Heavy Quarks in the Energy Correlator Spectrum

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Based on arXiv:2210.09311 and work on progress with

Evan Craft

Mark Gonzales

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Ian Moult

Heavy quarks at the LHC

- Deeper understanding of collider data at high energies requires detailed analysis of mass effects from heavy quarks

Include mass effects in perturbative calculations

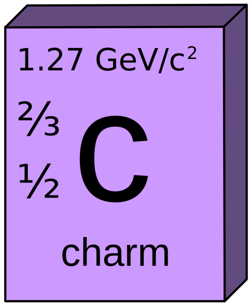
Next generation parton shower simulations

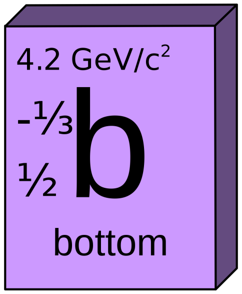
Heavy quarks at the LHC

Effort to better understand their hadronization

- **Many interesting processes include heavy quark measurements:** $h \to b\bar{b}, h \to c\bar{c}$

- **Monte Carlo work mostly well for light quarks.**

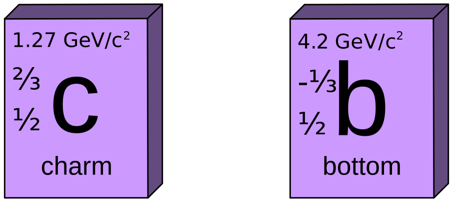




Heavy quarks at the LHC

Effort to better understand their hadronization

- **Many interesting processes include heavy quark measurements:** $h \to b\bar{b}, h \to c\bar{c}$
- **Monte Carlo work mostly well for light quarks.**
- **Heavy quarks such as b-(beauty) and c-(charm) quarks are less understood how they develop in the shower.**
- **Their mass is non-negligible and this introduces an extra scale in the problem!**



Application on Heavy Jets

Introduce an additional scale

- At the LHC energies there is access to the transition phase from massless to massive behavior ⇒ more complexity
- Complexity from transitioning from a massless to a massive regime for LHC at high energies
- Also very interesting!
    - Can probe intrinsic mass effects of quarks before confinement into hadrons

Heavy quarks at the LHC

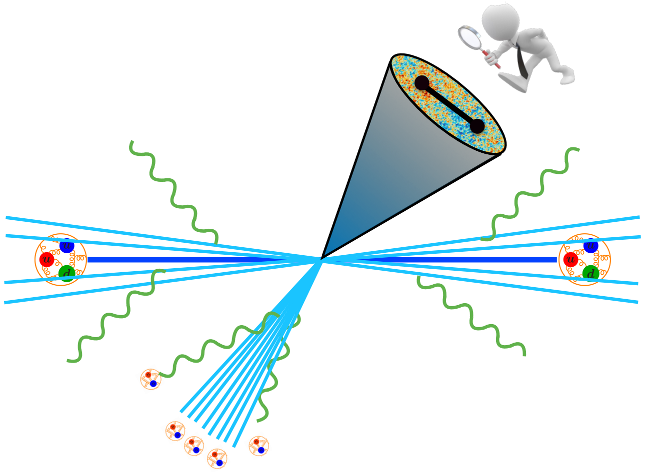
Effort to better understand their hadronization

$$\Downarrow$$

Robust Jet substructure observables that capture such effects systematically

In fact jet substructure was initiated by the studies of Higgs decays to heavy quarks!

[Butterworth, Davison, Rubin, Salam]



Energy Correlators for Jet Substructure

Energy Flow Inside the Jet

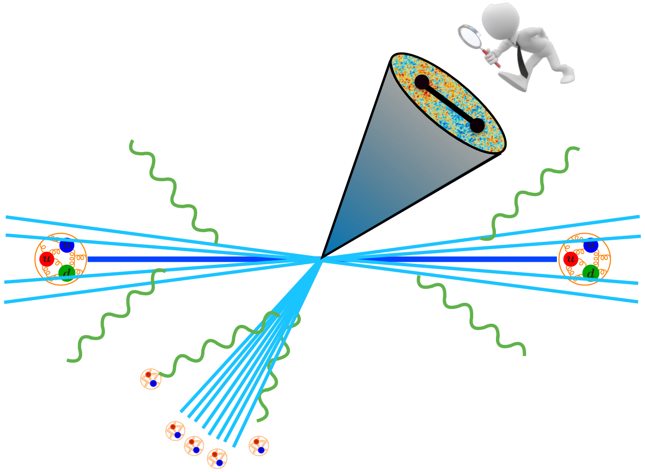
[Sterman; Basham, Brown, Ellis, Love]

- Distribution of energy inside the jet is described by correlation functions of the energy flow operators ⇒ Energy Correlators.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid \Psi \rangle$$

$$\boxed{\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\ r^2 n^i T_{0i}(t, r\vec{n})}$$

Defined from first principles in QFT!

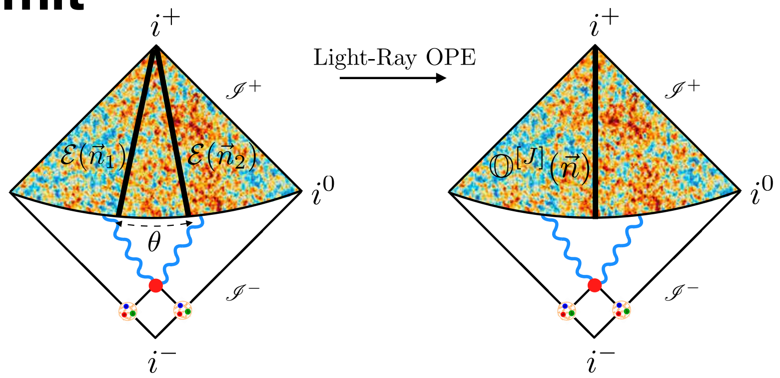


Any physics dynamics will be imprinted in the energy distributions inside the jet.

Scaling Behavior

Energy correlators inside high energy jets at the LHC

⇒small angle limit



- **Energy correlators admit an OPE:**

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle \sim \sum \theta^{\gamma_i}\mathcal{O}_i(\vec{n}_1)$$

[Hofman, Maldacena]
[Chang, Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Scaling Behavior

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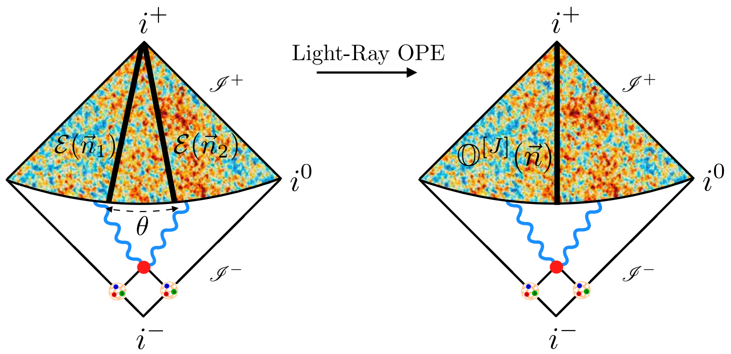

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⇒ Use LHC jets to test the leading QCD operators in this expansion

Scaling Behavior

Energy correlators inside high energy jets at the LHC

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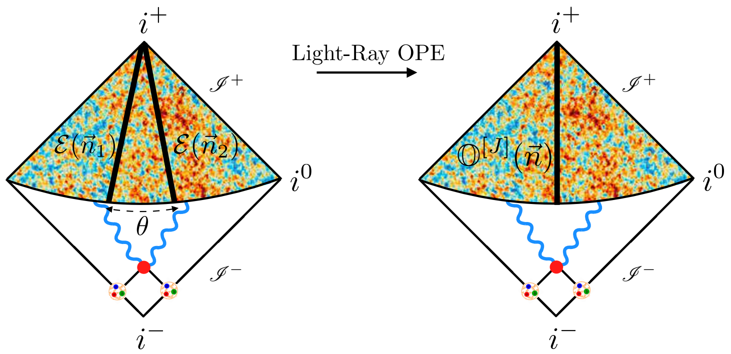

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⇒ Can such an OPE be tested for heavy states?

