Detectorology and its Phenomenological Applications

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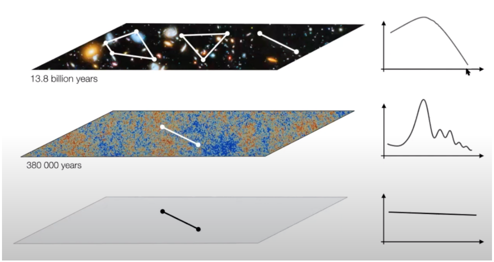
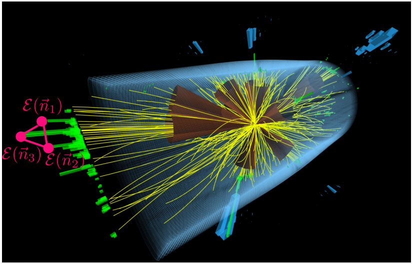
based on work with M Kologlu, G Korchemsky, K Lee, I Moult, and A Zhiboedov

July 19, 2024

What is a good collider observable?





Correlation functions

- Simple observable
  - Requires direct measurement of the system
- In collider physics
  - Detectors are situated far away and only see the final state
  - High multiplicity states - descriptions in terms of individual particles are difficult/impractical

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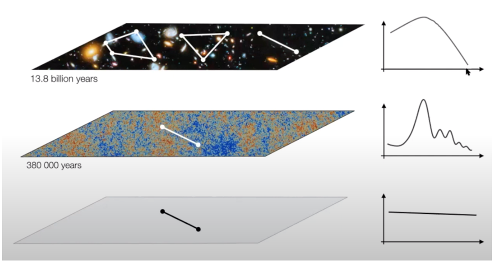

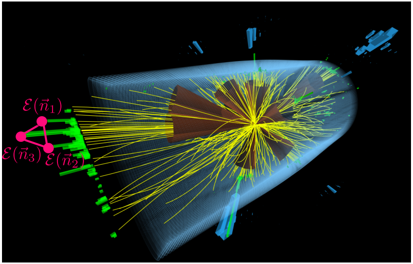

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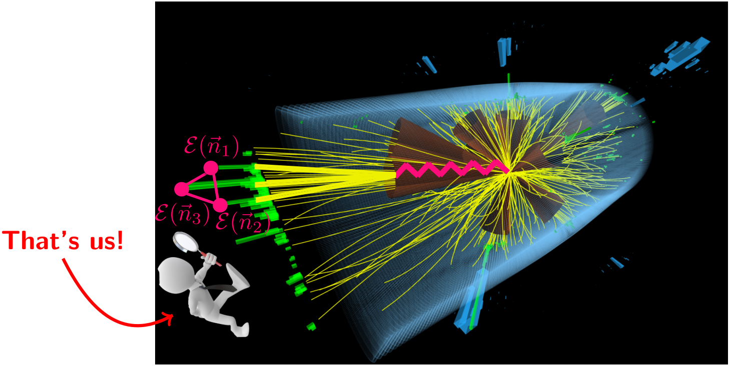
How should we study the system?

Asymptotic Observables

Our goal is to understand how one can characterize and study a quantum mechanical system using asymptotic observables

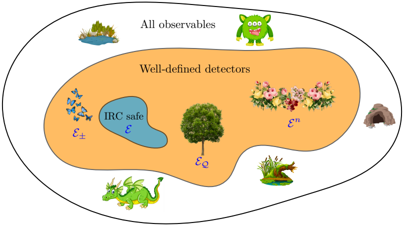


Applicable to any physical system

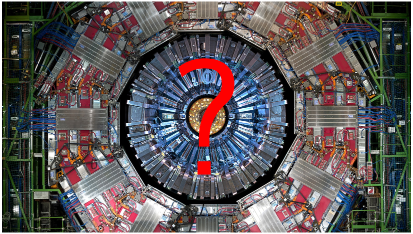
On hammers and cameras

[Caron-Huot Koloğlu Kravchuk Meltzer Simmons-Duffin '22]

- We understand the hammer in terms of local operators
- We can build cameras out of **well-defined** detector operators
- Particles have a number of properties such as energy and charge
- We want to understand detectors that can measure these various properties



Why build these objects in the first place?



