Hamiltonian Truncation and Effective Field Theory

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University of Geneva

based on work with Tim Cohen, Rachel Houtz, Markus Luty and Dorian Wenzel
arXiv: 2110.08273 and work in progress

Landscape of quantum field theory

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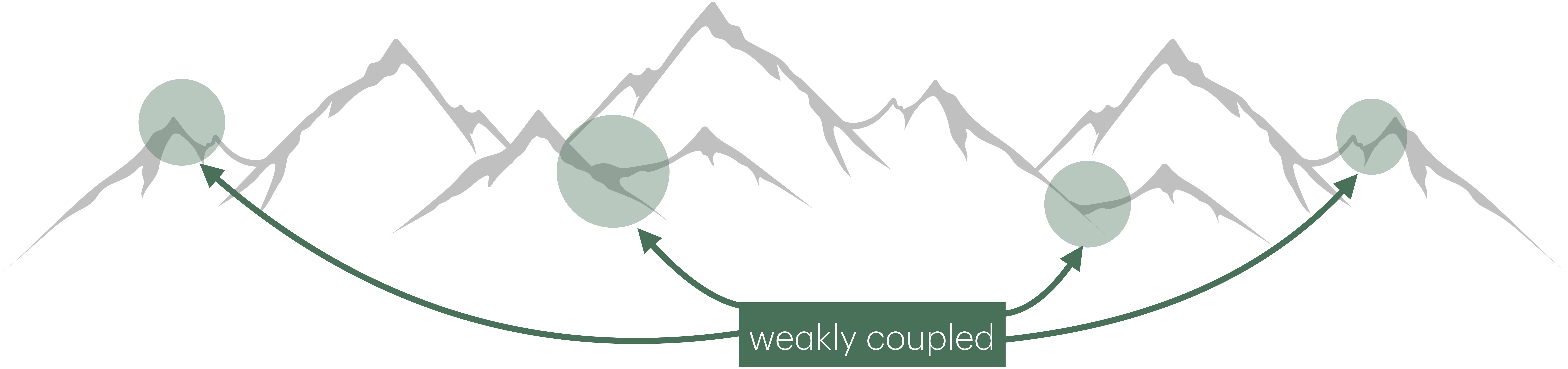

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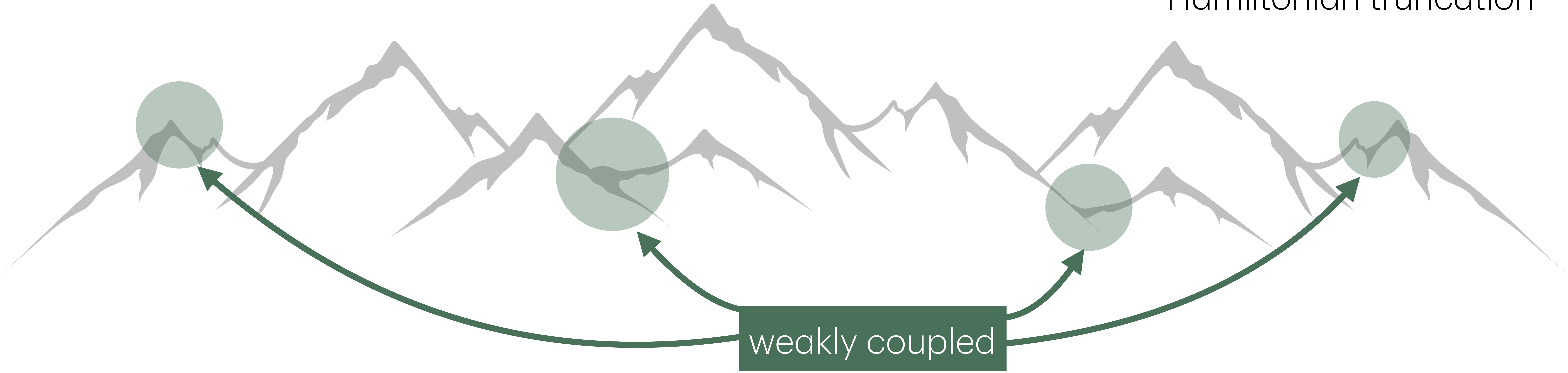
- need **new equipment** to explore whole landscape

perturbation theory

Lattice methods

Conformal bootstrap

Hamiltonian truncation



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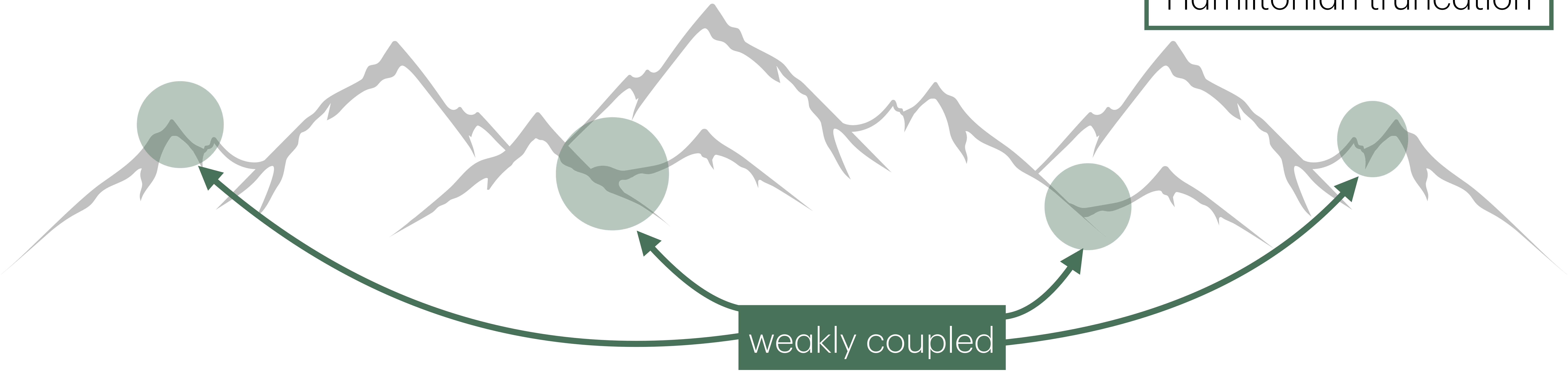

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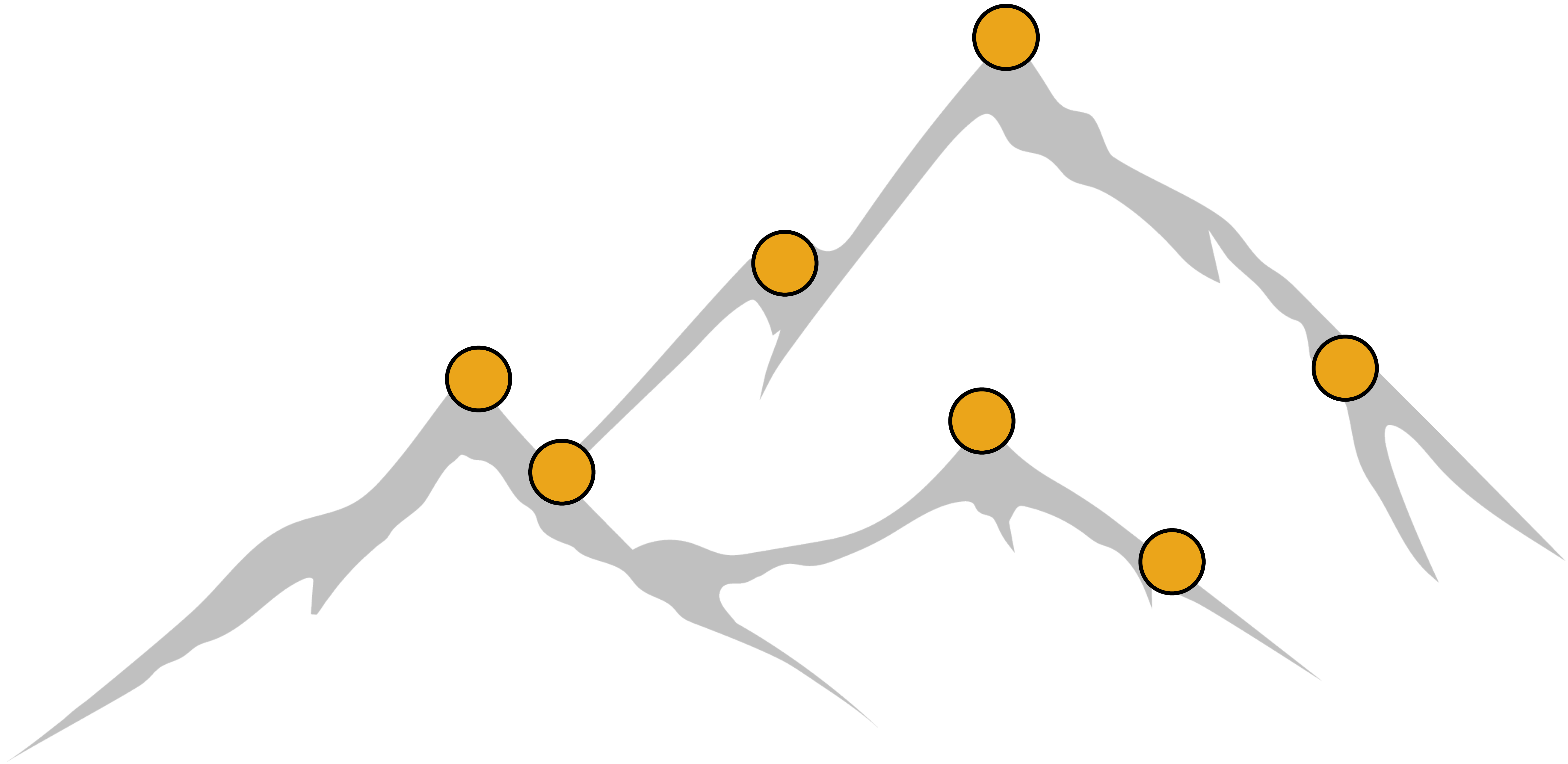

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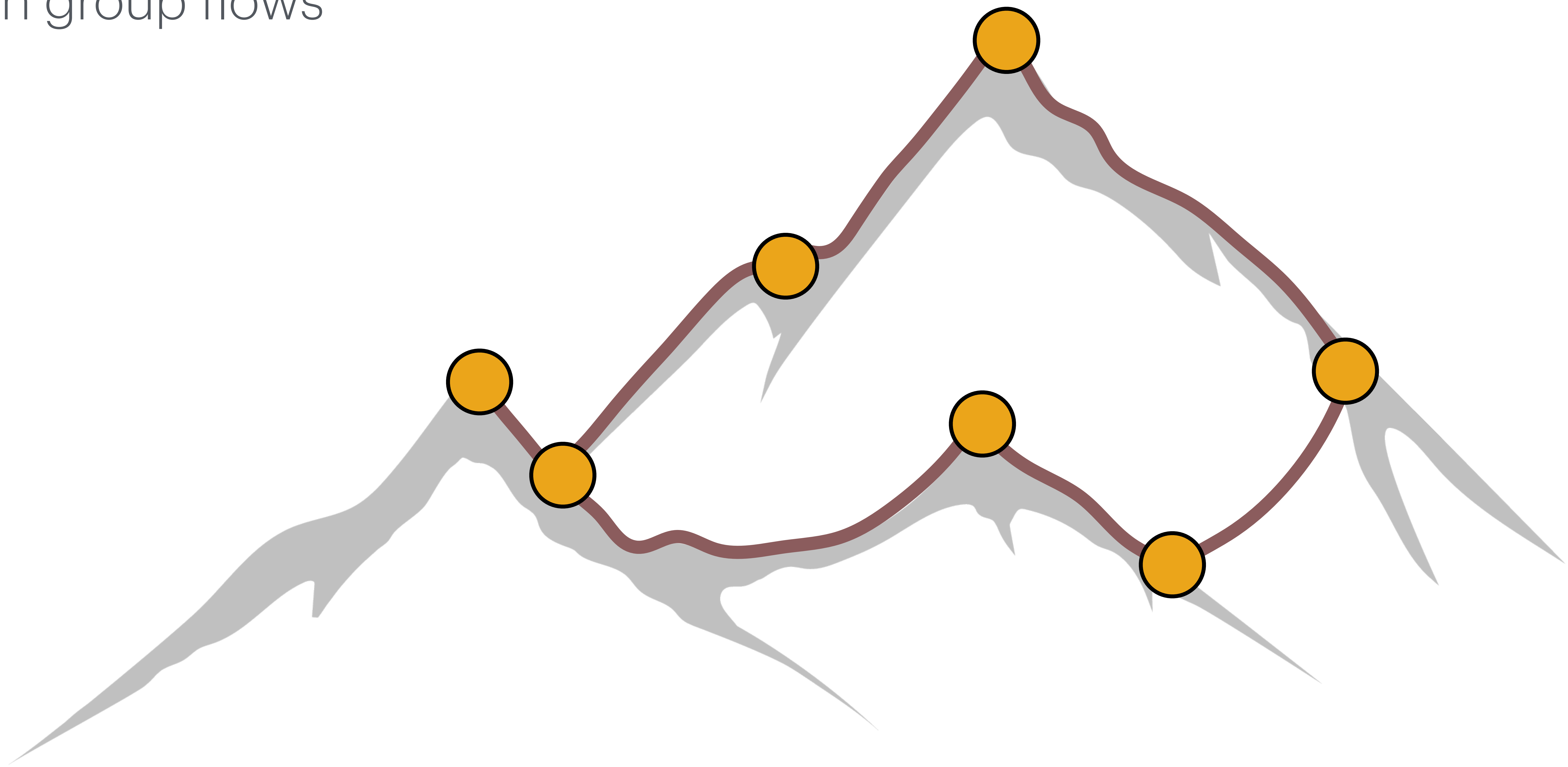

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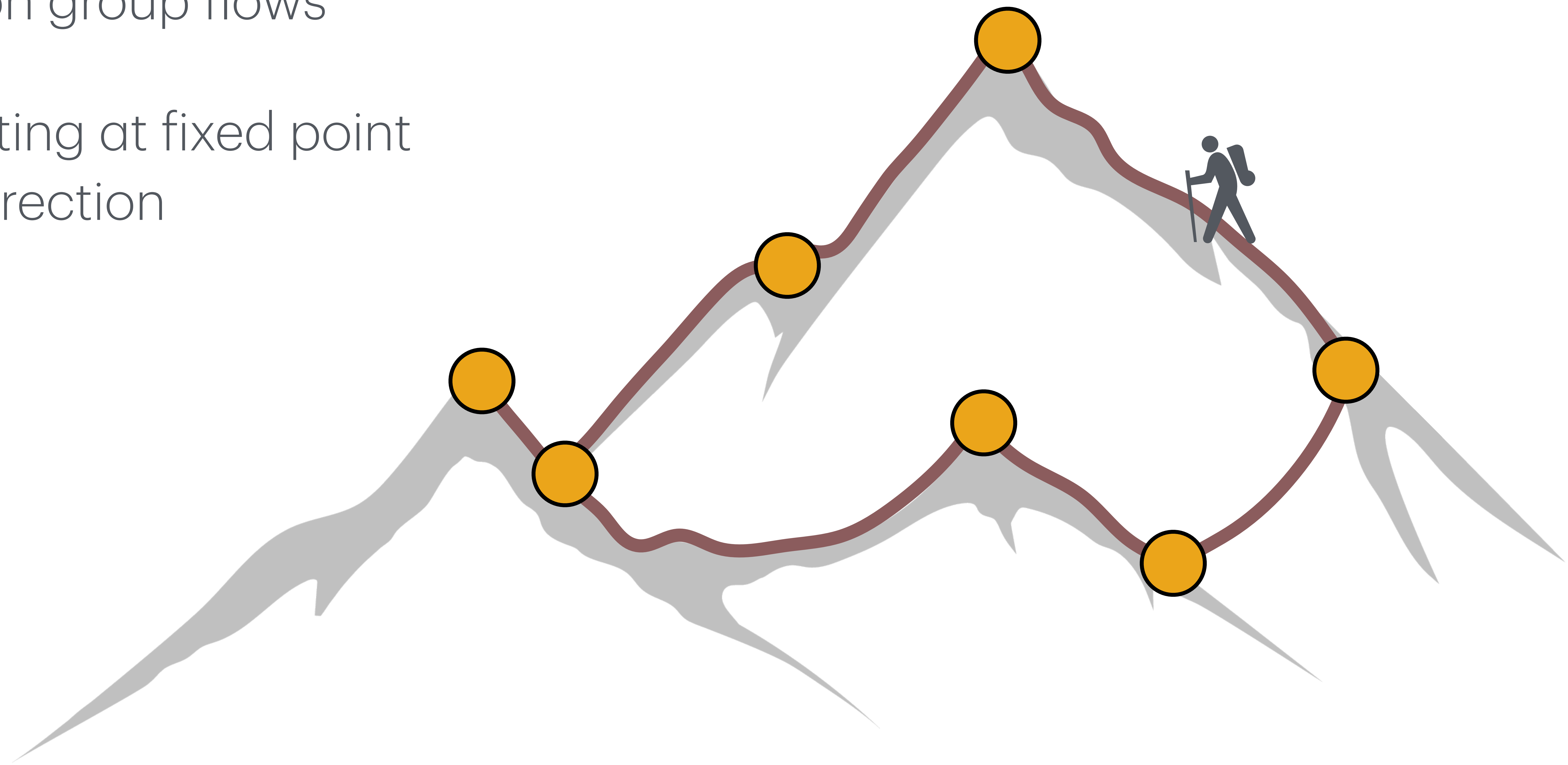

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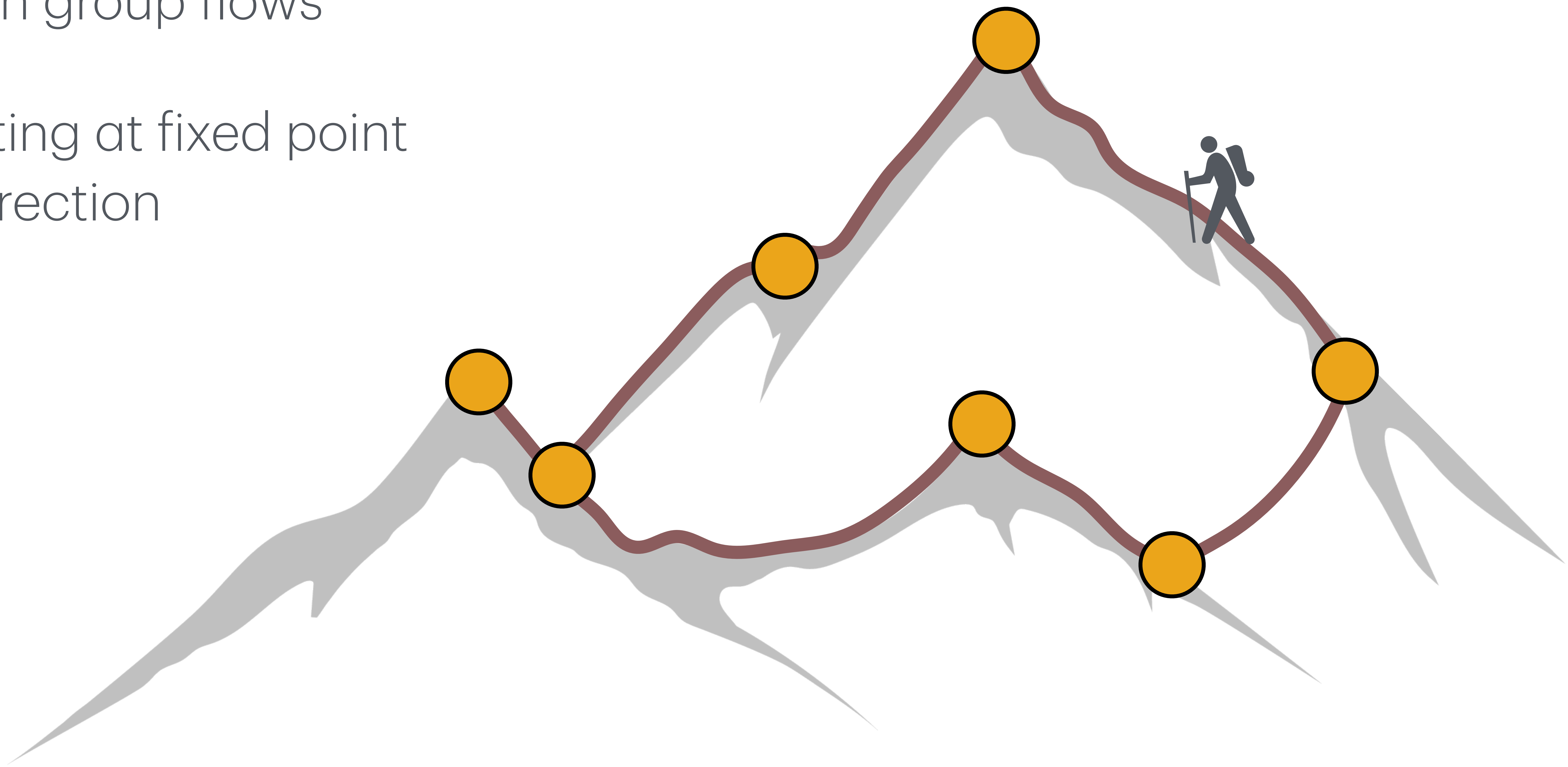
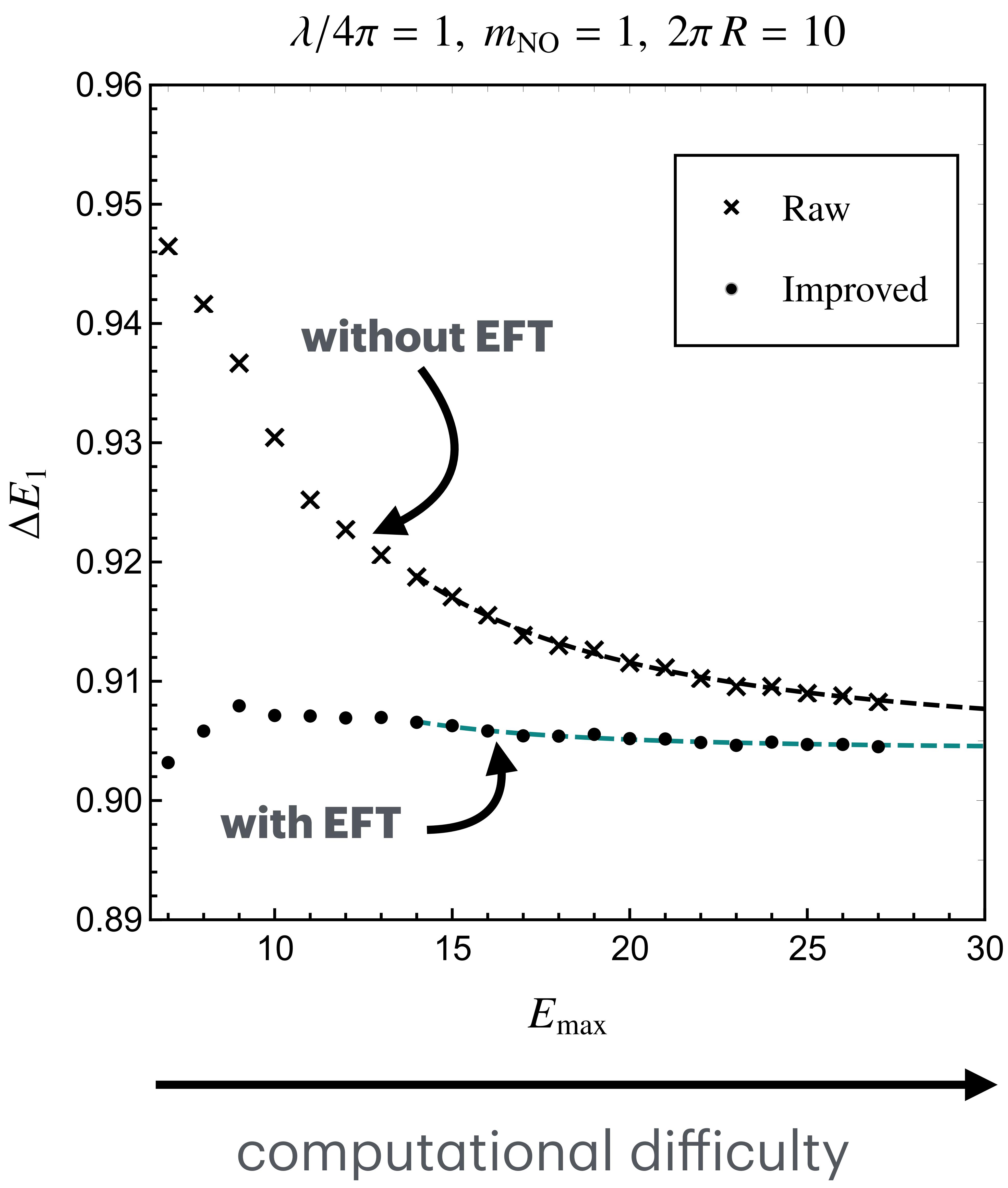
- special points in landscape = fixed points
- connected by renormalization group flows
- can think of **any** QFT as starting at fixed point and flowing in a particular direction
- captures intuition
  - universality
  - relevant vs. irrelevant