Energy Correlators on Heavy Quarks

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

$$\langle X \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\ldots\varepsilon(\vec{n}_n) \mid X\rangle$$

Measurement of correlation functions of $\varepsilon(\vec{n})$ between heavy quark jet states

Energy Correlators on Heavy Quarks

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Measurement of correlation functions of $\varepsilon(\vec{n})$ between heavy quark jet states

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{dz} = \frac{\int d^4x e^{iq\cdot x} \left\langle \mathcal{O}(x)\varepsilon\left(\vec{n}_1\right)\varepsilon\left(\vec{n}_2\right)\ldots\varepsilon\left(\vec{n}_n\right)\mathcal{O}^\dagger(0)\right\rangle}{\int d^4x e^{iq\cdot x}\left\langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\right\rangle}$$

⇒ **Weighted cross-sections: distribution of outgoing charges**

$$\sigma_{EEC}(q) = \sum_X (2\pi)^4\delta^{(4)}\left(q - k_X\right) E_{\{X\}}\,|\,\langle X\,|\,O(0)\,|\,0\rangle\,|^2$$

Energy Correlators on Heavy Quarks

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$\Rightarrow$ **Weighted cross-sections:distribution of outgoing charges**

$$\sigma_{EEC}(q) = \sum_X (2\pi)^4\delta^{(4)}\left(q - k_X\right) E_{\{X\}} \,|\langle X | O(0) | 0\rangle|^2$$

$E_{\{\Psi\}}$ are different permutations for all Ψ final states

Local operator that creates the state $|\Psi\rangle$ with momentum k_Ψ

Energy Correlators on Heavy Quarks

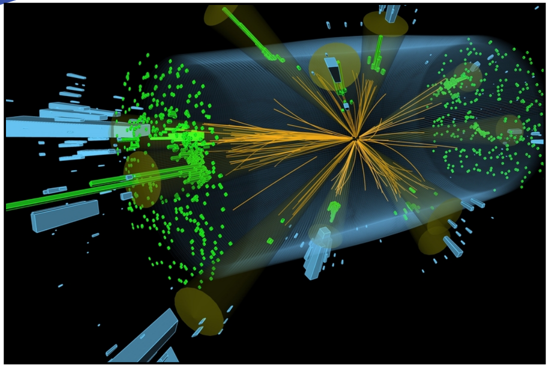
$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2)\dots\varepsilon($$

$$= \lim_{r\to\infty}\int_0^\infty dt\; r^2 n^i T_{0i}(t, r\vec{n})$$

Measurement of

Calculation for heavy quark final state jets requires derivation of a factorization theorem that ensures scale separation

$\Rightarrow$ Weighted cro … going charges

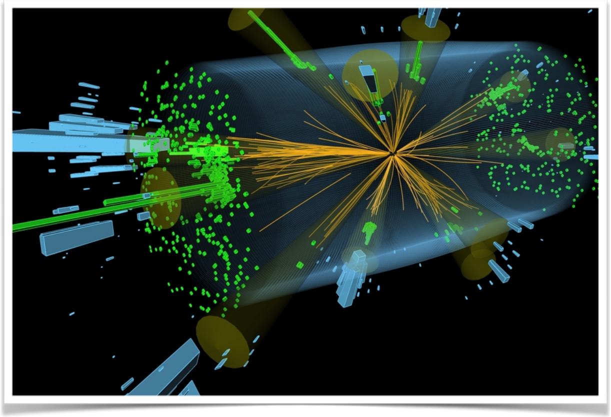


$$\sigma_{EEC}(q) = \sum_{\Psi} (2\pi)^4 \delta^{(4)}\left(q - k_\Psi\right) E_{\{\Psi\}} \,|\langle \Psi \,|\, O(0) \,|\, 0\rangle\,|^2$$

$E_{\{\Psi\}}$ are different permutations for all Ψ final states

Local operator that creates the state $|\Psi\rangle$ with momentum k_Ψ

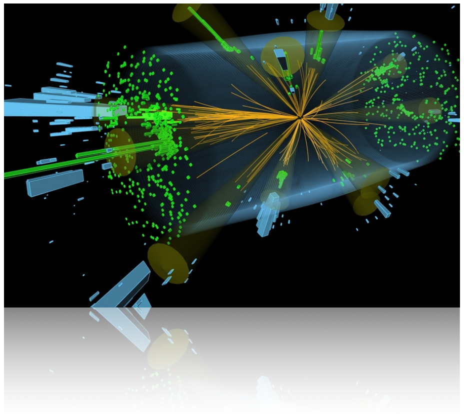
Energy Correlators at the LHC



Factorization theorem

[Craft, Lee, BM, Moult]

Can compute any higher point correlators on massive quarks at LHC at NLL

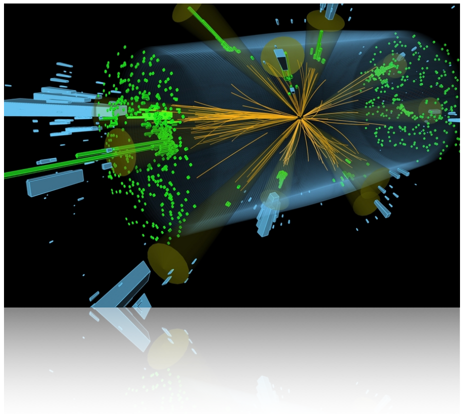


$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \underbrace{\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right)}_{\text{Describes the dependance on the observable}} \cdot \overbrace{\vec{H}\left(x, p_T^2, \mu\right)}^{\text{Describes the production of the collinear source}}$$

Factorization theorem

[Craft, Lee, BM, Moult]

Can compute any higher point correlators on massive quarks at LHC at NLL



Describes the production of the collinear source

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \underbrace{\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right)} \cdot \overbrace{\vec{H}\left(x, p_T^2, \mu\right)}$$

Describes the dependance on the observable

Three scales:

$\mu_H \sim p_T$

$\mu_J \sim p_T R$

m_Q

- **Factorization theorem derived within soft-collinear effective theory.**

[Bauer, Fleming, Pirjol, Stewart]

Factorization theorem

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \overrightarrow{H}\left(x, p_T^2, \mu\right)$$

$$\vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) = \left\{\begin{array}{l} \vec{J}_g^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_q^{[N]}\left(R_L, x, m_Q, \mu\right) \\ \vec{J}_Q^{[N]}\left(R_L, x, m_Q, \mu\right) \end{array}\right\}$$

$$\overrightarrow{H}\left(x, p_T^2, \mu\right) = \left\{\begin{array}{l} H_g\left(x, p_T^2, \mu\right) \\ H_q\left(x, p_T^2, \mu\right) \\ H_Q\left(x, p_T^2, \mu\right) = H_q\left(x, p_T^2, \mu\right) \end{array}\right\}$$

Factorization theorem

$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

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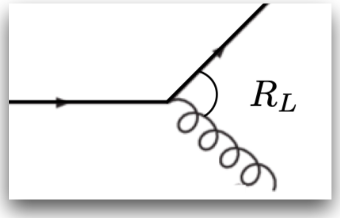

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Heavy Quark Jet Function

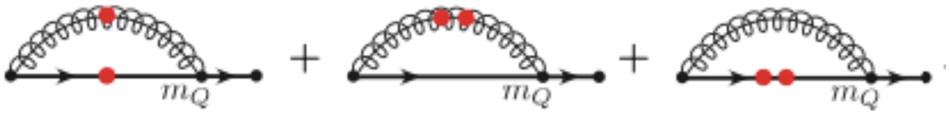
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$

$$W_{n_i}^{(A)}(x) = P \exp\left[ig_A\, t_A^a \int_{-\infty}^{0} ds\, \bar{n}_i \cdot A_{n_i}^a(x + s\bar{n}_i)\right]$$

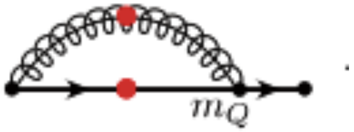
$$J_Q^{[N]}\left(R_L, m_Q\right) = \sum_X \sum_{i_1, i_2, \ldots, i_N \in X} \left\langle 0 \left| \bar{\chi}_n \right| X \right\rangle \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{p_T^N} \Theta\left(\max\left\{\theta_{ij}\right\} < R_L\right) \left\langle X \left| \chi_n \right| 0 \right\rangle$$

$$\chi_{n_i}(x) = \frac{\not{n}_i \not{\bar{n}}_i}{4} W_{n_i}^{\dagger}(x)\, \psi(x)$$

