Light-ray OPE of EEC and Collinear Limit

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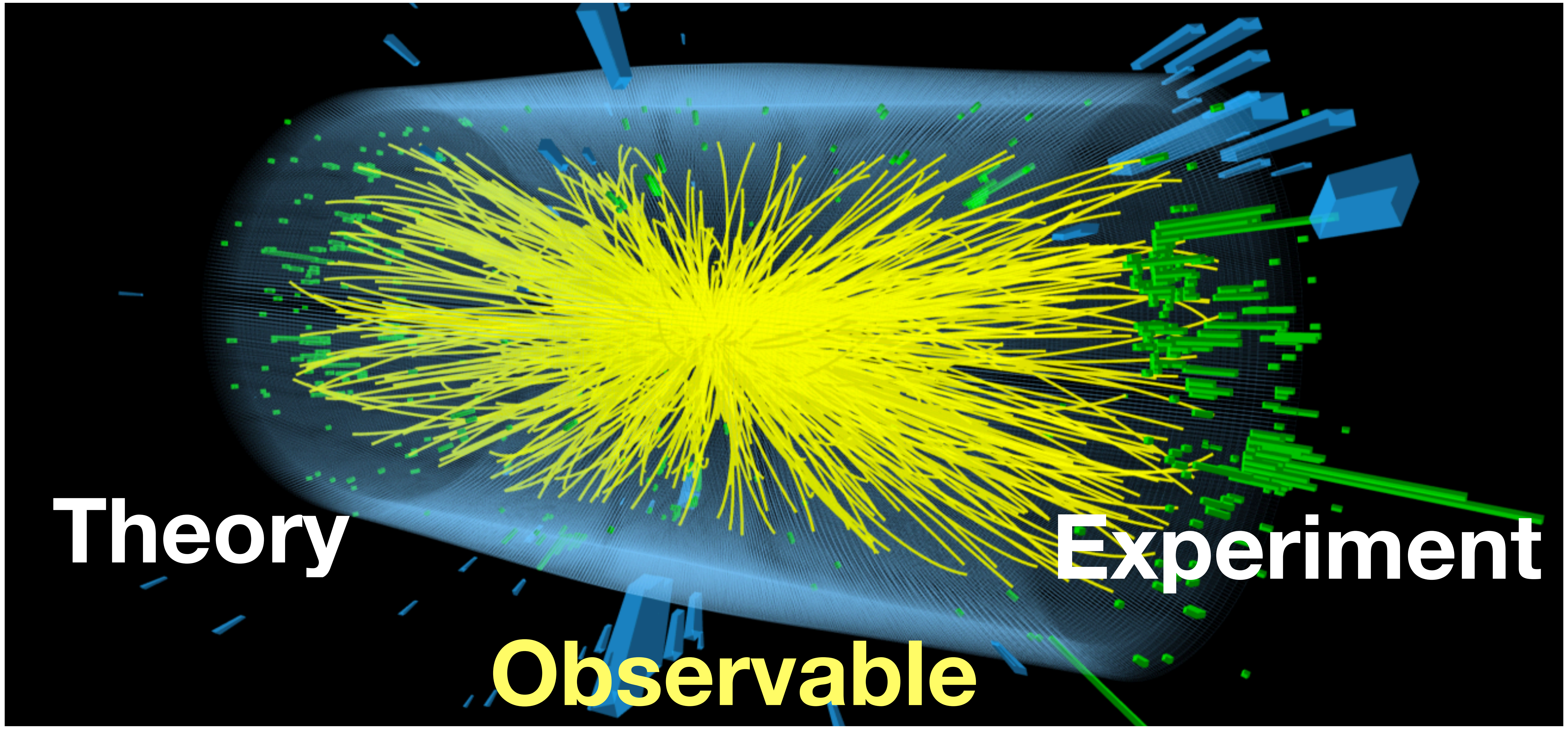
Energy Correlators at the Collider Frontier
July 18, 2024

Collider Physics

Probing microscopic physics from high-energy collision, through measurement at "infinity".

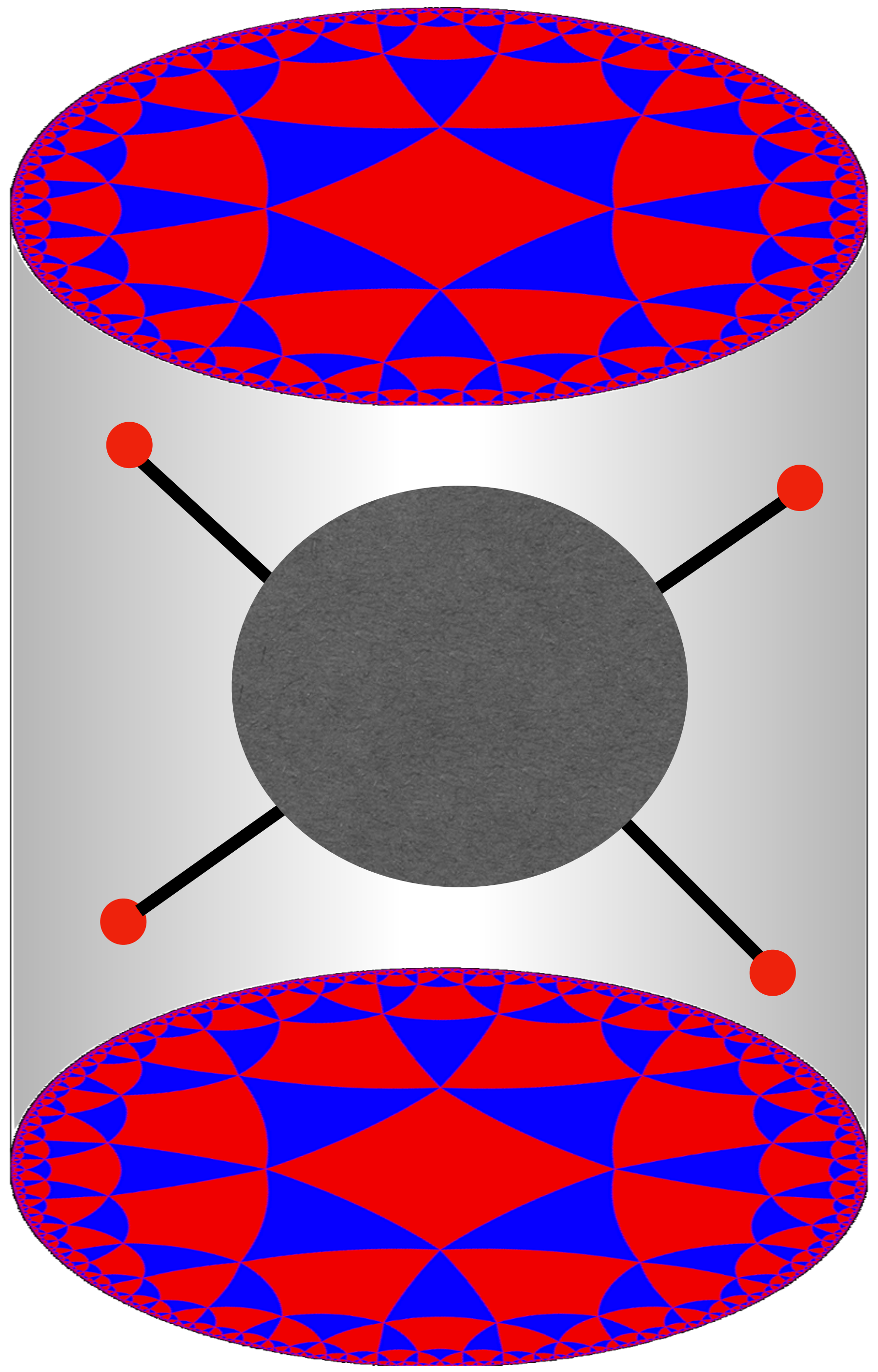


Examples of observables:
thrust, C-parameter, energy-energy correlator,…

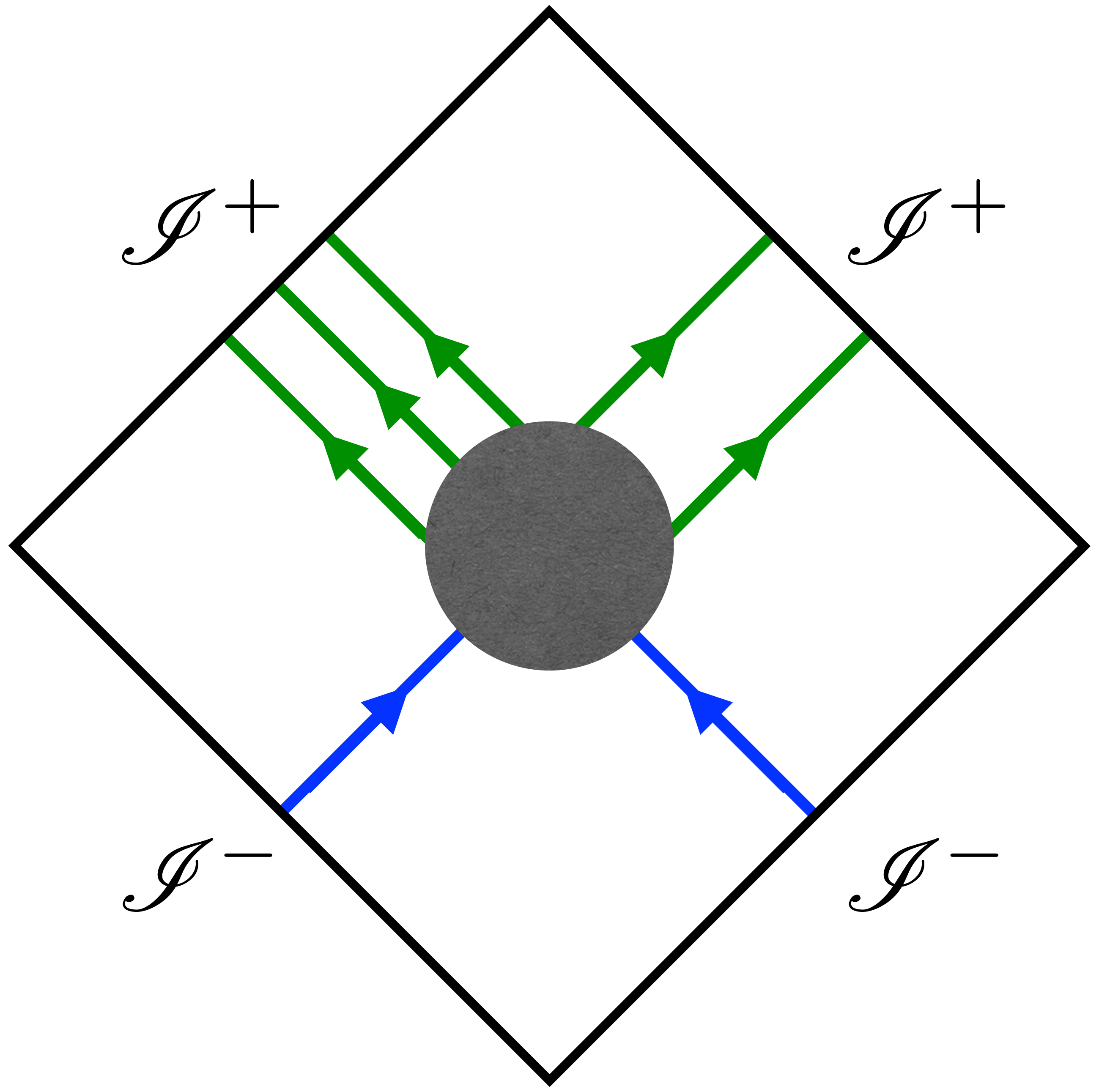
this workshop

Holography

Various theoretical interest in understanding bulk physics from the boundary of spacetime.



