Nonperturbative Corrections for Energy Correlators:

Confinement Transition and α_s Extraction

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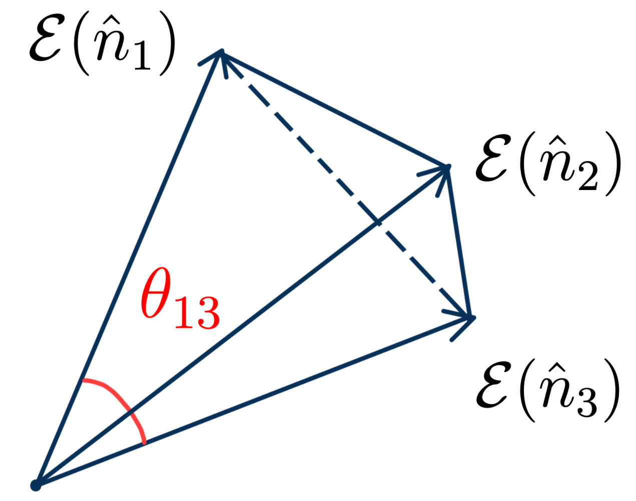
based on arXiv/2405.19396
In collaboration with: Kyle Lee, Aditya Pathak, Iain Stewart

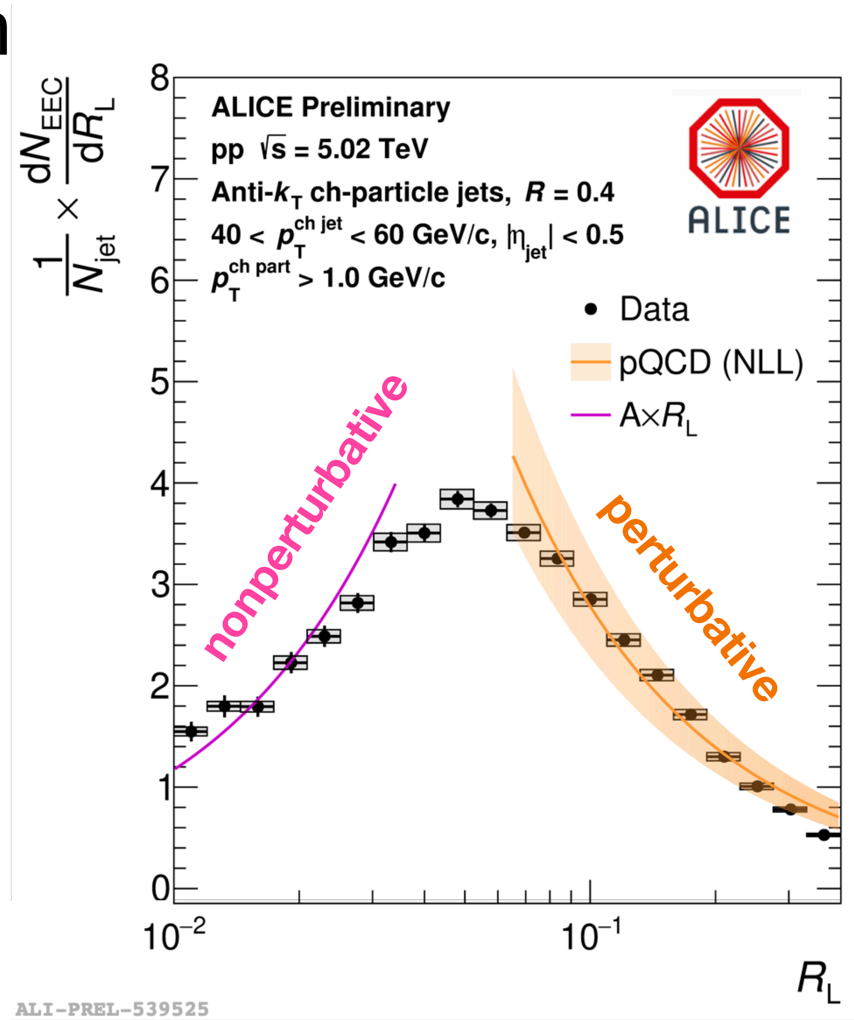


MITP Workshop, “Energy Correlators at the Collider Frontier”
July 11, 2024, Mainz, Germany

Outline



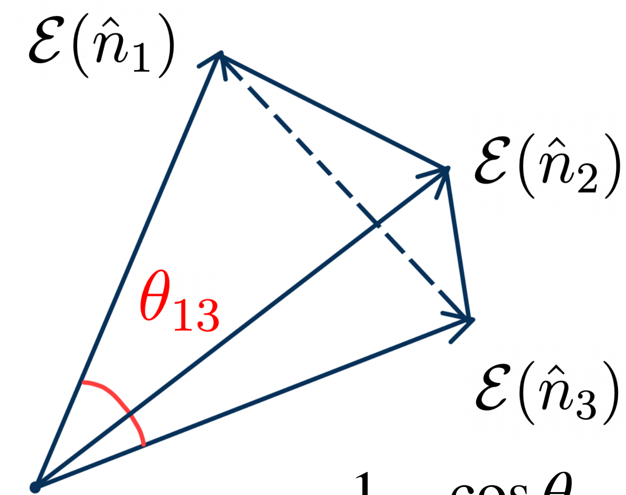
- Projected N-point energy correlator (pENC)
  - Leading nonperturbative correction
- pENC with R scheme
- Collinear region and small-angle resummation
  - Nonperturbative correction in the collinear limit (e^+e^-, pp)
  - Approaching the confinement transition
  - Impact on α_s extraction



Projected Energy Correlators

- N-point energy correlator:
  - Depends on $N(N-1)/2$ angles
- *Projected* N-point energy correlator (pENC):

H. Chen, I. Moult, X-Y. Zhang, H-X. Zhu, 2004.11381



$$z_{13} = \frac{1-\cos\theta_{13}}{2}$$

Integrate out information about the shape

$$\frac{d\Sigma^{[N]}}{dx_L} \equiv \sum_{i_1,\ldots,i_N} \int d\sigma \frac{\prod_{k=1}^{N} E_{i_k}}{Q^N} \delta\big(x_L - \max_{1\le l,m\le N}\{z_{lm}\}\big)$$

$$= \int d^4x \frac{e^{iq\cdot x}}{Q^N} \prod_{i=1}^{N} \int d\Omega_{\vec{n}_i} \delta\left(x_L - \frac{1-\min(\vec{n}_i\cdot\vec{n}_j)}{2}\right)$$

$$\times L_{\mu\nu} \langle 0 \big| J^{\mu\dagger}(x) \mathcal{E}(\vec{n}_1)\, \mathcal{E}(\vec{n}_2) \ldots \mathcal{E}(\vec{n}_N)\, J^{\nu}(0) \big| 0 \rangle$$

