Light-ray operators in gauge and gravity theory: new infrared finite local observables

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based on 2012.01406 with A.Pokraka
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Content

Why light-ray operators?

Open questions from Ian Moult: (talk 08/07)

- What other matrix elements of lightray operators are well defined? Relation to IR finite S-matrices?
- What are the implications of asymptotic symmetry algebras? [Cordova, Shao] [Korchemsky, Sokatchev, Zhiboedov]

- Light-ray operators naturally arise by integrating Einstein equations at null infinity along the light-cone time ("charge densities")

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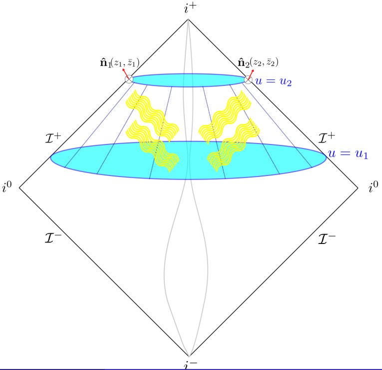
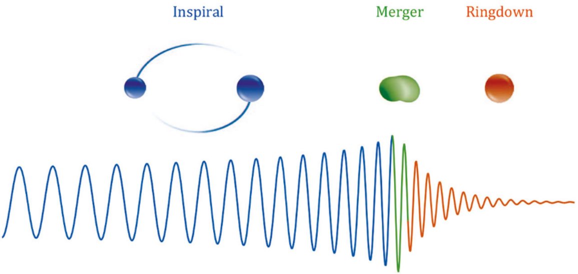
- Phenomenologically relevant for EM, QCD and gravity: one can define a notion of localized detector collecting quanta of radiation in a direction $\hat{\mathbf{n}}$



Gravitational waves: a theoretical lab for detector operators

- Recent interest in GWs emitted in the merging of compact binary systems (black holes, neutron stars, . . .): we need accurate waveform templates



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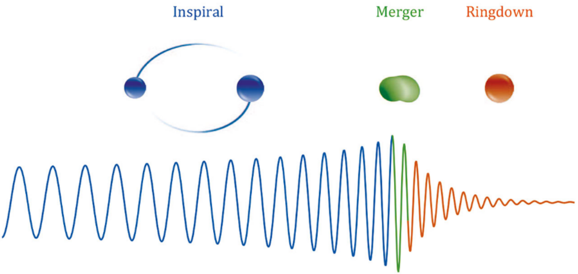

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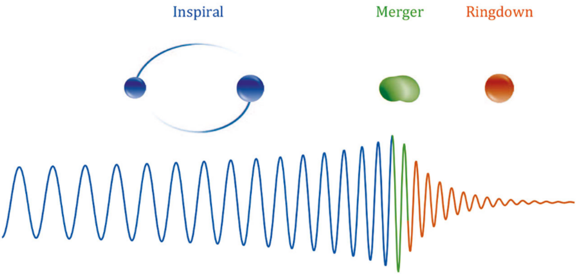

- The waveform is the most natural local observable! How about the gravitational energy and angular momentum flow?
- (Energy-energy) correlators are a smoking gun signature of quantum physics: classical physics ($\hbar \to 0$) implies factorization of $n-$pt correlation functions

Momentum and angular momentum flow in gauge theory

- Framework: Electromagnetism or Einstein gravity in $d = 4$

Momentum and angular momentum flow in gauge theory

- Framework: Electromagnetism or Einstein gravity in $d = 4$
- Let's start with QED. Using the standard mode expansion of gauge fields

$$\mathbb{A}_\mu(x) = \frac{1}{\sqrt{\hbar}} \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \left[a_\sigma(k) \varepsilon_\mu^{\sigma *}(k) e^{-i \frac{k \cdot x}{\hbar}} + a_\sigma^\dagger(k) \varepsilon_\mu^{\sigma}(k) e^{i \frac{k \cdot x}{\hbar}} \right],$$

local gauge-invariant momentum [Sveshnikov,Tkachov;Sterman,Korchemsky] and angular momentum [RG,Ilderton] flow $\mathcal{P}^\mu$, $\mathcal{N}^{\mu\nu}$ can be defined from $\mathcal{P}^\mu \sim \int \mathrm{d}^3 x \, \hat{\delta}^2 (\Omega - \Omega_{\hat{n}}) \, T^{\mu 0}$, $\mathcal{N}^{\mu\nu} \sim \int \mathrm{d}^3 x \, \hat{\delta}^2 (\Omega - \Omega_{\hat{n}}) \, x^{[\mu} T^{\nu]0}$,