At $\mathcal{O}(\alpha_s)$ the jet function is describes by the one-loop $1 \to 2$ splitting of a quark weighted by the energy of each particle in the loop



Heavy Quark Jet Function



$$\delta = \frac{im_Q}{p_T R_L}$$

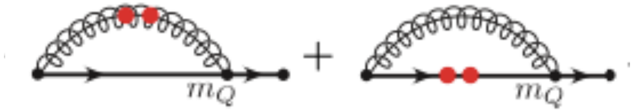
$$J_Q^{[N]}(R_L, m_Q)|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi} \int dx \frac{2(1-(1-x)^N - x^N)\left[2x^3 + (1+x^2)(x+\delta)(x+\bar{\delta})\ln\frac{\delta\bar{\delta}}{(x+\delta)(x+\bar{\delta})}\right]}{(-1+x)(x+\delta)(x+\bar{\delta})}$$

$$J_Q^{[2]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\delta^4 - 4\delta^3 + 2\delta^2 - 3\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(9\delta^2 + \frac{31}{6}\right)\right\} + c.c\,,$$

$$J_Q^{[3]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{3}{2}\delta^4 - 6\delta^3 + 3\delta^2 - \frac{9}{2}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{27}{2}\delta^2 + \frac{31}{4}\right)\right\} + c.c\,,$$

$$J_Q^{[4]}|_{R_L\neq 0} = \frac{\alpha_s C_F}{4\pi}\left\{\left[\frac{2}{3}\delta^6 - \frac{16}{5}\delta^5 - \delta^4 - \frac{20}{3}\delta^3 + 4\delta^2 - \frac{83}{15}\right]\ln\left(\frac{\delta}{1+\delta}\right) - \frac{1}{2}\left(\frac{106}{15}\delta^4 + \frac{74}{5}\delta^2 + \frac{1417}{150}\right)\right\} + c.c\,,$$

Heavy Quark Jet Function



$$J_Q^{[2]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-3\left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{M^2}\right] - \frac{49}{6}\right\},$$

$$J_Q^{[3]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{9}{2}\left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{M^2}\right] - \frac{47}{4}\right\},$$

$$J_Q^{[4]}|_{R_L=0} = \frac{\alpha_s C_F}{4\pi}\left\{-\frac{83}{15}\left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{M^2}\right] - \frac{6611}{450}\right\},$$

$$\delta = \frac{im_Q}{p_T R_L}$$

- **Mass regulates IR divergences!**
- **The remaining $\frac{1}{\epsilon}$ poles are UV poles regulated by renormalization.**

Comparison with massless jet functions

Two-point energy-energy correlator (EEC)

The mass should not affect the UV behavior of the jet function.
This can be seen from comparing the UV poles with the light quark jet function.

$$J_Q^{EEC}(z, M, \mu) = \delta(z)\left(1 + \frac{\alpha_s C_F}{4\pi}\left[-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\left(\frac{1}{\epsilon_{\text{UV}}} + \ln\frac{\mu^2}{M^2}\right) - \frac{19}{6}\right]\right) + \text{finite terms}$$

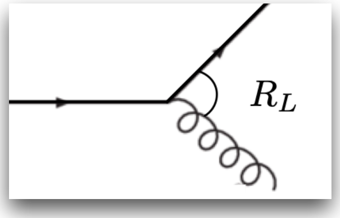
$$z = \frac{1 - \cos\theta_{ij}}{2}$$

$$J_q^{EEC} = \delta(z) + \frac{\alpha_s C_F}{4\pi}\left[\delta(z)\left(-\left(\gamma_{qq}^{(0)}(3) + \gamma_{gq}^{(0)}(3)\right)\frac{1}{\epsilon_{\text{UV}}} - \frac{37}{3}\right) + 3\frac{Q^2}{\mu^2}\mathscr{L}_0\left(\frac{Q^2}{\mu^2}z\right)\right]$$

$$\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} = \begin{pmatrix} \frac{25}{6}C_F & -\frac{7}{15}n_f \\ -\frac{7}{6}C_F & \frac{14}{5}C_A + \frac{2}{3}n_f \end{pmatrix}$$

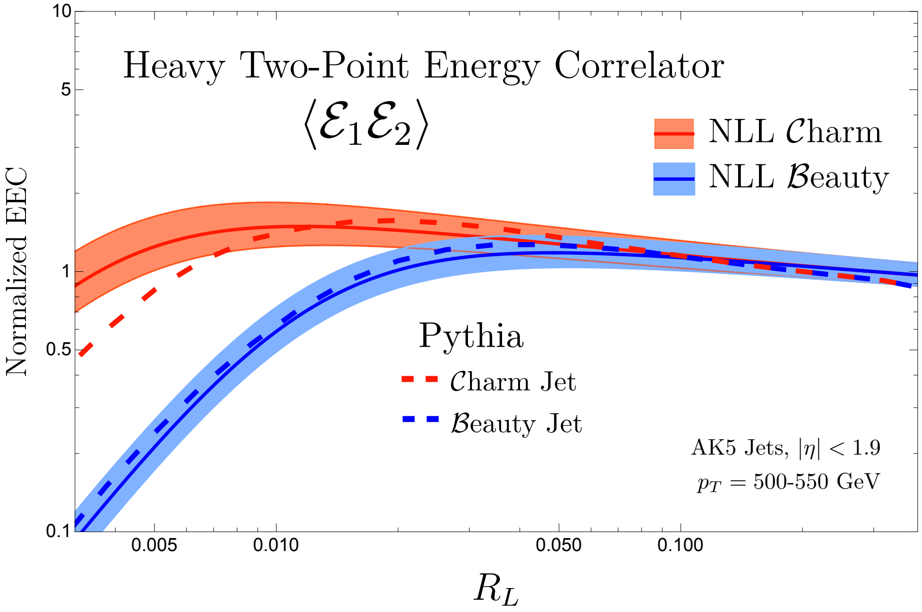
Massive two point correlator

First massive jet substructure observable at NLL



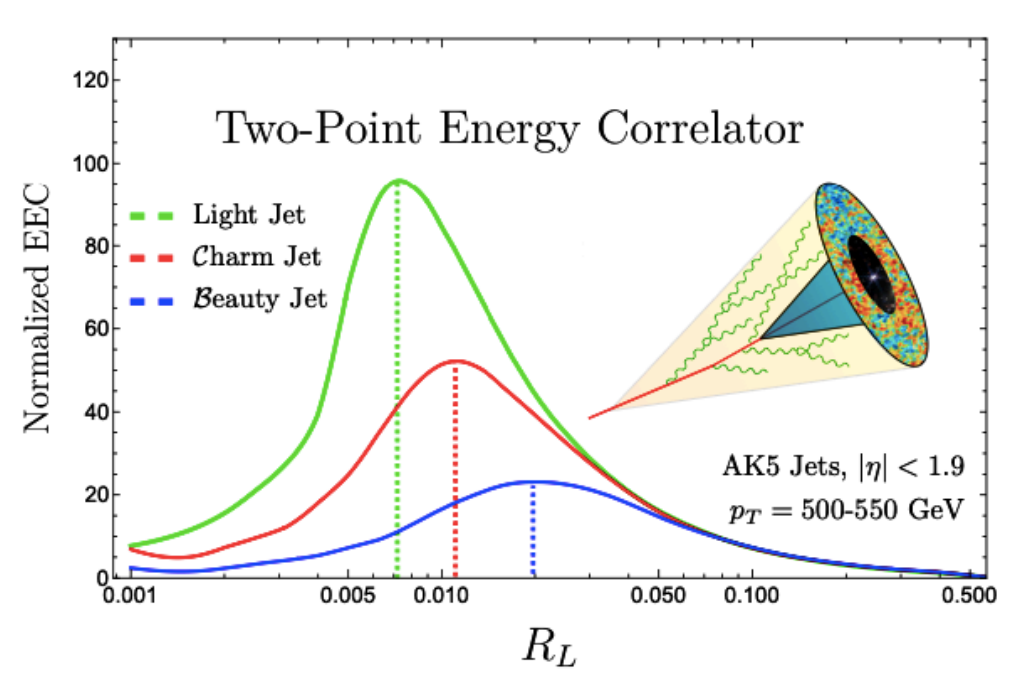
Virtuality $\sim p_T R_L + m_Q^2$

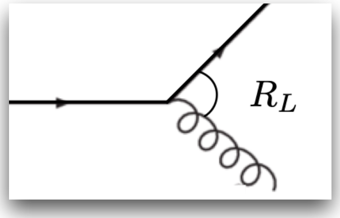
- **Scaling behaviour identical to massless case for larger scales.**
- **A turn-over for** $R_L \rightarrow m_Q/p_T$
- **The change in the slope is perturbative effect contrary to massless jets:** $R_L \rightarrow \Lambda_{QCD}/p_T$
- **The turn-over region is of interest for improving heavy quark description in parton shower.**