[https://cms.cern/detector]

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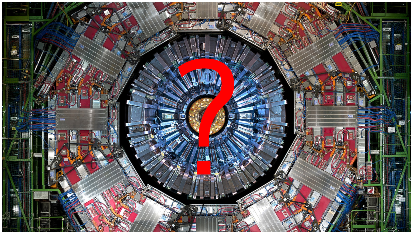
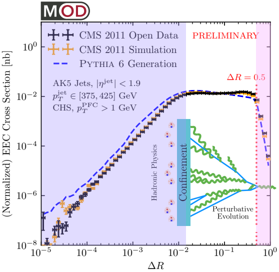
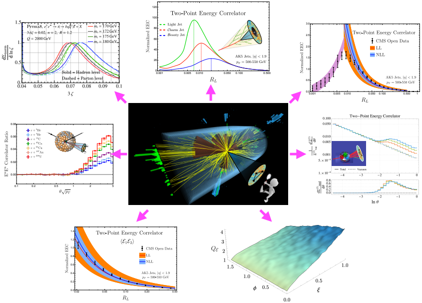
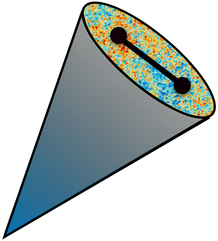


[https://cms.cern/detector]

Energy Correlators!

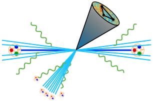


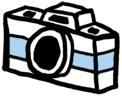


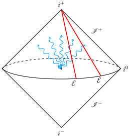


Outline

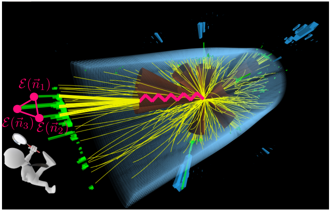
- The ANEC and Energy Correlators
- $\mathcal{E}_J$ and $\mathcal{E}_Q$ Detectors
- Event Shapes and "Renormalization"



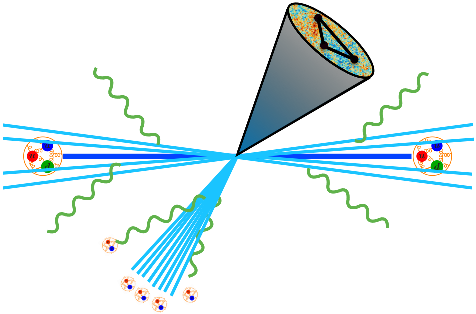




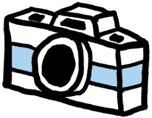
Phenomenological applications throughout



The ANEC and Energy Correlators



Asymptotic Energy Flux Operators



As an operator on multi-particle states $|X\rangle$

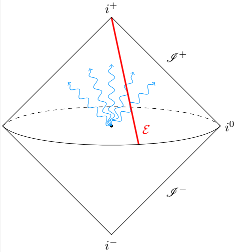
$$\mathcal{E}(\widehat{n})|X\rangle = \sum_i E_{k_i}\delta^{d-2}(\Omega_{\widehat{n}} - \Omega_{\widehat{k}_i})|X\rangle$$

- Sees particles along direction $\widehat{n}$ and measures their energy
  - Sensitive to asymptotic radiation along a particular direction - like a calorimeter cell!

