# Power Corrections to Energy Correlators from Light-ray OPE

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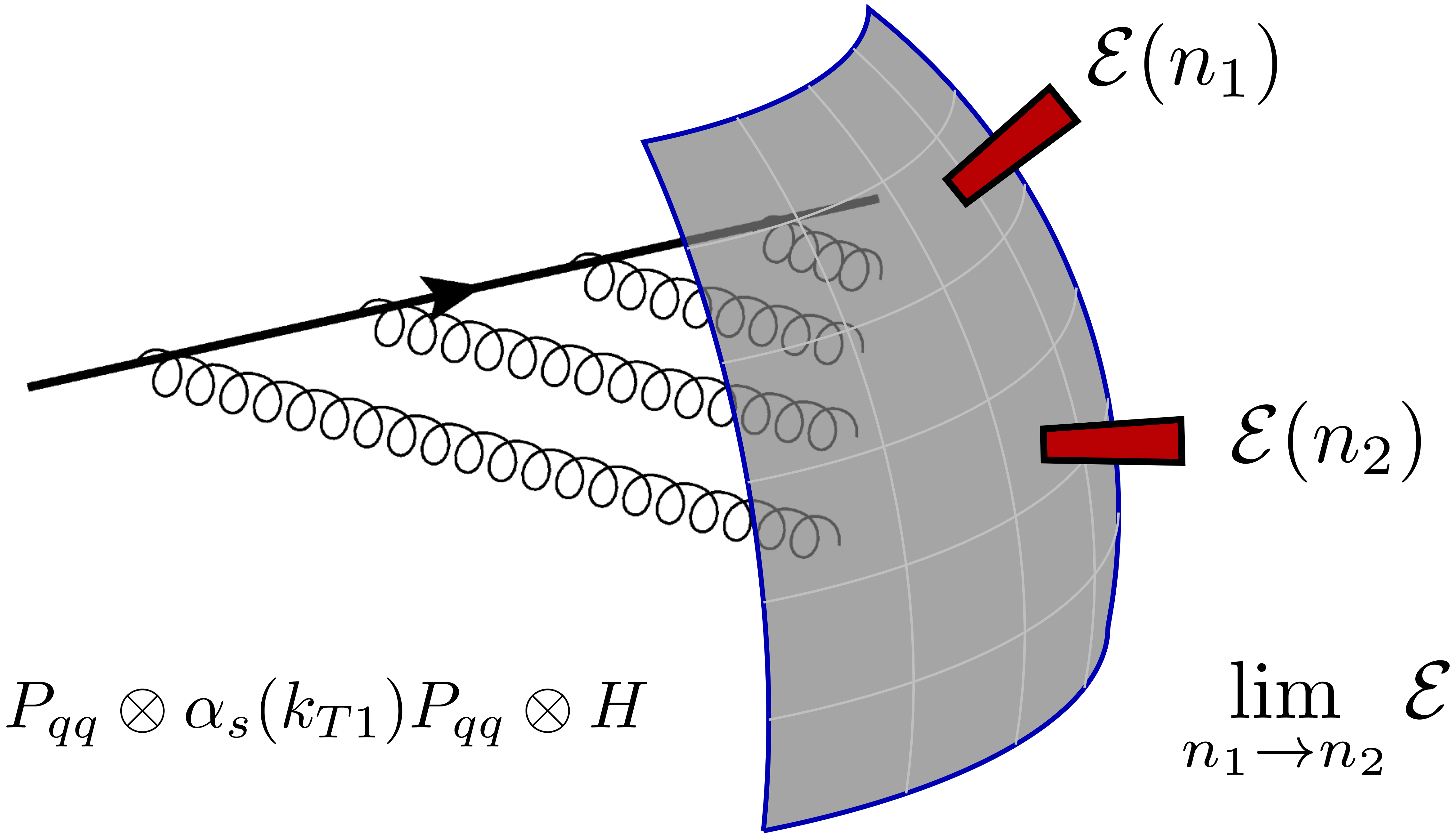
Energy Correlators at the Collider Frontier
July 11th, 2024, MITP, Mainz

# Dual view on energy correlators




the scaling interpretation makes the story appealing to broader community

$$\cdots \alpha_s(k_{T3}) P_{qq} \otimes \alpha_s(k_{T2}) P_{qq} \otimes \alpha_s(k_{T1}) P_{qq} \otimes H$$

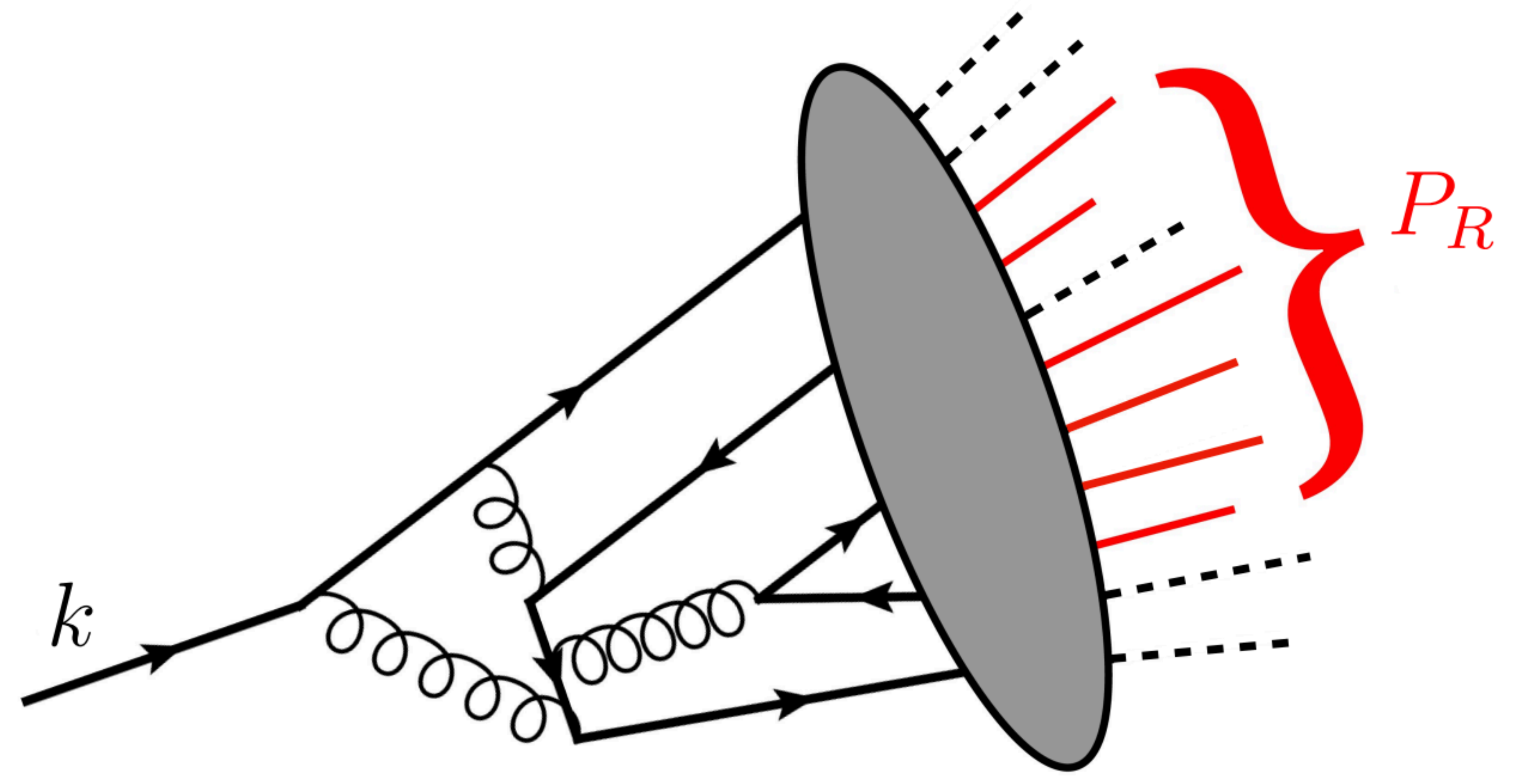
$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_i \frac{1}{\theta^{1-\gamma_i}} \mathbb{O}_i^{J=3}$$

| | |
|---|---|
| DGLAP evolution | Light-ray OPE |
| Time-like anomalous dimension | Space-like anomalous dimension |
| Running coupling | Smearing in spin |
| **Incorporating track** | ??? |
| Quantum interference in parton shower | **Evolution for non-diagonal density matrix** |
| nuclear structure | ??? |
| medium effects | ??? |
| massive quark | ??? |
| ??? | **P.T. and N.P. Power corrections** |

see also Iain and Zhiquan's talks

# The track function formalism

H.-M. Chang, M. Procura, J. Thaler, W. Waalewijn, 2013

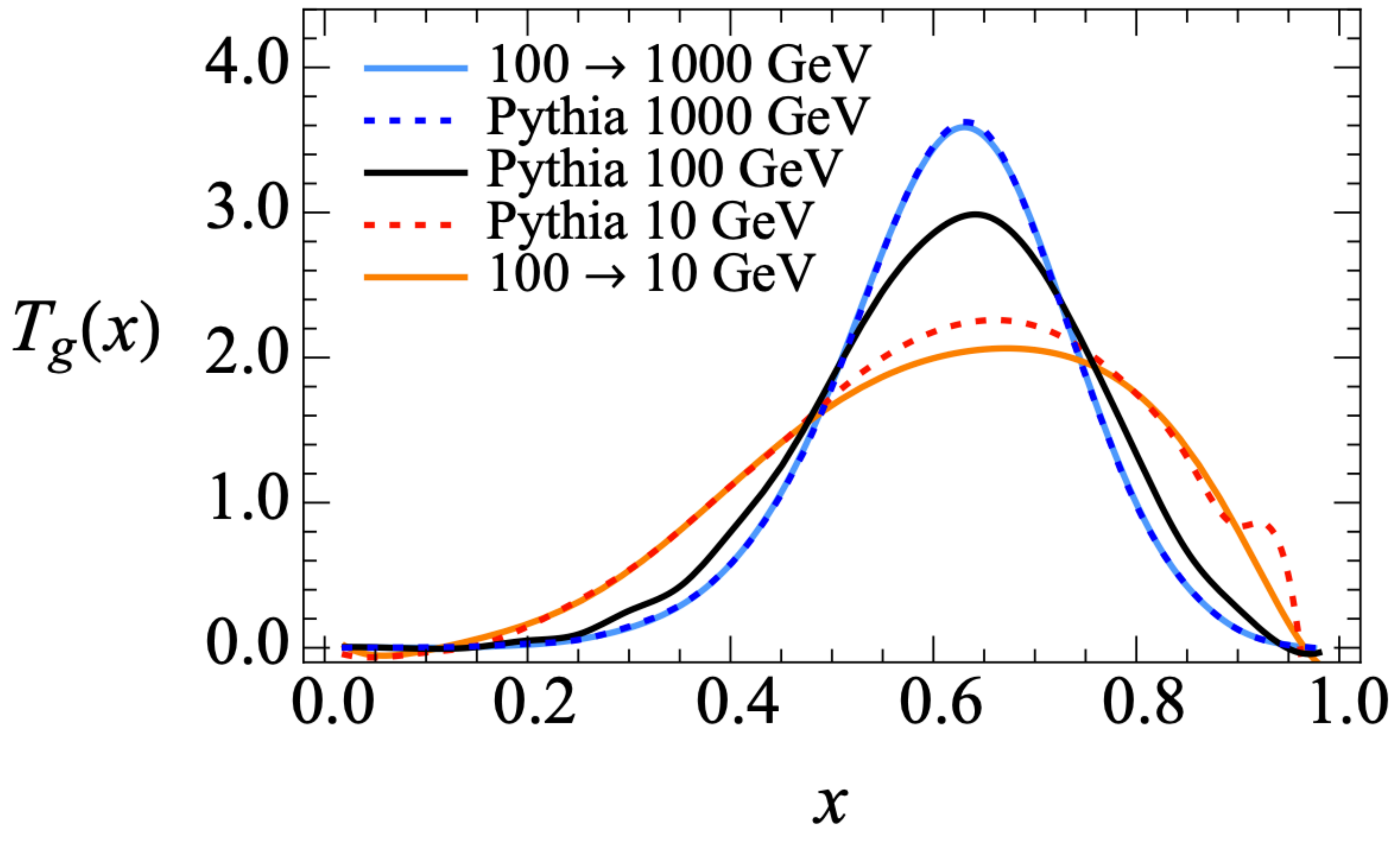



$$T_q(x) = \int \mathrm{d}y^+ \mathrm{d}^{d-2} y_\perp e^{\mathrm{i}k^- y^+/2} \sum_X \delta\Big(x - \frac{P_R^-}{k^-}\Big) \frac{1}{2N_c} \mathrm{tr}\Big[\frac{\gamma^-}{2} \langle 0|\psi(y^+,0,y_\perp)|X\rangle\langle X|\bar{\psi}(0)|0\rangle\Big],$$

$$T_g(x) = \int \mathrm{d}y^+ \mathrm{d}^{d-2} y_\perp e^{\mathrm{i}k^- y^+/2} \sum_X \delta\Big(x - \frac{P_R^-}{k^-}\Big) \frac{\langle 0|G^a_{-\lambda}(y^+,0,y_\perp)|X\rangle\langle X|G^{\lambda,a}_-(0)|0\rangle}{(2-d)(N_c^2-1)k^-}.$$

Track function describe a parton k converted to a subset of hadrons with quantum number R and total momentum fraction x.

One step closer to the true nature of hadronization than fragmentation function.



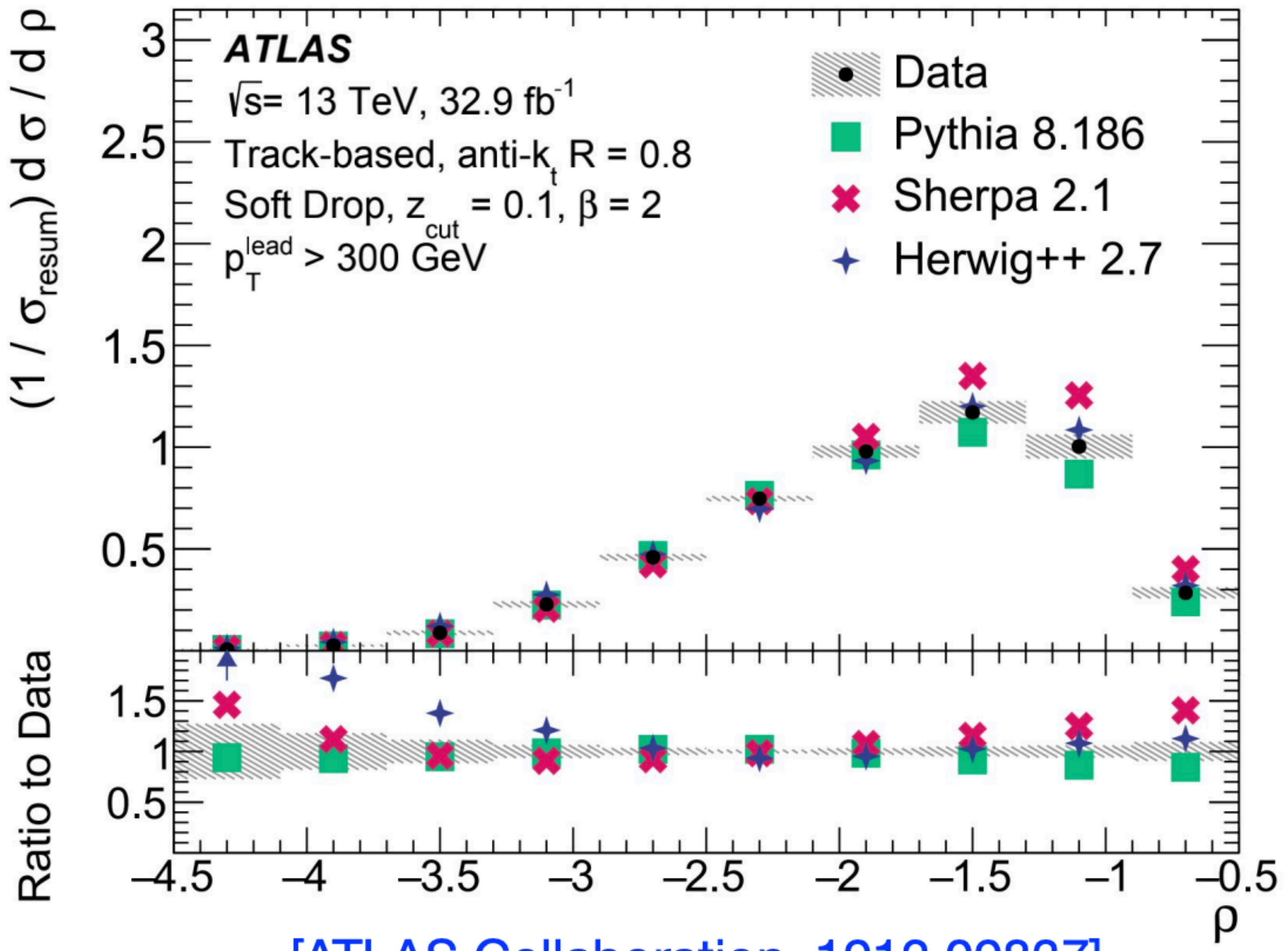
# The call for track-based calculation

observables. For all of these observables, the uncertainties for the track-based observables are significantly smaller than those for the calorimeter-based observables, particularly for higher values of $\beta$, where more soft radiation is included within the jet. However, since no track-based calculations exist at the present time, calorimeter-based measurements are still useful for precision QCD studies.

[ATLAS Collaboration, 1912.09837]

the selection of charged particle jets. Note that track-based observables are IRC-unsafe. In general, nonperturbative track functions can be used to directly compare track-based measurements to analytical calculations [67–69]; however, such an approach has not yet been developed for jet angularities. Two

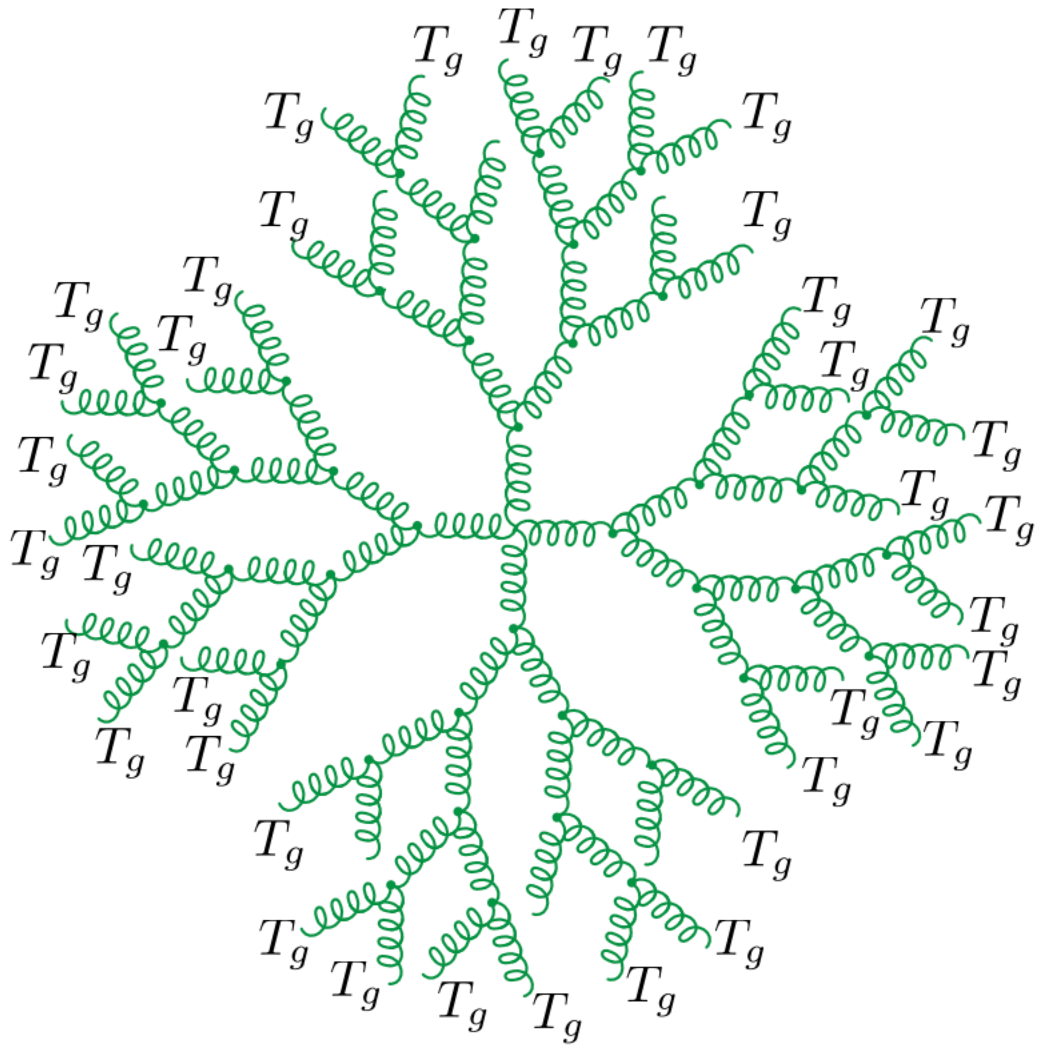
[ALICE Collaboration, 2107.11303]




[ATLAS Collaboration, 1912.09837]

Defined of track-based observable

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i\, T_i(x_i)\, \delta[\bar{e} - \hat{e}(\{x_i p_i^\mu\})]$$




e.g., jet mass: $\bar{e} = \sum_i (x_i p_i^\mu)^2$

# Track EEC is simple

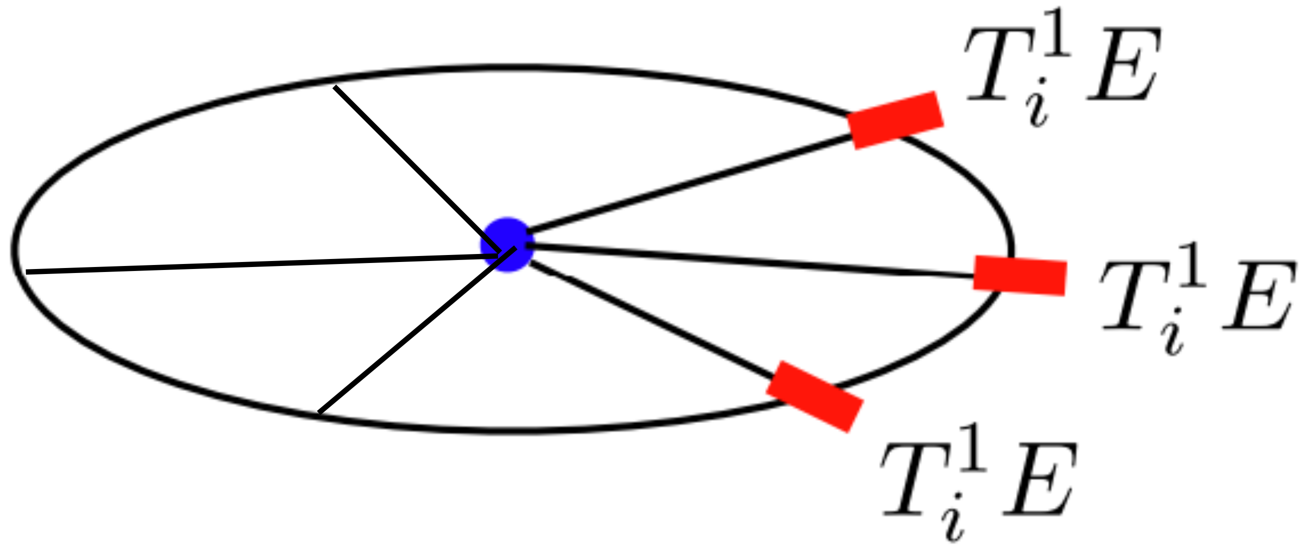
H. Chen, I. Moult, X.Y. Zhang, HXZ, 2020

