



# Energy Correlators, Heavy Flavor, & Precision QCD

Evan Craft — Yale University  
MITP



Based on work with K. Lee, B. Mecaj, I. Moulton, & M. Gonzalez



Yale University



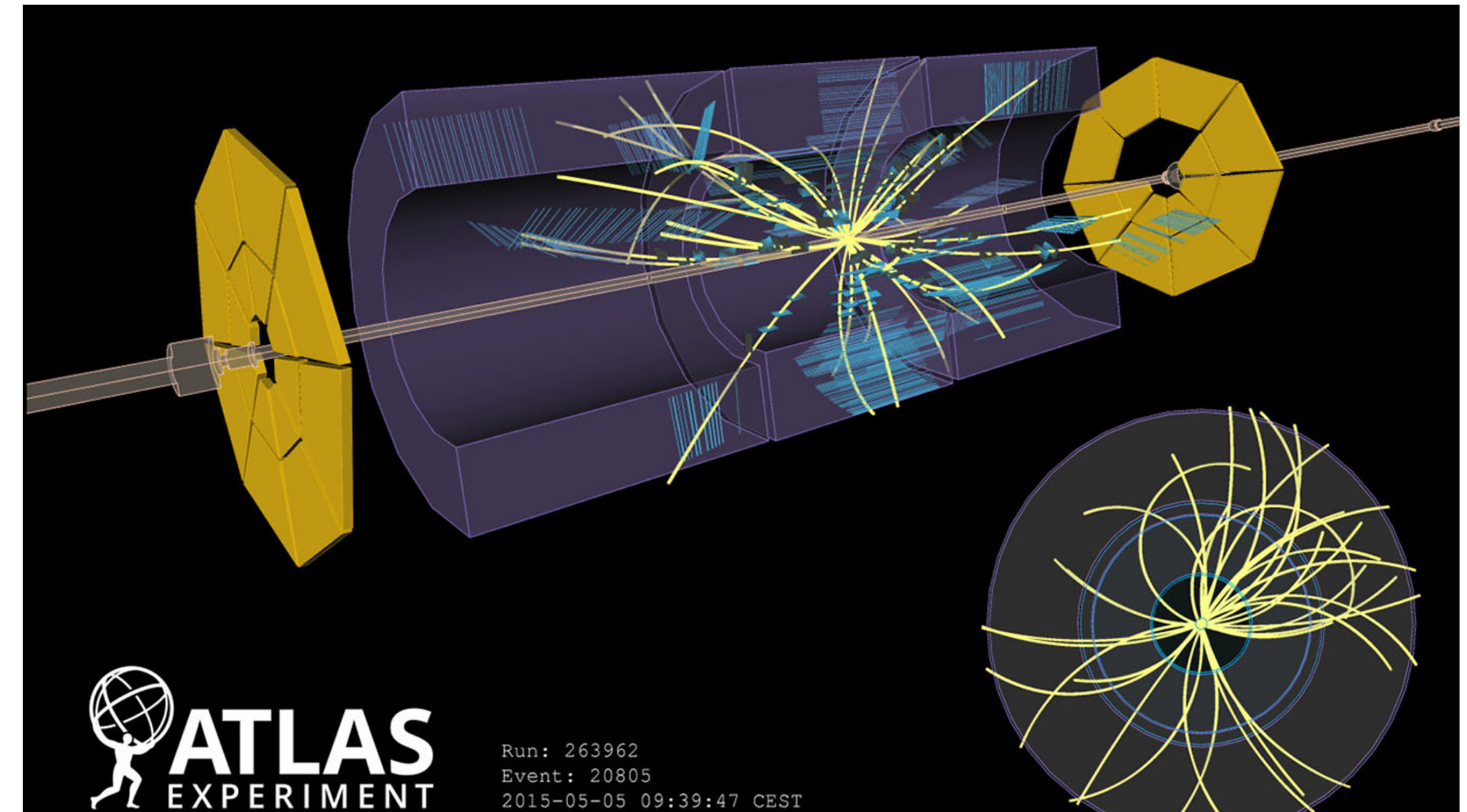
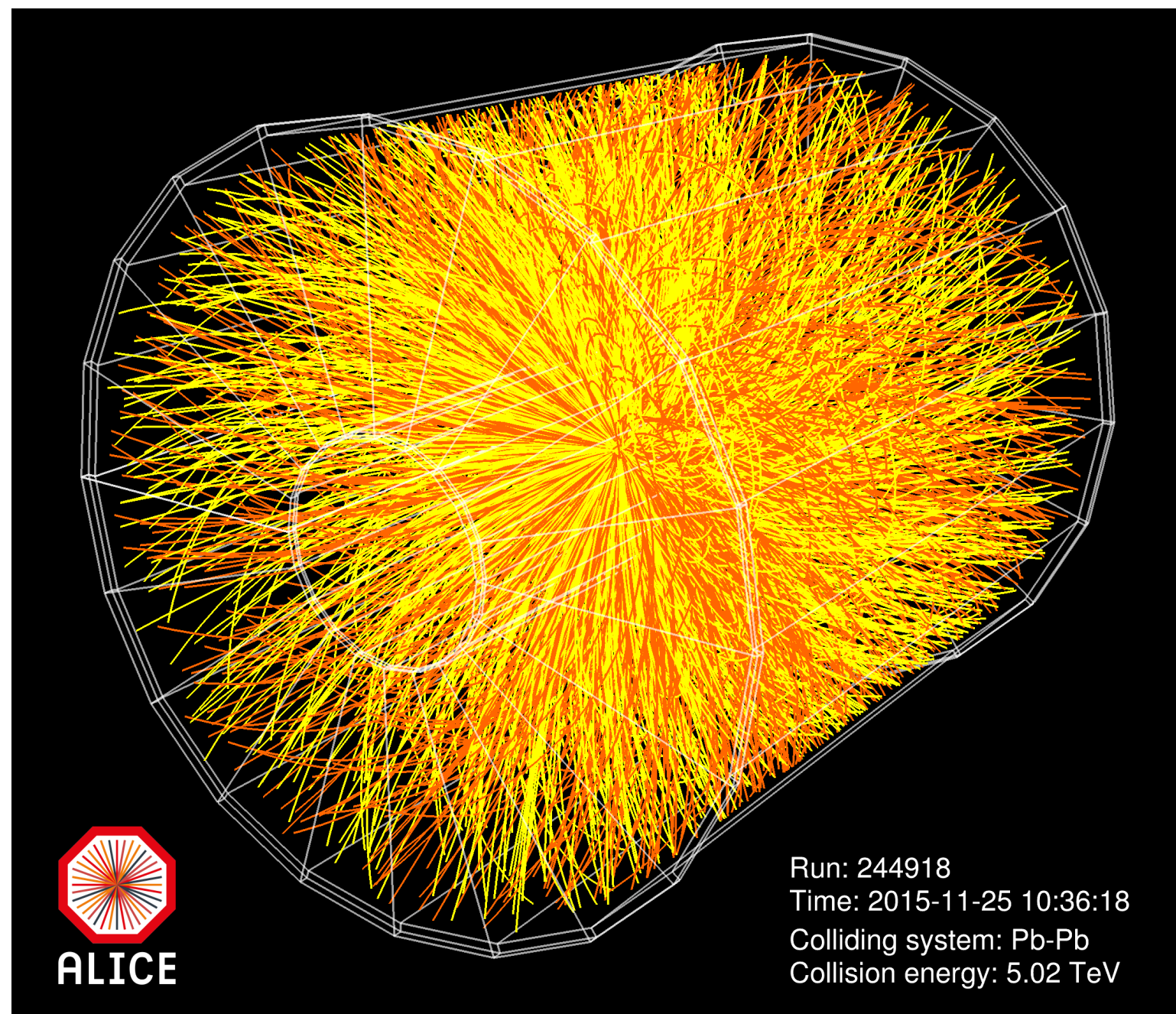
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# Collider Experiments

Many important questions have been addressed at **collider experiments**

→ Great historical success in verifying properties of the standard model



→ But the detailed structure of QCD produces immensely complicated datasets.

→ Need new tools for future success

A unique frontier for novel collaborations between both **theory and experiment**

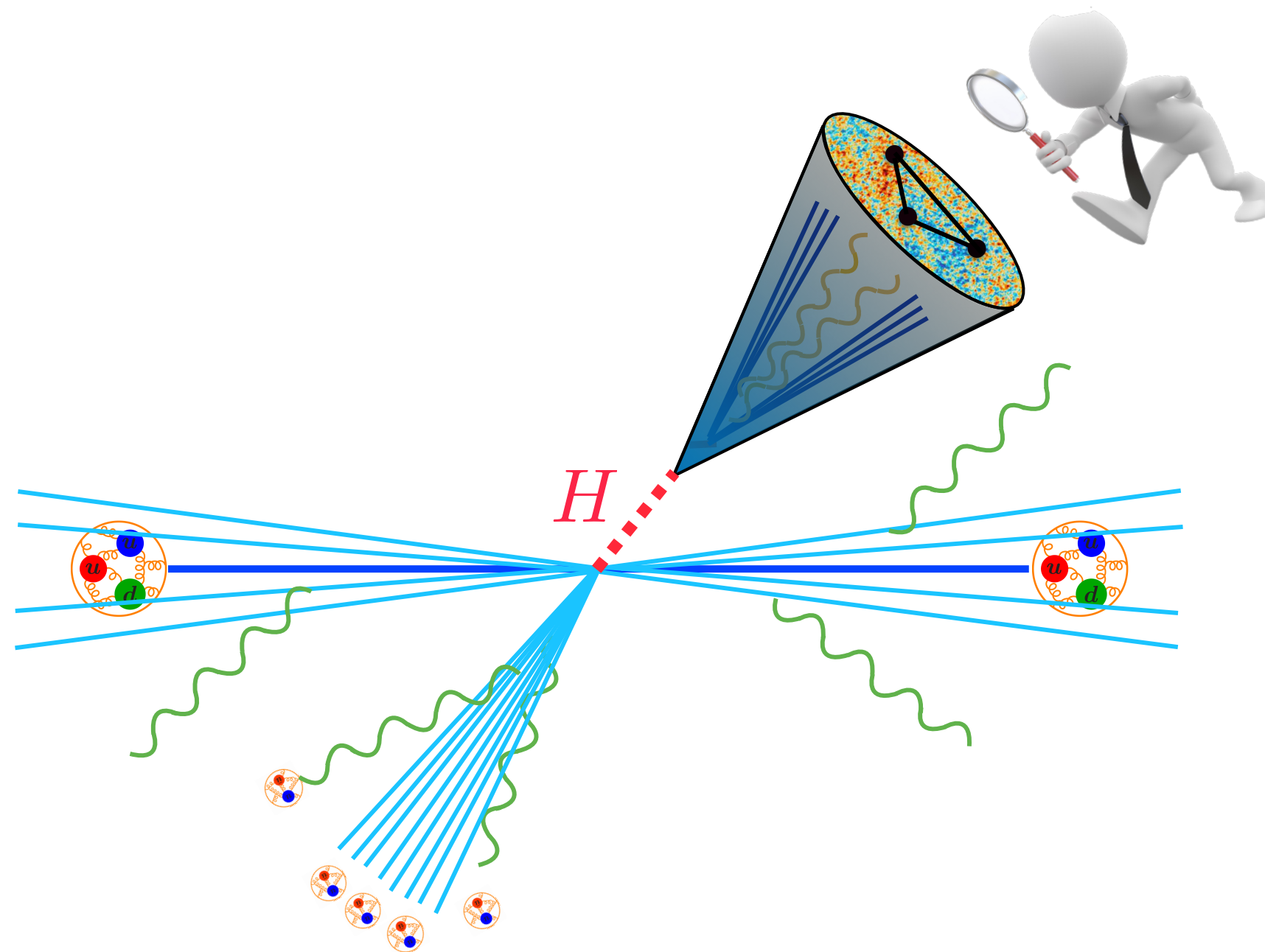


# Jet Substructure

In recent years, strong success in high energy physics has been driven by remarkable progress in **jet substructure**.

→ Uses the internal structure of jets to provide **qualitatively new** ways to study physics at colliders

→ Has impacted multiple collider experiments ranging from proton colliders all the way up to heavy ions.



## Jet substructure as a new Higgs search channel at the LHC

Jonathan M. Butterworth (University Coll. London), Adam R. Davison (University Coll. London), Mathieu Rubin (Paris, LPTHE), Gavin P. Salam (Paris, LPTHE)

Sep, 2008

3 pages

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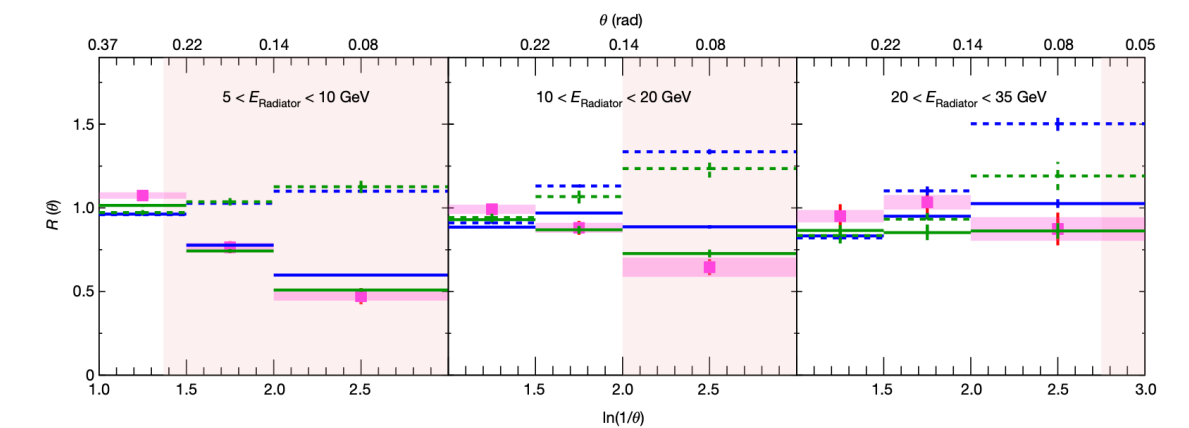
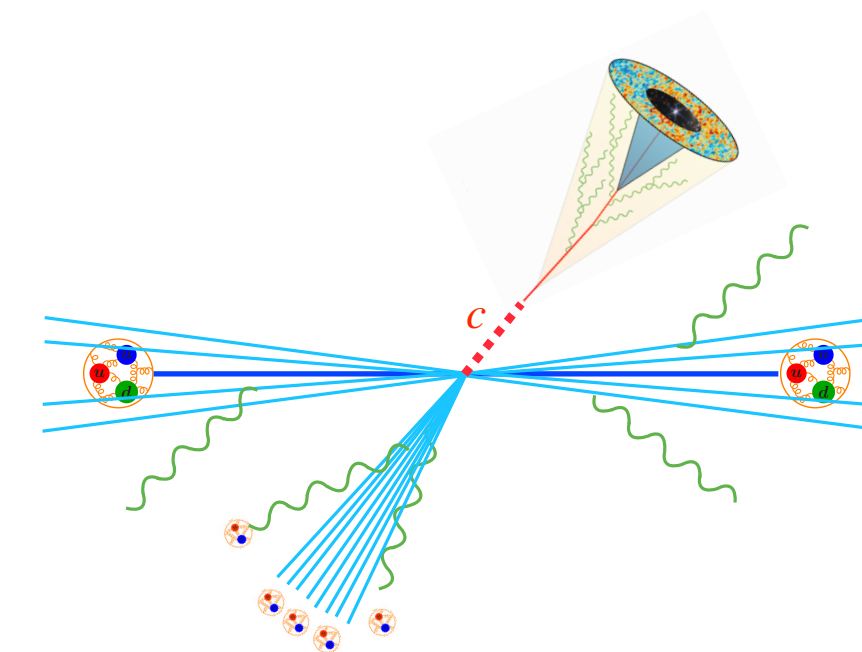
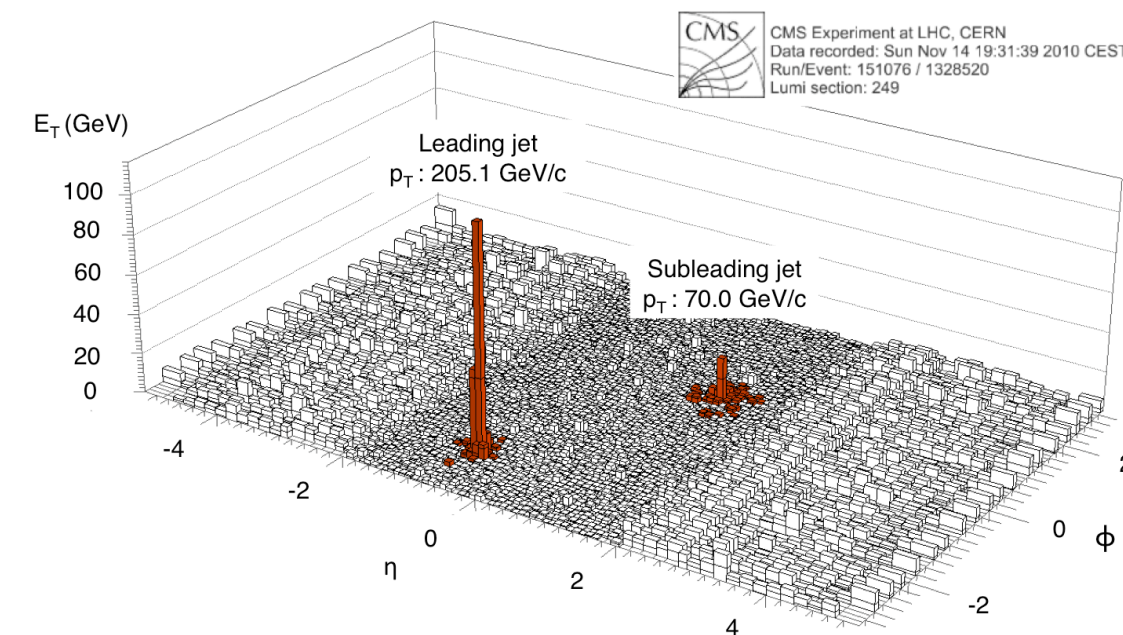
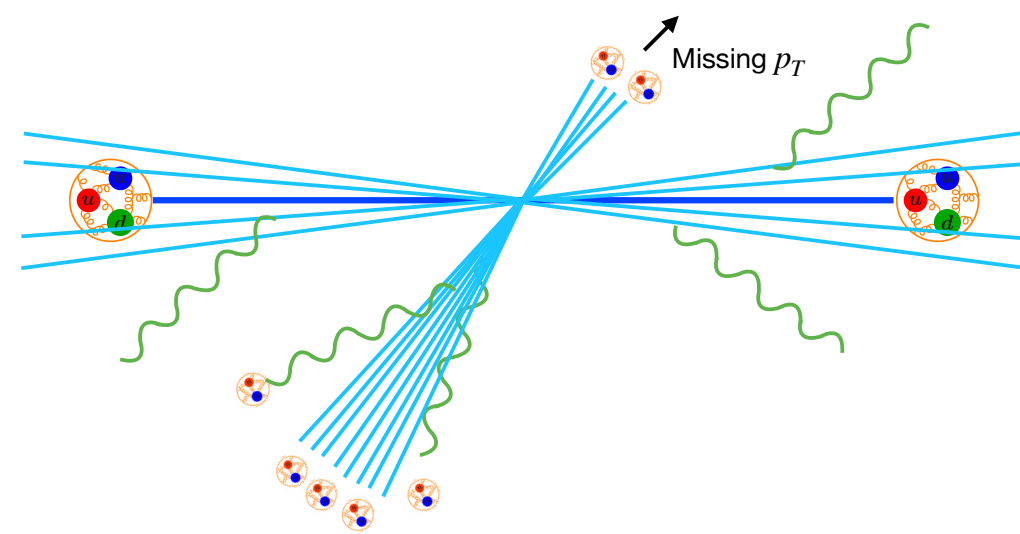
# How do we study collisions?

Along with **many** more exciting observations!

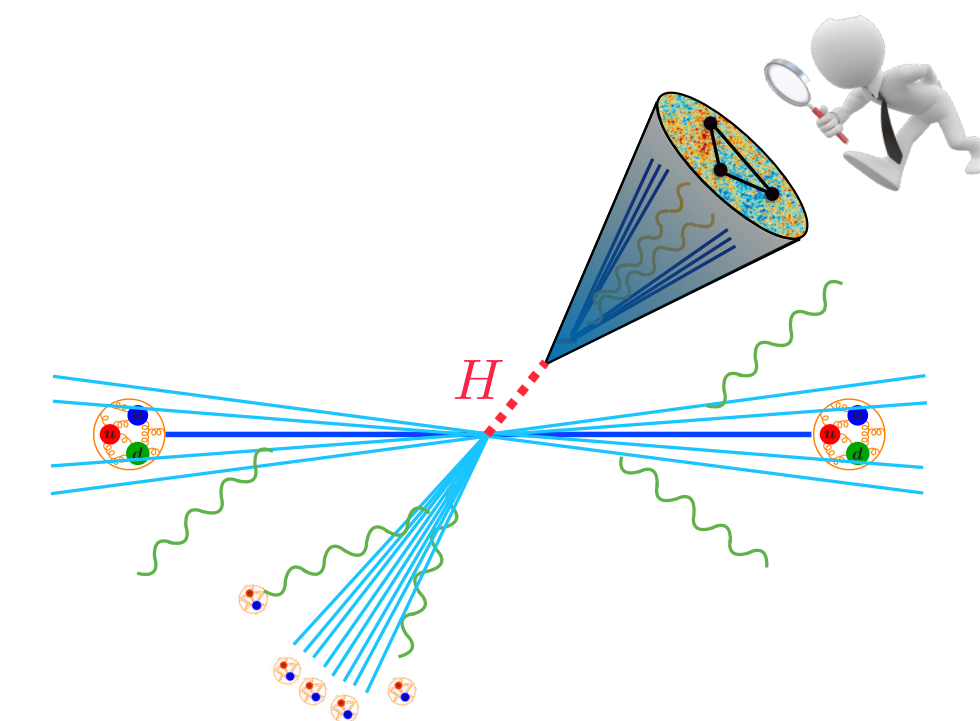
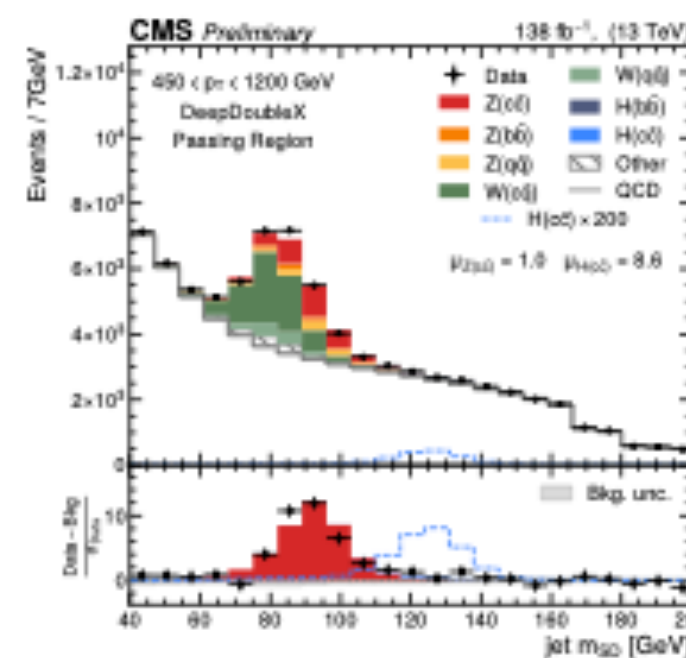
Several remarkable observations made by studying **jets**

→ provided initial evidence for the **existence** of the Quark Gluon Plasma

→ allowed first ever observation of the **“dead cone”** effect of QCD



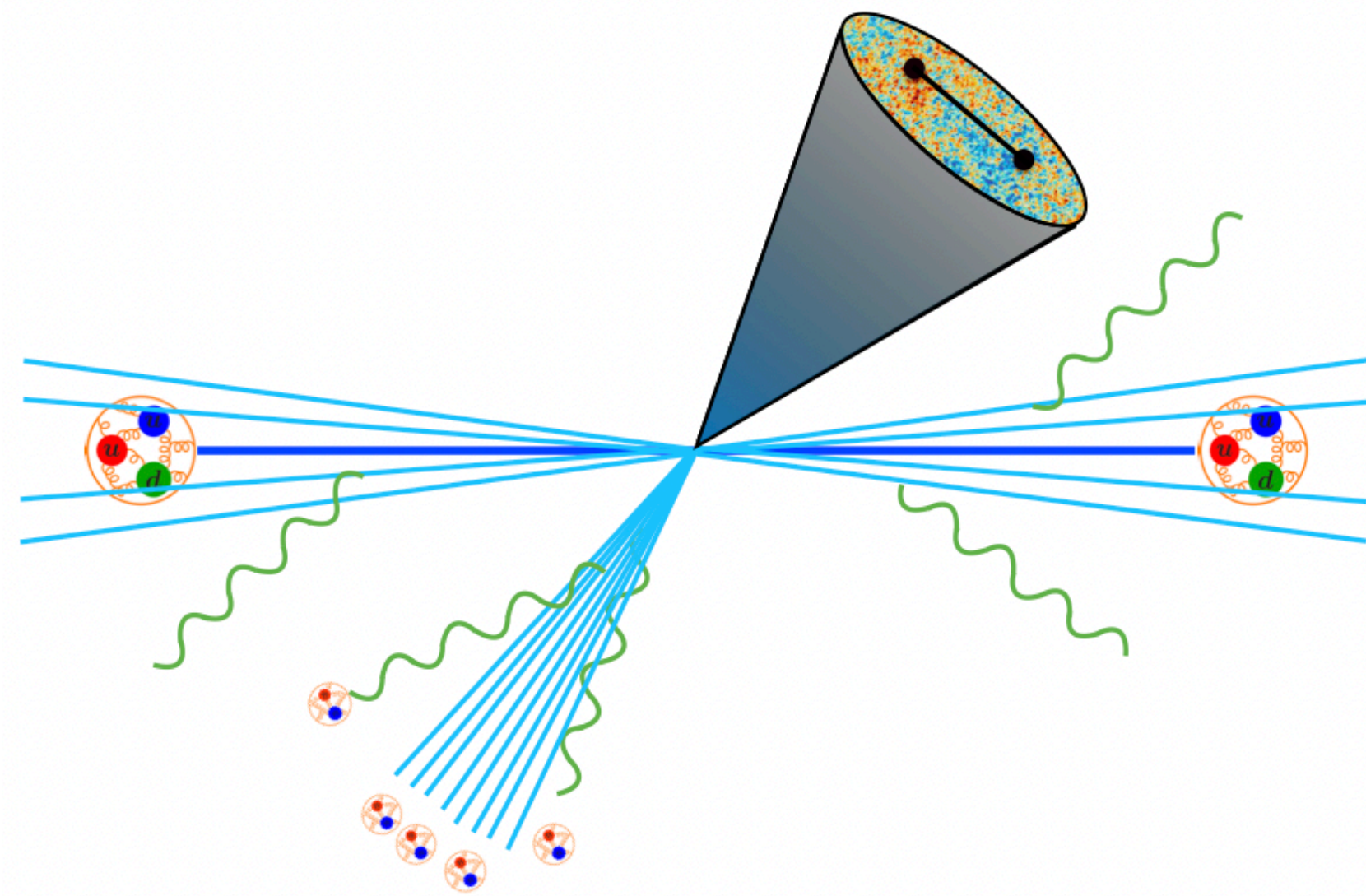
→ provides the most stringent bounds on **charm** Yukawa couplings