- Formally, integrate the stress tensor T along future null infinity $\mathscr{I}^+$
  - Averaged Null Energy Condition (ANEC) operator
  - Well-defined in field theory as a light-ray operator

$$\mathcal{E}(\widehat{n}) = \lim_{r\to\infty} r^{d-2}\int_0^\infty dt\, n^i T_{0i}(t, r\widehat{n}) \overset{\text{CFT}}{=} 2\mathbf{L}[T](\infty, z)$$

- Light transform $\mathbf{L}[\mathcal{O}_J]$: conformally invariant integral transform of local operator $\mathcal{O}_J$ (spin-J, dimension-Δ)



[Hofman Maldacena '08]

From operators to observables

We get observables by taking correlation functions of detectors
Two point correlator ⇒ Energy-energy correlator (EEC)

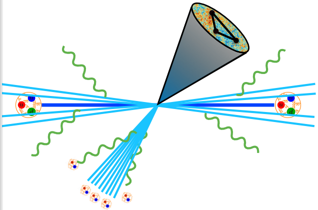
Perturbative calculation

- EEC is a weighted cross section
  - $d\sigma$: Phase space integral
  - Weighted by particle energies as a function of angular separation

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1 - \cos\chi_{ij}}{2}\right)$$

- Infrared and collinear (IRC) safe

[Basham Brown Ellis Love '78]
[Ore Sterman '80] [Korchemsky Sterman '99]
[Hofman Maldacena '08] [Belitsky et al. '13]



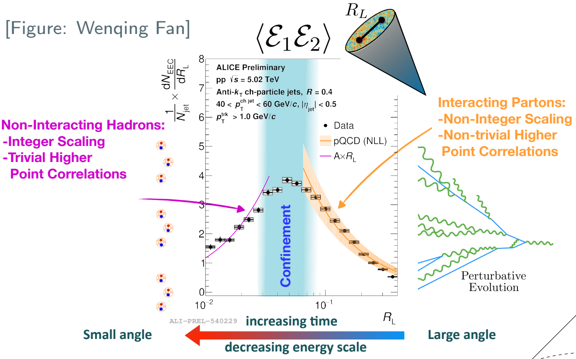
$$\frac{d\Sigma}{dp_T \, d\eta \, d\{\zeta\}} = \sum_i \mathcal{H}_i(p_T/z, \eta, \mu) \otimes \int_0^1 dx \, x^N \mathcal{J}_{ij}(z, x, p_T R, \mu) J_j^{[N]}(\{\zeta\}, x, \mu)$$

[Lee Mecaj Moult '22]

Factorization theorem in the collinear limit ⇒ **LHC jet substructure measurements!**

The Energy-Energy Correlator

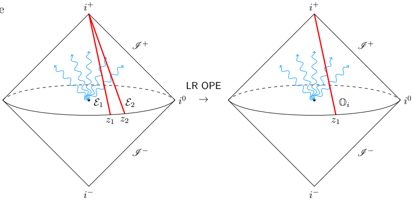
[Figure: Wenqing Fan]



Correlation function of detectors ⇒ jet substructure observable

A clear link between theory and experiment!

- Evolution of the jet goes from right to left
  - Distinct scaling regimes corresponding to partonic and hadronic physics
  - Transitions image the physical scales of QCD

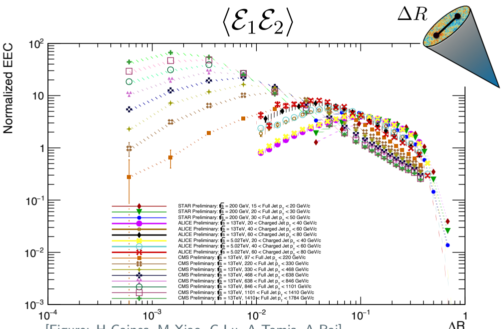


Perturbative scaling predicted by the light-ray OPE

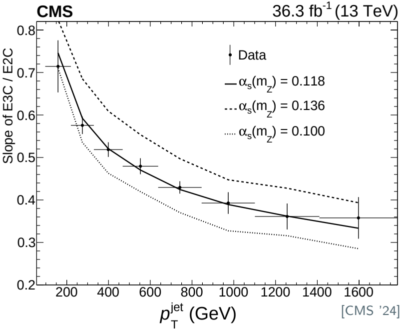
⇒ **Universal scaling behavior!**

$$\mathcal{E}(\widehat{n}_1)\mathcal{E}_2(\widehat{n}_2) \sim \sum_i \theta^{\tau_i - 4}\mathbb{O}_i(\widehat{n}_1)$$