AdS/CFT correspondence



flat space

AdS scattering = CFT correlation functions

less clear about <u>operators(?)</u> on the flat space boundary

Also see Sruthi's talk

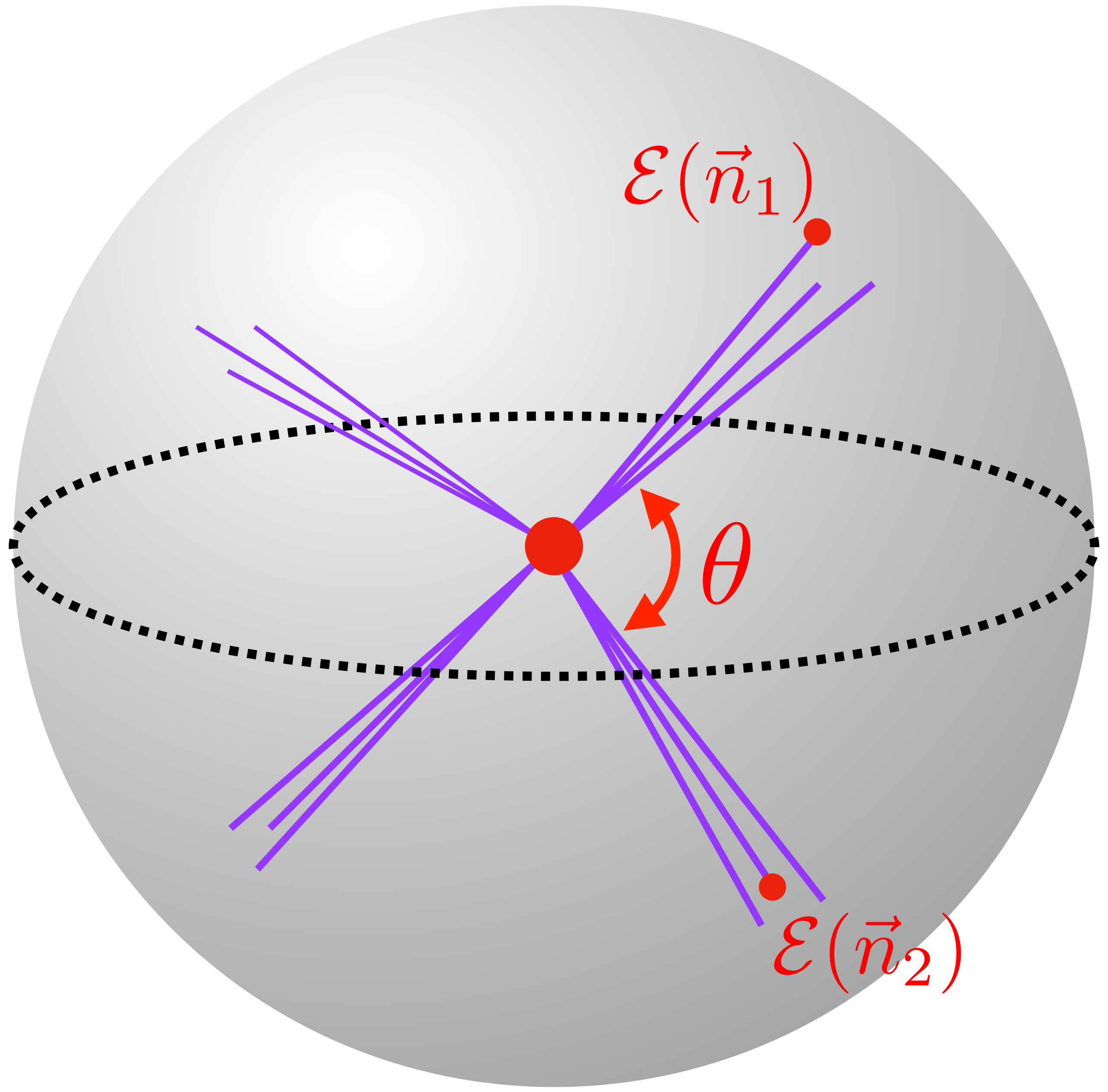
Correlators on the Celestial Sphere

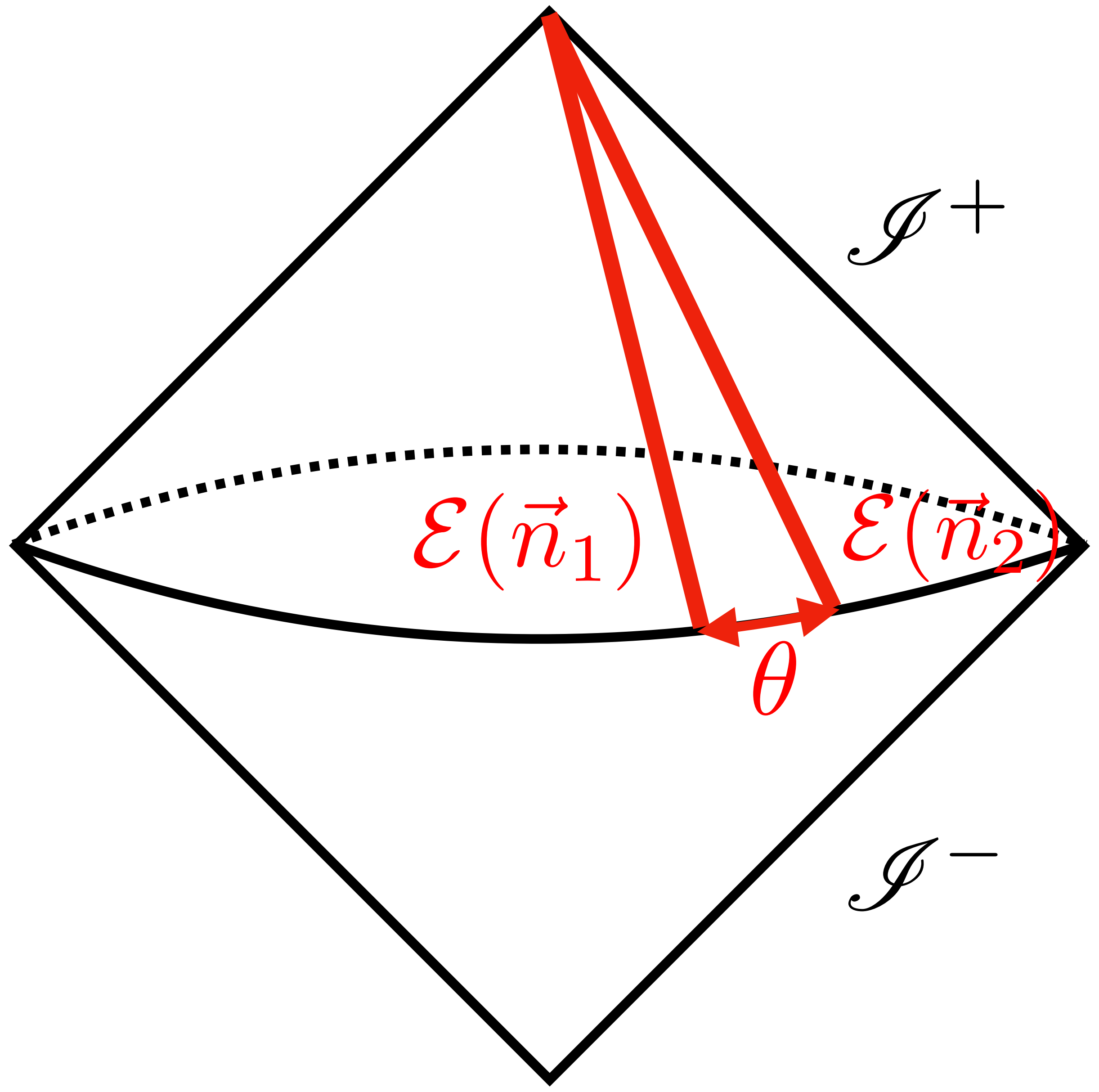
The experiment observables can serve as correlators on the boundary of flat space.

[Basham, Brown, Ellis and Love, 1978]

Example: energy-energy correlator (EEC)

$$\frac{d\Sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$





a point on the celestial sphere $\longleftrightarrow$ a light-ray at null infinity

calorimeter $\longleftrightarrow$ light-ray operator

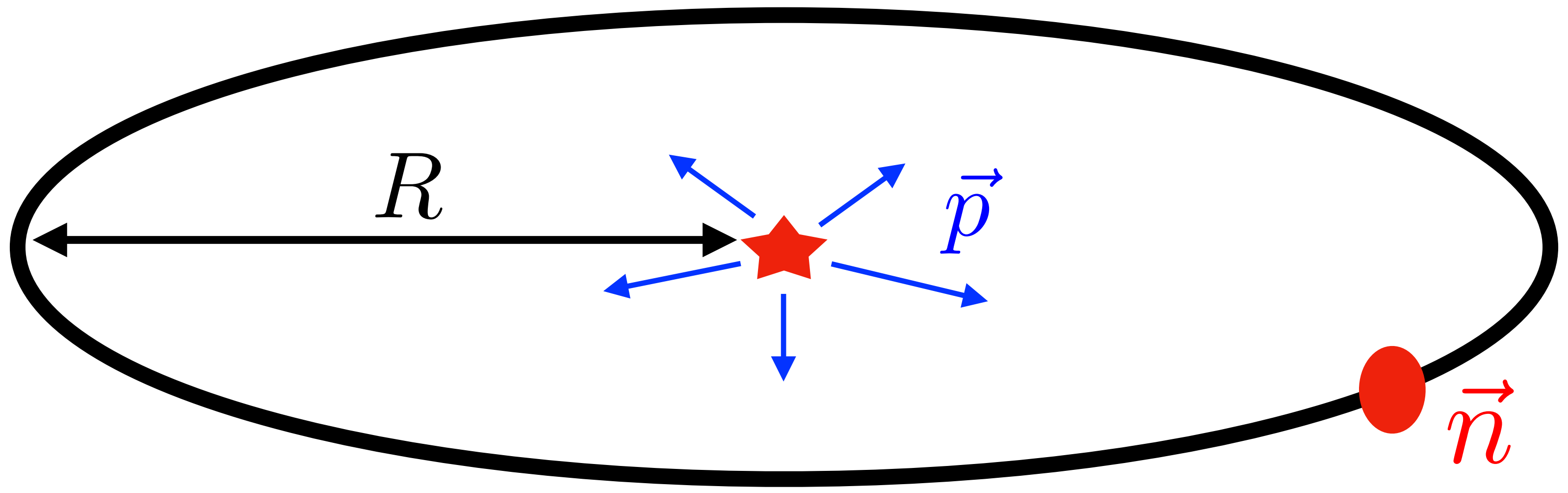
Energy Flow Operator $\mathcal{E}(\vec{n})$

- Building block of EEC, with which we can immediately generalize 2-point EEC to multi-point EECs.

"Perturbative" vs "Non-perturbative"

A calorimeter only detects particles flowing along direction $\vec{n}$, and weight with its energy E, e.g.

$$\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} E_{\vec{p}}\, a^\dagger_{\vec{p}}\, a_{\vec{p}}\, \delta^{(2)}(\vec{n}-\hat{p}) \longrightarrow \text{Amplitude Method}$$



Non-perturbative definition via energy-momentum tensor

$$\boxed{\mathcal{E}(\vec{n}) = \lim_{R\to\infty} R^2 \int_0^\infty dt\, n_i T^{0i}(t, R\vec{n})} \longrightarrow \text{Correlator Method}$$

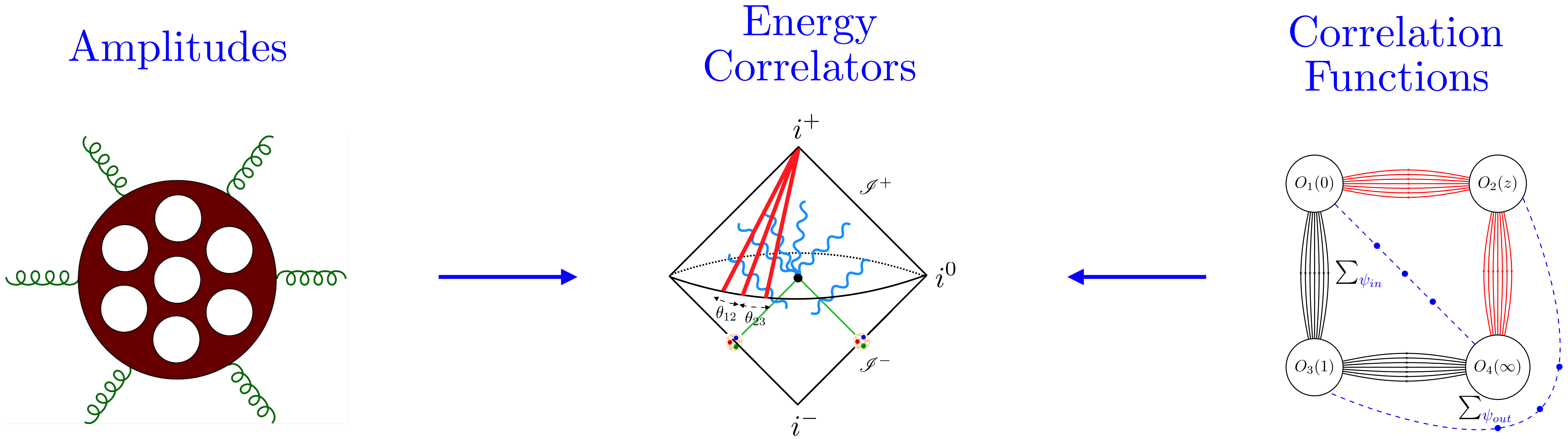
[Sveshnikov, Tkachov, 1996; Hofman, Maldacena, 2008]

- Integrate t to get the total received energy
- Detector is effectively located at infinity

Amplitudes v.s. Correlation Functions

in weakly coupled QCD and N=4 SYM

Also see Ian's and Kai's talks



EEC in QCD:

LO [Basham, Brown, Ellis, Love, 1978]

NLO [Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]

NNLO (numerically)

[Del Duca, Duhr, Kardos, Somogyi, Trócsányi, 2016]

Multi-point Energy Correlators in QCD and N=4 SYM:

[HC, Luo, Moult, Yang, Zhang, Zhu, 2019;
Yan, Zhang, 2022;Yang, Zhang, 2022 & 2024;
Chicherin, Moult, Sokatchev, Yan, Zhu, 2024]

