Energy Correlators & Beyond

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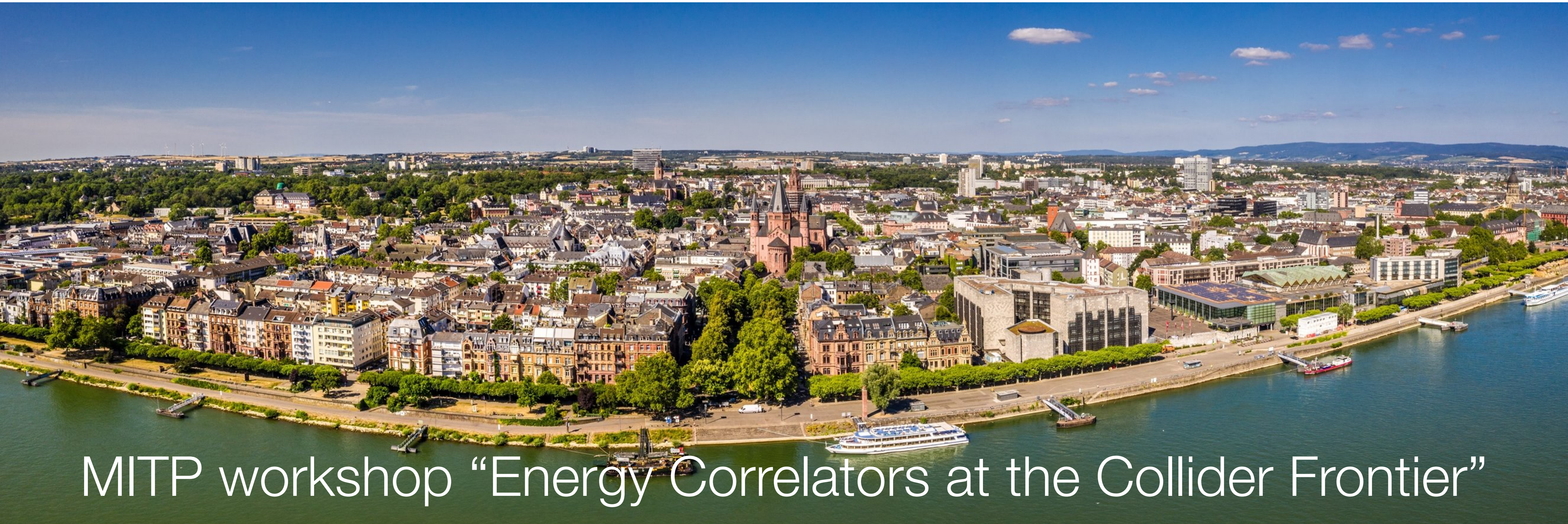


MITP workshop “Energy Correlators at the Collider Frontier”

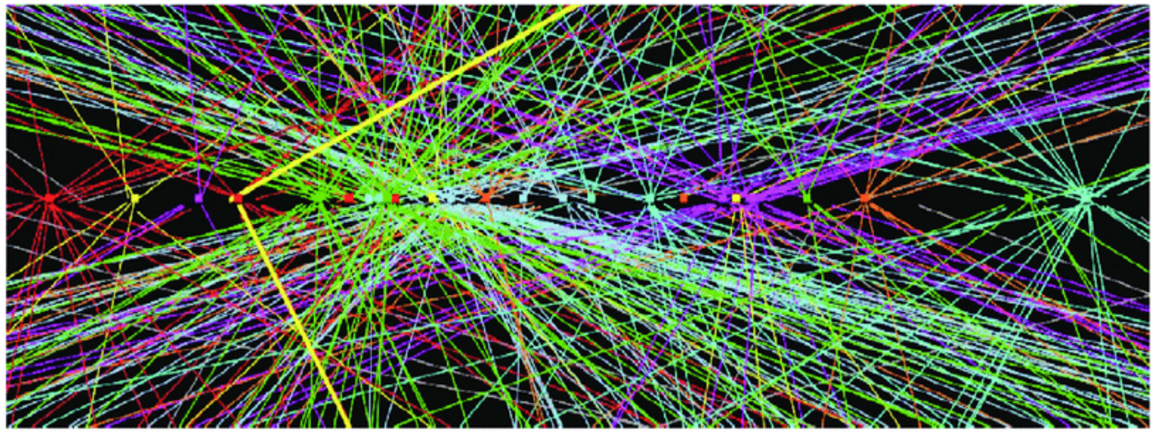
Outline

1. Energy correlators on tracks
2. Fast evaluation of energy correlators
3. & beyond

1. Energy correlators on tracks

1. Motivation for track-based measurements

- Pile-up removal
- Superior angular resolution
  → good for jet substructure



1. Motivation for track-based measurements

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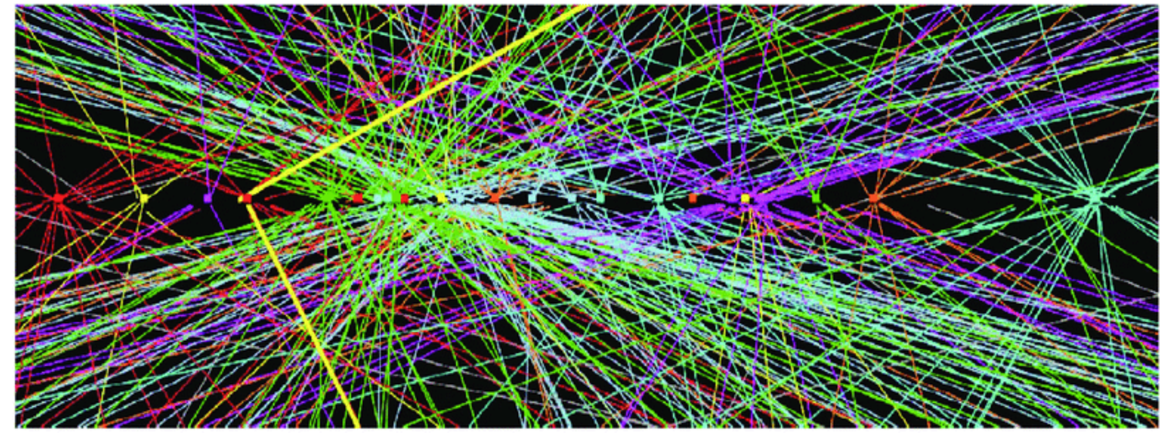
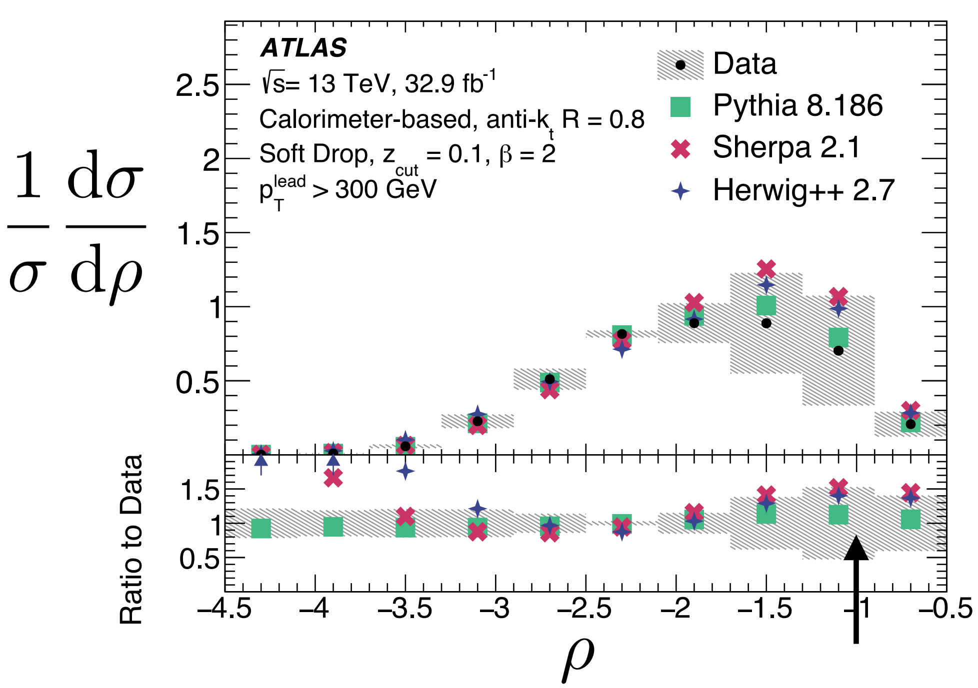


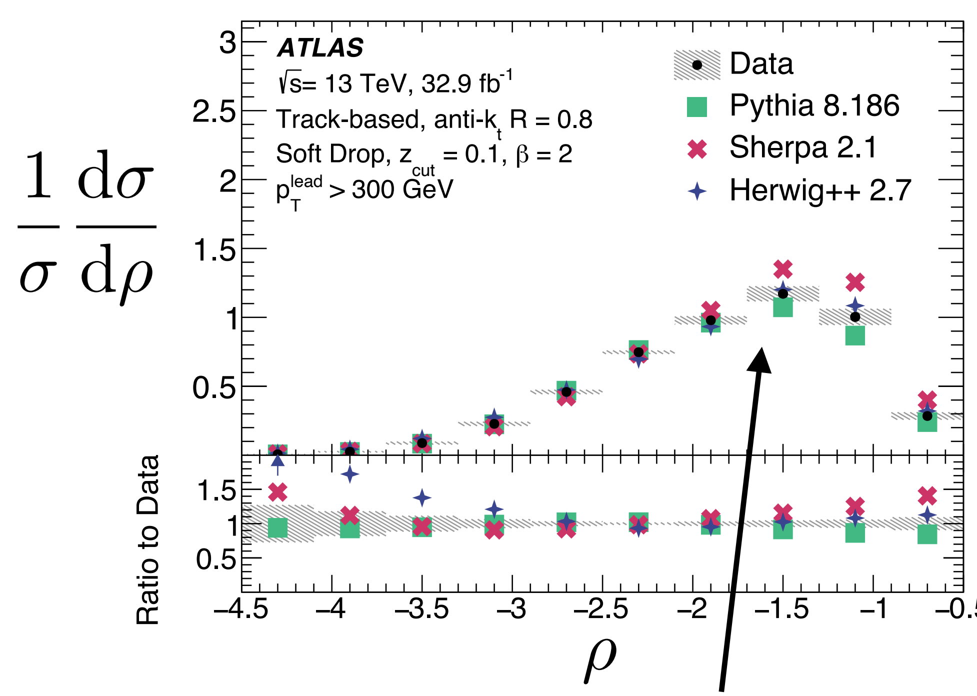
- E.g. for groomed $\rho = \ln(m^2/p_T^2)$

calorimeter:



Pile up

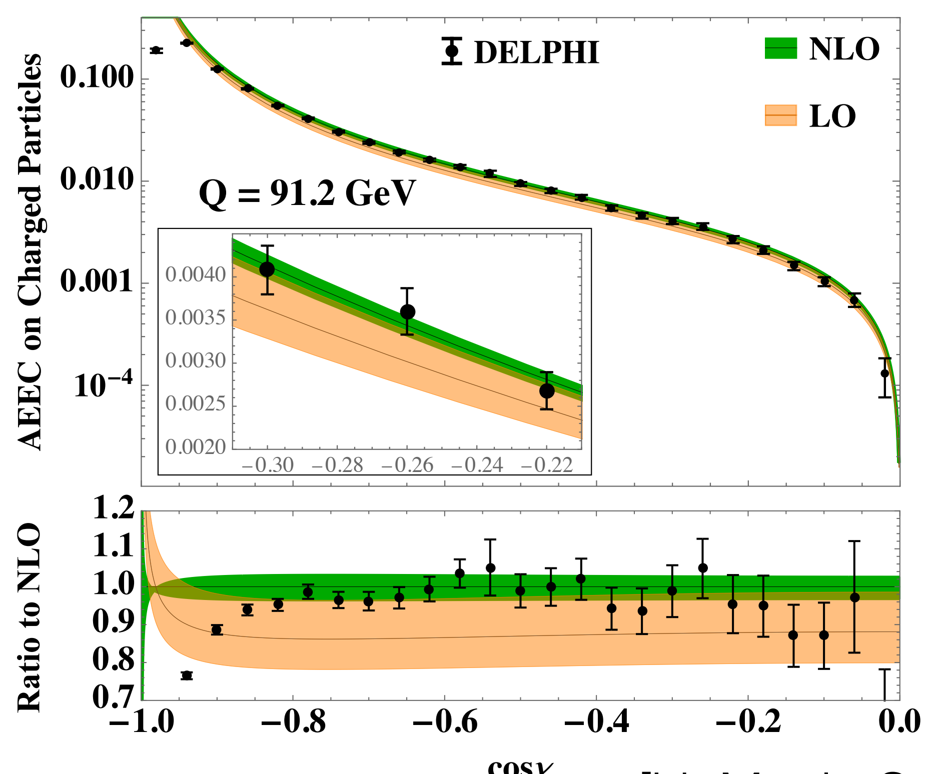
tracks:



Difference of parton showers
larger than exp. uncertainty

1. Main message on track-based predictions

- Track-based measurements are sensitive to hadronization.
- Instead of hadronization models in parton showers, track functions offer **systematically** improvable framework.
- Recently extended to $\mathcal{O}(\alpha_s^2)$ → high precision possible!
- For energy correlators, track functions are easy to implement (only moments)



[Li, Moult, Schrijnder van Velzen, WW, Zhu]

1. Some motivations for energy correlators

- Energy correlators probe correlations in energy flow:

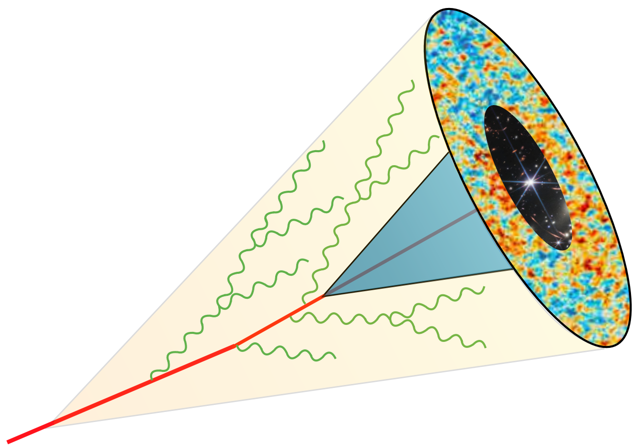
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \, \delta(\theta - \theta_{ij}) \sim \langle \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \rangle$$