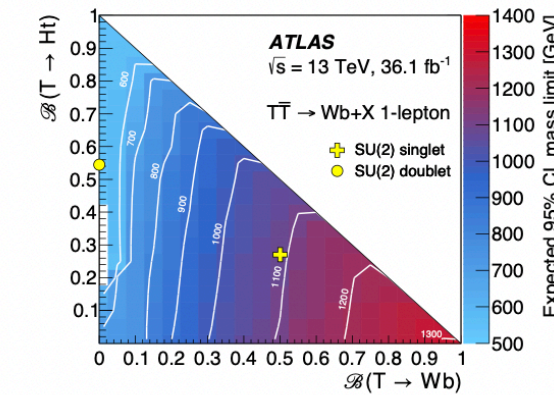
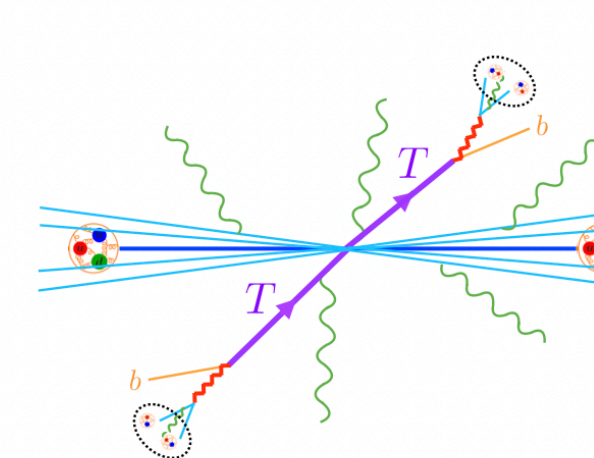
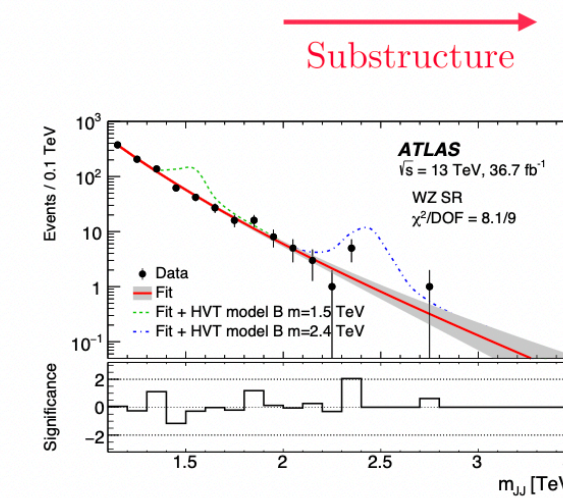
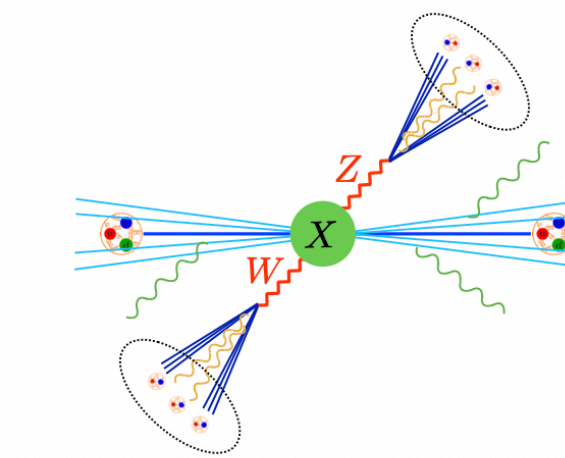
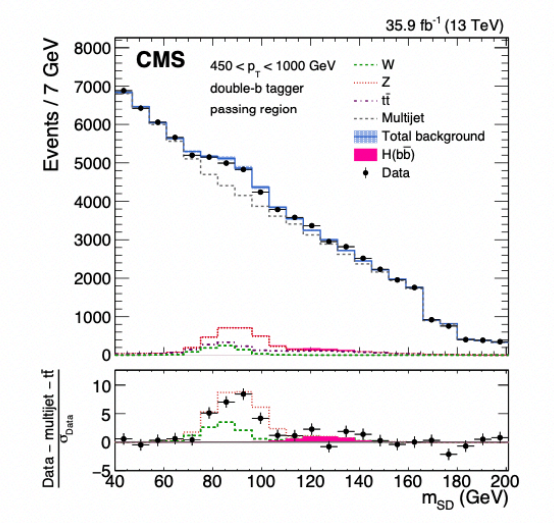
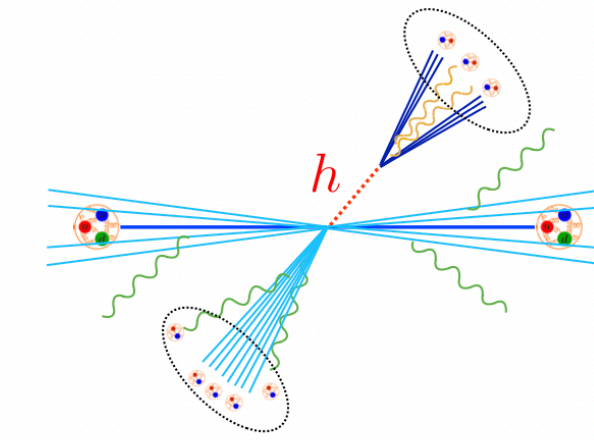
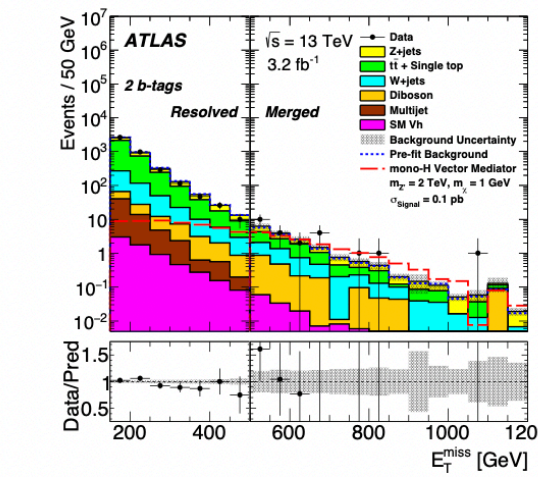
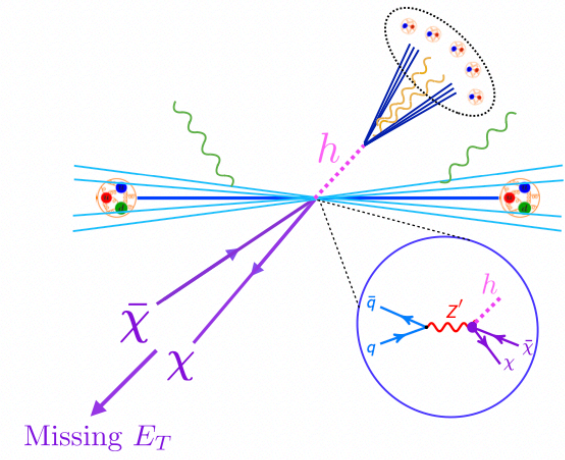
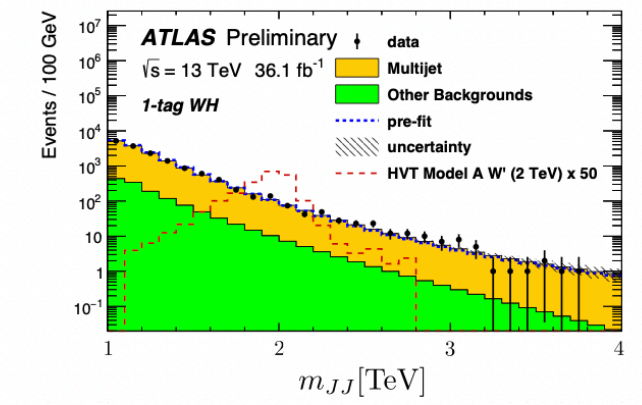
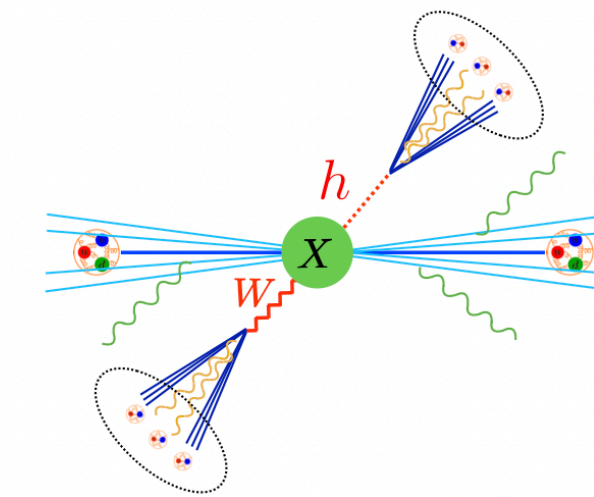
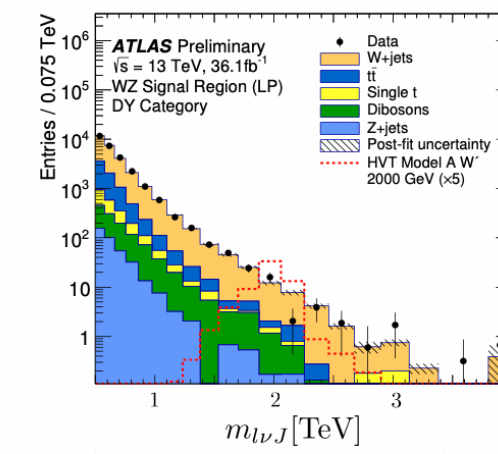
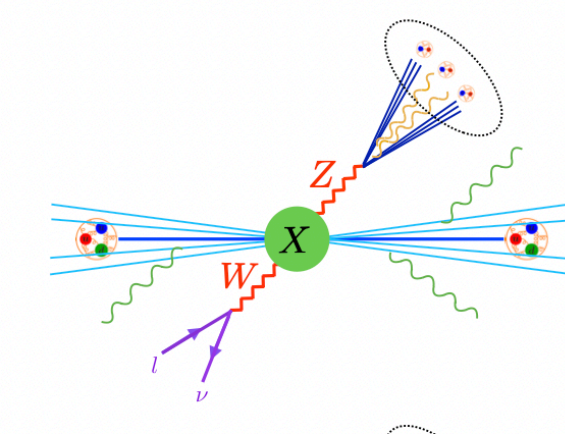


# Looking Forward

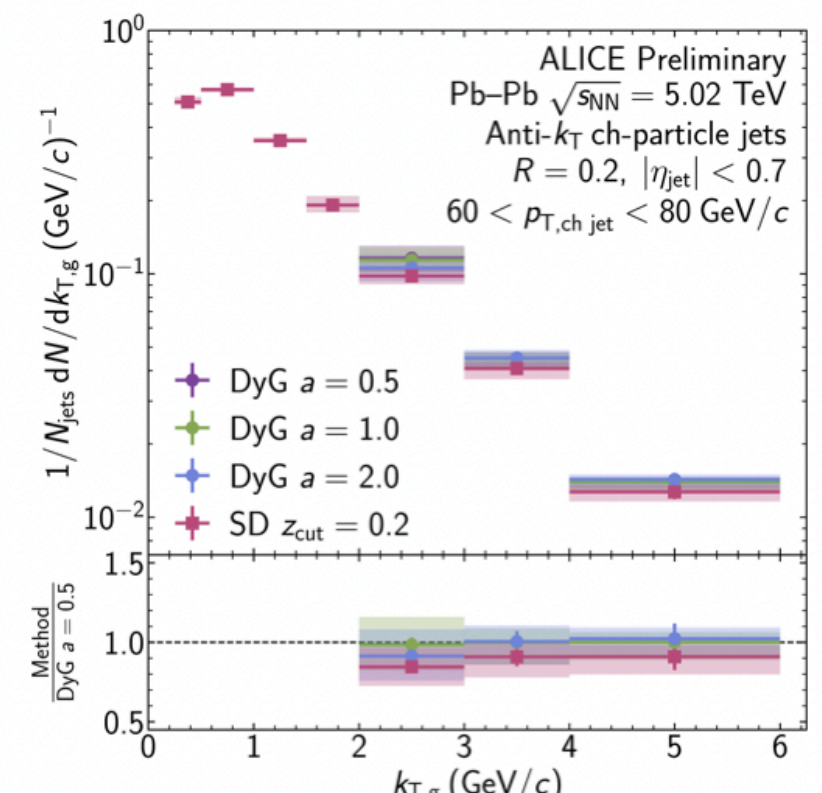
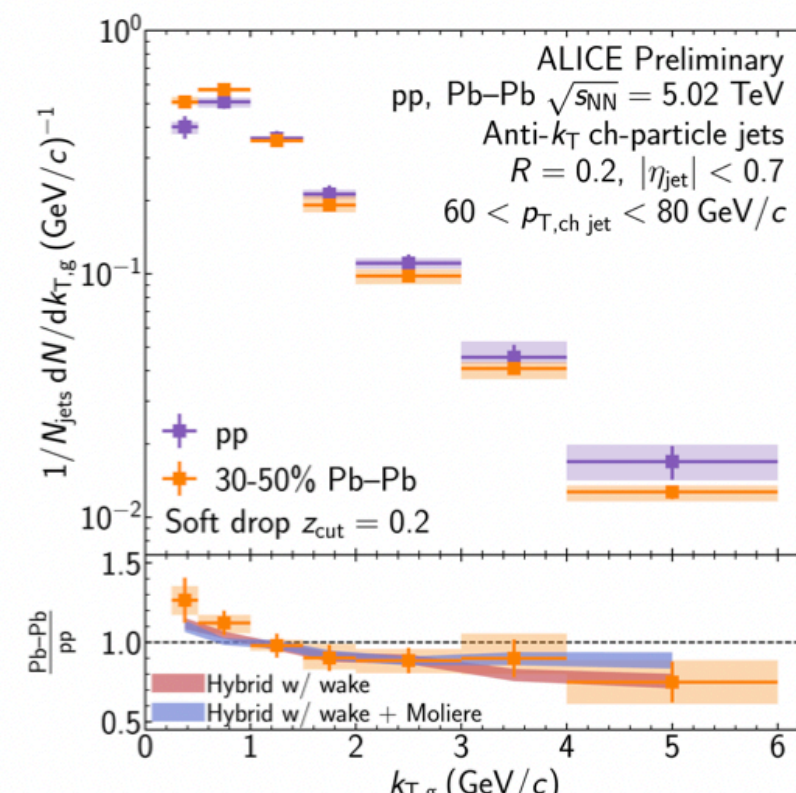
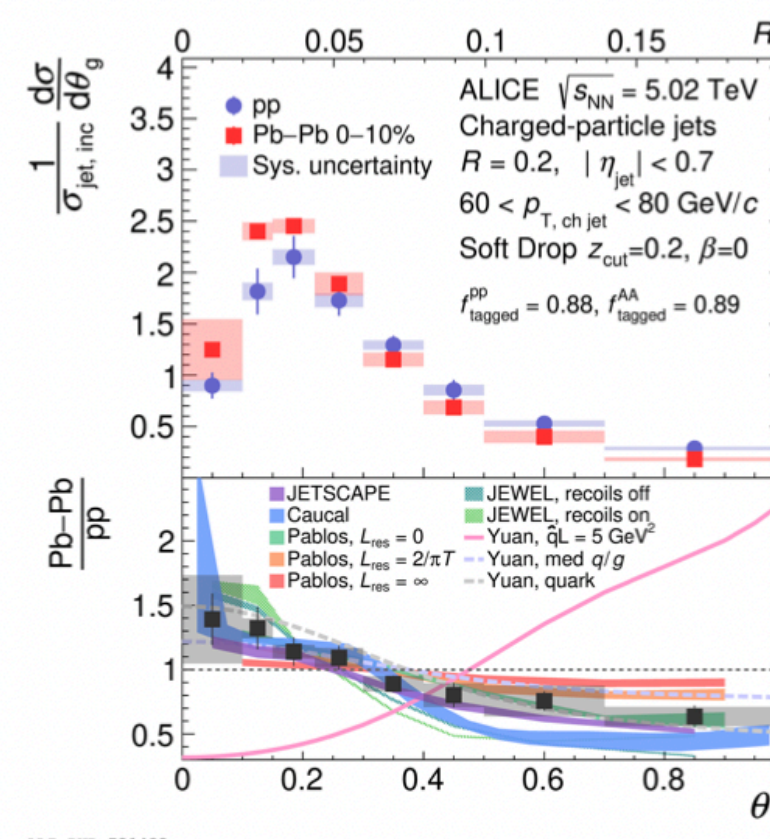
Large impact on **future** searches and probes across nuclear/particle physics



→ **Jet Substructure** is intrinsically tied to the future of collider studies



→ **(B)SM searches at the ATLAS and CMS**



→ **Quark Gluon Plasma studies at ALICE**

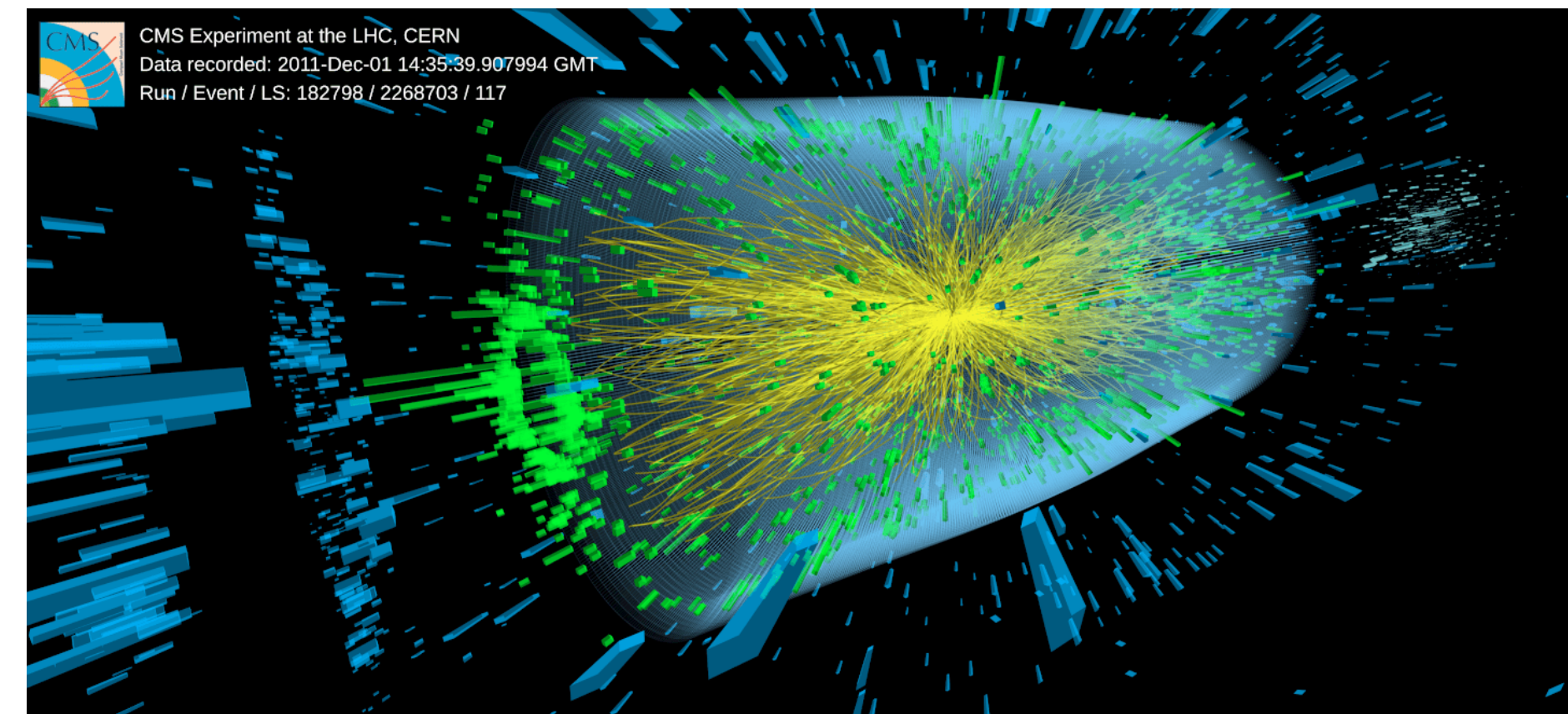
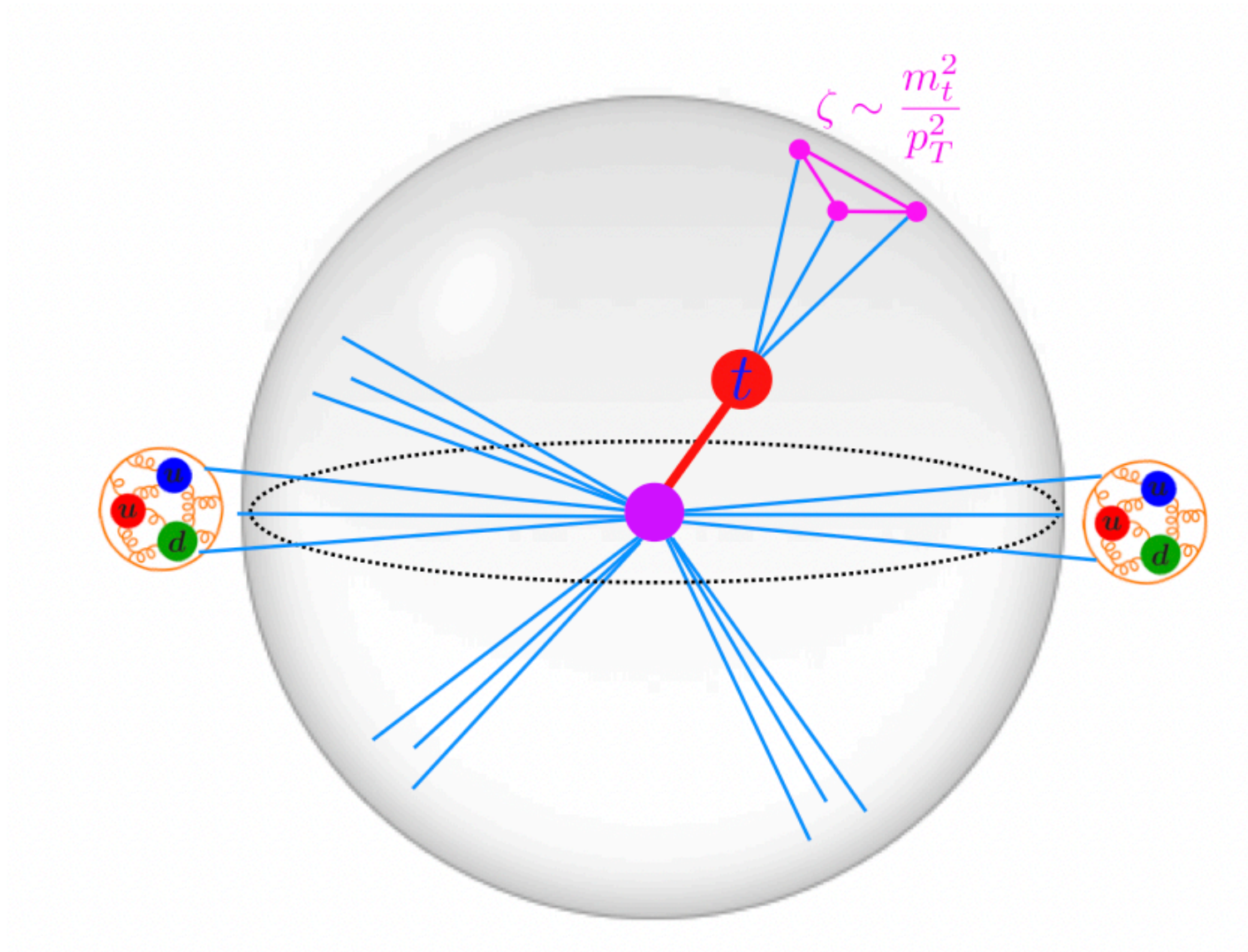
[H. Bossi]



# From Searches to Measurements

To fully take advantage of the LHC, it is necessary to bolster our current physics searches with **first principles theory calculations**

→ Many interesting opportunities to study QCD at high energies: understanding confinement, precision measurements, ...



Requires the development of a **new set of theoretical tools**



# Outline

## 1. Energy Flow Operators

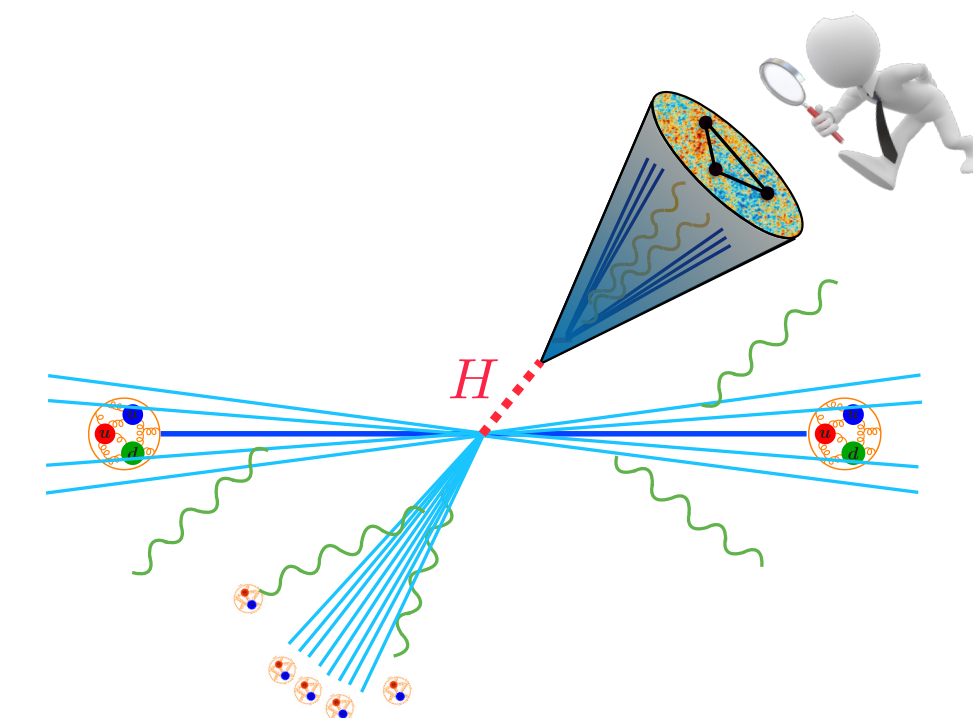
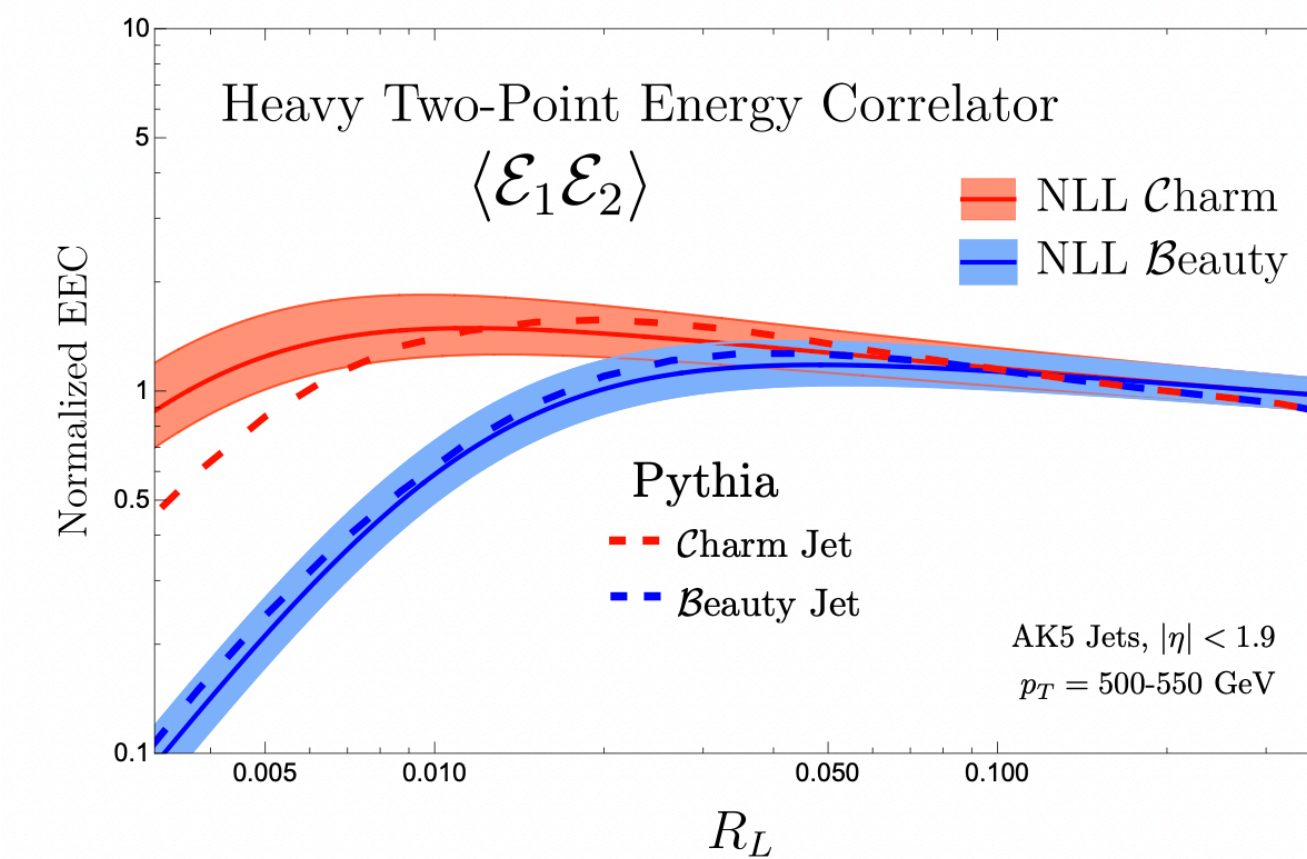
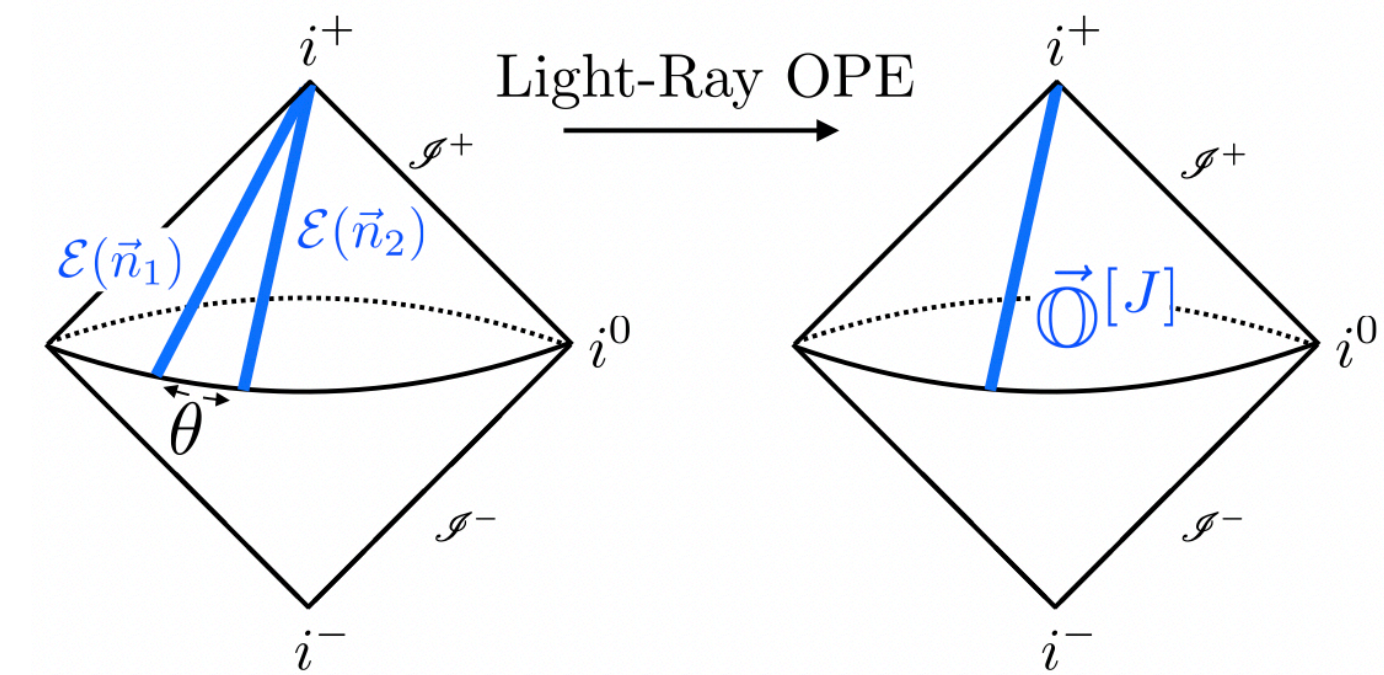
→ General Overview

