INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS

Energy-energy correlators in inclusive jets in heavy ion collisions

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Energy Correlators at the Collider Frontier,
MITP, Mainz, Germany,
July 9th, 2024.

In collaboration with C. Andres, J. Holguin, C. Marquet, and I. Moult

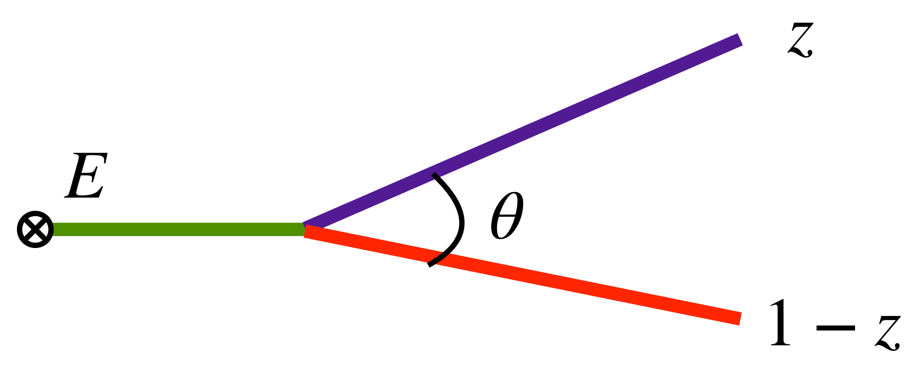
Cofinanciado por la Unión Europea

Fondos Europeos

Proof of principle of EEC in HIC

- Quark-initiated jet with known initial energy E (γ/Z-jet)
  - In this case energy loss effects are subleading and there is no need for resummation of soft emissions

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma_{qg}} \int dz \left(\frac{d\sigma_{qg}^{\text{vac}}}{d\theta dz} + \frac{d\sigma_{qg}^{\text{med}}}{d\theta dz} \right) z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$



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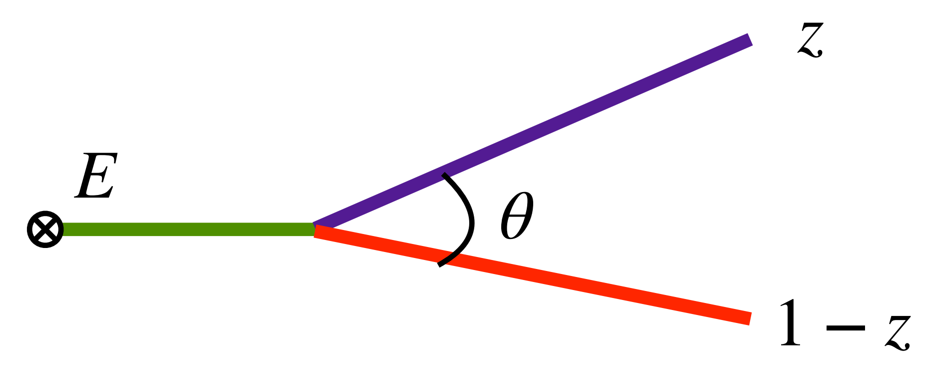
Includes collinear resummation



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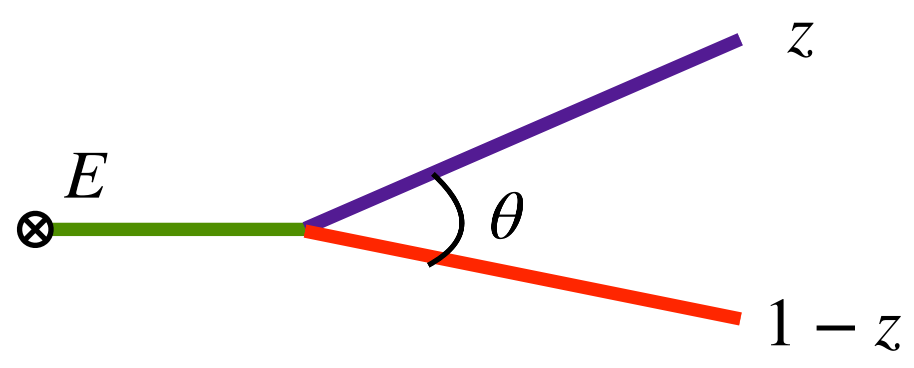
Leading order in the number of emissions



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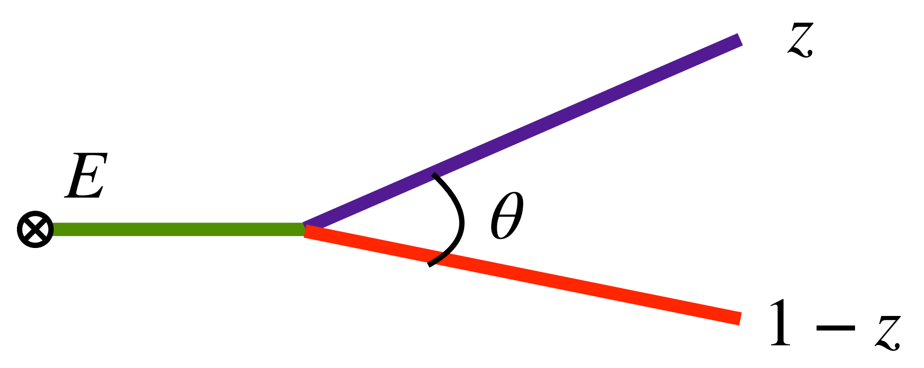


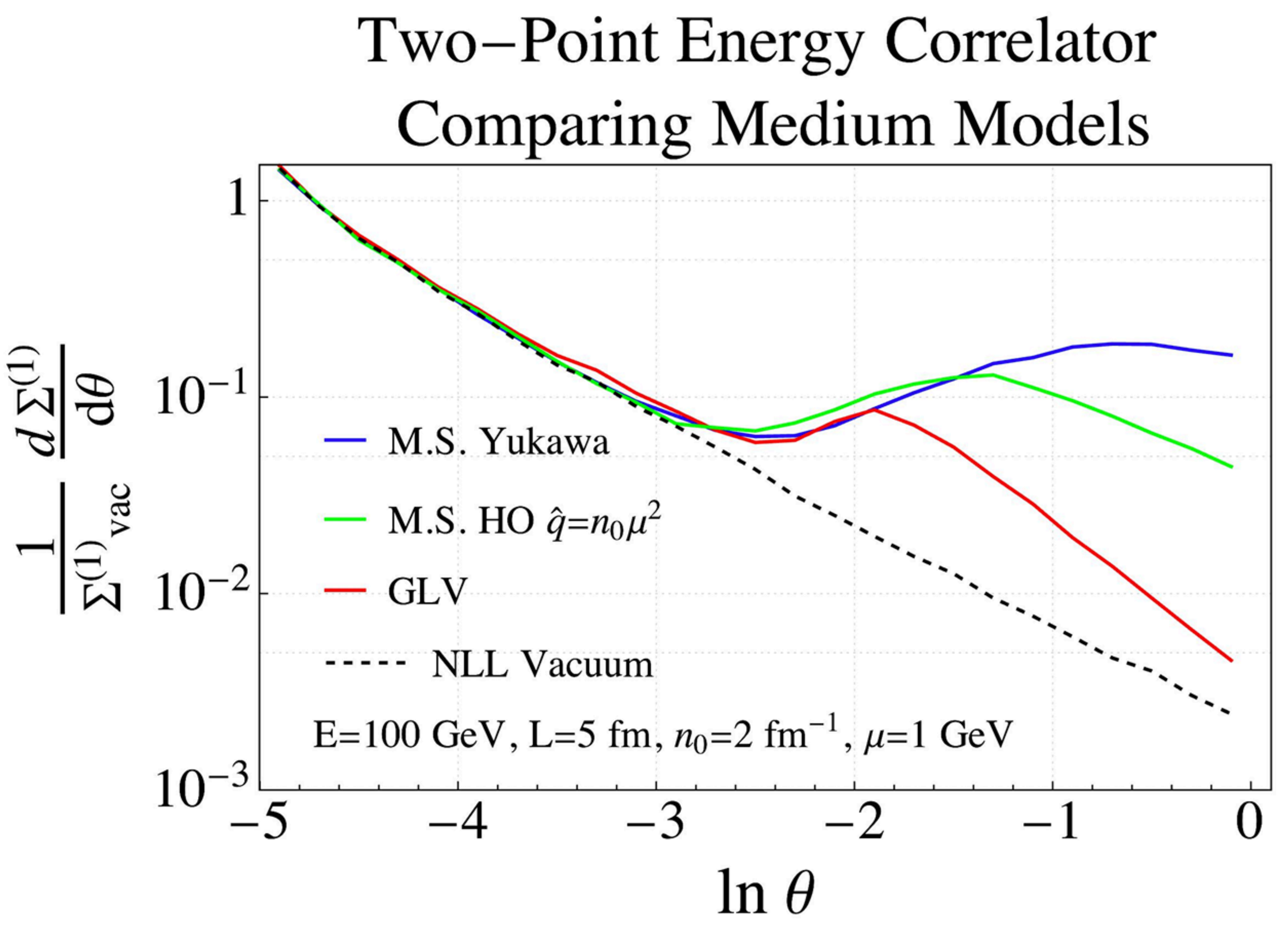
- Medium-splittings calculated on a brick

- Soft approximation ($z \to 0$, keeping zE finite) common in energy loss calculations not suitable here

- Two available approximations:
  - ✦ Semi-hard approximation: resums multiple scatterings, ignores momentum broadening in an eikonal approach
    FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653 Isaksen, Tywoniuk 2107.02542
  - ✦ Opacity expansion (GLV): possible unitarity issues, no eikonal assumptions involved
    Sievert, Vitev 1807.03799