EEC in N=4 SYM:

LO and NLO

[Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2014]

NNLO [Henn, Sokatchev, Yan, Zhiboedov, 2019]

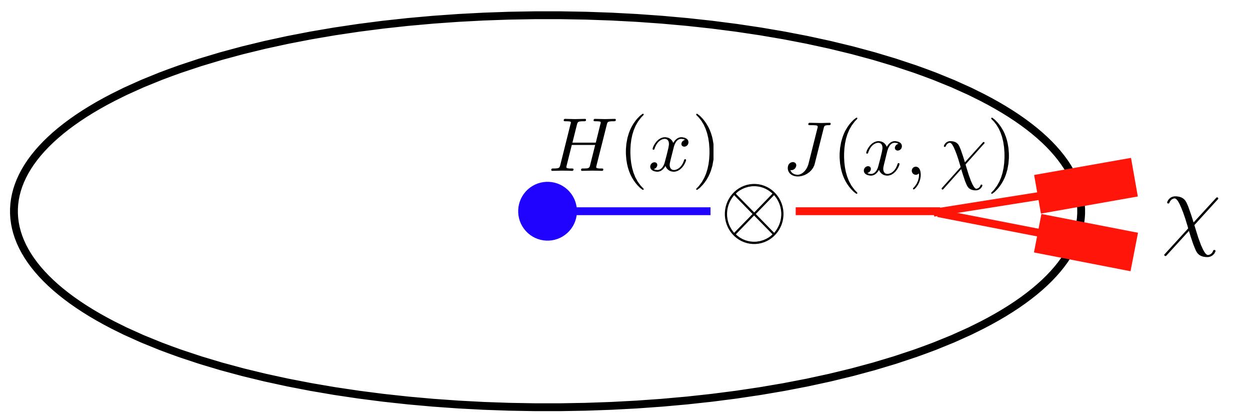
Charge-Charge Correlator in QCD:

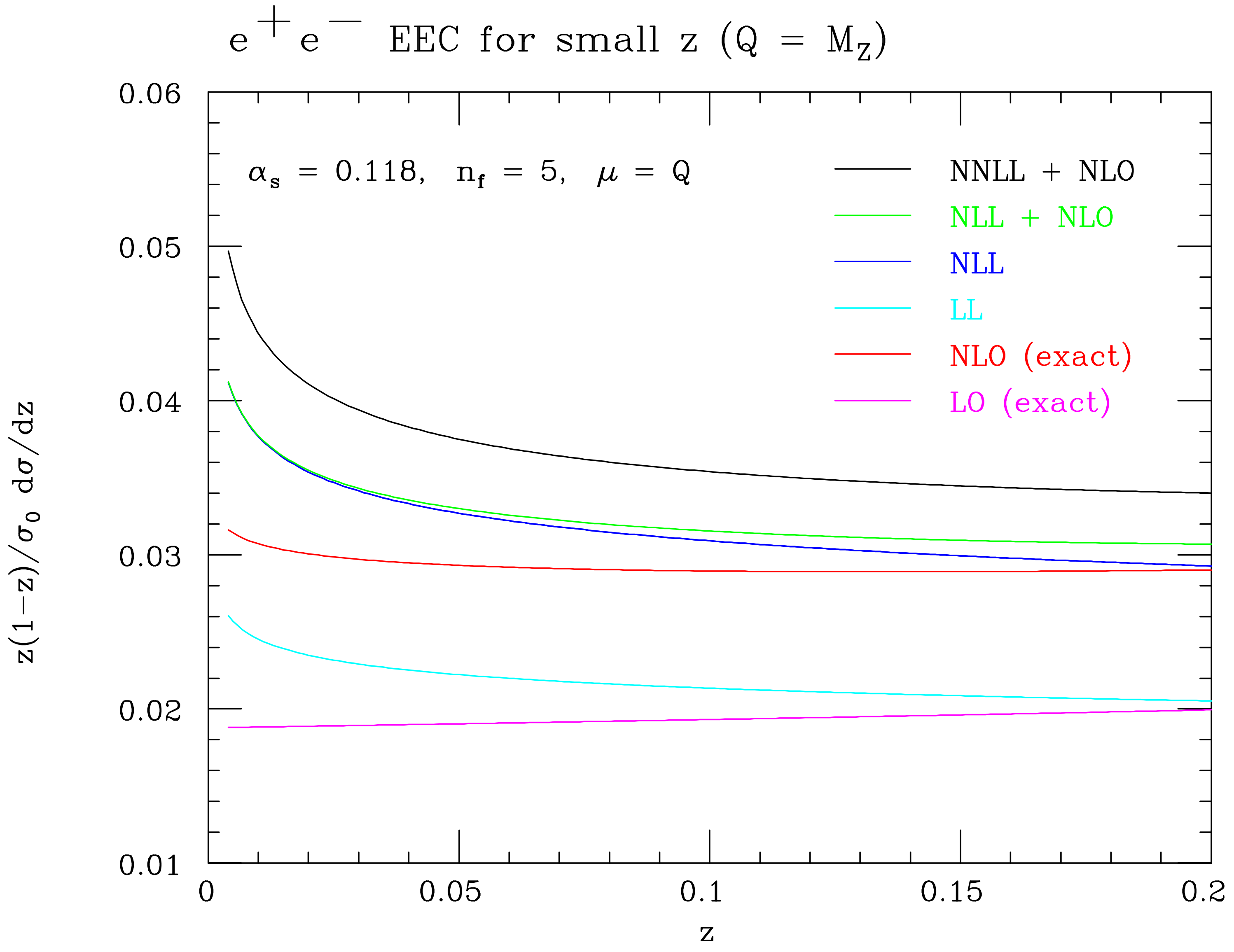
[Chicherin, Henn, Sokatchev, Yan, 2020]

End-point Regions in EEC

Collinear Factorization and Resummation

[Dixon, Moult, Zhu, 2019]

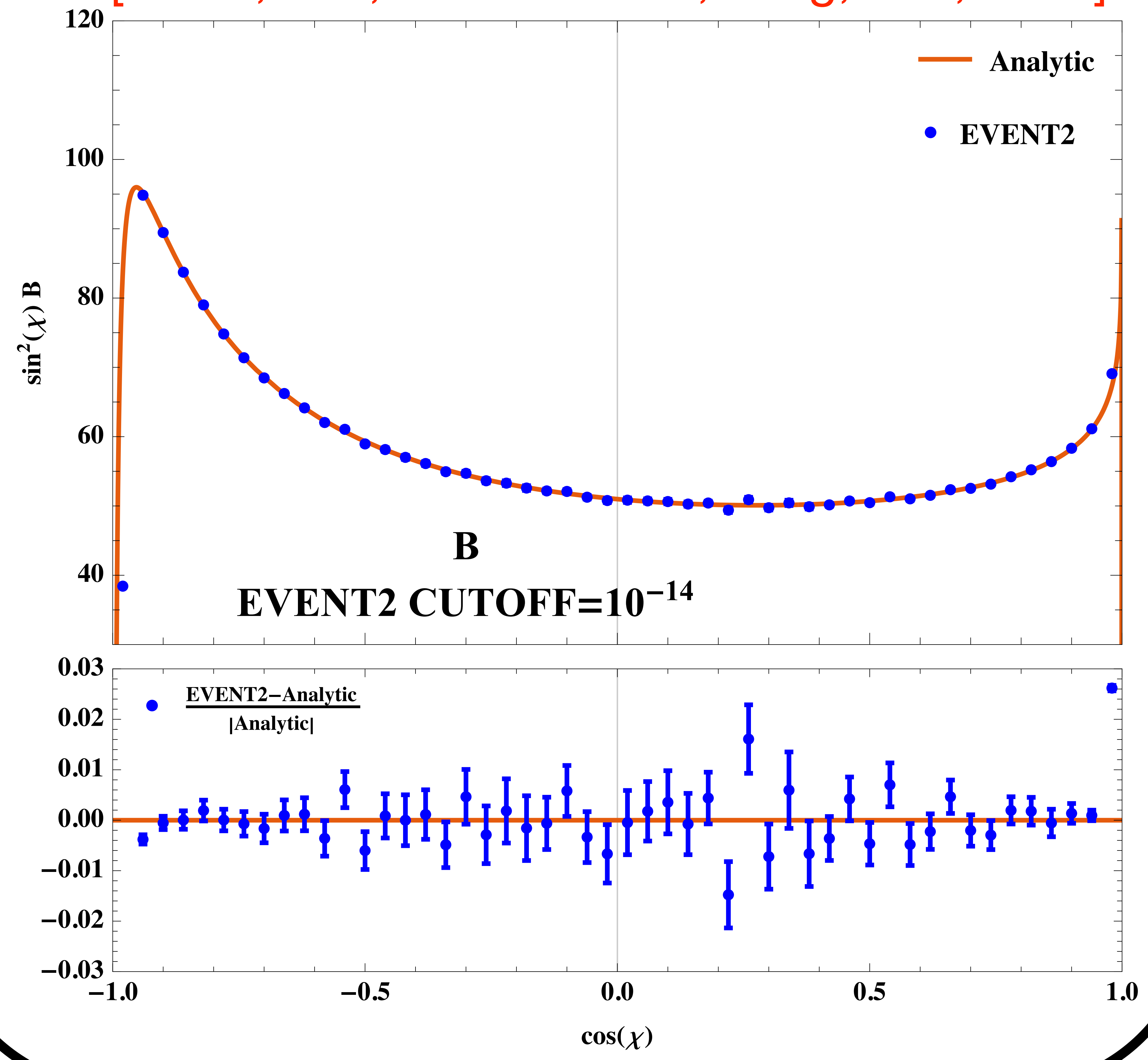




A lot of collinear stories in this workshop…

NLO Analytic Result

[Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]

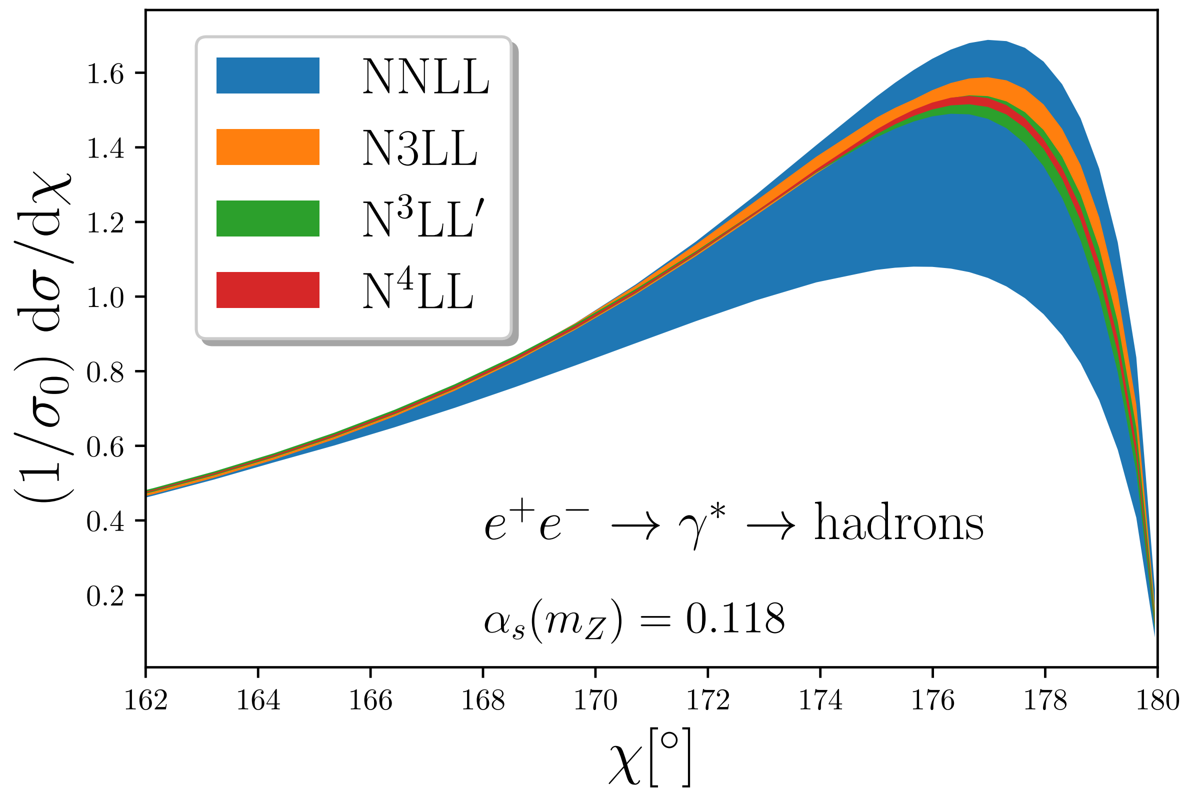


TMD Factorization and Resummation

[Moult, Zhu, 2018]

Recently achieved N^4LL accuracy

[Duhr, Mistlberger, Vita, 2022]