[Basham, Brown, Ellis, Love]

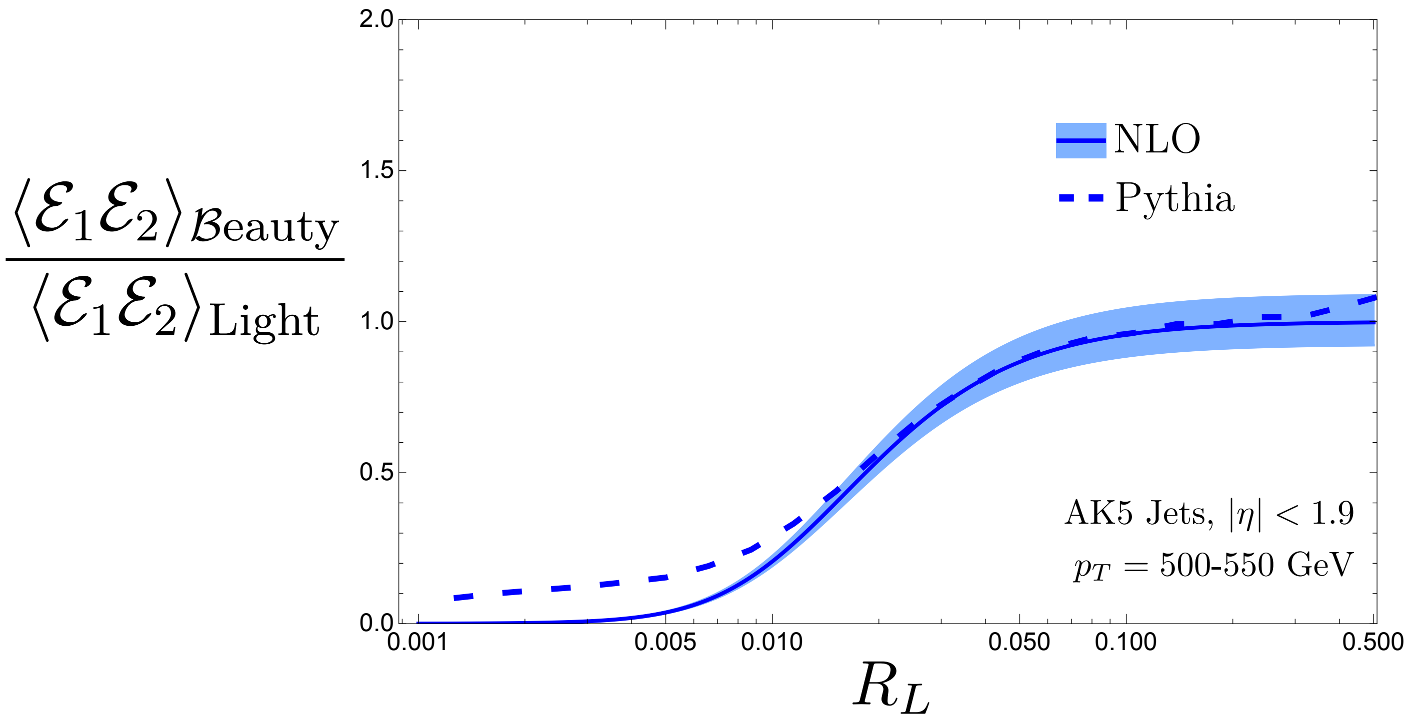
- **Theory:** energy weights suppress soft radiation → (simpler) collinear calculation with different (smaller) uncertainties.
- **Phenomenology:** separation of physics at different scales.
- **Applications:** α_s determination, hadronization, dead cone effect, quark-gluon plasma, top quark mass, …

1. Some pheno applications

- Dead cone



[Craft, Lee, Mecaj, Moult]



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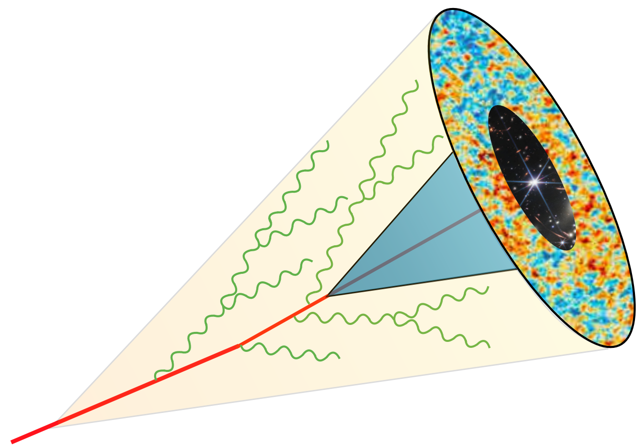

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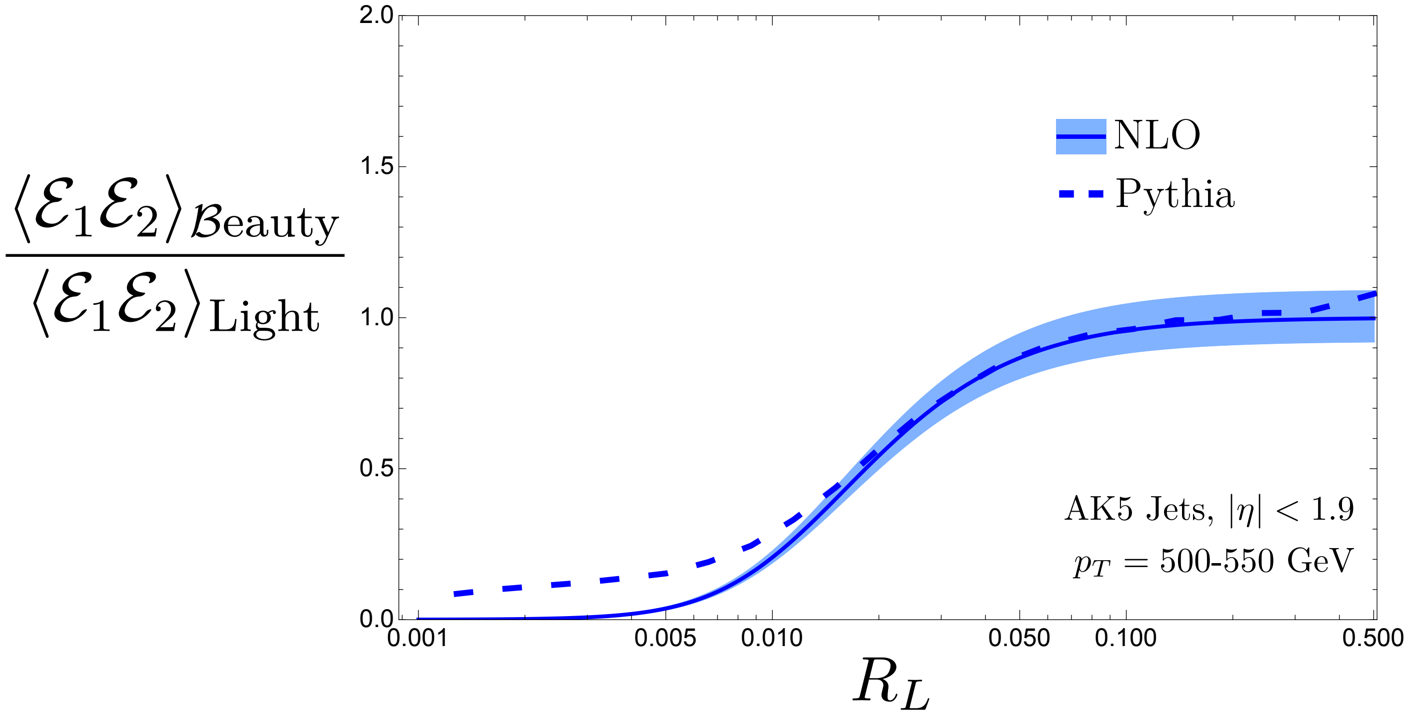
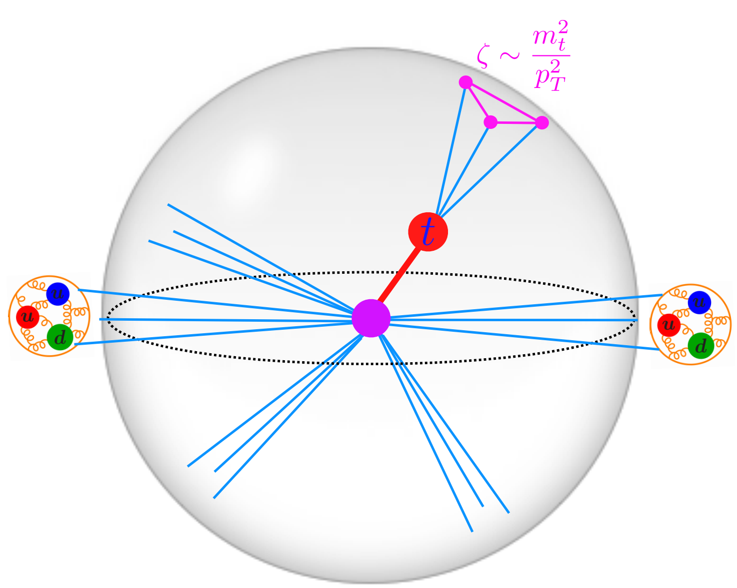
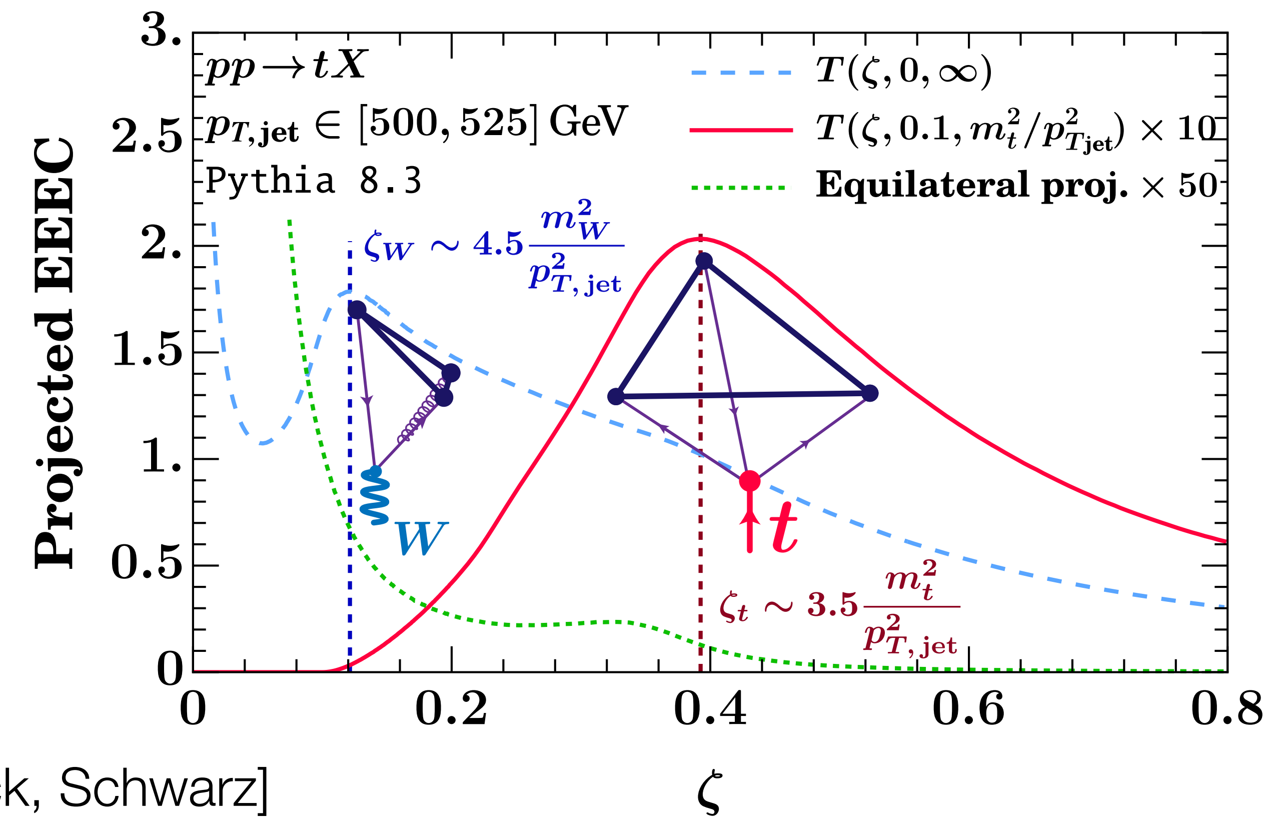


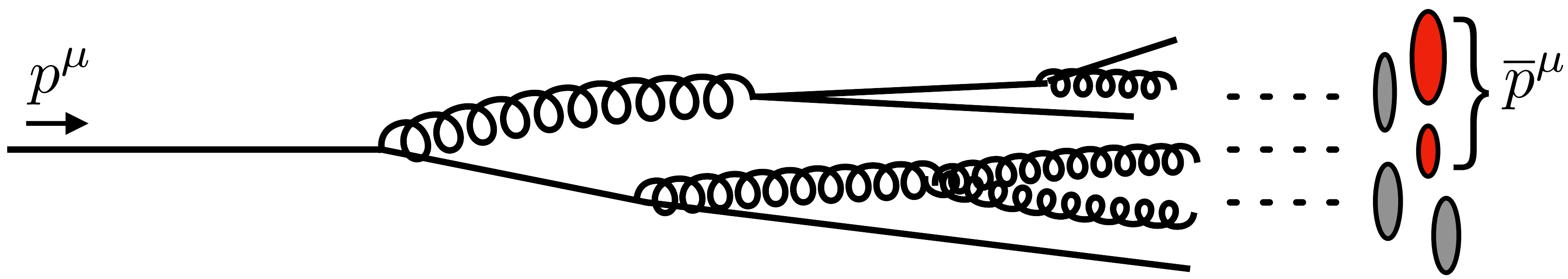
- Top quark mass





[Holguin, Moult, Pathak, Procura, Schöfbeck, Schwarz]

1. Calculations require track functions



[Chang, Procura, Thaler, WW]

- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton i converted to tracks, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\mathrm{QCD}})$
- Nonperturbative, process-independent function.