Energy flow operator $\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$

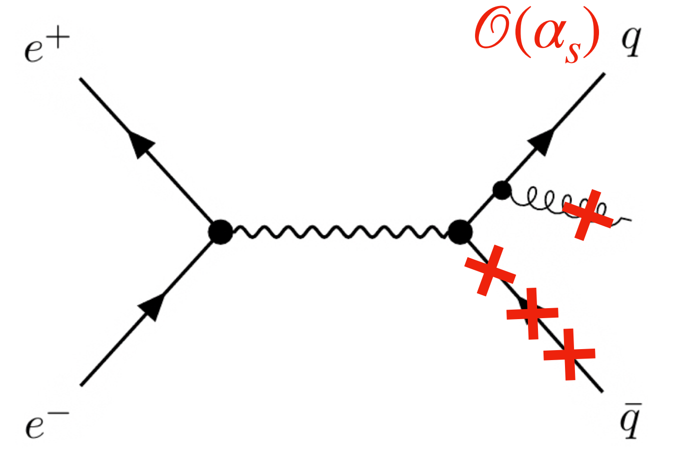
Leading NP Correction to pENC

C. Lee, G. F. Sterman, hep-ph/0603066, 0611061

- The **same** matrix element Ω_1 gives *leading* (in α_s) NP correction for pENC

$$\Omega_1 \equiv \frac{1}{N_c}\langle 0|\,\mathrm{tr}\,\overline{Y}^{\dagger}_{\bar n} Y^{\dagger}_{n} \underbrace{\mathcal{E}_T(0)} Y_n \overline{Y}_{\bar n}|0\rangle$$

Transverse energy flow operator $\mathcal{E}_T(\eta) = \cosh^{-3}\eta \int d\phi\, \mathcal{E}(\vec{n})$



- The same calculation as for the EEC gives x_L dependence; detector combinatorics gives factor of N

K. Lee, A. Pathak, I. Stewart, ZS (2024)

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{dx_L} = \frac{1}{\sigma}\frac{d\hat{\sigma}^{[N]}}{dx_L} + \frac{N}{2^N}\frac{\overline{\Omega}_1}{Q(x_L(1-x_L))^{3/2}}$$

$\overline{\mathrm{MS}}$ scheme

*At higher order in α_s, the x_L and N dependence can be different

Moving to R scheme

- Define a better scheme for both $d\hat{\sigma}^{[N]}/dx_L$ and $\overline{\Omega}_1$

Start with $\overline{\text{MS}}$ scheme:

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{dx_L} = \frac{1}{\sigma}\frac{d\hat{\sigma}^{[N]}}{dx_L} + \frac{N}{2^N}\frac{\overline{\Omega}_1}{Q(x_L(1-x_L))^{3/2}}$$

Move to R scheme:

$$\Omega_1(R) \equiv \overline{\Omega}_1 - R\sum_{n=1}^{\infty} d_n\left(\frac{\mu}{R}\right)\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

$$\frac{1}{\sigma}\frac{d\hat{\sigma}_{\rm R}^{[N]}(R)}{dx_L} \equiv \sum_{n=1}^{\infty}\left\{\underbrace{c_n\left(x_L,\frac{\mu}{Q}\right)}_{\overline{\text{MS}}\text{ series coefficients}} + \frac{N}{2^N}\frac{R}{Q}\frac{d_n(\mu/R))}{[x_L(1-x_L)]^{3/2}}\right\}\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

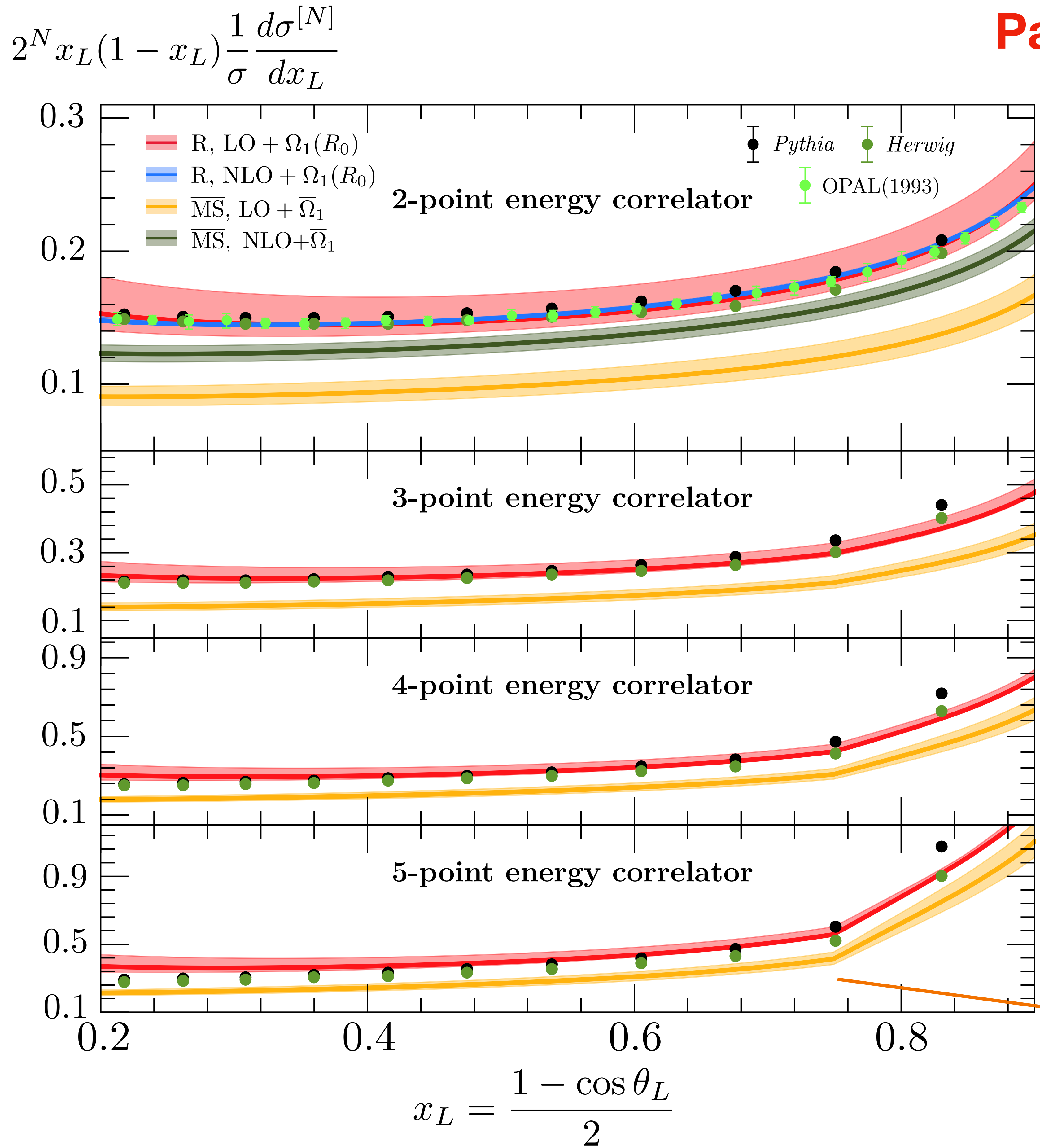
- Need to resum $\log(\mu/R)$ using R-RGE and incorporate hadron mass correction, similar to EEC

See Iain's talk

Parameter free predictions!