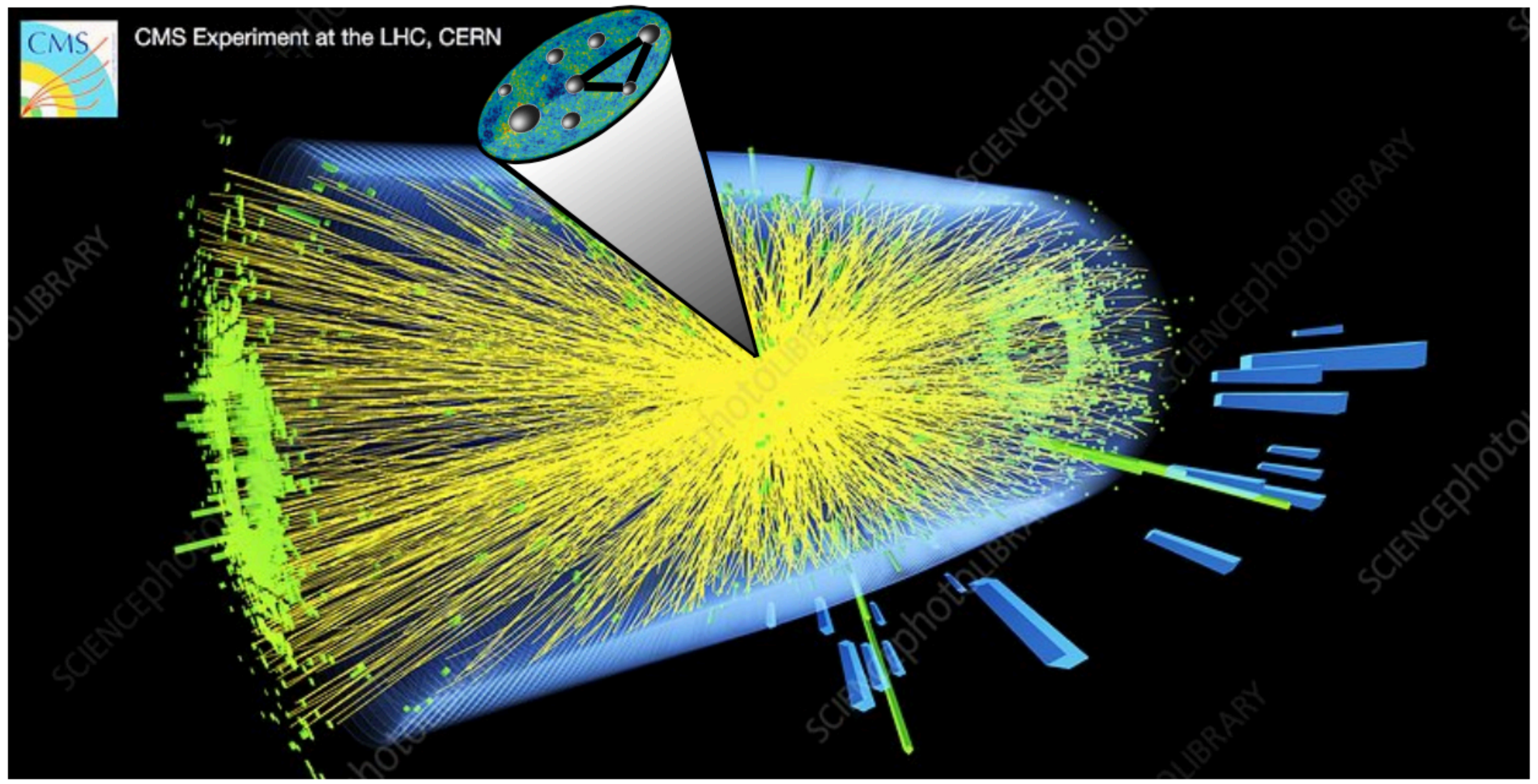
General expected features of EECs in heavy ion collisions

- Medium-modifications are not enhanced by large logs and can be computed to LO in the number of splittings
- Shape of correlators is not modified at very small angles
- Medium-enhancement at large angles deviates from vacuum power-law behavior
  - Amplitude of enhancement is model dependent



What is missing to have a meaningful comparison to data?

- In-medium splitting calculations must be improved
  - ✦ Go beyond the semi-hard approximation
- Implement evolving media
- Energy loss effects must be included for inclusive jets measurements given the dependence on the hard scale
- Wake, medium response — not included in this talk



Improvements to calculations of in-medium splittings

- Single scattering: allow medium parameters to change with evolving medium

- Multiple scatterings:
  - Computed first corrections to semi-hard approximation to account for transverse momentum broadening
  - Allow medium parameters to change with evolving medium
  - We can now completely remove the semi-hard approximation in the soft multiple scattering (harmonic) approximation in an evolving medium, but it takes much longer to run

Not yet implemented

Realistic medium evolution

- Express medium parameters entering our calculations in terms of temperature extracted from hydro simulations

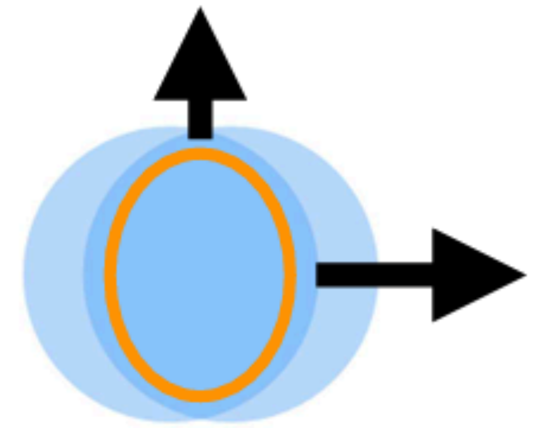
Multiple scatterings: $\hat{q}(t) = k_{\rm HO}\, T^3(\xi(t))$

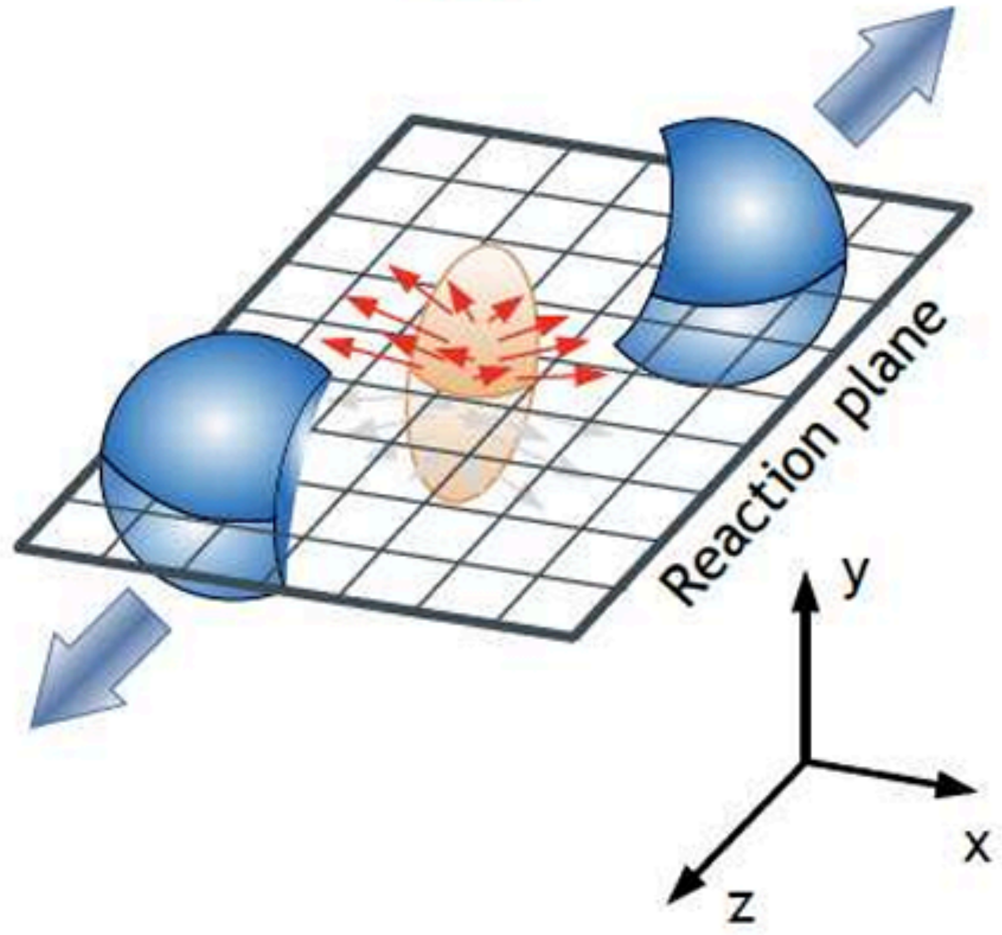
Single scattering: $n(t) = k_{\rm GLV} T(\xi(t))$ $\mu^2(t) = 6\pi\alpha_s\, T^2(\xi(t))$

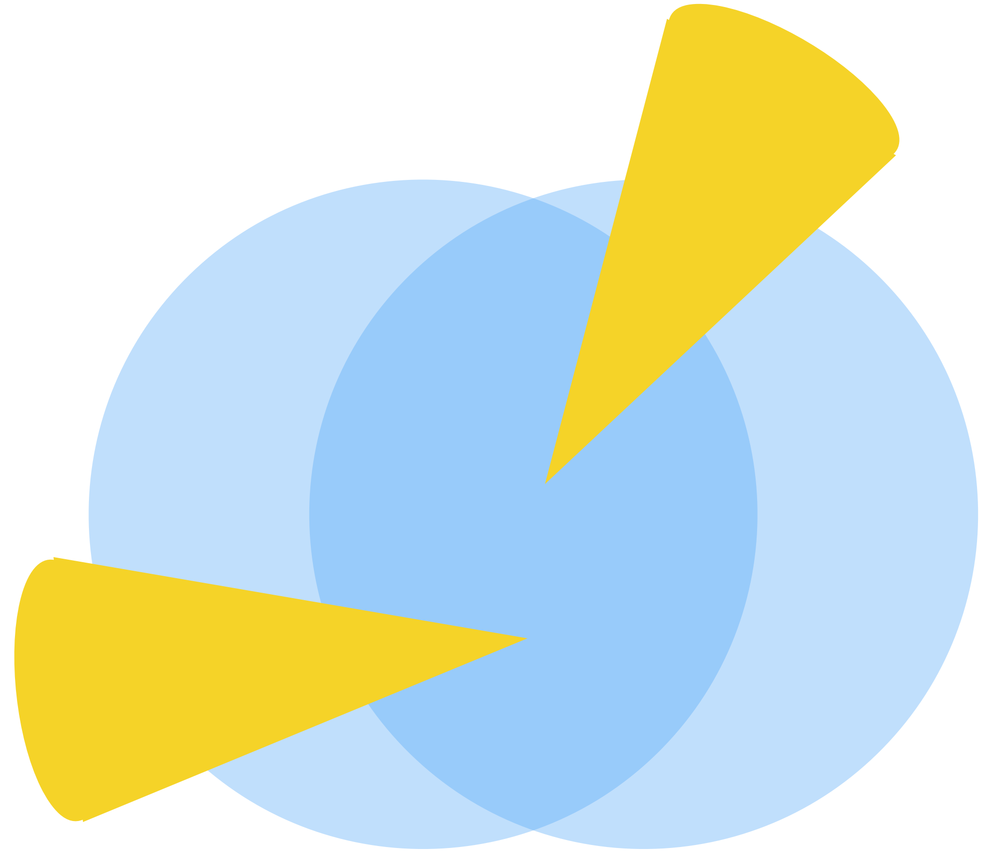
- Average over sample of possible trajectories for a given centrality class

$$\langle A_\xi \rangle_\xi = \frac{1}{N} \int {\rm d}\phi\, {\rm d}x_0\, {\rm d}y_0\, w(x_0, y_0)\, A_\xi$$

