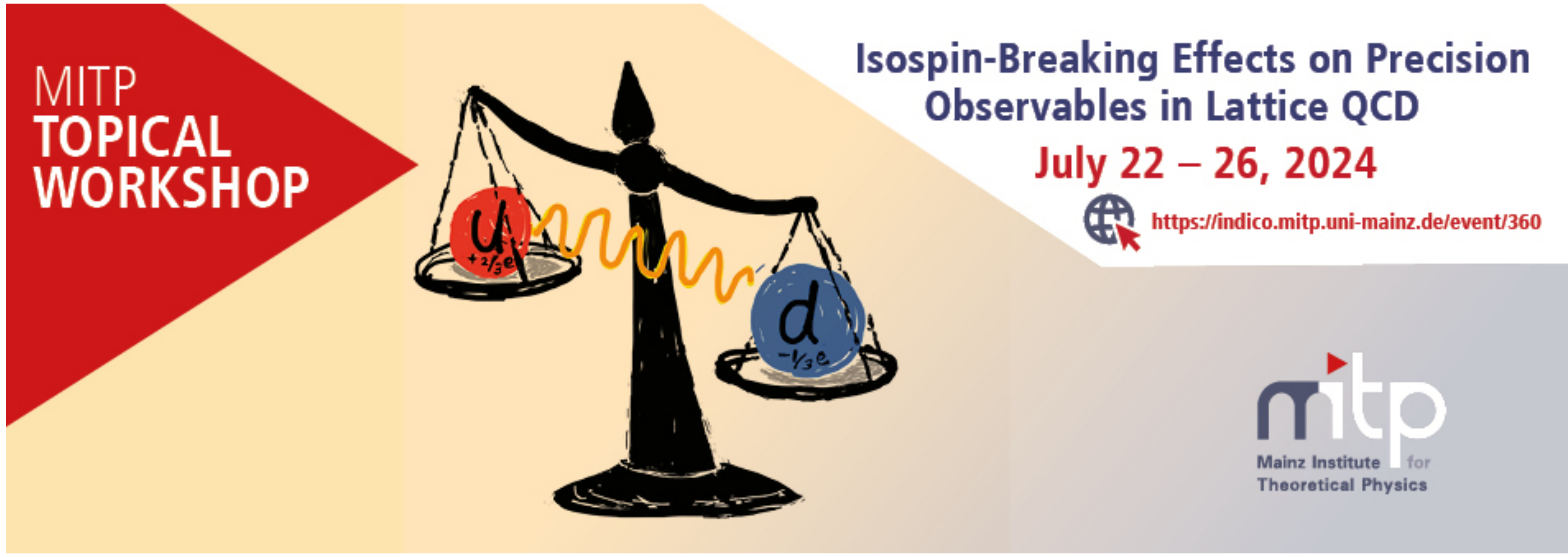
The Compton amplitude and towards electromagnetic mass shifts and νDIS

K. Utku Can

The University of Adelaide
(QCDSF/UKQCD/CSSM Collaboration)

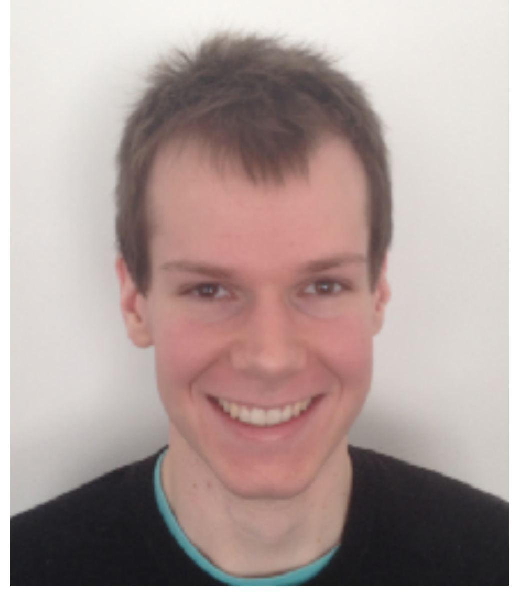

MITP
TOPICAL
WORKSHOP

Isospin-Breaking Effects on Precision Observables in Lattice QCD
July 22 – 26, 2024
https://indico.mitp.uni-mainz.de/event/360

CSSM/QCDSF/UKQCD Collaborations


Granada, Lattice 2017

R. Horsley (Edinburgh), Y. Nakamura (RIKEN, Kobe), H. Perlt (Leipzig), P. Rakow (Liverpool), G. Schierholz (DESY), H. Stüben (Hamburg), R. Young (Adelaide), **J. Zanotti (Adelaide)**


Alex Chambers
U.Adelaide
PhD 2018


Kim Somfleth
U.Adelaide
PhD 2020


Mischa Batelaan
U.Adelaide→W&M
PhD 2023


Alec Hannaford Gunn
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PhD 2023


Tomas Howson
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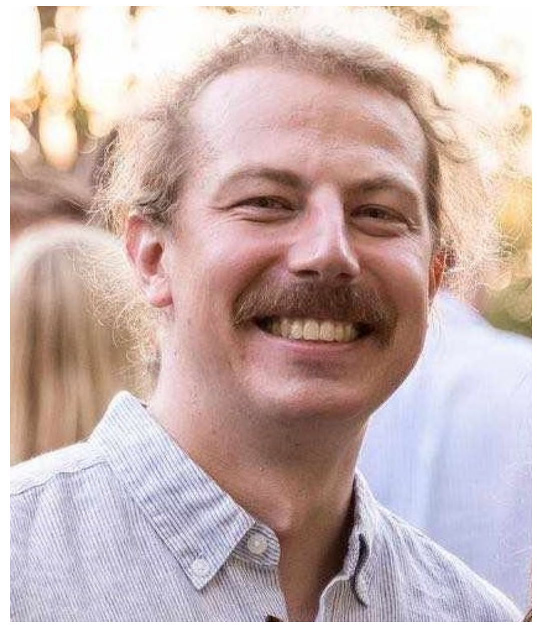
PhD 2023


Rose Smail
U.Adelaide
PhD 2024 (?)


Joshua Crawford
U.Adelaide
PhD ongoing

Motivation

- Nucleon structure (leading twist)
  - Parton distribution functions from first principles
  - Understanding the behaviour in the high- and low-x regions

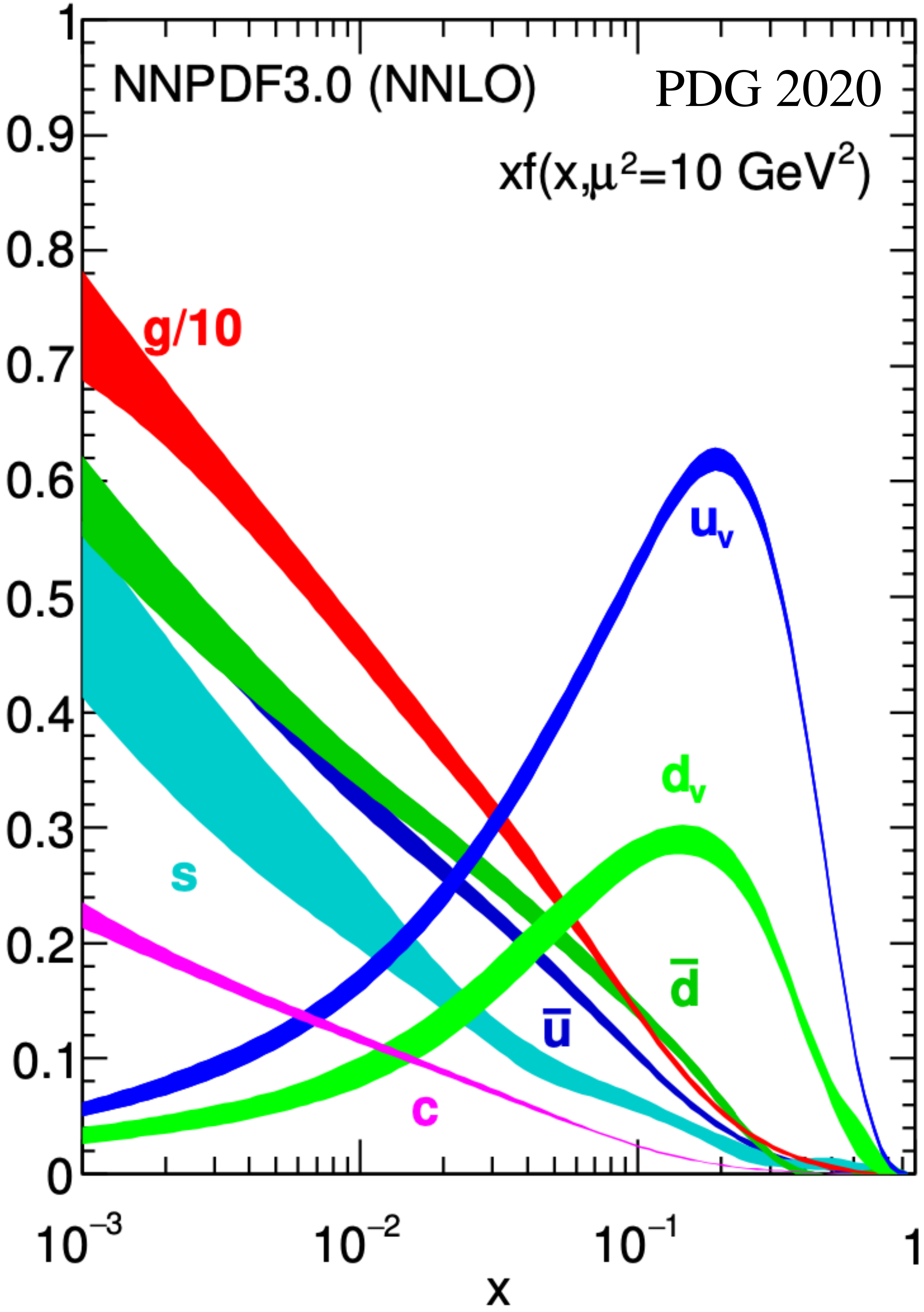
- Parton model

$$F_2 \propto (q + \bar{q})$$

$$F_3^{\gamma Z} \propto (q - \bar{q})$$

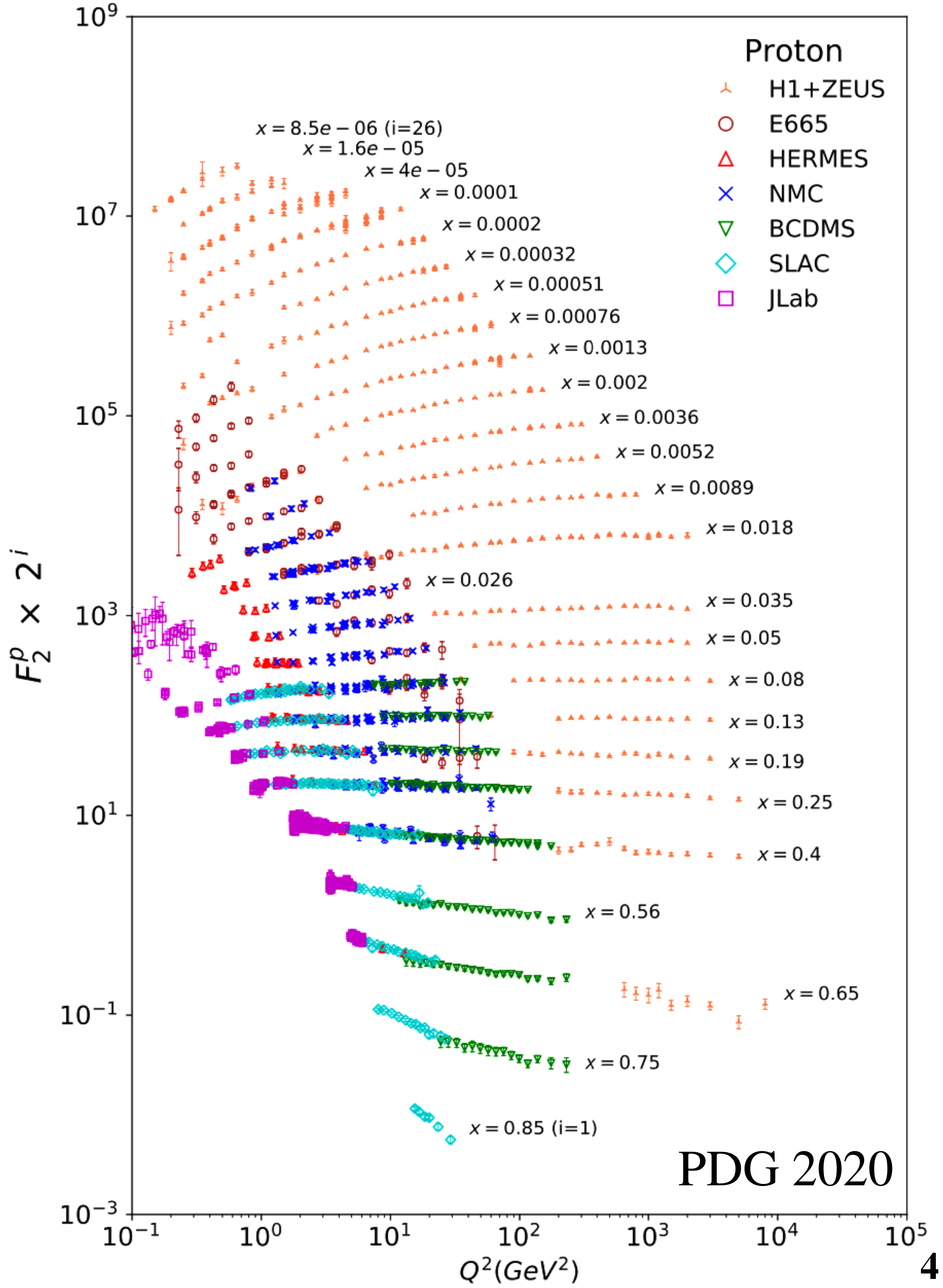
$$F_2^{W-} \propto u + \bar{d} + \bar{s} + c \dots$$

$$F_3^{W-} \propto u - \bar{d} - \bar{s} + c \dots$$



Motivation

- <u>Scaling</u>
  - Q^2 cuts of global QCD analyses
- <u>Power corrections / Higher twist effects</u>
  - Target mass corrections
  - Twist-4 contributions



PDG 2020

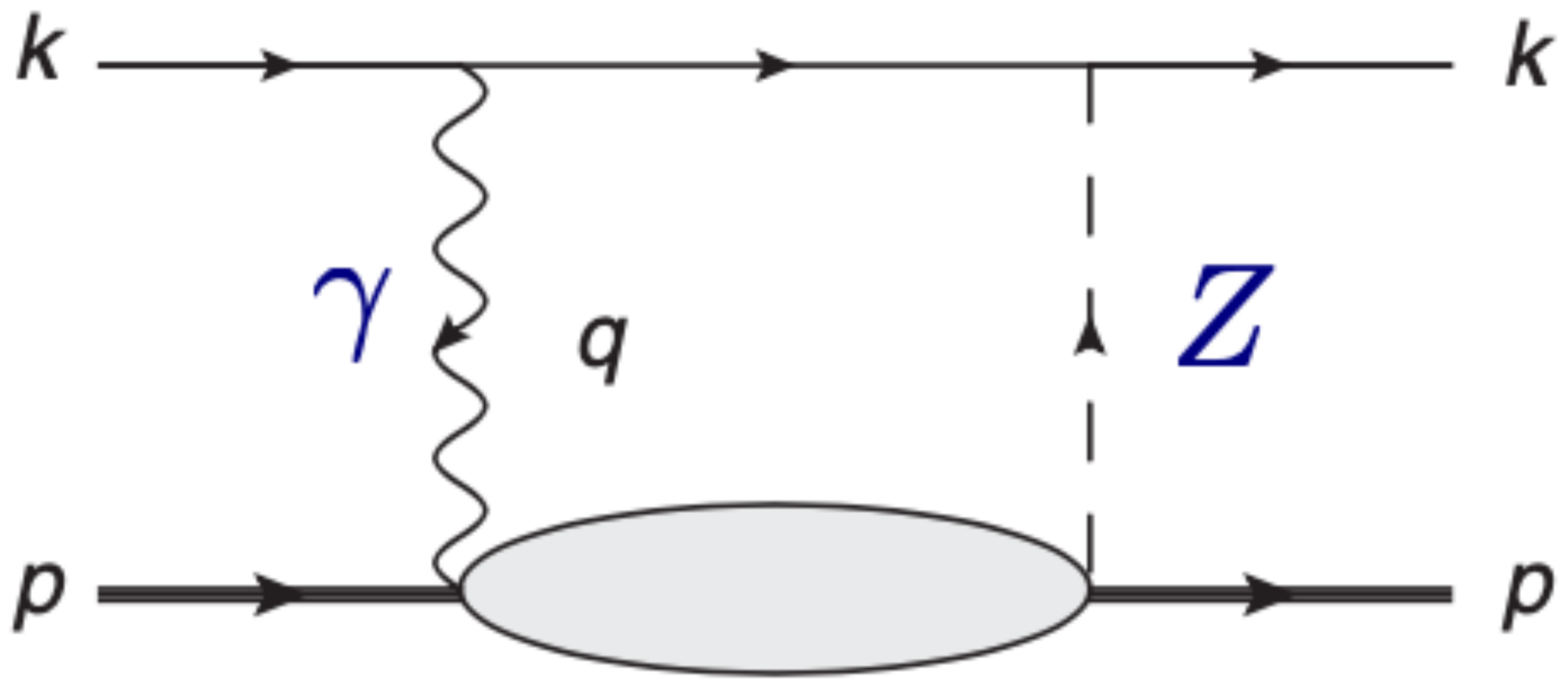
Motivation | EW Box

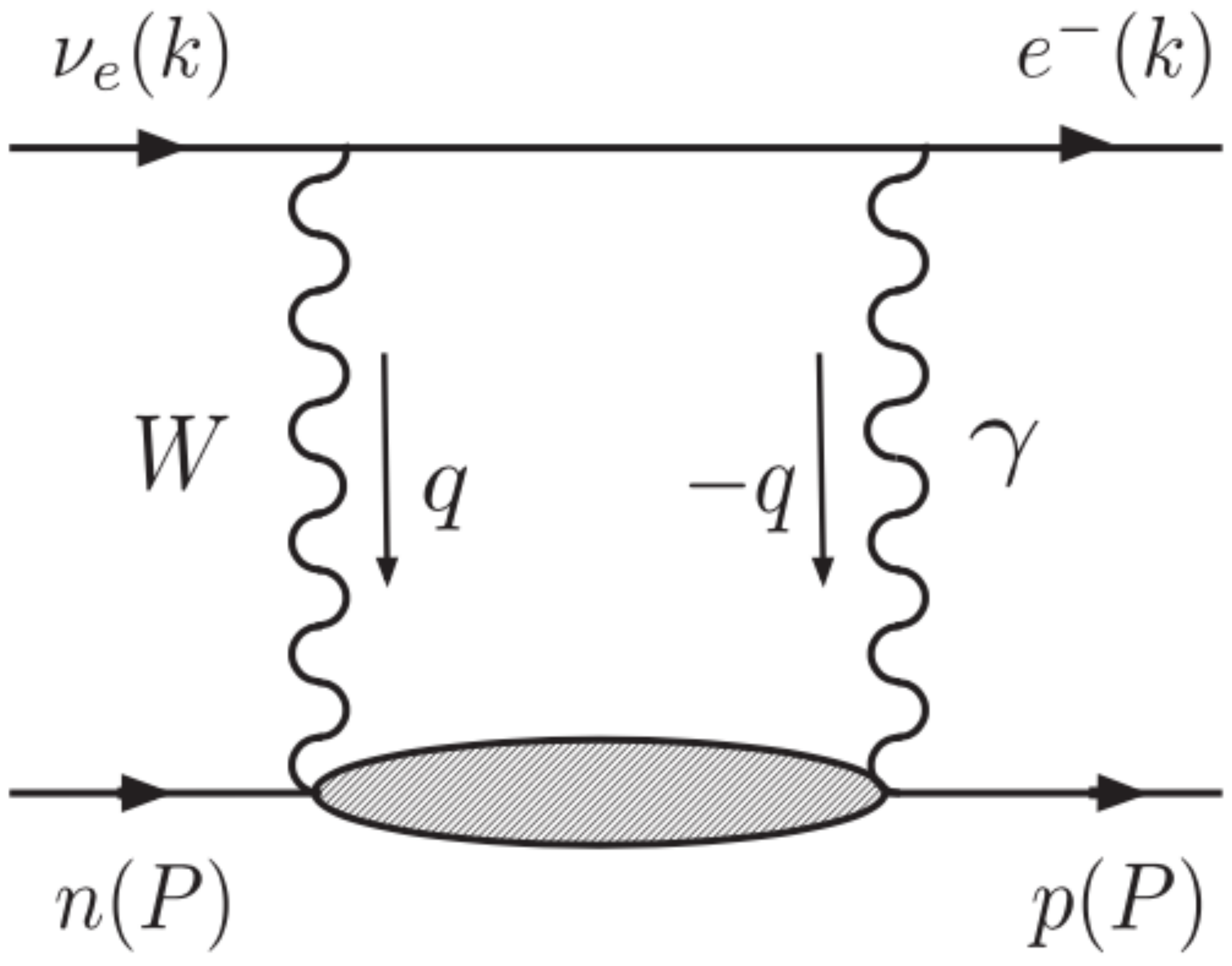
- Leading theoretical uncertainty in:
- Weak charge of the proton,

$$Q_W = (1 + \Delta_\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') + \Box^{WW}_{AA} + \Box^{ZZ}_{AA} + \Box^{\gamma Z}_{VA}$$

- CKM matrix element extracted from superallowed β decays,

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathscr{F}t(1+\Delta_R^V)} \longrightarrow \propto \Box^{\gamma W}_{VA}$$





Motivation | EW Box

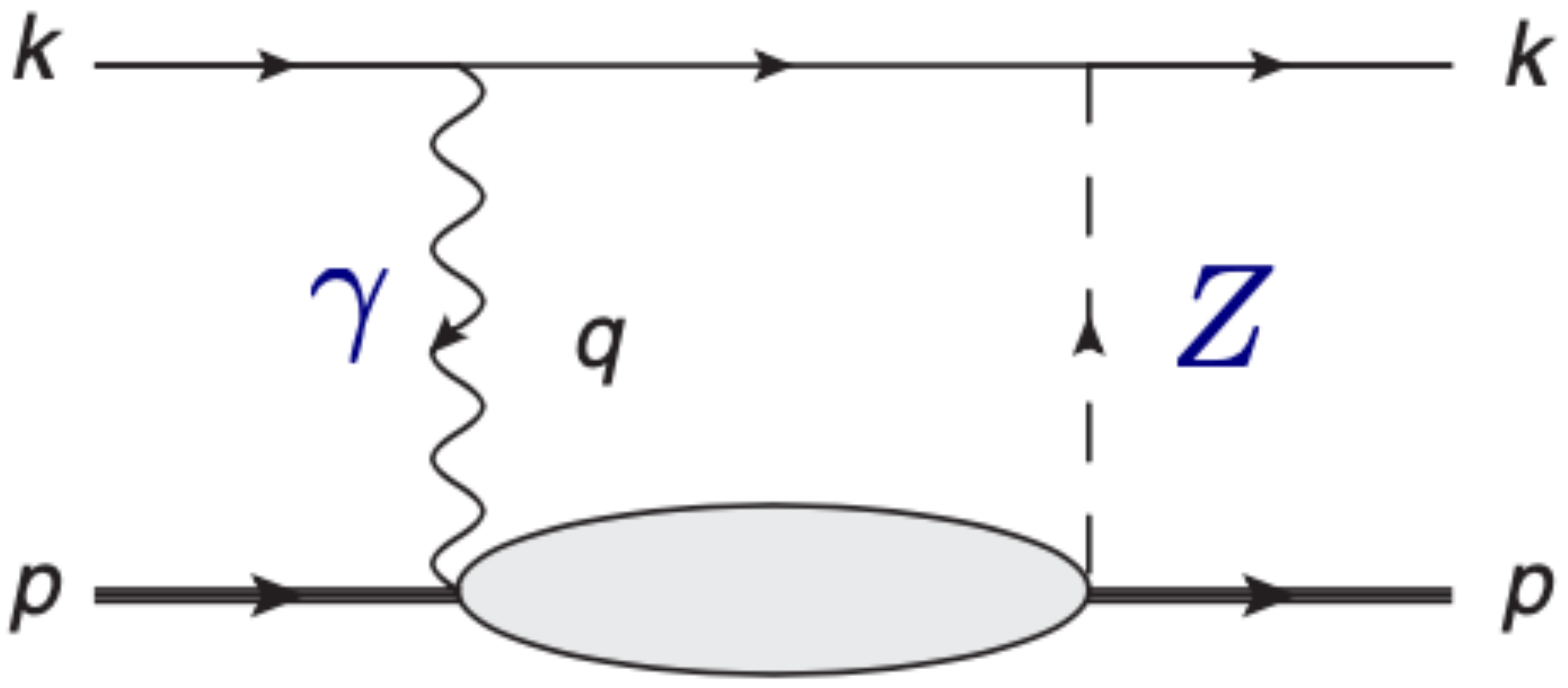
$$\Box_A^{\gamma Z} = \nu_e \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_Z^2}{M_Z^2 + Q^2} \underbrace{\int_0^1 dx\, C_N(x, Q^2)\, F_3^{\gamma Z}(x, Q^2)}$$