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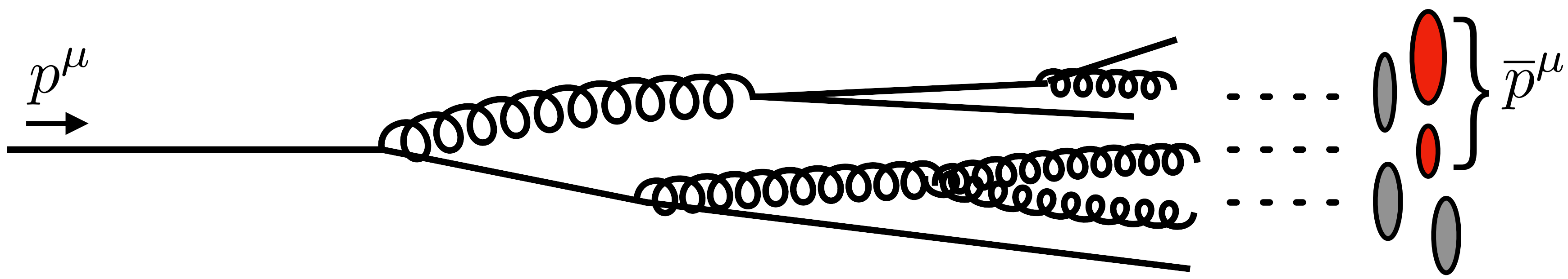

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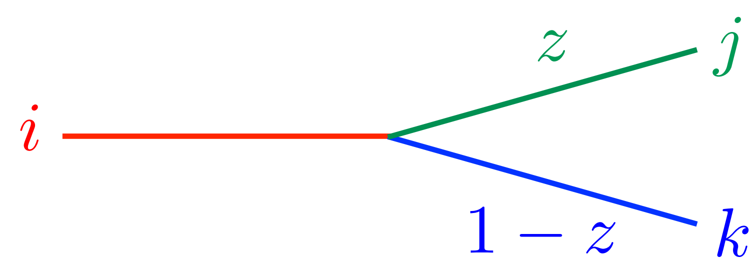
- $T_i(x, \mu)$ describes **total** momentum fraction x of initial parton i converted to tracks, i.e. $\bar{p}^\mu = x p^\mu + \mathcal{O}(\Lambda_{\mathrm{QCD}})$
- Nonperturbative, process-independent function.
- Conservation of probability: $\int_0^1 \mathrm{d}x\, T_i(x) = 1$
- Definition in light-cone gauge

all charged hadrons in X

$$T_q(x) = \int \mathrm{d}y^+ \mathrm{d}^2 y_\perp e^{ik^- y^+/2} \sum_X \delta\Big(x - \frac{\bar{p}^-}{k^-}\Big)$$
$$\times \frac{1}{2N_c} \mathrm{tr}\Big[\frac{\gamma^-}{2} \langle 0|\psi(y^+, 0, y_\perp)|X\rangle\langle X|\bar{\psi}(0)|0\rangle\Big]$$

1. Track function at order α_s



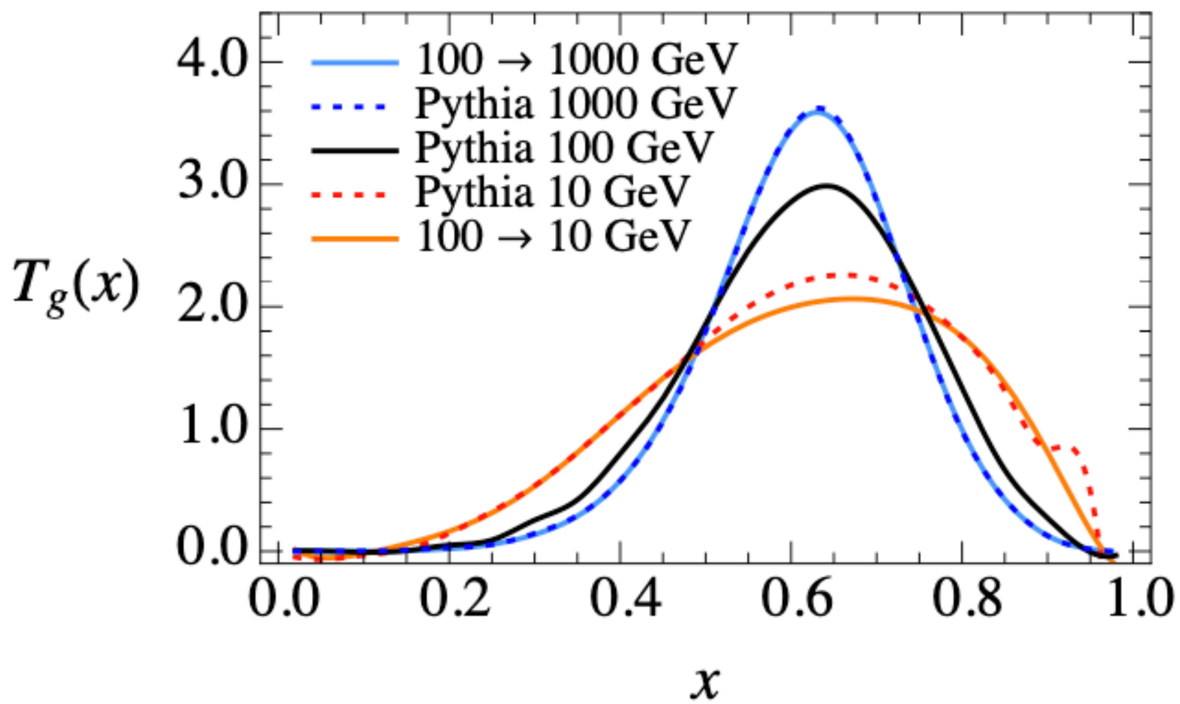
splitting function

$$T^{(1)}_{i,\mathrm{bare}}(x) = \sum_j \int \mathrm{d}z \left[\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right) P_{ji}(z)\right] \int \mathrm{d}x_1\, T^{(0)}_j(x_1,\mu)$$
$$\times \int \mathrm{d}x_2\, T^{(0)}_k(x_2,\mu)\,\delta\big[x - z x_1 - (1-z) x_2\big]$$

summing contribution of branches

- $1/\epsilon_{\mathrm{IR}}$ cancels against IR pole in partonic cross section.
- $1/\epsilon_{\mathrm{UV}}$ is renormalized, leads to evolution of track function.

1. Track function evolution at LO



$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} T_i(x,\mu) = \sum_{j,k} \int \mathrm{d}z\, \frac{\alpha_s}{4\pi} P_{ji}(z) \int \mathrm{d}x_1\, T_j(x_1,\mu) \int \mathrm{d}x_2\, T_k(x_2,\mu)$$

$$\times\, \delta[x - z x_1 - (1-z) x_2]$$

[Chang, Procura, Thaler, WW]

- Consistent with extraction from Pythia at different energies.

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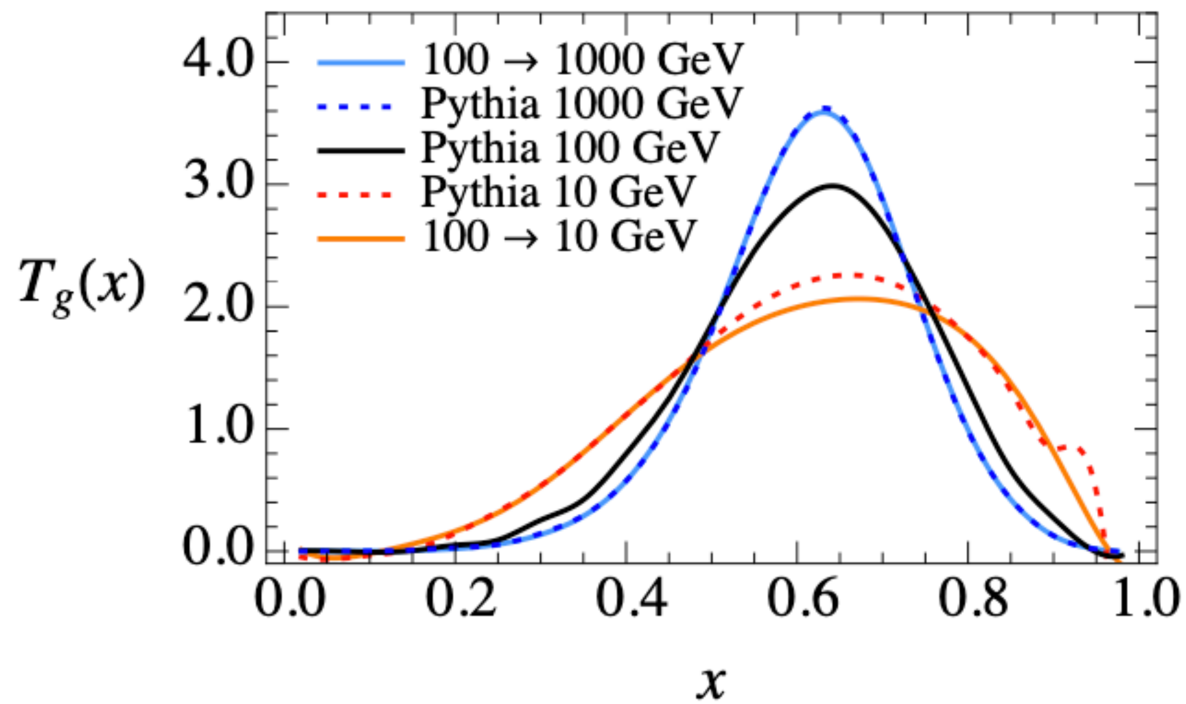

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$$\times\, \delta[x - zx_1 - (1-z)x_2]$$

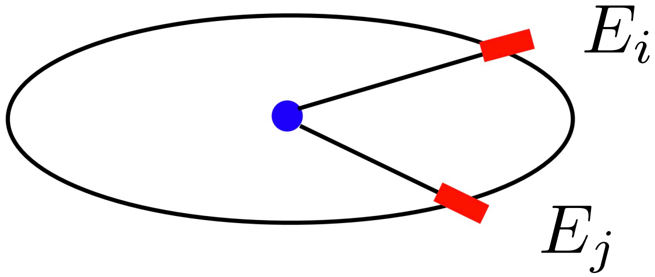
[Chang, Procura, Thaler, WW]

- Consistent with extraction from Pythia at different energies.
- Simplifies for **integer** moments: $x^N = [zx_1 + (1-z)x_2]^N$

binomial expansion

1. Track-based energy correlators

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \, \delta(\theta - \theta_{ij})$$



- Angular resolution of tracks is essential at small θ.
- Conversion to tracks is simple:

$$E_i \to \int \mathrm{d}x_i \, T_i(x_i) \, x_i E_i = T_i(1) E_i$$

[Chen, Moult, Zhang, Zhu]

- $\theta = 0$ involves $T_i(2)$, enters resummation of $\ln\theta$ for $\theta \ll 1$
- *N*-point energy correlators involve at most the *N*th moment

1. Track-based energy correlators beyond LO

- Beyond LO, there is a cancellation of IR poles between perturbative calculation and partonic track functions.

- Result is pretty simple. E.g. for finite part of gluon jet function

$$
j_g(z) = \delta(z) T_g(2) + \frac{\alpha_s}{4\pi}\left[\left(\frac{14}{5} C_A T_g(1)^2 + \frac{1}{5} n_f T_q(1)^2\right)\frac{1}{z}_+ + \delta(z)\left(-\frac{898}{75} C_A T_g(1)^2 - \frac{14}{25} n_f T_q(1)^2\right)\right]
$$

- Result on all particles is given by $T_i(n) \to 1$.

1. How to match onto track functions

- Lets take the example of PDFs, which we can write as:

$$\langle P|\mathcal{O}(Q)|P\rangle = C_i(Q) \otimes \langle P|O_i|P\rangle$$

hadronic cross section	partonic cross section	PDF (renormalized)

- Because $C_i(Q)$ holds independent of the states, we can calculate it by replacing $|P\rangle \to |q\rangle, |g\rangle$.