$$w(x_0, y_0) = T_A(x_0, y_0)\, T_A(\boldsymbol{b} - (x_0, y_0))$$

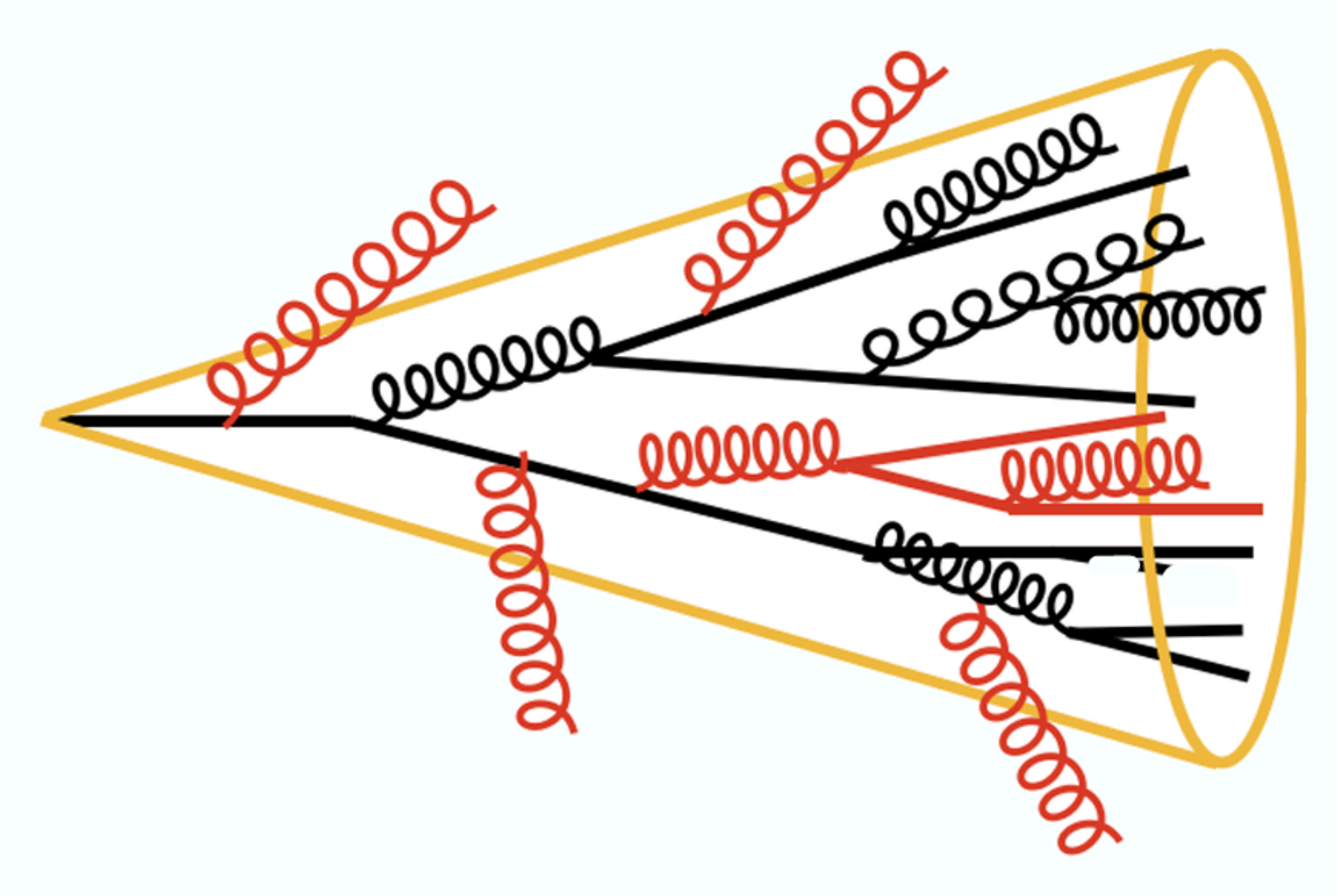


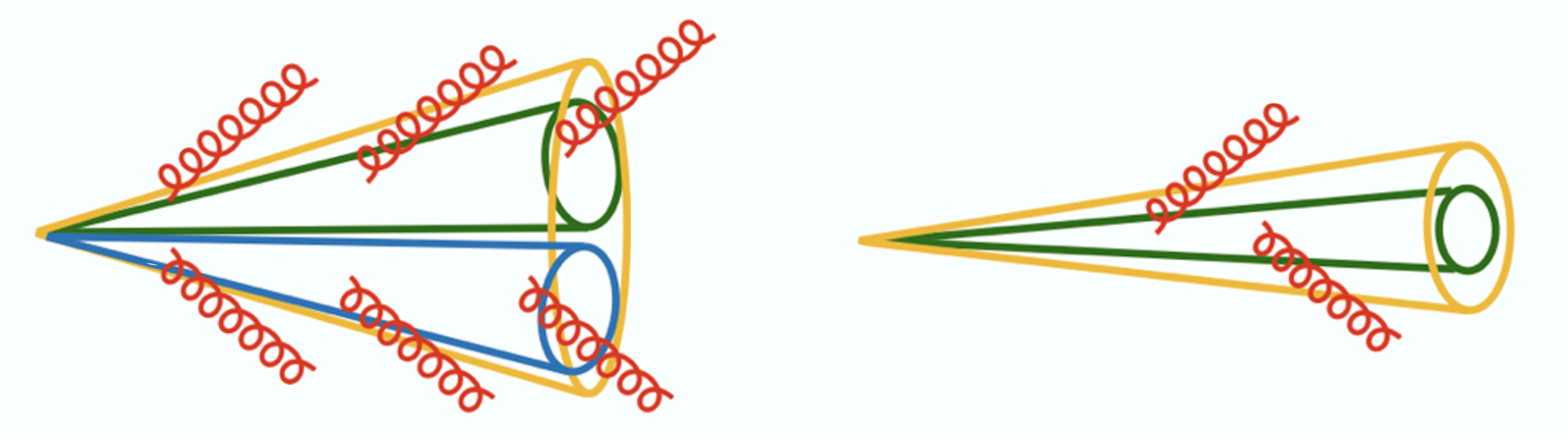




Energy loss in heavy ion collisions

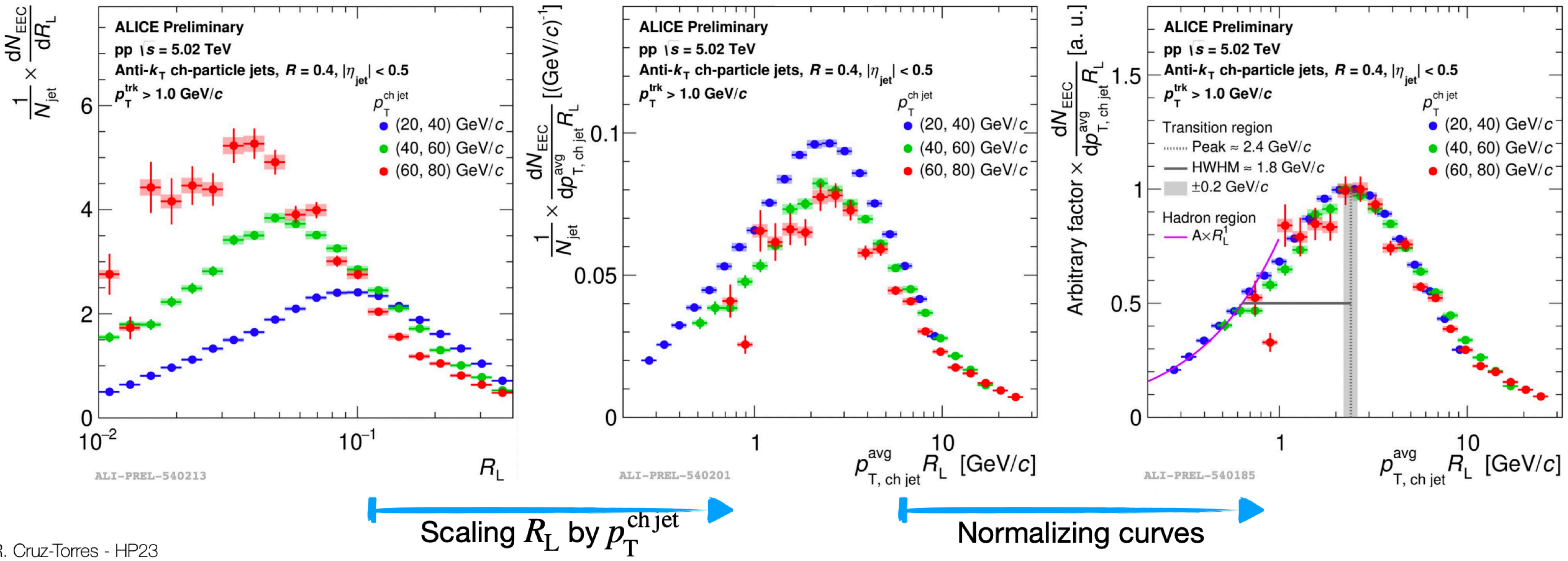
- Medium interactions enhance the probability of having soft emissions at large angles
- Reconstructed jet energy is lower than the initial parton energy
- Energy lost by a jet susceptible to jet-by-jet fluctuations
- Narrow jet expected to lose less energy than broader jets





Energy loss in EECs in heavy ion collisions

- For the γ/Z-jet case energy loss is expected to be subdominant and not change significantly the shape of the correlator
- For inclusive jets measurements, energy loss effects become important given that they shift the hard scale. Selection bias
- In particular, the hadronization transition will be shifted towards smaller angles



R. Cruz-Torres - HP23

Model for the hadronization transition in pp

- Interpolate smoothly between the hadron gas region and the perturbative result

$$\left.\frac{\mathrm{d}\Sigma}{\mathrm{d}R_L}\right|_{R_L \ll \theta_{\mathrm{NP}}} \sim R_L$$

$$\left.\frac{\mathrm{d}\Sigma}{\mathrm{d}R_L}\right|_{\theta_{\mathrm{NP}} \ll R_L \ll 1} \sim R_L^{-1+\mathcal{O}(\alpha_s)}$$

Y. L. Dokshitzer, G. Marchesini and B. R. Webber, hep-ph/9512336

$$\Rightarrow \quad \left.\frac{\mathrm{d}\Sigma}{\mathrm{d}R_L}\right|_{R_L<1} = \frac{R_L}{B_0 + B_2 R_L^2}\,, \qquad \text{with} \qquad \frac{B_0}{B_2} \sim \theta_{\mathrm{NP}}^2$$