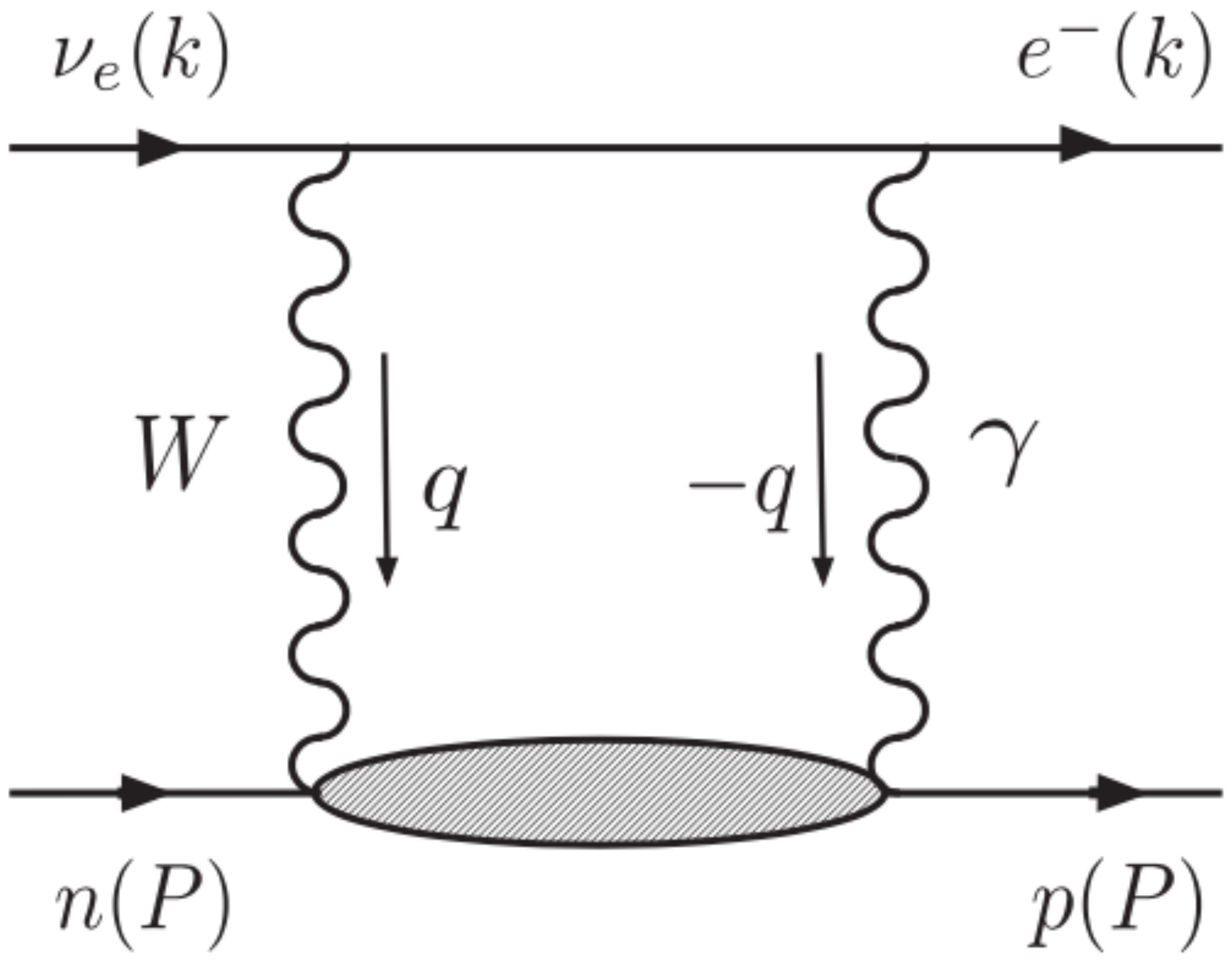
First Nachtmann moment of F_3

$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \overbrace{\int_0^1 dx\, C_N(x, Q^2)\, F_3^{(0)}(x, Q^2)}$$

$$F_3^{(0)} = F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z},$$

where $C_N(x, Q^2)$ is a known coefficient



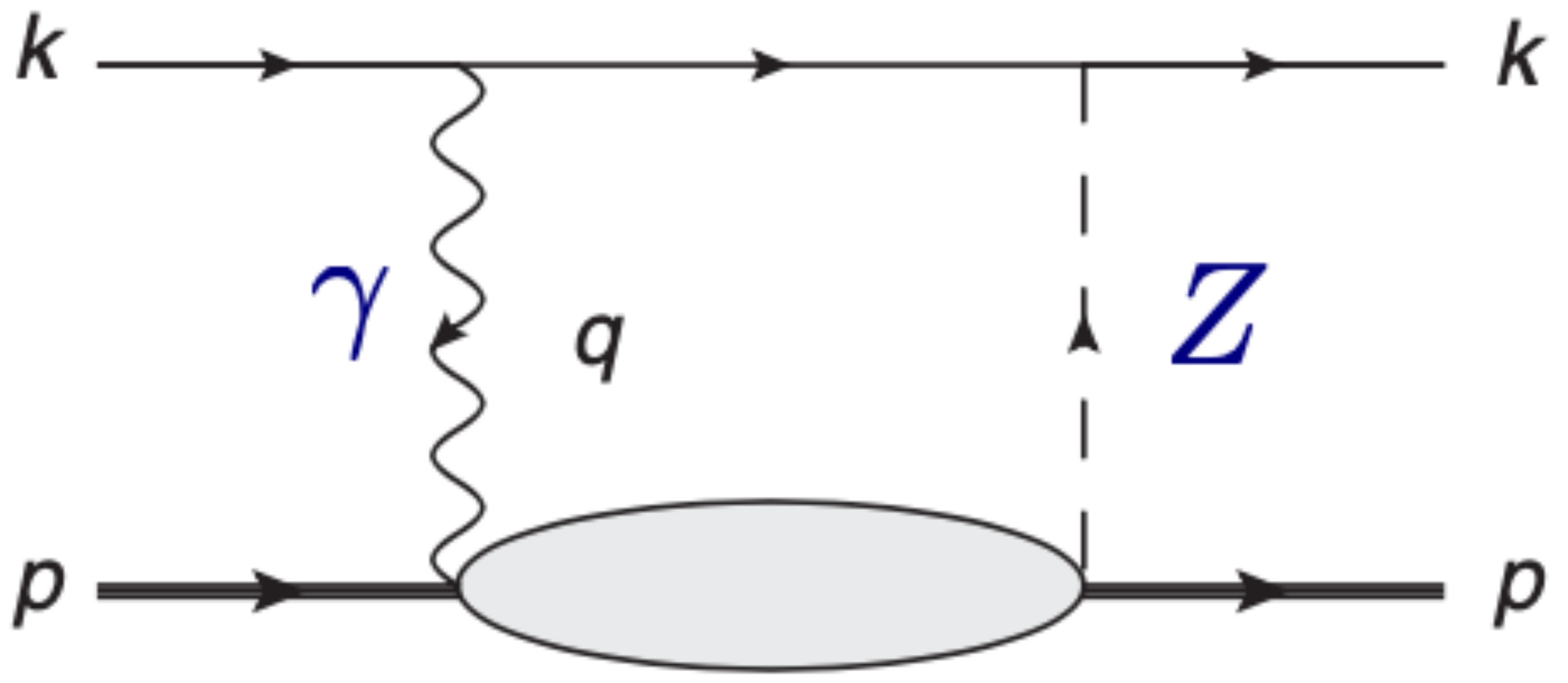


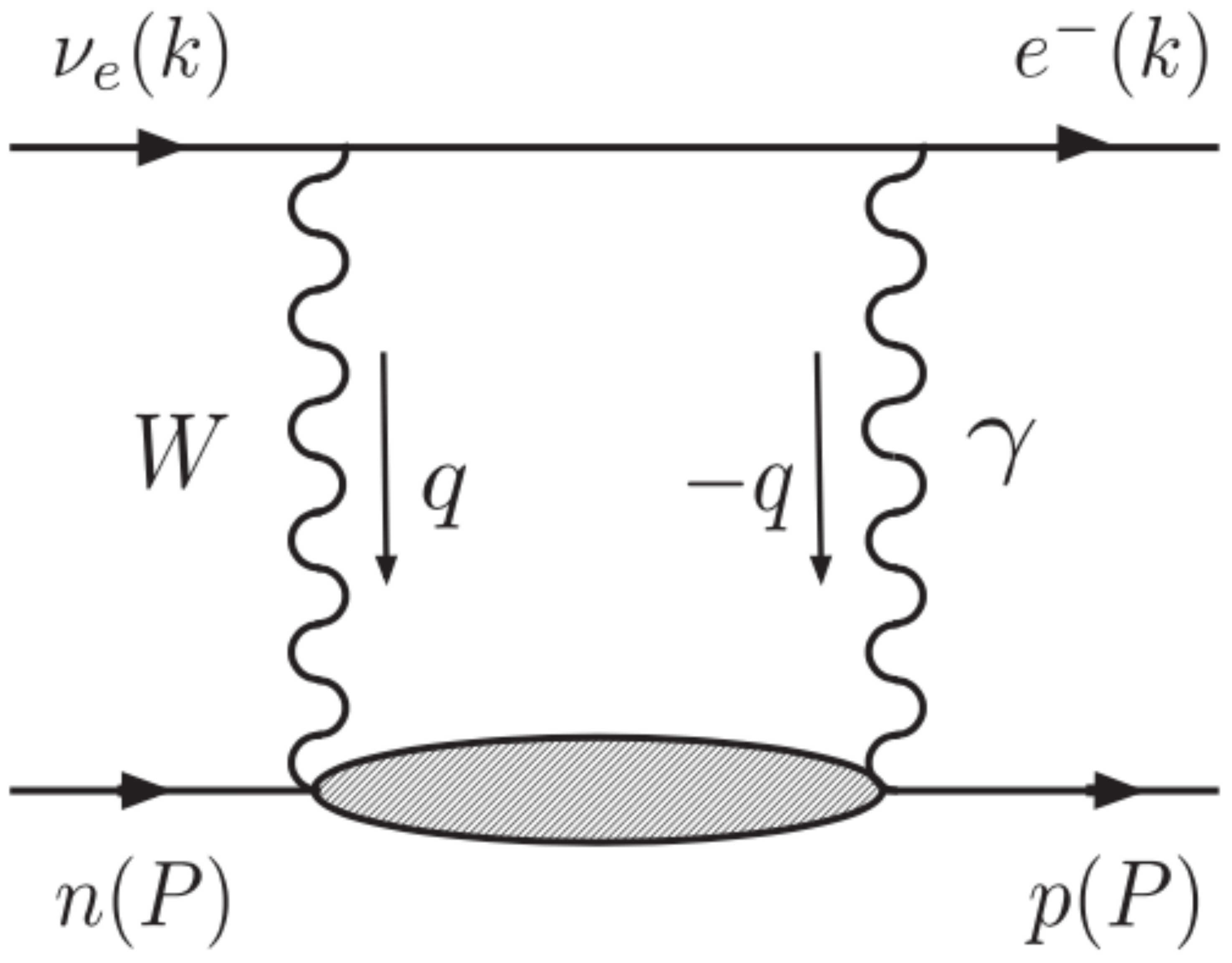
Motivation | EW Box

- Box diagrams proportional to an integral over the whole Q^2 range

$$\Box_A^{\gamma Z/W} \propto \int_0^\infty \frac{dQ^2}{Q^2}\,\mu_1^{(3)}(Q^2)\,(\ldots)$$

- Low-Q^2 (non-perturbative) regime dominates the integral
- F_3 is experimentally poorly determined in low Q^2
- Lattice approach is ideal for a high-precision determination of $\mu_1^{(3)}(Q^2)$ Nachtmann moment
- $\Box_A^{\gamma W}$ is ideal to study isospin breaking since $F_3^{(0)} = F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z}$

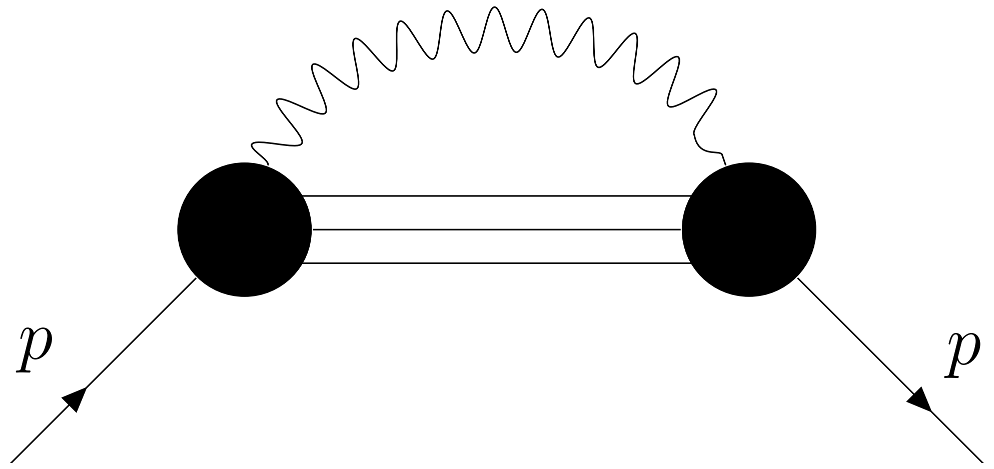




Motivation | Subtraction term

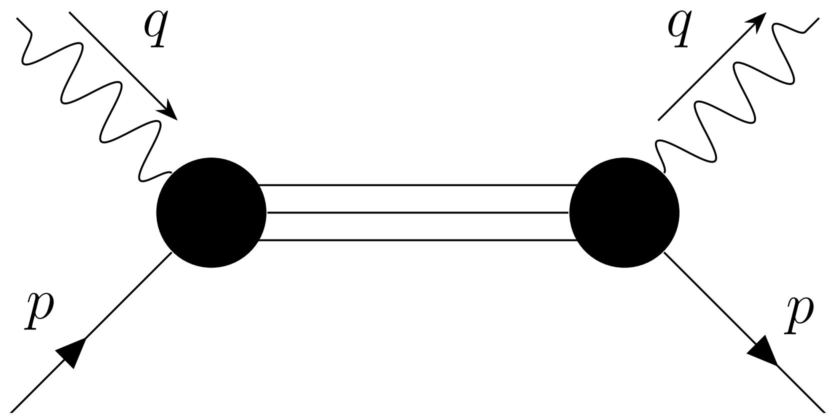
W.N. Cottingham, Annals Phys. 25, 424 (1963)
J. C. Collins, Nucl. Phys., B149:90–100, (1979)
[Erratum: Nucl. Phys.B915,392(2017)]
A. Walker-Loud, C. E. Carlson, G. A. Miller, PRL108, 232301 (2012)
A.W. Thomas, X.G. Wang, R.D. Young PRC91 (2015) 1, 015209

- **Cottingham formula:**



$$\delta M^{\gamma} = \delta M^{\rm el} + \delta M^{\rm inel} + \delta M^{\rm el}_{\rm sub} + \delta M^{\rm inel}_{\rm sub} + \delta \tilde{M}^{\rm ct}$$

$$\delta M^{\rm sub} \sim -\frac{3\alpha_{em}}{16\pi M}\int^{\Lambda_0^2} dQ^2 T_1^{p-n}(0,Q^2)$$

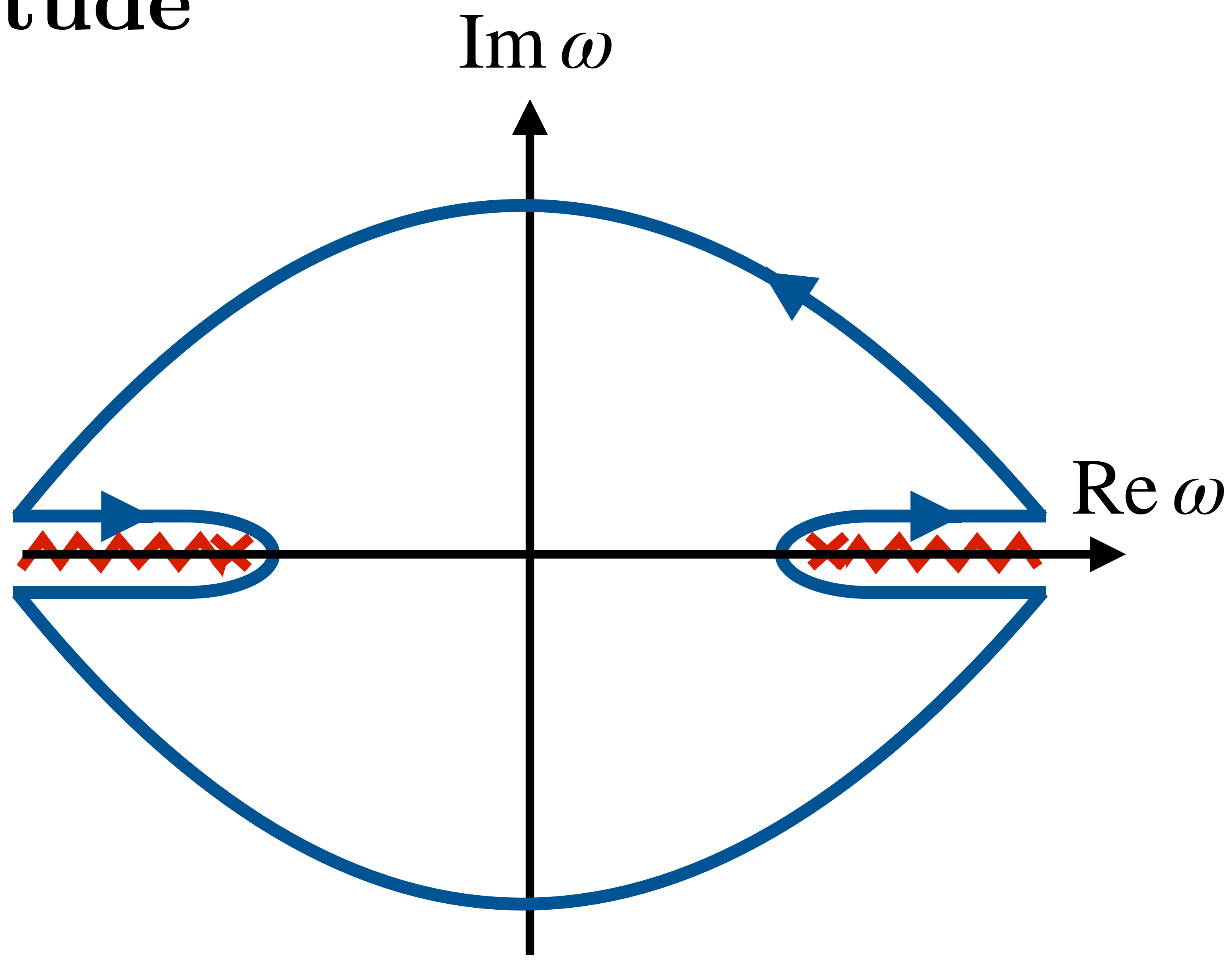


EM self energy is related to the spin-avg. forward Compton amplitude

- **Subtraction term $T_1(0,Q^2)$**

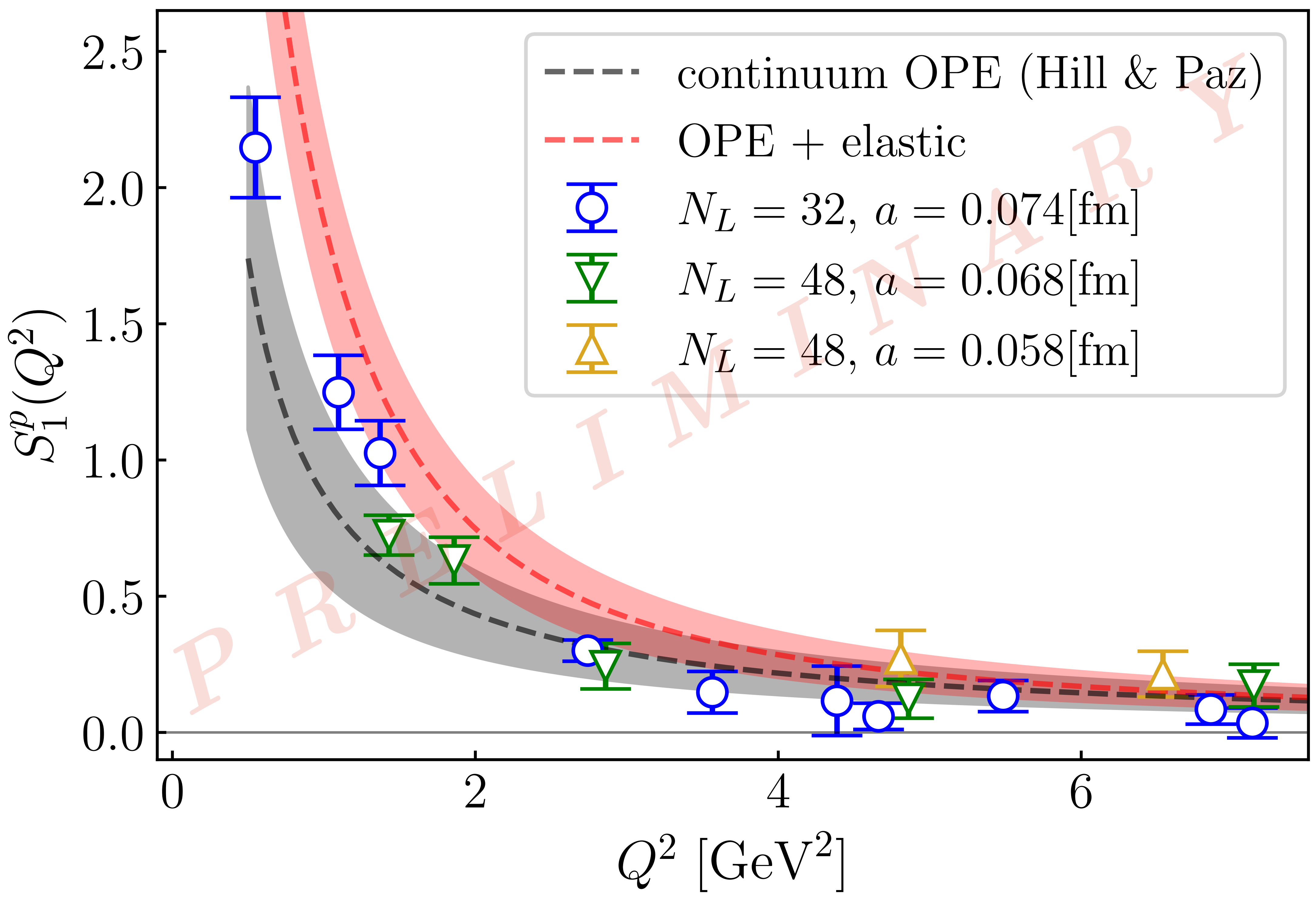
$$\mathcal{F}_1(\omega,Q^2) - \mathcal{F}_1(\omega=0,Q^2) = \frac{2\omega^2}{\pi}\int_1^{\infty} d\omega' \frac{\mathrm{Im}\,\mathcal{F}_1(\omega',Q^2)}{\omega'\left(\omega'^2-\omega^2-i\epsilon\right)}$$

  - dominant uncertainty
  - not accessible via experiments

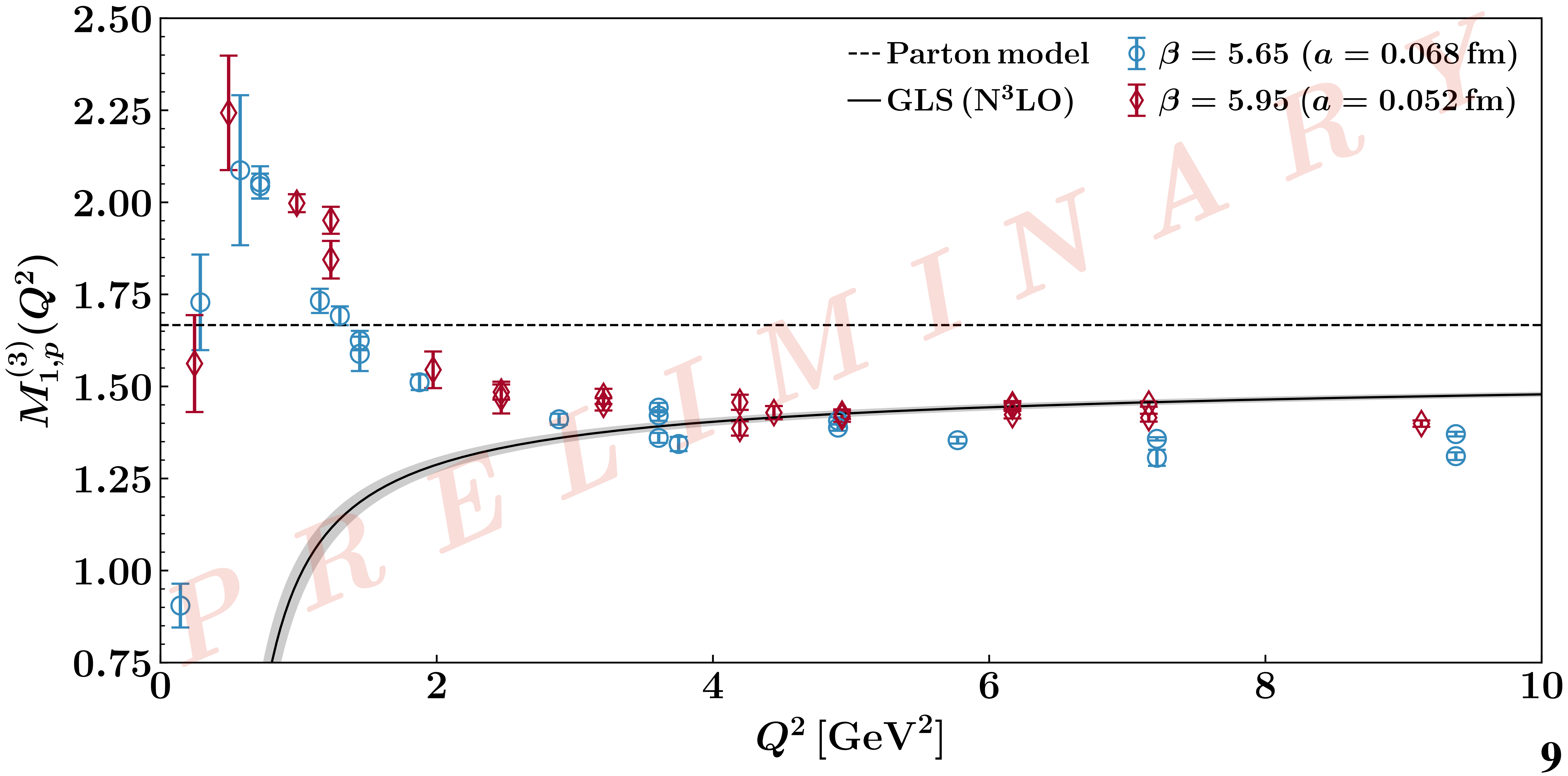


Highlights

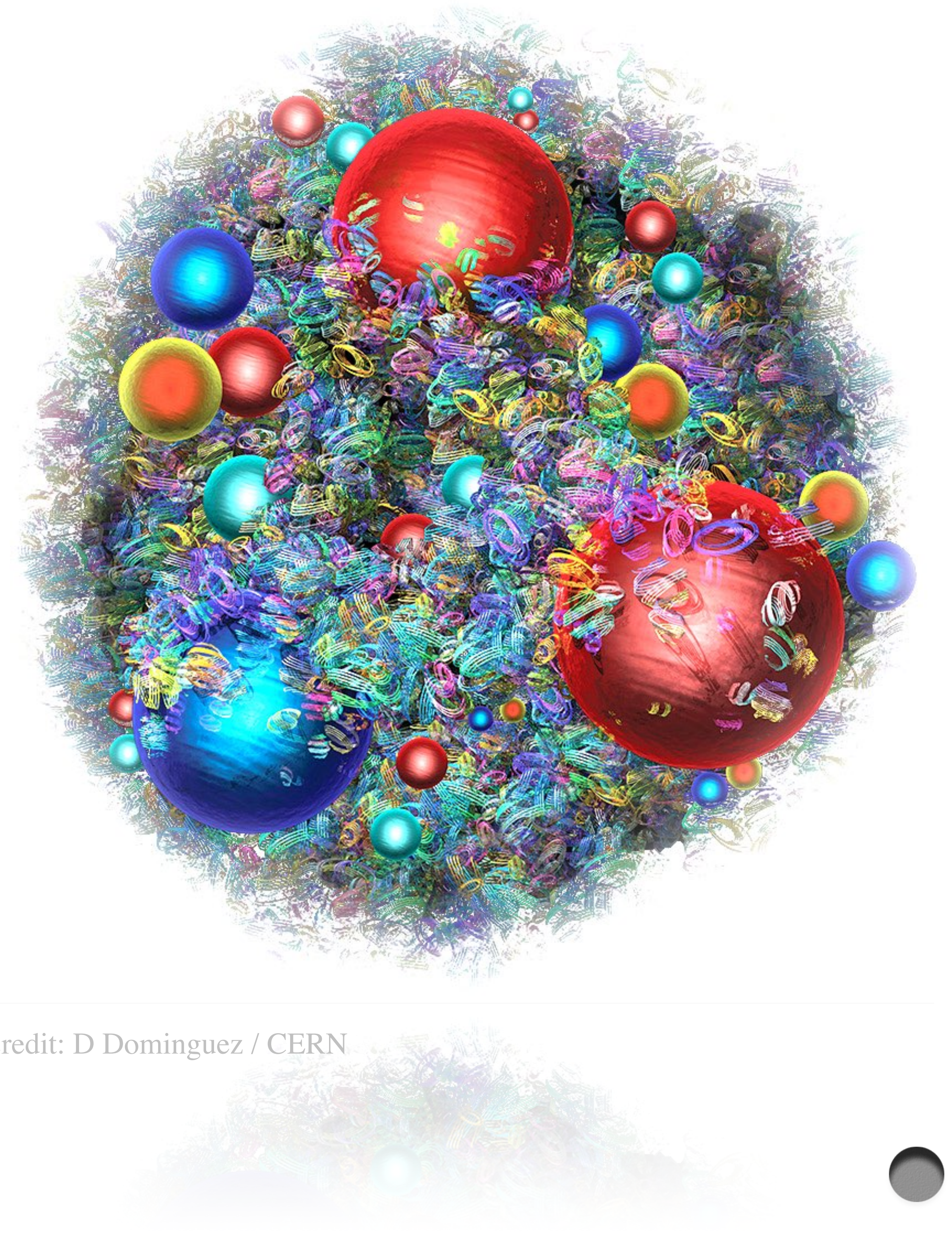
Subtraction function



Lowest odd moment of $F_3^{\gamma Z}(x, Q^2)$

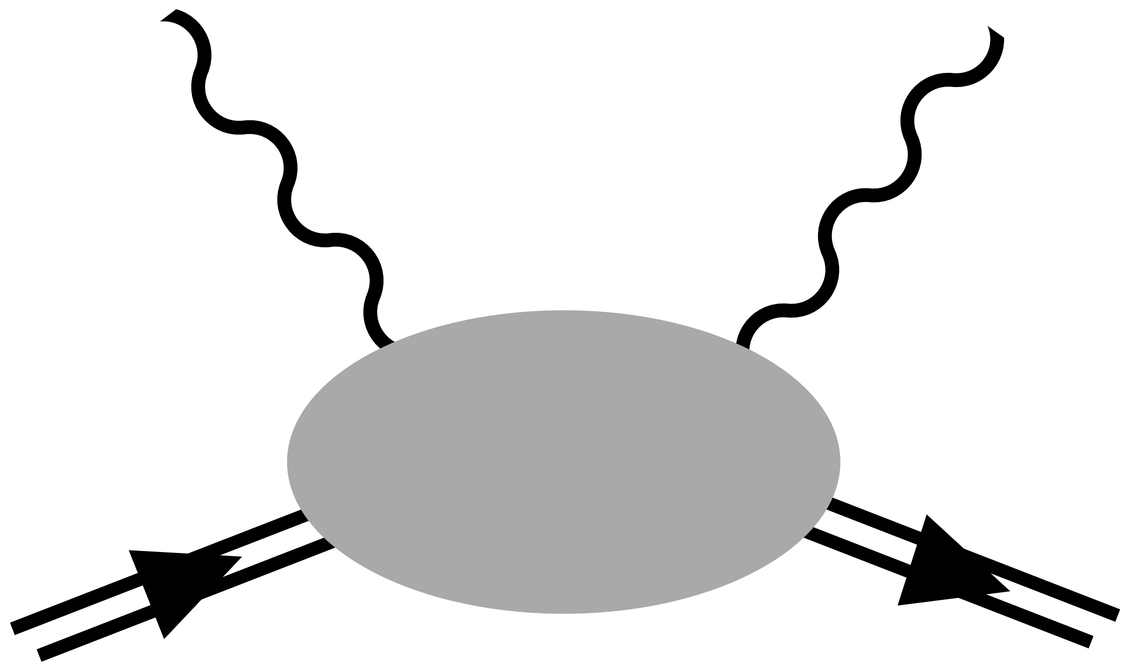


Outline


Credit: D Dominguez / CERN

- Forward Compton Amplitude
- Feynman-Hellmann Theorem on the Lattice
- F_1 subtraction function
- Parity-violating F_3
- Summary & Outlook

