Nucleon Energy Correlator

Xiaohui Liu
Beijing Normal University

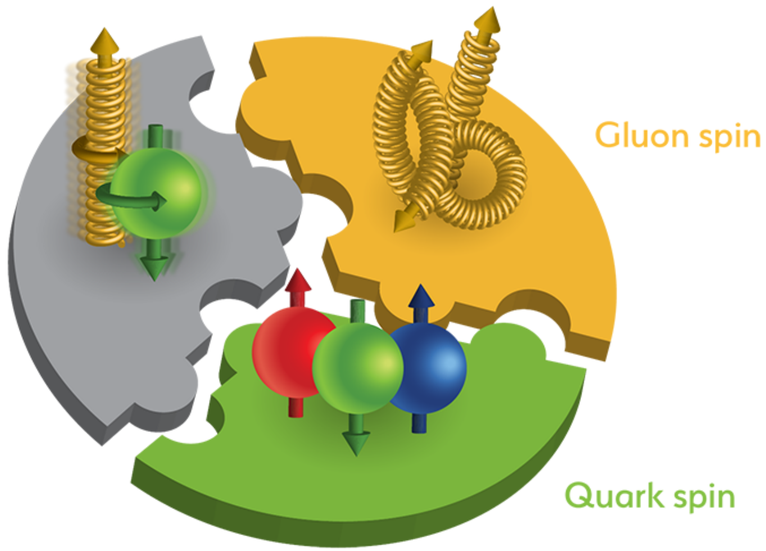
Energy Correlators at the Collider Frontier @ MITP Mainz
July 18, 2024

北京師範大學
BEIJING NORMAL UNIVERSITY

Outline

- Proton structure studies
- Nucleon energy Correlators (NECs)
  - Definition, measurement, factorization and properties
- Phenomenology and generalization
- New insights into the non-perturbative structures ???

Proton Structure



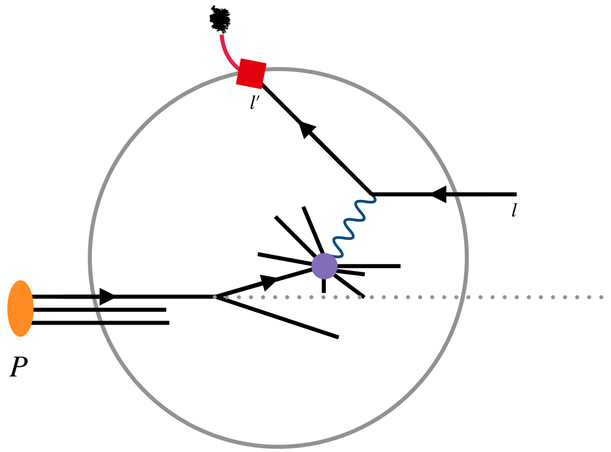
Major focus of the EIC …

Proton Structure

Collinear Parton Distribution Functions (PDFs)

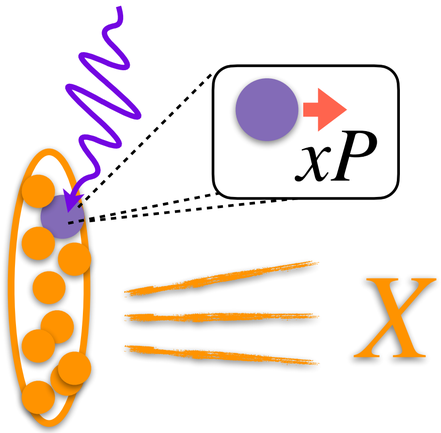
$$f_{q/p}(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P | \bar{\psi}(0) \mathscr{L} \psi(y^-) | P \rangle$$

$$\propto \delta(xP - p) \langle P | a_q^\dagger a_q | P \rangle$$



DIS

hard probe, e.g., DIS



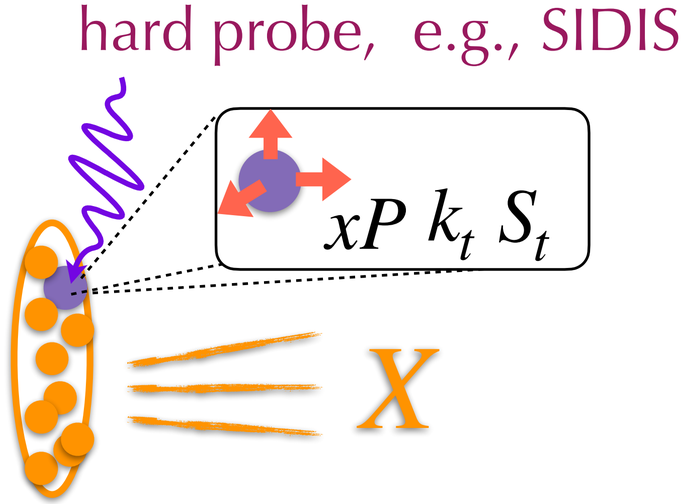
- inclusive over X, clean.
- not differential enough, lose information

Proton Structure

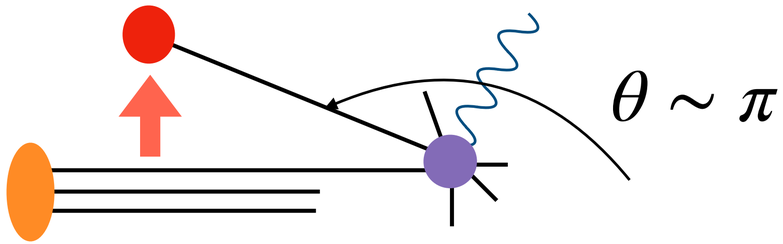
Transverse Moment Dependent-PDFs (TMDs)

See Dingyu's talk on Monday

$$f_{q/p}(x,k_t) = \int_{-\infty}^{\infty} \frac{dy^- dy_t}{(2\pi)^3} e^{ixp^+y^-} e^{ik_t\cdot y_t} \frac{\gamma^+}{2} \langle P|\bar{\psi}(0)\mathscr{L}\psi(y_t,y^-)|P\rangle$$



$$q_t \sim k_t \sim \Lambda_{\rm QCD}$$



SIDIS

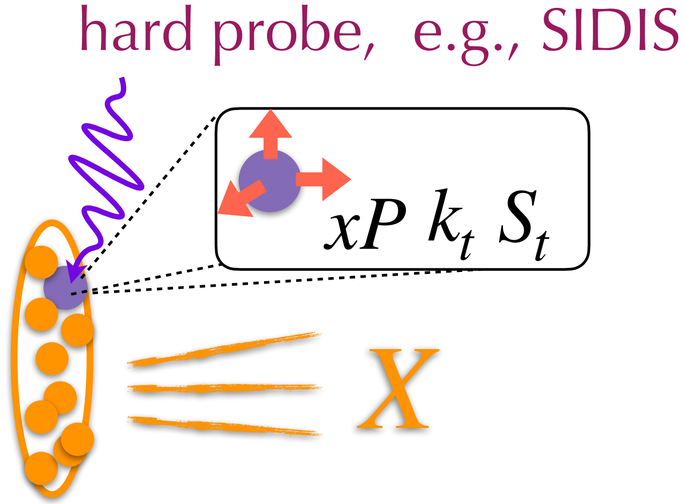
- Major tool for structure studies
  - Enforce the b-to-b configuration

Proton Structure

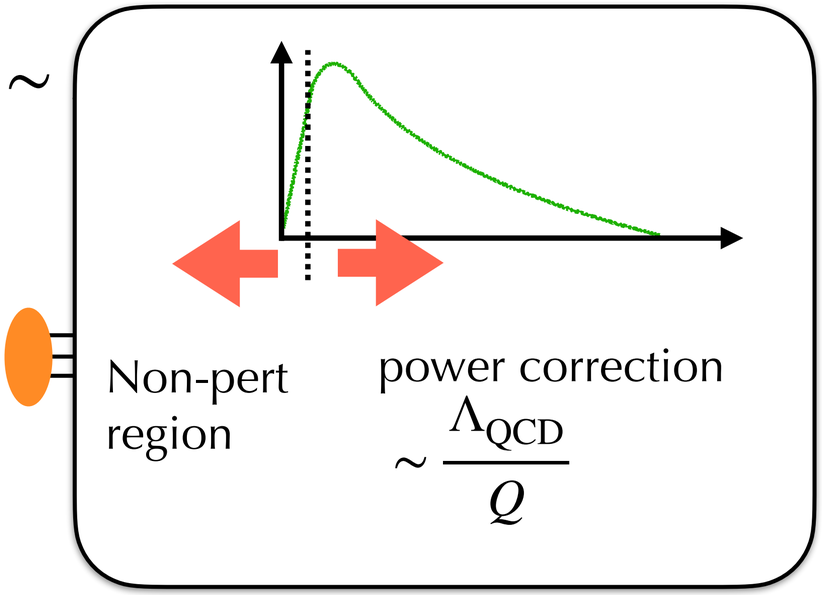
Transverse Moment Dependent-PDFs (TMDs)

See Dingyu's talk on Monday

$$f_{q/p}(x,k_t)=\int_{-\infty}^{\infty}\frac{dy^- dy_t}{(2\pi)^3}e^{ixp^+y^-}e^{ik_t\cdot y_t}\frac{\gamma^+}{2}\langle P|\bar{\psi}(0)\mathscr{L}\psi(y_t,y^-)|P\rangle$$



$q_t \sim k_t \sim$



- Major tool for structure studies
- Soft contamination
  - Sudakov suppression $\sigma(k_T) \propto \frac{1}{k_t^2} e^{-\frac{Q^2}{k_T^2}}$
  - Distort azimuthal asymmetry

Hatta, Xiao, Yuan, Zhou, PRL 2021

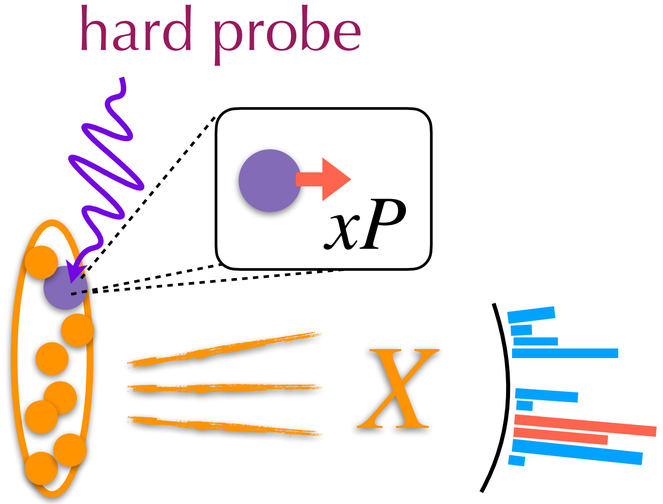
Nucleon Energy Correlator

Operator Definition

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

$$f_{q,EEC}(x,\theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P | \bar{\psi}(0) \mathscr{E}(\theta) \mathscr{L} \psi(y^-) | P \rangle$$

$$\mathscr{E}(n) = \int_0^{\infty} dt \lim_{r\to\infty} T_{0\vec{n}}(t, \vec{n}r) r^2$$

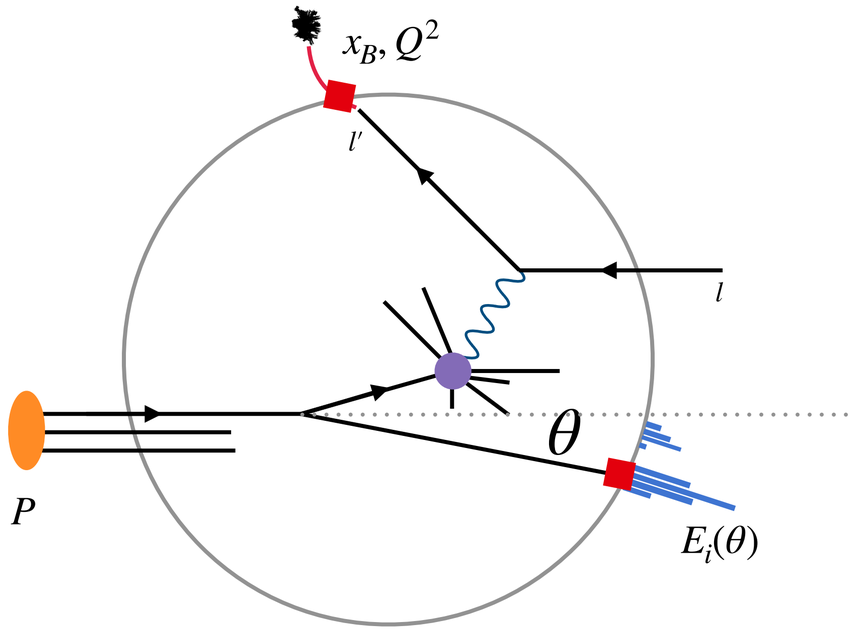


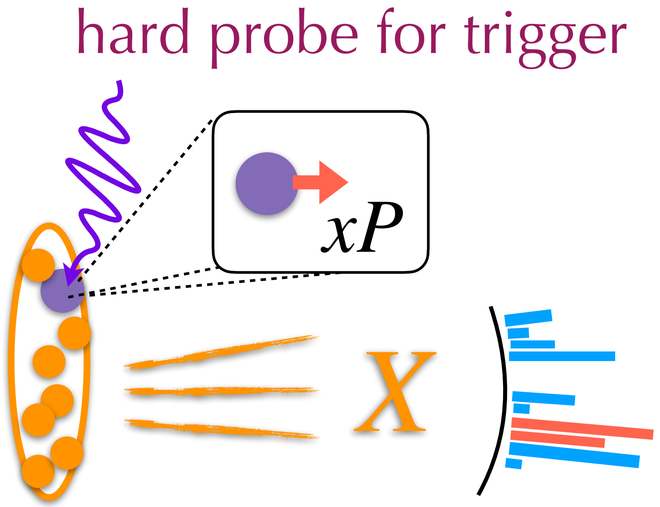
- Energy correlator in the forward region.
- Probe directly the broken proton
- Purely collinear object, insensitive to soft radiations, e.g. no Sudakov suppression
- Transverse dynamics through $\mathscr{E}(\theta)$
- Can be generalized to multiple-point correlation

Measurement, Factorization and Properties

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

- $\Sigma_N(Q^2,\theta) = \sum_i \int dx_B x_B^{N-1} \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i)\Theta(\theta - \theta_i)$