α_s and Ω_1 values are from thrust fit

R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu, I. Stewart, 1006.3080



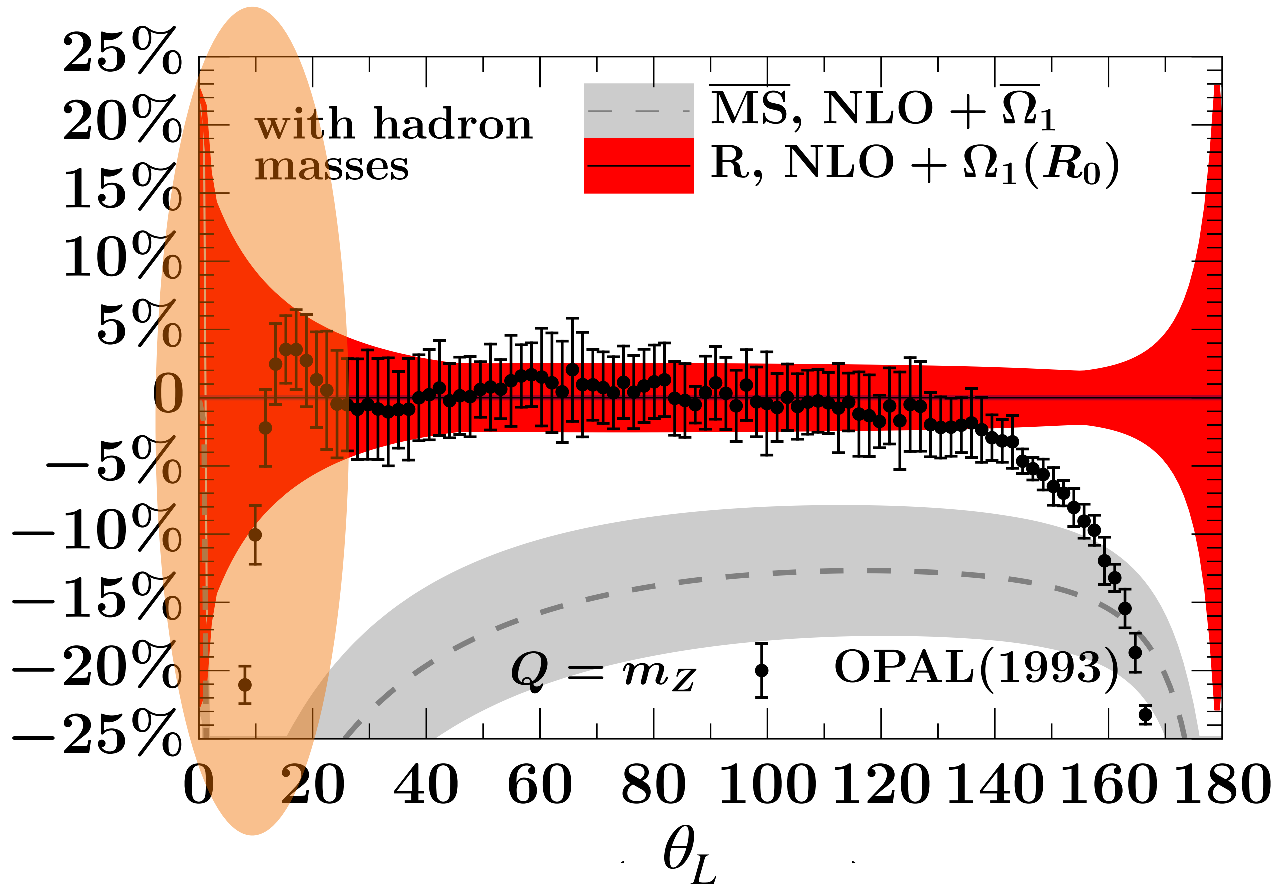
EEC results:
agree with MC and data

pENC results:
R scheme agrees with MC better than $\overline{\text{MS}}$

Kink at $x_{L_0} \leq \frac{1-\cos(2\pi/3)}{2}$
from 3-particle contribution

Small angle region

- Recall that fixed-order EEC results don't agree with data at endpoints



S. Schindler, I. Stewart, ZS (2023)

- Need to resum $\log(x_L)$ in collinear region $x_L = \frac{1 - \cos\theta_L}{2} \to 0$

*in back-to-back region, 3-particle contribution also gives leading NP correction, so the situation is more complicated for N>2

Collinear factorization for pENC

- Factorize into jet and hard function (true for e^+e^- and pp)

H. Chen, I. Moult, X-Y. Zhang, H-X Zhu, 2004.11381
W. Chen, J. Gao, Y-B. Li, Z. Xu, X-Y. Zhang, H-X Zhu, 2307.07510
K. Lee, B. Mecaj, I. Moult, 2205.03414

Cumulative distribution →

$$\Sigma^{[N]}(x_L) = \frac{1}{\sigma} \int_0^{x_L} dx'_L \frac{d\sigma^{[N]}}{dx'_L}$$
$$= \int_0^1 dx\, x^N \underbrace{\vec{J}^{[N]}\left(\ln \frac{x_L x^2 Q^2}{\mu^2}, \mu\right)} \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

pENC jet function captures x_L dependence

- Leading nonperturbative correction arises in $\vec{J}^{[N]}$ and the same Ω_1 will appear

NP Correction to Jet Function

- Scale of the jet function is $Q' = \sqrt{x_L}\, x\, Q$

coeff of power correction at LL accuracy

$$2^N\, J^{[N]}\left(\ln\frac{Q'}{\mu}, \mu\right) = 2^N\, \hat{J}^{[N]}\left(\ln\frac{Q'}{\mu}, \mu\right) + c^{\mathrm{LL}}\,\frac{\overline{\Omega}_1}{Q'} + \mathcal{O}\left(\frac{\alpha_s(Q')\Lambda_{\mathrm{QCD}}}{Q'}\right)$$

normalization

perturbative series in $\overline{\mathrm{MS}}$

same NP matrix element!