$$\mathcal{P}^\mu = \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \hat{\delta}^2 (\Omega - \Omega_{\hat{n}}) \, k^\mu a_\sigma^\dagger(k) a_\sigma(k),$$

$$\mathcal{N}^{\mu\nu}_{\mathrm{QED}} = \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \hat{\delta}^2 (\Omega - \Omega_{\hat{n}}) \, \varepsilon_\sigma^\alpha(k) a_\sigma^\dagger(k) \left[(\mathcal{J}_{\mathrm{QED}})^{\mu\nu}_{\alpha\beta} \right] \varepsilon_\sigma^{*\beta}(k) a_\sigma(k),$$

$$(\mathcal{J}_{\mathrm{QED}})^{\mu\nu}_{\alpha\beta} = -i \eta_{\alpha\beta} k^{[\mu} \frac{\overleftrightarrow{\partial}}{\partial k_{\nu]}} - i \delta_\alpha^{[\mu} \delta_\beta^{\nu]},$$

Non-trivial gauge-invariance under $\varepsilon_\sigma^\alpha(k) \to \varepsilon_\sigma^\alpha(k) + \xi k^\alpha$!

Momentum and angular momentum flow in gravity theory

- In gravity, the story is similar by expanding the metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa \mathfrak{h}_{\mu\nu}$

$$\mathfrak{h}_{\mu\nu}(x) = \frac{1}{\sqrt{\hbar}} \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \left[a_\sigma(k) \varepsilon^{\sigma *}_{\mu\nu}(k) e^{-i\frac{k\cdot x}{\hbar}} + a^\dagger_\sigma(k) \varepsilon^{\sigma}_{\mu\nu}(k) e^{i\frac{k\cdot x}{\hbar}} \right],$$

to define local momentum and angular momentum flow [RG,Ilderton]

$$\mathcal{P}^\mu = \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \hat{\delta}^2(\Omega - \Omega_{\hat{n}})\, k^\mu a^\dagger_\sigma(k) a_\sigma(k),$$

$$\mathcal{N}^{\mu\nu}_{\mathrm{GR}} = \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \hat{\delta}^2(\Omega - \Omega_{\hat{n}})\, \varepsilon^{\alpha\alpha'}_\sigma(k) a^\dagger_\sigma(k) \left[(\mathcal{J}_{\mathrm{GR}})^{\mu\nu}_{\alpha\alpha'\beta\beta'} \right] \varepsilon^{*\beta\beta'}_\sigma(k) a_\sigma(k),$$

$$(\mathcal{J}_{\mathrm{GR}})^{\mu\nu}_{\alpha\alpha'\beta\beta'} = -i\eta_{\alpha\beta}\eta_{\alpha'\beta'} k^{[\mu} \frac{\overleftrightarrow{\partial}}{\partial k_{\nu]}} - 2i\eta_{\alpha'\beta'} \delta^{[\mu}_{\alpha} \delta^{\nu]}_{\beta}$$

The global $\mathcal{J}^{\mu\nu}_{\mathrm{GR}} = \int \mathrm{d}\Omega\, \mathcal{N}^{\mu\nu}_{\mathrm{GR}}$ for the classical two-body problem has received a lot of attention (zero energy gravitons are subtle!) [Damour; Manohar,Ridgway,Shen; Heissenberg,Russo,Veneziano], much less the local!

Connection with asymptotic light-ray operators

- Projecting the local operators on the tetrad basis $(n^\mu, \bar{n}^\mu, m^\mu, \bar{m}^\mu)$ at $\mathcal{I}^+$ in Bondi-like flat null coordinates $(u, r, z, \bar{z})$, we obtain the light-ray basis [Korchemsky,Oderda,Sterman;Hofman,Maldacena;Cordova,Shao,RG,Pokraka]

$$\mathcal{E}(\hat{\mathbf{n}}) = \int_{-\infty}^{+\infty} \mathrm{d}u \lim_{r\to\infty} r^2 T_{uu}(u, r, z_{\hat{\mathbf{n}}}, \bar{z}_{\hat{\mathbf{n}}}) \overset{\text{saddle-point}}{\sim} \bar{n}_\mu \mathcal{P}^\mu,$$

$$\mathcal{K}(\hat{\mathbf{n}}) = \int_{-\infty}^{+\infty} \mathrm{d}u\, u \lim_{r\to\infty} r^2 T_{uu}(u, r, z_{\hat{\mathbf{n}}}, \bar{z}_{\hat{\mathbf{n}}}) \overset{\text{saddle-point}}{\sim} \bar{n}_\nu n_\mu \mathcal{N}^{\mu\nu},$$

$$\mathcal{N}_z(\hat{\mathbf{n}}) = \int_{-\infty}^{+\infty} \mathrm{d}u \lim_{r\to\infty} r^2 T_{uz}(u, r, z_{\hat{\mathbf{n}}}, \bar{z}_{\hat{\mathbf{n}}}) \overset{\text{saddle-point}}{\sim} \bar{n}_\mu m_\nu \mathcal{N}^{\mu\nu},$$

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- These operators appear by integrating Einstein's equations near $\mathcal{I}^+$: natural interpretation of local measurements of energy and angular momentum!
- Light-ray algebra consistent with complexified Cordova-Shao algebra!