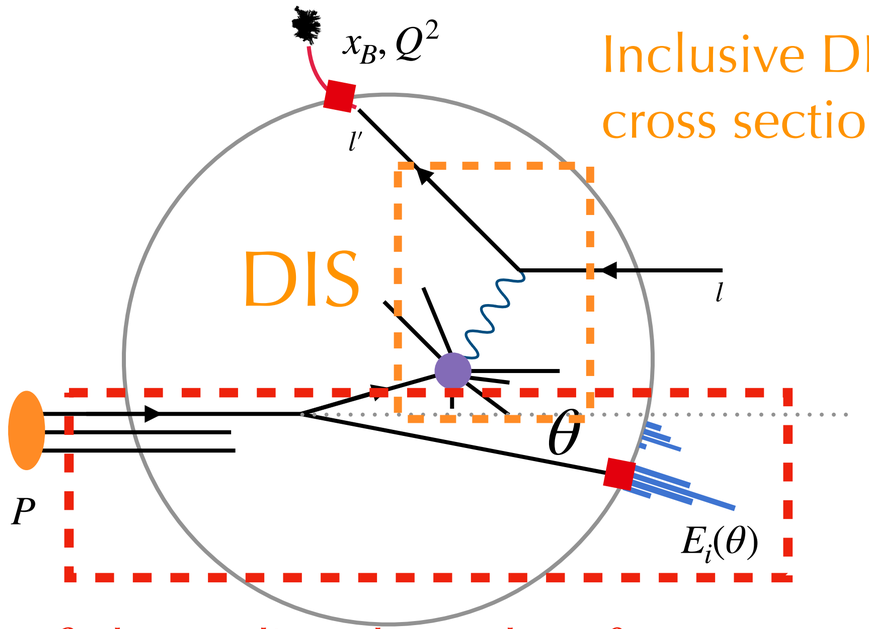


- Measurement in DIS
  - Full inclusive measurement, no jet/hadrons, weighted by E_i
  - Different θ's probe different physics

Measurement, Factorization and Properties

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

- $\Sigma_N(Q^2,\theta)=\sum_i\int dx_B x_B^{N-1}\frac{E_i}{E_P}d\sigma(x_B,Q^2,p_i)\Theta(\theta-\theta_i)$



Inclusive DIS partonic cross section

θ shape directly probes $f_{\rm EEC}$

- When $\theta Q \ll Q$, DIS type factorization

$$\Sigma_N(Q^2,\theta)=\int u^{N-1}\hat{\sigma}\left(u,Q^2,\mu\right)f_{\rm EEC}(N,\ln\frac{\theta Q}{u\mu})$$

  - Derived by SCET Cao, XL, Zhu, 2303.01530
  - rigorous QCD derivation by relating to the fracture function through sum rules

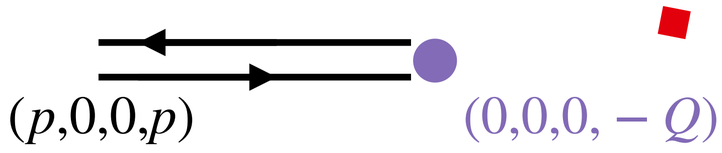
Chen, Ma, Tong, 2406.08559

Measurement, Factorization and Properties

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

- $\Sigma_N(Q^2,\theta) = \sum_i \int dx_B x_B^{N-1} \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i)\Theta(\theta-\theta_i)$

Breit Frame
LO



- When $\theta Q \ll Q$, DIS type factorization

$$\Sigma_N(Q^2,\theta) = \int u^{N-1} \hat{\sigma}\left(u, Q^2, \mu\right) f_{\mathrm{EEC}}(N, \ln\frac{\theta Q}{u\mu})$$

  - Derived by SCET Cao, XL, Zhu, 2303.01530
  - rigorous QCD derivation by relating to the fracture function through sum rules

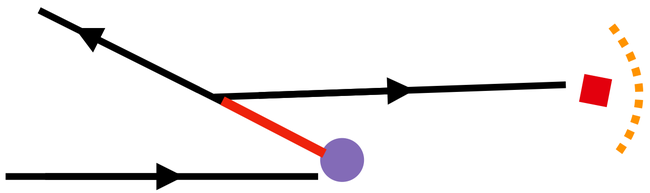
Chen, Ma, Tong, 2406.08559

Measurement, Factorization and Properties

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

- $\Sigma_N(Q^2,\theta) = \sum_i \int dx_B x_B^{N-1} \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i)\Theta(\theta-\theta_i)$

Breit Frame
NLO



$$\sim \frac{1}{Q} \times Q^2\theta^2 \to 0$$

- When $\theta Q \ll Q$, DIS type factorization

$$\Sigma_N(Q^2,\theta) = \int u^{N-1} \hat{\sigma}\left(u, Q^2, \mu\right) f_{\mathrm{EEC}}(N, \ln\frac{\theta Q}{u\mu})$$

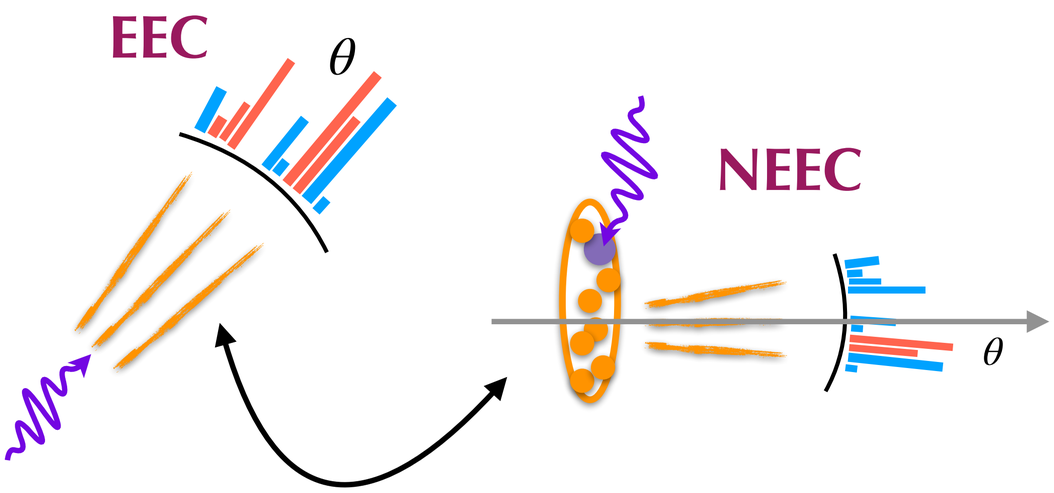
  - Derived by SCET Cao, XL, Zhu, 2303.01530
  - rigorous QCD derivation by relating to the fracture function through sum rules

Chen, Ma, Tong, 2406.08559

Measurement, Factorization and Properties

Nucleon EEC XL and Zhu, Phys. Rev. Lett. 130 (2023), 9, 9

- $\Sigma_N(Q^2,\theta) = \sum_i \int dx_B x_B^{N-1} \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i)\Theta(\theta-\theta_i)$



- When $\theta Q \ll Q$, DIS type factorization

$$\Sigma_N(Q^2,\theta) = \int u^{N-1}\hat{\sigma}\left(u, Q^2, \mu\right) f_{\mathrm{EEC}}(N, \ln\frac{\theta Q}{u\mu})$$

- Space like version of the EEC in e^+e^-

$$\Sigma = \int dx x^2 \sigma(x,\mu) J(\mu, \ln\frac{\theta x Q}{\mu})$$

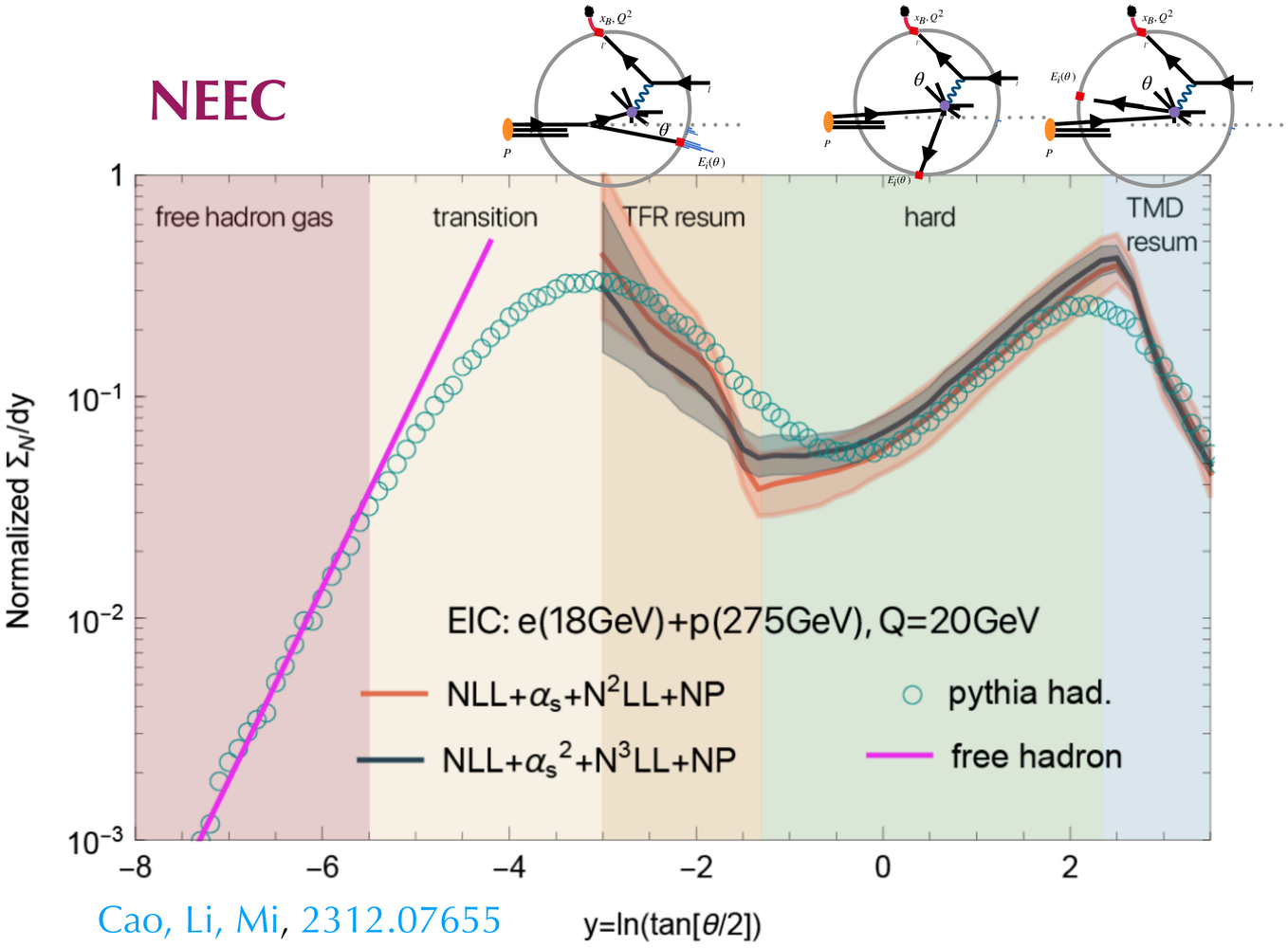
Dixon, Moult, Zhu, 2019

Chen, 2311.00350

Similar factorization form

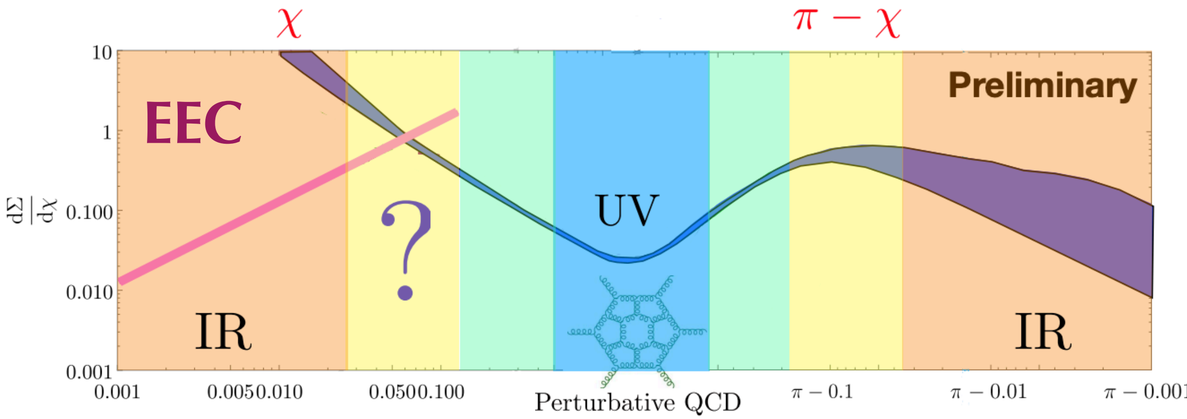
Measurement, Factorization and Properties