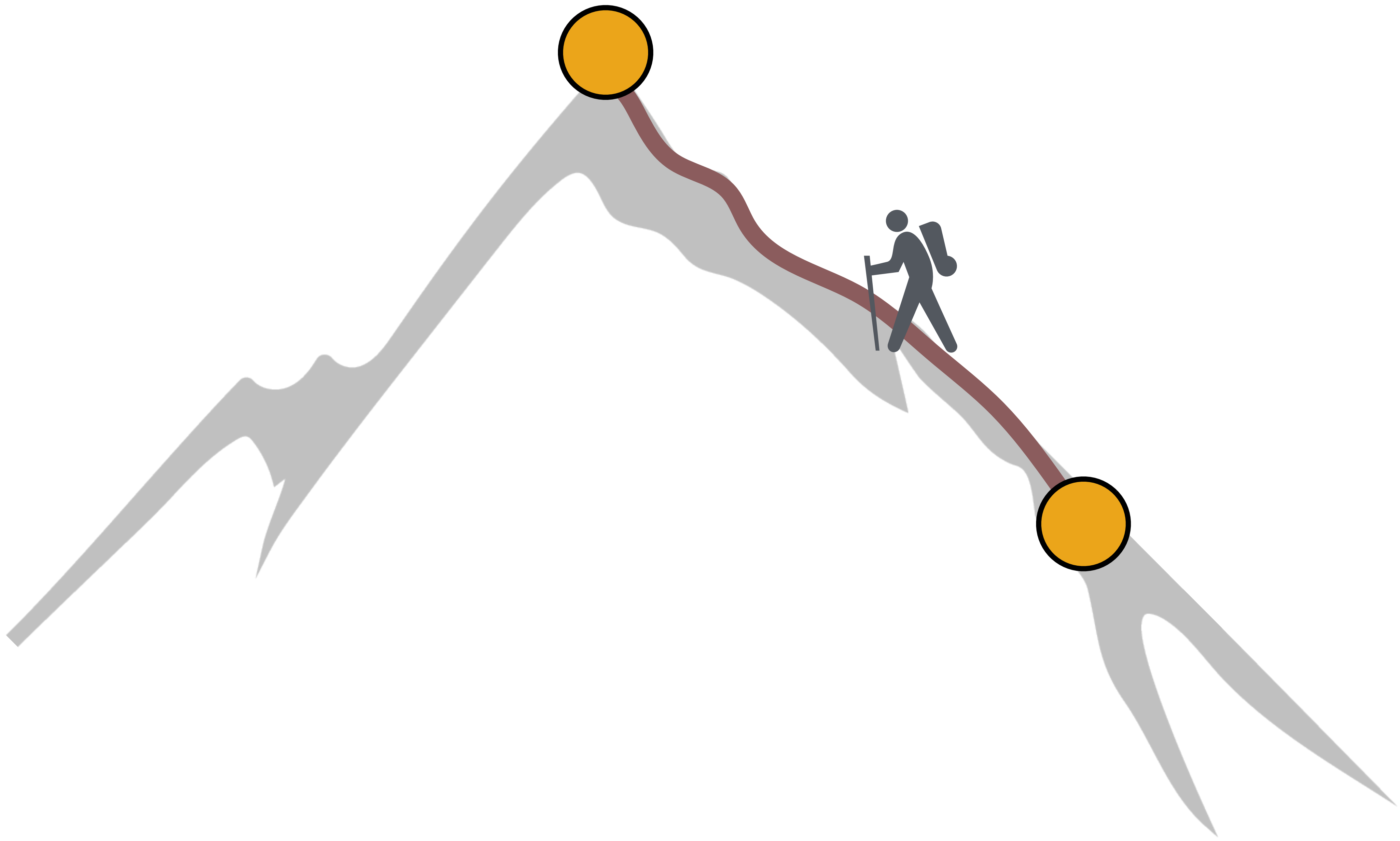
Hamiltonian truncation solidifies this conceptual picture

Punchline

- Hamiltonian truncation = non-perturbative method for computing observables in strongly coupled QFTs
- effective field theory = powerful tool for compensating for ignorance
- effective field theory techniques **drastically improve** convergence in Hamiltonian truncation calculations



Hamiltonian truncation



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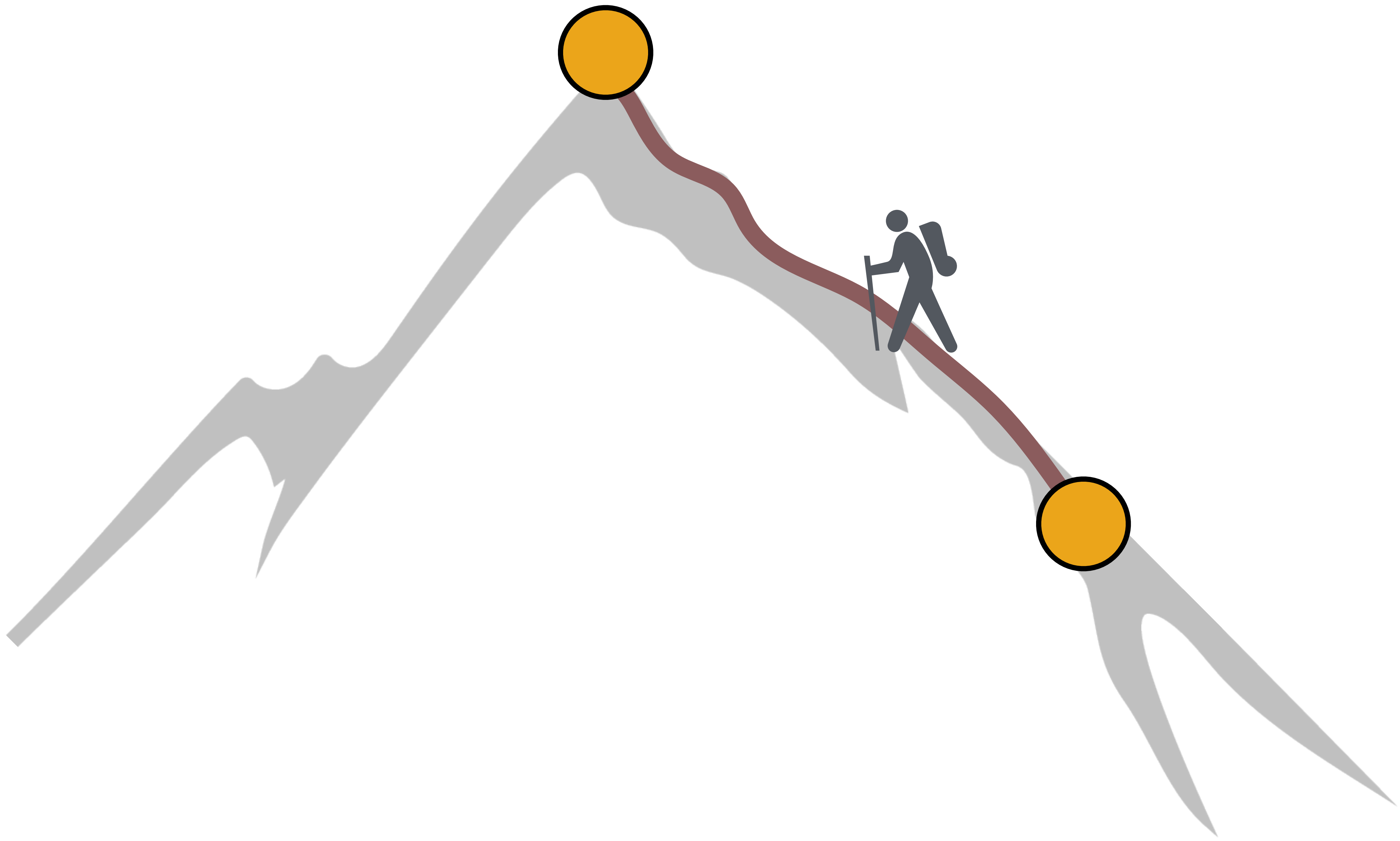

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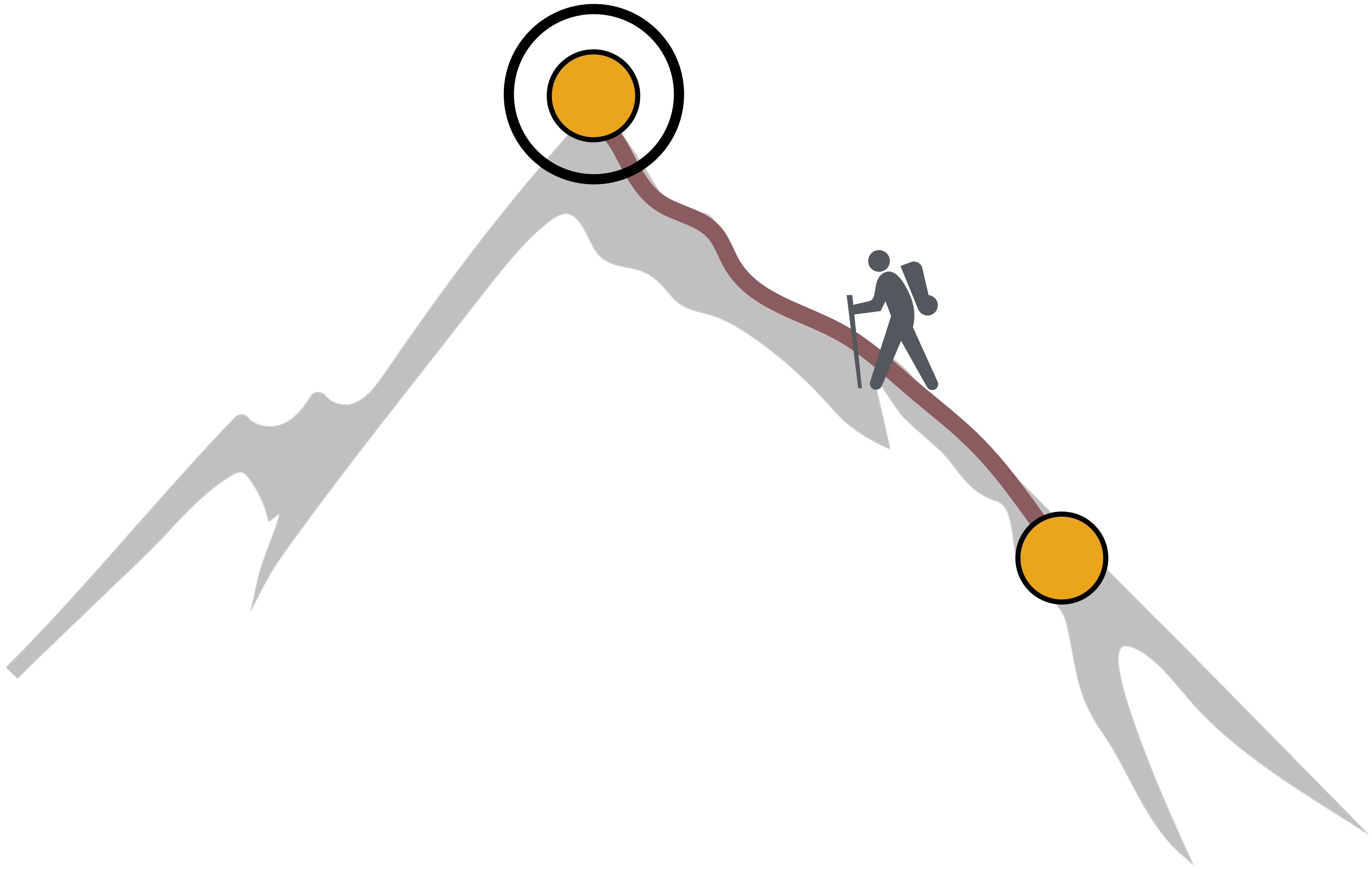

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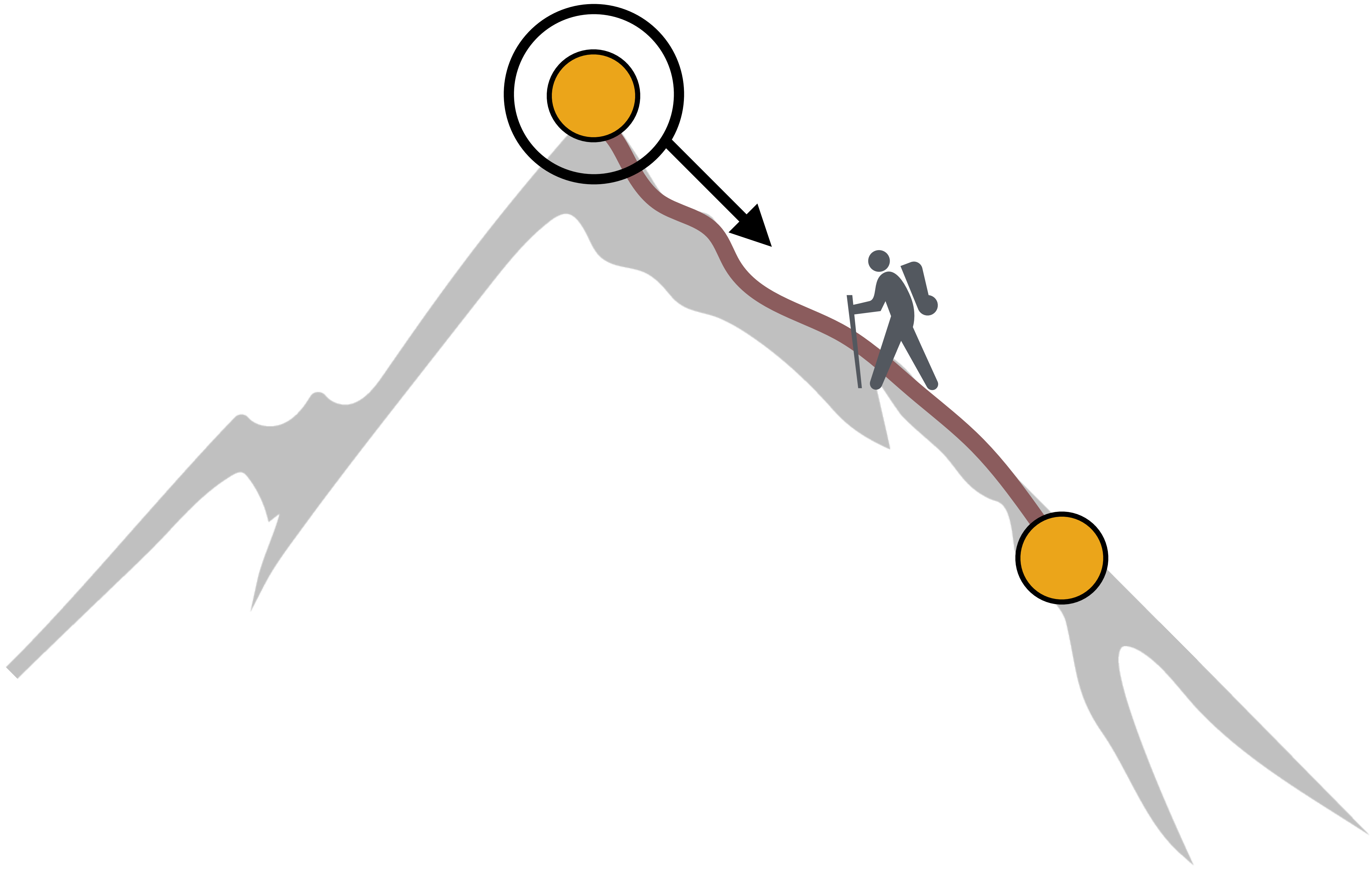

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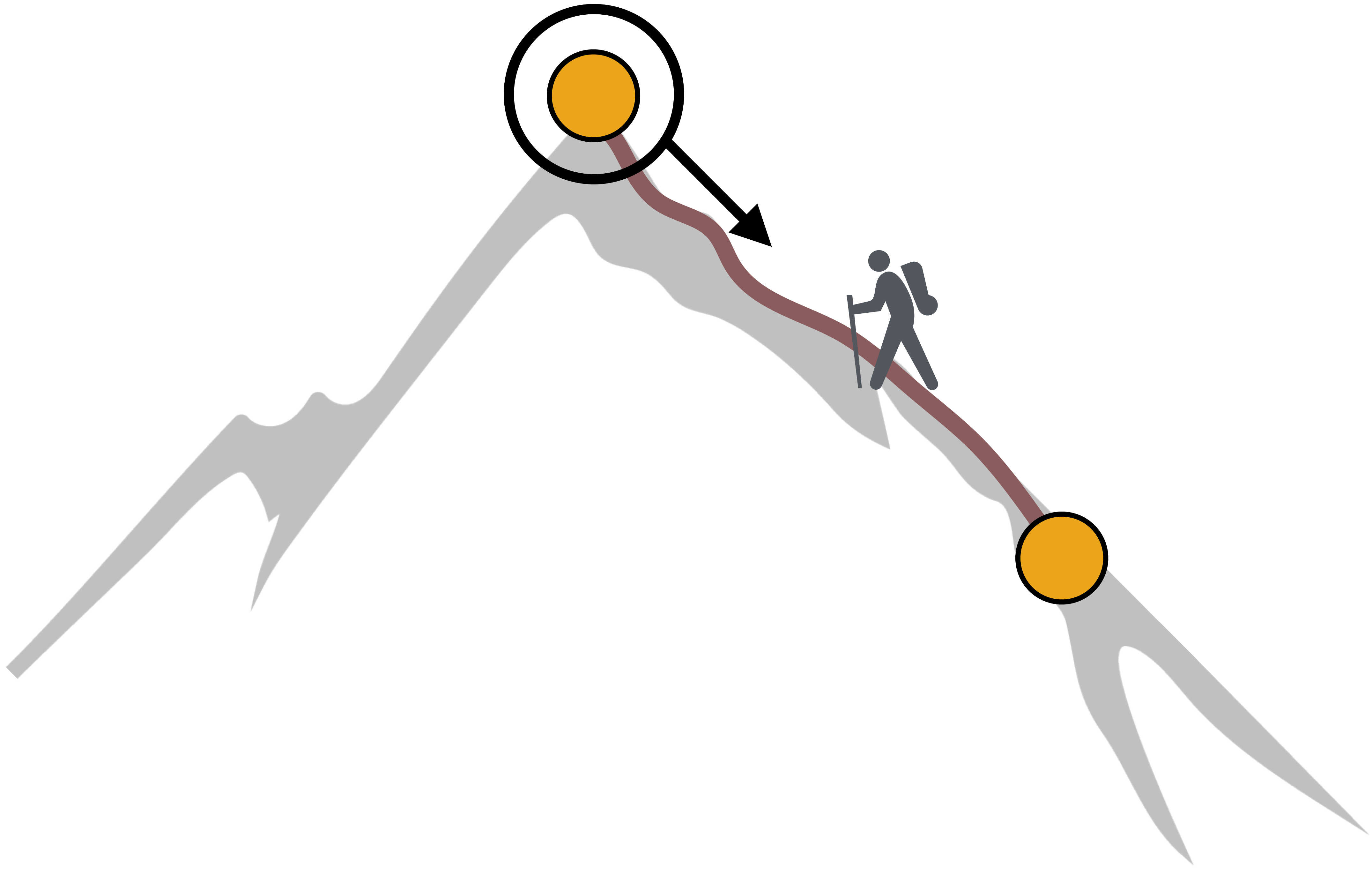

can't diagonalize full H ➡ truncate!

Hamiltonian truncation in practice

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- lots of approaches: DLCQ (Pauli et al ’85), TCSA (Yurov, ’90), Massive Fock Space (Brooks et al ’84, Rychkov et al ’14), LCT (Katz et al ’16), RCMPS (Tilloy ’21), ...