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$$\mathcal{P}^{\mu} = \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k) \hat{\delta}^2 \left(\Omega - \Omega_{\hat{n}}\right) k^{\mu} a_{\sigma}^{a\dagger}(k) a_{\sigma}^{a}(k),$$

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- Angular momentum flow? Subtle issue: mixing between radiative (on-shell) and coulombic (potential) modes of the field [Ashtekar,Bonga;RG,Pokraka]

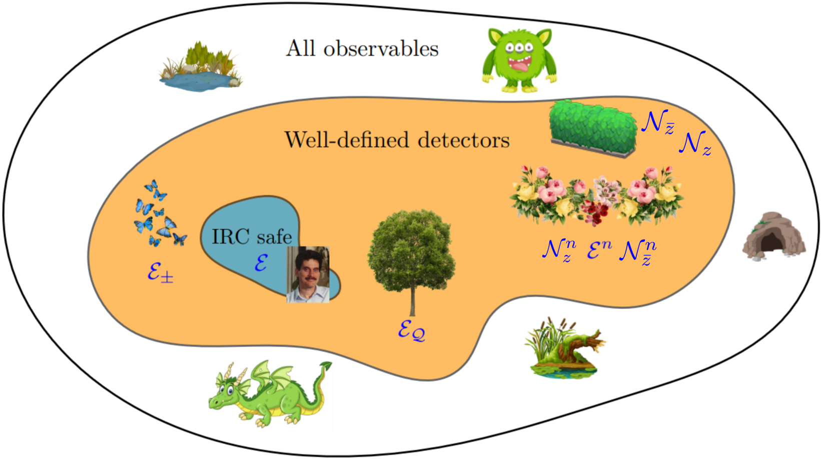
$$\mathcal{N}_z \sim \sum_{\sigma=\pm} \int \mathrm{d}\Phi(k)$$

$$\times \Big\{ \underbrace{\hat{\delta}^2 \left(z_{\hat{\mathbf{n}}} - z_{\hat{\mathbf{k}}}\right) a_{\sigma}^{a\dagger}(k) \left[i \overset{\leftrightarrow}{\partial}_{z_{\mathbf{n}}} \right] a_{\sigma}^{a}(k)}_{\sim\text{Orbital contribution}} + \underbrace{\sigma \left[i\partial_{z_{\hat{\mathbf{n}}}} \delta^2 \left(z_{\hat{\mathbf{n}}} - z_{\hat{\mathbf{k}}}\right)\right] a_{\sigma}^{a\dagger}(k) a_{\sigma}^{a}(k)}_{\sim\text{Spin contribution}}$$

$$+ 2\, g\, A_z^a(u = +\infty, z, \bar{z}) \underbrace{\left[-i f^{abc} a_{\sigma}^{b\dagger}(k) a_{\sigma}^{c}(k)\right]}_{\text{Density of color charge}} \Big\}$$

More to be understood! Potential implications for colliders?

Expanding the space of detector observables?



Compton scattering in EM and gravity: $\langle \mathcal{P}^\mu \rangle$ and $\langle \mathcal{N}^{\mu\nu} \rangle$

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- Classically, for a coherent state with real amplitude $A_{\pm}$ and a phase $\delta_{\pm}$

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- We find that $\langle \mathcal{P}^{\mu} \rangle$ encodes the amplitude of the outgoing wave while $\langle \mathcal{N}^{\mu\nu} \rangle$ is sensitive to both the amplitude and the phase

$$\langle \gamma | \mathcal{P}^{\mu} | \gamma \rangle = \sum_{\sigma=\pm} \int \frac{\mathrm{d}\omega\, \omega}{4\pi} \omega n^{\mu} |A_{\sigma}(\omega, \omega \hat{n})|^2 \ ,$$

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- What about quantum effects? N-pt correlators of local observables?