NEEC



Cao, Li, Mi, 2312.07655

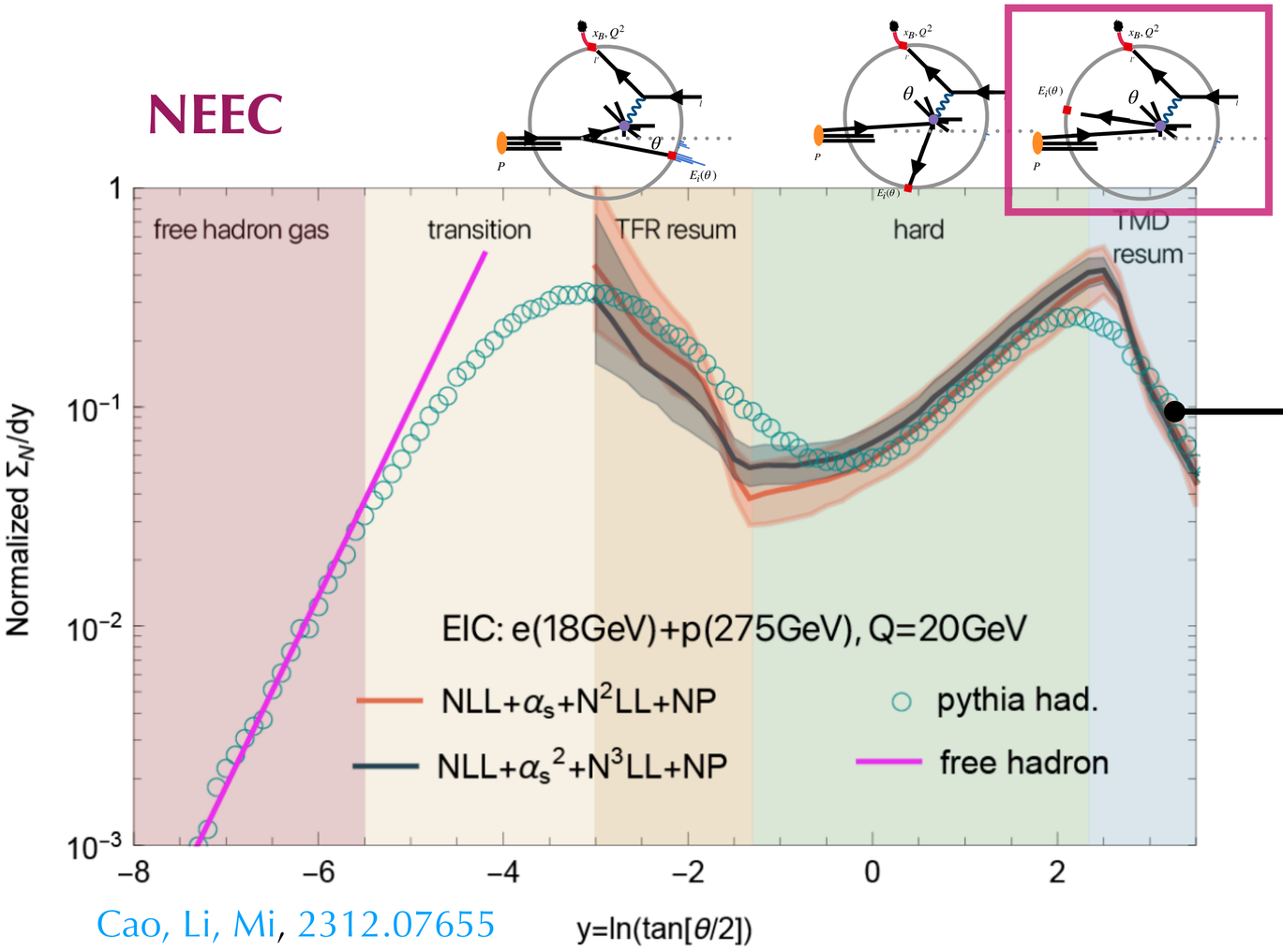
Share many similarities in the spectrum



Moult talk

Measurement, Factorization and Properties

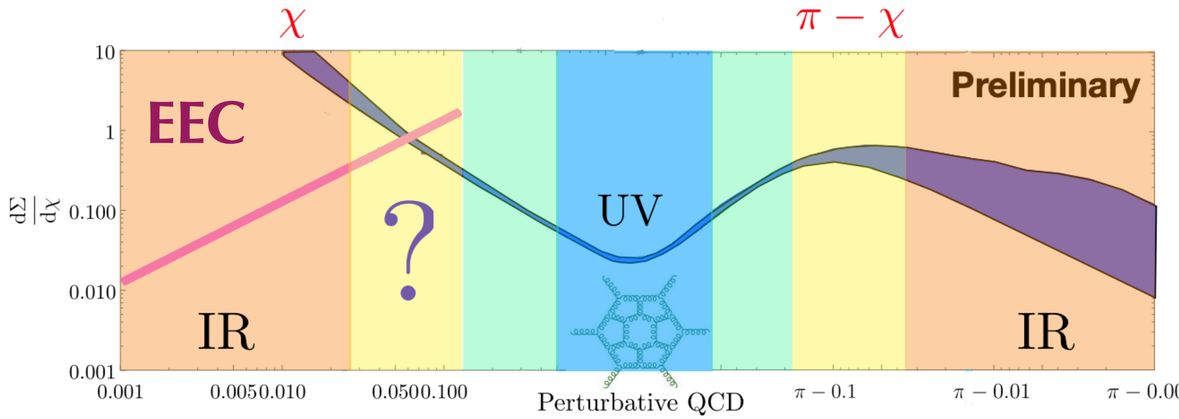
NEEC



Cao, Li, Mi, 2312.07655

TMD region

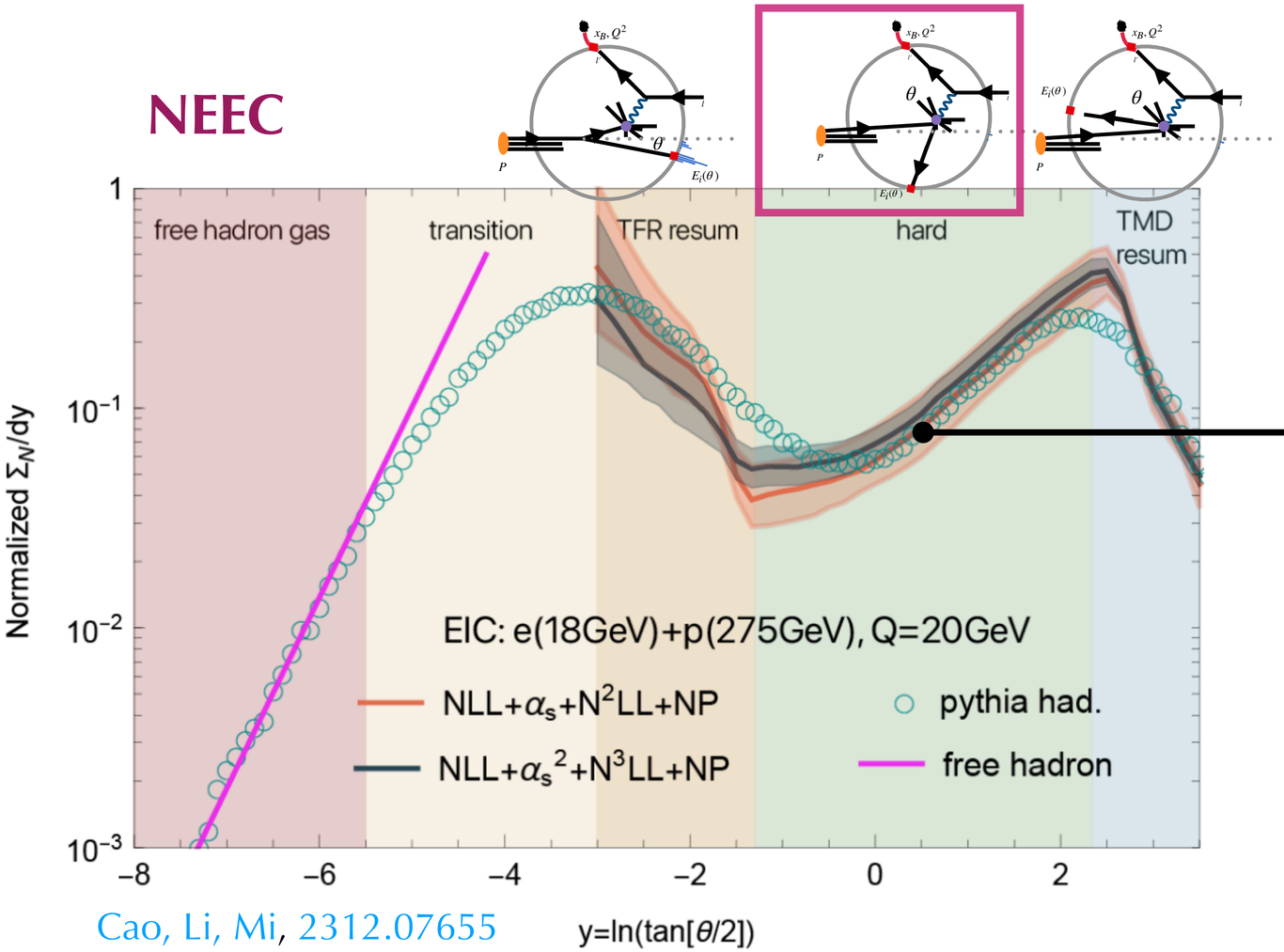
- conventional TMD physics



Moult talk

Measurement, Factorization and Properties

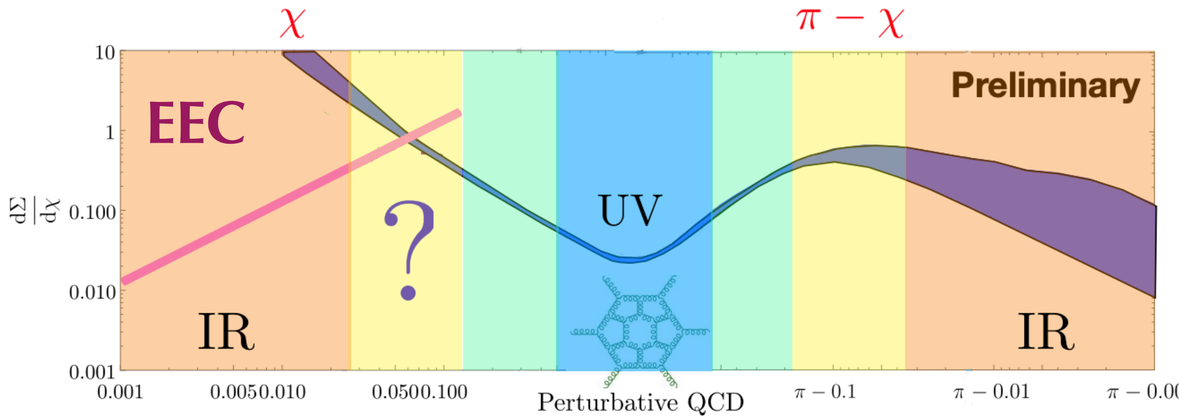
NEEC



Cao, Li, Mi, 2312.07655

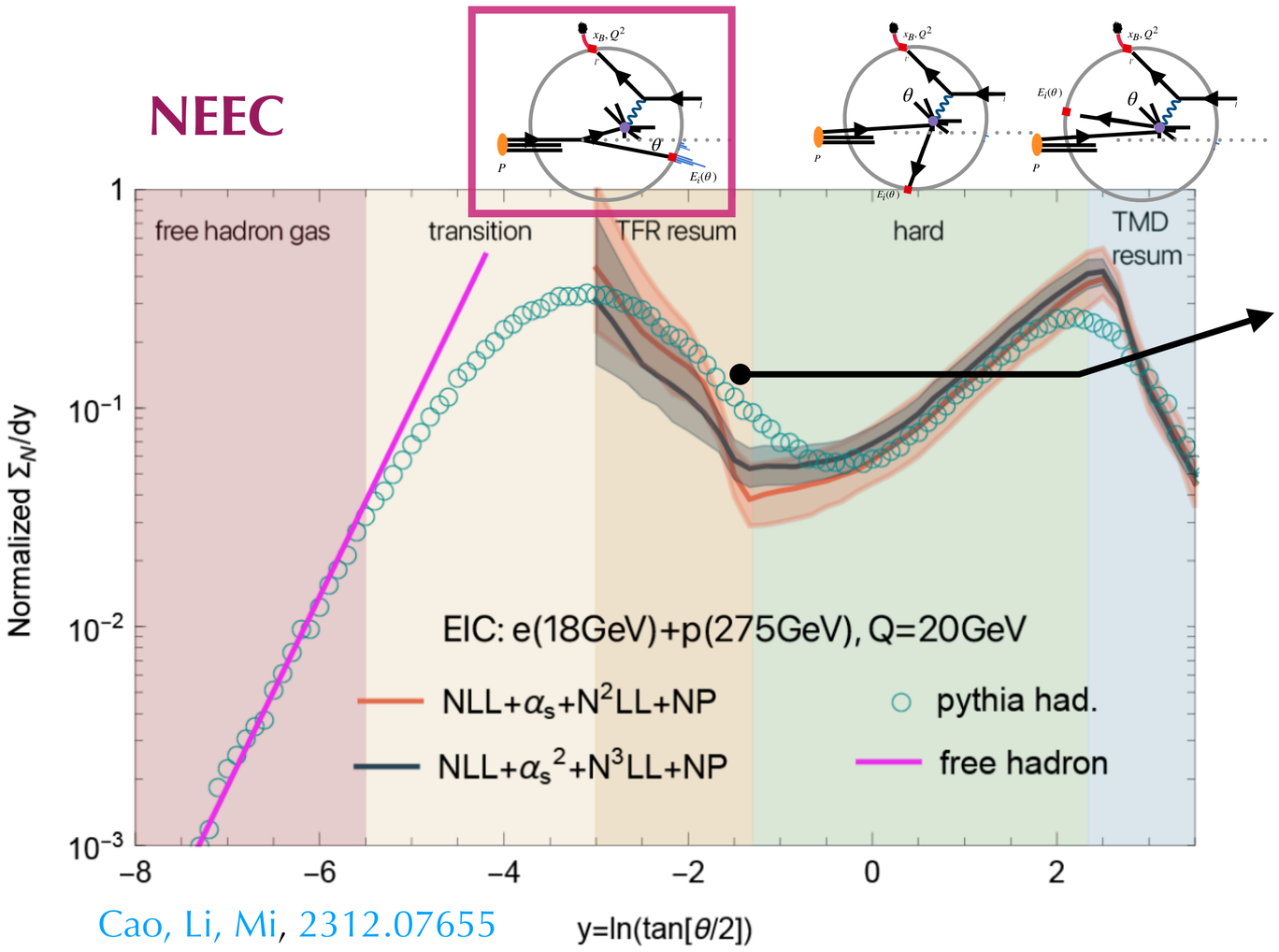
Hard region

- Fixed-order does the job



Moult talk

Measurement, Factorization and Properties



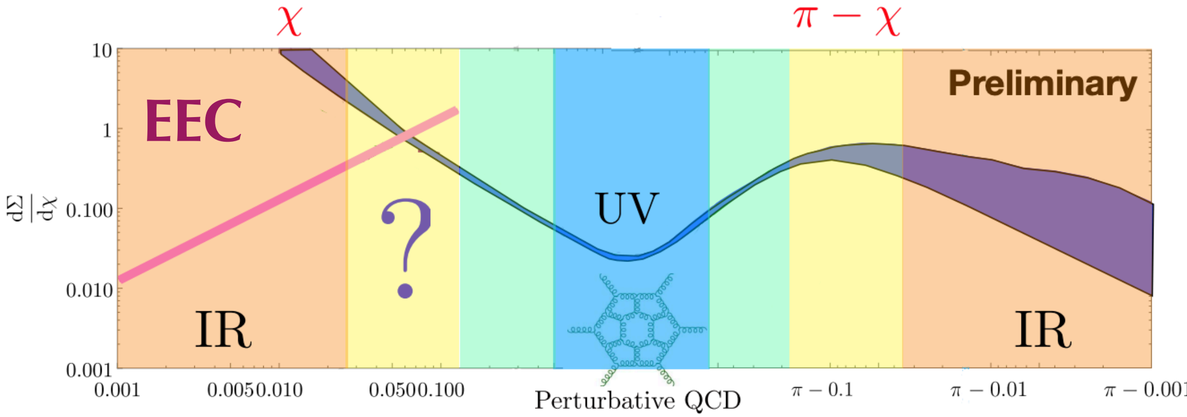
Cao, Li, Mi, 2312.07655

$$\Lambda_{\rm QCD} \ll \theta Q \ll Q$$

- Perturbatively calculable