EECs in Data



[Figure: H Caines, M Xiao, C Lu, A Tamis, A Rai]



[CMS '24]

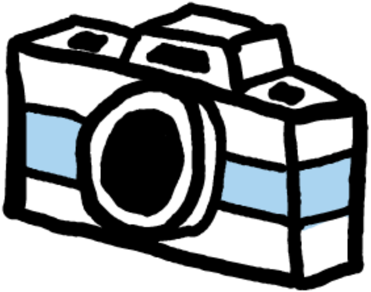
Many ongoing and future measurements

- Precision measurement of strong coupling constant
  - $\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$ [arXiv:2402.13864]
- In-medium (quark-gluon plasma)
- Higher point correlators
- Massive quark effects (see E Craft's talk)

Anomalous scaling at the LHC!

$$\frac{\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_{J-1} \rangle}{\langle \mathcal{E}_1 \mathcal{E}_2 \rangle} \sim R_L^{\gamma(J)-\gamma(3)}$$

$\mathcal{E}_J$ and $\mathcal{E}_Q$ Detectors

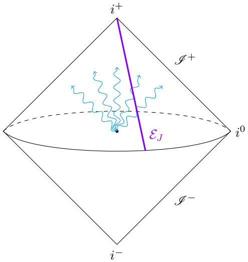


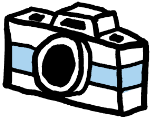
$\mathcal{E}_J$ Detectors

A new camera which measures the energy flux to an arbitrary power, E^{J-1}

$$\mathcal{E}_J(\widehat{n})|X\rangle = \sum_i E_{k_i}^{J-1}\delta^{d-2}(\Omega_{\widehat{n}} - \Omega_{\widehat{k}_i})|X\rangle$$

- In principle, allow $J \in \mathbb{C}$





Well-Defined Operators for all J?

- Expect light-ray operators
- For $J \in 2\mathbb{Z}_{\geq 0}$

$$\mathcal{E}_J(z) \sim \mathbf{L}[\mathcal{O}_J](\infty, z)$$

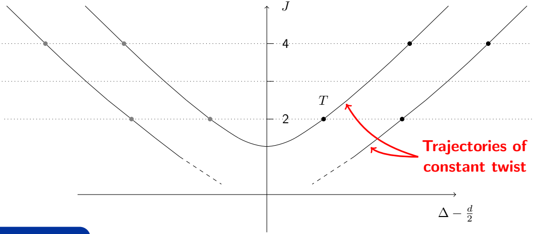
  - Light transform of a twist-2, spin-J, local operator
  - Energy weighting is related to spin
- What about generic J?

[Caron-Huot Koloğlu Kravchuk Meltzer Simmons-Duffin '22]

Analyticity in J

Analytically continue light transform in $J \Rightarrow$ light-ray operators $\mathbb{O}_J^+$

[Kravchuk Simmons-Duffin '18]



- Correspond to light transforms of local operators for even integer J

$$\mathbb{O}_J^+(x,z) = \mathbf{L}[\mathcal{O}_J](x,z)\,, \quad J \in 2\mathbb{Z}_{\geq 0}$$

Generalized energy flux operators are particular light-ray operators!

$$\mathcal{E}_J(z) \propto \mathbb{O}_J^+(\infty, z)$$

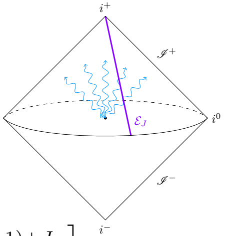
Non-local and interpolate between the local operators of the theory

$\mathcal{E}_J$ Properties

- "Even-spin" or "charge-even"
- Detector quantum numbers
  - $\mathbf{L}$: $(\Delta, J) \to (\Delta_L, J_L)$
  - $\Delta_L = 1 - J$; $J_L = 1 - \Delta$
  - Twist-2: $J_L = -1 - J$