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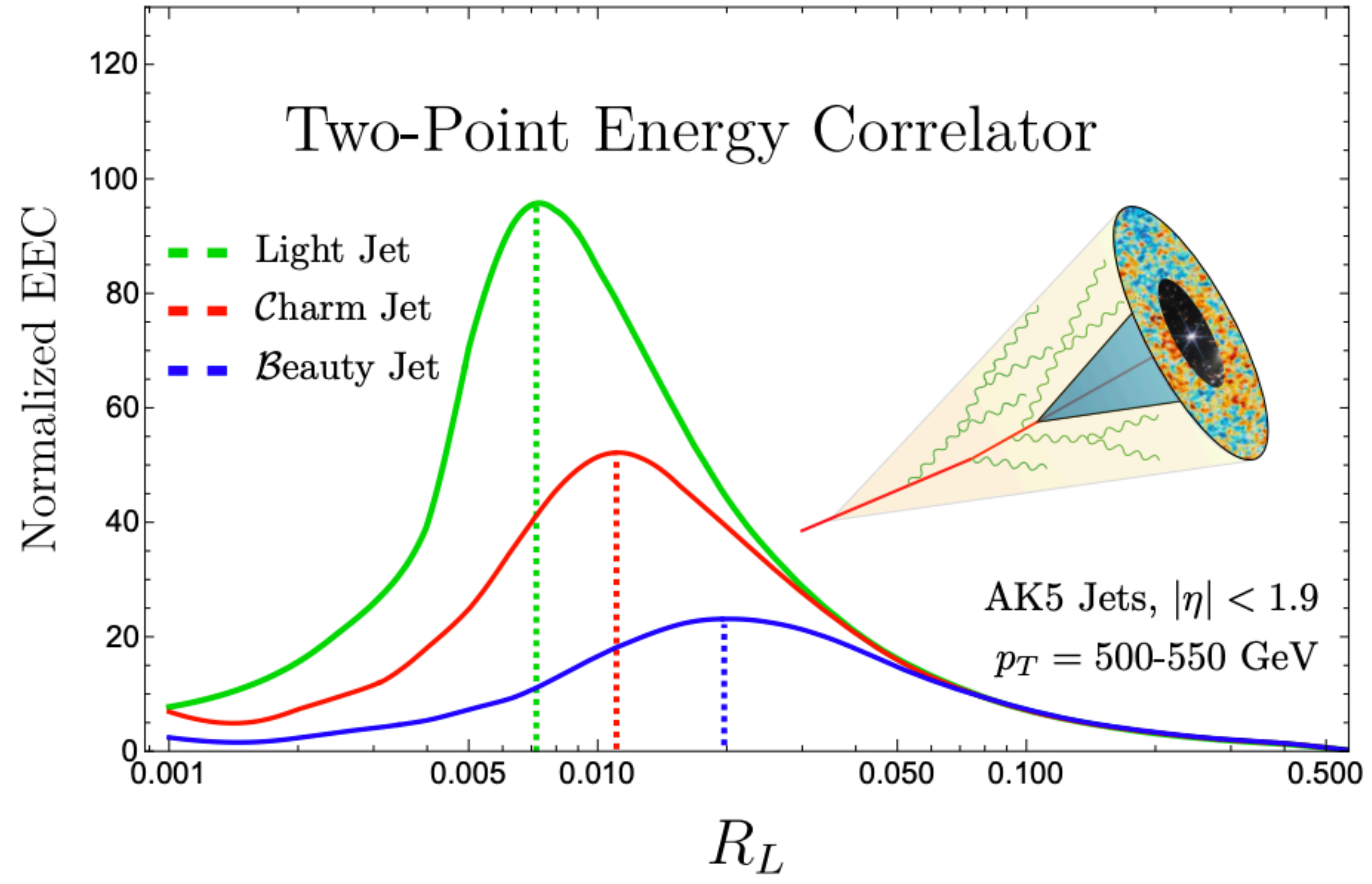
- Same result can be obtained from the small-angle limit of the cross-section for radiation off a massive particle

$$\left.\frac{\mathrm{d}\hat{\sigma}_{qg}}{\mathrm{d}R_L}\right|_{R_L<1} \sim \frac{R_L}{R_L^2 + \Theta^2}\,, \qquad \text{with} \qquad \Theta = \frac{m}{E}$$



E. Craft, K. Lee, B. Meçai, I. Moult 2210.09311

Energy loss through quenching weights

Salgado, Wiedemann hep-ph/0302184

- For sufficiently narrow jets, the medium does not resolve the jet substructure and it losses energy as a single source (totally coherent limit)

- Assuming independent emissions, the distribution of lost energy can be computed from the spectrum of soft medium-induced emissions

$$P_\xi(\epsilon) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{i=1}^{N} \left[\int \mathrm{d}\omega_i \, \frac{\mathrm{d}I_\xi}{\mathrm{d}\omega}\right] \delta\left(\epsilon - \sum_{i=1}^{N} \omega_i\right) \exp\left[-\int_0^\infty \mathrm{d}\omega \frac{\mathrm{d}I_\xi}{\mathrm{d}\omega}\right]$$

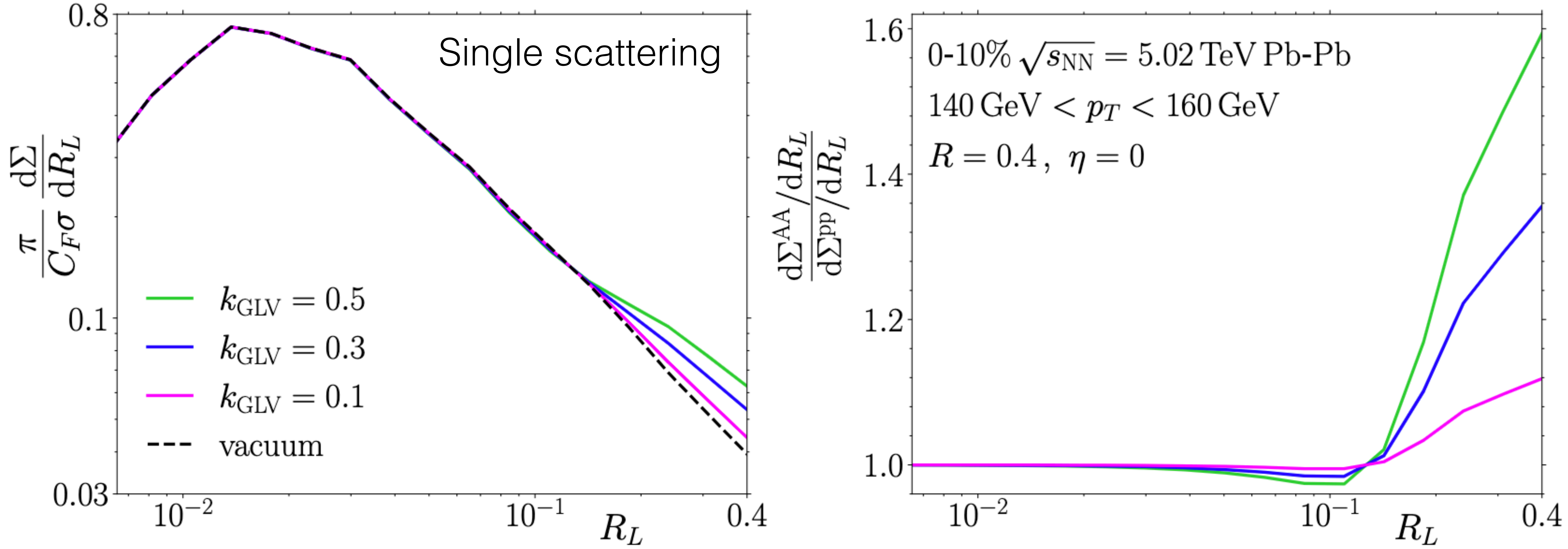
- This distribution is then convoluted with the observable at a higher hard scale and averaged over trajectories

$$\frac{\mathrm{d}\Sigma(Q)}{\mathrm{d}R_L} = \int \mathrm{d}\epsilon \, \left\langle P_\xi(\epsilon) \, \frac{\mathrm{d}\Sigma_\xi^{\mathrm{NP}}(Q+\epsilon)}{\mathrm{d}R_L} \right\rangle_\xi \, \frac{1}{\sigma_{q+X}} \, \frac{\mathrm{d}\sigma_{q+X}}{\mathrm{d}E_q}\bigg|_{E_q = Q+\epsilon}$$

- Corrections to this approach due to jet fluctuations can be calculated

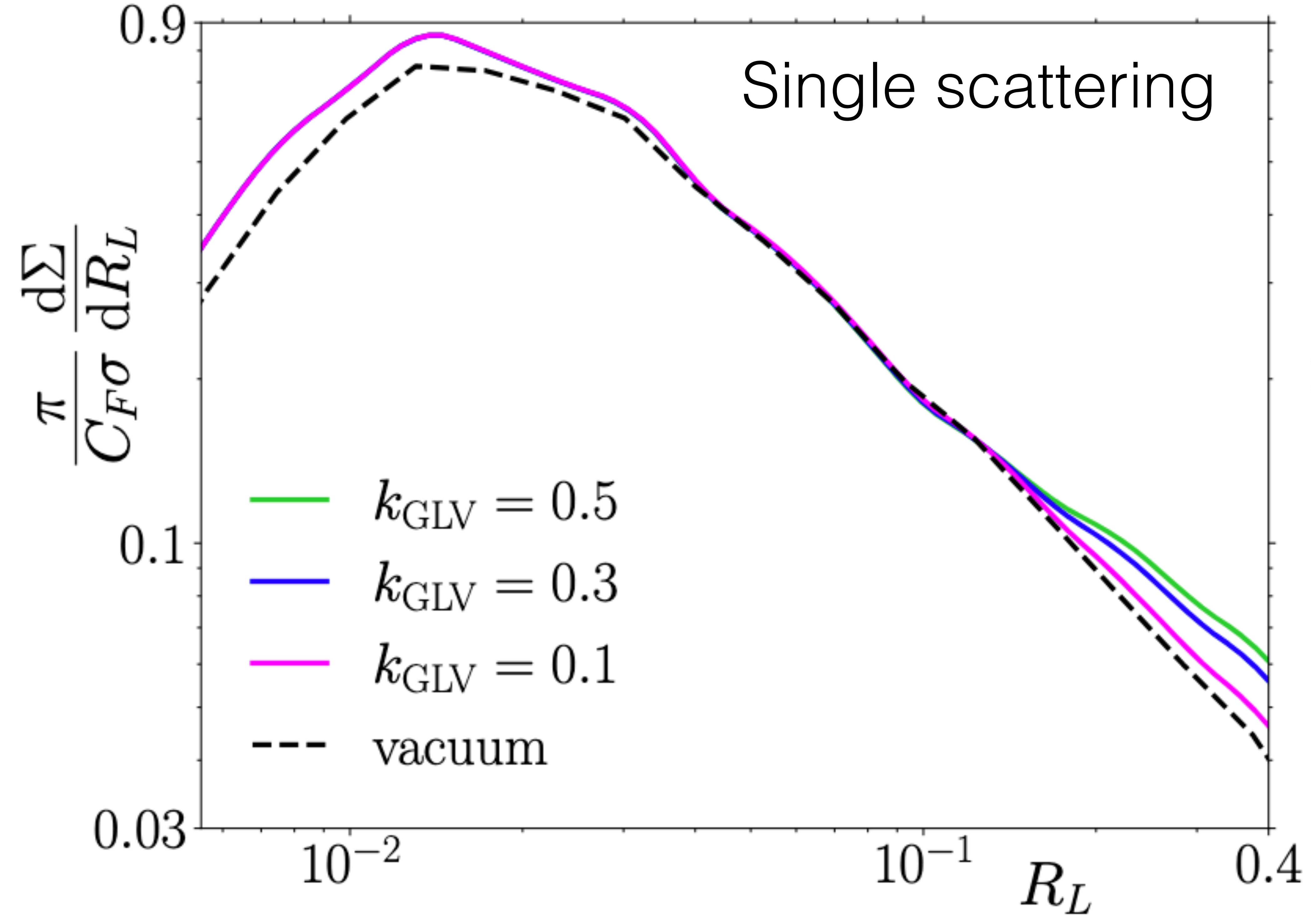
Mehtar-Tani, Tywoniuk 1707.07361 Barata, Caucal, Soto-Ontoso, Szafron 2312.12527