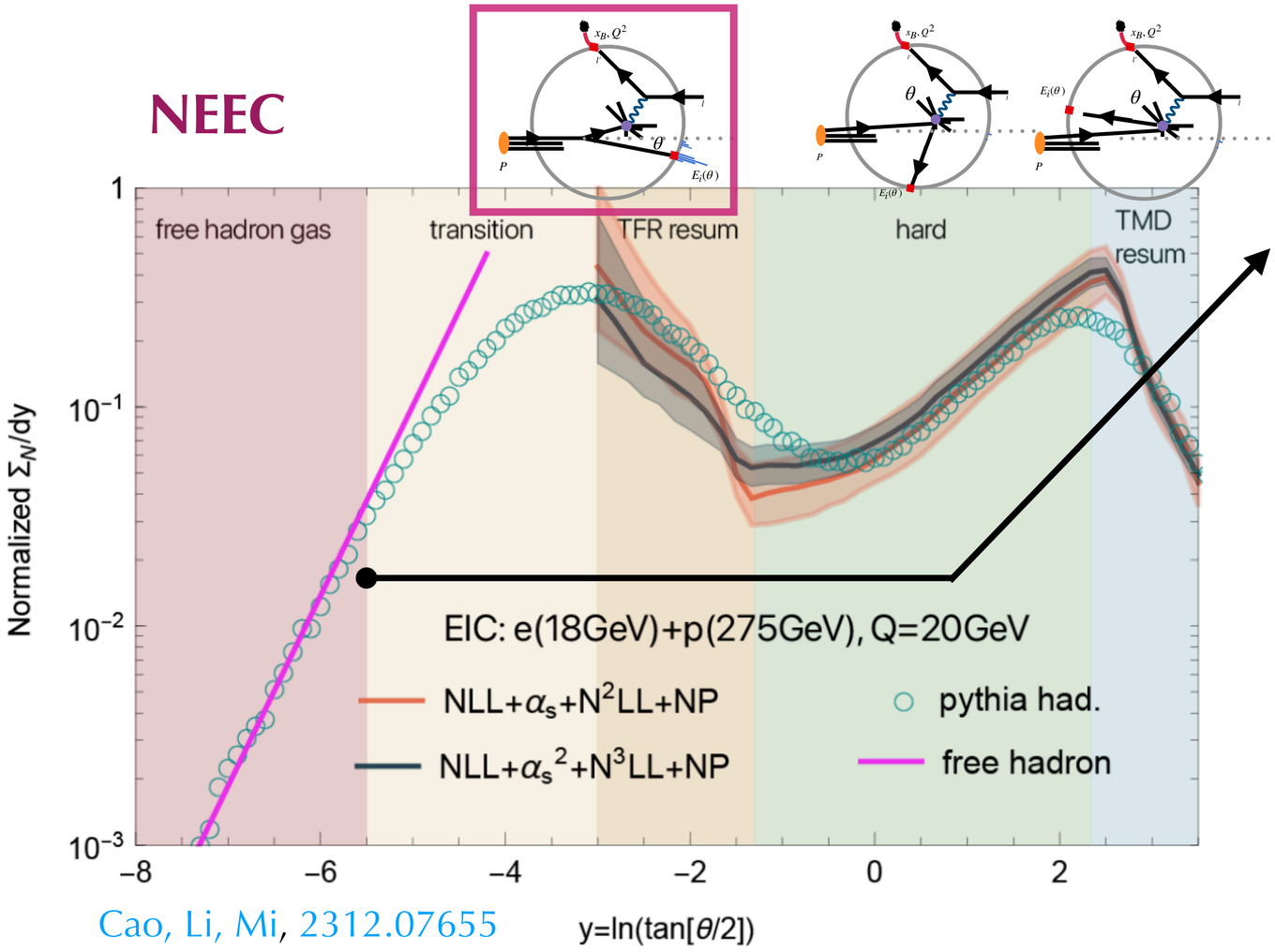
$$f^{(0)}_{\rm EEC}(\theta) \propto \left[\frac{1}{\theta^2}(1-x)P(x)\right] \times \left[\xi f(\xi)\right]$$

- Dynamics dominated by coll. splitting
- Power law: $\theta^{-2+\gamma}$, γ by $P(N)$ + coll. PDF



Moult talk

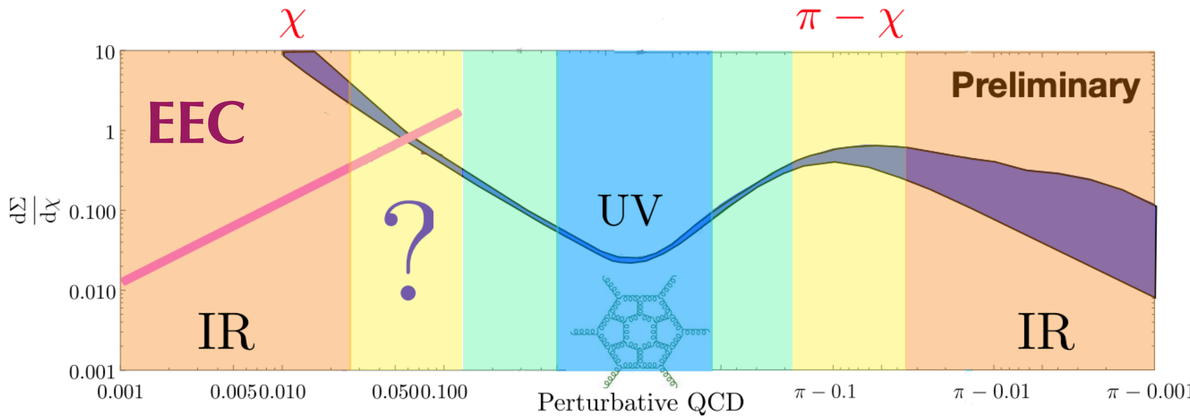
Measurement, Factorization and Properties



Cao, Li, Mi, 2312.07655

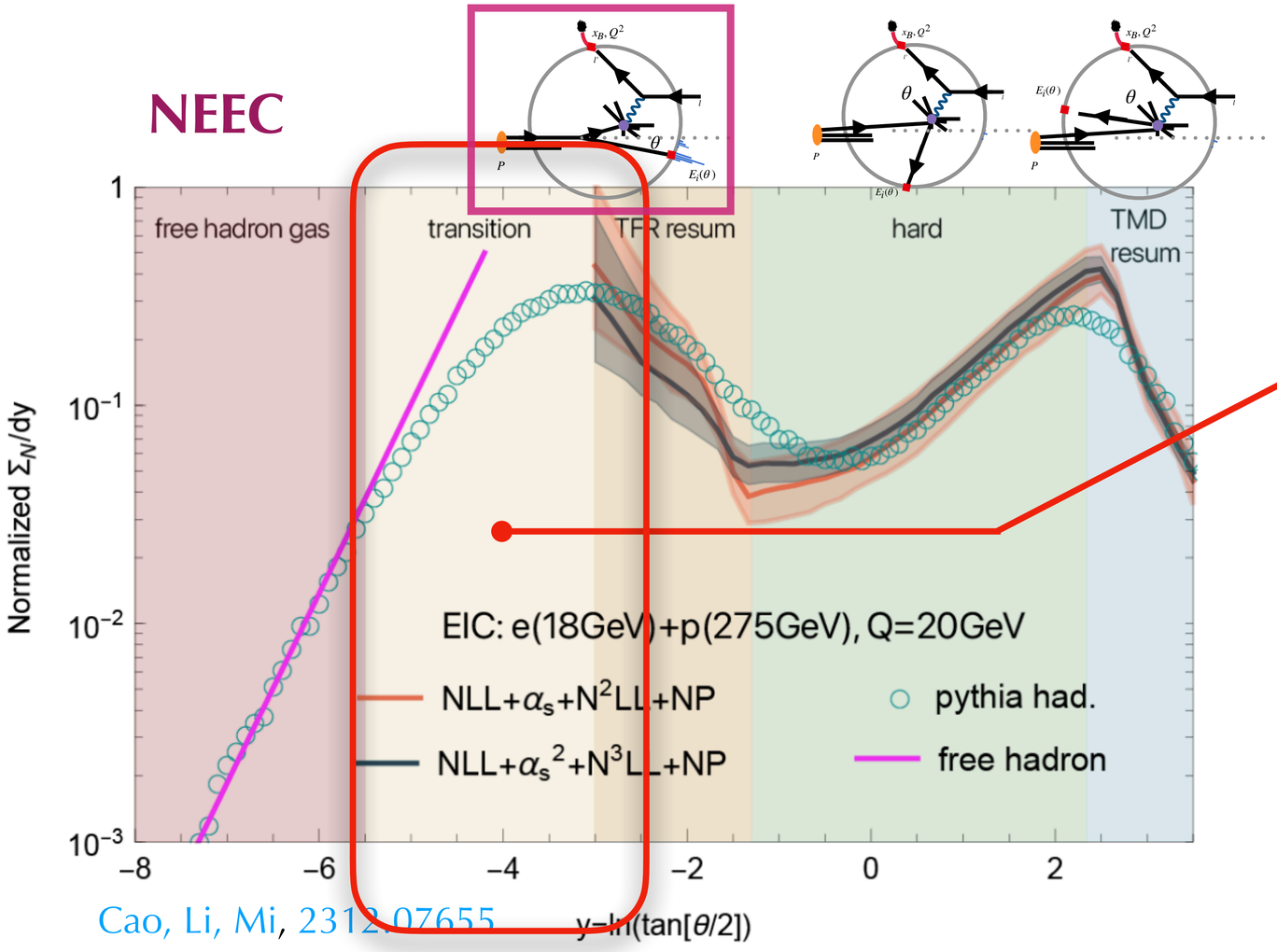
Deep NP region

- Un-correlated distribution



Moult talk

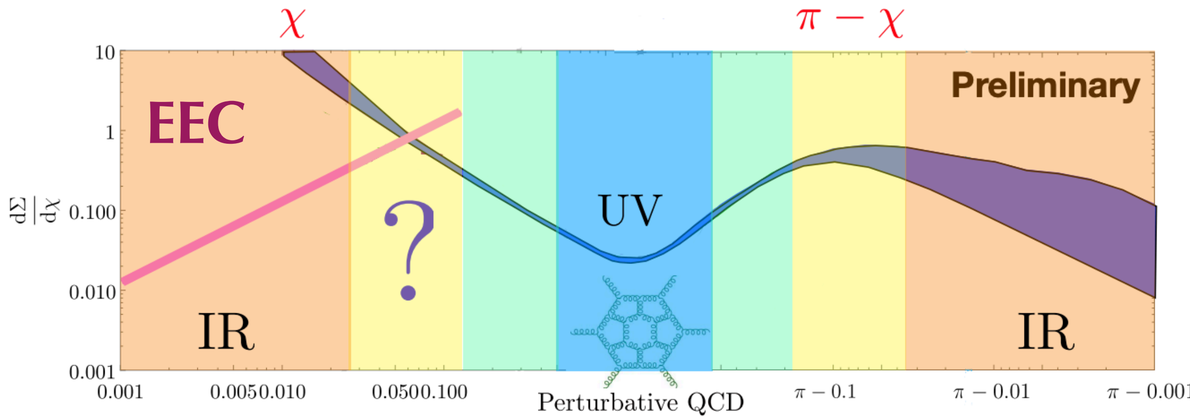
Measurement, Factorization and Properties



Cao, Li, Mi, 2312.07655

NP region

- Enhanced NP region, vs. TMD
- To be determined by future measurements
- Encodes info. on proton intrinsic structure and NP dynamics