All hadron ENC:

$$\mathrm{ENC} = \int d\Pi \frac{d\sigma}{d\Pi} \frac{E_1 E_2 \cdots E_N}{Q^N} \prod_{i,j} \delta(z_{ij} - \cos\theta_{ij})$$

track ENC:

$$\boxed{E_i \to \int dx_i\, x_i T_i(x) E_i = T_i(1) E_i}$$




$$\mathrm{ENC_{tr}} = \int d\Pi \frac{d\sigma}{d\Pi} \frac{T_1(1)E_1 T_2(1)E_2 \cdots T_N(1)E_N}{Q^N} \prod_{i,j} \delta(z_{ij} - \cos\theta_{ij})$$

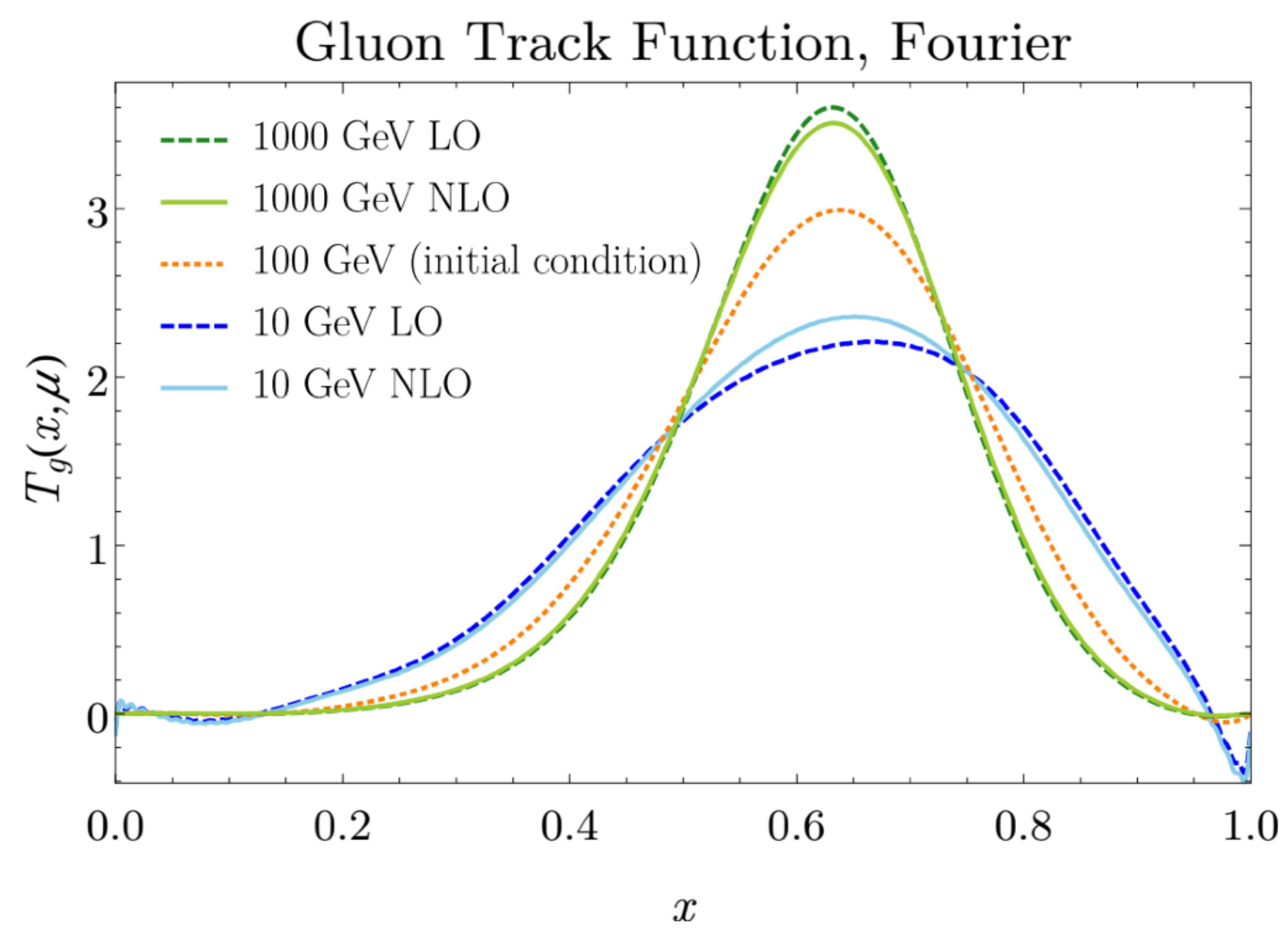
Complete factorization of measurement and weight

# RG flow of track function

RG equation for gluon track function

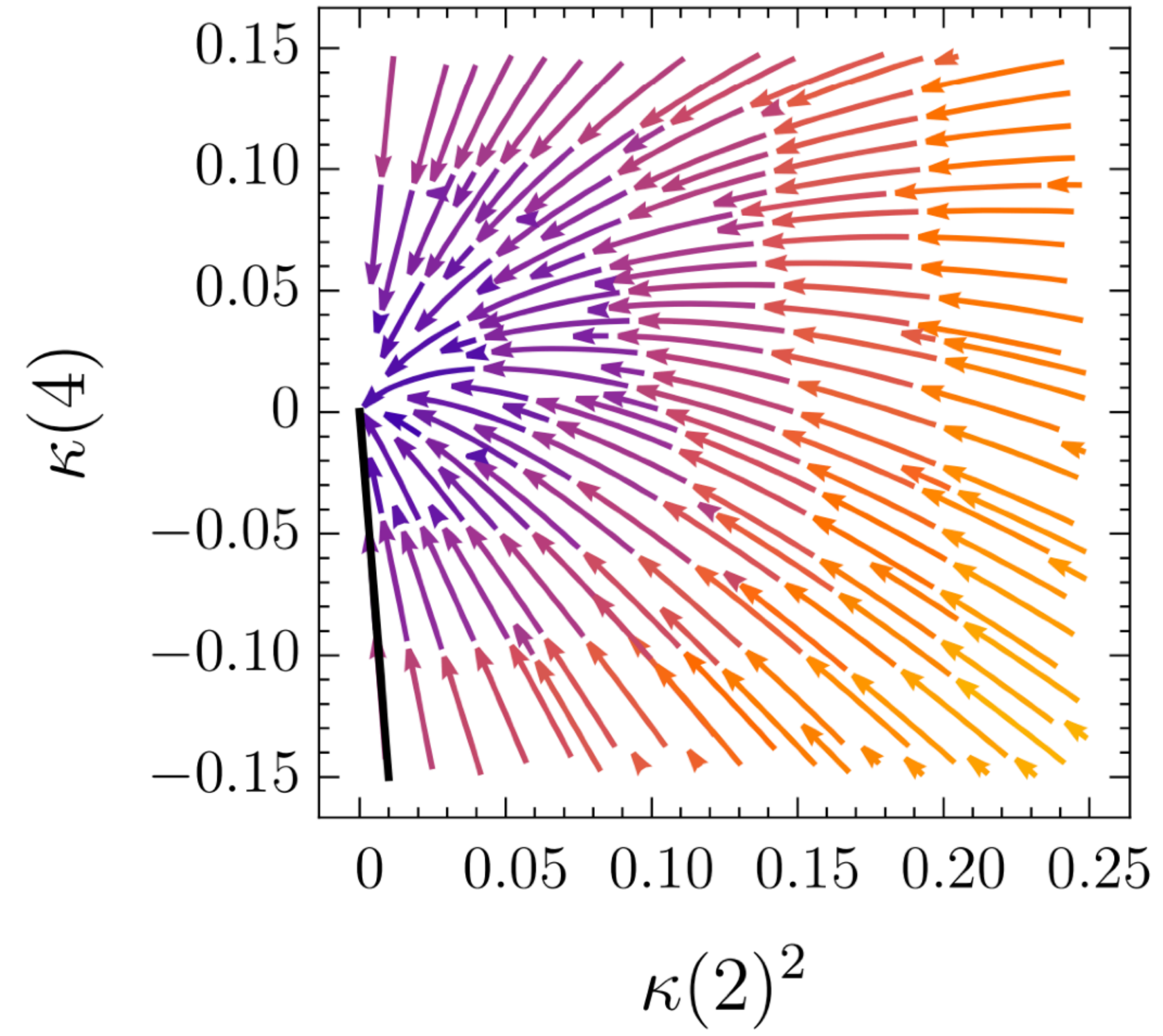
H. Chen et al., 2210.10058

$$
\begin{aligned}
\frac{d}{d\ln\mu^2} T_g(x) = {} & T_g(x)\; K_g^{(1)} \\
& + \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}z\, \delta\left(x - x_1 \frac{1}{1+z} - x_2 \frac{z}{1+z}\right) \Big[ T_g(x_1) T_g(x_2)\; K^{(1)}_{gg,1}(z) \\
& + \sum_q \left(T_q(x_1) T_{\bar{q}}(x_2) + T_q(x_2) T_{\bar{q}}(x_1)\right)\; K^{(1)}_{q\bar{q},1}(z) \Big] \\
& + \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}x_3 \int_0^1 \mathrm{d}z \int_0^1 \mathrm{d}t\; \delta\left(x - x_1 \frac{1}{1+z+zt} - x_2 \frac{z}{1+z+zt} - x_3 \frac{zt}{1+z+zt}\right) \\
& \times \Big\{ 6\, T_g(x_1) T_g(x_2) T_g(x_3)\; K^{(1)}_{ggg,1}(z,t) \\
& \quad + \sum_q \Big[ T_g(x_3) \left(T_q(x_2) T_{\bar{q}}(x_1) + T_q(x_1) T_{\bar{q}}(x_2)\right)\; K^{(1)}_{gq\bar{q},1}(z,t) \\
& \quad + T_g(x_2) \left(T_q(x_3) T_{\bar{q}}(x_1) + T_q(x_1) T_{\bar{q}}(x_3)\right)\; K^{(1)}_{gq\bar{q},2}(z,t) \\
& \quad + T_g(x_1) \left(T_q(x_3) T_{\bar{q}}(x_2) + T_q(x_2) T_{\bar{q}}(x_3)\right)\; K^{(1)}_{gq\bar{q},3}(z,t) \Big] \Big\}.
\end{aligned}
$$

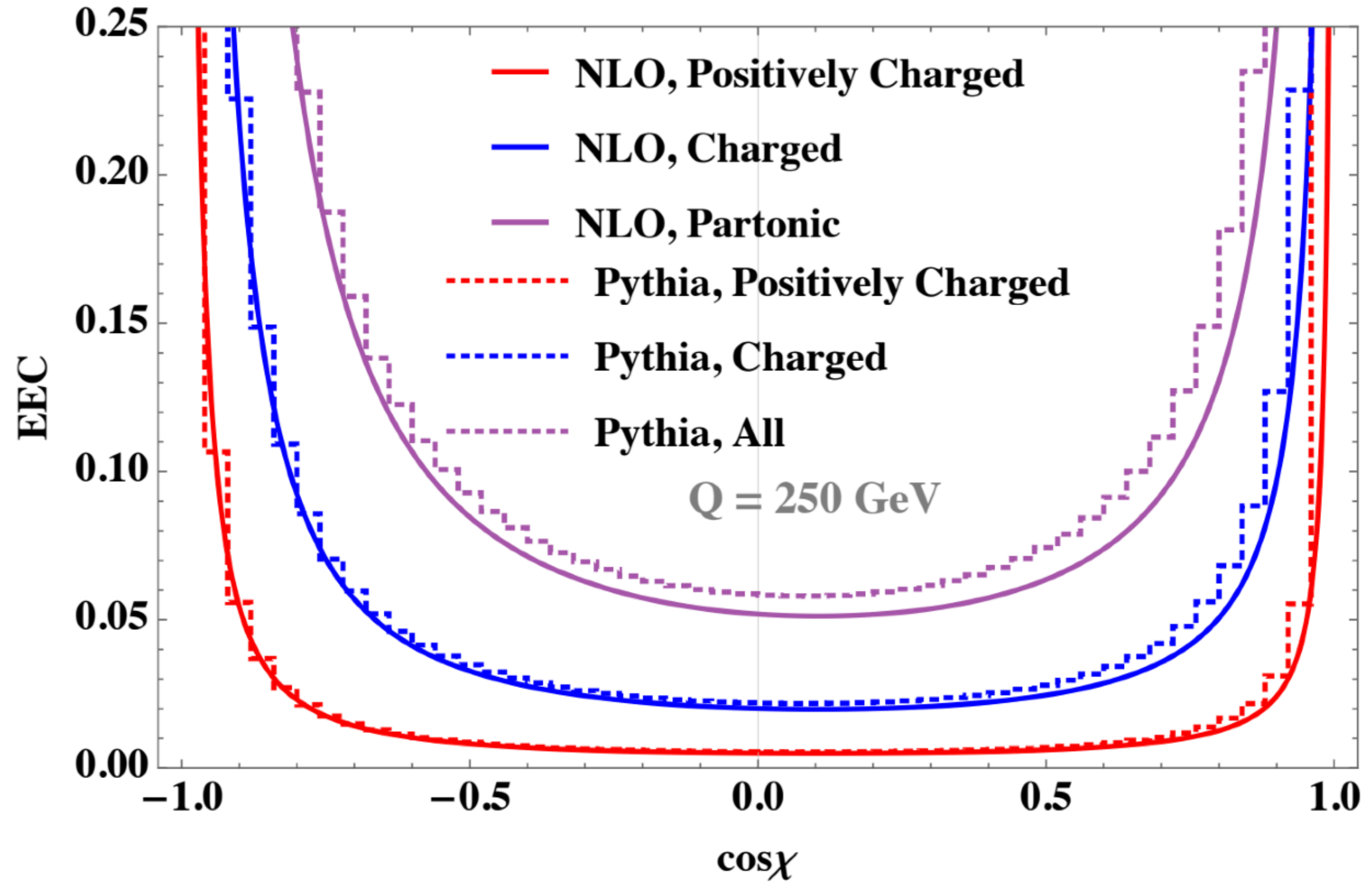



fifth central moment of gluon track func.

$$
\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\kappa(4) = -\gamma_{gg}(5)\kappa(4) + \gamma_{\kappa_2\kappa_2}\kappa^2(2)
$$



# Track EEC at NLO

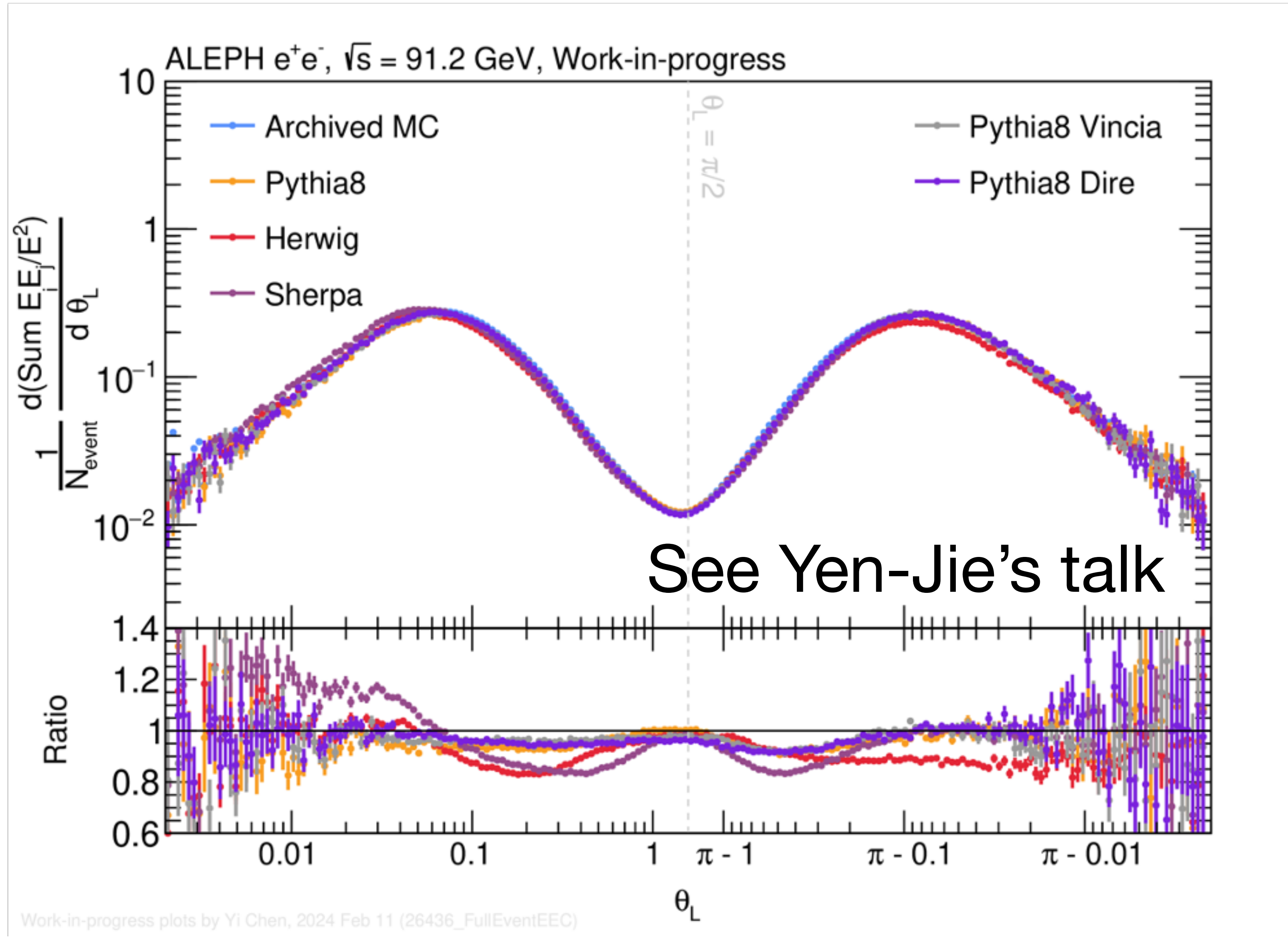


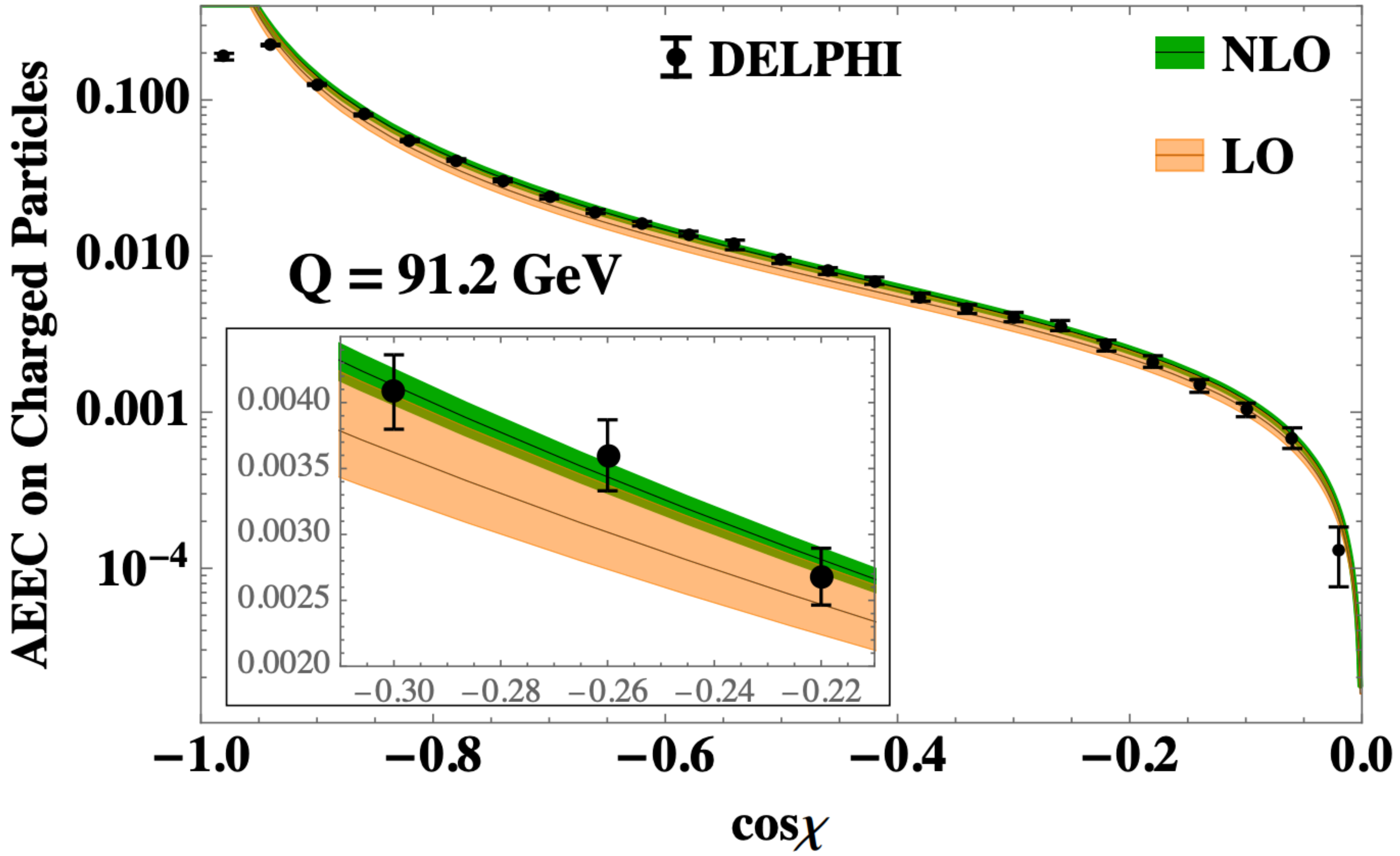

LO (1978): Basham, Brown, Ellis, Love

NLO (2018): L. Dixon et al. 1801.03219

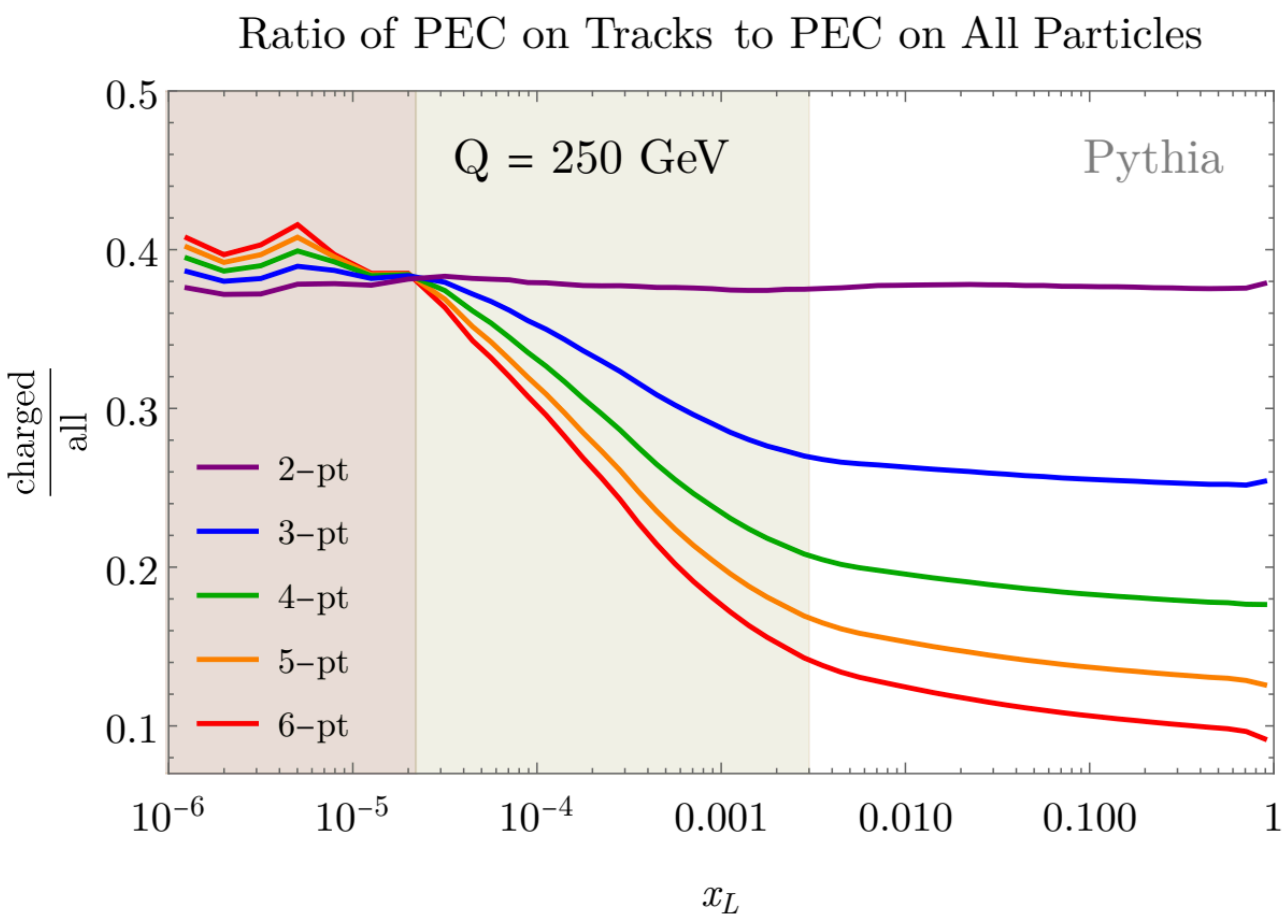
NLO on track (2018): Y.B. Li et al. 2108.01674

First ever track-based observable at NLO!