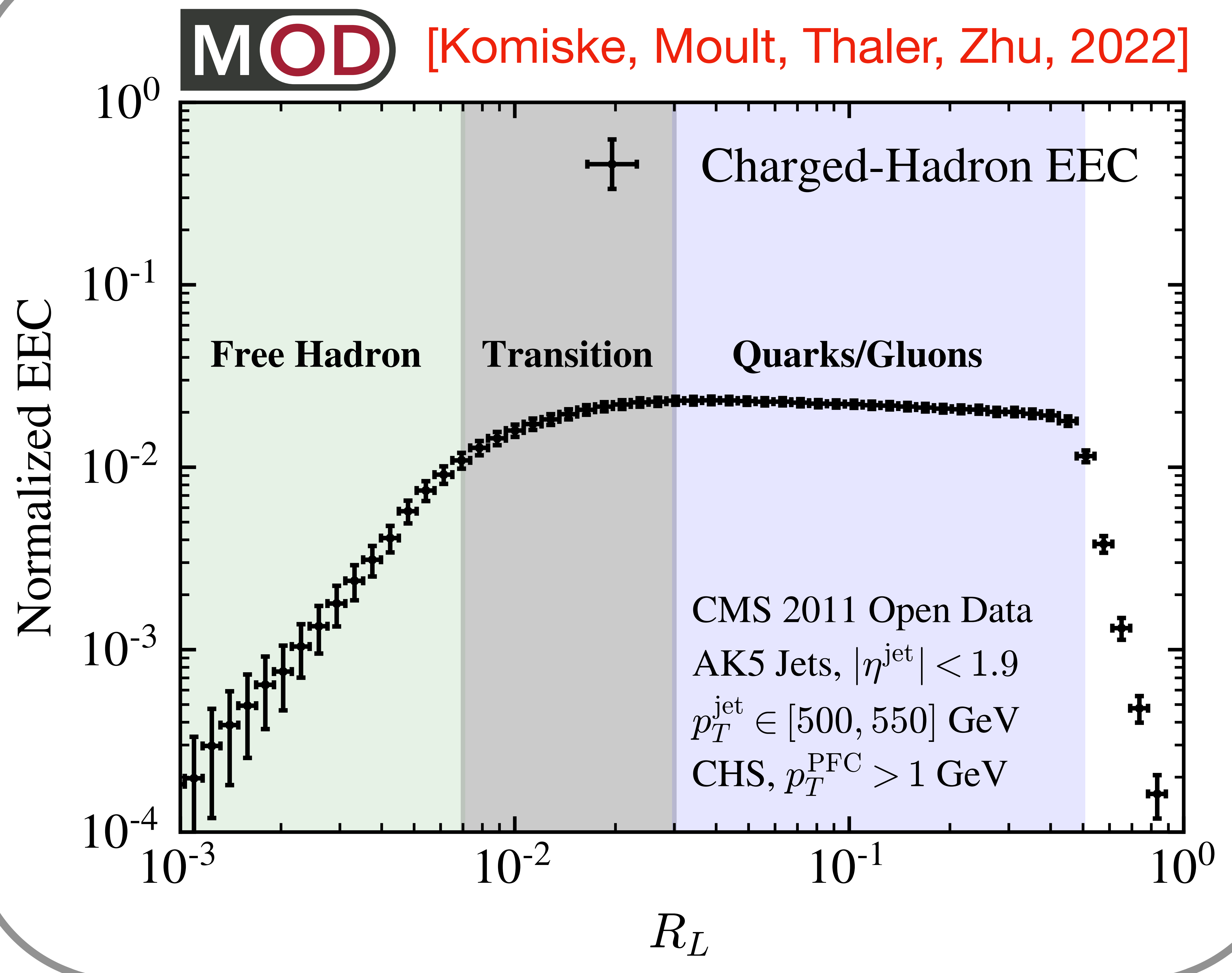


Thankfully, Dingyu gave a back-to-back story!

Collinear Limit Result from CMS

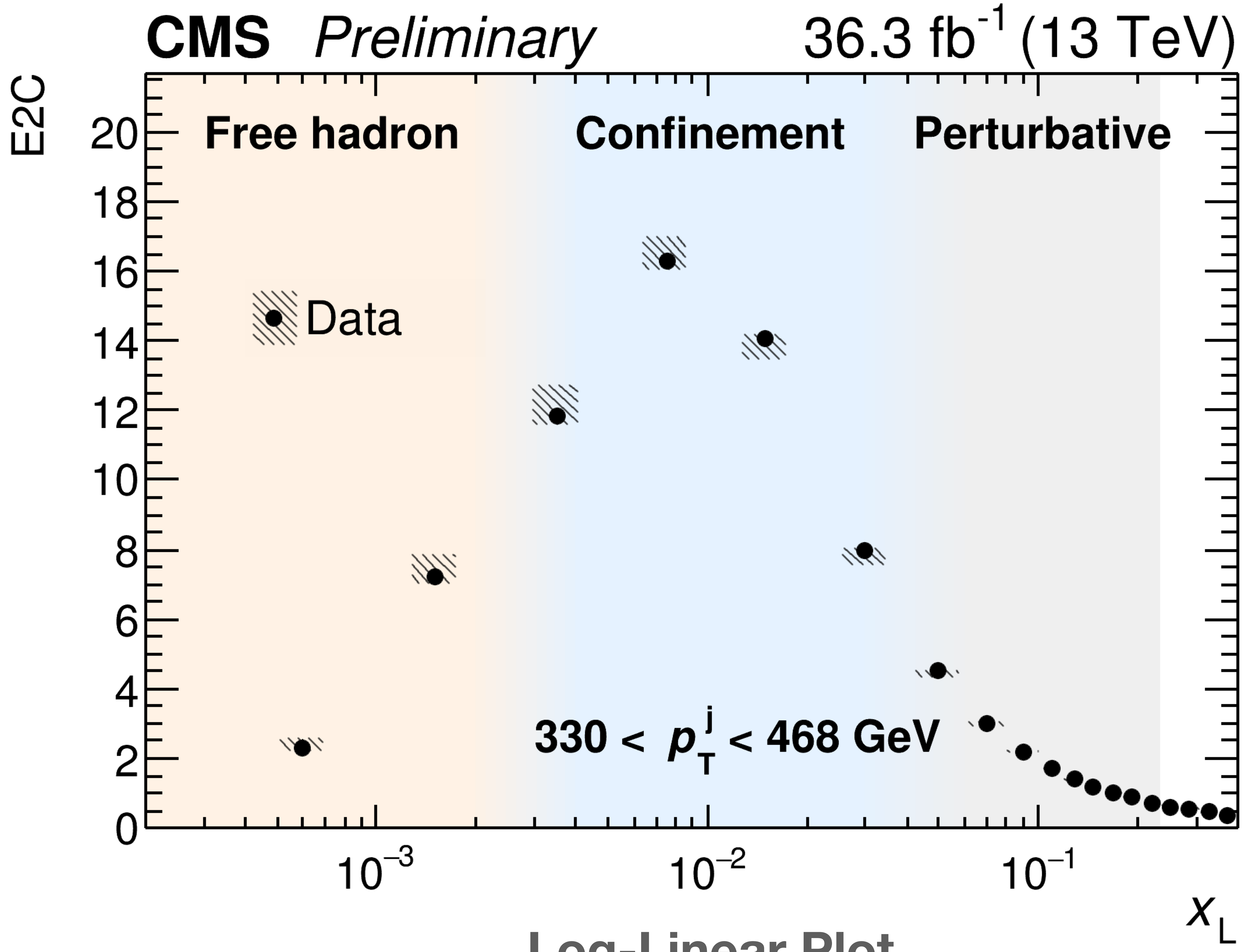
[Meng Xiao et al, 2023]

Previous exploration from CMS Open Data



Log-Log Plot

See illustration on
https://cms.cern/news/jets-elucidate-how-partons-evolve-hadrons



Log-Linear Plot

Enables precision measurement of α_s in jet substructure

Common interest on collinear limit from different perspectives (May 3, 2019):

CFT Perspective

The light-ray OPE and conformal colliders

Murat Kologlu (Caltech), Petr Kravchuk (CERN), David Simmons-Duffin (Caltech), Alexander Zhiboedov (CERN) (May 3, 2019)

Published in: *JHEP* 01 (2021) 128 • e-Print: 1905.01311 [hep-th]

pdf DOI cite claim reference search 136 citations

Light-ray OPE

Energy correlations in the end-point region

G.P. Korchemsky (IPhT, Saclay) (May 4, 2019)

Published in: *JHEP* 01 (2020) 008 • e-Print: 1905.01444 [hep-th]

pdf DOI cite claim reference search 62 citations

Mellin Representation

QCD Perspective

Collinear limit of the energy-energy correlator

Lance J. Dixon (SLAC), Ian Moult (UC, Berkeley and LBL, Berkeley), Hua Xing Zhu (Zhejiang U., Inst. Mod. Phys.) (May 3, 2019)

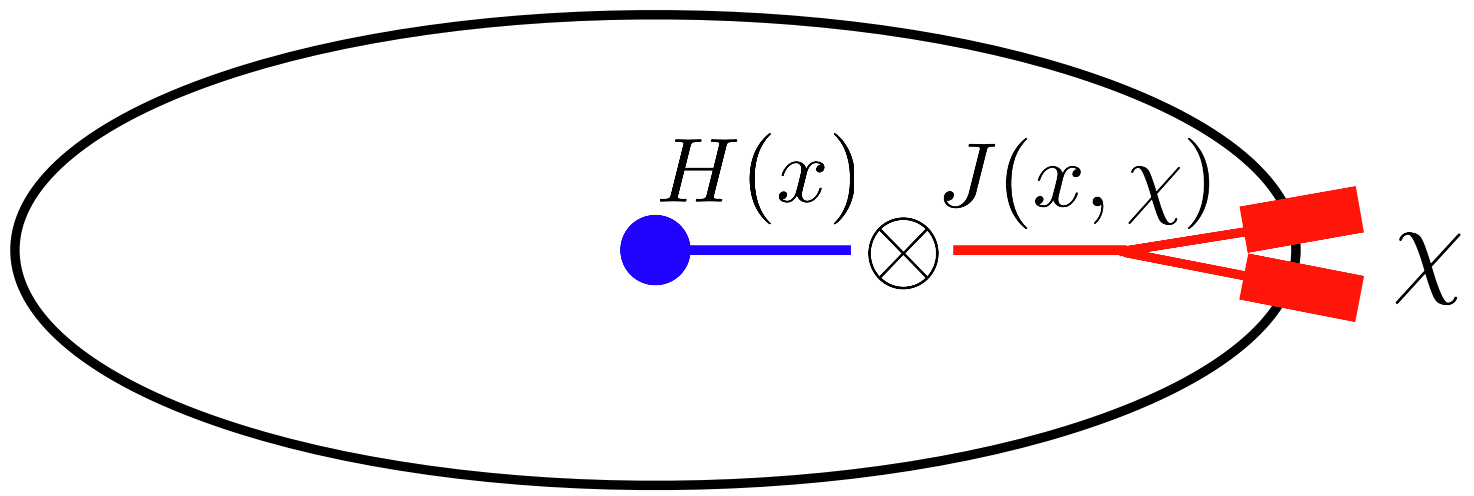
Published in: *Phys.Rev.D* 100 (2019) 1, 014009 • e-Print: 1905.01310 [hep-ph]

pdf DOI cite claim reference search 114 citations

QCD Factorization

Universality in the Collinear Limit

EEC exhibits collinear universality in pQCD, described by the factorization formula



Jet function Hard function

$$\Sigma(\zeta, \log\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2\, \vec{J}(\log\frac{\zeta x^2 Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

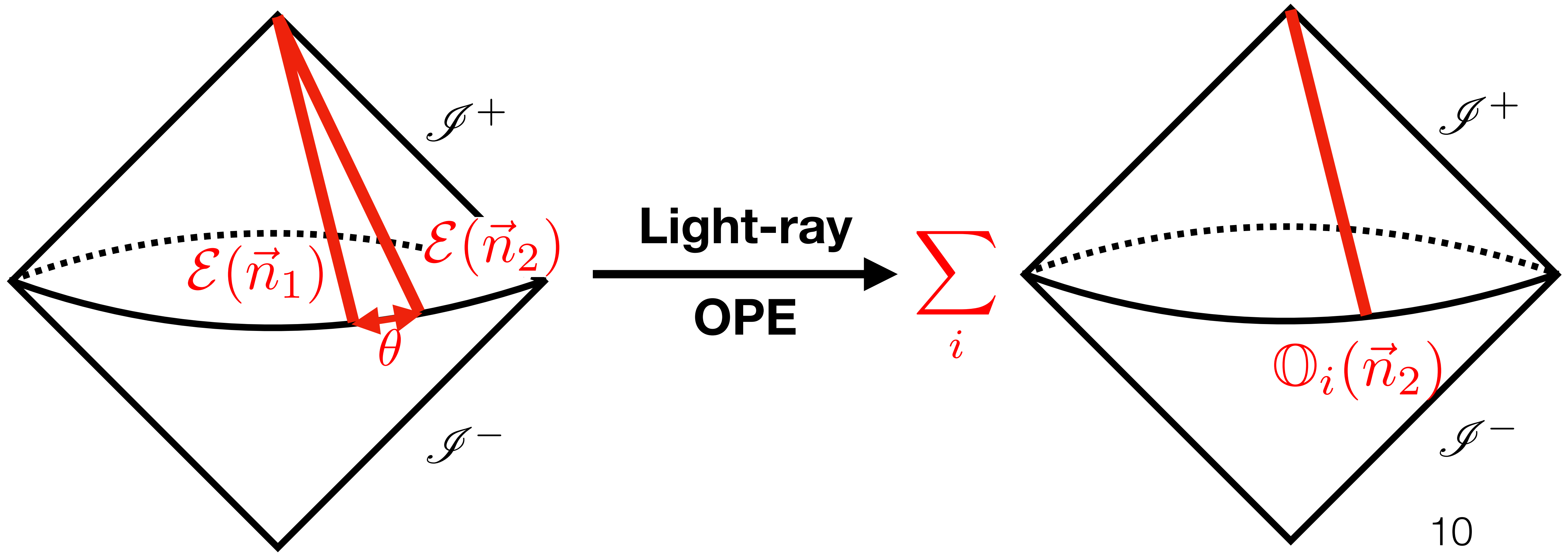
[Dixon, Moult, Zhu, 2019]

convolution in momentum fraction x

As a correlation function of light-ray operators,
collinear limit has the interpretation of light-ray OPE.