Forward Compton Amplitude



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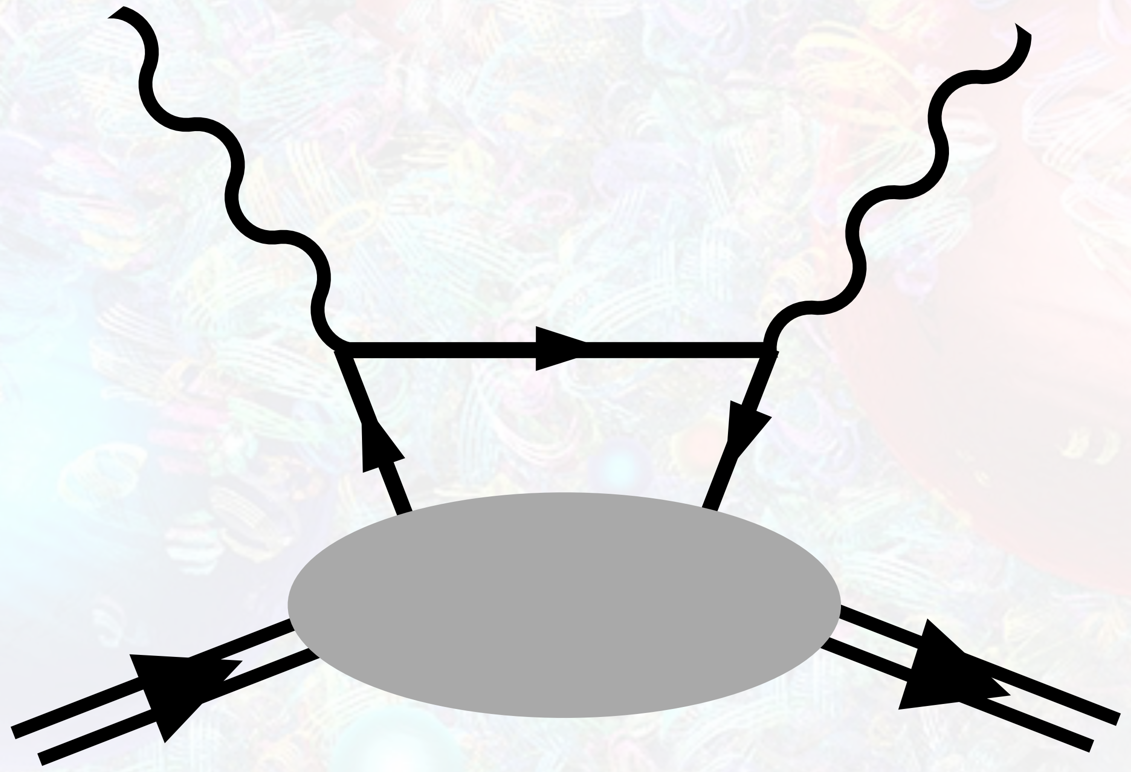
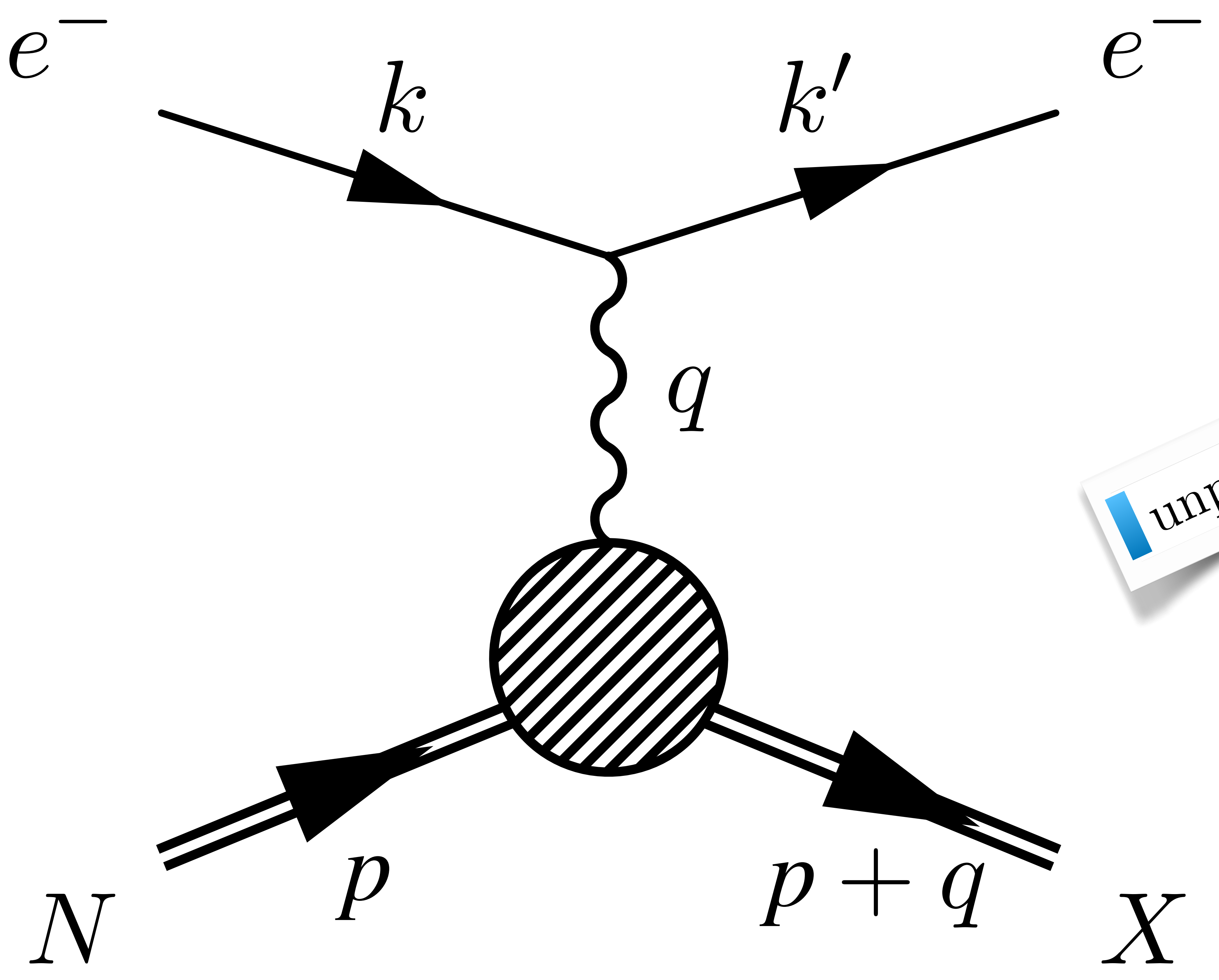


$$+ \ \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{1}{Q^2}\right)$$

DIS and the Hadronic Tensor

Deep ($Q^2 \gg M^2$) inelastic ($W^2 \gg M^2$) scattering (DIS)



$$d\sigma \sim L_j^{\mu\nu} W_{\mu\nu}^j$$

$j = \gamma, Z$, and γZ (neutral) or W (charged)

leptonic tensor **hadronic tensor**

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq\cdot z} \rho_{ss'} \langle p, s' | [J_\mu(z), J_\nu(0)] | p, s\rangle$$

$$\rho_{ss'} = \tfrac{1}{2}\delta_{ss'}$$

unpolarised

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) \frac{F_2(x,Q^2)}{p\cdot q}$$

Structure Functions

Forward Compton Amplitude

$$\omega = \frac{2p\cdot q}{Q^2}$$

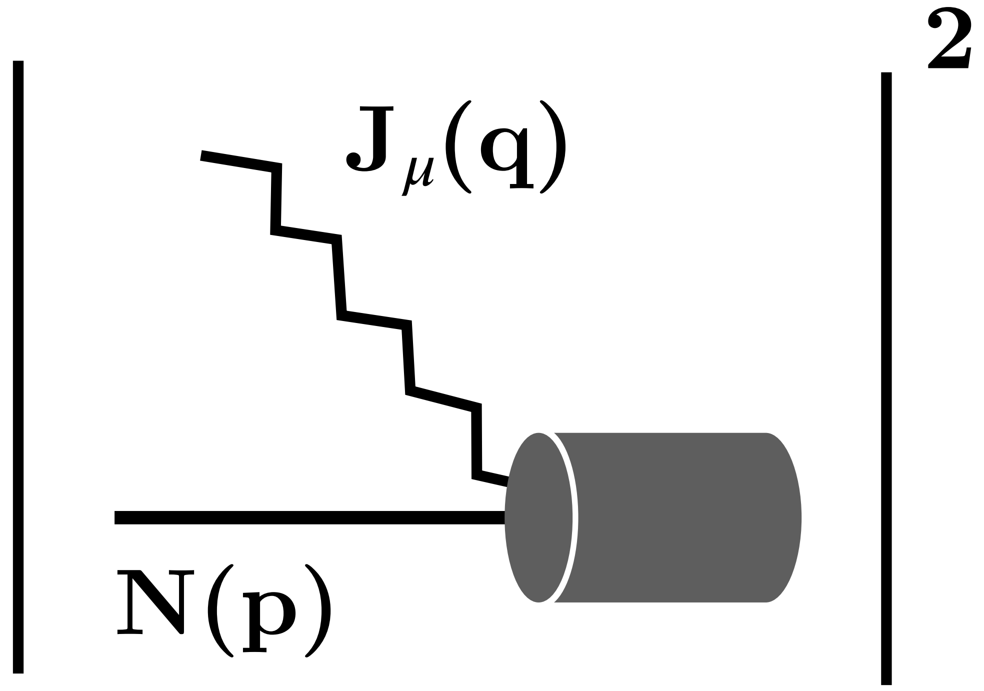
$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\,\mathscr{T}\{J_\mu(z)J_\nu(0)\}\,|p,s\rangle \quad \text{, spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

Same Lorentz decomposition as the Hadronic Tensor

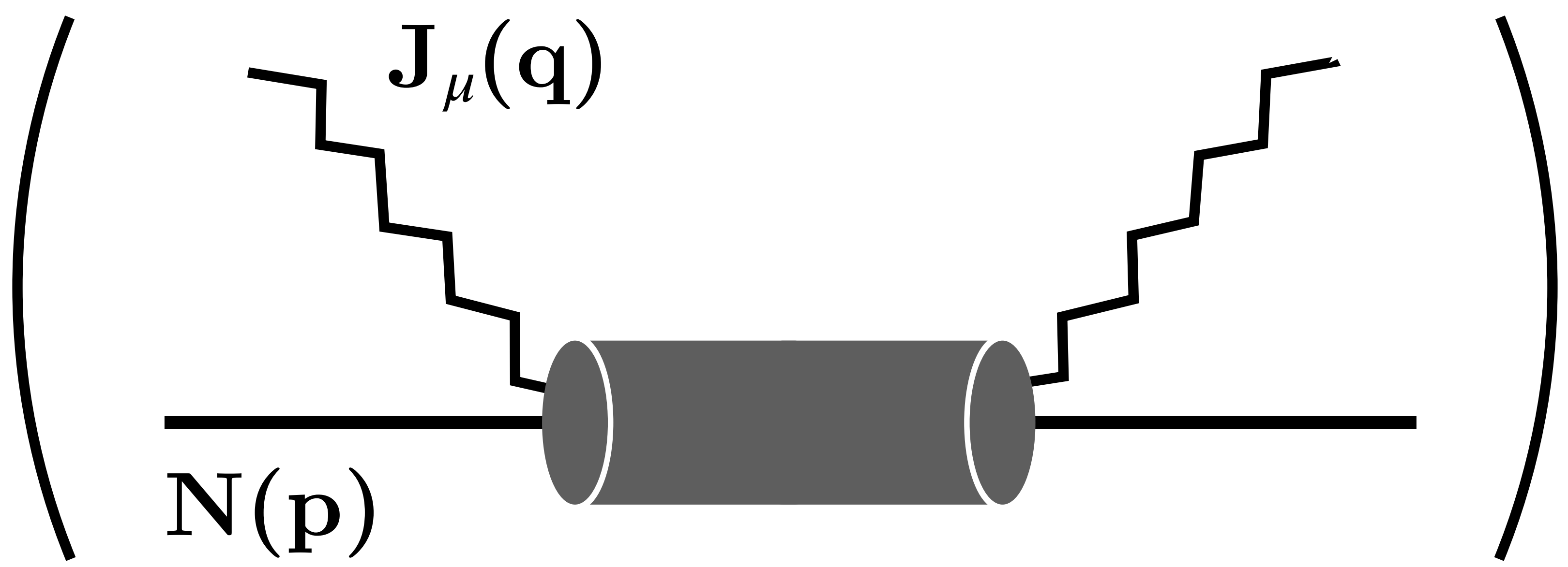
$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Optical theorem



$$\sim \quad 2\,\mathrm{Im}$$



$$W_{\mu\nu} \sim \int d^4x\langle p|\,[J_\mu(x), J_\nu(0)]\,|p\rangle$$

Structure Functions: $F_{1,2}(x,Q^2)$

$$T_{\mu\nu} \sim \int d^4x\langle p|\,T\{J_\mu(x)\,J_\nu(0)\}\,|p\rangle$$

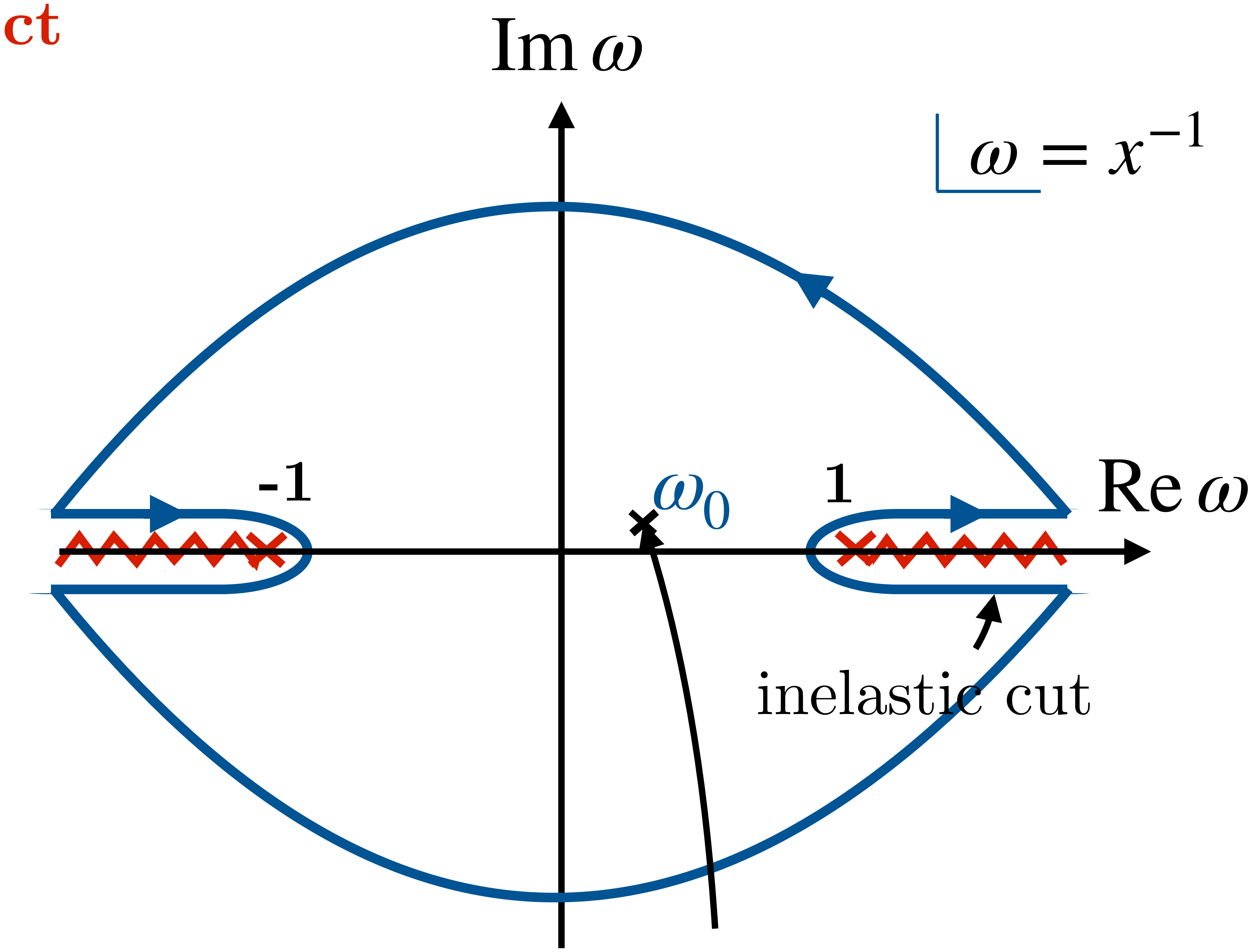
Compton Structure Functions: $\mathscr{F}_{1,2}(p\cdot q, Q^2)$

Nucleon Structure Functions

- **we can write down a dispersion relation and connect Compton SFs to DIS SFs:**

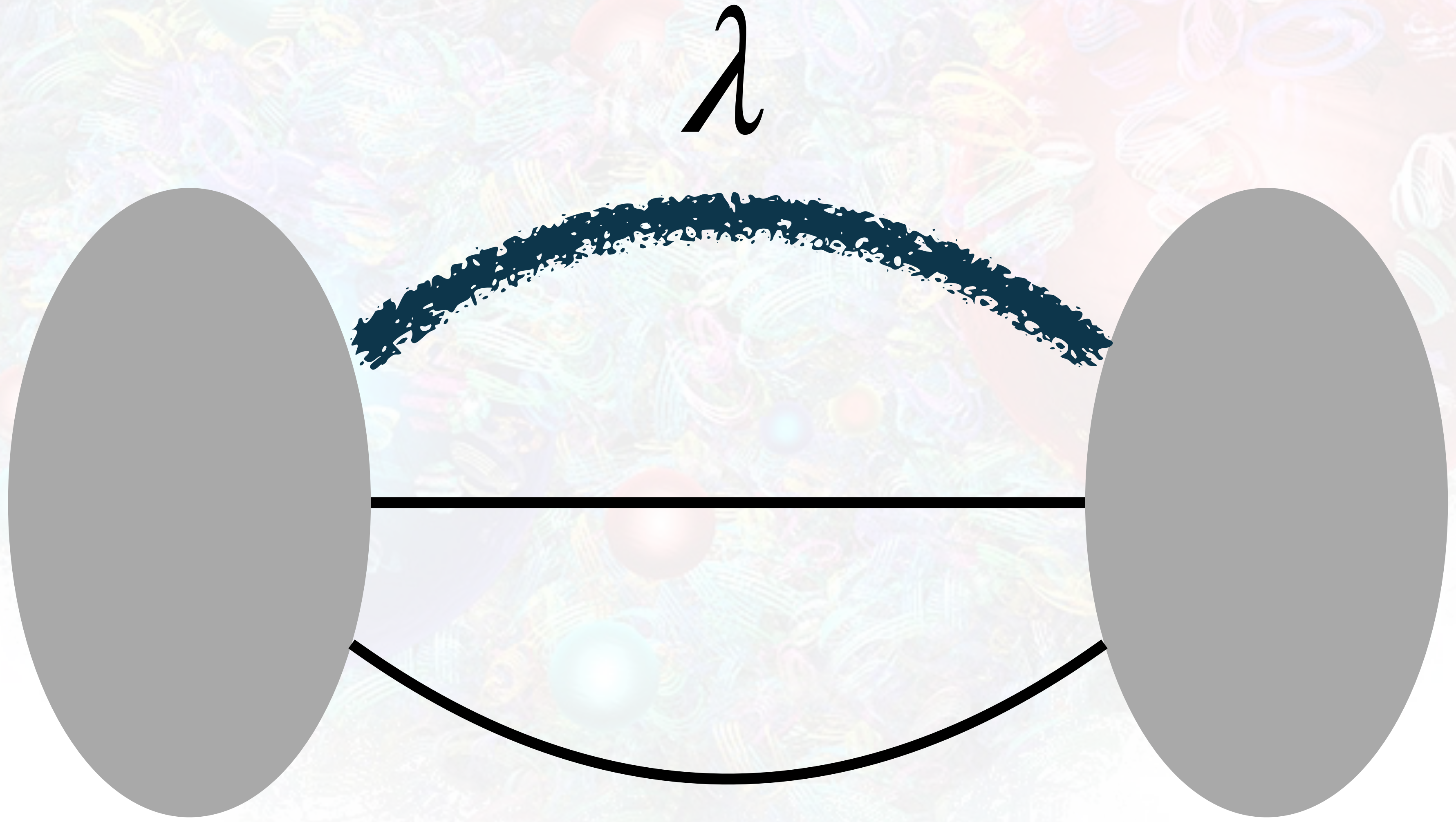
$$\mathscr{F}_1(\omega, Q^2) = \mathscr{F}_1(0,Q^2) + 2\omega^2 \int_0^1 dx \frac{2x\, F_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

$$\mathscr{F}_2(\omega, Q^2) = 4\omega \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Feynman-Hellmann Theorem on the Lattice



FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ: perturbed Hamiltonian of the system

E_λ: energy eigenvalue of the perturbed system

ϕ_λ: eigenfunction of the perturbed system

- **expectation value of the perturbed system is related to the shift in the energy eigenvalue**

in Lattice QCD: energy shifts in the presence of a weak external field

e.g. local bilinear operator

$$S \rightarrow S(\lambda) = S + \underset{\nwarrow}{\lambda} \int d^4x\, \mathcal{O}(x) \longrightarrow \bar{q}(x)\Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \ldots\}$$

real parameter

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda}\langle 0 | \mathcal{O} | 0 \rangle$$

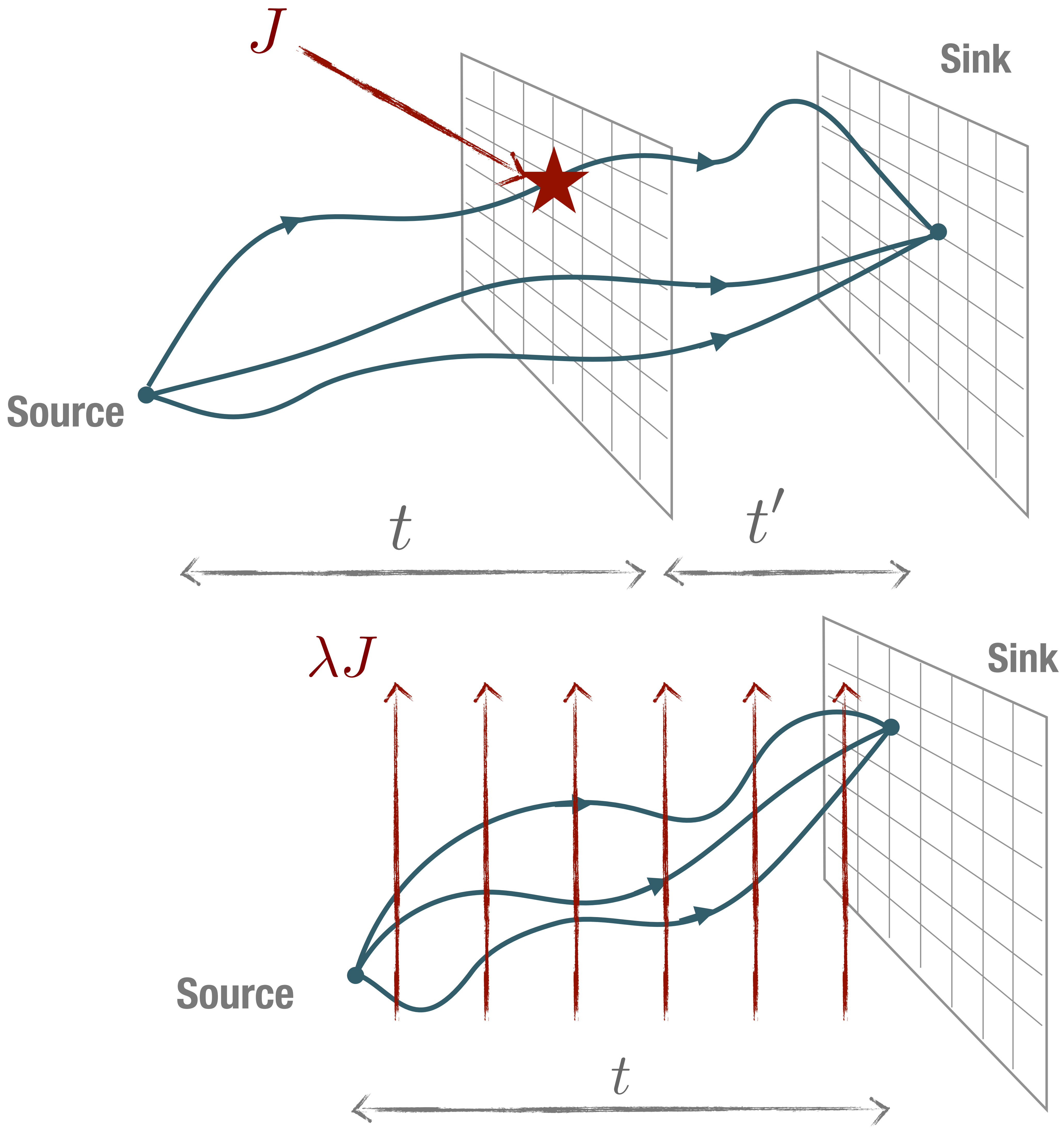
E_λ → **spectroscopy, 2-pt function**

$\langle 0 | \mathcal{O} | 0 \rangle$ → **determine 3-pt**

Applications:

- σ **- terms**
- **Form factors**

Matrix elements



- **3-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

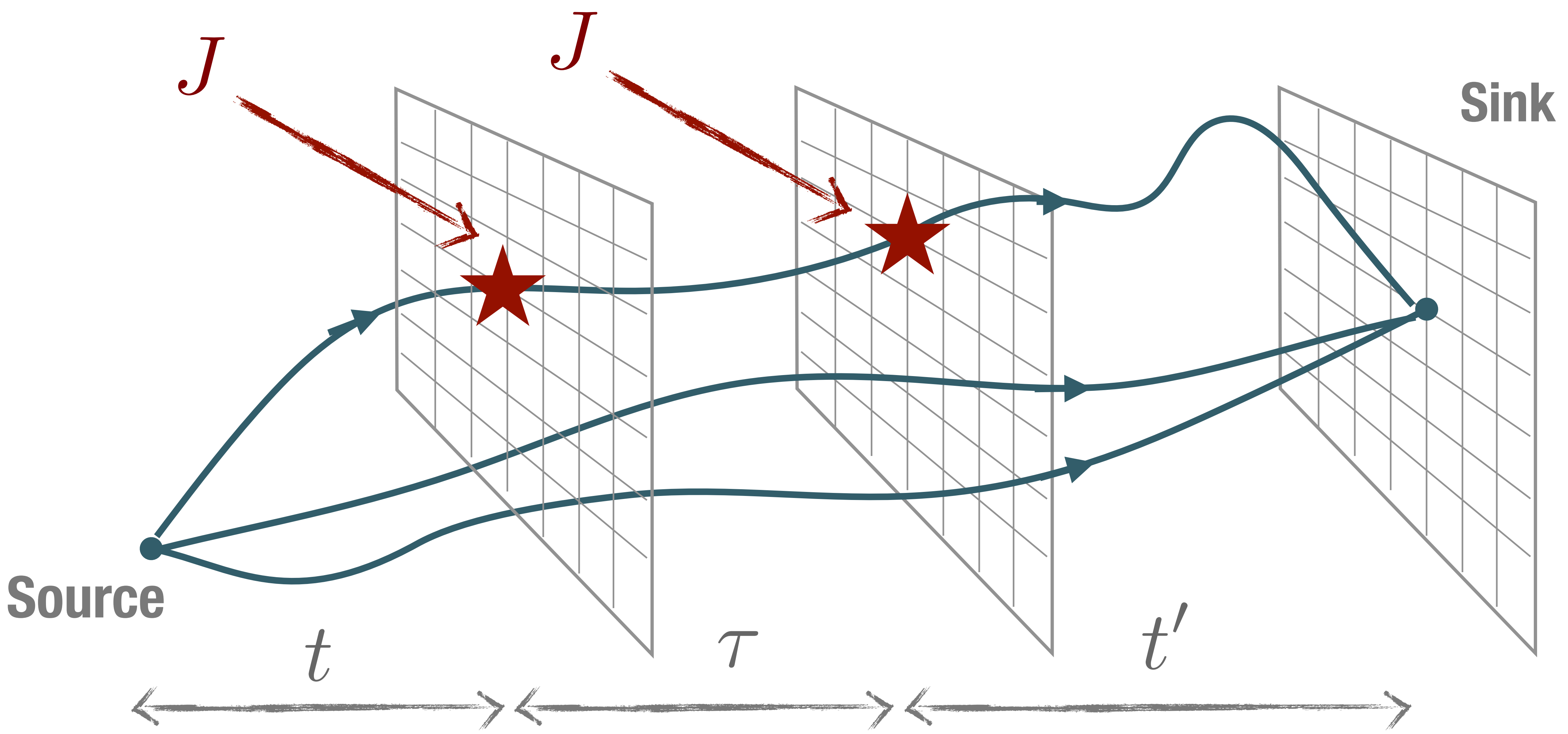
energy gap to the lowest excitation

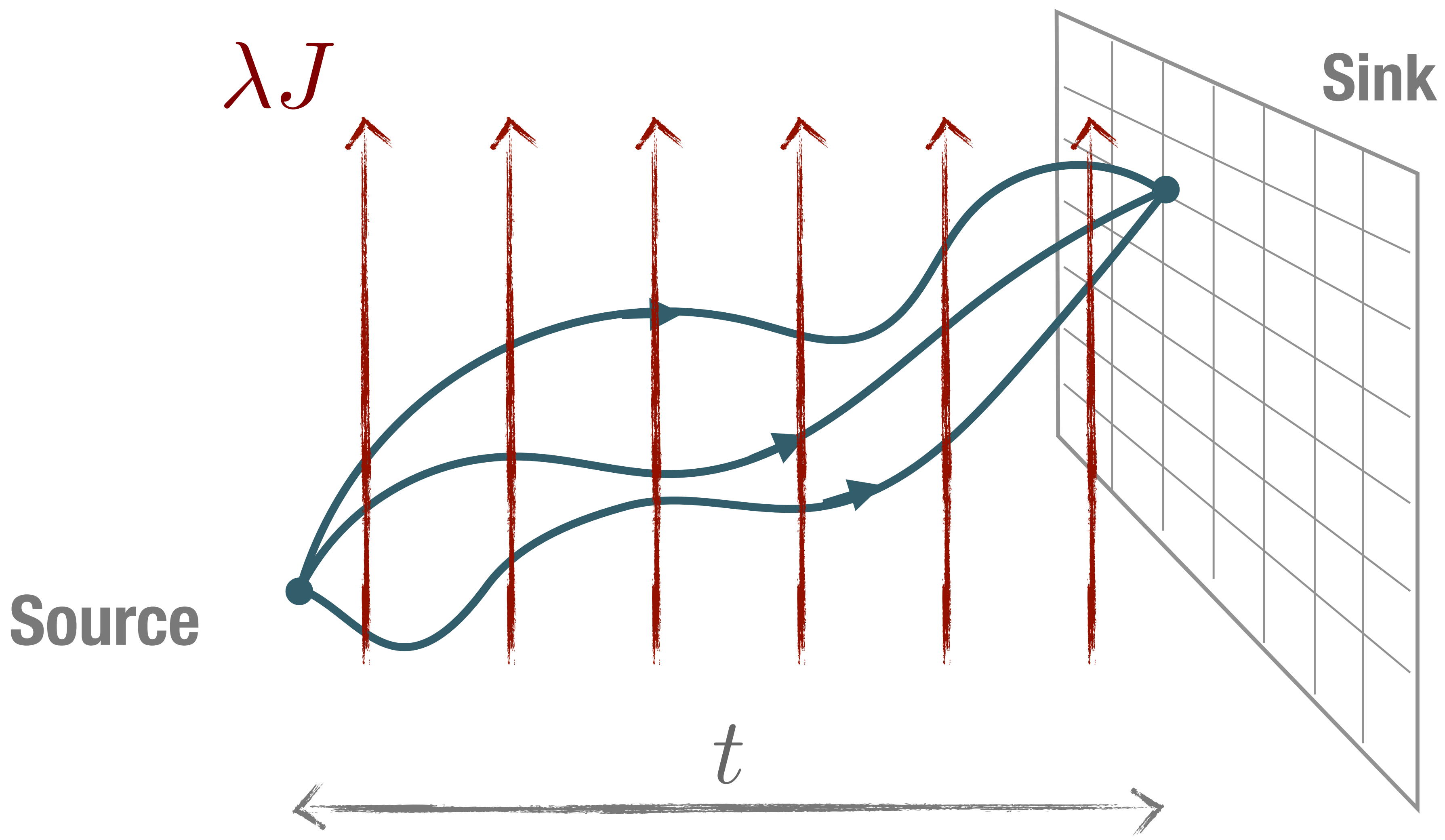
- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}$$

$$\left.\frac{\partial E}{\partial \lambda}\right|_{\lambda\to 0} \propto \langle N|J|N\rangle$$

Compton amplitude





- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N|J(\tau_E)J|N\rangle$$

$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$

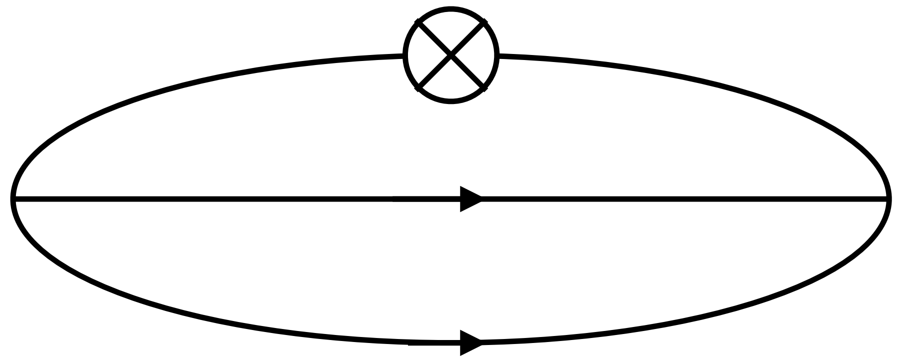
- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}, \qquad \left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda\to 0} \propto \langle N|JJ|N\rangle$$

QCDSF Applications of FH

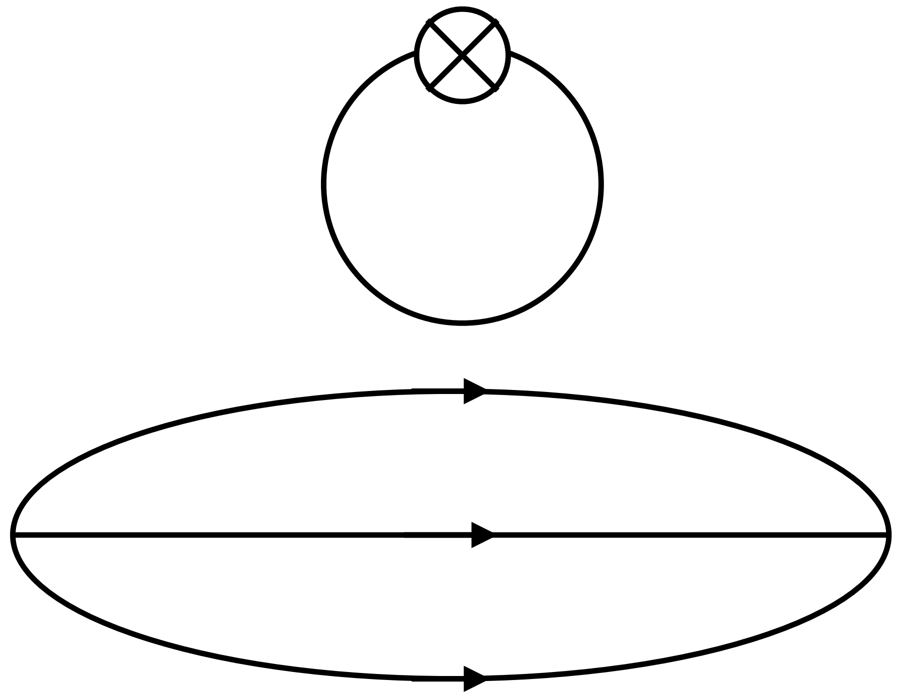
- Can modify fermion action in 2 places:

◉ quark propagators



Connected

◉ fermion determinant



Disconnected

(Requires new gauge configurations)

g_A, $\Delta\Sigma$ [PRD90 (2014)]

NPR [PLB740 (2015)]

G_E, G_M [PRD96 (2017)]

$F_{1,2}(\omega, Q^2)$ [PRL118 (2017), PRD102 (2020), PRD107 (2023)]

GPDs [PRD104 (2022), PRDXYZ (2024)]

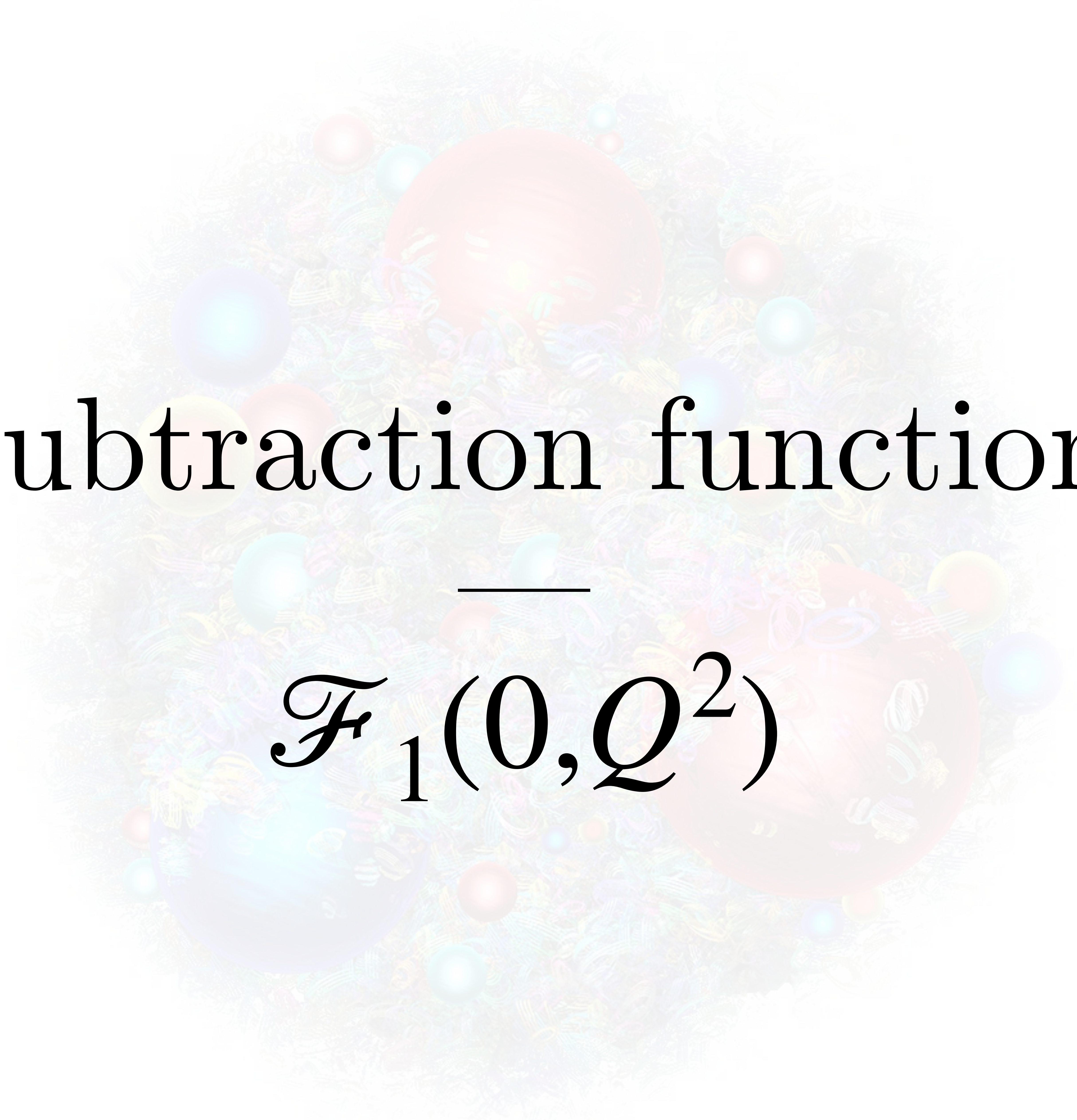
$\Sigma \to n$ [PRD108 (2023) 3, 034507]

g_A, g_T, g_S [PRD108 (2023) 9, 094511]

$\langle x \rangle_g$ [PLB714 (2012)]

NPR [PLB740 (2015)]

Δs [PRD92 (2015)]



Subtraction function

$$\mathscr{F}_1(0,Q^2)$$

Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle \quad \text{, spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

$\omega = \dfrac{2p\cdot q}{Q^2}$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Simplest kinematics to directly isolate $\mathscr{F}_1$

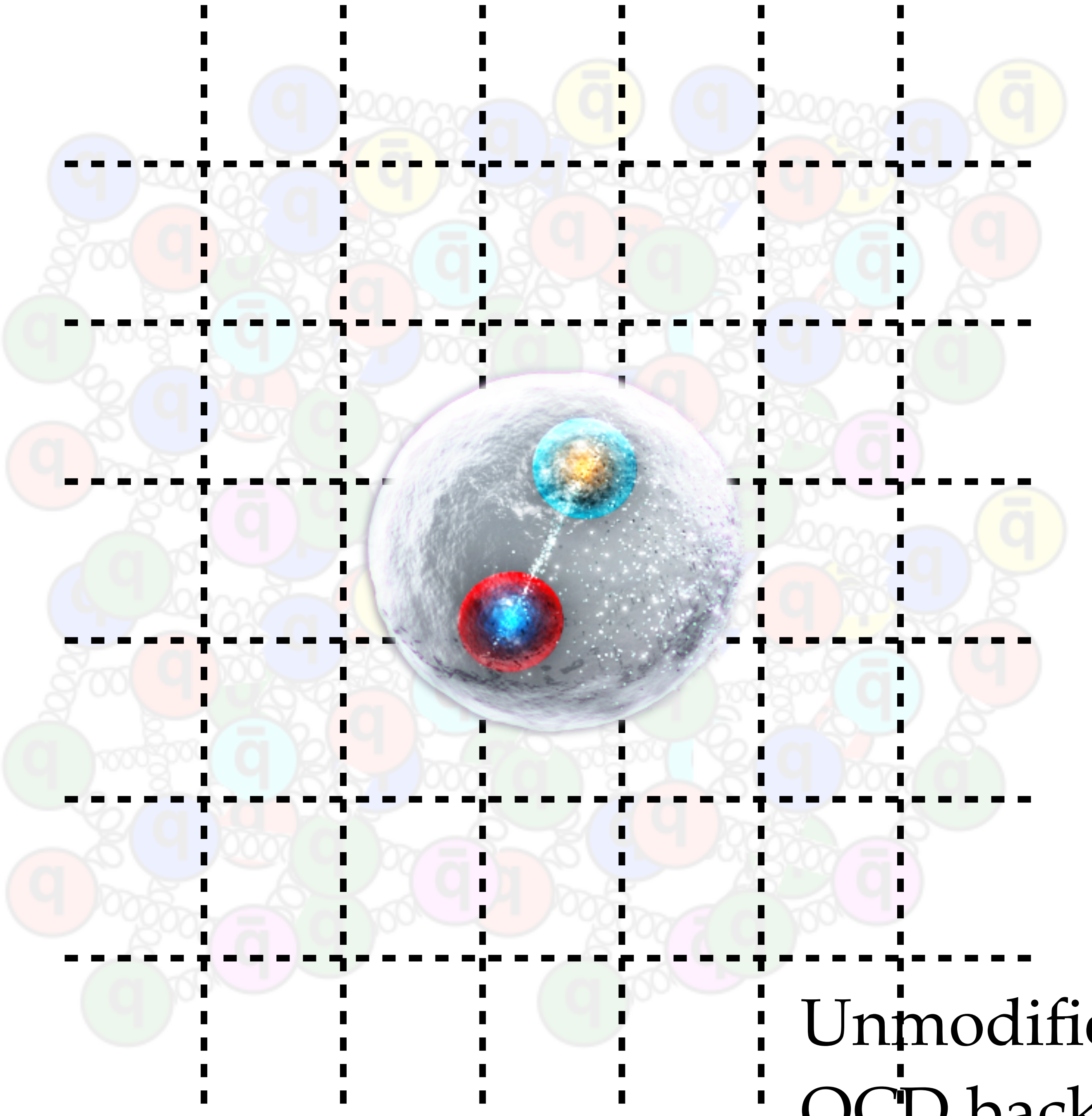
$J_3 J_3$ **and** $q_3 = 0, \vec{p} = \vec{0}$

$$T_{33}(\vec{0},q) = \mathscr{F}_1(\omega=0,Q^2) = T_1(0,Q^2) \equiv S_1(Q^2)$$

Calculation Details

QCDSF/UKQCD configurations
2+1 flavour (u/d+s)
NP-improved Clover action
PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

N_f	$L^3 \times T$	β	κ	a[fm]	m_π[MeV]	Z_V
$2+1$	$32^3 \times 64$	5.50	0.120900	0.074	470	0.86
$2+1$	$48^3 \times 96$	5.65	0.122005	0.068	410	0.87
$2+1$	$48^3 \times 96$	5.80	0.122810	0.058	430	0.88



- Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x)\gamma_\mu q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ
- Connected diagrams only

Strategy | Energy shifts

Isolate the 2nd-order energy shift

$$G^{(2)}_{\lambda}(\mathbf{p};t) \sim A_{\lambda}(\mathbf{p}) e^{-E_{N_\lambda}(\mathbf{p})t}$$

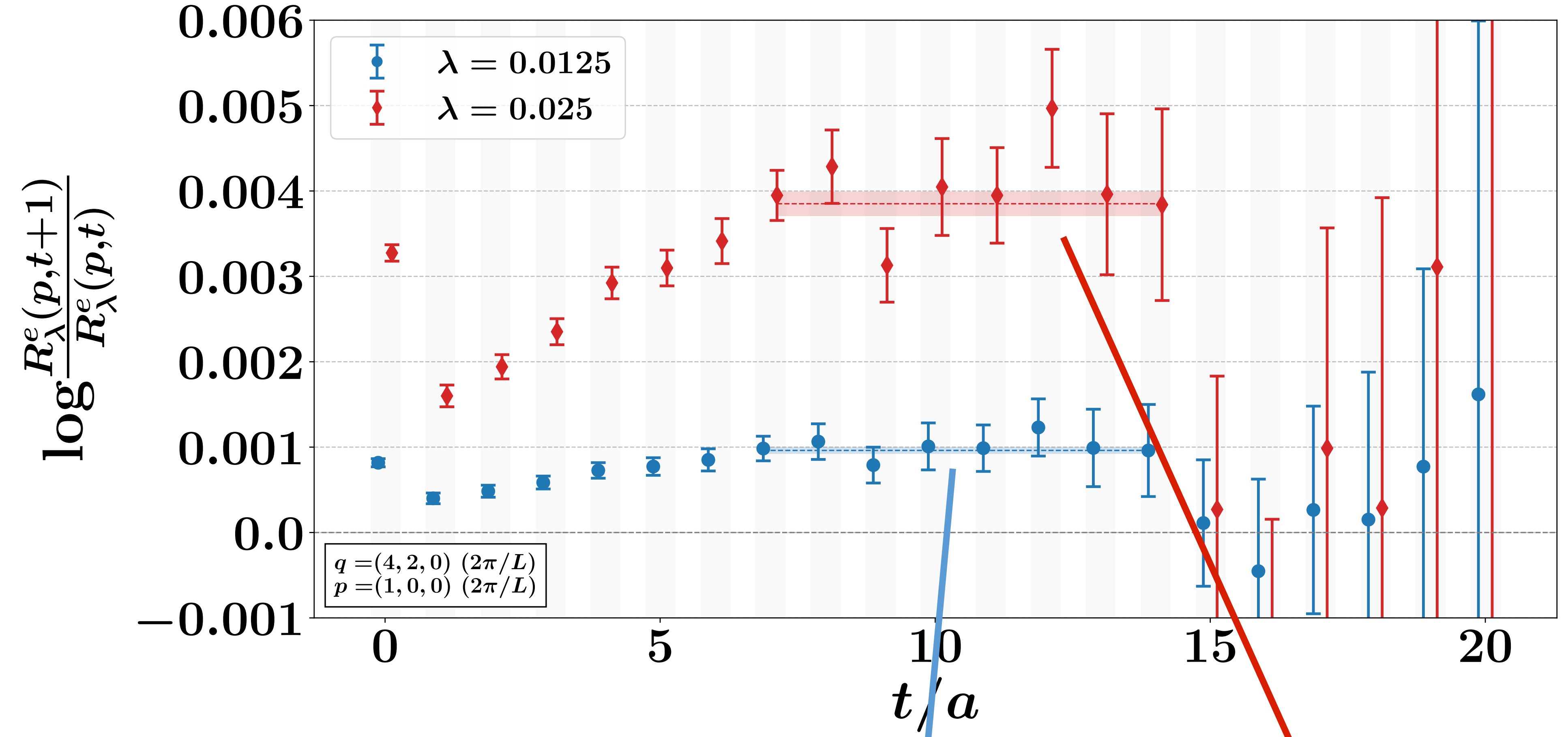
$$\begin{aligned} E_{N_\lambda}(\mathbf{p}) &= E_N(\mathbf{p}) + \lambda \left.\frac{\partial E_{N_\lambda}(\mathbf{p})}{\partial \lambda}\right|_{\lambda=0} + \frac{\lambda^2}{2!} \left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial^2 \lambda}\right|_{\lambda=0} + \mathcal{O}(\lambda^3) \\ &= E_N(\mathbf{p}) + \Delta E^o_N(\mathbf{p}) + \Delta E^e_N(\mathbf{p}) \end{aligned}$$

Ratio of perturbed to unperturbed 2-pt functions

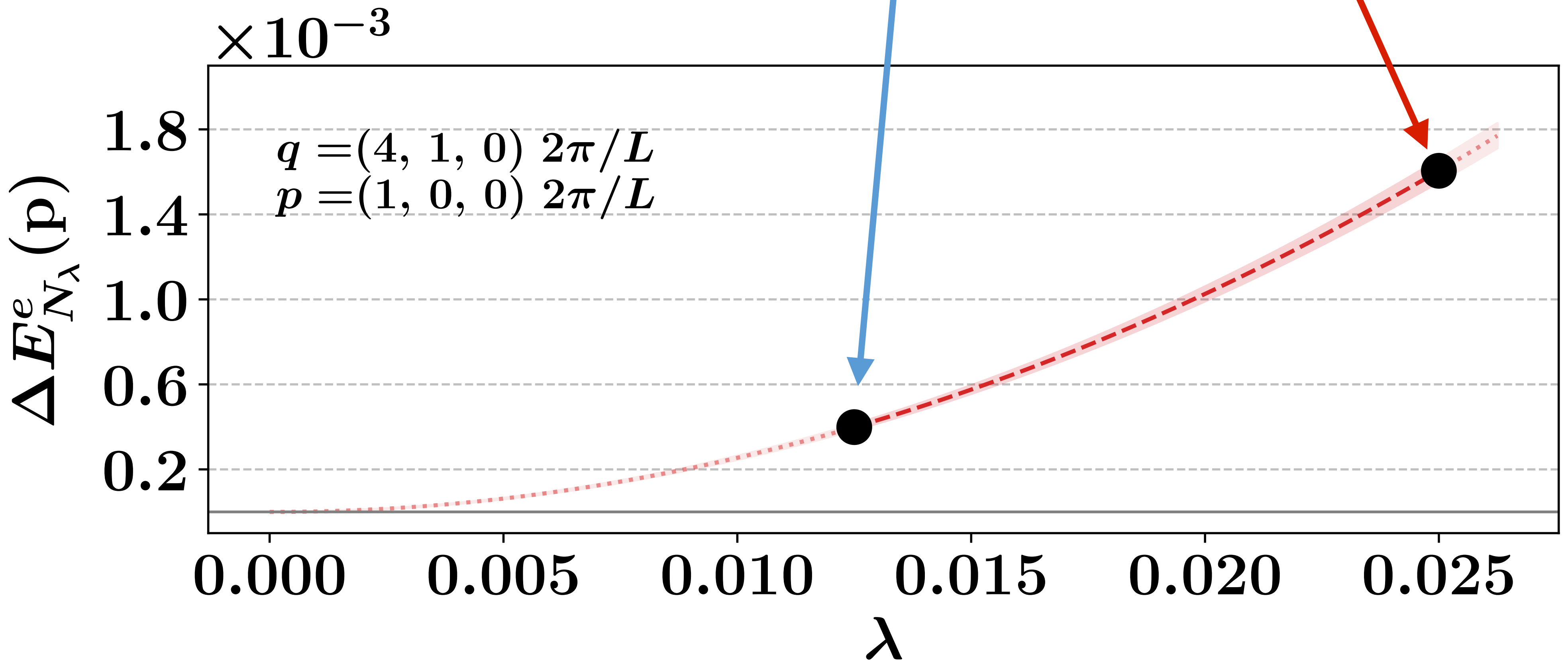
$$R^e_\lambda(\mathbf{p},t) \equiv \frac{G^{(2)}_{+\lambda}(\mathbf{p},t) G^{(2)}_{-\lambda}(\mathbf{p},t)}{\left(G^{(2)}(\mathbf{p},t)\right)^2}$$

$$\xrightarrow{t \gg 0} A_\lambda(\mathbf{p}) e^{-2\Delta E^e_{N_\lambda}(\mathbf{p})t}$$

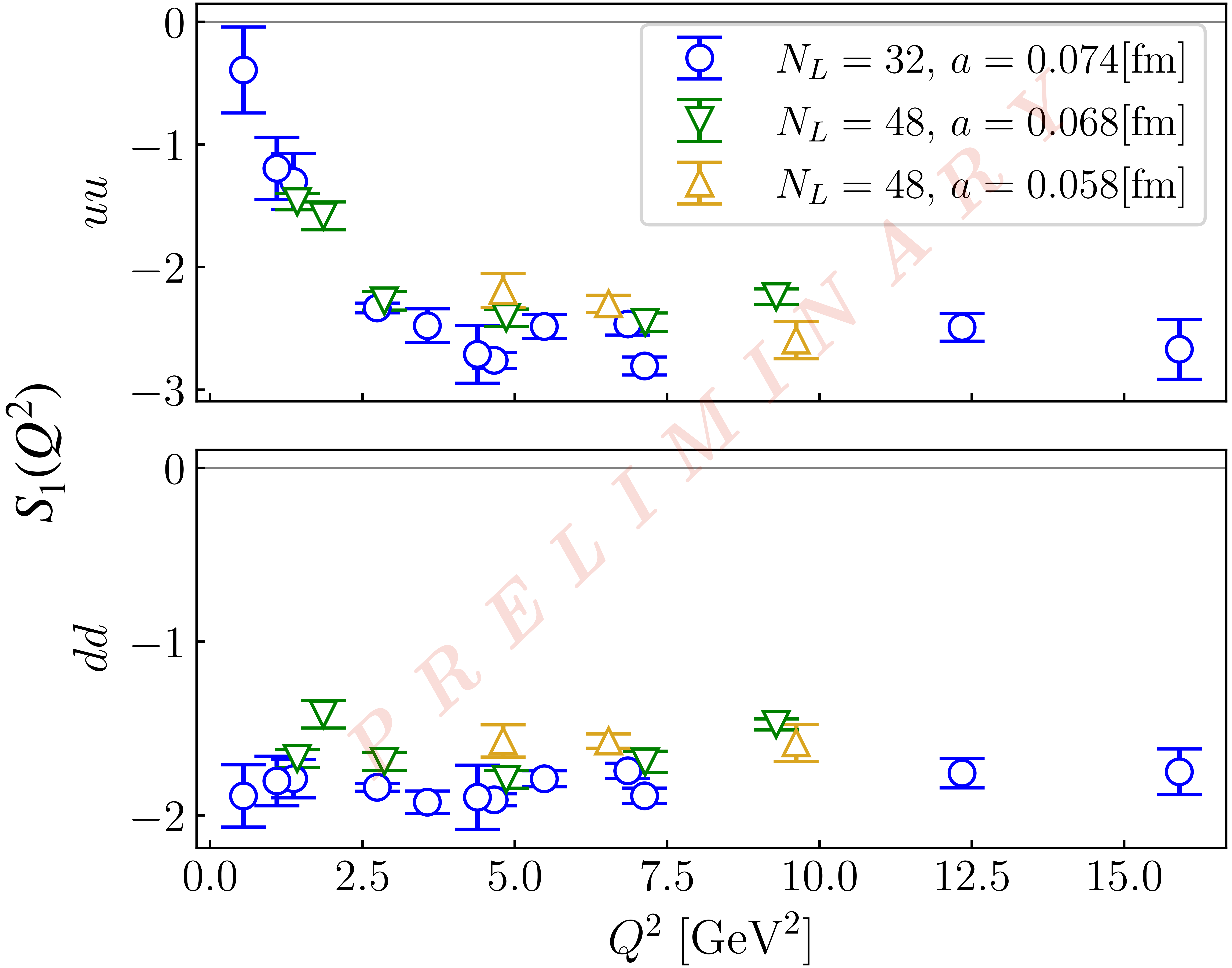
- **Extract energy shifts for each λ**



- **Get the 2nd order derivative**



S_1 | unimproved



- OPE expectation:

$$\lim_{Q^2\to\infty} S_1(Q^2) \sim 1/Q^2$$

- Did we confirm a fixed pole?
- Lattice artefacts maybe?