# Modification of scaling from track

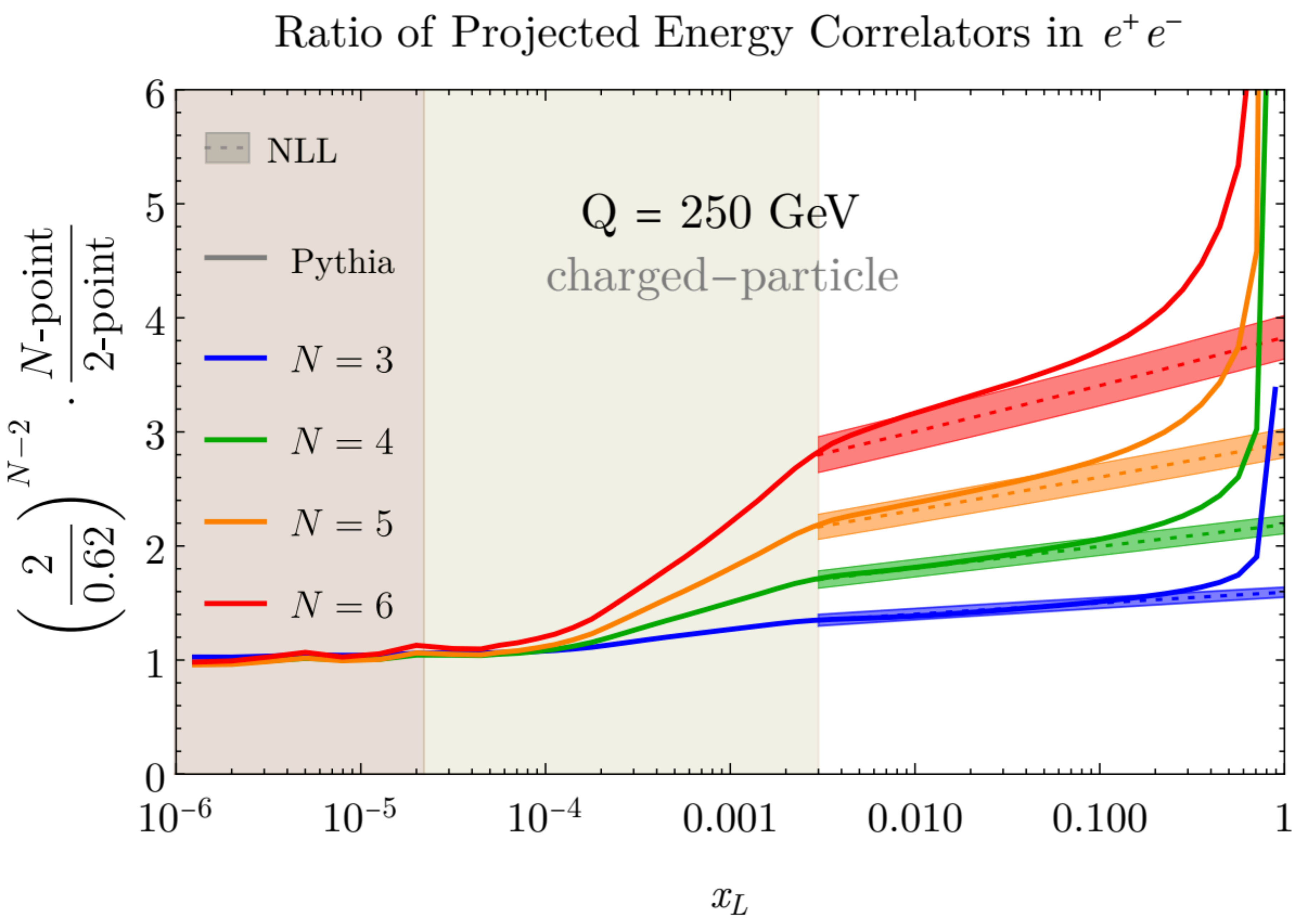



$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_q(1) = -\gamma_{qq}(2)\Delta$$
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_g(1) = -\gamma_{qg}(2)\Delta$$

$T(1)T(1)$

$T(2)T(1)$

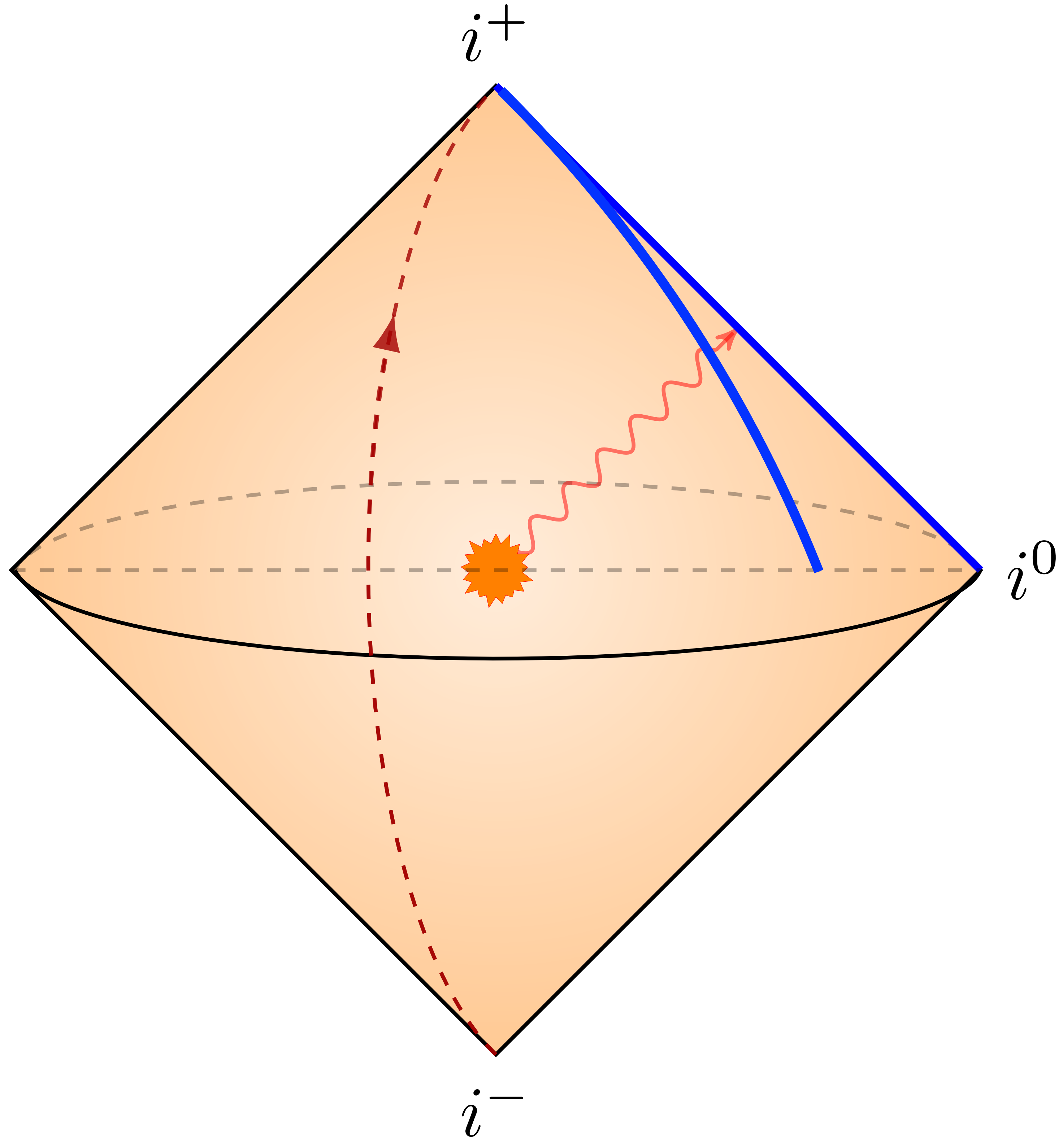
$T(3)T(1)$




- Modification to scaling from track is small at low point
- Two-point is almost identical on track and all hadrons in shape
- In the free hadron region, the charge-to-all hadron ratio approach $(2/3)^2$
- Monotonicity increasing in slope for ratio of track ENCs

# Light-ray operators

Hofman, Maldacena, 2008
Kravchuk, Simmons-Duffin, 2018

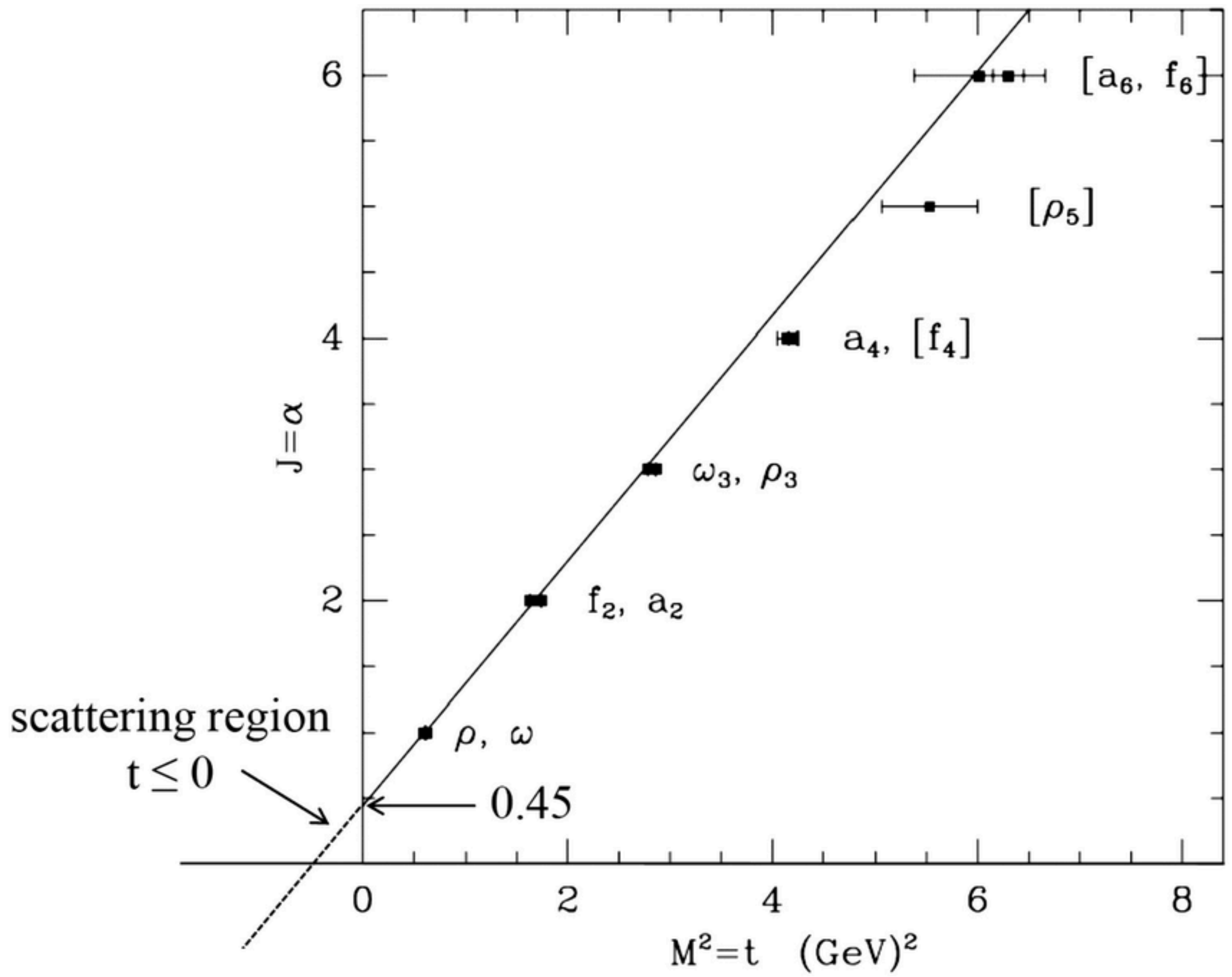


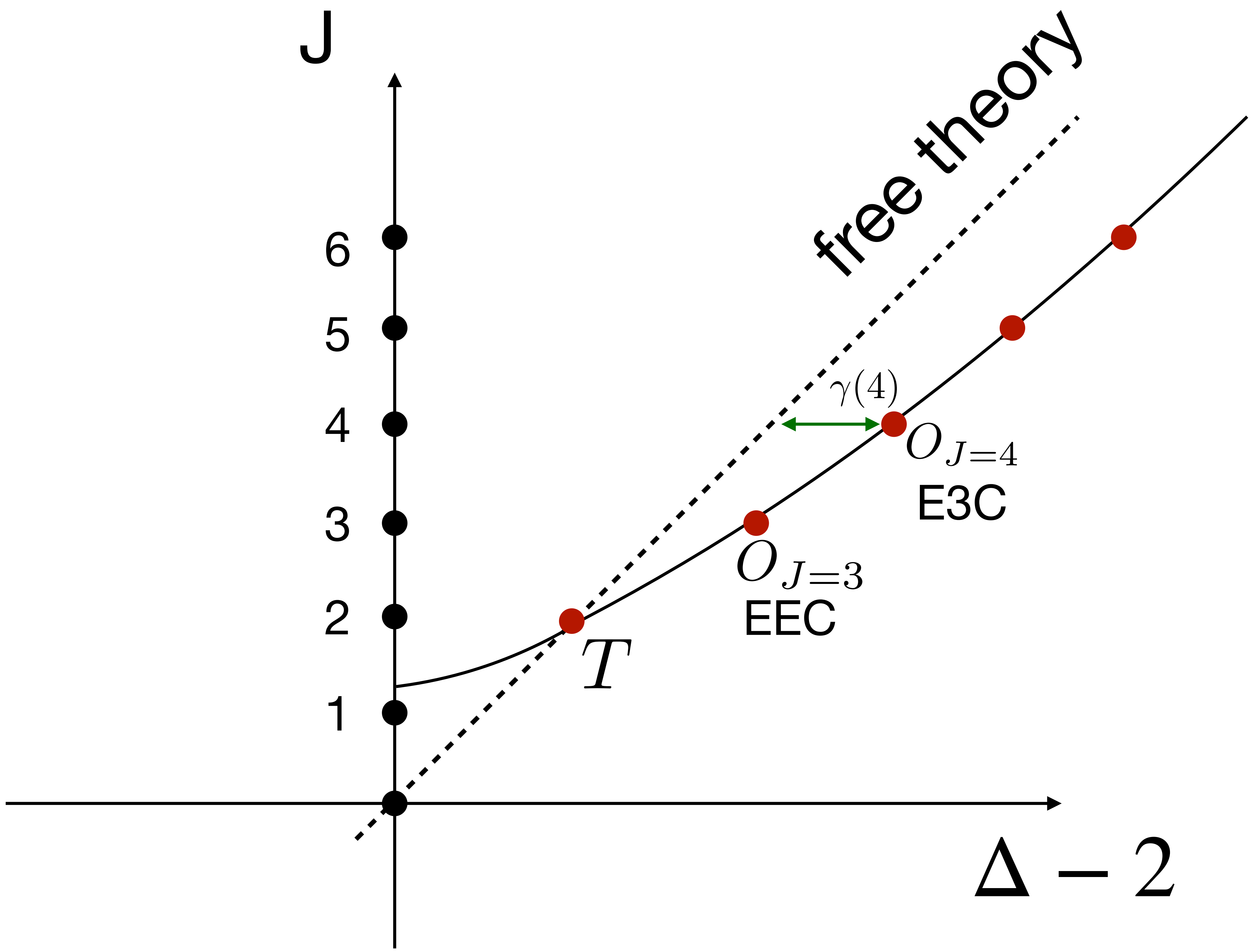

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} d(n\cdot x) \lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^2 \bar{n}^\mu \bar{n}^\nu T_{\mu\nu}(x)$$

The simplest light-ray operator measure the asymptotic energy flow at null infinity

# Regge trajectory for light-ray operator




$$\mathbb{O}_{J-1,1-\Delta}(\vec{n}) = \int_{-\infty}^{\infty} d(n\cdot x) \lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^{\Delta-J}\, \bar{n}_{\mu_1}\cdots\bar{n}_{\mu_J} O_{\Delta,J}^{\mu_1\cdots\mu_J}(x)$$

Energy correlators are the spectroscopy of high energy scattering

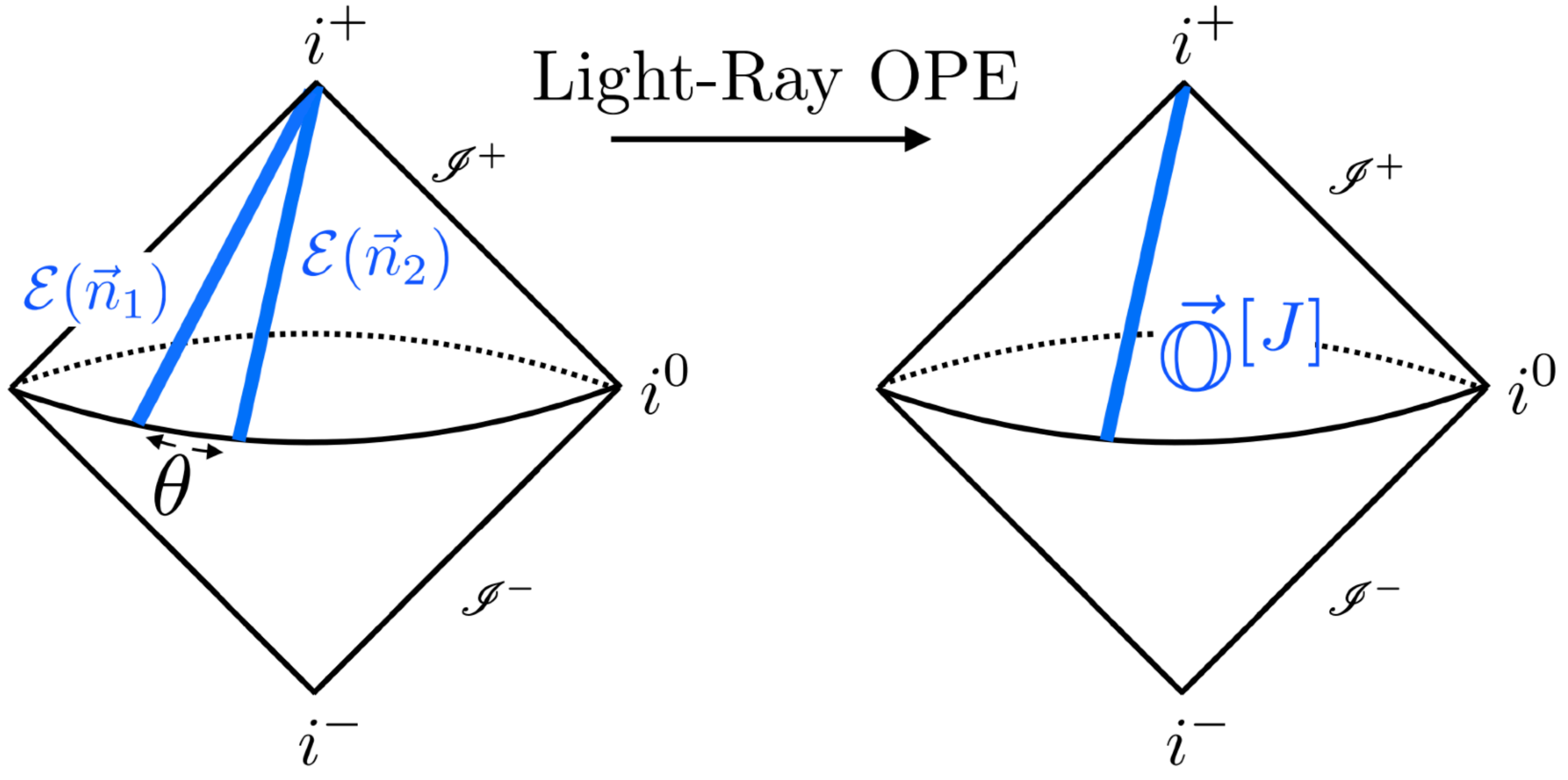
# Light-ray OPE: why important theoretically

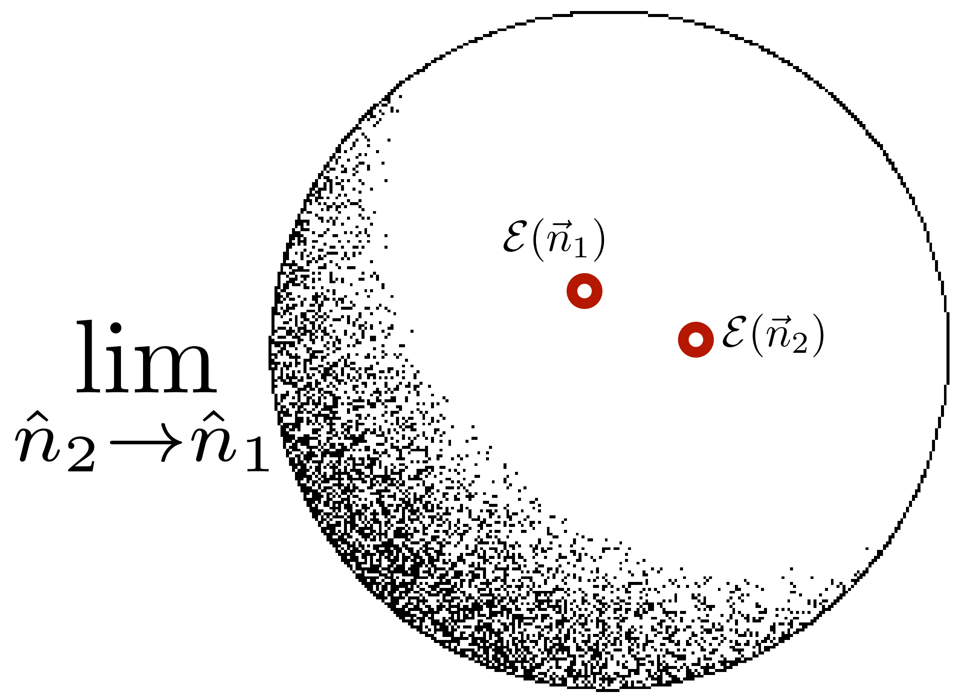
Euclidean OPE: $$O(x)O(0) = \sum_i x^{\gamma_i} c_i O_i \Rightarrow \langle O(x_1)O(x_2)\cdots O(x_n)\rangle$$

Light-ray OPE:

Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4}\mathbb{O}_i(\hat{n}_1)$$




form of an OPE in a fictitious two-dimensional Euclidean CFT

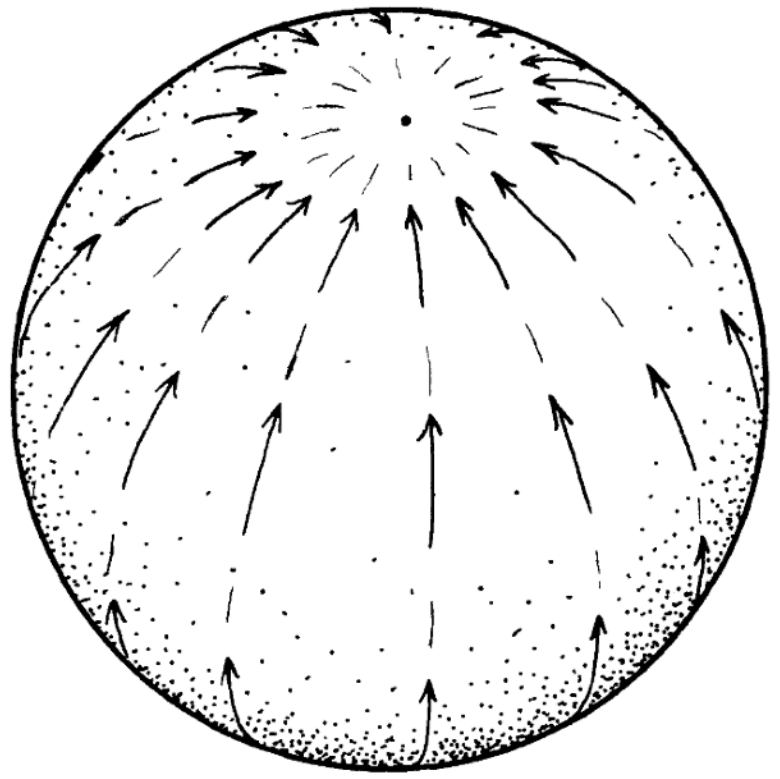
If the space of light-ray operators are fully carved out, then any N-point energy correlators can be build out from two-point

# Light-ray OPE: how it works

$$\mathbb{O}_{J-1,1-\Delta}(\vec{n}) = \underbrace{\int_{-\infty}^{\infty} d(n\cdot x) \lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^{\Delta-J}}_{\mathbb{L}_\tau} \bar{n}_{\mu_1}\cdots\bar{n}_{\mu_J} O^{\mu_1\cdots\mu_J}_{\Delta,J}(x)$$

$$\tau = \Delta - \overset{\text{twist-2}}{J} = 2+\gamma$$

| | $\mathbb{L}_\tau$ | $\vec{O}_\tau^{[J]}$ | $\vec{\mathbb{O}}_\tau^{[J]}$ | $\theta^2$ |
|---|---|---|---|---|
| coll. spin | $1-\tau$ | $-J$ | $1-(\tau+J)$ | 2 |
| dimension | $-\tau-1$ | $\tau+J$ | $J-1$ | 0 |

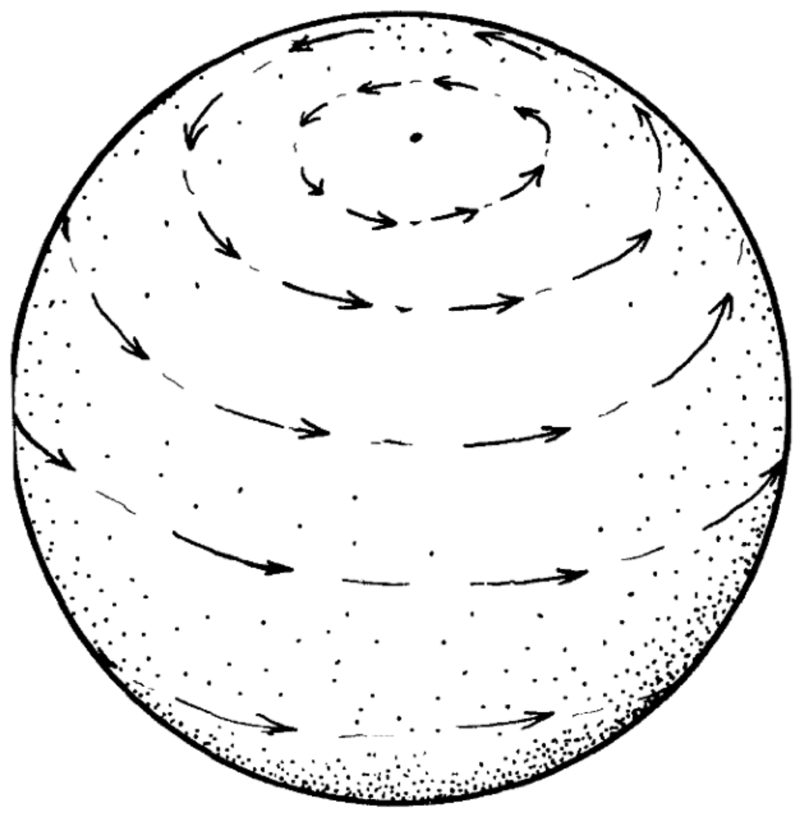



$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_i \frac{1}{\theta^{2-\gamma}} \mathbb{O}^{[J=3]}_{i,\tau=2}(n) + \text{power corrections}$$

dimension: 1 + 1 = J-1 ➡ J = 3

coll. spin: -3 + (-3) = x + 1-(2+$\gamma$+J) ➡ x = - 2 + $\gamma$

# Spinning light-ray operator

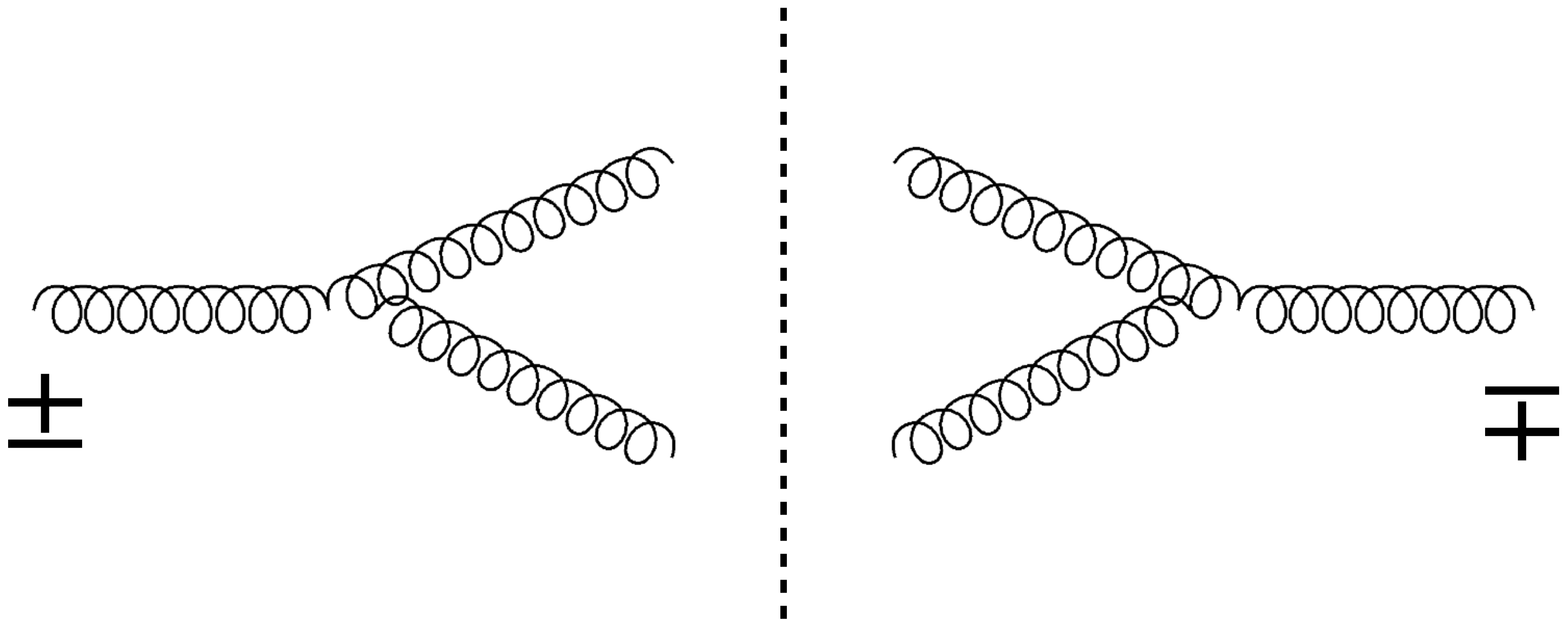


$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$



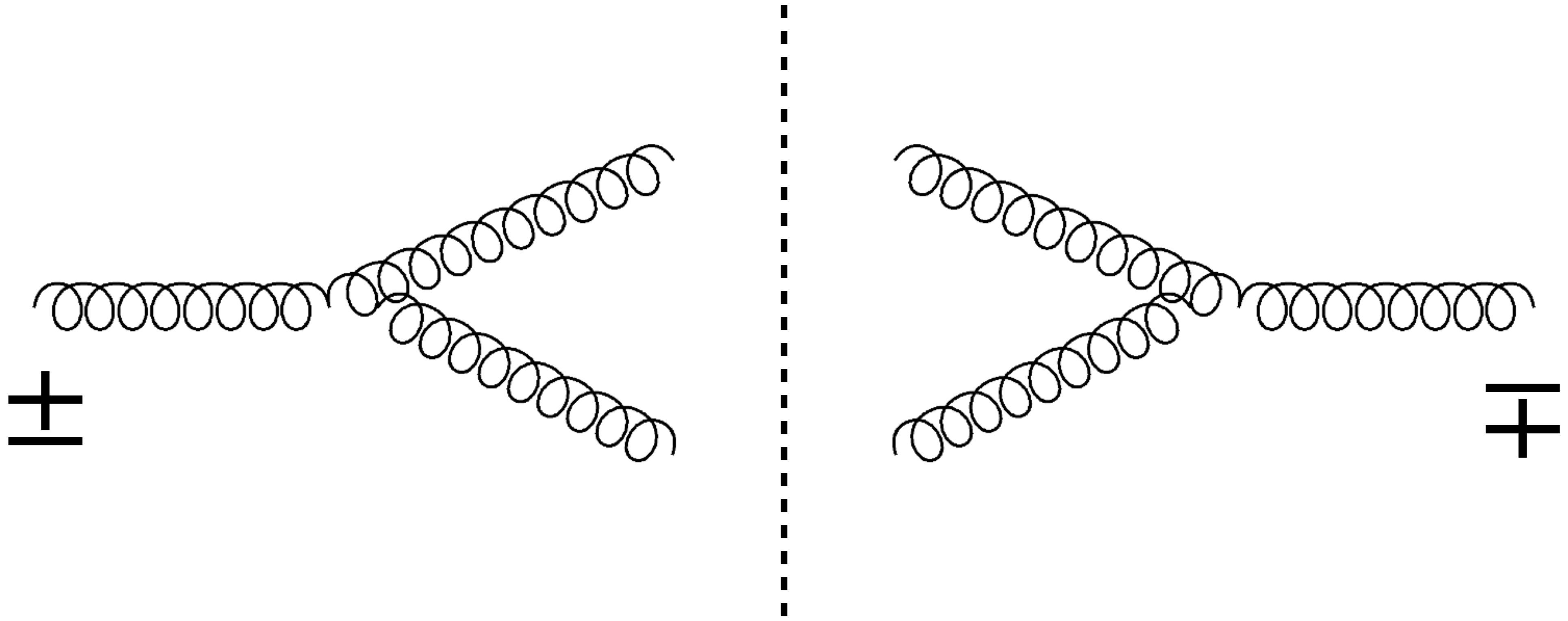

$$\mathcal{E}\mathcal{E} = \frac{c_1}{\theta^{2-\gamma_q}}\mathbb{O}_q^{J=3} + \frac{c_2}{\theta^{2-\gamma_g}}\mathbb{O}_g^{J=3} + e^{i2\phi}\frac{c_3}{\theta^{2-\gamma_{\tilde{g}}}}\mathbb{O}_{\tilde{g},+}^{J=3} + e^{-i2\phi}\frac{c_3}{\theta^{2-\gamma_{\tilde{g}}}}\mathbb{O}_{\tilde{g},-}^{J=3}$$

$$c_3 = \frac{\alpha}{15\pi}(C_A - n_f T_f)$$ ☹

# Different ways to polarize the gluon

un-pol. pol.

$$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$




$$A = \mathcal{E}\mathcal{E} = \begin{pmatrix} \mathbb{O}_g & e^{2i\phi}\mathbb{O}_{\tilde{g},+} \\ e^{-2i\phi}\mathbb{O}_{\tilde{g},-} & \mathbb{O}_g \end{pmatrix}$$

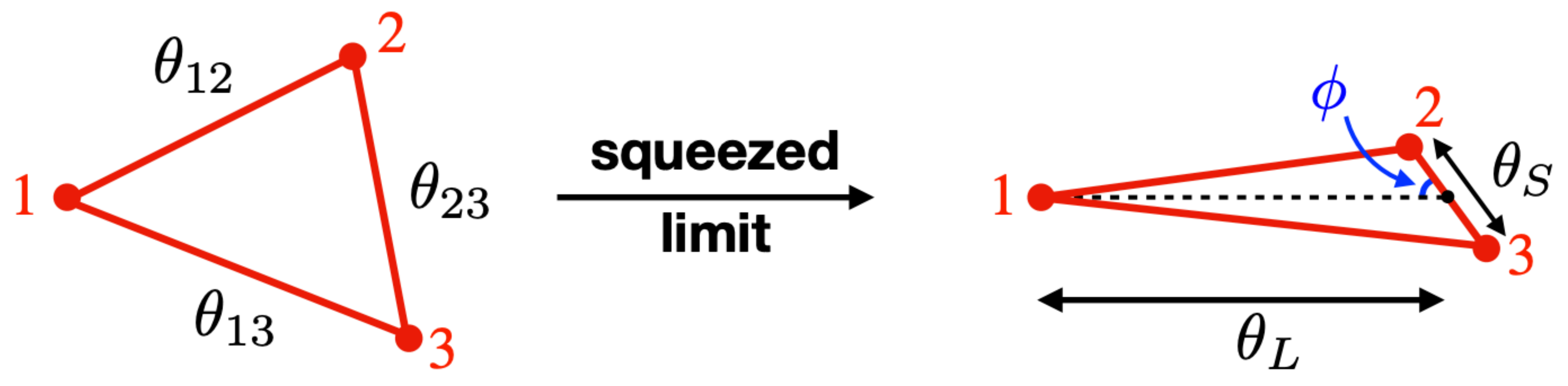
$$f(\phi) = \mathrm{Tr}[A\rho]$$

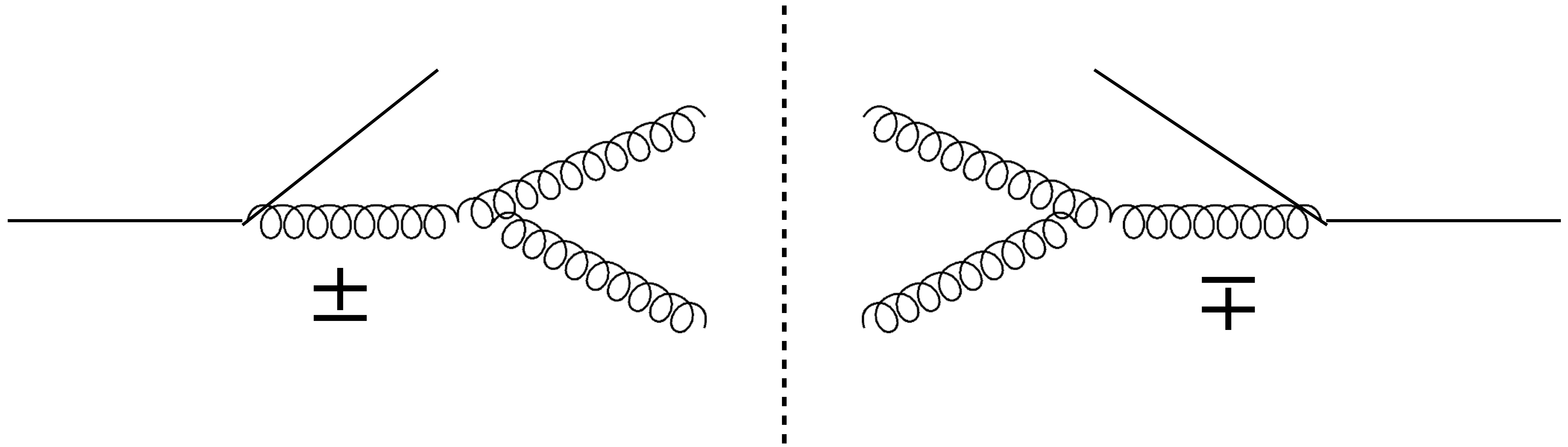
$$c_3 = \frac{\alpha}{15\pi}(C_A - n_f T_f)$$

How do we find the azimuthal asymmetry in experiment?

# The EEEC

H. Chen, I. Moult, HXZ, 2020




$$\rho = \begin{pmatrix} P_{gq} & P_{\tilde{g}q} \\ P_{\tilde{g}q} & P_{gq} \end{pmatrix}$$

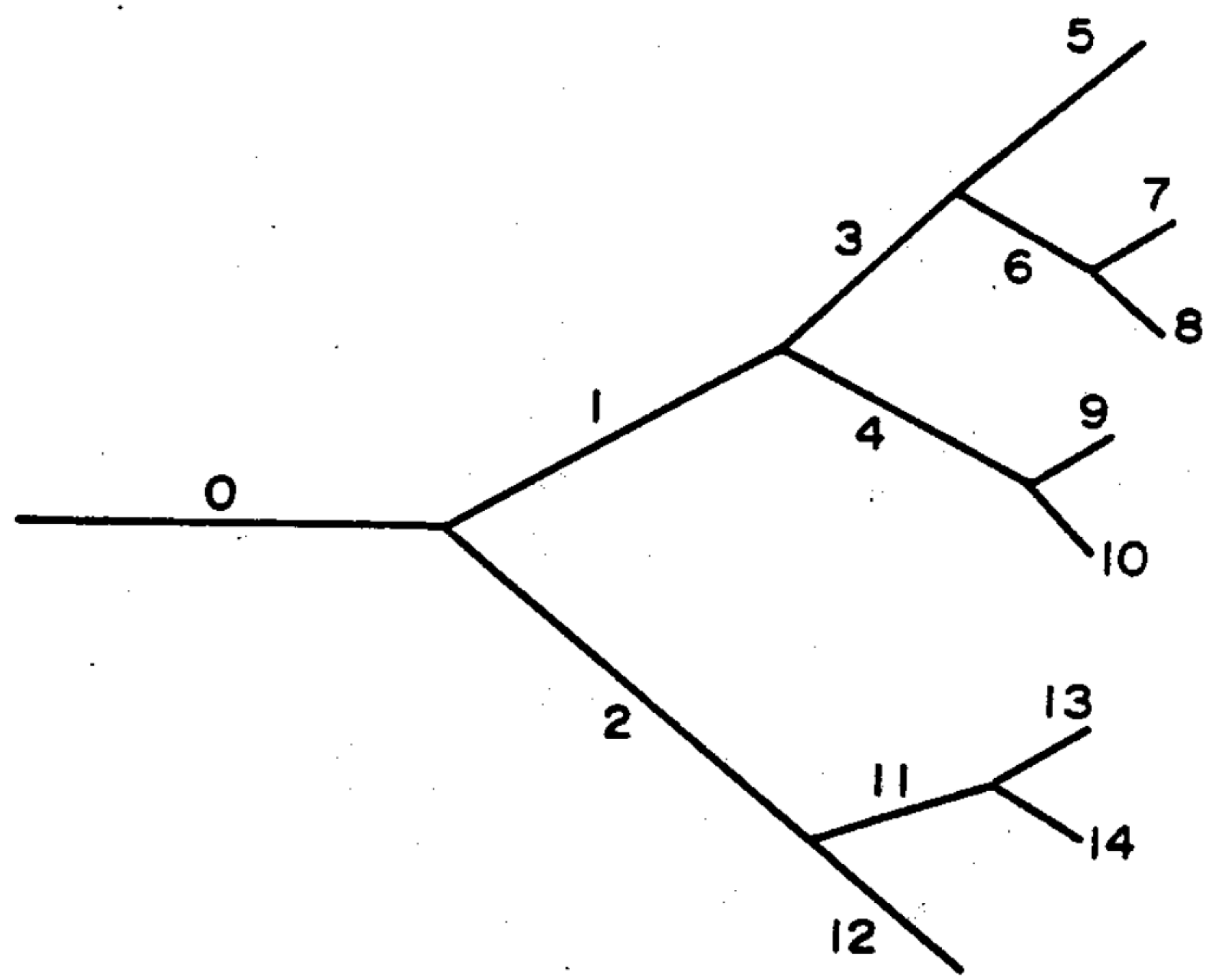
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3) = \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2) - \widehat{C}_{\phi_S}(3)\right]\left[\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right]^{\frac{\widehat{\gamma}^{(0)}(3)}{\beta_0}}\left[\widehat{C}_{\phi_L}(3) - \widehat{C}_{\phi_L}(4)\right]\left[\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right]^{\frac{\widehat{\gamma}^{(0)}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1) + \cdots$$

An analytic formula resumming the logs from light-ray OPE

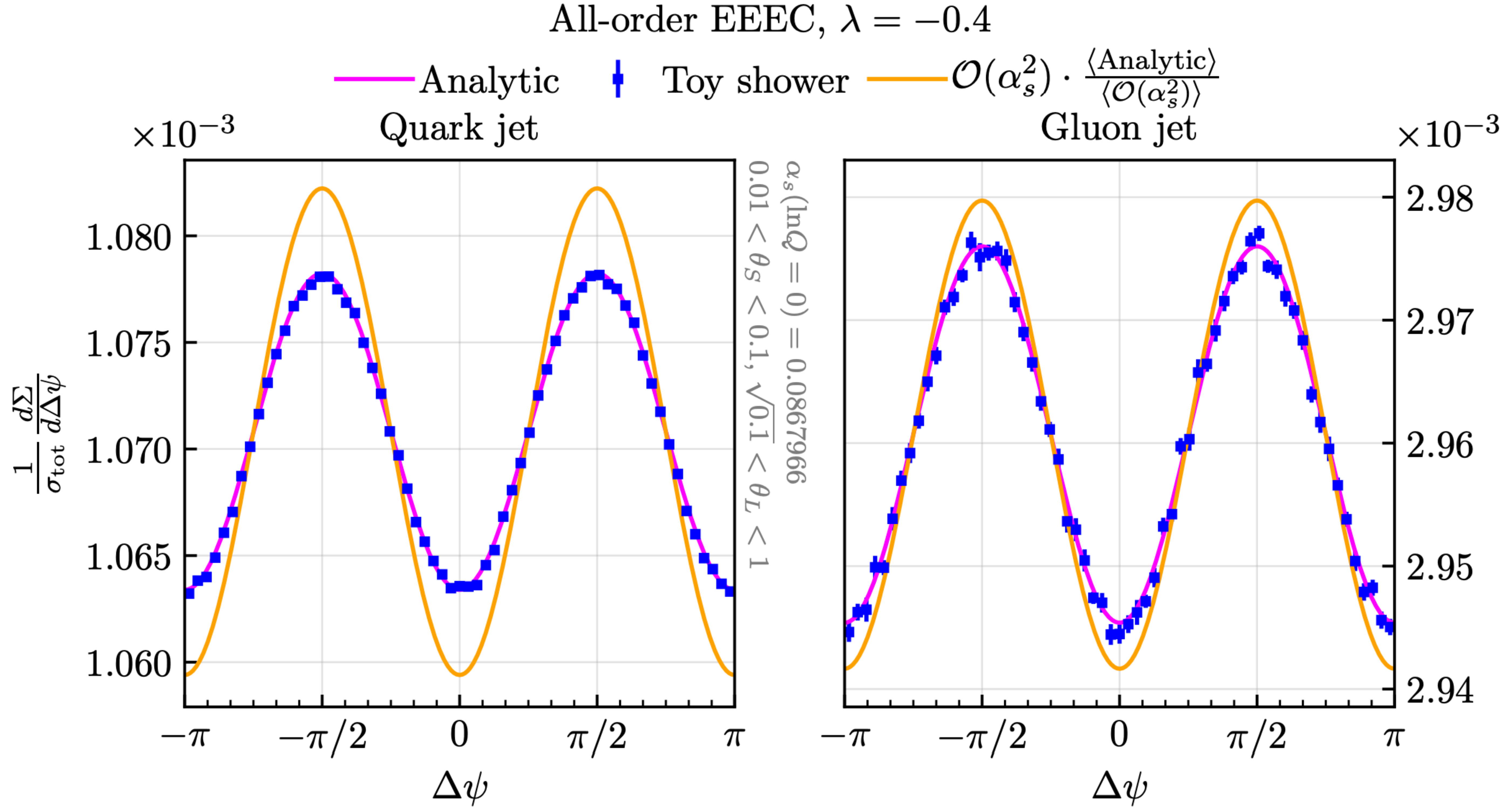
# Comparing with "quantum" parton shower

Algorithm for incorporating the full quantum mechanical effects of spin correlation exist for quite some time.