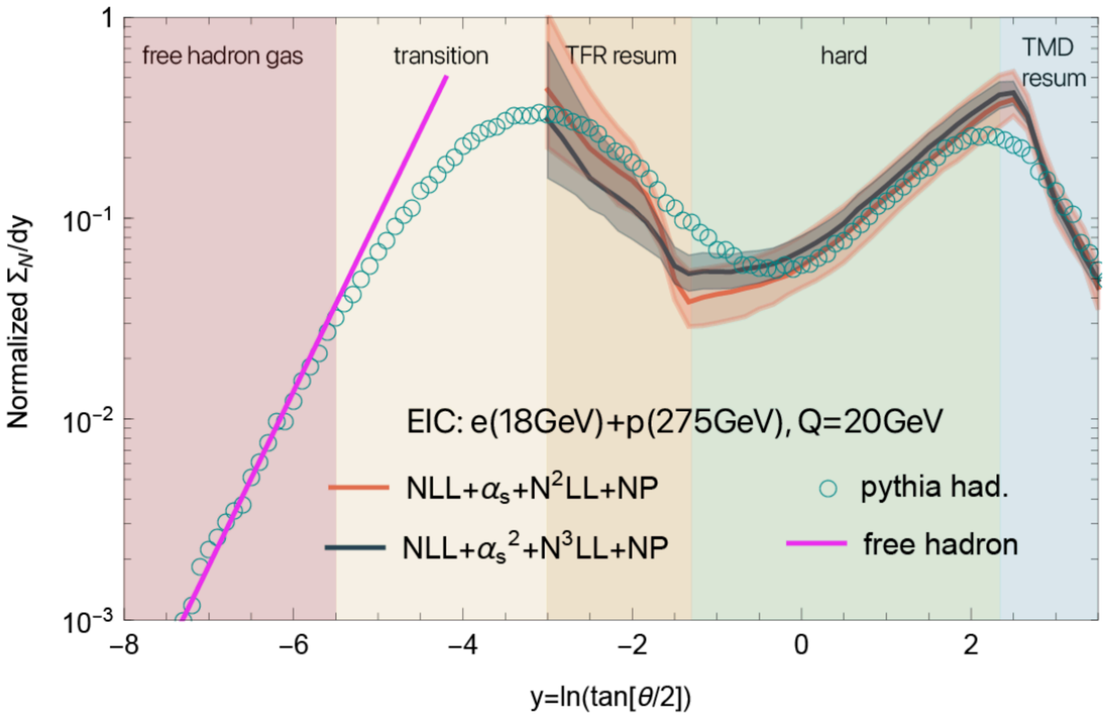


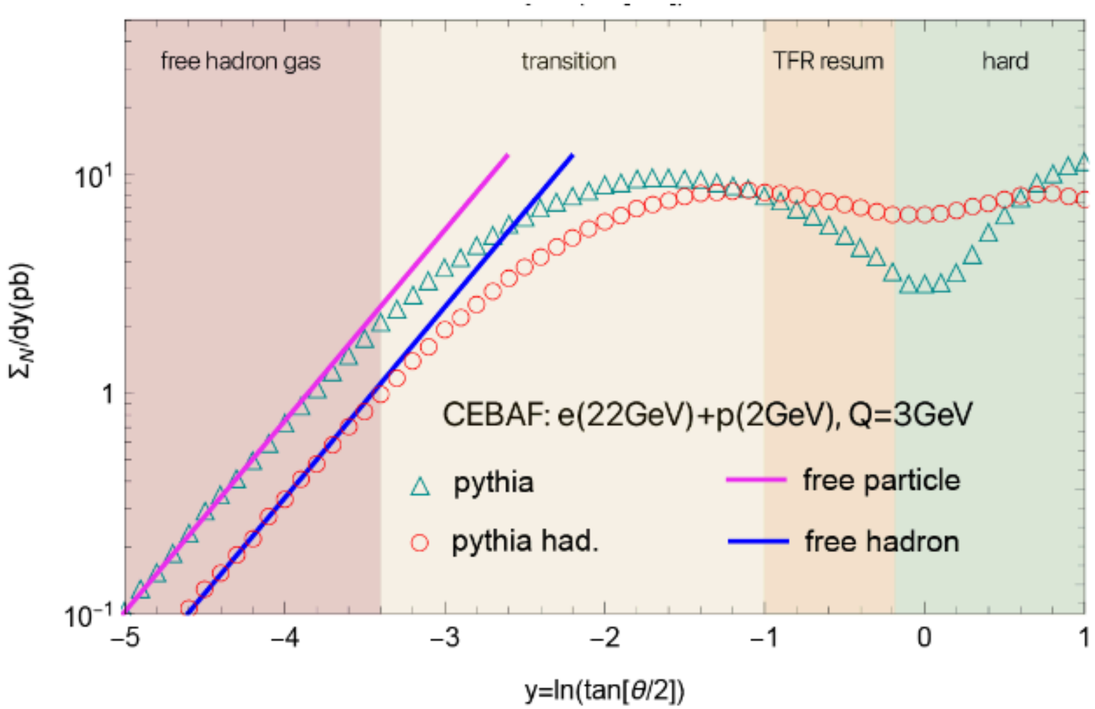
Moult talk

Phenomenology

NEC as a generating observable

XL, Zhu, **arxiv: 2403.08874**
XL, Shao, Zhu, **in preparation**

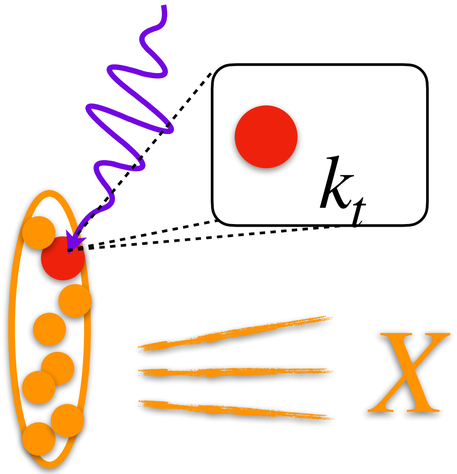




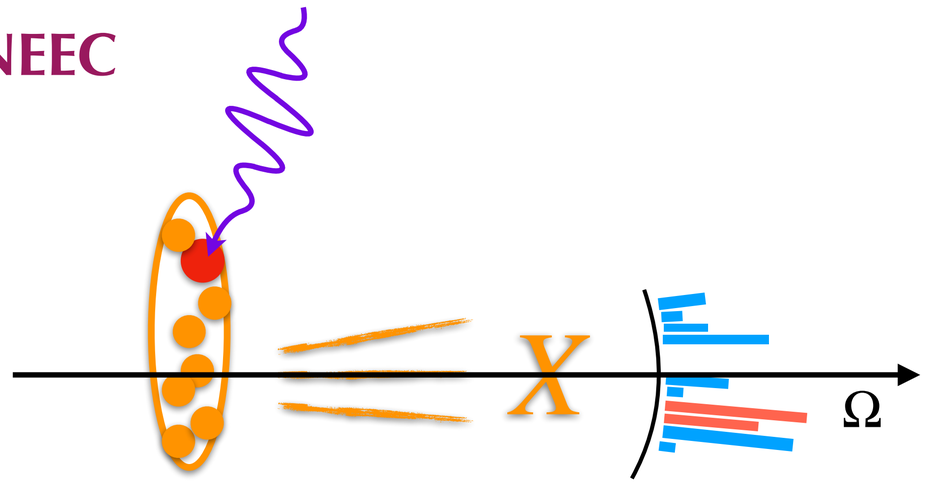
NEC as a generating observable

XL, Zhu, **arxiv: 2403.08874**
XL, Shao, Zhu, **in preparation**

TMD



NEEC



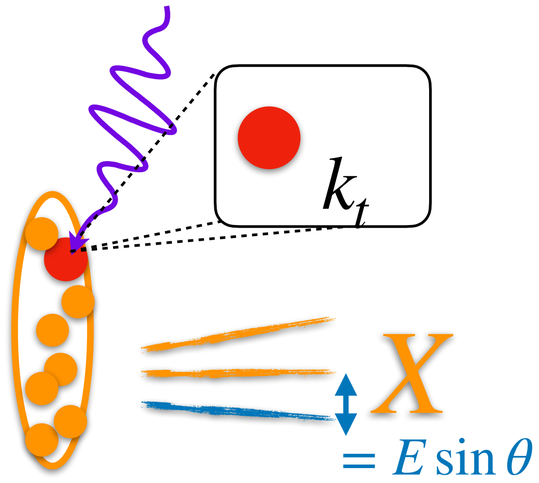
$$\vec{k}_t = -\sum_{i\in X} \vec{p}_{i,t} = -\sum_{i\in X} E_i \sin\theta_i(\cos\phi_i, \sin\phi_i)$$

$$\mathscr{E}(\Omega) = \sum_{i\in X} E_i \delta(\Omega - \Omega_i)$$

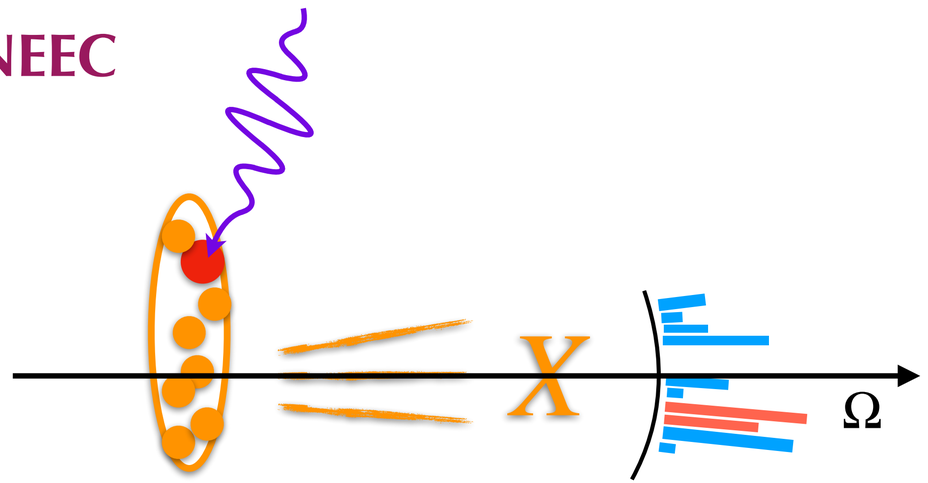
NEC as a generating observable

XL, Zhu, **arxiv: 2403.08874**
XL, Shao, Zhu, **in preparation**

TMD



NEEC



$$\vec{k}_t = -\int d\theta d\phi \ \sin\theta(\cos\phi, \sin\phi)\ \mathscr{E}(\Omega)$$

$$\mathscr{E}(\Omega) = \sum_{i\in X} E_i \delta(\Omega - \Omega_i)$$

$$\int^{\mu} dk_t k_t^n f(k_t) = (-)^n \int^{R} \prod_n d\Omega\, w(\Omega_1)\ldots w(\Omega_n)\langle P|\ldots\mathscr{E}(\Omega_1)\ldots\mathscr{E}(\Omega_n)\ldots|P\rangle$$

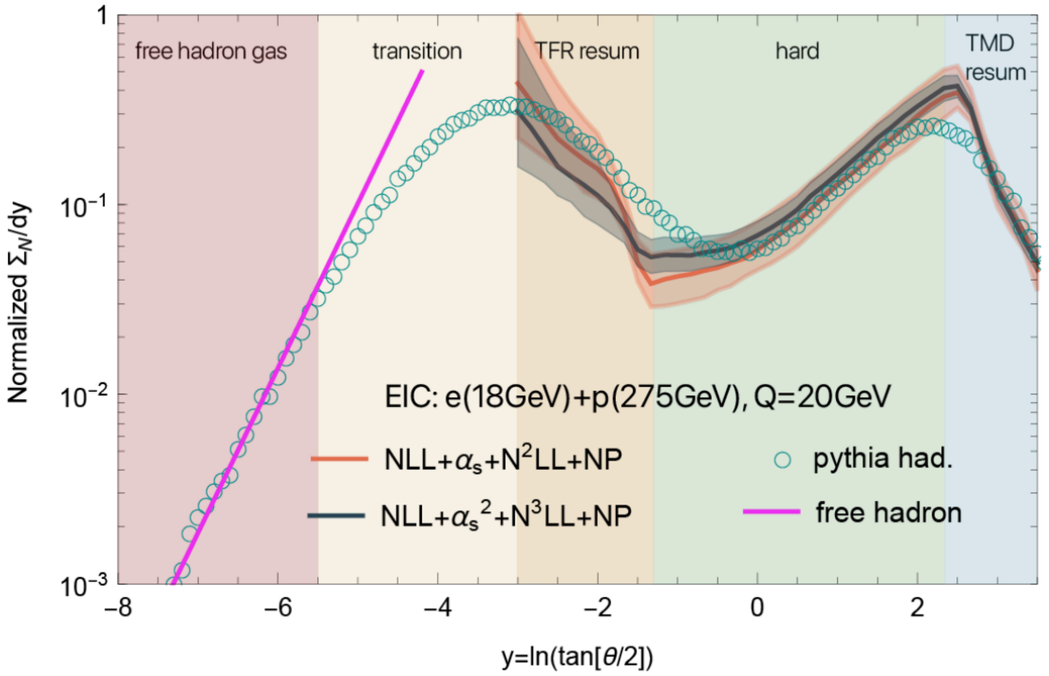
For TMD TMM see e.g.: del Rio, Prokudin, Scimemi, Vladimirov, arXiv:2402.01836v1

NEC as a generating observable

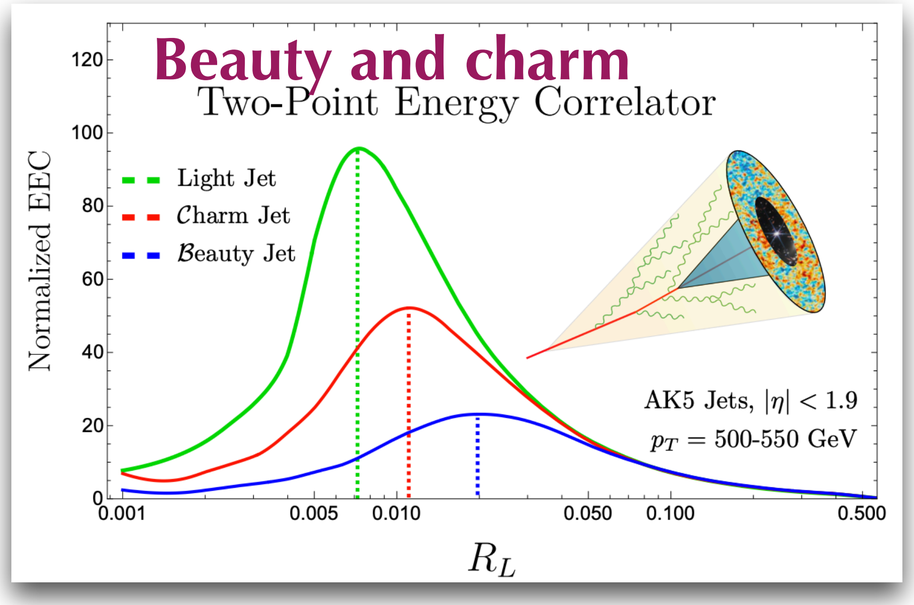
XL, Zhu, **arxiv: 2403.08874**
XL, Shao, Zhu, **in preparation**

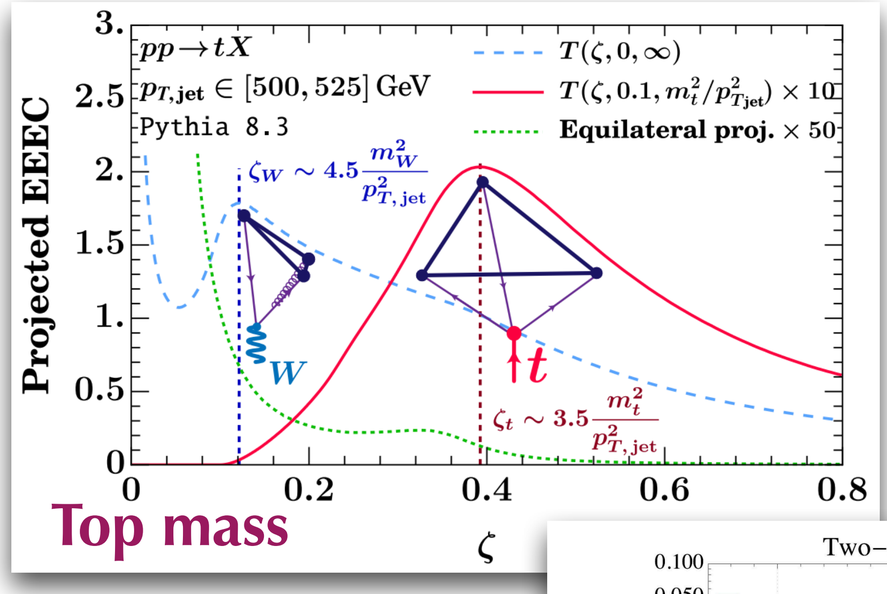
$$\int dk_t k_t^n f(k_t) = (-)^n \int \prod_n d\Omega\, w(\Omega_1)\ldots w(\Omega_n)\langle P|\ldots\mathscr{E}(\Omega_1)\ldots\mathscr{E}(\Omega_n)\ldots|P\rangle$$