[Hofman, Maldacena, 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

[this talk]



Light-ray Operators and OPE

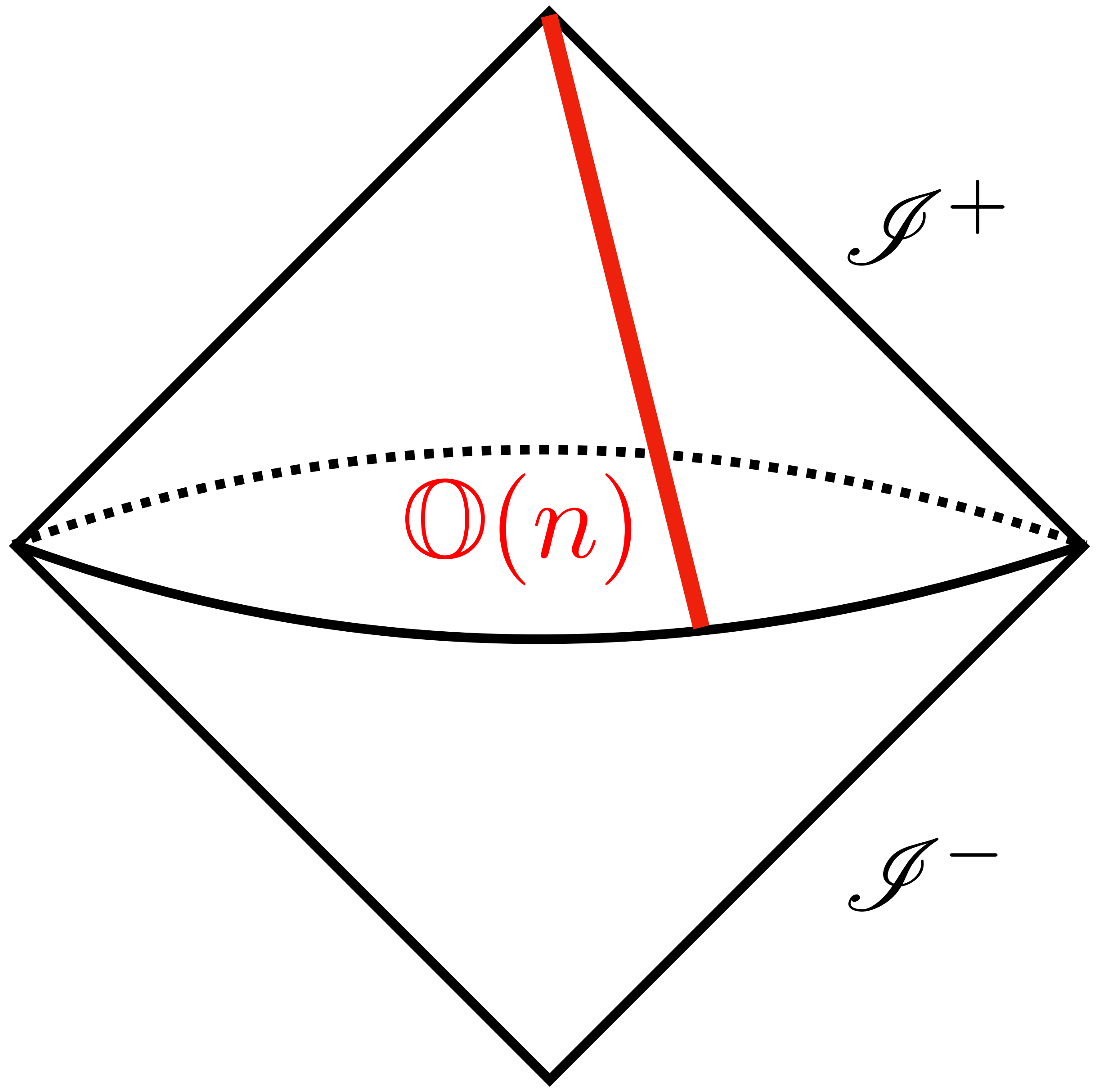
Light-ray Operators

light transform of **local operators**

Energy flow operator $$\mathcal{E}(\vec{n}) = \boxed{\lim_{r\to\infty} r^2 \int_0^\infty dt}\, \vec{n}_i T^{0i}(t, r\vec{n})$$

Energy radiation obeys inverse square law,

r^2 compensates this effect to be non-vanishing.



Generalization to other local operators

twist

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta - J} \int_0^\infty dt\, O^{\mu_1 \cdots \mu_J}(t, r\vec{n}) \bar{n}_{\mu_1} \ldots \bar{n}_{\mu_J}$$

In other contexts of physics, light-ray operators are not necessarily at null infinity—they can live on any light-ray.

direction

$$\mathbf{L}[\mathcal{O}](x, n) = \int_{-\infty}^{\infty} d\alpha (-\alpha)^{-\Delta - J} \mathcal{O}\left(x - \frac{n}{\alpha}, n\right)$$

starting point

[Kravchuk, Simmons-Duffin, 2018]

Examples of more general light-ray operators, see [Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020; Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin, 2022;…]

Spacetime Symmetry

Little group that fixes a light-ray in future null infinity consists of

$\underbrace{\text{translations, collinear boost, transverse rotations}}_{\text{Poincare group part}}$, dilatation

- Dimension = $J - 1$

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} \underbrace{\boxed{r^{\Delta - J}}}_{-(\Delta - J)} \underbrace{\boxed{\int_0^\infty dt}}_{-1} \underbrace{\boxed{O^{\mu_1\cdots\mu_J}(t, r\vec{n})}}_{+\Delta} \bar{n}_{\mu_1} \ldots \bar{n}_{\mu_J}$$

- Collinear Spin = $1 - \Delta$

Boost quantum number

$$\lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^{\Delta - J} \int_{-\infty}^{\infty} d(n\cdot x)\; O^{\mu_1\cdots\mu_J}(x)\bar{n}_{\mu_1} \ldots \bar{n}_{\mu_J}$$

Boost along $\vec{n}$ $\quad n^\mu \to \lambda n^\mu, \quad \bar{n}^\mu \to \lambda^{-1}\bar{n}^\mu \qquad \mathbb{O}(\vec{n}) \to \lambda^{1-\Delta}\mathbb{O}(\vec{n})$

- Transverse Spin = transverse spin of the local operator
- Momentum = 0 (invariant under translations)

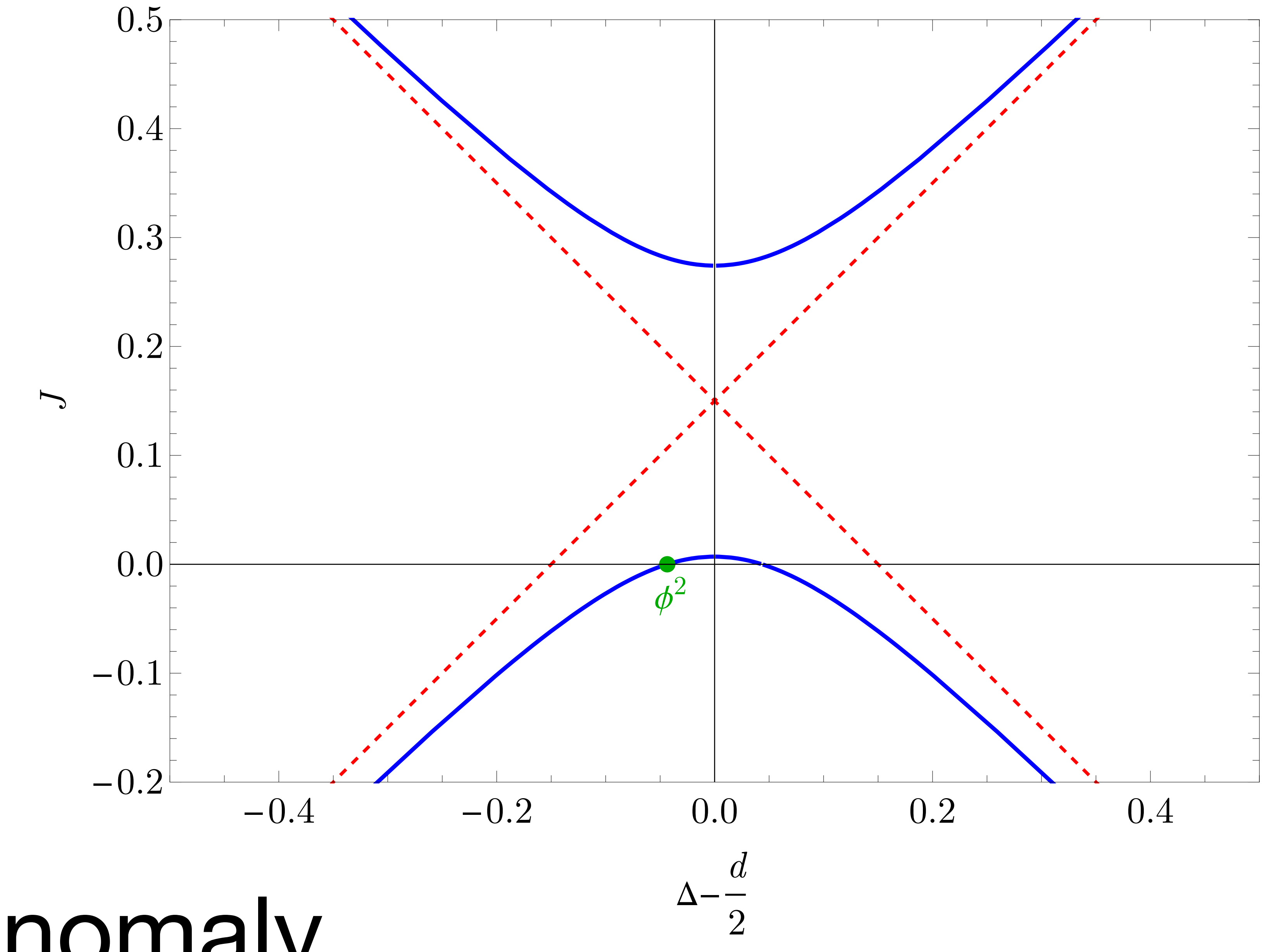
Interesting Aspects of Light-ray Operators

- Analyticity in spin

  [Caron-Huot, 2017; Kravchuk, Simmons-Duffin, 2018]

- Level crossing near Regge intercept

  [Caron-Huot, Kologlu, Kravchuk, Meltzer , Simmons-Duffin, 2022]



- ANEC and new perspective on a-theorem and c-anomaly

  [Hartman, Mathys, 2023 and 2024]