## 2. Applications to Heavy Flavor

→ Two Point Heavy Collinear EEC

## 3. The Frontiers of Jet Physics

→ Higher Point Correlators





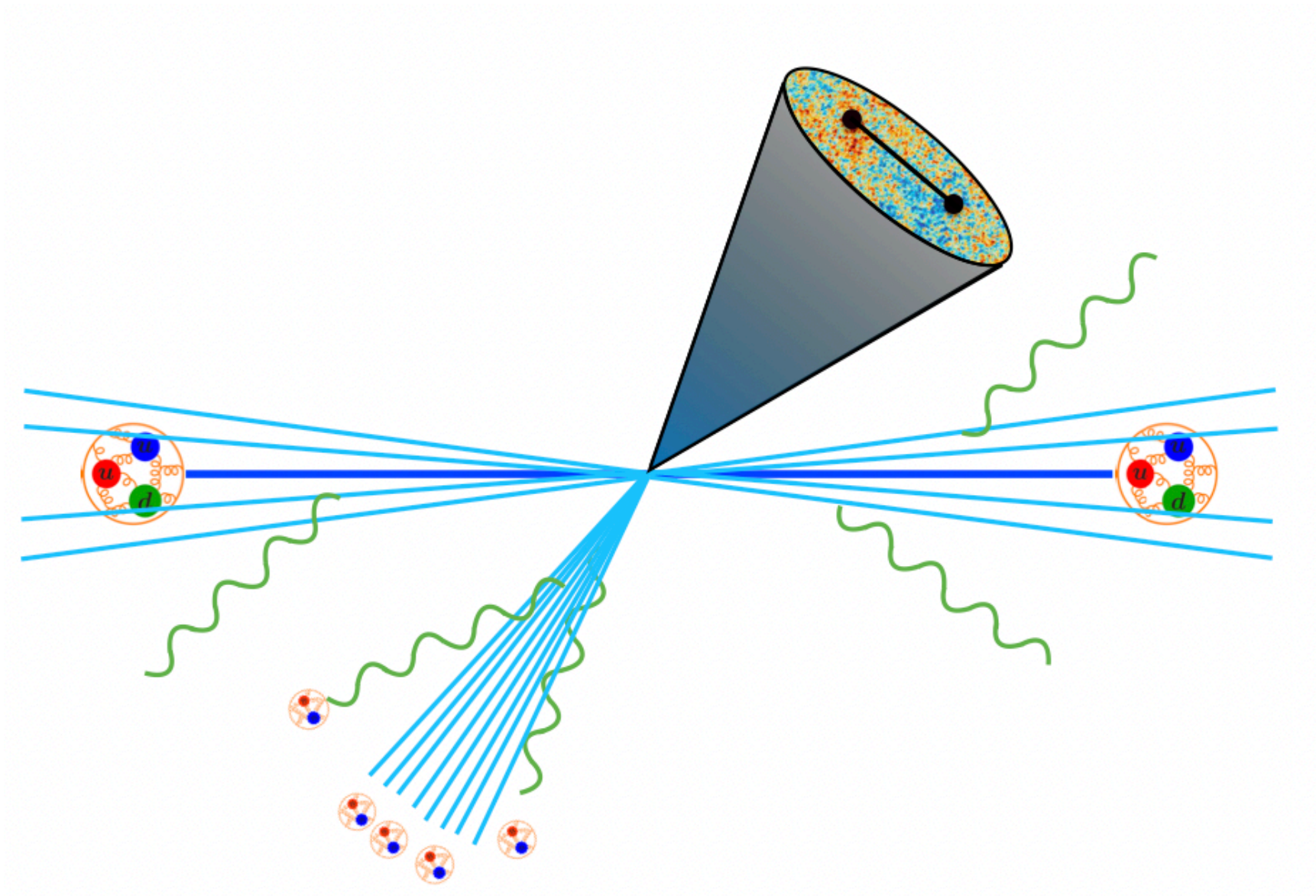
# Energy Flow Operators

Field Theoretic Foundations



# Energy Flow Operators

From the perspective of QFT, jet substructure is the study of **correlation functions** of energy flow operators



$$\mathcal{E}(\vec{n}) = \lim_{t \rightarrow \infty} t^3 \int_0^1 dv v^2 \left[ n^i T_i^0(t, tv \vec{n}) \right]$$

Sveshnikov, Tkachov (1995)

→ “Energy Flow Operator”

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

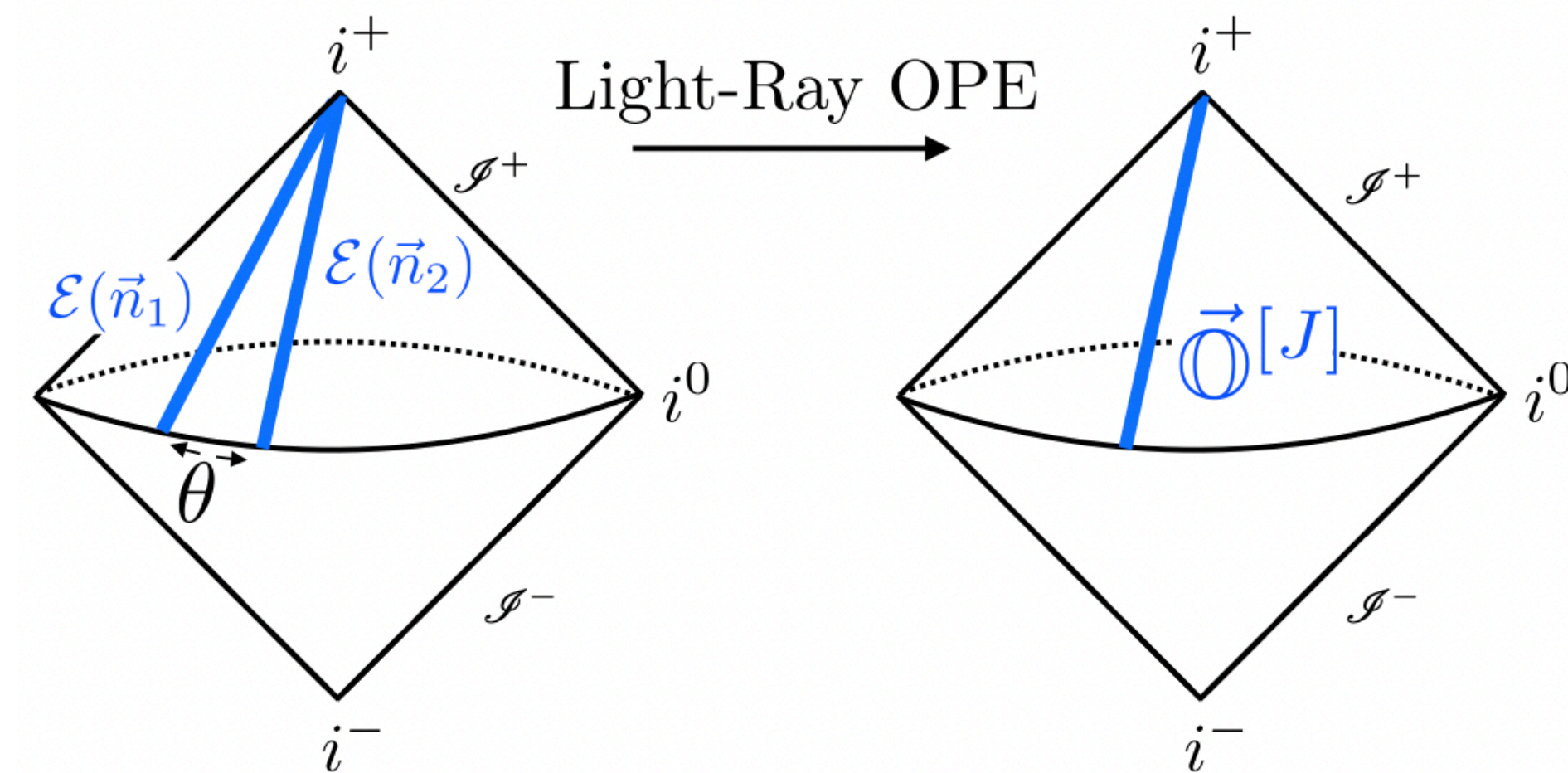
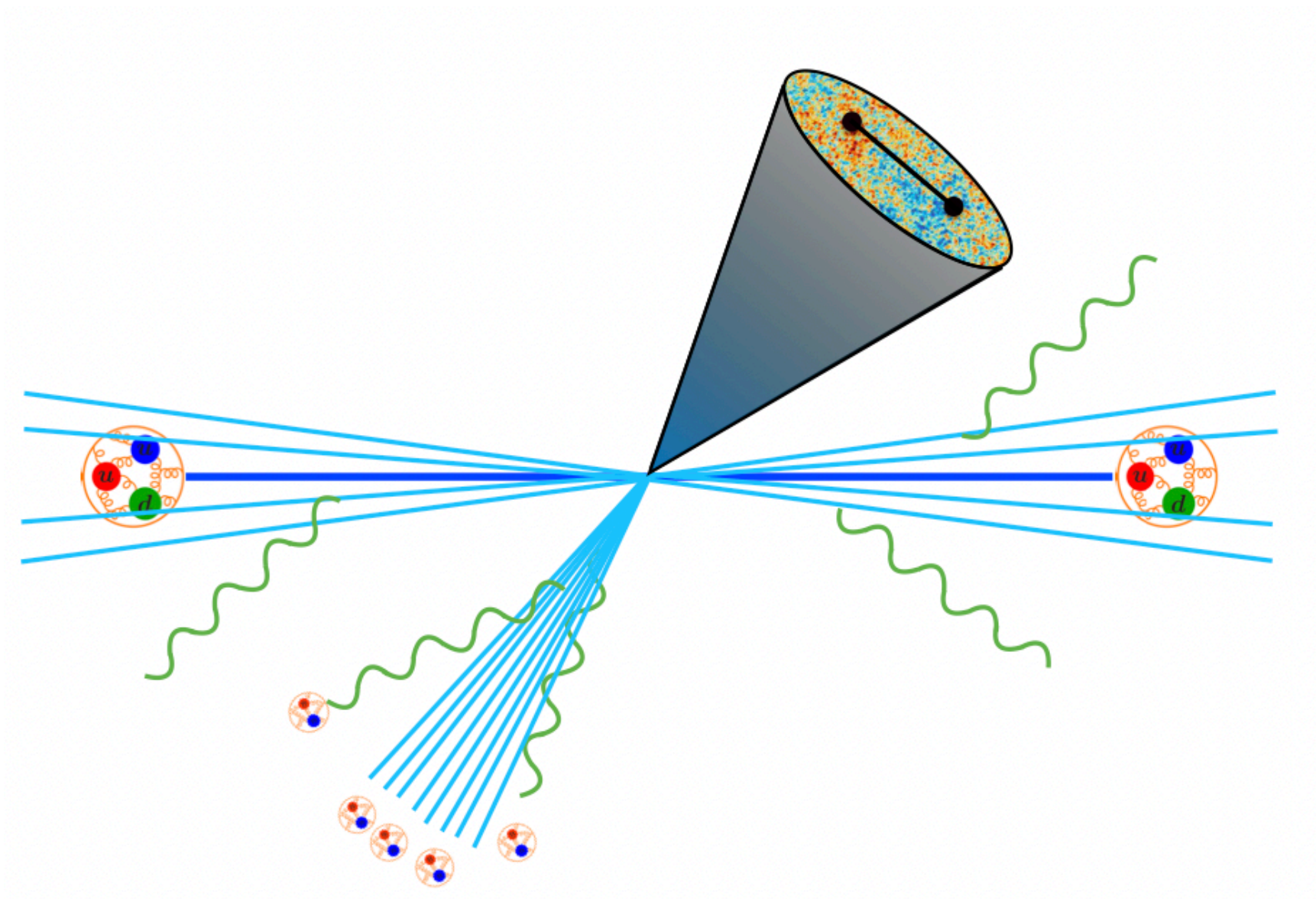
→ “Statistical Correlations”

These correlation functions measure the **flow** of energy at infinity.



# Energy Flow Operators

Situations of interest at the LHC involve non-generic configurations of lightray operators: **interested in the small angle (OPE) limit.**



$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

[Hofman, Maldacena]

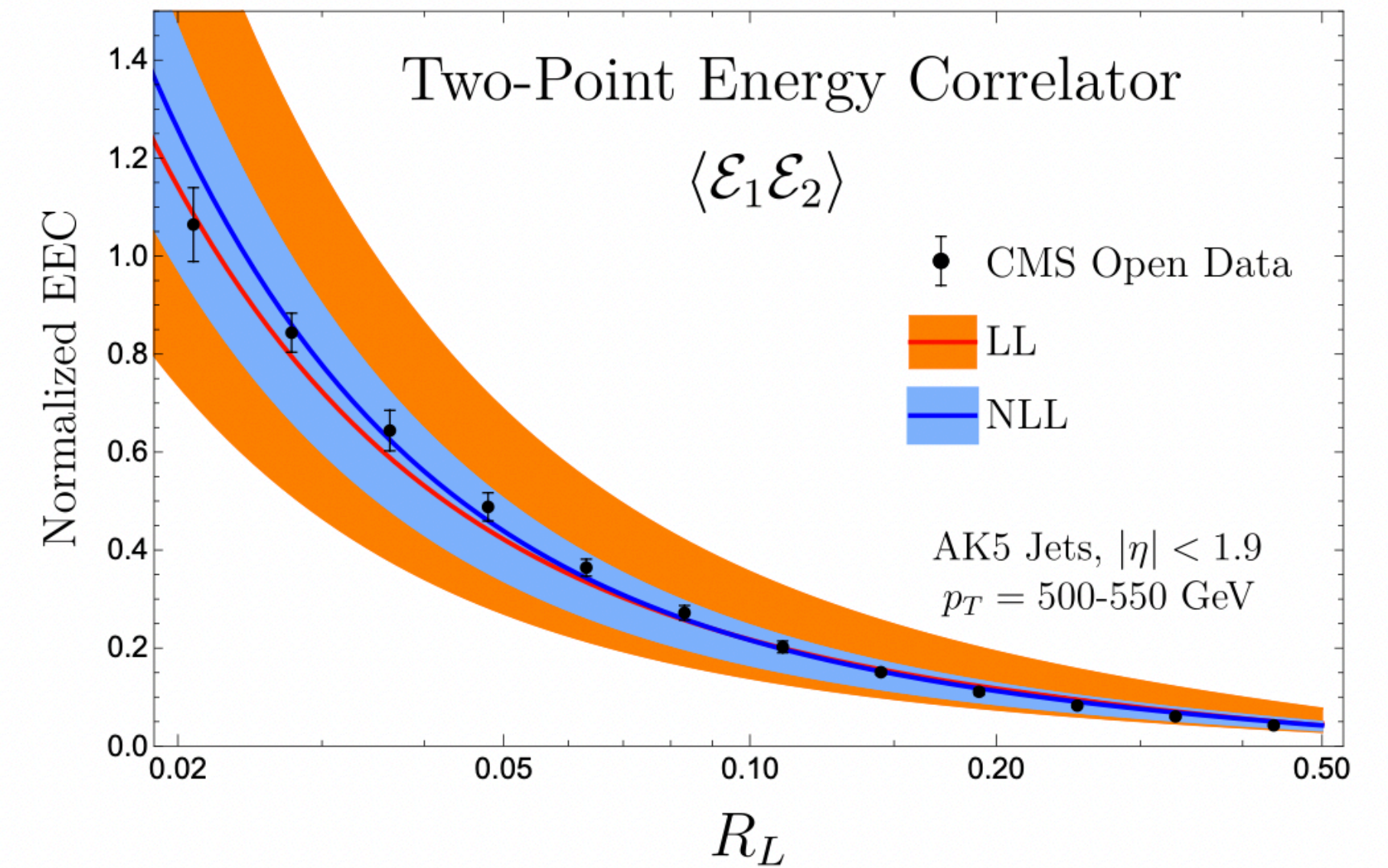
In the small angle limit, these lightray operators should exhibit the **universal behavior of QCD**



# Universal Behavior of QCD

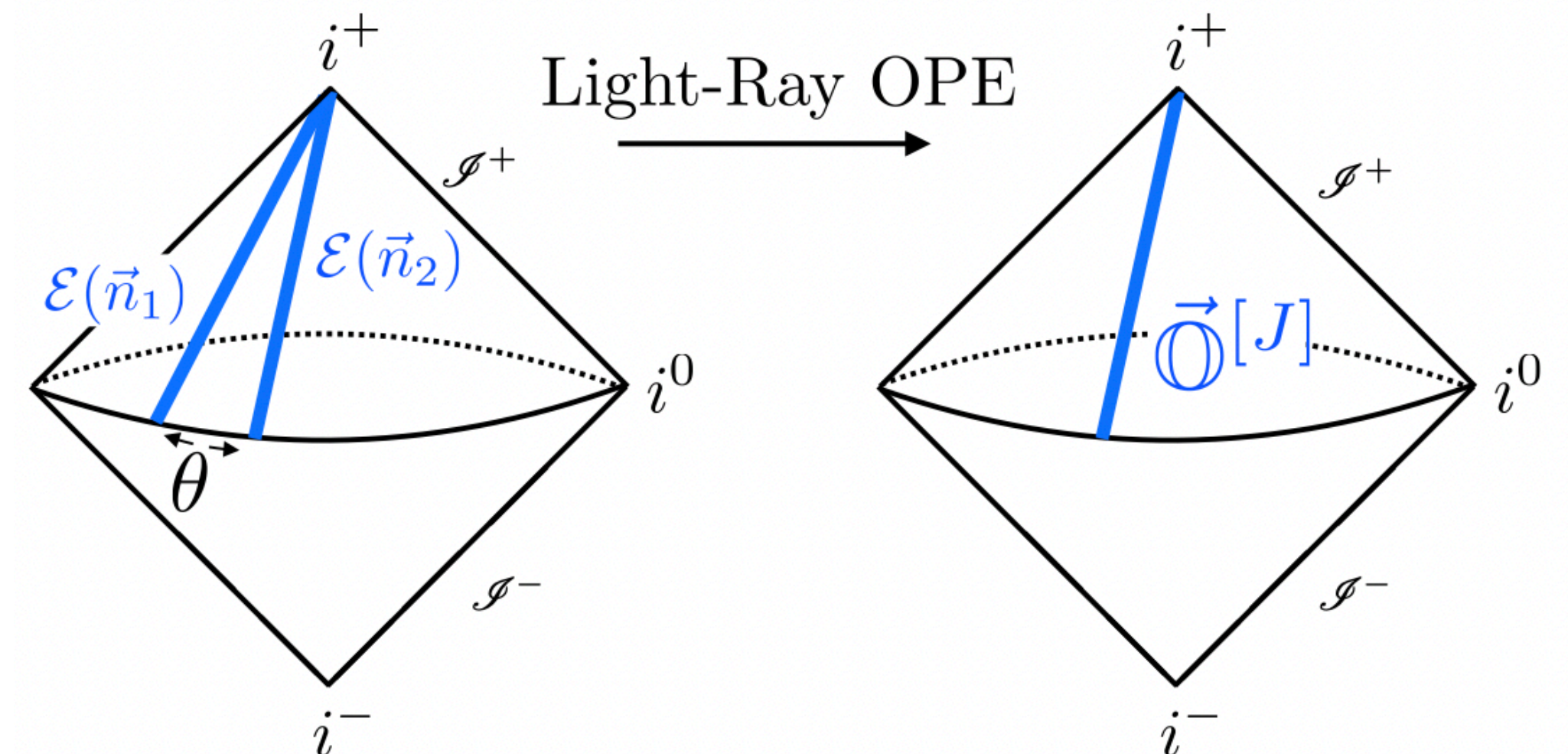
Energy flow operators exhibit universal scaling

→ The substructure of jets is completely determined by the OPE structure of **light-ray operators**



$$\mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1) \quad \begin{array}{l} \text{[Hofman, Maldacena]} \\ \text{[Moult, Zhu]} \end{array}$$

Reformulation of jet substructure using a **broader language** such that it can draw from diverse areas of physics





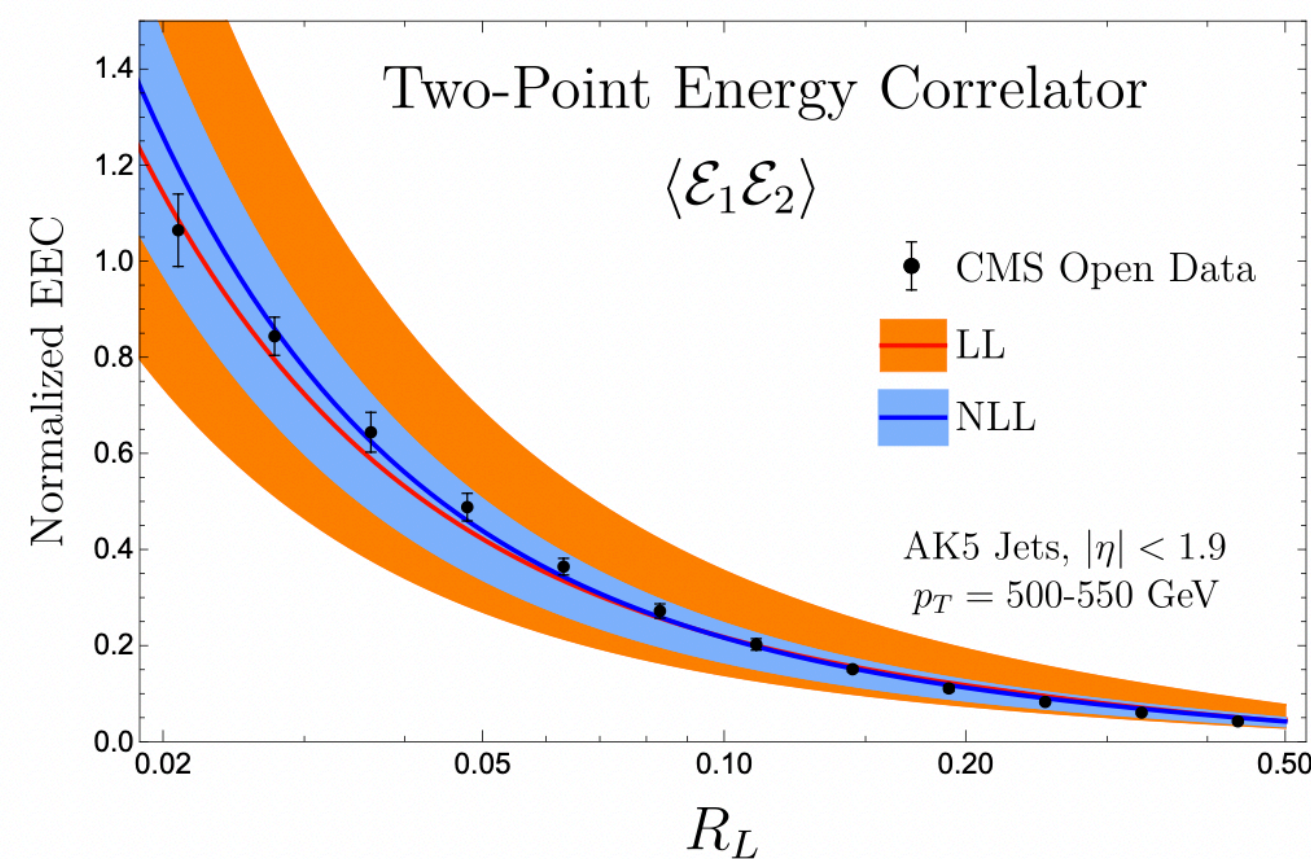
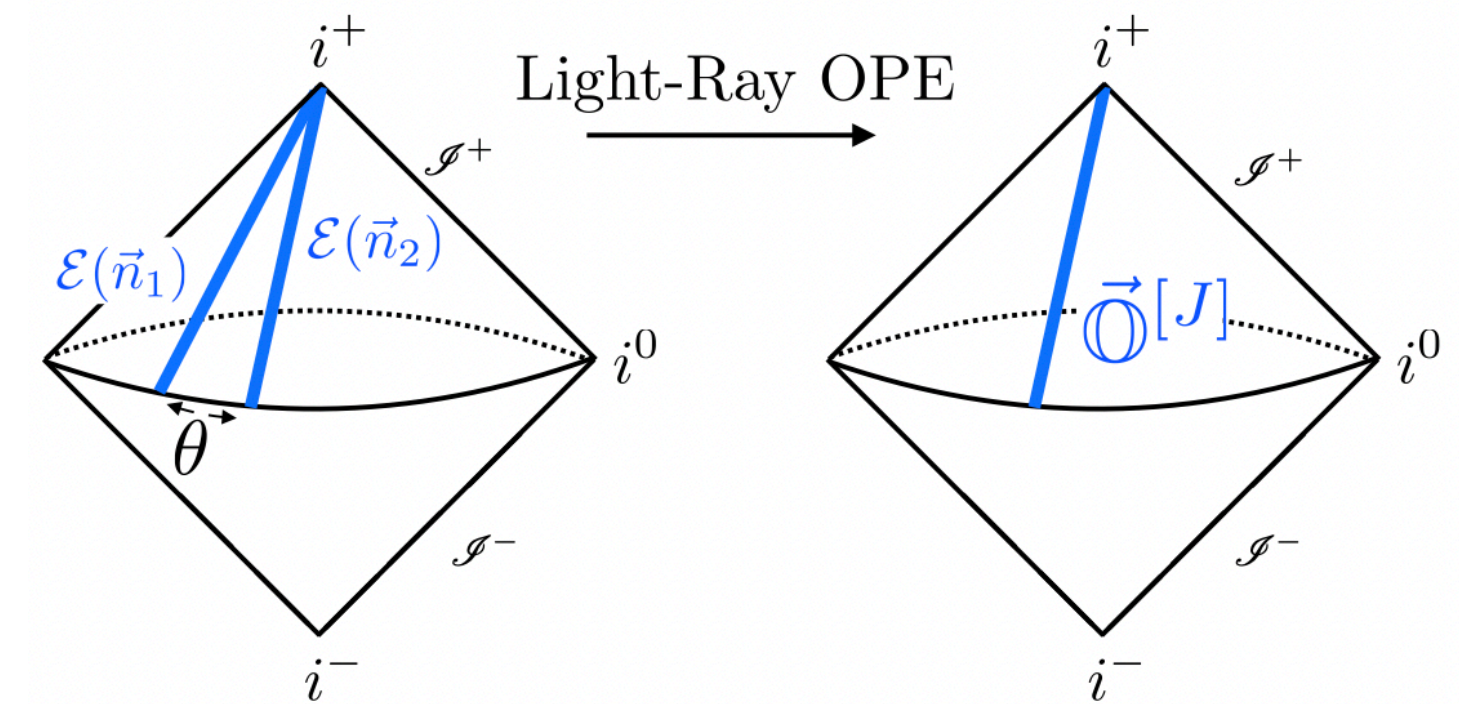
# Universal Behavior of QCD

In the **UV regime**, we can explicitly do the OPE and obtain the exact scaling

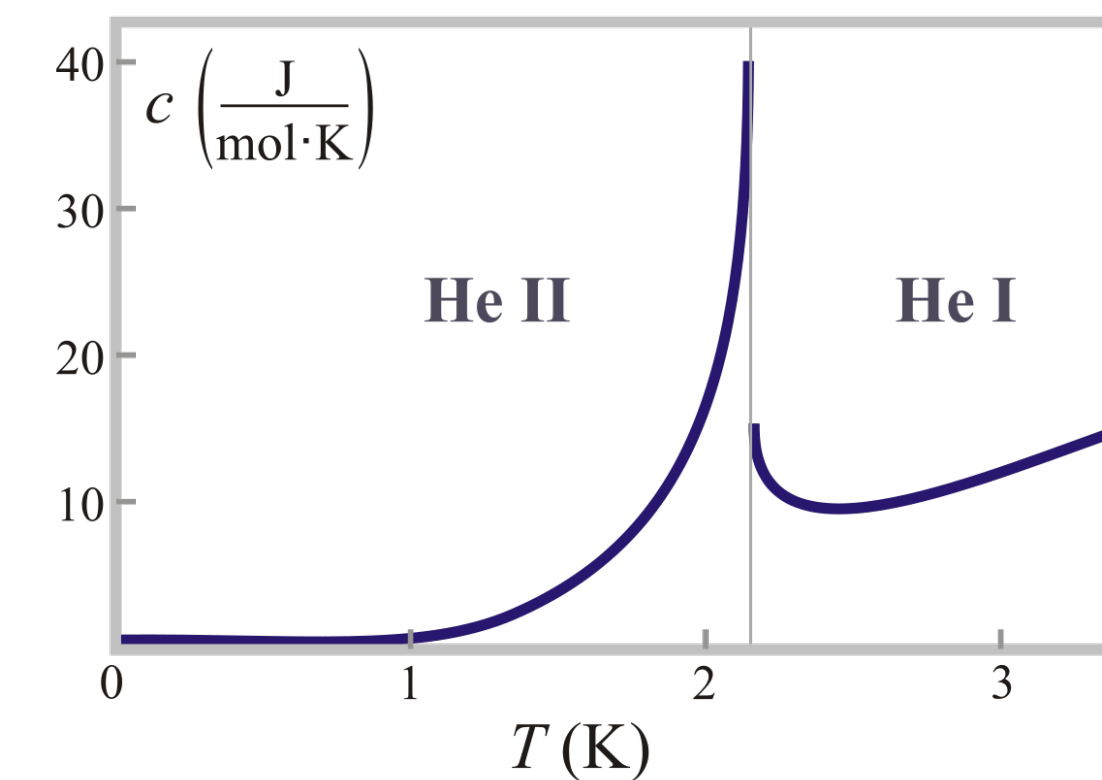
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi} \frac{2}{\theta_S^2} \hat{\mathcal{J}} [\hat{C}_{\phi_S}(2) - \hat{C}_{\phi_S}(3)] \overrightarrow{\mathbb{O}}^{[3]}(\hat{n}) + \dots \quad \text{[Moult, Zhu]}$$