J. Collins, 1988; I. Knowles, 1990; J. Forshaw, J. Holguin, S. Platzer, 2019




A. Karlberg, G. Salam, L. Scyboz, R. Verheyen, 2021




EEEC help validating the Monte-Carlo implementation

# Breaking degeneracy by PDFs

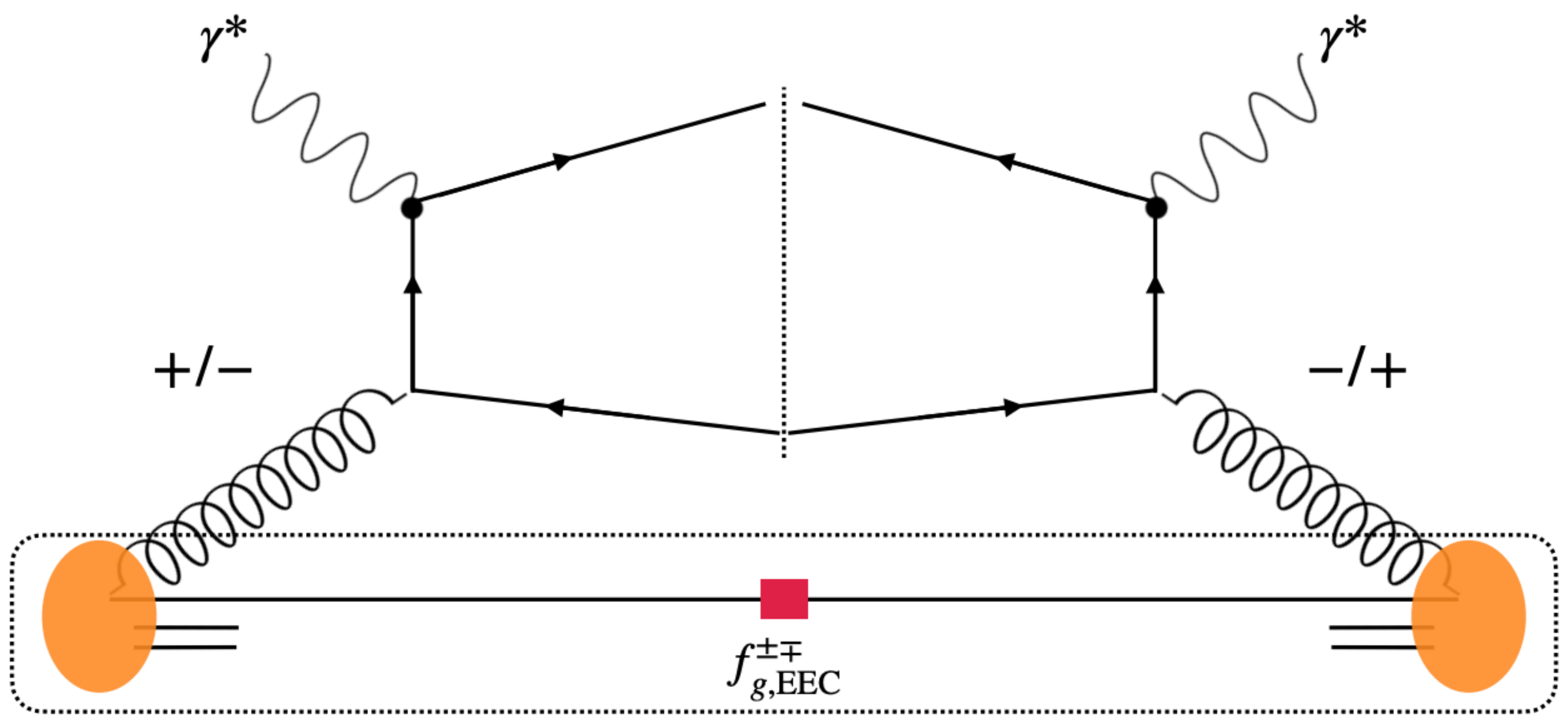
X.L. Li, X.H. Liu, F. Yuan, HXZ, 2023

$$f(\phi) = \mathrm{Tr}[A\rho]$$

$$\rho = \begin{pmatrix} P_{qg} & P_{q\tilde{g}} \\ P_{q\tilde{g}} & P_{qg} \end{pmatrix}$$

$$A = \mathcal{E}_{\text{proton}}\mathcal{E}$$

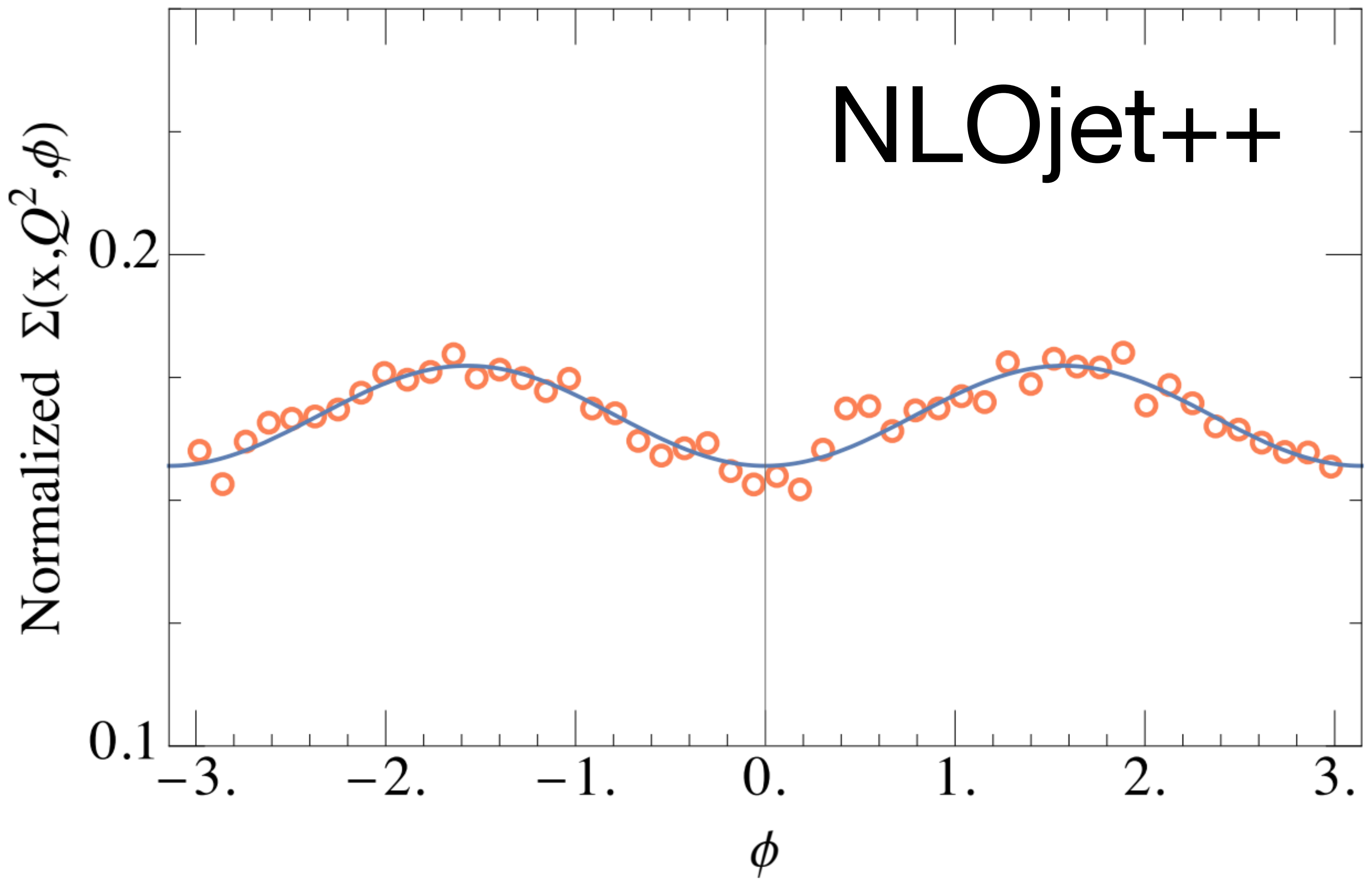
example of nuclear energy correlator




How to write a light-ray OPE for a proton state is an open question!

$$c_3 = f_g(x)C_A - n_f T_f f_q(x)$$

enhance the spin interference effects by exploiting the difference of PDFs



# Entangled by the Higgs

Y.X. Guo, X.H. Liu, F. Yuan, HXZ, 2024

$$f(\phi) = \mathrm{Tr}[A\rho]$$

$$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$

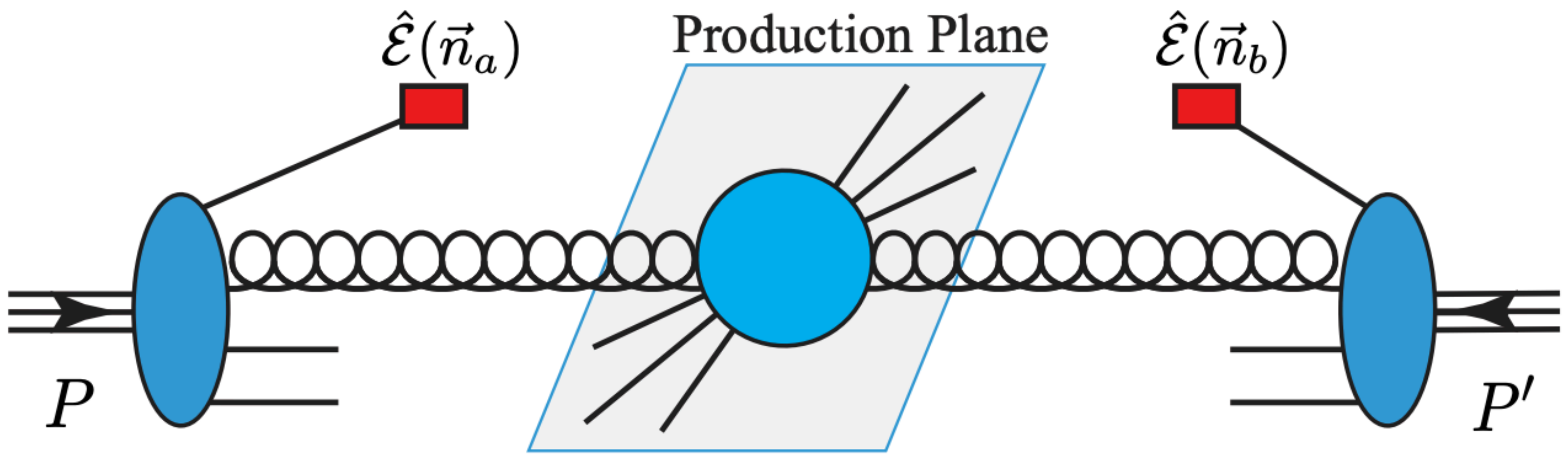
$$|\rho_{++}| = |\rho_{+-}|$$

Maximally entangled state

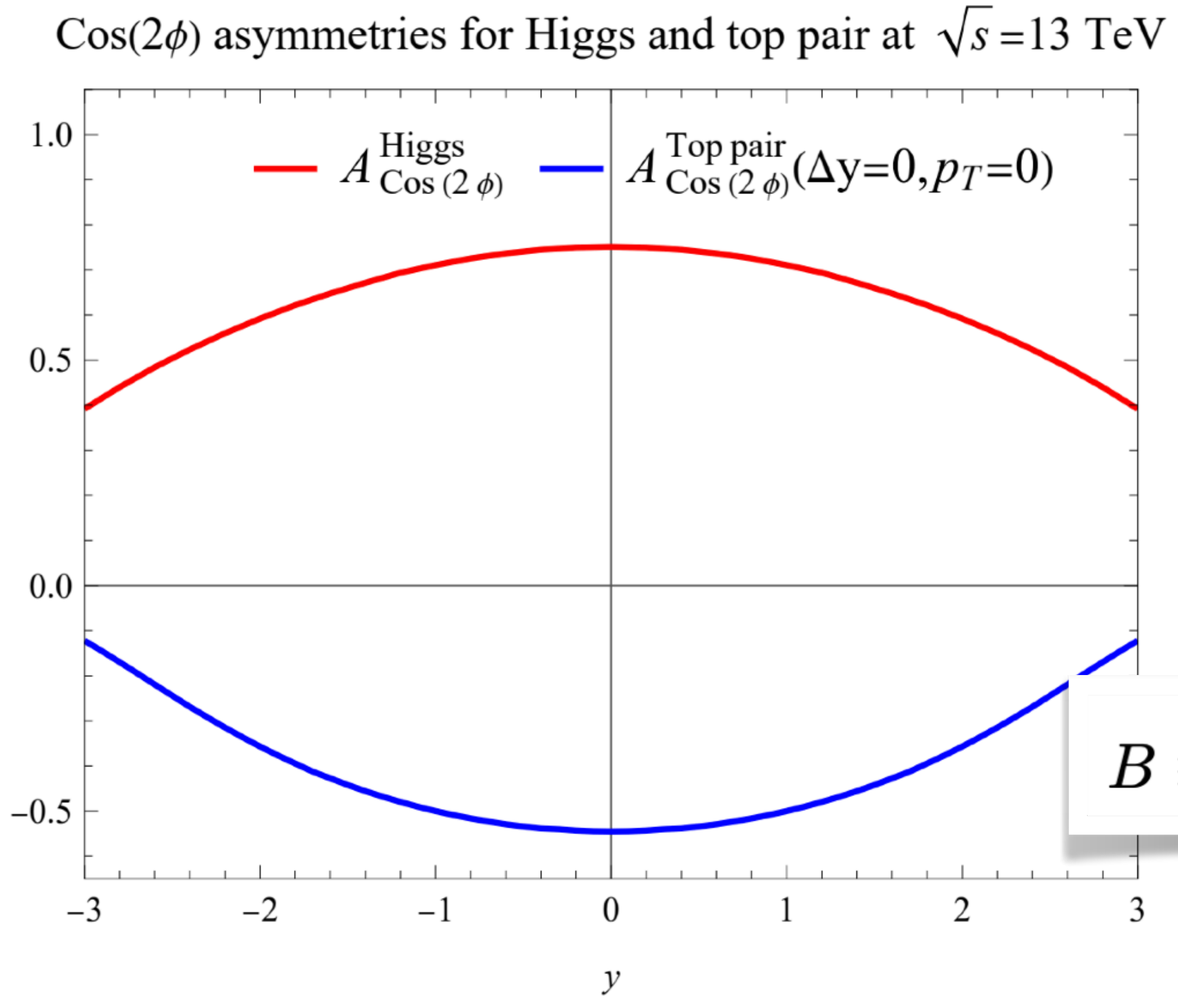
$$A = \mathcal{E}_{\mathrm{proton}}\mathcal{E} \otimes \mathcal{E}_{\mathrm{proton}}\mathcal{E}$$

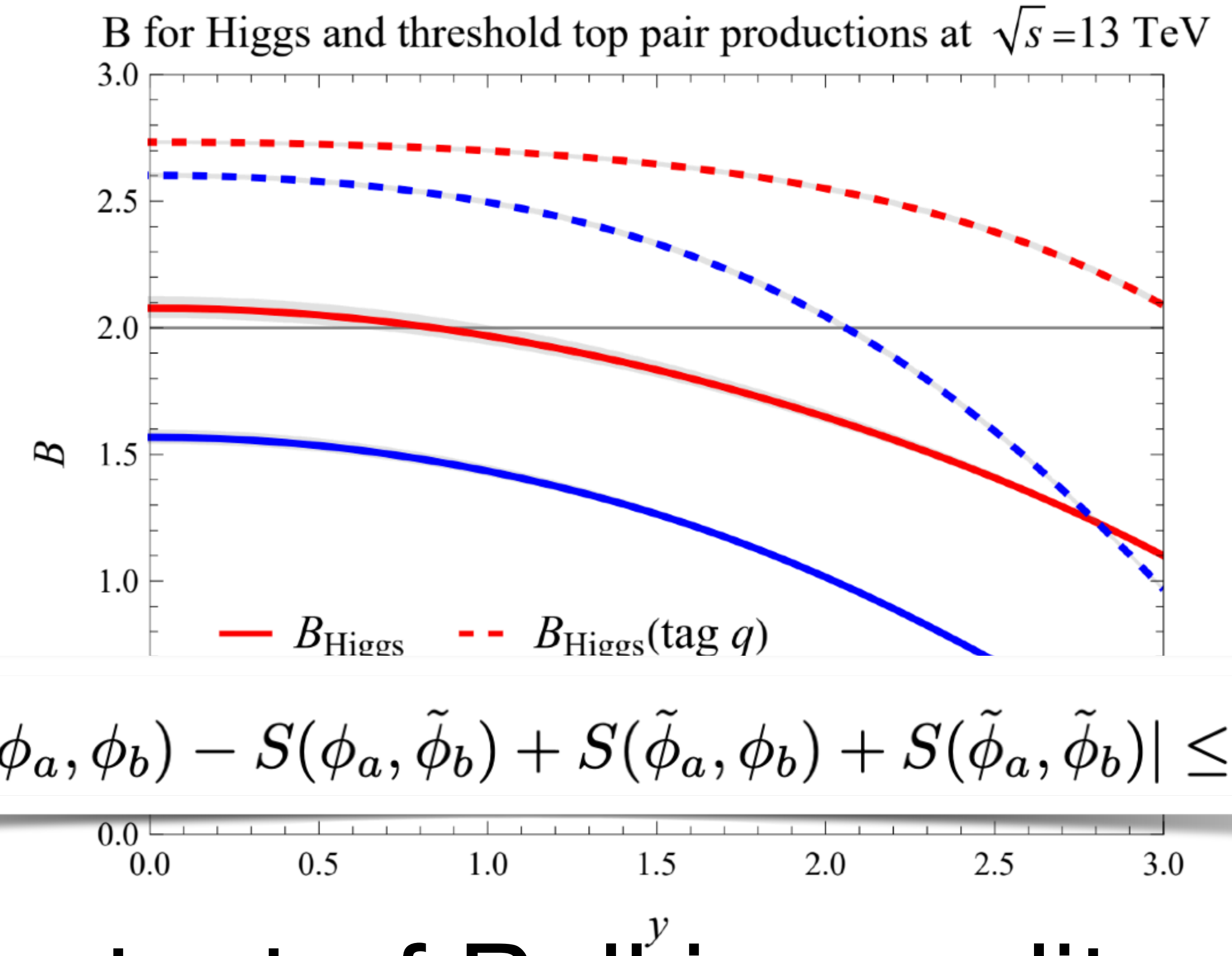
Similarly idea used to probe CP property of the Higgs

T. Plehn, D. Rainwater, D. Zeppenfeld, 2001




$$S(\phi_a, \phi_b) \equiv \frac{\Sigma(\phi_a,\phi_b) + \Sigma(\phi_a',\phi_b') - \Sigma(\phi_a',\phi_b) - \Sigma(\phi_a,\phi_b')}{\Sigma(\phi_a,\phi_b) + \Sigma(\phi_a',\phi_b') + \Sigma(\phi_a',\phi_b) + \Sigma(\phi_a,\phi_b')}$$




$$B \equiv |S(\phi_a,\phi_b) - S(\phi_a,\tilde{\phi}_b) + S(\tilde{\phi}_a,\phi_b) + S(\tilde{\phi}_a,\tilde{\phi}_b)| \le 2\,.$$

test of Bell inequality

# Perturbative and non-perturbative power expansion

twist expansion

$$\mathcal{E}\mathcal{E} = \frac{1}{\theta^2}\mathbb{O}_{\tau=2} + \theta^0\mathbb{O}_{\tau=4} + \theta^2\mathbb{O}_{\tau=6} + \cdots$$

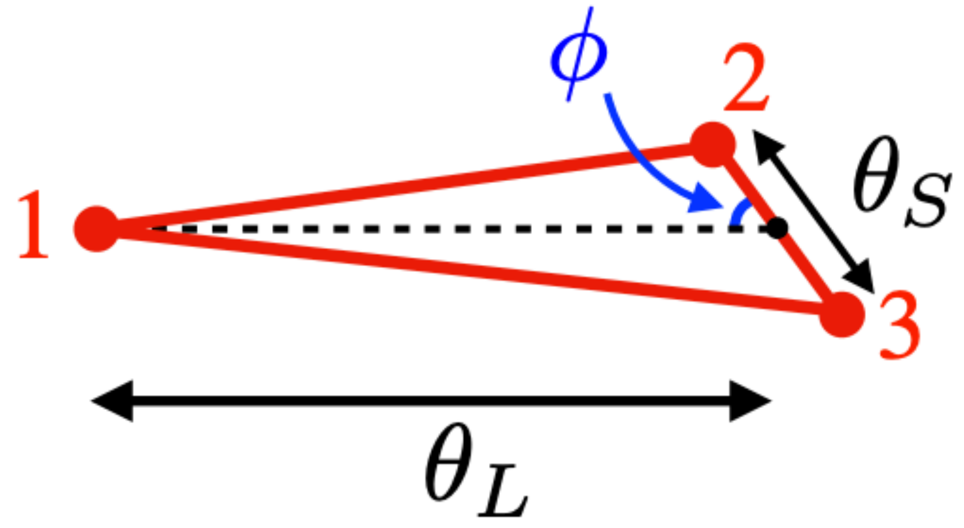
$$+\frac{\Lambda_{\mathrm{QCD}}}{Q}$$

N.P. Power corrections

$$+\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}$$

In both expansions, symmetry plays a prominent role

# Hidden structure in twist expansion




Perturbative data at higher order in twist expansion exhibit intriguing structure

$$z = \theta_S e^{i\phi}$$

**highest transverse spin series**

transverse twist = δ - j = 2

$$-z^3\bar{z} \qquad +\frac{39}{10}z^2\bar{z}^2 \qquad -z\bar{z}^3 \qquad \theta_S^{-2}$$

$\cos(2\phi)$

$$-z^4\bar{z} \qquad +\frac{39}{20}z^3\bar{z}^2 \qquad +\frac{39}{20}z^2\bar{z}^3 \qquad -z\bar{z}^4 \qquad \theta_S^{0}$$

$\cos(4\phi)$

$$-\frac{6}{7}z^5\bar{z} \qquad +\frac{229}{140}z^4\bar{z}^2 \qquad -\frac{211}{140}z^3\bar{z}^3 \qquad +\frac{229}{140}z^2\bar{z}^4 \qquad -\frac{6}{7}z\bar{z}^5 \qquad \theta_S^{2}$$

$\cos(6\phi)$

$$-\frac{5}{7}z^6\bar{z} \qquad +\frac{207}{140}z^5\bar{z}^2 \qquad -\frac{233}{140}z^4\bar{z}^3 \qquad -\frac{233}{140}z^3\bar{z}^4 \qquad +\frac{207}{140}z^2\bar{z}^5 \qquad -\frac{5}{7}z\bar{z}^6 \qquad \theta_S^{4}$$