[Craft, Lee, BM, Moult]

Heavy and Light Jets





Virtuality $\sim p_T R_L + m_Q^2$

Dead-cone effect in QCD

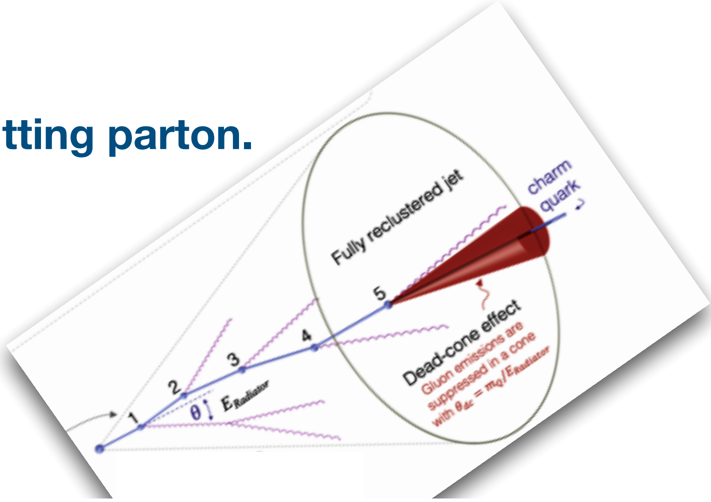
Fundamental phenomena

- **Parton-shower pattern depends on the mass of the emitting parton.**
- **Angular suppression** $\propto \frac{M}{E}$.

Observable used for the observation of the dead-cone effect in LHC data

$$R(\theta) = \frac{1}{N^{\mathrm{D^0\,jets}}} \frac{\mathrm{d}n^{\mathrm{D^0\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg/ \frac{1}{N^{\mathrm{inclusive\,jets}}} \frac{\mathrm{d}n^{\mathrm{inclusive\,jets}}}{\mathrm{d}\ln(1/\theta)} \Bigg|_{k_{\mathrm{T}},\, E_{\mathrm{Radiator}}}$$

- **Can we observe the dead-cone with EEC?**

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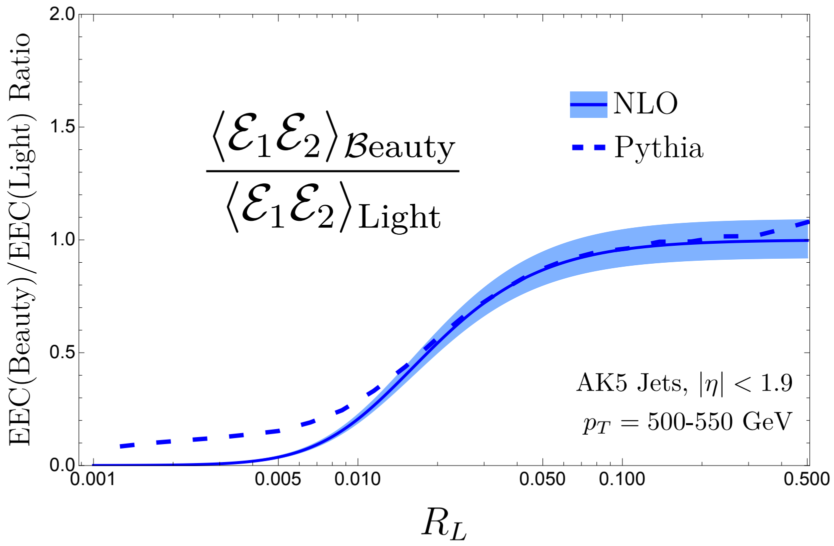
Direct observation of the dead-cone effect in quantum chromodynamics

ALICE Collaboration

Intrinsic mass effects

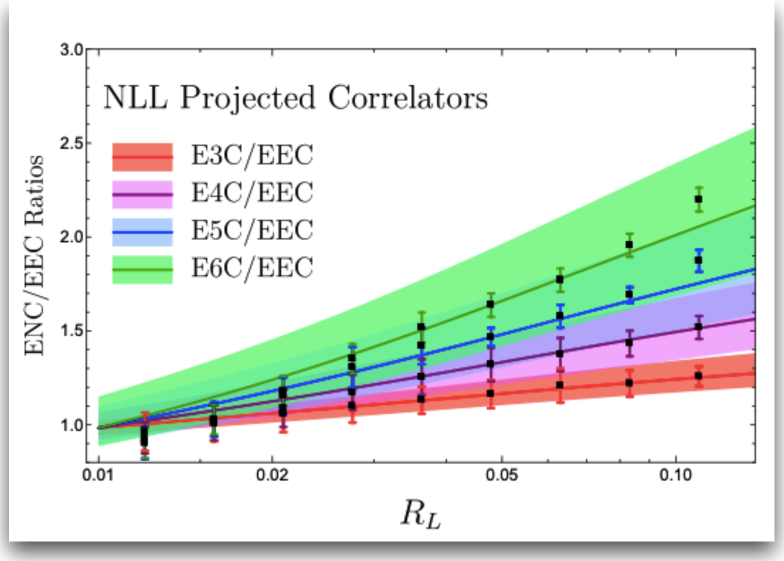
Dead-cone effect



[Craft, Lee, BM, Moult]

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**
- **A transition region related to the quark mass: perturbatively calculable.**
- **Excellent agreement with MC.**
- **Small angle suppression can be interpreted as a dead-cone effect.**

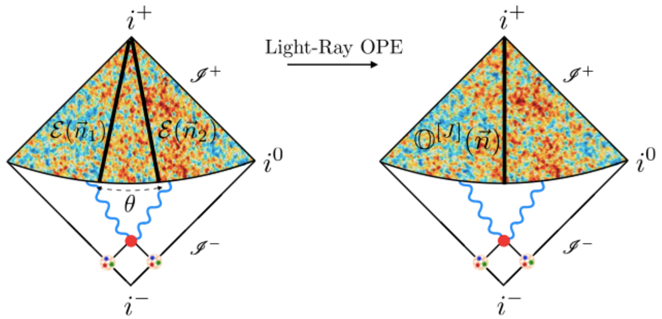
Higher point correlators



The light-ray OPE

- The leading scaling behavior at the LHC is described by the leading terms in the OPE: **twist two light-ray operators.**
- Light-ray OPE is a rigorous and convergent expansion in CFT.

$$\langle \Psi \mid \varepsilon(\vec{n}_1)\varepsilon(\vec{n}_2) \mid \Psi \rangle = \sum c_i \theta^{\gamma_i} \mathcal{O}_i(\vec{n}_1)$$



$$\langle \varepsilon\left(\vec{n}_1\right) \varepsilon\left(\vec{n}_2\right) \cdots \varepsilon\left(\vec{n}_k\right) \rangle = \frac{1}{R_L^2} \left\{ f_q^{[k]}\left(u_i, v_i\right) \mathbb{O}_q^{[k+1]}\left(\vec{n}_1\right) + f_g^{[k]}\left(u_i, v_i\right) \mathbb{O}_g^{[k+1]}\left(\vec{n}_1\right) \right\} + \mathcal{O}\left(R_L^0\right)$$

$$u_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_3}}{x_{i_1 i_3} x_{i_2 i_4}} \right)^2 \qquad v_i = \left(\frac{x_{i_1 i_2} x_{i_3 i_4}}{x_{i_1 i_4} x_{i_2 i_3}} \right)^2$$

$$\vec{\mathbb{O}}^{[J]} = \left(\mathbb{O}_q^{[J]}, \mathbb{O}_g^{[J]} \right)^T = \lim_{r \to \infty} r^2 \int_0^\infty dt \vec{\mathcal{O}}^{[J]}(t, r\vec{n})$$