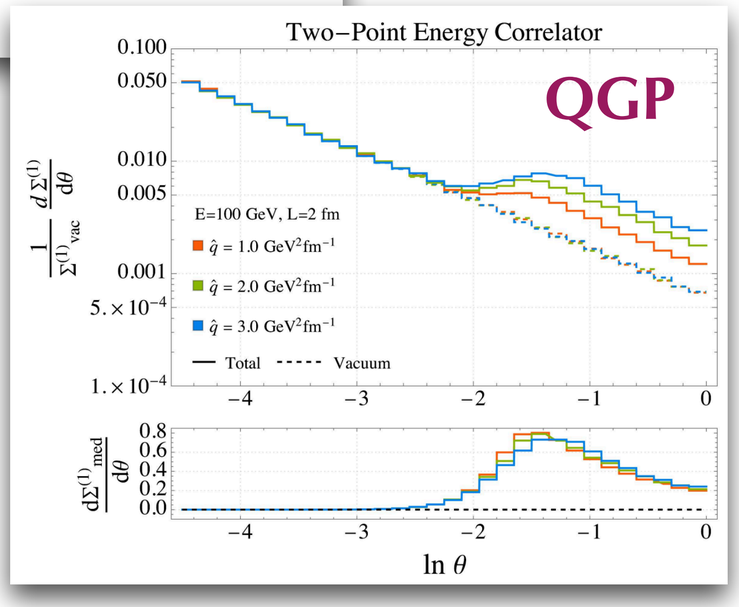
- TMD PDFs (moment) can be obtained by measuring N-pt Nucleon Energy Correlator, by suitably selecting $w(\Omega)$
- Inclusive measurement! Do not force b-to-b limit, no jets/fragmentation function involved.
- Nucleon Energy Correlator can be regarded as a generating observable, contains more comprehensive information



Revealing the gluon saturation

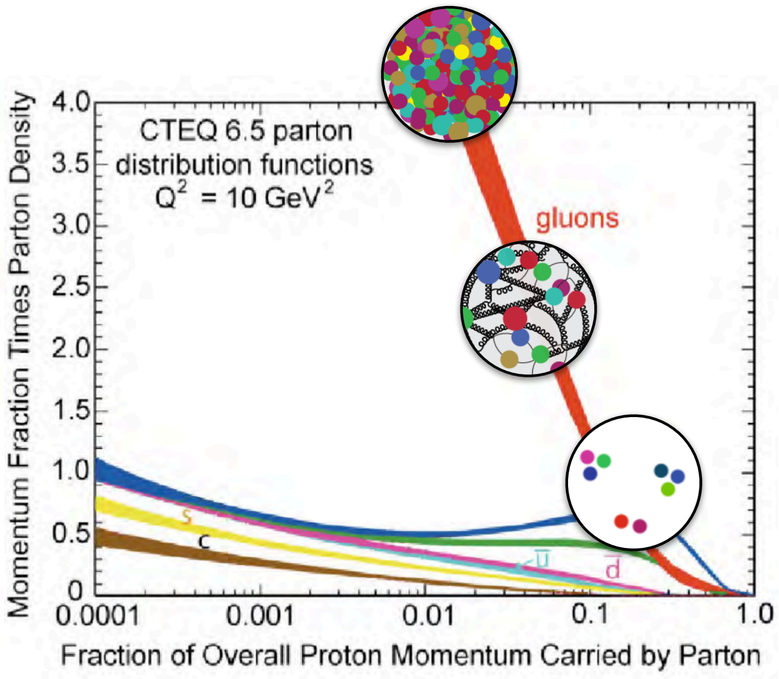






Exposing intrinsic mass scales

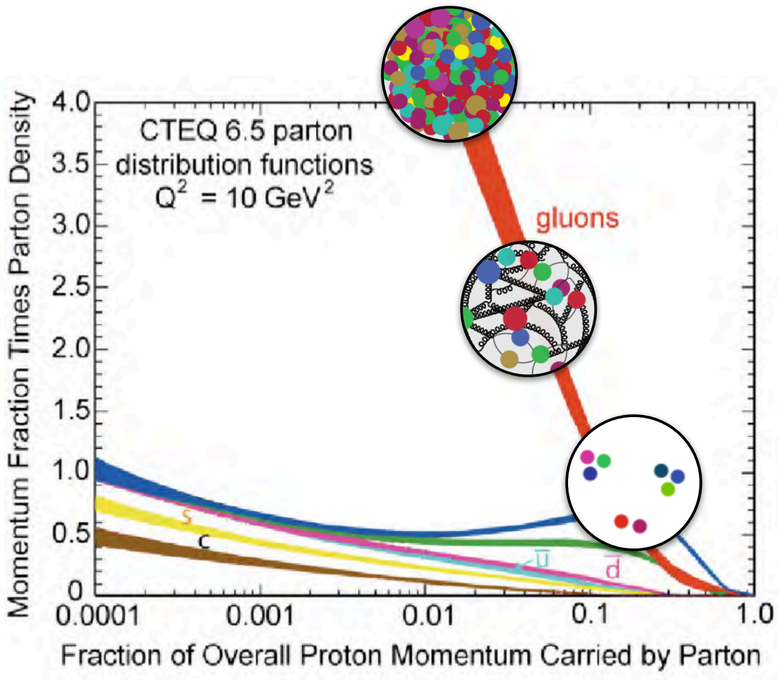
Revealing the gluon saturation



Gluon saturation at small x

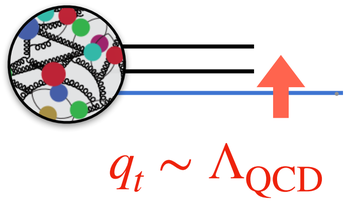
- Many hints but yet to be nailed down
- One major pillar of the EIC

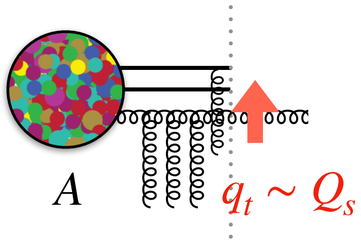
Revealing the gluon saturation



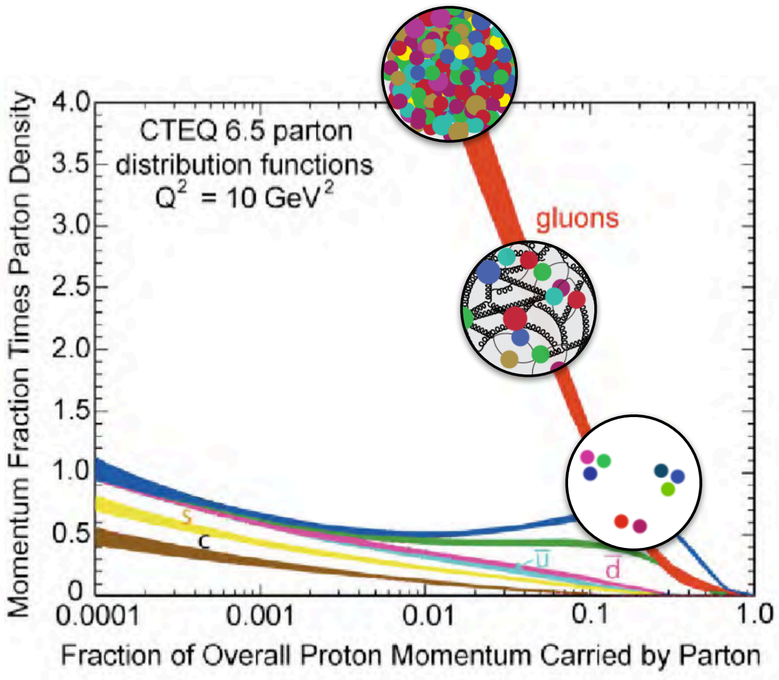
Gluon saturation at small x

- Saturation scale $q_t \sim Q_s \gg \Lambda_{\mathrm{QCD}}$



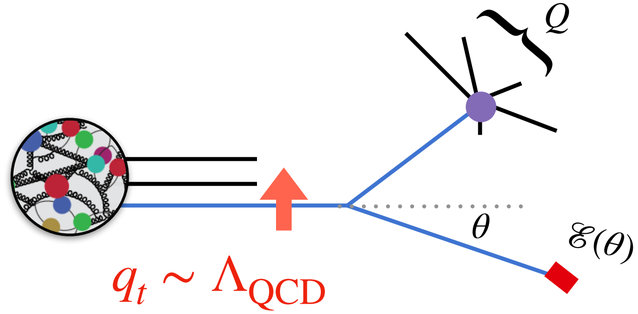


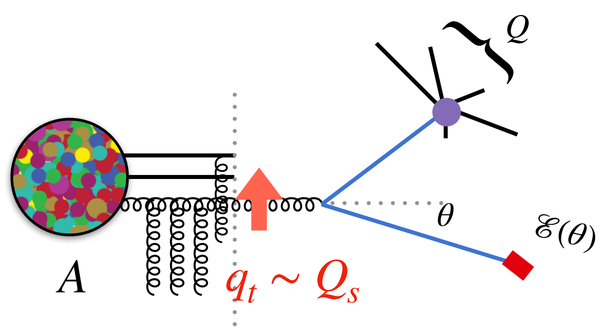
Revealing the gluon saturation

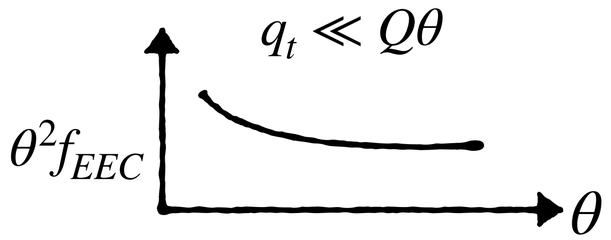


Gluon saturation at small x

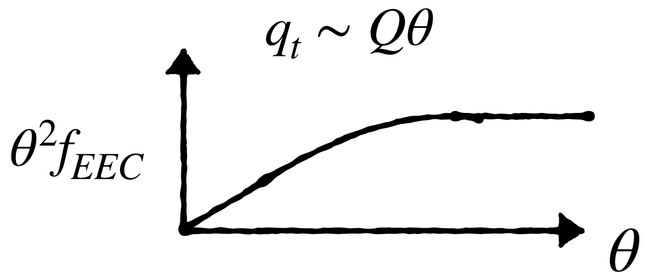
- Saturation scale $q_t \sim Q_s \gg \Lambda_{\rm QCD}$







$df_{\rm EEC}/d\ln\mu \propto P \cdot f_{\rm EEC}$

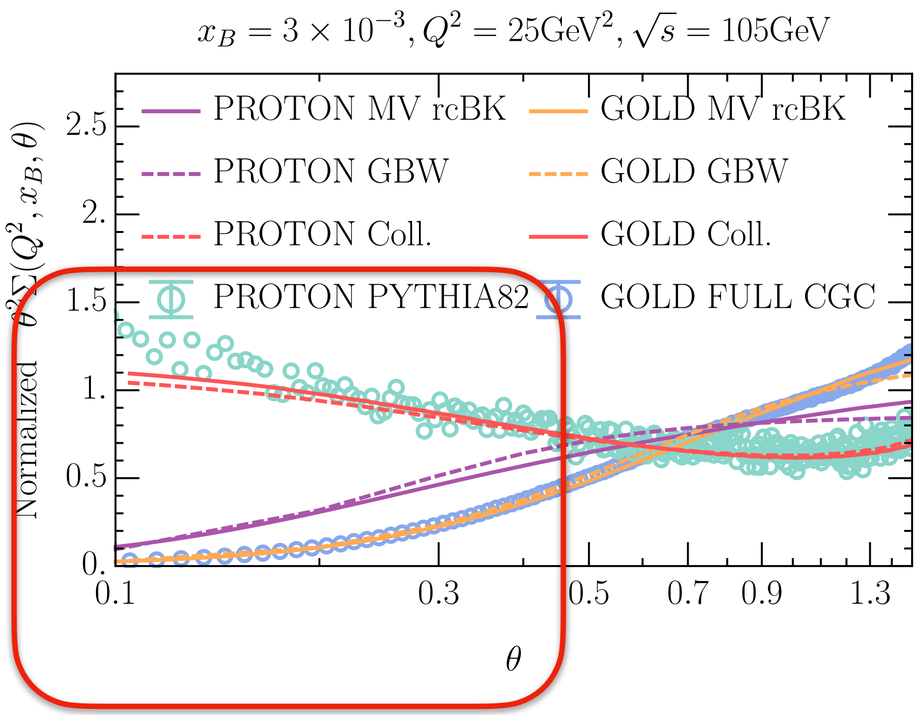


w/ large intrinsic kt

Liu, XL, Pan, Yuan, Zhu, PRL 2023 Physics

Revealing the gluon saturation

EIC



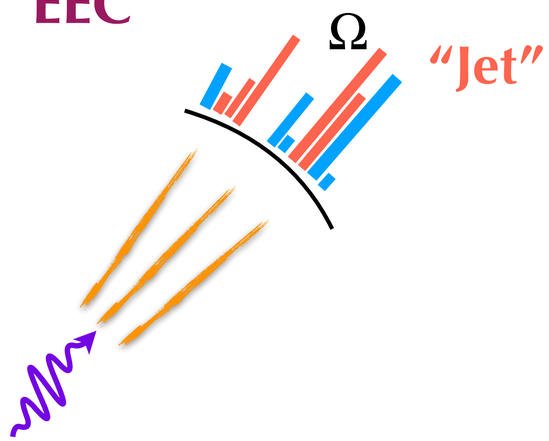
Gluon saturation at small x

- Absence of soft contamination guarantees the rising shape in the collinear factorization
- The shape is dramatically modified when gluons saturate
- NEEC as evident portal to the onset of gluon saturation