Hamiltonian truncation in practice

- discretize → finite volume
- truncate → separate Hamiltonian

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interaction (small in UV, big in IR) → V

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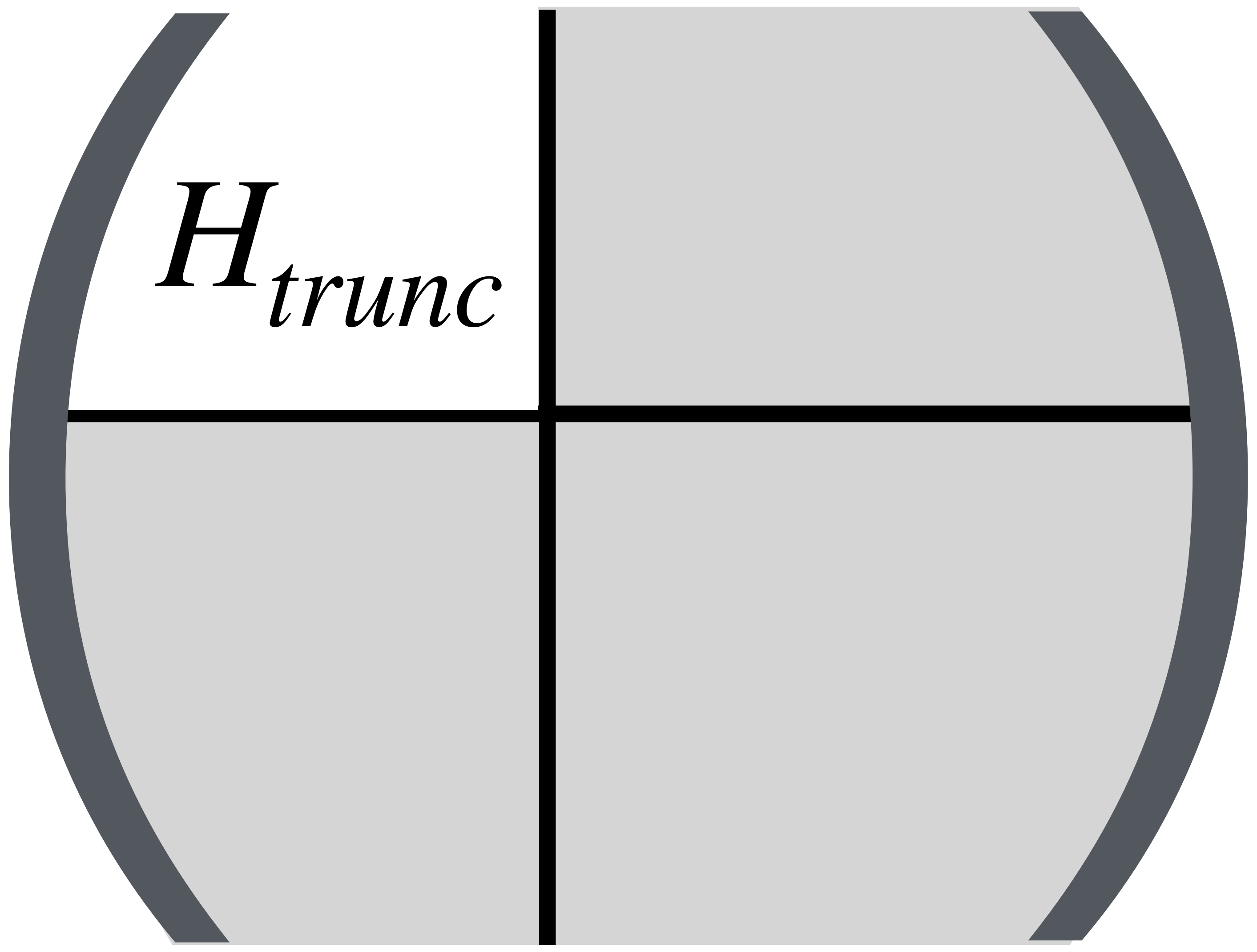

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- diagonalize → spectrum (finite volume)

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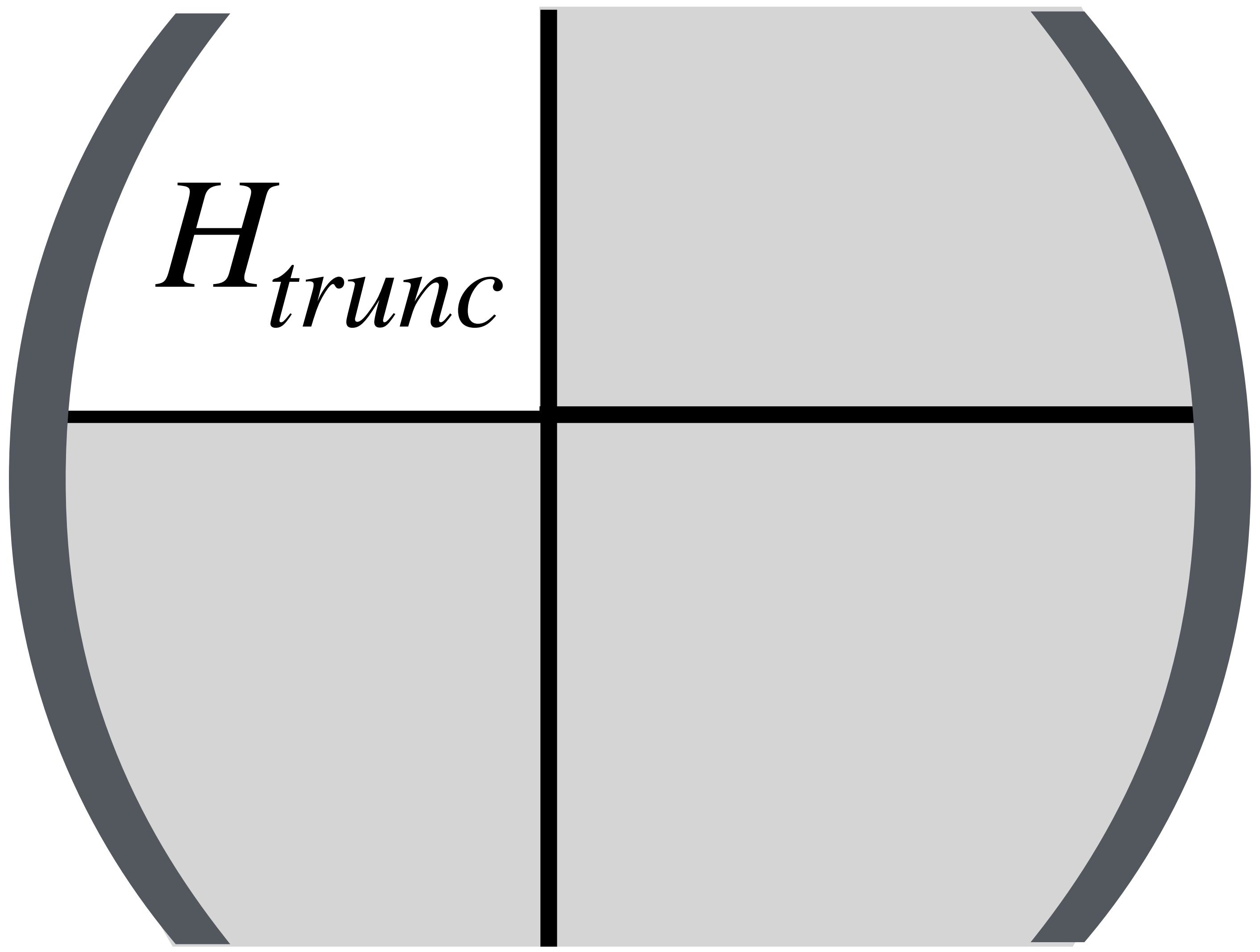

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- don't **need** extra symmetry (conformal, supersymmetry, etc.)
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- direct access to **dynamics** $(i\partial_t \Psi = H\Psi)$

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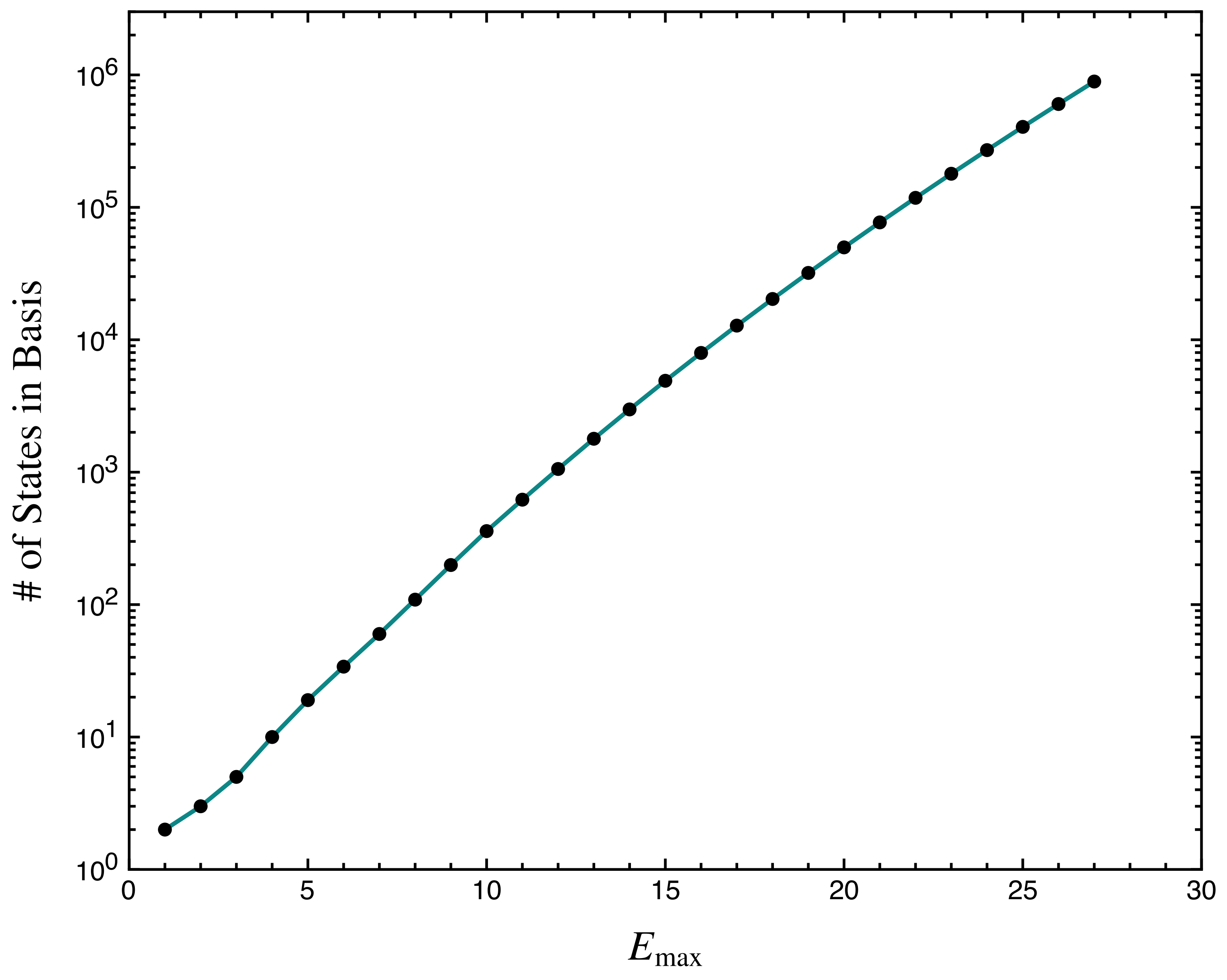

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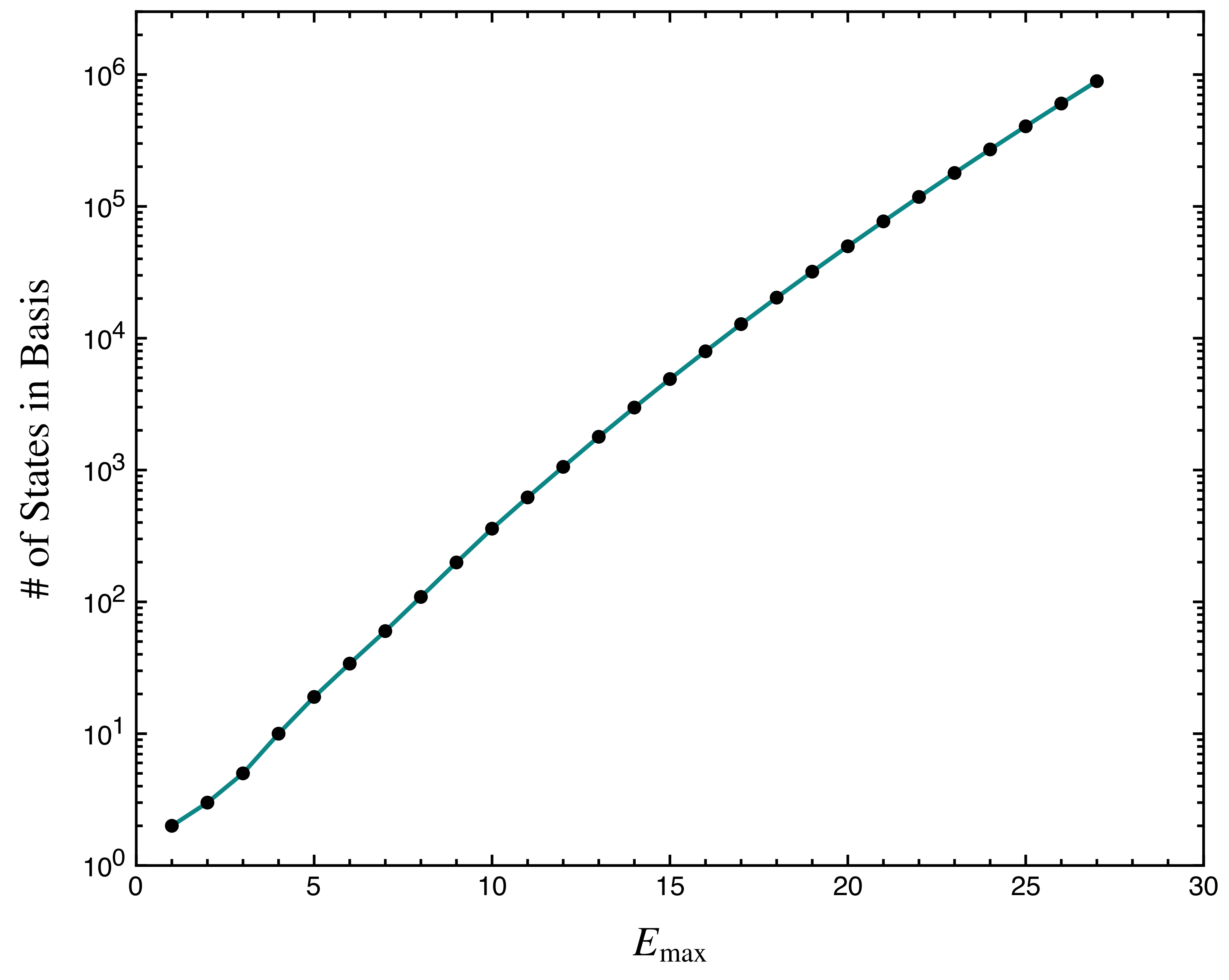
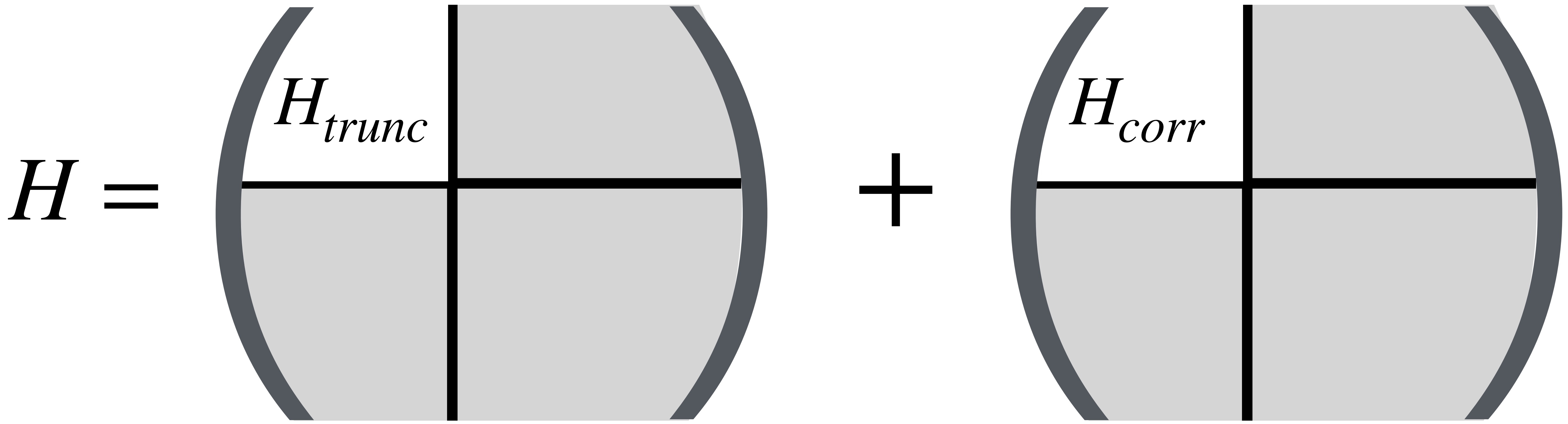
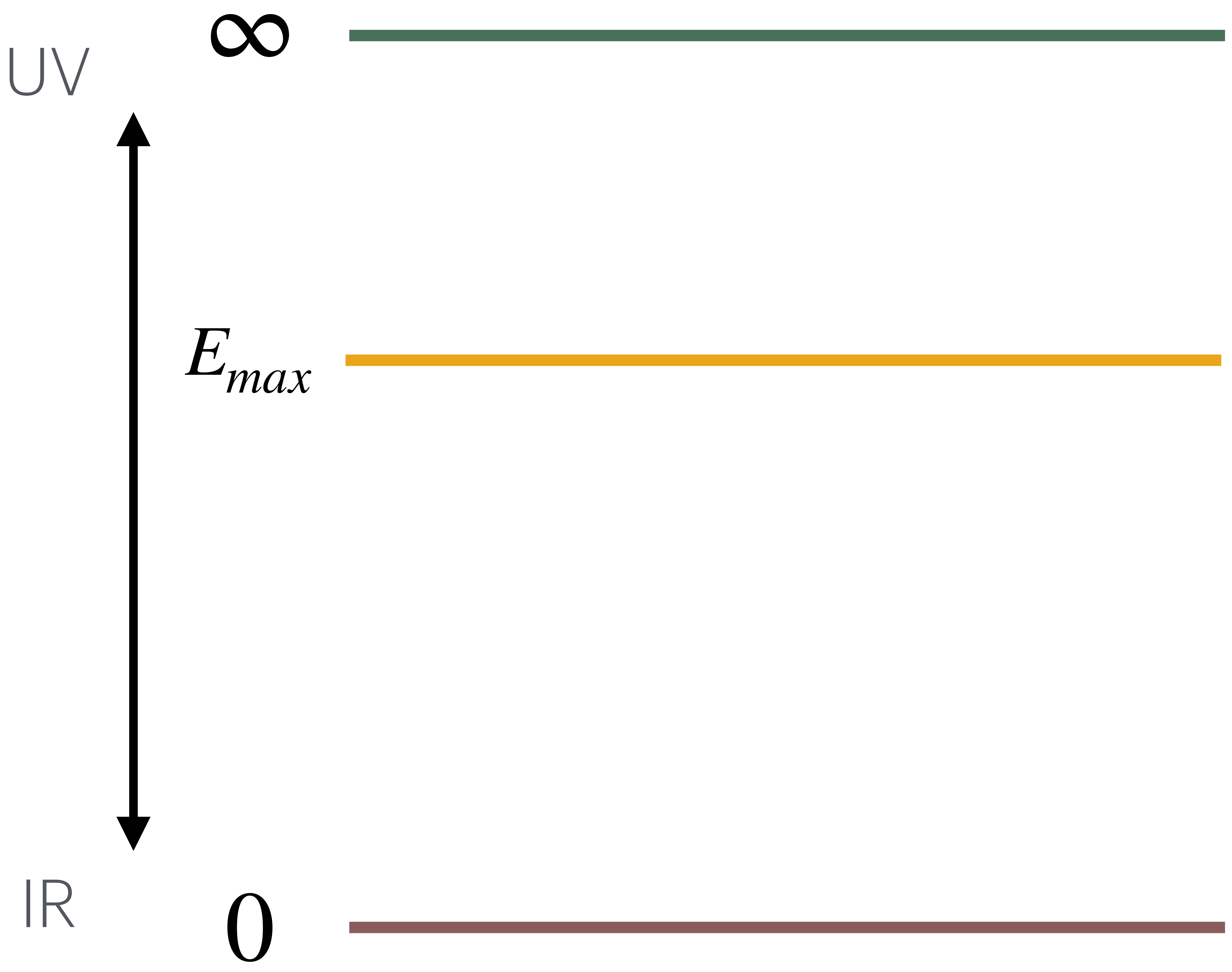


- Hilbert space grows exponentially with E_{max}
- add corrections to account for effects from states outside truncated Hilbert space ("integrate out")



- similar approaches: Feverati et al '06, Hogervorst et al '14, Elias-Miro et al '17, ...

Effective field theory for truncation



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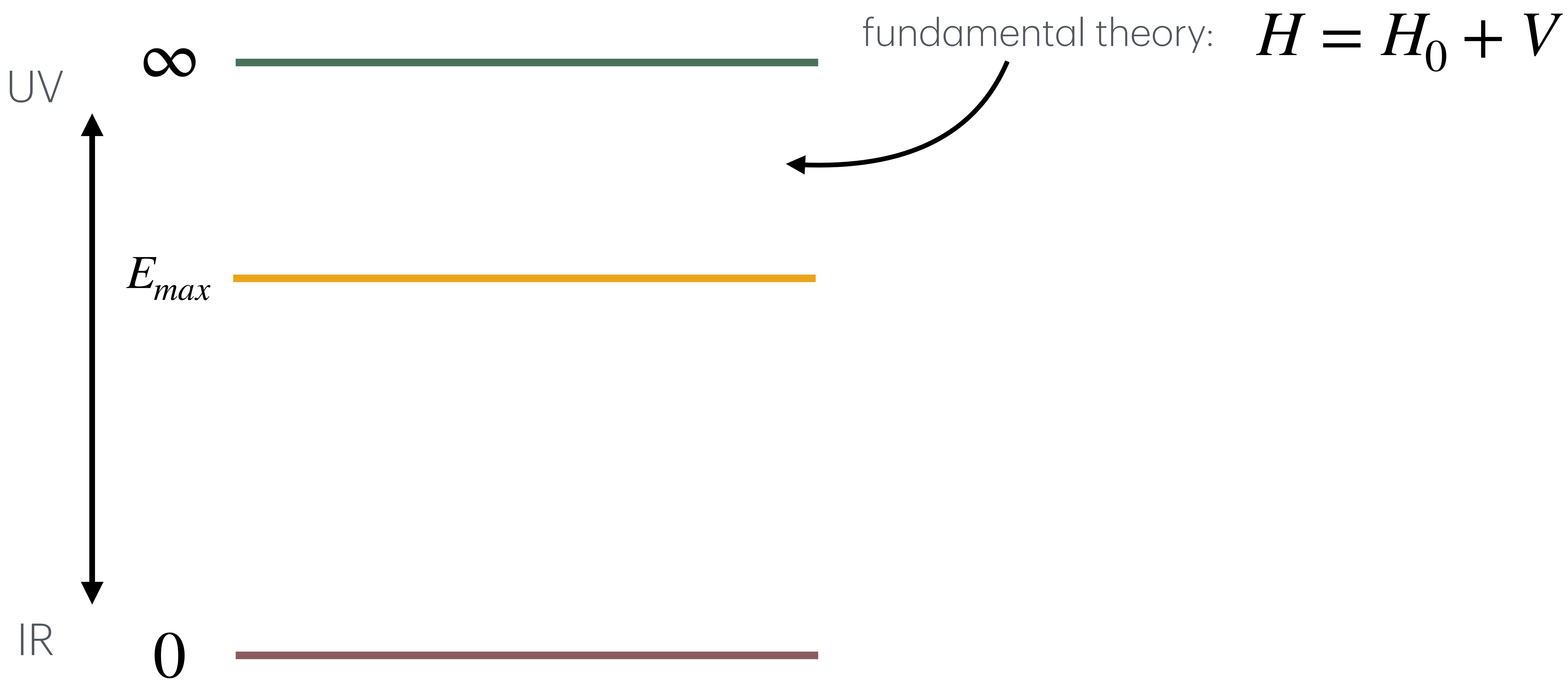