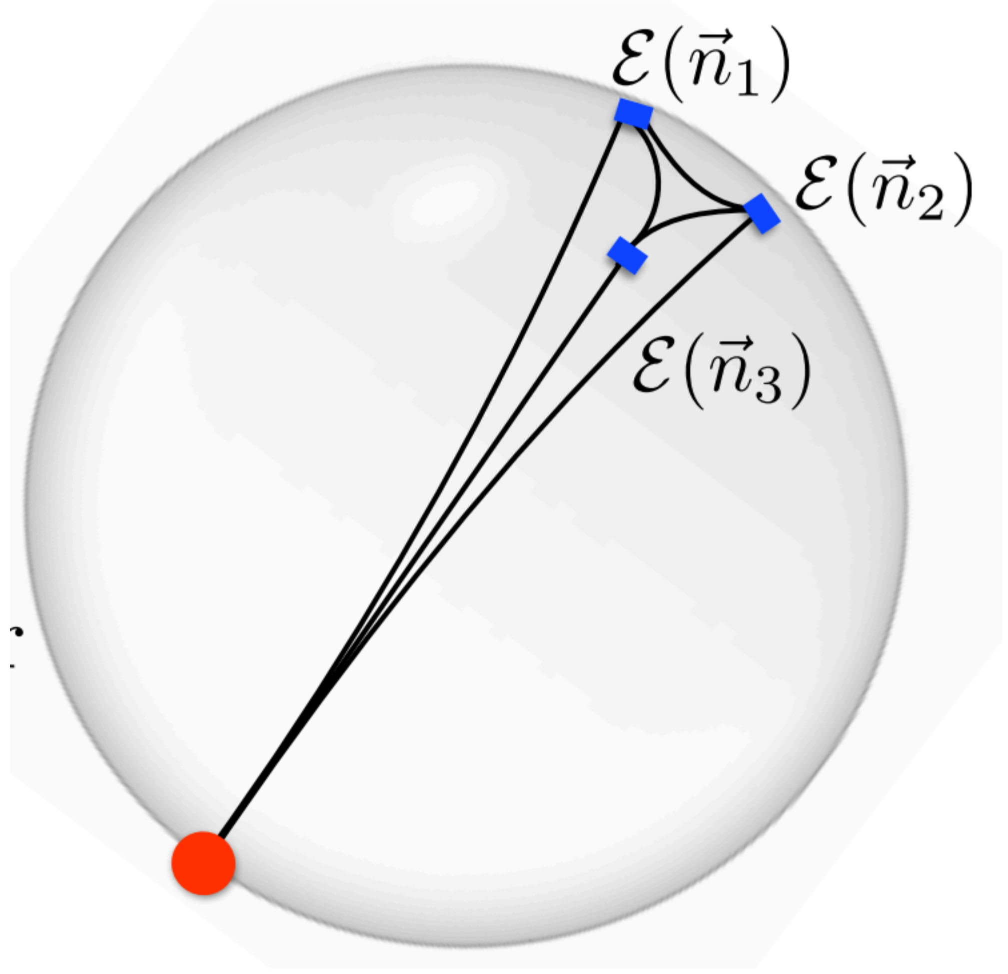
$\cos(8\phi)$

$$\ldots \qquad \ldots \qquad \ldots$$

twist expansion

$$\boxed{-z^3\bar{z}\ {}_2F_1(3,2,6,z) + \text{h.c.}}$$

# Conformal block expansion for EEEC

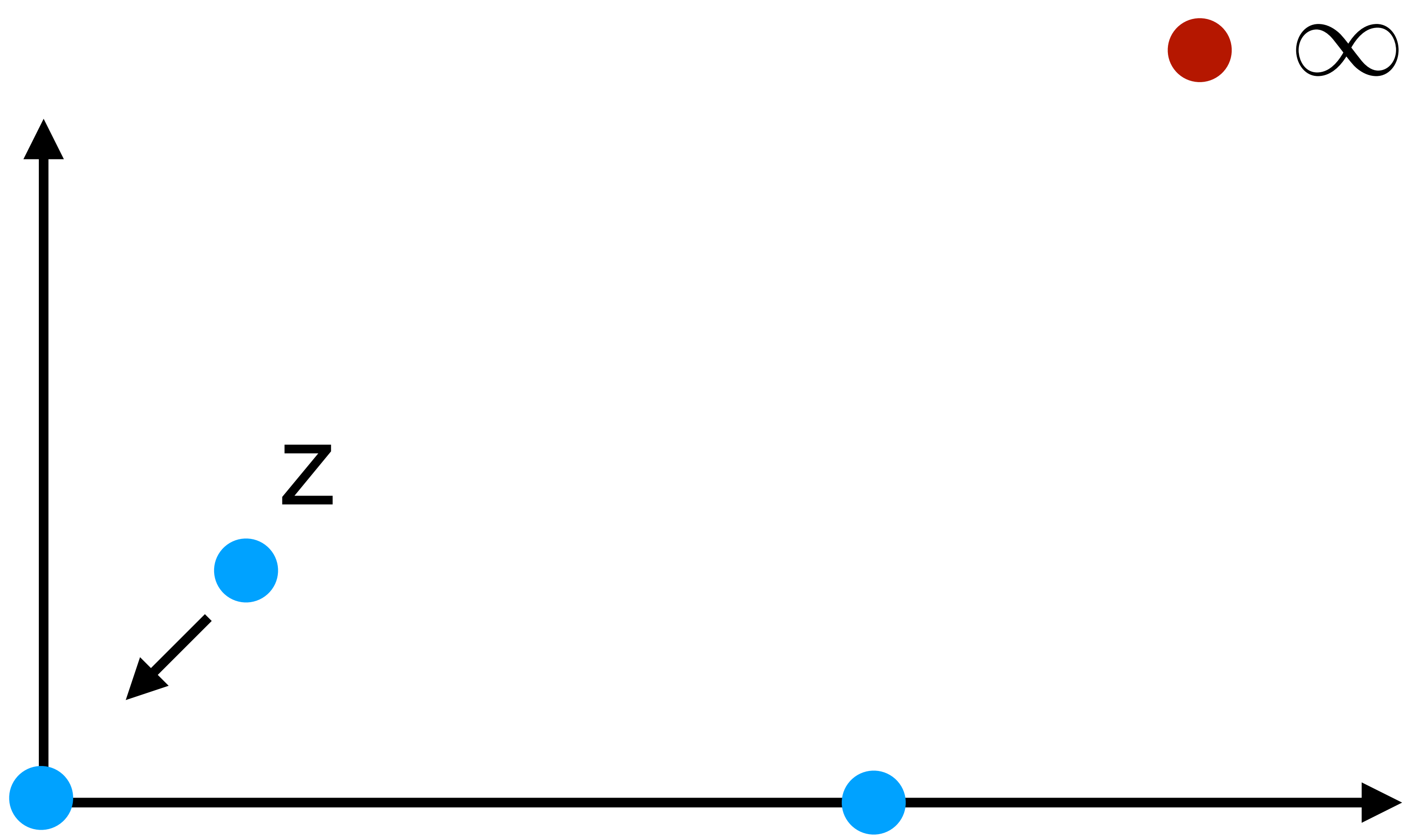



EEEC is secretly a four point function

$$\int dt\, e^{it\bar{n}\cdot P} \langle \Omega | \bar{\chi}(t\bar{n}) \frac{\not{\bar{n}}}{2} \mathcal{E}(n_1)\mathcal{E}(n_2)\mathcal{E}(n_3)\chi(0)|\Omega\rangle \equiv \langle \mathcal{E}_1\mathcal{E}_2\mathcal{E}_3\rangle_\chi$$

Lorentz invariance $\quad M_{\mu\nu}\langle \mathcal{E}_1\mathcal{E}_2\mathcal{E}_3\rangle_\chi = 0$

$$M_{\mu\nu} = -i\sum_j \left( p^j_\mu \frac{\partial}{\partial p^{j,\nu}} - (\mu \leftrightarrow \nu)\right) \qquad C_2 = \frac{1}{2} M_{\mu\nu}M^{\mu\nu}$$




$$\langle \mathcal{E}_1\mathcal{E}_2\mathcal{E}_3\rangle_\chi = \sum_{\delta,j} \underbrace{C_{\delta,j}(n_1,n_2,\partial_{n_2},\varepsilon)\langle \mathbb{O}^{J=3}_{\delta,j}(n_2,\varepsilon)\mathcal{E}_3(n_3)\rangle_\chi}_{\text{function of z}}$$

$$0 = \sum_{\delta,j} C_{\delta,j}\langle [\hat{C}_2, \mathbb{O}^{J=3}_{\delta,j}]\mathcal{E}_3\rangle_\chi + \sum_{\delta,j} \bar{C}_2 C_{\delta,j}\langle \mathbb{O}^{J=3}_{\delta,j}\mathcal{E}_3\rangle_\chi$$

eigenfunction of $C_2$

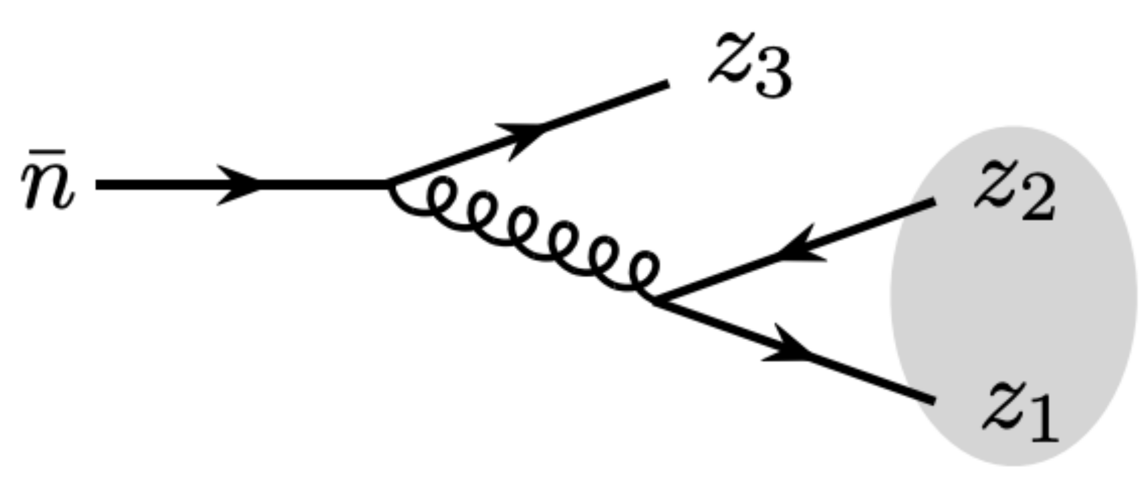
# Conformal block expansion

Chang, Simmons-Duffin, 2022
Chen, Moult, Sandor, HXZ, 2022

$$\langle \mathcal{E}_1 \mathcal{E}_2 \mathcal{E}_3 \rangle_\chi = \sum_{\delta,j} c_{\delta,j} G_{\delta,j}(z)$$

dynamical data

kinematical data
(conformal block in 2D CFT)

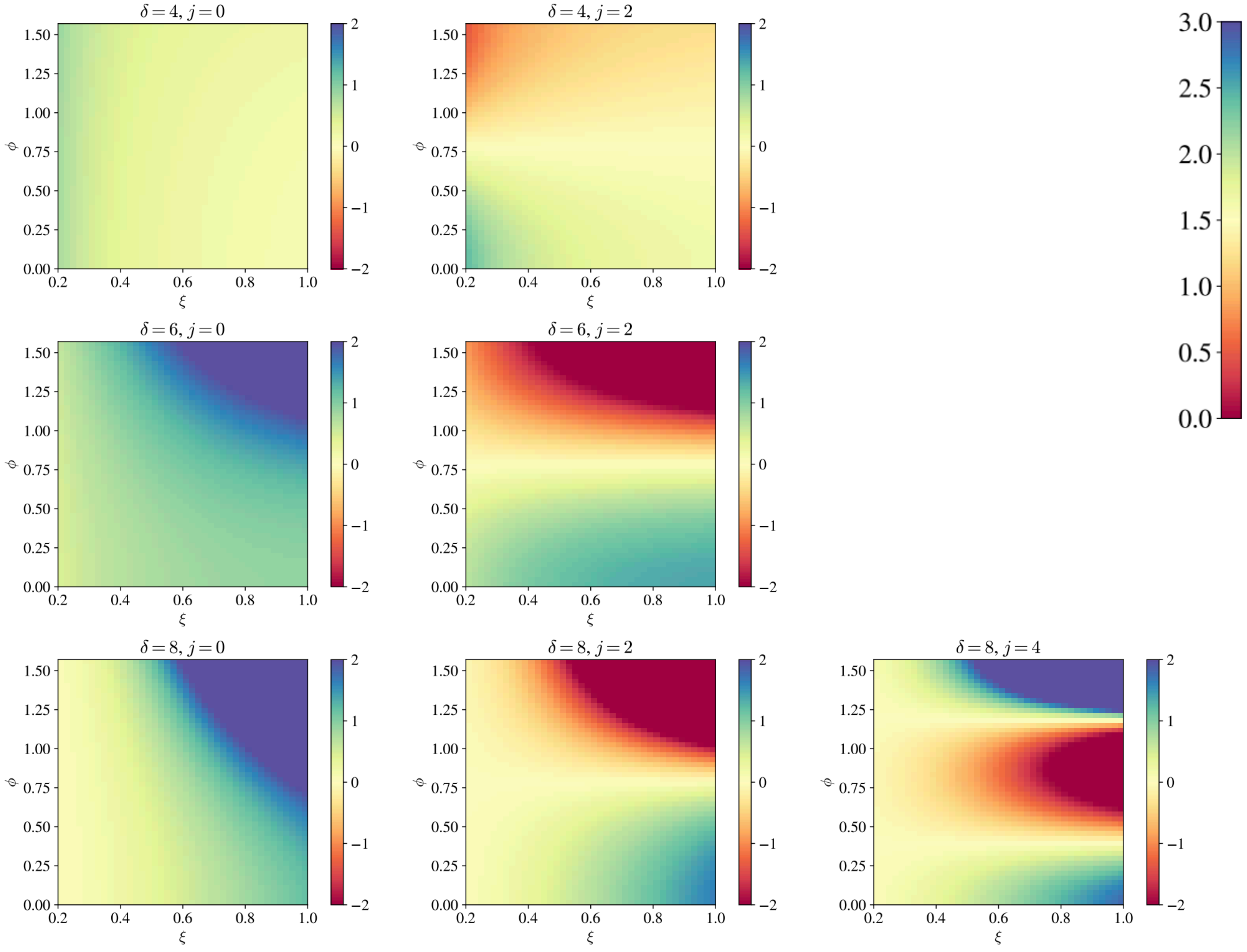


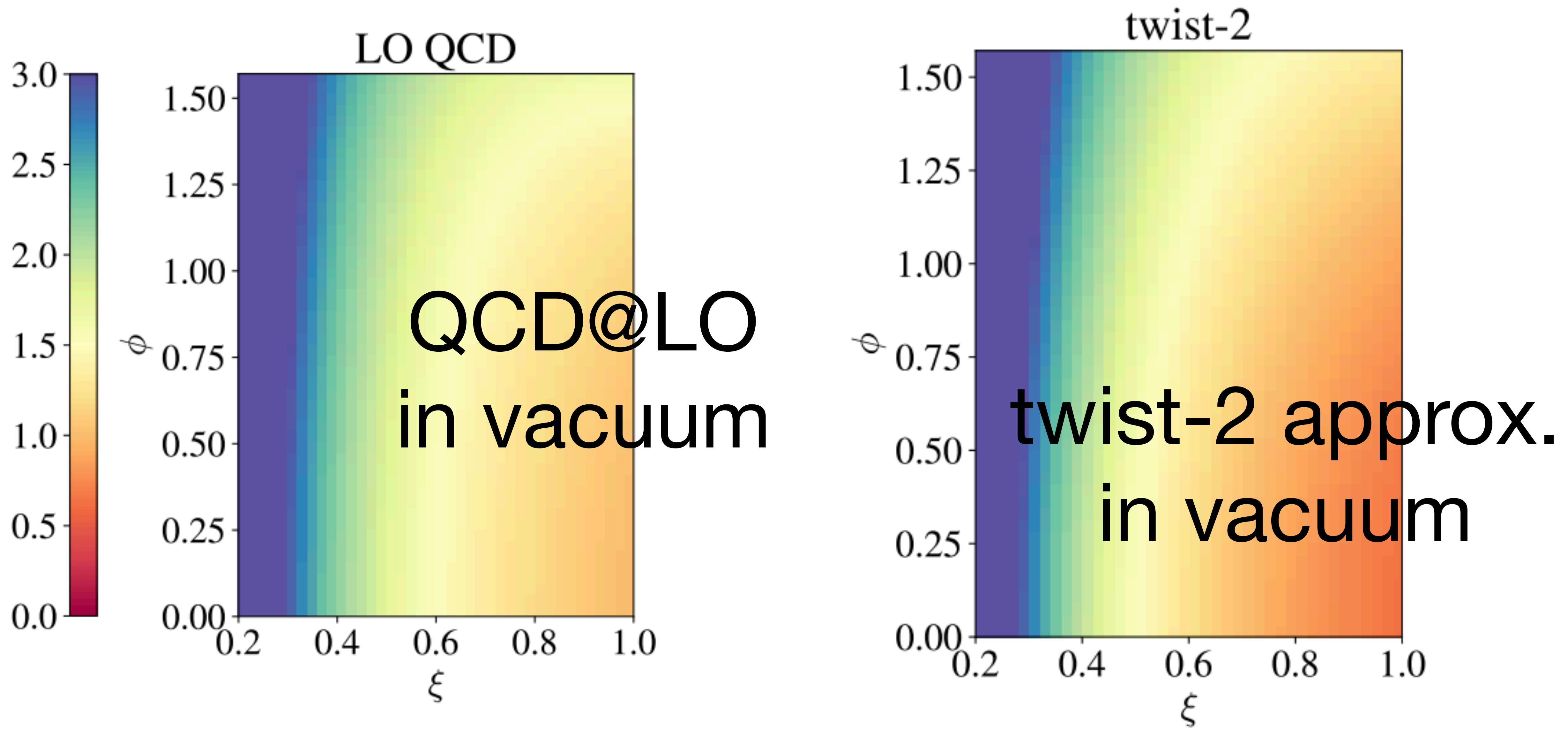

$$\begin{aligned} g_q(z) = C_F n_f T_F \Bigg[ & -\frac{1}{360} G_{4,2} + \frac{13}{1200} G_{4,0} + \frac{163}{126000} G_{6,2} + \left( \frac{111199}{33600} - \frac{\pi^2}{3} \right) G_{6,0} \\ & - \frac{67}{420} \partial_\delta G_{8,0} + \left( \frac{39243247}{2116800} - \frac{79\pi^2}{42} \right) G_{8,2} + \left( \frac{201264317}{8820000} - \frac{7\pi^2}{3} \right) G_{8,0} \\ & \boxed{- \frac{751}{4620} \partial_\delta G_{10,2} - \frac{12317}{18480} \partial_\delta G_{10,0}} + \left( \frac{9863251}{332640} - \frac{595\pi^2}{198} \right) G_{10,4} \\ & + \left( \frac{2801569019}{64033200} - \frac{40\pi^2}{9} \right) G_{10,2} + \left( \frac{168438023821}{3585859200} - \frac{937\pi^2}{196} \right) G_{10,0} + \cdots \Bigg] \end{aligned}$$