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = -\frac{1}{2\pi} \frac{2}{\theta_S^2} \frac{2}{\theta_L^2} \hat{\mathcal{J}} [\hat{C}_{\phi_S}(2) - \hat{C}_{\phi_S}(3)] [\hat{C}_{\phi_L}(3) - \hat{C}_{\phi_L}(4)] \overrightarrow{\mathbb{O}}^{[4]}(\hat{n}) + \dots$$

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$



The scaling behavior of the twist-2 light ray operators completely control the leading behavior of **jet substructure**



In **superfluid helium**, most precise measurements suggest  $C \sim |T - T_c|^{.009}$



# Universal Behavior of QCD

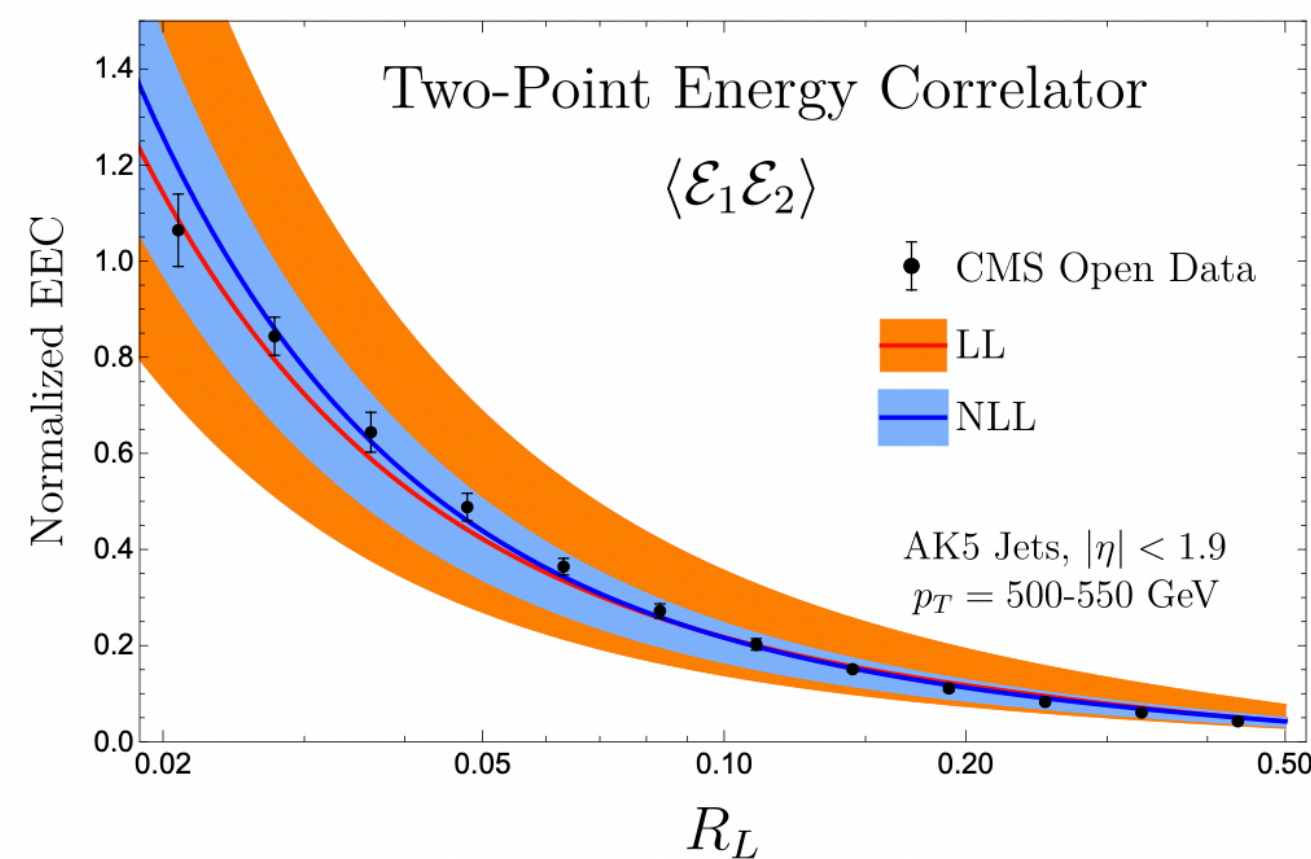
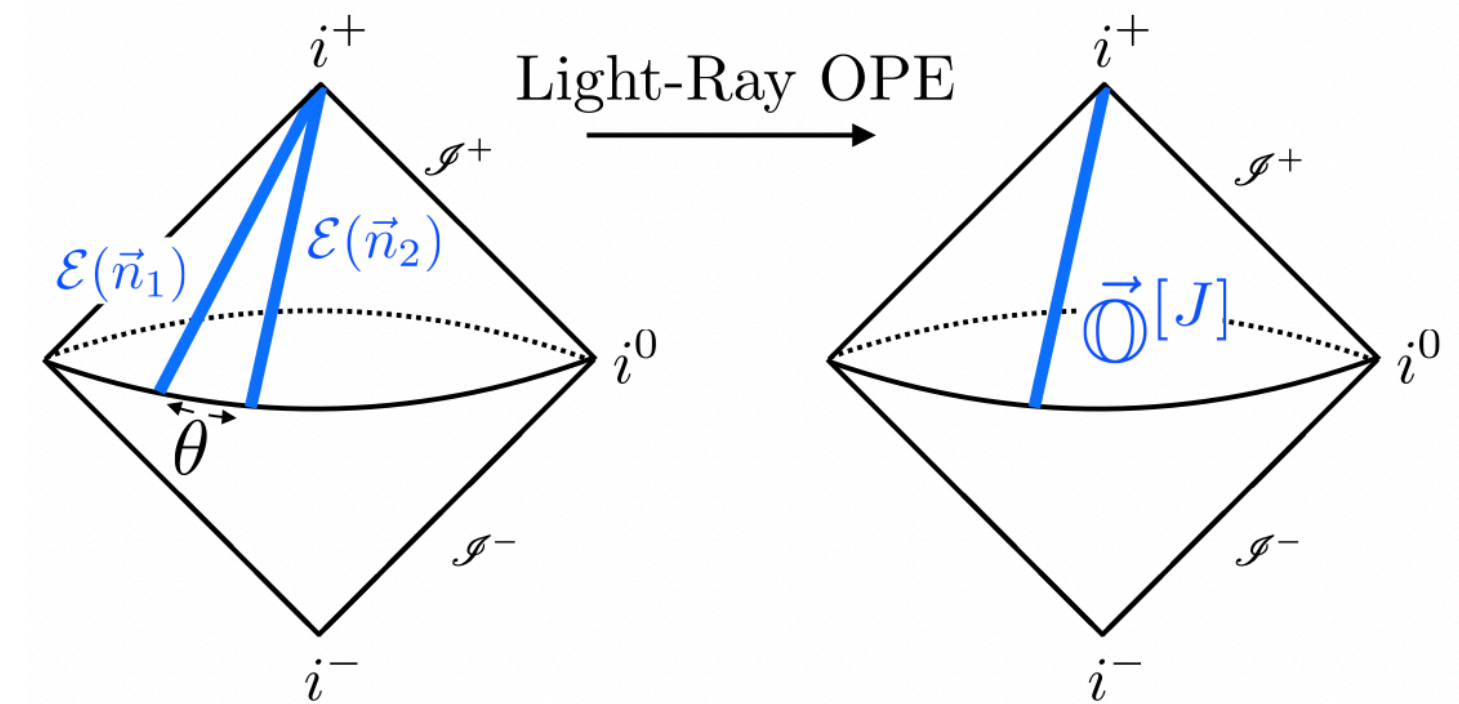
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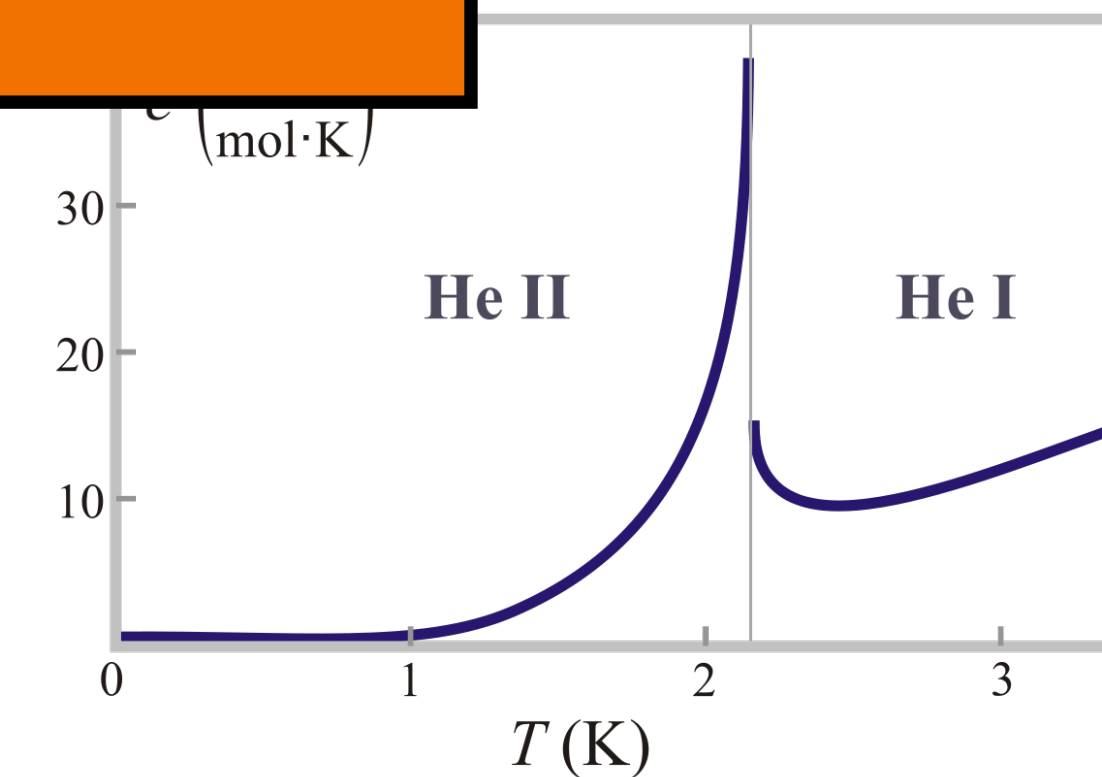
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = -\frac{1}{2\pi} \dots$$

All explained nicely in Hao Chen's talk

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$



The scaling behavior of the twist-2 light ray operators completely control the leading behavior of **jet substructure**

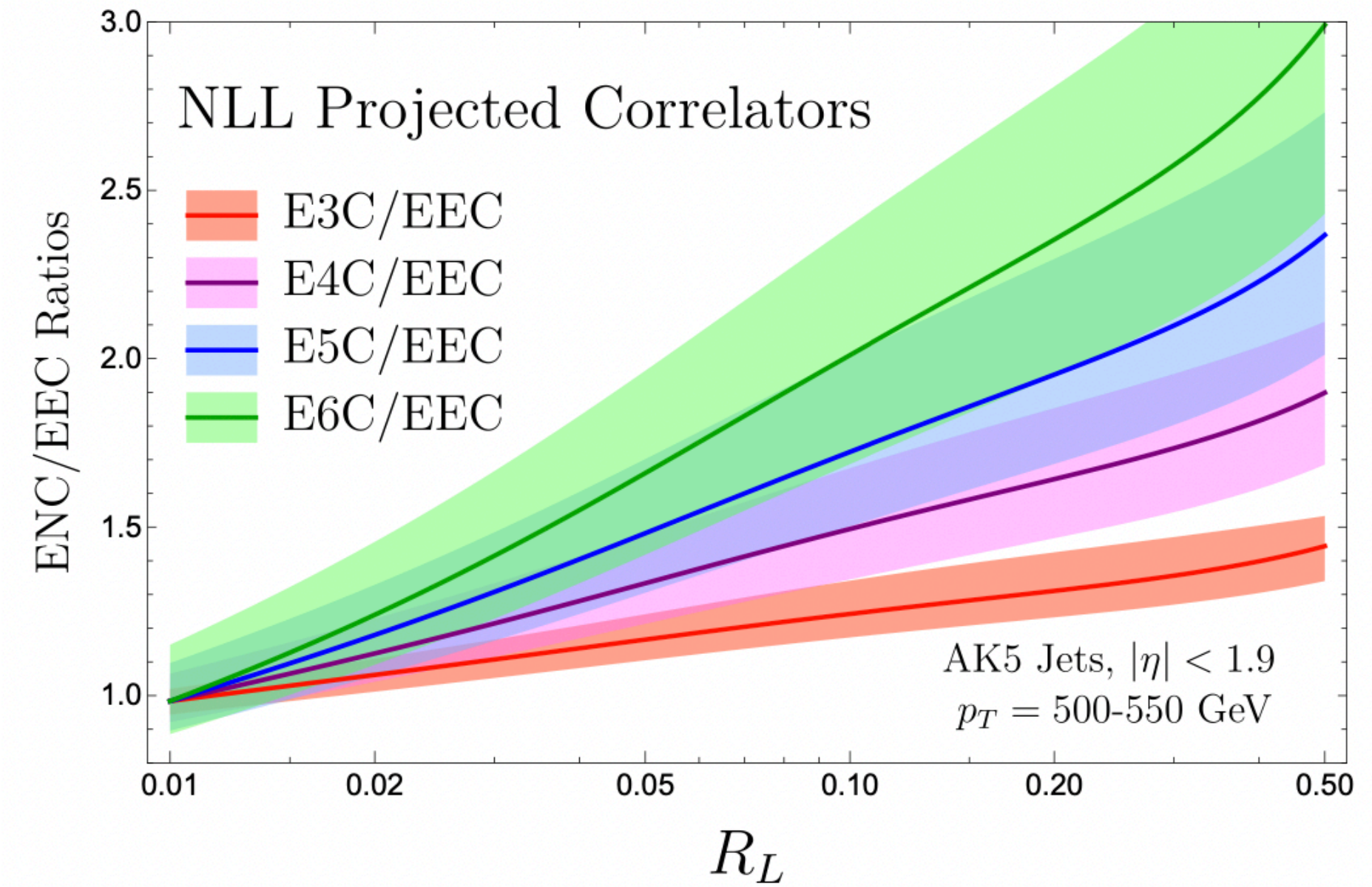
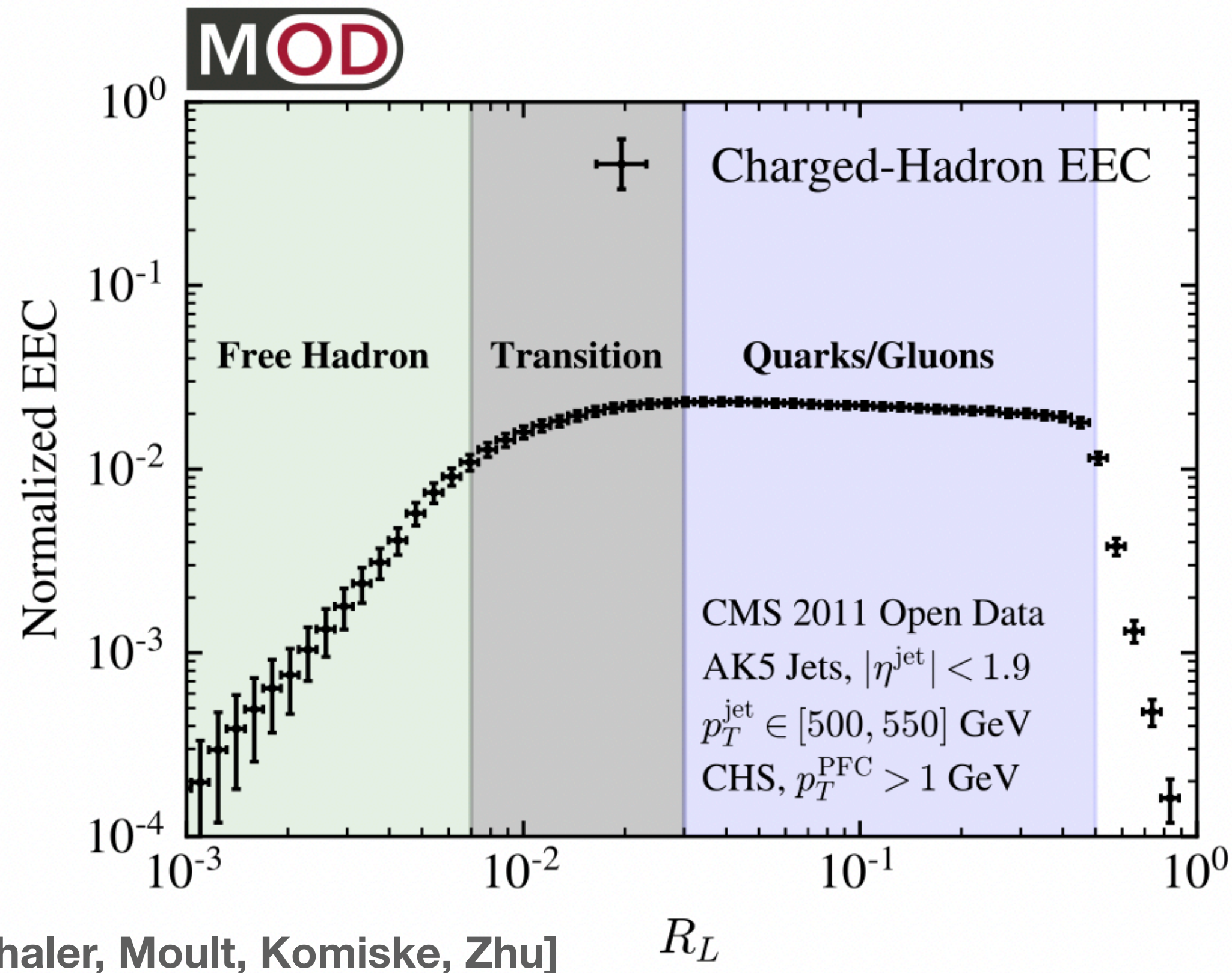


In **superfluid helium**, most precise measurements suggest  $C \sim |T - T_c|^{.009}$



# What can you do with this?

[Moult, Mecaj, Lee]



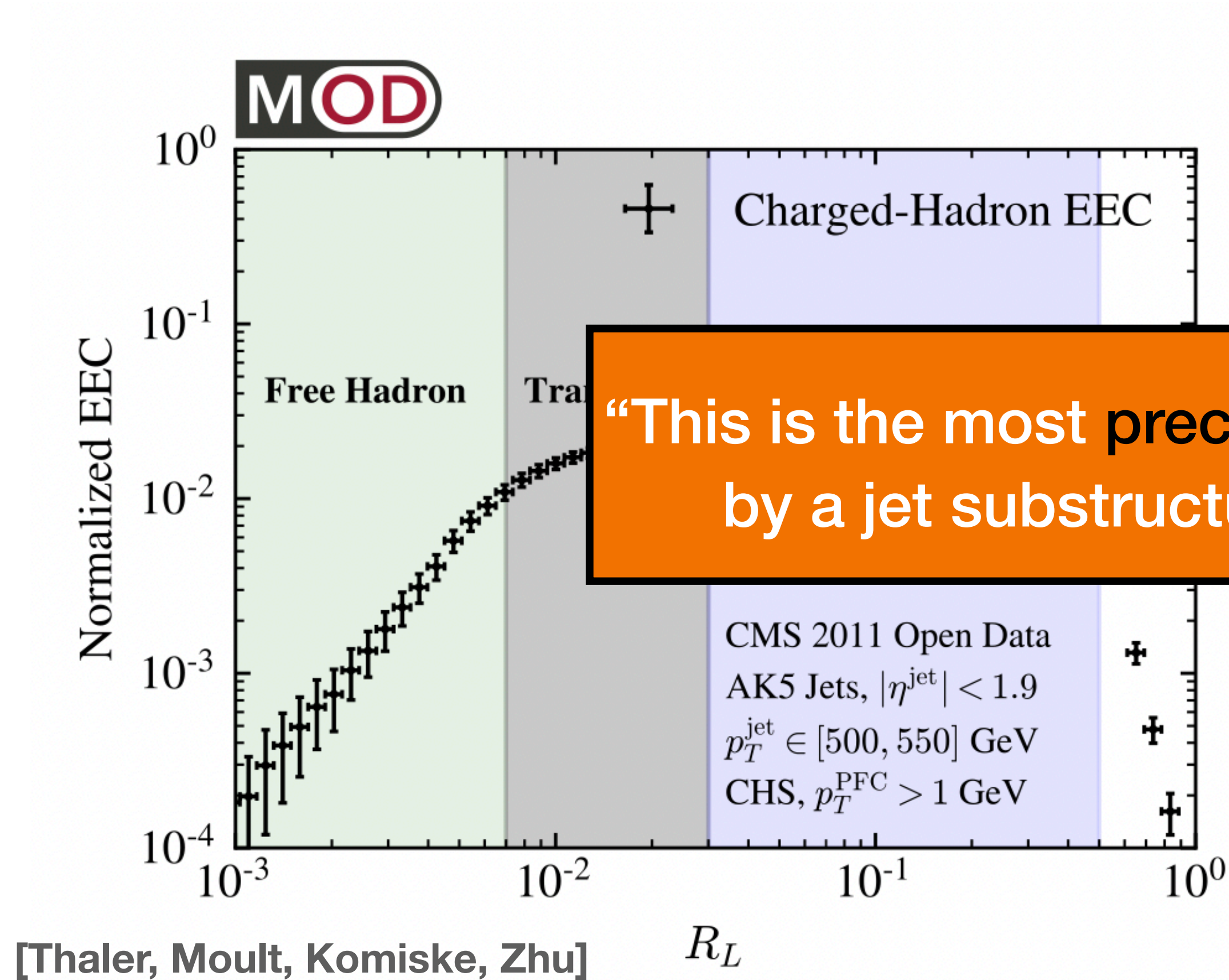
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

Access to **anomalous dimensions** of QCD twist-2 operators **directly** at the LHC

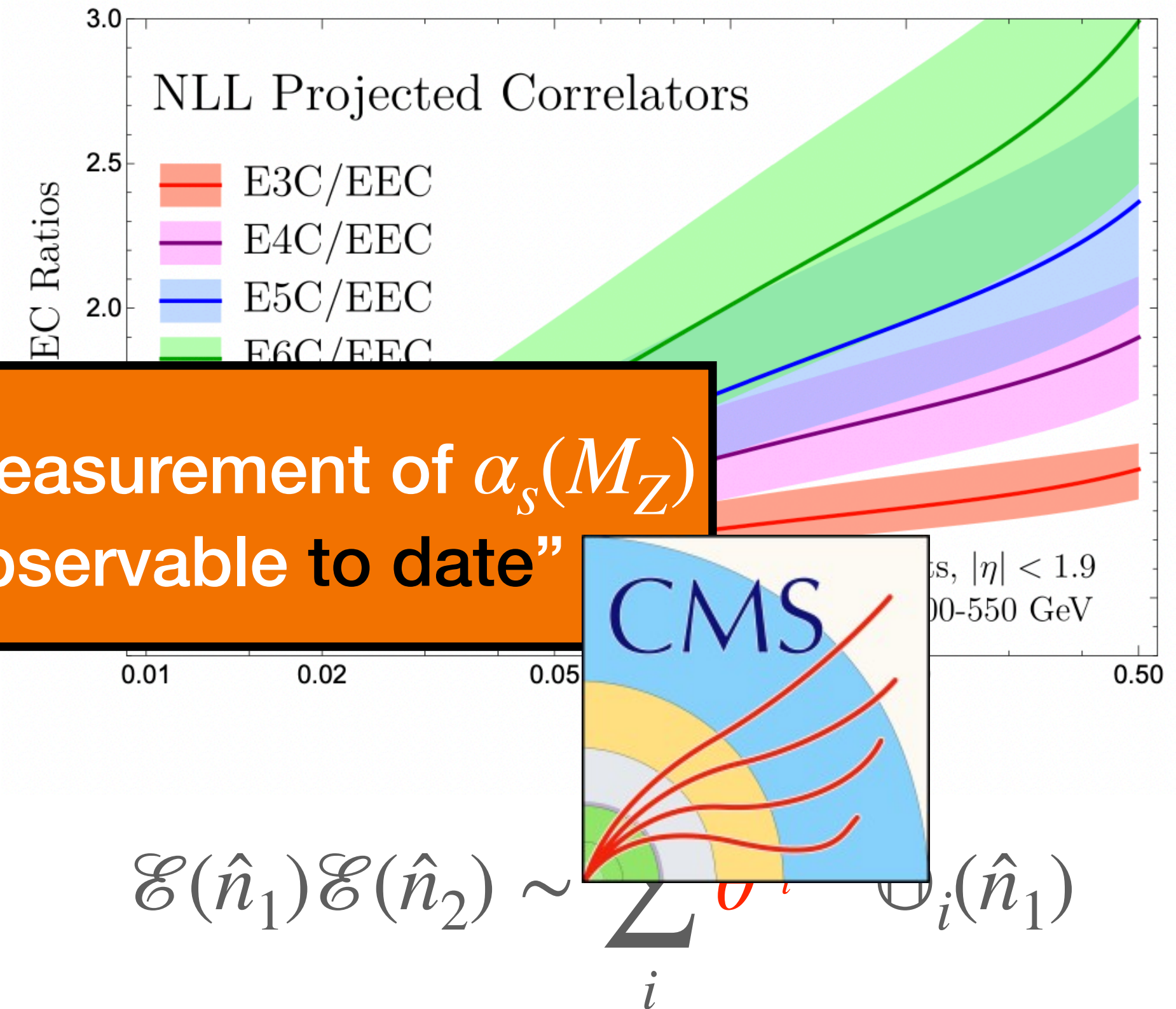


# What can you do with this?

[Moult, Mecaj, Lee]



“This is the most precise measurement of  $\alpha_s(M_Z)$  by a jet substructure observable to date”



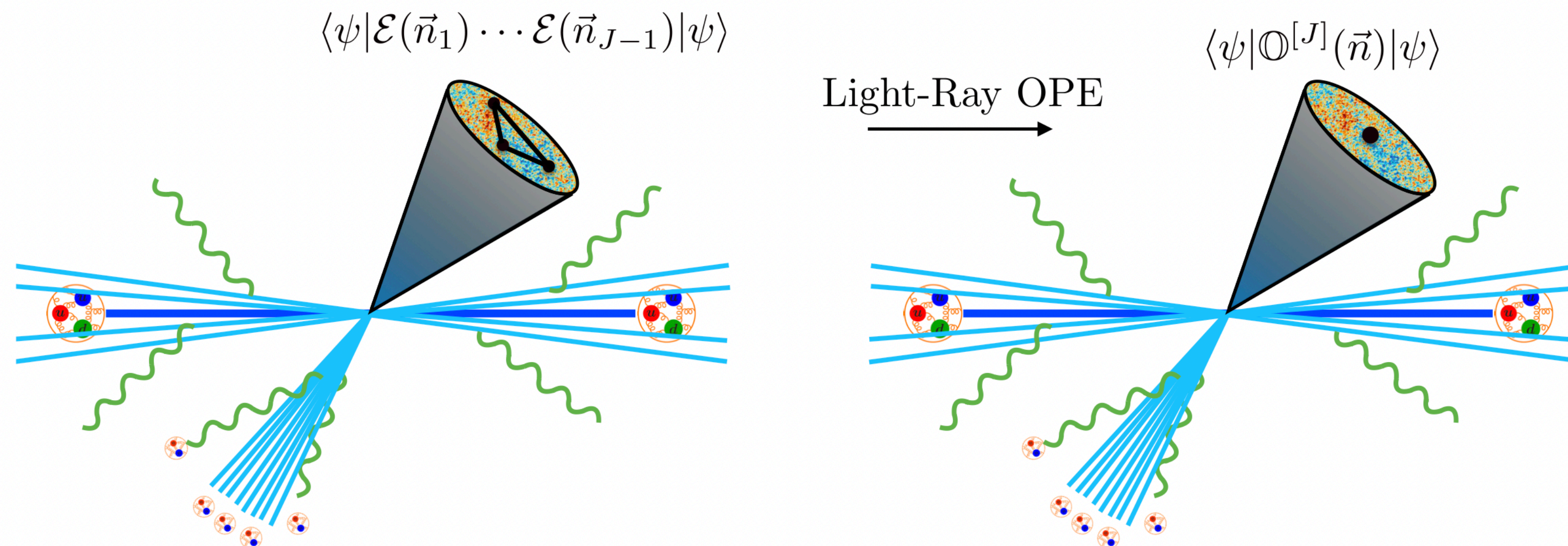
Access to **anomalous dimensions** of QCD twist-2 operators **directly** at the LHC



# Can we ask other questions?

Allows us to **replace heuristic jet shapes** with **field theoretic objects** controlling the underlying theory

- Can directly relate **observations** to **field theoretic quantities**
- Able to exploit new, **formal theory developments** to understand collider experiments







# Beautiful and Charming Energy Correlators

Evan Craft — Yale University  
arXiv: 2210.09311



Based on work with K. Lee, B. Mecaj, I. Moulton

Yale University

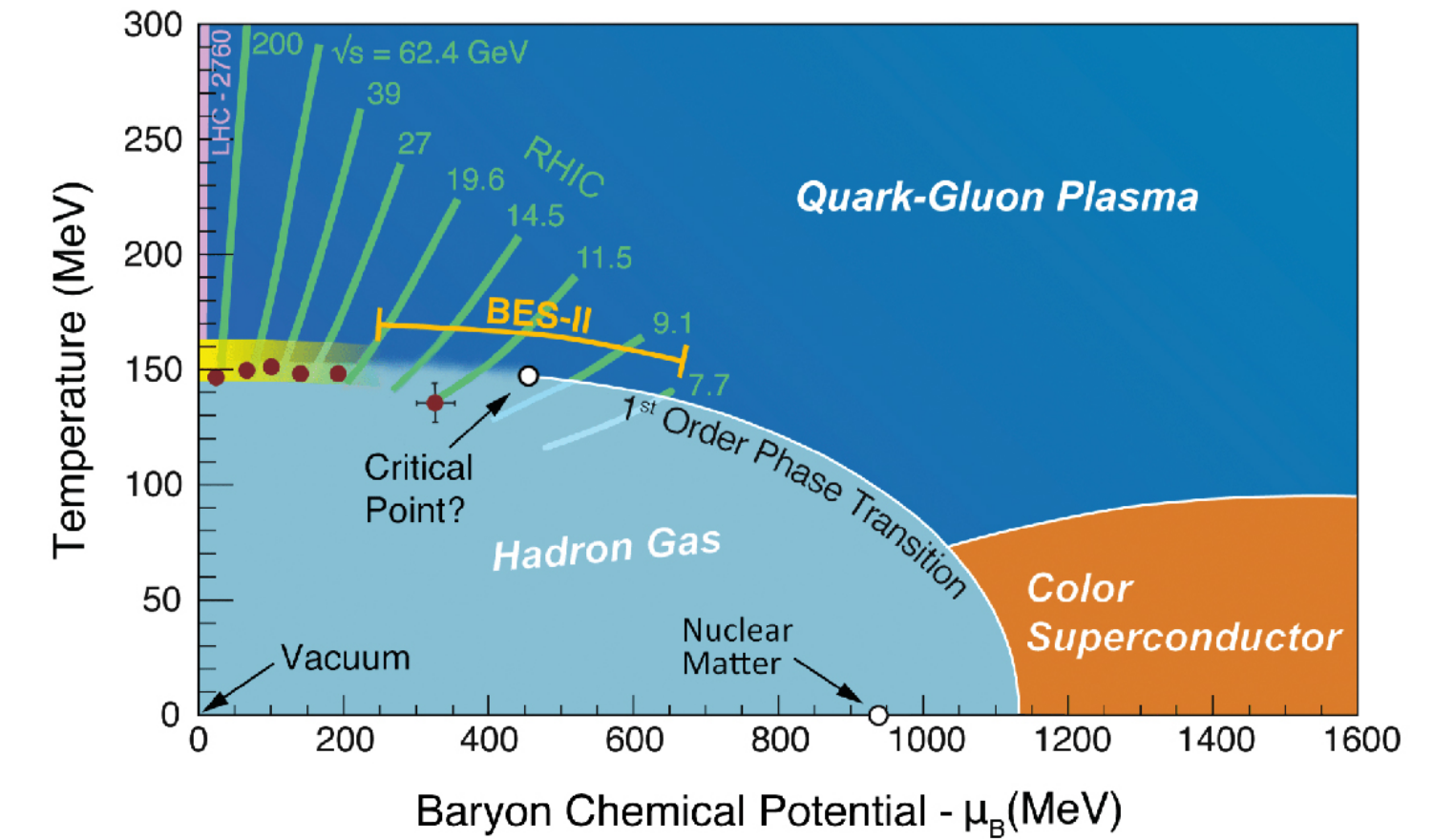
MIT



# Many Puzzles in QCD

Several **open questions** remain across both Particle and Nuclear Physics

→ Many of these open problems are deeply connected to **Quantum Chromodynamics**



→ Why is color charge so complicated?