NP Correction to Jet Function

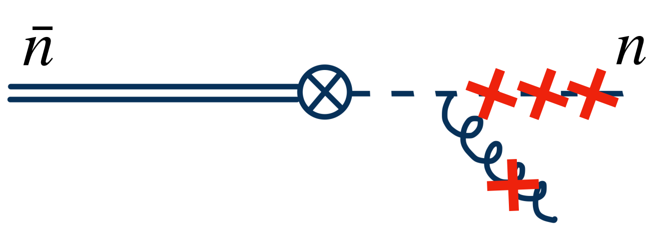
- Scale of the jet function is $Q' = \sqrt{x_L}\, x\, Q$

$$2^N\, J^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) = 2^N\, \hat{J}^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) + c^{\rm LL}\,\frac{\overline{\Omega}_1}{Q'} + \mathcal{O}\Big(\frac{\alpha_s(Q')\Lambda_{\rm QCD}}{Q'}\Big)$$

- Matrix element of jet function

$$\sum_X \sum_{i_1\ldots i_N \in X} \langle 0|\chi_n|X\rangle \frac{\Pi_{k=1}^N E_{i_k}}{Q^N}\Theta(\max_{1\le l,m\le N}\{z_{lm}\} < x_L)\langle X|\bar{\chi}_n|0\rangle$$

$W^\dagger(\bar{n}\cdot A_n)\xi_n$ $\qquad$ leading NP correction $\qquad$ $\bar{\xi}_n W(\bar{n}\cdot A_n)$



for c-soft $\Lambda_{\rm QCD}$ gluon

for c-soft $\Lambda_{\rm QCD}$ gluon

$$\langle 0|\overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \quad \mathcal{E}_T \quad Y_n \overline{Y}_{\bar{n}}|0\rangle$$

NP Correction to Jet Function

- Scale of the jet function is $Q' = \sqrt{x_L}\, x\, Q$

$$2^N\, J^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) = 2^N\, \hat{J}^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) + c^{\mathrm{LL}}\,\frac{\overline{\Omega}_1}{Q'} + \mathcal{O}\left(\frac{\alpha_s(Q')\Lambda_{\mathrm{QCD}}}{Q'}\right)$$

- Valid up to *leading log* accuracy

$$c^{\mathrm{LL}} = -N\, 2^{N-1}\, \hat{\mathcal{J}}^{[N-1]}\left(\ln\frac{Q'}{\mu},\mu\right)$$

additional x^{-1} from Q'

leading log coefficient

- RG consistency: $\hat{\mathcal{J}}^{[N]}$ has the same evolution structure as $\hat{J}^{[N]}$

$$\frac{\mathrm{d}\vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}\right)}{\mathrm{d}\ln\mu^2} = \int_0^1 \mathrm{d}y\, y^N\, \vec{J}^{[N]}\left(\ln\frac{x_L y^2 Q^2}{\mu^2}\right)\cdot\hat{P}(y)$$

H. Chen, I. Moult, X-Y. Zhang, H-X Zhu, 2004.11381

Splitting functions

NP Correction to Jet Function

- Putting everything together

K. Lee, A. Pathak, I. Stewart, ZS (2024)

$$2^N\, J^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right) = \underbrace{2^N\, \hat{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)}_{\text{perturbative series in }\overline{\text{MS}}} - \underbrace{\frac{N\,\overline{\Omega}_1}{\sqrt{x_L}xQ}\, 2^{N-1}\,\hat{\mathcal{J}}^{[N-1]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)}_{\text{leading NP correction at LL accuracy}}$$

- Move to R scheme

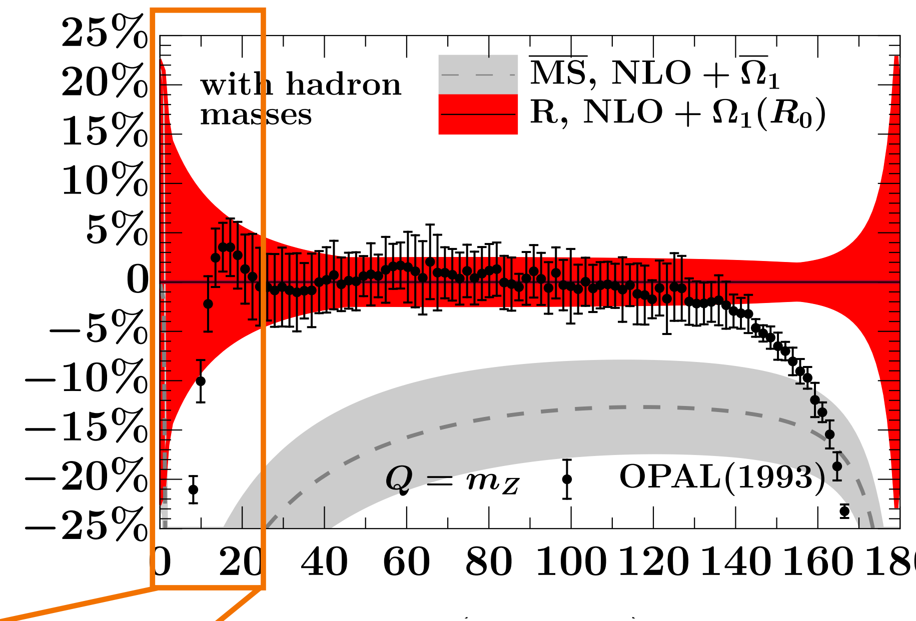
$$\Omega_1(R) \equiv \overline{\Omega}_1 \;-R\sum_{n=1}^{\infty} d_n\left(\frac{\mu}{R}\right)\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

$$2^N\,\hat{J}_{\mathrm{R}}^{[N]}\left(\ln\frac{x_L Q^2}{\mu^2},\mu\right) \equiv 2^N\,\hat{J}^{[N]}\left(\ln\frac{x_L Q^2}{\mu^2},\mu\right) - \sum_{n=1}^{\infty}\frac{N\,R}{Q\sqrt{x_L}}\, d_n(\mu/R)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$$