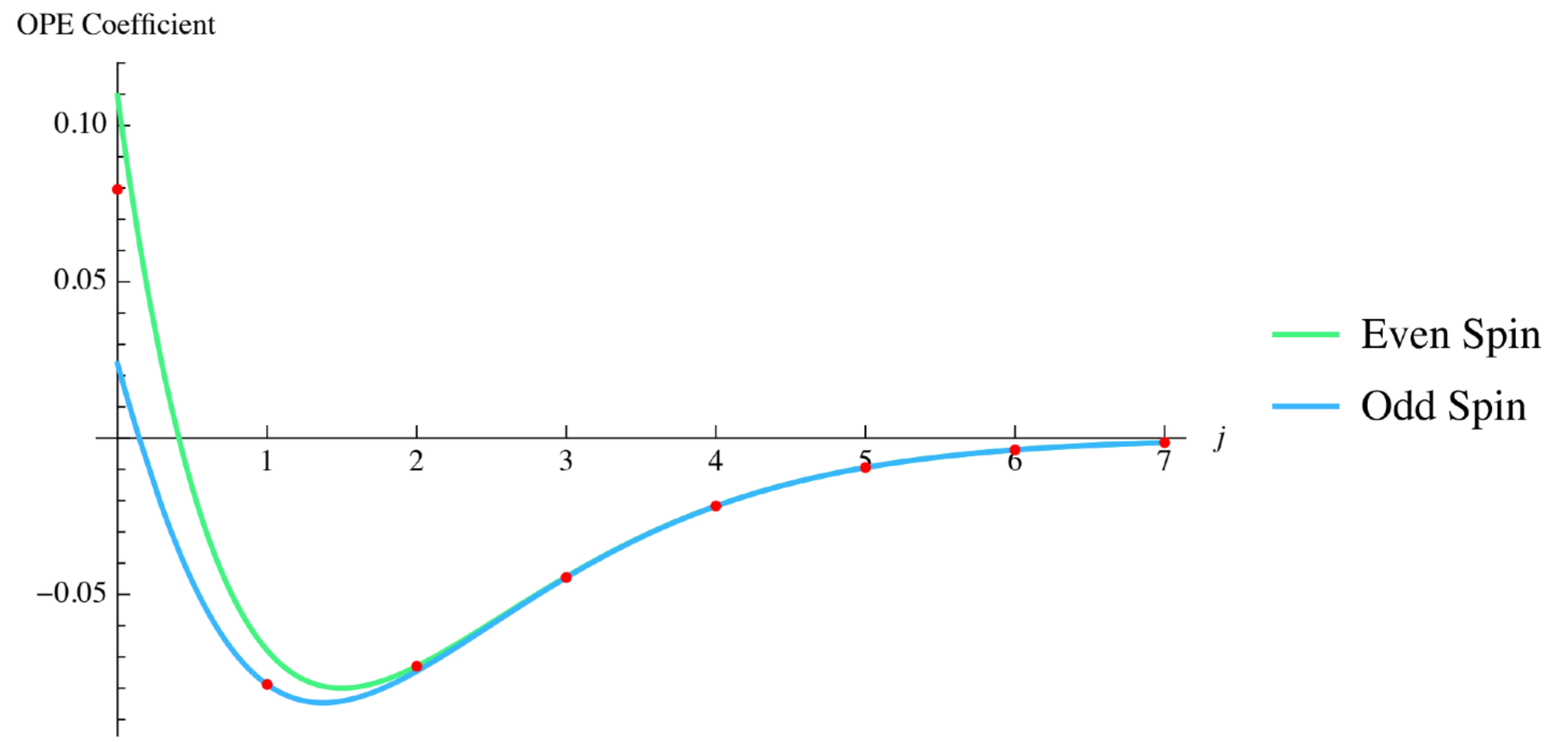
Appearance of log term in OPE expansion

Rapidly decaying coefficients

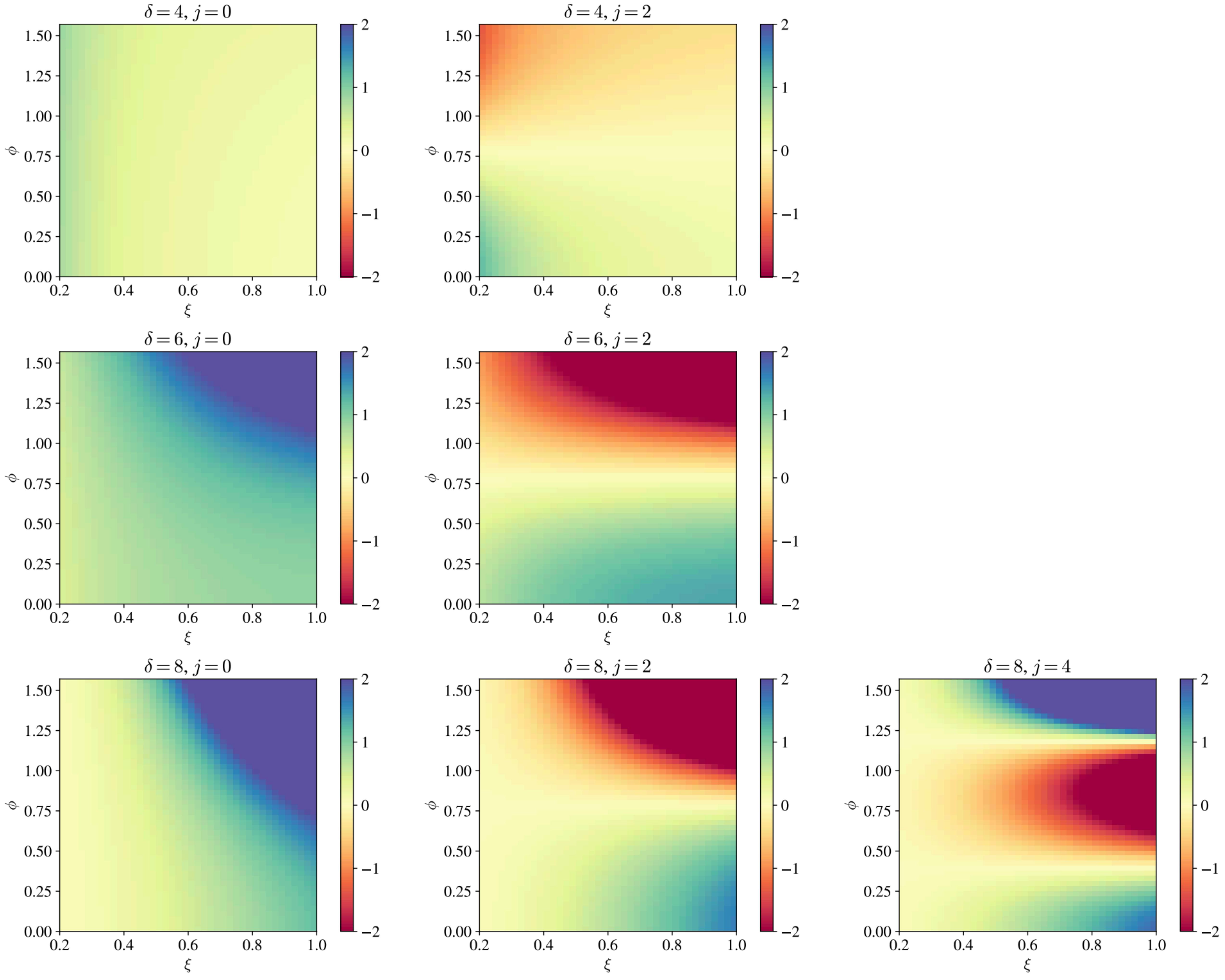
# Conformal block expansion



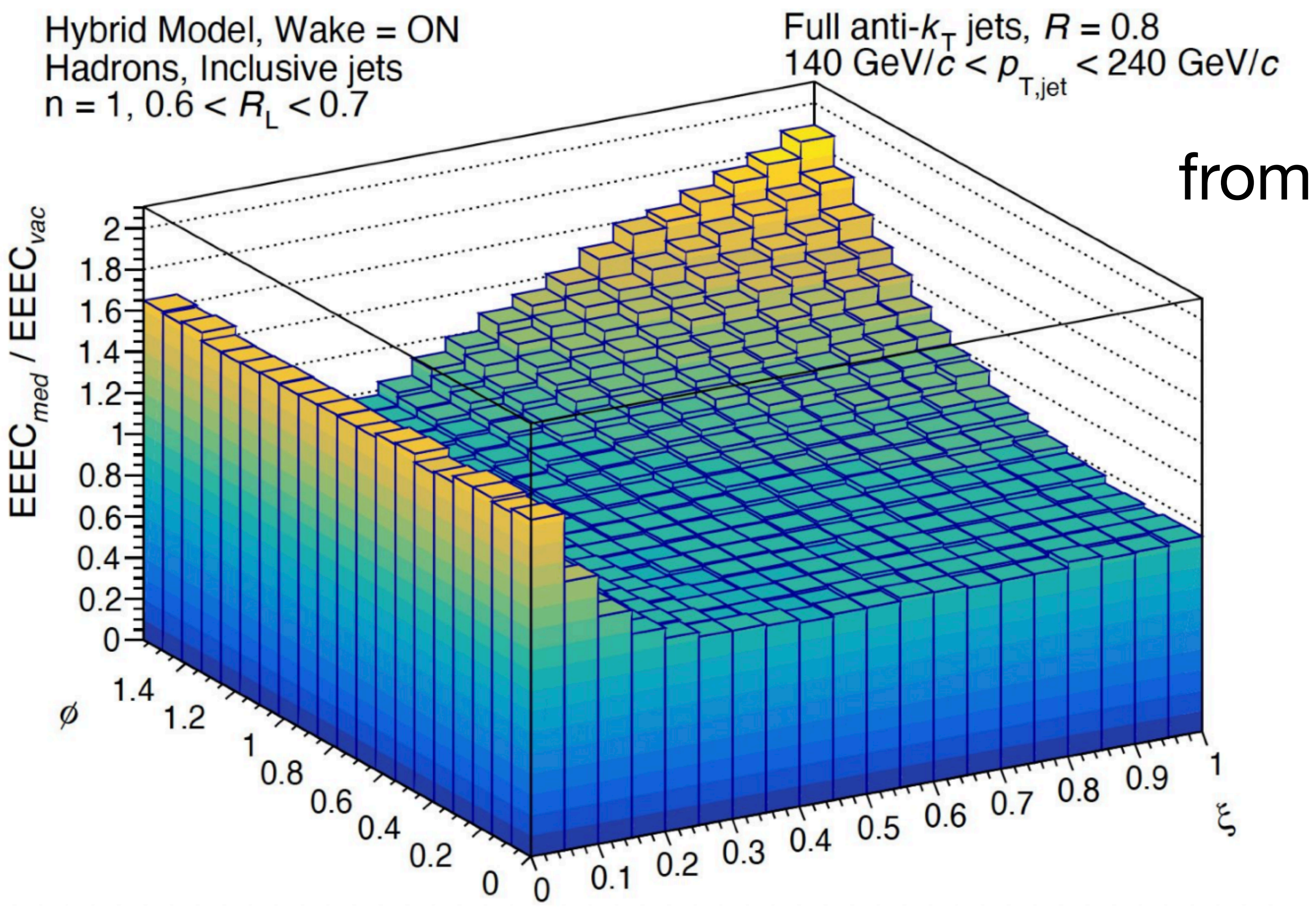



# Conformal block expansion




**Pb+Pb with wake / vacuum**




from Arjun's talk

Can a single $\delta = 6$ block fit the model?

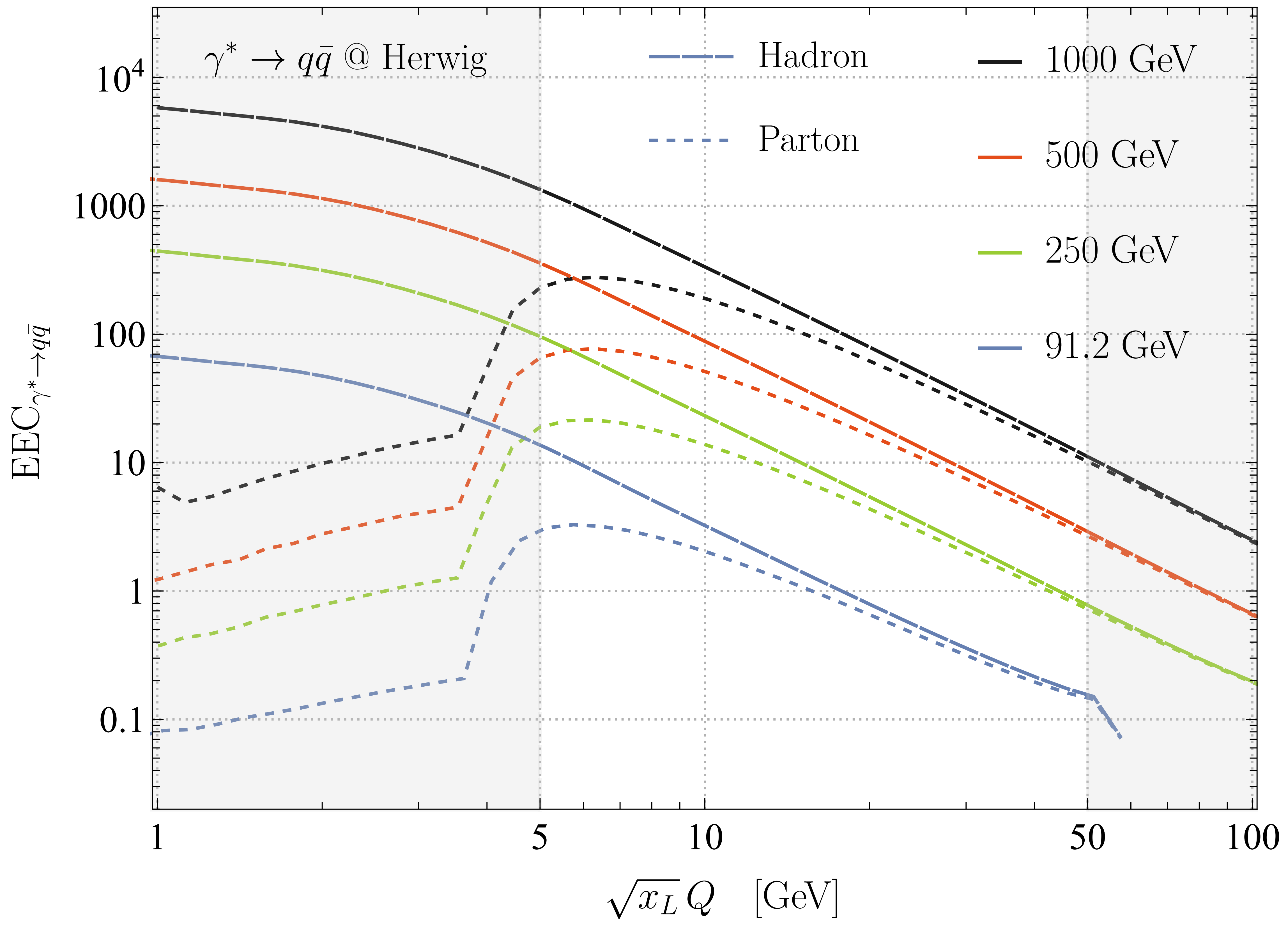
How to interpret jet wake using conformal block expansion?

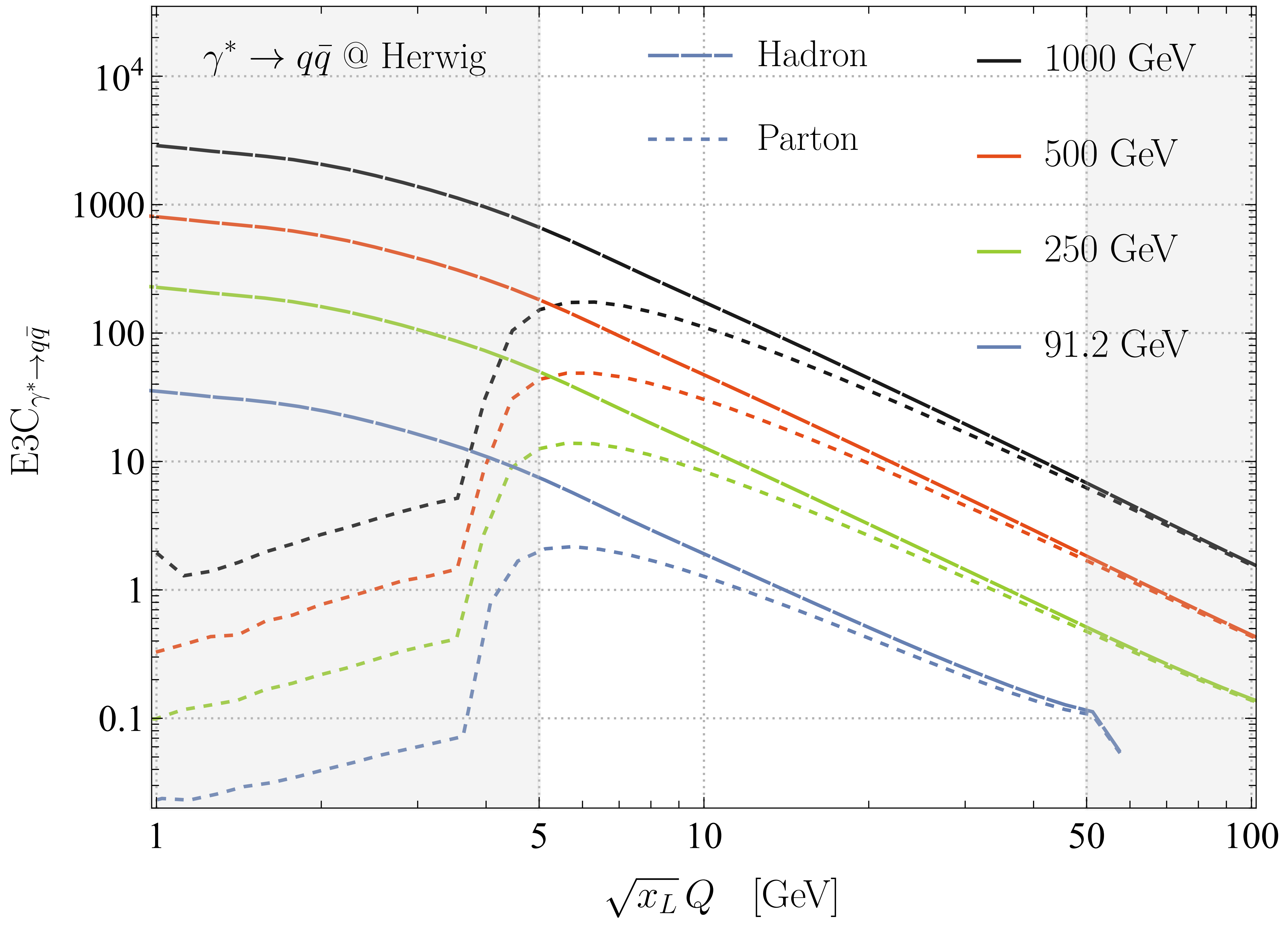
# Non-perturbative power corrections to energy correlators from light-ray OPE

2406.06668, with H. Chen, P. Monni, Z. Xu

# Significance of N.P. power corrections

power corrections = hadron - parton      better to extract from experiment




At % level, control over the power corrections are necessary

# Power expansion from symmetry

$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L} \vec{C} \cdot \vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}} \vec{D} \cdot \vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2) + \cdots$$

dimension: 1 + 1 = 1 + J-1 $\Rightarrow$ J=2

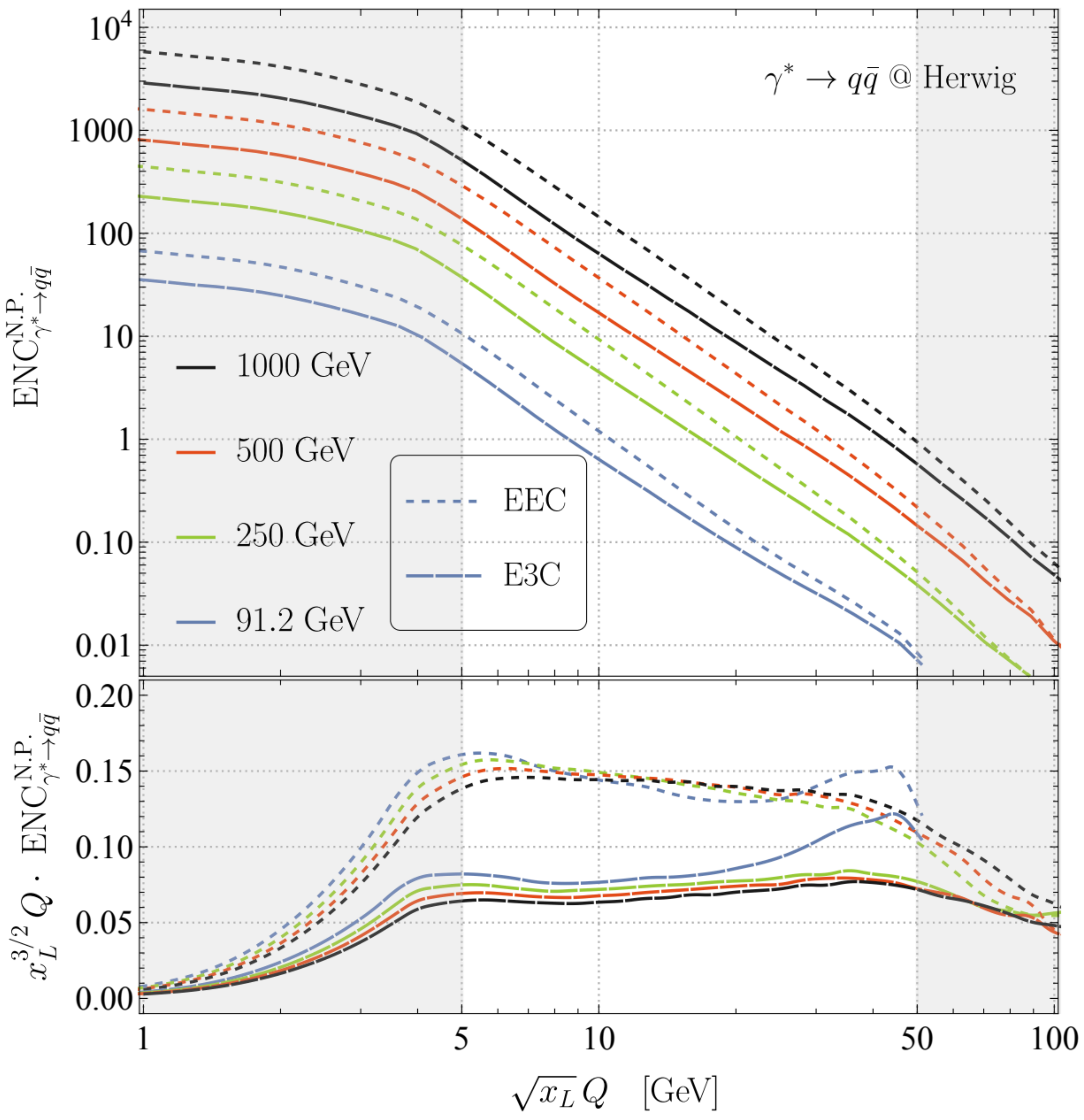
coll. spin: -3 + (-3) = y + -3 $\Rightarrow$ y=-3

$$x_L \sim \theta^2$$

| | $\mathbb{L}_\tau$ | $\vec{O}^{[J]}_\tau$ | $\vec{\mathbb{O}}^{[J]}_\tau$ | $x_L$ | $\Lambda_{\mathrm{QCD}}$ |
|---|---|---|---|---|---|
| coll. spin | $1-\tau$ | $-J$ | $1-(\tau+J)$ | 2 | 0 |
| dimension | $-\tau-1$ | $\tau+J$ | $J-1$ | 0 | 1 |

Key assumptions: the hadronization scale is boost invariant

# Scaling and its violation in power correction




$\theta Q$

- Power corrections also exhibits scaling law!
- Plotting in the correct variable $\theta Q$ manifests that the transition happen at a single transverse scale
- Removing the classical scaling behavior, leads to mild $\theta Q$ but still non-trivial dependence $\Rightarrow$ new non-perturbative function
- Also exists Q dependence

# Scaling violation from RG invariance

$$\lim_{n_1\to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L}\vec{C}\cdot\vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}}\vec{D}\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2)+\cdots$$

Define a transverse momentum scale $K_\perp = \theta Q$

$$\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q) = \Lambda_{\mathrm{QCD}} \times \underbrace{\vec{D}_N\left(\frac{K_\perp^2}{\mu^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\mu^2}\right)}_{\text{non-perturbative}} \cdot \underbrace{\frac{\langle \vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n;\mu)\rangle_{\Psi_q}}{(4\pi)^{-1}\sigma_{\Psi_q} Q^{N-1}}\left(\frac{Q^2}{\mu^2}\right)}_{\text{perturbative}}$$

In the absent of $\Lambda_{\mathrm{QCD}}$, $\theta$ dependence in D can be predicted from Q dependence

$$\mu\frac{d}{d\mu}\vec{\mathbb{O}}^{[J]}_{\tau=2}(n;\mu) = \gamma^{[J]}_{\tau=2}(\mu)\cdot\vec{\mathbb{O}}^{[J]}_{\tau=2}(n;\mu) \quad \Rightarrow \quad \mu\frac{d\vec{D}_N}{d\mu} = -\vec{D}_N\cdot\gamma^{[J=N]}_{\tau=2}$$

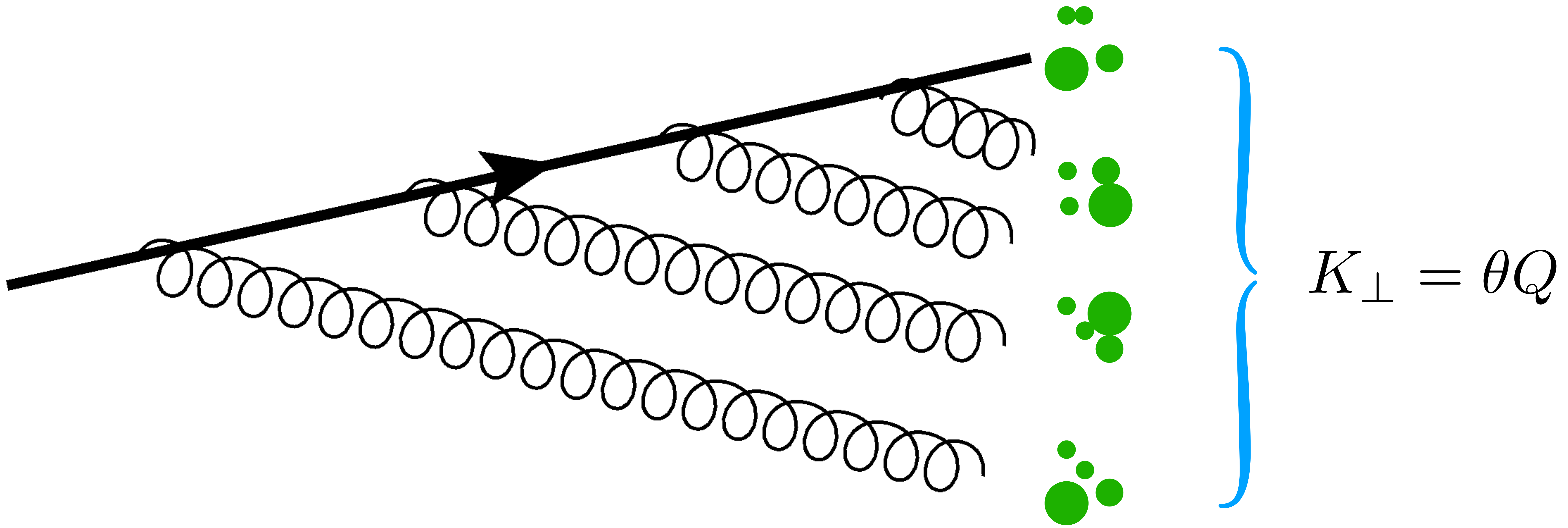
$$\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q) = \Lambda_{\mathrm{QCD}} \times \vec{D}_N\left(1, \frac{\Lambda^2_{\mathrm{QCD}}}{K_\perp^2}\right)\cdot U_N(K_\perp, Q)\cdot\frac{\langle \vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n;Q)\rangle_{\Psi_q}}{(4\pi)^{-1}\sigma_{\Psi_q} Q^{N-1}} \qquad U_N(K_\perp, Q) \equiv \mathbb{P}\exp\left(-\int_{K_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{[J=N]}_{\tau=2}(\mu)\right)$$

define a non-perturbative func. D($K_\perp$)

Q dependence predicted

# Universal N.P. functions

$$\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q) = \Lambda_{\mathrm{QCD}} \times \vec{D}_N\left(1, \frac{\Lambda^2_{\mathrm{QCD}}}{K^2_\perp}\right) \cdot U_N(K_\perp, Q) \cdot \frac{\langle \vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n;Q)\rangle_{\Psi_q}}{(4\pi)^{-1}\sigma_{\Psi_q} Q^{N-1}}$$