## Hot QCD

- Quark Gluon Plasma
- Hadronization
- Quarkonia

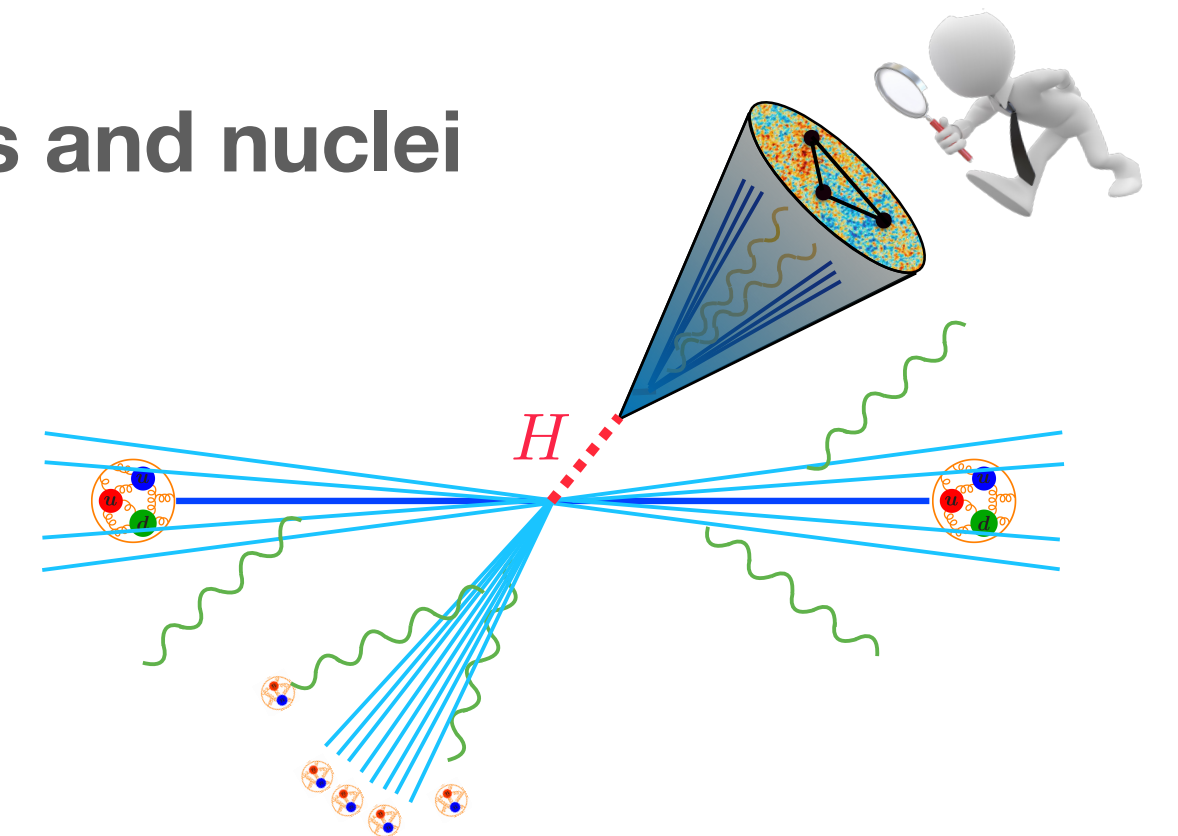
## Medium QCD

- Strong CP
- Rare Higgs Decays
- Confinement

## Cold QCD

- Gluon Saturation
- Proton Spin and Radius Puzzle
- 3D Structure of protons and nuclei

Numerous collider experiments spanning several continents working to resolve these **fundamental questions**

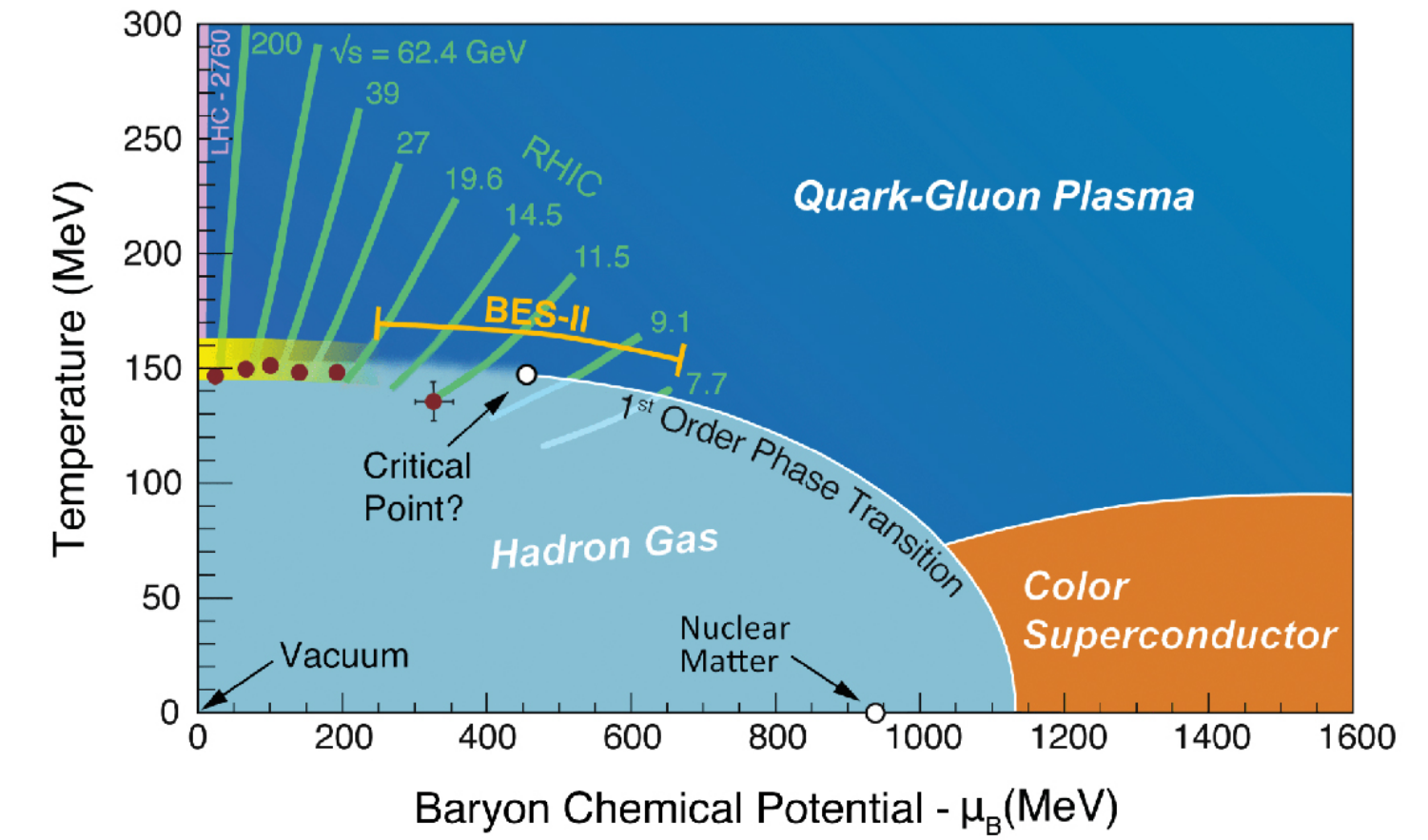




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## Hot QCD

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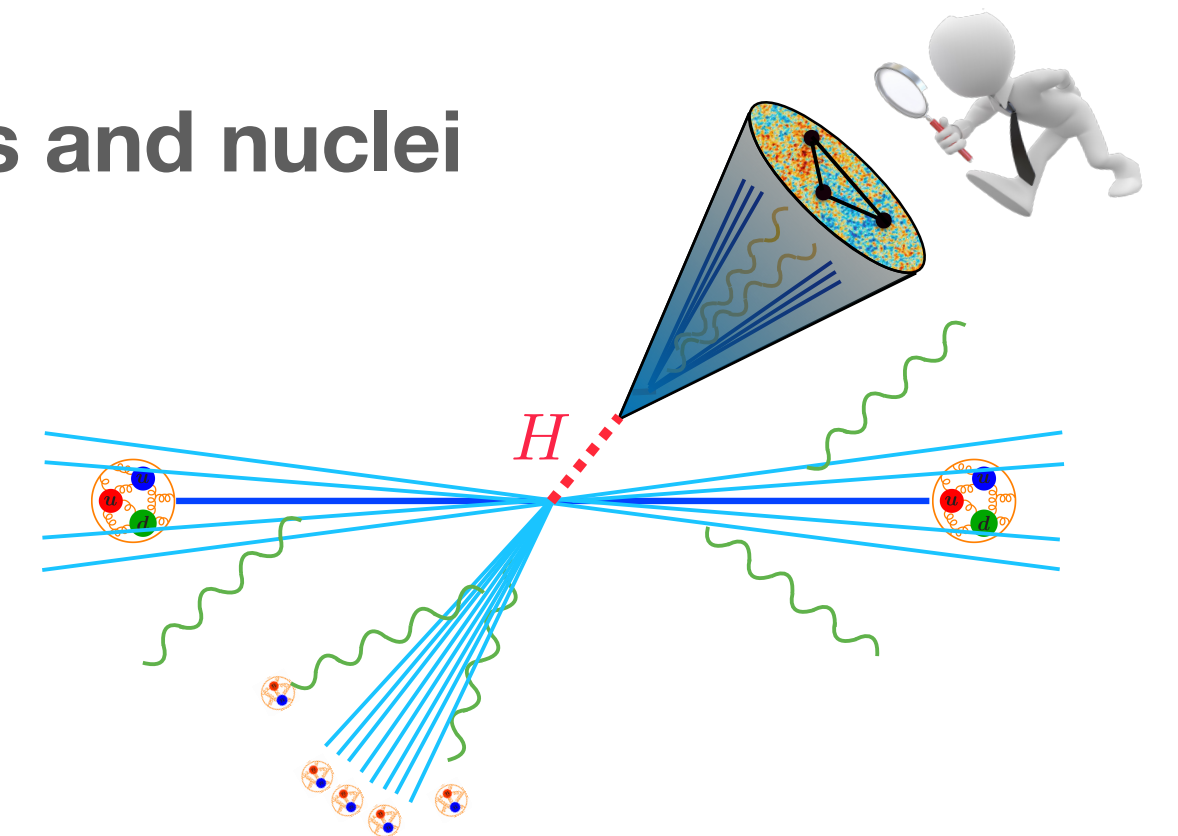
**Flavor Dead Cone Effect**

## Cold QCD

- Gluon Saturation
- Proton Spin and Radius Puzzle
- 3D Structure of protons and nuclei

[\(Nature Physics, 2021\)](#)

Numerous collider experiments spanning several continents working to resolve these **fundamental questions**





# Application: Confinement

[ALICE Collaboration, Nature Physics]

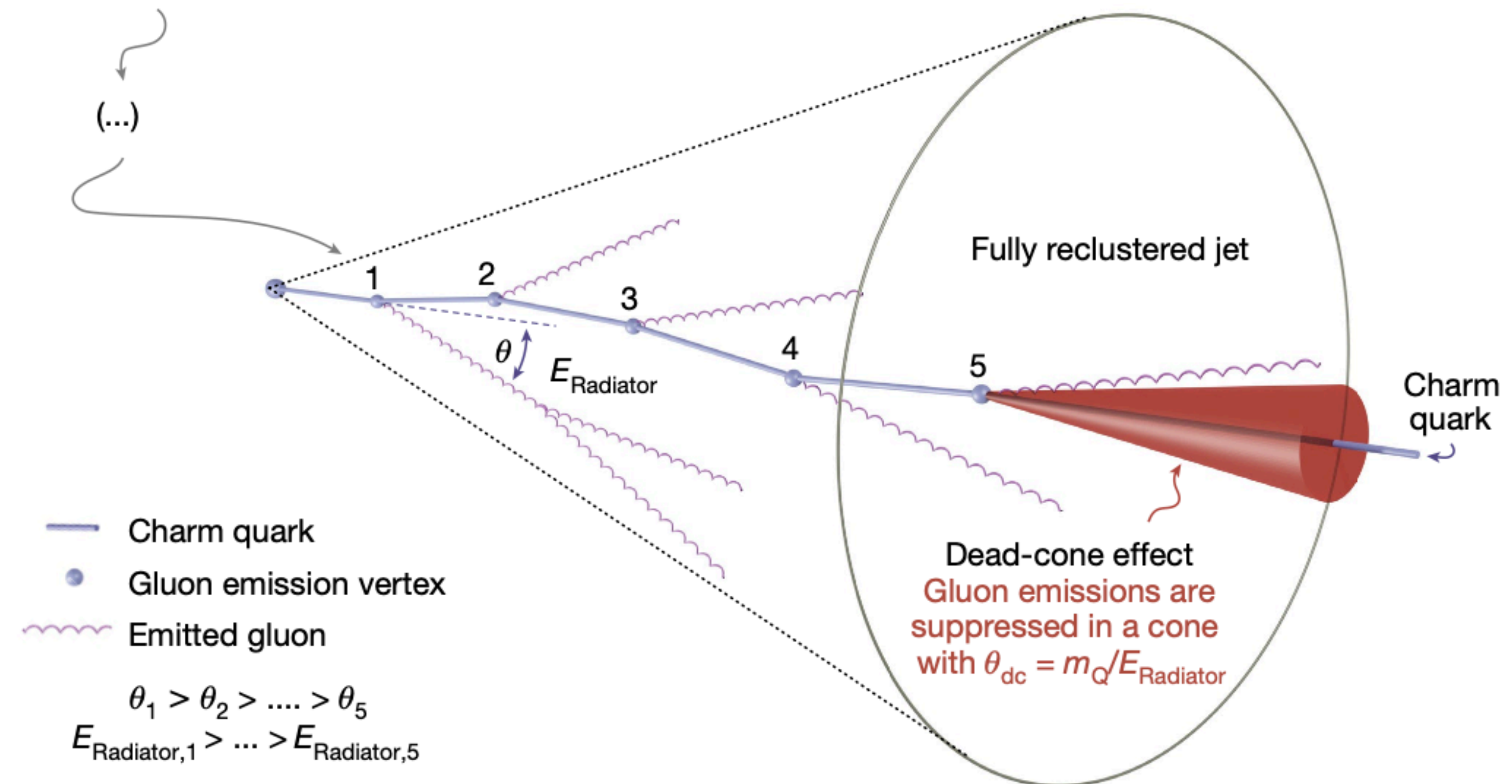
Dokshitzer, Khoze, Troyan (1991)

Heavy quark radiation of gluons must be **suppressed** within a cone of radius  $m_q/E_q$  around its center.

→ Fundamental property of all **gauge** field theories

→ Direct signature of intrinsic mass before **confinement**

We can access this effect simply with **statistical correlations (light-ray operators)** — providing a precise, **field theoretic** description of the dead cone.



Measured this year by ALICE using a complex **iterative declustering** technique

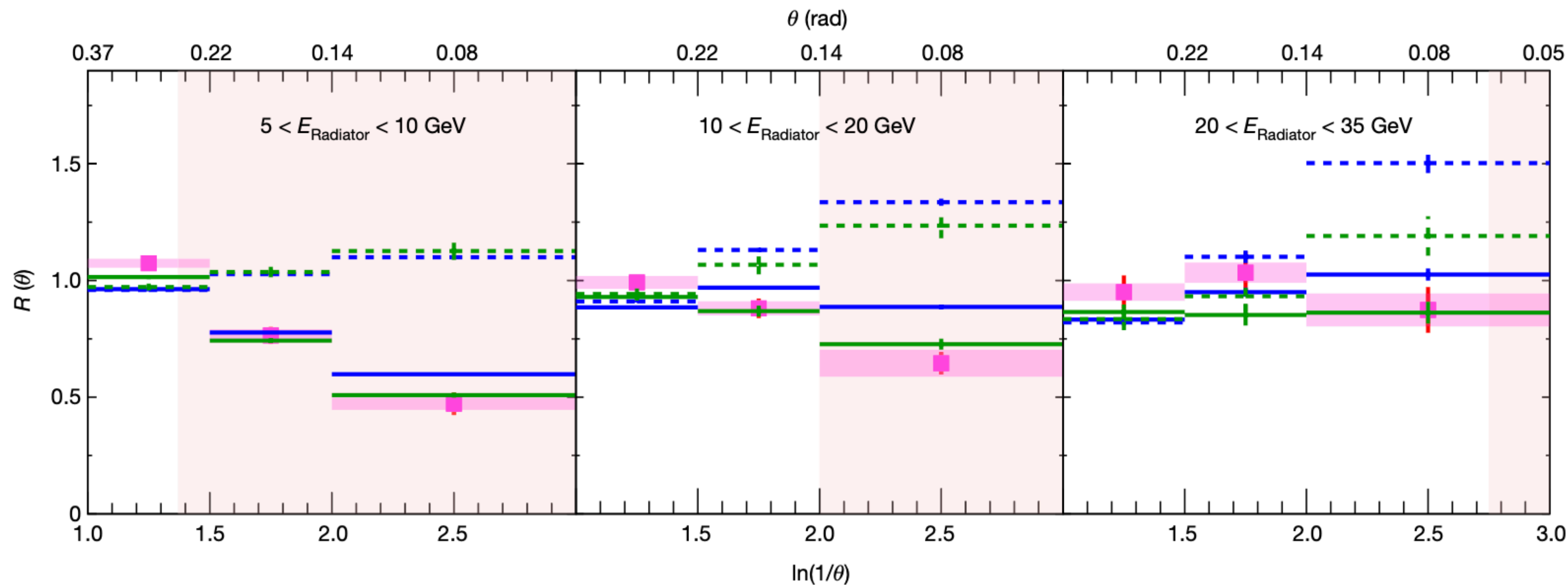
→ Inferred all gluon emissions *directly*

→ State of the art analysis techniques



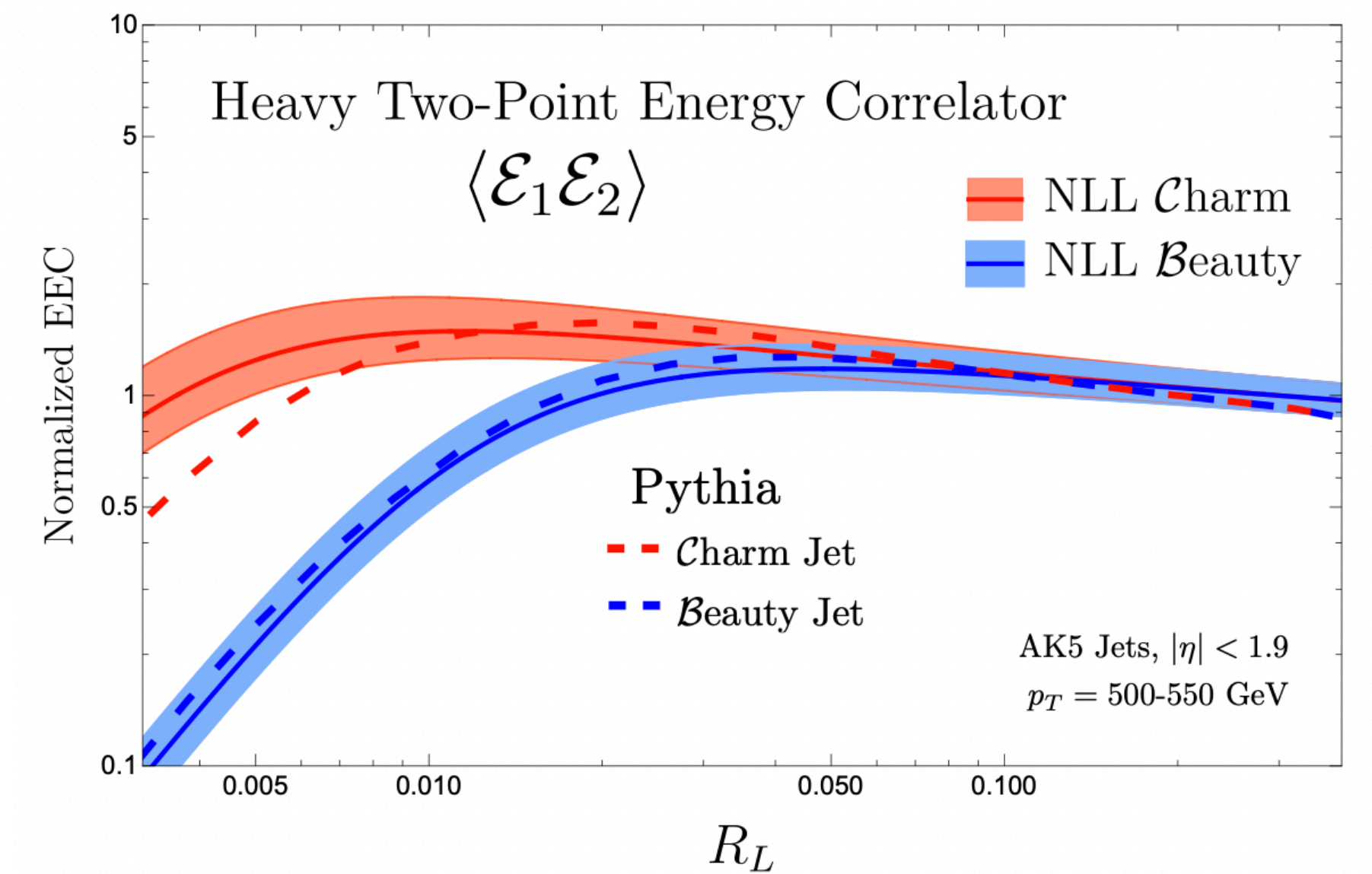
# Application: Confinement

ALICE measured this effect using an iterative declustering algorithm and the **Lund Plane**



[ALICE Collaboration, Nature Physics]

[EC, Lee, Mecaj, Moult]



→ EECs can be **systematically computed** in perturbation theory

→ Dead cone effect is **visible by eye** using light-ray operators

→ Can easily be **extended** to other heavy flavor based analyses



# Application: Confinement

In the **UV regime**, scaling should be independent of mass

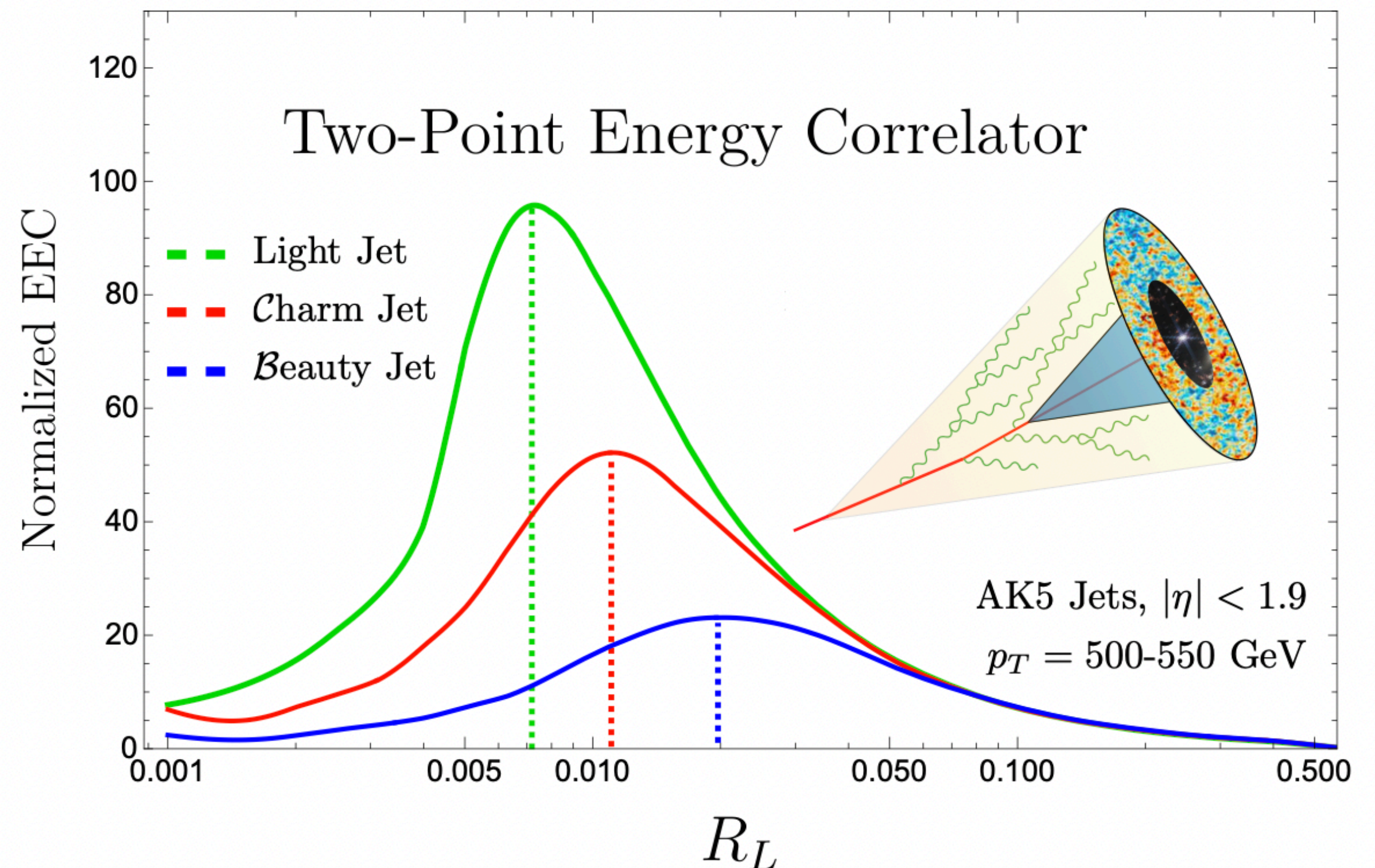
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