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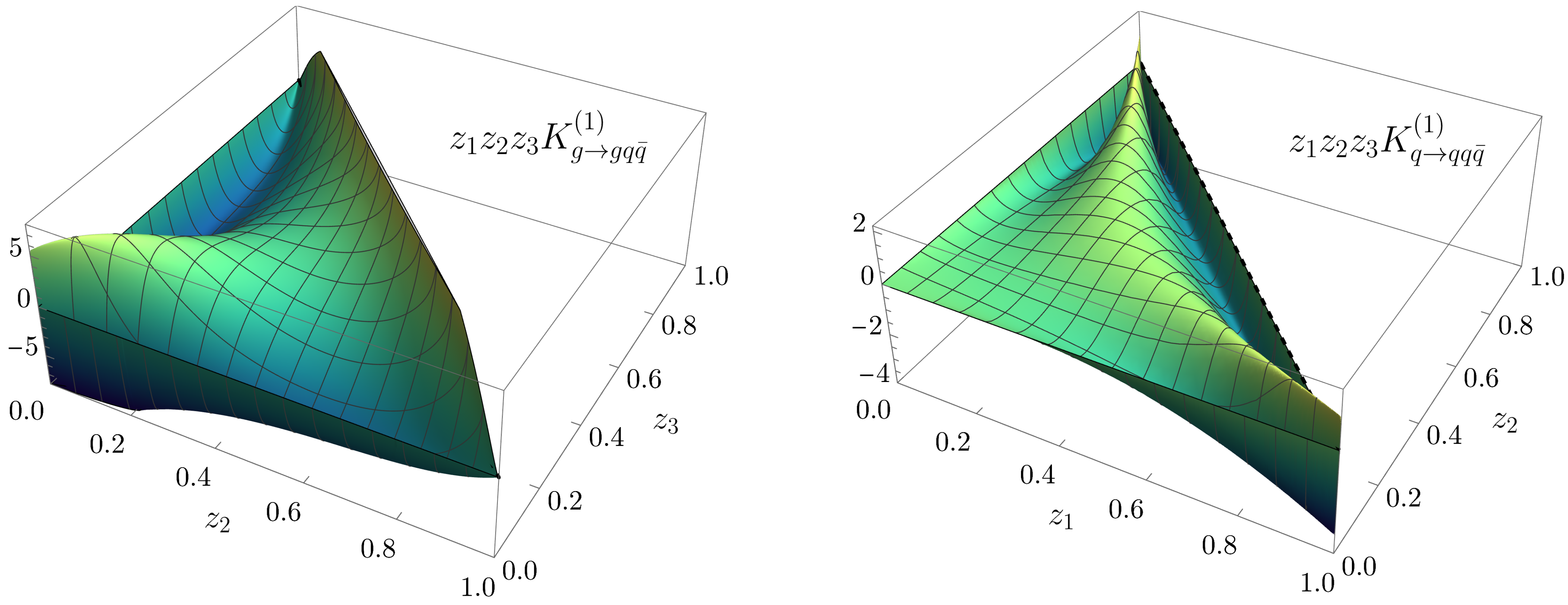
- Because $C_i(Q)$ holds independent of the states, we can calculate it by replacing $|P\rangle \to |q\rangle, |g\rangle$.
- Equivalently, we can calculate the LHS by "attaching" a tree-level PDF $f_i^{(0)}$ (partonic state). Using that in dim. reg.

$$f_i^{(0)} = f_i^{\text{bare}} = Z_{ij} \otimes f_j^{\text{ren}}$$

  the poles from Z must cancel against IR poles to give $C(Q)$.
- For track functions, you need to attach a $T_i^{(0)}$ to each parton i

1. Track function evolution at NLO



$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T(x,\mu) = a_s\Big[K^{(0)}_{1\to 1}\otimes T(x,\mu) + K^{(0)}_{1\to 2}\otimes TT(x,\mu)\Big]$$
$$+\, a_s^2\Big[K^{(1)}_{1\to 1}\otimes T(x,\mu) + K^{(1)}_{1\to 2}\otimes TT(x,\mu) + K^{(1)}_{1\to 3}\otimes TTT(x,\mu)\Big]$$

- Kernels are lengthy but available electronically.
- Projects onto DGLAP, also yields evolution of **multi-**hadron fragmentation functions

[Chen, Jaarsma, Li, Moult, WW, Zhu]

1. Track function evolution at NLO for moments

- Energy conservation implies evolution has symmetry $x \to x + a$
- Make manifest by using shift-invariant central moments

$$\Delta = T_q(1) - T_g(1), \quad \sigma_i(2) = T_i(2) - T_i(1)^2, \quad \ldots$$

leading to compact expressions:

$$\frac{\mathrm{d}\sigma_g(2)}{\mathrm{d}\ln\mu^2}\bigg|_{\alpha_s^2} = -\gamma_{gg}^{(1)}(3)\sigma_g(2) + \sum_i \bigg\{ -\gamma_{qg}^{(1)}(3)(\sigma_{q_i}(2) + \sigma_{\bar{q}_i}(2) + \Delta_{q_i}^2 + \Delta_{\bar{q}_i}^2)$$
$$+ T_F\Big[\Big(\frac{12413}{1350} - \frac{52}{45}\pi^2\Big)C_A + \frac{1528}{225}C_F - \frac{16}{25}n_f T_F\Big]\Delta_{q_i}\Delta_{\bar{q}_i}\bigg\}$$

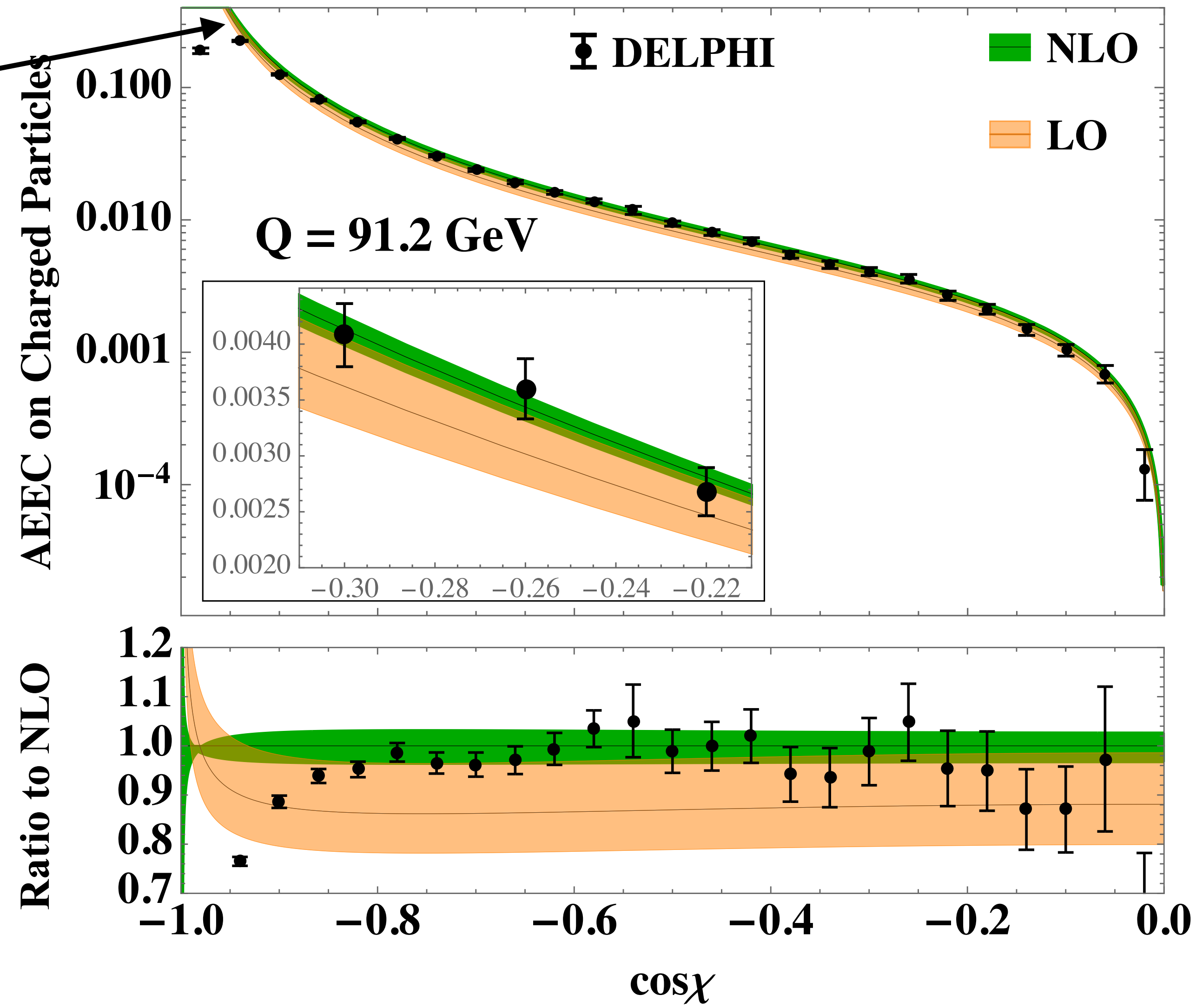
$$\frac{\mathrm{d}\sigma_g(3)}{\mathrm{d}\ln\mu^2}\bigg|_{\alpha_s^2} = -\gamma_{gg}^{(1)}(4)\sigma_g(3) + \sum_i \bigg\{ -\gamma_{qg}^{(1)}(4)(\sigma_{q_i}(3) + 3\sigma_{q_i}(2)\Delta_{q_i} + \Delta_{q_i}^3)$$
$$+ T_F\Big[\Big(-\frac{638}{45} + \frac{8}{3}\pi^2\Big)C_A - \frac{3803}{250}C_F\Big]\sigma_g(2)\Delta_{q_i}$$
$$+ T_F\Big[\Big(\frac{5321}{3000} - \frac{2}{5}\pi^2\Big)C_A + \frac{1523}{240}C_F - \frac{12}{25}n_f T_F\Big](\sigma_{q_i}(2)\Delta_{\bar{q}_i} + \Delta_{q_i}^2\Delta_{\bar{q}_i}) + (q \leftrightarrow \bar{q})\bigg\}$$

[Li, Moult, Schrijnder van Velzen, WW, Zhu]

1. Track-based EEC

- First $\mathcal{O}(\alpha_s^2)$ result for track-based measurement:

resummation needed

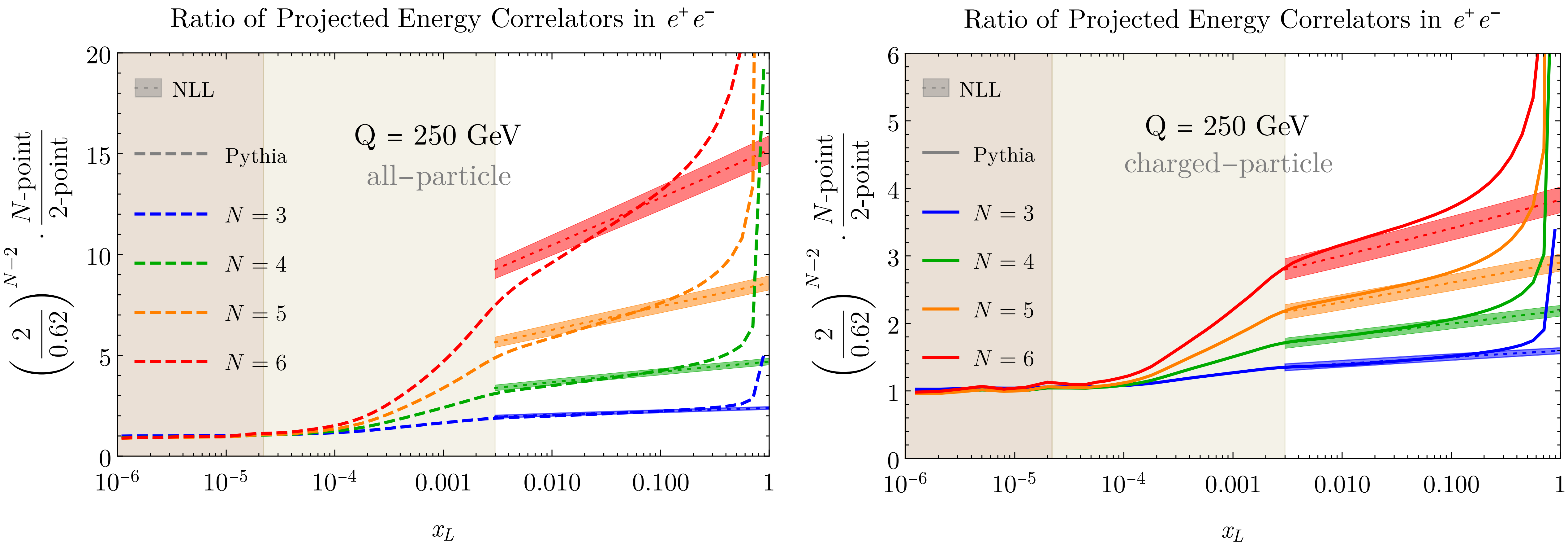


$$\mathrm{AEEC}(\cos\chi) = \mathrm{EEC}(\cos\chi) - \mathrm{EEC}(-\cos\chi)$$

- Uncertainty reduced at NLO, good agreement with data.

[Li, Moult, Schrijnder van Velzen, WW, Zhu]

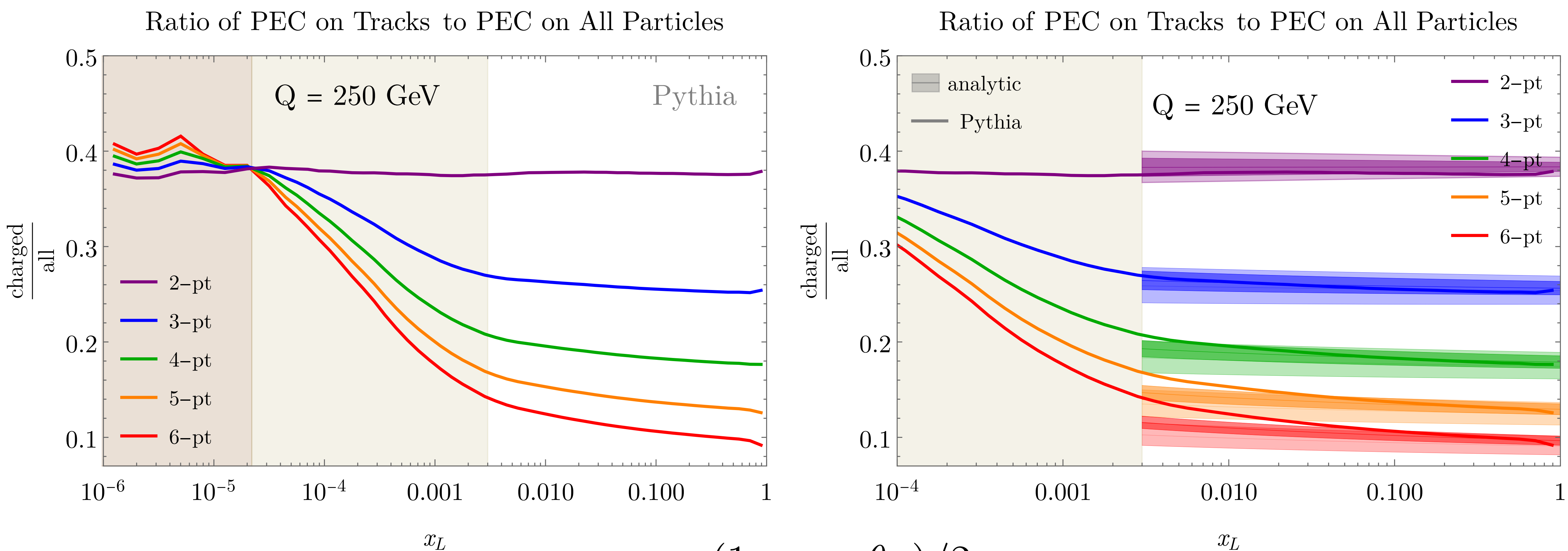
1. Projected *N*-point energy correlator



- All vs. charged particles are qualitatively similar, e.g. slope increases with *N*
- Quantitative difference is **calculable** with track functions!