- Applications in collider physics

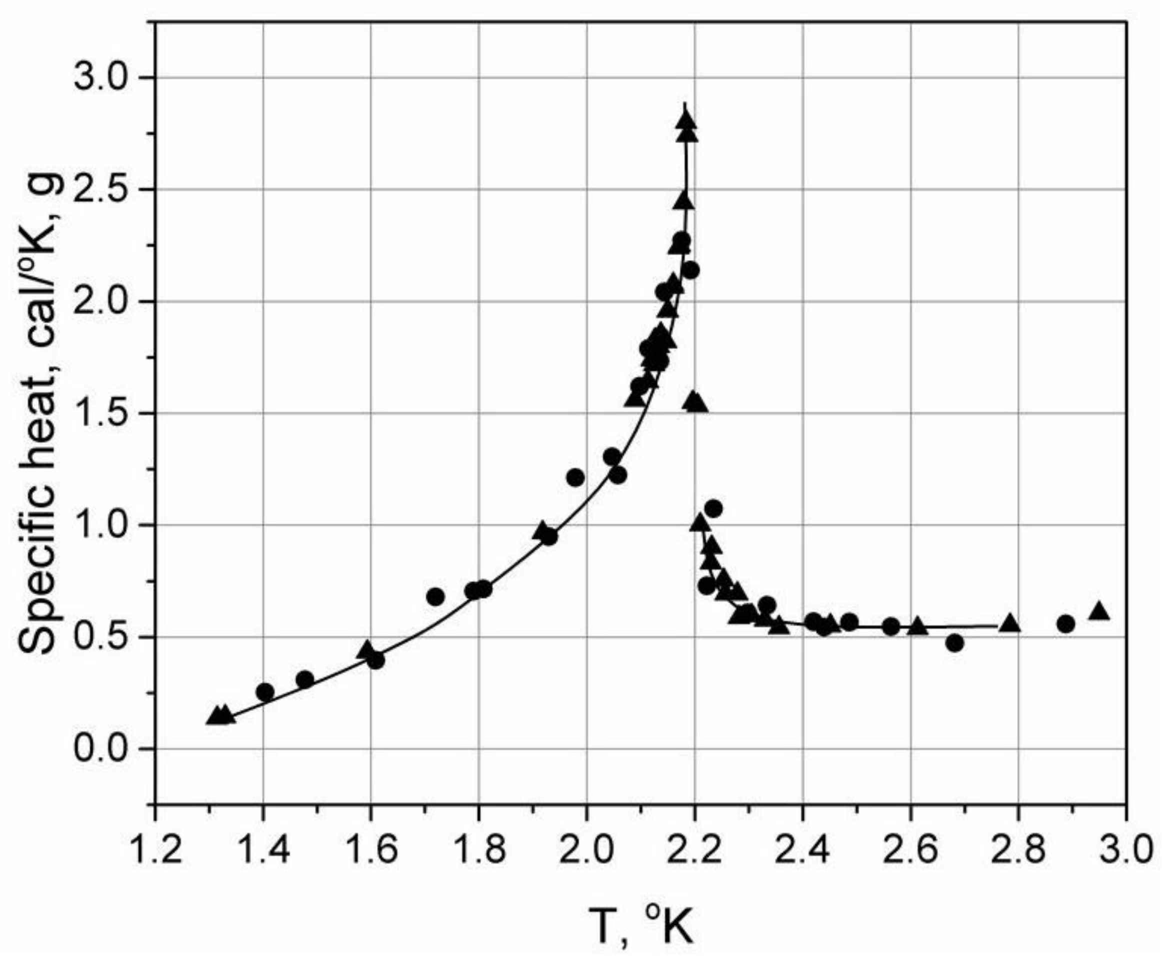
Light-ray OPE

Short distance scaling behavior is determined by local **Operator Product Expansion** (OPE).

Local Operator OPE

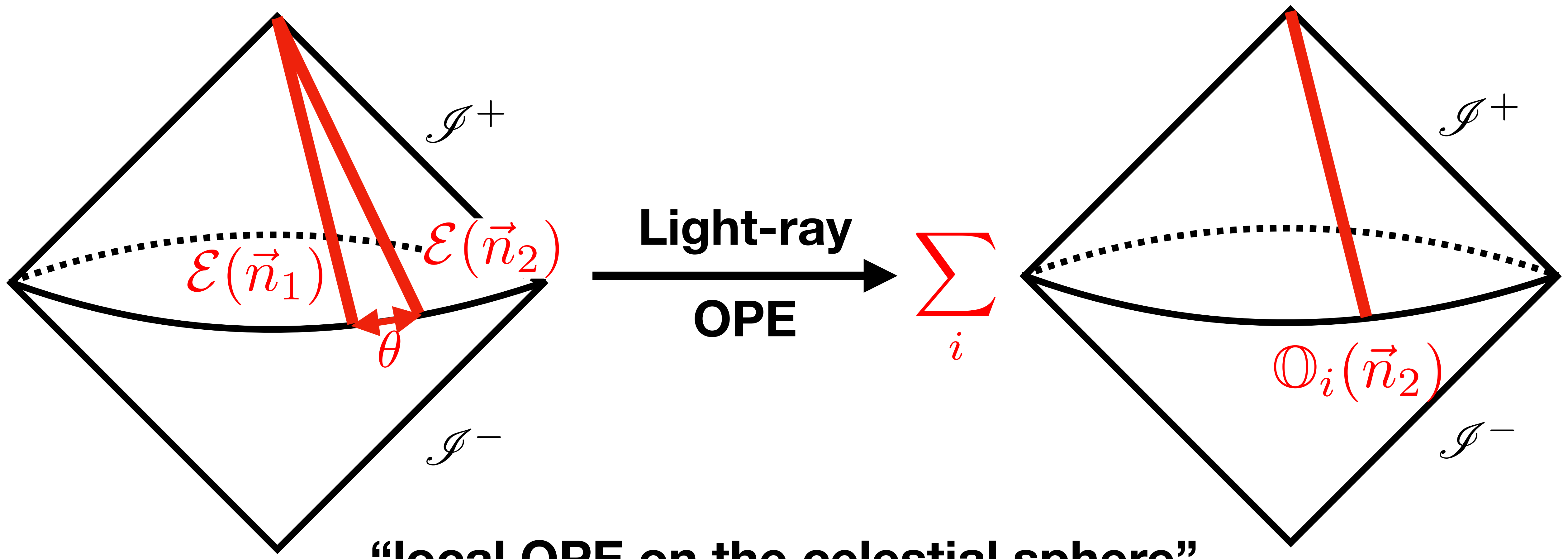
$$\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2) \sim \sum \mathcal{O}(x_1)$$

liquid helium critical behavior



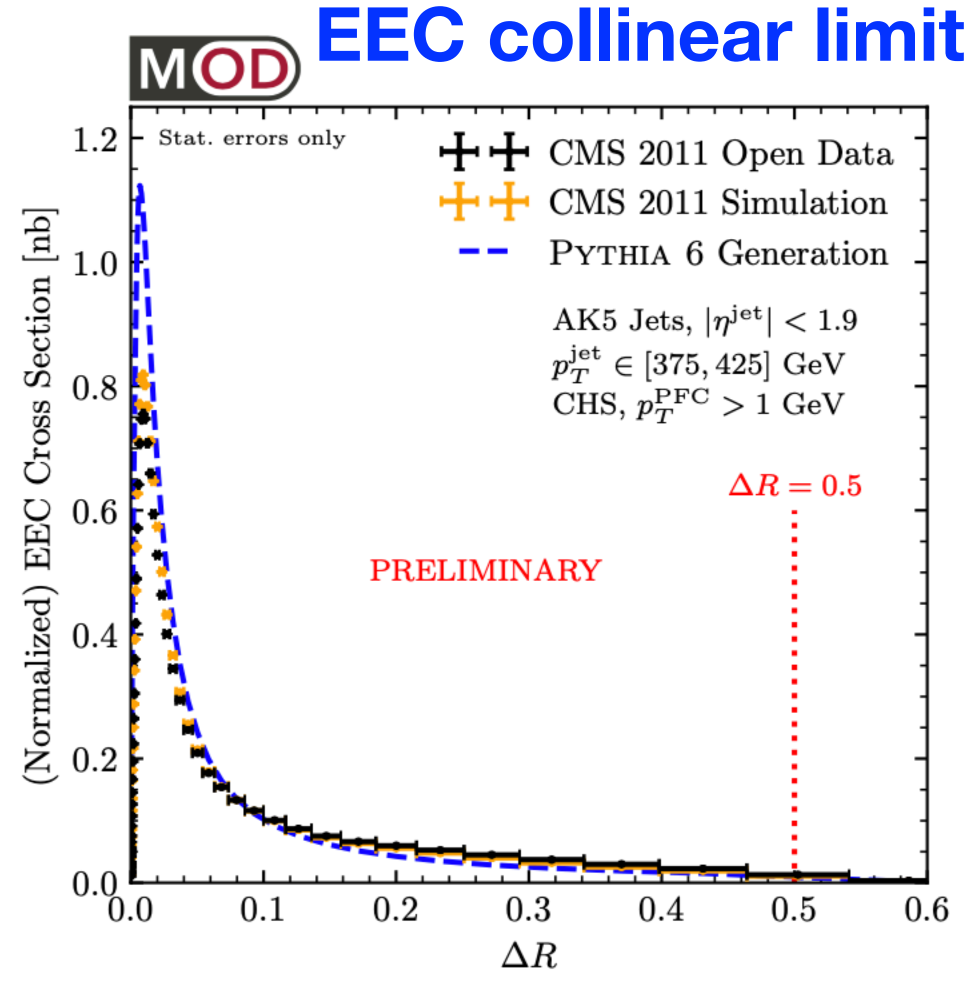
Small angle behavior is controlled by the OPE of these light-ray operators.

[Hofman, Maldacena, 2008]



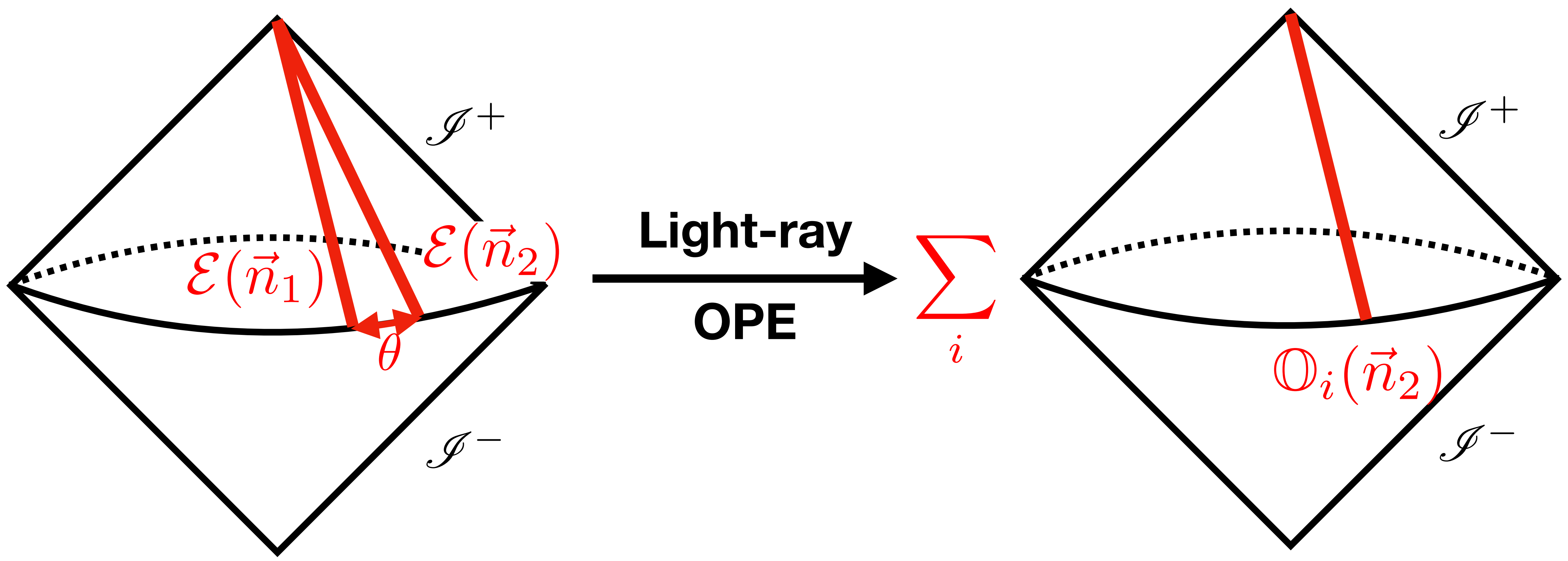
"local OPE on the celestial sphere"

EEC collinear limit



[Komiske, Moult, Thaler, Zhu, 2022]

Light-ray OPE in CFT



Light-ray OPE $\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i - 4}\, \mathbb{O}_i(\vec{n}_2)$ [Hofman, Maldacena, 2008]

See HuaXing's talk for details

Spin-3 light-ray operators

Small angle scaling is dominated by the leading twist operators. still true in pQCD

Conformal symmetry constrains the operators to have spin=3. broken by running coupling

Light-ray OPE in CFT is rigorous and convergent. [Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

Leading Twist Operators in QCD

For unpolarized cases, there are only two kinds of twist-2 operators in QCD

See HuaXing's talk for polarized case

Local Operators		**Light-ray Operators**
$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$	$\xrightarrow{\lim_{r\to\infty} r^2 \int_0^\infty dt}$	$\mathbb{O}_q^{[J]}(\vec{n})$
$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$	**Light Transform**	$\mathbb{O}_g^{[J]}(\vec{n})$

The analytic continuation of even spin branch is

Physics Interpretation
[in free theory]
Measuring E^{J-1}

$$\mathbb{O}_q^{[J]}(\vec{n}) = \sum_s \int \frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\, E^{J-1}(b^\dagger_{\vec{p},s}b_{\vec{p},s} + d^\dagger_{\vec{p},s}d_{\vec{p},s})$$

$$\mathbb{O}_g^{[J]}(\vec{n}) = \sum_{\lambda,c} \int \frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\, E^{J-1}\, a^\dagger_{\vec{p},\lambda,c}a_{\vec{p},\lambda,c}$$

Not IR-safe measurement

LP Light-ray OPE in QCD at LO

QCD in 4d is classically conformal

[HC, Moult, Zhu, 2020]