In the **IR regime**, mass is an intrinsic scale, and should be imprinted on the correlator

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$

EECs provide a precise, **field-theoretic description** of the dead-cone effect

[EC, Lee, Mecaj, Moutl]



$$\text{Transition Scale} \sim \frac{m_q}{p_{T,jet}}$$



# Application: Confinement

[EC, Lee, Mecaj, Moutl]

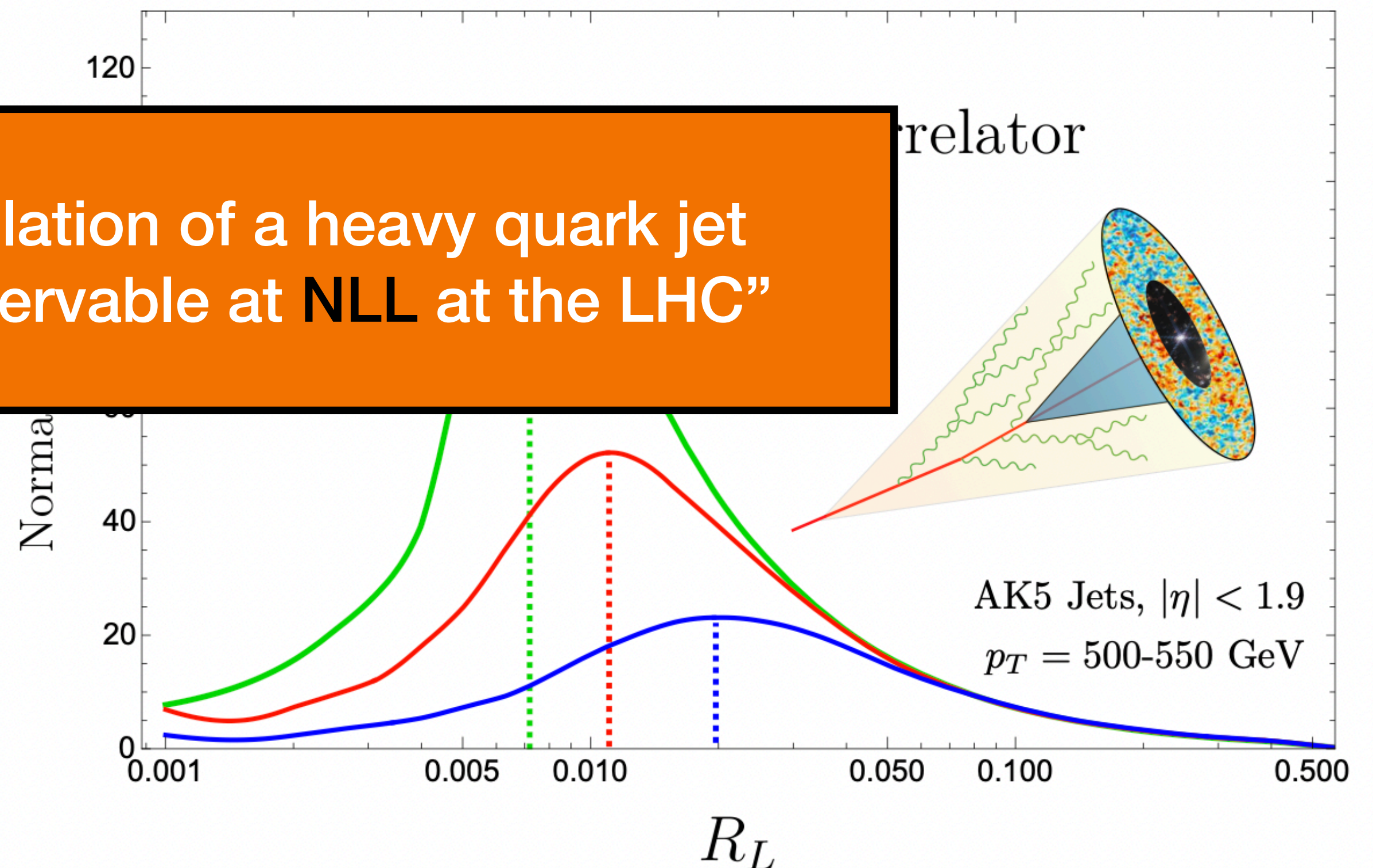
In the **UV regime**, scaling should be independent of mass

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \sim$$

“First ever calculation of a heavy quark jet substructure observable at **NLL** at the LHC”

In the **IR regime**, mass is an intrinsic scale, and should be imprinted on the correlator

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



EECs provide a precise, **field-theoretic description** of the dead-cone effect

$$\text{Transition Scale} \sim \frac{m_q}{p_{T,jet}}$$



# Application: Confinement

In the collinear limit, satisfies a **factorization theorem**

Formally, the EEC is defined in terms of **correlation functions**

$$\frac{d\Sigma}{dz} \sim \int d^4x e^{iQx} \langle 0 | \mathcal{O}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{O}^\dagger(0) | 0 \rangle$$

Source Term

Energy Flow

$$z \equiv \frac{1 - \cos \theta}{2}$$

$$\frac{d\Sigma}{dz} \sim \vec{J} \otimes \vec{H}$$

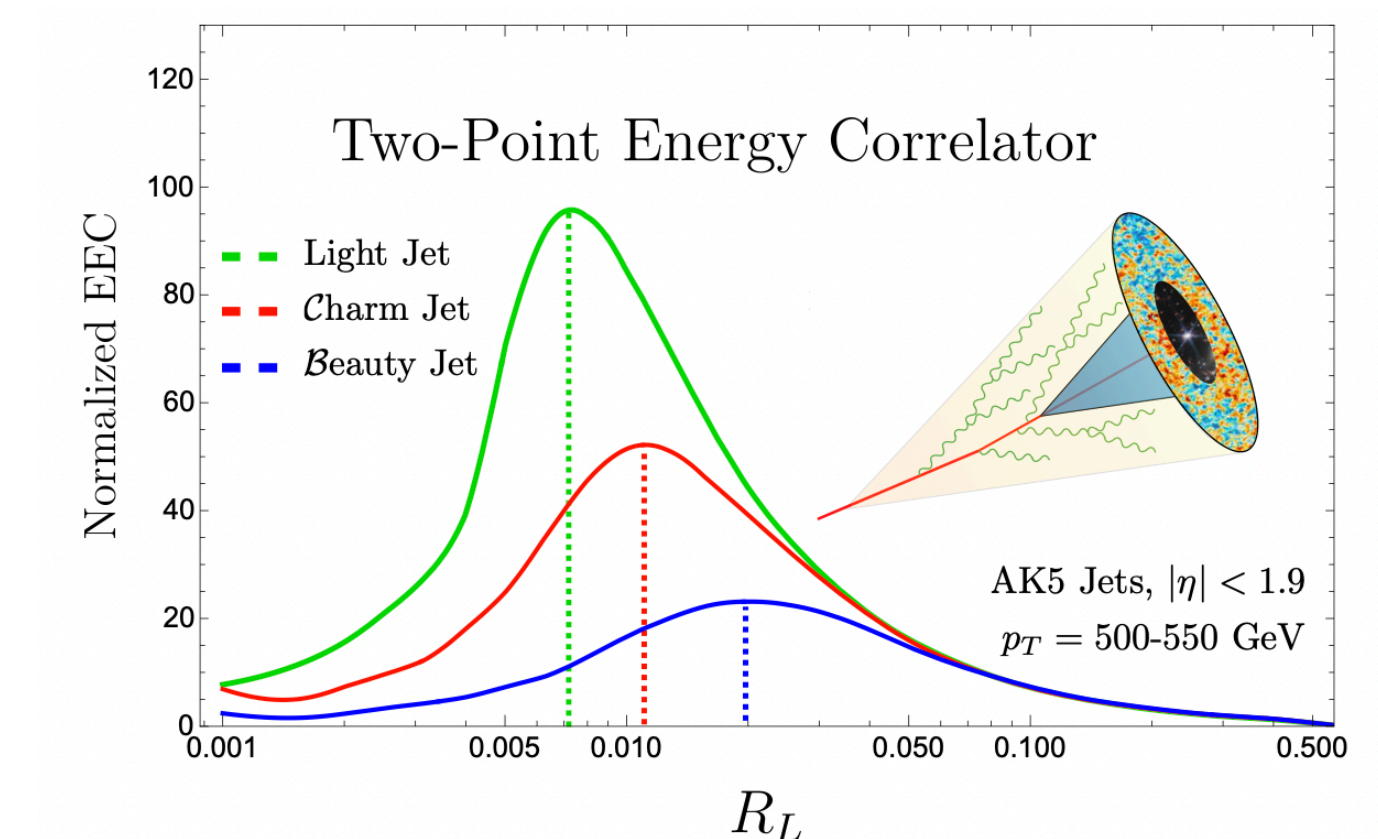
**Jet Function:**  
“quark evolution”

**Hard Function:**  
“initiates process”

## Quark Jet Function

$$J_{qg \leftarrow q} \sim \frac{1}{z} \left( \frac{3}{4} - \frac{5}{2} \delta^2 - \frac{\delta^4}{1 + \delta^2} + 3\delta^3 \tan^{-1} \left[ \frac{1}{\delta} \right] + \frac{1}{2} \delta^2 \left[ 1 - \delta^2 \right] \log \left[ \frac{\delta^2}{1 + \delta^2} \right] \right)$$

Depends only on the **ratio:**  $\delta = \frac{m}{Q\sqrt{z}}$

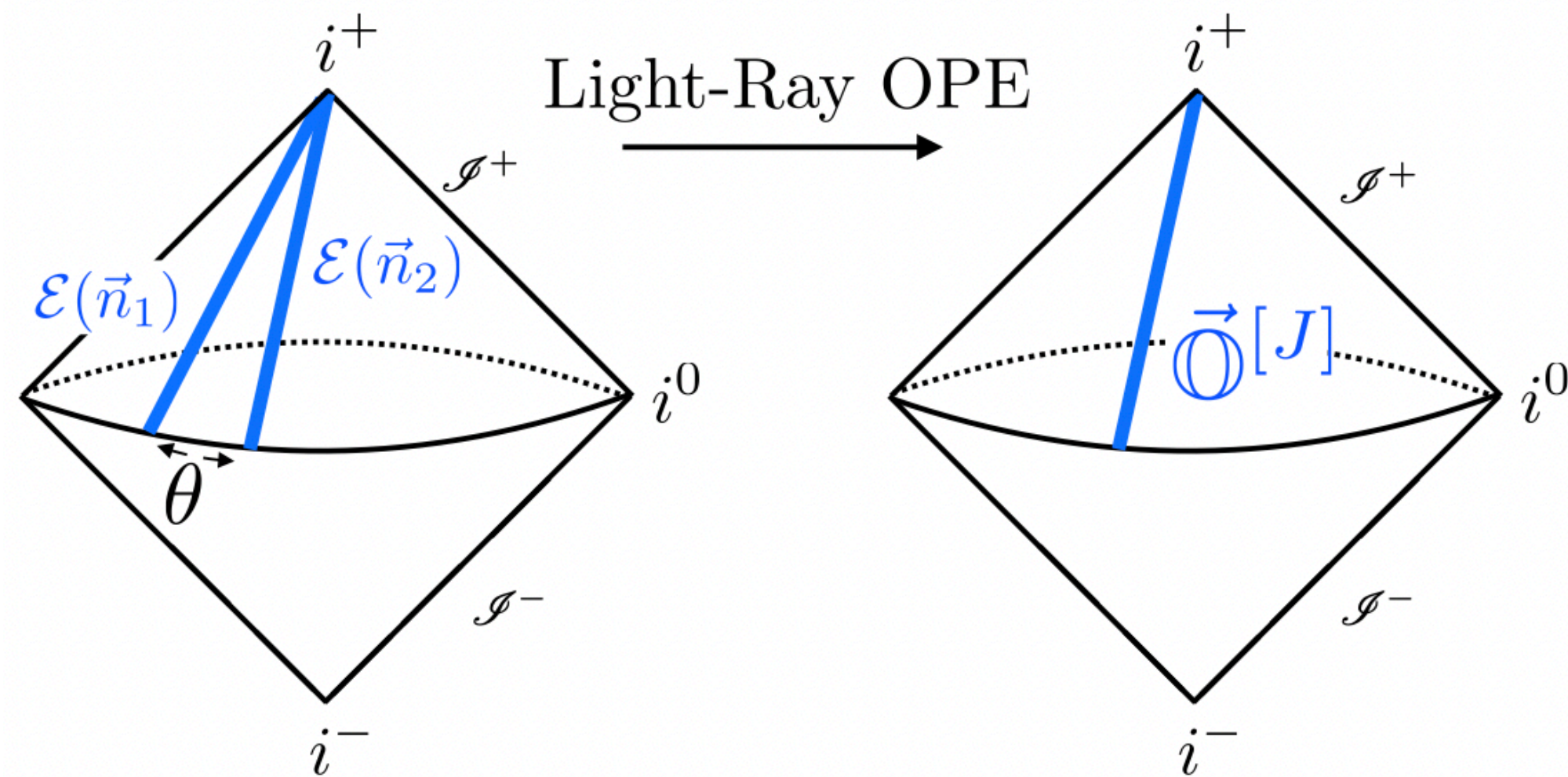




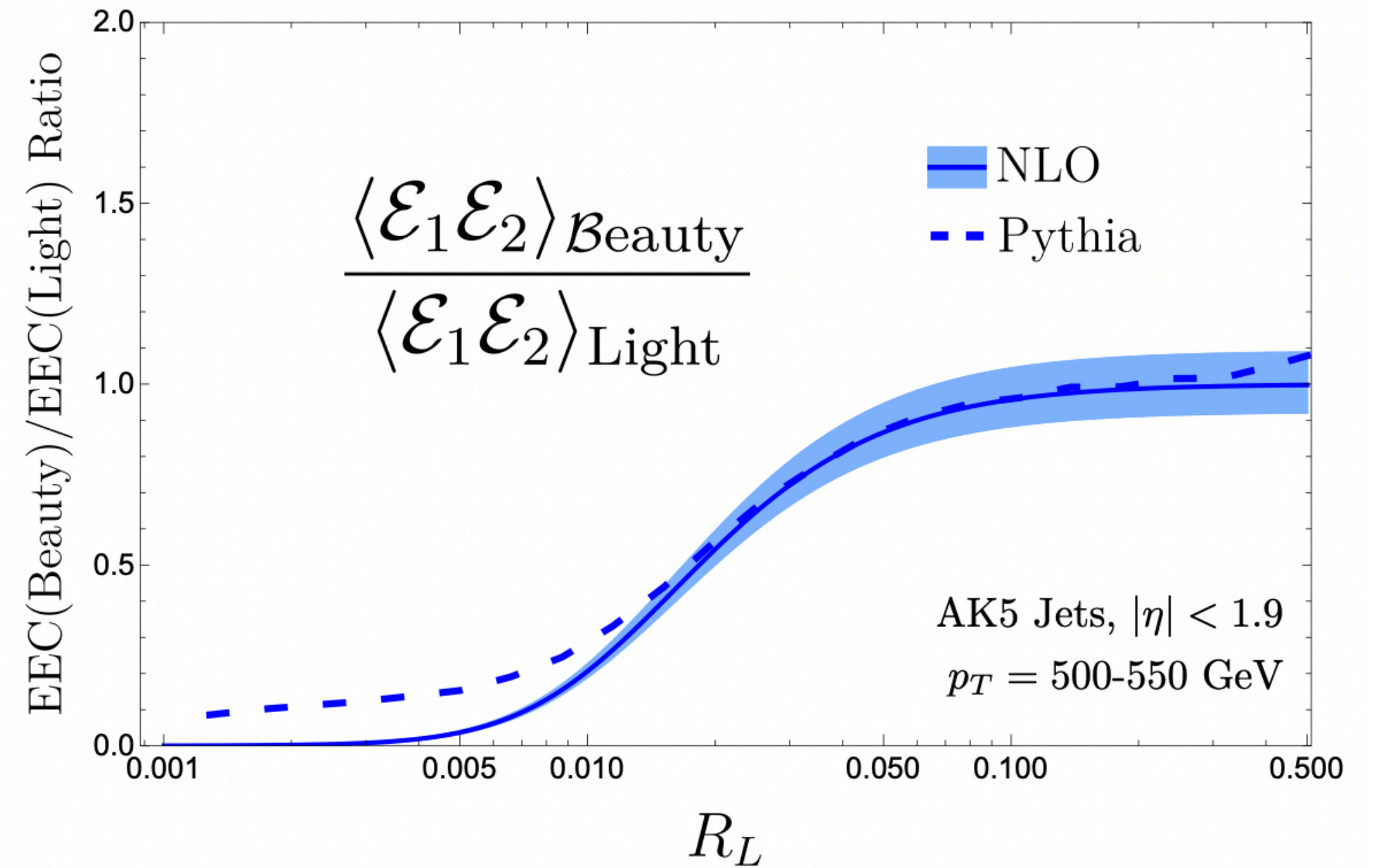
# Application: Confinement

No need for jet grooming to see this suppression!

$$\langle \Psi | \mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) | \Psi \rangle$$



[EC, Lee, Mecaj, Moult]



→ Highlights the role of formal theory in identifying the **correct field theoretic observables** for experiments





# Pushing the Boundaries of Jet Substructure

Evan Craft — Yale University



Work **in prep.** with K. Lee, B. Mecaj, I. Moulton, & M. Gonzalez



Yale University



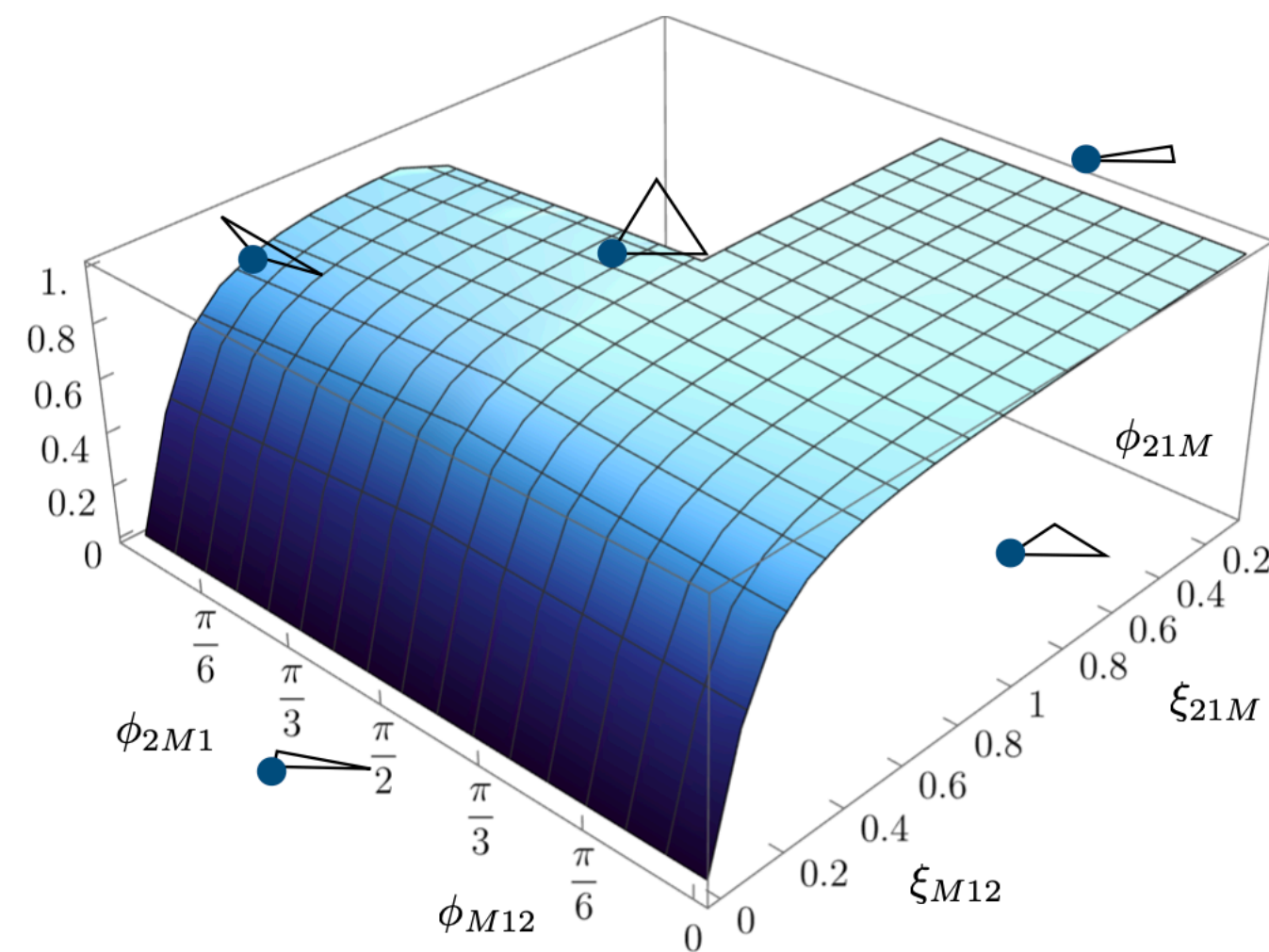
MIT



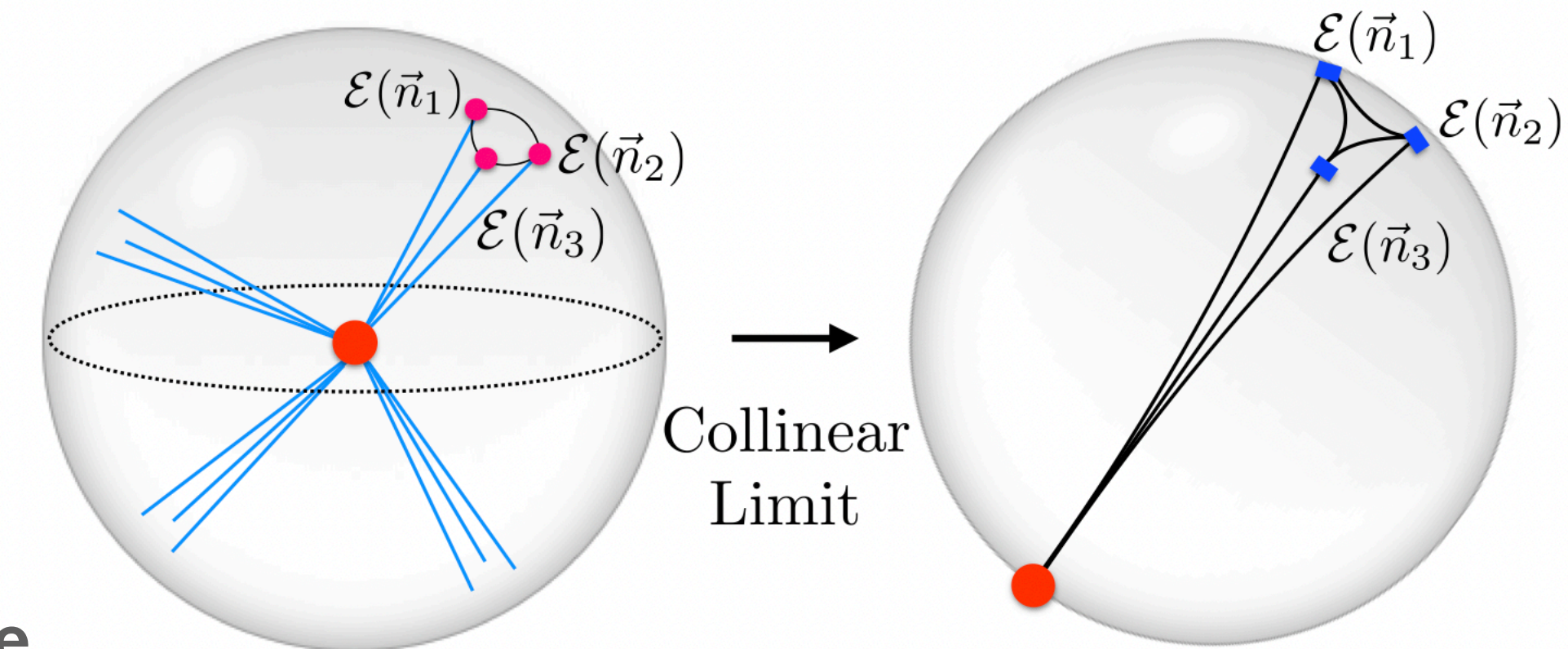
# Extension: Higher Points

Natural to also consider **higher point** correlators

## Experimental Side



3-point EEC allows access to the **shape** of the dead-cone!



## Theoretical Side

transverse spin 0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ (iD^+)^{J-1} \psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \gamma^+ (iD^+)^{J-2} F_a^{\mu+}$$

excited by **2-point**

transverse spin 2

$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \gamma^+ (iD^+)^{J-2} F_a^{\nu+} \epsilon_{\lambda\mu} \epsilon_{\lambda\nu}$$

↑  
helicity  $\pm 1$

excited by **3-point**

→ Access to **non-Gaussianities**

→ Full **Shape** Dependence

$$\mathcal{E}(\hat{n}_1) \dots \mathcal{E}(\hat{n}_k) \sim \sum_i \theta^{\tau_i-4} \mathbb{O}_i(\hat{n}_1)$$

→ Probe fundamental operators of **QCD**



# Elliptic Polylogarithms

In addition to elliptic integrals, one encounters further **complex structures** with the **EEEC**

→ Multiple Elliptic Polylogarithms (**MPLs**)

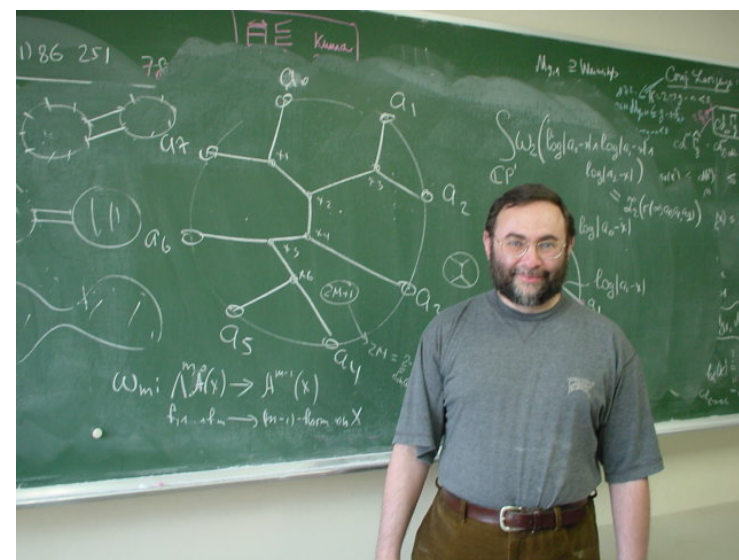
→ **Elliptic Curve** + Iterated **Logarithm**



C. Duhr

↓  
**Elliptic Polylogarithms**

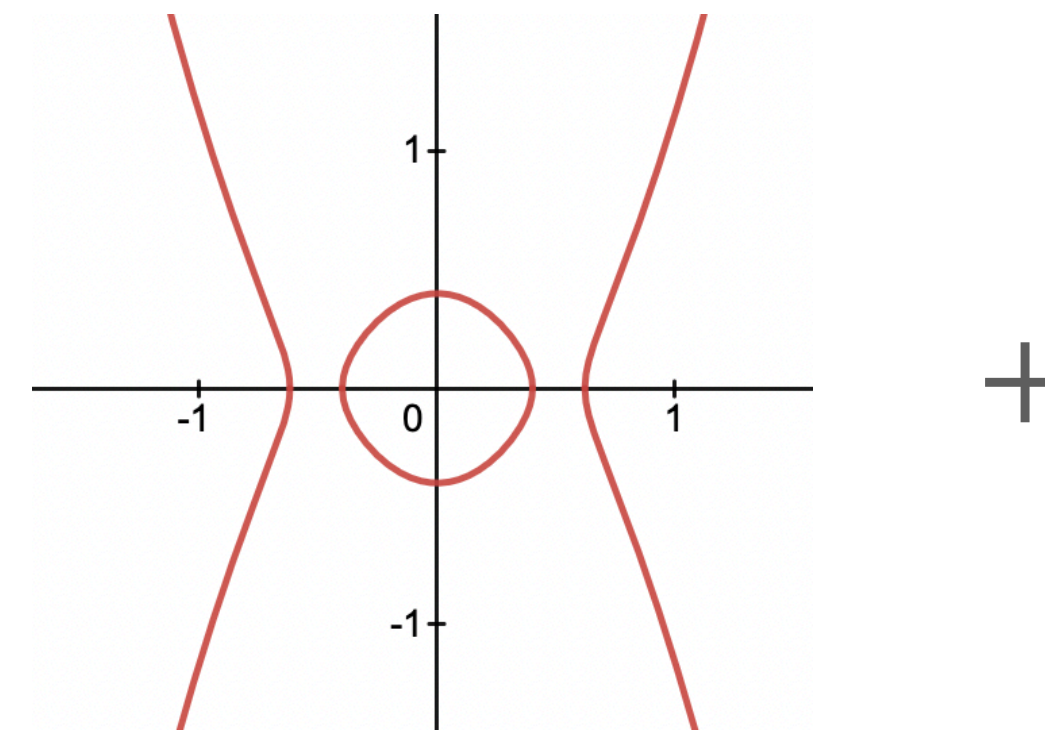
tools for **massive EEEC**



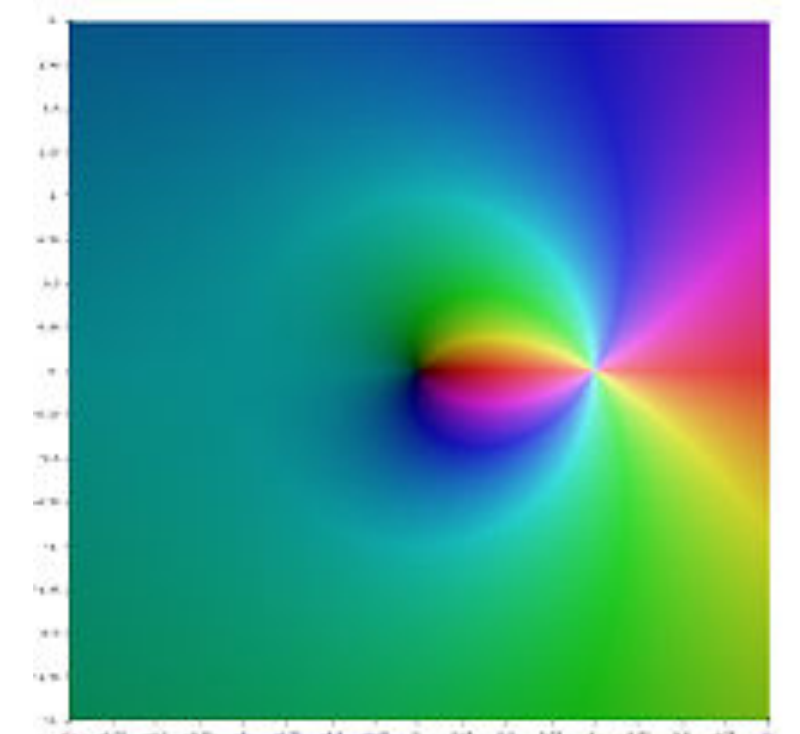
A. Goncharov

↓  
**Generalized Polylogarithms**

tools for **massless EEEC**



**elliptic structure**



**polylogarithmic structure**

→ Only understood within the **last 5 years**

→ **EEC** at the **frontiers** of math and physics

## Elliptic polylogarithms and iterated integrals on elliptic curves. Part I: general formalism

Johannes Broedel (Humboldt U., Berlin, Inst. Math. and Humboldt U., Berlin), Claude Duhr (CERN and Louvain U., CP3), Falko Dulat (SLAC), Lorenzo Tancredi (CERN)

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Report number: CERN-TH-2017-273, CP3-17-57, HU-EP-17-29, HU-Mathematik-2017-09, SLAC-PUB-17194

View in: [ADS Abstract Service](#), [CERN Document Server](#)



# Topological Aspects

Fixes the **structures** which appear in the result

## Beautiful Structures: **Elliptic Functions**

$$\frac{d\Sigma}{d \cos \chi} = \sum_{i < j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi)$$

Kinematic constraint gives rise to an **elliptic curve**

$$y^2 = 4x^3 - g_1 x - g_3$$

$g_1, g_3$  depend on the **kinematic configuration** (mass, angle, etc.)