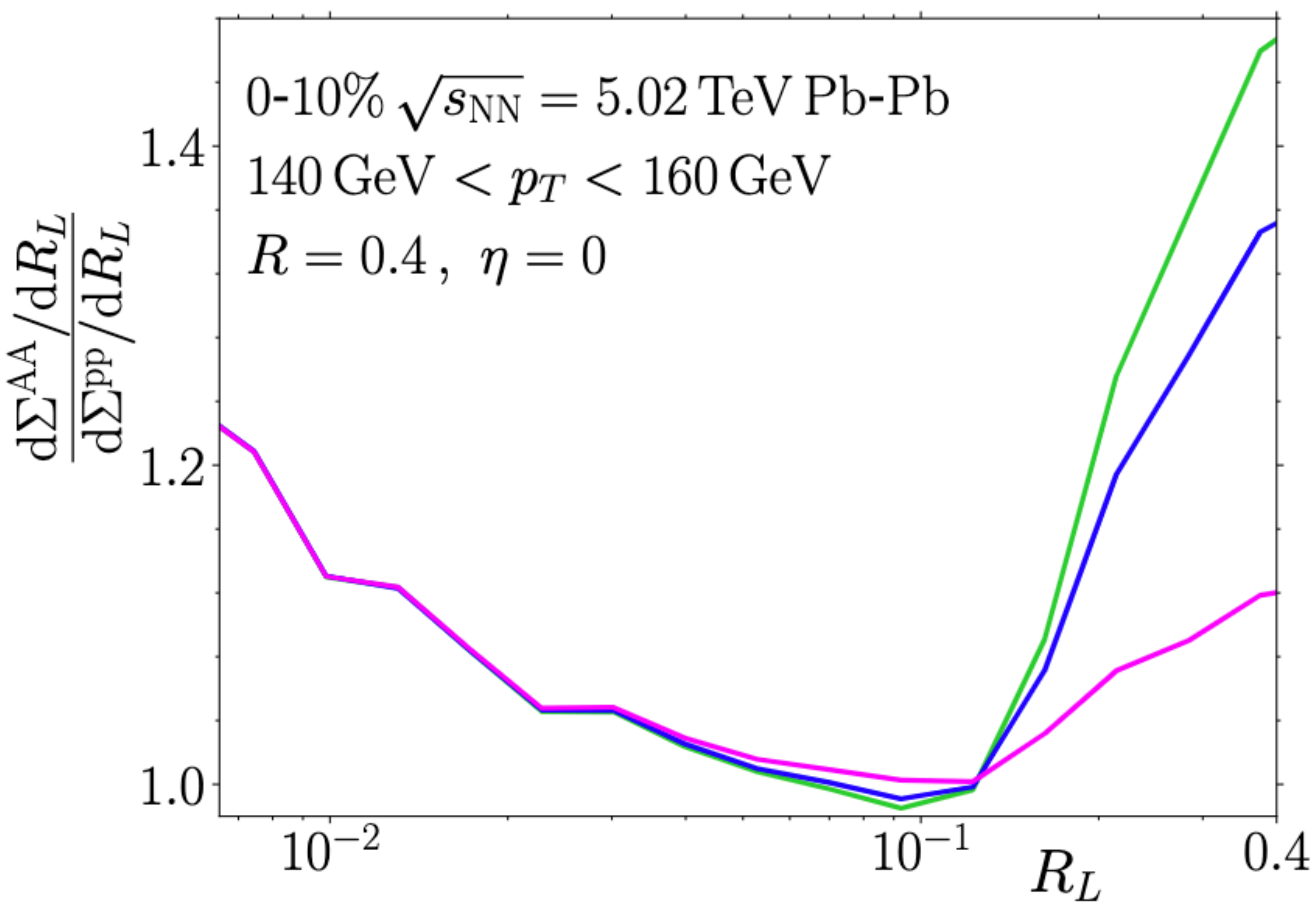
EECs for γ/Z-jets



- No modification to the shape of the correlators at small angles
- Normalization chosen such that quotient is 1 at small angles

EEC for inclusive jets





- Shift in the energy scale appears as not trivial structure at small angles, while large angle enhancement does not seem to change

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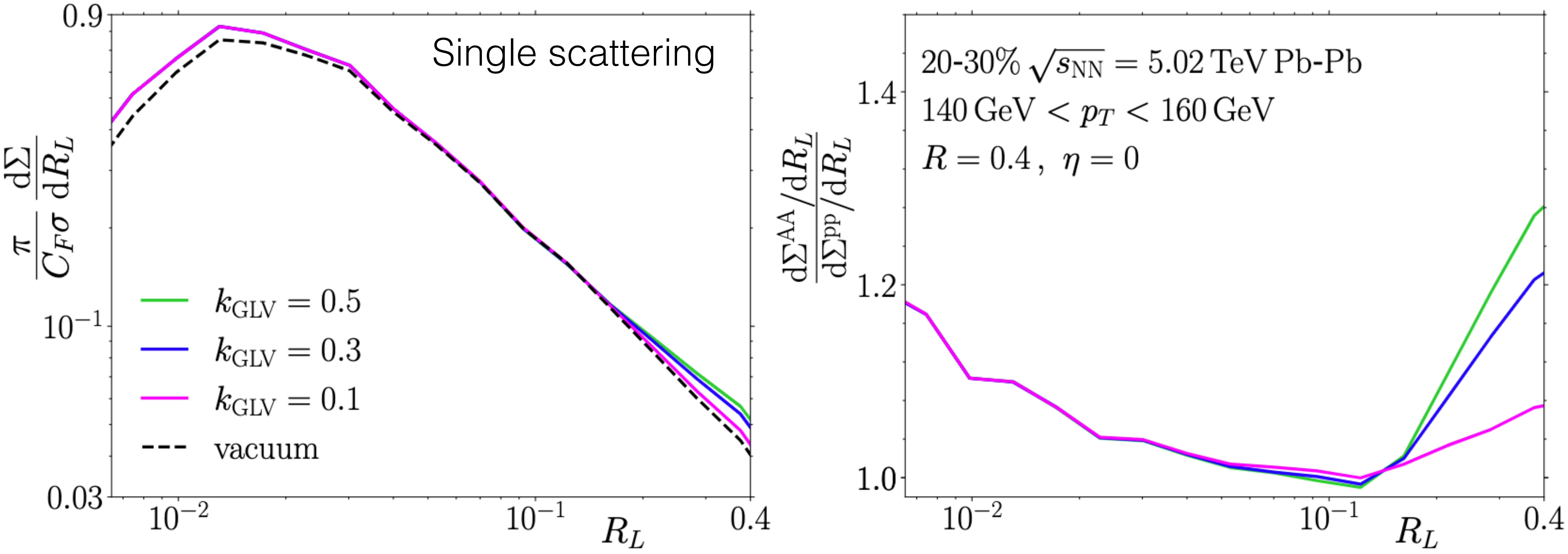