[Jaarsma, Li, Moult, WW, Zhu]

1. Projected *N*-point energy correlator



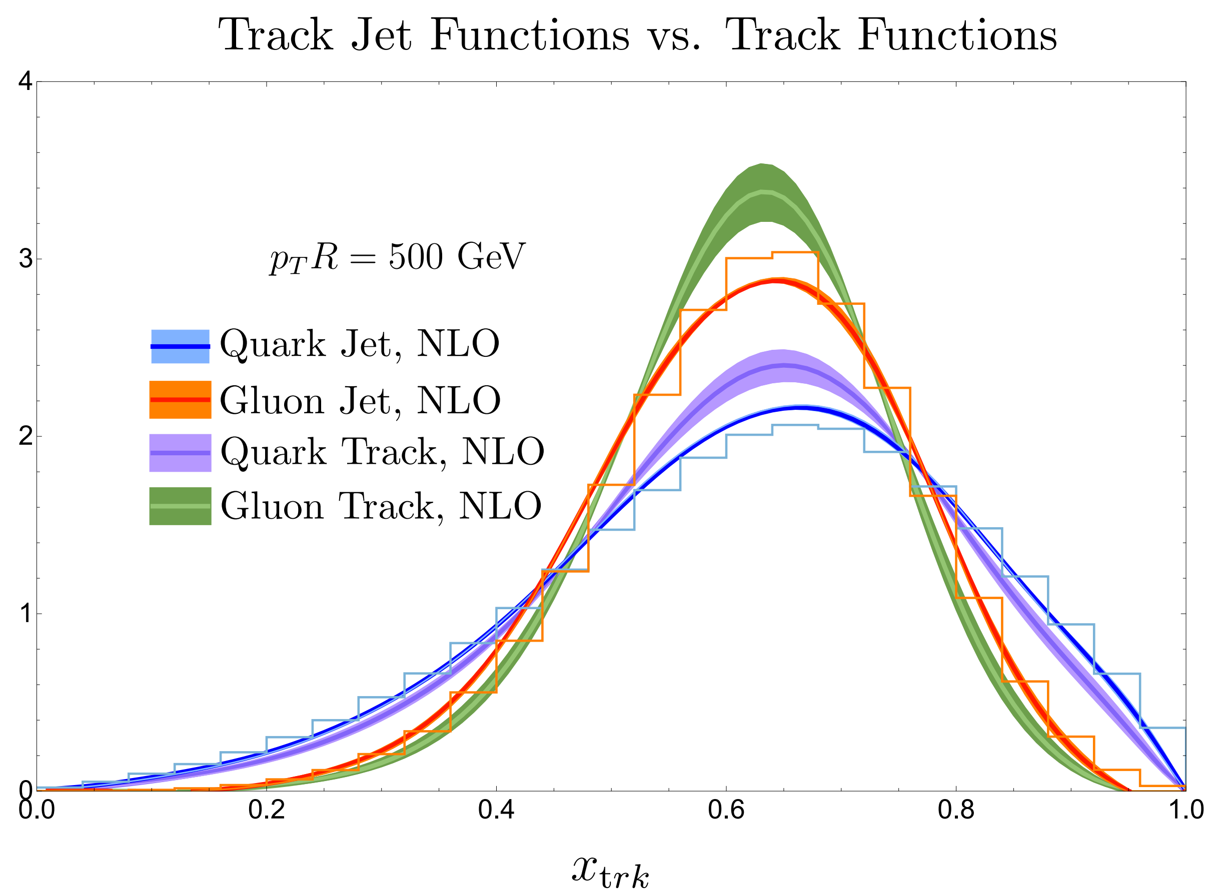
$$x_L = (1 - \cos\theta_L)/2$$

- Ratio of charged to all particles is constant for 2-point, but **not higher point.**
- In perturbative region ratio is $\approx T(m)T(N-M) \approx T(1)^N$
- In nonperturbative region ratio is $\approx (2/3)^2 \approx T_i(1)^2$ for all *N.*

[Jaarsma, Li, Moult, WW, Zhu]

1. Bonus: how to extract the track function

- The momentum fraction of charged particles in a jet is at LO the track function → use this to extract it!
- There are effects of hard scattering (quark vs. gluon) and jet formation:



[Lee, Moult, Ringer, WW]

2. Fast evaluation of energy correlators

2. The challenge

$$\frac{\mathrm{d}\sigma}{\mathrm{d}R_L} = \sum_{i_1,\ldots,i_N} \int \mathrm{d}\sigma \, \frac{p_{T,i_1} \cdots p_{T,i_N}}{P_T^N} \, \delta(R_L - \max\{R_{i_j i_k}\})$$

- Evaluating the (projected) *N*-point correlator for *M* particles, requires $\mathcal{O}(M^N)$ time. Prohibitive for *N* > 6.

2. The challenge

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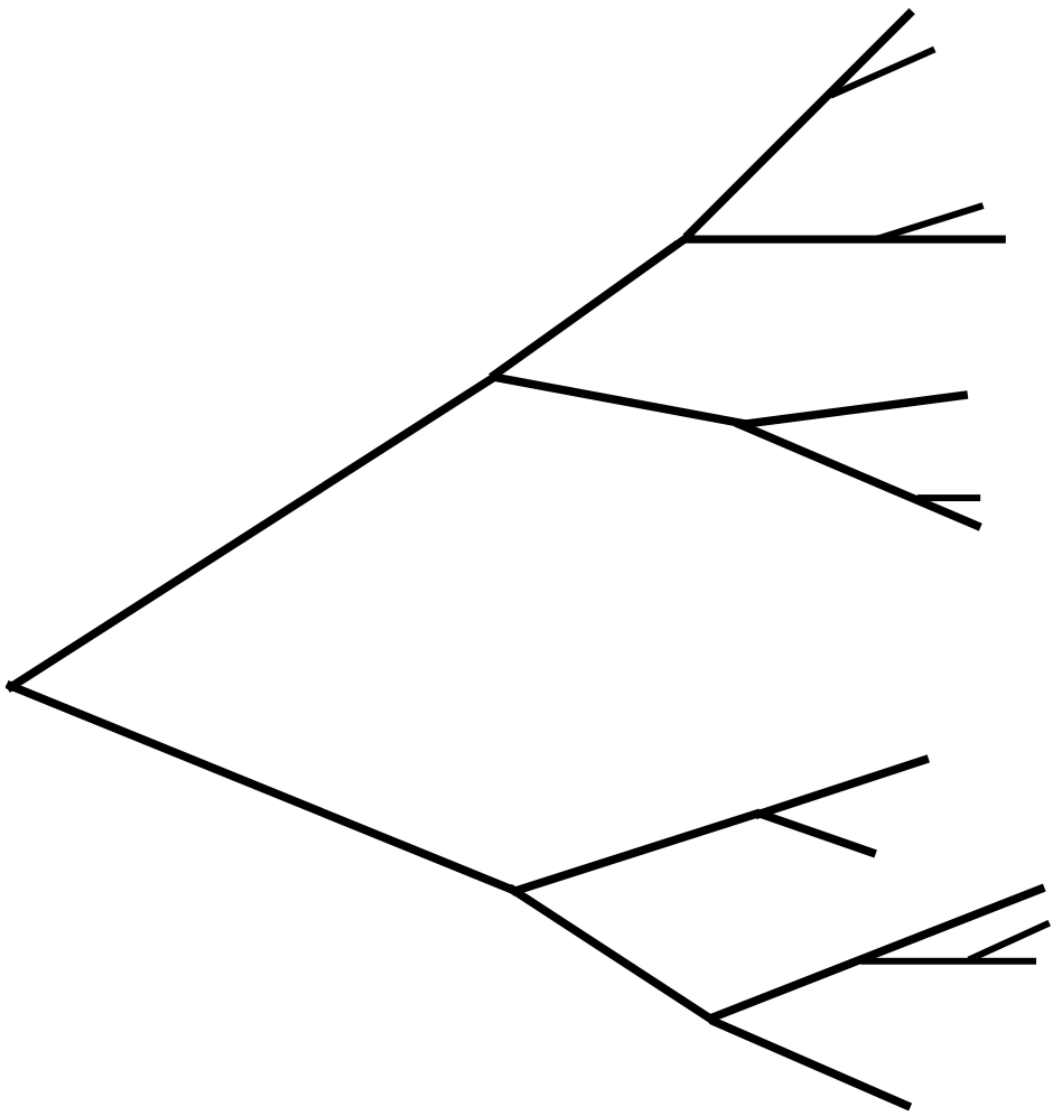
- Evaluating the (projected) *N*-point correlator for *M* particles, requires $\mathcal{O}(M^N)$ time. Prohibitive for *N* > 6.
- A simple solution is to (re)cluster using a (sub)jet radius r.
  - ✓ This speeds things up and gives reliable results for $R_L \gg r$.
  - x No results for $R_L \lesssim r$, and reducing r increases time.
- Our solution: use a dynamic subjet radius set by actual distances between particles in the event.

2. Our solution

- Recluster jet with C/A



[Budhraja, WW]

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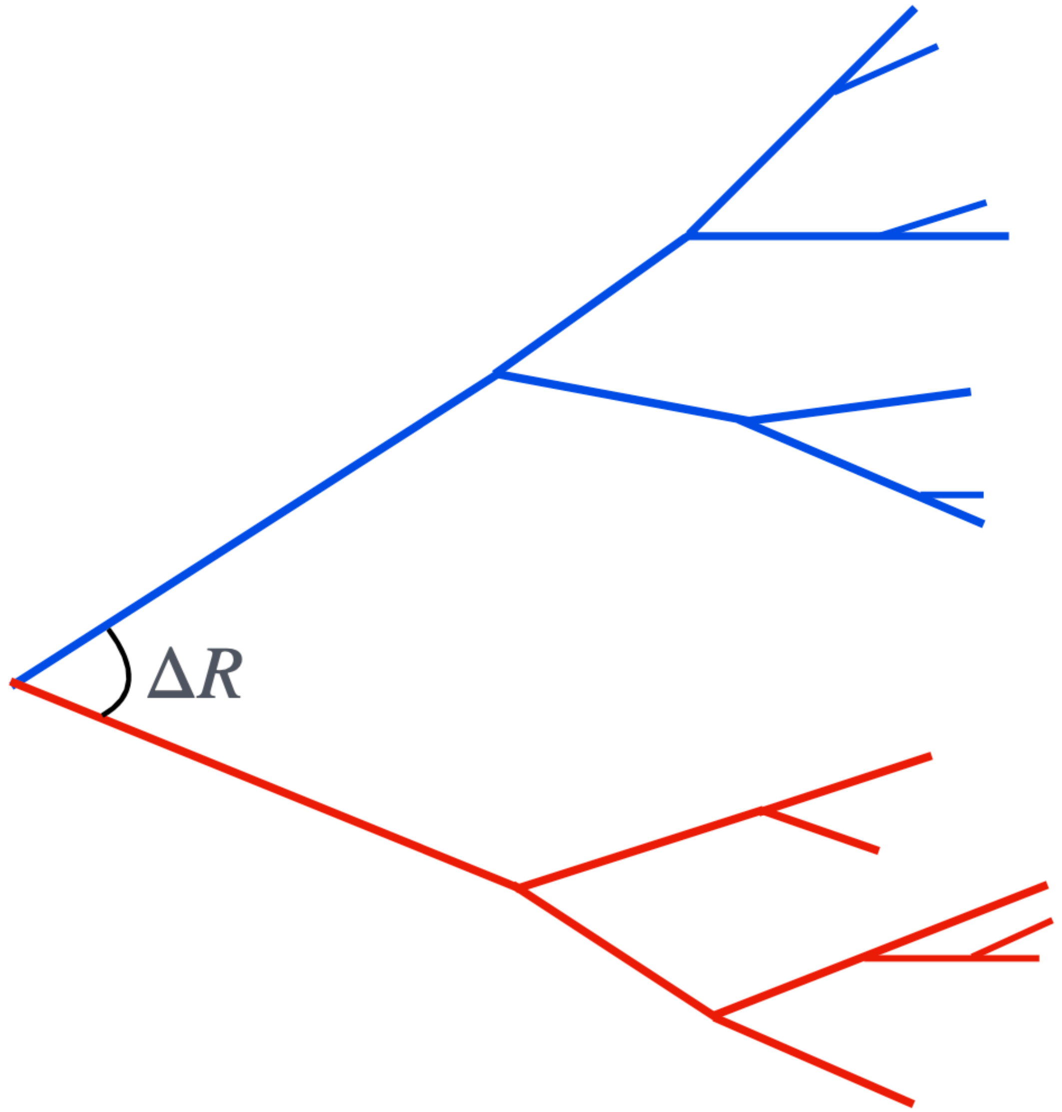
- Recluster jet with C/A
- Take first split, separation ΔR



[Budhraja, WW]