Two Point

$$\int \frac{1}{y}, \int \frac{x^2}{y}, \int \frac{1}{(x^2 - p^2)y}$$

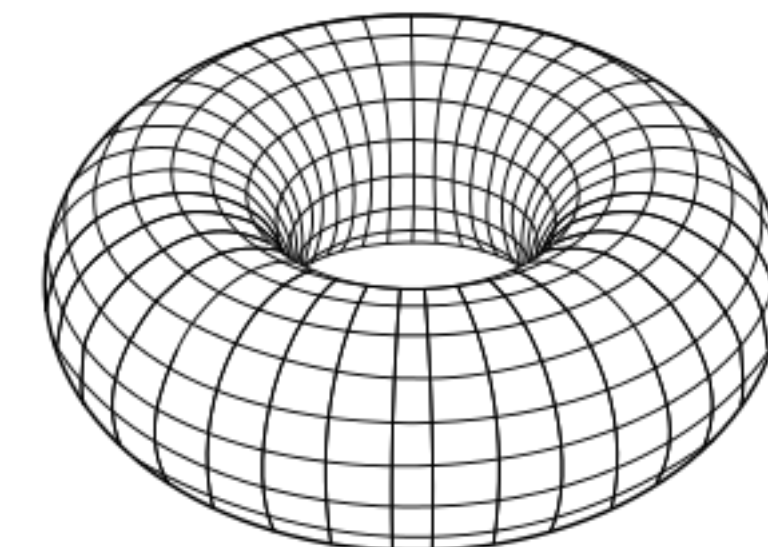
→ **Elliptic Integrals:  $E, F, \Pi$**

Three Point

$$\int \frac{1}{y} \{E, F, \Pi\}, \int \frac{x^2}{y} \{E, F, \Pi\}, \dots$$

→ **eMPL's, Dilogarithms, etc.**

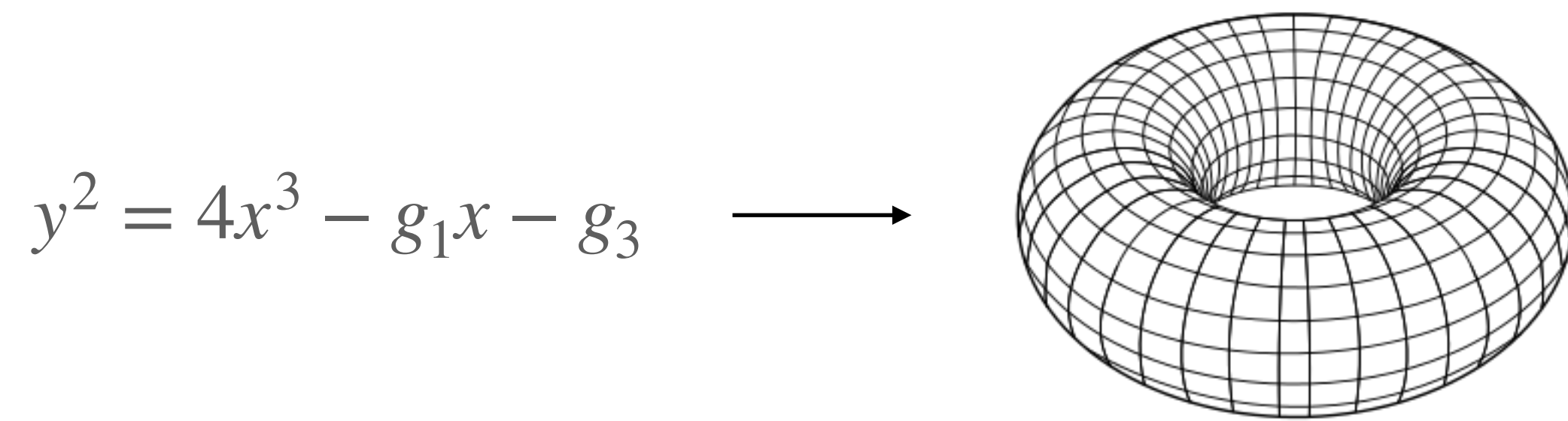
analytic **isomorphism** to a torus





# Topological Aspects

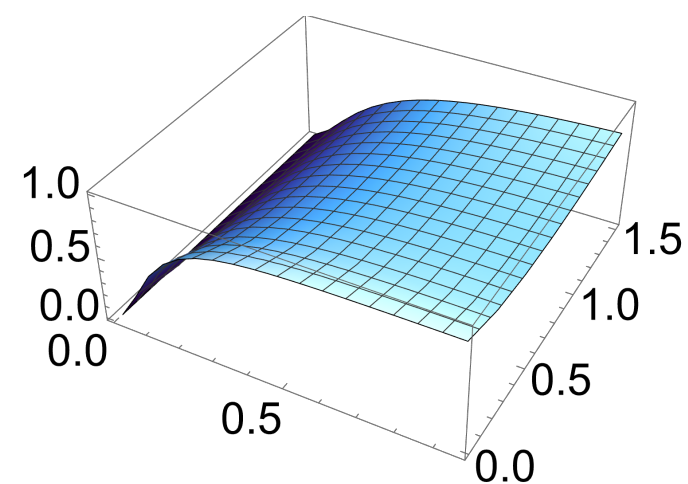
There is a direct mapping from the **kinematic configuration** of the **EEC**, to the torus



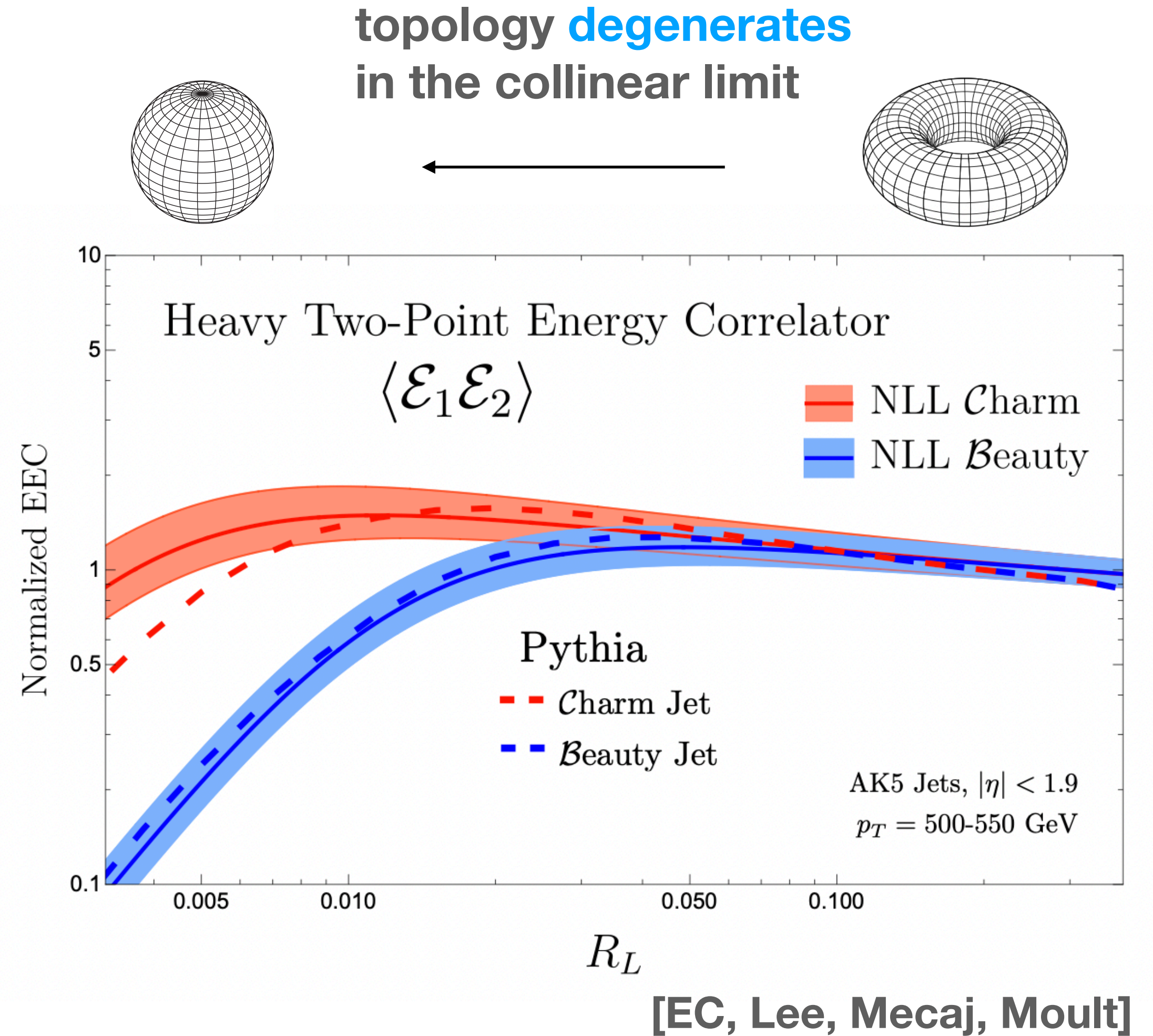
$$\omega_1 \sim {}_2F_1(1/2, 1/2, 1; \lambda)$$

$$\omega_2 \sim {}_2F_1(1/2, 1/2, 1; 1 - \lambda)$$

**periods** deformed by kinematics



Similar degeneration for the three point!

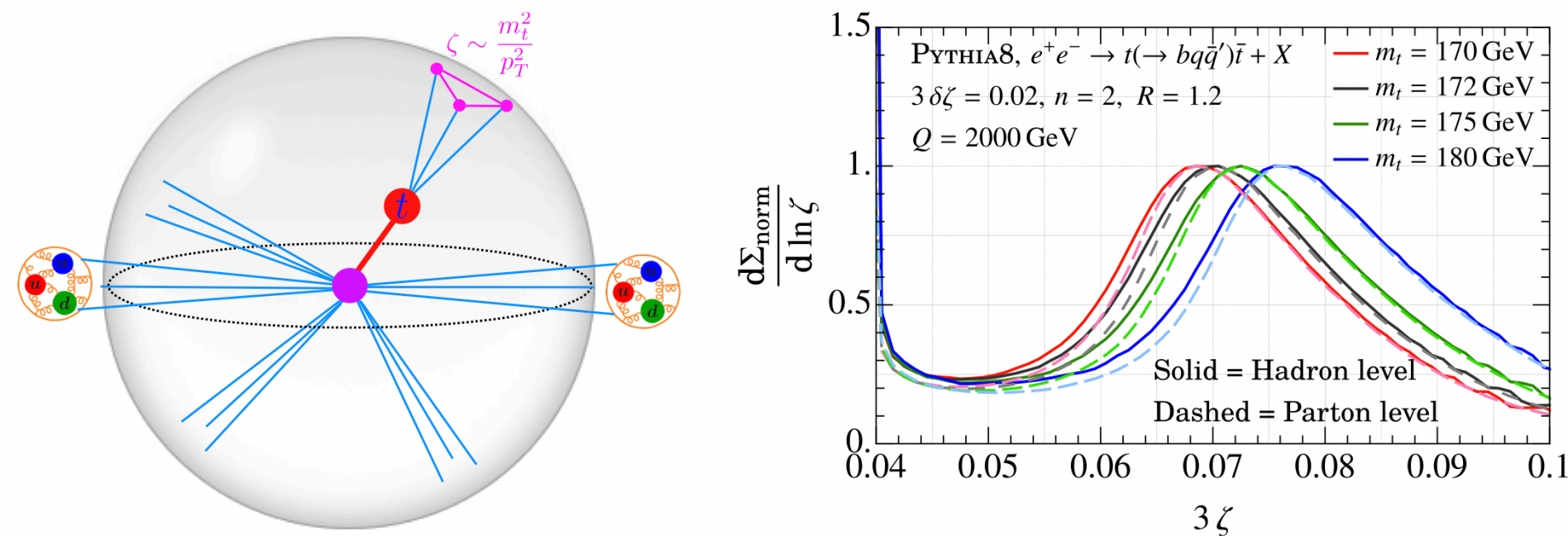


Interesting to study the **topological** aspects of the observable



# What can you do with this?

## Precision Measurements of the Top Mass

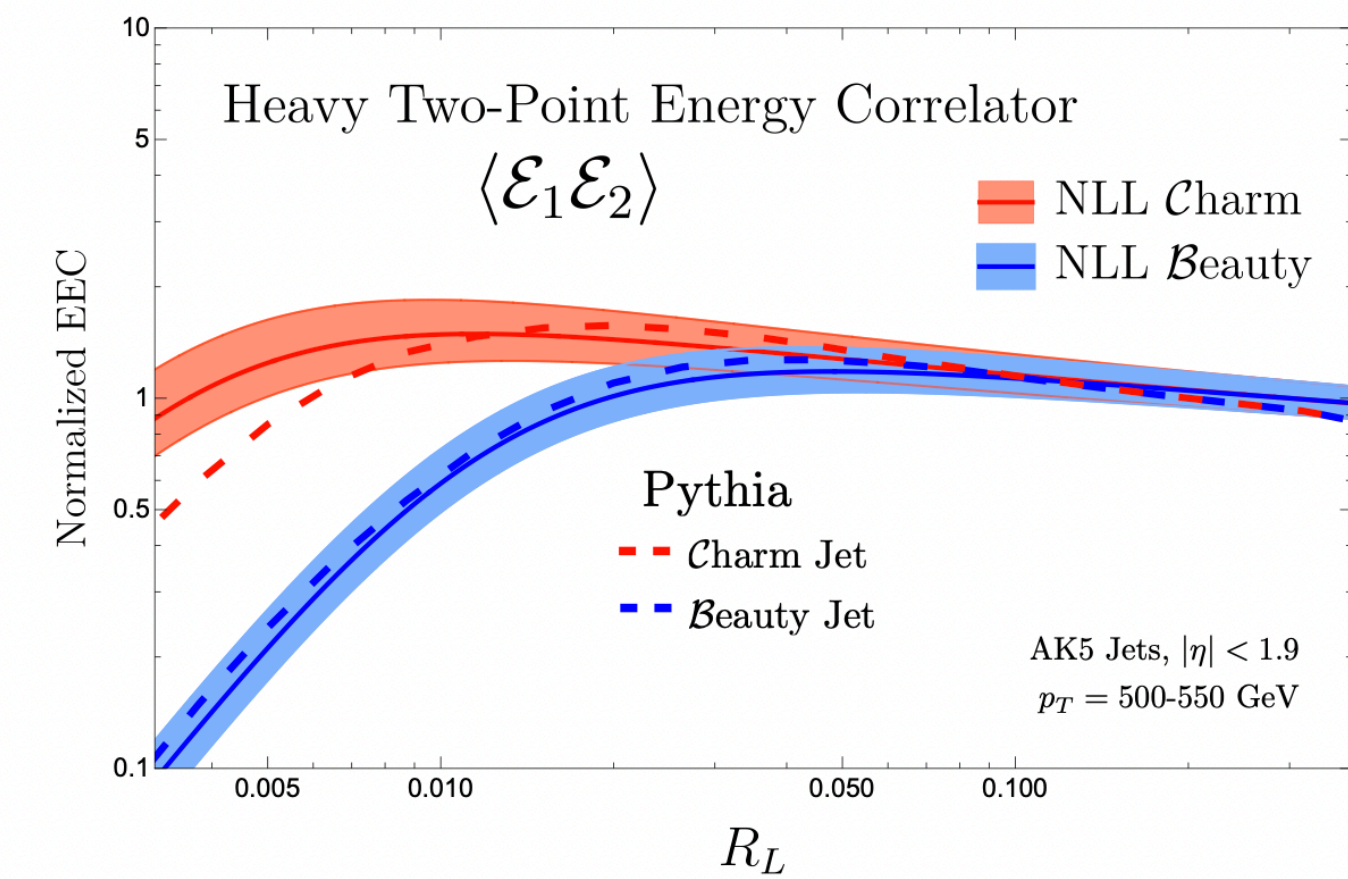


[Holguin, Mout, Pathak, Procura]

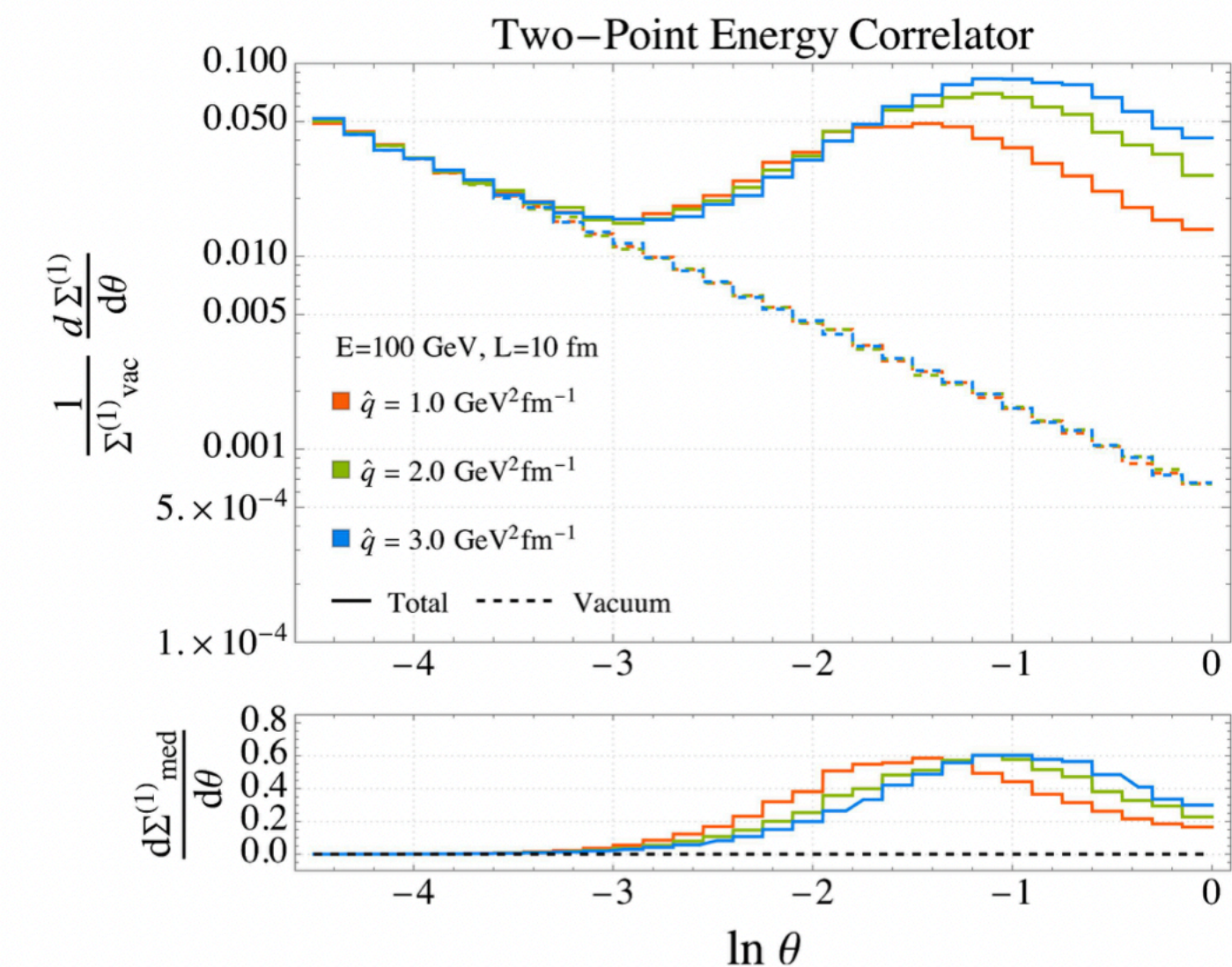
Because of its dual description, the EEC has been applied to a **diverse range of fundamental questions**

→ **Energy Correlators** present a promising avenue for bolstering our collider studies

[EC, Lee, Mecaj, Mout]



## Hadronization and Confinement



## Understanding the Quark Gluon Plasma

[Andres, Holguin, Mout, ...]



# What about the Heavy E3C?

## Formal

1. Understanding the **space of functions** generated by correlations of heavy energy flow

$$\int \frac{1}{y} \{E, F, \Pi\}, \quad \int \frac{x^2}{y} \{E, F, \Pi\}, \dots \subset \text{E3C}$$

2. Playground for heavy **SCET factorization** with clear applications to collider physics

$$\frac{d\Sigma}{d\psi_1 d\psi_2 d\psi_3} \sim \vec{J} \otimes \vec{H} \otimes \vec{S}$$

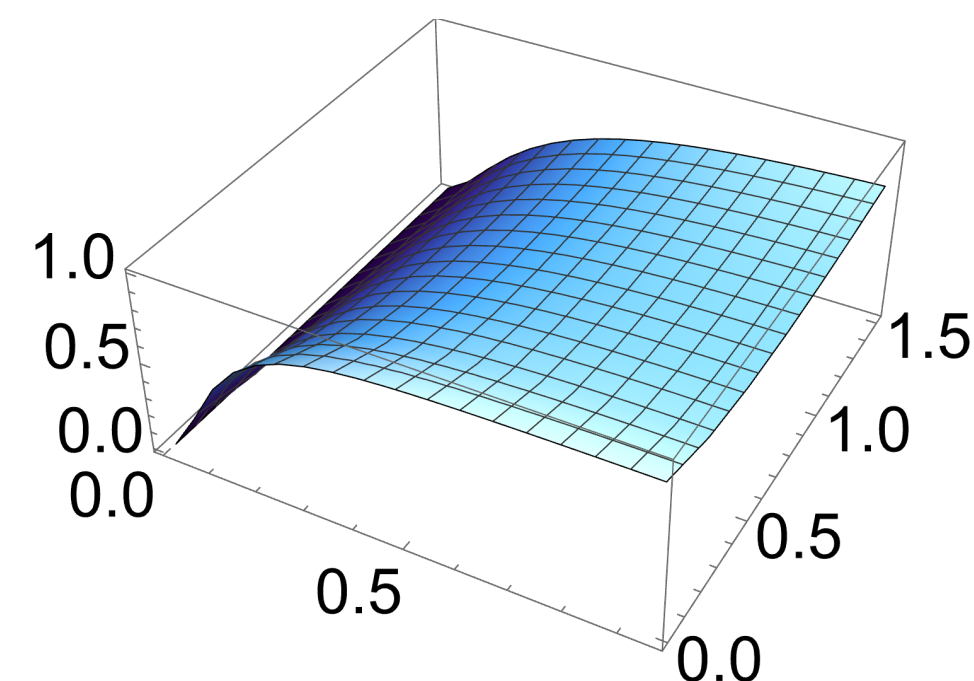
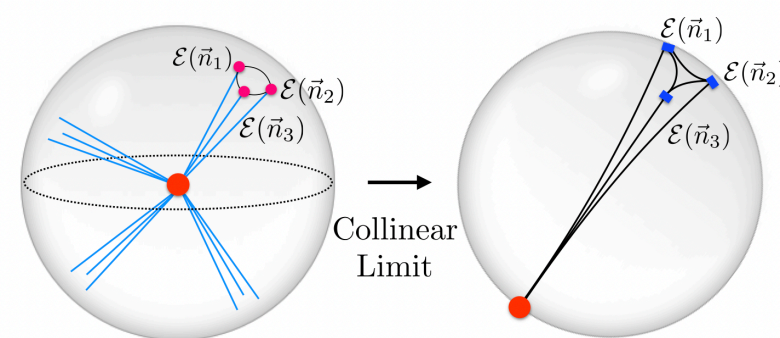
## Phenomenological

1. Access to the shape of the deadcone with applications to **precision tests of QCD**

2. An example of a **3 parameter, heavy** event shape observable

→ Useful for tuning event generators

→ Playground for testing ML techniques





# What about the Heavy E3C?

## Formal

1. Understanding the **space of functions** generated by correlations of heavy energy flow

2. Playground for heavy **SCET factorization** with clear applications to collider physics

$$\int \frac{1}{y} \{E, F, \Pi\}, \quad \int \frac{x^2}{y} \{E, F, \Pi\}$$

Both halves of this slide are in *symbiosis*

$$\vec{J} \otimes \vec{H} \otimes \vec{S}$$

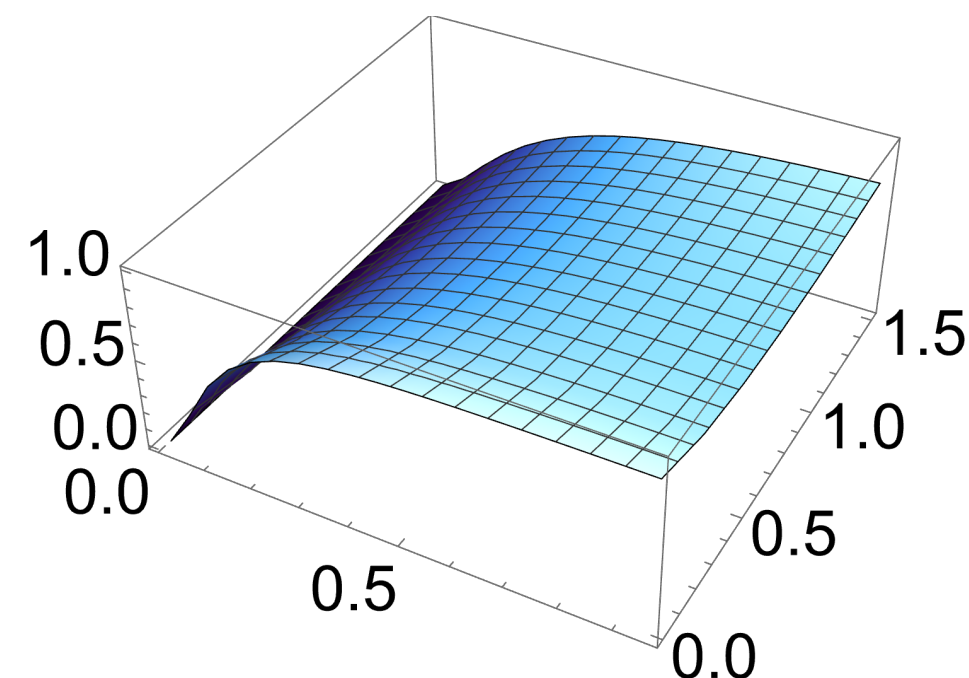
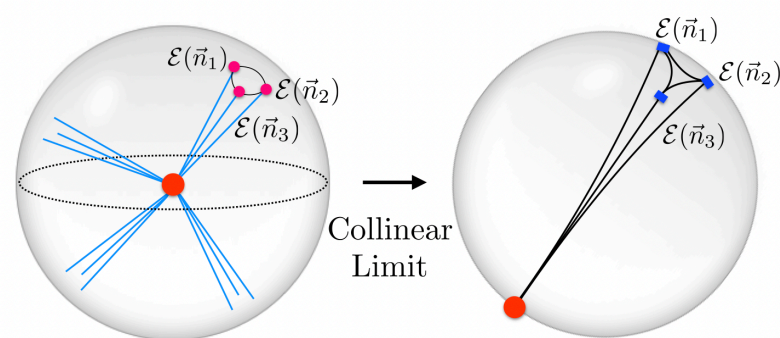
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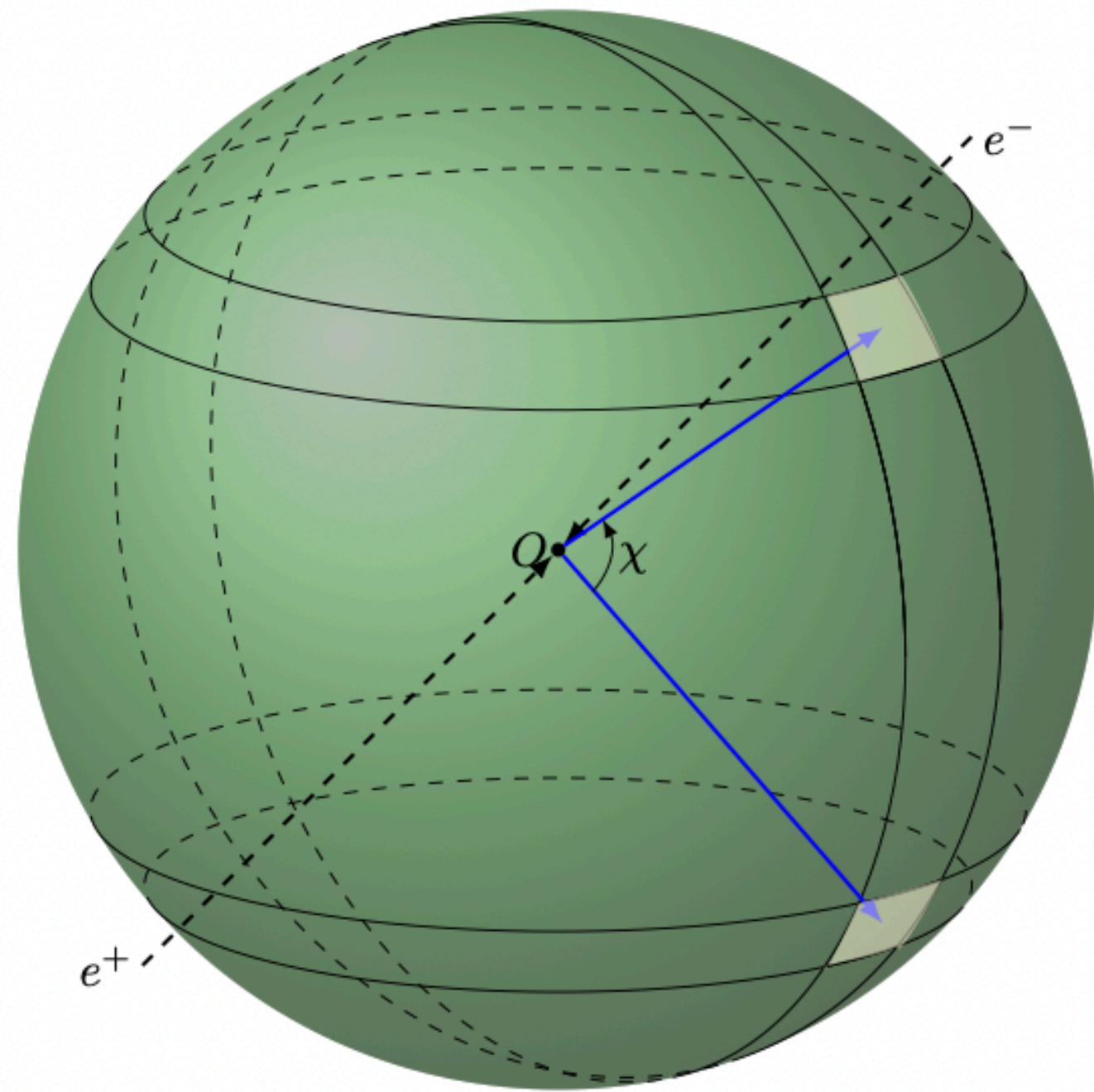