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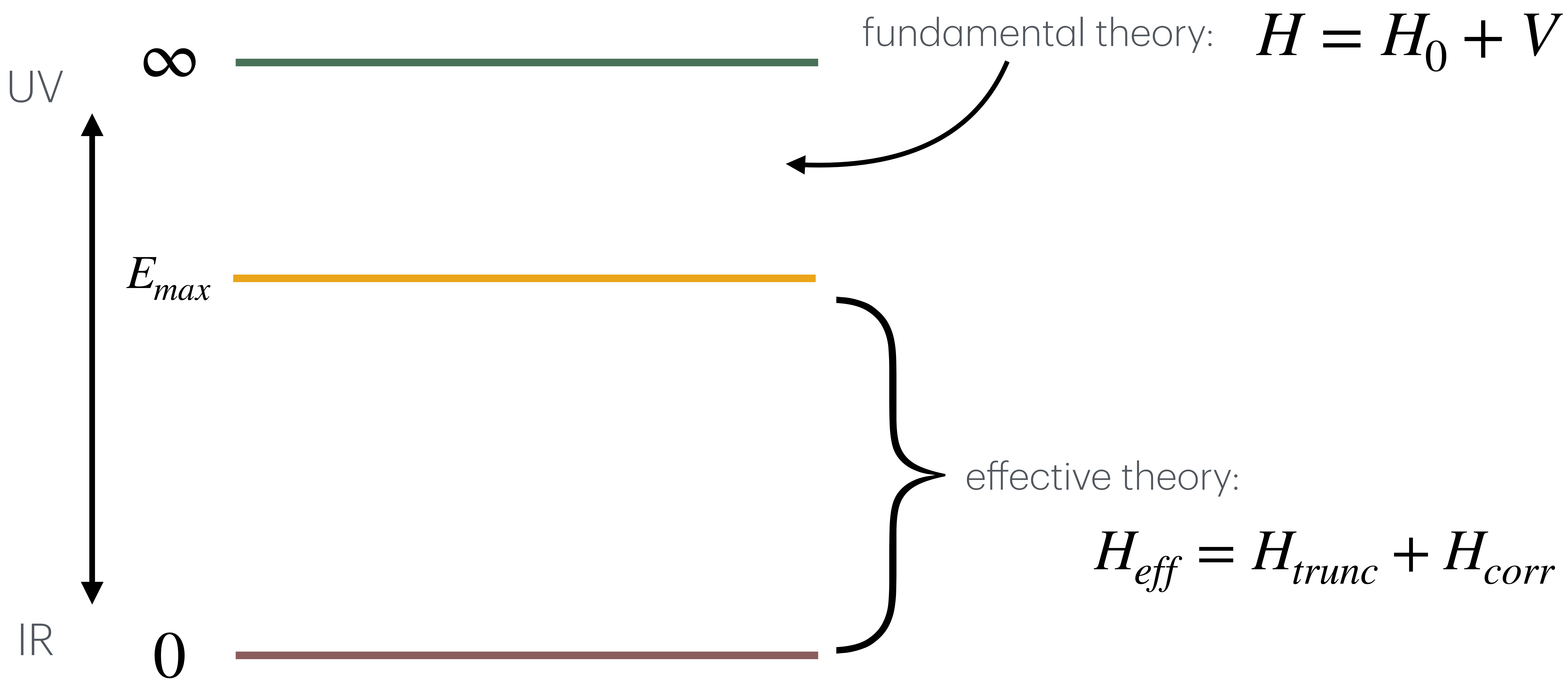

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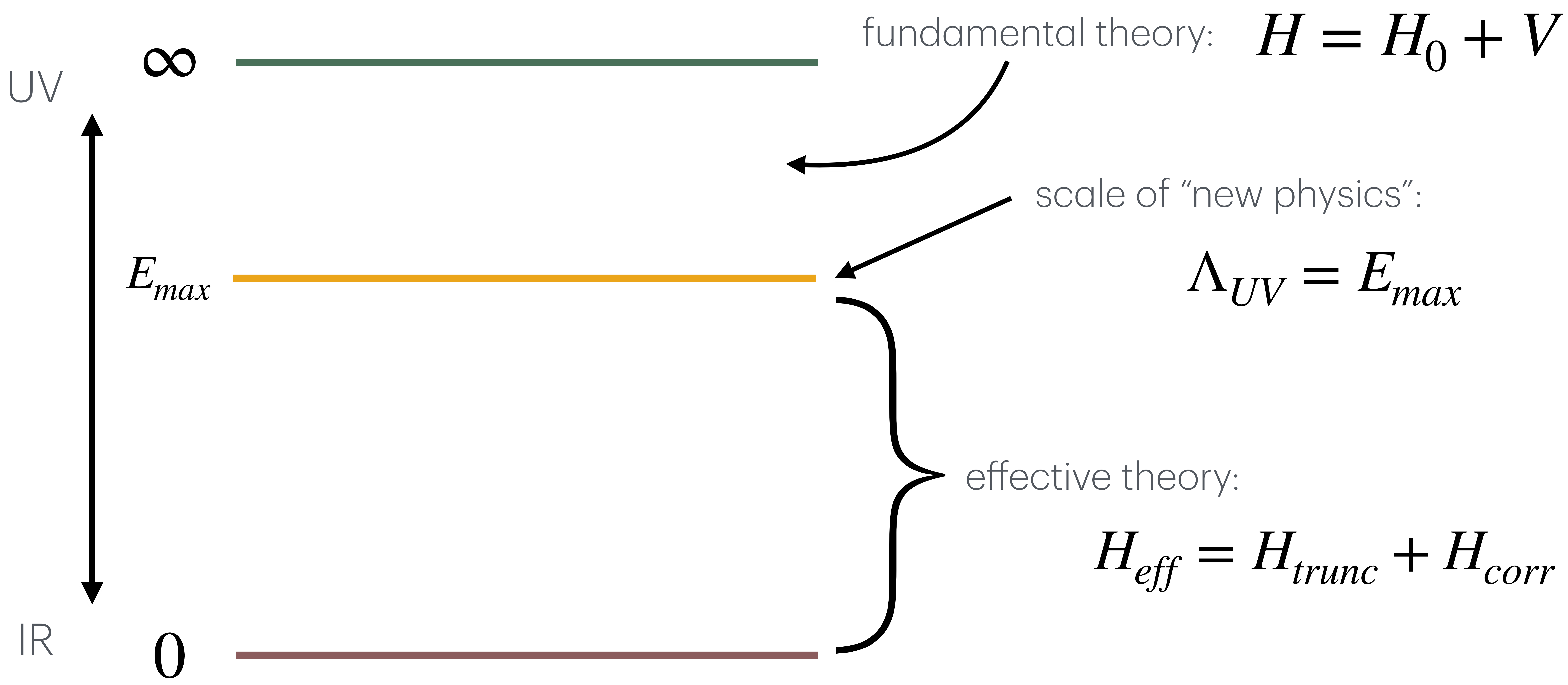

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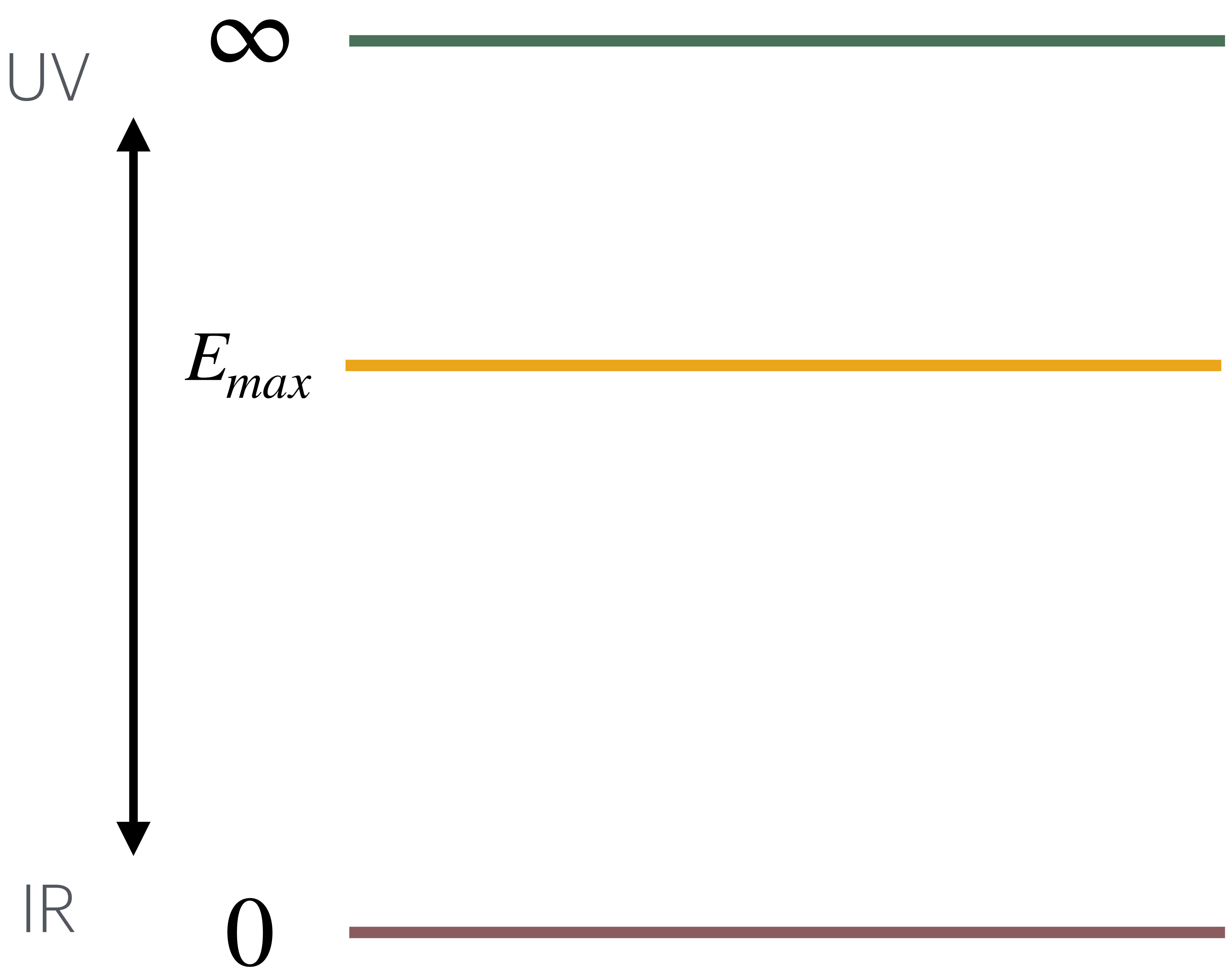

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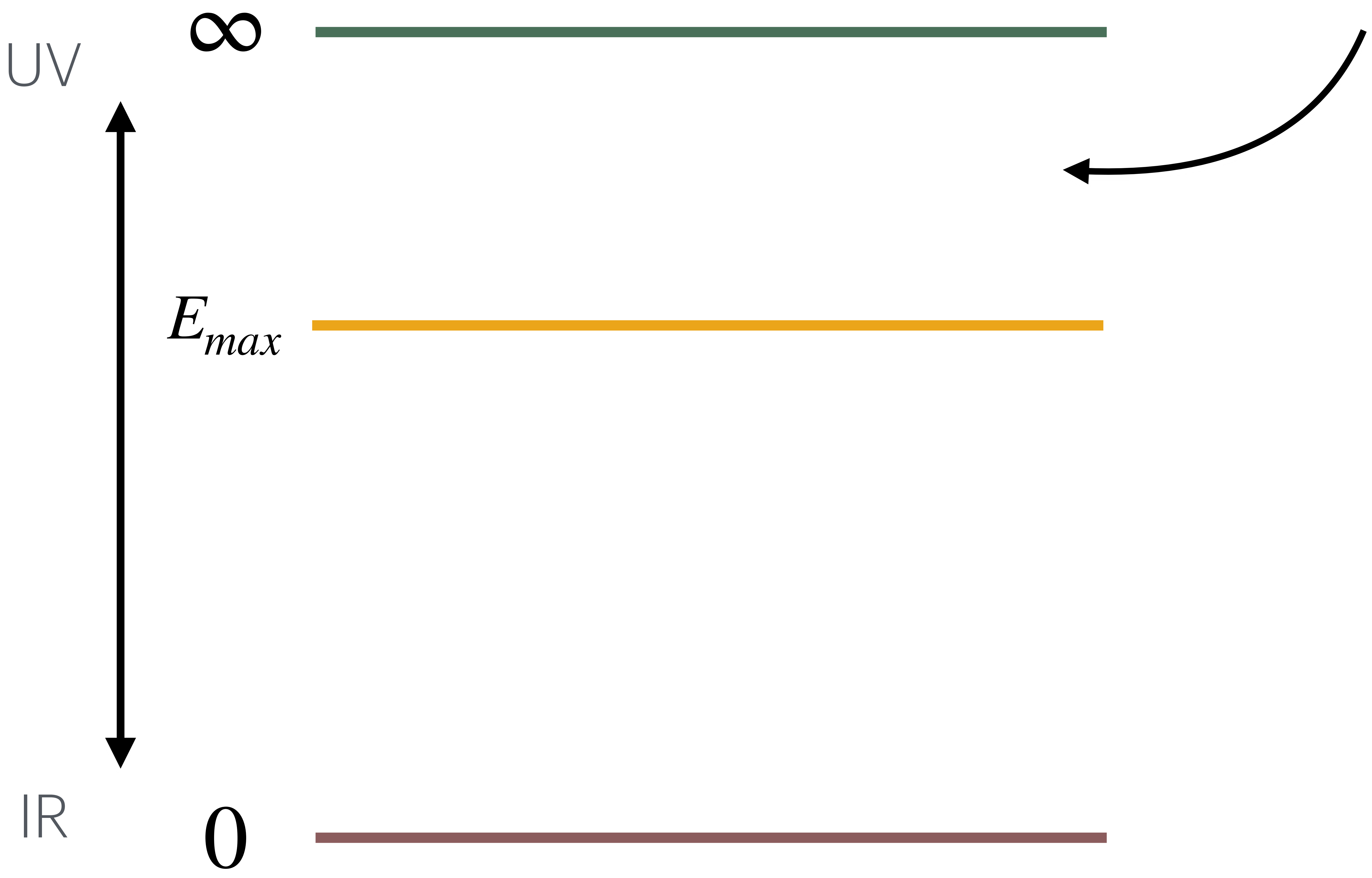

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$$H|\Psi_i\rangle = \mathscr{E}_i|\Psi_i\rangle$$

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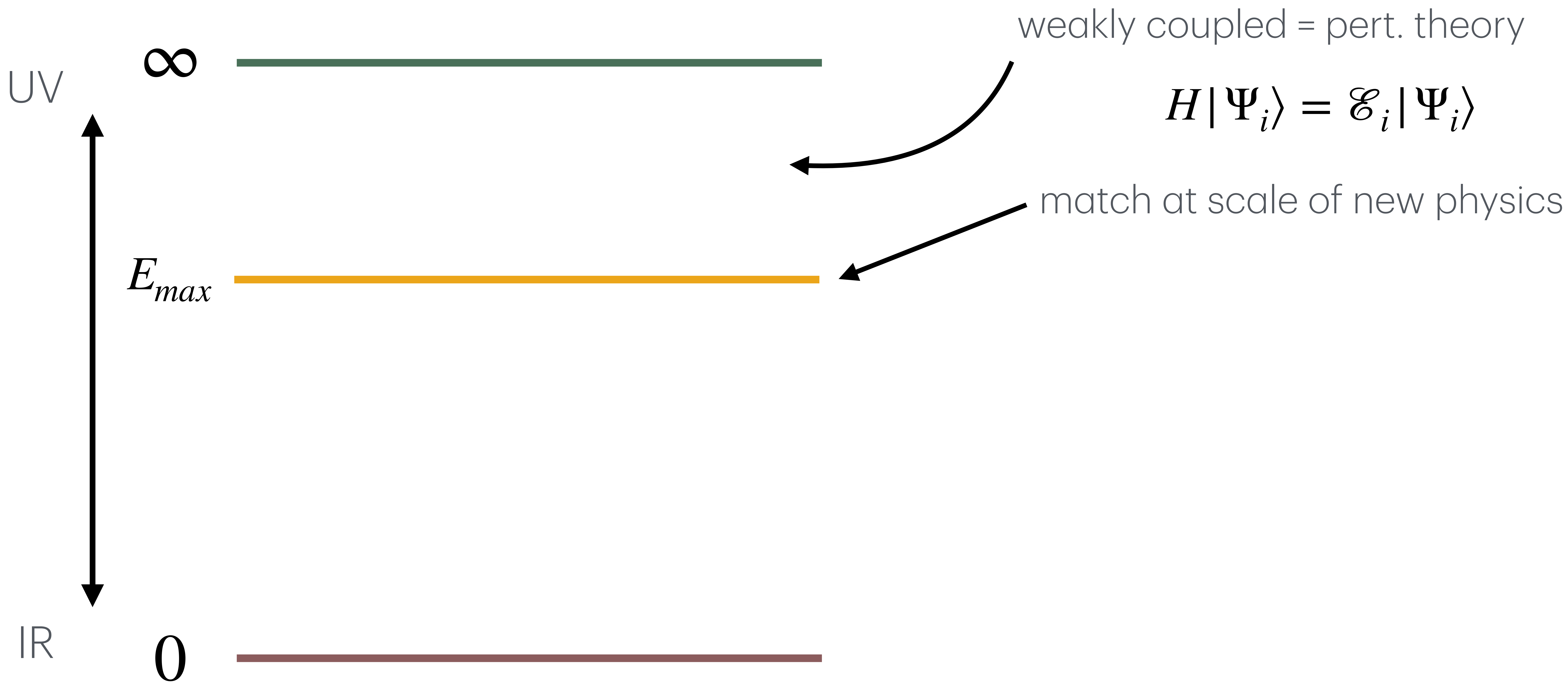

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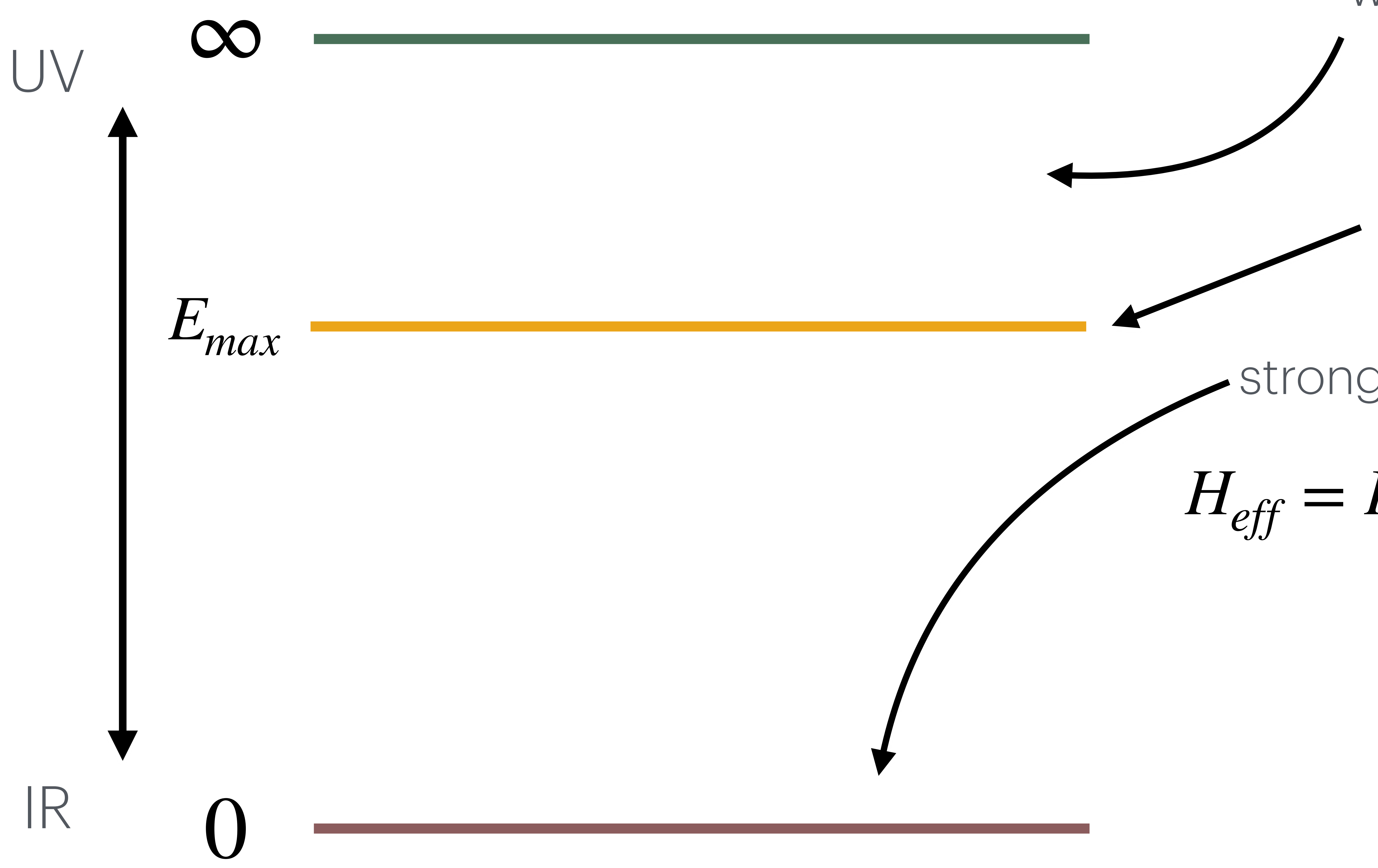

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strongly coupled = effective description

$$H_{eff} = H_0 + H_1 + H_2 + \cdots \qquad H_n = \mathcal{O}(V^n)$$

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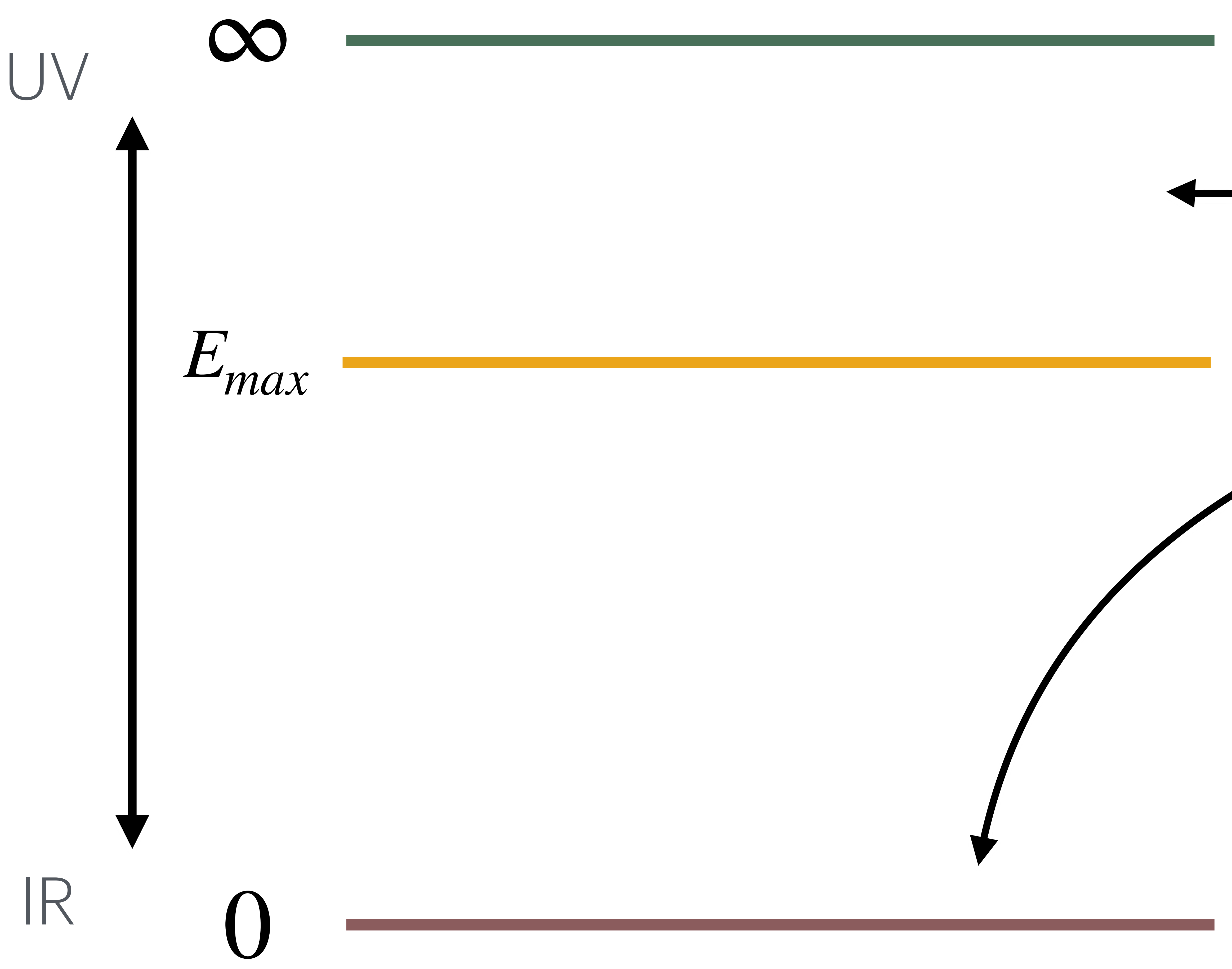