Gravitational event shapes: energy-energy correlators

- The action of the "shear-inclusive ANEC operator" at infinity [Wall]

$$\mathcal{E}(\hat{\mathbf{n}}) \prod_{\alpha \in \text{species}} |X_\alpha\rangle = \int_{-\infty}^{+\infty} \mathrm{d}u \left[\lim_{r\to\infty} r^2 T_{uu}^{\text{matter}} + \frac{1}{16\pi G} N_{\zeta\zeta} N^{\zeta\zeta} \right] \prod_{\alpha \in \text{species}} |X_\alpha\rangle$$

$$= \underbrace{\sum_{i=1}^{n} E_i \, \hat{\delta}^2 \left(\Omega_{\hat{k}_i} - \Omega_{\hat{n}} \right)}_{w_{\mathcal{E}}(\hat{\mathbf{n}})} \prod_{\alpha \in \text{species}} |X_\alpha\rangle .$$

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- Gravitational energy event shapes are defined as ($|\text{out}\rangle = S\,|\text{in}\rangle$) [RG,Pokraka]

$$\langle \text{out}|\, \mathcal{E}(\hat{\mathbf{n}})\, |\text{out}\rangle = \sum_X \langle \text{in}|\, S^\dagger \mathcal{E}(\hat{\mathbf{n}})\, |\mathbf{X}\rangle \langle \mathbf{X}|\, S\, |\text{in}\rangle = \sum_X w_{\mathcal{E}}(\hat{\mathbf{n}}) |\langle \mathbf{X}|\, S\, |\text{in}\rangle|^2,$$

where $\sum_X$ is the phase space integration over the final states.

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where $\sum_X$ is the phase space integration over the final states.

- Upshot: we can study $\langle \text{out}| \, \mathcal{E}(\hat{\mathbf{n}}_1)\mathcal{E}(\hat{\mathbf{n}}_2) \, |\text{out}\rangle$ not only in gauge theory but also in gravity: power-suppression guarantees infrared-finiteness

Factorization of gravity EEC in the classical limit $\hbar \to 0$

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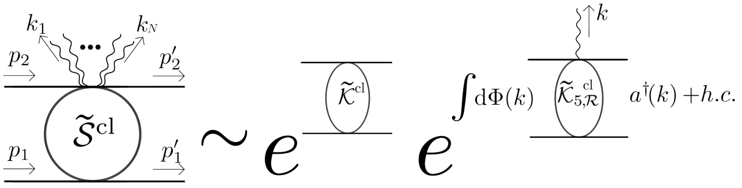

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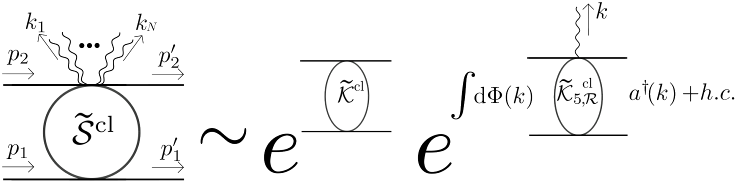

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- Upshot: quantum effects in EECs $\leftrightarrow$ deviation from GWs coherence! Leading quantum effects related to $\mathcal{M}_6^{(0)}(\phi\phi \to \phi\phi h h)$! [Britto,RG,Jehu]

Summary and future directions

- We provide novel gauge-invariant expressions for the momentum and angular momentum flow, defining a space of local observables in gauge and gravity

Theory	Global radiative observables	Local radiative observables
QED	$\sigma, \mathbb{K}^{\mu}_{\text{QED}}, \mathbb{J}^{\mu\nu}_{\text{QED}}$	$\frac{d\sigma}{d\Omega}, \mathcal{P}^{\mu}_{\text{QED}}, \mathcal{N}^{\mu\nu}_{\text{QED}}$
GR	$\sigma, \mathbb{K}^{\mu}_{\text{GR}}, \mathbb{J}^{\mu\nu}_{\text{GR}}$	$\frac{d\sigma}{d\Omega}, \mathcal{P}^{\mu}_{\text{GR}}, \mathcal{N}^{\mu\nu}_{\text{GR}}$

- We made contact with the light-ray basis in the BMS framework (via the saddle-point method at $r \to +\infty$), verifying Cordova-Shao algebra and highlighting some subtleties for the angular momentum flow in QCD
- We study the properties of $\langle \mathcal{P}^{\mu} \rangle$ and $\langle \mathcal{N}^{\mu\nu} \rangle$: classical factorization of correlation functions, relation with the amplitude and phase of the waveform, singularity structure at high-energy (better than global operators!)
- Lots to do: Angular momentum in QCD? IR-finite S-matrix for gravity and related observables? Quantum effects for gravity correlators of local observables (probing non-coherence of GWs)? Collider OPE tools for gravity?