Nonperturbative Corrections for Energy Correlators:

R-scheme for Precision Predictions

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Energy Correlators at the Collider Frontier workshop
MITP, Mainz
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arXiv:2305.19311 S.Schinder, IS, Z.Sun = S.^{3} ‘23

arXiv:2405.19396 K.Lee, A.Pathak, IS, Z.Sun

Massachusetts Institute of Technology

Outline

My Talk: Formalism and basic EEC results

Board
- Operator Expansion for Nonperturbative Effects
- Universality Classes for Hadronization in e^+e^-
- Defining Nonperturbative parameters:
  renormalization schemes and renormalons

- Results for EEC in e^+e^-

Part 2 by Zhiquan Sun: extension to projected N-point Correlators,
small angle limit (e^+e^- and pp), and cool results

Notes from Blackboard

"Nonperturbative Effects in ECs: R-scheme for Precision Predictions"

arXiv: 2305.19311 S. Schindler, IS, Z. Sun = S.S.S. '23

2405.19396 K. Lee, A. Pathak, IS, Z. Sun

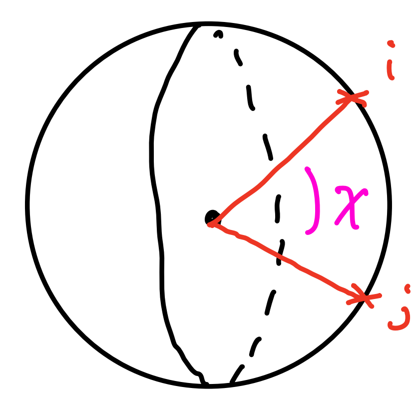
1. Operator methods
2. Universality
3. Definitions / Schemes
4. Results in e^+e^-

PtZ Zhiquan Sun

- N-point correlators
- small angle e^+e^-, pp
- cool results

Consider EEC in e^+e^-
$$\frac{d\sigma}{dz} = \sum_{i,j} d\sigma_{ij} \frac{E_i E_j}{Q^2} \delta(z - z_{ij})$$

$$z_{ij} = \frac{1-\cos\theta_{ij}}{2}, \quad z = \frac{1-\cos\chi}{2} \in [0,1]$$



$$d\sigma \sim \langle 0 | J^\dagger \mathcal{E}(\vec{n}_1)\, \mathcal{E}(\vec{n}_2)\, J | 0 \rangle$$

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

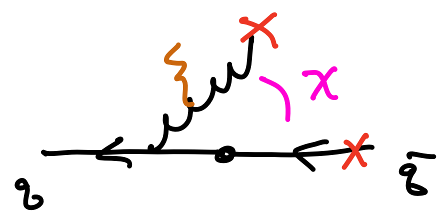
Field Theory: ψ, A @ Q $\longleftrightarrow$ ψ, A @ Λ_{QCD}

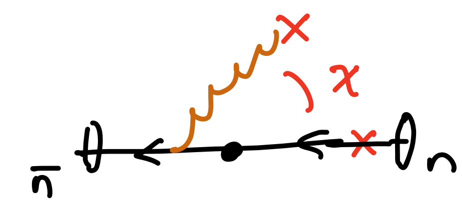
Nice for analytic calculations, NNLO here.

① What about Non Pert. Corr ?

① $d\sigma$ ② $d\sigma$ ③ leading

$d\sigma_{ij}\, E_i E_j \qquad \delta(z - z_{ij}) \qquad d\sigma_{ij}\, E_i E_j$





dijet

$z = 1 \rightarrow z \in (0,1)$

Wilson lines for soft approx

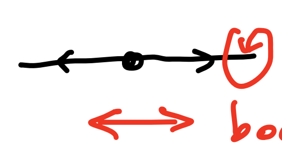
$$\langle \mathcal{O}(z) \rangle = \frac{1}{N_c} \mathrm{tr} \langle 0 | Y_{\bar{n}}^{+} Y_n^{+} \, \mathcal{E}(\hat{n}) \, Y_n Y_{\bar{n}} | 0 \rangle$$

Korchemsky – Sterman '99 (hep-ph/9902341)

$$Y_n = P \exp\left(i g \int_0^\infty ds \; n \cdot A(ns) \right)$$

1-gluon $\quad \langle \mathcal{O}(z) \rangle \sim \dfrac{1}{[z(1-z)]^{3/2}} \; \Lambda_{QCD} \qquad$ ie $\dfrac{1}{\sin^3 \chi}$