S_1 | lattice artefacts

- **tree-level mass-dependent lattice operator product expansion (LOPE)**

$$\mathcal{T}_{\mu\nu} = \bar{\psi}\gamma_\mu S_W(p+q)\gamma_\nu\psi + \bar{\psi}\gamma_\nu S_W(p-q)\gamma_\mu\psi,$$

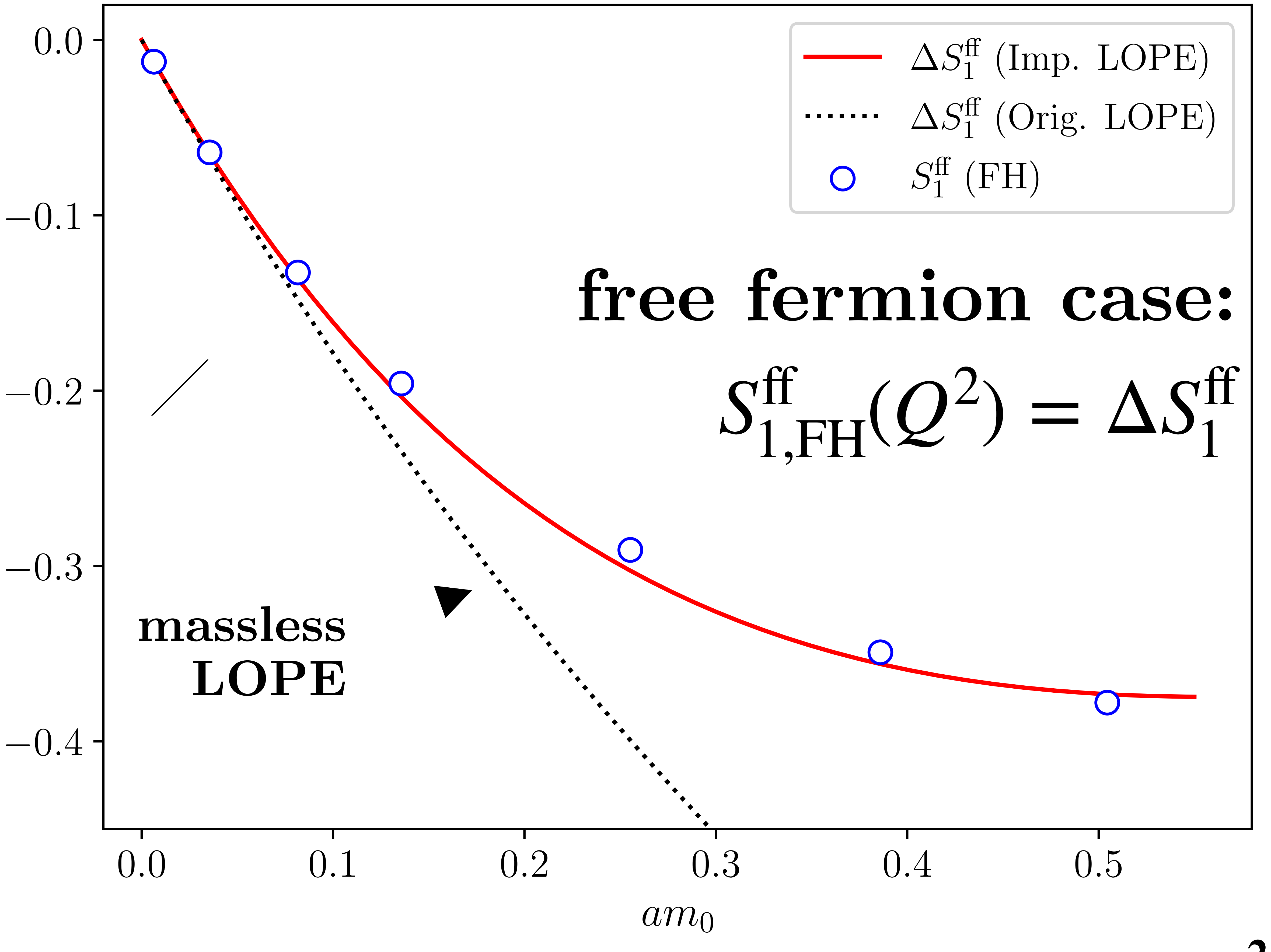
with the Wilson quark propagator,

$$S_W(k) = a\frac{M(k) - i\gamma_\mu \sin(ak_\mu)}{M(k)^2 + \sum_\mu \sin^2(ak_\mu)},$$

where $M(k) = am_0 + \sum_\rho \left[1 - \cos(ak_\rho)\right],$

- **giving the correction**

$$\Delta S_1 = \frac{4m_p a Z_V^2 g_S^{\text{bare}} \sum_\rho \left[\cos(aq_\rho) - 1\right]}{\sum_\rho \sin^2(aq_\rho) + M^2(m_0, q)}$$

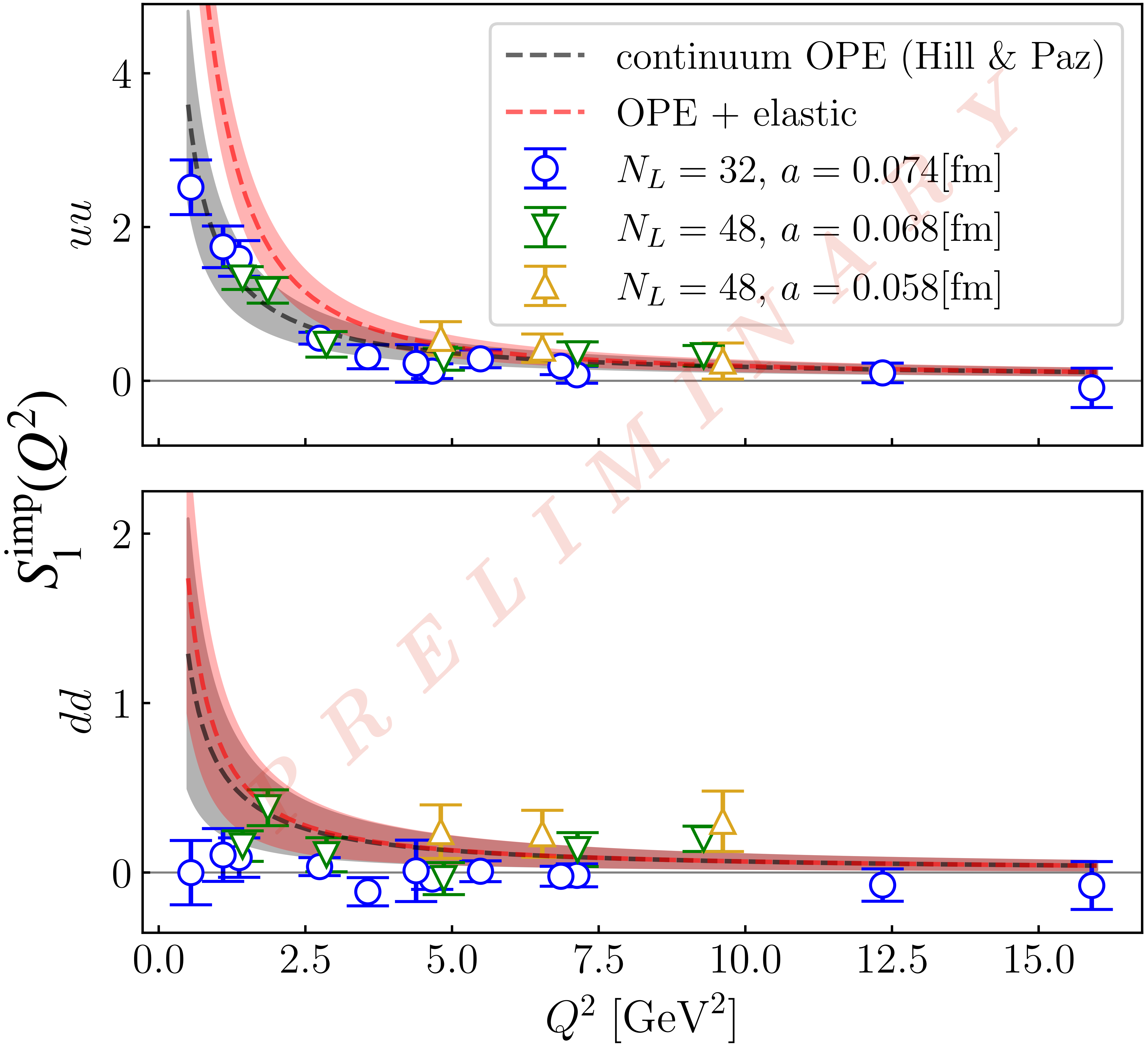


free fermion case:

$$S_{1,\text{FH}}^{\text{ff}}(Q^2) = \Delta S_1^{\text{ff}}$$

massless LOPE

S_1 | improved



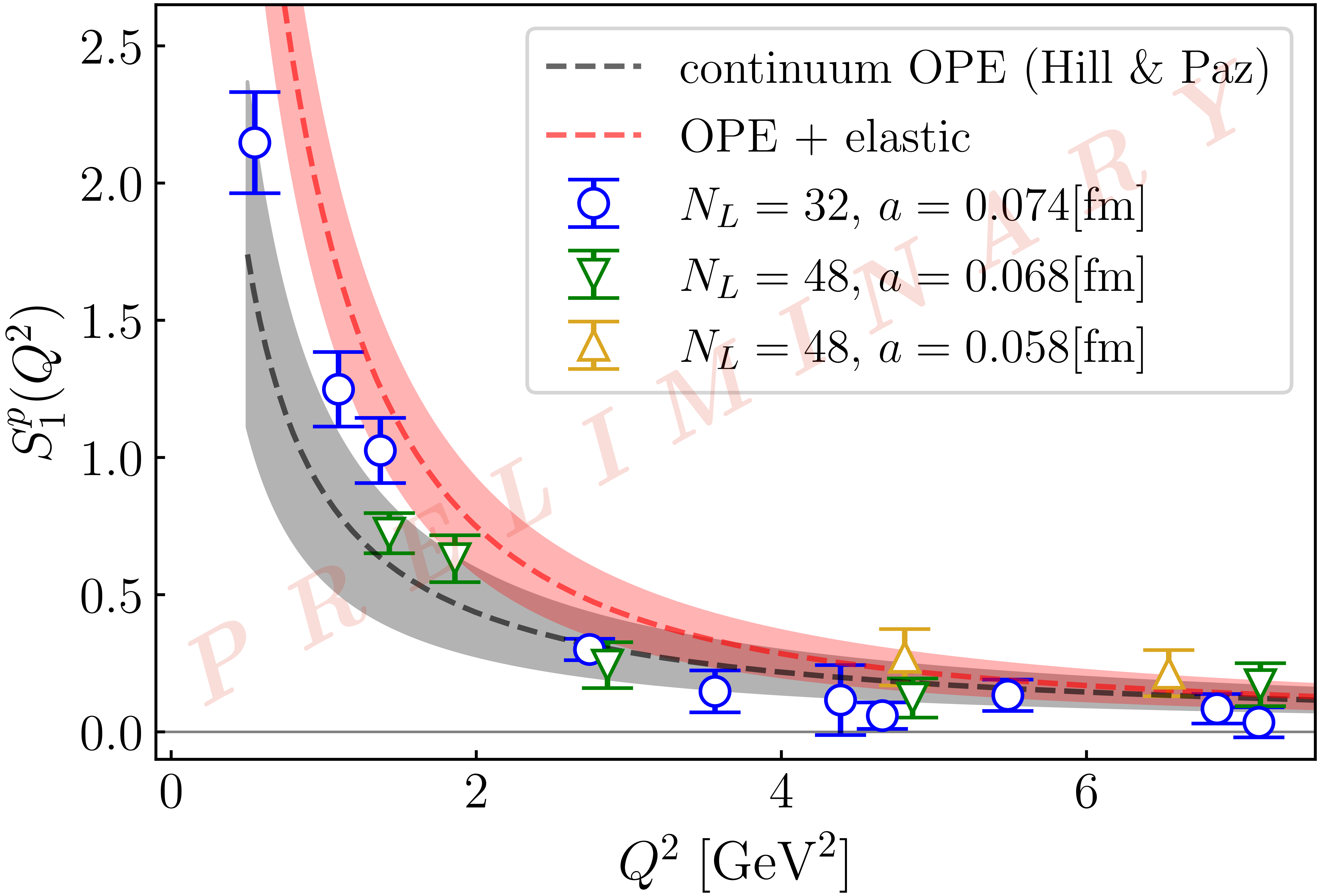
- $S_1^{\text{imp}}(Q^2) = S_1^{\text{latt}}(Q^2) + \Delta S_1$

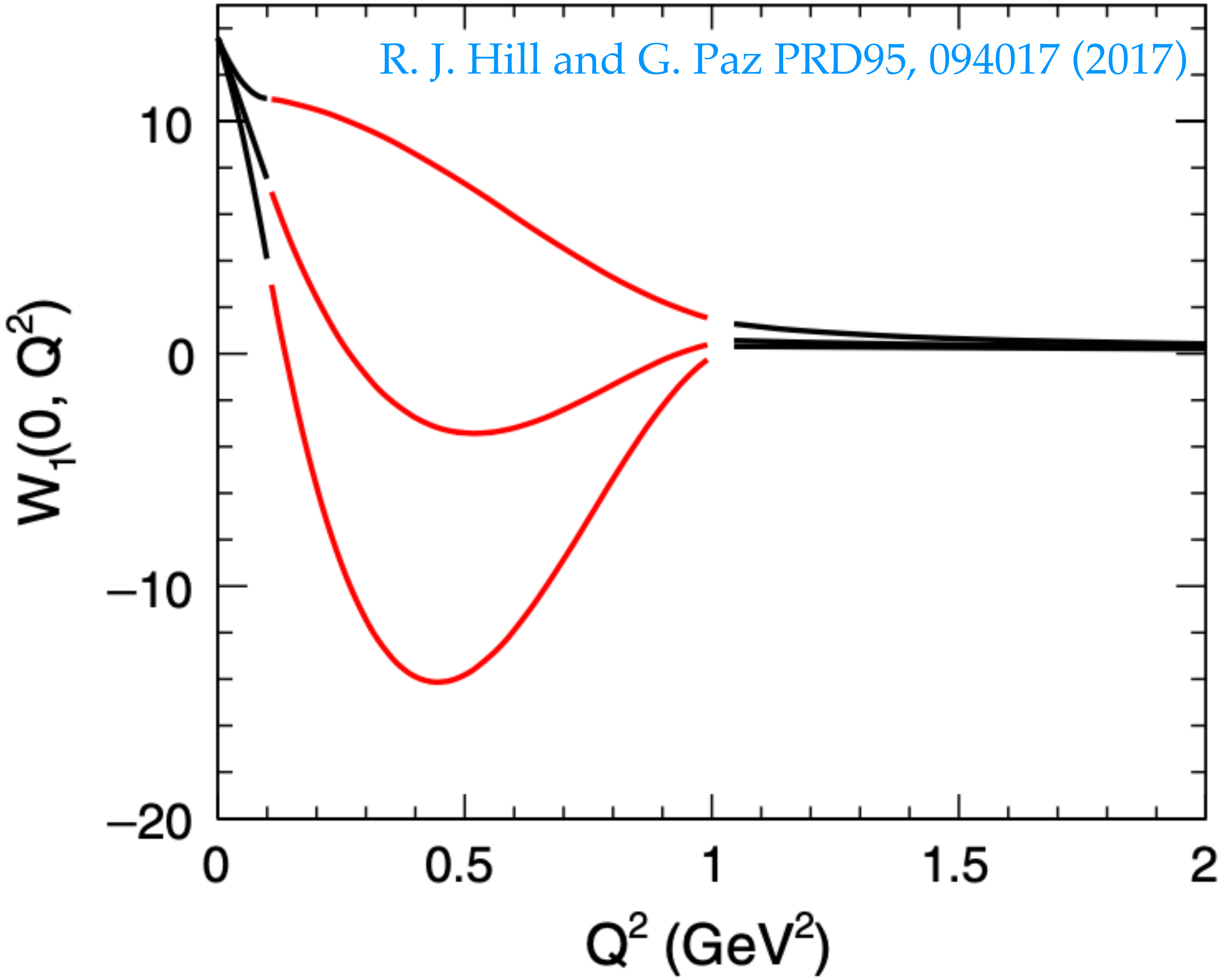
$$\Delta S_1 = \frac{4 m_p a Z_V^2 g_S^{\text{bare}} \sum_\rho \left[\cos(a q_\rho) - 1\right]}{\sum_\rho \sin^2(a q_\rho) + M^2(m_0, q)}$$

- g_s^{bare} calculated on the same set of ensembles
- Good agreement with OPE
- $$S_1^{\text{OPE}}(Q^2) = \frac{4m_p^2}{Q^2} \sum_q e_q^2 \left(a_2^q - \frac{m_q}{m_p} g_S^q \right)$$

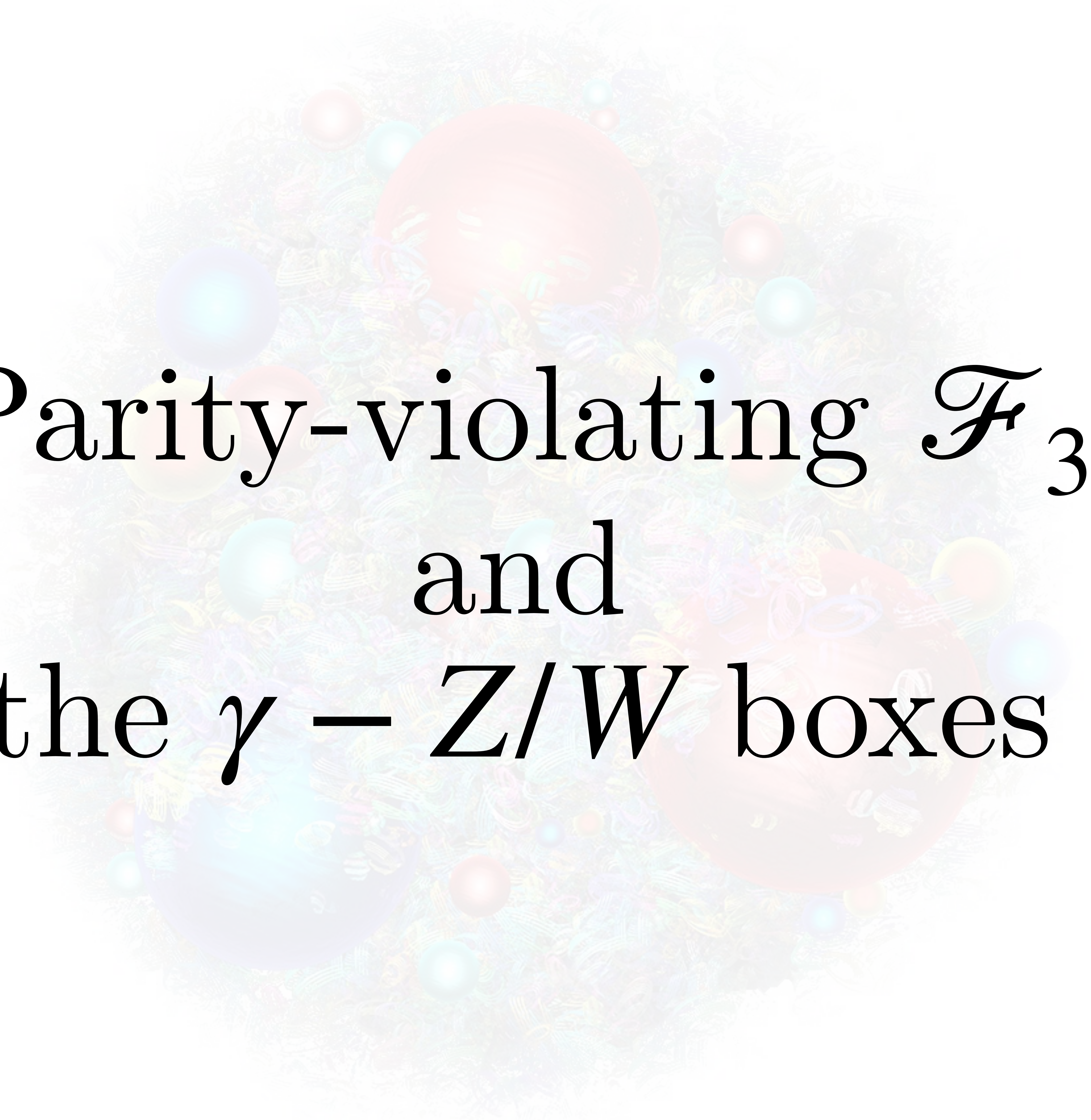
S_1 | impact

- Low- and high-Q^2 regions are known
- Possible to constrain the mid-Q^2 region





R. J. Hill and G. Paz PRD95, 094017 (2017)



Parity-violating $\mathscr{F}_3$
and
the $\gamma - Z/W$ boxes

Parity Violating Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\,\mathcal{T}\left\{J_\mu(z)J_\nu(0)\right\}|p,s\rangle \quad \text{, spin avg. } \rho_{ss'} = \frac{1}{2}\delta_{ss'}$$

$$= -g_{\mu\nu}\mathcal{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathcal{F}_2(\omega,Q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2)$$

$$+\frac{q_\mu q_\nu}{p\cdot q}\mathcal{F}_4(\omega,Q^2) + \frac{p_{\{\mu}q_{\nu\}}}{p\cdot q}\mathcal{F}_5(\omega,Q^2) + \frac{p_{[\mu}q_{\nu]}}{p\cdot q}\mathcal{F}_6(\omega,Q^2)$$

$\varepsilon^{0123} = 1$

allowed terms
because parity
is violated

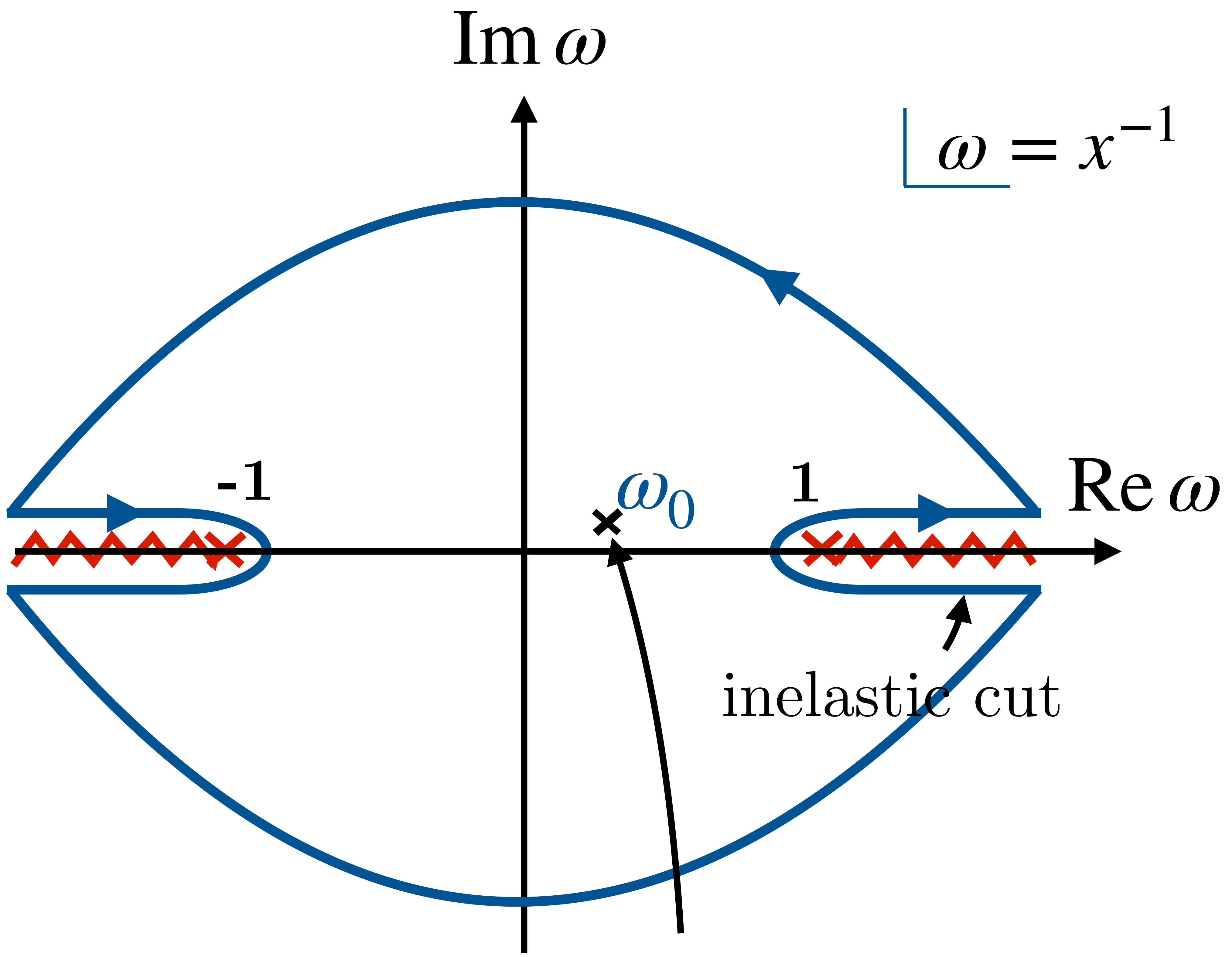
Nucleon Structure Functions | F_3

- **for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,**

$$T_{\mu\nu}(p,q) = i\,\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega, Q^2)$$

- **we can write down dispersion relations and connect Compton SFs to DIS SFs:**

$$\mathcal{F}_3(\omega, Q^2) = 4\omega\int dx \frac{F_3(x,Q^2)}{1-x^2\omega^2}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Parity Violating Forward Compton Amplitude

- **The 1st moment**

$$M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x, Q^2) = \left.\frac{\mathscr{F}_3(\omega, Q^2)}{4\omega}\right|_{\omega=0}$$

allows for a test of the Gross-Llewellyn-Smith sum rule $(a_s = \alpha_s(Q^2)/\pi)$

$$M_{1,uu}^{(3)}(Q^2) = \int_0^{1^-} dx F_3(x, Q^2) = 2\left(1 + \sum_{i=1}^{3} a_s^i c_i(n_f)\right) + \frac{\Delta_{HT}}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

known coeffs.