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J} \bar{\psi} \gamma^+ \left(iD^+\right)^{J-1} \psi,$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J} F_a^{\mu+} \left(iD^+\right)^{J-2} F_a^{\mu+}$$

Leading twist light-ray OPE

Control scaling at leading power

- **Twist-2 operators in QCD are characterized by a spin J and transverse spin j=0,2.**
- **They can be transformed to a twist-2 light-ray operator vector parametrized by J**

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$$\mathcal{O}_{\bar{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}\left(iD^+\right)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

$$\xrightarrow{\lim_{r\to\infty} r^2\int_0^\infty dt}$$

$$\vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\bar{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

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Unpolarized

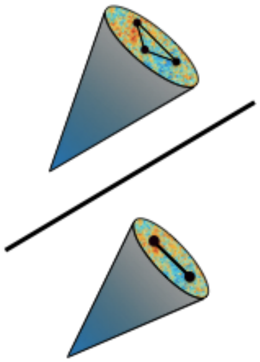
Polarized

Unpolarized Scaling

LHC scenario

- Probe the unpolarized spin $j = 0$ operators
- The leading scaling behavior is determined by the anomalous dimension $\gamma(N+1)$ for an operator of spin $N+1$.

→ can isolate the anomalous dimensions!

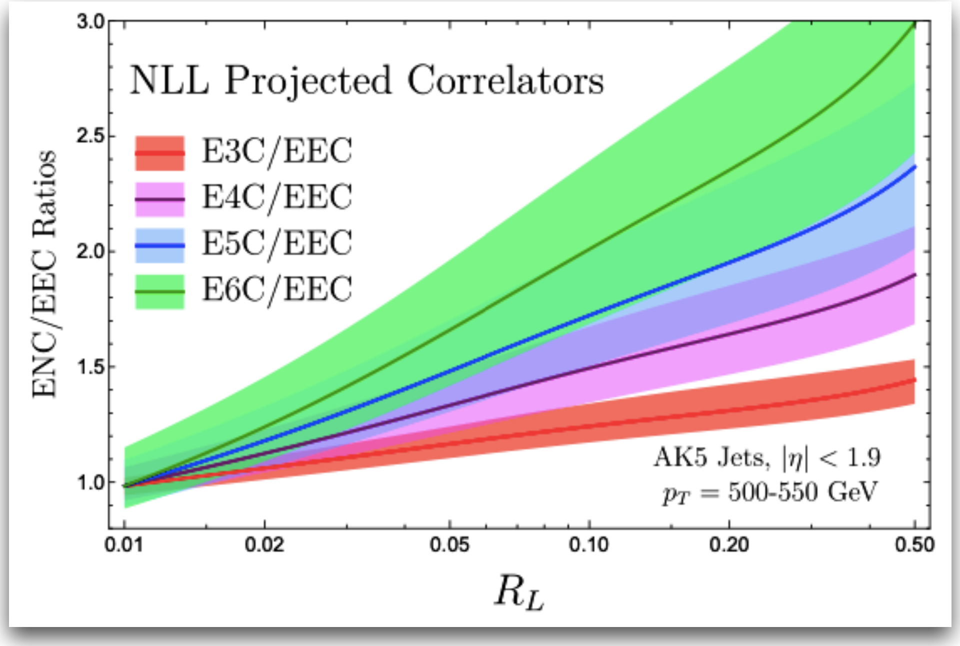


$$\frac{\langle \mathcal{E}_1\mathcal{E}_2\cdots\mathcal{E}_{J-1}\rangle}{\langle \mathcal{E}_1\mathcal{E}_2\rangle} \sim \frac{\langle \mathbb{O}^{[J]}\rangle}{\langle \mathbb{O}^{[3]}\rangle}$$

The jet spectrum

Higher-point correlators

- **Asymptotic energy flux directly probes the spectrum of (twist-2) lightray operators at the quantum level**
- **Ratio of the higher-point correlators with the two-point isolates anomalous scaling!**
- **The anomalous scaling behavior depends on N (slope increases with N)**
- **First hand probe of the anomalous dimensions of QCD operators.**



[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

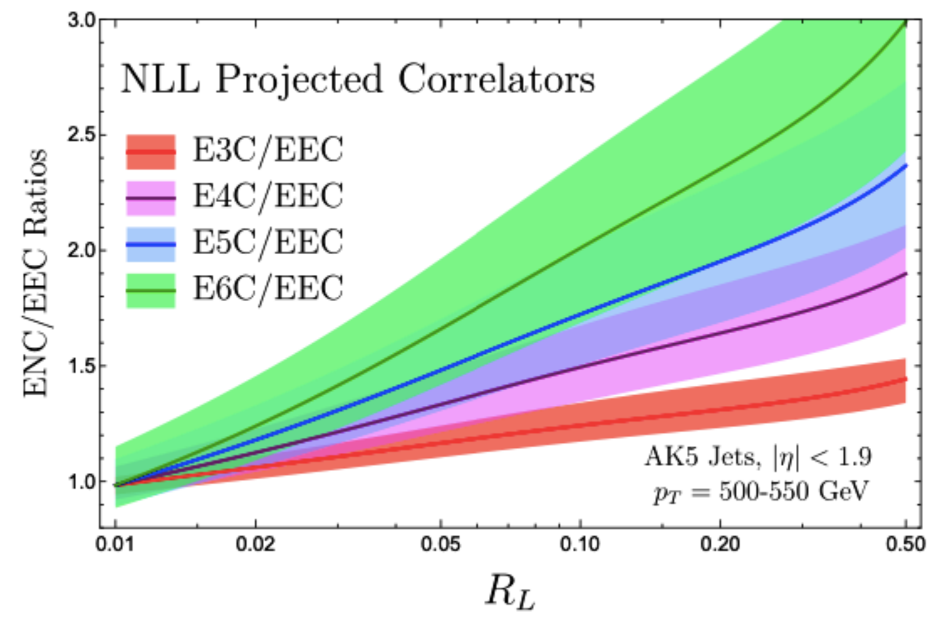
The jet spectrum

Higher-point correlators

- **Non-perturbative effects cancel in the ratio**
- **A clean measurement of strong coupling**

$$\theta^{\gamma} \to \exp\left(\frac{\hat{\gamma}}{2\beta_0} \ln \frac{\alpha_s(\theta Q)}{\alpha_s(Q)}\right)$$

- **Can be observed at the high energies at the LHC at high precision**



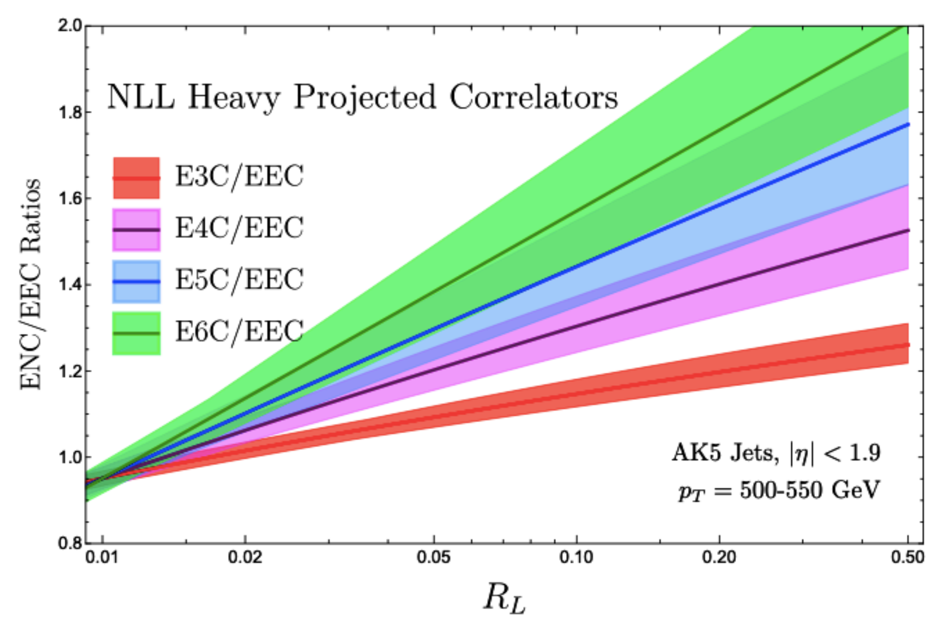
[Lee, BM, Moult]