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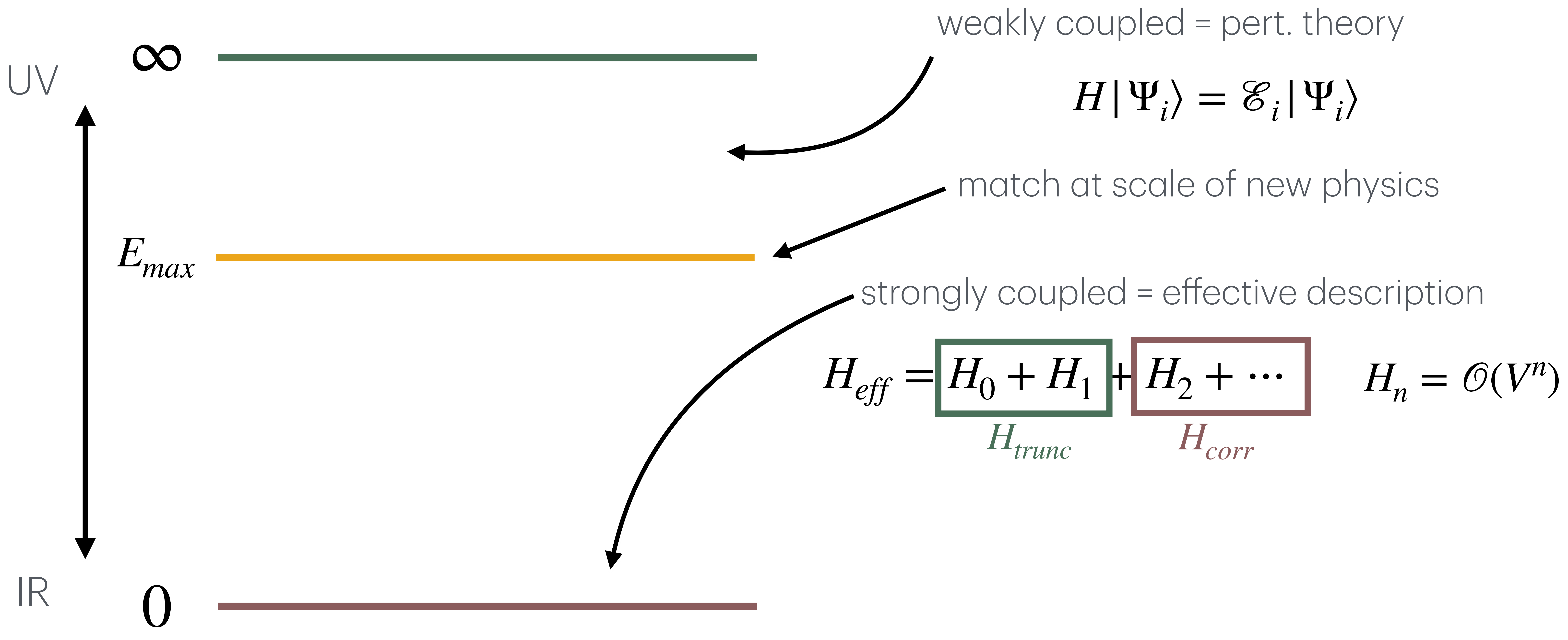

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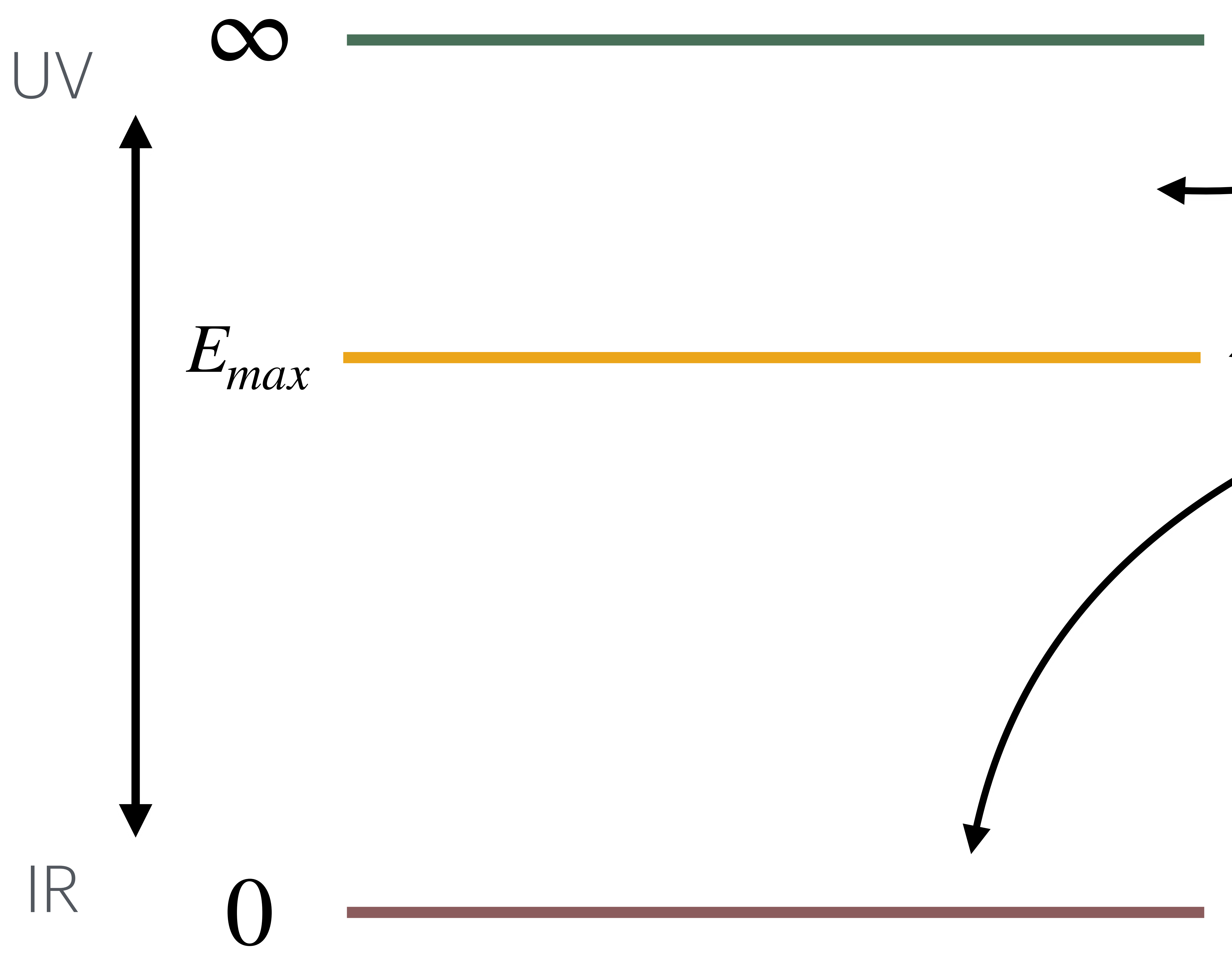

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- consistent power counting
- written in terms of local operators

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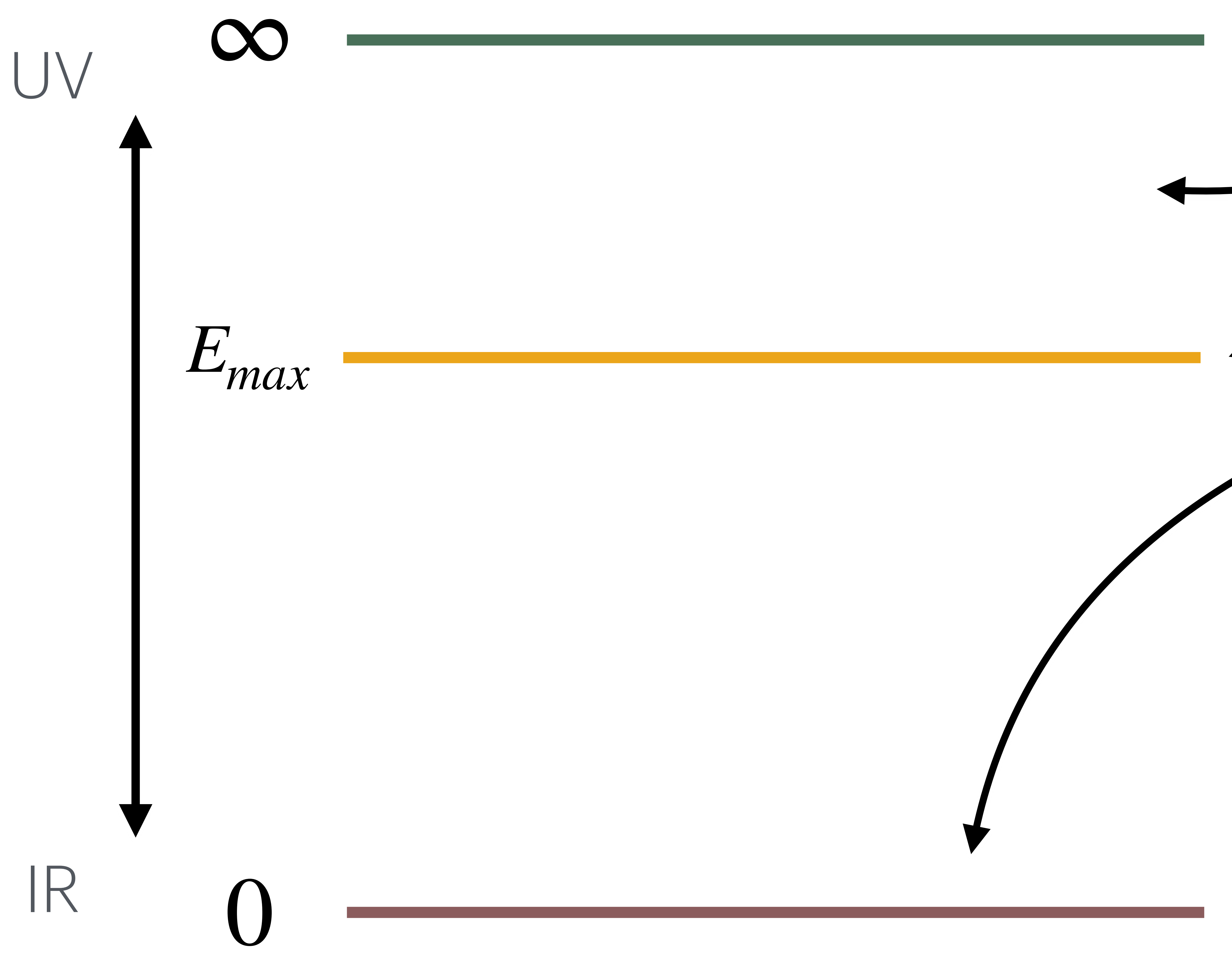

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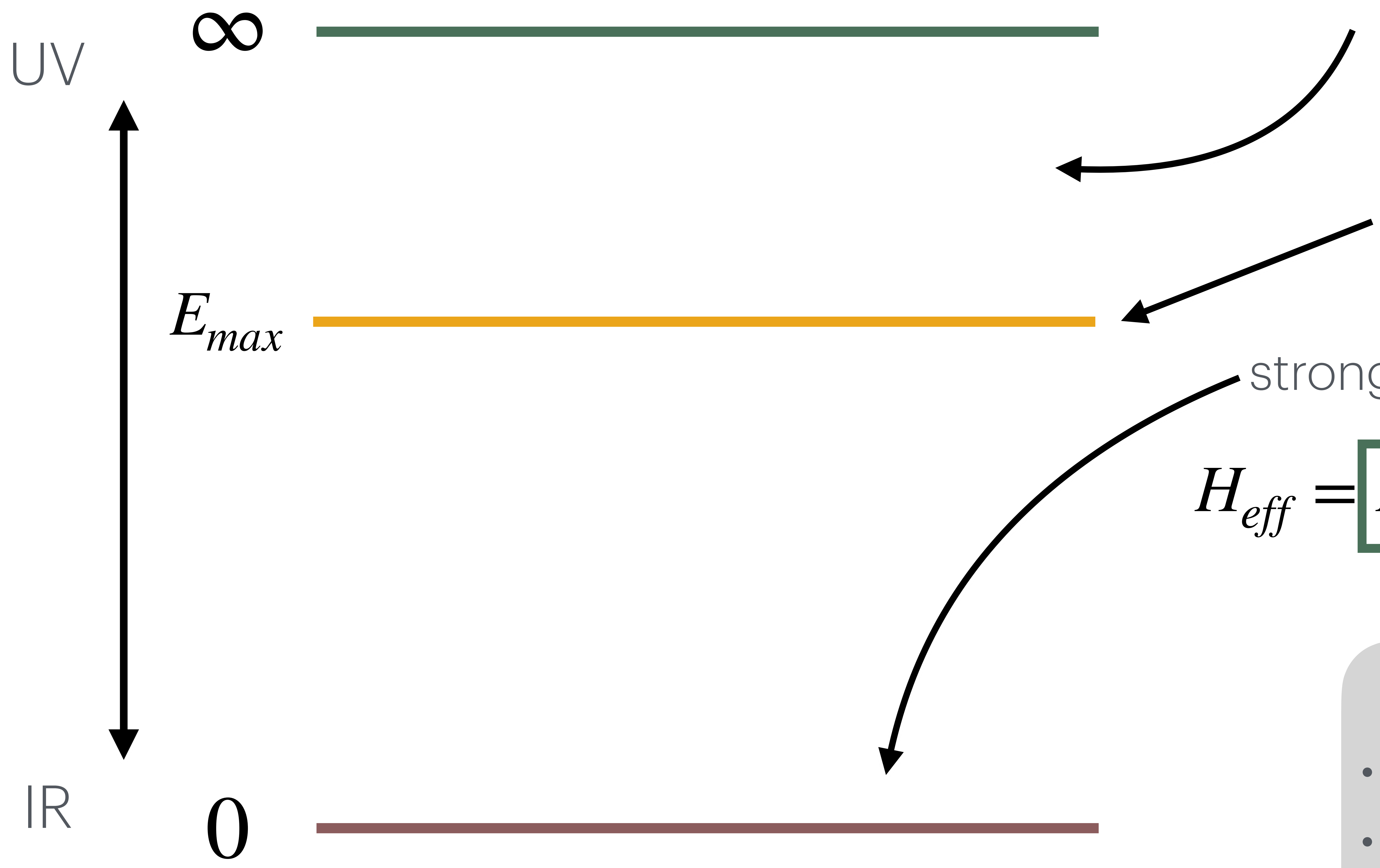

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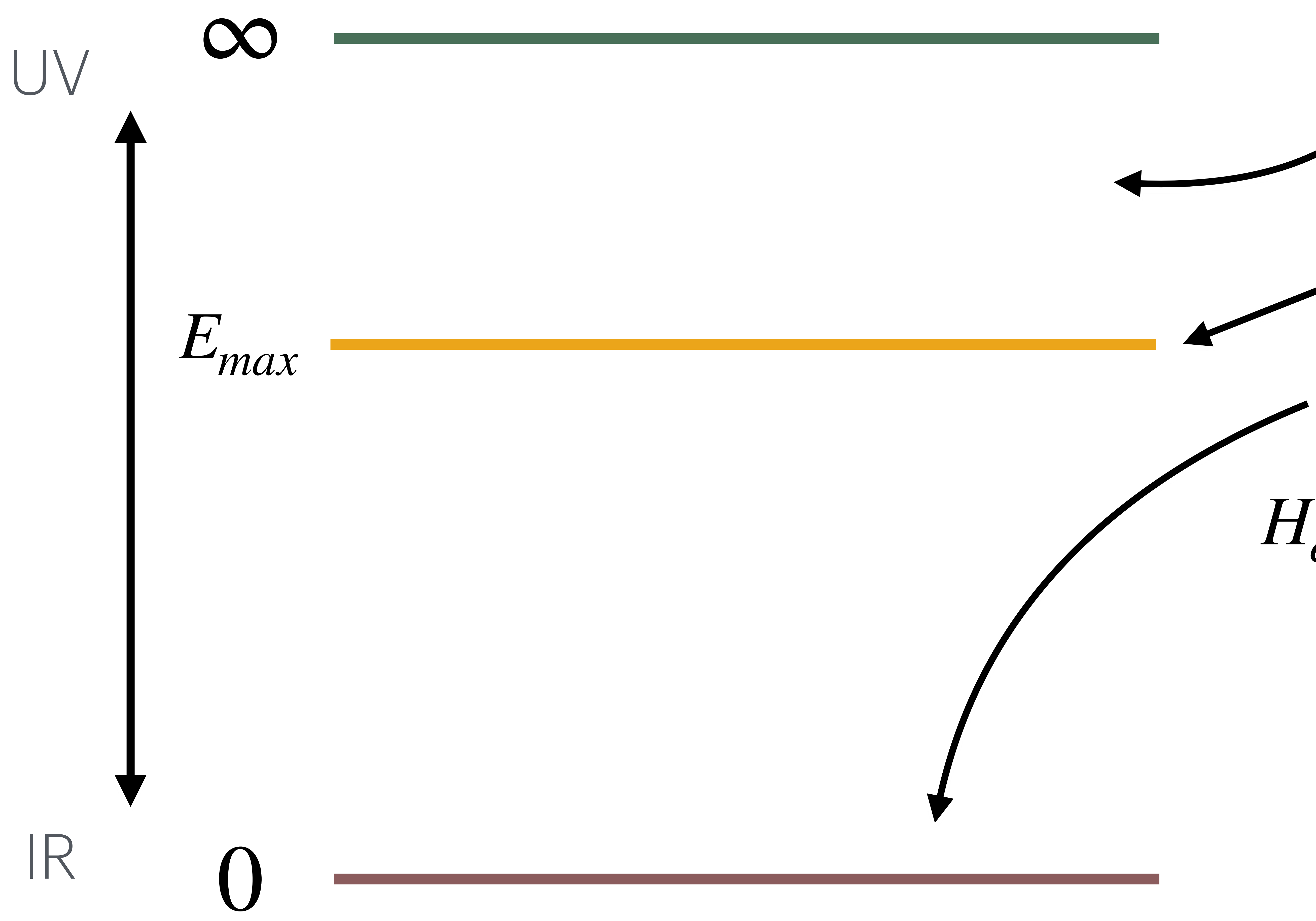

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Cohen, KF, Houtz, Luty, arXiv: 2110.08273

Example Theory: 2D $\lambda\phi^4$

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- strongly relevant interaction
- $\mathbb{Z}_2$ ($\phi \to -\phi$) symmetry broken at strong coupling = phase transition
  - in universality class of Ising Model: good check

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Power Counting for 2D $\lambda\phi^4$

$$[\lambda] = 2, \quad [\phi] = 0$$

$$H_n \simeq \frac{\lambda^n}{E_{max}^{2n-2}} \int dx \, (\mathit{dimensionless})$$

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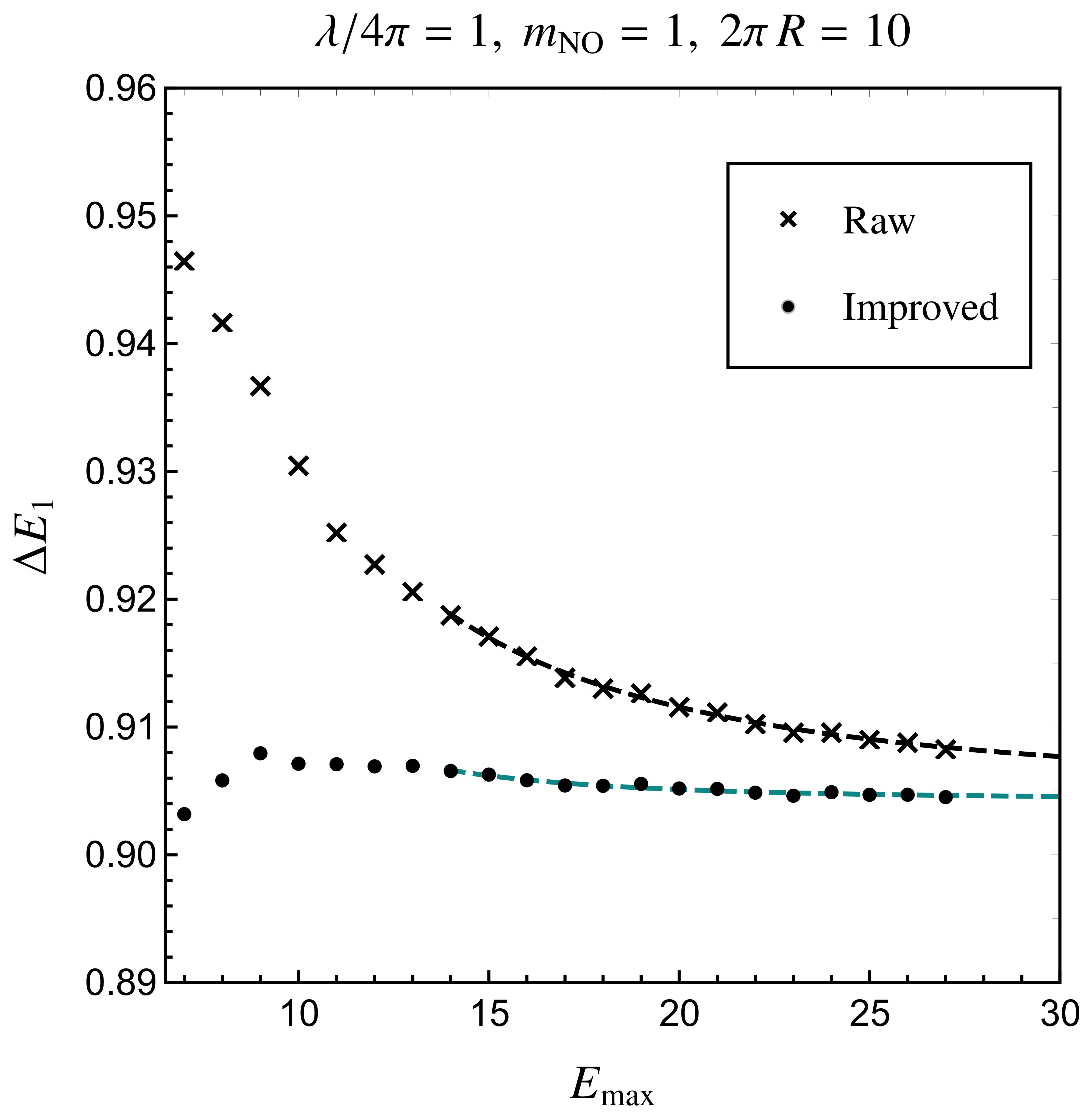
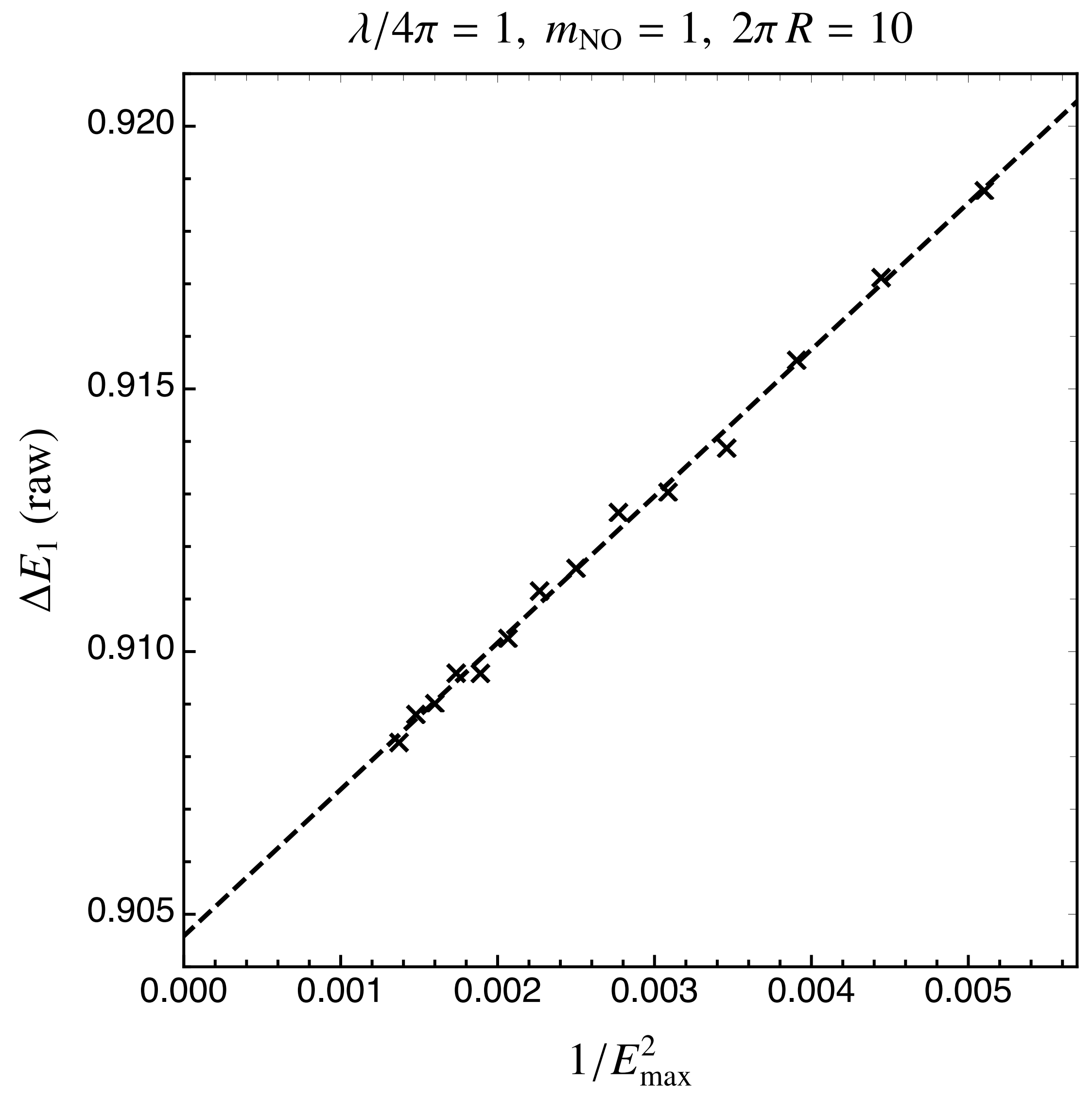
What we expect:

- error $\sim 1/E_{max}^2$ for raw truncation
- error $\sim 1/E_{max}^3$ after including corrections
- phase transition:
  - $\mathbb{Z}_2$ symmetry breaking at critical coupling
  - 2D Ising model

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

Scaling raw vs. improved





without EFT

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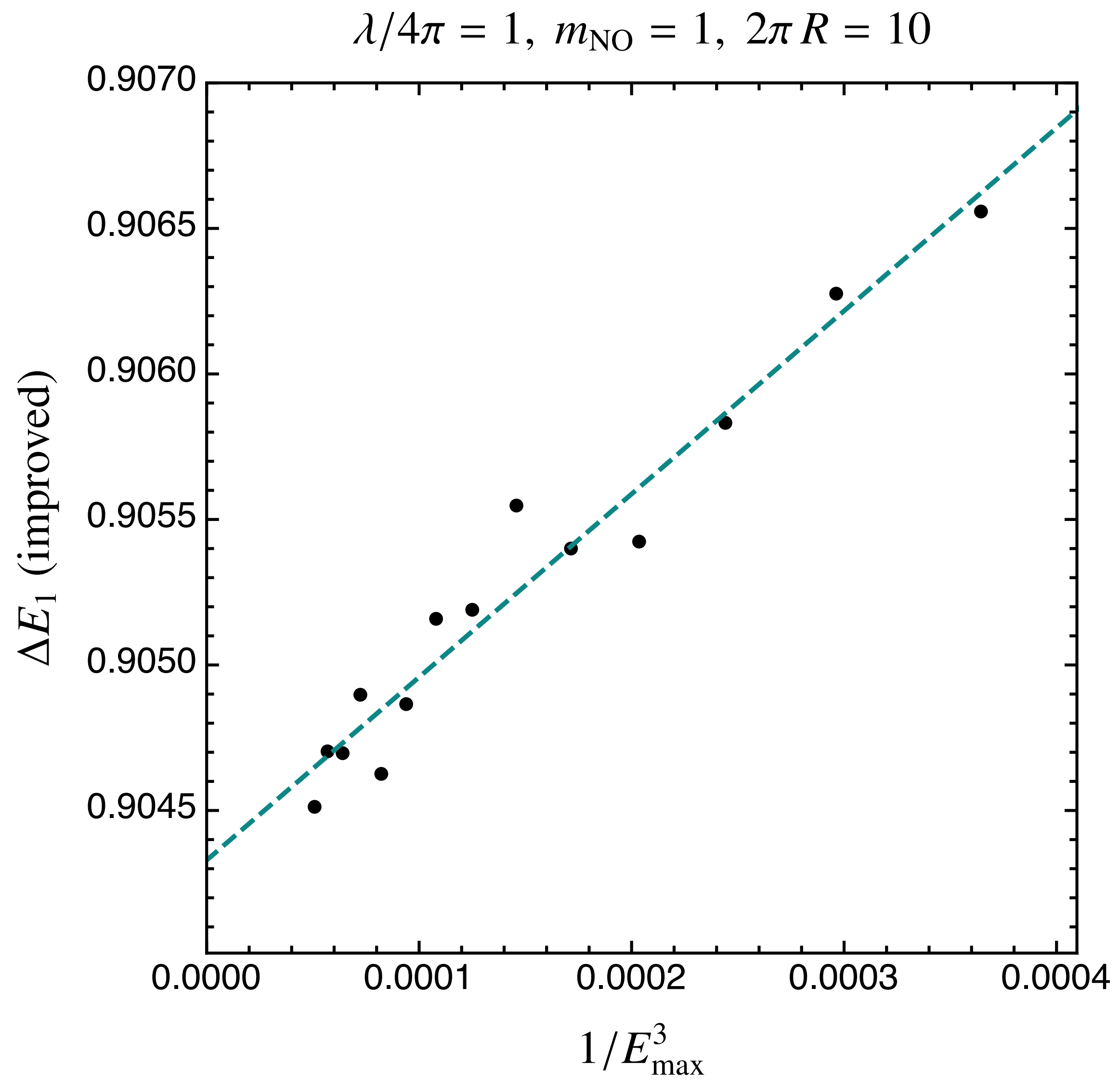

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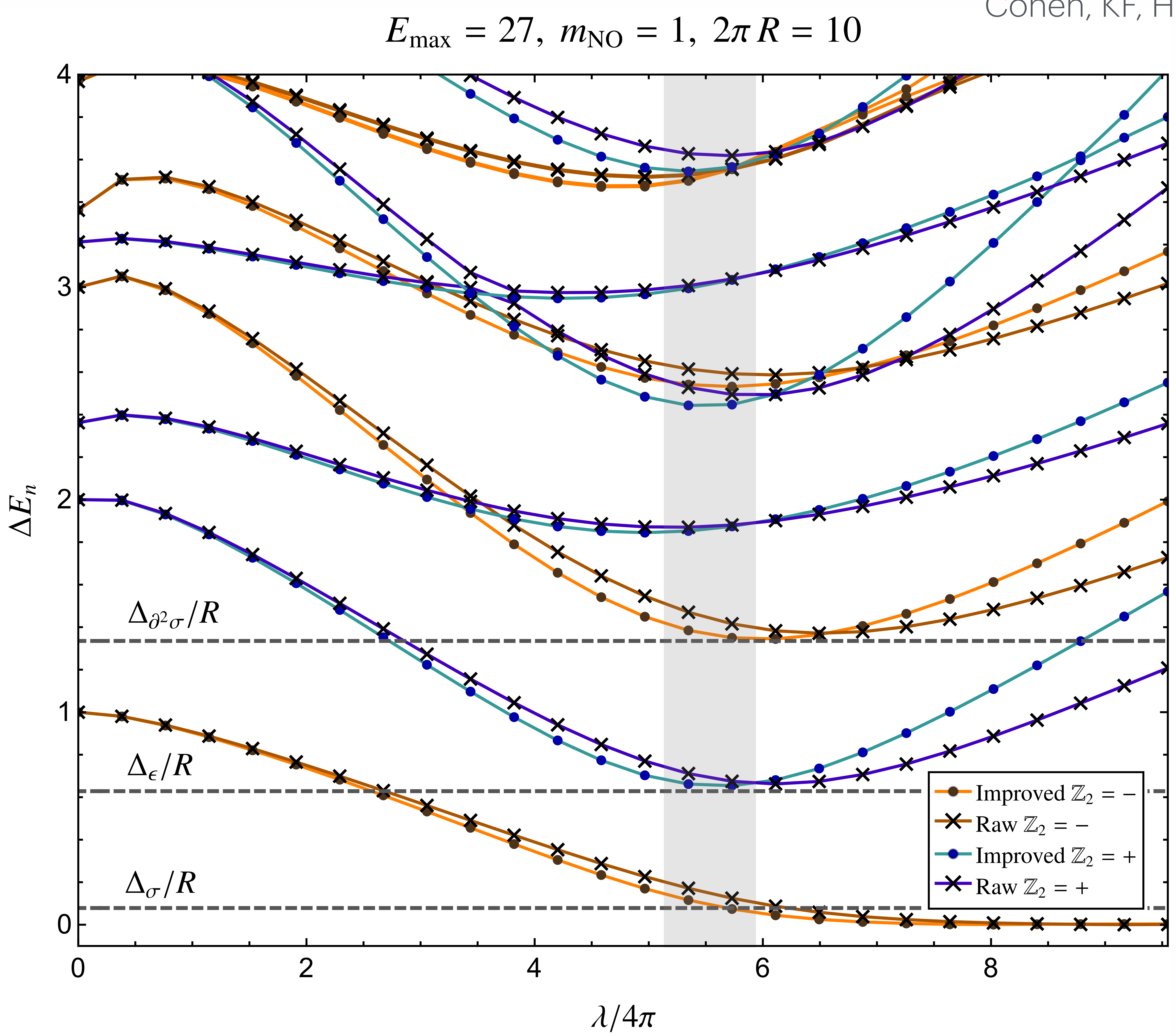

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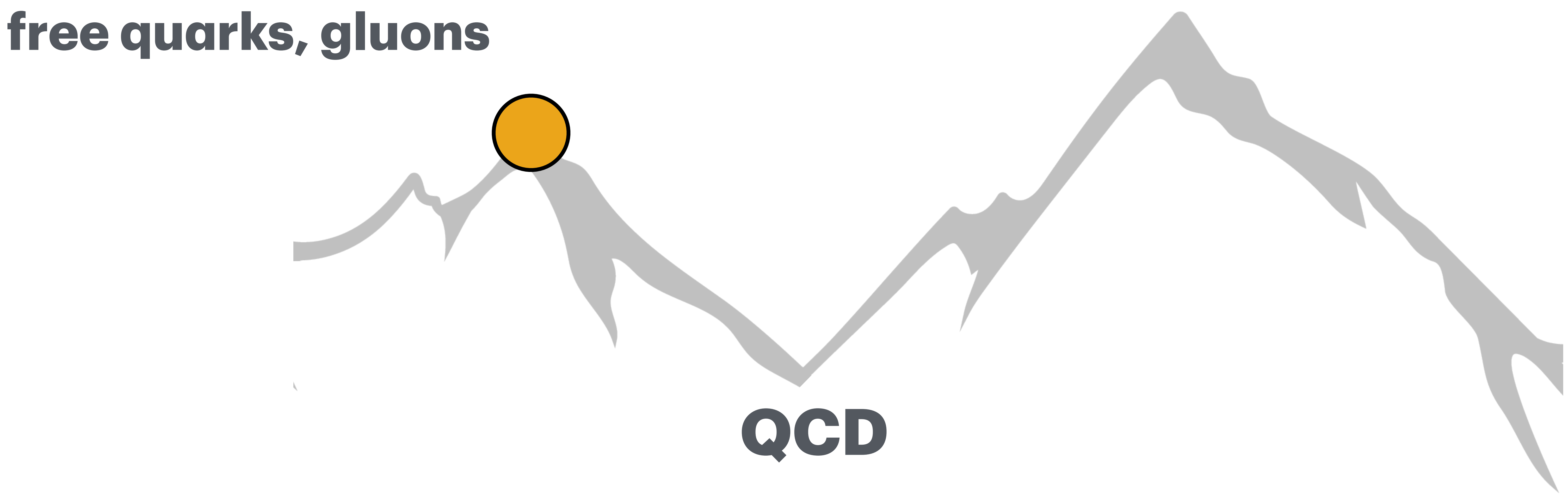

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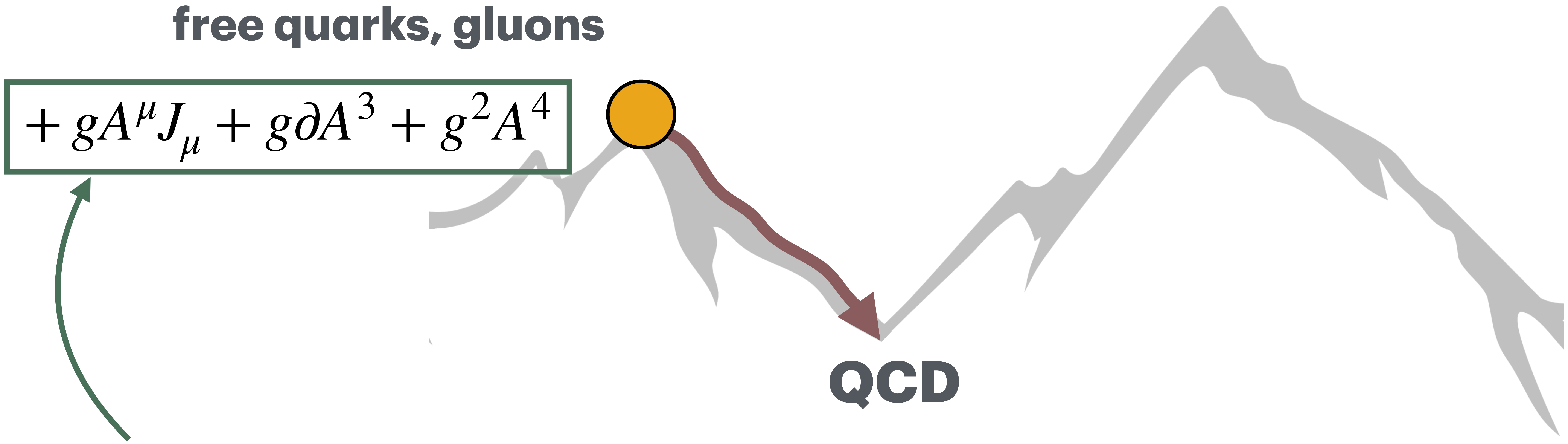

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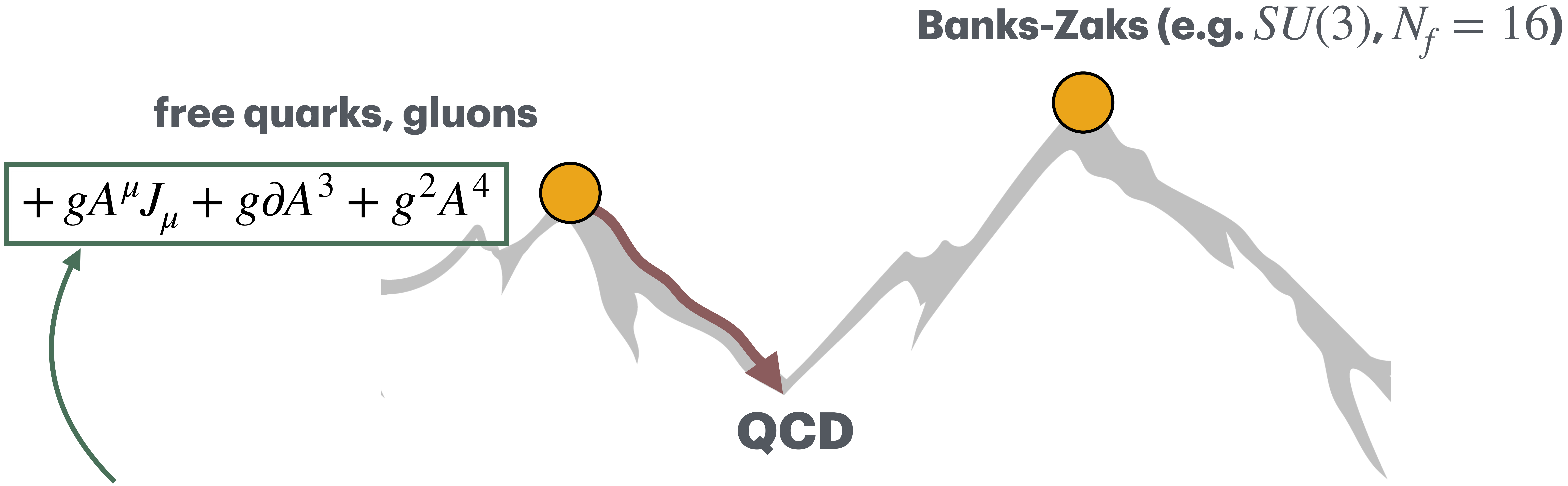

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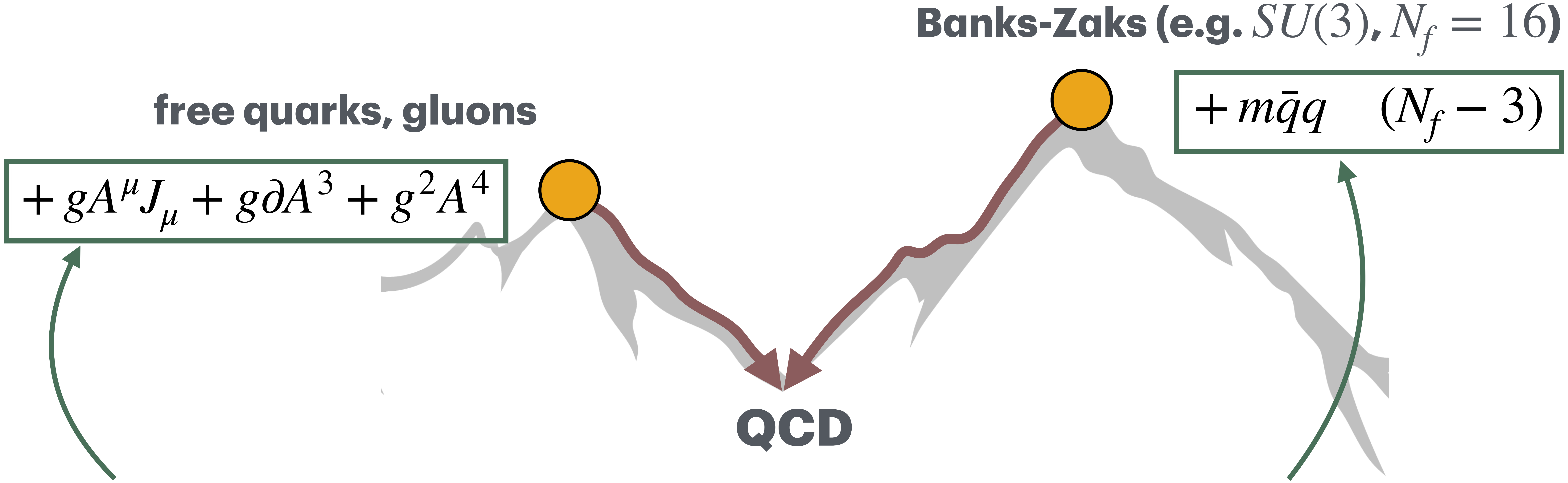

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- relevant (convergence under control)
- gauge-invariant

First step: 2D Gross-Neveu

in progress with Dorian Wenzel

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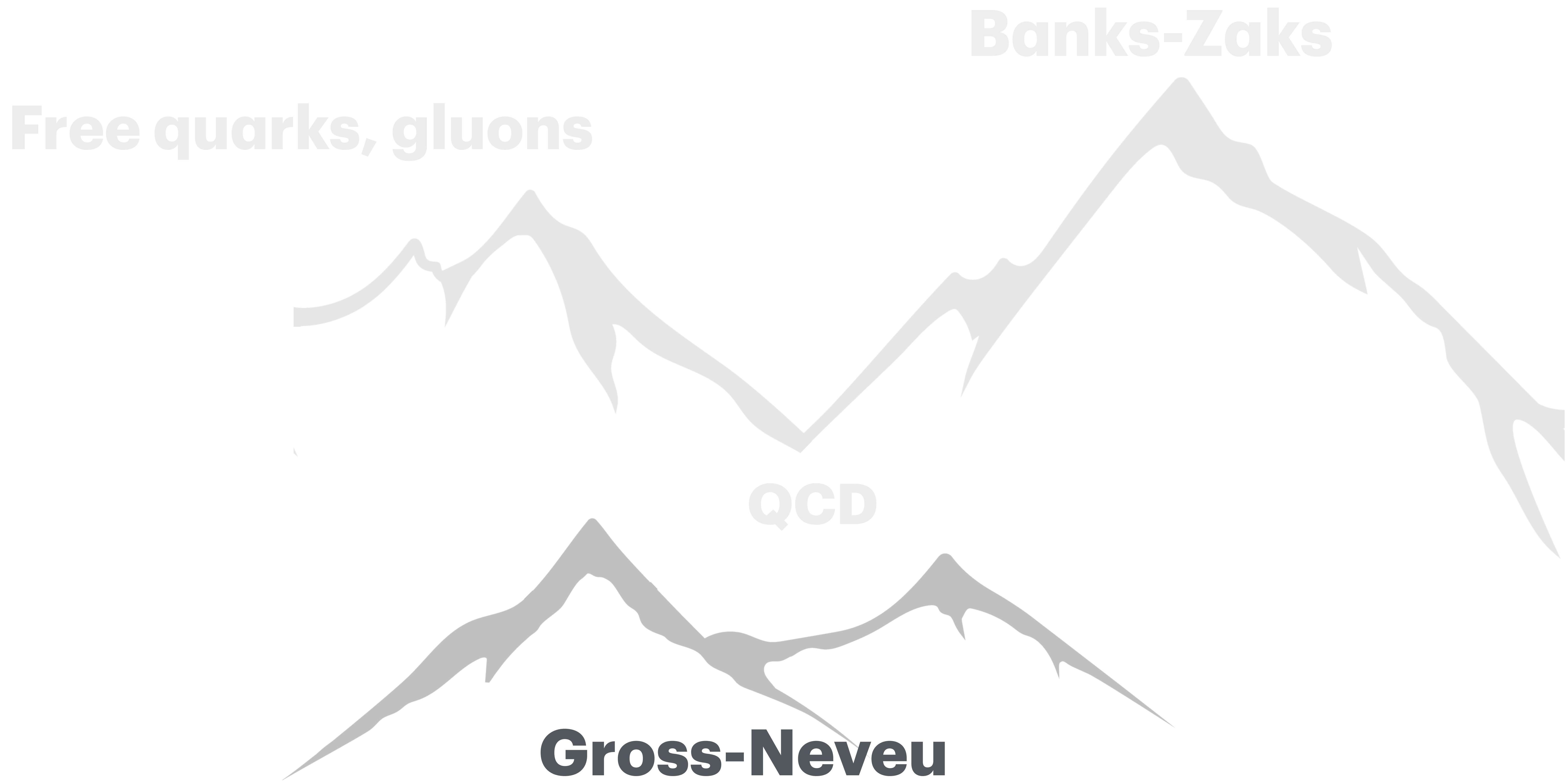