Higher-twist

- **Also allows for a determination of the EW box diagram**

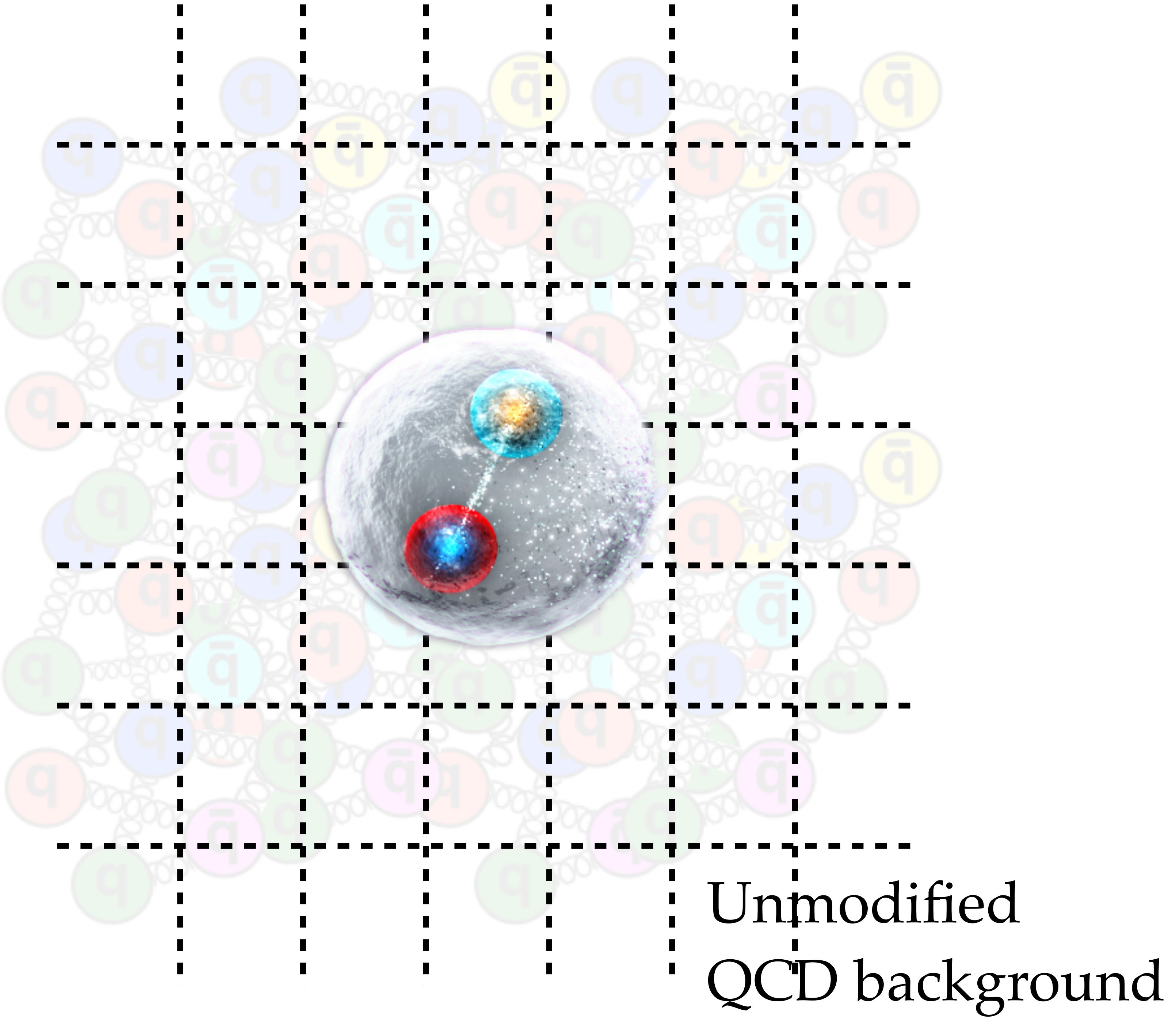
$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2 + Q^2} M_1^{(3)}(Q^2)$$

Calculation Details

QCDSF/UKQCD configurations
$48^3 \times 96$, 2+1 flavour (u/d+s)
$\beta = \begin{pmatrix} 5.65 \\ 5.95 \end{pmatrix}$, NP-improved Clover action
PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$$m_\pi \sim 410 \text{ MeV}, \sim\text{SU(3) sym.}$$

$$m_\pi L \sim \begin{bmatrix} 6.9 \\ 5.3 \end{bmatrix} \qquad a \sim \begin{bmatrix} 0.068 \\ 0.052 \end{bmatrix} \text{ fm}$$



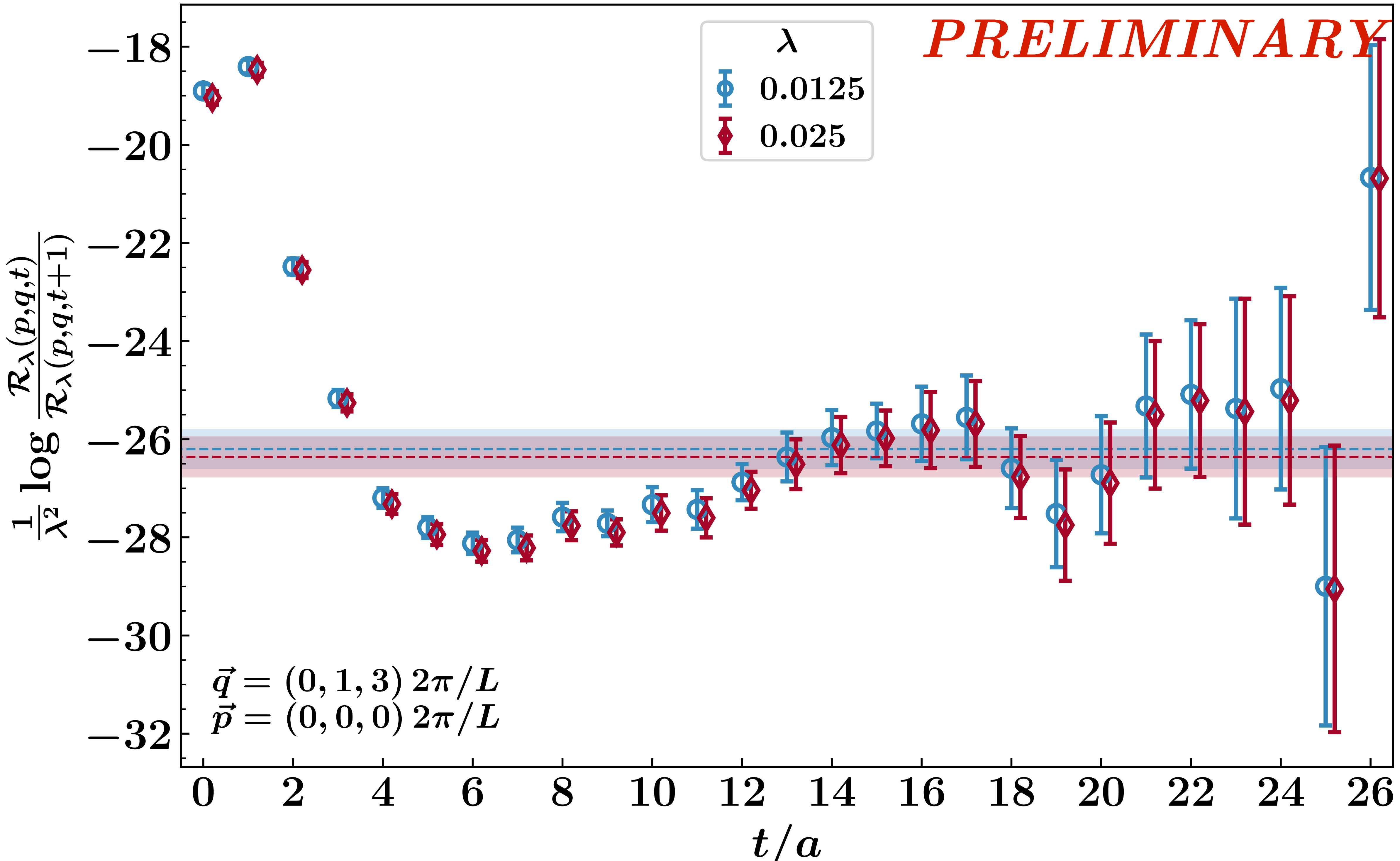
- Local EM and axial current insertion, $J_\mu^{V[A]}(x) = Z_{V[A]}\bar{q}(x)\gamma_\mu[\gamma_5]q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Current momenta $0.1 \lesssim Q^2 \lesssim 10 \text{ GeV}^2$
- Roughly 500 measurements
- Nucleon at rest: $\vec{p} = (0,0,0)$ thus $\omega = 0$, varying $\vec{q}$
- Connected 2-pt only, no disconnected since F_3 is non-singlet

Energy shifts

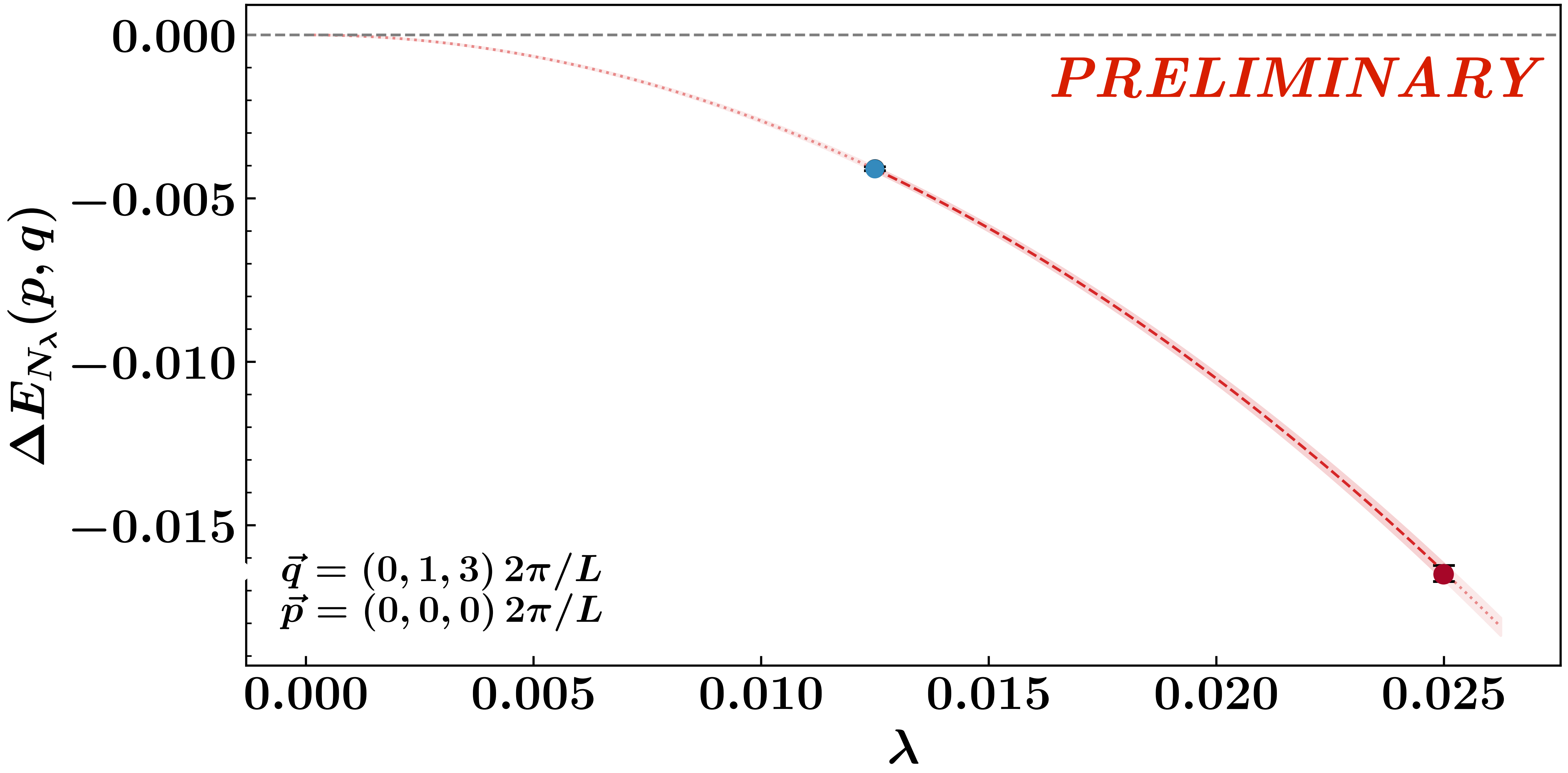
- Ratio of perturbed 2-pt functions

$$\mathscr{R}^{qq}_{\lambda}(p,t) \equiv \frac{G^{(2)}_{+\lambda_1^q,+\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,-\lambda_2^q}(p,t)}{G^{(2)}_{+\lambda_1^q,-\lambda_2^q}(p,t)\, G^{(2)}_{-\lambda_1^q,+\lambda_2^q}(p,t)} \to A_\lambda e^{-4\Delta E_{N_\lambda}(p)\,t}$$

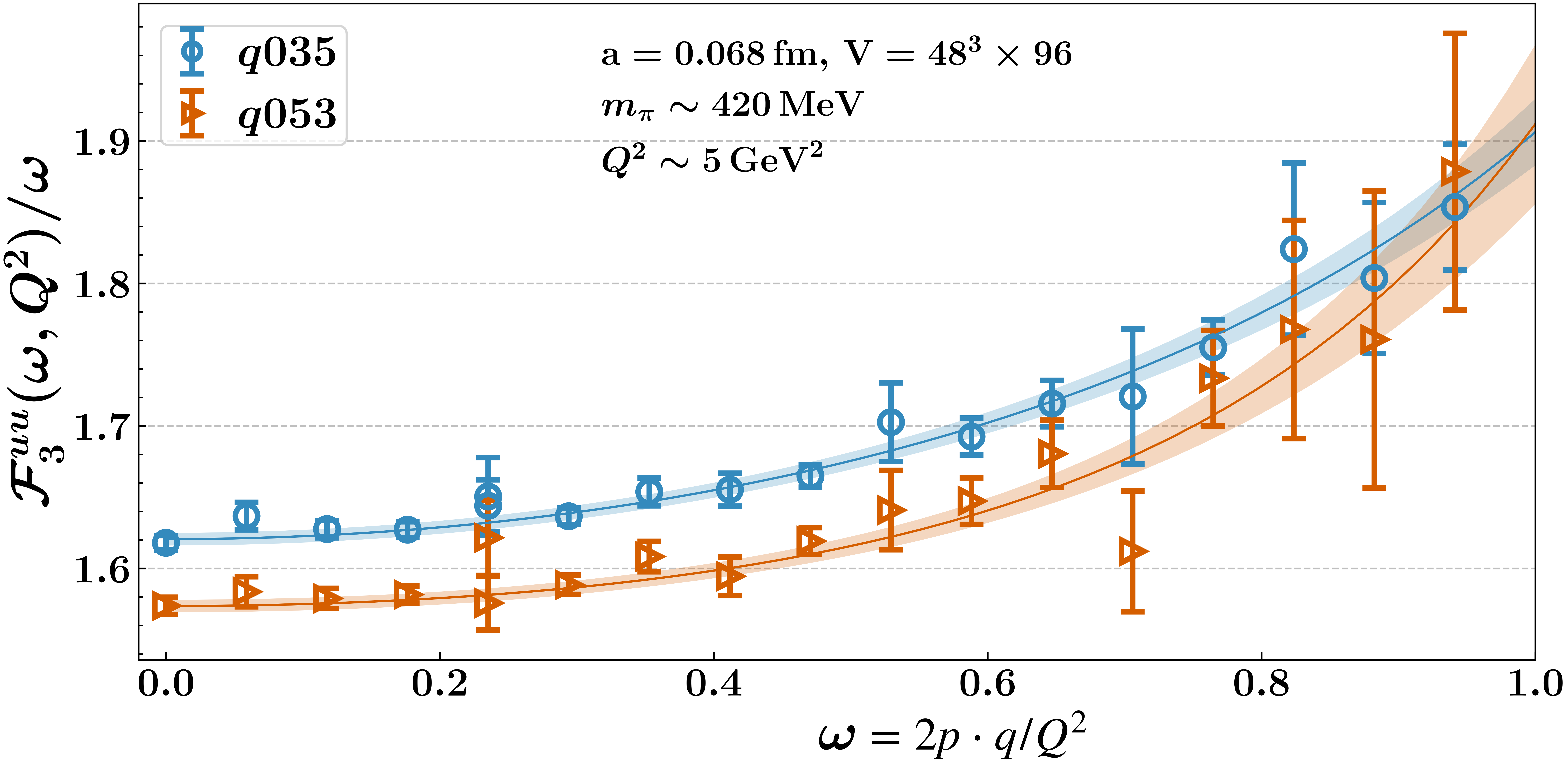
- Extract energy shifts for each $|\lambda|$



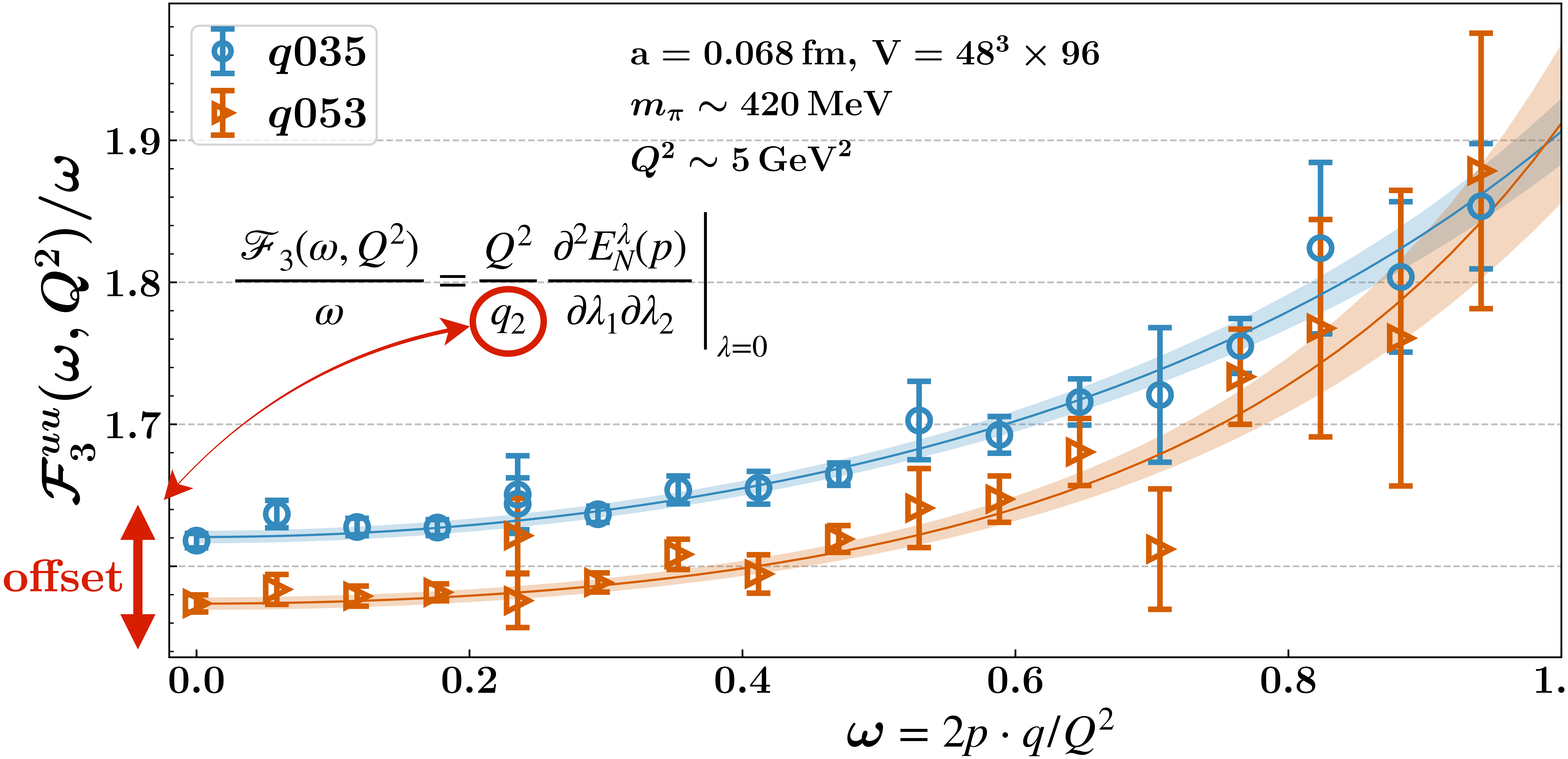
- Get the 2nd order derivative $\left.\dfrac{\partial^2 E_N^{\lambda}(p)}{\partial\lambda_1\partial\lambda_2}\right|_{\lambda=0}$



$\mathcal{F}_3$ | unimproved



$\mathcal{F}_3$ | unimproved



$$\frac{\mathcal{F}_3(\omega,Q^2)}{\omega} = \frac{Q^2}{\boxed{q_2}} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1 \partial\lambda_2}\bigg|_{\lambda=0}$$

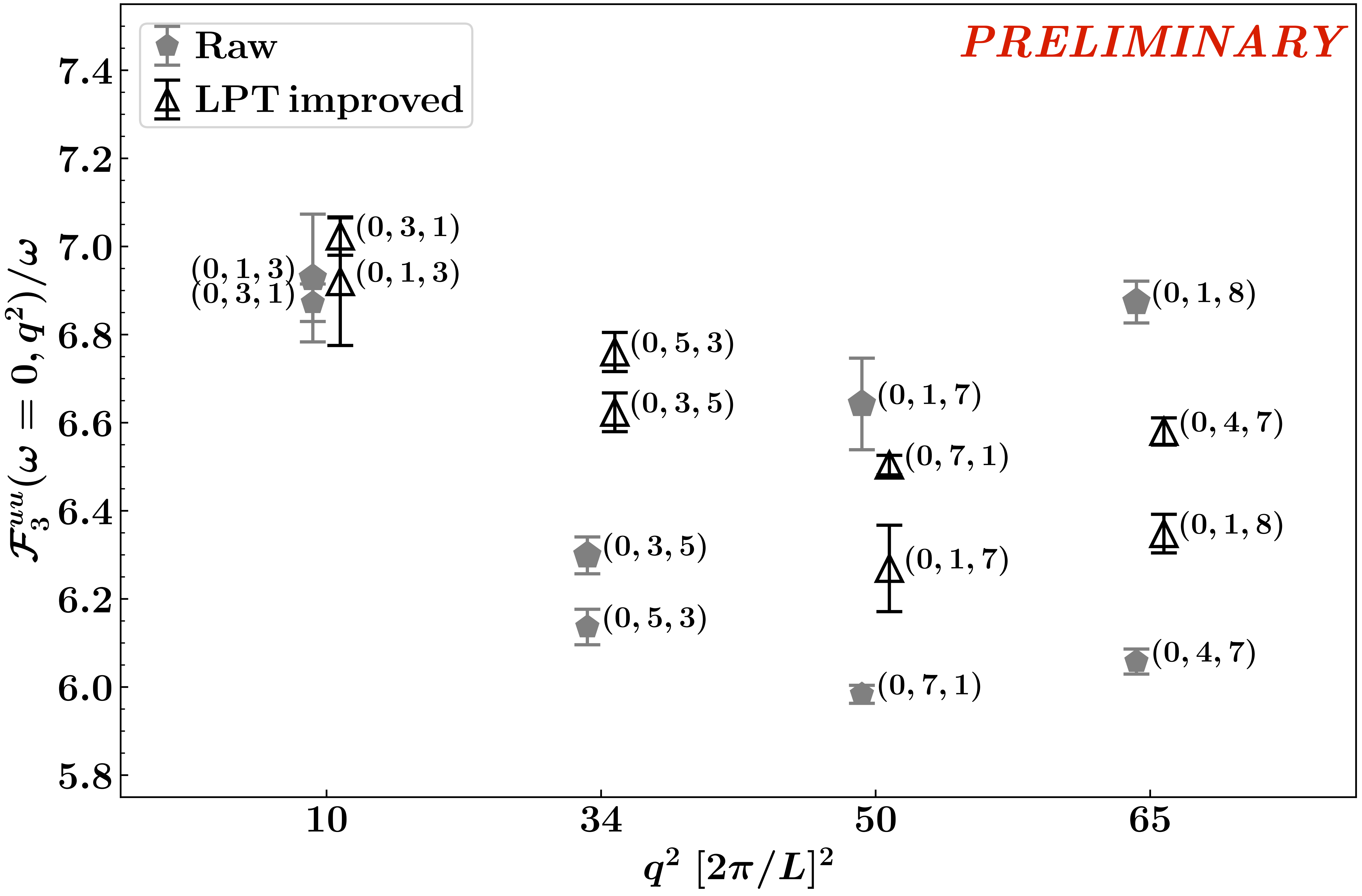
Syst. 1: LPT improvement

$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{q_2} \frac{\partial^2 E_N^\lambda(p)}{\partial\lambda_1\partial\lambda_2}\bigg|_{\lambda=0}$$

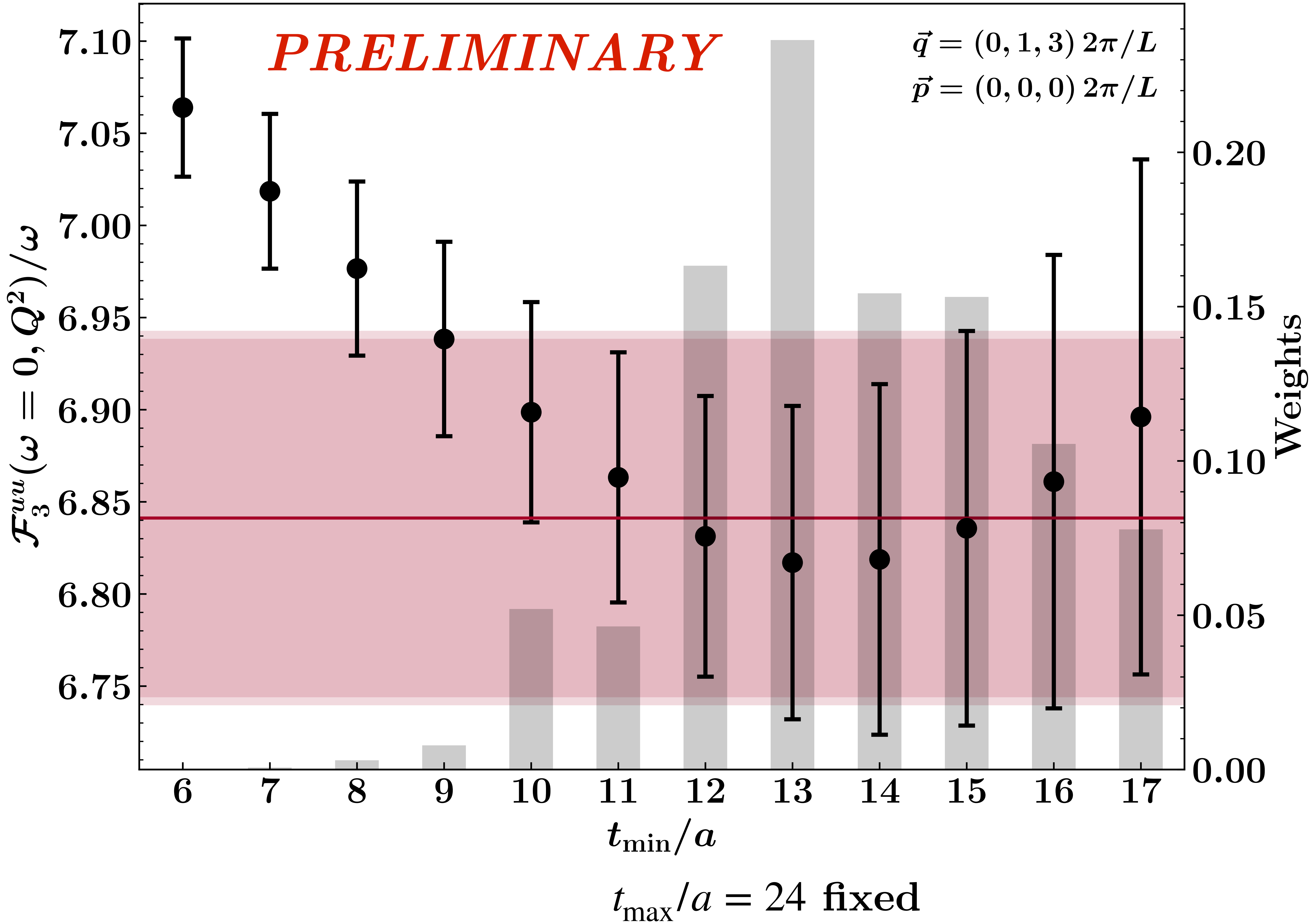
introduces discretisation error due to broken rotational symmetry

- **Replace the kinematic factor by a lattice OPE motivated factor**

$$\frac{Q^2}{q_2} \to \frac{\sum_i \sin^2 q_i + \left[\sum_i (1 - \cos q_i)\right]^2}{\sin q_2}$$



Syst. 2: Weighted averaging



- **Red line (mean):** $\bar{\mathcal{O}} = \sum_f w^f \mathcal{O}^f$
- **Red band (total uncertainty):**

$$\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 = \sum_f w^f (\delta\bar{\mathcal{O}}^f)^2$$

$$\delta_{\mathbf{sys}}\bar{\mathcal{O}}^2 = \sum_f w^f (\mathcal{O}^f - \bar{\mathcal{O}})^2$$