# Two Symbiotic Perspectives

## Experimental View

$$\frac{d\sigma}{d\cos\chi} = \sum_{i < j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi)$$

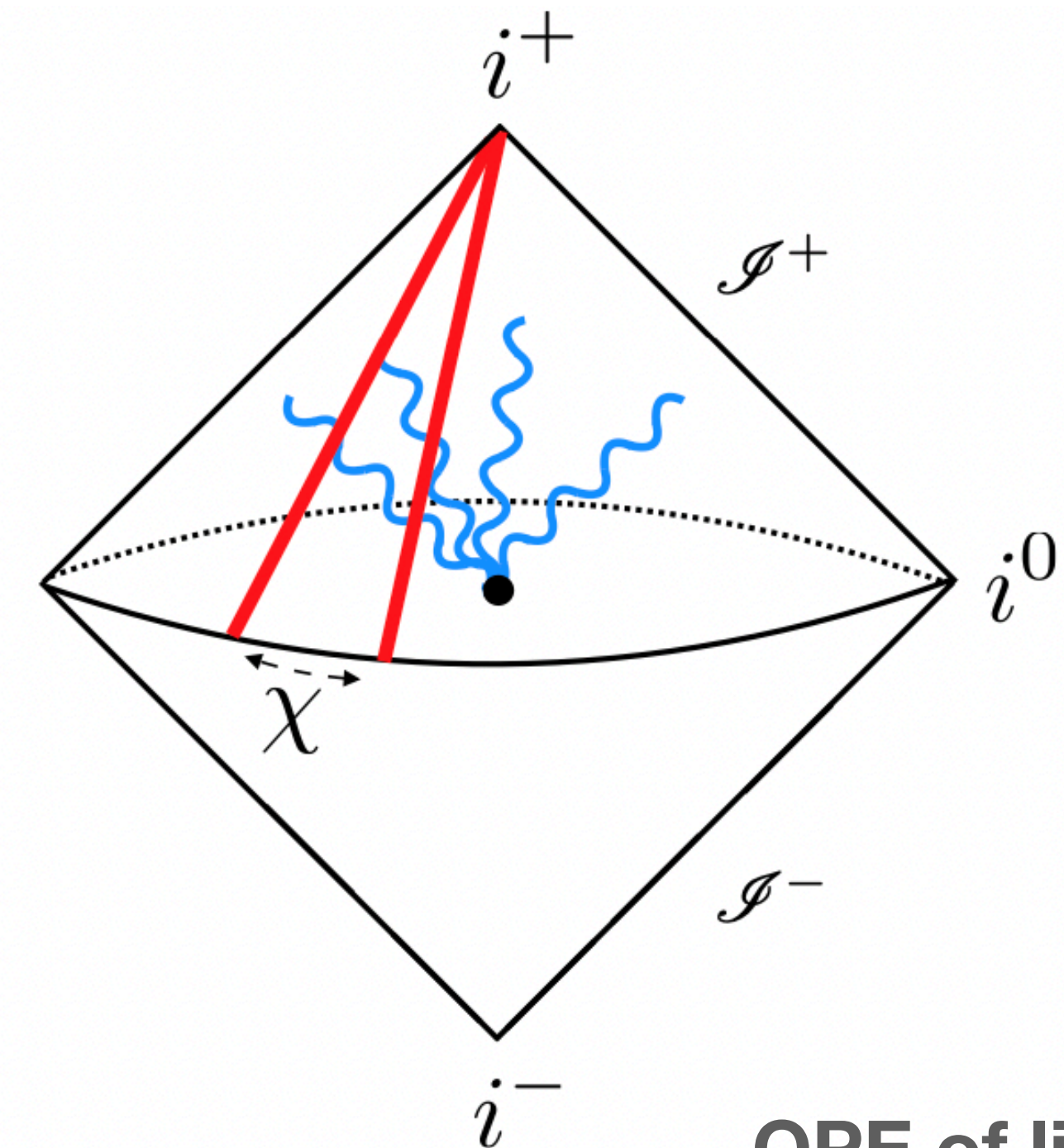


Calorimeter cells at null infinity

## Theoretical Foundation

$$\frac{d\sigma}{d\cos\chi} = \frac{\int d^4x e^{iq \cdot x} \langle \mathcal{O}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{O}^\dagger(0) \rangle}{\int d^4x e^{iq \cdot x} \langle \mathcal{O}(x) \mathcal{O}^\dagger(0) \rangle}$$

Beautiful Correspondence



OPE of lightray operators



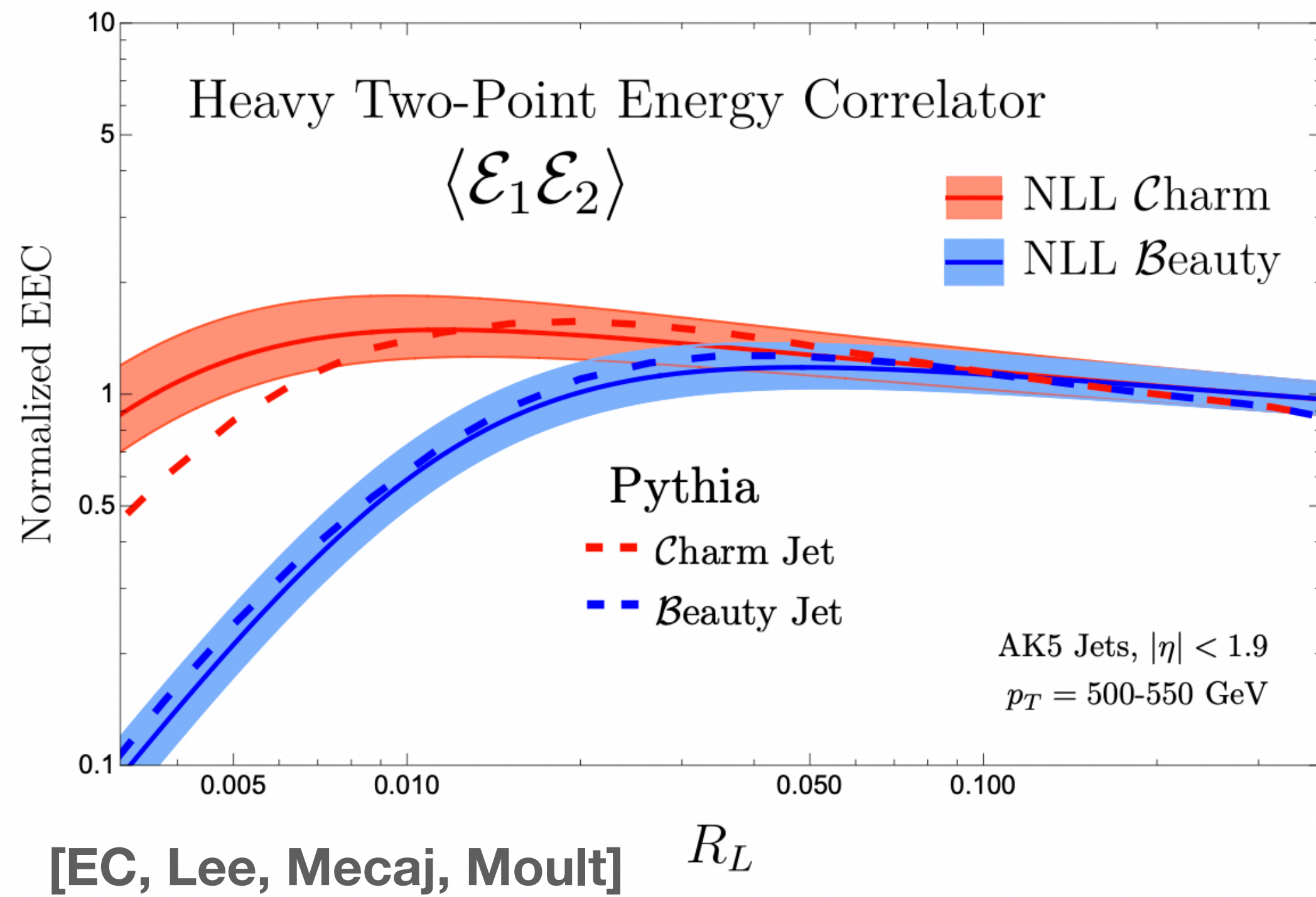
# Concluding Remarks

Unifying Theory and Experiment

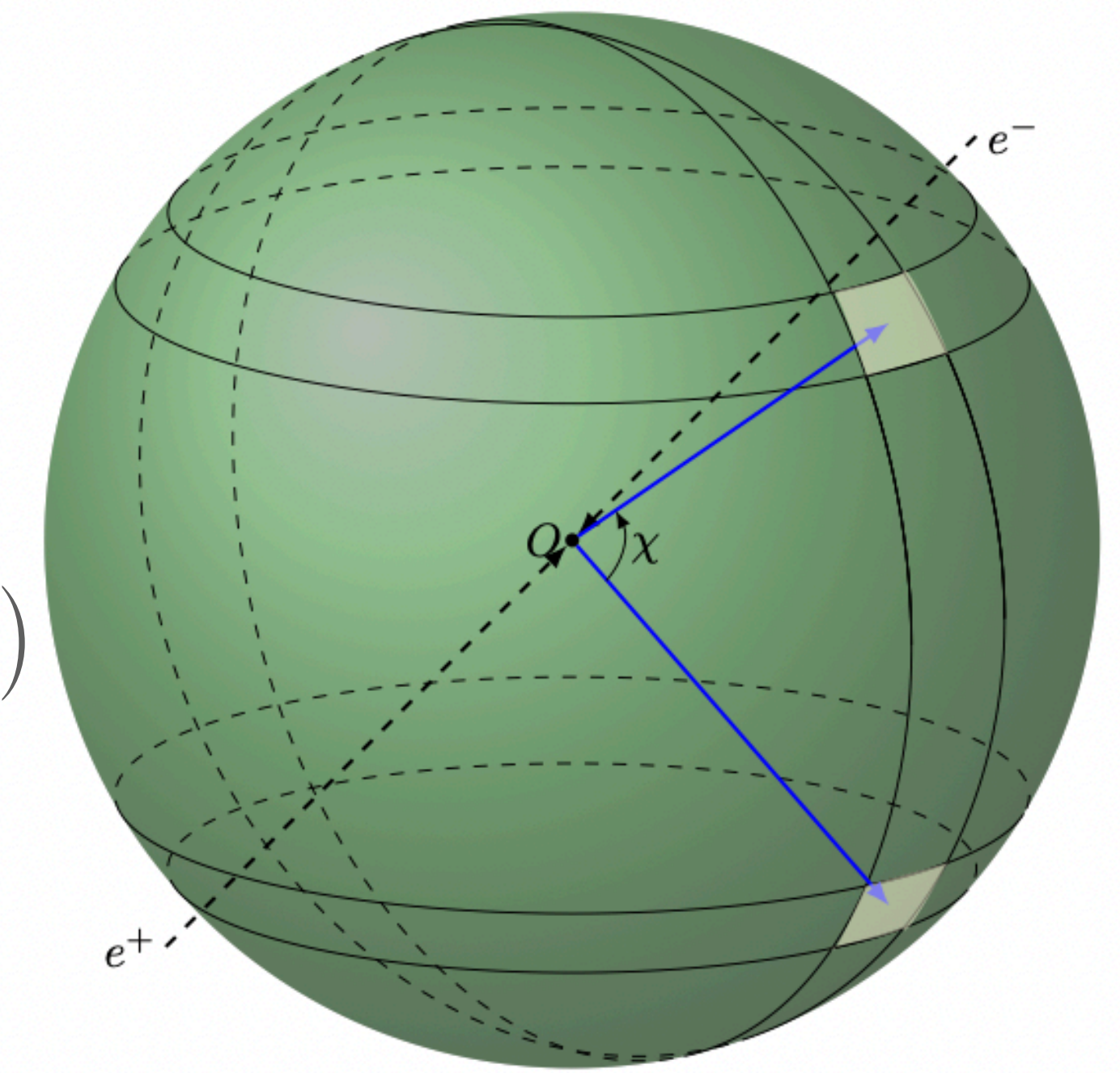


# Two Symbiotic Perspectives

Beautiful and Charming Interplay!



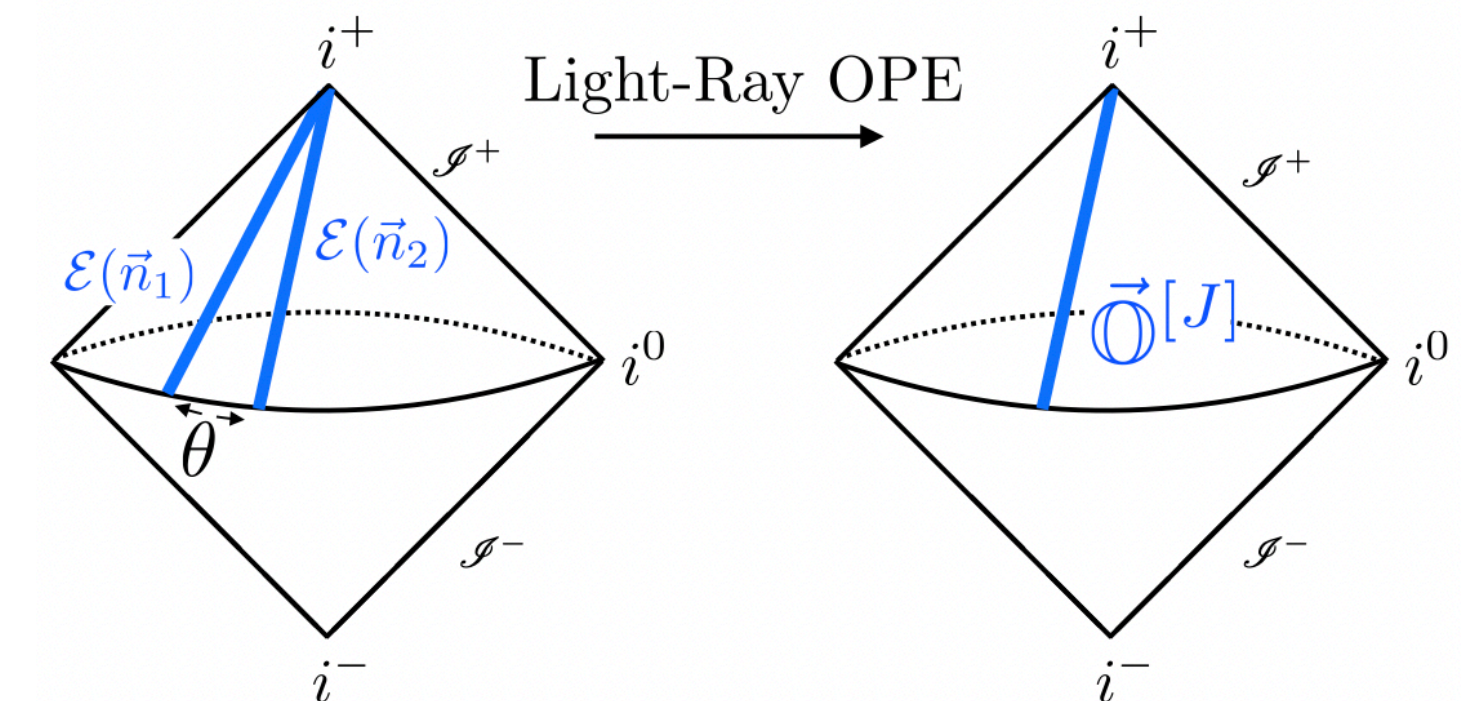
$$\frac{d\sigma}{d \cos \chi} = \sum_{i < j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi)$$



Experiment

$$\mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \sim \sum_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$

Theory



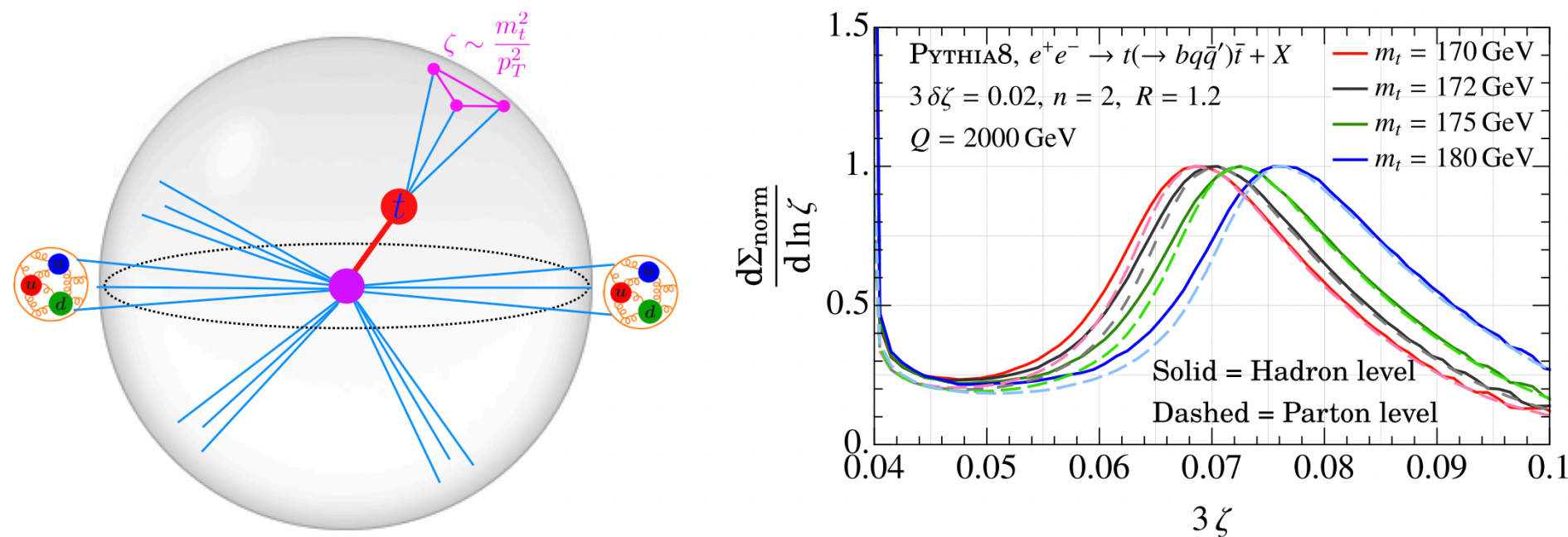
New Observables

This sort of collaboration is crucial for the success of future collider studies



# Symbiosis in Action

## Precision Measurements of the Top Mass

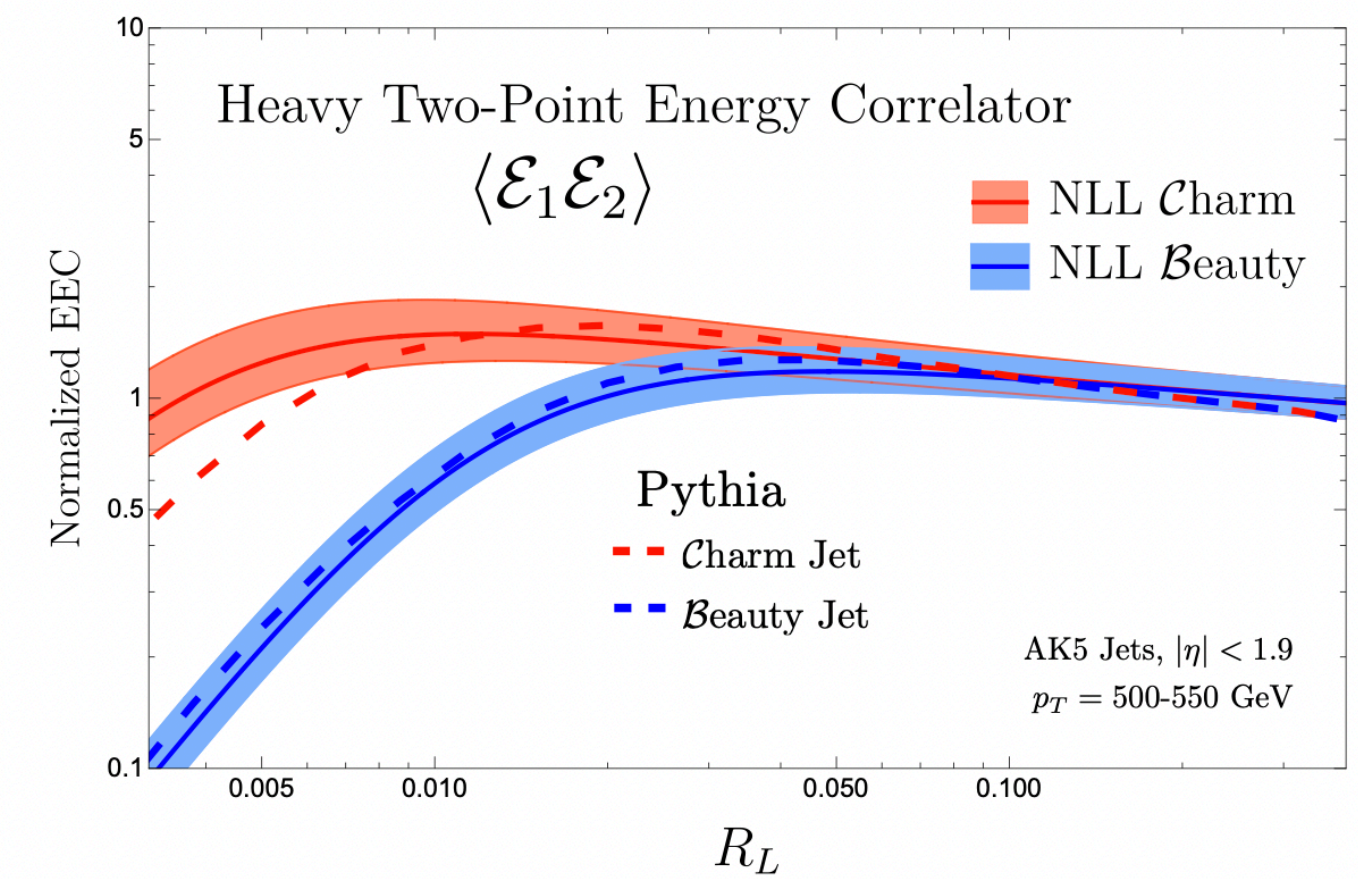


[Holguin, Mout, Pathak, Procura]

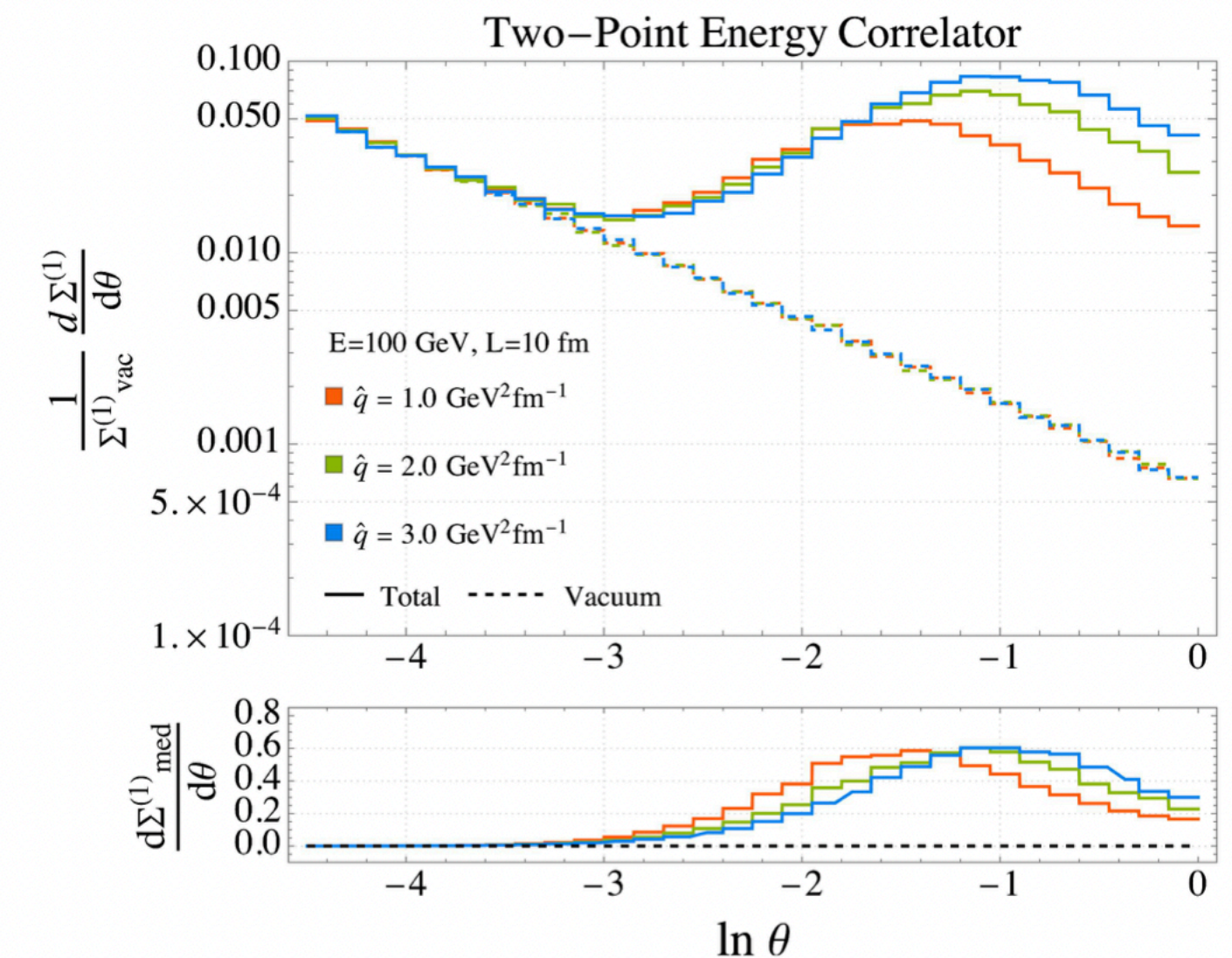
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# Summary

Jet substructure provides a physical realization of the OPE limit of **light-ray operators**

→ Direct **bridge** between recent theoretical advancements and QCD Phenomenology

Creates an unprecedented symbiosis between **theory** and **experiment**

→ Allowing for sharp probes of interesting physics, new and old