② Lee, Sterman '06 (hep-ph/0611061) $\qquad e \in$ dijet config



calculable!

$$\Omega_1^{e} = C_e\, \Omega_1$$

universality

$$\Omega_1 = \frac{1}{N_c} \mathrm{tr} \langle 0 | Y_{\bar{n}}^{+} Y_n^{+} \, \mathcal{E}_T(0) \, Y_n Y_{\bar{n}} | 0 \rangle$$

$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3 \eta} \int d\phi \; \mathcal{E}(\vec{n}) \, , \qquad p_T = \frac{E}{\cosh \eta}$$

AFHMS '10 (Abbate, Fickinger, Hoang, Mateu, IS) (1006.3080)

e^+e^- thrust data $\quad \Omega_1 = 0.32 \pm .05$ GeV $\quad$ (in "R-scheme")

Mateu, IS, Thaler '12 (1209.3781) (motivated by Wicke & Salam '01, hep-ph/0102343)

hadron masses $m_H \sim \Lambda_{QCD}$ are leading effects

$$\Omega_1^e = c_e \, \Omega_1^{g_e}, \qquad \Omega_1^{g_e} = \int_0^1 dr \, g_e(r) \, \Omega_1(r) \qquad r = \frac{p_\perp}{\sqrt{p_\perp^2 + m_H^2}} = \frac{p_\perp}{m_\perp} \in [0,1]$$

EEC (S.S.S '23) $\quad 2\hat{E}\,\delta(z - \ldots) = \hat{m}_\perp \, f(\hat{r},\hat{y})$

$$f(r,y) = 2\cosh y \, \delta\Big(z - \frac{(1-\cos\theta)}{2}\Big) = 2\cosh y \, \delta\left(z - \frac{1}{2} + \frac{\sinh y}{2\sqrt{r^2 + \sinh^2 y}}\right)$$

$$c_e = \int dy \, f(1,y) = \frac{1}{2[z(1-z)]^{3/2}} \quad ✓$$

$$g_e = \frac{1}{c_e} \int dy \, f(r,y) = r \qquad \text{"E-scheme" universality class}$$

Upshot $\Rightarrow$ understanding of hadron mass corrections

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \underbrace{\frac{1}{\sigma}\frac{d\hat{\sigma}}{dz}}_{\sum_n C_n(z, \mu/Q)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n}^{\overline{MS}} + \frac{1}{2[z(1-z)]^{3/2}} \frac{\overline{\Omega}_1}{Q} + \mathcal{O}\left(\alpha_s(Q)\frac{\Lambda_{QCD}}{Q}\right)$$

$\overline{MS}$

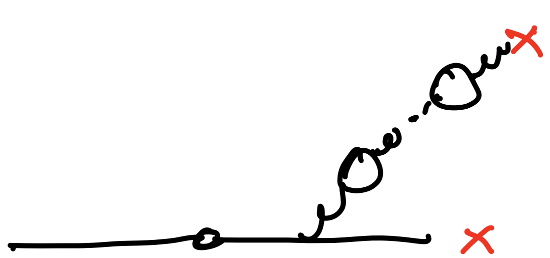
③ Ω_1 scheme

- issue of $\overline{MS}$, integrate to zero, <u>an</u> <u>issue</u>, corresponds to not properly seperating power law IR divergences
- two terms are not fully seperated by UV = pert & IR = nonpert

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{1}{\sigma}\frac{d\hat{\sigma}(R)}{dz} + \frac{1}{2[z(1-z)]^{3/2}}\frac{\Omega_1(R)}{Q} \qquad \text{cutoff}$$

$p^2 \gtrsim R^2$ $\qquad\qquad$ $p^2 \lesssim R^2$

- How do we see this issue? "renormalon probe"



$\sim (2\beta_0)^n \, n! \, \alpha_s^{n+1}$ asymptotic series

Hoang & IS '07 for $\bar{\Omega}_1$ (0709.3519)

S.S.S. '23 for EEC

- leading $\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$ ambiguity in $\overline{MS}$ $d\hat{\sigma}$
- universality ✓

• Define a scheme $\sum_n d_n \alpha_s^n$ without this ambiguity.