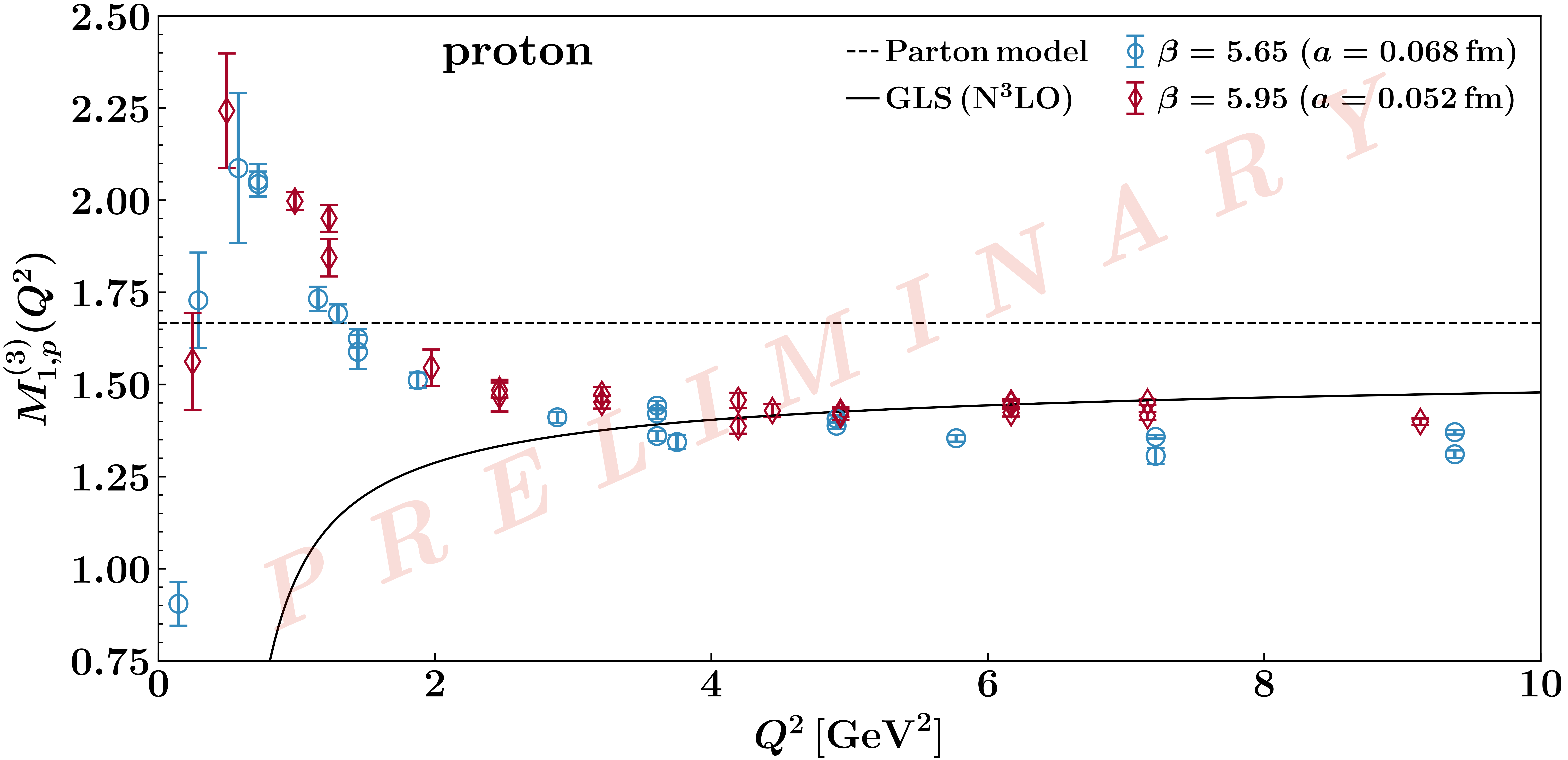
$$\delta\bar{\mathcal{O}} = \sqrt{\delta_{\mathbf{stat}}\bar{\mathcal{O}}^2 + \delta_{\mathbf{sys}}\bar{\mathcal{O}}^2}$$

- **Weights:** $w^f = \dfrac{p_f(\delta\mathcal{O}^f)^{-2}}{\sum_{f'} p_{f'}(\delta\mathcal{O}^{f'})^{-2}}$

where p_f is the one sided p-value of the ratio fits

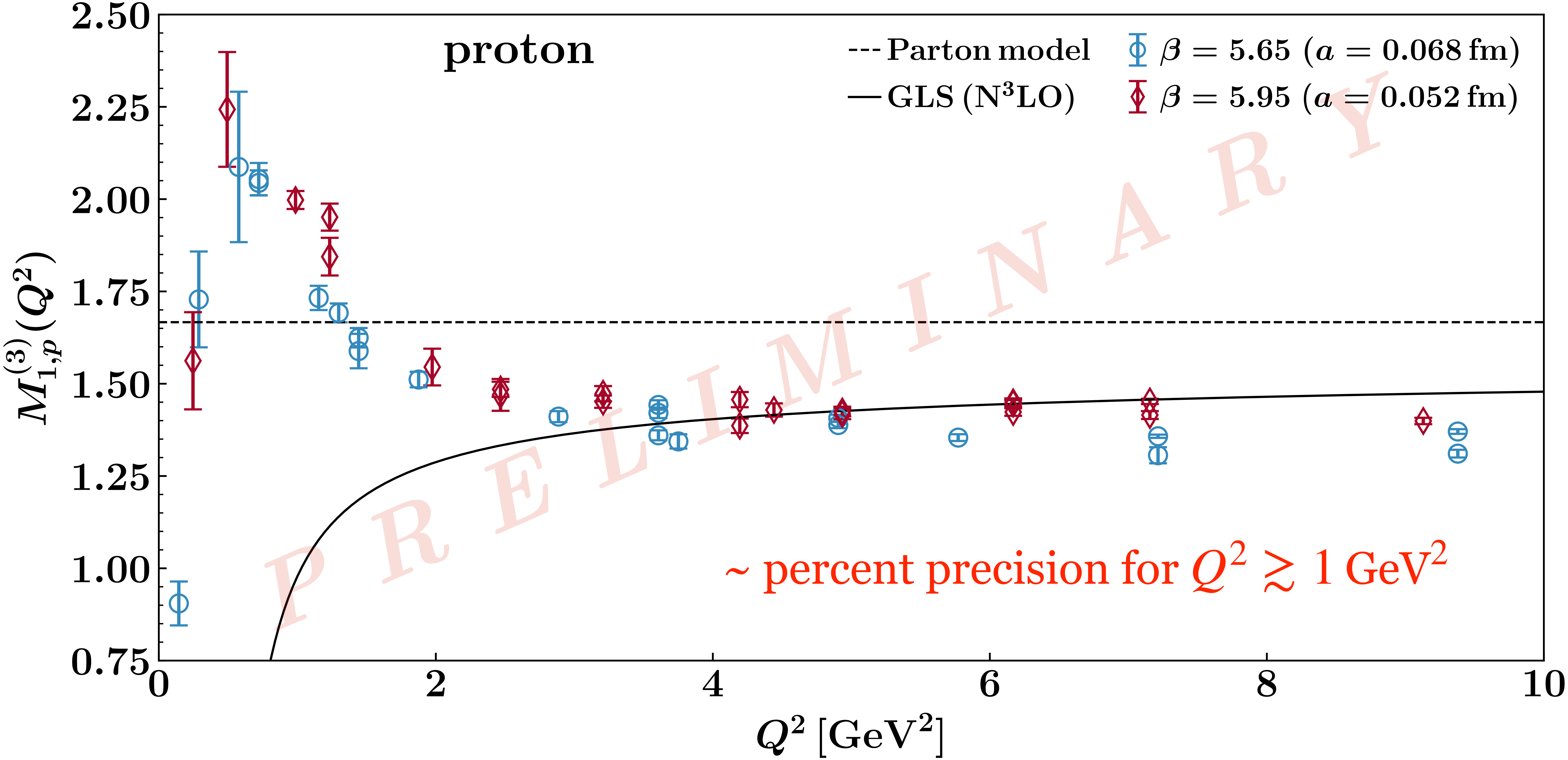
$\mathscr{F}_3^{\gamma Z}$ | First moment

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48³x96, 2+1 flavour



$\mathscr{F}_3^{\gamma Z}$ | First moment

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour



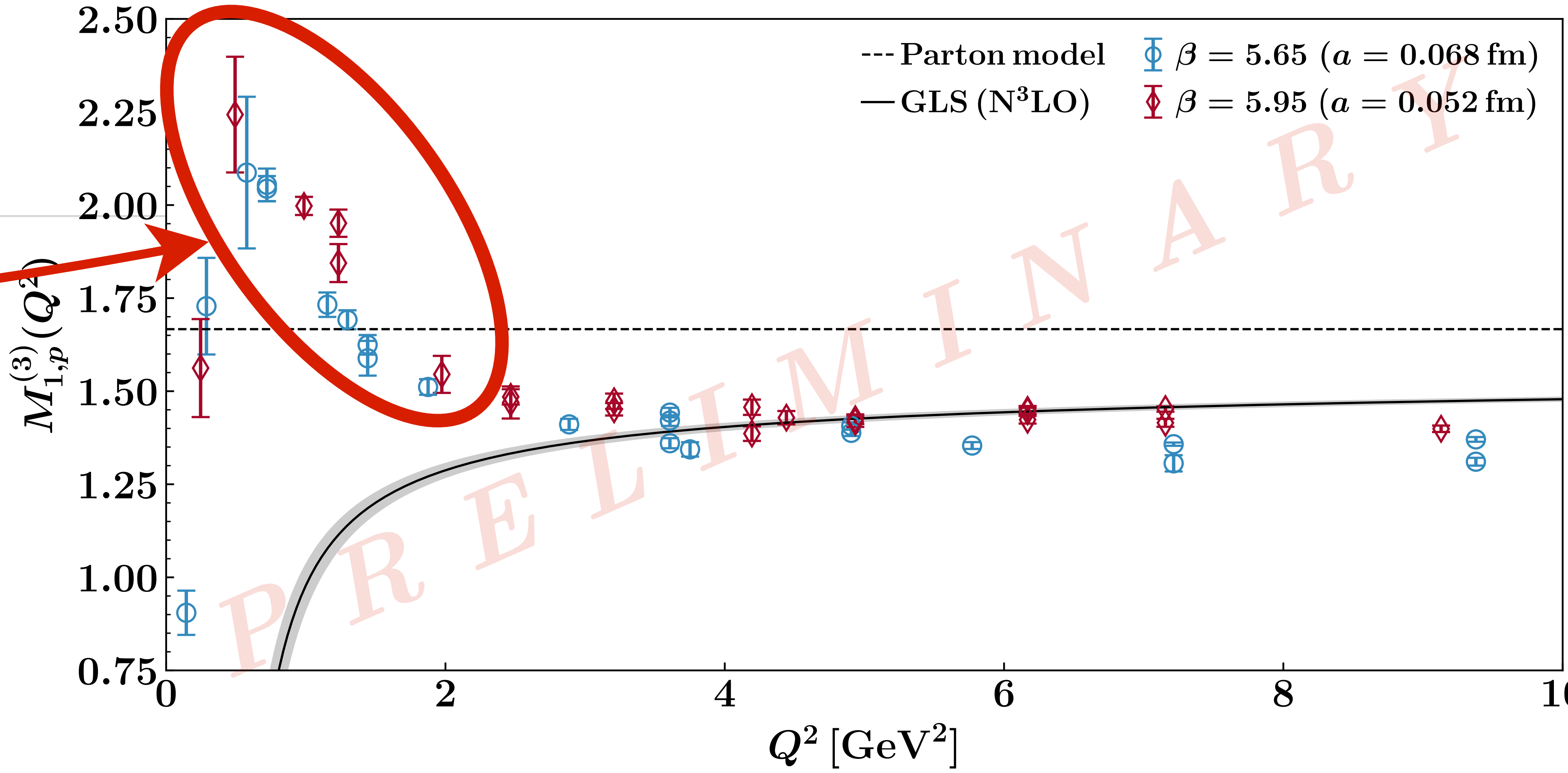
~ percent precision for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

Elastic contribution

- **Peak is mostly elastic**
- subtract elastic contribution:

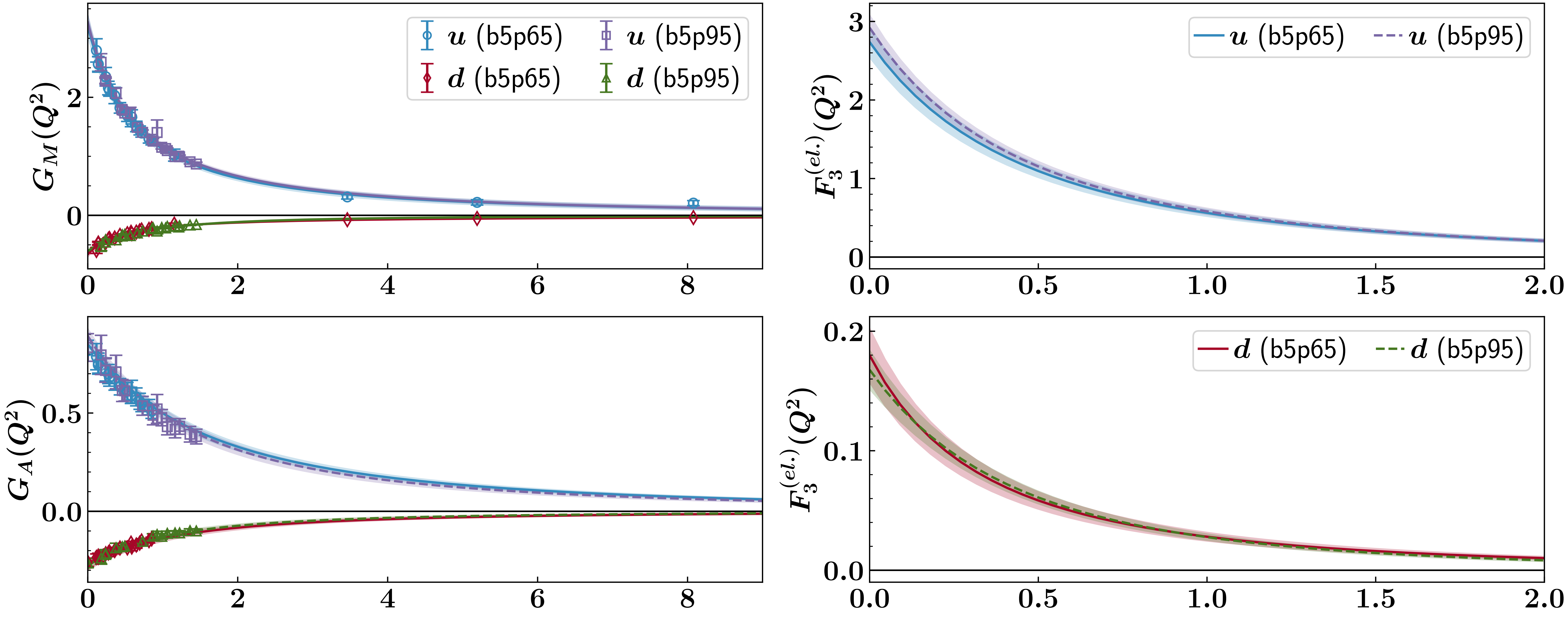
$$F_3^{(el.)} = -G_M(Q^2)G_A(Q^2)x\delta(1-x)$$

- provides insights into higher twist contributions



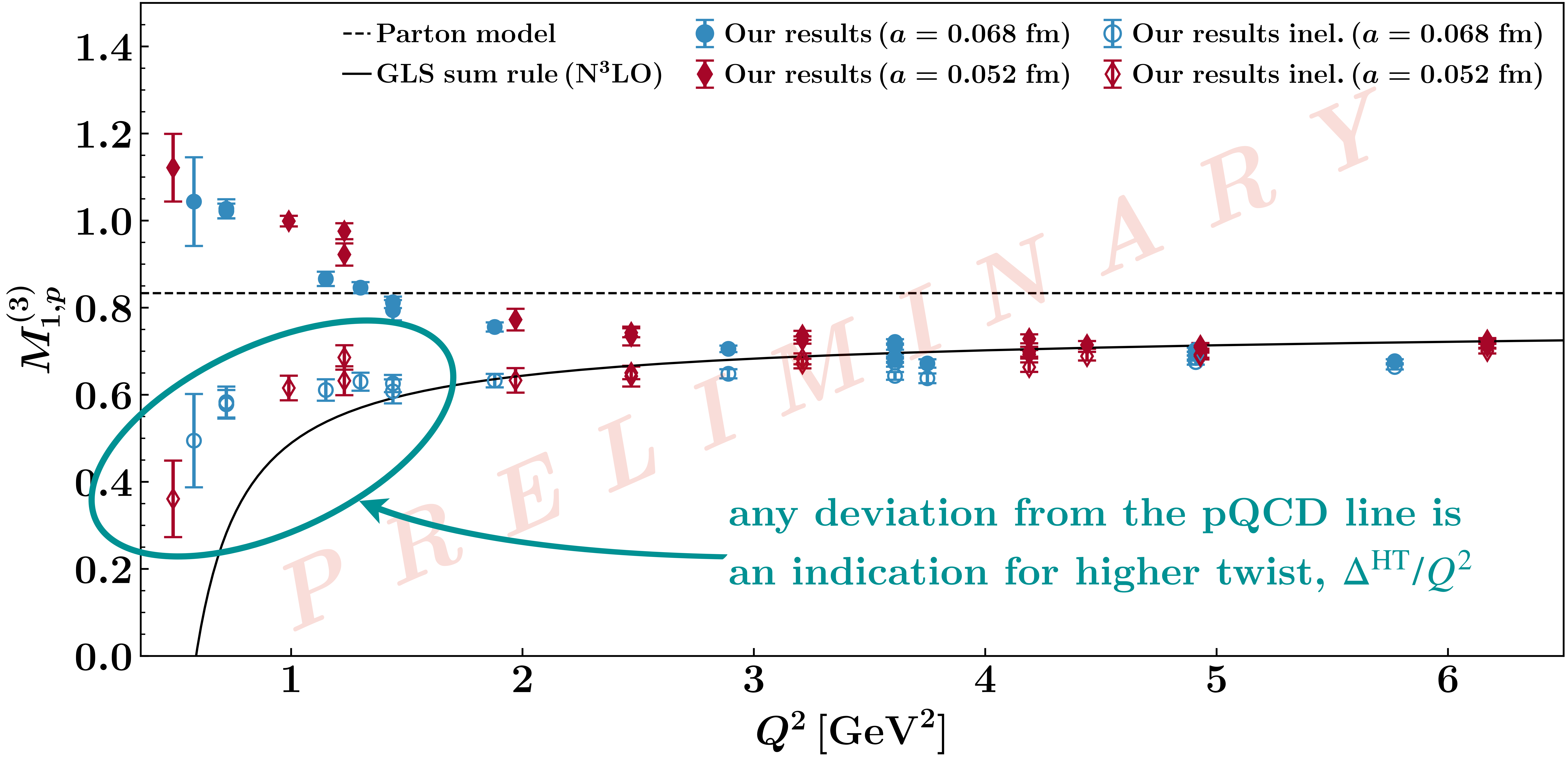
low-Q^2: 3-pt functions
high-Q^2: Feynman-Hellmann

low-Q^2: 3-pt functions
dipole parametrisation



$\mathscr{F}_3^{\gamma Z}$ | Higher-twist

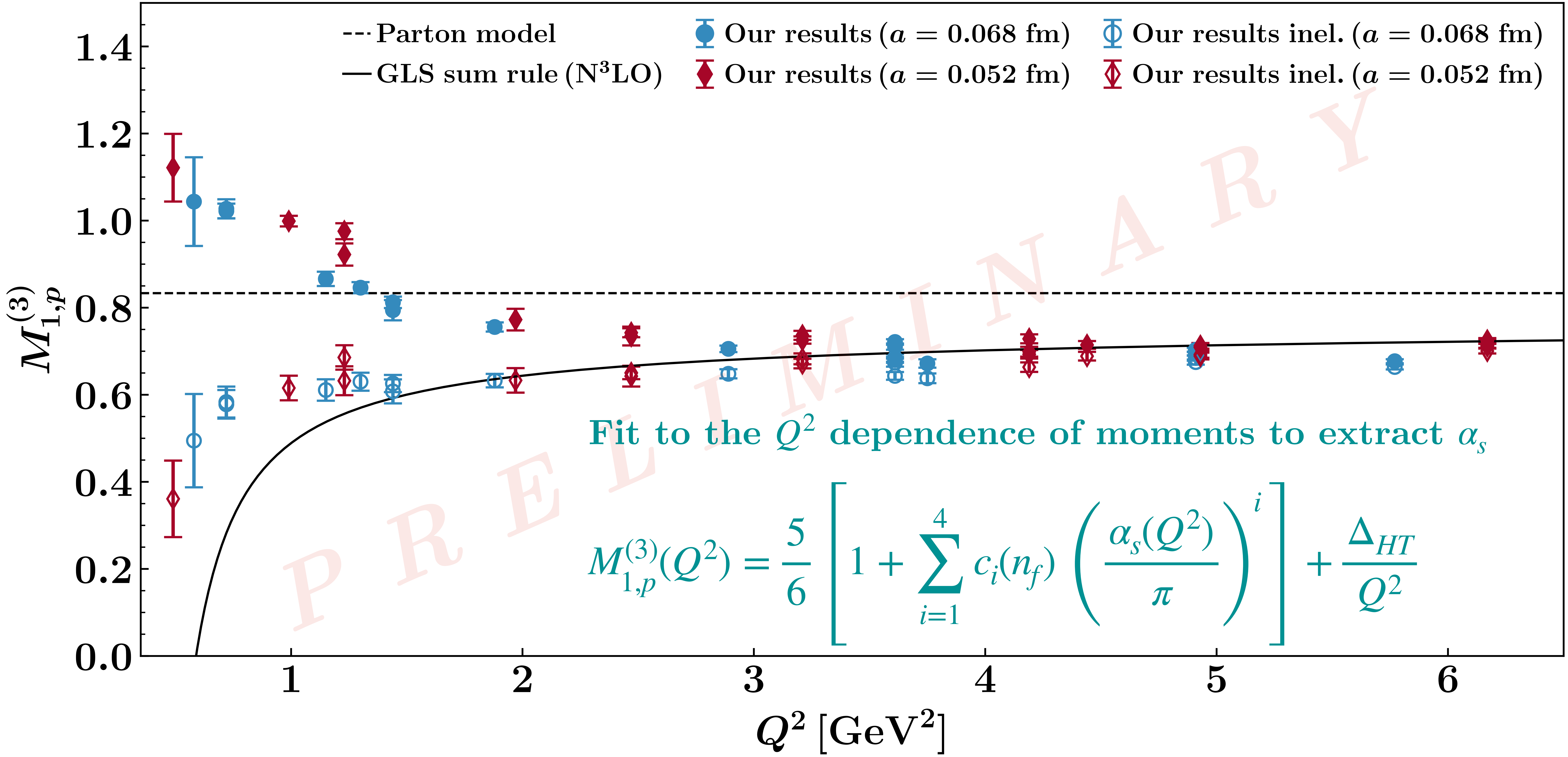
$a = 0.068, 0.052\,\text{fm}$
$m_\pi \sim 410\,\text{MeV}$
48^3x96, 2+1 flavour



any deviation from the pQCD line is an indication for higher twist, Δ^{HT}/Q^2

$\mathcal{F}_3^{\gamma Z}$ | determining α_s

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
48^3x96, 2+1 flavour



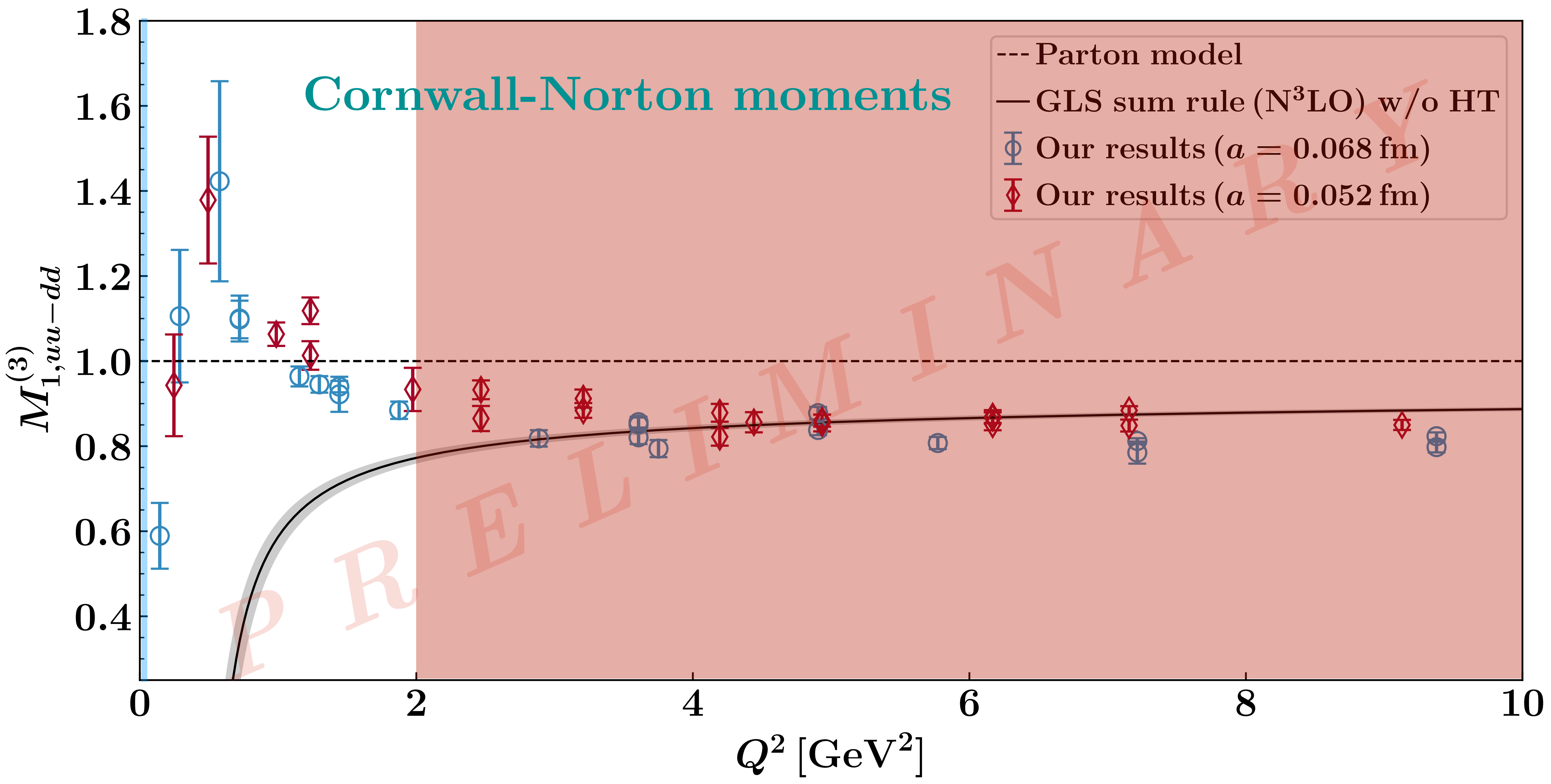
Fit to the Q^2 dependence of moments to extract α_s

$$M_{1,p}^{(3)}(Q^2) = \frac{5}{6}\left[1 + \sum_{i=1}^{4} c_i(n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^i\right] + \frac{\Delta_{HT}}{Q^2}$$