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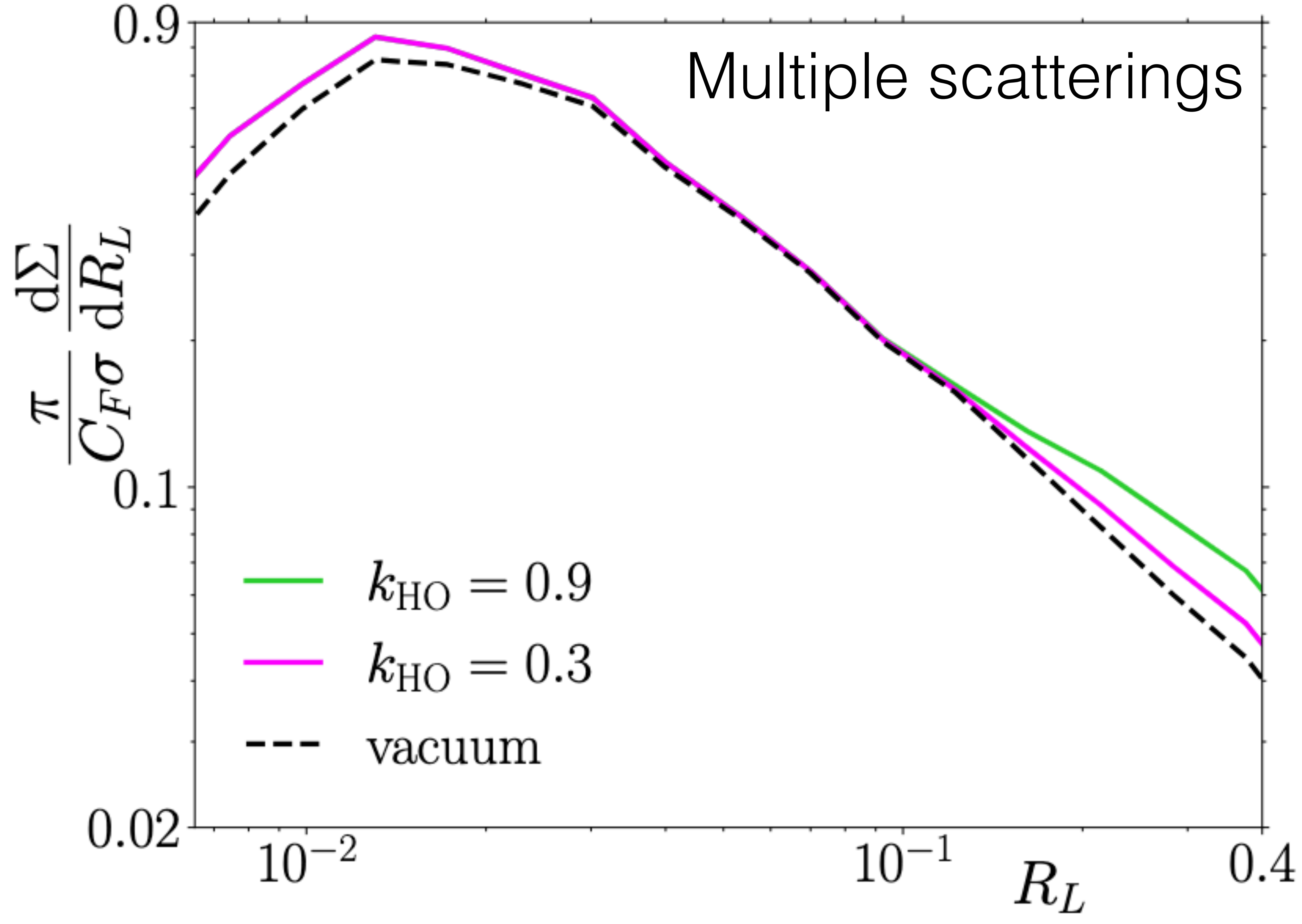
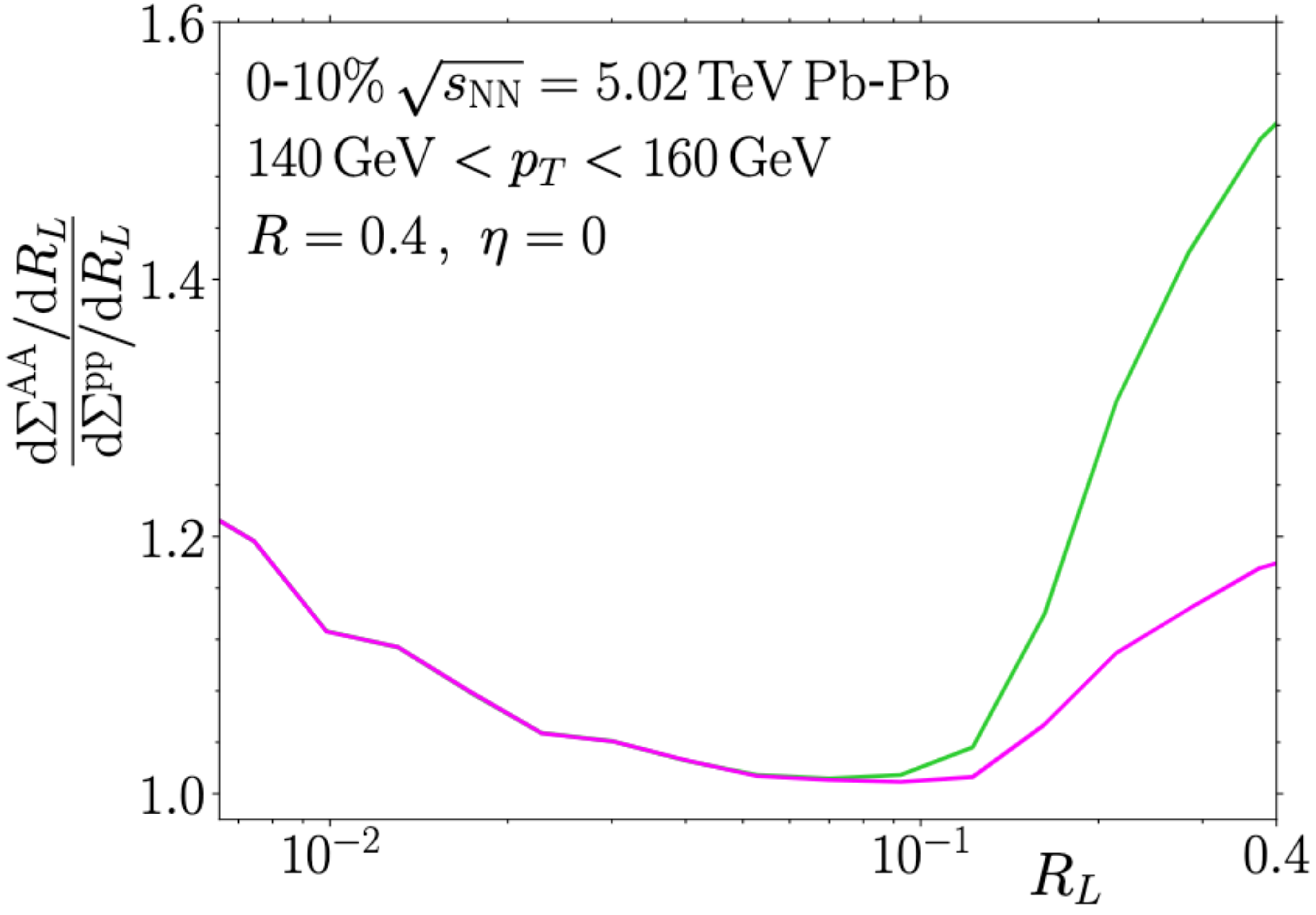




- Shift in the energy scale appears as not trivial structure at small angles, while large angle enhancement does not seem to change

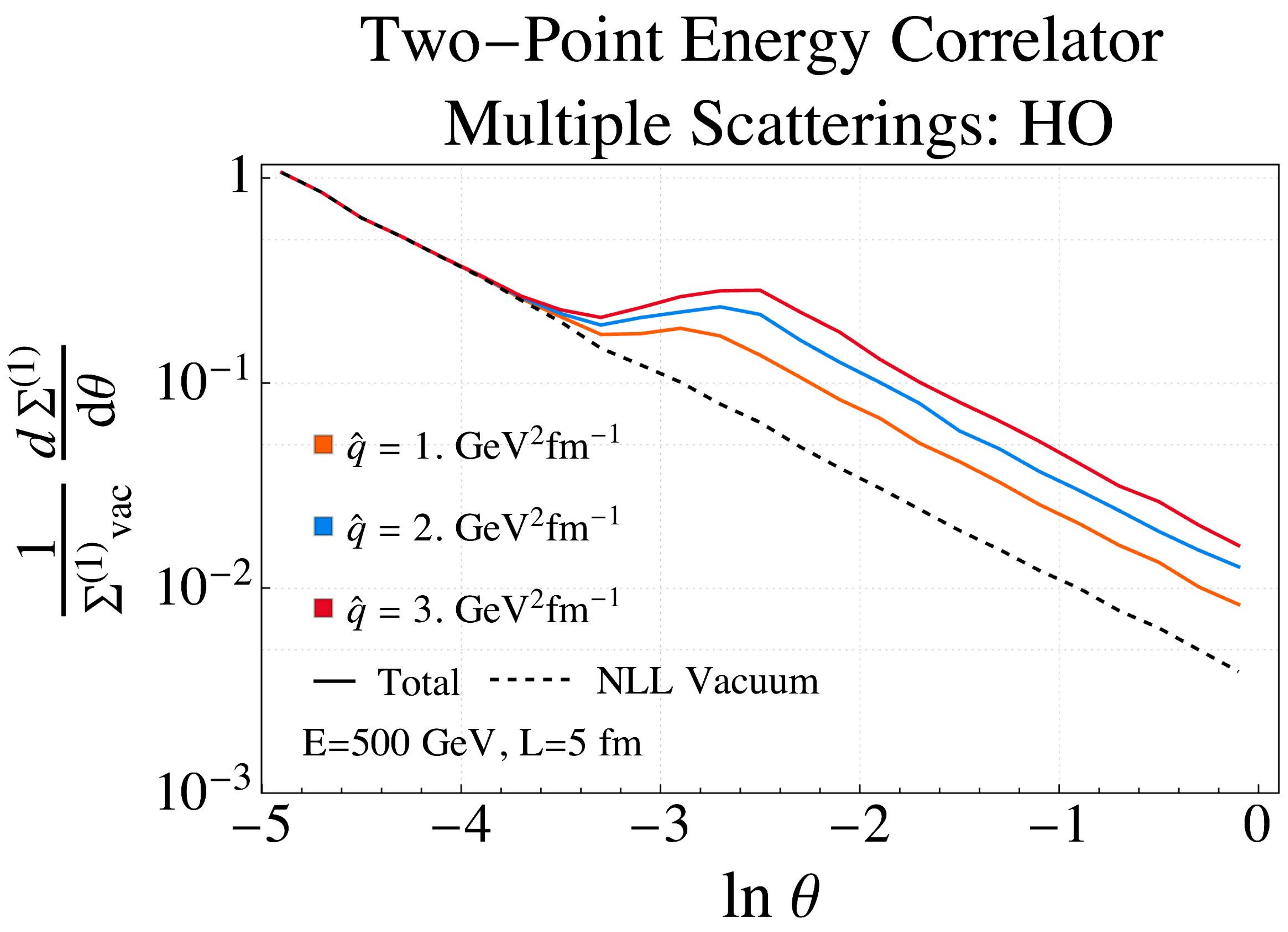
Summary and outlook

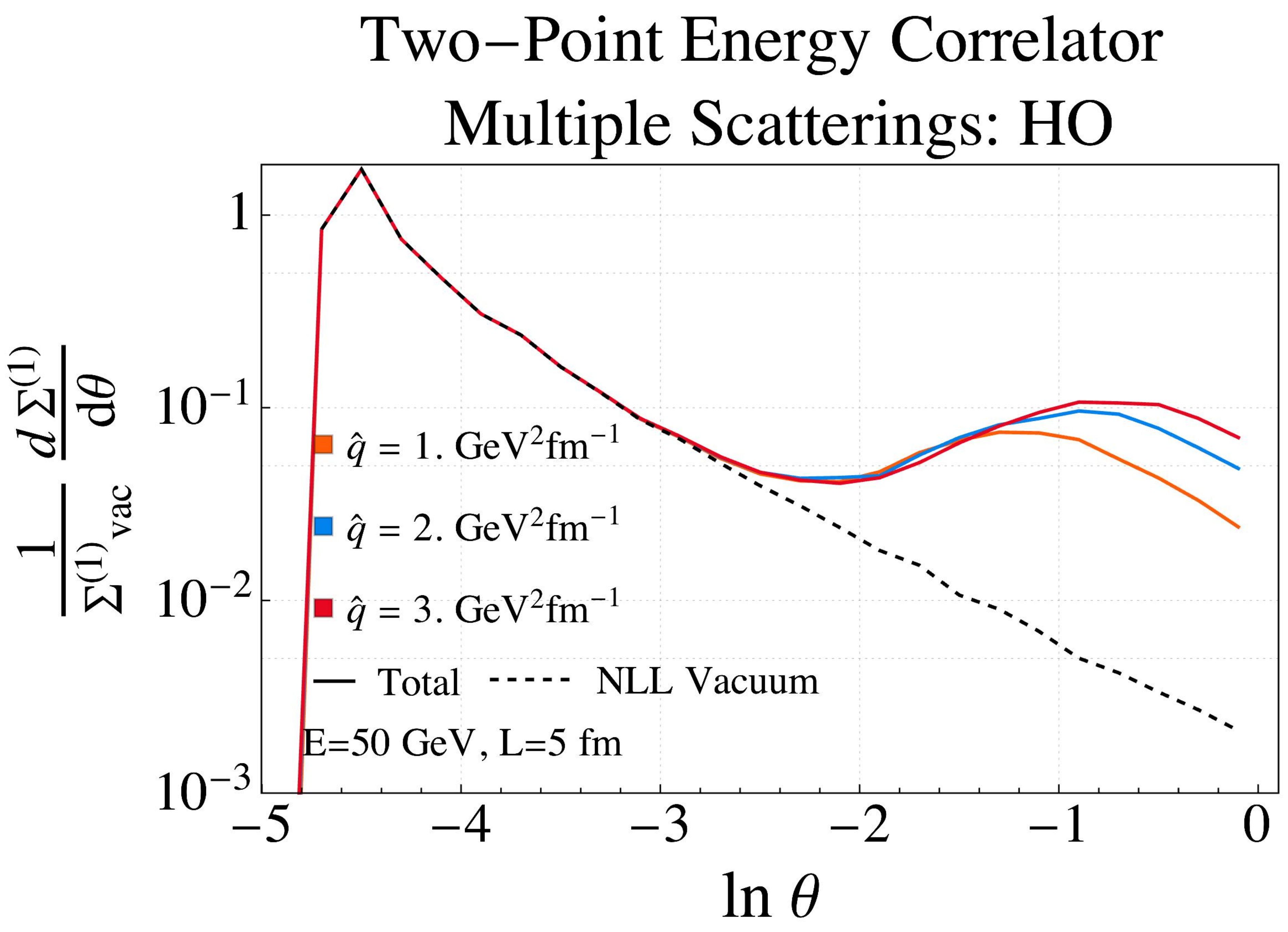
- Selection bias appears in the correlators as a negative slope in the small angle region
- Medium-induced splittings create an enhancement at larger angles
- One free parameter + normalization is not fixed

- Further improvements in the calculation will allow us to fix the free parameter using other measurements
- Comparisons with several p_T bins and centralities will provide a more complete picture

Thank you!