Definition does not rely any renormalon calculation!!

fully non-abelian

ambiguity $\propto$ $\cancel{\Lambda_{QCD}}$ $-$ $R\,\cancel{\frac{\Lambda_{QCD}}{R}}$

$$\Omega_1(R) = \overline{\Omega}_1 - R \sum_n d_n\left(\frac{\mu}{R}\right)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$$

$\overline{MS}$ convert to R-scheme

$$\frac{1}{\sigma}\frac{d\hat{\sigma}(R)}{dz} = \sum_n \left\{ C_n\left(z,\frac{\mu}{Q}\right) + \frac{R}{2Q}\frac{d_n(\mu/R)}{[z(1-z)]^{3/2}} \right\}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$$

ambiguity $\propto$ $\cancel{-\frac{\Lambda_{QCD}}{2Q[z(1-z)]^{3/2}}}$ $+$ $\cancel{\frac{R}{2Q}\frac{\Lambda_{QCD}/R}{[z(1-z)]^{3/2}}}$

• must expand in <u>some</u> $\alpha_s(\mu)$ for two $d\hat{\sigma}(R)$ terms

• d_n determined from perturbative soft function $\hat{S}(y,\mu)$ for dijets.

Hoang & Kluth '08 (0806.3852)

Bachu, Hoang, Mateu, Pathak, IS '2020 (2012.12304)

$$\hat{S}(y,\mu) = FT\; S(k,\mu) = FT\; \frac{1}{N_c}\mathrm{tr}\langle 0|Y_{\bar{n}}^\dagger Y_n^\dagger\, \delta[k - \mathcal{E}_T(0)]\, Y_n Y_{\bar{n}}|0\rangle$$

Fourier transform

- issue about scales

  some μ

  $R \sim Q$ for $d\hat{\sigma}(R)$ to avoid large logs, $\log\left(\frac{\mu}{Q}\right)$, $\log\left(\frac{\mu}{R}\right)$

  $R \sim 1\,\text{GeV}$ for $\Omega_1(R)$ to only shift by $\alpha_s R \sim \Lambda_{QCD}$

  $\Rightarrow$ solved by R-RGE

  see Hoang, Jain, Scimemi, IS '08, '09 (0803.4214) (0908.3189)

AFHMS '10

$$\Omega_1(R = 2\,\text{GeV}) = 0.32 \pm 0.05\ \text{GeV},$$

$$\alpha_s(M_Z) = 0.114 \pm .001$$

values used to make plots

Perturbative Results: $\overline{\text{MS}}$ scheme versus R scheme

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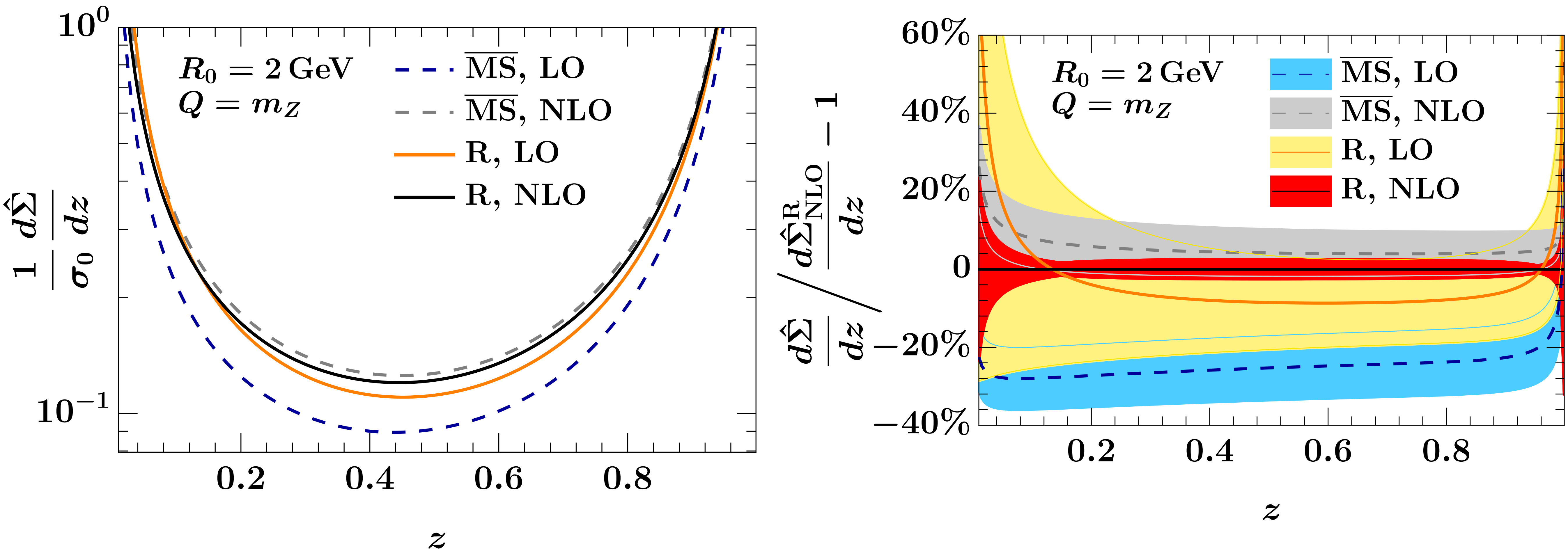