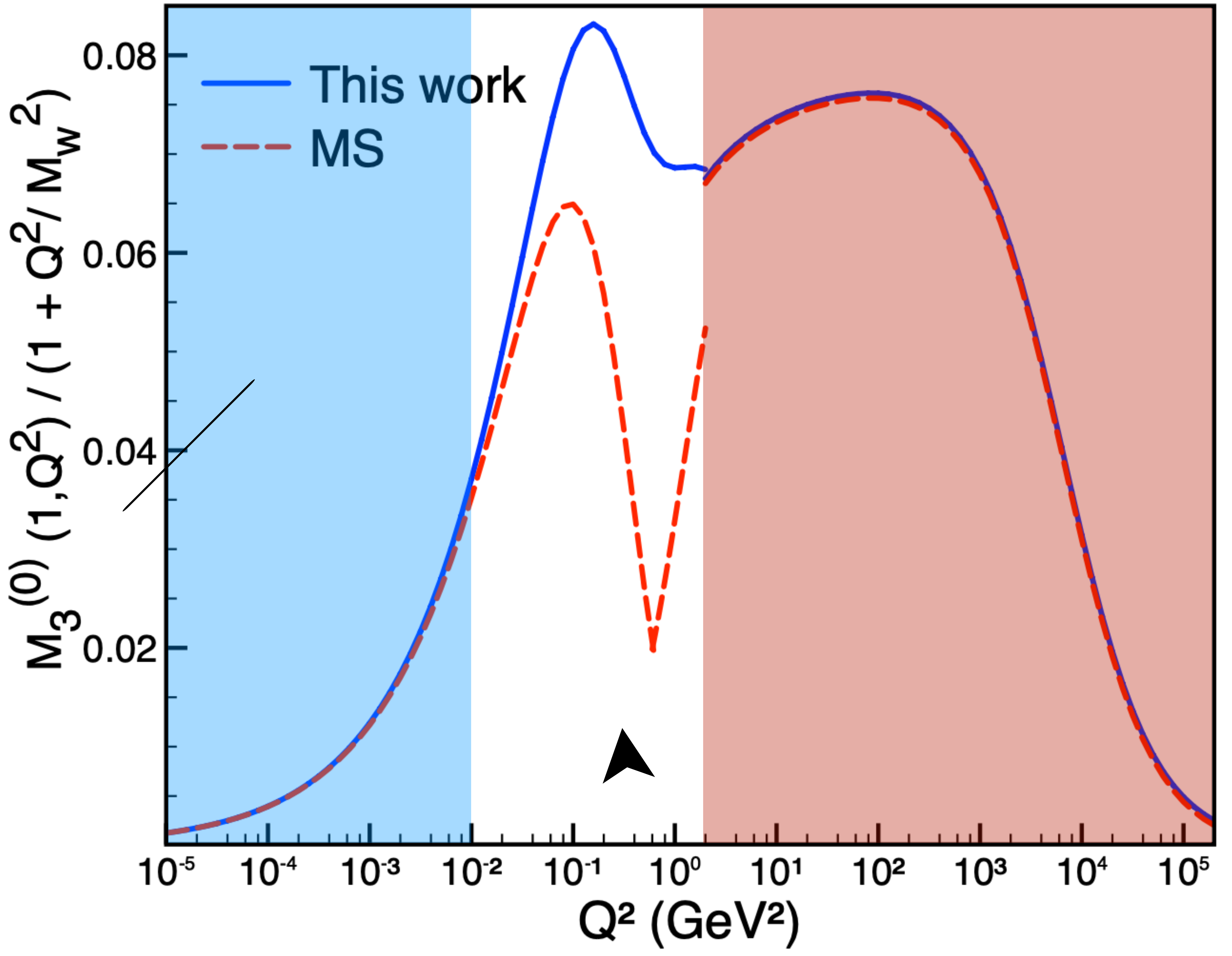
$\mathscr{F}_3^{\gamma W}$ | EW box

- Electroweak box diagrams need Nachtmann moments

$$\Box^{\gamma W}_{VA} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\, C_N(x,Q^2)\, F_3^{(0)}(x,Q^2)$$



C.-Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, PRL 121, 241804 (2018)



form factors

perturbation theory

input from LQCD required

$0.01 \lesssim Q^2 \lesssim 2\ \text{GeV}^2$

$\mathscr{F}_3^{\gamma W}$ | EW box

- Electroweak box diagrams need Nachtmann moments

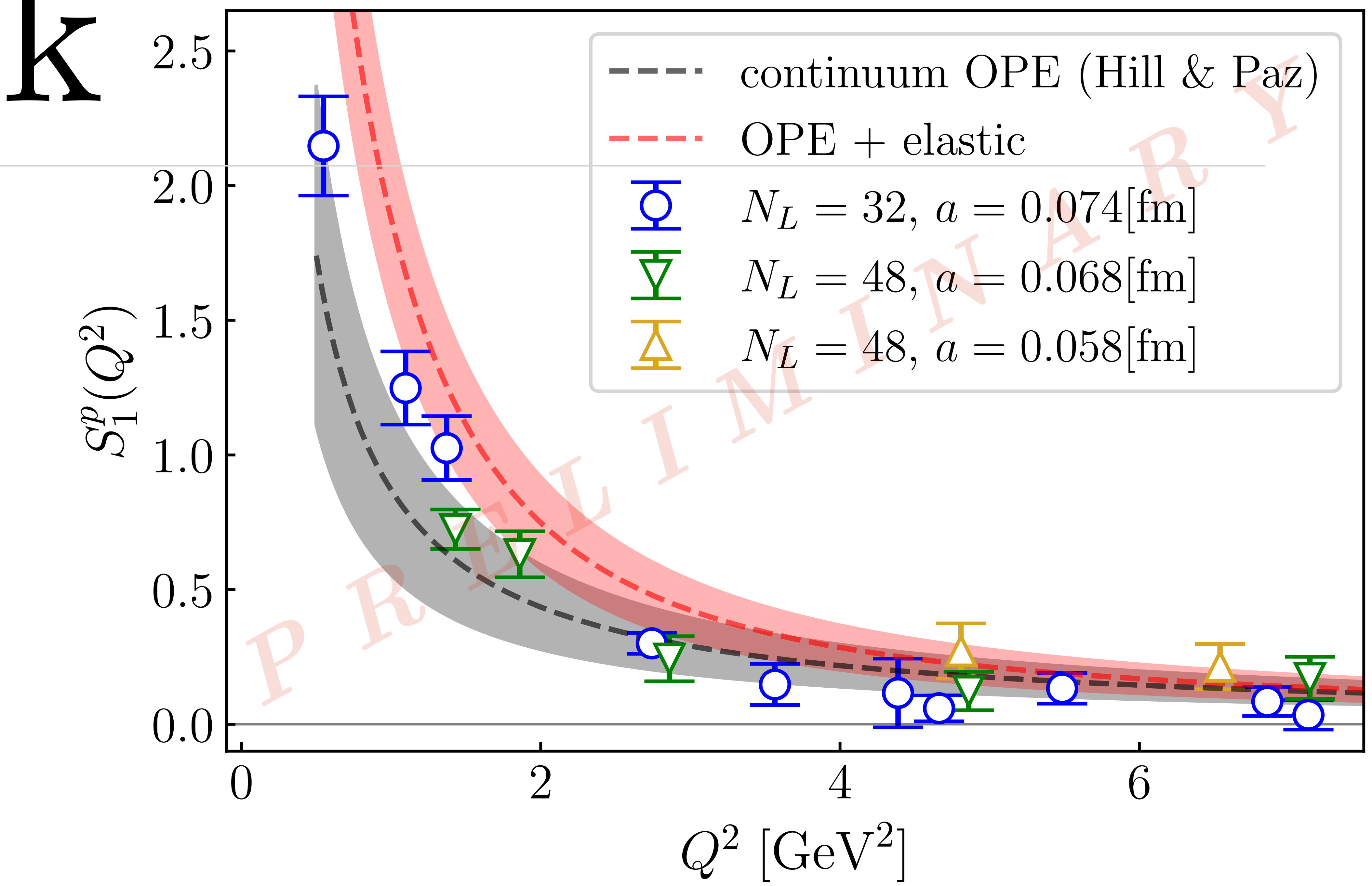
$$\Box_{VA}^{\gamma W} = \frac{3\alpha_{EM}}{2\pi}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\, C_N(x,Q^2)\, F_3^{(0)}(x,Q^2)$$

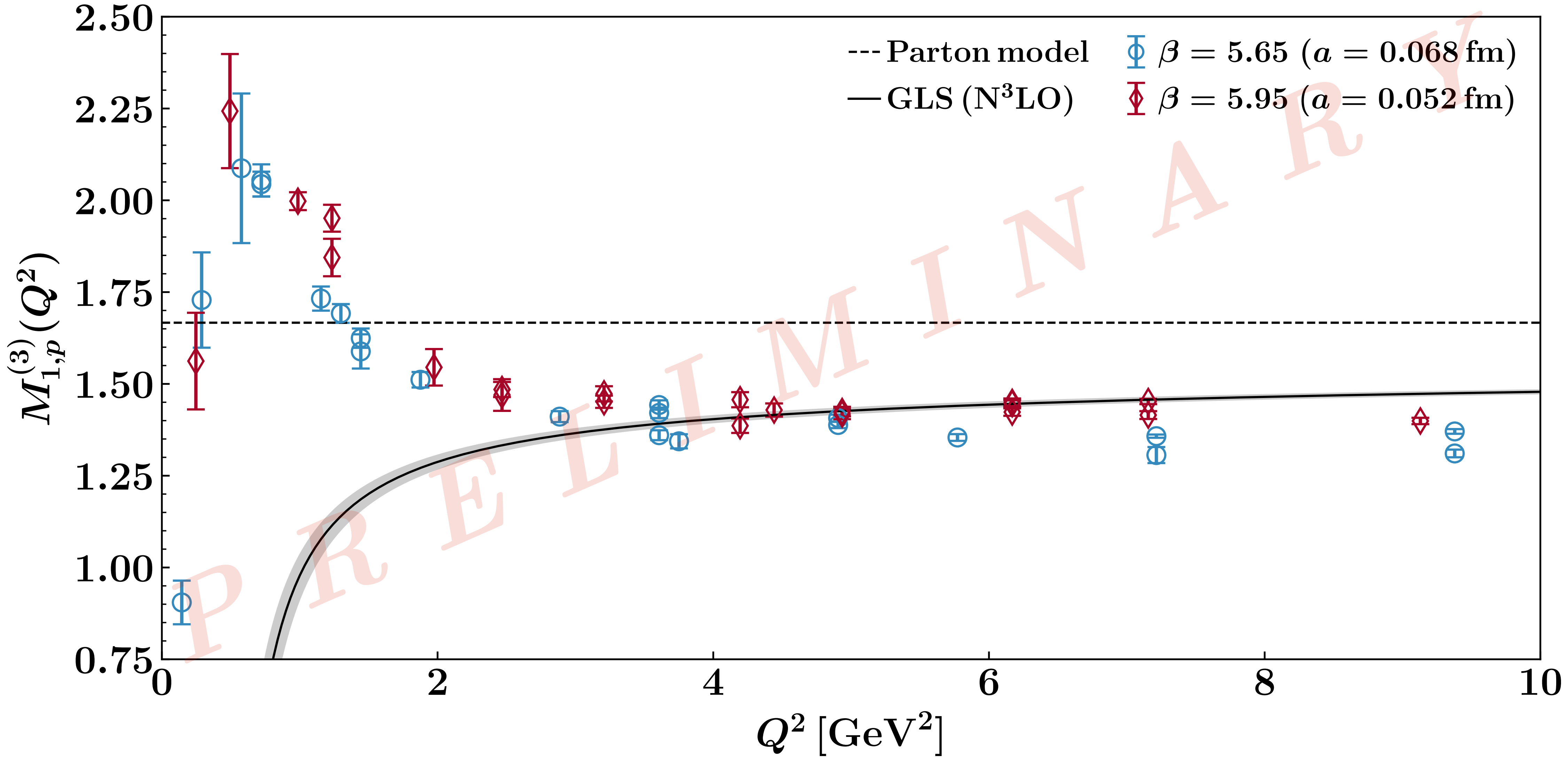
where $F_3^{(0)} = F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z} = \frac{1}{6}\left(F_{3,uu}^{\gamma Z} - F_{3,dd}^{\gamma Z}\right)$

- Short-term: $C_N(x,Q^2)$ can be approximated which allows a precise approximation of Nachtmann moments from lowest 3 Cornwall-Notron moments
- Mid/long-term: Working towards a direct calculation of the moments of $\mathscr{F}_3^{\gamma W}$ to test isospin breaking effects

Summary & Outlook

- ☑ $\mathcal{O}(a)$-improved results
- ☑ Good agreement with OPE/pQCD
- ☑ Clear indication of higher-twist and non-perturbative effects
- ☐ Working towards:
  - ☐ Full control over lattice artefacts, e.g. a, M_π, V dependence
  - ☐ Constraining the subtraction function over a wide range of Q^2
  - ☐ Estimating EW box contribution and effects of isospin breaking





Acknowledgements

- The numerical configuration generation (using the BQCD lattice QCD program)) and data analysis (using the Chroma software library) was carried on the
  - DiRAC Blue Gene Q and Extreme Scaling (EPCC, Edinburgh, UK) and Data Intensive (Cambridge, UK) services,
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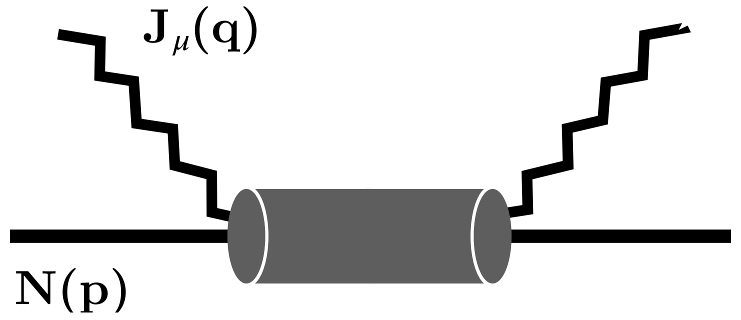
Backup

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p,q) = \int d^4z\, e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p)\,|\,\mathscr{T}\{J_\mu(z)J_\mu(0)\}\,|\,N(p)\rangle$$



- **Action modification**

local EM current

$$J_\mu(z) = \sum_q e_q \bar{q}(z)\gamma_\mu q(z)$$

$$S \to S(\lambda) = S + \lambda \int d^4z\,(e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}})\,J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G^{(2)}_\lambda(\mathbf{p};t)$, in the presence of the external field**

$$\left.\frac{\partial^2 G^{(2)}_\lambda(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial\lambda^2} - tA(\mathbf{p})\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right)\, e^{-E_N(\mathbf{p})t}$$

from spectral decomposition

$$\left.\frac{\partial^2 G^{(2)}_\lambda(\mathbf{p};t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z (e^{iq\cdot z} + e^{-iq\cdot z})\langle N(\mathbf{p})|\mathcal{T}\{\mathcal{J}(z)\mathcal{J}(0)\}|N(\mathbf{p})\rangle$$

from path integral

- **equate the time-enhanced terms:**

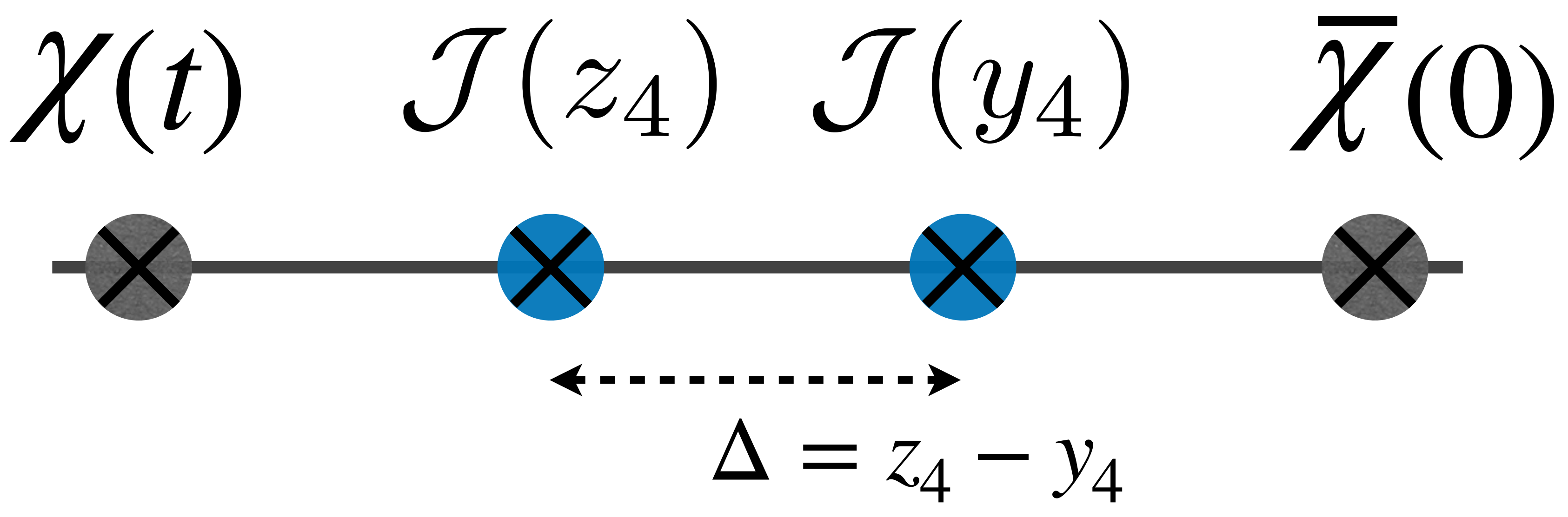
$$\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial\lambda^2}\right|_{\lambda=0} = -\frac{1}{2E_N(\mathbf{p})}\overbrace{\int d^4z (e^{iq\cdot z} + e^{-iq\cdot z})\langle N(\mathbf{p})|\mathcal{J}(z)\mathcal{J}(0)|N(\mathbf{p})\rangle}^{T_{\mu\mu}(p,q)} + (q \to -q)$$

<u>**Compton amplitude is related to the second-order energy shift**</u>

kuc et al. (CSSM/QCDSF/UKQCD) PRD102, 114505 (2020), arXiv:2007.01523 [hep-lat]

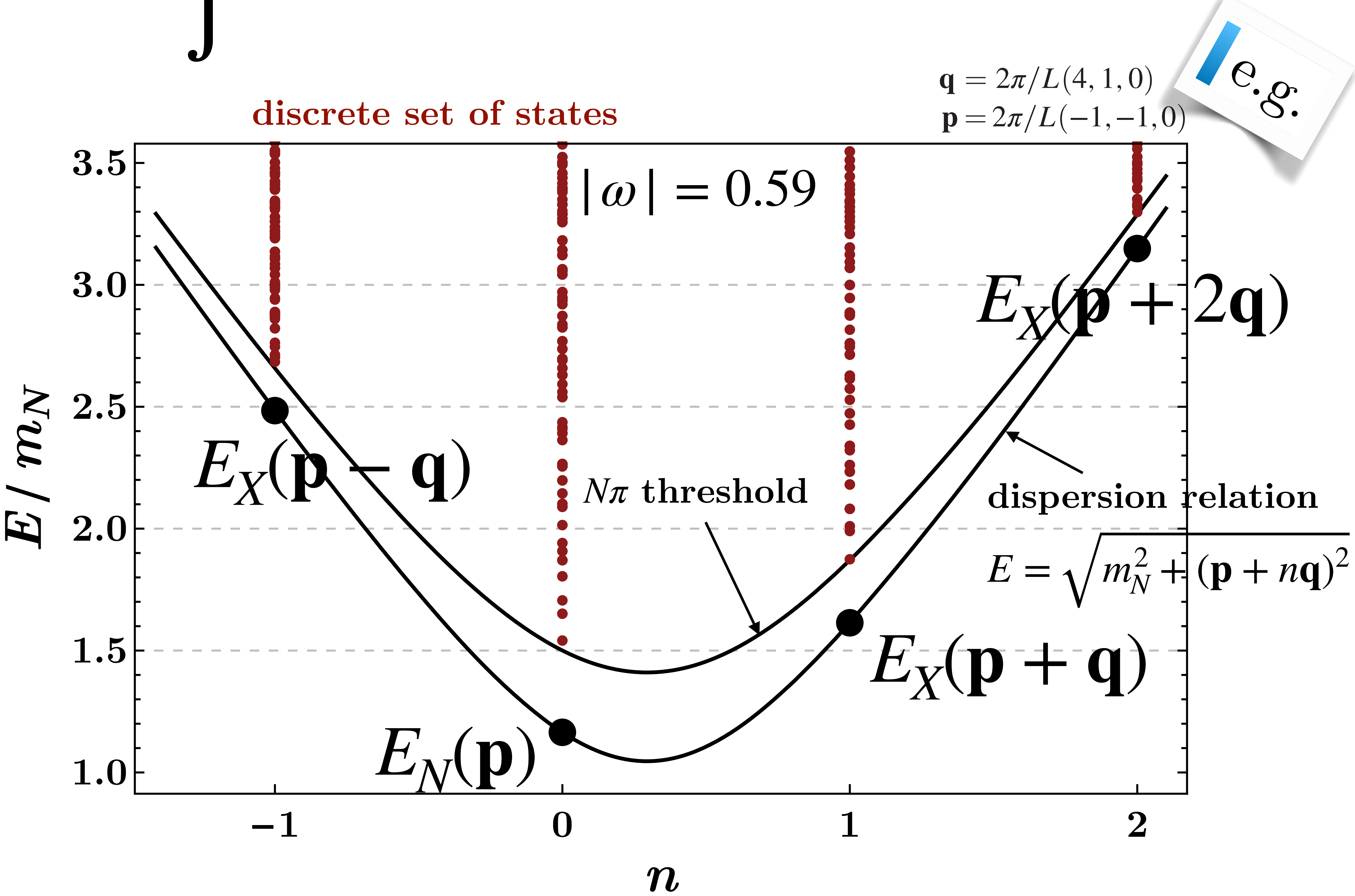
Compton amplitude via the FH relation at 2nd order

- **relevant contribution comes from the ordering where the currents are sandwiched**



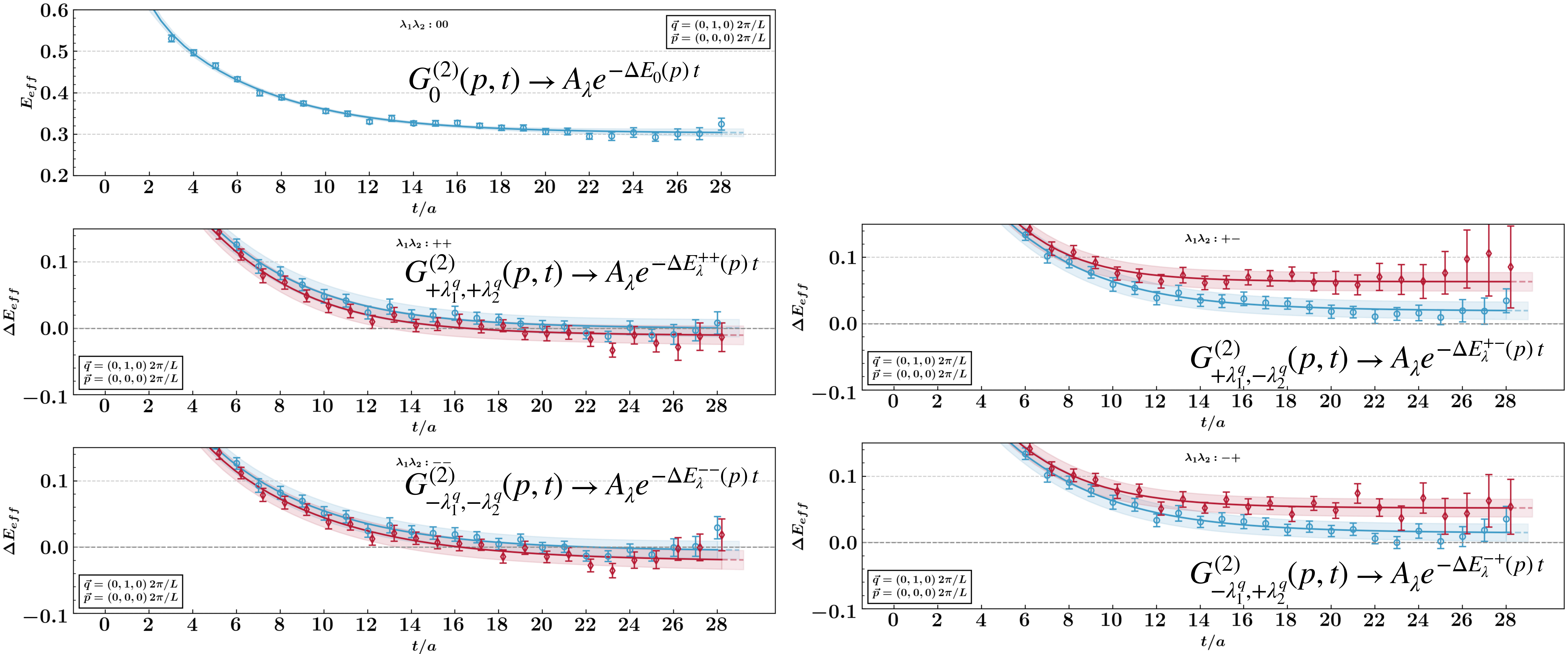
$$\sim e^{-E_N(\mathbf{p})t} \int^t d\Delta\, e^{-\left(E_X(\mathbf{p}+\mathbf{q}) - E_N(\mathbf{p})\right)\Delta} (t-\Delta)$$

- under the condition $|\omega| < 1$, $E_X(\mathbf{p}+n\mathbf{q}) \gtrsim E_N(\mathbf{p})$, so the intermediate states <u>cannot go on-shell</u>
- ground state dominance is ensured in the large time limit



Multi-exp fits ($Q^2 \lesssim 1\ \mathrm{GeV}^2$)

Second order energy shift: $\Delta E_{N_\lambda}(p) = \frac{1}{4}\left[\Delta E_\lambda^{++}(p) + \Delta E_\lambda^{--}(p) - \Delta E_\lambda^{+-}(p) - \Delta E_\lambda^{-+}(p)\right] - E_0(p)$



Future lattices

Currently thermalising/generating

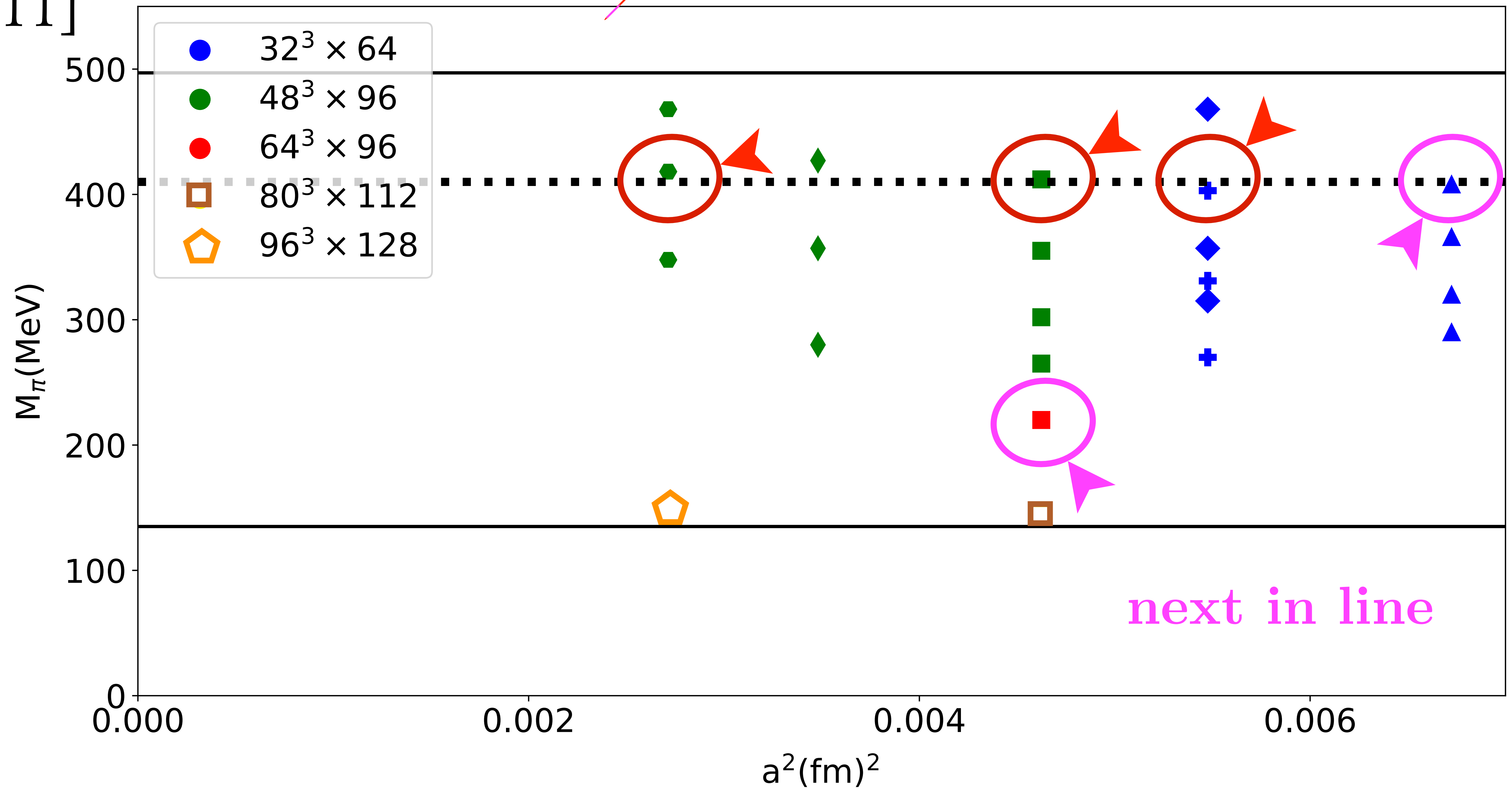
- $64^3 \times 96$, $a = (0.068, 0.052)$ fm, $m_\pi = (220, 270)$ MeV *(completed - early 2024)*
- $80^3 \times 114$, $a = 0.068$ fm, $m_\pi = 150$ MeV *(still thermalising)*
- $96^3 \times 128$, $a = 0.052$ fm, $m_\pi = 140$ MeV *(thermalised + O(50) trajectories)*

Using BQCD [EPJ Web Conf. 175 (2018) 14011]

on

- JUWELS (Jülich, Germany)
- CSD3 (Cambridge, UK)
- Tursa (Edinburgh, UK)

Compton results so far



next in line