[Caron-Huot Koloğlu Kravchuk Meltzer Simmons-Duffin '22]

Even-Spin Detectors



As an operator in a perturbative scalar field theory

$$\mathcal{D}^+_{J_L}(z) = \frac{1}{C_{J_L}} \int d\alpha_1\, d\alpha_2 : \overline{\varphi}(\alpha_1, z)\varphi(\alpha_2, z) :$$
$$\times \left[(\alpha_1 - \alpha_2 + i\epsilon)^{2(\Delta_\varphi - 1)+J_L} + (\alpha_2 - \alpha_1 + i\epsilon)^{2(\Delta_\varphi - 1)+J_L}\right]$$

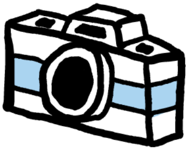
- Twist-2, spin-J_L
  - One operator for the whole trajectory
- Bi-local integral over retarded time α
  - Collapses to a single null integral for $J_L \in 2\mathbb{Z}_{\geq 0}$
- Gives detector vertex factor

$$\langle \overline{\varphi}(-q)|\mathcal{D}^+_{J_L}(z)|\varphi(p)\rangle = (2\pi)^d \delta^d(p-q) V_{J_L}(z;p)$$
$$V_{J_L}(z;p) = \int_0^\infty d\beta\, \beta^{-1-J_L} \delta^d(p - \beta z)$$

Observables are no longer collinear safe due to energy weighting

Access to universal non-perturbative physics through multi-hadron fragmentation functions

$\mathcal{E}_Q$ Detectors



We can also build cameras that probe different quantum numbers!

Sensitivity to a charge Q (times the energy)

$$\mathcal{E}_Q(\hat{n})|X\rangle = \sum_i E_{k_i} Q_{k_i} \delta^{d-2}(\Omega_{\hat{n}} - \Omega_{k_i})|X\rangle$$

Example: null integral of a $U(1)$ current j

$$\mathcal{Q}(\hat{n}) = \lim_{r\to\infty} r^{d-2} \int_0^\infty dt\, n^i j_i(t, r\hat{n})$$

- Charge/spin-odd
- Local current j lies on twist-2 trajectory

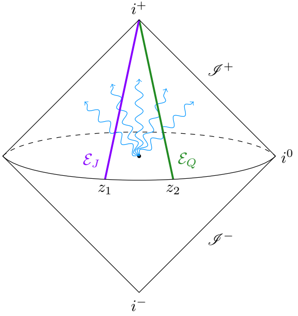
$\mathcal{E}_Q$ should be some charge-odd version of $\mathcal{E}_J$

[Hofman Maldacena '08]

Leading-Twist Detectors

Perturbative definitions of $\mathcal{E}_J$ and $\mathcal{E}_Q$ are nearly identical!

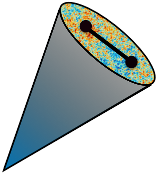
$$\mathcal{D}^{\pm}_{J_L}(z) = \frac{1}{C'_{J_L}} \int d\alpha_1\, d\alpha_2 : \overline{\varphi}(\alpha_1, z)\varphi(\alpha_2, z) : \times \left[(\alpha_1 - \alpha_2 + i\epsilon)^{2(\Delta_\varphi - 1) + J_L} \pm (\alpha_2 - \alpha_1 + i\epsilon)^{2(\Delta_\varphi - 1) + J_L} \right]$$

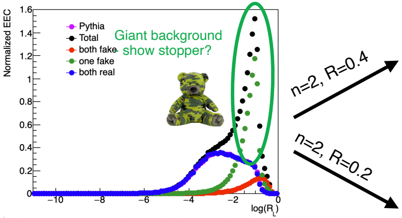


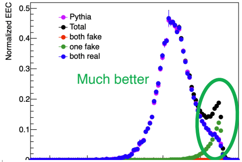
- Encodes both charge even and odd, leading-twist detector operators for $J_L \in \mathbb{C}$!
  - Same perturbative vertex factor V_{J_L} (up to signs)
- $\pm$ sign: Definite sign under charge conjugation
- Valid for any global $U(1)$ symmetry