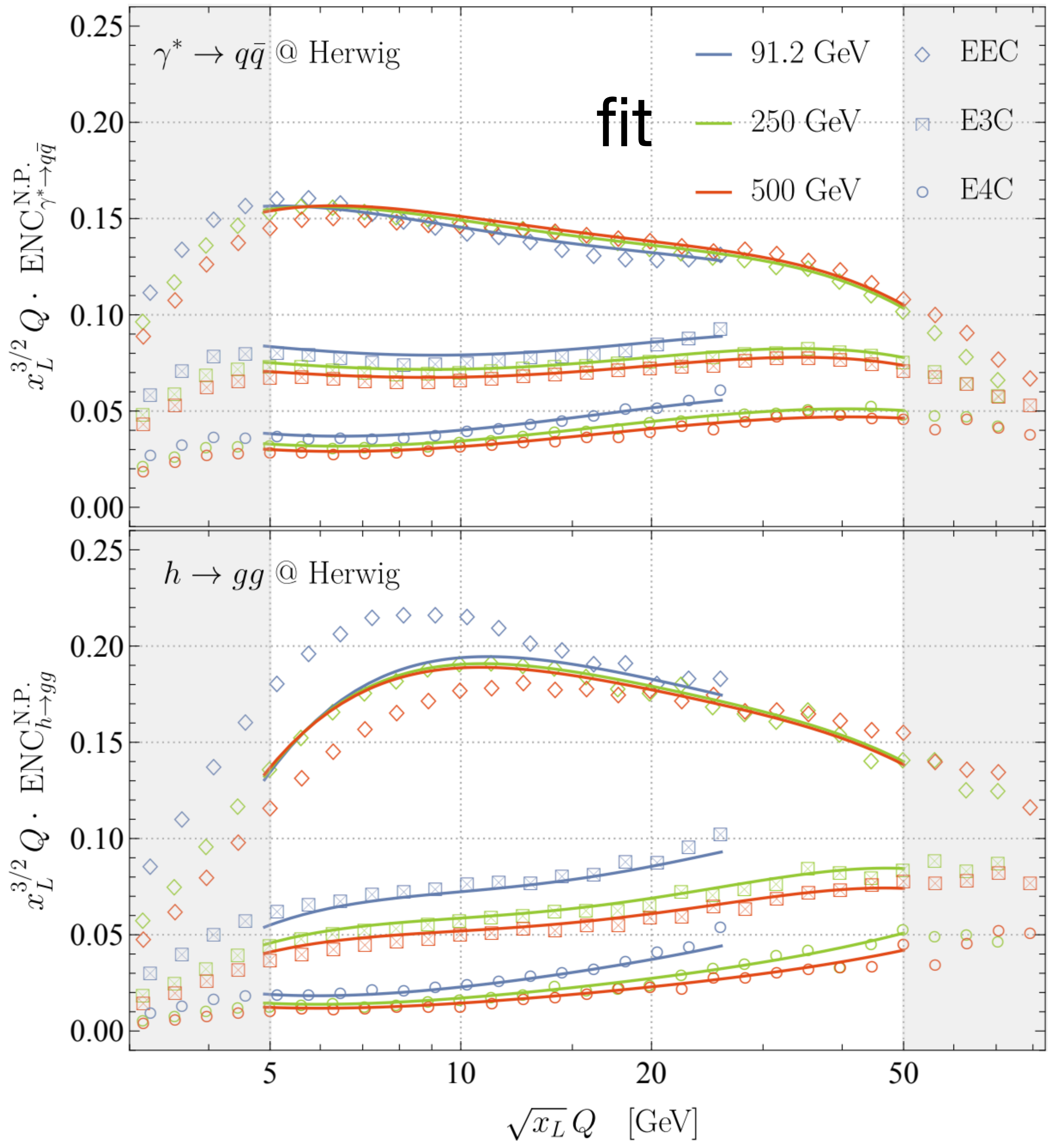


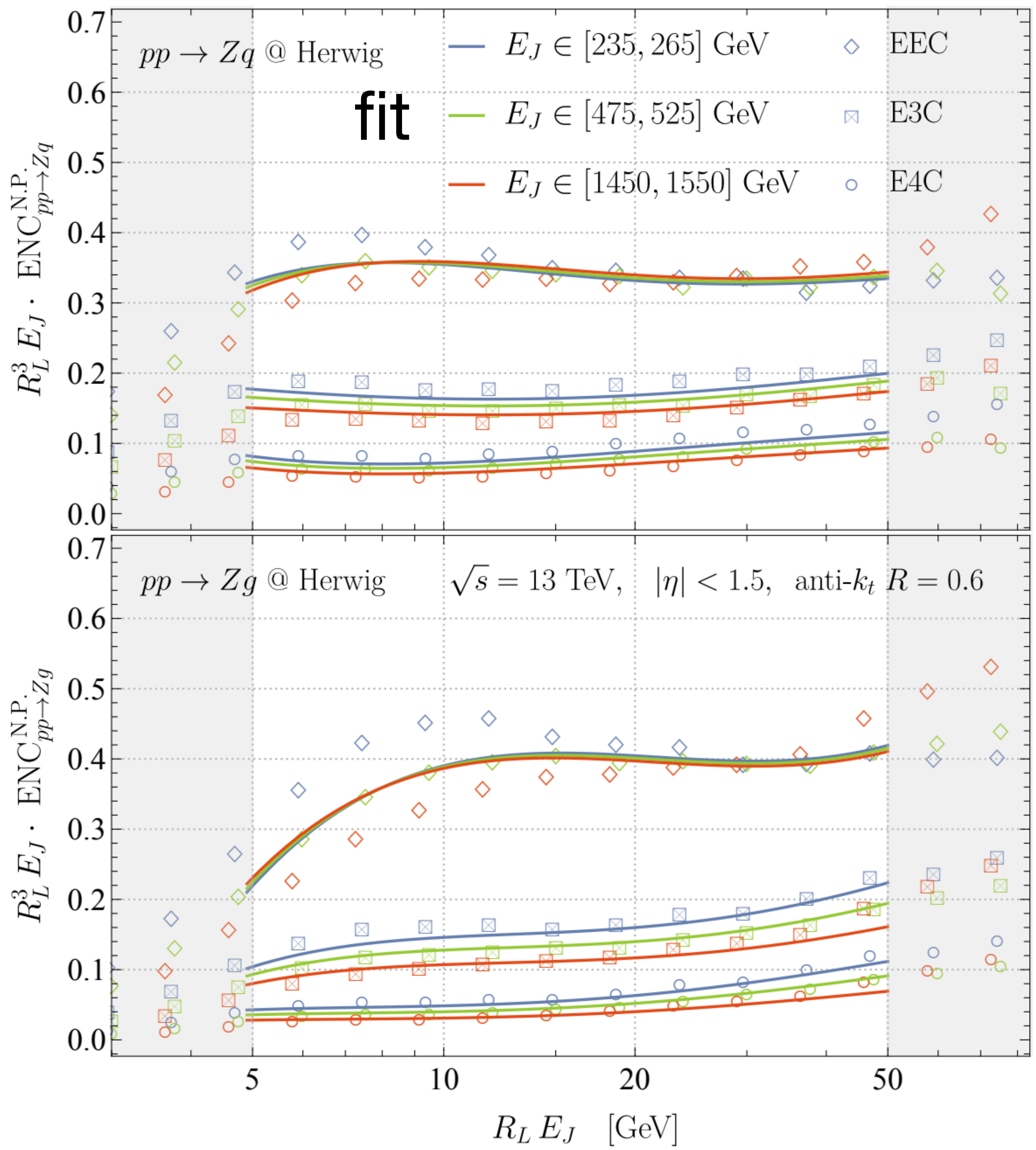

Non-perturbative scale enters the formula as a boost invariant scale

Justify the use of light-ray OPE

The Q dependence of power corrections as a whole is predicted

# Validating against event generator



# What's happening to EEC?

Expanding to the next-to-next-to-leading power

$$\lim_{n_1 \to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L}\vec{C}\cdot\vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}}\vec{D}\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}^2}{x_L^2}\vec{E}\cdot\vec{\mathbb{O}}^{[J=1]}_{\tau=2}(n_2) + \cdots$$

$$\begin{array}{ccc} & \text{qq} & \text{qg} \\ \gamma^{[J],(0)}_{\tau=2} = & \left(\begin{array}{c} 2C_F\left(4H_J - \frac{2}{J(J+1)} - 3\right) \\ -C_F\frac{4(J^2+J+2)}{(J-1)J(J+1)} \end{array}\right. & \left.\begin{array}{c} -T_F\frac{4(J^2+J+2)}{J(J+1)(J+2)} \\ 8C_A\left(H_J - \frac{2(J^2+J+1)}{(J-1)J(J+1)(J+2)}\right) - 2\beta_0 \end{array}\right) \\ & \text{gq} & \text{gg} \end{array}$$

Pole at J=1 due to small-x singularity

Large anomalous dimension could enhance the sensitivities to NNLP

# An better scheme for e+e-

$$\vec{F}_1^{[N]}(K_\perp, Q)\cdot \vec{v}_i^{[N]} = \left[\frac{\alpha_s(Q)}{\alpha_s(K_\perp)}\right]^{\lambda_i^{[N]}/(2\beta_0)} \left[\Lambda_{\mathrm{QCD}}\vec{D}_N\left(1, \frac{\Lambda_{\mathrm{QCD}}^2}{K_\perp^2}\right)\cdot \vec{v}_i^{[N]}\right]$$

the evolution factor

$$\vec{F}_1^{[N]}(K_\perp, Q) = \frac{R_N(Q,Q_2)}{R_N(Q_1,Q_2)}\vec{F}_1^{[N]}(K_\perp, Q_1) + \frac{R_N(Q_1,Q)}{R_N(Q_1,Q_2)}\vec{F}_1^{[N]}(K_\perp, Q_2)$$

$$R_N(Q_1,Q_2) = \det\begin{pmatrix} \alpha_s(Q_1)^{\lambda_1^{[N]}/(2\beta_0)} & \alpha_s(Q_1)^{\lambda_2^{[N]}/(2\beta_0)} \\ \alpha_s(Q_2)^{\lambda_1^{[N]}/(2\beta_0)} & \alpha_s(Q_2)^{\lambda_2^{[N]}/(2\beta_0)} \end{pmatrix}$$

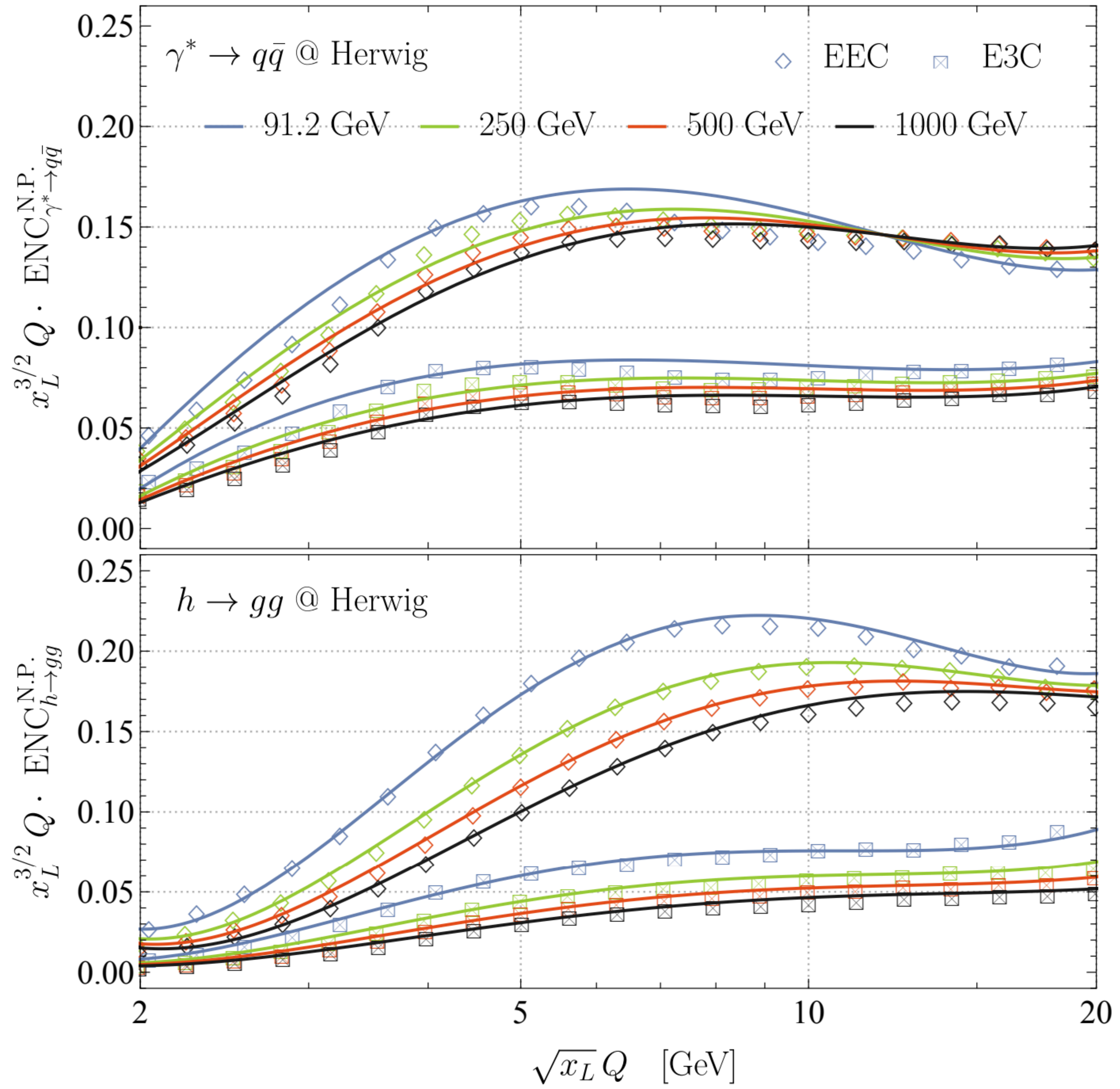
```
Log[Q] - Log[Q2]       αs (2 β0 + λ1 + λ2) (Log[Q] - Log[Q1]) (Log[Q] - Log[Q2])
----------------  -  ------------------------------------------------------------
Log[Q1] - Log[Q2]                  8 π (Log[Q1] - Log[Q2])
```

$$x_L^{3/2} Q\,\mathrm{ENC}_{\Psi_q}^{\mathrm{N.P.}}(K_\perp, Q) \approx \frac{R_N(Q,Q_2)}{R_N(Q_1,Q_2)} x_L^{3/2} Q_1\,\mathrm{ENC}_{\Psi_q}^{\mathrm{N.P.}}(K_\perp, Q_1) + \frac{R_N(Q_1,Q)}{R_N(Q_1,Q_2)} x_L^{3/2} Q_2\,\mathrm{ENC}_{\Psi_q}^{\mathrm{N.P.}}(K_\perp, Q_2)\,,$$

contains $\mathcal{O}(\Lambda_{\mathrm{QCD}}) + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2) + \cdots$

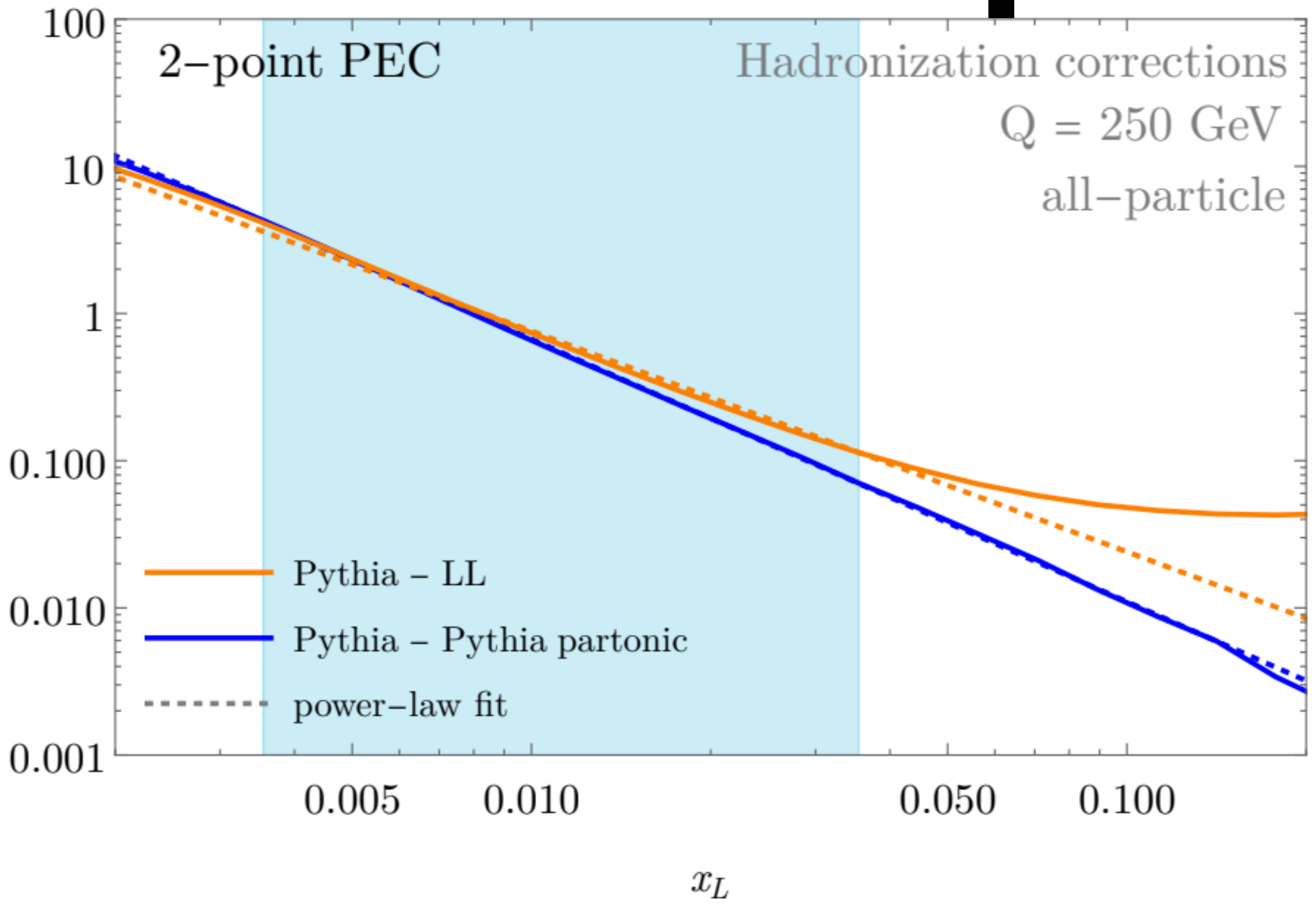
requires input at two energy scales

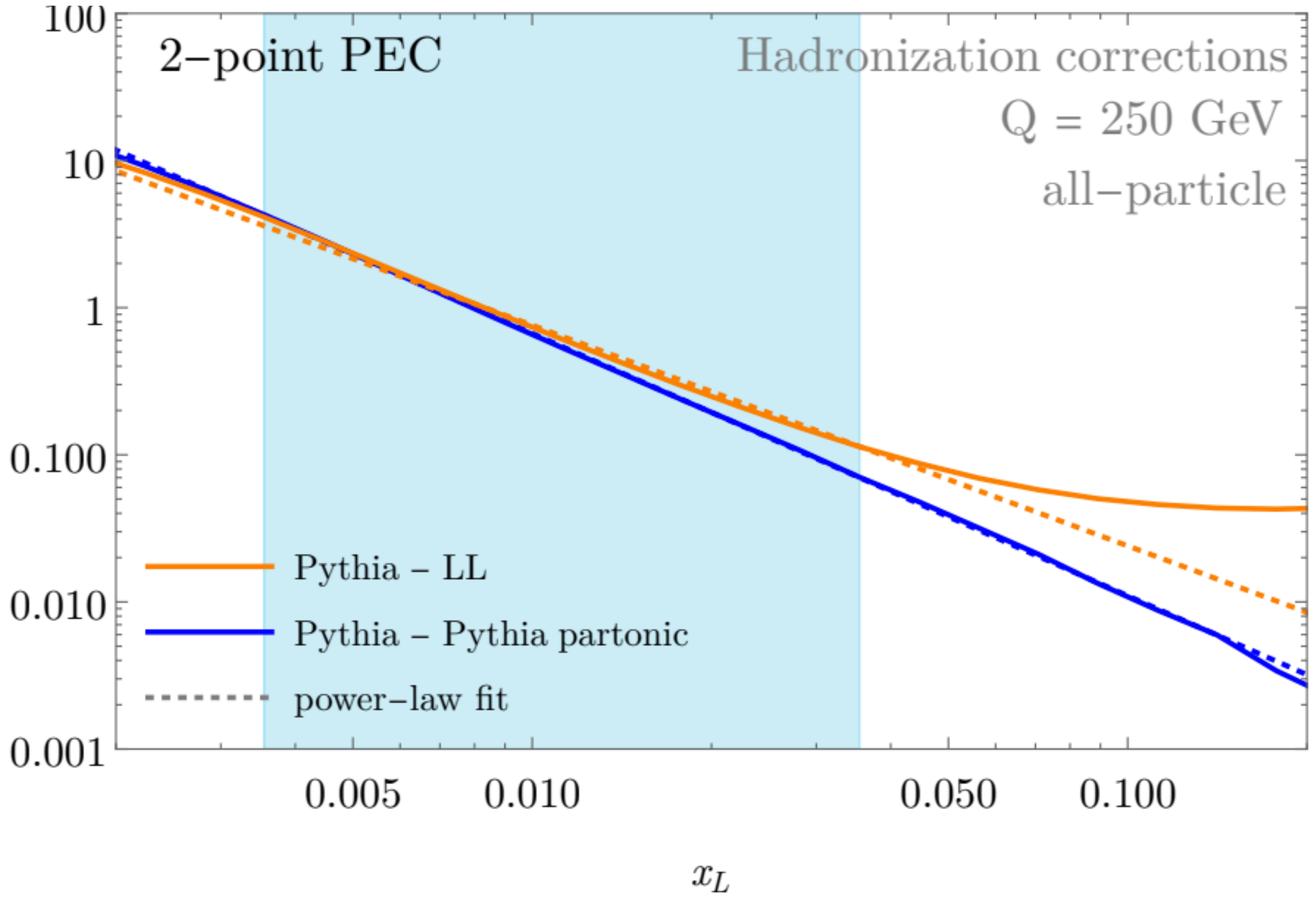
# Results for the alternative scheme




- For e+e-, fit for 250 and 500 GeV from Monte-Carlo and predict other energy
- Same for Higgs->gg
- Now good agreement is found even for EEC
- Agreement is even found deep into the non-perturbative region

# Incorporating track again




power corrections on tracks is defined as Pythia - perturbative LL

$$\mathrm{PNC}^{\mathrm{tr}}(x_L) = \mathrm{PNC}^{\mathrm{tr}}_{\mathrm{pert}}(x_L) + \frac{\Lambda^{(n)}_{\mathrm{tr},1}}{x_L^{1.5}} + \frac{\Lambda^{(n)}_{\mathrm{tr},2}}{x_L}$$

| | 2-point | 3-point | 4-point |
|---|---|---|---|
| all-particle | $\frac{\Lambda_1^{(2)}}{x_L^{1.5}} + \frac{\Lambda_2^{(2)}}{x_L}$ | $\frac{\Lambda_1^{(3)}}{x_L^{1.5}} + \frac{\Lambda_2^{(3)}}{x_L}$ | $\frac{\Lambda_1^{(4)}}{x_L^{1.5}} + \frac{\Lambda_2^{(4)}}{x_L}$ |
| $\Lambda_1^{(n)}$ | $0.00076 \pm 0.00006$ | $0.00044 \pm 0.00004$ | $0.000229 \pm 0.000013$ |
| $\Lambda_2^{(n)}$ | $7 \times 10^{-9} \pm 0.0005$ | $1.0 \times 10^{-9} \pm 0.00028$ | $0.000031 \pm 0.00011$ |
| charged-particle | $\frac{\Lambda_{\mathrm{tr},1}^{(2)}}{x_L^{1.5}} + \frac{\Lambda_{\mathrm{tr},2}^{(2)}}{x_L}$ | $\frac{\Lambda_{\mathrm{tr},1}^{(3)}}{x_L^{1.5}} + \frac{\Lambda_{\mathrm{tr},2}^{(3)}}{x_L}$ | $\frac{\Lambda_{\mathrm{tr},1}^{(4)}}{x_L^{1.5}} + \frac{\Lambda_{\mathrm{tr},2}^{(4)}}{x_L}$ |
| $\Lambda_{\mathrm{tr},1}^{(n)}$ | $0.000266 \pm 0.000023$ | $0.000106 \pm 0.000013$ | $0.000044 \pm 0.000004$ |
| $\Lambda_{\mathrm{tr},2}^{(n)}$ | $0 \pm 0.00019$ | $0 \pm 0.00011$ | $0 \pm 0.00004$ |
| $\Lambda_{\mathrm{tr},1}^{(n)}/\Lambda_1^{(n)}$ | 0.35 | 0.24 | 0.19 |

M. Jaarsma, et al., 2307.15739

# Summary

- Energy correlators provide a dual view of high energy scattering
- Light-ray operators make symmetry manifest:
    - spin physics
    - perturbative and non-perturbative power corrections
- Phenomenology and experimental driven measurement can bring in new challenging for field theory: e.g. tracks
- Simple, but not simpler