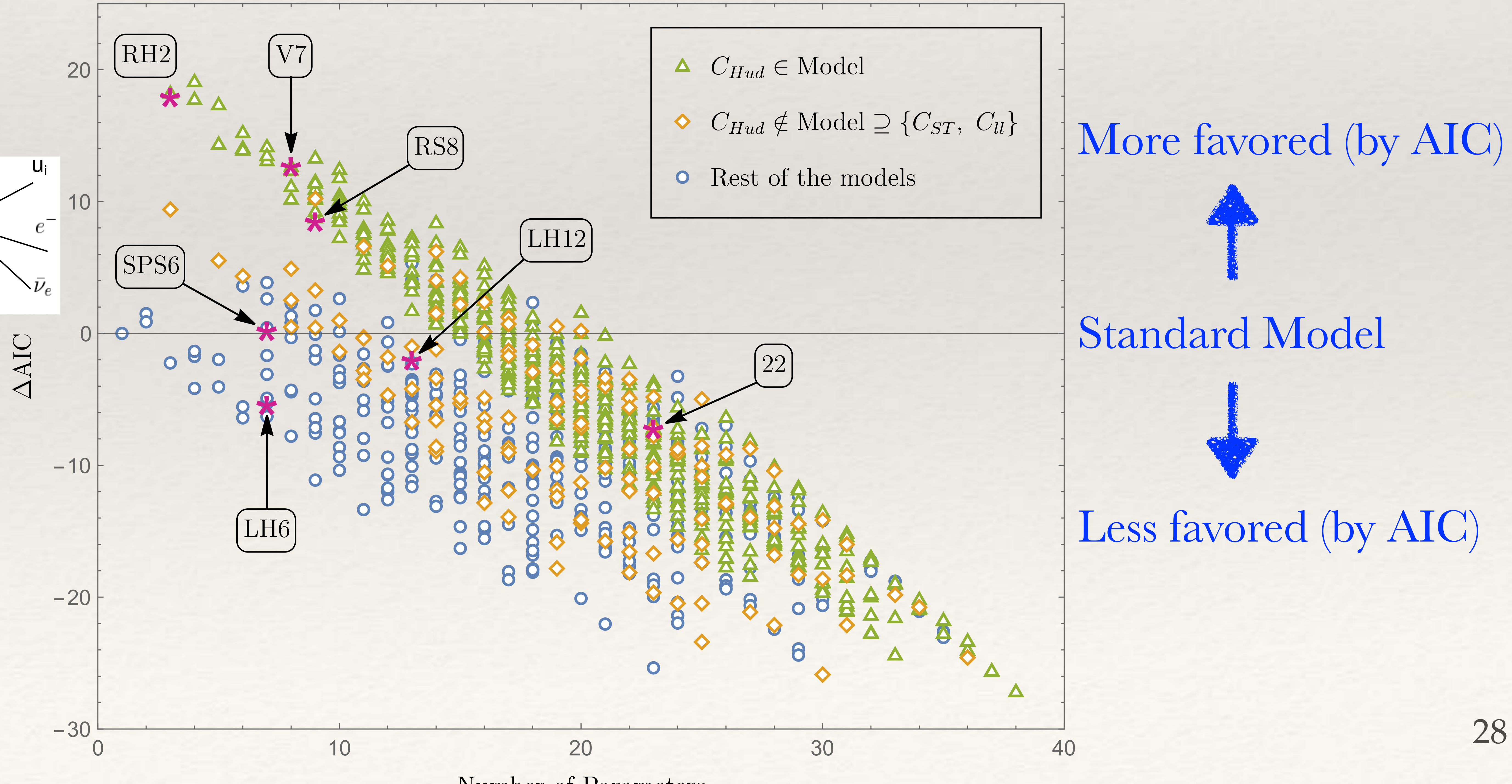
- "But you just told me there is an unknown O(2%) QED correction to g_A, so why should I care?"
  - Presumably, we will figure out how to determine this QED correction, which will allow us to utilize our high-precision iso-symmetric LQCD determination of g_A by applying the QED correction in a correlated way

Cirigliano, Crivellin, Hoferichter, Moulson
PLB 838 (2023) [2208.11707]





- Global analysis of first-row CKM constraints, including collider constraints, favors BSM Right-handed currents
  Cirigliano, Dekens, de Vries, Mereghetti, Tong, JHEP 03 (2024) [2311.00021]



Thank You