Results in the semi-hard approximation

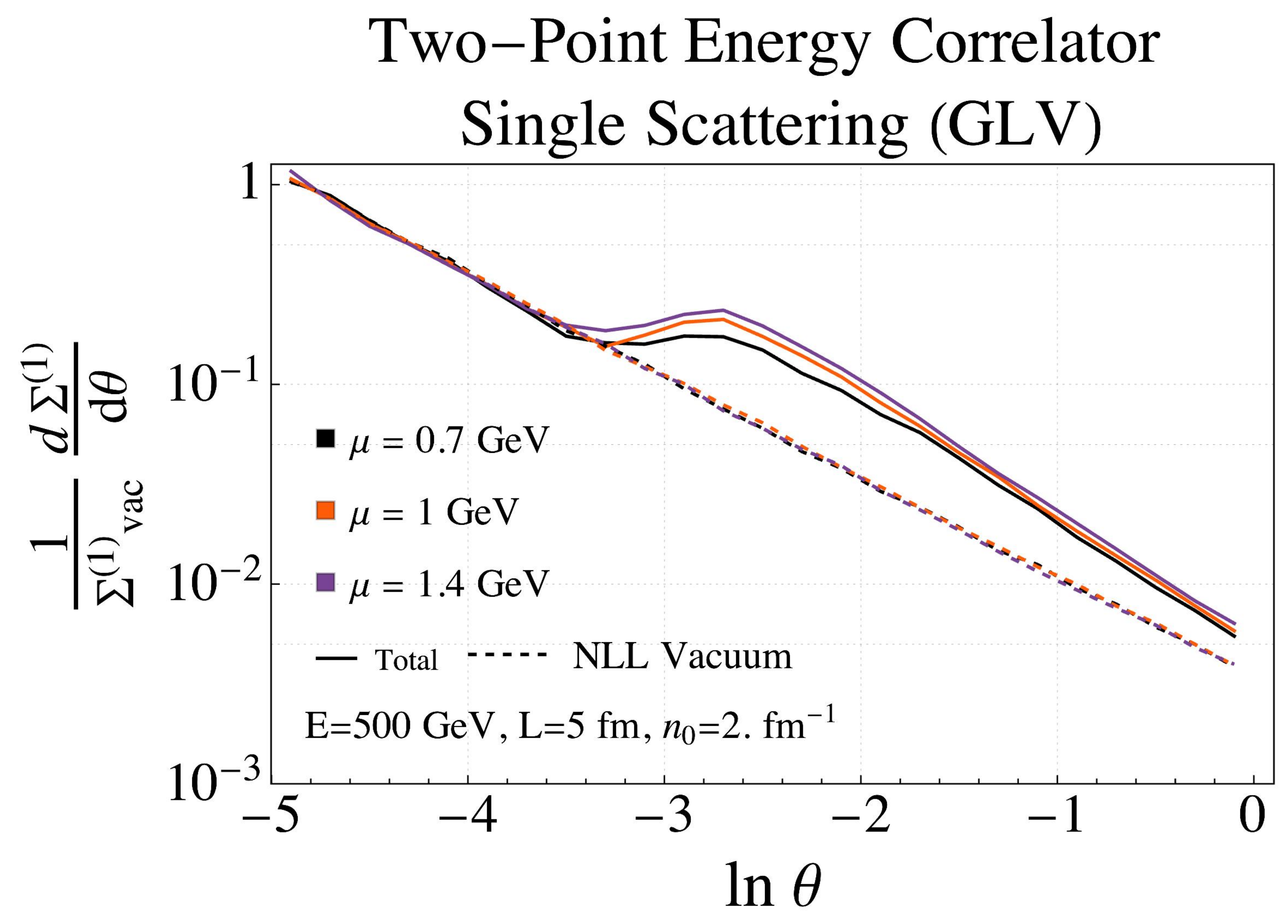


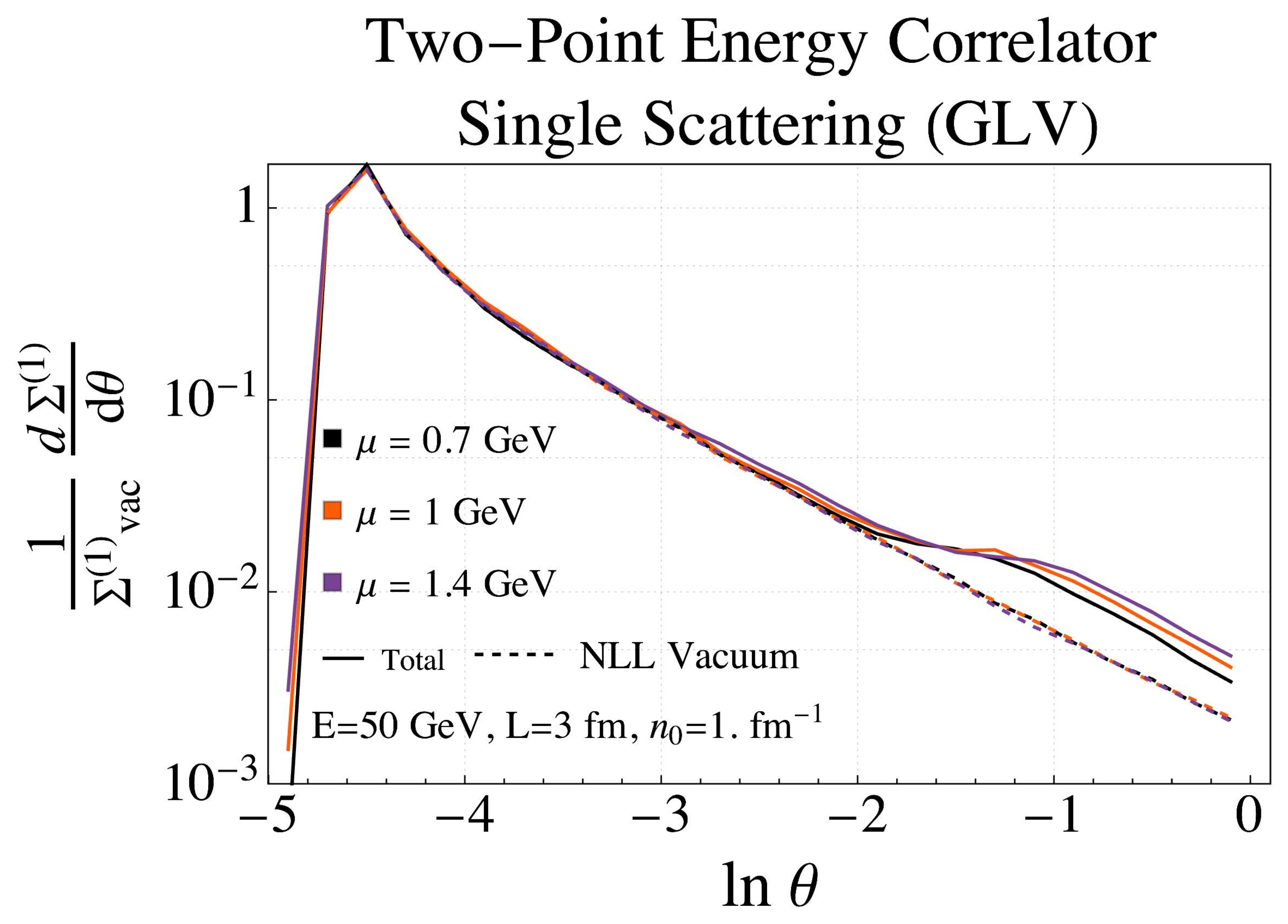


- No modification at small angles
- Transition towards medium-induced enhancement at larger angles
- Varying $\hat{q}$ has different effects depending on medium resolution

C. Andres, FD, R. K. Elayavalli, J. Holguin, C. Marquet, I. Moult, arXiv:2209.11236
C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2303.03413

Results for first order in opacity





- Similar qualitative features compared to the semi-hard approach
- Transition towards medium-enhancement not so well defined
- Enhancement at large angles has a much smaller amplitude

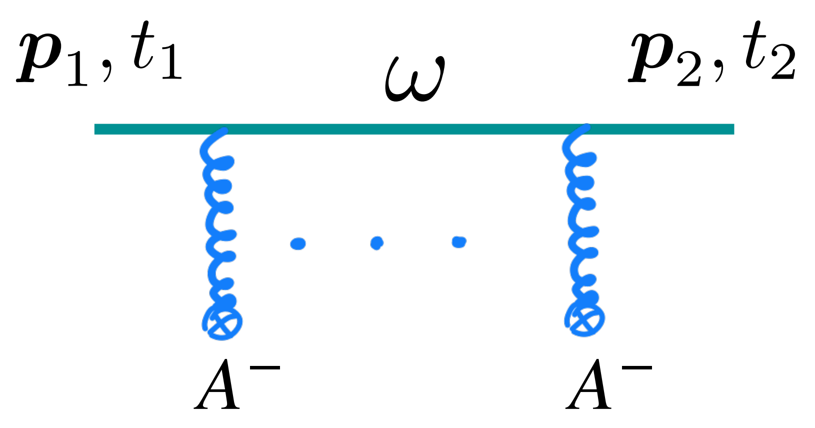
C. Andres, FD, J. Holguin, C. Marquet, I. Moult, arXiv:2303.03413

Calculation of in-medium splittings

- Collinear (high-energy) limit: All particles have a large longitudinal momentum compared to their transverse momenta (small angles, DGLAP limit)

- Decoupling of transverse and longitudinal dynamics: Effects coming from the transverse (with respect to the direction of the jet) structure of the medium are suppressed by powers of the energy