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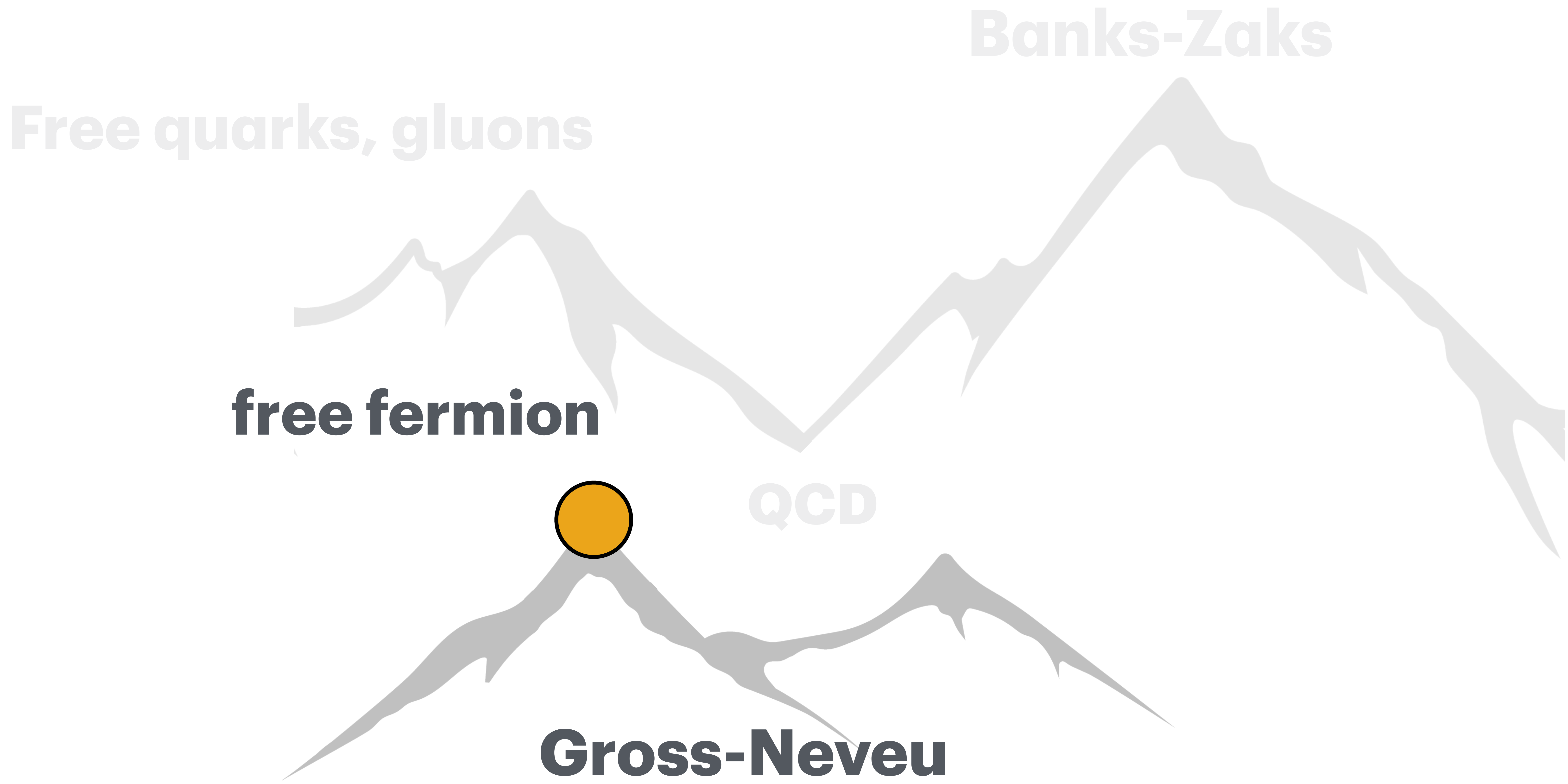

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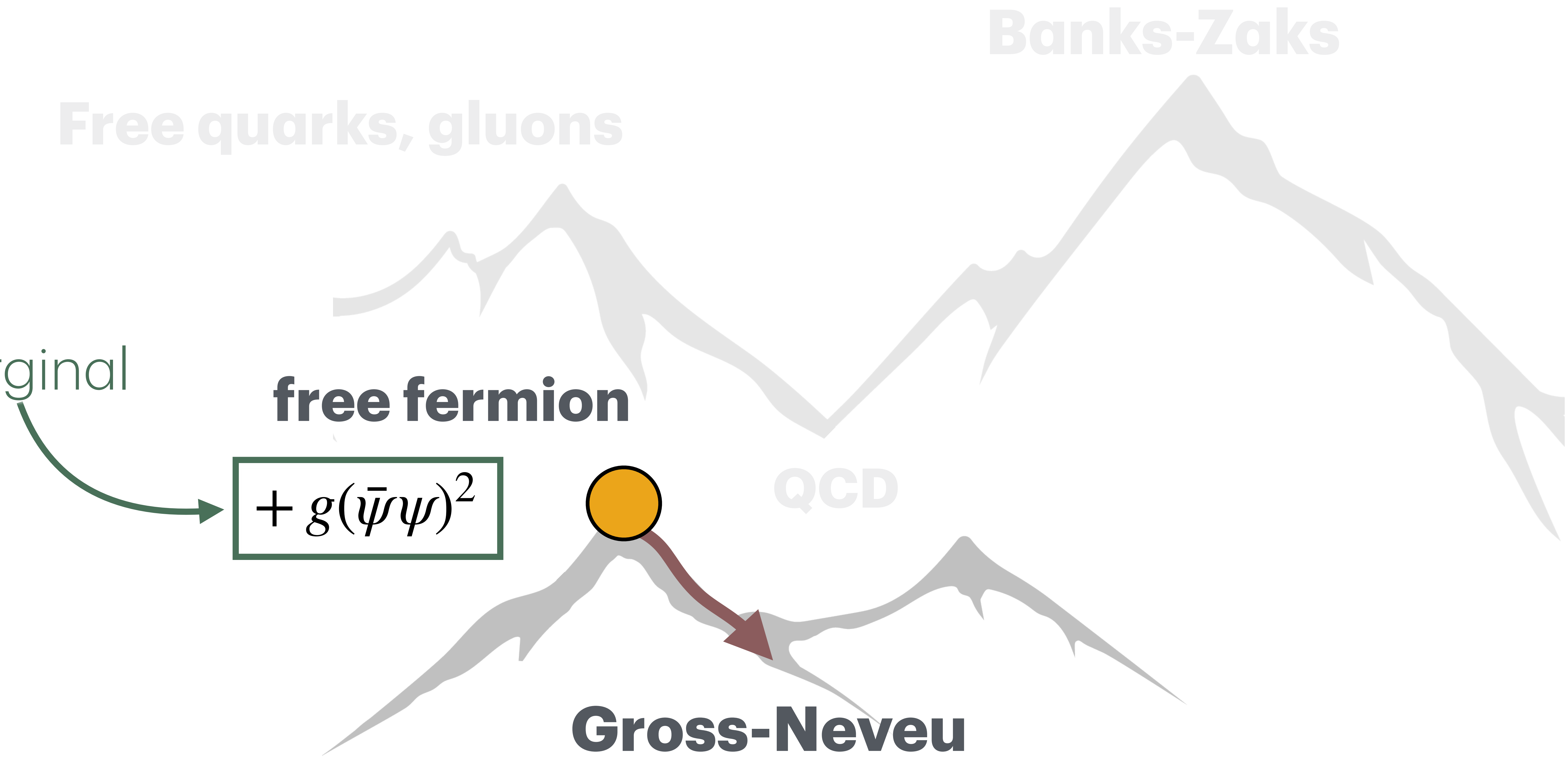

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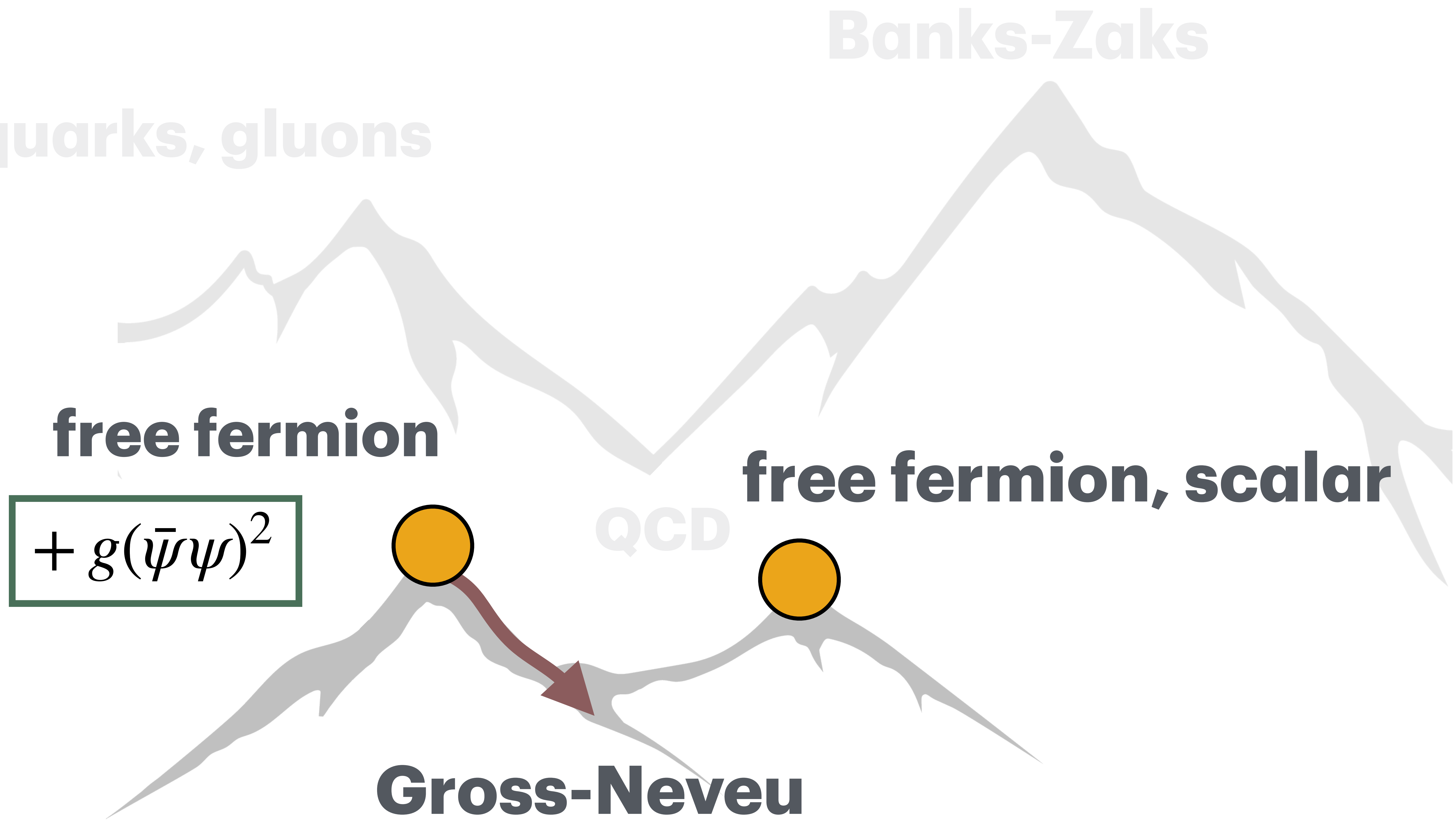
- marginal



in progress with Dorian Wenzel

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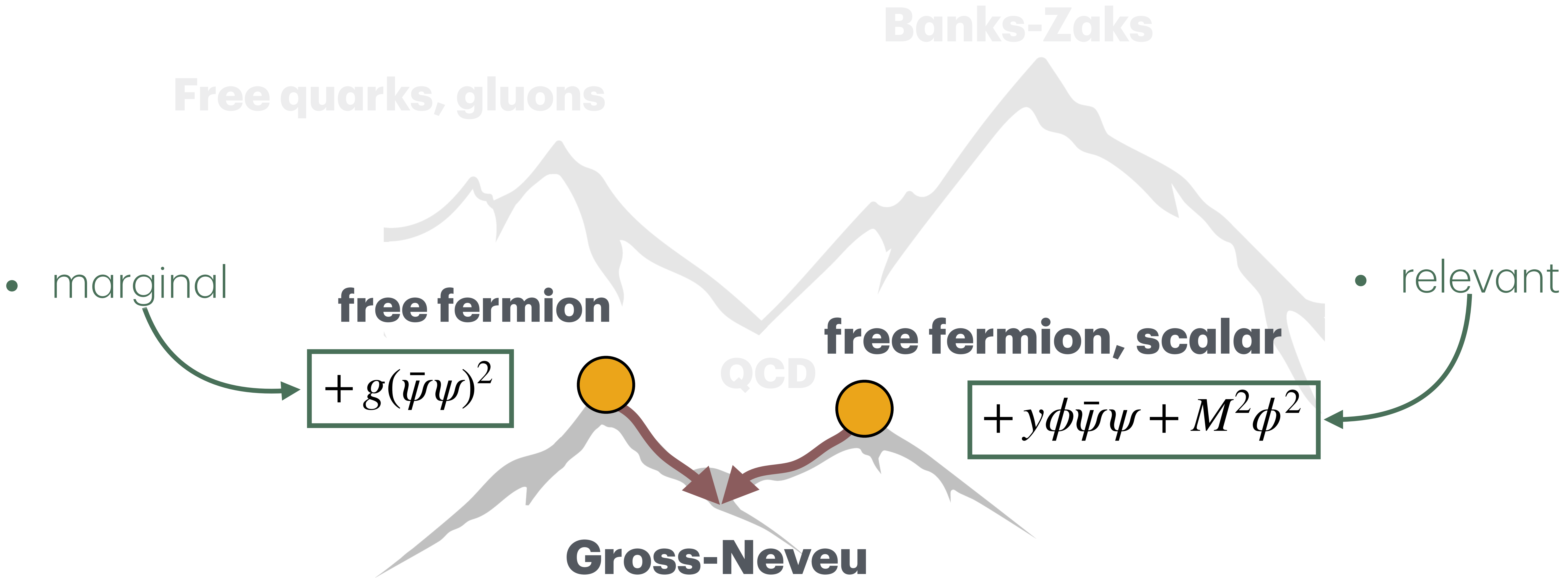


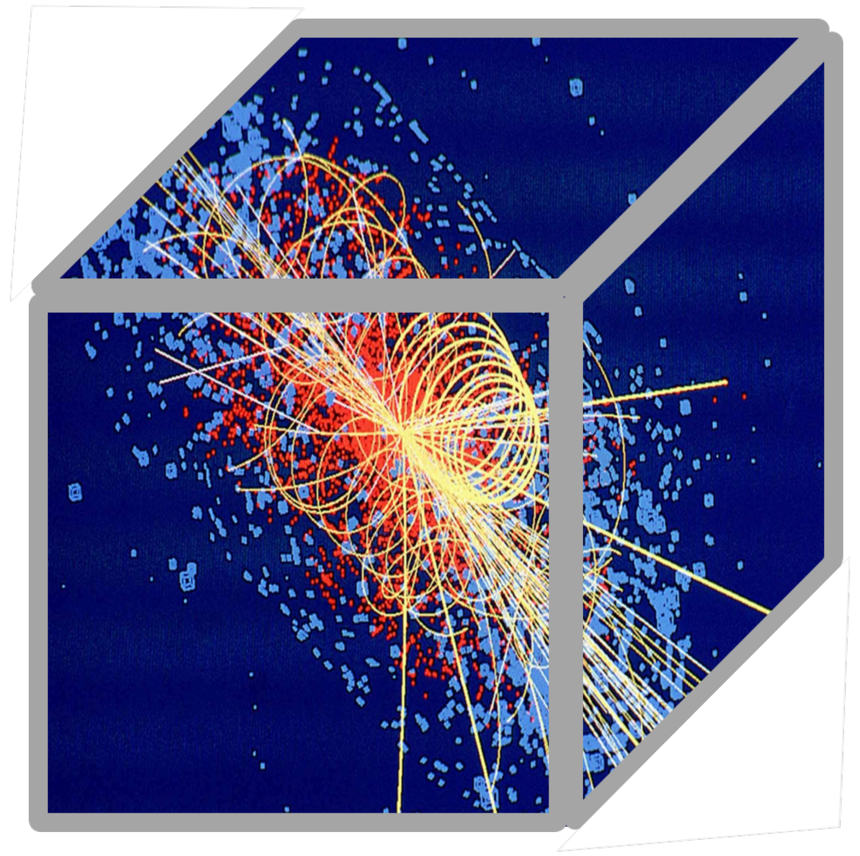
Simpler testing ground on way to QCD

in progress with Dorian Wenzel

Future directions

- **improve this method** (next order, include more UV divergences)
  - work in progress with Rachel Houtz
- look at **new observables** (Wilson loops, entanglement entropy, **energy correlators**)
  - finite volume S-matrix, work in progress with Carl Beadle, Francesco Riva, Matthew Walters
- move to **higher dimensions**
- include **new fields** (gauge bosons)
- **curved spacetime** (cosmology, S-matrix)
- **connection** to other non-perturbative methods
- ... you tell me!

$$S_{fi} = \langle \Psi_f | \Psi_i \rangle \, ?$$



Landscape of quantum field theory

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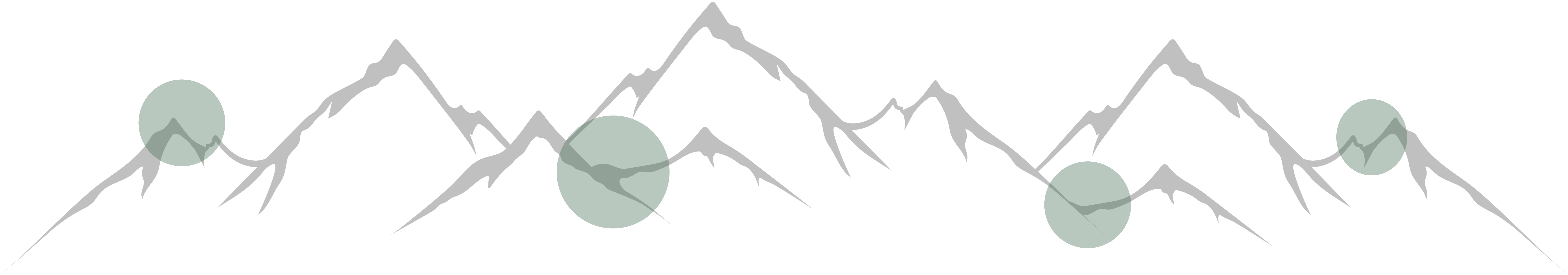