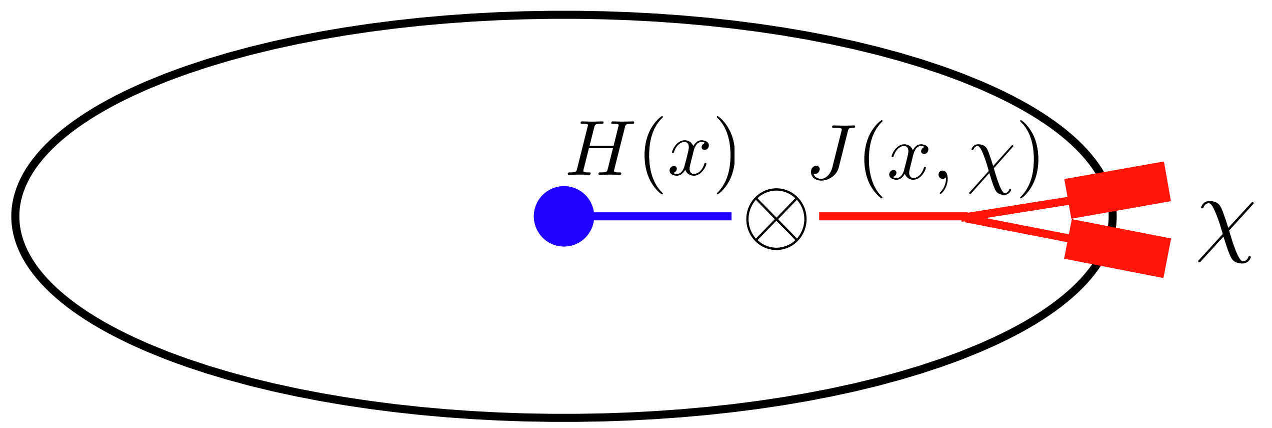
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\vec{\mathcal{J}}\left[\widehat{C}_\phi(2) - \widehat{C}_\phi(3)\right]\vec{\mathbb{O}}^{[3]}(\hat{n}_1) + \text{higher twist}$$

$$\vec{\mathcal{J}} = (1,1,0,0)$$

OPE coefficient matrix

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

Apply light-ray OPE to the (2-point) EEC with unpolarized/scalar source, it is equivalent to the factorization approach in the collinear limit



at LL accuracy

$$\Sigma(\zeta, \log\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}(\log\frac{\zeta \cancel{x^2} Q^2}{\mu^2}, \mu)\cdot\vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

[Dixon, Moult, Zhu, 2019]

“Bootstrapping” Factorization Formula

[2311.00350]

Beyond Conformal Symmetry

Renormalization of Light-ray Operators

In perturbation theory, the light-ray operators have divergences.

$\longrightarrow$ require renormalization [Caron-Huot, Kologlu, Kravchuk, Meltzer, Simmons-Duffin, 2022]

bare twist-2 local operators

$$\mathbb{O}^{[J]}_{a;\mathrm{bare}}(n) = \lim_{\bar{n}\cdot x\to\infty}\left(\frac{\bar{n}\cdot x}{2}\right)^2 \int_{-\infty}^{\infty} d(n\cdot x)\, \mathcal{O}^{[J]}_{a;\mathrm{bare}}(x;\bar{n})$$

Introducing a renormalization factor and define renormalized light-ray operators

$$\mathbb{O}^{[J]}_{a;\mathrm{bare}} = \mathbb{Z}^{[J]}_{ab}\mathbb{O}^{[J]}_{b;\mathrm{ren}}$$

[IR behaviors of detectors]

contains scale dependence

Only $\mathbb{O}^{[J]}_{q}$ and $\mathbb{O}^{[J]}_{g}$ can mix, as long as J is large enough.

Time-like anomalous dimension

RG equation: $$\frac{d}{d\ln\mu^2}\mathbb{O}^{[J]}_{a;\mathrm{ren}}(n;\mu) = \underline{\gamma^{T}_{ab}(J;\alpha_s(\mu))}\mathbb{O}^{[J]}_{b;\mathrm{ren}}(n;\mu)$$

different from the space-like anomalous dimension for the local operators. But they are related by **reciprocity relation.**

Generalizing Conformal Light-ray OPE

Recall the conformal case: $\mathcal{E}(n)\mathcal{E}(n') \sim \sum_i c_i \theta^{\tau_i - 4} \mathbb{O}_i^{[J=3]}(n)$

To balance the dimension

In non-conformal theories, dimension is not a good quantum number.

However, QCD in 4d is classically conformal and the running coupling effect is a higher order effect —> We expect the breaking is a small perturbation

$z = n \cdot n'$

Ansatz: $$\mathcal{E}(n)\mathcal{E}(n') = \sum_{k=0}^{\infty} C_a^{(k)}(z;\mu) \left[\underline{\partial_J^k \mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu)} \right]\Big|_{J=3} + \text{higher twists}$$

assume **analyticity** in perturbation theory

Goal: constrain the ansatz from general principles

Lorentz Symmetry

$$\mathcal{E}(n)\mathcal{E}(n') = \sum_{k=0}^{\infty} C_a^{(k)}(z;\mu) \left[\partial_J^k \mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu)\right]\Big|_{J=3} + \text{higher twists}$$

choose a frame

$$(1,0,0,1) \qquad (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

Act boost generator on both sides of the ansatz

collinear approx $-2(3+z\partial_z)$

$$\text{L.H.S.}: \quad -i[\vec{n}\cdot\vec{\mathbf{K}}, \mathcal{E}(n)\mathcal{E}(n'(\theta,\phi))] = (-3(1+\cos\theta) - \sin\theta\partial_\theta)\mathcal{E}(n)\mathcal{E}(n'(\theta,\phi))\,,$$

$$\text{R.H.S.}: \quad -i[\vec{n}\cdot\vec{\mathbf{K}}, \partial_J^k \mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu)] = -(J+1)\partial_J^k \mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu) - k\partial_J^{k-1}\mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu)\,,$$

Recursion Relation: $$\left(\frac{\partial}{\partial \ln z} + 1\right) C_a^{(k)}(z;\mu) = \frac{k+1}{2} C_a^{(k+1)}(z;\mu)$$

Solution: $$C_a^{(k)}(z;\mu) = \frac{1}{z}\frac{2^k}{k!}\left(\frac{\partial}{\partial \ln z}\right)^k \widetilde{C}_a(z;\mu)$$

- Only single undetermined function
- Contains logarithms in perturbation theory

classical scaling behavior

RG invariance

The RG evolution of coefficients are determined by the RG of operators

$$\mathcal{E}(n)\mathcal{E}(n') = \sum_{k=0}^{\infty} C_a^{(k)}(z;\mu)\left[\partial_J^k \mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu)\right]\Bigg|_{J=3} + \text{higher twists}$$

RG invariant (vanishing anom. dim.)

$$\frac{d}{d\ln\mu^2}[\mathcal{E}(n)\mathcal{E}(n')] = 0$$

known RG behavior

$$\frac{d}{d\ln\mu^2}\mathbb{O}_{a;\mathrm{ren}}^{[J]}(n;\mu) = \gamma_{ab}^T(J;\alpha_s(\mu))\mathbb{O}_{b;\mathrm{ren}}^{[J]}(n;\mu)$$

Impose RG consistency condition on coefficients

$$\frac{d}{d\ln\mu^2}C_a^{(k)}(z;\mu) = -\sum_{m=0}^{\infty}\frac{(k+m)!}{k!\,m!}C_b^{(k+m)}(z;\mu)\partial_J^m\gamma_{ba}^T(J;\alpha_s(\mu))$$

They are compatible with the Lorentz symmetry constraint

$$\frac{d}{d\ln\mu^2}\widetilde{C}_a(z;\mu) = -\sum_{m=0}^{\infty}\frac{2^m}{m!}\left(\frac{\partial}{\partial\ln z}\right)^m\widetilde{C}_b(z;\mu)\left[\partial_J^m\gamma_{ba}^T(J;\alpha_s(\mu))\right]\Bigg|_{J=3}$$

Constraint from Physical Observables

Consider EEC with the center of mass energy Q.

In perturbative massless QCD, we expect the functional from

$$\langle \mathcal{E}(n)\mathcal{E}(n')\rangle_Q = Q^2 f(z, \ln\frac{Q^2}{\mu^2}, \alpha_s(\mu))$$

But the light-ray OPE does not manifest this property

$$Q^{J-1} h_a(J, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu))$$

$$\langle \mathcal{E}(n)\mathcal{E}(n')\rangle_Q = \frac{1}{z}\sum_{k=0}^{\infty}\frac{2^k}{k!}\left[\left(\frac{\partial}{\partial \ln z}\right)^k \widetilde{C}_a(z;\mu)\right]\left[\partial_J^k \boxed{\langle \mathbb{O}^{[J]}_{a;\mathrm{ren}}(n;\mu)\rangle_Q}\right]\Big|_{J=3}$$

Rearrange $\longrightarrow$ $$\frac{Q^2}{z}\sum_{m=0}^{\infty}\frac{2^m}{m!}\left[\partial_J^m h_a(J, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu))\right]\Big|_{J=3}\left(\frac{\partial}{\partial \ln z}\right)^m \boxed{\sum_{k=0}^{\infty}\frac{2^k}{k!}(\ln Q)^k\left(\frac{\partial}{\partial \ln z}\right)^k \widetilde{C}_a(z;\mu)} = \widetilde{C}_a(zQ^2;\mu)$$

Constraint: $$\boxed{\widetilde{C}_a(z;\mu) = \widetilde{C}_a(\ln\frac{z}{\mu^2}; \alpha_s(\mu))}$$

Deriving Factorization from Light-ray OPE

$$\frac{\langle \mathcal{E}(n)\mathcal{E}(n')\rangle_Q}{Q^2} = \frac{1}{z}\sum_{m=0}^{\infty}\frac{1}{m!}\left[\partial_J^m h_a(J, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu))\right]\Bigg|_{J=3} \times 2^m \left(\frac{\partial}{\partial \ln z}\right)^m \widetilde{C}_a(\ln\frac{zQ^2}{\mu^2}; \alpha_s(\mu))$$

Mellin moment $\quad h_a(J, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu)) = \int_0^1 dx\, x^{J-1}\widetilde{h}_a(x, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu))$

$$= \frac{1}{z}\int_0^1 dx\, x^2 \widetilde{h}_a(x, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu)) \sum_{m=0}^{\infty} \boxed{\frac{2^m}{m!}(\ln x)^m \left(\frac{\partial}{\partial \ln z}\right)^m} \widetilde{C}_a(\ln\frac{zQ^2}{\mu^2}; \alpha_s(\mu))$$

$$\boxed{= \frac{1}{z}\int_0^1 dx\, x^2 \widetilde{h}_a(x, \ln\frac{Q^2}{\mu^2}; \alpha_s(\mu)) \widetilde{C}_a(\ln\frac{zx^2Q^2}{\mu^2}; \alpha_s(\mu))\,,}$$

hard function

jet function

QCD factorization formula for EEC

Back to Conformal Case

For simplicity, let's assume there is no degeneracy and mixing

RG equation $$\frac{d}{d\ln\mu^2}\widetilde{C}(\ln\frac{z}{\mu^2};\alpha_s(\mu)) = -\sum_{m=0}^{\infty}\frac{2^m}{m!}\left(\frac{\partial}{\partial\ln z}\right)^m \widetilde{C}(\ln\frac{z}{\mu^2};\alpha_s(\mu))\left[\partial_J^m\gamma^T(J;\alpha_s(\mu))\right]\Big|_{J=3}$$

Conformal case $$\frac{d}{d\ln\mu^2} = \frac{\partial}{\partial\ln\mu^2} + \frac{\beta(\alpha_s)}{2}\frac{\partial}{\partial\alpha_s} \longrightarrow \frac{\partial}{\partial\ln\mu^2} = -\frac{\partial}{\partial\ln z}$$