- Medium interactions are resummed through in-medium propagators



$$= \mathcal{G}_R(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1; \omega)$$

- Cross section are expressed in terms of medium averages of products of propagators

Semi-hard approximation

FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653 Isaksen, Tywoniuk 2107.02542

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- Take in-medium propagators in the extreme eikonal limit where they can be written as a Wilson line in a straight trajectory in coordinate space

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$$\to \mathcal{G}_{0,R}(t_2, \boldsymbol{x}_2; t_1, \boldsymbol{x}_1; \omega)\, V_R(t_2, t_1; [\boldsymbol{x}_{\mathrm{cl}}])$$

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$$\mathcal{G}_R(t_2, \boldsymbol{p}_2; t_1, \boldsymbol{p}_1; \omega) \to e^{-i\frac{\boldsymbol{p}_2^2}{2\omega}(t_2-t_1)} \int_{\boldsymbol{x}} e^{-i(\boldsymbol{p}_1-\boldsymbol{p}_0)\cdot\boldsymbol{x}}\, V_R(t_2, t_1; [\boldsymbol{x}+\boldsymbol{n}t])$$

$$\to (2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)\, e^{-i\frac{\boldsymbol{p}_2^2}{2\omega}(t_2-t_1)} V_R(t_2, t_1; [\boldsymbol{n}t])$$

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- The harmonic approximation (HO) is usually employed when using this approach but it is not necessary

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- The harmonic approximation (HO) is usually employed when using this approach but it is not necessary

- Angular and time scales are easily identifiable from the analytic formulas

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- Neglect additional transverse momentum broadening

$$\mathcal{G}_R(t_2,\boldsymbol{p}_2;t_1,\boldsymbol{p}_1;\omega) \to e^{-i\frac{\boldsymbol{p}_2^2}{2\omega}(t_2-t_1)} \int_{\boldsymbol{x}} e^{-i(\boldsymbol{p}_1-\boldsymbol{p}_0)\cdot\boldsymbol{x}}\, V_R(t_2,t_1;[\boldsymbol{x}+\boldsymbol{n}t])$$

$$\to (2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)\, e^{-i\frac{\boldsymbol{p}_2^2}{2\omega}(t_2-t_1)} V_R(t_2,t_1;[\boldsymbol{n}t])$$

- The harmonic approximation (HO) is usually employed when using this approach but it is not necessary

- Angular and time scales are easily identifiable from the analytic formulas

- Numerical evaluations are straightforward in the large-N_c limit

Relaxing approximations

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FD, Milhano, Salgado, Tywoniuk, Vila 1907.03653

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Blaizot, FD, Iancu, Mehtar-Tani 1209.4585

Apolinario, Armesto, Milhano, Salgado 1407.0599

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Blaizot, FD, Iancu, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

- Going beyond the large-N_c limit requires a much more complex setup where a system of coupled differential equations must be numerically solved

Isaksen, Tywoniuk 2303.12119

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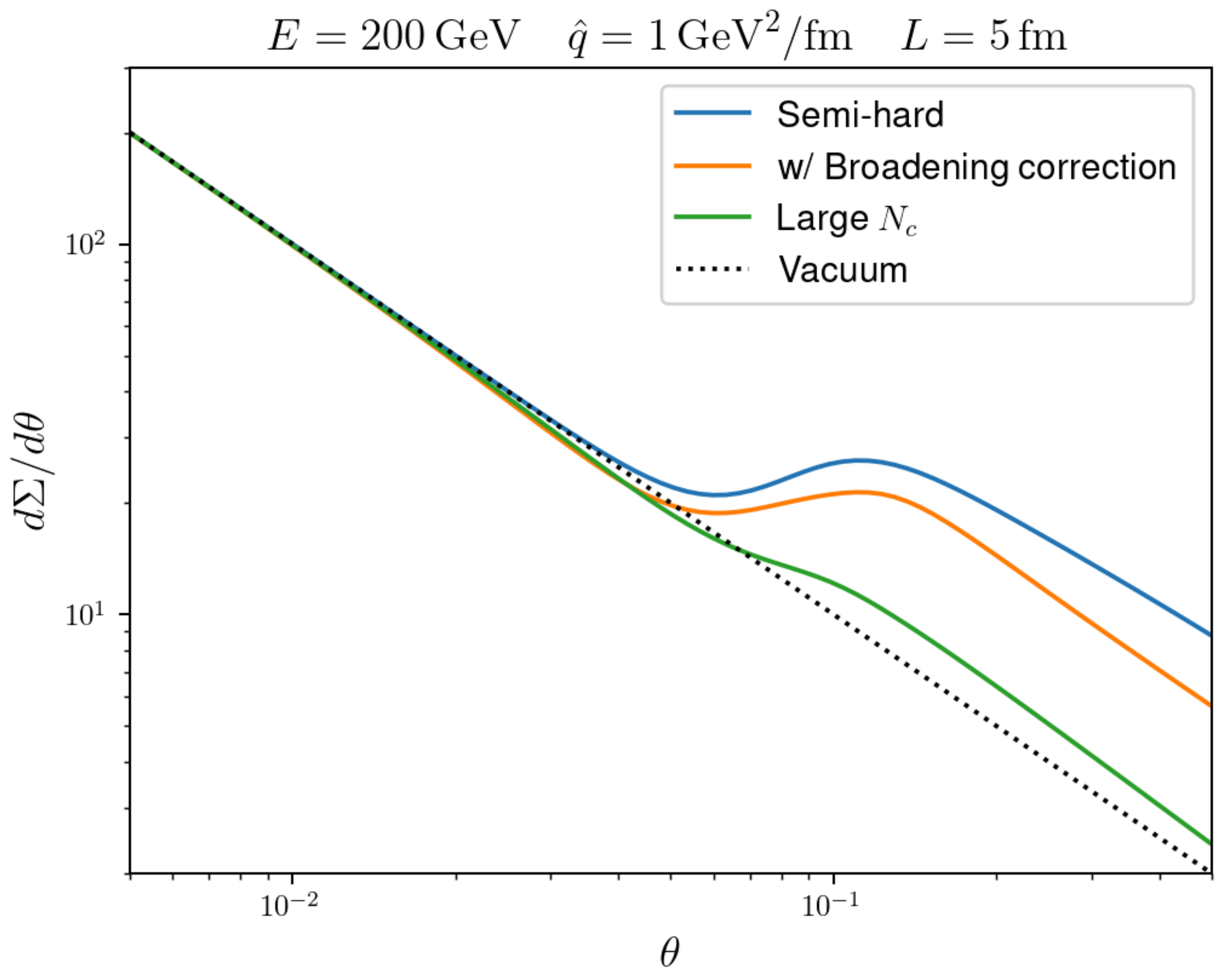
Blaizot, FD, Iancu, Mehtar-Tani 1209.4585

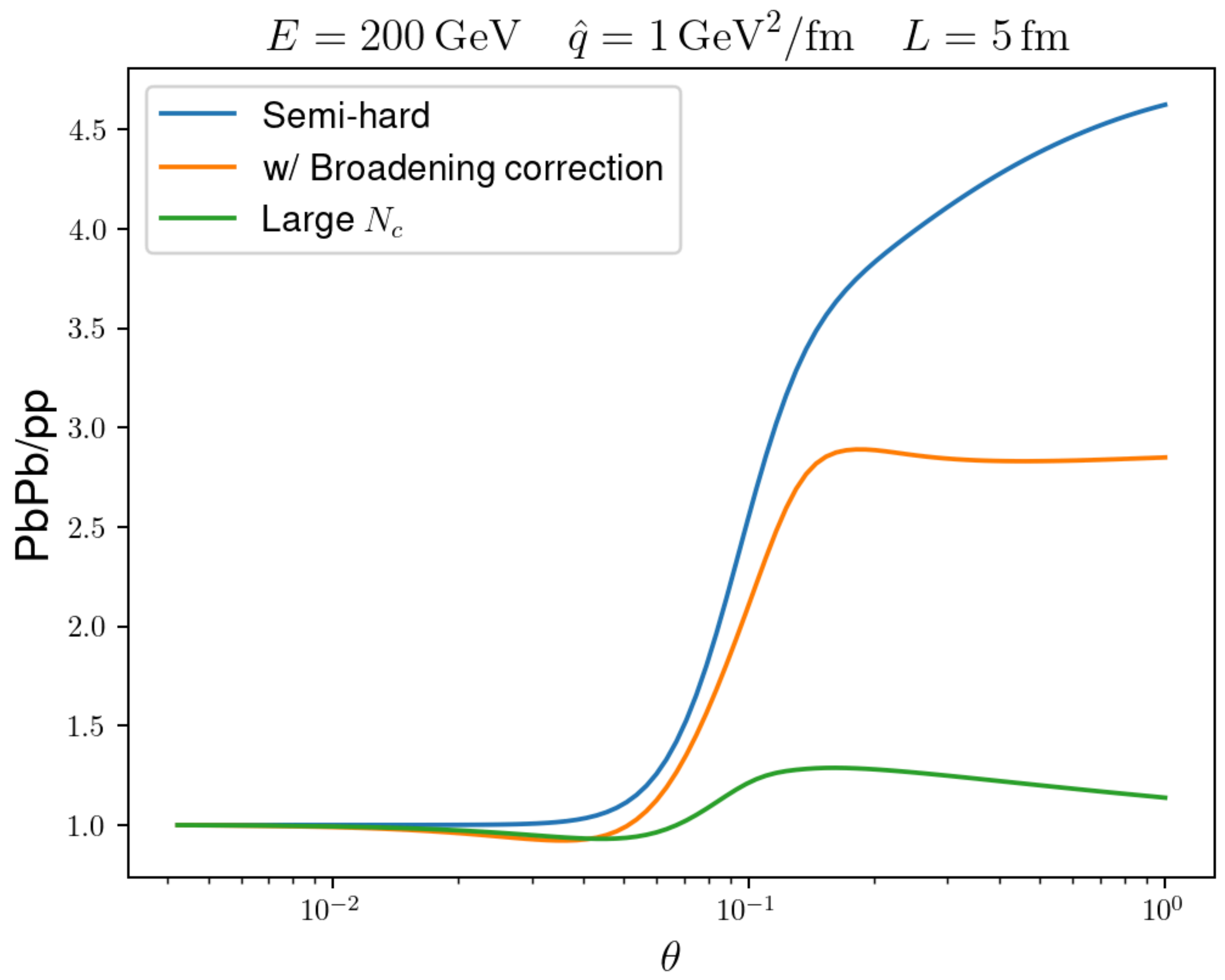
Apolinario, Armesto, Milhano, Salgado 1407.0599

- Goin … tup wher … ally solve … 3.12119

Done only for $\gamma \to q\bar{q}$ and the code for this solution has stability problems beyond a restricted area in parameter space and will not be used for this talk

Results for a brick with the harmonic approximation





- Large reduction of the enhancement amplitude
- Small dip before transitioning to medium-enhancement, already present when the broadening correction is included
- Overall picture of no modification at small angles followed by a transition to medium-enhancement at large angles still valid