- Construct cross section *in collinear limit* with factorization

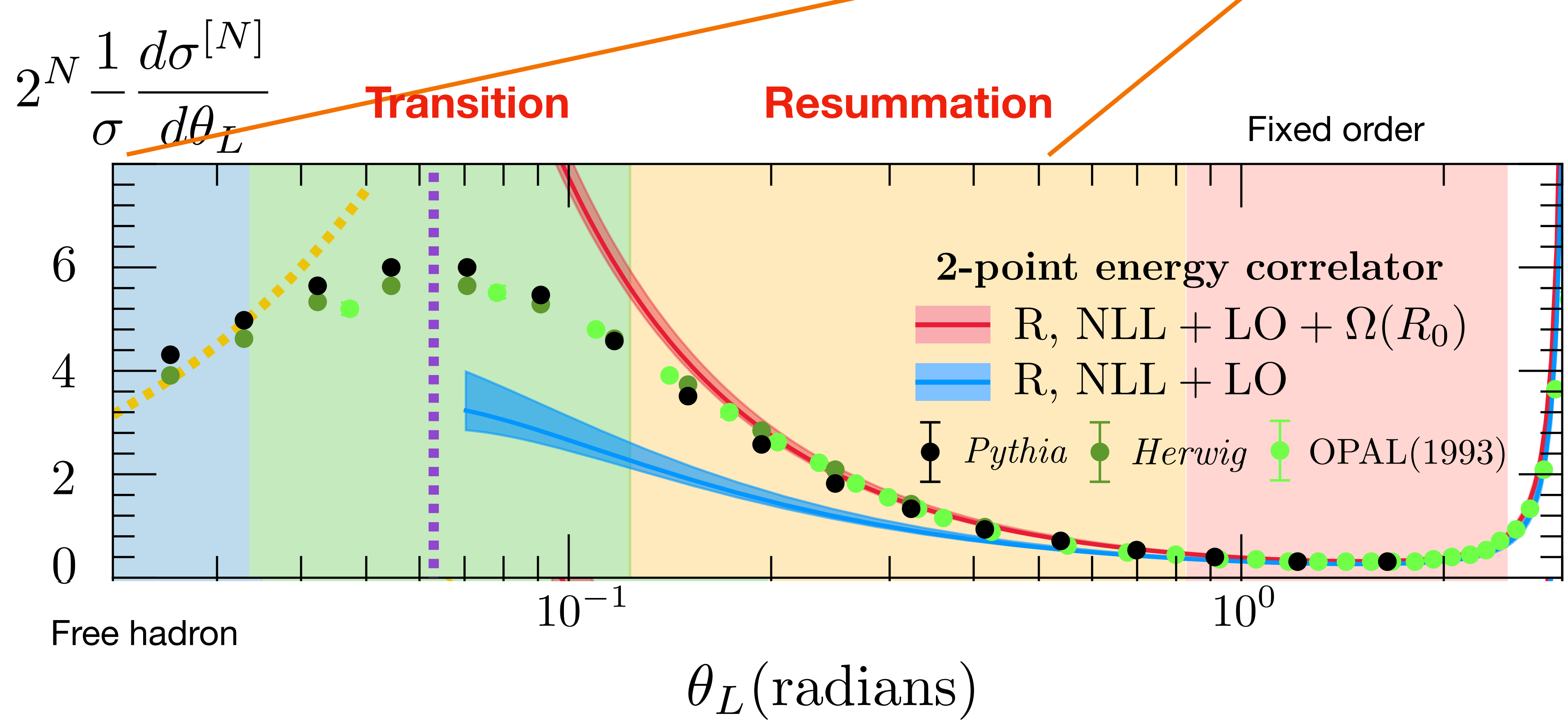
$$\Sigma^{[N]}(x_L) = \int_0^1 dx\, x^N\, \vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)\cdot\vec{H}\left(x,\frac{Q^2}{\mu^2},\mu\right)$$

Confinement transition $x_L \to 0$



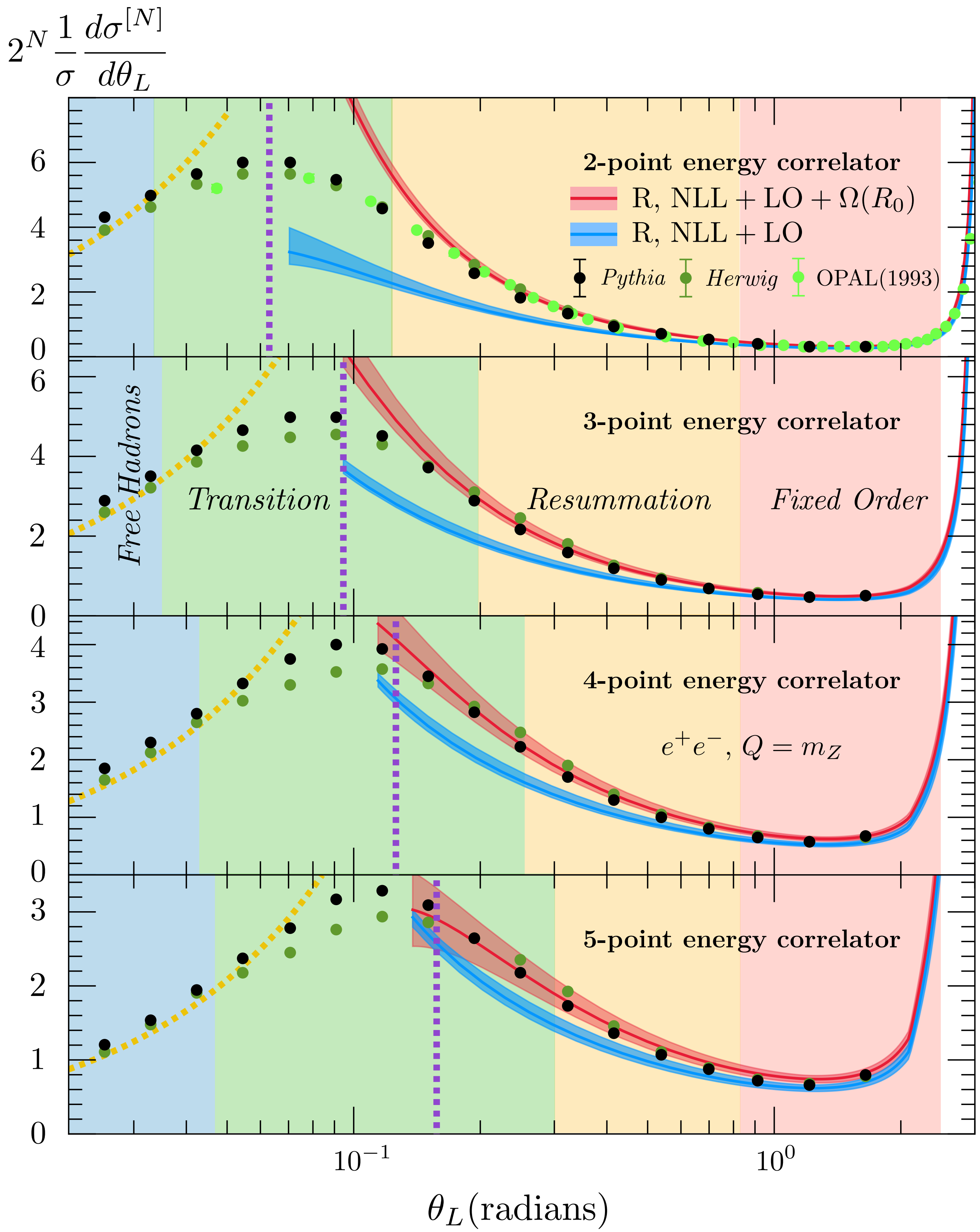
- Including resummation significantly improves our description of the approach to the transition region