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- Recluster jet with C/A
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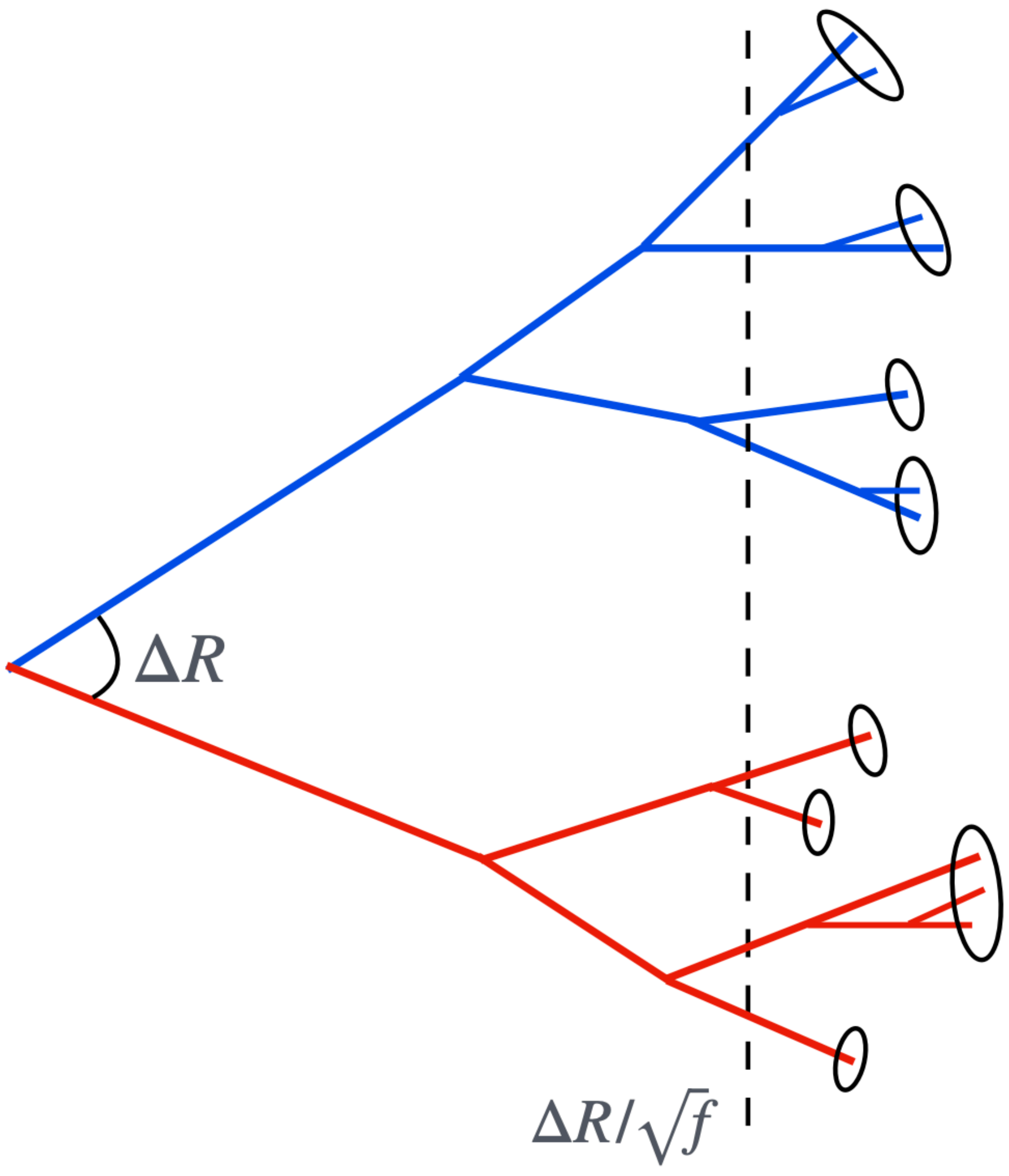
- Decluster until subjets with radius $r = \Delta R/\sqrt{f}$



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- Recluster jet with C/A
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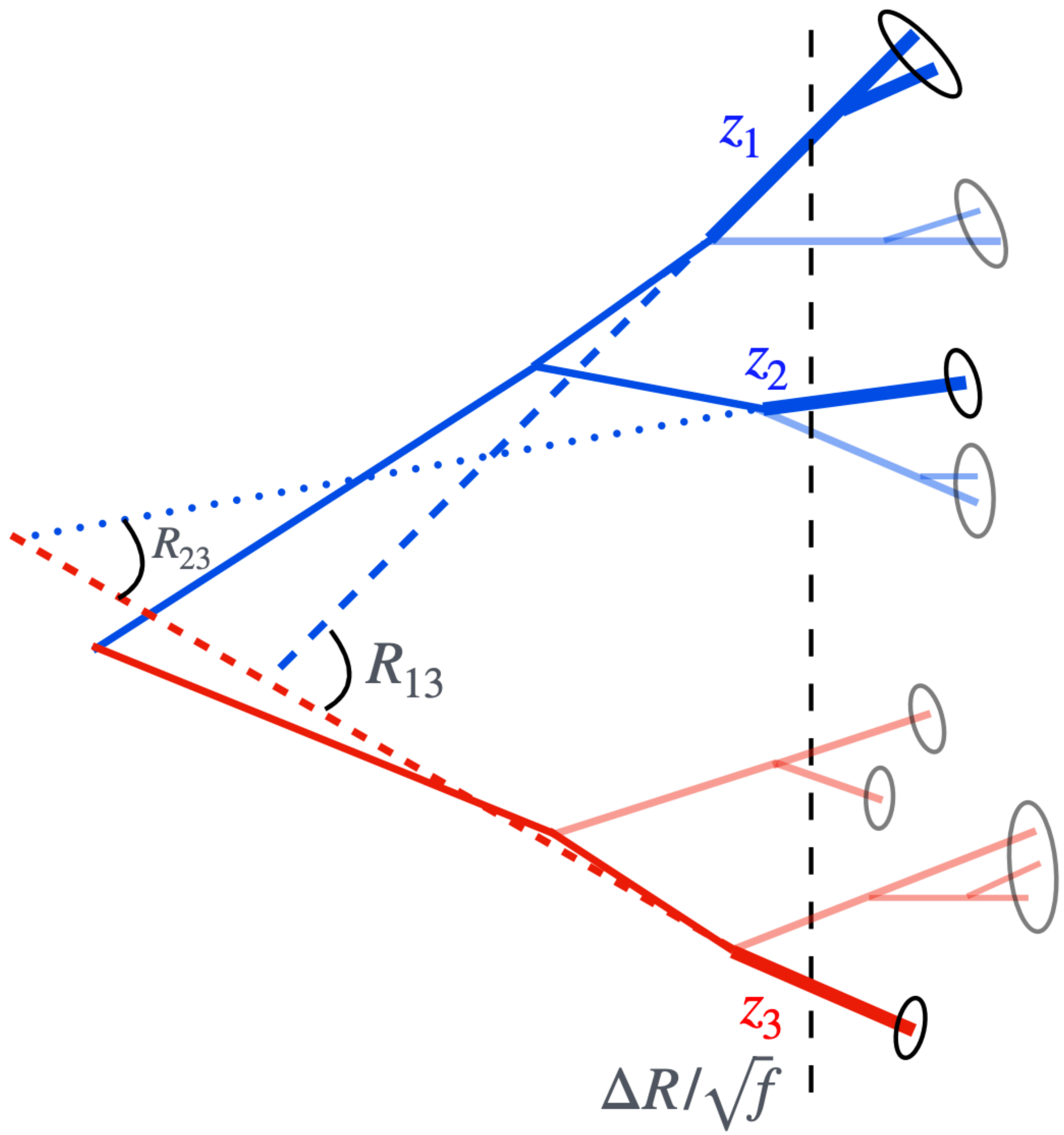
- Obtain correlator for terms involving both sides of the split:

$$\sum_i p_{T,i} \sum_j p_{T,j}\, \delta(R_L - R_{ij})$$



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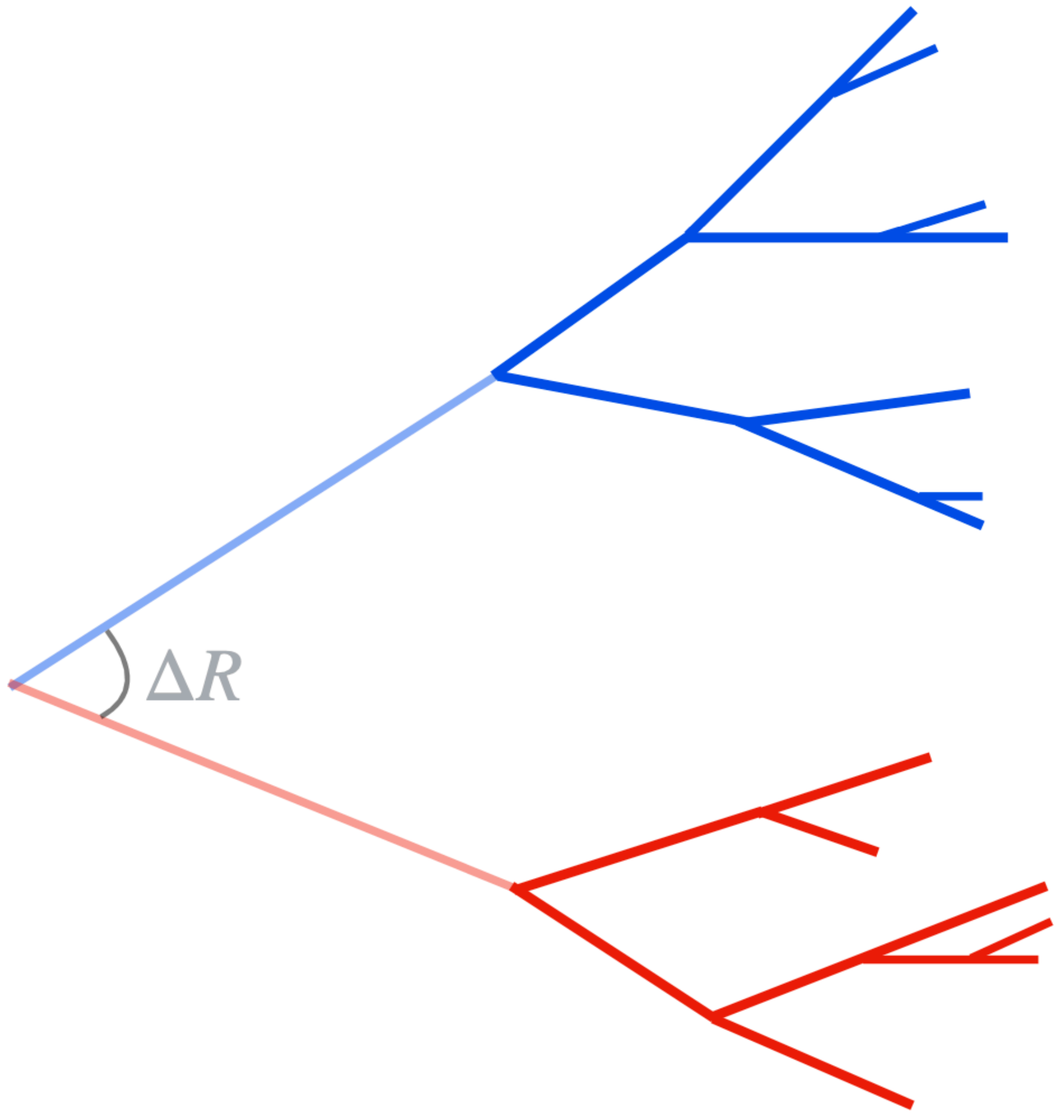
- Recurse on each branch to get correlations at smaller scales.



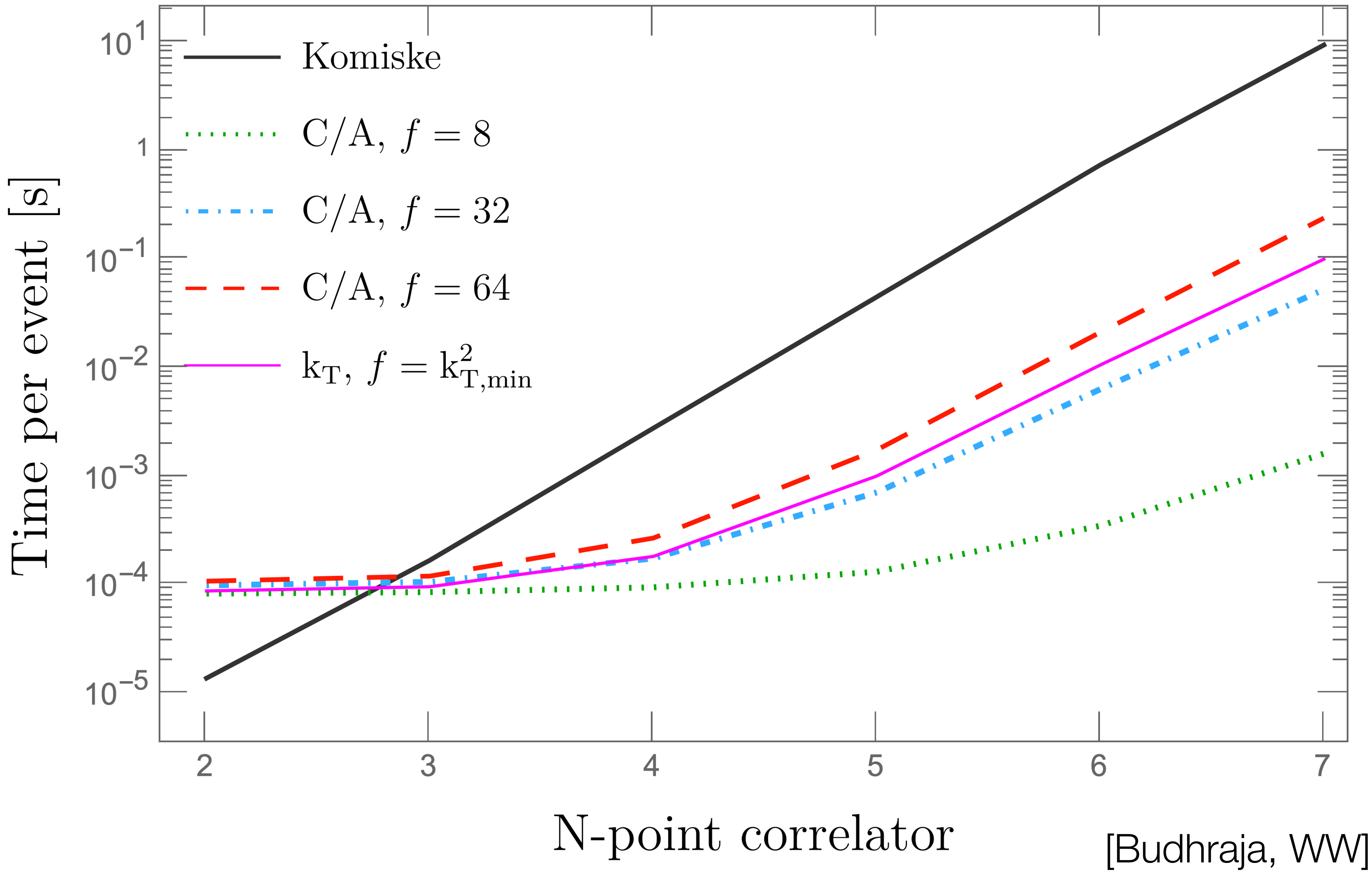
[Budhraja, WW]