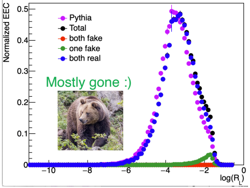
Detector Applications

[Figure: L Havener, A Rai]





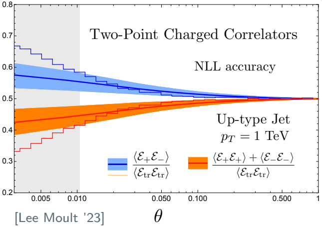




$\mathcal{E}_J$ Detectors

Large (small) powers of energy suppress (enhance) soft physics:

Applications in hot and cold QCD

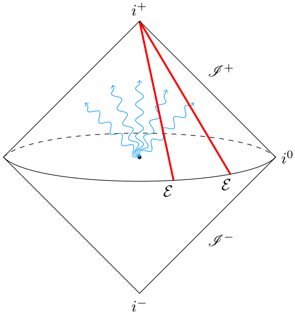


[Lee Moult '23]

$\mathcal{E}_Q$ Detectors

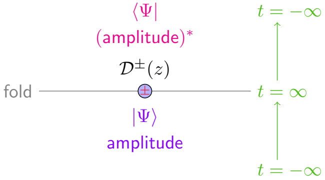
- Great resolution on charged tracks
- **More hadronization/ non-perturbative information**

Event Shapes and “Renormalization”



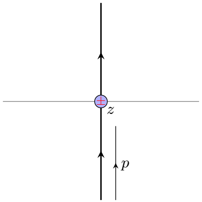
Event Shapes in Perturbation Theory

- Use Schwinger-Keldysh (in-in) formalism
- Compute diagrams order-by-order
- Choose single particle initial states $|\varphi(p)\rangle$
  - Charged under the $U(1)$ symmetry



Particles (+ charge):

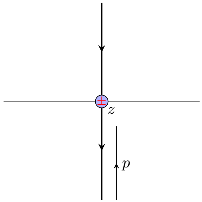
$$\langle \overline{\varphi}(-q)|\mathcal{D}^{\pm}_{J_L}(z)|\varphi(p)\rangle =$$



$$=(2\pi)^d \delta^d(p-q) V_{J_L}(z;p)$$

Anti-particles (− charge):

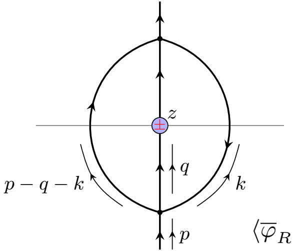
$$\langle \varphi(-q)|\mathcal{D}^{\pm}_{J_L}(z)|\overline{\varphi}(p)\rangle =$$



$$= \pm(2\pi)^d \delta^d(p-q) V_{J_L}(z;p)$$

Identical up to signs!

Two-Loop Corrections



- Compute in terms of renormalized fields

$$\varphi_R(p) = Z_\varphi^{-1/2}\varphi(p)$$

- Encounter IR divergences proportional to tree-level result

$$\langle\overline{\varphi}_R(-p)|\mathcal{D}^+_{J_L}(z)|\varphi_R(p)\rangle = Z_\varphi^{-1}\left(V_{J_L}(z;p) + \mathcal{F}^{(2)}_{J_L}(z;p)\right) + O(\lambda^3)$$

$$= Z_\varphi^{-1}\left(1 - \frac{1}{\epsilon}\frac{\lambda^2}{2(4\pi)^4}\frac{1}{J_L(J_L+1)}\right)V_{J_L}(z;p) + \mathcal{O}(\lambda^3)$$

"Renormalization"