- Nonperturbative effects are essential



Identify location of peak at $2\Omega_1/(Q\sqrt{x_{L,\text{peak}}}) \sim 2/3$

Confinement transition $x_L \to 0$



- Predict location of peak for pENC

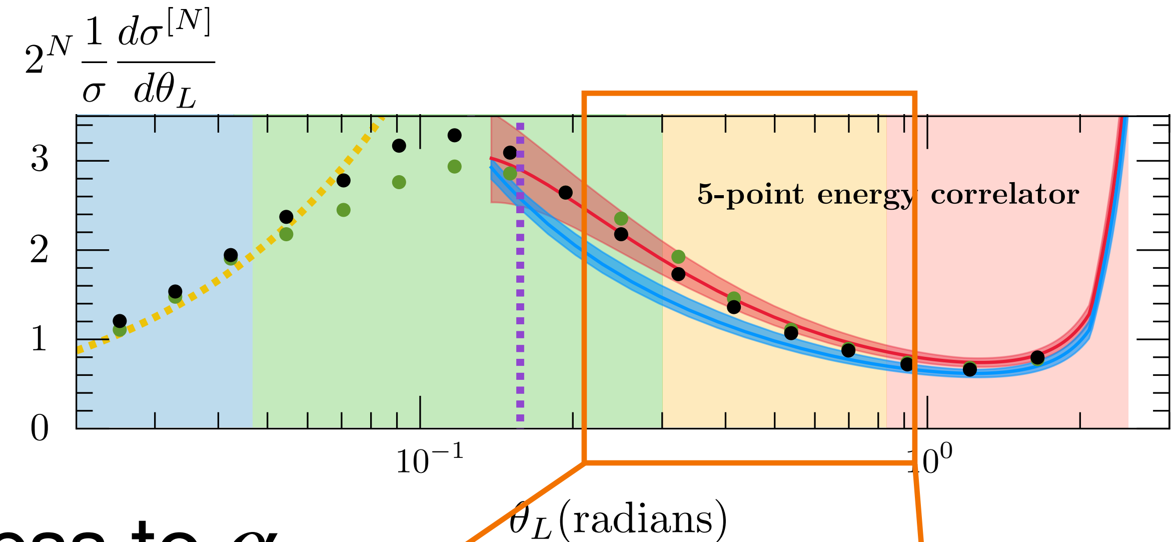
$$N\Omega_1/(Q\sqrt{x_{L,\text{peak}}}) \sim 2/3$$

- **Including NP correction is essential** to agree with hadron-level MC
- Normalizing to the total cross section (not just perturbative region!)

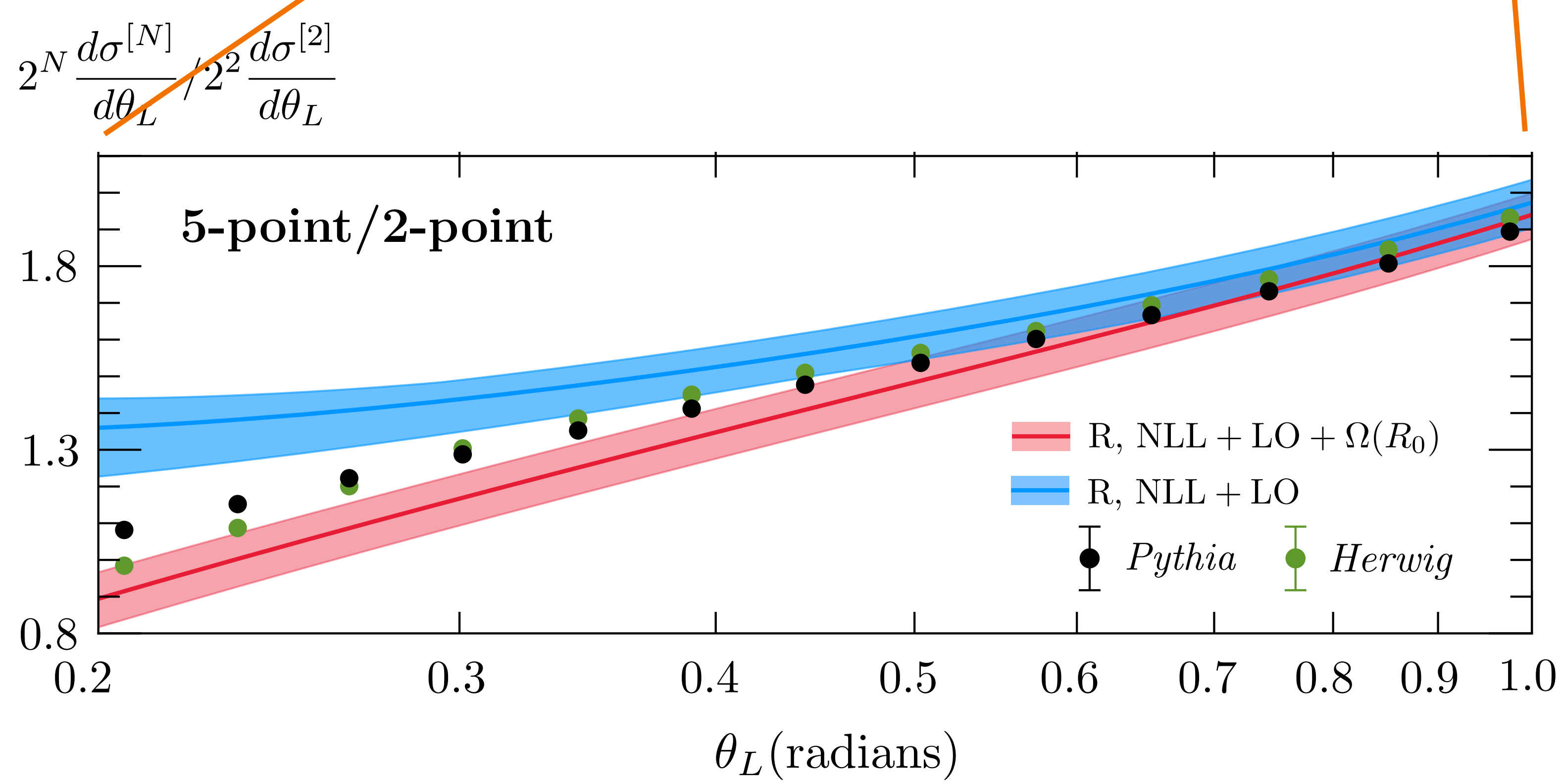
Ratio of pENC/EEC

- In collinear region: $$\frac{d\hat{\sigma}^{[N]}}{dx_L} \sim x_L^{\gamma(N)-1}$$
- Ratios of energy correlators give access to α_s