2. Some comments

- These approximations preserve the sum rule exactly.
- Our default is to cluster with C/A with fixed resolution f. Surprisingly, k_T with $f = k^2_{T,\mathrm{min}}$ works well.
- We use MIT Open Data that utilizes CMS 2011A reprocessed data on jets. This is a sample of jets with

$$p_T \in [500, 550]\,\mathrm{GeV}, \quad |\eta| < 1.9$$

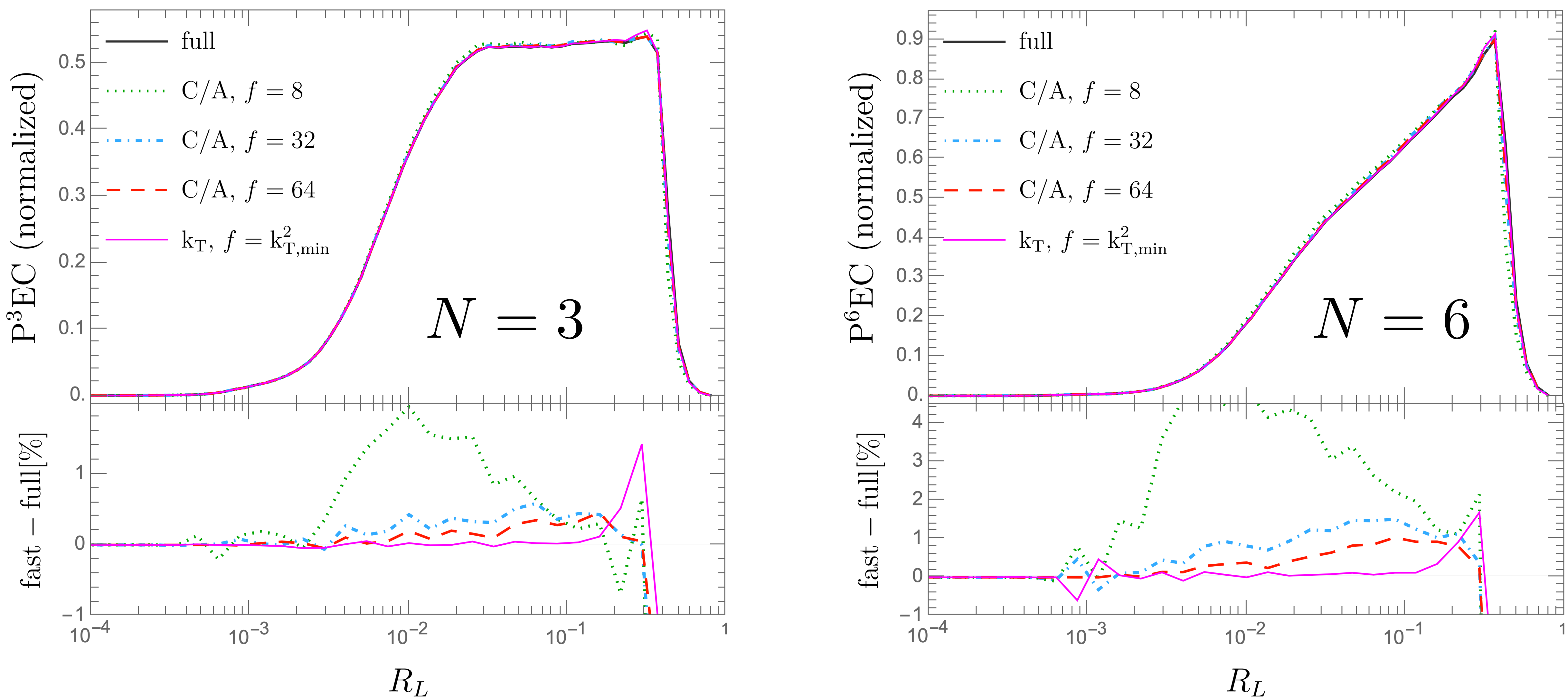
2. Performance: computation time



[Budhraja, WW]

- Up to **several orders of magnitude** speed up over Komiske's code, depending on desired accuracy.
- Time per event using single core on MacBook Air M1.

2. Performance: accuracy



- Going from 3- to 6-point increases error from 2 to 5% for $f = 8$
- k_T method has sub-permille accuracy across most of range.
- All methods perform less well at jet boundary.

2. FastEEC public code

- Available at:

  https://github.com/abudhraj/FastEEC/releases/tag/0.1

- Includes:
  - C++ code, only dependency is FastJet (for reclustering).
  - Example input and output files.
  - Mathematica notebook that converts output into a plot of our paper.

When the code is executed from the command line the following inputs are needed

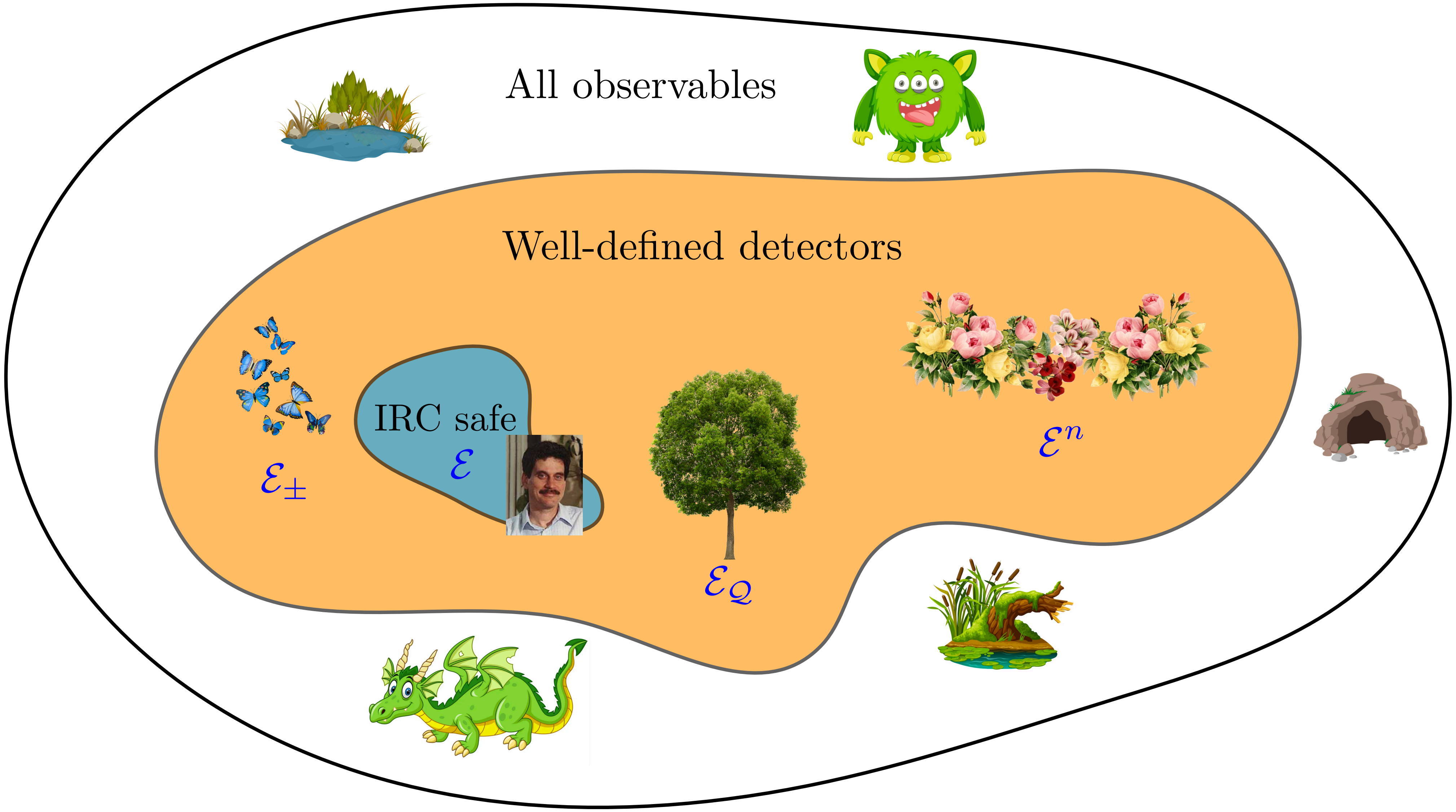
```
./eec_fast input_file events N f minbin nbins output_file
```

The above command line parameters require

> The input_file from which events should be read
> events > 0 is the number of events
> 1 < N < 9 specifies which point correlator to compute

3. & beyond

3. The space of observables



- Coming from Soft-Collinear Effective Theory/Jet Substructure, energy correlators are actually quite special.
- Most observables (e.g. jet mass) are sensitive to soft radiation → messy, grooming, …

3. Collinear sensitive observables

- Observables that are only sensitive to collinear radiation are usually associated with nonperturbative effects: PDFs, FFs…
- Momentum fractions of (sub)jets instead of hadrons can be considered [Dasgupta, Dreyer, Salam, Soyez; Kang, Ringer, Vitev; …]

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- Momentum fractions of (sub)jets instead of hadrons can be considered [Dasgupta, Dreyer, Salam, Soyez; Kang, Ringer, Vitev; …]
- Recoil-free jet axes also work, e.g. jet shape with respect to the winner-take-all axis [Neil, Scimemi, WW]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\theta} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \, \delta(\theta - \theta_{ij})$$

3. Collinear sensitive observables

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- What about correlations in observables other than angle?

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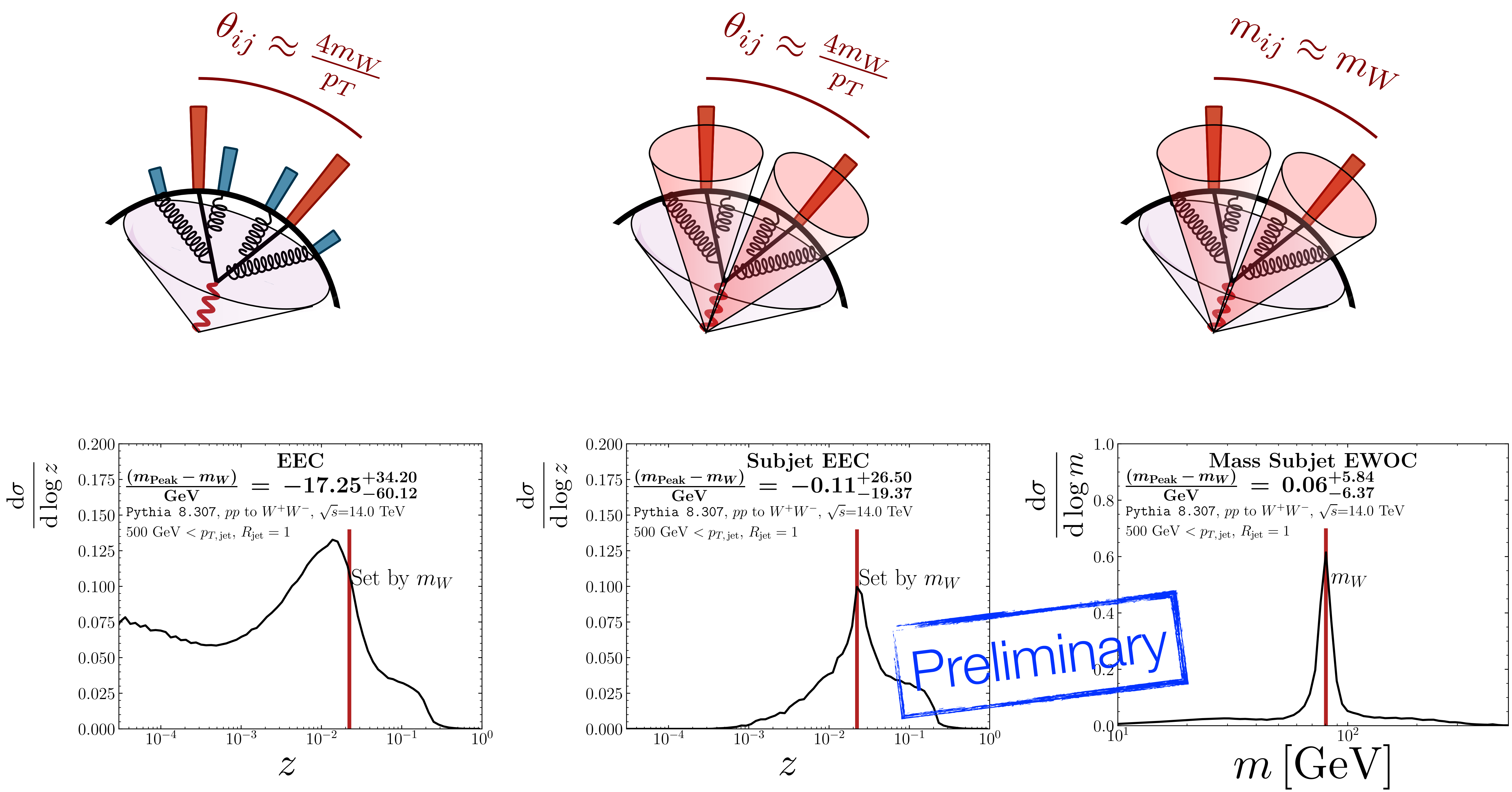
3. Energy Weighted Observable Correlations

- **Motivation:** instead of correlations in angle, we may prefer mass (of a resonance), formation time (QGP), …
- **Challenge:** collinear unsafe → regularize using (sub)jet radius.
- **Example:** mass EWOC

$$\frac{\mathrm{d}\sigma}{\mathrm{d}m} = \sum_{\text{subjets } i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \, \delta(m - m_{ij})$$