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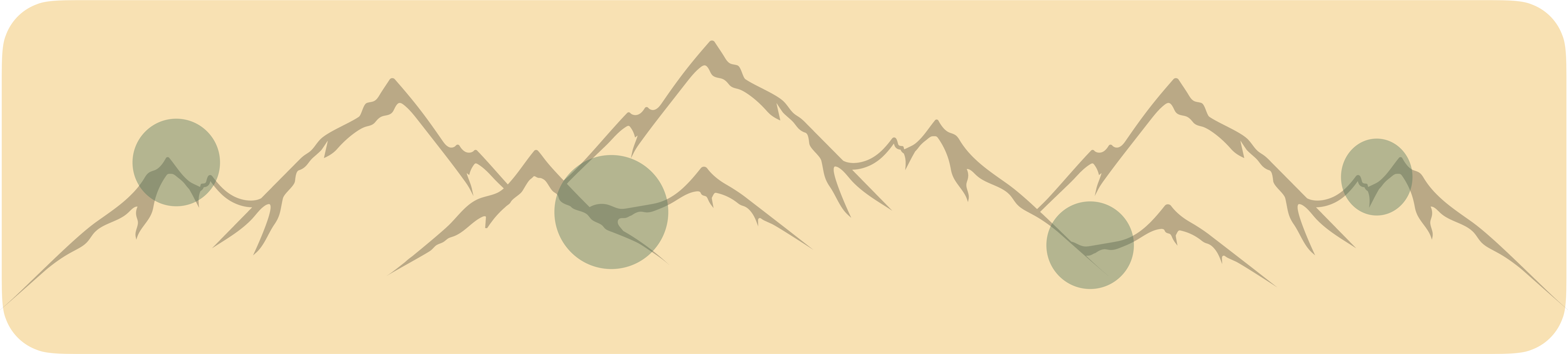
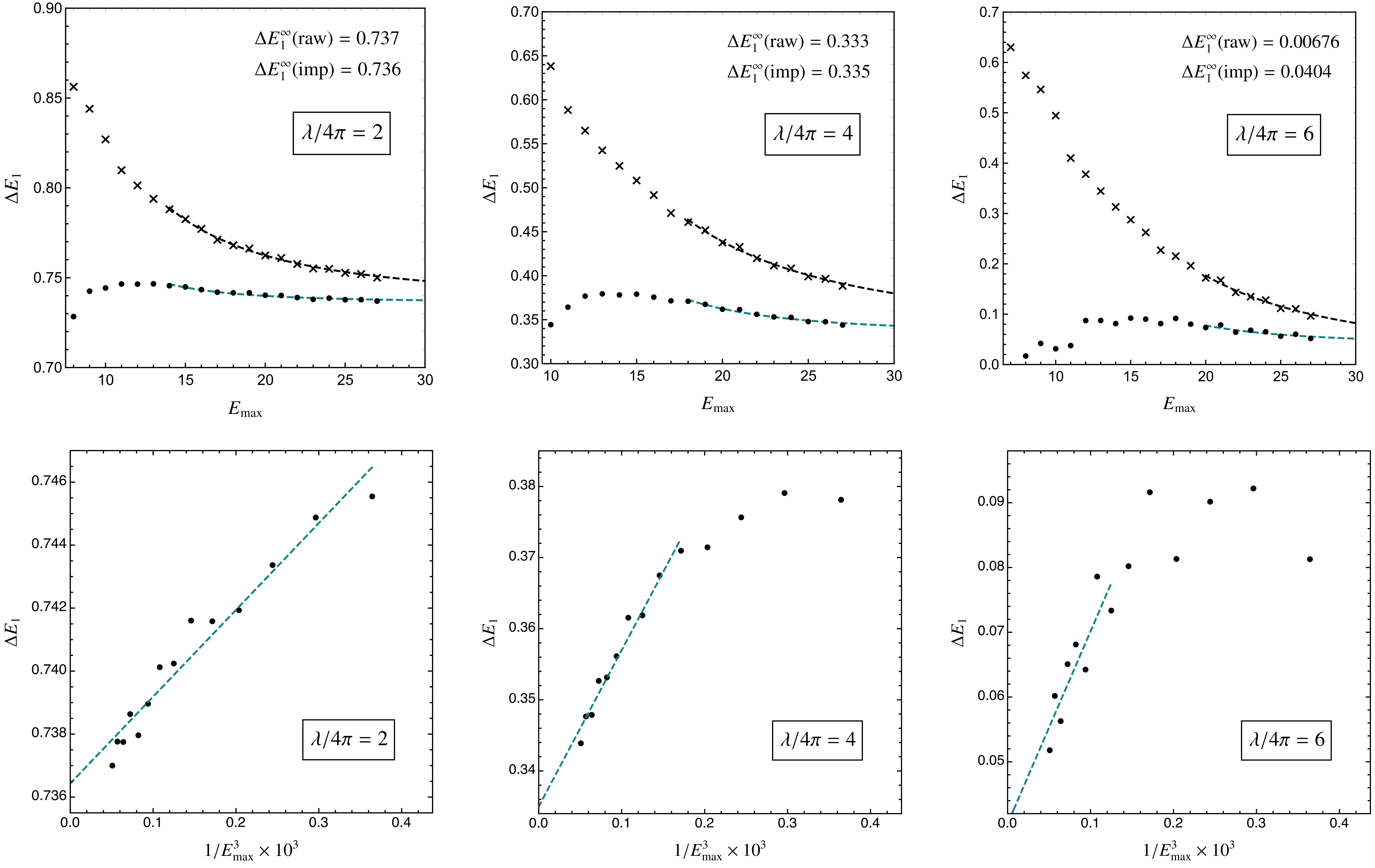
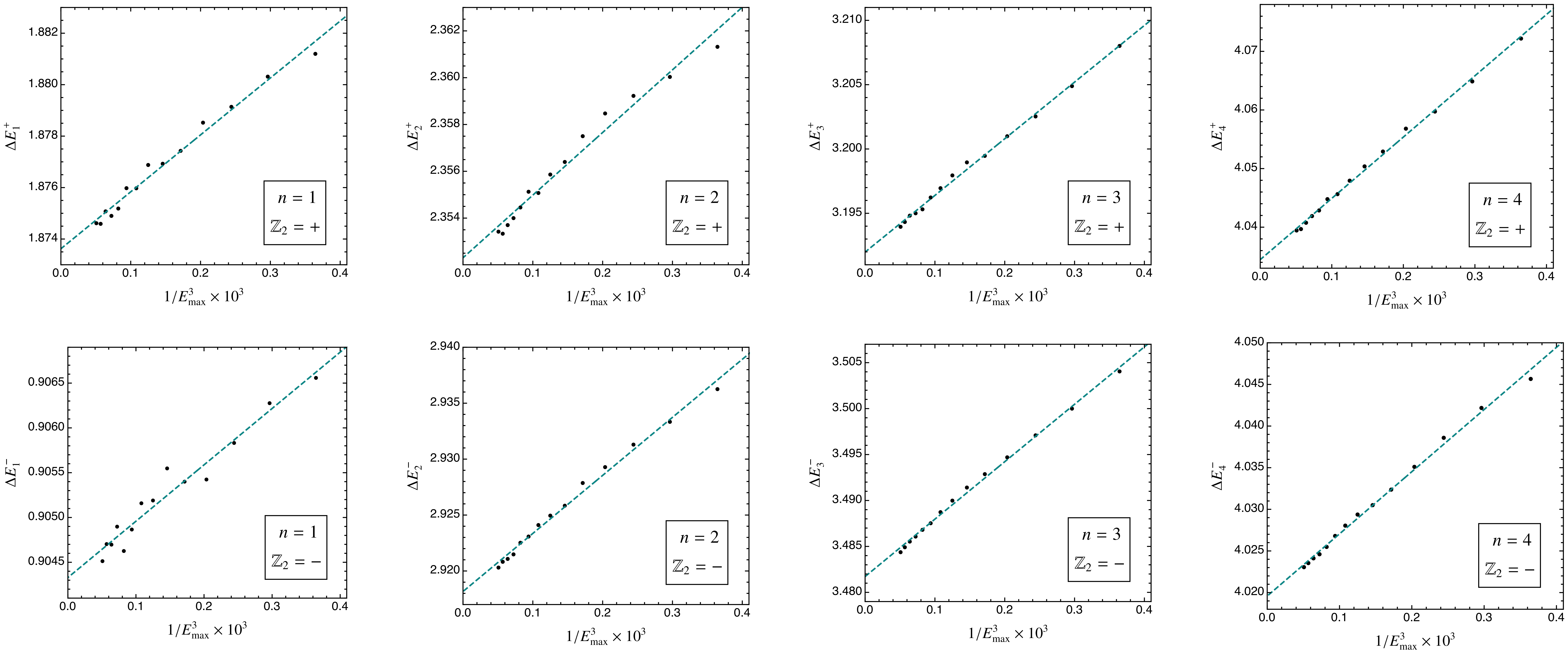


Thank You!

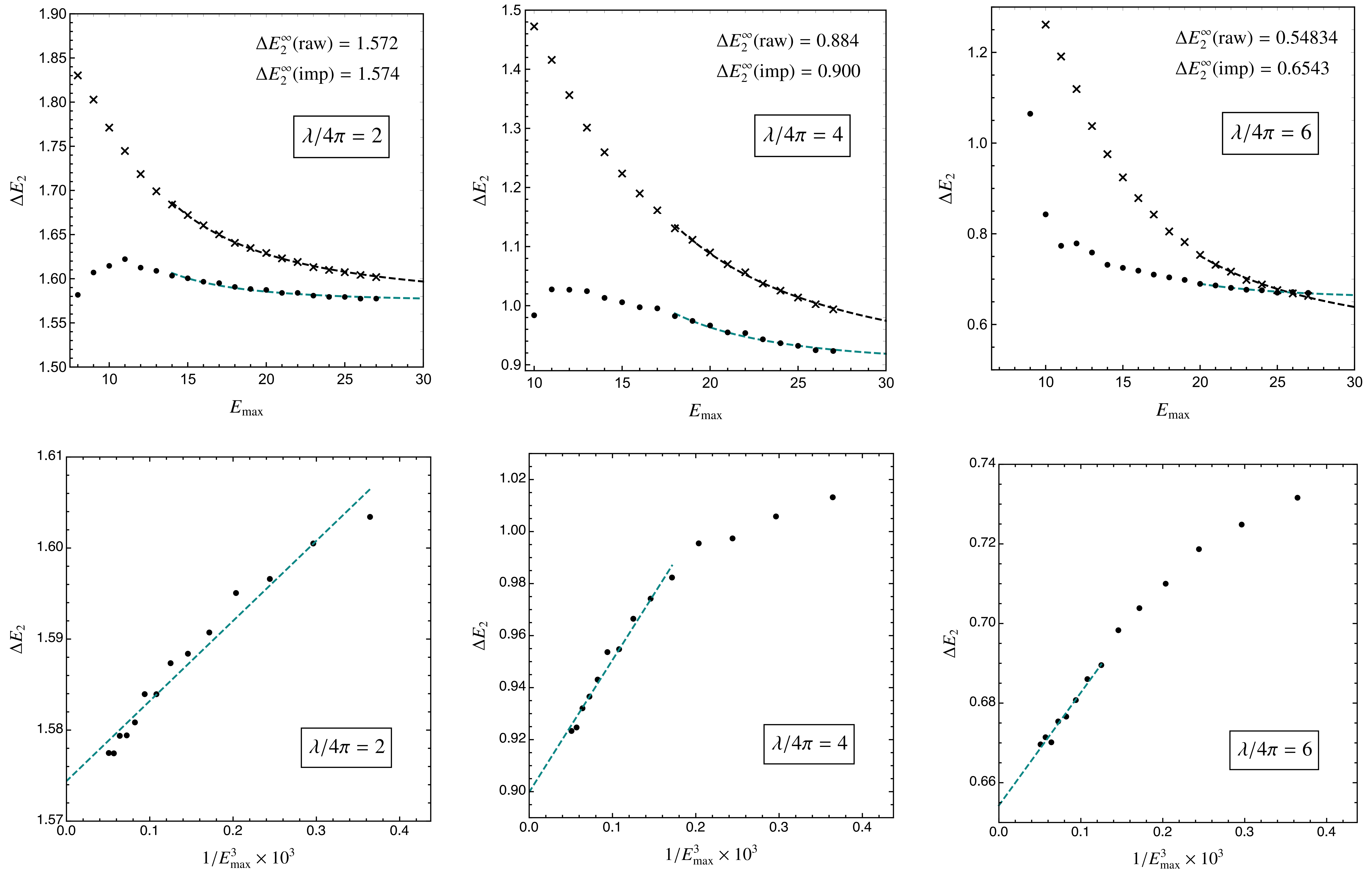
Higher coupling



Higher Excited States



Backup: Higher coupling ΔE_2



Cohen, KF, Houtz, Luty, arXiv: 2110.08273

What to match?

$$H_0 | i \rangle = E_i | i \rangle$$

full theory: $H | \Psi_i \rangle = \mathscr{E}_i | \Psi_i \rangle$ $\quad \mathscr{E}_i = E_i + \mathscr{E}_{1i} + \mathscr{E}_{2i} + \cdots$

$$\mathscr{E}_{ni} = \mathcal{O}(V^n)$$

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eigenvectors? $\langle \Psi_f|i\rangle \Rightarrow \langle f|H_1|i\rangle = \langle f|V|i\rangle$ uniquely fixes H_{eff}

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

What to match? "S-matrix"?

Cohen, KF, Houtz, Luty, arXiv: 2110.08273

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$$|i\rangle \xrightarrow{U_{IP}(0,-\infty)} |\Psi_i\rangle$$

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$$\langle\Psi_f|i\rangle = \lim_{t_f\to\infty}\langle f|U_{IP}(t_f,0)|i\rangle = \delta_{fi} + \frac{\langle f|T|i\rangle}{E_f - E_i + i\epsilon}$$

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Example Theory 1: 2D $\lambda\phi^4$

$$\langle f | H_2 | i \rangle = \sum_{\alpha \neq i}^{>} \frac{\langle f | V | \alpha \rangle \langle \alpha | V | i \rangle}{E_f - E_\alpha}$$

$$\omega_i > 0$$

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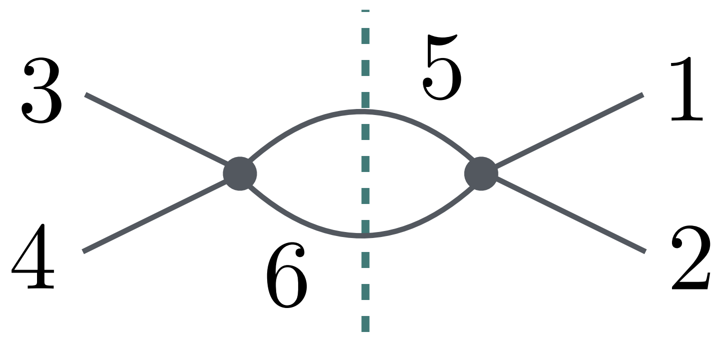
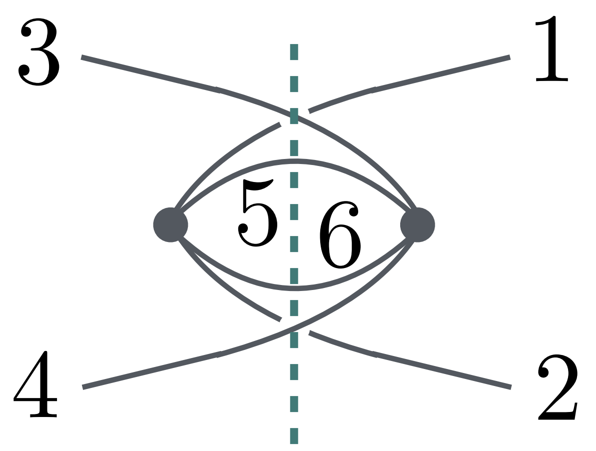


$$\Bigg|_{fund\,-\,eff} \sim \lambda^2 \sum_{5,6} \frac{\Theta(E_\alpha - E_{max})}{\omega_5\, \omega_6\, (\omega_3 + \omega_4 - \omega_5 - \omega_6)}$$



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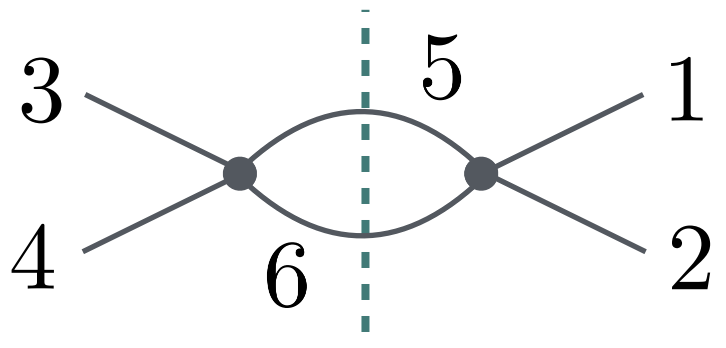

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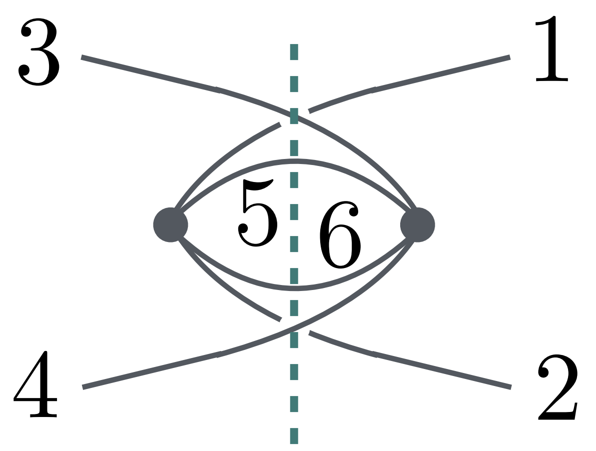

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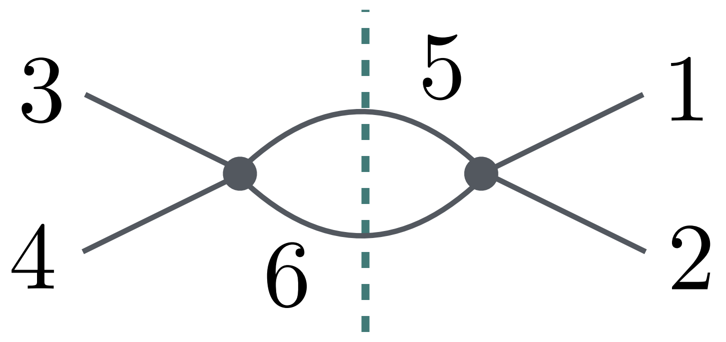

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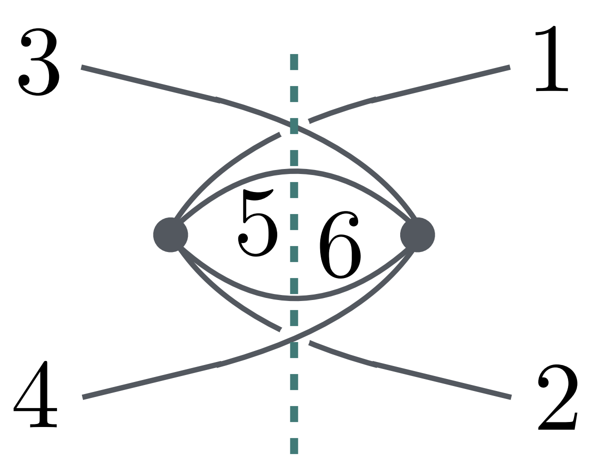

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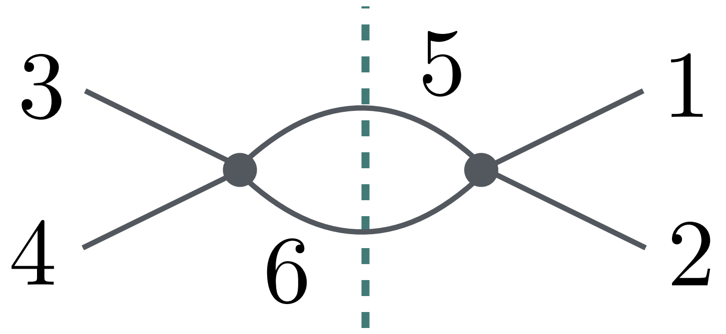

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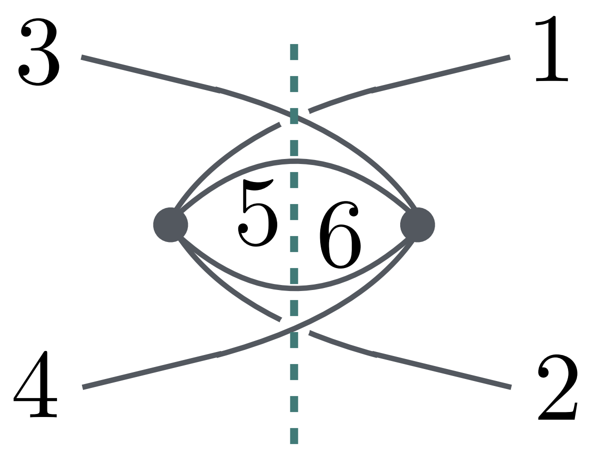

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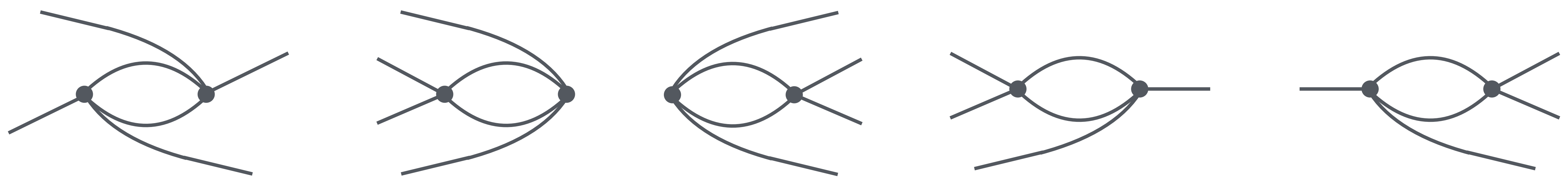

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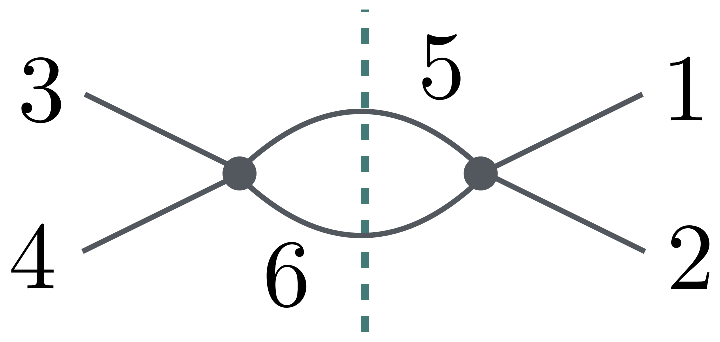

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local approximation

$$\omega_{1,2,3,4} \lesssim E_{i,f} \ll E_{max}$$

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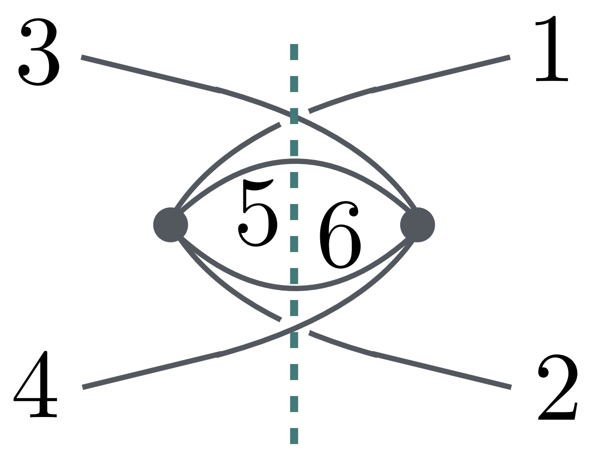

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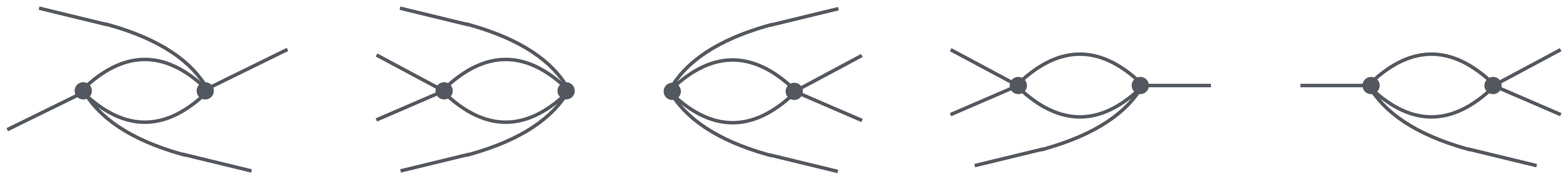

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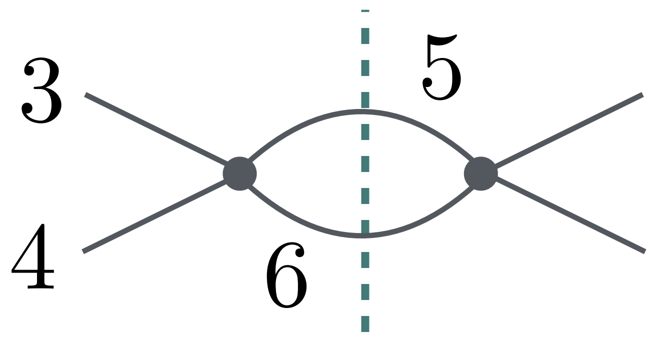

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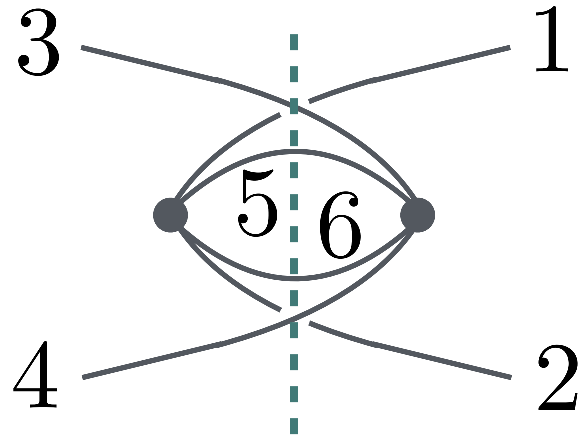

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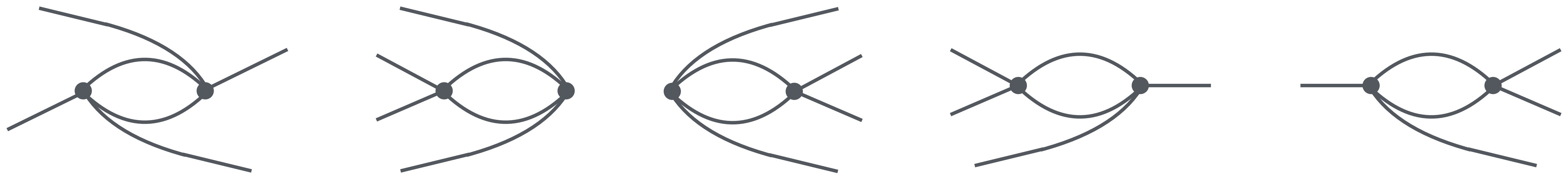

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$$\sim \lambda^2 \sum_k \frac{\Theta(2\omega_k - E_{max})}{\omega_k^2\, (-2\omega_k)} \int dx\, \phi^4$$

$$\omega_i > 0$$