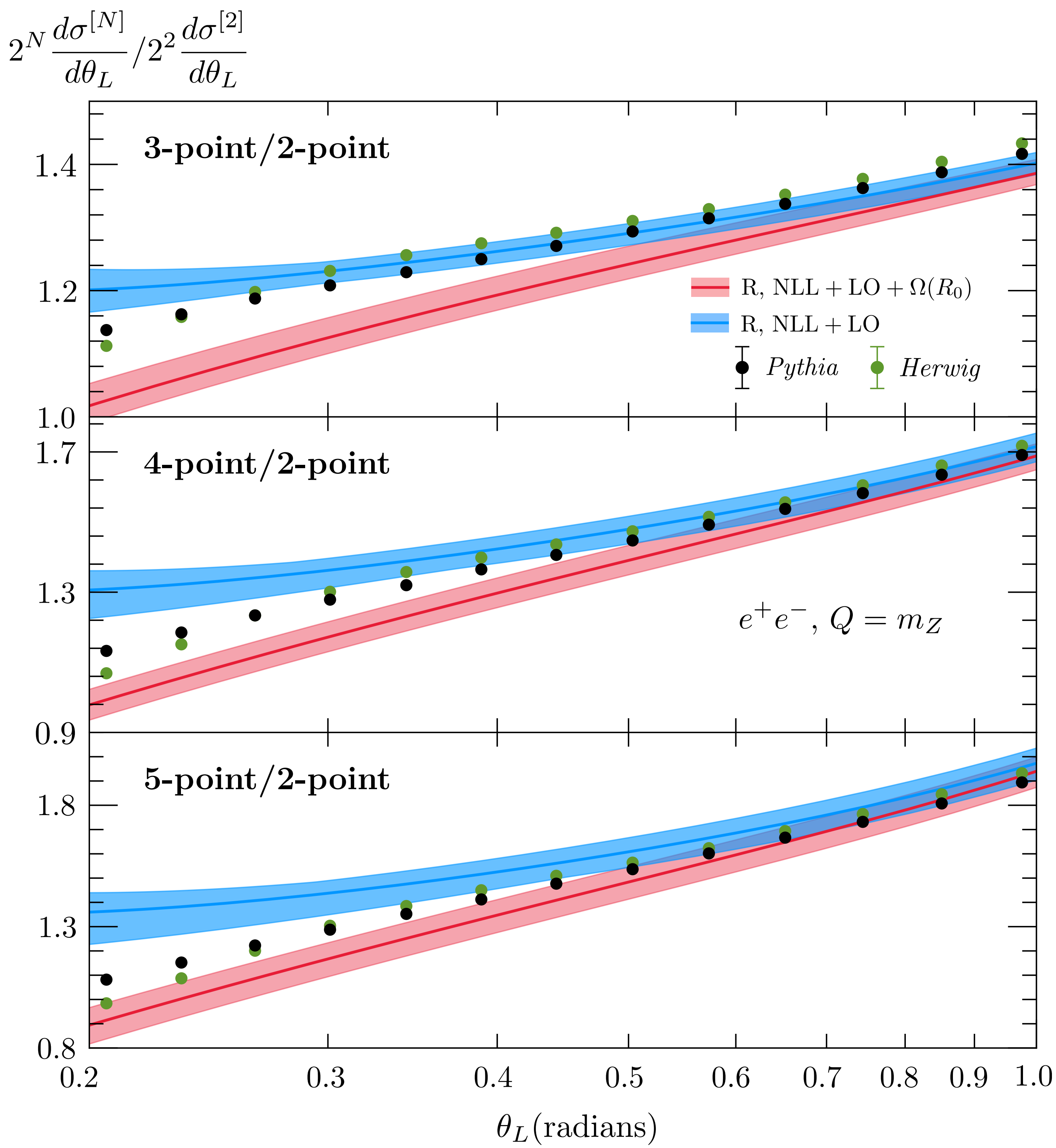
$$\frac{d\hat{\sigma}^{[N]}/dx_L}{d\hat{\sigma}^{[M]}/dx_L} \sim x_L^{\gamma(N)-\gamma(M)} \sim x_L^{\alpha_s[\gamma^{(0)}(N)-\gamma^{(0)}(M)]}$$



- NP correction changes linear scaling behavior!