Diagonalizing the evolution equation with $\widetilde{C}(\ln z;\alpha_s) = \int\frac{d\nu}{2\pi i}z^\nu\mathcal{C}(\nu;\alpha_s)$

$$0 = (\nu - \gamma^T(J+2\nu;\alpha_s))\mathcal{C}(\nu;\alpha_s) \xrightarrow{\textbf{solution}} \mathcal{C}(\nu;\alpha_s) \propto \delta(\nu-\gamma^T(J+2\nu;\alpha_s)) \propto \delta(\nu-\gamma^S(J))$$

$$\boxed{\gamma^S(J)-\gamma^T(J+2\gamma^S(J))=0}$$

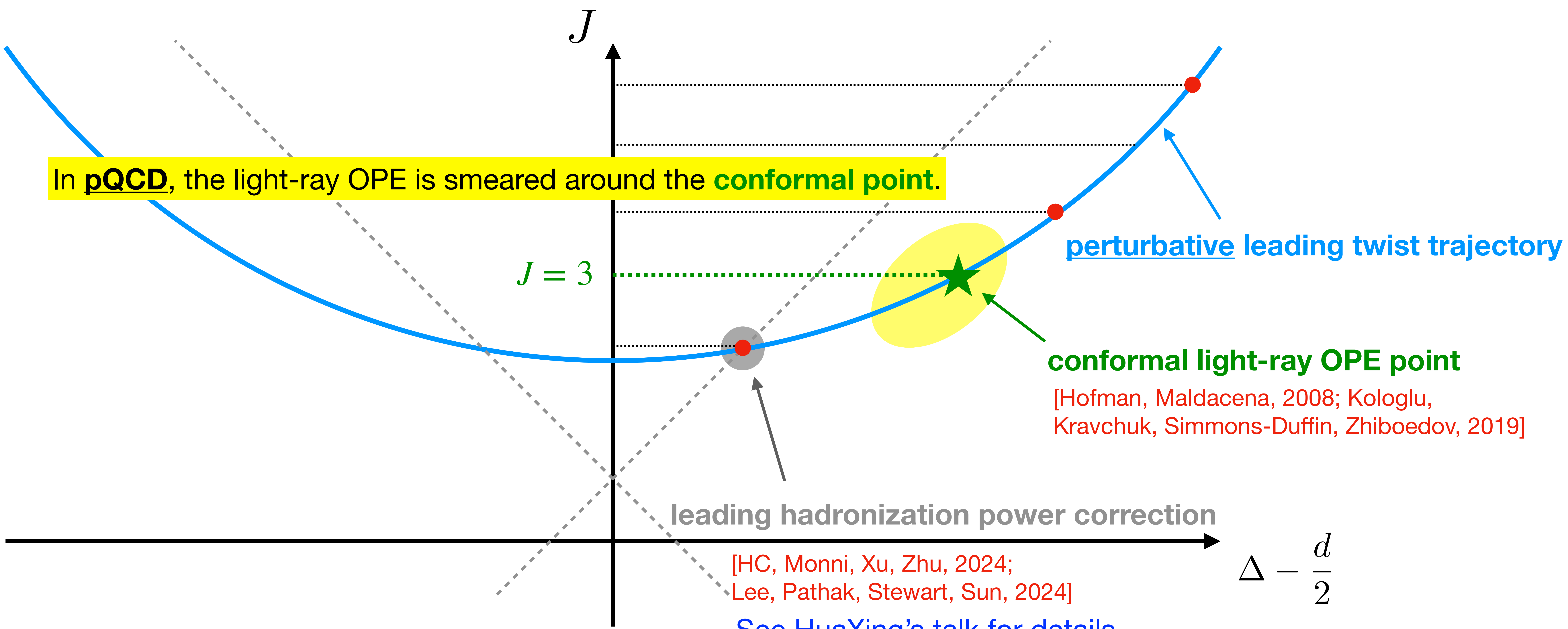
Reciprocity Relation

Exact scaling behavior $\widetilde{C}(\ln z;\alpha_s) \propto z^{\gamma^S(3;\alpha_S)}$

Light-ray OPE and Regge Trajectory

Light-ray operators are expected to be the **analytic continuation** of local operators.

[Kravchuk, Simmons-Duffin, 2018]



See HuaXing's talk for details

Also see Iain, Zhiquan, Kyle's talks for an alternative perspective

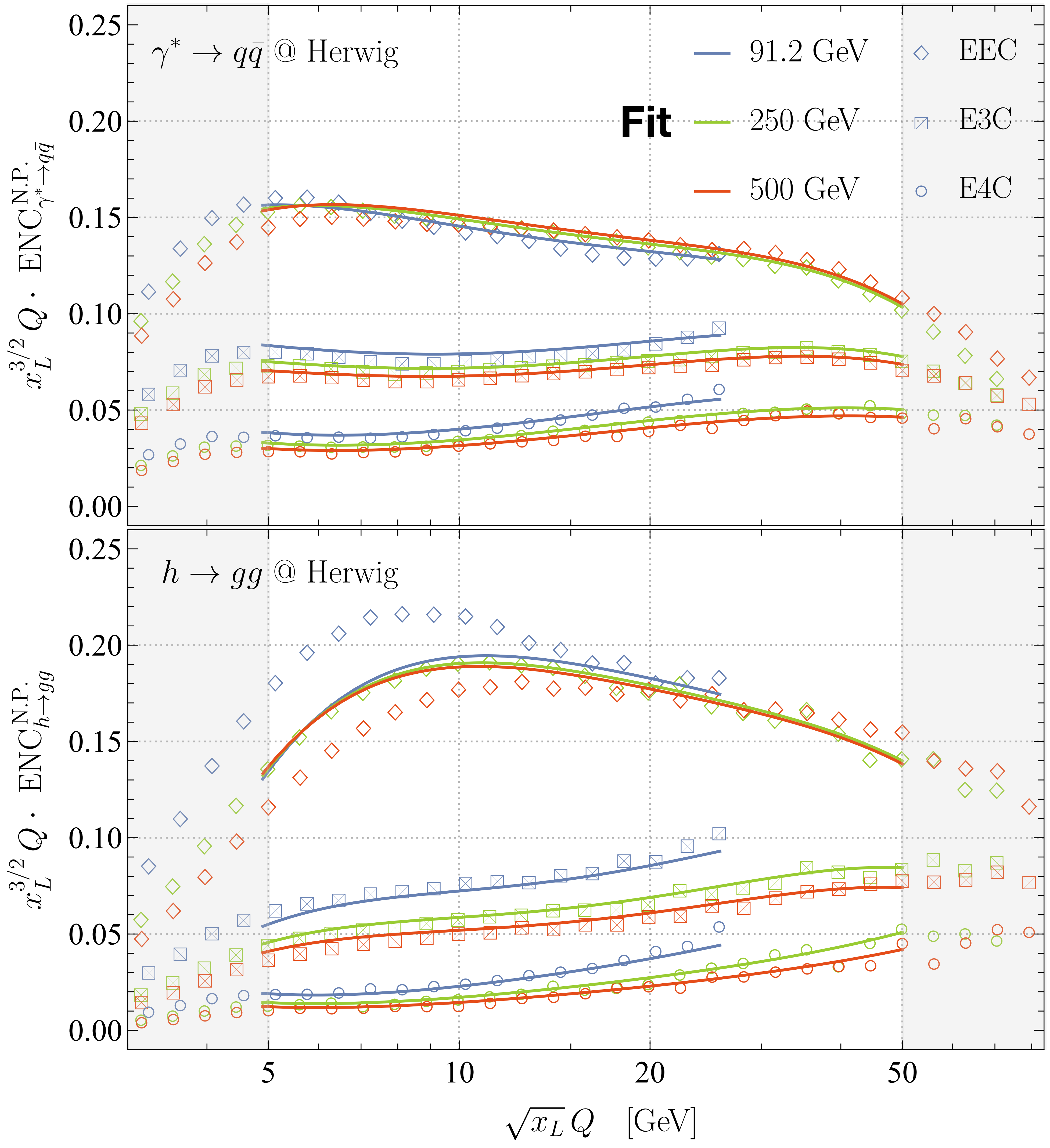
$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L} \vec{C} \cdot \vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}} \vec{D} \cdot \vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2) + \cdots$$

Evolution of Leading Power Correction

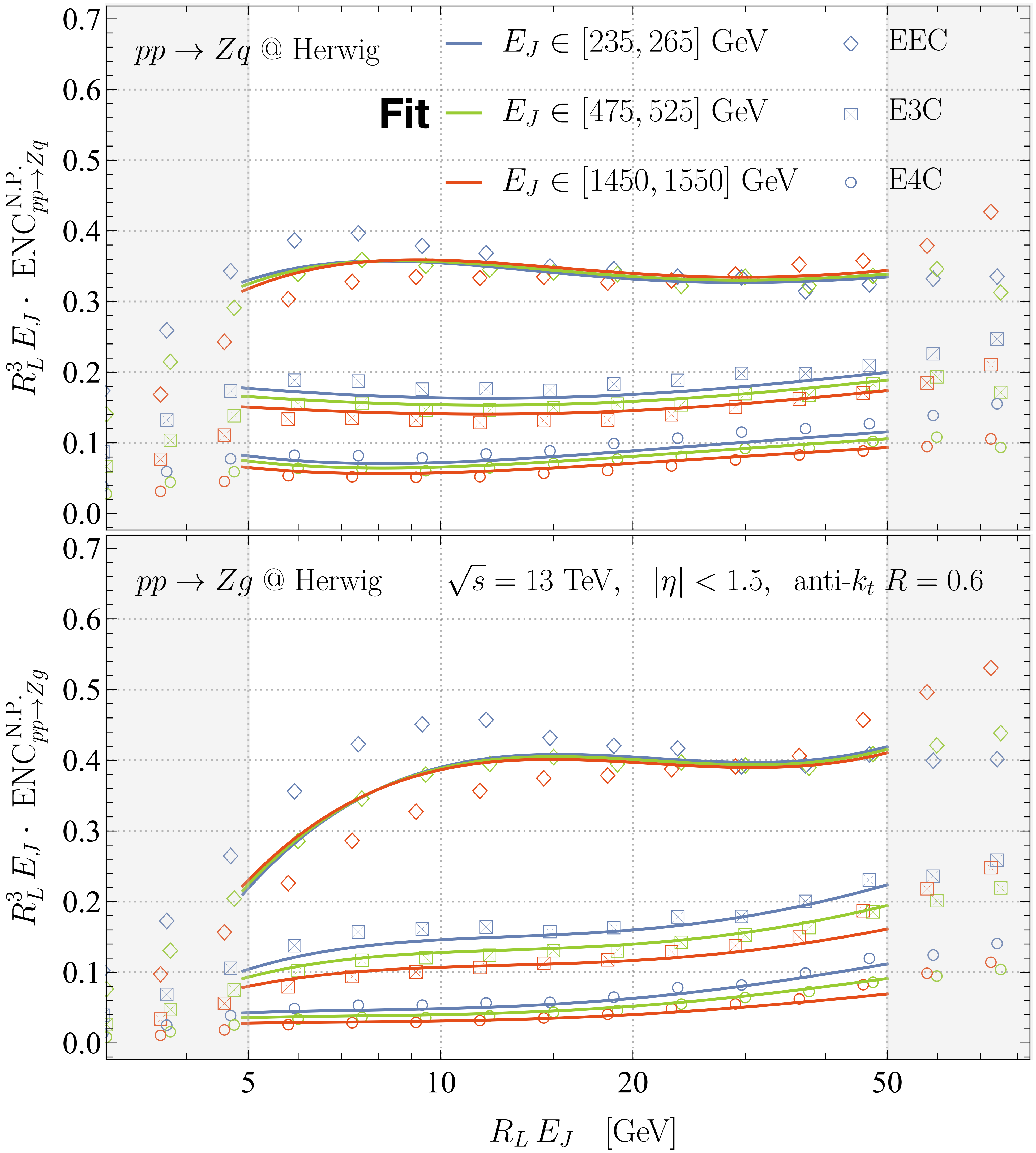
See HuaXing's talk

Assume power corrections = (hadron minus parton) results from Monte Carlo generator.

Lepton Collider



Hadron Collider



Strong Coupling Limit

Strong coupling limit in generic QFTs are hard. For holographic CFTs, we can access this limit through AdS/CFT correspondence.

Example: N=4 SYM and Type II B superstring theory on $AdS_5 \times S^5$

Classical GR <—— large λ and N_c limit:

Expansion in $1/\lambda$: Stringy Correction

Expansion in $1/N_c$: QG Correction

't Hooft coupling: $\lambda = g_{YM}^2 N_c$

string length: $\ell_s = \lambda^{-1/4} R_{AdS}$

$G_N \sim N_c^{-2} \sim 1/c_T$

EEC in the strong coupling limit:

[Hofman, Maldacena, 2008]

uniform distribution in the strong coupling limit $\langle \mathcal{E}\mathcal{E} \rangle \sim \langle \mathcal{E} \rangle \langle \mathcal{E} \rangle$

Also see Riccardo's and Matthew's talks

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle = \left(\frac{p^0}{4\pi}\right)^2 \left[1 + \frac{1}{\lambda} 4\pi^2 (1 - 6z + 6z^2) + \ldots\right]$$

Energy detectors correspond to shockwaves in AdS

Leading QG Correction: $-384 \left(\frac{p^0}{4\pi}\right)^2 \frac{\log c_T}{c_T} \log z \left(1 - 36z + 216z^2 - 400z^3 + 225z^4\right) + \cdots$ [HC, Karlsson Zhiboedov, 2024]

Enhancement property compared to the local correlator: $\langle \mathcal{O}^\dagger T_{\mu\nu} T_{\rho\sigma} \mathcal{O} \rangle_c = \text{Tree-level Sugra} + \frac{1}{\lambda^{3/2}} \text{Stringy} + \frac{1}{c_T} \text{One-loop Sugra} + \ldots$

Summary

- Light-ray operators play an important role in collider physics.

- Light-ray OPE, organized as twist expansion, governs the small angle scaling behavior.

- By generalizing light-ray OPE to non-conformal theory, we are able to derive the QCD factorization formula.

- Light-ray OPE is an interesting formalism that has fruitful phenomenological applications, including hadronization effects.