- Define the renormalized detector

$$[\mathcal{D}^+_{J_L}(z)]_R = \left(Z^+_{J_L}\right)^{-1}\mathcal{D}^+_{J_L}(z)$$

$$Z^+_{J_L} \equiv Z_\varphi^{-1}\left(1 - \frac{1}{\epsilon}\frac{\lambda^2}{2(4\pi)^4}\frac{1}{J_L(J_L+1)}\right) + O(\lambda^3)$$

  - Defines a "good" detector which gives finite event-shapes

[Caron-Huot Koloğlu Kravchuk Meltzer Simmons-Duffin '22]

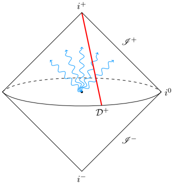
Renormalization and Anomalous Spin

- Z_{J_L} gives the detector's "anomalous spin"

$$-\gamma_L(J_L) = \frac{\partial \ln Z_{J_L}}{\partial \lambda} \beta(\lambda)$$

- $J(\Delta) = \Delta - (d-2) - \gamma_L(1-\Delta)$
  - When solved for $\Delta(J)$, reproduces anomalous dimensions of twist-2 local operators in Wilson-Fisher and $O(N)$ CFTs

local operator $\mathcal{O}$	detector $\mathcal{D}$
"measure at a point"	"measure in cross-sections"
UV divergence	IR divergence
need to renormalize	need to renormalize
theory-dependent	theory-dependent
OPE	light-ray OPE
radial quantization	?



The takeaway: renormalizing detectors is familiar!

[Caron-Huot Koloğlu Kravchuk Meltzer Simmons-Duffin '22]

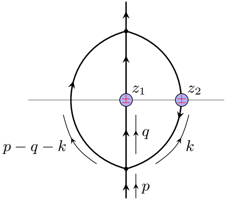
Multi-Detector Event Shapes

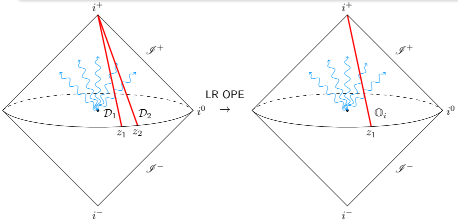
- Event shapes with multiple renormalized detectors have contact term singularities in the collinear limit

$$\left\langle \overline{\varphi}_R(-p) \right| \left[\mathcal{D}^{+}_{J_{L_1}}(z_1)\right]_R \left[\mathcal{D}^{+}_{J_{L_2}}(z_2)\right]_R \left|\varphi_R(p)\right\rangle \propto \frac{1}{\epsilon}\delta^{d-2}(z_1, z_2) V_{J_{L_1}+J_{L_2}+d-2}(z_1; p)$$

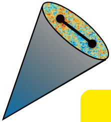
  - Proportional to the tree level vertex V_{J_L}
  - Spin is consistent with $E^n \times E^m \sim E^{n+m}$

- Renormalization of individual detectors is not sufficient to renormalize the product

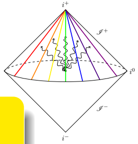




Renormalization perturbatively probes the structure and operators of the light-ray OPE
Operator definition of fragmentation functions

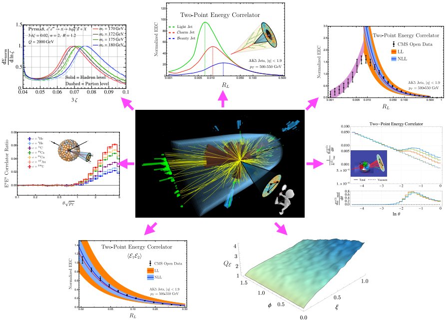


Discussion and Future Directions



Study the system in terms of asymptotic observables built out of well-defined operators

Allows for a link between operators in a field theory and phenomenologically useful observables!



Thanks!

Looking forward:

- What other cameras are out there?
  - What is the full space of detectors?
  - What can these tell us about:
    - Non-perturbative physics
    - The light-ray OPE
- How can we use these?
  - Precision jet substructure
  - Hot and cold nuclear environments
- All the other exciting things we've heard about here!