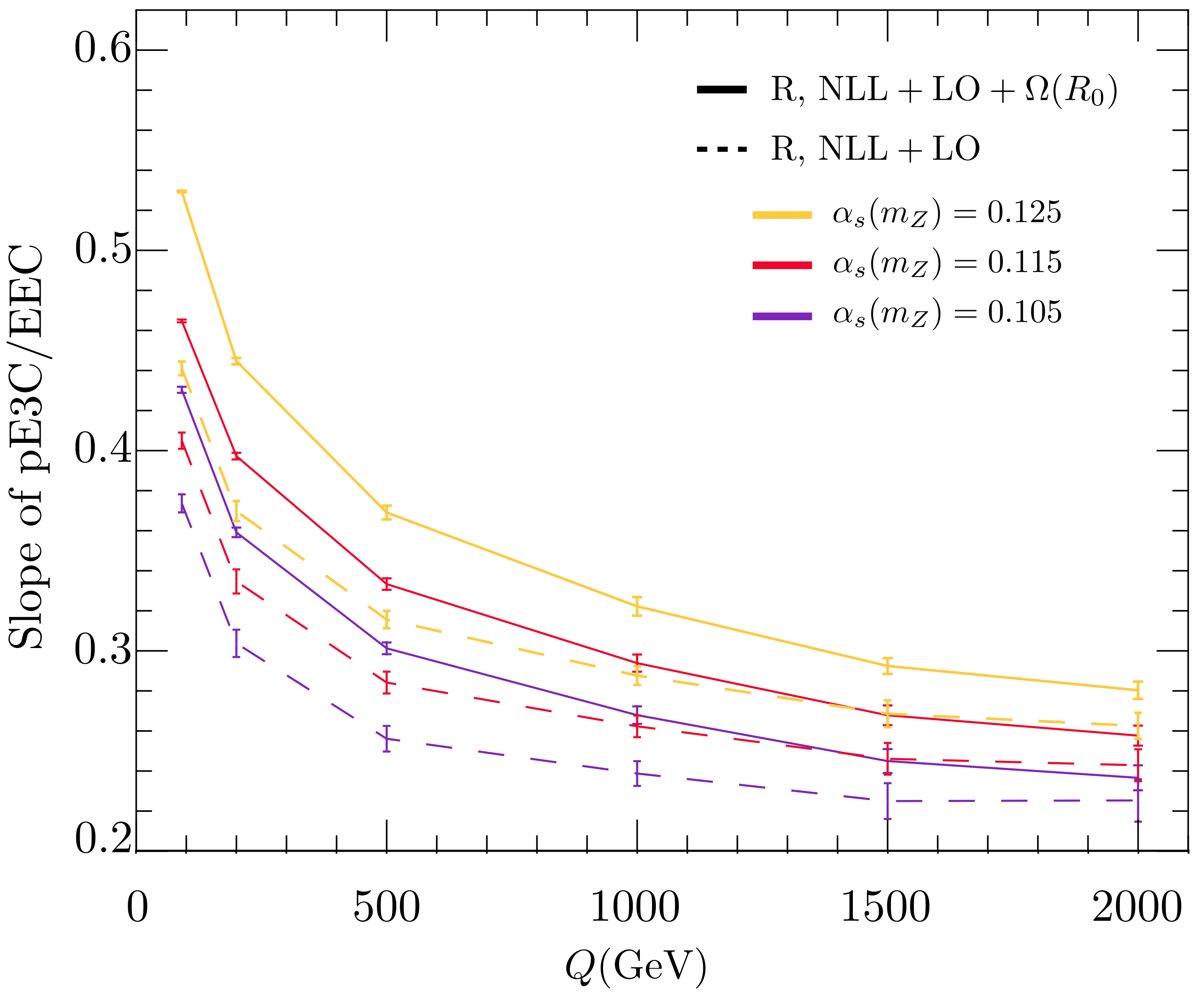


Scaling behavior



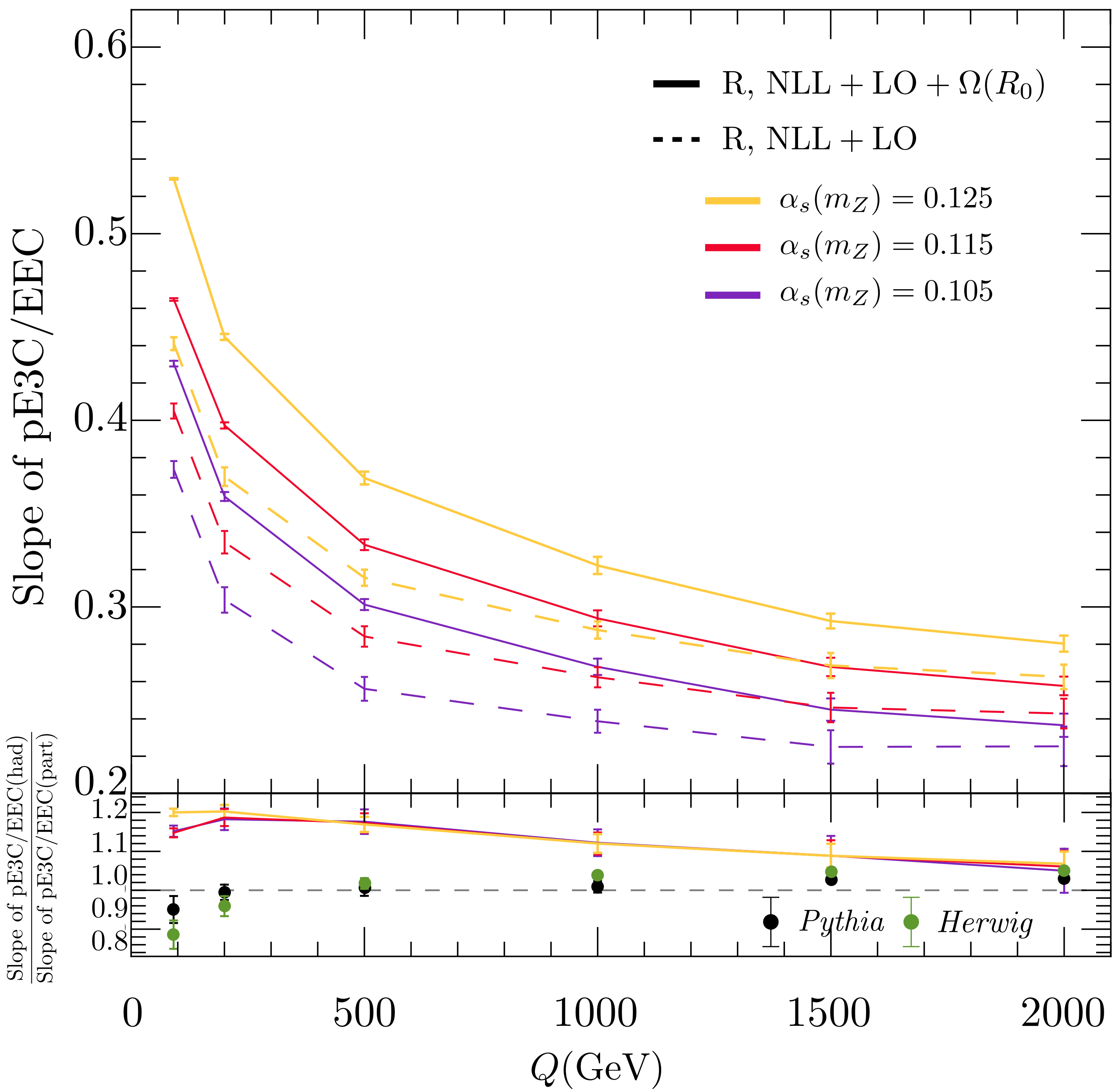
- Close to transition region, hadron-level MC deviates from linear slope
- Our model-independent NP correction captures this deviation well
- NP effects **do not completely cancel** in ratios!

Impact on α_s extraction



- Extract slope away from transition region
- Asymptotic freedom as α_s decrease with high energy
- Including NP corrections **significantly impact the slope**!
- Degenerate effect between including NP correction and decreasing α_s

Impact on α_s extraction



- NP effects **do not completely cancel** in ratios!
- Lower panel: difference between parton and hadron level prediction
- **Hadronization effects are larger than standard prediction from MC(had - part)!**

Summary

- We give a model-independent prediction of the leading nonperturbative correction to the projected energy correlator
  - Derive N and x_L dependence
  - The universal matrix element Ω_1 shows up for all angle cross section and collinear jet function
- We show that the x_L-resummed R scheme series with nonperturbative correction better captures the approach to confinement transition
- We demonstrate in a model-independent way that including nonperturbative corrections has a more significant impact on α_s extraction than predicted by MCs