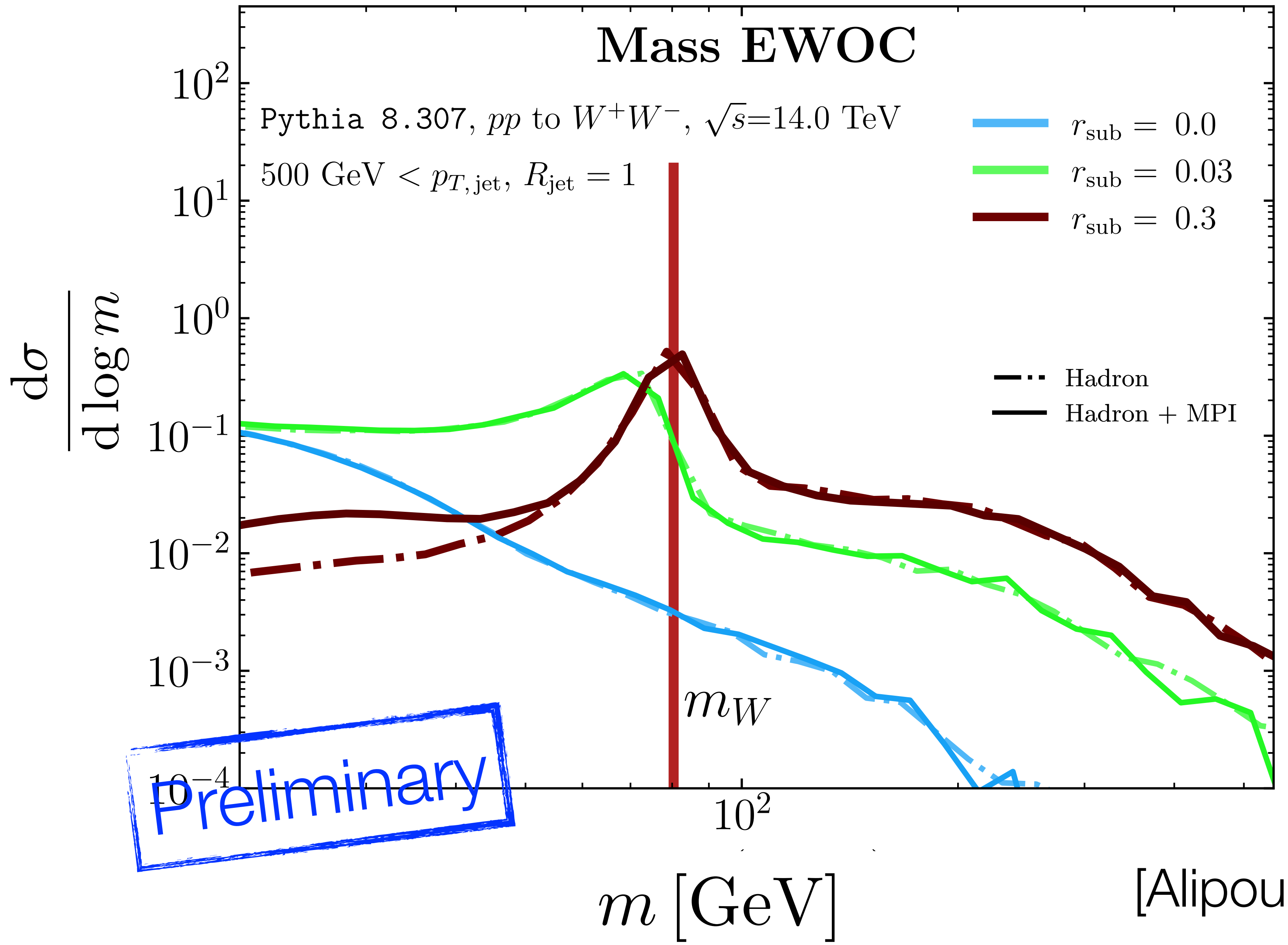
3. Mass EWOC for hadronic *W* decay

EEC | EEC on subjets | Mass EWOC



[Alipour-Fard, WW]

3. Choice of subjet radius



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- Choosing $r_{\mathrm{sub}} > 0$ is crucial for identifying W mass peak.

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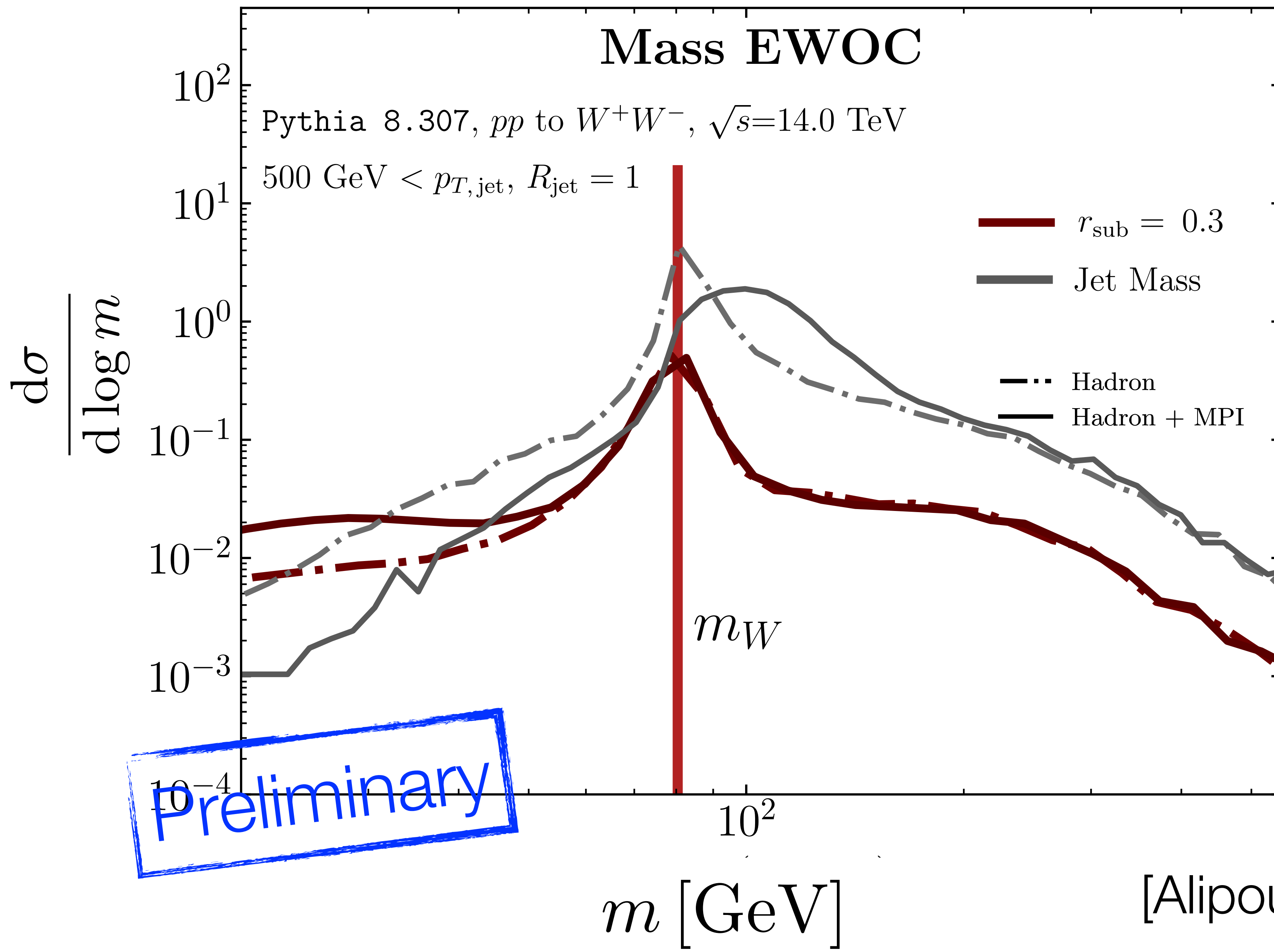
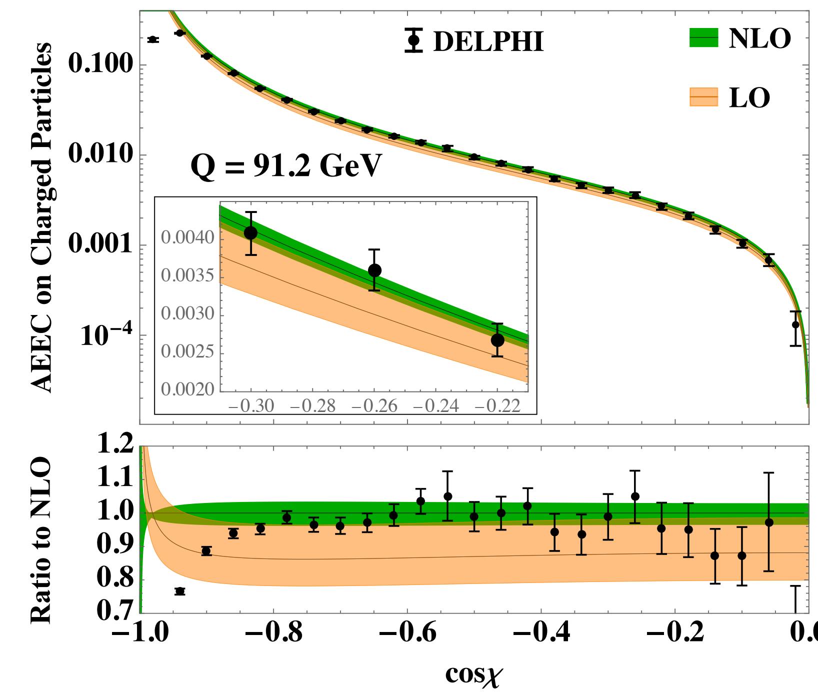


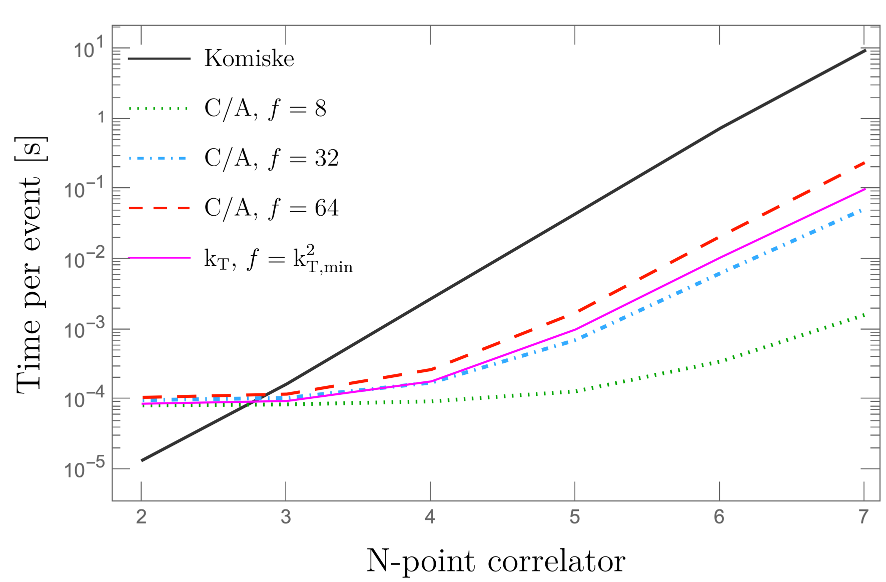
[Alipour-Fard, WW]

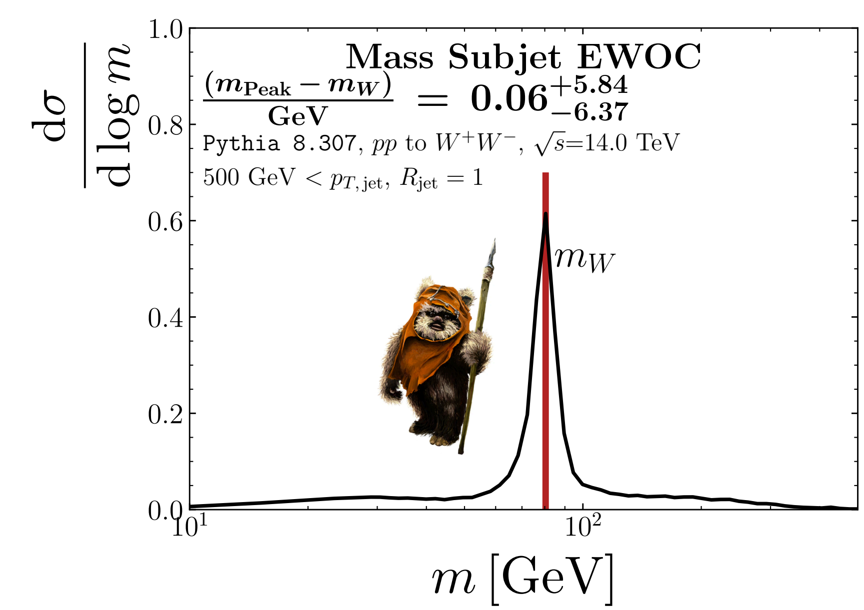
- Choosing $r_{\text{sub}} > 0$ is crucial for identifying W mass peak.
- Mass EWOC similar to jet mass, but much robuster to MPI.

Conclusions and outlook

- Tracks & energy correlators are perfect match:
  - Superior angular resolution essential.
  - Only track function moments enter.
- FastEEC: orders of magnitude speed up for higher-point energy correlator.
- Correlations in other observables are possible and promising (using subjets).







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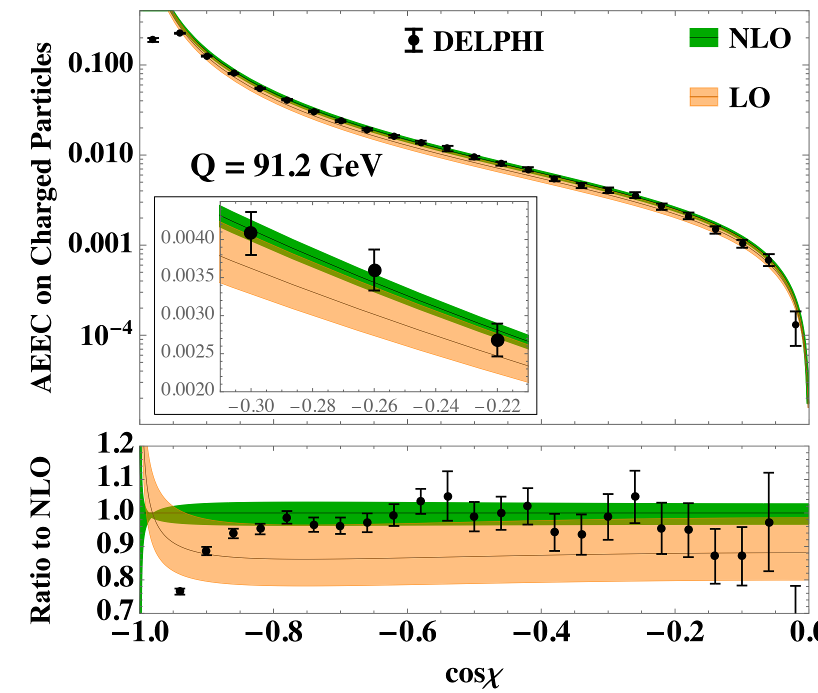

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Danke!