[Chen, Moult, Zhang, Zhu]

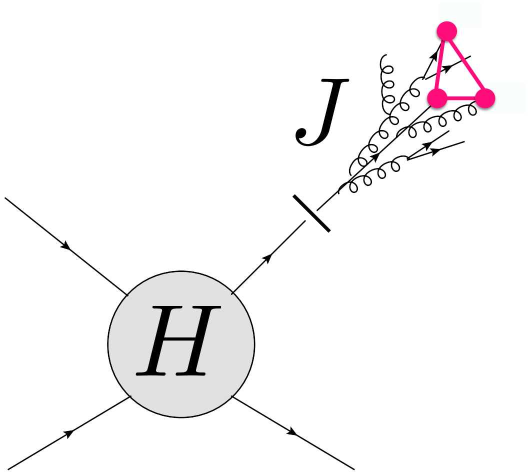
Heavy Projected Energy Correlators

Resolve the UV scaling behaviour

- **Ratios of higher point correlators with the two point EEC are independent of IR effects, including quark mass.**
- **The exact behavior as the massless case.**
- **Non-trivial cross check of the factorization theorem!**
- **Anomalous dimensions should not be affected by the IR physics.**



[Craft, Lee, BM, Moult]



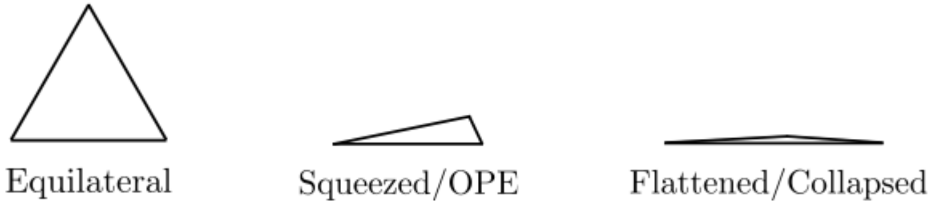
Massive three-point EEC for $1 \to 3$ splittings

Higher point correlators in $1 \to 3$ splittings

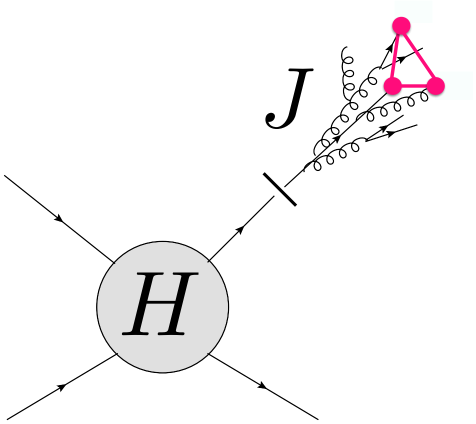
Shape dependance

- Different “shapes” for higher point functions are distinguished by different physics.
- It is interesting to probe such shape dependance.
- First time calculated in the context of cosmological three point function.

[Maldacena and Pimentel]



Higher Point Splittings



$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{dz} = \frac{\int d^4x e^{iq\cdot x}\left\langle \mathcal{O}(x)\varepsilon\left(\vec{n}_1\right)\,\varepsilon\left(\vec{n}_2\right)\,\varepsilon\left(\vec{n}_3\right)\,\mathcal{O}^\dagger(0)\right\rangle}{\int d^4x e^{iq\cdot x}\left\langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\right\rangle}$$

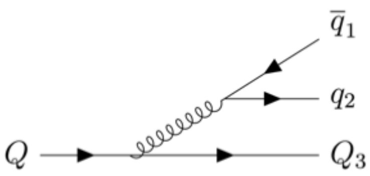
$$\frac{1}{\sigma_{\text{tot}}}\frac{d^3\Sigma}{dx_1dx_2dx_3} = \sum_{i,j,k}\int\frac{E_iE_jE_k}{Q^3}d\sigma\delta\left(x_1-\frac{1-\cos\theta_{ij}}{2}\right)\delta\left(x_2-\frac{1-\cos\theta_{jk}}{2}\right)\delta\left(x_3-\frac{1-\cos\theta_{kl}}{2}\right).$$

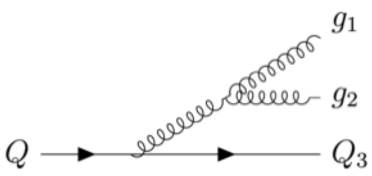
- **Presence of a mass term: analytic integration becomes more complex.**
- **At the same time more interesting analytic structures to study.**

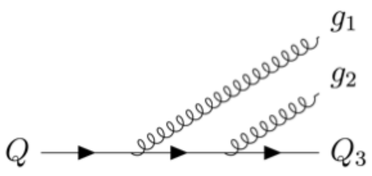
Higher Point Splittings

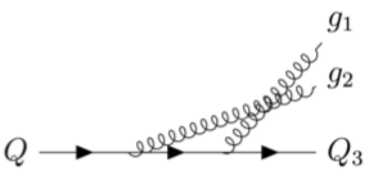
[Craft, Gonzales, Lee, BM, Moult]

[Dhani, Rodrigo, Sborlini]









- **Compute first the massive $1 \to 3$ QCD splitting functions**
- **There are four distinct denominator dependencies on the kinematics**

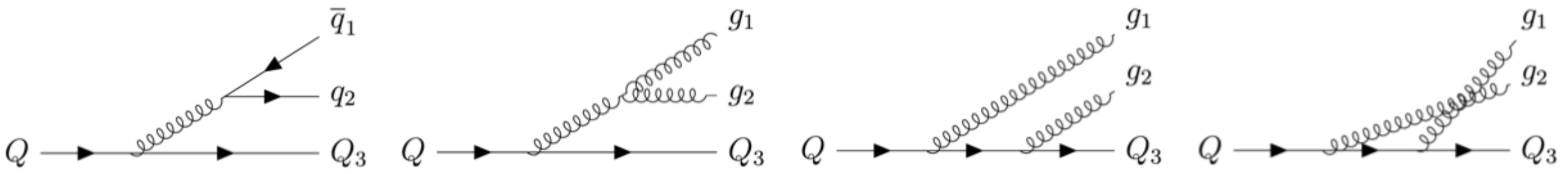
$$\frac{1}{s_{ij}s_{ik}}, \frac{1}{s_{ij}s_{jk}}, \frac{1}{s_{ik}s_{jk}}, \frac{1}{s_{ijk}}$$

$$s_{ijk} = (p_i + p_j + p_k)^2$$

Higher Point Splittings

[Craft, Gonzales, Lee, BM, Moult]

[Dhani, Rodrigo, Sborlini]



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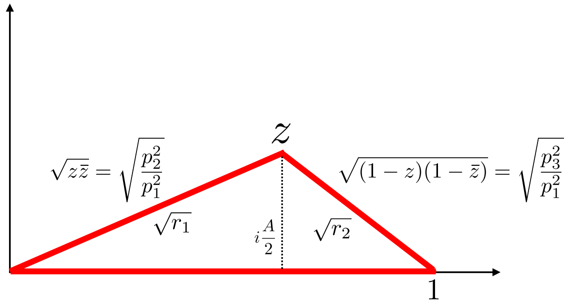
$$s_{ijk} = (p_i + p_j + p_k)^2$$

Source of non-trivial elliptic structure

Other cases are multi-polylogs!