Liu, XL, Pan, Yuan, Zhu, PRL 2023

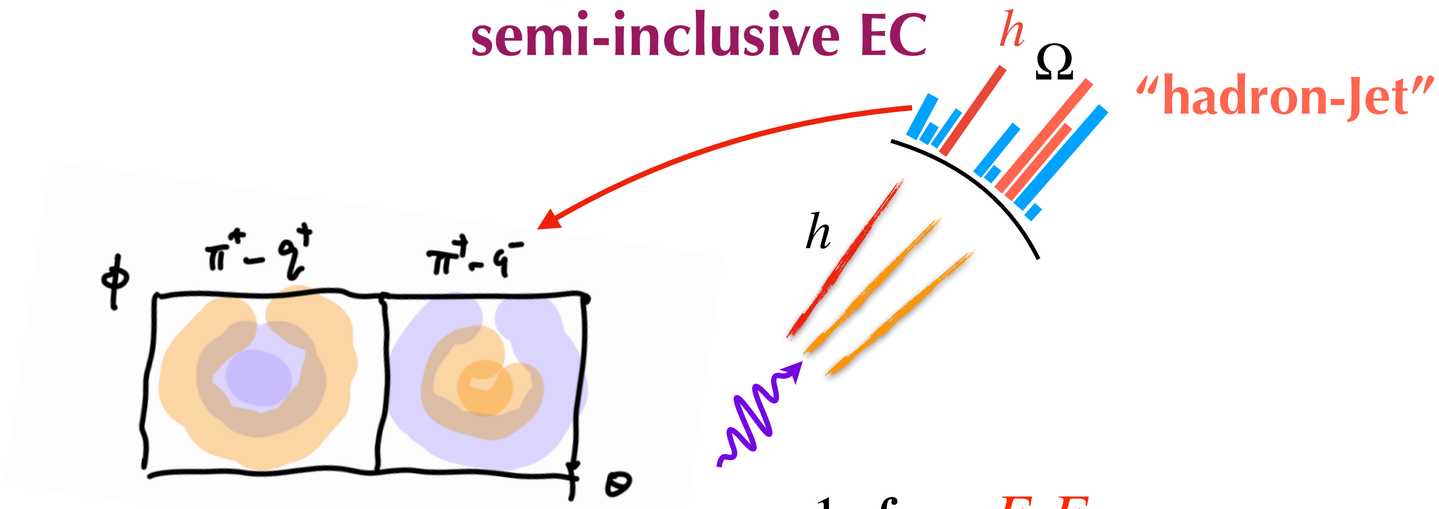
Generalization

EEC



$$\Sigma_{\rm EEC} \propto \frac{1}{\sigma}\int d\sigma \frac{E_i E_j}{Q^2}\delta(\Omega - \Omega_{ij})$$

fragmenting EC/
semi-inclusive EC



$$\Sigma_{\rm FEC} \propto \frac{1}{\sigma_h}\int d\sigma_h \frac{E_h E_i}{Q^2}\delta(\Omega - \Omega_i)$$

- $D_{EEC}(x,\theta) \propto \langle 0|\bar{\psi}(y^-)\mathscr{E}(\Omega)a_h^\dagger(P)a_h(P)\psi(0)|0\rangle$
- Provides a comprehensive picture for light hadron hadronization, Collins …
- Fit well to light hadron studies at EiC

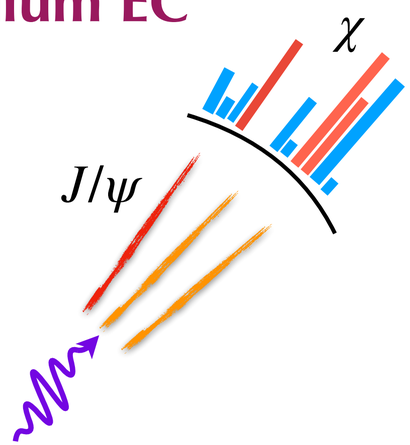
XL, Zhu, **arxiv: 2403.08874**

Generalization

Chen, XL, Ma, **2405.10056**

Quarkonium EC may provide new venue to the hadronization of a $Q\bar{Q}$ into the Quarkonium

Quarkonium EC



"quarkonim-Jet"

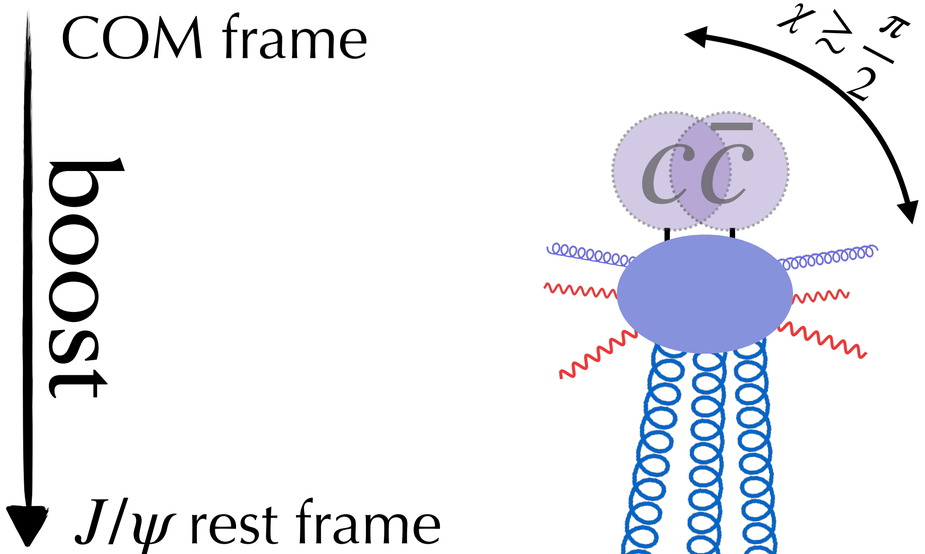
Rest frame!!

$$\Sigma_{@\mathrm{EC}} \propto \frac{1}{\sigma_{J/\psi}} \int d\sigma_{J/\psi} \frac{E_i}{M} \delta(\chi - \chi_i)$$

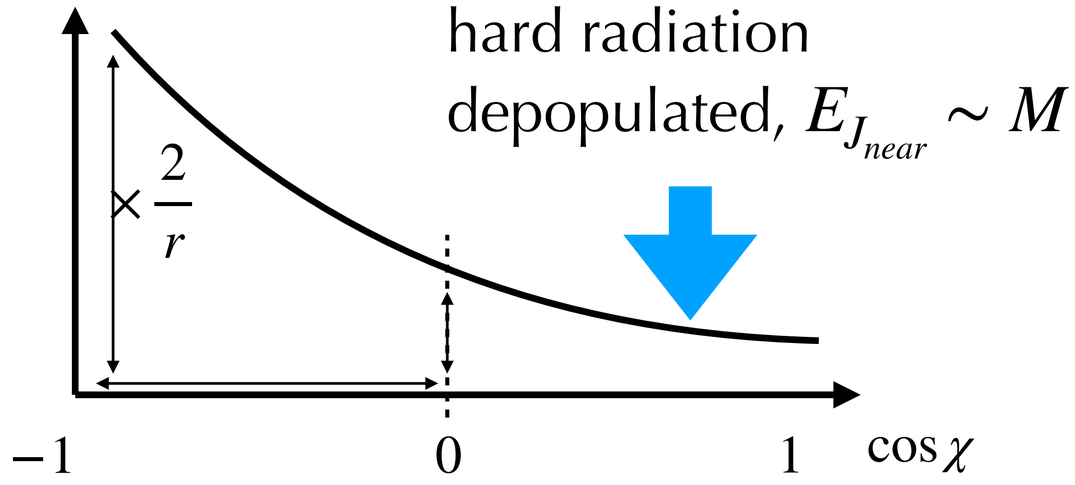
Generalization

Chen, XL, Ma, **2405.10056**

Generic J/ψ production configuration in pQCD



$$E_{J_{away}}/E_{J_{near}} \sim \frac{1}{2}\text{boost factor}^2 \sim \frac{2}{r}$$



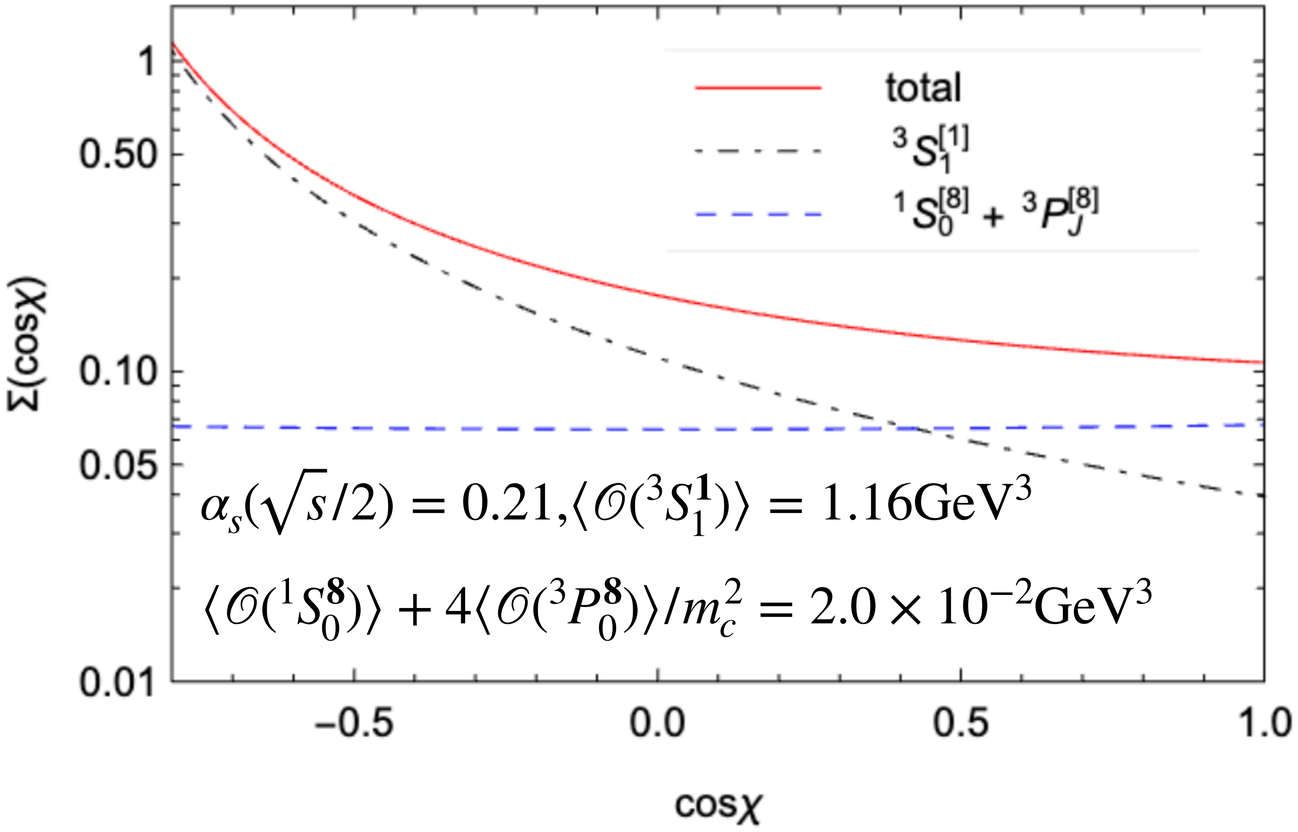
$$E_{J_{near}} \sim E_s \sim \mathcal{O}(M), E_{J_{away}} \sim \mathcal{O}\left(\frac{\hat{s}}{2M}\right)$$

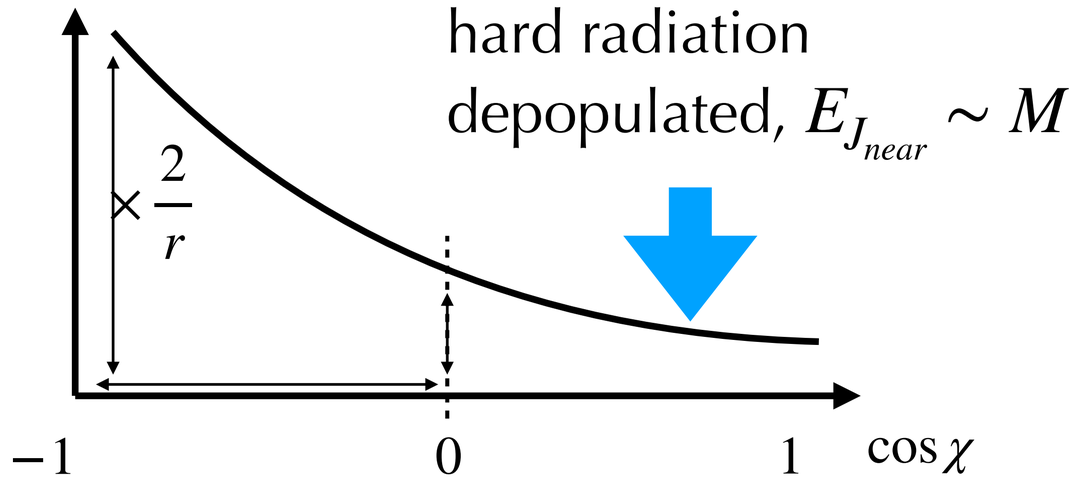
Generalization

Chen, XL, Ma, **2405.10056**

Sizable hadronization effect!!