- improved convergence in R scheme (vs. $\overline{\text{MS}}$ scheme)
- smaller perturbative uncertainty

$\overline{\text{MS}}$ NLO (analytic): Dixon, Luo, Shtabovenko, Yang, Zhu (2018)

Including Leading Nonperturbative Correction:

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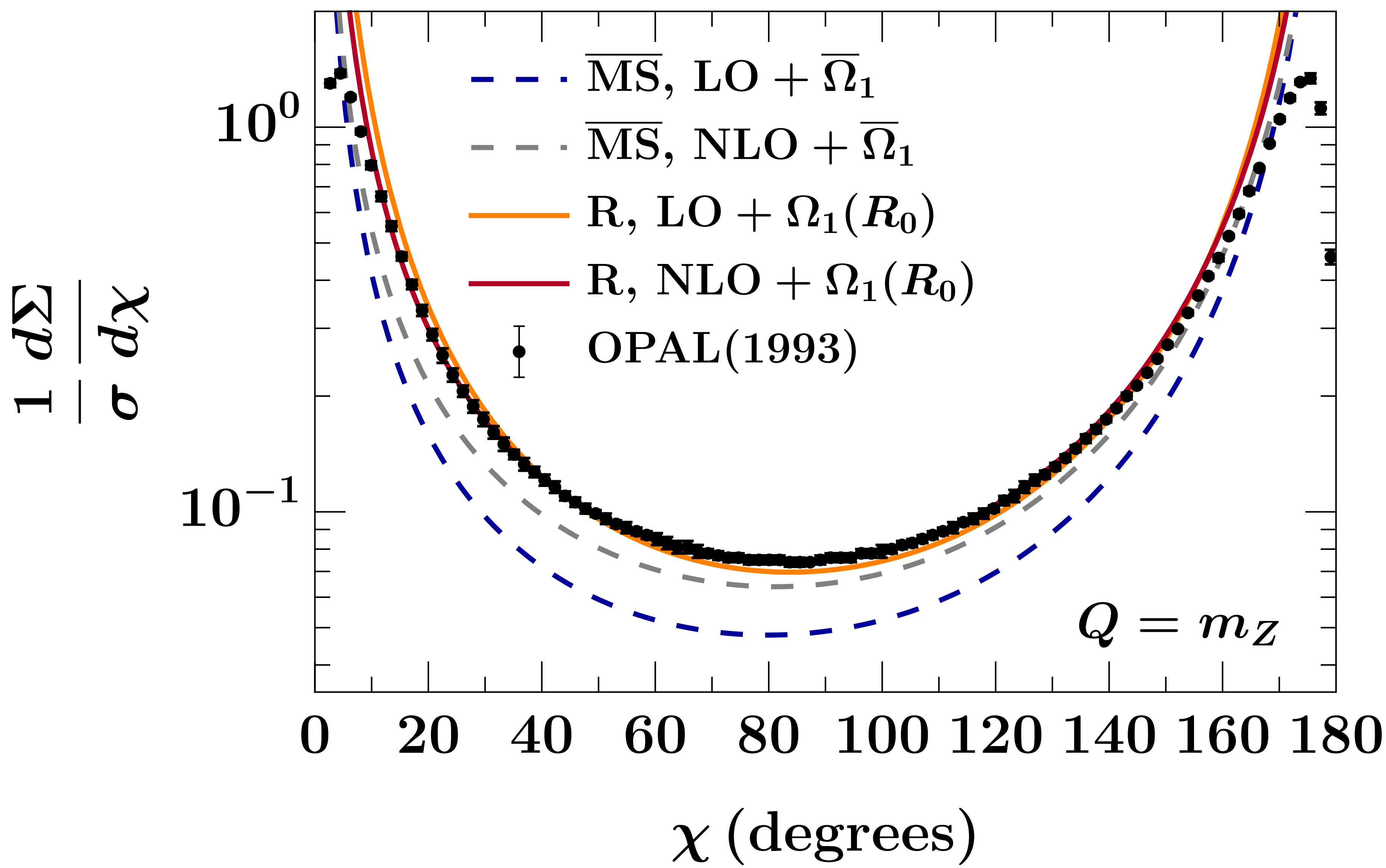
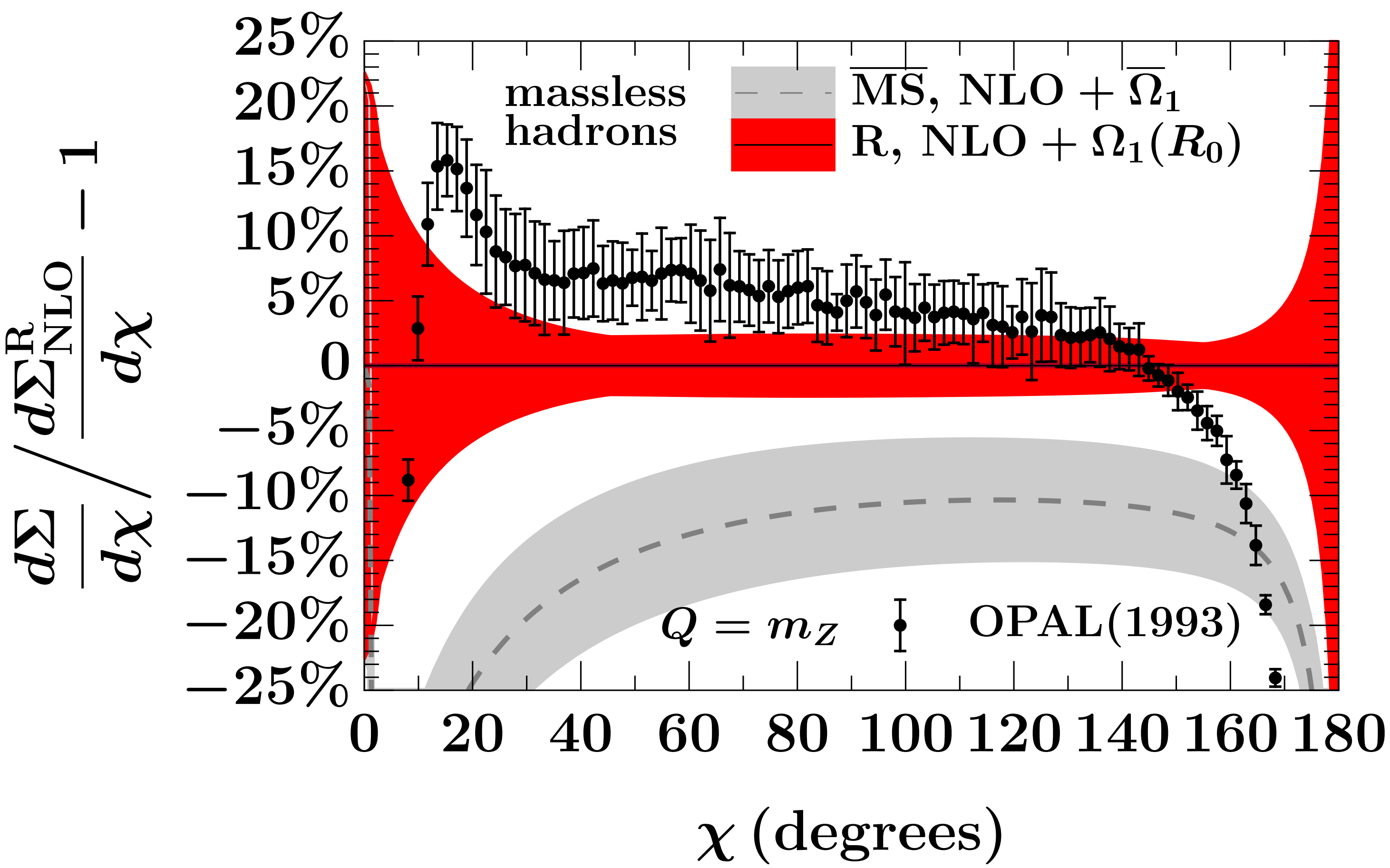
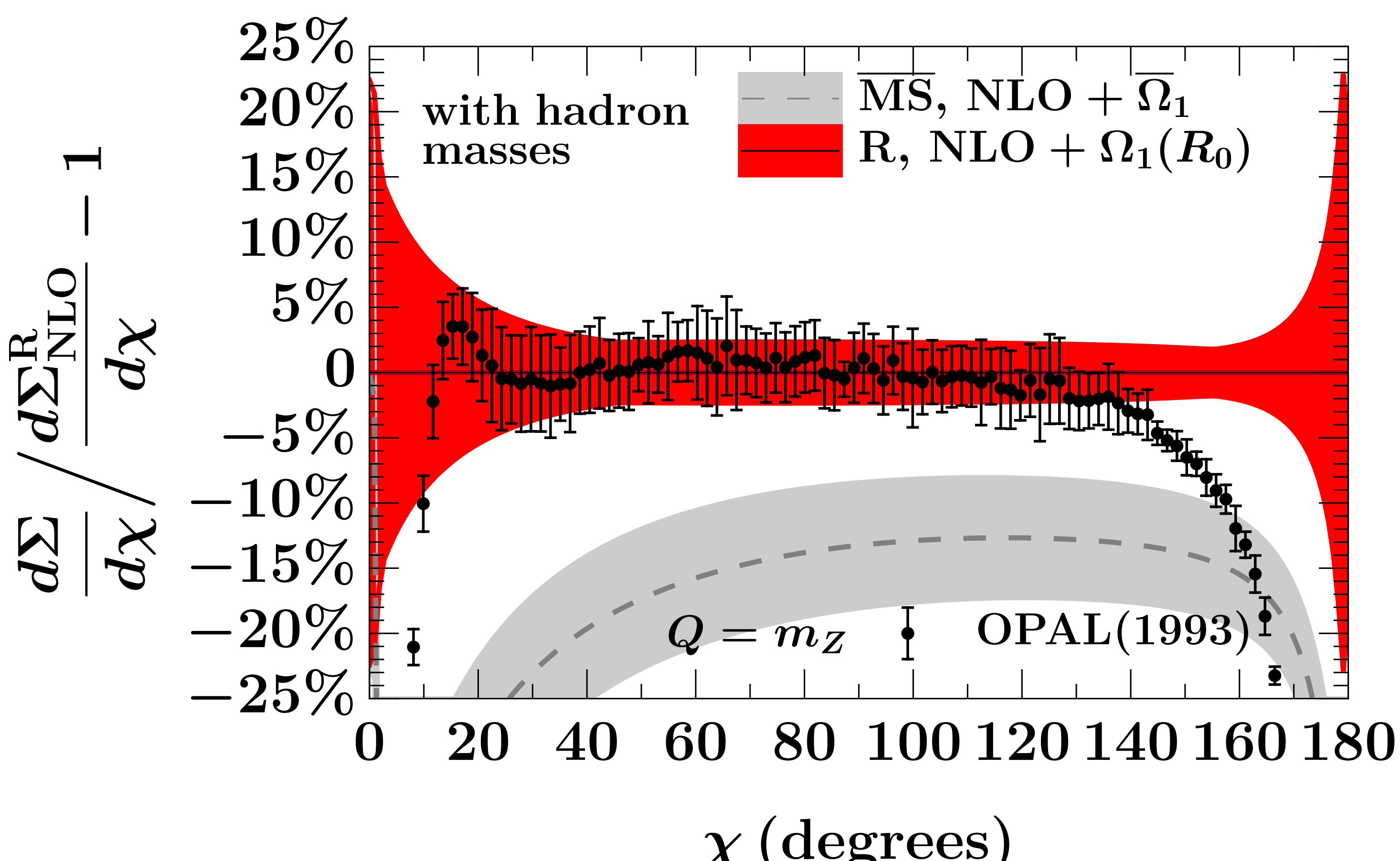


- no fit parameters!
- model independent
- good agreement with data

S.^{3} ‘23



- with thrust parameters (assuming massless hadrons)



- include +20% hadron mass correction to Ω_1
- better agreement