Higher Point Splittings



- The integrant can be written in the compact form

$$G\left(z, \bar{z}, x_L, m/p\right) = \int \frac{d\xi_1 d\xi_2 d\xi_3 \left(\xi_1\xi_2\xi_3\right)^2}{\left[\xi_1\xi_2 + \xi_1\xi_3(1-z)(1-\bar{z}) + \xi_2\xi_3 z\bar{z} + \frac{m^2}{4p^2x_L}\left(\frac{\xi_1+\xi_2}{\xi_3}\right)\right]^2} P_{ijk}\delta\left(1 - \sum_i \xi_i\right)$$

$$\frac{1-\cos\theta_{23}}{1-\cos\theta_{12}} = z\bar{z}$$

$$\frac{1-\cos\theta_{23}}{1-\cos\theta_{13}} = (1-z)(1-\bar{z})$$

$$\xi_i = \frac{E_i}{p}$$

- Integration of any two of the energy fractions reveals a square root of a quartic polynomial in the third variable in the denominator

$$P_4(\xi_3; z, \bar{z}, \alpha) = x_L^2[(z-\bar{z})^2\,\xi_3^4 + 2\,(2z\bar{z} - z - \bar{z})\,\xi_3^3 + (1-\alpha)\,\xi_3^2 + \alpha\,\xi_3]$$

Higher Point Splittings

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- **Integration of any two of the energy fractions reveals a square root of a quartic polynomial in the third variable in the denominator**

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Elliptic Integrals!

Elliptic Structure

- The final result for the three-point massive EEC takes the compact form:

$$\boxed{\int_0^1 d\xi_3 \left[R_1(\xi_3) + \frac{R_2(\xi_3)}{\sqrt{P_4}} + \sum_i \left(R_3^i(\xi_3) + \frac{R_4^i(\xi_3)}{\sqrt{P_4}}\right) \log\left(R_5^i(\xi_3) + \frac{R_6^i(\xi_3)}{\sqrt{P_4}}\right)\right]}$$

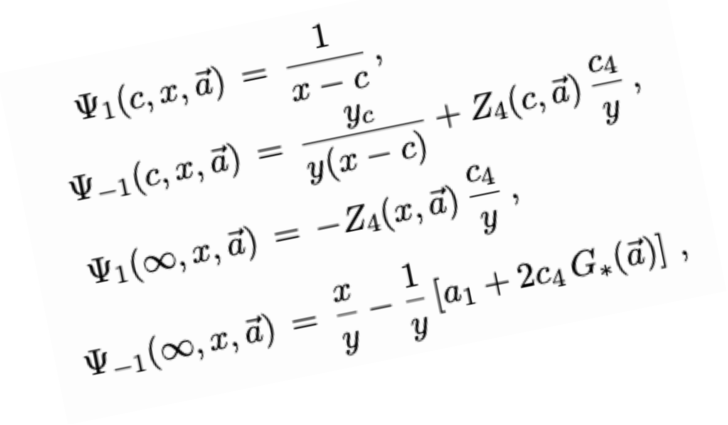
- Can use a basis of kernels to rewrite the result in terms of eMPLs.
- **Similar analogy to the higher loop massive amplitudes structure, but for a jet observable**

Elliptic Structure

- **Can use a basis of kernels to rewrite the result in terms of eMPLs.**

$$\int_0^1 d\xi_3 \left[R_1\left(\xi_3\right) + \frac{R_2\left(\xi_3\right)}{\sqrt{P_4}} + \sum_i \left(R_3^i\left(\xi_3\right) + \frac{R_4^i\left(\xi_3\right)}{\sqrt{P_4}} \right) \log \left(R_5^i\left(\xi_3\right) + \frac{R_6^i\left(\xi_3\right)}{\sqrt{P_4}} \right) \right]$$

[Broedel, Duhr, Dulat, Penante, Tancredil]



$$\begin{aligned} \Psi_1(c,x,\vec{a}) &= \frac{1}{x-c}, \\ \Psi_{-1}(c,x,\vec{a}) &= \frac{y_c}{y(x-c)} + Z_4(c,\vec{a})\,\frac{c_4}{y}, \\ \Psi_1(\infty,x,\vec{a}) &= -Z_4(x,\vec{a})\,\frac{c_4}{y}, \\ \Psi_{-1}(\infty,x,\vec{a}) &= \frac{x}{y} - \frac{1}{y}\left[a_1 + 2c_4\,G_*(\vec{a})\right], \end{aligned}$$

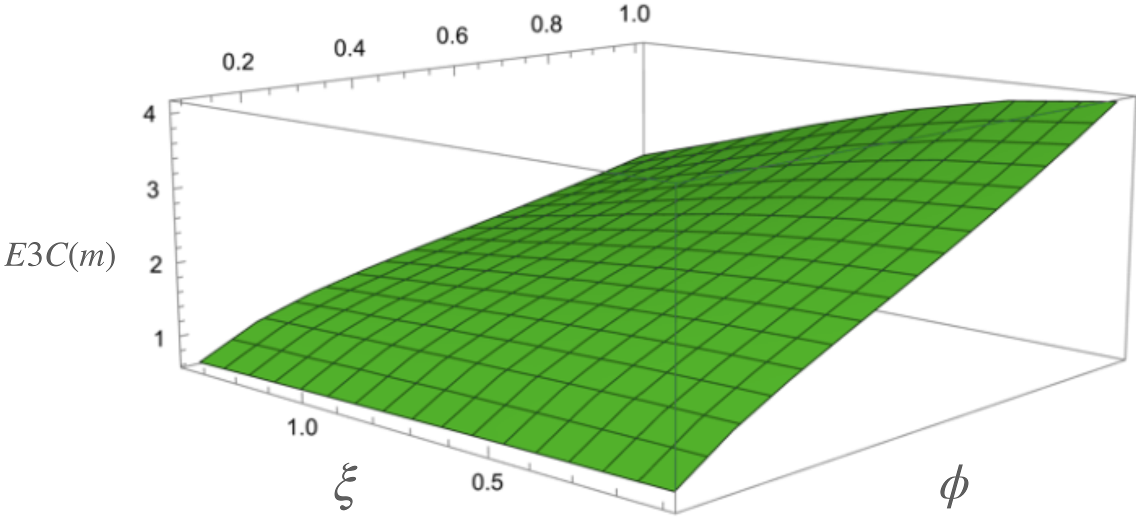
- **Similar analogy to the higher loop massive amplitudes structure, but for a jet observable.**
- **Kernel Basis:**

$$\int \frac{1}{\sqrt{P4}}, \quad \int \frac{\xi}{\sqrt{P4}}, \quad \int \frac{\xi^2}{\sqrt{P_4}}, \quad \int \frac{1}{(\xi - p)\sqrt{P_4}}$$

Numerical results

Non-Gaussianity for the massive case

- **Probe effects of the splitting functions for massive particles.**
- **Small angle: little difference between $1 \to 2$ and $1 \to 3$ splitting**
- **Non-Gaussianity more pronounced for larger ξ**

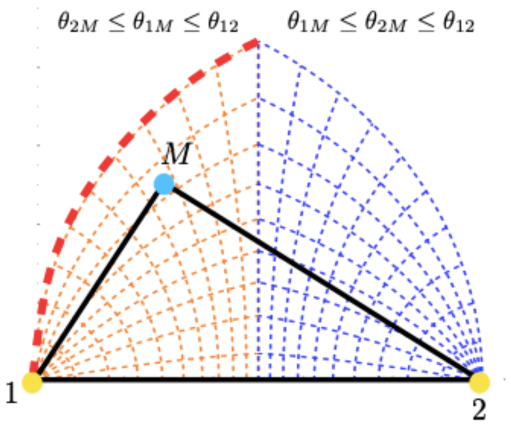


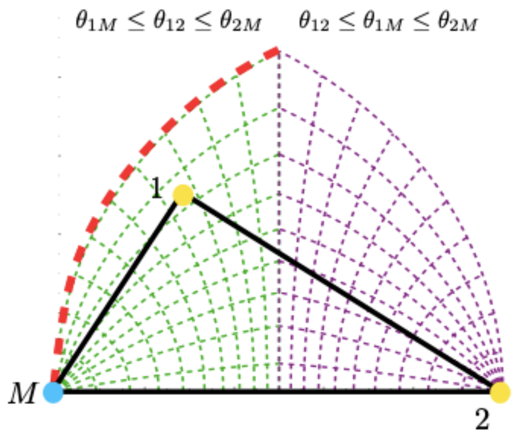
Isolating mass effects

- **Different types of detector combinations**

$$\langle \mathcal{E}\mathcal{E}\mathcal{E}_M\rangle, \quad \langle \mathcal{E}\mathcal{E}_M\mathcal{E}_M\rangle, \quad \langle \mathcal{E}_M\mathcal{E}_M\mathcal{E}_M\rangle, \quad \langle \mathcal{E}_{M'}\mathcal{E}_{M'}\mathcal{E}_M\rangle.$$