$$e^+e^- \to J/\psi + X$$



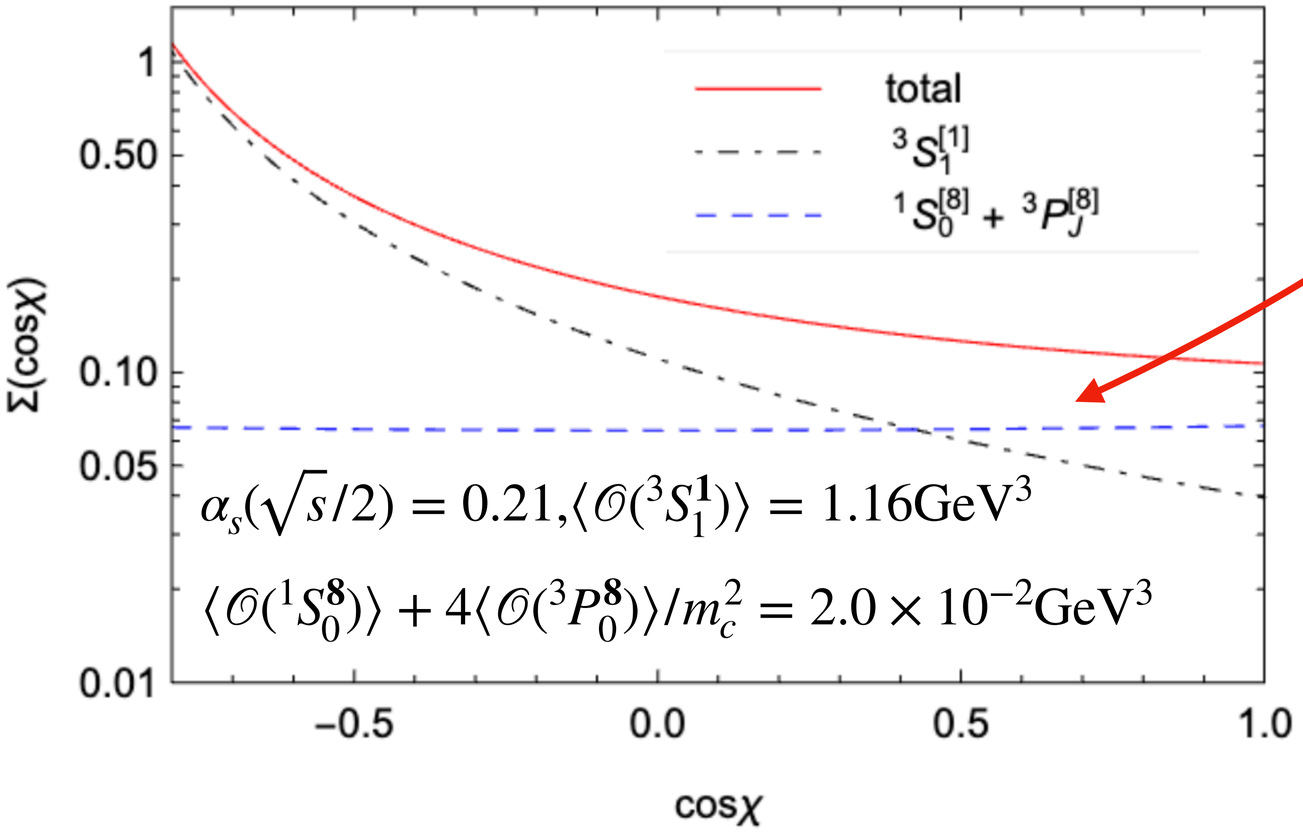


Generalization

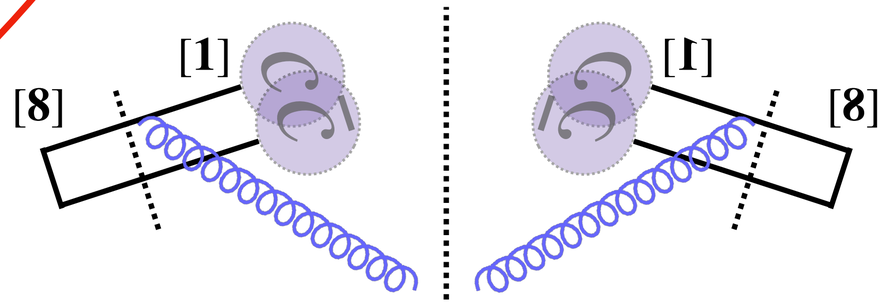
Chen, XL, Ma, **2405.10056**

Sizable hadronization effect!!

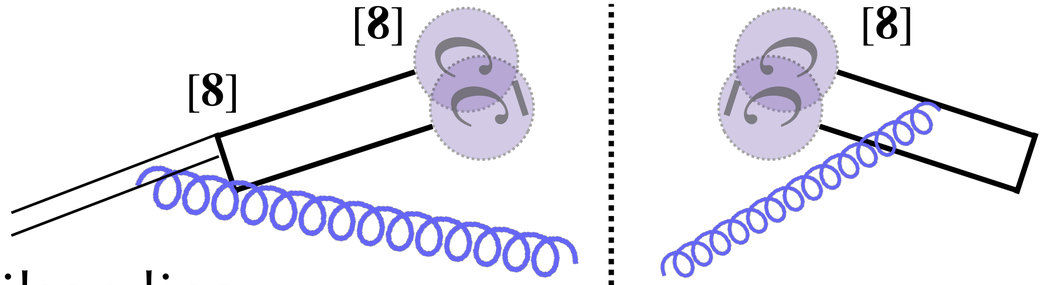
$e^+e^- \to J/\psi + X$



NRQCD: rotational covariant



interference, boost covariant

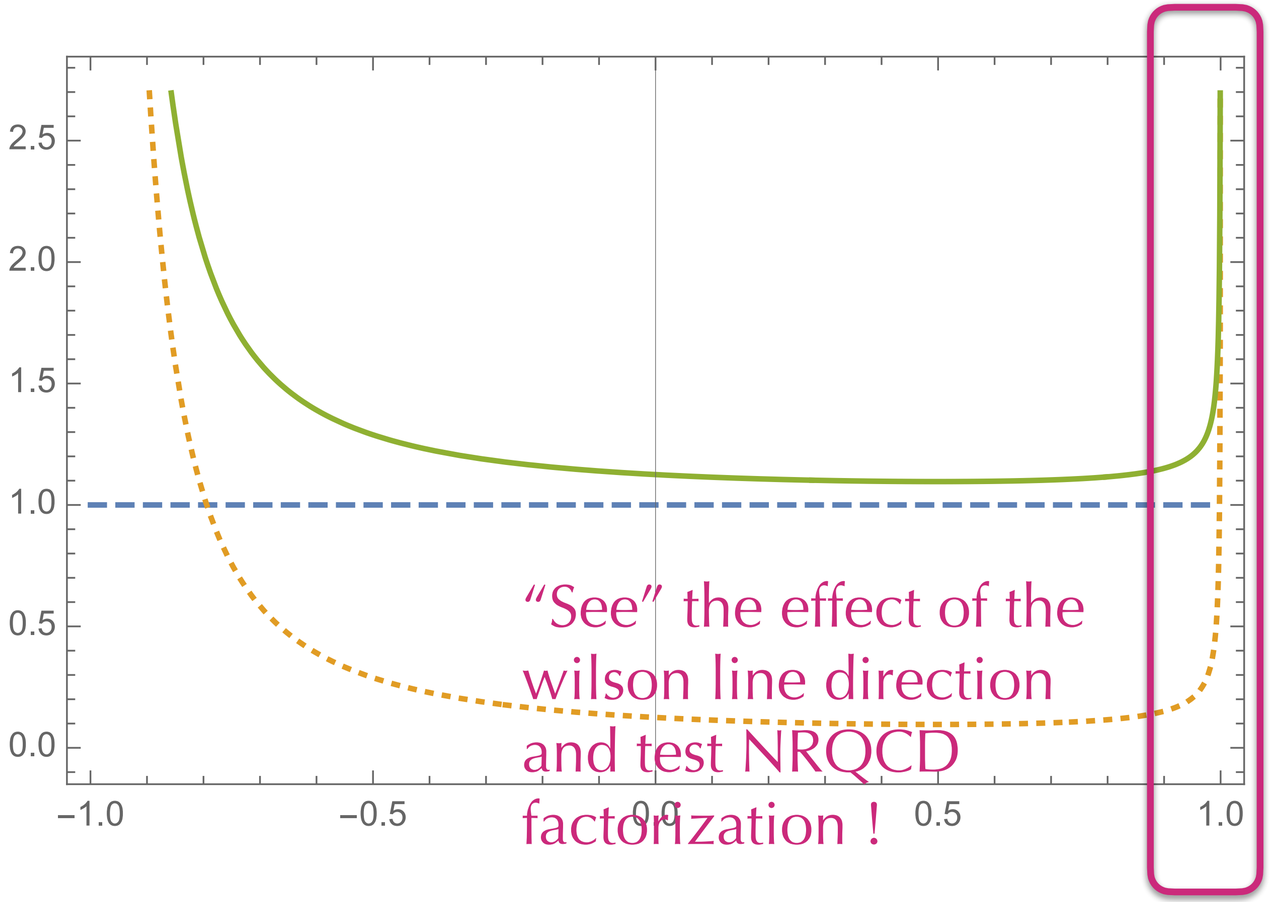


Wilson line

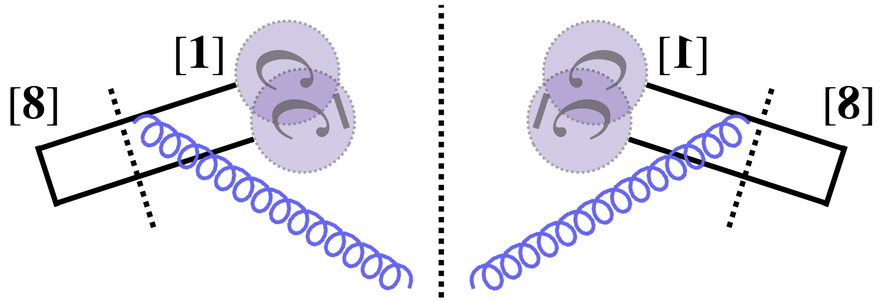
Generalization

Chen, XL, Ma, **2405.10056**

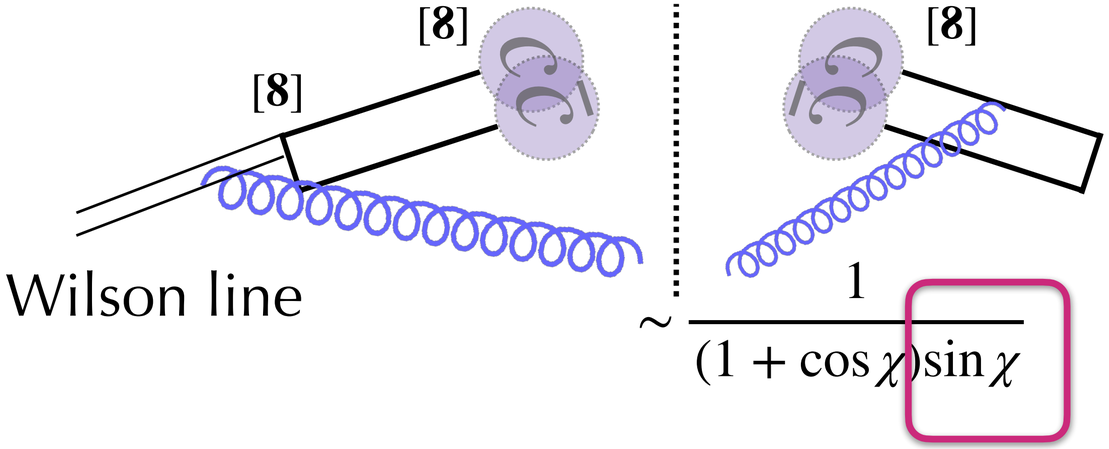
Relative size between non-inter vs interference



NRQCD: rotational covariant



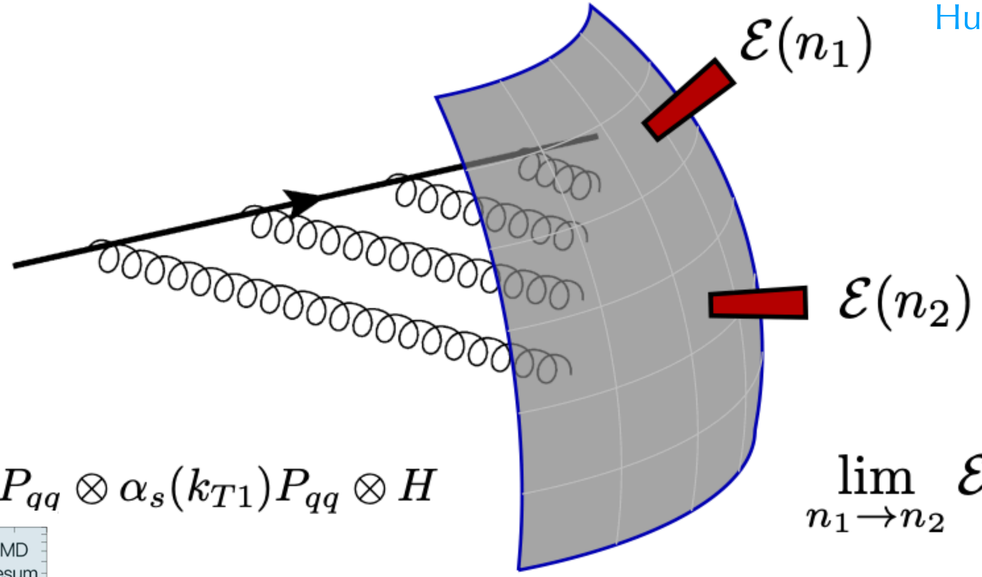
interference, boost covariant



New insights???

Dual view on energy correlators

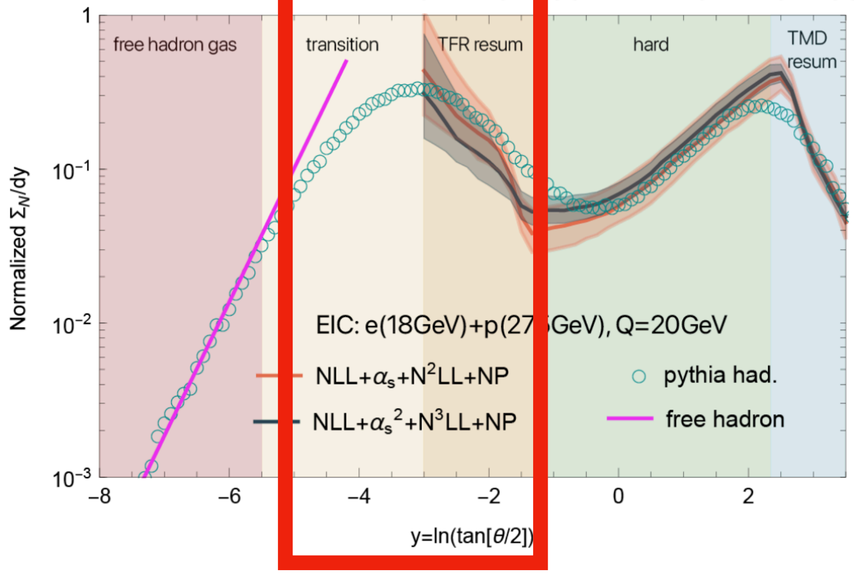
Hua Xing's talk



the scaling interpretation makes the story appealing to broader community

$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_i \frac{1}{\theta^{1-\gamma_i}} \mathbb{O}_i^{J=3}$$

$$\cdots \alpha_s(k_{T3}) P_{qq} \otimes \alpha_s(k_{T2}) P_{qq} \otimes \alpha_s(k_{T1}) P_{qq} \otimes H$$



DGLAP evolution	Light-ray OPE
Time-like anomalous dimension	Space-like anomalous dimension
Running coupling	Smearing in spin
Incorporating track	???
Quantum interference in parton shower	**Evolution for non-diagor**
nuclear structure	???
[illegible]	???
massive quark	???
???	**P.T. and N.P. Power corrections**

New perspective to the NP hadron structures

?

Conclusions

- Adapt EEC to the proton structures physics
- A new non-pert. structure nucleon energy corrector is introduced
- Have already seen interesting applications
- Theoretical implication on hadron structures?

Thanks