- **We isolate the case with two identical detectors and one massive one**





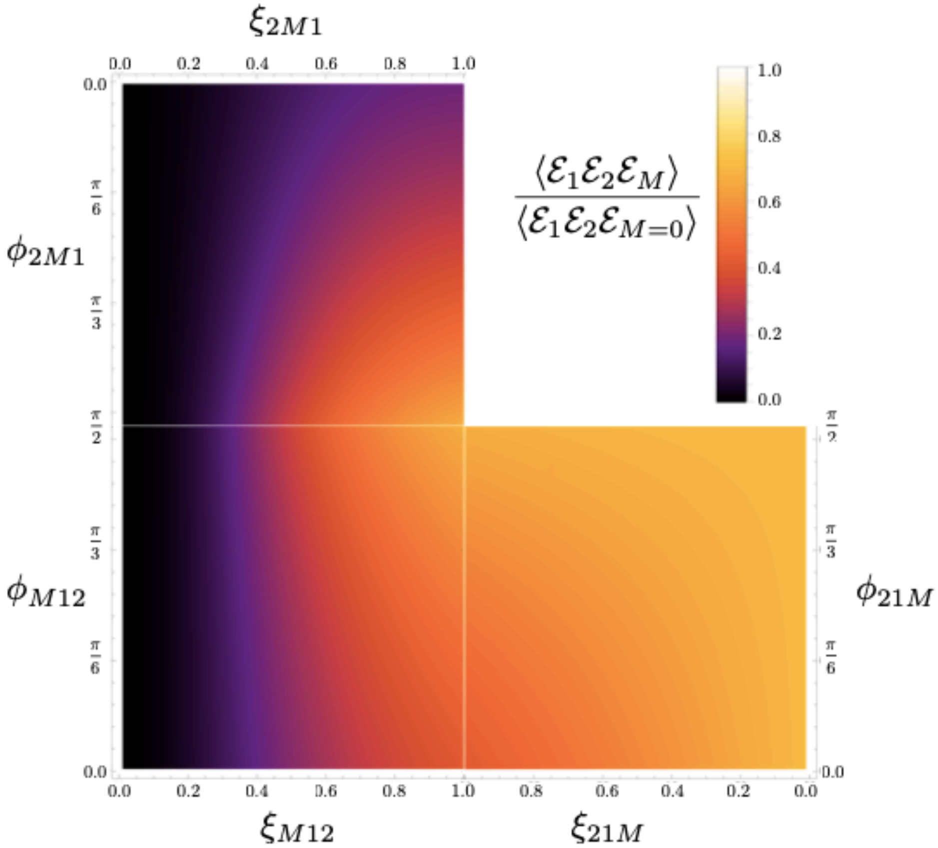
Numerical results

Dead-cone in the triple collinear splitting

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**

- $$\xi = \frac{\theta_s}{\theta_m}, \quad \phi = \arcsin\sqrt{1 - \frac{(\theta_l - \theta_m)^2}{\theta_s^2}},$$

[Work in progress Craft, Gonzalez, Lee, BM, Moult]

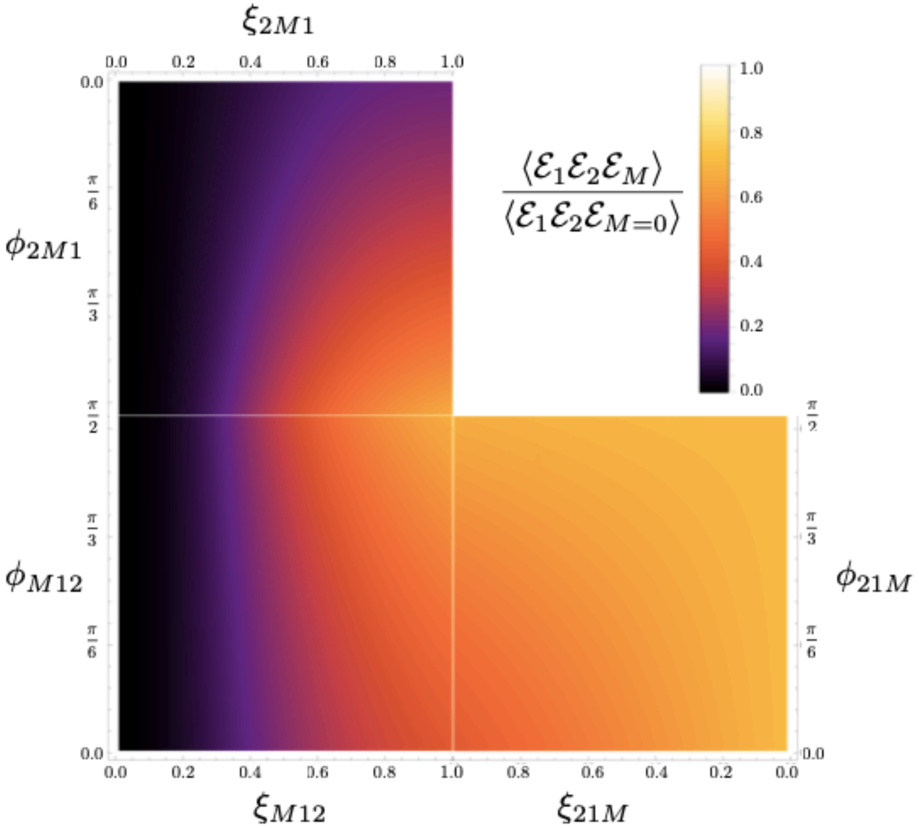


Numerical results

Dead-cone in the triple collinear splitting

- **Ratios of the massive and massless EEC isolate mass (IR) effects.**
- **A transition region related to the quark mass at small angles is also visible for the 3-point function.**
- **Small angle suppression can be interpreted as a dead-cone effect.**

[Work in progress Craft, Gonzalez, Lee, BM, Moult]



Future Prospects

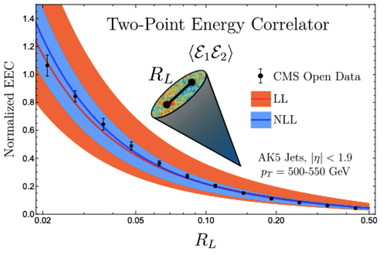
- Jet modeling in MC simulations: heavy flavours
- Precision in parton showers: “reference resummation” for testing DGLAP finite moments.
- Understand properties of the QGP: multi-scale problem too, global properties of plasma.

[Andres, Dominguez, Kunnawalkam Elayawalli, Holguin, Marquet, Moult,…]

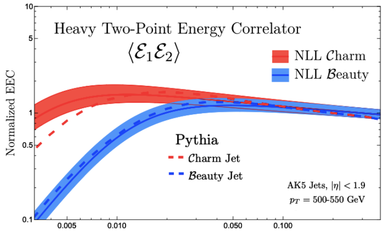
Conclusions

- **Factorization formula for calculating energy correlators for heavy jets at the LHC.**

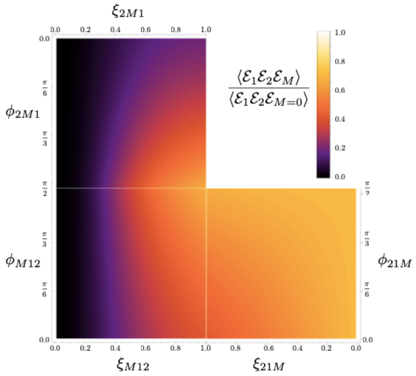
$$\Sigma^{[N]}\left(R_L, p_T^2, m_Q, \mu\right) = \int_0^1 dx x^N \vec{J}^{[N]}\left(R_L, x, m_Q, \mu\right) \cdot \vec{H}\left(x, p_T^2, \mu\right)$$



- **Intrinsic mass effects of strongly interacting elementary particles.**



- **Higher-point correlators can be calculated for LHC and probe anomalous scaling dimension of QCD operators.**



Thank You!